Rectifying the long-term climate fluctuations in the Milankovitch Bands

by

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Abstract

In this thesis we address the ice age problem in light of our recent knowledge of both geological evidence on climate spanning the late Pleistocene and the intensity of Hadley circulation in modulating the mid-latitude wave activity and high-latitude temperature.

Since the radiative forcing at the top of the atmosphere is mathematically modulated by the eccentricity and the global ice volume measured by the $\delta O^{18}$ is inversely correlated with the eccentricity[30], we argue that the 100kyr period in the Late Pleistocene $\delta O^{18}$ records is the result of rectification of the radiative forcing. Based on the physics of the climate system, we have constructed a new rectifier which takes into account the Hadley winter heat flux.

This rectifier has only two free adjustable parameters, and under certain range it can well detect the low frequencies from the high frequency radiative forcing by comparing the rectified time series with those of the eccentricity. It is further compared with a commonly used rectifier due to Imbrie and Imbrie[31]. By combining the rectified Hadley winter heat flux with the traditional Milankovitch high-latitude summer insolation into a linear differential model, we simulated the ice volume fluctuations during the last 700,000 years. The results are further compared with the SPECMAP stack data[32].

The results demonstrated the importance of rectification mechanism in understanding the late Pleistocene ice ages while the role of the Hadley circulation is manifested through the simple physically based rectifier.

Thesis Supervisor: Richard S. Lindzen
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Chapter 1

Introduction

1.1 The specific problems

It is been a long time unsolved scientific mystery regarding the mechanism of climatic fluctuations in the Milankovitch bands, namely:

1. How can we explain the glacial cycles with 100kyr dominant periodicity over the late Pleistocene[23][30]?

2. What determines the transition in the period(from dominant \(41\)kyr period in the early Pleistocene and late Pliocene to 100kyr period in the late Pleistocene), amplitude(\(\sim 50\%\) increase in late Pleistocene), and the mean value of the ice fluctuations occurring at about 700kyr B.P., i.e., near Matuyama and Brunhes magnetic polarity epochs boundary[43][62]?

3. What triggers the initiation of northern hemisphere significant glaciation around 2.5ma (1 ma refers to one million years ago, hereafter)[2][75][88][60]?

Most of the theoretical research in palaeoclimate has been concentrated on the late Pleistocene, i.e., regarding the first problem above, because we have more available data during this period. For the second and third problems, emphases are still around observational analyses and associated hypotheses. However, any successful theory about the ice age problem has to answer the three questions together. In the following
we will briefly review the proposed mechanisms on the three questions mentioned above.

1.2 Review of current research involving the first problem

The most accepted theory regarding the first problem is the astronomical one, which attributes the climatic fluctuations to the changing of the Earth's orbital parameters, i.e., eccentricity (~100kyr, 400kyr periods), obliquity (~41kyr period), and the precession of the equinoxes (~22kyr period). Although by comparing the time series of ice volume with that of the eccentricity there is an inverse correlation between eccentricity of the orbit and the global ice volume[30], it is commonly agreed that the direct insolation forcing in the eccentricity period (~100kyr) is too weak to explain the dominant Brunhes 100kyr glacial-interglacial response (The change of eccentricity will only account for ~0.2% change in the total received radiation; cf. Chapter 3.2).

The essential question here is why we still have the dominant 100kyr periodicity in the climate records. Does the 100kyr period have anything to do with the external radiative forcing? Or is it the only result of the internal oscillation due to the complexity of the climate system? Around these questions a lot of works have been done. To summarize the enormous works on this issue, it is useful to collect them according to the following three categories:

1. Free oscillation of an internally driven, nonlinear system:

   The amplitude and period are determined by the inherently nonlinear interactions in the air-sea-ice system, with orbital variations serving only to phase-lock the variations at the preferred time scales. The argument is that low-frequency (red noise) variance can be produced when white noise forcing, from a time series with a short response time, is applied to another system with a relatively long response time.
The studies fitting the second category are Sergin[72][73], Saltzman et al.[66], Saltzman and Sutera[67], Chalikov and Verbitsky[8][9]. It is worth noting that Saltzman's approach is fundamentally different from some other schemes. Instead of being based on the standard deductive modelling method, i.e., starting from the fundamental statements of conservation of mass, energy, and momentum and successive averaging and parameterization of flux processes, it is based on an inductive method through formulating a closed set of equations and tuning the modelling output to match the observations.

2. Resonance of a nonlinear system driven both externally and internally:

The proposed models are from Kallen et al.[33], Ghil and LeTreut[19], LeTreut and Ghil[38] and LeTreut et al.[39], where the free response of the system is dominated by oscillations near 10kyr. A nonlinear resonance mechanism that is based on feedbacks between the model's ice sheet and other parts of the system transfers energy from the precession band (where radiative forcing is strong) into the 100kyr band (where this forcing is very weak).

The thing we must keep in mind is that even though sophisticated statistical modelling is useful, it never forms a real alternative to physics-based models.

3. Ice volume fluctuations forced by secular change of incident solar radiation:

- Suarez & Held[79][80]: including ice-albedo feedback but the resulting impact is too small.
- Weertman[86], Pollard[57], and Pollard et al.[59]: ice-sheet model without containing explicit representations of the physical processes, i.e., ice sheet flow and accumulation and glacial isostatic adjustment.
- Oerlemans[50][51][52], Birchfield et al.[4][5], Pollard[58] and Hyde & Peltier[28][29]: including the nonlinear interactions between accumulation and ablation of the ice sheet flow, elastic lithosphere and viscoelastic mantle. Oerlemans[50] was the first to demonstrate that the feedback between ice physics and glacial isostatic adjustment could lead to long period os-
cillations in predicted ice volume only when applying a relaxation time of 10-20kyr which is approximately an order of magnitude longer than is known to be reasonable on geophysical grounds. Birchfield[4][5] employed a similar model with a more appropriate isostatic adjustment time scale of 3kyr but failed to produce significant power in 100kyr-period. The third one incorporated a more realistic representation of the glacial isostatic adjustment process and managed to get a promising result. However, the model result is very sensitive to the choice of tuning parameters.

