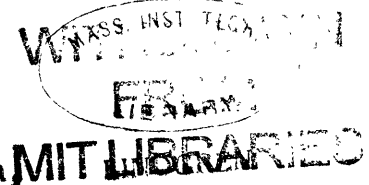VERTICAL MOTION IN THE ATLANTIC
CIRCULATION PATTERN

by

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A.B., University of California
Los Angeles
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SUBMITTED IN PARTIAL FULFILLMENT
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CIRCULATION PATTERN

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William R. Holland

Submitted to the Department of Meteorology
on 19 May 1961 in partial fulfillment of the
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ABSTRACT

A simple model of oceanic circulation is established
on the basis of the distribution of radiocarbon in the
Atlantic Ocean. This radioactive isotope, formed in the
upper atmosphere by cosmic rays and introduced into the
surface ocean by way of the carbon dioxide exchange,
decays with a half life of 5570 years. The differences in
radiocarbon concentration then should be related to the
mixing rates.
In view of the low temperature of the deep waters
it is apparent that downward diffusion of heat from the
warm surface layers must be balanced by a slow upward
transport of water from depth. The radiocarbon model is
established with the aim of studying these motions.
The values of vertical transport obtained are found
to be comparable to horizontal exchanges and therefore
very important in the Atlantic circulation pattern.
Average vertical velocities of the order of one centimeter
per day are found, thus substantiating estimates made by
Robinson and Stommel from a consideration of theoretical
thermocline models.

Thesis Supervisor:
Henry G. Houghton

Title: Professor of Meteorology

ACKNOWLEDGEMENT

TABLE OF CONTENTS

I. EXPERIMENTAL AND THEORETICAL BACKGROUND

A. Traditional Oceanographic Studies

1. Atlantic Water Masses. The general circulation
in the Atlantic Ocean and the major water masses
comprising it have been explored extensively by
oceanographic expeditions during the last half century.
Traditional studies based upon density distributions and
patterns of salinity and temperature have made possible
the definition of distinct water masses as well as a
description of the general circulation in the North and
South Atlantic. These findings have been summarized by
Sverdrup, Johnson, and Fleming (1942).

The Atlantic Ocean connects with the other major
oceans through the Antarctic Ocean to the south but
otherwise is isolated from them. Several smaller
adjacent seas, the Caribbean, the Mediterranean, and the
Arctic Sea, contribute dynamically to the Atlantic
circulation and importantly affect the distribution of
conservative properties in the sea.

Five, well defined, sub-surface water masses are
found in the Atlantic. The first, defined by nearly linear
T-S relationship, has been called the South Atlantic Central
Water. This mass lies just below the hot saline surface
water and comprises a layer 450 meters thick. The
temperature ranges from 6° C at the bottom of the layer

to about 18° C just below the surface mixed layer, and the salinity increases from 34.5 to 36.0 per mil. This vertical temperature - salinity relationship is similar to the horizontal T-S relationship found in the region of the Subtropical Convergence and hence is thought to outcrop there. The mass can be traced throughout the South Atlantic from 40° S to the equator.

Below the Central Water lies the Antarctic Intermediate Water characterized by a salinity minimum at about 800 meters over the greater part of the ocean. The mass appears to be considerably mixed with other masses as one traces it into the North Atlantic. This layer, characterized by a salinity of less than $34.65°/_{oo}$, is about 750 meters thick and overlies the great bulk of the deep and bottom water.

In the North Atlantic one finds only a single mass sandwiched between the thin surface layer and the deep and bottom water. This mass, the North Atlantic Central Water, is characterized by a nearly straight T-S curve between $T = 8°$, $S = 35.10°/_{oo}$, and $T = 19°$, $S = 36.70°/_{oo}$ and is seen to be similar, although warmer and more saline, to the South Atlantic Central Water.

At intermediate depths in the North Atlantic three other types of water are found but are not nearly as extensive or influential as the major water masses. One of these is the remainder of the Antarctic Intermediate

Water which becomes rapidly mixed with over and under-
lying waters as it moves north. Another is water of
high salinity which results from spreading out of
Mediterranean water flowing out through the Strait of
Gibraltar. The third represents small quantities of water
of Arctic origin which is somewhat analogous to the
intermediate water of the Antarctic.

By far the most extensive water mass is the <u>Atlantic</u>
Deep Water which occupies the depths between 1500 m and
4000 m throughout the Atlantic. This water is thought to
form in the Arctic regions where high salinity Atlantic
surface waters mix with cool waters from the north. The
mixture, having a temperature of about 2.5° C and a
salinity of 34.9 per mil, sinks and moves southward,
extending nearly to the Antarctic region.

The fifth major water mass which we identify is the
<u>Antarctic Bottom Water</u> which constitutes the heaviest of
all waters. This water is formed in the Weddell Sea where
it sinks rapidly to fill the greatest depths in the
South Atlantic. The mass is found at variable depths
depending upon the bottom topography but ranges from
3600 meters at 40° S to 4500 meters at 25° N.

2. <u>The Shallow Circulation</u>. It is in the upper regions
of the ocean that the strongest currents are found. The
form which this shallow circulation takes is that of a
huge gyre, rotating clockwise in the Northern Hemisphere

and counterclockwise in the Southern Hemisphere.

The most striking feature of the South Atlantic gyre is the Benguela Current which flows north along the west coast of Africa. At about 20° S the current leaves the coast to turn westward along the equator. This part of the southern gyre, lying between latitutes 0° and 20° S, is known as the South Equatorial Current. As this current approaches the South American coast it divides, part flowing across the equator into the North Atlantic and the remainder turning south along the South American coast. This southward flow of water, the Brazil Current, constitutes the western arm of the South Atlantic gyre. Along the extreme southern coast of the continent there is a northward flowing branch of the Falkland Current which turns away from the coast to join with the remnants of the Brazil Current. These waters form the South Atlantic Current which flows eastward to complete the southern gyre.

The North Atlantic circulation is dynamically connected to that of the South Atlantic by virtue of the flow of southern surface water across the equator. The water joins with part of the North Equatorial Current and enters the Caribbean Sea. There thorough mixing takes place so that temperature, salinity, and oxygen data indicate water of intermediate characteristics. The continuation of these currents represents the beginning of the Gulf Stream system which dominates the

circulation of a great part of the North Atlantic ocean.

The first segment of this system, known as the Florida Current, flows out of the Caribbean through the Florida Straits and turns north along the continental slope of North America. The Gulf Stream leaves the continental slope north of Cape Hatteras and continues as a well-defined, narrow current as far north as the Grand Banks. In this region the stream begins to branch and form the more diffuse North Atlantic Current which gradually swings eastward to form the northern branch of the North Atlantic gyre. Numerous branches send North Atlantic water into the Arctic regions to mix there with the colder, less saline waters of the Arctic Sea. When the surface waters are cooled, vertical convection currents develop leading to a renewal of the Atlantic Deep Water. Thus the surface circulation is coupled importantly to the deep circulation by way of the Arctic Sea. We shall be interested in how importantly the two are coupled in lower latitudes.

