VERTICAL MOTION IN THE ATLANTIC CIRCULATION PATTERN

by

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ABSTRACT

A simple model of oceanic circulation is established on the basis of the distribution of radiocarbon in the Atlantic Ocean. This radioactive isotope, formed in the upper atmosphere by cosmic rays and introduced into the surface ocean by way of the carbon dioxide exchange, decays with a half life of 5570 years. The differences in radiocarbon concentration then should be related to the mixing rates.

In view of the low temperature of the deep waters it is apparent that downward diffusion of heat from the warm surface layers must be balanced by a slow upward transport of water from depth. The radiocarbon model is established with the aim of studying these motions.

The values of vertical transport obtained are found to be comparable to horizontal exchanges and therefore very important in the Atlantic circulation pattern. Average vertical velocities of the order of one centimeter per day are found, thus substantiating estimates made by Robinson and Stommel from a consideration of theoretical thermocline models.

Thesis Supervisor:
Henry G. Houghton
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I. EXPERIMENTAL AND THEORETICAL BACKGROUND

A. Traditional Oceanographic Studies

1. Atlantic Water Masses. The general circulation in the Atlantic Ocean and the major water masses comprising it have been explored extensively by oceanographic expeditions during the last half century. Traditional studies based upon density distributions and patterns of salinity and temperature have made possible the definition of distinct water masses as well as a description of the general circulation in the North and South Atlantic. These findings have been summarized by Sverdrup, Johnson, and Fleming (1942).

The Atlantic Ocean connects with the other major oceans through the Antarctic Ocean to the south but otherwise is isolated from them. Several smaller adjacent seas, the Caribbean, the Mediterranean, and the Arctic Sea, contribute dynamically to the Atlantic circulation and importantly affect the distribution of conservative properties in the sea.

Five, well defined, sub-surface water masses are found in the Atlantic. The first, defined by nearly linear T-S relationship, has been called the South Atlantic Central Water. This mass lies just below the hot saline surface water and comprises a layer 450 meters thick. The temperature ranges from 6° C at the bottom of the layer
to about 18° C just below the surface mixed layer, and the salinity increases from 34.5 to 36.0 per mil. This vertical temperature - salinity relationship is similar to the horizontal T-S relationship found in the region of the Subtropical Convergence and hence is thought to outcrop there. The mass can be traced throughout the South Atlantic from 40° S to the equator.

Below the Central Water lies the Antarctic Intermediate Water characterized by a salinity minimum at about 800 meters over the greater part of the ocean. The mass appears to be considerably mixed with other masses as one traces it into the North Atlantic. This layer, characterized by a salinity of less than 34.65°/oo, is about 750 meters thick and overlies the great bulk of the deep and bottom water.

In the North Atlantic one finds only a single mass sandwiched between the thin surface layer and the deep and bottom water. This mass, the North Atlantic Central Water, is characterized by a nearly straight T-S curve between $T = 8°, S = 35.10°/oo$, and $T = 19°, S = 36.70°/oo$, and is seen to be similar, although warmer and more saline, to the South Atlantic Central Water.

At intermediate depths in the North Atlantic three other types of water are found but are not nearly as extensive or influential as the major water masses. One of these is the remainder of the Antarctic Intermediate
Water which becomes rapidly mixed with over and underlying waters as it moves north. Another is water of high salinity which results from spreading out of Mediterranean water flowing out through the Strait of Gibraltar. The third represents small quantities of water of Arctic origin which is somewhat analogous to the intermediate water of the Antarctic.

By far the most extensive water mass is the Atlantic Deep Water which occupies the depths between 1500 m and 4000 m throughout the Atlantic. This water is thought to form in the Arctic regions where high salinity Atlantic surface waters mix with cool waters from the north. The mixture, having a temperature of about 2.5° C and a salinity of 34.9 per mil, sinks and moves southward, extending nearly to the Antarctic region.

The fifth major water mass which we identify is the Antarctic Bottom Water which constitutes the heaviest of all waters. This water is formed in the Weddell Sea where it sinks rapidly to fill the greatest depths in the South Atlantic. The mass is found at variable depths depending upon the bottom topography but ranges from 3600 meters at 40° S to 4500 meters at 25° N.

2. The Shallow Circulation. It is in the upper regions of the ocean that the strongest currents are found. The form which this shallow circulation takes is that of a huge gyre, rotating clockwise in the Northern Hemisphere.
and counterclockwise in the Southern Hemisphere.

The most striking feature of the South Atlantic gyre is the Benguela Current which flows north along the west coast of Africa. At about 20° S the current leaves the coast to turn westward along the equator. This part of the southern gyre, lying between latitudes 0° and 20° S, is known as the South Equatorial Current. As this current approaches the South American coast it divides, part flowing across the equator into the North Atlantic and the remainder turning south along the South American coast. This southward flow of water, the Brazil Current, constitutes the western arm of the South Atlantic gyre. Along the extreme southern coast of the continent there is a northward flowing branch of the Falkland Current which turns away from the coast to join with the remnants of the Brazil Current. These waters form the South Atlantic Current which flows eastward to complete the southern gyre.

The North Atlantic circulation is dynamically connected to that of the South Atlantic by virtue of the flow of southern surface water across the equator. The water joins with part of the North Equatorial Current and enters the Caribbean Sea. There thorough mixing takes place so that temperature, salinity, and oxygen data indicate water of intermediate characteristics. The continuation of these currents represents the beginning of the Gulf Stream system which dominates the
circulation of a great part of the North Atlantic ocean.

