SEISMIC WAVE-PROPAGATION IN A LAYER
OVER A HALF-SPACE

by

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Abstract

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Seismic wave propagation in the near field of a point
source is studied theoretically. Barker (1970) formulation
is extended to include the horizontally polarized shear (SH)
waves for the case of a layer over a half-space. Synthetic
seismograms are then calculated for P, SV and SH sources
using an approximation in the Cagniard-deHoop technique.
Seismograms are displayed to show the effects of the varia-
tions of source type, source depth, range and structure on
the wave-forms so obtained. Near surface P or SV sources
are efficient generators of Rayleigh waves. Dispersion of
Love waves become more prominent as the contrast between
layer and half-space parameters increase.

Thesis Supervisor: M. Nafi Toksoz
Title: Professor of Geophysics
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Chapter I

INTRODUCTION

With the advent of high-speed computers there has been an upsurge of interest in theoretical seismology. The extensive computations needed in this field seem to be less formidable now than in the past. Further, with the accumulation of large amounts of data, obtaining better and more accurate seismograms has become almost essential. Here in this thesis an attempt has been made in this direction.

Helmberger (1967) has shown how to compute seismic records in the high frequency range for long distances. He was mainly interested in the refractions of P and SV waves from the oceanic Mohorovicic discontinuity, well known as head waves. Harkrider (1964) also calculates theoretical seismograms for long ranges. Filson (1970), utilizing the technique developed by Pekeris et. al. (1963) computes long period SH_{n} records for long ranges.

Here in this thesis, an attempt has been made to compute synthetic seismograms for a simple case of a layer over a half-space. We consider a point source responsible for generating P, SV or SH wave types to be embedded in the layer. Barker (1970) has solved this problem using Helmberger's (1967)
approach for the case of P and SV sources. The author here incorporates an SH source into this problem and then computes seismograms for different models which could be synthesized by the use of the above sources. The Cagniard-deHoop method has been utilized to put the solution into operational form.

In order to compare synthetic seismograms with the observed ones we require a double convolution process with the response of the model. The source-function, a function describing the behavior of the source, is first convolved with the response of the model which in turn is again convolved with the instrument response to give us the synthetic seismograms. However, in this thesis we have only convolved the impulse response of the model with a source time function, a triangle of base 0.4 and area unity. This would allow us to see the effects of source type, source depth, range and structure on the wave forms thus obtained which is the main aim of this study.

The technique exposed in Barker (1970) and here requires less computer time and the results obtained are accurate for both high and low frequency ranges. Near field P, SV and SH wave seismograms computed in this thesis show the accuracy of our results. It should be remarked, however, that in order to obtain a more exact seismogram
at a distance of about 50 or 100 km for P, SV case a greater number of rays have to be incorporated, which increases the computation time considerably. As a further test, comparisons have been made with the results of Pekeris et al. (1963,65) for P, SV and SH sources.

Chapter II deals with the statement and the formulation of the problem. Here we solve the inhomogeneous wave equation by the Laplace Transform technique, which is simple and straightforward. Chapter III deals with the computation of seismograms. Finally in Chapter IV we outline the conclusions reached by undertaking this study.
Chapter II
THEORETICAL FORMULATION OF THE PROBLEM

Let us consider a homogeneous, linearly isotropic, elastic half-space overlain by a homogeneous, linearly isotropic elastic layer. Further, let there be a point source of P, SV and SH waves in the layer. The geometry of the assumed model is shown in Figure 1. We would like to formulate the problem in circular cylindrical coordinates \((r, \theta, z)\). The angle \(\theta\) would not appear in the final form of the expression for the displacement because of the axial symmetry of the source. Let \(\lambda\) and \(\mu\) define Lamé's constants, \(\rho\) be density, \(\alpha\) and \(\beta\) denote compressional and shear (SV and SH both) wave velocities.

The response at the observation point \(A\) can be calculated by solving the inhomogeneous wave equation, e.g., for SH waves

\[
\mu_1 V^2 u_\theta(r, z, t) - \rho \frac{\partial^2}{\partial t^2} u_\theta(r, z, t) = \frac{H(t) \delta(r) \delta(z-h)}{2\pi r}
\]

where \(u_\theta(r, z, t)\) denotes the component of the displacement in the \(\theta\) direction. \(H(t)\) is the Heaviside unit step-function defined as

\[
H(t) = \begin{cases} 
0 & t < 0 \\
1 & t > 0
\end{cases}
\]
FREE SURFACE

Observation Point

Layer

HALF SPACE

$[0,0,0]$ to $r$ to $A$

$H$

$h$

$z$

$\alpha_1, \beta_1, \rho_1, \lambda_1, \mu_1$

$\alpha_2, \beta_2, \rho_2, \lambda_2, \mu_2$

FIG. 1
and \( \delta \) denotes the Dirac-delta function. The quantity on the right-hand side of Equation 1 represents the force produced by the point source of torque about the \( z \)-axis in the layer. We then exploit Laplace Transform technique to solve Equation 1, as done by Helmberger (1967). The details have been given in Appendix A.

The once-transformed in time displacement due to the \( n \)th generalized ray can be written [Barker(1970)] as

\[
\bar{u}_{z,n}(r,0,s) = s \text{ Im} \int_{0}^{\infty} pF_{n,z}(p)K_0(spr) e^{-s\gamma_n(p)} dp
\] (2)

\[
\bar{u}_{r,n}(r,0,s) = s \text{ Im} \int_{0}^{\infty} pF_{n,r}(p)K_1(spr) e^{-s\gamma_n(p)} dp
\] (3)

and

\[
\bar{u}_{\theta,n}(r,0,s) = s \text{ Im} \int_{0}^{\infty} pF_{n}(p)K_0(spr) e^{-s\gamma_n(p)} dp
\] (4)

where a bar has been placed over the quantity whose Laplace transform has been taken. In the above equations, \( s \) denotes the transform variable in complex frequency, \( p \) is the complex integration variable, and the \( \text{Im} \) symbol means that only the imaginary part of the complex function is considered. Further, \( K_0(spr) \) and \( K_1(spr) \) are the modified Bessel functions of zero and first order respectively. The notation \( u_{z,n}(r,z,t) \) and
$u_{r,n}(r,z,t)$ represent the vertical and the radial components of displacement for P or SV source and similarly $u_{\theta,n}(r,z,t)$ is the component of the displacement in the $\theta$ direction for an SH source. However, in Equations 2, 3 and 4 these displacements have been Laplace transformed in time. The functions $F_{n,z}(p)$, $F_{n,r}(p)$ and $F_{n}(p)$ are

\[
F_{n,z}(p) = S R_z(p) \zeta_n(p)
\]

\[
F_{n,r}(p) = S R_r(p) \zeta_n(p)
\]

and

\[
F_{n}(p) = S R \zeta'_n(p)
\]

where

\[
\begin{cases}
S = \frac{1}{2\pi^2} \eta_1 (\lambda_1 + 2\mu_1) & \text{for P source} \\
S = \frac{1}{2\pi^2} \eta_1 !\mu_1 & \text{for SV & SH source}
\end{cases}
\]

and

\[
\eta_1 = \left( \frac{1}{\alpha_1^2} - p^2 \right) \psi^2, \quad \eta_2 = \left( \frac{1}{\beta_1^2} - p^2 \right) \psi^2
\]

where $\alpha_1$ and $\beta_1$ are the compressional and shear (SV and SH) wave velocities in the layer. The functions $R_z(p)$, $R_r(p)$ and $R$ are the responses due to the presence of the free surface for an upgoing P or SV and SH wave. $\zeta_n(p)$ and $\zeta'_n(p)$ are the reflection coefficient for the $n$th generalized ray for P or SV and SH sources. These functions have been written out explicitly in the Appendix B. Finally, the function $g_n(p)$ and $j_n(p)$ are given by
For P and SV waves:

\[ g_n(p) = (1-HK_{ud}-h)((1-K_{sp})n_1n_1 + K_{sp}n_1n_1) + l_s^{(n)}n_1H + l_p^{(n)}n_1H \]

and for SH waves

\[ j_n(p) = [2h-H]K_{ud}n_1 + [H-h]n_1 + l_s^{(n)}n_1H \]

where

\[ K_{ud} = \begin{cases} 
1 & \text{if, at the source, the ray is directed upward} \\
0 & \text{if, at the source, the ray is directed downward} 
\end{cases} \]

and

\[ K_{sp} = \begin{cases} 
1 & \text{if the source emits P waves} \\
0 & \text{if the source emits SV waves} 
\end{cases} \]

\[ l_s^{(n)} \] denotes the number of traverses made by the nth ray as an SV or SH wave. Similarly, \[ l_p^{(n)} \] is the number of traverses as a P wave.

We choose to approximate \( K_0(spr) \) and \( K_1(spr) \) by its asymptotic expansion for large arguments and making use of the Laplace inversion formulae [See Barker (1970)] to get

\[ u_{z,n}(r,0,t) = \text{Im} \ \frac{3}{\sqrt{\tau}} \left\{ \frac{1}{\sqrt{2\pi r}} F_{n,z}(p) \frac{dp}{dt} - \frac{1}{8} H(t) \right\} \]

\[ \left[ \frac{1}{\sqrt{2\pi r}} F_{n,z}(p) \frac{dp}{dt} + \frac{9}{128} \tau \left( \frac{1}{\sqrt{2\pi r}} F_{n,z}(p) \frac{dp}{dt} \right) \right] \]

and

\[ u_{r,n}(r,0,t) = \text{Im} \ \frac{3}{\sqrt{\tau}} \left\{ \frac{1}{\sqrt{2\pi r}} F_{n,r}(p) \frac{dp}{dt} - \frac{3}{8} H(t) \right\} \]

\[ \left[ \frac{1}{\sqrt{2\pi r}} F_{n,r}(p) \frac{dp}{dt} + \frac{3}{128} \tau \left( \frac{1}{\sqrt{2\pi r}} F_{n,r}(p) \frac{dp}{dt} \right) \right] \]
\[ \bar{u}_{\theta, n}(r, 0, t) = \text{Im} \left( \frac{\partial}{\partial t} \right) \left( \frac{1}{\sqrt{2\pi}} \int \frac{F_n(p)}{p} F_n(p) \frac{dp}{dt} - \frac{1}{8} H(t) \right) \]

where * denotes convolution. We have done calculations based on Equations 5, 6 and 7. We use only the first two terms in the modified Bessel function which holds good for large arguments, i.e. for long ranges. It seems that for long ranges we get most of the contribution in Bessel function's asymptotic expansion from the first term as it should be. A computer program for evaluating Equations 5, 6 and 7 has been given in Appendix C with a detailed listing. However, we would like to add that the computations for all cases discussed in this thesis, we have taken 64 generalized rays. Pekeris et al. (1963, 65) have used almost as much as 5 times the number of rays we have used.