The eastern North Atlantic has no distinct currents but is characterized by an irregular flow toward the south. Some of this water enters the Mediterranean as a surface current, but the major portion continues toward the south to form the westward flowing North Equatorial Current.

3. The Deep Circulation. The deep circulation poses

one of the most difficult problems to the oceanographer.
Direct measurement of deep currents has only recently
been attempted (Swallow and Worthington, 1957) and little
success in understanding the large scale pattern may be
expected from this source for a number of years. One
must resort to indirect evidence supplied by a
distribution of the various properties of sea water and
to simple theories of wind-driven or thermally driven
currents.

As has been noted, the South Atlantic shallow water
enters the North Atlantic along the coast of South
America. As a result of this net transport of surface
waters to the north there must be a compensating return
flow in the deep water. Sverdrup concludes that the
amount of sinking of surface waters which takes place in
the Labrador Sea and off southern Greenland plus the
contribution of heavy Mediterranean water must equal in
magnitude the surface flow across the equator. However,
one must not fail to take into account the possibility of
rising or sinking motions in other regions. It has been
found, in fact (Robinson and Stommel, 1959), that an
average upward transport from the deep water into the
shallow surface layer plays an integral part in the
thermohaline circulation. Hence we cannot at this time
accept the rates of sinking specified by Sverdrup but
shall determine them for ourselves using independent

evidence.

Water mass analysis indicates that deep water of
high density is formed in the subarctic and in the
antarctic regions of the Atlantic Ocean. In the North
Atlantic, Atlantic Deep Water flows to the south. High
oxygen content decreasing in the direction of flow
confirms this. This water crosses the equator and
continues to flow south above the Antarctic Bottom Water.

The bottom water of Antarctic origin is colder than
the deep water and forms a relatively thin bottom layer
moving slowly northward. Large amounts of this water
return to the Antarctic after being mixed with southward
moving deep water.

In summary, the large scale circulation pattern may
be described in the following way: there is a net north-
ward transport of surface waters from the South Atlantic
to the North Atlantic to the Arctic region where cold
saline water sinks. This deep water flows southward
transporting an amount equal to that of the shallow
water across each latitude circle. Superimposed upon this
large scale flow we have the possibility of vertical
mixing between shallow and deep waters. We shall use
these broad features to establish a simple model which
will give us some information about vertical mixing and
some idea of the magnitude of the sinking which takes
place in the Arctic.

B. Box Model Investigations

The chemical and physical properties of sea water and the manner in which they vary from region to region have been used extensively in studies of ocean movements. The introduction and interpretation of the temperature-salinity diagram for instance has provided a valuable tool for determining the vertical and horizontal extents of the major water masses. Early attempts to use the conservative properties of sea water in determining mixing rates between certain regions demonstrated some of the difficulties inherent in the problem. In particular, it was found that small scale variations are considerable and that such distributions are indicative only of large scale movements.

As a simple example of the general method used in box model investigations, consider an isolated, steady-state model in which three, internally mixed reservoirs may exchange water among themselves (Fig. 1). For each
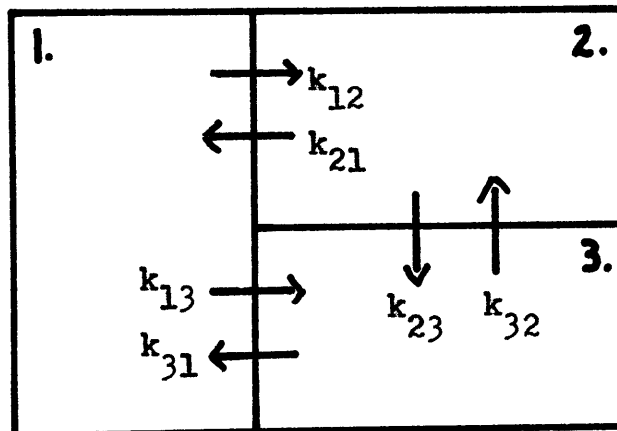

Figure 1. Simple Box Model.

physical and chemical property which is conserved in
the mixing process, two independent equations may be
written. Mass must certainly be conserved, thus
providing two additional equations. If, for instance,
the water in each reservoir is characterized by two
independent properties (such as temperature and salinity),
then six independent equations may be obtained for the
system and six unknowns may be determined at least within
a constant factor. In figure 1 we have shown six unknown
mixing rates, k_{ij}, which represents the rate of flow of
water from reservoir i to reservoir j. The conservation
equations may now be written:

Conservation of mass

$$k_{12} + k_{32} - k_{21} - k_{23} = 0$$

$$k_{23} - k_{32} + k_{13} - k_{31} = 0$$

Conservation of S

$$S_1 k_{12} - S_2 k_{21} + S_3 k_{32} - S_2 k_{23} = 0$$

$$S_2 k_{23} - S_3 k_{32} + S_1 k_{13} - S_3 k_{31} = 0$$

Conservation of T

$$T_1 k_{12} - T_2 k_{21} + T_3 k_{32} - T_2 k_{23} = 0$$

$$T_2 k_{23} - T_3 k_{32} + T_1 k_{13} - T_3 k_{31} = 0$$

where S_1, S_2, S_3, T_1, T_2, T_3 are the known concentrations
of the properties S and T in the reservoir indicated by

the subscript. In this particular case the six
equations are homogeneous and the rates of flow can be
determined only within a common factor.

This simple example may be complicated in various
ways to illustrate important aspects of the method.
Consider first the effect of knowing only one property
of the fluid rather than two. Then there are only four
independent equations and only four unknowns may be
determined (again within a common factor). This means
that we cannot get information about all possible modes
of mixing and must choose to investigate certain ones.
This is equivilent to assuming that two of the mixing
rates are negligible. The determination of the four
allowed rates is only as good as this assumption.

As a second variation in this model consider the
effect of knowing the concentrations of three properties
of sea water. Since there are only six possible mixing
rates any two properties serve to determine them. The
third property then may be used as a check upon the
results; that is, one can check to see how close the
third property comes to being conserved using the mixing
rates determined by the other properties. Then, a
knowledge of the accuracy with which the various
properties were measured gives an indication of the
accuracy of the determination of the mixing rates.