The first segment of this system, known as the Florida Current, flows out of the Caribbean through the Florida Straits and turns north along the continental slope of North America. The Gulf Stream leaves the continental slope north of Cape Hatteras and continues as a well-defined, narrow current as far north as the Grand Banks. In this region the stream begins to branch and form the more diffuse North Atlantic Current which gradually swings eastward to form the northern branch of the North Atlantic gyre. Numerous branches send North Atlantic water into the Arctic regions to mix there with the colder, less saline waters of the Arctic Sea. When the surface waters are cooled, vertical convection currents develop leading to a renewal of the Atlantic Deep Water. Thus the surface circulation is coupled importantly to the deep circulation by way of the Arctic Sea. We shall be interested in how importantly the two are coupled in lower latitudes.

The eastern North Atlantic has no distinct currents but is characterized by an irregular flow toward the south. Some of this water enters the Mediterranean as a surface current, but the major portion continues toward the south to form the westward flowing North Equatorial Current.

3. The Deep Circulation. The deep circulation poses
one of the most difficult problems to the oceanographer. Direct measurement of deep currents has only recently been attempted (Swallow and Worthington, 1957) and little success in understanding the large scale pattern may be expected from this source for a number of years. One must resort to indirect evidence supplied by a distribution of the various properties of sea water and to simple theories of wind-driven or thermally driven currents.

As has been noted, the South Atlantic shallow water enters the North Atlantic along the coast of South America. As a result of this net transport of surface waters to the north there must be a compensating return flow in the deep water. Sverdrup concludes that the amount of sinking of surface waters which takes place in the Labrador Sea and off southern Greenland plus the contribution of heavy Mediterranean water must equal in magnitude the surface flow across the equator. However, one must not fail to take into account the possibility of rising or sinking motions in other regions. It has been found, in fact (Robinson and Stommel, 1959), that an average upward transport from the deep water into the shallow surface layer plays an integral part in the thermohaline circulation. Hence we cannot at this time accept the rates of sinking specified by Sverdrup but shall determine them for ourselves using independent
evidence.

Water mass analysis indicates that deep water of high density is formed in the subarctic and in the antarctic regions of the Atlantic Ocean. In the North Atlantic, Atlantic Deep Water flows to the south. High oxygen content decreasing in the direction of flow confirms this. This water crosses the equator and continues to flow south above the Antarctic Bottom Water.

The bottom water of Antarctic origin is colder than the deep water and forms a relatively thin bottom layer moving slowly northward. Large amounts of this water return to the Antarctic after being mixed with southward moving deep water.

In summary, the large scale circulation pattern may be described in the following way: there is a net northward transport of surface waters from the South Atlantic to the North Atlantic to the Arctic region where cold saline water sinks. This deep water flows southward transporting an amount equal to that of the shallow water across each latitude circle. Superimposed upon this large scale flow we have the possibility of vertical mixing between shallow and deep waters. We shall use these broad features to establish a simple model which will give us some information about vertical mixing and some idea of the magnitude of the sinking which takes place in the Arctic.
B. **Box Model Investigations**

The chemical and physical properties of sea water and the manner in which they vary from region to region have been used extensively in studies of ocean movements. The introduction and interpretation of the temperature-salinity diagram for instance has provided a valuable tool for determining the vertical and horizontal extents of the major water masses. Early attempts to use the conservative properties of sea water in determining mixing rates between certain regions demonstrated some of the difficulties inherent in the problem. In particular, it was found that small scale variations are considerable and that such distributions are indicative only of large scale movements.

As a simple example of the general method used in box model investigations, consider an isolated, steady-state model in which three, internally mixed reservoirs may exchange water among themselves (Fig. 1). For each

![Diagram](image)

**Figure 1. Simple Box Model.**
physical and chemical property which is conserved in the mixing process, two independent equations may be written. Mass must certainly be conserved, thus providing two additional equations. If, for instance, the water in each reservoir is characterized by two independent properties (such as temperature and salinity), then six independent equations may be obtained for the system and six unknowns may be determined at least within a constant factor. In figure 1 we have shown six unknown mixing rates, \( k_{ij} \), which represents the rate of flow of water from reservoir \( i \) to reservoir \( j \). The conservation equations may now be written:

Conservation of mass

\[
\begin{align*}
\text{Conservation of mass} & \\
\quad k_{12} + k_{32} - k_{21} - k_{23} &= 0 \\
\quad k_{23} - k_{32} + k_{13} - k_{31} &= 0
\end{align*}
\]

Conservation of \( S \)

\[
\begin{align*}
S_{1}k_{12} - S_{2}k_{21} + S_{3}k_{32} - S_{2}k_{23} &= 0 \\
S_{2}k_{23} - S_{3}k_{32} + S_{1}k_{13} - S_{3}k_{31} &= 0
\end{align*}
\]

Conservation of \( T \)

\[
\begin{align*}
T_{1}k_{12} - T_{2}k_{21} + T_{3}k_{32} - T_{2}k_{23} &= 0 \\
T_{2}k_{23} - T_{3}k_{32} + T_{1}k_{13} - T_{3}k_{31} &= 0
\end{align*}
\]

where \( S_{1}, S_{2}, S_{3}, T_{1}, T_{2}, T_{3} \) are the known concentrations of the properties \( S \) and \( T \) in the reservoir indicated by
the subscript. In this particular case the six equations are homogeneous and the rates of flow can be determined only within a common factor.

This simple example may be complicated in various ways to illustrate important aspects of the method. Consider first the effect of knowing only one property of the fluid rather than two. Then there are only four independent equations and only four unknowns may be determined (again within a common factor). This means that we cannot get information about all possible modes of mixing and must choose to investigate certain ones. This is equivalent to assuming that two of the mixing rates are negligible. The determination of the four allowed rates is only as good as this assumption.