In order to get synthetic seismograms the impulse response obtained from evaluating Equations 5, 6 and 7 is convolved with a suitable source-time-function. (Figure 2a).

The step-function with rounded shoulders defined by Figure 2a is mathematically equivalent to (where \( \Delta \) is the digitization interval)

\[ f(t) = \begin{cases} 0 & \text{if } t < 0 \\ \frac{1}{2} t^2 & \text{if } 0 < t < \Delta \end{cases} \]
**FIG. 2**
\[ f(t) = \frac{1}{2} t^2 - (t-\Delta)^2 \quad \Delta < t < 2\Delta \]

\[ = \frac{1}{2} t^2 - (t-\Delta)^2 + \frac{1}{2} (t-2\Delta)^2 = \Delta^2 \quad 2\Delta < t \]  

At long distances Equation 8 produces saw-tooth-like displacements, as shown in Figure 2b. The impulse response obtained by evaluating Equations 5, 6 and 7 was convolved with a triangle of base 4, and an area of unity. This is true of all the plots shown in this thesis.

We should be careful in using equations 5, 6 and 7 because they hold good for sufficiently long ranges and large times, otherwise we cannot justify the use of asymptotic expansion for the modified Bessel functions \( K_0(spr) \) and \( K_1(spr) \).
Chapter III

COMPUTATION OF SEISMOGRAMS

Synthetic seismograms for a layer over a half-space have been computed in the distance range zero to 100 km using the numerical technique described in the previous chapter. Barker (1970) computer program was modified to take into consideration SH wave option. Seismograms have been calculated to investigate the effects of structure, source-type, source-depth and distance range. In all the synthetic seismograms presented in this thesis, the impulse response is convolved with a triangle of base 0.4 sec and area unity. For structure we chose two separate crustal models listed in Table I. Model I represents a 10 km thick granitic layer over a gabbroic half-space. Model II represents a sedimentary basin with a 2 km thick sedimentary layer overlying hard basement rock. To see the effect of source-type, we computed theoretical seismograms for P, SV and SH wave sources. We embedded each of these sources in case of Model I at 5, 2, and 5 km, while in Model II the source-depth was 1 km. We have made computations for 50 and 100 km distance range and in one case for 20 km.

Now let us examine each of the above-mentioned factors in detail. We start our discussion with source effects
beginning with a P source at different depths in the Model I (Figures 3, 4). The seismograms show that the record begins with P wave refractions, which is later followed by reflections and Rayleigh waves. Here it is very clear that shallow P sources generate Rayleigh waves quite efficiently. As we increase the source depth we see a change in the surface wave amplitude and spectral characteristics. Now to determine the range effect we use a P source and Model II. In Figures 5 and 6 we see that as we increase the range, we get well-defined P reflections arriving at \( t = 25.3 \) sec. and also at later times.

To illustrate the effect of source type we go back to Model I but this time use an SV source at different depths for range 50 km. This is shown in Figures 7 and 8. Here we see that Rayleigh wave is an outstanding feature. In Figures 9 and 10 we consider the same model as has been used above, but now at the 100 km range. The first of these figures represents the early part of the seismogram showing the refracted and reflected arrivals. We did this just to show the similarity in body wave-forms for different source depths. Later in Figure 10 we display the generation of Rayleigh waves. These are so strong and large in amplitude that on a scale such as the one used in Figure 10, we could not see any other type of waveforms. In the real world, when we are actually recording the events, the response of the instrument can be
controlled, which enables us to see different sorts of waveforms, though they differ greatly in amplitudes. We computed for an SV source, Model II and a range of 50 km, both the early part and the later part of the seismograms (Figure 11a,b).

Further, we also compare records obtained for different ranges using Model II. This is shown in Figure 12, for body waves. Then we compare the z component of the seismogram for Model II for SV and P sources for a range of 50 km (Figure 13). We see a resemblance between the two waveforms which we should expect. We can also see that the Rayleigh wave generated by an SV source has a larger amplitude than for a P source. It is clear than an SV source is a more efficient generator of Rayleigh waves than a P source, for the simple reason that the integration contour comes closer to the real axis in the case of the SV source.

Following our earlier procedure we examine the source-depth effect on the computed model seismograms for an SH source. To begin with, we look at Model I and range 50 km. This is shown in Figure 14. There is a very close resemblance in waveform shape for source-depths 0.5, 2 and 5 kms. We can clearly see the $SH_n$ arrival (in Figure 14 b,c), which is followed by SH reflections. There is no indication of dispersed Love waves. We again consider the same model but increase the range to 100 km (Figure 15). We see a long quiet period
beginning with refracted SH arrivals followed by SH reflections. Finally we consider Model II and here we investigate the range effect. Figure 16 shows the synthetic seismograms for the ranges 20, 50 and 100 km. In all of these seismograms we can see the generation of Love waves, which clearly stand out with their relatively larger amplitudes than the body waves.
In this thesis, we investigated the effects of source type, source depth, range and structure or seismic wave propagation in a layer over a half-space. We considered compressional sources, vertical torque (SV) and horizontal torque (SH) as point source models. A close examination of the synthetic seismograms discussed in the last chapter revealed the following:

(a) A relatively thin high-contrast layer accentuates the surface waves on the seismogram by increasing the dispersive effects in near-fields.

(b) Source-depth plays a very important role in determining the relative amplitude of the surface waves. Frequency content of these waves are also affected by the source-depth.

(c) For a given structure and source depth, SV type sources generate Rayleigh waves more efficiently than purely compressional (P type) sources.

(d) Body wave response (early portion of the seismogram) changes more rapidly with distance than the surface wave response.
(e) An extension of this work is to compare the theoretical seismograms with those of field data and to investigate earthquake source characteristics. However, before this can be done we need to incorporate the instrument response and realistic source-time function in the calculation of synthetic seismograms.
Table I
MODELS FOR COMPUTATION

Model I
Layer Thickness $H = 10$ km.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$\alpha$ (km/sec)</th>
<th>$\beta$ (km/sec)</th>
<th>$\rho$ (gm/cc)</th>
<th>$\sigma$</th>
</tr>
</thead>
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<tr>
<td>Layer</td>
<td>5.0</td>
<td>2.9</td>
<td>2.7</td>
<td>.25</td>
</tr>
<tr>
<td>Half-Space</td>
<td>7.2</td>
<td>4.2</td>
<td>2.9</td>
<td>.25</td>
</tr>
</tbody>
</table>

Model II
Layer Thickness $H = 2$ km.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$\alpha$ (km/sec)</th>
<th>$\beta$ (km/sec)</th>
<th>$\rho$ (gm/cc)</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer</td>
<td>4.0</td>
<td>2.9</td>
<td>2.2</td>
<td>.25</td>
</tr>
<tr>
<td>Half-Space</td>
<td>6.0</td>
<td>3.46</td>
<td>2.4</td>
<td>.25</td>
</tr>
</tbody>
</table>

In both Models I and II $\alpha$ denotes a compressional wave velocity, $\beta$ denotes shear wave velocity, $\rho$ denotes density and $\sigma$ denotes Poisson's ratio. Unless otherwise stated, source-depth is one-half the layer thickness.
In the figure captions, the first entry refers to the source type (P, SV, SH and C₀, where the last symbol means simultaneous comparison of two source types), second to component (Z and R for P and SV cases, otherwise omitted in SH case where only radial component can be plotted), third to the model (I or II) and finally fourth, to the range (20, 50 or 100 km or C, in which case the seismograms are simultaneously compared at two ranges for the same model). Unless otherwise stated, source-depth is one-half the layer thickness. However, while examining the plots we should note the variation of the amplitude scale. It should also be emphasized that the horizontal (time) scale is not always uniform in all the plots.
Fig. 3.  P, Z, I, 50
Fig. 4. P, R, I, 50
Fig. 5.  P, Z, II, C
r = 50 km.

Fig. 6. P, R, II, C
Fig. 7. SV, Z, I, 50
Fig. 8. SV,R,1,50
Fig. 9: SV, Z, I, 100
Fig. 10  SV, Z, I, 100
Fig. 11.  SV, Z, II, 50
Fig. 12. $SV, Z, II, C$
Fig. 14. SH, 1, 50
Fig. 16. SH, II, C
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Appendix A

SOLUTION OF THE INHOMOGENEOUS WAVE EQUATION

Here we would like to solve the inhomogeneous wave equation

\[ \mu \nabla^2 u_\theta (r,z,t) - \rho \frac{\partial^2}{\partial t^2} u_\theta (r,z,t) = \frac{-H(t) \delta (r) \delta (z-h)}{2\pi r} \]  \hspace{1cm} (A1)

The Laplace transform of (A1) in time domain is

\[ \nabla^2 \tilde{u}_\theta (r,z,s) - \frac{s^2}{\beta^2} \tilde{u}_\theta (r,z,s) = \frac{-\delta (r) \delta (z-h)}{2\pi ur} \]  \hspace{1cm} (A2)

Note that

\[ \nabla^2 \equiv \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right) \equiv \nabla_r^2 + \frac{\partial^2}{\partial z^2} \]

And now let us define the bilateral Laplace transform as

\[ \tilde{u}_\theta (r,\nu, s) = \int_{-\infty}^{+\infty} \tilde{u}_\theta (r,z,s) e^{\nu z} \, dz \]

When this is applied to (A2) we get as a consequence

\[ \nabla_r^2 \tilde{u}_\theta (r,\nu, s) - (K^2 - \nu^2) \tilde{u}_\theta (r,\nu, s) = \frac{-\delta (r) e^{-\nu h}}{2\pi ur} \]  \hspace{1cm} (A3)

Define \( \zeta^2 = K^2 - \nu^2 \), where \( K^2 = s^2/\beta^2 \), radiation condition becomes \( \text{Re} \, \zeta \geq 0 \) which has to be satisfied. Consider the equation
\[
\frac{\nu^2 v - \zeta^2 v}{r} = -\frac{\delta(x-r_0)}{r}
\]  (A4)

which has as its homogeneous solutions

\[ V_1 = I_0(\zeta, r) \quad V_2 = K_0(\zeta, r) \]

which for large \( r \) behaves as

\[ V_1 \approx \frac{e^{\zeta r}}{\sqrt{2\pi \zeta r}} \quad V_2 \approx \frac{\pi}{\sqrt{2\zeta r}} e^{-\zeta r} \]

\[ V = cI_0(\zeta r_<)K_0(\zeta r_>) \]  (A5)

where \( r_> \equiv \max(r, r_0) \), \( r_< \equiv \min(r, r_0) \) and \( c \), a constant, to be determined from the continuity condition of \( V \) at \( r = r_0 \).