A third variation which must be considered is that in which one or more of the properties of the fluid is nonconservative and varies with time in some known manner. Dissolved oxygen and radiocarbon are two such properties. Oxygen is absorbed at the sea surface as is carbon dioxide containing radiocarbon. Biological processes in the sea influence the oxygen content in such a way as to lead to a consumption of oxygen, and Carbon 14 decays at a certain known rate; hence the processes of advection must lead to replenishment if the steady state is to prevail. The effect of this nonconservation is to make the governing equations nonhomogeneous. That is, the net amount of the substance being brought in is no longer zero but equal to the amount which decays within the reservoir. Since the equations are now nonhomogeneous, the rates of flow are uniquely determined, and we are in a position to learn something about the absolute magnitude of the mixing.

In view of these statements regarding the example of a three reservoir model, we can generalize to an N reservoir model. For each property of the fluid, (N-1) independent equations may be found. In addition (N-1) equations can be gotten by requiring conservation of mass. Then if k equals the number of properties for which data is available, (salinity, radiocarbon, etc.)

we have

$$a = (k + 1)(N - 1) \qquad (1)$$

where a is the number of independent equations.

An important parameter to be considered in such box model investigations is the number of possible modes of mixing. This depends upon the number of boundaries between adjacent reservoirs. If this number is n, then there are, in general, 2n possible mixing rates (mixing can take place in both directions). Hence if a system is to be uniquely and completely determined,

$$2n = (k + 1)(N - 1). \qquad (2)$$

If 2n is greater then $(k + 1)(N - 1)$ then all possible modes of mixing cannot be examined and one must assume, on the basis of other knowledge, that $2n - (k + 1)(N - 1)$ of the mixing rates may be neglected. Such a model is only as good as this assumption.

In the model which will be established in the present investigation, there will be four reservoirs $(N = 4)$ and five possible modes of mixing $(n = 5)$. Thus if $k = 1$ (carbon 14), $2n - (k + 1)(N - 1) = 10 - 6 = 4$; i.e. we must assume that four mixing rates are negligble. (As stated above, this assumption must be justified from other considerations). We shall also examine the case $k = 2$ (carbon 14 and temperature). Then $2n - (k + 1)(N - 1)$

equals 10 - 9 = 1, and only one mixing rate need be
assumed zero.

C. The Dynamic Carbon Cycle and the Role of Radiocarbon

Much interest has been shown in recent years con-
cerning the carbon cycle in nature. The manner and size
of the exchange between atmosphere, biosphere, and oceans
is of fundamental importance not only to the biologist
but also to the geochemist, the meteorologist, and the
oceanographer. Carbon is taken from the atmosphere as
CO_2 by plant life and is given back to it by the processes
of respiration and decay. The oceans absorb CO_2 from
the atmosphere and give it back in a never ending cycle.
A knowledge of these exchanges can provide valuable
information to a score of independent problems in several
related fields.

The total amount of carbon in the atmosphere is small
compared to that dissolved in sea water. Because of the
interchange between atmosphere and ocean across the sea
surface, any change in CO_2 concentration in the atmosphere
could be compensated for by the ocean, depending upon the
rate of mixing. In fact, large amounts of CO_2 have been
added to the atmosphere through the combustion of fossil
fuels, and the rate of combustion had continually increased.
Despite this fact a comparison of measurements made in the
19th century and in recent years does not indicate a

significant increase in atmospheric CO_2 (Suess and Revelle, 1957). Suess (1955) estimates that the world wide contamination of the earth's atmosphere by industrial CO_2 probably amounts to less than 1%. This would suggest that the CO_2 exchange rate between atmosphere and ocean is quite large, and much of the excess carbon has been absorbed in the ocean reservoir.

An important tool in the study of the carbon cycle is the radioactive isotope, Carbon 14, which has a half-life of 5570 years. Radiocarbon, as this isotope is known, is formed by cosmic ray interaction with OO_2 in the upper atmosphere. If this is a steady state process, that is, the amount produced in such interactions is just balanced by the amount which decays throughout the world reservoirs, then one would expect to find a certain ratio of C^{14} atoms to C^{12} atoms in any one locality. This ratio and its variation from place to place provide the basis for the radiocarbon dating method.

It is immediately apparent from the preceding discussion concerning the "industrial effect," i.e. the addition of CO_2 to the atmosphere and oceans by the burning of fossil fuels, that the steady state assumption has been upset for at least a hundred years. The carbon being added in this way is free of C^{14} and hence tends to reduce the observed C^{14}/C^{12} ratio. Corrections for this effect

must be applied if one is to use this ratio as an indica-
tion of mixing rates between reservoirs.

A second effect must be considered and an appropriate
correction made if one is to consider rates of mixing
based upon C^{14} data. Neutrons produced in nuclear tests
interact with nitrogen in the atmosphere to produce radio-
carbon. Much C^{14} had been added to the earth's dynamic
carbon reservoir in this way. The C^{14} concentration has
been found to have increased 2.1% a year since 1955 in
the Southern Hemisphere tropospheric CO_2 (Rafter and
Ferfusson, 1957). Broecker and Walton (1959) have found
a 5% per year increase in the Northern Hemisphere from
1955 to 1958. It is apparent, therefore, that this effect
will seriously affect the steady state assumption, and
appropriate corrections must be applied to all data used.

In summary then, any study of the carbon cycle must
take into account two important factors: (1) the "industrial
effect," the decrease in C^{14} concentration due to the
addition of C^{14} free CO_2 produced in the combustion of oil
and coal for the past 100 years; (2) the "bomb C^{14} effect,"
the increase in C^{14} concentration brought about by the
production of artifical C^{14} in nuclear tests.

Recently a group of investigators at Lamont Geological
Laboratory published results of the determination of the
radiocarbon concentration in the dissolved biocarbonate

of 135 samples representing the major water masses of the
Atlantic Ocean (Broecker et al, 1960). They found that
while the standard deviation within a given water mass
was only slightly larger than the experimental error
between water masses, the total range of the C^{14}/C^{12} ratio
was 10%.

As might be expected from traditional studies of the
Atlantic circulation the surface water showed a
progressive increase from south to north as the upper
mixed layer came more and more into equilibrium with the
atmospheric value. The C^{14}/C^{12} ratios ranged from 120 per
mil lower than the preindustrial atmospheric value in the
Antarctic to 50 per mil lower in the North Atlantic. The
deep water mass C^{14}/C^{12} ratios also showed significant
variations which could be used for a quantitative
determination of the rates of mixing.

The above investigation included a steady state
circulation model based upon a south to north surface
transport of water with a return flow at depth. Mixing
across the main thermocline in the North and South Atlantic
was considered to be negligible, the deep waters being
formed from vertically mixed reservoirs at the extremes
of the ocean. The model does not allow for either sinking
or upwelling of water except in high latitudes. We shall
show in the present investigation that such an assumption
is not valid and that vertical mixing throughout the oceans,

not just in polar regions, contributes importantly to
the total ocean circulation. The study and determination
of such vertical motions represents the major aim of
this paper.

