Empirical Study of New Keynesian Model using Cointegrated VAR: What New Zealand data tell us

by

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BCom(Hons), University of Auckland (2003)

Submitted to the Department of Economics
in partial fulfillment of the requirements for the degree of

Master of Science

at the

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Abstract
Econometric analysis of rational expectations models has been a widely studied topic in the macro-econometric literature. This thesis looks in particular at evaluating Neokeynesian model (NKM) with respect to its conformity with the data. Among the available econometric techniques, this thesis investigates what cointegrated VAR can illuminate about how close the NKM gets to the data. This project closely follow the approach taken by Mikael Juselius (2008) and extends the analysis to the New Zealand data. The findings from the thesis lend support to Juselius’ conclusions but in a limited way. The results from this thesis question the robustness of his claims based on US data supporting inexact rational expectations models.

Thesis Supervisor: Michael Piore
Title: Professor of Political Economy
INTRODUCTION

A critical issue in the empirical investigation of rational expectations model is the cross-equation restrictions implied by the hypothesis of rational expectations. This is bound to complicate the econometric efforts to take the theory to the data. Hansen and Sargent (1980) derive the formula for the cross-equation rational expectations restrictions, with agents’ decision rules modelled as time invariant stochastic difference equation. Nason and Smith (2005) studies the identification problem of new Keynesian Phillips curve within three-equation new Keynesian model context. From construction of tests and confidence intervals they find little evidence of forward looking inflation dynamics. Peersman and Straub (2006) use a VAR with sign restrictions that are robust to model and parameter uncertainty to estimate the effects of various shocks to the macroeconomic variables. There have been various efforts to estimate new Keynesian models. Smets and Wouters (2003) and Lubik and Schorfheide (2004) employ the Bayesian methodology to estimate several versions of NKM. McCallum and Nelson (1998) and Ireland (2001) obtain instrumental variables and carry out maximum likelihood estimation. Rotemberg and Woodford (1998), Christiano, Eichenbaum, and Evans (2001), and Boivin and Giannoni (2003) conduct the estimation of structural New Keynesian models by minimizing a measure of distance between empirical VARs and their models. This thesis looks in particular at evaluating Neokeynesian model (NKM) with respect to its conformity with the New Zealand data. Among the available econometric techniques, this thesis investigates what multivariate cointegration (cointegrated VAR) can illuminate about how close the NKM gets to the data. This thesis closely follows the approach taken by Mikael Juselius (2008) and extends the analysis to the New Zealand data. Juselius shows that the multivariate cointegration methodology can be applied to the NKM case in an implementable way.

The motivation behind this direction of investigation comes in a few fronts. Most of the empirical analysis on the rational expectations models assume that the variables are stationary when in fact many variables show unit root features in the actual
Another point to consider is the fact that the cross equation restrictions of the theoretical model will end up constraining the short run dynamics to a considerable extent. As a result, models that could be found useful might be rejected. Cointegration implications are necessary conditions for the theoretical model to hold: it is a subset of the restrictions on the data that the theoretical model imposes, in particular it ensures the consistency with the long run properties of the data without putting restrictions on the short term dynamics.² This thesis reveals that the extension to the New Zealand economy brings somewhat different findings to the original paper: that the version of the NKM with marginal costs approximated by labor cost rather than output gap is more consistent with the long run properties of the data.

²Juselius (2008), p.7
MODEL SPECIFICATION

The following general form of the linear rational expectations model is assumed in this analysis:

$E_t A_f(L^{-1})A_b(L)y_t + B(L)x_t + \phi D_t = 0 \quad (1)$

$C(L)x_t = c_t \quad (2)$

The first equation can be treated as a linearized version of the non-linear Euler equation. In this thesis, the NewKeynesian model of the following representation is considered:

$\bar{y}_t = \phi_1 E_t y_{t+1} + \phi_2 y_{t-1} - \phi_3 (i_t - E_t \pi_{t+1}) \quad (3)$

$\pi_t = \phi_4 E_t \pi_{t+1} + \phi_5 \pi_{t-1} + \phi_6 \bar{y}_t \quad (4)$

$i_t = \phi_7 i_{t-1} + (1 - \phi_7)(\phi_8 (E_t \pi_{t+1} - \pi^*_t) + \phi_9 \bar{y}_t) \quad (5)$

The first and second equations are an optimizing IS curve and a newKeynesian Phillips curve, respectively. The last equation captures a Taylor-type policy rule together with interest rate smoothing behaviour.