Multiplying (A4) by \( r \) and integrating from \( r_0-\epsilon \) to \( r_0+\epsilon \) we get

\[ \lim_{\xi \to 0} \left[ r \frac{dV}{dr} \right]_{r_0-\epsilon}^{r_0+\epsilon} = -1 \]  (A6)

Put (A5) into (A6) to get

\[ c\zeta r_0 [I_0(\zeta r_0)K_0'(\zeta r_0) - I_0'(\zeta r_0)K_0(\zeta r_0)] = -1 \]  (A7)

where the prime (') denotes differentiation with respect to \( r_0 \). The quantity in parenthesis in (A7) denotes the Wronskian of \((I_0, K_0)\) and is found to be \(-1/\zeta r_0\). Hence \( c = 1 \), giving the particular solution as
\[ V = I_0(\zeta r)K_0(\zeta r) \]  

(A8)

From (A3) we get \( r_0 = 0 \) which gives us

\[ u_\theta = K_0(\zeta r) \quad I_0(0) = 1 \]

and therefore the particular solution of (A3) is

\[ \bar{u}_\theta(r,\nu,s) = K_0(\zeta r)e^{-\nu h/(2\pi \mu)} \]

The inversion from \( \nu \) to \( z \) gives us

\[ \bar{u}_\theta(r,z,s) = \frac{1}{2\pi i} \frac{1}{2\pi \mu} \int_{-i\infty}^{+i\infty} K_0(\zeta r)e^{\nu(z-h)}d\nu \]  

(A9)

Alternatively we can write (A9) as

\[ \bar{u}_\theta(r,z,s) = \frac{1}{2\pi^2 \mu} \text{Im} \int_0^{i\infty} K_0(\zeta r)e^{\nu(z-h)}d\nu \]  

(A10)

or

\[ \bar{u}_\theta(r,z,s) = -\frac{1}{2\pi^2 \mu} \text{Im} \int_0^{i\infty} K_0(\zeta r)e^{\nu(z-h)}d\nu \]  

(A11)

Keeping in mind the condition \( \text{Re} \, \zeta \geq 0 \) has to be satisfied, we cut the \( \nu \) plane as shown in Figure A1.

Shifting the path of integration in Equation A10 to the tip of the cut in the left half of the \( \nu \) plane for \( z > h \), we get
\[ \bar{u}_0(r, z, s) = \frac{1}{2\pi^2 \mu} \text{Im} \int_{-\infty}^{\infty} K_0(\zeta r) e^{\nu|z-h|} d\nu \]  
\text{(Al2)}

or making a change in the variable of integration gives us

\[ \bar{u}_0(r, z, s) = -\frac{1}{2\pi^2 \mu} \text{Im} \int_{0}^{\infty} K_0(\zeta r) \frac{\zeta}{\nu} e^{\nu|z-h|} d\zeta \]  
\text{(Al3)}

for \( \text{Re } \nu \leq 0 \).

If we put \( \nu = -\nu \) in (Al2) we get

\[ \bar{u}_0(r, z, s) = \frac{1}{2\pi^2 \mu} \text{Im} \int_{0}^{\infty} K_0(\zeta r) \frac{\zeta}{\nu} e^{-\nu|z-h|} d\zeta \]  
\text{(Al4)}

for \( \text{Re } \nu \geq 0 \). If we again consider for \( z > h \) an appropriate equation corresponding to (Al0), i.e. (Al1), we would see that we get (Al4). This shows that (Al4) holds true for all \( z \). Let \( \zeta = sp, \nu = sn \) in (Al4) to get

\[ \bar{u}_0(r, z, s) = \frac{s}{2\pi^2 \mu} \text{Im} \int_{0}^{\infty} \frac{p}{\eta} K_0(spr) e^{-sn|z-h|} dp \]  
\text{(Al5)}

which is the solution for the homogeneous half-space. This formulation can easily be extended for layered media if we introduce a function \( F_n(p) \) in (Al5) which takes into consideration the reflection coefficient at the interface, the receiver directivity function and the source radiation function to get
\[ \bar{u}_{\theta,n}(r,0,s) = s \, \text{Im} \int_{0}^{\infty} p F_n(p) K_0(s r) \, e^{-sj_n(p)} \, dp \]  

where \( j_n(p) \) describes the time taken by the \( n \)th generalized ray to traverse the layer. This is Equation 4 in the main text of this thesis. Proceeding in exactly the same way, we can obtain Equations 2 and 3 for \( P \) and/or \( SV \) sources.
Appendix B

CALCULATION OF REFLECTION COEFFICIENTS AND RECEIVER FUNCTIONS

P and SV Waves. The reflection function \( \tau_n(p) \) is a measure of the attenuation of energy due to internal reflections and is given by

\[
\tau_n(p) = \frac{1}{D} \left( \frac{1}{D_{PP}} + \frac{1}{D_{PS}} + \frac{1}{D_{SP}} + \frac{1}{D_{SS}} + \frac{1}{U_{PP}} + \frac{1}{U_{PS}} + \frac{1}{U_{SP}} + \frac{1}{U_{SS}} \right)
\]

where \( D \) denotes the number of \( P \) to \( P \) reflections off the lower boundary and similarly, \( u_{UPP}(n) \) is the number of \( P \) to \( P \) reflections off the free surface. The functions \( R_{DPP}, R_{DPS}, \) etc. denote the reflection coefficients and are given by

[See for details Helmberger (1967), Barker (1970)]

\[
R_{DPP} = \frac{(-[1] + [2] + [3] - [4] - [5] + [6])/D_1}{D_1}
\]

\[
R_{DPS} = \frac{2p\eta_1[(K_2-p^2)(K_3-p^2) - \eta_2\eta_2'(K_1-p^2)]/D_1}{D_1}
\]

\[
R_{DSP} = \frac{2p\eta_1'[K_2-p^2)(K_3-p^2) - \eta_2\eta_2'(K_1-p^2)]/D_1}{D_1}
\]

\[
R_{DSS} = \frac{(-[1] + [2] + [3] - [4] + [5] - [6])/D_1}{D_1}
\]

where

\[
\]

\[
[1] = p^2(K_3-p^2)^2 \quad [2] = \eta_1\eta_2\eta_1'\eta_2'p^2
\]
\[ [3] = \eta_1 \eta_1 (K_3 - p^2)^2 \]
\[ [4] = \eta_2 \eta_2 (K_1 - p^2)^2 \]
\[ [5] = \eta_1 \eta_2 K_1 K_2 \]
\[ [6] = \eta_2 \eta_1 K_1 K_2 \]

and

\[ K_1 = \frac{1}{2} \left( \frac{\rho_1}{\mu_2 - \mu_1} \right) \]
\[ K_2 = \frac{1}{2} \left( \frac{\rho_2}{\mu_2 - \mu_1} \right) \]
\[ K_3 = K_1 + K_2 \]

Also

\[ R_{UPP} = R_{USP} = [4\beta_1^4 p^2 \eta_1 \eta_1' - (1 - 2\beta_1^2 p^2)^2] / D_2 \]
\[ R_{UPS} = 4p \eta_1 (1 - 2\beta_1^2 p^2) / D_2 \]
\[ R_{USP} = -4p \eta_1' (1 - 2\beta_1^2 p^2) / D_2 \]
\[ D_2 = 4\beta_1^4 p^2 \eta_1 \eta_1' + (1 - 2\beta_1^2 p^2)^2 \]

where

\[ \eta_1 = \sqrt{1 / \alpha_1^2 - p^2} \quad \eta_1' = \sqrt{1 / \beta_1^2 - p^2} \]
\[ \eta_2 = \sqrt{1 / \alpha_2^2 - p^2} \quad \eta_2' = \sqrt{1 / \beta_2^2 - p^2} \]

where \( \alpha_1 \) and \( \beta_1 \) are velocities of compressional and shear waves in the layer, and \( \alpha_2 \) and \( \beta_2 \) that in the half-space.

\( \rho \) and \( \mu \) denote density and rigidity respectively and \( p \) is complex integration variable which has dimensions of sec/km.
For SH Waves. Taking Figure 1 of the main text into consideration, we have the following wave equations

\[ \nabla^2 \chi_1(x,z,t) - \frac{1}{\beta_1^2} \frac{\partial^2 \chi_1}{\partial t^2} = -\delta(t) \delta(x) \delta(z-h) \]  
\( (B1) \)

and

\[ \nabla^2 \chi_2(x,z,t) - \frac{1}{\beta_2^2} \frac{\partial^2 \chi_2}{\partial t^2} = 0 \]  
\( (B2) \)

where \( \chi_1 \) and \( \chi_2 \) are SH-potentials in the layer and half-space respectively.

We define unilateral and bilateral Laplace Transforms with respect to \( t \) for the function \( g(t) \) and similarly with respect to \( x \) for \( g(x) \) as follows

\[ \tilde{g}(s) = \int_0^\infty g(t)e^{st} dt \text{ where } s \text{ is positive real } \]  
\( (B3) \)

and

\[ \hat{g}(\zeta) = \int_{-\infty}^{+\infty} g(x)e^{-\zeta x} dx \text{ where } \zeta \text{ is complex } \]  
\( (B4) \)

After transforming (B1) and (B2) with respect to \( s \) and \( \zeta \), the solutions for SH wave displacements after Gilbert and Knopoff (1961) in the layer and the half-space respectively are given by

\[ \chi_1 = \frac{1}{2\nu_1} \left[ e^{-\nu_1|z-h|} + Ae^{-\nu_1z} \right] \]  
\( (B5) \)

\[ \chi_2 = \frac{B}{2\nu_1} e^{\nu_2z} \]  
\( (B6) \)

where

\[ \nu_1 = \sqrt{\frac{1}{\beta_1^2} - p^2} \quad \nu_2 = \sqrt{\frac{1}{\beta_2^2} - p^2} \]
Boundary conditions at the interface require continuity of displacement and stress, viz.

\[ \chi_2 = \chi_2 \]  
\[ P_{zy} \bigg|_1 = \mu_1 \frac{\partial \chi_1}{\partial z} \bigg|_2 = P_{zy} \bigg|_2 = \mu_2 \frac{\partial \chi_2}{\partial z} \bigg|_2 \]

in order to determine A and B in (B5) and (B6). If we change the origin of the coordinate system to the top of the interface we get

\[ e^{-\nu_1 h} + A = B \]

\[ \nu_1 \mu_1 e^{-\nu_1 h} - \mu_1 \nu_1 A = \mu_2 \nu_2 B \]

\[ A = R_{DSH} = \frac{\mu_1 \nu_1 - \mu_2 \nu_2}{\mu_1 \nu_1 + \mu_2 \nu_2} e^{-\nu_1 h} \]

If we put \( \nu_1 = s\eta_1 \) \( \nu_2 = s\eta_2 \)

\[ A = \frac{b\eta_1 - \eta_2}{b\eta_1 + \eta_2} e^{-s\eta_1 h} \quad \text{where} \quad b = \frac{\mu_1}{\mu_2} \]  
(B10)

and

\[ B = \frac{2\nu_1 \mu_1}{\mu_1 \nu_1 + \mu_2 \nu_2} e^{-\nu_1 h} = \frac{2\nu_1 \mu_1}{\mu_1 \eta_1 + \mu_2 \eta} e^{-s\eta_1 h} \]  
(B11)

The reflection coefficient given by (B10) turns out to be of the same form as given by Pekeris et al. (1963) and Mitra (1963). Similarly, we can find out the reflection coefficient (c) for the free surface by requiring stress \( \tilde{P}_{z\theta} \) to vanish at that surface.
The reflection function $\zeta_n(p)$ is, therefore given by

$$
\zeta_n'(p) = \frac{l_{\text{USH}}^{(n)}}{R_{\text{USH}}} \frac{l_{\text{DSH}}^{(n)}}{R_{\text{DSH}}}
$$

(B13)

where $R_{\text{USH}}$ and $R_{\text{DSH}}$ are given by (B12) and (B10) respectively. $l_{\text{USH}}^{(n)}$ is the number of SH to SH reflections off the free surface and similarly $l_{\text{DSH}}^{(n)}$ is the number of SH to SH reflections off the lower boundary.