As a second variation in this model consider the effect of knowing the concentrations of three properties of sea water. Since there are only six possible mixing rates any two properties serve to determine them. The third property then may be used as a check upon the results; that is, one can check to see how close the third property comes to being conserved using the mixing rates determined by the other properties. Then, a knowledge of the accuracy with which the various properties were measured gives an indication of the accuracy of the determination of the mixing rates.
A third variation which must be considered is that in which one or more of the properties of the fluid is nonconservative and varies with time in some known manner. Dissolved oxygen and radiocarbon are two such properties. Oxygen is absorbed at the sea surface as is carbon dioxide containing radiocarbon. Biological processes in the sea influence the oxygen content in such a way as to lead to a consumption of oxygen, and Carbon 14 decays at a certain known rate; hence the processes of advection must lead to replenishment if the steady state is to prevail. The effect of this nonconservation is to make the governing equations nonhomogeneous. That is, the net amount of the substance being brought in is no longer zero but equal to the amount which decays within the reservoir. Since the equations are now nonhomogeneous, the rates of flow are uniquely determined, and we are in a position to learn something about the absolute magnitude of the mixing.

In view of these statements regarding the example of a three reservoir model, we can generalize to an N reservoir model. For each property of the fluid, (N-1) independent equations may be found. In addition (N-1) equations can be gotten by requiring conservation of mass. Then if k equals the number of properties for which data is available, (salinity, radiocarbon, etc.)
we have

$$a = (k + 1)(N - 1)$$  \hspace{1cm} (1)

where \( a \) is the number of independent equations.

An important parameter to be considered in such box model investigations is the number of possible modes of mixing. This depends upon the number of boundaries between adjacent reservoirs. If this number is \( n \), then there are, in general, \( 2n \) possible mixing rates (mixing can take place in both directions). Hence if a system is to be uniquely and completely determined,

$$2n = (k + 1)(N - 1).$$  \hspace{1cm} (2)

If \( 2n \) is greater then \((k + 1)(N - 1)\) then all possible modes of mixing cannot be examined and one must assume, on the basis of other knowledge, that \( 2n - (k + 1)(N - 1) \) of the mixing rates may be neglected. Such a model is only as good as this assumption.

In the model which will be established in the present investigation, there will be four reservoirs \((N = 4)\) and five possible modes of mixing \((n = 5)\). Thus if \( k = 1 \) (carbon 14), \( 2n - (k + 1)(N - 1) = 10 - 6 = 4; \) i.e. we must assume that four mixing rates are negligible. (As stated above, this assumption must be justified from other considerations). We shall also examine the case \( k = 2 \) (carbon 14 and temperature). Then \( 2n - (k + 1)(N - 1) \)}
equals $10 - 9 = 1$, and only one mixing rate need be assumed zero.

C. The Dynamic Carbon Cycle and the Role of Radiocarbon

Much interest has been shown in recent years concerning the carbon cycle in nature. The manner and size of the exchange between atmosphere, biosphere, and oceans is of fundamental importance not only to the biologist but also to the geochemist, the meteorologist, and the oceanographer. Carbon is taken from the atmosphere as CO$_2$ by plant life and is given back to it by the processes of respiration and decay. The oceans absorb CO$_2$ from the atmosphere and give it back in a never ending cycle. A knowledge of these exchanges can provide valuable information to a score of independent problems in several related fields.

The total amount of carbon in the atmosphere is small compared to that dissolved in sea water. Because of the interchange between atmosphere and ocean across the sea surface, any change in CO$_2$ concentration in the atmosphere could be compensated for by the ocean, depending upon the rate of mixing. In fact, large amounts of CO$_2$ have been added to the atmosphere through the combustion of fossil fuels, and the rate of combustion had continually increased. Despite this fact a comparison of measurements made in the 19th century and in recent years does not indicate a
significant increase in atmospheric CO₂ (Suess and Revelle, 1957). Suess (1955) estimates that the world wide contamination of the earth's atmosphere by industrial CO₂ probably amounts to less than 1%. This would suggest that the CO₂ exchange rate between atmosphere and ocean is quite large, and much of the excess carbon has been absorbed in the ocean reservoir.

An important tool in the study of the carbon cycle is the radioactive isotope, Carbon 14, which has a half-life of 5570 years. Radiocarbon, as this isotope is known, is formed by cosmic ray interaction with CO₂ in the upper atmosphere. If this is a steady state process, that is, the amount produced in such interactions is just balanced by the amount which decays throughout the world reservoirs, then one would expect to find a certain ratio of C¹⁴ atoms to C¹² atoms in any one locality. This ratio and its variation from place to place provide the basis for the radiocarbon dating method.

It is immediately apparent from the preceding discussion concerning the "industrial effect," i.e. the addition of CO₂ to the atmosphere and oceans by the burning of fossil fuels, that the steady state assumption has been upset for at least a hundred years. The carbon being added in this way is free of C¹⁴ and hence tends to reduce the observed C¹⁴/C¹² ratio. Corrections for this effect
must be applied if one is to use this ratio as an indication of mixing rates between reservoirs.

A second effect must be considered and an appropriate correction made if one is to consider rates of mixing based upon $^{14}C$ data. Neutrons produced in nuclear tests interact with nitrogen in the atmosphere to produce radiocarbon. Much $^{14}C$ had been added to the earth's dynamic carbon reservoir in this way. The $^{14}C$ concentration has been found to have increased 2.1% a year since 1955 in the Southern Hemisphere tropospheric CO$_2$ (Rafter and Ferusscon, 1957). Broecker and Walton (1959) have found a 5% per year increase in the Northern Hemisphere from 1955 to 1958. It is apparent, therefore, that this effect will seriously affect the steady state assumption, and appropriate corrections must be applied to all data used.

In summary then, any study of the carbon cycle must take into account two important factors: (1) the "industrial effect," the decrease in $^{14}C$ concentration due to the addition of $^{14}C$ free CO$_2$ produced in the combustion of oil and coal for the past 100 years; (2) the "bomb $^{14}C$ effect," the increase in $^{14}C$ concentration brought about by the production of artificial $^{14}C$ in nuclear tests.