This is a Clarida et al (1999) and Woodford (2003) benchmark version once we set a particular choice of parameters, marginal cost measures, and shock structures. More recent models such as Ireland (2004) and Bekaert et al (2005) also can fit into this mold.

The above three equation NKM model is a special case of the rational expectations model with,
According to Hansen and Sargent (1980, 1981), there are two ways of introducing a disturbance term in the first rational expectations model and make it suitable for econometric analysis: exact and inexact modelling. In the exact modelling case, the investigator observes an information set, $\Lambda_t$, that consists of past and current values of $x_t$ and $y_t$. But $\Lambda_t \subset \Omega_t$, where $\Omega_t$ is the agent’s information set relevant for forecasting $x_t$. Hence, the disturbance term is interpreted as an omitted information in this case. The inexact option partitions $x$ into $(x_1, t, x_2, t)'$ where the investigator only observes $x_{1,t}$. Therefore, in contrast to the previous case, the disturbance term is interpreted as a missing variables here. This analysis investigates cointegration implication of both exact and inexact rational expectations, in particular NKM, based on New Zealand data.

The alternative versions of the NKM models are distinguished here by the corresponding measures of marginal costs, namely, the output gap and the labor share. First, we consider the NKM with flexible price output gap as marginal costs, i.e., $\bar{x}_t = \bar{y}_t = y_t^f - y_t^p$. Flexible price output level is approximated to be some measure of potential output. So with $y_t^f = y_t^p$, the NKM can be written as:

$$y_t = (\bar{y}_t, \pi_t, i_t)' = (x_t, \pi_t^*)'$$

$$A(L) = \begin{pmatrix}
1 - \phi_1 L^{-1} - \phi_2 L & -\phi_3 L^{-1} & \phi_3 \\
0 & 1 - \phi_4 L^{-1} - \phi_5 L & 0 \\
-\phi_9 (1 - \phi_7) & -\phi_8 (1 - \phi_7) L^{-1} & -\phi_7 L
\end{pmatrix}$$

and

$$B(L) = \begin{pmatrix}
0 & 0 \\
-\phi_6 & 0 \\
0 & \phi_8 (1 - \phi_7)
\end{pmatrix}$$

$$y_t = y_t^p + \phi_1 E_t(y_{t+1}^p - y_{t+1}^p) + \phi_2 (y_{t-1}^p - y_{t-1}^p) - \phi_3 (i_t - E_t \pi_{t+1})$$

$$\pi_t = \phi_4 E_t \pi_{t+1} + \phi_5 \pi_{t-1} + \phi_6 \bar{x}_t$$

$$i_t = \phi_7 i_{t-1} + (1 - \phi_7)(\phi_8 (E_t \pi_{t+1} - \pi_t^*) + \phi_9 (y_t^* - y_t^p))$$
We identify two possible sources of stochastic trends in the above specification: a technology shock originating in \( y_t^f \) (Ireland (2004)) and a time varying central bank inflation target \( \pi_t^* \) as in Kozicki and Tinsley (2005). The exact rational expectations model has the feature that the only source for a stochastic trend is through potential output since true central bank inflation target is unknown to the public. This means that the real output shares this stochastic trend and hence the output gap must also be stationary. Along with this, inflation and interest rate must be stationary with no other sources of stochastic trends. Therefore, with the data matrix \( z_t = (y_t, i_t, \pi_t, y_t^n)' \), the exact version has three cointegrating vectors: stationary output gap, stationary inflation, and stationary nominal interest rate. Next, we assume that there are permanent changes in the unobserved central bank inflation target. Real output can now contain stochastic trends in both the central bank inflation target as well as the stochastic trend in technology and hence output gap is no longer guaranteed to be stationary. Now, the choice of parameter values in NKM determines which variables share stochastic trends in \( y_t^n \) and \( \pi_t^* \). This is the inexact version of NKM and the corresponding sub-cointegration space in terms of the observable variables, \( z_t = (y_t, i_t, \pi_t, y_t^n)' \) is given by:

\[
\tilde{A}_\theta = \begin{pmatrix}
1 - \phi_1 - \phi_2 & -\phi_3 & \phi_3 & \phi_1 + \phi_2 - 1 \\
-\phi_6 & 1 - \phi_4 - \phi_5 & 0 & \phi_6
\end{pmatrix}
\]