The receiver function is calculated from stress-free boundary condition at the free surface (Phinney, 1967). The once transformed in time displacement due to an upgoing SH wave is

$$
\tilde{u}_{\theta,n}(r,o,s) = \frac{1}{2\pi^2 \mu} \text{Im} \int_0^{i\infty} \frac{s p}{n_1^2} K_0(s p r) \{1+R_{\text{USH}}\} e^{-n_1^2 h} \, dp
$$

(B14)

The function in braces is our receiver function $R$ which can be easily seen to be identical to

$$
R = 2
$$

(B15)
Appendix C

COMPUTER PROGRAM

Here we give a printout of the program that we have used in evaluating Equations 5, 6 and 7. We have incorporated into this listing a detailed explanation of various features of the program. It was originally written by Don Helmberger. Barker (1970) extended it for the P-SV problem valid for both high and low frequency ranges. We included the SH wave option in it. The main program produces the impulse response. A secondary program then convolves this with a suitable source-function to give the final results.
This program is for determining the theoretical response to a point source of an elastic layer over an elastic half space. It revises the original model of Terry Barker to include an SH source.

The program is divided into main and the following subprograms in alphabetical order:

- AdjustContur
- CrcurayDelpsfa
- Find2
- Helphighinterp
- Pln2
- Psicti
- Raydef
- Recr
- Refft
- Setup
- Sf2
- Time2
- Ts

The function of each subprogram will be made clear in the body of the program and in the comments included in each.

All calculations are double precision unless otherwise noted.
THE SUBPROGRAMS SETUP, CONTOR, TIME2, PSICO, REFFT, AND RECVR HAVE BEEN REVISED SINCE THE LIBRARY WAS CREATED AND ARE NOW RUN AS SOURCE DECKS.

ADDITIONAL PROGRAMS

CONVOLUTION WITH A SPECIFIED SOURCE FUNCTION AND PLOTTING ON CALCOMP ARE PERFORMED IN ANOTHER PROGRAM.

THE LIBRARY WAS CREATED AND ARE NOW RUN AS SOURCE DECKS.

ADDITIONAL PROGRAMS

THE INPUTS TO THIS PROGRAM ARE AS FOLLOWS: THE NUMBER OF LAYERS, NMOD, IN FORMAT (I10) ON THE FIRST CARD. IN THIS CASE NMOD IS ALWAYS 3, INCLUDING THE HALF SPACE ABOVE THE LAYER WHICH IS A VACUUM.


THE NEXT CARD CONTAINS THE RANGE IN FORMAT (F10.0).

THE NEXT CARD CONTAINS THE DESIRED INTERVAL BETWEEN RECORD POINTS (DP) IN SECONDS, THE LENGTH OF RECORD CALCULATED FOR EACH RAY (TMX, IN SEC), AND THE NUMBER OF RECORD POINTS (NN) IN THE FINAL OUTPUT IN FORMAT (2F10.0, 110).


THE NEXT CARDS CONTAIN THE RAY PARAMETERS FOR (19, LAST RAY USED), THE FIRST EIGHTEEN RAYS ARE CURRENTLY DEFINED IN RAYDEF. AS THE SUBROUTINE RAYDEF HAS NO OTHER FUNCTION, IT WOULD BE CONVENIENT TO READ IN ALL THE RAY PARAMETERS IN FINAL FORM AND ELIMINATE THIS SUBROUTINE. THE FIRST SET OF PARAMETER CARDS CONTAIN THE FOLLOWING IN FORMAT (4I11): KUDIJ (A FOR UPGOING RAY, F FOR DOWNGOING), TS (J), THE NUMBER OF S-S REFLECTIONS;
LTP(J), THE NUMBER OF P-P REFLECTIONS, RSP(J), THE MODE OF
THE RECEIVER (=0 FOR S AND 1 FOR P)
C THE NEXT SET OF CARDS CONTAIN THE VALUES OF THE TWO DIMENSIONAL ARRAY
C LREF(I,J) WHICH DEFINE THE NATURE OF THE REFLECTIONS AT THE TOP
C AND THE BOTTOM OF THE LAYER, EIGHT VALUES FOR EACH RAY DEFINING GIVING
C THE NEXT NUMBER OF P TO P, S TO P, P TO S, AND S TO S INTERACTIONS
C AT THE BOTTOM INTERFACE, NEXT THE SAME VALUES FOR THE FREE SURFACE.
C THE SH PROGRAM SETS LTP AND RSP ALWAYS EQUAL ZERO AND HAS NO PP, PS,
C OR SP INTERACTIONS
C
C**********OUTPUT***************************************************************00000820 MAIN0083
C THIS PROGRAM GENERATES PUNCHED OUTPUT FOR THE THEORETICAL RESPONSE
C UNCONVOLVED WITH ANY SOURCE FUNCTION, PRINTING FIRST THE STARTING TIME
C FOR THE RECORD IN SECONDS, TT(1), THE TIME INTERVAL BETWEEN DATA POINTS
C IN SECONDS, DP, AND THE NUMBER OF DATA POINTS IN THE RECORD, NN IN THE
C FORMAT(2E15.6, I10). THIS HEADING CARD IS FOLLOWED BY THE SURFACE
C RESPONSE IN FORMAT(5E15.6).
C
C THERE IS ALSO PRINTED OUTPUT, SOME OPTIONAL SOME NOT: FIRST A LISTING
C OF MODEL PARAMETERS; THEN FOR EACH RAY, THE VALUES OF P AT THE POINT
C OF REFLECTION (AN INTEGRATION PARAMETER) AND DP/DT AS COMPLEX NUMBERS;
C THEN THE TIME (IN SEC) OF THE REFLECTION ARRIVAL OF THE SURFACE RESPONSE
C -S, FOLLOWED BY THE SURFACE RESPONSE AT EVERY FIFTH CALCULATED VALUE.
C NOTE THE DESIRED RESPONSE IS HERE GIVEN BY PHIZ, THE VERTICAL COMPONENT.
C -T, AS THAT COMPONENT INVOLVES THE K0 BESSEL FUNCTION REQUIRED FOR AN
C SH SOURCE, THE PHIR (RADIAL) COMPONENT USES THE K1 BESSEL FUNCTION.
C IN THIS APPROXIMATION ONLY THE FIRST TWO TERMS IN THE EXPANSION OF
C THESE FUNCTIONS ARE RETAINED, THEREFORE THE TWO ITEMS DIFFER ONLY IN
C THE SECOND TERM, AND IN THE FIRST-ORDER NOT AT ALL. 
C THE RAY OUTPUT IS FOLLOWED BY A LISTING OF THE TIME SERIES OF THE
C OUTPUT SURFACE RESPONSE. NOTE THAT THIS PRINTED RECORD IS PADDED WITH
C ZEROES AT THE BEGINNING TO 10% OF ITS LENGTH
C
C**********THE ORDER OF EXECUTION**************************************************00000850 MAIN0086
C
C THE PROGRAM IS EXECUTED APPROXIMATELY AS FOLLOWS:
C MODEL PARAMETERS ARE READ IN OR OTHERWISE SPECIFIED IN MAIN. CONTROL
C IS THEN PASSED TO SUBROUTINE SETUP.
C IF THIS CALL IS FIRST TO SETUP, THE SUBROUTINE RAYDEF IS CALLED TO READ000001110 MAIN0112
C IN OR OTHERWISE ESTABLISH THE RAY PARAMETERS, THE ARRAYS WHICH CONTAIN000001120 MAIN0113
C THE OUTPUT ARE INITIALIZED, AND THE FUNCTION IS USED TO DETERMINE
C THE TIME OF FIRST ARRIVAL, THE BEGINNING OF THE RECORD. THE PROGRAM THEN000001140 MAIN0115
C ITERATES OVER EACH RAY. FOR EACH RAY THE FIRST SUBROUTINE CALLED IS
C HIGH. IN HIGH WE EMPLOY THE SUBROUTINE FIND2 TO DETERMINE THE VALUES
C CF PC AND TO, THAT IS THE VALUE OF THE INTEGRATION PARAMETER P AND THE
C TIME T AT THE REFLECTION ARRIVAL FOR THIS RAY. IN GENERAL EACH RAY
C RESPONSE IS DIVIDED INTO TWO PARTS, AN INTEGRATION ALONG THE REAL
C AXIS FOR THE REFRACTED RESPONSE, AND AN INTEGRATION ALONG THE COMPLEX 000001200 MAIN0121
C CONTOUR FOR THE RESPONSE AFTER REFLECTION. FOR A DISCUSSION OF THE
C METHOD SEE THE REFERENCES. THE POINT OF REFLECTION IS FOUND WHERE THE
C CONTOUR LEAVES THE REAL AXIS AND DP/DT HAS A POLE.
C BEFORE THE REFRACTION ARRIVAL, THAT IS BEFORE THE FIRST BRANCH POINT
C AS GIVEN BY P=1/(LARGEST VELOCITY OF PROPAGATION IN THE HALF SPACE),
C THE INTEGRAL IS ZERO. THE FIRST BRANCH CUT MAY NOT OCCUR UNTIL AFTER
C THE REFLECTION ARRIVAL. IF THIS IS SO THERE IS NO REFRACTED SIGNAL
C THE SUBROUTINE HELP IS NOW CALLED TO FIND THE TIME OF ARRIVAL OF THE
C REFRACTED SIGNAL AND THE VALUE OF DP/DT THERE. WE NOW COMPARE THIS
C REFRACTED ARRIVAL TIME WITH THE REFLECTED ARRIVAL TIME AND CALCULATE
C THE INTEGRATION INTERVAL CELP FOR THE REFRACTED RECORD, THAT IS UP TO
C BUT NOT INCLUDING THE POINT OF REFLECTION, DEPENDING ON THE SEPERATION000001320 MAIN0133
C BETWEEN THE BRANCH CUT (REFRACTION ARRIVALS) AND THE POLE (REFLECTION 000001330 MAIN0134
C ARRIVAL) THE INTERVAL SIZE MAY BE CONSTANT OR CALCULATED IN SUBROUTINE000001340 MAIN0135
C DELPS USING A SINE RELATIONSHIP. IF THERE IS A REFRACTED ARRIVAL
C CONTROL IS TRANSFERRED TO SUBROUTINE PLN1 TO PERFORM THIS PART OF THE
C INTEGRATION. NOTE THAT IF THERE IS A REFRACTED ARRIVAL THE FIRST VALUE
C OF THE TIME SERIES OF SURFACE RESPONSE IS ALWAYS SET TO ZERO. IN PLN1
C WE USE THE ALREADY CALCULATED INTERVALS DELP AND THE SUBROUTINE HELP
C TO CALCULATE THE TIME AND DP/DT FOR EACH VALUE OF THE INTEGRATION
C PARAMETER P. FOR EACH P WE THEN CALL THE SUBROUTINE PSICOJ IN PSICOJ
C WE PERFORM THE INTEGRATION, CALCULATING THE TWO COMPLEX TERMS IN THE
C EXPANSION USED BY TERRY BARKER TO APPROXIMATE THE INTEGRAL. FIRST WE
CALL THE SUBROUTINE REFFT TO DETERMINE THE VALUE OF THE REFLECTION FUNCTION, THEN THE SUBROUTINE RECUR TO DETERMINE THE RECEIVER FUNCTION.