Recently a group of investigators at Lamont Geological Laboratory published results of the determination of the radiocarbon concentration in the dissolved bicarbonate
of 135 samples representing the major water masses of the Atlantic Ocean (Broecker et al, 1960). They found that while the standard deviation within a given water mass was only slightly larger than the experimental error between water masses, the total range of the $^{14}C/^{12}C$ ratio was 10%.

As might be expected from traditional studies of the Atlantic circulation the surface water showed a progressive increase from south to north as the upper mixed layer came more and more into equilibrium with the atmospheric value. The $^{14}C/^{12}C$ ratios ranged from 120 per mil lower than the preindustrial atmospheric value in the Antarctic to 50 per mil lower in the North Atlantic. The deep water mass $^{14}C/^{12}C$ ratios also showed significant variations which could be used for a quantitative determination of the rates of mixing.

The above investigation included a steady state circulation model based upon a south to north surface transport of water with a return flow at depth. Mixing across the main thermocline in the North and South Atlantic was considered to be negligible, the deep waters being formed from vertically mixed reservoirs at the extremes of the ocean. The model does not allow for either sinking or upwelling of water except in high latitudes. We shall show in the present investigation that such an assumption is not valid and that vertical mixing throughout the oceans,
not just in polar regions, contributes importantly to the total ocean circulation. The study and determination of such vertical motions represents the major aim of this paper.

D. Oceanic Upwelling

Before investigating vertical motions using radio-carbon data, let us examine past studies of vertical upwelling in the ocean. Until recently not much interest has been shown in the abyssal circulation. Stommel (1956) made a numerical calculation of the depth of no meridional motion and found the associated vertical component of velocity using actual oceanographic data from two stations. However, the result depended strongly upon a knowledge of the wind stress at the sea surface and must be considered doubtful.

Stommel points out (1958) that it seems likely that the low temperature of the deep water is maintained in the face of downward diffusion of heat from the warm surface layer by a slow upward component of velocity in the deep water. Using a very primitive model of convection he finds the maximum vertical velocity to be of the order $3 \times 10^{-5}$ cm/sec.

In 1959 a more comprehensive model of the thermohaline circulation was proposed by Robinson and Stommel. On the basis of this theoretical model they computed the average
amplitude of the maximum vertical velocity to be $4 \times 10^{-5}$ cm/sec. Two assumptions, however, are difficult to justify; first, that the eddy mixing parameter does not vary horizontally or vertically and second, that horizontal mixing is negligible.

The weaknesses in the above studies have led Stommel and Arons (1960) to adopt an intermediate approach, one which combines the Robinson-Stommel theory with oceanographic data taken from various sources. These lead to values for the vertical velocity under the thermocline varying between 0.5 and 3.0 cm/day ($0.6 \times 10^{-5}$ to $3.3 \times 10^{-5}$ cm/sec). These then give us order of magnitude values for the vertical components of velocity with which we can compare values computed on the basis of a steady state radiocarbon model.

It is important to realize that actual measurements of such small velocities is impossible, and therefore other means must be used. One such means is that described above; that is, a theoretical study based upon the existence of the strong vertical temperature gradient which must be maintained by upwelling. Independent approaches to the problem are necessary. The distribution of radioactive elements, such as radiocarbon, provides an excellent means to this end. It was on this premise that the present investigation was begun.
II THE MODEL AND ITS IMPLICATIONS

A. The Data

Let us now construct a simple, steady state model for the Atlantic circulation in which vertical transports between shallow and deep water masses are allowed. Radiocarbon data to be used in this investigation has been gathered and published by a group at Lamont Geological Laboratory (Broecker et al, 1960). Since the steady state condition of the $^{14}C/^{12}C$ ratio has been upset by the industrial effect and by nuclear bomb tests, these investigators have applied corrections to the measured data. The results, expressed in parts per mil below the atmospheric value, are summarized in Table 1. No correction has been made for samples from the deep water masses, since our model indicates that turnover times are much greater than the length of time the steady state assumption has been upset.

The Atlantic Ocean has been divided into four internally homogeneous reservoirs as shown diagramatically in Figure 2.

---

<table>
<thead>
<tr>
<th>SOUTH ATLANTIC SHALLOW WATER</th>
<th>NORTH ATLANTIC SHALLOW WATER</th>
<th>ATLANTIC DEEP WATER</th>
</tr>
</thead>
</table>

Figure 2. The Atlantic Reservoir System.
<table>
<thead>
<tr>
<th>Geographic Region</th>
<th>Uncorrected Average</th>
<th>Bomb Correction</th>
<th>Industrial Correction</th>
<th>Corrected Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arctic</td>
<td>-35°/oo</td>
<td>-3°/oo</td>
<td>+10°/oo</td>
<td>-28°/oo</td>
</tr>
<tr>
<td>North Atlantic</td>
<td>-49</td>
<td>-3</td>
<td>+17</td>
<td>-35</td>
</tr>
<tr>
<td>Shallow Water</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>South Atlantic</td>
<td>-57</td>
<td>-6</td>
<td>+17</td>
<td>-46</td>
</tr>
<tr>
<td>Shallow Water</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Antarctic</td>
<td>-120</td>
<td>-3</td>
<td>+10</td>
<td>-113</td>
</tr>
<tr>
<td>Atlantic Deep Water</td>
<td>-104</td>
<td>---</td>
<td>---</td>
<td>-104</td>
</tr>
<tr>
<td>World Ocean</td>
<td>-160</td>
<td>---</td>
<td>---</td>
<td>-160</td>
</tr>
</tbody>
</table>
The transfer of CO₂ from ocean to atmosphere is assumed to occur at a uniform rate everywhere. Transfer through the north end of the ocean (through the Bering Strait) is assumed to be negligible. The locations and depth ranges of each reservoir are tabulated in Table 2. (as defined by Broecker et al.)