D. Oceanic Upwelling

Before investigating vertical motions using radio-
carbon data, let us examine past studies of vertical
upwelling in the ocean. Until recently not much interest
has been shown in the abyssal circulation. Stommel (1956)
made a numerical calculation of the depth of no
meridional motion and found the associated vertical
component of velocity using actual oceanographic data from
two stations. However, the result depended strongly upon
a knowledge of the wind stress at the sea surface and must
be considered doubtful.

Stommel points out (1958) that it seems likely that
the low temperature of the deep water is maintained in the
face of downward diffusion of heat from the warm surface
layer by a slow upward component of velocity in the deep
water. Using a very primitive model of convection he finds
the maximum vertical velocity to be of the order
3×10^{-5} cm/sec.

In 1959 a more comprehensive model of the thermohaline
circulation was proposed by Robinson and Stommel. On the
basis of this theoretical model they computed the average

amplitude of the maximum vertical velocity to be 4×10^{-5} cm/sec. Two assumptions, however, are difficult to justify; first, that the eddy mixing parameter does not vary horizontally or vertically and second, that horizontal mixing is negligible.

The weaknesses in the above studies have led Stommel and Arons (1960) to adopt an intermediate approach, one which combines the Robinson-Stommel theory with oceanographic data taken from various sources. These lead to values for the vertical velocity under the thermocline varying between 0.5 and 3.0 cm/day (0.6×10^{-5} to 3.3×10^{-5} cm/sec). These then give us order of magnitude values for the vertical components of velocity with which we can compare values computed on the basis of a steady state radiocarbon model.

It is important to realize that actual measurements of such small velocities is impossible, and therefore other means must be used. One such means is that described above; that is, a theoretical study based upon the existence of the strong vertical temperature gradient which must be maintained by upwelling. Independent approaches to the problem are necessary. The distribution of radioactive elements, such as radiocarbon, provides an excellent means to this end. It was on this premise that the present investigation was begun.

II THE MODEL AND ITS IMPLICATIONS

A. The Data

Let us now construct a simple, steady state model for the Atlantic circulation in which vertical transports between shallow and deep water masses are allowed. Radiocarbon data to be used in this investigation has been gathered and published by a group at Lamont Geological Laboratory (Broecker et al, 1960). Since the steady state condition of the C^{14}/C^{12} ratio has been upset by the industrial effect and by nuclear bomb tests, these investigators have applied corrections to the measured data. The results, expressed in parts per mil below the atmospheric value, are summarized in Table 1. No correction has been made for samples from the deep water masses, since our model indicates that turnover times are much greater than the length of time the steady state assumption has been upset.

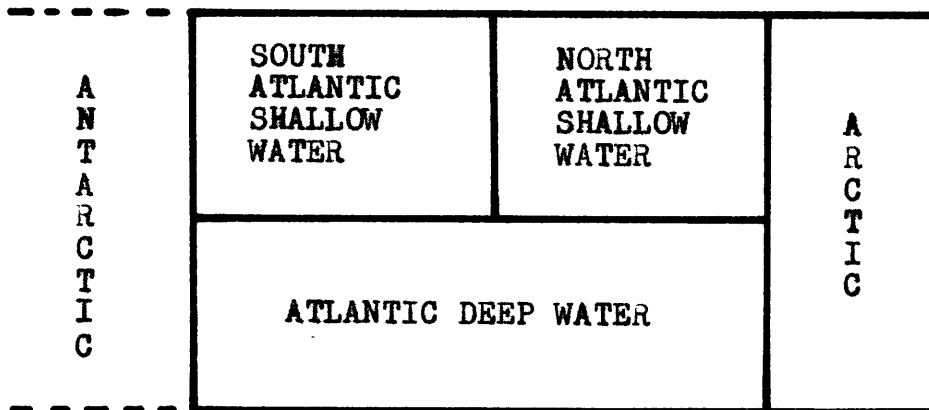The Atlantic Ocean has been divided into four internally homogeneous reservoirs as shown diagramatically in Figure 2.

Figure 2. The Atlantic Reservoir System.

TABLE I

Geographic Region	Uncorrected Average	Bomb Correction	Industrial Correction	Corrected Average
Arctic	$-35°/_{oo}$	$-3°/_{oo}$	$+10°/_{oo}$	$-28°/_{oo}$
North Atlantic Shallow Water	-49	-3	+17	-35
South Atlantic Shallow Water	-57	-6	+17	-46
Antarctic	-120	-3	+10	-113
Atlantic Deep Water	-104	----	----	-104
World Ocean	-160	----	----	-160

The transfer of CO_2 from ocean to atmosphere is assumed to occur at a uniform rate everywhere. Transfer through the north end of the ocean (through the Bering Strait) is assumed to be negligible. The locations and depth ranges of each reservoir are tabulated in Table 2. (as defined by Broecker et al.)

On the basis of these assumptions one notes that L, the rate of exchange of CO_2 between the atmosphere and ocean, is determined by the average C^{14}/C^{12} ratio in the world ocean and by the difference in the C^{14}/C^{12} ratios between the surface ocean and the atmosphere. It is independent of the internal mixing. Similarly the mixing rate R , the rate of exchange of water between the Atlantic Ocean and the Antarctic - Indian - Pacific Ocean system, is determined by the amount of C^{14} which decays in the Atlantic and is independent of the arrangement of reservoirs and the mode of mixing between them. Therefore we can immediately determine these values without specifying the manner in which internal mixing takes place within the Atlantic.

The governing equations are simply the mathematical statements which describe the balance of C^{14}; the net amount of C^{14} entering a reservoir equals the amount which decays therein. Hence for the whole world ocean:

$$L A(C_a^{14} - C_s^{14}) = \lambda T C_t^{14} \tag{3}$$

where L is the rate of exchange of CO_2 between atmosphere and surface ocean (in moles $CO_2/m^2/year$), A is the area of

TABLE 2

DEFINITION OF RESERVOIRS [*]

Reservoir	Depth (m)	Surface Area (m^2)	CO_2 Content (moles)	Δc^{14} ($^o/_{oo}$)	c^{14} [**]
South Atlantic Shallow Water	100	40×10^{12}	9×10^{15}	-46	.954
North Atlantic Shallow Water	100	40×10^{12}	9×10^{15}	-35	.965
Arctic	entire	15×10^{12}	40×10^{15}	-28	.972
Atlantic Deep Water	100	-----	655×10^{15}	-104	.896
World Ocean	entire	360×10^{12}	3220×10^{15}	-160	.840
Atmosphere	entire	360×10^{12}	57×10^{15}	0	1.00

[*] As defined by Broecker et al (1960).