WE THEN MULTIPLY THE REFLECTION FUNCTION BY ITSELF ACCORDING TO THE NUMBER OF REFLECTIONS SPECIFIED IN ARRAY LREF. THE TWO COMPLEX TERMS IN THE EXPANSION FOR THE RADIAL AND THE VERTICAL SURFACE RESPONSE ARE THEN CALCULATED. CONTROL RETURNS TO PLN1. WE THEN COMPLETE SIMILAR CALCULATIONS FOR GRADUALLY DECREASING INTERVALS APPROACHING BUT NOT INCLUDING THE POINT OF REFLECTION. CONTROL IS RETURNED TO HIGH. THE SUBROUTINE CONTR IS NOW CALLED TO DEFINE THE COMPLEX CONTOUR OF INTEGRATION FOR THE SIGNAL AFTER THE REFLECTION ARRIVAL. FIRST A SERIES OF DECREASING INTERVALS IS CALCULATED ABOUT THE POINT OF REFLECTION. THE INTEGRATION PARAMETER P IS NOW COMPLEX AND THIS COMPLEX CONTOUR IS DEFINED BY REQUIRING THAT ANOTHER PARAMETER, THE TIME, BE REAL. THEREFORE THE CONTOUR IS DEFINED BY MINIMIZING THE IMAGINARY PART OF THE TIME PARAMETER IN SUBROUTINE TIME2. THE INTEGRATION INTERVALS ABOUT THE SEVERAL BRANCH POINTS ARE DEFINED USING THE SUBROUTINE DEPS0.


IN THE SERIES, WE NOW ADJUST THIS DESIRED SERIES SO THAT ONE POINT OF IT COINCIDES WITH THE REFLECTION ARRIVAL AT TO. WE NOW USE THE SUBROUTINE INTERP TO INTERPOLATE THE OUTPUT TO THE DESIRED TIME SERIES. THE SUBROUTINE DDGT3 IS THEN USED TO TAKE THE TIME DERIVATIVE, AND THE OUTPUT IS PRINTED AND Punched ON CARDS. CONTROL RETURNS TO MAIN AND PROGRAM ENDS.
C
C ********************************************** 00001800 MAIN0181
C THE MAIN PROGRAM 00001810 MAIN0182
C 00001820 MAIN0183
C 00001830 MAIN0184
C THE MAIN PROGRAM SERVES THREE FUNCTIONS, READ-IN OF PARAMETERS DESCRIP 00001840 MAIN0185
C TING THE MODEL, SETTING THE VALUES OF CONSTANTS, AND CALLING ONCE THE 00001850 MAIN0186
C SUBROUTINE SETUP WHICH CONTROLS ALL ACTUAL CALCULATIONS. IT IS 00001860 MAIN0187
C SUGGESTED THAT ALL THE PARAMETER READ-INS BE INCLUDED IN MAIN ELIMINATING IN THE MAIN PROGRAM FOR CONVENIENCE, 00001870 MAIN0188
C THAT THE SUBROUTINES MAY BE PLACED IN A DISC LIBRARY AND THE LENGTH OF THE 00001880 MAIN0190
C MAIN PROGRAM MINIMIZED. 00001900 MAIN0191
C THE MAIN PROGRAM CALLS THE SUBPROGRAMS CURAY AND SETUP 00001910 MAIN0192
C ********************************************** 00001920 MAIN0193
C COMMON/ORSTF/C(100), S(100), D(100), TH(100), X 00001930 MAIN0194
C COMMON/CONFIX/DEL, NN, HDP, TMX, XDIM, YDIM, DP, KJ 00001940 MAIN0195
C COMMON/STUFF/CC(100), SS(100), DD(100), TTH(100), XX, RCSQ(100), RSSQ(100), 00001950 MAIN0196
C COMMON/THY/T(1000), PP(1000), RP(600) 00001960 MAIN0197
C COMMON/PLOT/CON, NNF, NPT, NSYN 00001970 MAIN0198
C COMMON/FIXP/CDN(100), APN(100), FLAT 00001980 MAIN0199
C COMMON/STOR/P(1000), TD(1000) 00001990 MAIN0200
C COMMON/THZ/TT(1000), PPZ(1000), PPR(1000) 00002000 MAIN0201
C COMMON/TNIP/DELM, DLTM, NDA, MTD, DLTP, NDB, JO, NDIRT 00002010 MAIN0202
C COMMON/LPRINT/PRNT, PRNTS, KST, KEND, PRNTC, NDC, DET 00002020 MAIN0203
C COMMON/CRSTF/NCASE, NPRAY, NYT, NDUM, VRL2, VRL3 00002030 MAIN0204
C COMMON/MYER/DTI 00002040 MAIN0205
C DIMENSION R(10) 00002050 MAIN0206
C LOGICAL PRNT, PRNTS, PRNTC, FLAT 00002060 MAIN0207
C FLAT = TRUE. 00002070 MAIN0208
C PRNT = .FALSE. 00002080 MAIN0209
C PRNTS = .FALSE. 00002090 MAIN0210
C PRNTC = .FALSE. 00002100 MAIN0211
C FLAT IS USED IN CURAY AND FOR THIS PROGRAM IS ALWAYS TRUE, INDICATING 00002110 MAIN0212
C
C THAT THE MODEL USES A FLAT SECTION RATHER THAN AN ACTUAL SPHERICAL
C MODEL OF THE EARTH
C THESE THREE VARIABLES CONTROLS THE PRINTING OF INTERMEDIATE RESULTS
C OF SOME DIAGNOSTIC IMPORTANCE. IN PARTICULAR PRNTC=TRUE PRINTS ALL THE
C ITERATION STEPS IN SUBROUTINE TIME2 FOR DETERMINING THE IMAGINARY
C PART OF THE INTEGRATION VARIABLE T, THAT IS TIME ALONG THE COMPLEX
C CONTOUR AFTER REFLECTION. DO NOT USE THIS OPTION IF RUNNING FOR MORE
C THAN ONE RAY AS A PROHIBITIVE VOLUME OF OUTPUT IS PRODUCED.
C IN GENERAL IT IS BEST NOT TO SET THESE OPTIONS TO TRUE FOR A RUN
C INVOLVING MORE THAN A FEW RAYS
C
C INPUT MODEL PARAMETERS
C
READ (5,100) NMOD
DD 200 J=1,NMOD
200 READ (5,300) C(J),S(J),D(J),TH(J)
100 FORMAT (I10)
300 FORMAT (4F10.0)
600 READ(5,500) X,IGJ,KPEK
500 FORMAT (F10.0,2I10)
501 FORMAT (5,501) DP,TMX,NN

C ASSIGN VALUES TO CONSTANTS
C
NNF=1
NPT=0
JO=3
C DESIGNATES A THREE LAYER MODEL
CALL CURAY (JO)
C CURAY IS APPARENTLY DESIGNED TO ALLOW FOR USE OF A SPHERICAL SECTION
C OF THE EARTH AND IS REALLY UNNECESSARY HERE, IT CALCULATES A FEW
C CONSTANTS SUCH AS THE SQUARES OF THE VELOCITIES. THESE CALCULATIONS
C COULD EASILY BE INTEGRATED INTO THE MAIN PROGRAM
CON =1.0/(2.*(3.141592**2)*D(2)*S(2)**2)
C CON IS A FACTOR USED IN PSICO TO CALCULATE THE TERMS OF THE SURFACE
C RESPONSE.
  XX=x
C XX IS THE RANGE
  MTD=2
C MTD IS USED IN CONTOUR TO INCREASE THE INTERVAL SIZE BETWEEN THE LAST
  DELTM=.0025
C DELTM IS THE MINIMUM SIZE P(INTEGRATION VARIABLE) STEP,SEE CONTOUR
C SUBROUTINE
  DMAX=1.E-4
  DELTM=.1
  NDP=30
C NDP IS A PARAMETER USED IN SUBROUTINE DELPS IN CALCULATING INTEGRATION
C INTERVALS USING A SINE RELATIONSHIP
  DET=1.E-5
  DTIM=DP
C IN CALCULATING INTEGRATION INTERVALS ON THE COMPLEX CONTOUR SIDE OF
C REFLECTION POINT, THE CALCULATION IS TERMINATED WHEN IT IS WITHIN
  DEL=DP
C DEL IS USED AS THE DELTA INTERVAL FOR THE TIME SERIES OF SURFACE
C RESPONSE
  DLTP=DP
  NYT=3
  VRL2=.92*S(2)
  VRL3=.92*S(3)
  NPPAY=40
  K=NN/2
C NCASE REFERS TO THE SOURCE AND MODEL COMBINATION, NCASE=0 DESIGNATES
C SH SOURCE AND THREE LAYER MODEL AS HERE, =1 P, SV, SOURCE AND SAME MODEL
C =2 IS APPARENTLY USED FOR SOLID HALF SPACE RUNS, NOT RELEVANT HERE
C SETUP EXECUTES THE PROGRAM AND GENERATES OUTPUT
  CALL SETUP(1,0,01,36,C,MPLUT,1)
  END
ADJT
SUBROUTINE ACJUST(NN,NFIX)
IMPLICIT REAL*8 (A-H,O-Z)
COMMON/EXACT/PHIZ(1000),PHIR(1000),TD(1000),NEND,NM
COMMON/THZ/ T(1000),PPZ(1000),PPR(1000)
M = NN+1
TR = TC(M)
I = 0
80 I = I+1
IF(I.GT.NN) GO TO 70
IF(T(I).GT.TR) GO TO 81
GO TO 80
81 DNE = TR-T(I-1)
DPL = T(I)-TR
IF(DABS(DNE).GT.DABS(DPL)) GOT O 83
DELTA = -DNE
NFIX = I-1
GO TO 85
83 DELTA = DPL
NFI = I
85 DO 84 J=1,NEND
TD(J) = TD(J)+DELTA
84 CONTINUE
RETURN
70 NFIX = 0
RETURN
END
SUBROUTINE CONTOR(TM, M, KN, N, MO)

**********************************************************************

C THIS SUBROUTINE SPECIFIES THE COMPLEX CONTOUR OF INTEGRATION FOR THE
C REFLECTED ARRIVAL.
C THE SUBPROGRAMS TIME2 AND DELPS ARE CALLED

C CALL LIST: TMX IS THE LENGTH OF RECORD (IN SEC) TO BE CALCULATED AFTER
C THE REFLECTED ARRIVAL, M HAS NO APPARENT FUNCTION, NCR DOES KN, N IS THE
C RAY NUMBER, AND MO THE NUMBER OF POINTS FIRST FOLLOWING THE REFLECTION.
C ARRIVAL.