On the basis of these assumptions one notes that L, the rate of exchange of CO₂ between the atmosphere and ocean, is determined by the average \( \frac{^{14}C}{^{12}C} \) ratio in the world ocean and by the difference in the \( \frac{^{14}C}{^{12}C} \) ratios between the surface ocean and the atmosphere. It is independent of the internal mixing. Similarly the mixing rate R, the rate of exchange of water between the Atlantic Ocean and the Antarctic - Indian - Pacific Ocean system, is determined by the amount of \( ^{14}C \) which decays in the Atlantic and is independent of the arrangement of reservoirs and the mode of mixing between them. Therefore we can immediately determine these values without specifying the manner in which internal mixing takes place within the Atlantic.

The governing equations are simply the mathematical statements which describe the balance of \( ^{14}C \); the net amount of \( ^{14}C \) entering a reservoir equals the amount which decays therein. Hence for the whole world ocean:

\[
L \cdot A (C_{a}^{14} - C_{s}^{14}) = \Lambda T C_{t}^{14}
\]

where \( L \) is the rate of exchange of CO₂ between atmosphere and surface ocean (in moles CO₂/m²/year), \( A \) is the area of
### TABLE 2

**DEFINITION OF RESERVOIRS***

<table>
<thead>
<tr>
<th>Reservoir</th>
<th>Depth (m)</th>
<th>Surface Area (m²)</th>
<th>CO₂ Content (moles)</th>
<th>$\Delta^{14}C$ (%)</th>
<th>$^{14}C$ **</th>
</tr>
</thead>
<tbody>
<tr>
<td>South Atlantic</td>
<td>100</td>
<td>$40 \times 10^{12}$</td>
<td>$9 \times 10^{15}$</td>
<td>-46</td>
<td>.954</td>
</tr>
<tr>
<td>Shallow Water</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>North Atlantic</td>
<td>100</td>
<td>$40 \times 10^{12}$</td>
<td>$9 \times 10^{15}$</td>
<td>-35</td>
<td>.965</td>
</tr>
<tr>
<td>Shallow Water</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arctic</td>
<td>entire</td>
<td>$15 \times 10^{12}$</td>
<td>$40 \times 10^{15}$</td>
<td>-28</td>
<td>.972</td>
</tr>
<tr>
<td>Atlantic Deep Water</td>
<td>100</td>
<td>-----</td>
<td>$655 \times 10^{15}$</td>
<td>-104</td>
<td>.896</td>
</tr>
<tr>
<td>World Ocean</td>
<td>entire</td>
<td>$360 \times 10^{12}$</td>
<td>$3220 \times 10^{15}$</td>
<td>-160</td>
<td>.840</td>
</tr>
<tr>
<td>Atmosphere</td>
<td>entire</td>
<td>$360 \times 10^{12}$</td>
<td>$57 \times 10^{15}$</td>
<td>0</td>
<td>1.00</td>
</tr>
</tbody>
</table>

* As defined by Broecker et al (1960).

** $^{14}C = 1 + (\Delta^{14}C/1000)$ equals the ratio of the $^{14}C/^{12}C$ value for that reservoir to the $^{14}C/^{12}C$ value of the atmosphere.
the world ocean \( (m^2) \), \( T \) is the volume of the world ocean (in moles of \( CO_2 \)), \( C_{a}^{14} \) is the radiocarbon concentration of the atmosphere, \( C_{s}^{14} \) is the average radiocarbon concentration of the world surface waters (equal to \(-41^\circ/_{oo} \)), \( C_{t}^{14} \) is the average radiocarbon concentration of the total world ocean, and \( \lambda \) is the decay constant of \( C_{a}^{14} \) (equal to \( 1.2 \times 10^{-4} \) years\(^{-1} \)). Broecker determines \( L \) to be 22 moles \( CO_2/m^2/\)year. (See appendix for all calculations).

For the Atlantic Ocean:

\[
R_{o}(C_{o} - C_{4}) + L \left[ A_{1}(1.0 - C_{1}) + A_{2}(1.0 - C_{2}) + A_{3}(1.0 - C_{3}) \right]
\]

where \( R_{o} \) is the rate of flow of Antarctic water into the surface South Atlantic region (with a compensating return flow at depth), \( C_{1} \) is the radiocarbon concentration in the \( i \)-th reservoir, \( A_{i} \) is the surface area of the \( i \)-th reservoir \( (m^2) \), \( T_{i} \) is the volume of the \( i \)-th reservoir (moles of \( CO_2 \)), and \( L \) and \( \lambda \) are as defined above. \( R_{o} \) is determined to be \( 3.7 \times 10^{14} \) moles \( CO_2/\)year which is equivalent to \( 5.5 \times 10^{6} \) m\(^3\) of water/\)sec. Thus, \( L \) and \( R_{o} \) are constants independent of internal mixing.

B. The Radiocarbon Model

The equations governing internal mixing between the
four reservoirs of the Atlantic depend upon which mixing rates are allowed. There are five boundaries between adjacent reservoirs and hence ten possible mixing rates (mixing is possible in both directions). Then, ten independent equations would be required for complete generality. However, equation (1), developed in our consideration of general models, indicates that there are only six independent equations. Since \( N \), the number of reservoirs, is four and \( k \), the number of properties considered, is one, the number of independent equations, \( a \), is given by

\[
a = (k + 1)(N - 1) = 6.
\]

Hence only six unknowns can be determined, and four modes of mixing must be assumed negligible.