- Denton & Hughes[12], Ledley[40], Denton et al.[13], Peltier[55]: global ice-sheet model (sea-level linkage between terrestrial ice-sheet model and marine model), wherein the linkage occurs by direct atmospheric cooling and by the impact of high-elevation ice sheets in the North Atlantic on the westerly jet-stream flow.

1.3 Review of current research involving the second and third problems

For the mechanisms regarding the onset of northern hemisphere glaciation there are many proposed explanations. One group of the explanations looks to changes in atmospheric composition, such as $CO_2$, ozone or trace gasses[56], or in solar strength[54], where the first one partly verifiable from geological data while the second one is untestable. While Kennett and Thunell[34] suggested the increased volcanism during the latest Cenozoic as a possible cause of glaciation.

Most of the other explanations call on tectonic changes as the trigger of onset of significant northern hemisphere glaciation. Among them there are polar wandering theory proposed by Ewing and Donn[17]; possible land-sea distribution link by [49]; changes in susceptibility to glaciation due to epeirogenic uplift of highland regions in northern Canada by[18], Emiliani and Geiss[16], and Birchfield et al.[5]; changes in oceanic heat and moisture fluxes induced by shoaling or deepening of critical oceanic
gateways such as the Panama Isthmus or the Bering Straits by Emiliani[15].

Birchfield and Weertman[4] proposed that the large-scale uplift in two key regions, i.e., Tibet and Western North America, would enhance albedo-temperature feedback on a globally significant scale. The cold air over the large, newly created region of high-standing topography at middle latitudes provides a temperature discontinuity that accelerates the normal southward movement of the snowline in autumn-winter. This enlarged high-albedo surface in turn may lead to global cooling during the transitional and winter seasons and trigger the onset of significant northern hemisphere glaciation.

Ruddiman et al.[64] and Ruddiman & Raymo[65] proposed that the late Neogene(Pliocene-Miocene) uplift in southeast Asia and the American southwest results in greater meridionality of northern hemisphere winter jet stream and subsequently cooling over large areas of Canada and northwestern Europe, which can set the stage for the growth of large continental ice sheets.

Raymo et al.[60] have also suggested the connections between orogeny and climate/ocean chemistry can trigger the initiation of northern hemisphere glaciation. Namely, significant increases in uplift rates observed in the late Neogene have resulted in a global increase in chemical weathering rates over the last 5my. The associated increase in river dissolved fluxes would have increased ocean alkalinity and, therefore, lowered oceanic and atmospheric $CO_2$ levels. Decreased atmospheric $CO_2$ may have resulted in a cooling sufficient for the growth of northern hemisphere ice sheets.

However, Molnar and England[46] argue that most of the evidence used to infer late Cenozoic uplift could, in large part, be a consequence of the very climate changes that this supposed uplift is thought to have caused. They propose the possibility that climate change, weathering, erosion and isostatic rebound might interact in a system of positive feedback. Suppose increased weathering and erosion could enhance a negative "greenhouse effect" as aforementioned; then, if climate change accelerated erosion and weathering, the associated withdrawal of carbon dioxide from the atmosphere might cause further global cooling. Increased rates of erosion and isostatic rebound would alter the distribution of elevations by making the crests of
ranges higher than before. The prolonged duration of winter and snow cover in such higher areas might lead to further reductions in temperature. In addition, mountain ranges with high crests and deep valleys will perturb atmospheric circulation more strongly than those with the same mean elevation but lower crests. Thus, if the rise of crests of mountain ranges were to perturb the climate in such a way as to enhance erosion, isostatic rebound would cause a further rise, and further perturbations to the climate. By such feedback mechanism, the gradual regional uplift of the Earth's surface in Asia, due to Cenozoic tectonics, might have led to accelerating climate change and the illusion of the accelerated uplift of mountain ranges throughout the world.

In addition, Rind and Chandler[61] emphasize the important role of the oceans in understanding the climate changes. After employing an atmospheric GCM to the oceanic simulation using two special techniques, namely, fixing sea surface temperatures at whatever values seem to fit a past or potential future climate and then letting the atmospheric GCM run to equilibrium in order to get the oceanic heat transport; altering the form of atmospheric GCM's lower boundary conditions to specify oceanic heat transport in order to assess the response of climate to a given amount of oceanic heat transport, they suggest that increased topography during the late Cenozoic might have altered the surface wind stress in such a manner that led to reduced oceanic heat transports; this effect would then need to be considered in understanding the beginning of glaciations(Figure 1-1) provided this effect is not fully compensated by changes in the atmosphere.

Both paleobotanical data and oxygen isotopes indicate a monotonic, but not steady, cooling of the Earth over the past 50myr, which might be the consequence of the tectonic processes that built Tibet, the Himalayas and other high terrains in Asia. Abrupt decreases in temperature occurred near Eocene/Oligocene boundary(~ 36ma), at ~ 15ma in the Miocene epoch, and at ~ 2.5ma near the end of the Pliocene epoch, when continental glaciation in the northern hemisphere began(Figure 1-2). Moreover, the decrease in temperature has been much greater at high latitudes than at low latitudes. For example, oxygen isotopes from planktonic
foraminifera deposited at sub-Antarctic latitudes indicate a decrease in surface water temperature of $\sim 12^\circ C$ since late Eocene time [76][74], but those from equatorial latitudes indicate hardly any change in such temperature [70][69]; also see Figure 1-2. The combined effect of global cooling trend from the late Cenozoic and the increased latitudinal temperature gradient resulting from complex interactions within the climate system determines the onset of northern hemisphere significant glaciation around 2.5ma, although we do not know how these interactions set the latitudinal temperature gradient of the Earth's surface. For example the regional response to the late Cenozoic global cooling due to the tectonic processes might be quite different because of the differences in regional land-sea distribution, emissivity, opacity, etc., thus setting the secular change of meridional temperature gradient as well as affecting the global cooling itself. Increased temperature gradient leads to more accumulation of ice at high latitudes due to both atmospheric and oceanic moisture fluxes, while the global cooling trend prevents it from ablation in the warm seasons. Thus we think the change in latitudinal temperature gradient is most important because it promotes an altered Hadley circulation, a changed eddy transport of latent heat and sensible heat from low to high latitudes, a reorganized ocean heat transport due to wind-driven circulation as well as thermohaline circulation.
Figure 1-2: Oxygen isotope records (and inferred temperature) for the past 70 Myr from low-latitude surface waters of the Pacific (Δ) and from ocean deep waters as observed in the South Atlantic (+). Because ocean deep waters are formed at high latitude, the lower curve can be taken as representing the isotope composition of high latitude surface water. The temperature scale is valid only in the absence of an Antarctic ice sheet—roughly up to mid-Miocene time. Including a correction for the effect of the ice sheet would raise more recent temperatures by \( \sim 3^\circ C \). (Figure courtesy of Molnar and England[43].)
As for the transition in period, amplitude and the mean value of the ice fluctuation around 700kyr, there are only a few proposed explanations. Ruddiman and McIntyre[63] suggest that the transition may be linked to the onset of larger ice sheets, whose southern limits in North America and Europe must have extended farther into middle latitudes, and thus come under the influence of stronger precessional forcing. Kutzbach[37] proposes that it might in some way reflect an increased sensitivity of ice sheets to precessional insolation heating of low- and mid-latitude land masses at latitudes south of the ice sheets. While Saltzman[68] implied a model of a nonlinear oscillator driven by internal variabilities and demonstrated the possibility of mid-Quaternary climatic transition.