[**] $c^{14} = 1 + (\Delta c^{14}/1000)$ equals the ratio of the c^{14}/c^{12} value for that reservoir to the c^{14}/c^{12} value of the atmosphere.

the world ocean (m^2), T is the volume of the world ocean (in moles of CO_2), C_a^{14} is the radiocarbon concentration of the atmosphere, C_s^{14} is the average radiocarbon concentration of the world surface water (equal to $-41°/_{\bullet\bullet}$), C_t^{14} is the average radiocarbon concentration of the total world ocean, and λ is the decay constant of C^{14} (equal to 1.2×10^{-4} years^{-1}). Broecker determines L to be 22 moles CO_2/m^2/year. (See appendix for all calculations).

For the Atlantic Ocean:

$$R_o(C_o - C_4) + L\left[A_1(1.0 - C_1) + A_2(1.0 - C_2)\right.$$

$$\left. + A_3(1.0 - C_3)\right] \tag{4}$$

$$= \lambda\,(T_1 C_1 + T_2 C_2 + T_3 C_3 + T_4 C_4)$$

where R_o is the rate of flow of Antarctic water into the surface South Atlantic region (with a compensating return flow at depth), C_i is the radiocarbon concentration in the i-th reservoir, A_i is the surface area of the i-th reservoir (m^2), T_i is the volume of the i-th reservoir (moles of CO_2), and L and λ are as defined above. R_o is determined to be 3.7×10^{14} moles CO_2/year which is eqivalent to 5.5×10^6 m^3 of water/sec. Thus, L and R_o are constants independent of internal mixing.

B. The Radiocarbon Model

The equations governing internal mixing between the

four reservoirs of the Atlantic depend upon which mixing rates are allowed. There are five boundaries between adjacent reservoirs and hence ten possible mixing rates (mixing is possible in both directions). Then, ten independent equations would be required for complete generality. However, equation (1), developed in our consideration of general models, indicates that there are only six independent equations. Since N, the number of reservoirs, is four and k, the number of properties considered, is one, the number of independent equations, a, is given by

$$a = (k + 1)(N - 1) = 6.$$

Hence only six unknowns can be determined, and four modes of mixing must be assumed negligible.

It is in this assumption that our knowledge of previous oceanographic studies makes possible judicious choice. First, we wish to have an overall movement of water northward in the shallow layers and southward in the deep water. This specifies the direction of three rates, R_2, R_4, and R_5 (See Figure 3). Second, based upon the thermocline theory of Robinson and Stommel, there must be upward vertical motions beneath the thermocline to balance the downward diffusion of heat. This determines the direction of R_1 and R_3. We see, therefore, that five of the six rates are specified as to location and direction. The sixth rate, which is not specified by these conditions, may be inserted in various ways in order to gain
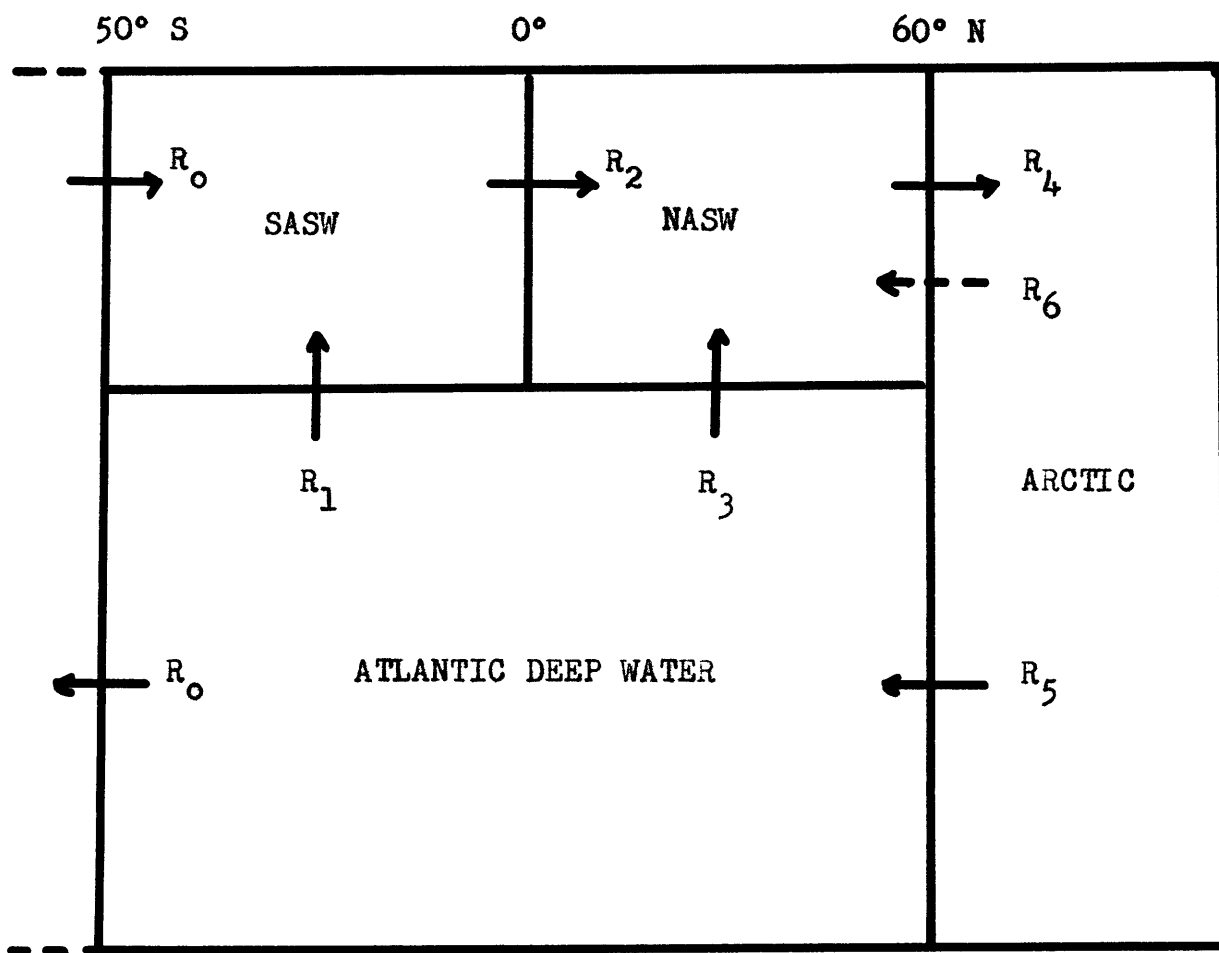
Figure 3. The Radiocarbon Model.

additional information about back mixing between various
reservoirs.