It is in this assumption that our knowledge of previous oceanographic studies makes possible judicious choice. First, we wish to have an overall movement of water northward in the shallow layers and southward in the deep water. This specifies the direction of three rates, \( R_2 \), \( R_4 \), and \( R_5 \) (See Figure 3). Second, based upon the thermocline theory of Robinson and Stommel, there must be upward vertical motions beneath the thermocline to balance the downward diffusion of heat. This determines the direction of \( R_1 \) and \( R_3 \). We see, therefore, that five of the six rates are specified as to location and direction. The sixth rate, which is not specified by these conditions, may be inserted in various ways in order to gain
Figure 3. The Radiocarbon Model.
additional information about back mixing between various reservoirs.

As a first example, let us determine the various rates for the model shown diagramatically in Figure 3, in which the sixth rate, $R_6$, is back mixing from the Arctic to the North Atlantic Shallow Water. The general circulation described by Sverdrup, Johnson, and Fleming indicates that this is the most probable occurrence.

The governing equations are:

**Radiocarbon Balance**

$$R_0C_0 + R_1C_4 + L A_1(1.0 - C_1) = R_2C_1 + \lambda T_1C_1 \quad \text{Res 1 (SASW)}$$

$$R_2C_1 + R_3C_4 + R_6C_3 + L A_2(1.0 - C_2) = R_4C_2 + \lambda T_2C_2 \quad \text{Res 2 (NASW)}$$

$$R_4C_2 + L A_3(1.0 - C_3) = R_6C_3 + R_5C_3 + \lambda T_3C_3 \quad \text{Res 3 (Arctic)}$$

**Mass Conservation**

$$R_0 + R_1 = R_2 \quad \text{Res 1 (SASW)}$$

$$R_2 + R_3 + R_6 = R_4 \quad \text{Res 2 (NASW)}$$

$$R_4 = R_5 + R_6 \quad \text{Res 3 (Arctic)}$$

These six linear, nonhomogeneous equations determine uniquely the six unknowns $R_1, R_2, R_3, R_4, R_5,$ and $R_6$. These values are tabulated in Table 3. (See appendix for calculations).
### TABLE 3

Transports Based Upon the Radiocarbon Model

<table>
<thead>
<tr>
<th>Transports</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
</tr>
<tr>
<td>R₀</td>
</tr>
<tr>
<td>R₁</td>
</tr>
<tr>
<td>R₂</td>
</tr>
<tr>
<td>R₃</td>
</tr>
<tr>
<td>R₄</td>
</tr>
<tr>
<td>R₅</td>
</tr>
<tr>
<td>R₆</td>
</tr>
</tbody>
</table>
For reference, \( R_o \) and \( L \) are also inserted.

On the basis of these results we find a surface transport of 5.1 million cubic meters of Antarctic water entering the South Atlantic Shallow Reservoir each second. Vertical transport from the Deep Water contributes another 3.4 million cubic meters per second. If this vertical transport is distributed uniformly over the South Atlantic region (which we have hypothesized extends from 0° to 50° S), then \( 4/10 \) or \( 1.5 \times 10^6 \) m\(^3\)/sec of this water joins the northward flowing surface water south of latitude 30° S for a total of \( 6.6 \times 10^6 \) m\(^3\)/sec crossing that latitude. Sverdrup (The Oceans, p. 629) calculates the transport across 30° S to be due to the Benguela Current \( (16 \times 10^6 \) m\(^3\)/sec northward), the Brazil Current \( (10 \times 10^6 \) m\(^3\)/sec southward), and the central part of the South Atlantic gyre \( (7 \times 10^6 \) m\(^3\)/sec northward, \( 7 \times 10^6 \) m\(^3\)/sec southward). The net result is 6 million cubic meters per second northward transport which compares very well with the \( 6.6 \times 10^6 \) m\(^3\)/sec predicted by our model.

Sverdrup calculates that 6 million m\(^3\)/sec of Upper water crosses the equator to enter the North Atlantic. An additional \( 2 \times 10^6 \) m\(^3\)/sec of Intermediate water contributes to the northward transport. Although our model does not take into account northward moving water below 100 meters, our value of \( R_2 \), equal to \( 8.5 \times 10^6 \) m\(^3\)/sec, seems to suggest that such effects are taken into account. This is understandable in view of the intimate coupling between deep and shallow
water suggested by the rather large magnitudes of vertical transport.

The water budget of the North Atlantic is determined to be as follows: \(8.5 \times 10^6\) m\(^3\)/sec of South Atlantic Surface Water crossed the equator, \(4.6 \times 10^6\) m\(^3\)/sec of deep water is contributed from below, and \(18.2 \times 10^6\) m\(^3\)/sec flows northward across 50° N of which \(5.1 \times 10^6\) m\(^3\)/sec returns back across this latitude. These compare favorably with estimates of surface transport indicated for the Gulf Stream System. The Gulf Stream, as it leaves the Straits of Florida, transports 26 million m\(^3\)/sec of which \(6 \times 10^6\) m\(^3\)/sec is South Atlantic Water and the remainder North Atlantic Water. As the current flows northward it is joined by the Antilles Current, which according to Wüst (1924) carries 12 million m\(^3\)/sec. It is estimated that off Woods Hole the total transport has increased to about 72 million m\(^3\)/sec but that off the Grand Banks has decreased to less than \(40 \times 10^6\) m\(^3\)/sec. Beyond this region the Gulf Stream loses its well defined character and divides into branches, part mixing with cold northern waters and the remainder returning southward. Our model indicates that 18.2 million cubic meters of North Atlantic Water mix with cold Arctic Waters each second, that 5.1 million cubic meters return to the south in the shallow layer while the rest sinks to contribute to the formation of the Atlantic Deep Water.