A similar approach but based on a stochastic resonance model has been recently proposed by Matteucci[45]. By slowly changing the only free parameter of the model, the system can undergo a pitchfork bifurcation and the bifurcation point separates a linear regime (identified with the early Pleistocene) from a strongly nonlinear regime (the late Pleistocene) where the stochastic resonance mechanism produces rapid and symmetric transitions between the stable steady states of the system. Both Saltzman[68] and Matteucci[45] postulate tectonic forcing has altered the climatic state resulting in a change of the stability properties of the system.

1.4 Outline of the thesis

In this thesis I focused myself on the first problem because we have better climatic proxy data both in quantity and quality for the late Pleistocene than for the earlier times. In Chapter 2 recent work on the determination of Hadley circulation and its role in modulating mid-latitude wave activity and high-latitude temperature is reviewed. Also we pointed out the insufficiencies in the present ice-climate link approaches. In Chapter 3 a simple parameterization formula of the intensity of the Hadley circulation is given based on both numerical results and analytical arguments. In Chapter 4 a new nonlinear rectifier is constructed based on the physics of high-latitude snow accumulation and ablation, and by incorporating it with the traditional
Milankovitch radiative forcing into a differential model we simulated the global ice volume fluctuations for the entire 700,000 years. Implications and summary of this thesis are given in the last chapter.
Chapter 2

The importance of the Hadley circulation in modulating the mid-latitude baroclinic wave activity and the winter extratropical temperature

2.1 The importance of the Hadley circulation in the present climate

The highly effective role of dynamic transport in determining the equator-to-pole temperature distribution can be demonstrated using the equilibrium Energy Balance Model (EBM):

\[ I + F[T(y)] = QS(y)A, \]  

(2.1)

with \( y = \sin(\text{latitude}) \), \( T \) surface temperature(\(^\circ\text{C} \)), \( Q \) the solar constant divided by four \((340.0 \text{Wm}^{-2})\), \( S(x) \) the latitudinal distribution of solar radiation, \( A \) the absorptivity of the surface to incoming solar radiation.
Taking the annual average distribution of solar radiation,

\[ S(y) = 1 + 0.45P_2(y) \]

and the ice free absorptivity, \( A = 0.68 \), together with the present infrared radiation flux linearization,

\[ I = A + BT \]

where \( A(= 203.3 W/m^2) \), \( B(= 2.09 W/m^2 C) \) are empirical coefficients taken from satellite data[78], gives an annual average equator temperature of 42° and pole temperature of -35°, in the absence of heat flux divergence, that is with \( F[T] = 0 \). This temperature difference of 77° indicates the importance of dynamic heat flux both in reducing the equator-to-pole temperature difference to the present value near 40° and in reducing the equator temperature to the present value near 27°.

The work of Charney[10] and Eady[14] identified the meridional temperature gradient and the associated available potential energy as the energy source for unstable baroclinic wave growth. Recent studies indicate that the subtropical jet is determined by mean-flow accelerations by the Hadley circulation and decelerations by midlatitude eddies. Theoretical models of the Hadley circulation showed that in the absence of eddies, the poleward angular momentum advection by the Hadley circulation could lead to a strong subtropical jet while the effect of midlatitude eddies is to reduce the jet intensity to the observed one[25]. GCM study[1] and a series of observational studies[6][35][36][87] supported the relationship between tropical heating maxima, subtropical winter jet maxima, and extratropical baroclinic activities.

Using a simplified zonally symmetric model Hou and Lindzen[27] showed that, in the absence of waves, a stronger Hadley winter cell resulting from tropical heating concentration can dramatically increase the baroclinicity and potential vorticity gradients in the winter extratropics. Very recently Hou[26] employed a simplified GCM and showed that “a shift of the prescribed tropical heating toward the summer pole, on time scales longer than a few weeks, leads to a more intense cross-equatorial winter Hadley circulation, enhanced upper-level tropical easterlies, and a slightly stronger
subtropical winter jet, accompanied by a warming at the winter middle and high latitudes as a result of increased dynamical heating.”

Although the above conclusions are based on simplified models with prescribed latent heating and need to be further tested in a more complex model, the importance of the Hadley circulation in modulating the mid-latitude baroclinic wave activity and the winter extratropical temperature is manifested. Before considering this hypothesis in our paleoclimate research, we have to say something about the determination of the Hadley intensity.

2.2 The determination of the Hadley intensity

As mentioned in Hack[21], there appear to be two conceptual views of the Hadley circulation and the ITCZ. The first, considers the Hadley circulation and the ITCZ as monthly (or seasonally) and zonally averaged phenomena. The second view is that the Hadley circulation and the ITCZ are phenomena that can be identified on individual weather maps and whose fluctuations can be followed day to day. Regardless of which conceptual view is adopted, the intensity of Hadley circulation is physically determined by the tropical diabatic heating concentration and displacement with respect to the equator.