As a first example, let us determine the various rates
for the model shown diagramatically in Figure 3, in which
the sixth rate, R_6, is back mixing from the Arctic to the
North Atlantic Shallow Water. The general circulation as
described by Sverdrup, Johnson, and Fleming indicates that
this is the most probable occurance.

The governing equations are:

Radiocarbon Balance

$$R_0C_0 + R_1C_4 + L\, A_1(1.0 - C_1) = R_2C_1 + \lambda\, T_1C_1 \qquad \text{Res 1 (SASW)}$$

$$R_2C_1 + R_3C_4 + R_6C_3 + L\, A_2(1.0 - C_2) = R_4C_2 + \lambda\, T_2C_2 \qquad \text{Res 2 (NASW)}$$

$$R_4C_2 + L\, A_3(1.0 - C_3) = R_6C_3 + R_5C_3 + \lambda\, T_3C_3 \qquad \text{Res 3 (Arctic)}$$

Mass Conservation

$$R_0 + R_1 = R_2 \qquad\qquad\qquad\qquad \text{Res 1 (SASW)}$$

$$R_2 + R_3 + R_6 = R_4 \qquad\qquad\qquad \text{Res 2 (NASW)}$$

$$R_4 = R_5 + R_6 \qquad\qquad\qquad\qquad \text{Res 3 (Arctic)}$$

These six linear, nonhomogeneous equations determine uniquely
the six unknowns R_1, R_2, R_3, R_4, R_5, and R_6. These values
are tabulated in Table 3. (See appendix for calculations).

TABLE 3

Transports Based Upon the Radiocarbon Model

Transports	Magnitude (moles CO_2/year)	Magnitude (m^3 H_2O/second)
L	22/ m^2	-----
R_0	3.7×10^{14}	5.1×10^6
R_1	2.4×10^{14}	3.4×10^6
R_2	6.1×10^{14}	8.5×10^6
R_3	3.3×10^{14}	4.6×10^6
R_4	13.0×10^{14}	18.2×10^6
R_5	9.3×10^{14}	13.1×10^6
R_6	3.7×10^{14}	5.1×10^6

For reference, R_0 and L are also inserted.

On the basis of these results we find a surface transport of 5.1 million cubic meters of Antarctic water entering the South Atlantic Shallow Reservoir each second. Vertical transport from the Deep Water contributes another 3.4 million cubic meters per second. If this vertical transport is distributed uniformly over the South Atlantic region (which we have hypothesized extends from 0° to 50° S), then 4/10 or 1.5×10^6 m³/sec of this water joins the northward flowing surface water south of latitude 30° S for a total of 6.6 $\times 10^6$ m³/sec crossing that latitude. Sverdrup (The Oceans, p. 629) calculates the transport across 30° S to be due to the Benguela Current (16×10^6 m³/sec northward), the Brazil Current (10×10^6 m³/sec southward), and the central part of the South Atlantic gyre (7×10^6 m³/sec northward, 7×10^6 m³/sec southward). The net result is 6 million cubic meters per second northward transport which compares very well with the 6.6×10^6 m³/sec predicted by our model.

Sverdrup calculates that 6 million m³/sec of Upper water crosses the equator to enter the North Atlantic. An additional 2×10^6 m³/sec of Intermediate water contributes to the northward transport. Although our model does not take into account northward moving water below 100 meters, our value of R_2, equal to 8.5×10^6 m³/sec, seems to suggest that such effects are taken into account. This is understandable in view of the intimate coupling between deep and shallow

water suggested by the rather large magnitudes of vertical transport.

The water budget of the North Atlantic is determined to be as follows: 8.5×10^6 m^3/sec of South Atlantic Surface Water crossed the equator, 4.6×10^6 m^3/sec of deep water is contributed from below, and 18.2×10^6 m^3/sec flows north-ward across 50° N of which 5.1×10^6 m^3/sec returns back across this latitude. These compare favorably with estimates of sur-face transport indicated for the Gulf Stream System. The Gulf Steam, as it leaves the Straits of Florida, transports 26 million m^3/sec of which 6×10^6 m^3/sec is South Atlantic Water and the remainder North Atlantic Water. As the current flows northward it is joined by the Antilles Current, which according to Wüst (1924) carries 12 million m^3/sec. It is estimated that off Woods Hole the total transport has increased to about 72 million m^3/sec but that off the Grand Banks has decreased to less than 40×10^6 m^3/sec. Beyond this region the Gulf Stream loses its well defined character and divides into branches, part mixing with cold northern waters and the remainder return-ing southward. Our model indicates that 18.2 million cubic meters of North Atlantic Water mix with cold Arctic Waters each second, that 5.1 million cubic meters return to the south in the shallow layer while the rest sinks to contribute to the formation of the Atlantic Deep Water.

The Deep circulation, then, consists of 13.1×10^6 m^3/sec of Atlantic Deep Water being formed in the Arctic regions,

4.6×10^6 m^3/sec being drawn upward in the North Atlantic and 3.4×10^6 m^3/sec being drawn upward in the South Atlantic in the maintenance of the thermocline, and the remainder, 5.1×10^6 m^3/sec, leaving the Atlantic to mix with Antarctic water and/or contribute to the formation of the Pacific Common water. Stommel and Arons (1960) have calculated, on the basis of the distribution of temperature, salinity, and radiocarbon, that the Atlantic Deep Water contributes 3.9×10^6 m^3/sec to the formation of the Common Water.

An added item of interest to be gotten from this model is the mean residence time for each reservoir. This is a measure of the rate at which the waters are renewed. These times, calculated by dividing the total volume of the reservoir by the rate of addition of new water to that reservoir, are summarized in Table 4. Stommel (1957) calculated a mean

TABLE 4

RESIDENCE TIMES

Reservoir	Volume (moles CO_2)	Transport In (moles CO_2/yr)	Residence Time (years)
SASW	9×10^{15}	6.1×10^{14}	15
NASW	9×10^{15}	13.0×10^{14}	7
ARCTIC	40×10^{15}	13.0×10^{14}	30
NADW	655×10^{15}	9.3×10^{14}	700

residence time of about 500 years for deep Atlantic water (below 2000 meters). This is in good accord with the 700 year value determined for our model. Other investigators have found residence times for the deep Atlantic varying between 100 and 2000 years. New, independent approaches, such as the present one, will help settle this question.

The average vertical velocities at the 100 meter level may be gotten from the transports by simply dividing by the area across which the water flows. Then

$$v_1 = \frac{R_1}{A_1} \qquad \text{South Atlantic}$$

$$= .85 \times 10^{-5} \text{ cm/sec}$$

and

$$v_2 = \frac{R_3}{A_2} \qquad \text{North Atlantic}$$

$$= 1.15 \times 10^{-5} \text{ cm/sec.}$$

These velocities are roughly one centimeter a day. As noted earlier in this paper, Stommel and Arons (1960) have determined values ranging from 0.6×10^{-5} cm/sec to 3.3×10^{-5} cm/sec for the vertical velocity under the thermocline. Our values fall within this range.