C**********************************************************************

IMPLICIT REAL*8 (A-H, J-Z)

COMMON/MAGIC/PP(1200), DDPT(1200), TT(1200)
COMMON/SPE/DEL(800), DD1, DD2, DD3, DD4, NU
COMMON/PATHC/PD, TD, K
COMMON/TINP/DELTM, DELT, NDA, MTO, DLTP, NDB, JO, NDRT

COMMON/TFIX/TN1, TN2, TN3, TN4, JN1, JN2, JN3, JN4

COMMON/TRSF/NCASE, NPRAY, NYT, NDM, VRL2, VRL3

DIMENSION DER(400)

LOGICAL PRNT, PRNTS, PRNTC

COMMON/LPRINT/PRNTS, PRNTS, KST, KEND, PRNTC, NDC, DET

COMMON/MYER/DTIM

COMPLEX*16 PP, DDPT, P, CTDEV

DET = 1.D-5

JN = 10

Q = PC

I = MO-1

PIL = 1.E-6

PP(I) = PC

TT(I) = TO

KM = 100

IF(DELPT(1) .LE. DELTM) DELPT(1) = DELTM

L = KM-1

L = 0

DO 10 J=1,JN

L = L+1
I = I+1

Q = Q*DELP(J)
PI = PI*0.2
DL = PI*0.45
CALL TIME2(0,PI,DL,P,DEV,CT,KN,N,PIL)
RTIME = CT
TIMEI = CT*(0.,-1.)
CR = DEV
DDPT(I) = DEV
TT(I) = RTIME
PP(I) = P

IF(TTI(I)-TO.LE.DTIM) GO TO 32
IF(DR.LE.0.) GU TO 30
IF(DABS(TIMEI).GT.1.E-3) GO TO 30
IF(RTIME.TO.LT.0.01) GO TO 30
JJ = J+1
DELP(JJ) = (((PO-PP(I))/2.)

CONTINUE

I = I-1
DO 31 J=1,JJ
LL = MO+J-1
NN = I-J+1
TT(LL) = TT(NN)
PP(LL) = PP(NN)
DDPT(LL) = DDPT(NN)

CONTINUE

I = LL
DELP(JN) = (PP(LL)-PP(LL-1))
J = JN
MM = 1
TM = TMX+TO
PI = PP(LL)*(0.,-1.)  
0 = PP(LL)  
JF = LL  
DEP(JF) = PP(LL) - PP(LL-1)  
IF(NCASE.EQ.0) GO TO 12  
IF(NCASE.EQ.1) RG=DABS(1./VRL2-Q)  
IF(NCASE.EQ.2) RG=DABS(1./VRL3-Q)  
CALL DELPS(NPRAY,RG,1,NYT)  
J1 = LL  
J2 = J1+NO  
IJ = 0  
DO 21 J=J1,J2  
IJ = IJ+1  
21 DER(J) = DELP(IJ)  
JF = J2  
IF(NCASE.EQ.1) GO TO 23  
PG =DABS(1./VRL2-1./VRL3)  
CALL DELPS(NPRAY,RG,1,NYT)  
J1 = J2+1  
J2 = J1+NO  
IJ = 0  
DO 24 J=J1,J2  
IJ = IJ+1  
24 DER(J) = DELP(IJ)  
JF = J2  
IF(PRNTC) WRITE(6,1CO) (DER(J),J=J1,J2)  
100 FORMAT(6E12.4)  
DO 25 J=LL,JF  
Q = Q*DER(J)  
I = I+1  
DELPR = (PP(I-1)-PP(I-2))  
DELP = (PP(I-1)-PP(I-2)) *(0.,-1.)  
PI = (DELP*DER(J)) /DELPR +PI  
DL = PI*.5  
CALL TIME2(Q,PI,DL,P,DEV,CT,KN,N,PIL)  
PP(I) = P
RTIME = CT
DDPT(I) = DEV
TT(I) = RTIME
IF(TT(I) GT TM) GO TO C 13

25 CONTINUE
12 Q = Q*MM*DER(JF)
I = I+1
DELP R = (PP(I-1)-PP(I-2))
DELP I = (PP(I-1)-PP(I-2)) *(O.,-1.)
PI = (DELP I*MM*DER(JF))/DELP R +PI
DL = PI*.5
CALL TIME2(Q,PI,DL,P,CEV,CT,KN,N,PIL)
PP(I) = P
RTIME = CT
DDPT(I) = DEV
TT(I) = RTIME
IF(TT(I) GT TM) GO TO 13

14 IF(TT(I)-TT(I-1).LE.CLTM) MM=MTD*MM
GO TO 12

13 CONTINUE
M = I
RETURN
END
COMPLEX FUNCTION CR*16 (P,C)
IMPLICIT REAL*8 (A-H,O-Z)
COMPLEX*16 P,CZ
CZ=1.0/C**2-P*P
U=CZ
X=CZ*(0.,-1.)
R=DSQRT(X*X+U*U)
W1=DABS(R+U)/2.
W2=DABS(R-U)/2.
R1=DSQRT(W1)
R2=DSQRT(W2)
CR=R1-R2*(0.,1.)
RETURN
END
SUBROUTINE CUPAY(JO)
IMPLICIT REAL*8 (A-H,O-Z)

COMMON /SENSE/ DRCSQ(100), DRSSQ(100)
COMMON/STUFF/C(100), S(100), D(100), TH(100), X, RCSQ(100), RSSQ(100)
COMMON/ORSTF/CC(100), SS(100), DD(100), TTH(100), XX
COMMON/FIXP/CDN(100), ARN(100), FLAT

INTEGER DEPTH(JO)

DIMENSION DEPTH(100)
DIMENSION DT(100)

PRINT 2, X

2 FORMAT (1H1,10X,'CURAY',/11X,'RANGE',F10.0/16X,'THICKNESS',9X,'DEPTH',5X,'P-VELOCITY',5X,'S-VELOCITY',8X,'DENSITY')

DEPTH(1) = TTH(1)/2.0

DO 10 J = 2, JO

10 DEPTH(J) = DEPTH(J-1)+(TTH(J)+TTH(J-1))/2.

DO 5 J = 1, JC

Q = 6371.0 / (6371.0-DEPTH(J))

IF(FLAT) Q = 1.

ARN(J) = 1./Q

C(J) = CC(J)*Q

S(J) = SS(J)*Q

D(J) = DD(J)*Q

TH(J) = TTH(J)*Q

DRCSQ(J) = 1.0 / C(J) **2

DRSSQ(J) = 1.0 / S(J)**2

RCSQ(J) = DRCSQ(J)

RSSQ(J) = DRSSQ(J)

CONTINUE

DT(1) = TTH(1)

DO 20 J=2,JO

20 CT(J) = DT(J-1)+TTH(J)

CONTINUE

DO 25 J=1,JC

IF(FLAT) DT(J) = 0.0

DDN(J) = (6371.0- DT(J))/6371.
CONTINUE
PRINT 1, (J, TH(J), DEPTH(J), C(J), S(J), D(J), ARN(J), DDN(J), J=1,JO)
1 FORMAT (15,5X,7G15.4)
RETURN
END
SUBROUTINE DELPS (NNN, RG, NN, N)
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION PP(200)
COMMON/SPEC/DELP(800), CD1, DD2, DD3, DD4, NO
PI = 3.141593
AN = PI/(NNN*2.)
J = NN
A = AN
DELP(J) = RG*(DSIN(A)**N)
TU = DELP(J)
A = A+AN
K = 1
PP(1) = DELP(1)
1 J = J+1
K = K+1
PP(K) = RG*DSIN(A)**N
DELP(J) = PP(K)-PP(K-1)
DELP(J) = DABS (DELP(J))
TU = TU+DELP(J)
A = A+AN
IF (TU.LT.RG) GO TO 1
2 NO = J-1
RETURN
END
SUBROUTINE FIND2 (Q, K, DEL, DET, PQ, TQ, NRY)

IMPLICIT REAL*8 (A-H, O-Z)

DIMENSION E(100)
COMMON/RAYPAR/KUD(100), KRSP(100), LTS(100), LTP(100), LREF(100, 4)
COMMON/PLACE/THIC, H, KSSP
COMMON/STUFF/C(100), S(100), T(100), TH(100), X
COMMON /SENSE/ RCSQ(100), RSSQ(100)
COMMON /PLACE/THIC, H, KSSP

COMMTI/PRI/PRT, PRI/PRTS

LOGICAL PRNT, PRNTS
TH(1) = (1. - KUD(NRY)) * (THIC - H) + KUD(NRY) * H
TH(3) = 0.
TH(4) = 0.
IF (KSSP * EQ. 1) TH(4) = TH(1)
IF (KSSP * EQ. 0) TH(3) = TH(1)
KF = 1
KOUNT = 0
TDE = DEL
8 P = Q
KOUNT = KCUNT + 1
5 P = P * DEL
PSQ = P ** 2
E(2) = DSQRT(DABS(RCSQ(2) - PSQ))
E(3) = DSQRT(DABS(RSSQ(2) - PSQ))
BLTEM = -TH(2) * LTP(NRY) / E(2) - TH(2) * LTS(NRY) / E(3)
BLTEM = BLTEM - TH(4) / E(2)
BLTEM = BLTEM - TH(3) / E(3)
BL = X + BLTEM * P
IF (DABS (DEL) * LE. 1. * E-18) GO TO 1
6 IF (DABS(BL) * LE. X / DET) GO TO 1
2 IF (BL) = DABS (DEL * 5)
GO TO 5
3 DEL = DABS (DEL * 5)
GO TO 5
1 IF (DABS(BL) * LT. 1. * E-8) GO TO 7
IF (KOUNT * GE. 5) GO TO 7