The difference in the strengths of the Hadley winter and summer cells when the maximum heating being off the equator was first discussed by Lindzen and Hou[41]. This can be illustrated by comparing the June-August and the December-February meridional circulation fields. In June, July and August most of the latent heat release associated with the maximum heating occurs north of the equator. The meridional circulation associated with this latent heat release exhibits a distinct preference for the winter hemisphere cell. In December, January and February most of the latent heat release occurs south of the equator, and the low level air is drawn into the ascending region primarily from the north. This effect is clearly illustrated in the observational analyses of Newell et al. (Figure 3-19)[47] and Oort(Figure F-44)[53].

The sensitivity of the Hadley circulation to tropical heating displacement is care-
Figure 2-1: Numerical model results for streamfunction for (a) $\phi_0 = 0 \text{ deg}$, (b) $\phi_0 = 2 \text{ deg}$, and (c) $\phi_0 = 6 \text{ deg}$. Units are in $10^{10} \text{ kgs}^{-1}$ and the contour interval is $0.1 \times 10^{10} \text{ kgs}^{-1}$ for (a) and (b); twice this value for (c)[41].

fully studied in [41]. They find that moving peak heating even 2 degrees off the equator leads to profound asymmetries in the Hadley circulation, with the winter cell amplifying greatly and the summer cell becoming negligible. An example of this is shown in Figure 2-1, where the numerical model results for the streamfunction are plotted for three cases of displacements.

The influence of concentrated heating on the Hadley circulation is investigated in [27]. They found that in the case of heating symmetrically centered on the equator, concentration unambiguously increases the intensity of the Hadley circulation by up to a factor of 5. For heating centered off the equator, concentration of heating primarily from the winter side of maximum heating, which is consistent with the explicit
Figure 2-2: Equal-area results for asymmetric redistributions within the winter cell, $\bar{\theta}_e/\theta_0$ is the heavy lines, and $\bar{\theta}/\theta_0$ the thin line[26].

Figure 2-3: Calculated meridional streamfunctions in units of $10^{10} kgs^{-1}$ for $\phi_0 = 6^\circ$. (a) $A = 0$, (b) $A = 20/300$ with an asymmetric redistribution over both cells[26].

calculation and from ECMWF analyses, also leads to pronounced intensification of the Hadley circulation. Further they found that agreement between the calculated and observed Hadley intensity is achieved with mild concentration, consistent with the observed zonally averaged precipitation. The nature of the concentrations is shown in Figure 2-2, and Figure 2-3 shows the numerically calculated streamline distributions for various degrees of concentration. The strength of the circulation for $A = 20/300$ is nearly 5 times that for $A = 0$. 

23
2.3 The insufficiencies in the present ice-climate link approaches

The several flaws in the ice-climate link approaches (cf. Chapter 1.2.3) stem from the fact that they can not explain the following facts:

- Southern hemisphere paleoclimatic evidence. GCM results indicate that the climatic influence of large North Atlantic and European ice sheets is restricted to Northern hemisphere, and that the expansion of the Antarctic ice sheets was not large enough to influence the Southern hemisphere[44][7].

- Observation of last termination about 14000 B.P. indicates near-synchronocity in both hemispheres, which does not allow much time lag.

- A number of pre-Pleistocene climate records exhibit significant fluctuations at 100kyr, when there is little evidence for the presence of extensive ice sheets[11].

The above flaws in the ice sheet models might be related to the neglecting of global heat transport. In fact, most of the previous ice sheet models assume that the climatic response is forced by local radiative forcing at the top of the atmosphere and has nothing to do with the meridional heat transport and a variety of processes relating the surface temperature with the radiative forcing. Although the radiation field sets the fluxes of energy at the top of the atmosphere and provides overall constraints upon the behavior of the climate system, the climate system is for the most part heated from the surface. Both observational and modelling results indicate that the high latitude climate is determined by the combination of local radiative forcing, meridional heat transport from lower latitudes by the atmosphere and oceans, vertical heat transport, as well as the ocean memory through storing and releasing both heat and chemical species(e.g., $CO_2$).

However, it is a well-known observational fact that the winter hemisphere is dynamically more active than the summer hemisphere. The GCM experiments[20] showed that the thermal tendency in the summer hemisphere depends more strongly
on the diabatic heating and dynamics, while it depends on horizontal advection in the winter hemisphere. Based on linear model results for axisymmetric basic states, it is suggested that the winter hemisphere is close to the "advective limit", while the summer hemisphere close to "diabatic limit"[83]. Very crudely speaking, for example, the high latitude surface is in near radiative-convective equilibrium during summer, while it is in a high eddy regime during winter([47][22][81][82]).

Based on the above discussion, we assume that the high latitude winter surface temperature is dominantly determined by eddy heat transport, which may be forced by the tropical Hadley circulation; while the summer surface temperature is dominantly set up by diabatic heating, which is related to the summer radiative forcing. One simple but realistic approach to modelling the high latitude ice-sheet response to the external forcing is to take into account both winter heat transport and summer insolation. Before doing this, we have to parameterize the intensity of Hadley circulation first.
Chapter 3

Parameterization of the intensity of the Hadley circulation

The aim of this chapter is to parameterize the intensity of the Hadley circulation in terms of the displacement of the surface temperature maximum. In the following we are going to employ a simple seasonal EBM to translate the radiative forcing at the top of the atmosphere to the surface temperature, which can be used to parameterize the intensity of the Hadley circulation.

3.1 Seasonal energy balance model and the results

3.1.1 Model choice

There are several reasons for choosing North’s seasonal energy balance model (SEBM, hereafter)[48] rather than annual mean energy balance model or other kinds of SEBMs. Firstly, the answer may lay in the Milankovitch hypothesis itself, namely, that the latitudinal distribution of seasonal component of insolation would be more important than annual component to the accumulation or ablation of ice sheets.

Moreover, in order to calculate the heat flux based on the asymmetric Hadley circulation model rather than the traditional parameterization of heat flux involving
a relaxation toward the global mean or diffusion, we need a model that can resolve
the seasonal variation of surface temperature. At minimum we need a model with an
ocean mixed layer in order to have the heat capacity appropriate to seasonal heating.