C. Variations of the Model

There are two possible kinds of variations to the radiocarbon model. The first, which was mentioned earlier,

is to place the unknown, R_6, in various other regions in which there is the possibility of back mixing. Three such possibilities will be examined in this section. The second kind of variation is to take into consideration an additional property of sea water, such as temperature. We shall find that such complications in the model are at present not easily handled due mainly to the lack of knowledge of the heat storage in regions other than the North Atlantic. A more comprehensive investigation, using bathymetric data which has accumulated for many years, would make possible the determination of the annual mean heat storage data required to compute the surface heating terms.

With regard to the first method of variation, let us examine Figures 4, 5, and 6. Since little is known regarding vertical mixing, we certainly wish to examine the possibility of sinking of waters in the North and South Atlantic reservoirs. These are illustrated respectively by Figures 4 and 5. The third figure illustrates an attempt to gain some knowledge of how strongly Antarctic Bottom Water influences the characteristics of Deep Atlantic reservoir. These modes of mixing represent the most probable, large scale back mixing based on our knowledge of traditional oceanographic studies.

As before the various rates can be determined for each of these models. One finds, however, that in each case one or more of the rates turns out to be negative. What this means

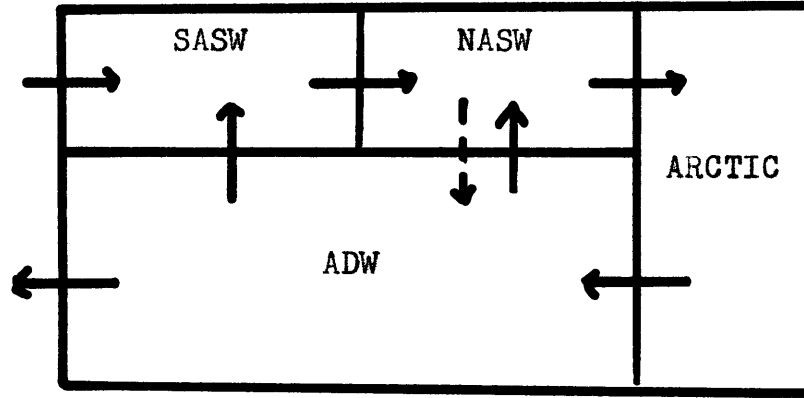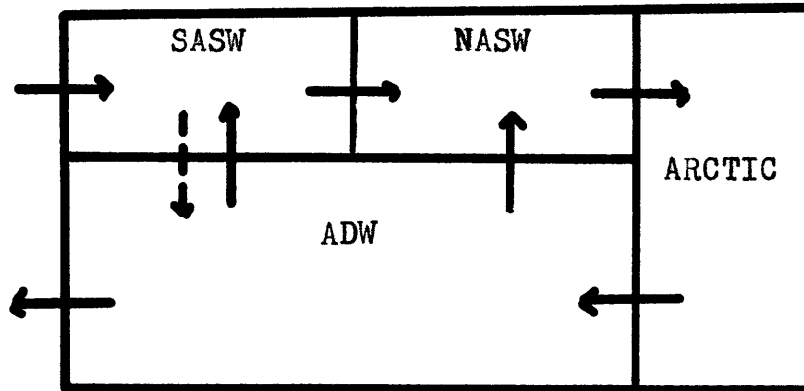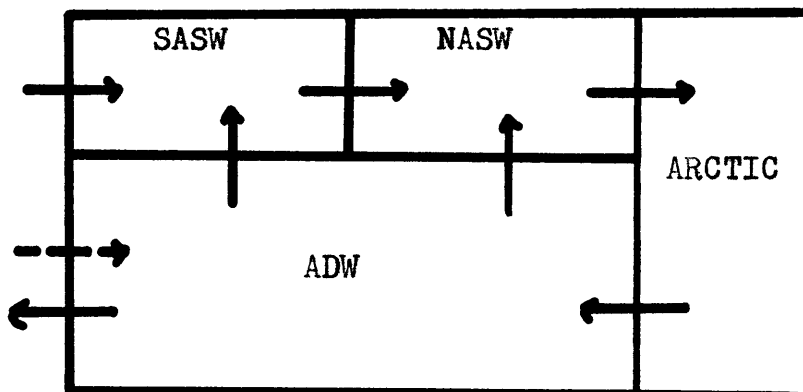Variations in the Model.

Figure 4.

Figure 5.

Figure 6.

in terms of the model is not immediately obvious, but further investigation shows that it indicates a basic inadequacy. It means that the conditions of radiocarbon balance cannot be satisfied with the rates drawn in the directions shown. Hence, we must discard these modes of mixing as being untenable without other modes of mixing being allowed.

This result leads directly to the second variation in the model. In view of the inability of the radiocarbon model alone to determine the less important mixing rates a more complicated model must be established. Let us look, for instance, at a two property model, radiocarbon and temperature. Then the number of independent equations is given by $a = (k + 1)(N - 1) = 9$. Since there are still only ten possible mixing rates, only one mode of mixing need be neglected. If we neglect transport from the North Atlantic shallow water to the South Atlantic shallow water (see Figure 7), the governing equations
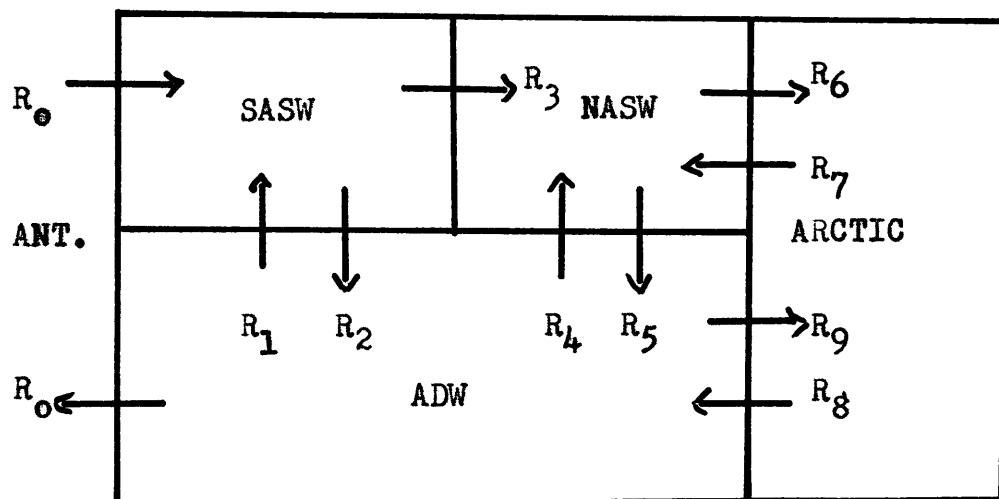
Figure 7. A Two Property Model.