00005240 FND200001
00005250 FND200002
00005250 FND200003
00005270 FND200004
00005280 FND200005
00005290 FND200006
00005300 FND200007
00005310 FND200008
00005320 FND200009
00005330 FND200010
00005340 FND200011
00005350 FND200012
00005360 FND200013
00005370 FND200014
00005380 FND200015
00005390 FND200016
00005400 FND200017
00005410 FND200018
00005420 FND200019
00005430 FND200020
00005440 FND200021
00005450 FND200022
00005460 FND200023
00005470 FND200024
00005480 FND200025
00005490 FND200026
00005500 FND200027
00005510 FND200028
00005520 FND200029
00005530 FND200030
00005540 FND200031
00005550 FND200032
00005560 FND200033
00005570 FND200034
00005580 FND200035
00005590 FND200036
G = Q/10.0
DEL = TDE
G = T0 = B
P0 = P

TOTEM = F(2)*TH(4)+E(2)*LTP(NRY)*TH(2)+E(3)*LTS(NRY)*TH(2)
        + E(3)*TH(3)

T0 = P*X + TOTEM
P0 = P0
T0 = T0
IF (DAHS(BL)*LT*1.0E-6) RETURN
IF (.NOT.PRNT) RETURN
WRITE (6,17) P0,T0,BL

17 FORMAT (1HO,4X,'P0 ',E18.6,10X,'TO ',E18.6,10X,'BL ',E18.6)
RETURN
END
SUBROUTINE HELP(K,N,P,TTP,DTP,NRY)
IMPLICIT REAL*8 (A-H,O-Z)
COMMON/STUFF/C(100),S(100),TH(100),X,RCSQ(100),RSSQ(100)
COMMON/RAYPAP/KUD(100),KSP(100),LTS(100),LTP(100),LREF(100,4)
COMMON/LPRINT/PRNT,PRNTS,KST,KEND,PRNTC,NDC,DET
LOGICAL PRNT,PRNTS,PRNTC,FLAT

FORMAT(10X,'SUB. HELP,P,E,TOTEM,BTEM,DTP,TTP',/6(E16.6))
PSQ = P**2
J = 2
E = DSQRT(DABS(RCSQ(J)-PSQ))
TOTEM = E*(TH(4)+TH(2)*LTP(NRY))
BLTEM = -(TH(4)+TH(2)*LTP(NRY))/E
IF(PRNTC) WRITE(6,100) P,E,TOTEM,BTEM,DTP,TTP
E = DSQRT(DABS(RSSQ(J)-PSQ))
TOTEM = TOTEM+TH(2)*LTS(NRY)*E +TH(3)*E
BLTEM = BLTEM-TH(2)*LTS(NRY)/E +TH(3)/E
BL = X + P*BLTEM
TO = P*X + TOTEM
DTP = 1./BL
TTP = TO
IF(PRNTC) WRITE(6,100) P,E,TOTEM,BTEM,DTP,TTP
RETURN
END
SUBROUTINE HIGH(NDPTMX,K,KI,N)
C******************************************************************************
C THE SUBPROGRAM HIGH CALCULATES THE SURFACE RESPONSE FOR EACH RAY
C NDPTMX IS USED IN CALCULATING THE INTEGRATION STEPS ABOUT REFLECTION,
C TMX IS THE LENGTH OF THE RECORD READ IN MAIN,K AND KI DO NOT SEEM TO
C BE USED, N IS THE RAY NUMBER
C THE SUBPROGRAM FIND2,HELP,DELPS,PLN1,CONTOR, AND PLN2 ARE CALLED
C THE SURFACE RESPONSE IS PRINTED FOR EVERY FIFTH RAY
C******************************************************************************
IMPLICIT REAL*8 (A-H,O-Z)
COMMON/RAYPAR/KUD(100),KRSP(100),LTS(100),LTP(100),LREF(100,4)
COMMON/PLACE/THIC,H,KSSP
COMMON/EXACT/PHIZ(1000),PHIR(1000),TD(1000),NEND,NM
COMMON/MAGIC/PP(1200),DDPT(1200),TT(1200)
COMM CN/SPE/DEL P(800),CD1,DD2,DD3,DD4,NO
COMMON/PATHC/PO,TOKK
COMMON/T INP/DEL TM,DL TM,ND A,MTD,DLTP,NDB,JO,ND R T
COMMON/ORSTF/CC(100),SS(100),DD(100),TTH(100),XX
COMMON/LPRINT/PRNT,PRNTS
LOGICAL PRNT,PRNTS
COMPLEX*16 PP,DDPT
JN1 = 12
JN2 = 10
JN3 = 8
JN4 = 100
TN1 = .8
TN2 = .2
TN3 = .1
TN4 = .001
C THESE EIGHT PARAMETERS ARE USED IN CALCULATING THE INTEGRATION
C INTERVAL ABOUT PU, THE POINT OF REFLECTION
V2 = SS(3)
C V2 SHOULD BE THE LARGEST PROPAGATION VELOCITY USED IN THE MODEL,
C FOR P SV MODEL, CC(3)
NRY = N
C NRY BECOMES THE RAY NUMBER

DEL IS THE INITIAL ITERATION STEP IN THE INTEGRATION PARAMETER, FOR
USE IN FIND2 IN DETERMINING THE VALUES OF P AND TIME AT THE REFLECTION
ARRIVAL IF THE SIGNAL IS AT ANY TIME IN P OR COMPRESSIONAL MODE, DEL =
1.0/CC(1), IF NOT DEL = 1.0/SS(2)