North's model has limited adjustable parameters which are always explicit, and
standard efficiency typical of spectral models which start with the largest and slowest
scales and add on as corrections the finer scale information. More specifically, in the
model the latitudinal distribution of the zonal average surface temperature is repres-
ted by a series of Legendre polynomials, while its time- dependence is represented
by a Fourier sine-cosine series. However for the present purpose, only mixed layer
is at issue and time harmonics are adequate. The Legendre polynomial representa-
tion of the latitudinal distribution of the surface temperature is certainly not a must
here. Although the relaxation time over the oceans is on the order of years, and to
investigate a perturbation one must wait through many e-folding times by numerical
integration, here we may extract analytical "steady-state" solutions directly by just
solving algebraic equations.

3.1.2 Model description

In order to translate the solar forcing at the top of the atmosphere into the seasonal
surface temperature, we apply North's seasonal energy balance climate model, taking
into account the 75m ocean mixing layer and an idealized geometry of land-ocean
distribution.

The model equation is given by

$$ C(x, \phi) \frac{\partial T(x, \phi, t)}{\partial t} - F[T(x, \phi, t)] + I = QS(x, t)A, $$

(3.1)

with $x$ denoting sine of latitude, $C(x, \phi)$ the latitude- and longitude-dependent heat
capacity per unit area, $T(x, \phi, t)$ surface temperature ($^\circ C$), $Q$ the solar constant
divided by four (340.0$Wm^{-2}$), $S(x, t)$ the fraction of the incident solar flux received
by latitude $x$ at time $t$ and normalized so that the mean annual fraction integrated
over the hemisphere is unity, and $A$ an absorptivity of the surface to incoming solar
radiation.

Since we will be primarily interested in the displacement of the solstitial temperature maximum from the equator, there is no point in including the parameterized meridional heat flux divergence.

The outgoing infrared radiation is simply parameterized linearly in terms of surface temperature,

\[ I = A + BT, \]  
(3.2)

with \( A = 203.3\text{Wm}^{-2}, B = 2.09\text{Wm}^{-2}\text{C}^{-1}. \)

For the heat capacity, we follow North and Coakley[48] and take the thermal response over land to be the heat capacity of an atmospheric column divided by the radiation constant, \( C_L/B = 0.16 \text{ year}. \) Over water we take the thermal response to be the heat capacity of the ocean 75m mixing layer divided by the radiation constant, \( C_W/B = 4.7 \text{ years}. \)

The interaction between land and ocean can be represented as \( \nu \frac{f_L}{f_L}(T_L - T_W) \) and \( \nu \frac{f_W}{f_W}(T_W - T_L), \) where \( T_L \) and \( T_W \) are the surface temperatures over land and ocean respectively, \( \nu \) is the interaction coefficient. Both North & Coakley’s experiment and ours show that by increasing \( \nu \) one effectively couples the land to a large thermal reservoir, and the ocean temperature amplitude is hardly affected while the land amplitude is reduced. By tuning \( \nu \) we can bring the \( T_L \) to some observed phase and amplitude. Since as shown in Figure 3-4(b) the coupling between land and ocean has small effect on the magnitude of the displacement and does not affect the variation and secular pattern of the zonally averaged displacement, for simplicity we will neglect the land-sea interaction and consider \( T_L \) and \( T_W \) as functions only of latitude and time. The zonally averaged temperature is given by

\[ T(x,t) = f_LT_L + f_WT_W, \]  
(3.3)

where \( f_L \) and \( f_W \) are fractions of land and ocean areas.

For the absorptivity \( A \) we take \( A=0.68 \) and don’t include the albedo feedback mechanism since it is irrelevant to the displacement.
3.1.3 Model limitations

The limitations in the above model are obvious. The model is one-level zonally averaged energy balance model without any vertical and longitudinal structure as well as detailed snow budget, and it's not been developed enough to incorporate the ocean dynamics. Also we don't invoke such mechanisms as albedo feedback and atmosphere's composition changes because at this stage we are only interested in exploring the role of the displacement in the poleward heat flux. In addition, the linearization of infrared radiation tends to overestimate the sensitivity in low latitudes and underestimate it in high latitudes. These latitudinal differences in sensitivity are related to changes in the tropospheric static stability[24].

3.2 Orbital parameters and the incident solar radiation at the top of the atmosphere

The energy available at any given latitude at the top of the atmosphere is a single-valued function of the solar constant $S_o$, the semi-major axis $a$ of the ecliptic, its eccentricity $e$, its obliquity $\epsilon$ and the longitude of the perihelion $\omega$ measured from the moving vernal equinox. To determine the evolution of the external forcing thus requires computing the long-term variations of three orbital parameters $e$, $\epsilon$ and $\omega$. We use the formulae given by Berger[3] to make the calculation. The resulting time series for $e$, $\epsilon$, and $\epsilon \sin \omega$ and their power spectrums are plotted in Figure 3-1 and Figure 3-2 respectively.

The daily-mean incident solar flux at the top of the atmosphere at time $t$ and latitude $\phi$ is

$$S(t, \phi; e, \epsilon, \omega) = \frac{S_o a^2}{4r^2} \tilde{S}[\omega(t; e, \epsilon, \phi; \epsilon)]$$

(3.4)

where $\omega$ is the longitude of the earth with respect to the moving vernal equinox, $\frac{2}{4} \tilde{S}$ is the incident flux for a circular orbit, i.e., $r = a$, $\omega = t$; and

$$\frac{1}{2} \int_{-\pi/2}^{\pi/2} \tilde{S}(\omega, \phi; \epsilon) \cos \phi d\phi = 1.$$  

(3.5)
Figure 3.1: Time series of the three orbital parameters from 3myr B.P. to 1kyr A.P.
Figure 3.2: Power spectral density for the time series shown in Fig. 3.1.
The formulas for calculating $S(t, \phi; e, \epsilon, \omega)$ are given in [3]. There are several properties in Equation 3.4 which need to be pointed out in order to understand the evolution of $S$.

1. The annual global mean flux is only a function of $e$ at a given time. Because $e$ varies from 0 to $\sim 0.06$, the variation of annual global mean flux is about $\sim 0.2\%$ of $\frac{S_0}{4}$.

2. The annual mean flux at a given latitude is a function of $\epsilon$ and $e$. The annual mean flux received by the atmosphere is further dependent on planetary albedo which is a function of latitude and $\omega$.