may be written

Conservation of Mass

$$R_0 + R_1 - R_2 - R_3 = 0$$

$$R_3 + R_4 + R_7 - R_5 - R_6 = 0$$

$$R_6 + R_9 - R_7 - R_8 = 0$$

Conservation of Heat

$$C_p (R_0 t_0 + R_1 t_4 - R_2 t_1 - R_3 t_1) + Q_1 = 0$$

$$C_p (R_3 t_1 + R_4 t_4 + R_7 t_3 - R_5 t_2 - R_6 t_2) + Q_2 = 0$$

$$C_p (R_6 t_2 + R_9 t_4 - R_7 t_3 - R_8 t_3) + Q_3 = 0$$

Radiocarbon Balance

$$R_0 C_0 + R_1 C_4 - R_2 C_1 - R_3 C_1 = \lambda T_1 C_1 - \mathsf{L} A_1 (1.0 - C_1)$$

$$R_3 C_1 + R_4 C_4 + R_7 C_3 - R_5 C_2 - R_6 C_2 = \lambda T_2 C_2 - \mathsf{L} A_2 (1.0 - C_2)$$

$$R_6 C_2 + R_9 C_4 - R_7 C_3 - R_8 C_3 = \lambda T_3 C_3 - \mathsf{L} A_3 (1.0 - C_3)$$

where t_i is the temperature of the i-th reservoir, Q_i is the net rate of heat added to the i-th reservoir, and C_p is the specific heat of sea water. The rates R_j, of course, must be put in the proper units, m^3 H_2O/sec for the conservation of mass and heat equations and moles CO^2/year for the radiocarbon balance equations.

The main difficulties of this approach are (1) the lack of knowledge of the rates of heat addition and (2) the very complicated temperature structure even within a single reservoir. The problem, however, is not untractable but merely very tedious. The determination of heat storage as a function of time and latitude for the Arctic and South Atlantic requires the processing of very large amounts of data. This task has not been undertaken here, but the method suggested indicates the direction which further research should take. Such future work will then allow a more complicated model to be established.

III DISCUSSION OF RESULTS

Vertical transports at all latitudes, derived from a simple model based upon the distribution of radiocarbon in the Atlantic Ocean, are found to be extremely important in the overall circulation pattern. The deep waters contribute 3.4 million cubic meters per second to South Atlantic shallow water and 4.6 million cubic meters per second to the North Atlantic shallow layer. This represents over half of the transport in the deep Atlantic; that is, half of the water which is formed by sinking in the high northern latitudes returns to the shallow layers at low latitudes.

It was found that these transports were of the right order of magnitude to balance downward diffusion of heat from the hot surface layers as determined by a theory of the thermocline. Because of several simplifications and idealizations in this theory, values of vertical velocity can only be estimated roughly. Hence it is of value to have an independent estimate of this important element of the thermohaline circulation in order to confirm the theory's prediction. The simple model presented in this paper has accomplished this aim.

The inadequacies of this model must be kept in mind. The present inability to study all possible modes of mixing is a major weakness, and we must rely upon traditional studies for our choice of the major mixing modes. We have

found that this leads to results which compare favorably with those derived from geostrophic calculations. This gives us some confidence in the method.

It must be kept in mind that the study is concerned only with the very large scale water movements. The division of the Atlantic into four reservoirs is in itself a gross simplification, and the model can be expected to be indicative only of the major exchanges between them. Such simplification, however, is a fundamental feature of box models and must be accepted as part of the method.

In conclusion then, there are weaknesses inherent in the method used in this investigation. Nevertheless, important information can be obtained regarding mixing rates and renewal times. Further studies of this type will continue to be useful in oceanographic studies, particularly when more information on the distribution of radioactive elements is available.

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APPENDIX

A. Balance in the whole world ocean and the determination of:

$$LA\ (C_a^{14} - C_s^{14}) = \lambda\ TC_t^{14}$$

or

$$L = \frac{\lambda TC_t}{A(C_a - C_s)}$$

$$= \frac{(1.2 \times 10^{-4})(3220 \times 10^{15})(.840)}{(360 \times 10^{12})(.041)}$$

$$= 22 \text{ moles } CO_2/m^2/year.$$

B. Balance in the Atlantic Ocean and the determination of R_o:

$$R_o(C_o - C_4) + LA_1(1.0 - C_1) + LA_2(1.0 - C_2)$$

$$+ LA_3(1.0 - C_3) = \lambda(T_1C_1 + T_2C_2 + T_3C_3 + T_4C_4)$$

or

$$R_o = \frac{1.2 \times 10^{11})(9 \times .954 + 9 \times .965 + 40 \times .972 + 655 \times .896)}{(.887 - .896)} -$$

$$- \frac{22(40 \times .046 + 40 \times .035 + 15 \times .028) \times 10^{12}}{(.887 - .896)}$$

$$= 3.7 \times 10^{14} \text{ moles } CO_2/year.$$

C. Balance in the South Atlantic Shallow reservoir and the determination of R_1 and R_2:

$$R_1 C_4 + R_0 C_0 + LA_1(1.0 - C_1) = \lambda T_1 C_1 + R_2 C_1$$

$$R_0 + R_1 = R_2$$

Eliminating R_2 we have

$$R_1 = \frac{R_0(C_1 - C_0) + \lambda T_1 C_1 - LA_1(1.0 - C_1)}{(C_4 - C_1)}$$

$$R = \frac{3.7 \times 10^{14}(.954 - .887) + 1.2 \times 10^{11}(.954)9 -}{(.896 - .954)}$$

$$- \frac{22 \times 40 \times 10^{12}(.046)}{(.896 - .954)}$$

$$= 2.4 \times 10^{14} \text{ moles } CO_2/\text{year.}$$

D. Balance in the North Atlantic and Arctic reservoirs
and the determination of R_3, R_4, R_5, and R_6:

$$R_2 C_1 + R_3 C_4 + R_6 C_3 + LA_2(1.0 - C_2) = \lambda T_3 C_3 + C_2 R_4$$

$$R_2 + R_3 + R_6 = R_4$$

$$R_4 C_2 + LA_3(1.0 - C_3) = \lambda T_3 C_3 + C_3 R_6 + C_3 R_5$$

$$R_4 = R_5 + R_6$$

These four equations may be solved for each of the four
remaining unknowns in terms of the known quantities.
Then, it is found

$$R_3 = 3.3 \times 10^{14}$$
$$R_4 = 13.0 \times 10^{14}$$
$$R_5 = 9.3 \times 10^{14}$$
$$R_6 = 3.7 \times 10^{14}$$