The above indicates two cointegration vectors and two common trends. This cointegration space describes an optimising IS curve and a new Keynesian Phillips curve with the nominal interest rate exogenously given. The restrictions, \( \phi_4 + \phi_5 = 1 \) and \( \phi_6 \neq 0 \) implies that the output gap is stationary and if \( \phi_1 + \phi_2 = 1 \) and \( \phi_3 \neq 0 \), the real interest rate is stationary.

The next variant of NKM we consider is when we measure marginal cost as labor's share, \( s_t \). The following represents such variant model:

\[
y_t = y_t^n + \phi_1 E_t(y_{t+1}^r - y_{t+1}^n) + \phi_2(y_{t-1}^r - y_{t-1}^n) - \phi_3(i_t - E_t\pi_{t+1})
\] (9)
\[ \pi_t = \phi_4 E_t \pi_{t+1} + \phi_5 \pi_{t-1} + \phi_6 s_t \]  
\[ i_t = \phi_7 i_{t-1} + (1 - \phi_7)(\phi_8 (E_t \pi_{t+1} - \pi_t^*) + \phi_9 (y_t^n - y_t^n) + \phi_{10} s_t) \]

And assume that \( y_t^n \) and \( s_t \) have separate stochastic trends. The exact model: we assume a central bank inflation target to be constant, \( \pi_t^* = \pi^* = 0 \). The cointegration space with \( y_t = (y_t^n, \pi_t, i_t)' \) and \( x_t = (y_t^n, s_t)' \) is:

\[
\bar{A} = \begin{pmatrix}
1 - \phi_1 - \phi_2 & -\phi_3 & \phi_3 & \phi_1 + \phi_2 - 1 & 0 \\
0 & 1 - \phi_4 - \phi_5 & 0 & 0 & -\phi_6 \\
-\phi_9 (1 - \phi_7) & -\phi_8 (1 - \phi_7) & -\phi_7 & \phi_9 (1 - \phi_7) & -\phi_{10} (1 - \phi_7)
\end{pmatrix}
\]

This implies three cointegration vectors and two common stochastic trends.

The inexact model: we assume \( \pi_t^* I(1) \). Given \( y_t = (y_t^n, \pi_t, i_t)' \), \( x_{1,t} = \pi_t \), and \( z = (y_t^n, \pi_t, i_t, y_t^n, s_t)' \) as observable variables, the cointegration space is:

\[
\bar{A} = \begin{pmatrix}
1 - \phi_1 - \phi_2 & -\phi_3 & \phi_3 & \phi_1 + \phi_2 - 1 & 0 \\
0 & 1 - \phi_4 - \phi_5 & 0 & 0 & -\phi_6
\end{pmatrix}
\]

This implies two cointegration vectors and three stochastic trends.

Table 1 summarises different models considered.

Table 1

<table>
<thead>
<tr>
<th>Model</th>
<th>Type</th>
<th>( y_t )</th>
<th>( x_{1,t} )</th>
<th>( x_{2,t} )</th>
<th>MC</th>
<th>( r+p_1 )</th>
<th>CI-rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>E</td>
<td>( (y_t^n, \pi_t, i_t)' )</td>
<td>( y_t^n )</td>
<td>-</td>
<td>( y_t^n - y_t^n )</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>I</td>
<td>( (y_t^n, \pi_t, i_t)' )</td>
<td>( y_t^n )</td>
<td>( i_t^* )</td>
<td>( y_t^n - y_t^n )</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>E</td>
<td>( (y_t^n, \pi_t, i_t)' )</td>
<td>( y_t^n, s_t)' )</td>
<td>-</td>
<td>( s_t )</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>I</td>
<td>( (y_t^n, \pi_t, i_t)' )</td>
<td>( y_t^n, s_t)' )</td>
<td>( i_t^* )</td>
<td>( s_t )</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