3. For an eccentric orbit there is a symmetry for $S$:

$$S(\omega, \phi; e, \epsilon, \omega) = S(\omega + 180^\circ, -\phi; e, \epsilon, \omega + 180^\circ). \quad (3.6)$$

This tells us that at two given latitudes symmetric about the equator, the incident solar fluxes at the top of the atmosphere for opposite seasons are exactly out of phase. That is reasonable because the precessional effect only acts to redistribute the incident solar radiation among different latitudes so that it does not affect the annual mean flux at any given latitude[80][87].

Following North and Coakley[48], we decompose $S(x, t)$ by Legendre polynomials in latitude and Fourier series in time with the form of

$$S(x, t) = \sum_{l=0}^{4} \sum_{k=0}^{4} (a_{lk} \cos 2\pi kt + b_{lk} \sin 2\pi kt)P_l(x), \quad (3.7)$$

where $x$ denoting sine of latitude $\phi$ same as in Equation 3.1.

### 3.3 Model results

By decomposing $T(x, t)$ in the same manner as for $S(x, t)$ we can solve Equation 3.1 simultaneously for all components.
In Figure 3-4 we show the time series of $\phi_0$ (the displacement of $T_{max}$) and $C$ (the curvature of the temperature profile at $T_{max}$) for constant land ocean distribution, i.e., $f_L : f_W = 0.2 : 0.8$ for the tropical region, which are approximated from Figure 3-3. In Figure 3-4(b) we also show the time series of $\phi_0$ with land and ocean interaction coefficient $\nu = 0.472$ as in [48]. Here $T_{max}$ is the averaged surface temperature maximum for the northern hemisphere winter (December, January and February).

There are several aspects from the diagram needed to be emphasized:

- The displacement of maximum temperature in the northern hemisphere winter always occur south of the equator, varying from about $-2.5^\circ$ to $-7.0^\circ$ with a variation of $\sim 100\%$, indicating significant secular shift of the displacement of maximum heating.

- The variation of pole to equator temperature gradient (approximated proportional to the curvature at maximum heating as defined in Appendix) for the calculated time domain are much less than that of the displacement of maximum heating, i.e., $\sim 7\%$, indicating no significant change of the external forcing.
in terms of the radiative equilibrium pole-to-equator surface temperature gra-
dient. It should be mentioned that in reality the curvature depends mostly on
easterly waves and monsoons and is not at issue here but might be if monsoons
were altered.

- Finally the dominant frequencies for the time series of displacement of maximum
temperature are 23kyr and 19kyr in precessional bands (near equator, insensitive
to obliquity), while 41kyr for the curvature at maximum temperature (mainly
depends on obliquity).

3.4 Parameterization of the intensity of Hadley circulation

3.4.1 The dependence of the Hadley intensity on the dis-
placement of the maximum temperature

In this section we will apply the symmetric model results[41] to formulate an em-
pirical formula relating the intensity of Hadley winter cell in terms of the maximum
streamfunction $\psi_{\text{max}}$ with the displacement of maximum temperature $\phi_0$ when the
concentration of the temperature profile is fixed.

Figure 3-5 is a plot of maximum streamfunction in the Hadley winter cell, $\psi_{\text{max}}$, vs.
the latitude of the temperature maximum, $\phi_0$, based on[41]. The empirical relation
between $\psi_{\text{max}}(\text{unit}:10^9\text{kg/s})$ and $\phi_0$ is $\psi_{\text{max}}(\text{unit}:10^9\text{kg/s})$ and $\phi_0$ is

$$\psi_{\text{max}} \approx 4.60 + 1.23\phi_0 + 1.67 \times 10^{-2}\phi_0^2 + 7.80 \times 10^{-2}\phi_0^3 - 5.73 \times 10^{-3}\phi_0^4. \quad (3.8)$$

The resulting time series of normalized $\psi_{\text{max}}$ only due to the change of $\phi_0$ using
Equation 3.8 are plotted in Figure 3-6, where we see $\psi_{\text{max}}(t)$ has large variation
($\sim 100\%$) in amplitude.
Figure 3-4: Time series of (a) the displacement of $T_{max}$ with $\nu = 0$; (b) the displacement of $T_{max}$ with $\nu = .472$; (c) the curvature at $T_{max}$ for the period of 1000 kyr B.P. to 100 kyr A.P.
Figure 3-5: Numerical calculation for the maximum streamfunction of the Hadley winter cell vs. the displacement of maximum temperature during northern hemisphere winter[99].

Figure 3-6: The time series of normalized \( \psi_{\text{max}} \) only due to the change of \( \phi_0 \) using Equation 3.8.
3.4.2 Parameterization of the intensity of the maximum Hadley winter heat flux

One simple way to consider the dependence of $F_{\text{max}}$ (the maximum Hadley winter heat flux) on the displacement of the maximum heating is to just take Equation (3.8) as in Appendix:

$$F_{\text{max}} \propto (-\sigma)^{5/2} f(\phi_0).$$  \hspace{1cm} (3.9)

In Figure 3-7 we show the time series of so parameterized maximum Hadley winter heat flux from 1000kyr B.P. to 100kyr A.P..

It is true that in order to take into account more accurately the change of Hadley intensity due to the displacement and concentration of the tropical heating, we need some kind of model which can model the secular change of tropical deep convection pattern. As pointed out by Lindzen and Nigam[42] the tropical rainfall distributions are tightly related to mixed moist boundary layer and are essential to the calculation of the circulation aloft. However the secular variations of the tropical rainfall pattern is hard to resolve in our simple model; and we assume that for the most part it is linearly related to the radiative equilibrium temperature distribution.
Chapter 4

A new nonlinear rectifier

4.1 Equilibrium models vs. differential models

In the literature of paleoclimate modeling, there exist two conceptually different groups: equilibrium models and differential models. Equilibrium models take the form of $y = f(x)$, where a system function $(f)$ relates the equilibrium climatic state $(y)$ with the orbital boundary condition $(x)$. In the simplest forms, the system function is used such as the input being a linear combination of insolation curves. This approach has been widely used by geologists searching for correlations between geological records and Milankovitch orbital forcing. In more complex models, the system function is produced from a set of differential equations which is assumed to control the fast physics of the response process. The equilibrium climatic state is obtained by integrating the equations to equilibrium at fixed values of $x$[79][71]. However, as would be expected with an equilibrium model, the geological record of climate lags significantly behind the model’s output.