The Deep circulation, then, consists of \(13.1 \times 10^6\) m\(^3\)/sec of Atlantic Deep Water being formed in the Arctic regions,
4.6 x 10^6 m^3/sec being drawn upward in the North Atlantic and 3.4 x 10^6 m^3/sec being drawn upward in the South Atlantic in the maintenance of the thermocline, and the remainder, 5.1 x 10^6 m^3/sec, leaving the Atlantic to mix with Antarctic water and/or contribute to the formation of the Pacific Common water. Stommel and Arons (1960) have calculated, on the basis of the distribution of temperature, salinity, and radiocarbon, that the Atlantic Deep Water contributes 3.9 x 10^6 m^3/sec to the formation of the Common Water.

An added item of interest to be gotten from this model is the mean residence time for each reservoir. This is a measure of the rate at which the waters are renewed. These times, calculated by dividing the total volume of the reservoir by the rate of addition of new water to that reservoir, are summarized in Table 4. Stommel (1957) calculated a mean

<table>
<thead>
<tr>
<th>Reservoir</th>
<th>Volume (moles CO₂)</th>
<th>Transport In (moles CO₂/yr)</th>
<th>Residence Time (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SASW</td>
<td>9 x 10^{15}</td>
<td>6.1 x 10^{14}</td>
<td>15</td>
</tr>
<tr>
<td>NASW</td>
<td>9 x 10^{15}</td>
<td>13.0 x 10^{14}</td>
<td>7</td>
</tr>
<tr>
<td>ARCTIC</td>
<td>40 x 10^{15}</td>
<td>13.0 x 10^{14}</td>
<td>30</td>
</tr>
<tr>
<td>NADW</td>
<td>655 x 10^{15}</td>
<td>9.3 x 10^{14}</td>
<td>700</td>
</tr>
</tbody>
</table>
residence time of about 500 years for deep Atlantic water (below 2000 meters). This is in good accord with the 700 year value determined for our model. Other investigators have found residence times for the deep Atlantic varying between 100 and 2000 years. New, independent approaches, such as the present one, will help settle this question.

The average vertical velocities at the 100 meter level may be gotten from the transports by simply dividing by the area across which the water flows. Then

\[ v_1 = \frac{R_1}{A_1} \]

= \(0.85 \times 10^{-5}\) cm/sec

and

\[ v_2 = \frac{R_3}{A_2} \]

= \(1.15 \times 10^{-5}\) cm/sec.

These velocities are roughly one centimeter a day. As noted earlier in this paper, Stommel and Arons (1960) have determined values ranging from \(0.6 \times 10^{-5}\) cm/sec to \(3.3 \times 10^{-5}\) cm/sec for the vertical velocity under the thermocline. Our values fall within this range.

C. Variations of the Model

There are two possible kinds of variations to the radiocarbon model. The first, which was mentioned earlier,
is to place the unknown, $R_6$, in various other regions in which there is the possibility of back mixing. Three such possibilities will be examined in this section. The second kind of variation is to take into consideration an additional property of sea water, such as temperature. We shall find that such complications in the model are at present not easily handled due mainly to the lack of knowledge of the heat storage in regions other than the North Atlantic. A more comprehensive investigation, using bathymetric data which has accumulated for many years, would make possible the determination of the annual mean heat storage data required to compute the surface heating terms.

With regard to the first method of variation, let us examine Figures 4, 5, and 6. Since little is known regarding vertical mixing, we certainly wish to examine the possibility of sinking of waters in the North and South Atlantic reservoirs. These are illustrated respectively by Figures 4 and 5. The third figure illustrates an attempt to gain some knowledge of how strongly Antarctic Bottom Water influences the characteristics of Deep Atlantic reservoir. These modes of mixing represent the most probable, large scale back mixing based on our knowledge of traditional oceanographic studies.

As before the various rates can be determined for each of these models. One finds, however, that in each case one or more of the rates turns out to be negative. What this means
Variations in the Model.

Figure 4.

Figure 5.

Figure 6.
in terms of the model is not immediately obvious, but further investigation shows that it indicates a basic inadequacy. It means that the conditions of radiocarbon balance cannot be satisfied with the rates drawn in the directions shown. Hence, we must discard these modes of mixing as being untenable without other modes of mixing being allowed.

This result leads directly to the second variation in the model. In view of the inability of the radiocarbon model alone to determine the less important mixing rates a more complicated model must be established. Let us look, for instance, at a two property model, radiocarbon and temperature. Then the number of independent equations is given by \( a = (k + 1)(N - 1) = 9 \). Since there are still only ten possible mixing rates, only one mode of mixing need be neglected. If we neglect transport from the North Atlantic shallow water to the South Atlantic shallow water (see Figure 7), the governing equations

\[
\begin{align*}
R_0 &\rightarrow \text{SASW} \\
\text{ANT.} &\rightarrow R_3 \rightarrow \text{NASW} \\
&\rightarrow R_6 \\
R_7 &\leftarrow \text{ARCTIC} \\
R_1 &\downarrow \text{ADW} \\
&\downarrow R_2 \\
R_4 &\downarrow \text{ADW} \\
&\downarrow R_5 \\
&\downarrow R_9 \\
R_8 &\leftarrow \text{ARCTIC}
\end{align*}
\]

**Figure 7.** A Two Property Model.
may be written

**Conservation of Mass**

\[ R_0 + R_1 - R_2 - R_3 = 0 \]
\[ R_3 + R_4 + R_7 - R_5 - R_6 = 0 \]
\[ R_6 + R_9 - R_7 - R_8 = 0 \]

**Conservation of Heat**

\[ C_p (R_{o_0} + R_1 t_4 - R_2 t_1 - R_3 t_1) + Q_1 = 0 \]
\[ C_p (R_3 t_1 + R_4 t_4 + R_7 t_3 - R_5 t_2 - R_6 t_2) + Q_2 = 0 \]
\[ C_p (R_6 t_2 + R_9 t_4 - R_7 t_3 - R_8 t_3) + Q_3 = 0 \]