The DATA

\textsuperscript{3}Juselius (2008), p.17
The New Zealand data used in this analysis are obtained from the Reserve Bank of New Zealand and Infoshare from Statistics New Zealand. The real GDP for a measure of real output, the inflation rate, and short-term interest rate, and labor cost (for a measure of marginal costs) are used in the analysis. The measure of potential output is obtained from filtering real GDP by Hodrick Prescott filter. The quarterly sample covers the period 1987:3 to 2008:3, comprising of 85 observations for each variable.
Real and potential output

Inflation rate

Interest Rate

Labor Cost
Statistical Modelling

The cointegrated VAR model is used to test implications for the long run properties of the data: the number of common trends and the structure of the cointegration space. In particular the following statistical model is applied:

$$\Delta X_t = \sum_{i=1}^{k-1} \Delta t_{-i} + \Pi X_{t-1} + \Psi D_t + \epsilon_t$$

The p-dimensional vector process $X_t$ is assumed to be at most I(1), $D_t$ collects the deterministic components, and $\epsilon_t \sim N_p(0, \Sigma)$. The cointegration is tested as a reduced rank hypothesis on the $\Pi$ matrix. With $r = \text{rank of } \Pi$, $X_t$ is I(0), i.e., stationary if $r = p$. $\Pi = \alpha \beta'$ and $X_t$ is I(1) and it is cointegrated with $r$ cointegration vectors and $p-r$ common trends if $0 < r < p$. $X_t$ is I(0) and is not cointegrated if $r = 0$. In this project, we use Johansen’s trace test (1991) as a test of reduced rank of $\Pi$. The null hypothesis of the test is: the rank of $\Pi$ is less than or equal to $r$. With $\Pi = \alpha \beta'$, the general linear hypotheses on $\beta$ can be tested in the form: $H_\beta : \beta = (H_1 \varphi_1, \ldots, H_r \varphi_r)$. The likelihood ratio test of the hypotheses is approximately $\chi^2$.

Empirical Analysis and Results

Cointegration Rank Test:

The data vector for models 1 and 2 is $X_{12,t} = (y_t^r, \pi_t, i_t, y_t^n)'$ and for models 3 and 4, $X_{34,t} = (y_t^r, \pi_t, i_t, y_t^n, s_t)'$. After fitting cointegrated VAR model to the two data vectors, the reduced rank hypotheses of the two statistical models are tested.
Table 2 summarises the results. The test concludes that the appropriate choice of cointegration rank is two or three in both models. The choice of two is borderline accepted and the choice of three is more likely. This implies that the exact model variants 1 and 3 seem to be more consistent with the long run properties of the data than the inexact models. This deviates from the conclusions drawn by Juselius in his original paper. This means both exact models 1 and 3 and inexact models 2 and 4 are possible options and exact models are more likely to conform to long run properties of the data.

Table 2

<table>
<thead>
<tr>
<th>Models 3.2</th>
<th>Models 3.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cointegration: 3 alternatives against the alternative that there are r-1 cointegration vectors</td>
<td>Cointegration: 3 alternatives against the alternative that there are r-1 cointegration vectors</td>
</tr>
<tr>
<td>Critical values</td>
<td>Critical values</td>
</tr>
<tr>
<td>r</td>
<td>critical values</td>
</tr>
<tr>
<td>0</td>
<td>1.6</td>
</tr>
<tr>
<td>1</td>
<td>3.2</td>
</tr>
<tr>
<td>2</td>
<td>10.2</td>
</tr>
<tr>
<td>3</td>
<td>17.2</td>
</tr>
<tr>
<td>4.3</td>
<td>27.5 32.5 34.2</td>
</tr>
</tbody>
</table>

Stationarity and weak exogeneity tests:

Table 3

<table>
<thead>
<tr>
<th>X_t</th>
<th>Test</th>
<th>y_t</th>
<th>π_t</th>
<th>i_t</th>
<th>y_t^n</th>
<th>s_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_{12,t}</td>
<td>Unit root</td>
<td>Not rejected</td>
<td>Not rejected</td>
<td>Not rejected</td>
<td>Not rejected</td>
<td>-</td>
</tr>
<tr>
<td>X_{12,t}</td>
<td>Weak Exogeneity</td>
<td>rejected</td>
<td>rejected</td>
<td>rejected</td>
<td>Not rejected</td>
<td>-</td>
</tr>
<tr>
<td>X_{34,t}</td>
<td>Unit root</td>
<td>Not rejected</td>
<td>Not rejected</td>
<td>Not rejected</td>
<td>Not rejected</td>
<td>Not rejected</td>
</tr>
<tr>
<td>X_{34,t}</td>
<td>Weak Exogeneity</td>
<td>rejected</td>
<td>rejected</td>
<td>rejected</td>
<td>Not rejected</td>
<td>Not rejected</td>
</tr>
</tbody>
</table>
Next, we test the null hypotheses of unit root and weak exogeneity on the data variables. The ADF and PP tests for unit root and Granger causality tests for weak exogeneity are carried out and the results are summarised in Table 3. The unit root cannot be rejected in all variables while weak exogeneity cannot be rejected in potential output and labor’s share. This is evidence in support of inexact model. This means that the assumptions of a non-stationary technology shock and a non-stationary central bank inflation target are consistent with the long-run properties of the data.
**Unit root tests (Tests for stationarity)**

**H0**: Unit root  
**H1**: stationarity

(Real output)

**ADF test**:

<table>
<thead>
<tr>
<th>OLS estimate</th>
<th>t-value</th>
<th>Asymptotic critical regions:</th>
</tr>
</thead>
<tbody>
<tr>
<td>z(t-1)</td>
<td>-0.0039</td>
<td>-0.2310 &lt; -2.89 (5%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt; -2.58 (10%)</td>
</tr>
</tbody>
</table>

\[ p-value = 0.93000 \]

**PP test**

**Test statistic**:  1.08

\[ p-value = 0.99000 \]

- **5% Critical region**: <-14.51
- **10% Critical region**: <-11.65

**Test result**:  
H0 is not rejected at the 10% significance level
(inflation)

**ADF test**

<table>
<thead>
<tr>
<th>OLS estimate</th>
<th>t-value</th>
<th>Asymptotic critical regions</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.7508</td>
<td>-1.9839</td>
<td>&lt; -2.89 (5%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt; -2.58 (10%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>p-value = 0.29000</td>
</tr>
</tbody>
</table>

**PP test**

- Test statistic: -89.93
- p-value = 0.00000
- 5% Critical region: < -14.51
- 10% Critical region: < -11.65

**Test result:**

Unit root not rejected based on ADF test.

(interest rate)

**ADF test**

<table>
<thead>
<tr>
<th>OLS estimate</th>
<th>t-value</th>
<th>Asymptotic critical regions</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.2327</td>
<td>-2.7603</td>
<td>&lt; -3.40 (5%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&lt; -3.13 (10%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>p-value = 0.21000</td>
</tr>
</tbody>
</table>
PP test

Test statistic: \(-7.08\)

p-value: \(0.66000\)

5% Critical region: \(< -21.78\)

10% Critical region: \(< -18.42\)

Test result:

H0 is not rejected at the 10% significance level

(potential output)

ADF test:

OLS estimate: 0.0000

t-value: -0.9811

Asymptotic critical regions:

\(< -2.89\) (5%)

\(< -2.58\) (10%)

p-value = 0.76000

PP test

Test statistic: 1.56

p-value: 1.00000

5% Critical region: \(< -14.51\)

10% Critical region: \(< -11.65\)

Test result:
H0 is not rejected at the 10% significance level (labor cost)

ADF test

<table>
<thead>
<tr>
<th>OLS estimate</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0126</td>
<td>1.7272</td>
</tr>
</tbody>
</table>

Asymptotic critical regions:

< -2.89 (5%)
< -2.58 (10%)

p-value = 1.00000

PP test

Test statistic: 0.33

p-value = 0.97000

5% Critical region: < -14.51
10% Critical region: < -11.65

Test result:

H0 is not rejected at the 10% significance level

Weak exogeneity test:

Pairwise Granger Causality Tests
Null Hypothesis:

<table>
<thead>
<tr>
<th>Obs</th>
<th>F-Statistic</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>83</td>
<td>1.29475</td>
<td>0.27979</td>
</tr>
<tr>
<td>0.71564</td>
<td>0.49206</td>
<td></td>
</tr>
<tr>
<td>83</td>
<td>4.63116</td>
<td>0.01257</td>
</tr>
<tr>
<td>0.82626</td>
<td>0.44148</td>
<td></td>
</tr>
<tr>
<td>83</td>
<td>11.6348</td>
<td>3.8E-0.5</td>
</tr>
<tr>
<td>33.6753</td>
<td>2.9E-11</td>
<td></td>
</tr>
<tr>
<td>83</td>
<td>2.13597</td>
<td>0.12499</td>
</tr>
<tr>
<td>2.13597</td>
<td>0.01640</td>
<td></td>
</tr>
</tbody>
</table>

Structural tests on the cointegration spaces:

Table 4

<table>
<thead>
<tr>
<th>H</th>
<th>Restrictions</th>
<th>β̂</th>
<th>LR, χ² (df)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{IS,1}$</td>
<td>-</td>
<td>$\tilde{\beta}_1 = (1 - \phi_1 - \phi_2, -\phi_3, \phi_1 + \phi_2 + 1)'$</td>
<td>33.70 (3)</td>
<td>0.00</td>
</tr>
<tr>
<td>$H_{IS,2}$</td>
<td>$\phi_1 + \phi_2 = 1$</td>
<td>$\tilde{\beta}_3 = (0, -\phi_3, 0, 0)'$</td>
<td>28.14 (3)</td>
<td>0.00</td>
</tr>
<tr>
<td>$H_{PC,1}$</td>
<td>-</td>
<td>$\tilde{\beta}_2 = (-\phi_5, 1 - \phi_4 - \phi_5, 0, \phi_6)'$</td>
<td>33.70 (3)</td>
<td>0.00</td>
</tr>
<tr>
<td>$H_{PC,2}$</td>
<td>$\phi_4 + \phi_5 = 1$</td>
<td>$\tilde{\beta}_3 = (-\phi_6, 0, 0, \phi_6)'$</td>
<td>33.70 (3)</td>
<td>0.00</td>
</tr>
<tr>
<td>$H_{IS,1} \cap H_{PC,1}$</td>
<td>-</td>
<td>$\tilde{\beta}_1 = (1 - \phi_1 - \phi_2, -\phi_3, \phi_1 + \phi_2 + 1, 0)'$</td>
<td>36.92 (4)</td>
<td>0.00</td>
</tr>
<tr>
<td>Models 3,4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_{IS,1}$</td>
<td>-</td>
<td>$\tilde{\beta}_1 = (1 - \phi_1 - \phi_2, -\phi_3, \phi_1 + \phi_2 + 1, 0)'$</td>
<td>0.96 (4)</td>
<td>0.92</td>
</tr>
<tr>
<td>$H_{IS,2}$</td>
<td>$\phi_1 + \phi_2 = 1$</td>
<td>$\tilde{\beta}_3 = (0, -\phi_3, 0, 0)'$</td>
<td>349.39 (4)</td>
<td>0.00</td>
</tr>
<tr>
<td>$H_{PC,1}$</td>
<td>-</td>
<td>$\tilde{\beta}_2 = (0, 1 - \phi_4 - \phi_5, 0, 0, \phi_6)'$</td>
<td>346.67 (4)</td>
<td>0.00</td>
</tr>
<tr>
<td>$H_{IS,1} \cap H_{PC,1}$</td>
<td>-</td>
<td>$\tilde{\beta}_1 = (1 - \phi_1 - \phi_2, -\phi_3, \phi_1 + \phi_2 + 1, 0)'$</td>
<td>7.11 (6)</td>
<td>0.31</td>
</tr>
<tr>
<td>Exact model 3</td>
<td></td>
<td></td>
<td>7.08 (6)</td>
<td>0.31</td>
</tr>
</tbody>
</table>

In this empirical analysis, the individual cointegrating vectors (corresponding to the optimising IS curve, and a new Keynesian Phillips curve) are first tested separately and then jointly. The results are summarised in Table 4. The second index