Differential models take the form of $\frac{dy}{dt} = f(x, y)$, where $y$ is usually used to measure the ice-sheet response. One example of this approach is presented by Imbrie and Imbrie[31], where the input $x$ is parameterized as

$$x = \epsilon + \alpha e \sin(\omega - \phi)$$
where $\alpha$ and $\phi$ are adjustable parameters and $\epsilon$, $e$, and $\omega$ are the orbital elements previously defined. The parameter $\phi$ controls the phase of the precession effect, and $\alpha$ controls the ratio of precession and obliquity effects. More examples can be found in Chapter 1.2.3. It should be pointed out that most of the previous work assumes that the ice sheet fluctuation in the high latitudes is determined by the local summer insolation alone, a consistent consideration with Milankovitch's original hypothesis. However, as we discussed in Chapter 2.2.3, the high-latitude snow cover is determined by both accumulation during the cold season and melting during the warm season. While it is safe to say that the melting of snow fall depends strongly on the summer insolation, the accumulation in the winter season depends strongly on the dynamics and moisture supply. It is expected that too much or too little heat flux both reduce the accumulation of snow fall in the high latitude winter, because they tend to favor a warmer winter or a dryer winter. An intermediate optimal heat flux maximizes the accumulation of snow cover. Based on such consideration, we are going to offer one simple rectifier in the next section.

4.2 Constructing a new rectifier

As shown in [31], it is necessary to introduce some form of nonlinearity in order to produce the 100kyr period in the model response. However, the nature of why we need such nonlinearities is not clear, so far as we know. There exist several forms of nonlinearity in the literature. The most popular used one is that associated with different time scales for ice sheet growth and decay. This nonlinearity is supported by both the theoretical arguments, suggesting that growth times of land-based ice sheets are considerably longer than shrinkage times[85], and the isotopic curves suggesting that major glaciations terminate faster than they begin. However, in order to produce enough 100kyr power in the output, the relaxation time scales for the model are set to be 42.5kyr for the accumulation and 10.6kyr for the melting[31], which are much longer than the theoretical values[84].

We argue that the nature of nonlinearity in producing the lower frequency is
Figure 4-1: Eccentricity and global ice volume over the past 730,000 years. (A) Variations in orbital eccentricity calculated by Berger[3]. (B) Oxygen isotope curve for deep-sea core V28-238 from the Pacific Ocean.

rectification, analogous to that used in electronics, because the radiation forcing is mathematically modulated by the eccentricity and also because of the inverse correlation between eccentricity and the global ice volume[30]. Figure 4-1[31] shows the time series of eccentricity and global ice volume over the past 730,000 years. It is clear that the timing of major deglaciations coincides with the timing of peak eccentricity. Although as we mentioned in Chapter 1.2, the direct insolation forcing in the eccentricity period is too weak to explain the dominant 100kyr period in the late Pleistocene proxy data, rectification can channel the energy into the 100kyr period band. The question is what is the proper rectifier in the nature. A proper rectifier must not only be physically reasonable but also effective in extracting the lower frequency information from the input. As we already discussed in the previous section, a good candidate is the nonlinear interaction between the dynamical transport and ice sheet accumulation and decay. We argue that only optimal winter heat flux ensures the accumulation of snow cover, while too much of that will cause well above normal warmer winter and therefore deteriorate the accumulation of snow fall, and too little of that will cause drier winter and therefore cut the source of snow fall. The wide variation in $\psi_{\text{max}}$ associated with eccentricity suggests that the extremes are relevant.
Therefore the rectifier based on the above consideration is the following:

\[ R[F] = \exp \left( -\frac{F - F_0}{c} \right)^2 \]  

(4.1)

where \( R \) is called "glacial potential", \( F \) the maximum Hadley winter heat flux as determined by Equation 3.9, \( F_0 \) the optimal \( F \) corresponding to the maximum of \( R \), and \( c \) the e-folding half-width. There are two control parameters in Equation 4.1: \( F_0 \) and \( c \). Here the larger \( R \) is associated with maximum accumulation.

In Figure 4-2-Figure 4-6 we show the resulting time series of \( R[F(t)] \) and their corresponding power spectral density for different choices of control parameters.

Although the input of \( F \) as shown in Figure 3-7 only has the powers in the obliquity and precession bands, the glacial potential \( R[F(t)] \) displays different power spectral density patterns, varying from lacking of lower frequencies at 100kyr/400kyr periods to lacking of the frequencies in the entire precession bands based on different choice of free parameters of \( F_0 \) and \( c \). Moreover, \( F_0 \) is more effective to change the power spectrum of the input than \( c \).

4.3 Rectifying the lower frequencies of orbital forcing

To illustrate the effectiveness of this rectifier in detecting the lower frequencies of orbital forcing, we take a simple differential model and use the glacial potential as the model input. The model equation is:

\[ \frac{dy}{dt} + y/a = R[F(t)] \]  

(4.2)

where \( a \) is the relaxation time scale for the integration and \( t \) the time in kiloyear.

In Figure 4-7 we show the time series for \( R \) and the corresponding power spectral density with \( F_0 = 3.442 \) for four choices of \( c \). In Figure 4-8 the time series of \( y \) and the corresponding power spectral density are shown with \( a = 17.\text{kyr} \). In Figure 4-9 we
Figure 4-2: The time series of glacial potential (left column) and their corresponding power spectral density (right column) with $F_0 = 2.0$ for four choices of $c$. The units for the horizontal coordinates are kiloyears in time for the left column diagrams and 1/kiloyears in frequency for the right column diagrams.
Figure 4.3: Same as in Figure 4.2 except with $F_0 = 2.5$. 
Figure 4-4: Same as in Figure 4.2 except with $F_0 = 3.0$. 
Figure 4.5: Same as in Figure 4.2 except with $F_0 = 3.5$. 
Figure 4-6: Same as in Figure 4.2 except with $F_0 = 4.0$. 
plot the time series of both eccentricity and $y$ and their corresponding power spectral density. For the convenience of comparison, we normalized the time series of both eccentricity and $y$. It is clear that the rectifier effectively detects the lower frequencies in the eccentricity bands.