**Radiocarbon Balance**

\[ R_0 C_0 + R_1 C_4 - R_2 C_1 - R_3 C_1 = \lambda T_1 C_1 - \Lambda A_1 (1.0 - C_1) \]
\[ R_3 C_1 + R_4 C_4 + R_7 C_3 - R_5 C_2 - R_6 C_2 = \lambda T_2 C_2 - \Lambda A_2 (1.0 - C_2) \]
\[ R_6 C_2 + R_9 C_3 - R_7 C_3 - R_8 C_3 = \lambda T_3 C_3 - \Lambda A_3 (1.0 - C_3) \]

where \( t_i \) is the temperature of the \( i \)-th reservoir, \( Q_i \) is the net rate of heat added to the \( i \)-th reservoir, and \( C_p \) is the specific heat of sea water. The rates \( R_i \), of course, must be put in the proper units, \( m^3 H_2O/sec \) for the conservation of mass and heat equations and moles \( CO_2/year \) for the radiocarbon balance equations.
The main difficulties of this approach are (1) the lack of knowledge of the rates of heat addition and (2) the very complicated temperature structure even within a single reservoir. The problem, however, is not untractable but merely very tedious. The determination of heat storage as a function of time and latitude for the Arctic and South Atlantic requires the processing of very large amounts of data. This task has not been undertaken here, but the method suggested indicates the direction which further research should take. Such future work will then allow a more complicated model to be established.
III DISCUSSION OF RESULTS

Vertical transports at all latitudes, derived from a simple model based upon the distribution of radiocarbon in the Atlantic Ocean, are found to be extremely important in the overall circulation pattern. The deep waters contribute 3.4 million cubic meters per second to South Atlantic shallow water and 4.6 million cubic meters per second to the North Atlantic shallow layer. This represents over half of the transport in the deep Atlantic; that is, half of the water which is formed by sinking in the high northern latitudes returns to the shallow layers at low latitudes.

It was found that these transports were of the right order of magnitude to balance downward diffusion of heat from the hot surface layers as determined by a theory of the thermocline. Because of several simplifications and idealizations in this theory, values of vertical velocity can only be estimated roughly. Hence it is of value to have an independent estimate of this important element of the thermohaline circulation in order to confirm the theory's prediction. The simple model presented in this paper has accomplished this aim.

The inadequacies of this model must be kept in mind. The present inability to study all possible modes of mixing is a major weakness, and we must rely upon traditional studies for our choice of the major mixing modes. We have
found that this leads to results which compare favorably with those derived from geostrophic calculations. This gives us some confidence in the method.

It must be kept in mind that the study is concerned only with the very large scale water movements. The division of the Atlantic into four reservoirs is in itself a gross simplification, and the model can be expected to be indicative only of the major exchanges between them. Such simplification, however, is a fundamental feature of box models and must be accepted as part of the method.

In conclusion then, there are weaknesses inherent in the method used in this investigation. Nevertheless, important information can be obtained regarding mixing rates and renewal times. Further studies of this type will continue to be useful in oceanographic studies, particularly when more information on the distribution of radioactive elements is available.
REFERENCES


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APPENDIX

A. Balance in the whole world ocean and the determination of:

\[ L_A \left( C_a^{14} - C_s^{14} \right) = \lambda T C_t^{14} \]

or

\[ L = \frac{\lambda T C_t}{A(C_a - C_s)} \]

\[ = \frac{(1.2 \times 10^{-4})(3220 \times 10^{15})(.840)}{(360 \times 10^{12})(.041)} \]

\[ = 22 \text{ moles CO}_2/\text{m}^2/\text{year}. \]

B. Balance in the Atlantic Ocean and the determination of \( R_0 \): 

\[ R_0(C_0 - C_4) + L A_1(1.0 - C_1) + L A_2(1.0 - C_2) + L A_3(1.0 - C_3) = \lambda(T_1 C_1 + T_2 C_2 + T_3 C_3 + T_4 C_4) \]

or

\[ R_0 = 1.2 \times 10^{11}(9x.954+9x.965+40x.972+655x.896) - \frac{(887 - .896)}{.887 - .896} \]

\[ = 22(40x.046+40x.035+15x.028) \times 10^{12} \]

\[ = 3.7 \times 10^{14} \text{ moles CO}_2/\text{year}. \]

C. Balance in the South Atlantic Shallow reservoir and the determination of \( R_1 \) and \( R_2 \):
R_1C_4 + R_0C_0 + LA_1(1.0 - C_1) = \lambda T_1C_1 + R_2C_1

R_0 + R_1 = R_2

Eliminating R_2 we have

R_1 = \frac{R_0(C_1 - C_0) + \lambda T_1C_1 - LA_1(1.0 - C_1)}{(C_4 - C_1)}

R = 3.7 \times 10^{14}(\cdot 954 - \cdot 887) + 1.2 \times 10^{11}(\cdot 954)\cdot 9 - (\cdot 896 - \cdot 954)

- 22 \times 4 \cdot 10^{12}(\cdot 046)

= 2.4 \times 10^{14} \text{ moles CO}_2/\text{year.}

D. Balance in the North Atlantic and Arctic reservoirs and the determination of R_3, R_4, R_5, and R_6:

R_2C_1 + R_3C_4 + R_6C_3 + LA_2(1.0 - C_2) = \lambda T_3C_3 + C_2R_4

R_2 + R_3 + R_6 = R_4

R_4C_2 + LA_3(1.0 - C_3) = \lambda T_3C_3 + C_3R_6 + C_3R_5

R_4 = R_5 + R_6

These four equations may be solved for each of the four remaining unknowns in terms of the known quantities. Then, it is found

R_3 = 3.3 \times 10^{14}

R_4 = 13.0 \times 10^{14}

R_5 = 9.3 \times 10^{14}

R_6 = 3.7 \times 10^{14}