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4Juselius (2008), p.21
on $H_{\cdot\cdot}$ indicates whether restrictions are placed on the parameters or not. $H_{IS,1}$ is rejected, indicating that the optimising IS curve does not reflect the long run behaviours of the data well when no restrictions are placed on the coefficients. This is in contrast to the findings by Juselius. With the restriction $\phi_1 + \phi_2 = 1$ in place, $H_{IS,2}$ is rejected, implying non-stationary real interest rates. Similar results come out from testing $H_{PC,1}$ and $H_{PC,2}$. $H_{PC,1}$ is rejected. $H_{PC,2}$ is rejected indicating that the restriction $\phi_4 + \phi_5 = 1$ is rejected. Hence, the output gap is non-stationary. When the labor share is used as marginal costs, IS restrictions $H_{IS,1}$ is not rejected, indicating that the optimising IS curve is in line with the long run behaviours of the data when no restrictions are placed on the coefficients. Both $H_{IS,2}$ and $H_{PC,1}$ are rejected. Overall, NKPC is rejected. The joint test of the cointegration space implied by models 2 ($H_{IS,1} \cap H_{PC,1}$) is rejected while it is not rejected for model 4. The exact model 3 when labor costs are used as marginal costs is not rejected.
Case 1

LR test: Test statistic = 33.70. Null distr.: Chi-square(3)

Significance levels: 10% 5%

Critical values: 6.25 7.81

Conclusions: reject reject

p-value = 0.00000

Case 2


Significance levels: 10% 5%

Critical values: 6.25 7.81

Conclusions: reject reject

p-value = 0.00000

Case 3

LR test: Test statistic = 33.70. Null distr.: Chi-square(3)

Significance levels: 10% 5%

Critical values: 6.25 7.81

Conclusions: reject reject

p-value = 0.00000

Case 4

LR test: Test statistic = 33.70. Null distr.: Chi-square(3)
Significance levels:  
10% 5%

Critical values:  
6.25 7.81

Conclusions:  
reject reject

p-value = 0.00000

Case 5

LR test: Test statistic = 36.92. Null distr.: Chi-square(4)

Significance levels:  
10% 5%

Critical values:  
7.78 9.49

Conclusions:  
reject reject

p-value = 0.00000

Case II (Labor Cost as a measure of MC)

Case 1

LR test: Test statistic = 0.96. Null distr.: Chi-square(4)

Significance levels:  
10% 5%

Critical values:  
7.78 9.49

Conclusions:  
accept accept

p-value = 0.91596

Case 2

LR test: Test statistic = 349.39. Null distr.: Chi-square(4)
Significance levels: 10% 5%
Critical values: 7.78 9.49
Conclusions: reject reject
p-value = 0.00000

Case 3
LR test: Test statistic = 346.67. Null distr.: Chi-square(4)
Significance levels: 10% 5%
Critical values: 7.78 9.49
Conclusions: reject reject
p-value = 0.00000

Case 4
LR test: Test statistic = 7.11. Null distr.: Chi-square(6)
Significance levels: 10% 5%
Critical values: 10.64 12.59
Conclusions: accept accept
p-value = 0.31045

Case 5
LR test: Test statistic = 7.08. Null distr.: Chi-square(6)
Significance levels: 10% 5%
Critical values: 10.64 12.59
Conclusions: accept accept
p-value = 0.31316
CONCLUSION

The empirical analysis carried out on New Zealand data seems to present somewhat different conclusions from the original analysis done on US data, lending little support to the original conclusions drawn from evaluating new Keynesian models. It turns out that only model 4 is consistent with the long run properties of the New Zealand data, departing from the conclusion drawn from Juselius’ paper. Hence in contrary to Juselius’ empirical findings, New Zealand data seems to support labor cost as a measure of marginal cost rather than output gap. This empirical exercise in particular derives the cointegration implications of the exact and inexact linear rational expectations model, in this case, NKM version. The use of this particular framework will be on shedding some light on data relevance of different models. It also elicits some guidance from the data as to the appropriate shock structure of the models. The analysis done on this project on New Zealand data questions the robustness of Juselius’ claim regarding the appropriateness of the inexact models. In this study, it seems not as clear as to which model the New Zealand data indicates to and the evidence seems to speak slightly more in favour of exact models.
Bibliography

Bekaert, G., Cho, S., and Moreno, A., (2005), "New Keynesian Macroeconomics and Term Structure", School of Economics and Business Administration, University of Navarra, Faculty Working Papers 04/05


in cointegrated vector autoregressive models", *Journal of Econometrics*, 93, 73-91