4.4 Modeling the global ice volume fluctuations

In this section we assume that the global ice volume in the late Pleistocene is determined by both the rectified Hadley winter heat flux and northern hemisphere high-latitude summer insolation. The governing equation for the global ice volume $G$ is:

$$\frac{dG}{dt} = (c_1 * R[F(t), F_0, c] - c_2 * I(t) - G)/a$$  \hspace{1cm} (4.3)$$

where $I(t)$ is the averaged summer insolation (JUNE-AUGUST) from $60^\circ N$ to the North Pole at time $t$ (in kiloyear B.P.). $c_1$ and $c_2$ are two parameters controlling the relative importance of $R$ and $I$.

In Figure 4-10 we show the time series for the averaged summer insolation from $60^\circ N$ to the North Pole and its scaled power spectral density. Figure 4-11 shows the time series of modeling result $G$ and $\delta O^{18}$ of SPECMAP for the last 700kyr. Figure 4-12 shows their corresponding power spectral density. The data are normalized such that their standard deviations are unity for the convenience of comparison.

Despite the simplicity of our model, the result is promising in terms of both the secular pattern and the power spectral density. We didn’t make fine tuning of the parameters because at this stage our goal is to illustrate the conceptual feasibility of our approach rather than to exactly match the result with the data. Indeed our simple model lacks a lot of potentially important processes which may determine the detail response of the ice sheet. To name only a few of them, the process which translates both the Hadley winter heat flux and the high-latitude summer insolation into the budget of snow accumulation and ablation and the feedbacks within the climate system are potentially important in our further modeling effort.
Figure 4-7: The time series of $R$ and their corresponding power spectral density with $F_0 = 3.442$ for four choices of $c$. The units for the horizontal coordinates are the same as in Figure 4-2.
Figure 4.8: The time series of $y$ and their corresponding power spectral density with $F_0 = 3.442$, $a = 17. kyr$ for four choices of $c$. The units for the horizontal coordinates are the same as in Figure 4.2.
Figure 4-9: The time series of normalized eccentricity and its power spectral density (a); and normalized $y$ and its corresponding power spectral density with $F_0 = 3.442, c = 0.5, a = 17\text{ kyr}$ (b). The units for the horizontal coordinates are the same as in Figure 4-2.

Figure 4-10: Time series of the averaged summer insolation from $60^\circ N$ to the North Pole for the period of 1000 kyr B.P. to 100 kyr A.P. and its associated power spectral density. The units for the horizontal coordinates are the same as in Figure 4-2.
Figure 4.11: Time series of the normalized modeling result (dash line) and $\delta^{18}\text{O}$ of SPECMAP (solid line) for the past 700,000 years with $a = 17\text{kyr}$, $c_1 = 1.0$, $c_2 = 0.012$, $F_0 = 3.442$.

Figure 4.12: Power spectral density of the modeling result (dash line) and the SPECMAP record (solid line) for the past 700,000 years.
Chapter 5

Summary and implications

A study of the ice age problem has been presented in this thesis in light of the recent knowledge of both geological evidence on climate spanning the late Pleistocene and the intensity of Hadley circulation in modulating the wave transport and dynamic heating at the mid- and high-latitude.

Based on the seasonal energy balance model and the Hadley circulation theory, we find that there is considerable large variation in the tropical forcing in terms of the intensity of the Hadley winter heat flux, which may determine the mid-latitude wave activity and high-latitude temperature. A new rectifier based on the dependence of high-latitude snow accumulation and ablation on the Hadley winter heat flux is tested in detecting the 100kyr period from the orbital forcing and in simulating the global ice volume fluctuation for the entire late Pleistocene. The modeling result is compared with the SPECMAP $\delta O^{18}$ data in both time and frequency domains.

The implication of this study is that the tropical forcing may be very important in modeling the late Pleistocene ice volume fluctuation. And the final success might lie in the appropriate treatment of translating both high latitude summer insolation and the winter heat flux induced by the tropical Hadley circulation into high-latitude snow accumulation and ablation. In that case a more sophisticated model which includes the budget of high-latitude snow cover is a must.
Appendix

Here we consider $\phi_0 = 0$, and the radiative equilibrium temperature distribution is

$$T(y) = T_0(1 + \Delta/3 - \Delta y^2)$$

(5.1)

where $T_0$ is the reference temperature, $y = \sin(latitude)$, $\Delta = -\frac{1}{2T_0} \left( \frac{\partial^2 T}{\partial y^2} \right) \bigg|_{y=0}$ = $- \frac{C}{2T_0} (C$ is the curvature at $T_{\text{max}}$).

The mean temperature within the Hadley domain is

$$\bar{T} \approx \int_{-y_H}^{y_H} T(y) dy / (2y_H).$$

(5.2)

The heat flux $F$ satisfies

$$\frac{\partial F}{\partial y} = \frac{T(y) - \bar{T}}{\tau}$$

(5.3)

$$F(y = \pm y_H) = 0.$$  

(5.4)

So we have the latitudinal distribution of $F(y)$

$$F(y) = -\frac{\Delta T_0}{3\tau} (y^3 - y_H^2 y)$$

(5.5)

where $\Delta > 0$ and thus $F(y) > 0 (0 < y < y_H)$.

Also $\Delta T_0 = -C/2$ so Equation 5.5 becomes

$$F(y) = \frac{C}{6\tau} (y^3 - y_H^2 y).$$

(5.6)
The $F_{\text{max}}$ occurs when $\frac{\partial F(y)}{\partial y} = 0$, i.e., $y = \frac{\sqrt{3}}{3}y_H$. So the resulting $F_{\text{max}}$ is

$$F_{\text{max}} \propto (-C)^{5/2}$$

(5.7)

where noting in the symmetric case $y_H \propto \Delta^{1/2} \propto (-C)^{1/2}[25]$.

Combining the above formula with $f(\phi_0)$ (normalized empirical relation between $\psi_{\text{max}}$ and $\phi_0$, cf. Equation 3.8 in Chapter 3.4.1, we have the highly empirical formula

$$F_{\text{max}}(\phi_0, C) \propto (-C)^{5/2}f(\phi_0).$$

(5.8)
Bibliography