KNYR = NRY*KSSP
DEL = 1.0/SS(2)
IF(LTP(NRY).GT.0) GO TO 82
IF(KSSP.EQ.0) GO TO 81
82 DEL = 1.0/CC(2)
81 P = -1.0E-9
DAT = 1.0E+12
CALL FIND2(P, KK, DEL, CAT, PO, T0, N)
NNN = NDP
NK = 2
IF(NRY.EQ.1) V2 = 0.9*SS(2)
P = 1.0/V2
RG = DABS(PO-P)
CALL HELP(K, N, P, TTP, DTP, N)
TC = TTP
TG = TC-TTP
IF(PO.LE.1.0/V2) GO TO 6
IF(TG.GT.TN1) GO TO 6
JN = JN1
IF(TG.GT.TN2) GO TO 18
JN = JN2
IF(TG.GT.TN3) GO TO 18
JN = JN3
QZ = RG/(JN+1)
DO 15 J = 1, JN
DELP(J) = QZ
15 CONTINUE
NO = JN
IF(TG.LT.TN4) GO TO 2
GO TO 19
CALL DELPS(NNN, RG, 1, NK)
IF (.NOT. PRNT) GO TO 19
PRINT 7, V2, XM, PO, RG, TC, TJ, (DELP(J), J=1, NO)
7 FORMAT (1HO, 4X, 'V2=', G13.6, 5X, 'XM=', G13.6, 5X, 'PO=', G13.6/5X, 'RG=', G13.6, 5X, 'TC=', G13.6, 5X, 'TJ=', G13.6/5X, 'DELP(*/)
2(G15.6))
19 IF (P3.LE.1./V2) GO TO 2
CALL PLN1(PG, TG, K, N, TT, N, V2)
2 M0 = NO+2
2 IF (TG.LT.TN4) M0=2
2 IF (PG.LT.1./V2) M0=2
2 CALL CJNTOR(TMX, M, KN, N, MO)
2 IF (.NOT.PRT) GO TO 620
WRITE (6,5)
5 FORMAT (1HO, 13X, 'PP', 27X, 'DDPT', 24X, 'TT')
5 JJ = MC
5 WRITE(6,200) (PP(J), DDPT(J), TT(J), J=JJ, M)
200 FORMAT (5E15.4)
6 CALL PLN2(PG, TG, K, MO, M, N)
20 NEND = M
20 NM = NC
20 IF (PG.LT.1./V2) NM=0
20 IF (TG.LT.TN4) NM= 0
20 LK = -19
20 WRITE (6,98)
20 KJMP=5
20 LK = 1-KJMP
20 LK = LK+KJMP
97 IF (LK .GT. NEND) GC TO 99
97 WRITE (6,100) TD(LK), PHIZ(LK), PHIR(LK), LK
99 GO TO 97
98 CONTINUE
98 FORMAT (12X, 'SUR. HIGH TDPHIZPHI ', 2X)
100 FORMAT (3E18.6,110)
RETURN
END
SUBROUTINE INTEPP(XP, YP, N, X, Y)
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION XP(1000), YP(1000)
1 IF (X .GT. XP(N)) GO TO 6
IF (X .LT. XP(1)) GO TO 6
2 DO 10 I = 1, N
IF (XP(I) - X) 10, 102, 2
10 CONTINUE
3 K = I - 1
DIF1 = XP(I) - XP(K)
DIF2 = XP(I) - X
RATIO = DIF2 / DIF1
DIFY = DABS (YP(I) - YP(K))
DR = DIFY * RATIO
IF (YP(I) .GT. YP(K)) GO TO 4
5 Y = YP(I) + DR
RETURN
4 Y = YP(I) - DR
RETURN
102 Y = YP(I)
RETURN
6 Y = 0.
RETURN
END
SUBROUTINE PLN1(P0,T0,K,N,TC,NRY,V2)
IMPLICIT REAL*8 (A-H,O-Z)
COMMON/T,INP/DLM,T,NDM,MTD,LTP,NDB,JO,NDIRT
COMMON/SPE/DELP(800),DD1,DD2,DD3,DD4,ND
COMMON/DFSTF/C100),S100),D100),TH100),X
COMMON/TSRSL/PZI(500),PRI(500)
COMMON/MAGIC/PQ(1200),DPST(1200),TT(1200)
COMMON/EXACT/PHIZ(1000),PHIR(1000),TT(1000),END,NS
COMMON / LPRINT/ PRNT,PRNTS
LOGICAL PRNT,PRNTS
COMPLEX*16 PQ,DPST,FNZ,FNR,FNZ1,FNR1,Q
P = 1.*V2
DO 80 I=2,NO
J = I-1
P = P+DELP(J)
Q = P+Q*(0.,1.)
CALL HELP(KN,P,TTP,CTP,NRY)
IF(PRNT) PRINT 100,Q,P,DPST,TTP
TT(I) = TTP
DPST(I) = DPST
CALL PSICO(Q,FNZ,FNR,FNZ1,FNR1,I,NRY)
IF(PRNT) PRINT 100,FNZ,FNR,FNZ1,FNR1
PHIZ(I) = FNZ*(0.,-1.)
PHIR(I) = FNR*(0.,-1.)
PZI(I) = FNZ1*(0.,-1.)
PRI(I) = FNR1*(0.,-1.)
IF((TJ-TTP) .LT. DLTP) GO TO 5
8C CONTINUE
100 FORMAT(8E15.4)
WRITE(6,100) P3,TJ
5 NC = J+1
I = NC
4 IF(TJ-TTP,LT,DLTP) GO TO 3
PP = PC - P
P = P + PP/2.*0
I = I+1
NC  =  I
Q   =  P
CALL HELP(K,N,P,TTP,DTP,NRY)
TT(I) = TTP
DTP(I) = DTP
CALL PSIC(JQ,FNZ,FNR,FNZ1,FNR1,I,NRY)
PHIZ(I) = FNZ * (0.,-1.)
PHIP(I) = FNR * (0.,-1.)
PZ1(I) = FNZ1 * (0.,-1.)
PR1(I) = FNR1 * (0.,-1.)
IF(PRNT) PRINT 100,FNZ,FNR,FNZ1,FNR1
GO TO 4
3
TT(I) = TC
PHIZ(I) = 0.
PHIR(I) = 0.
PR1(I) = 0.
PZ1(I) = 0.
RETURN
END
SUBROUTINE PLN2(P,D,T,C,K,M,NRY)
IMPLICIT REAL*8 (A-H,O-Z)
COMMON/FIXP,CDN(100),ARN(100)
COMMON/TINP,DELTM,DLTP,NDP,MTL,DLTP,NDB,JLJ,NDIRT
COMMON/MAGIC/PP(1200),DDPT(1200),TT(1200)
COMMON/EXACT/PHIZ(1000),PHIR(1000),TTT(1000),NEND,NM
COMMON/TRESL/PZ(500),PR(500)
COMMON/URSTF/C(100),S(100),D(100),TH(100),X
DIMENSION FF(50),GG(50)
COMMON / LPRINT/ PRINTS
LOGICAL PRINT,PRINTS
COMPLEX*16 PP,BT,DDPT,RP,RPP,GC,P
COMPLEX*16 FNZ,FNR,FNZ1,FNRI
DO 5 I=MO,M
      TTT(I) = TT(I)
      P = PP(I)
      CALL PSICO(P,FNZ,FNR,FNZ1,FNRI,I,NRY)
      PHIZ(I) = FNZ *(0.,-1.)
      PHIR(I) = FNR *(0.,-1.)
      PR(I) = FNRI *(0.,-1.)
      PZ(I) = FNRI *(0.,-1.)
      IF(PRINT) PRINT 100,FNZ,FNR,FNZ1,FNRI
      CONTINUE
5 CONTINUE
100 FORMAT (8E15.4)
3  ND = MD-2
   DP = DLTP
   P = P0*(1.,0.)*0. *(0.,1.)
   I = MD-1
   Q = P0
   DDPT(I) = SF2(Q,K,NRY,DP)
   TTT(I) = TO
   WRITE(6,100) P,DDPT(I)
   CALL PSICO(P,FNZ,FNR,FNZ1,FNRI,I,NRY)
   IF(PRINT) PRINT 100,FNZ,FNR,FNZ1,FNRI
   PHIZ = FNZ
   PIMZ = FNZ *(0.,-1.)
PRER = FMR
PIMR = FNR * (0.,-1.)
IF (PRNT) PRINT 100,PREZ,PIMZ,PRER,PIMR
F1 = 0.
G1 = 0.
SUM = 0.
TUM = 0.
IF(MJ.LE.3) GO TO 46
TNN = TO-DO
CALL INTERP(TTT,PHIZ,M,TNN,Y)
F1 = Y
FF(1) = Y
CALL INTERP(TTT,PHIR,M,TNN,Y)
G1 = Y
GG(1) = Y
SUM = SUM+2.*PIMZ*DSQRT(TTO)
TUM = TUM+2.*PIMR*DSQRT(TTO)
IF (PRNT) PRINT 4, SUM
IF (PRNT) PRINT 7, TUM
DO 41 J=2,6
TT(J) = TTT(J-1) +DELL
CALL INTERP(TTT,PHIZ,M,TT(J),Y)
FF(J) = Y
CALL INTERP(TTT,PHIR,M,TT(J),Y)
GG(J) = Y
TUM = TUM+(GG(J-1)+GG(J))/2.*DELL
SUM=SUM+(FF(J-1)+FF(J))/2.*DELL
IF (PRNT) PRINT 7, TUM
IF (PRNT) PRINT4, SUM
CONTINUE
46 TTPP = TTT(MO)
TT(1) = TTT(MO)
IF (TTT(MO)-TO,GT.*DP) GO TO 43
SUM = SUM+2.*PREZ*DSQRT(TTT(MO)-TO)
TUM = TUM+2.*PRER*DSQRT(TTT(MO)-TO)
IF (PRNT) PRINT4, SUM
IF (PRNT) PRINT 7, TUM
CALL INTERP(TTT,PHIZ,*,TPP,Y)
FF(1) = Y
CALL INTERP(TTT,PHIR,*,TPP,Y)
GG(1) = Y
DELL = (TO+DP-TTT(MO))/5.
DO 42 J=2,6
IF (PRNT) PRINT4, SUM
TT(J) = TT(J-1) +DELL
CALL INTERP(TTT,PHIZ,*,TT(J),Y)
FF(J) = Y
CALL INTERP(TTT,PHIR,*,TT(J),Y)
GG(J) = Y
TUM = TUM+(GG(J-1)+GG(J))/2.*DELL
SUM = SUM+(FF(J-1)+FF(J))/2.*DELL
IF (PRNT) PRINT 4, SUM
IF (PRNT) PRINT 7, TUM
CONTINUE
42 F3 = FF(6)
G3 = GG(6)
PHIZ(I) = (3.*SUM/DP-F1-F3)/4.
PHIR(I) = (3.*TUM/DP-G1-G3)/4.
GO TO 44
43 TTT(MJ) = TO+DP
PHIZ(MJ) = PREZ/(DP**.5)
PHIR(MJ) = PRER/(DP**.5)
F3 = PHIZ(MJ)
G3 = PHIR(MJ)
SUM = SUM + 2. * PREZ * DSQRT(DP)
TUM = TUM + 2. * PRER * DSQRT(DP)
IF (PRNT) PRINT 4, SUM
IF (PRNT) PRINT 7, TUM
CONTINUE

PREZ1 = FNZ1
PIMZ1 = FNZ1 * (0., -1.)
PRER1 = FNR1
PIMR1 = FNR1 * (0., -1.)
IF(PRNT) PRINT 100, PREZ1, PIMZ1, PRER1, PIMR1
IF(MO . LE. 3) GO TO 47
DO 81 J = 2, NO
AREA = (PR1(J) + PR1(J-1)) * (TTT(J) - TTT(J-1))
PHIR(J) = PHIR(J) + AREA / 2.
AREA = (PZ1(J) + PZ1(J-1)) * (TTT(J) - TTT(J-1))
PHIZ(J) = PHIZ(J) + AREA / 2.
CONTINUE

PHIZ(I) = PHIZ(I) + 2. * PIMZ1 * DSQRT(TTT(I) - TTT(NO))
PHIR(I) = PHIR(I) + 2. * PIMR1 * DSQRT(TTT(I) - TTT(NO))
CONTINUE

PHIZ(MO) = PHIZ(MO) + 2. * PREZ1 * DSQRT(TTT(MO) - TTT(I))
PHIR(MO) = PHIR(MO) + 2. * PRER1 * DSQRT(TTT(MO) - TTT(I))
JM = MO + 1
DO 82 J = JM, M - 1
AREA = (PR1(J) + PR1(J-1)) * (TTT(J) - TTT(J-1))
PHIR(J) = PHIR(J) + AREA / 2.
AREA = (PZ1(J) + PZ1(J-1)) * (TTT(J) - TTT(J-1))
PHIZ(J) = PHIZ(J) + AREA / 2.
CONTINUE
WRITE (6, 111) TTT(I), PHIZ(I), PHIR(I)
111 FORMAT (2X, 'CRITICAL TIME', G20.8, 5X, 'PHIZ', G20.8, 'PHIR', G20.8)
RETURN
END
SUBROUTINE PSICO (P, FNZ, FNR, FNZ1, FNR1, I, NRY)

*C*****************************************************************************
C PSICO CALCULATES THE TWO TERMS IN BARKER'S EXPANSION OF THE SOLUTION
C THE SUBPROGRAM REFFT, RECVR ARE CALLED
C VERTICAL RESPONSE, FNR, SIMILAR FOR THE RADIAL, FNZ1, SECOND TERM IN
C VERTICAL RESPONSE, FNR1, SAME FOR RADIAL; I INDICATES WHICH VALUE OF P
C ALONG THE CONTOUR (STORAGE ARRAY INDEX ), NRY IS THE RAY NUMBER
C*****************************************************************************

IMPLICIT REAL*8 (A-H,J-Z)

COMMON/RAYPAR/KUD(100), KRSP(100), LTS(100), LTP(100), LREF(100,4)
COMMON/ORSTF/C(100), S(100), T(100), TH(100), X
COMMON/MAGIC/PP(1200), DDPT(1200), TT(1200)
COMMON/PLOTC/CON, NNF, NPT
COMMON/PLACE/THICH, H, KSSP
COMMON/LPRINT/PRNT, PRNTS
LOGICAL PRNT, PRNTS
DIMENSION RF(16)

COMPLEX*16 P, RF, DR, G1, G2, G3, FNZ, FNR, FNZ1, FNR1, RC, DDPT, PP, RDS, DZ
ROD = D(2)/D(3)

CALL REFFT(P, S(2), S(3), ROD, RDS)
RF(2) = RDS
CALL RECVR(P, S(2), KRSP(NRY), DZ, DR, KSSP, RSS)
RF(1) = RSS
RC = (1., 0., 1)
CJ 100 J = 1, 2
IF (LREF(NRY, J) .EQ. 0) GOTO 200
RC = RC*RF(J)**LREF(NRY, J)

200 CONTINUE
CJ 100 CONTINUE
IF (KSSP .EQ. 1) GOTO 3
IF (NRY .EQ. 1) KRSP(NRY) = 0
3 CONTINUE
G1 = DDPT(I)*RC*CUN
G2 = G1*CDSQRT(P/(2.*X))
FNZ = DZ*G2
FNR = DR*G2
G3 = G1/(-8.*CDSQRT(2.*P*X**3))
FNZ1 = G3*DZ
FNR1 = G3*DR*(-3.)
IF (PRNT) WRITE (6,30C) P,C(3),C(2),S(3),S(2),KUD(NRY),KRSP(NRY), 00009620 PSIC0037
1 LTS(NRY),LTP(NRY) 00009630 PSIC0038
IF (PRNT) WRITE(6,400) RF(J),LREF(NRY,J),J=1,4) 00009640 PSIC0039
400 FORMAT (1X,'RECVR',4(2(E13.4),12)) 00009650 PSIC0040
IF (PRNT) WRITE(6,500) G1,G2,G3,RC 00009660 PSIC0041
300 FORMAT (1X,'RECVR',2(E15.4),4(F10.3),2X,4(12)) 00009670 PSIC0042
500 FORMAT (1X,'RECVR',8(E13.4)) 00009680 PSIC0043
RETURN 00009690 PSIC0044
END 00009700 PSIC0045
REAL FUNCTION PTIM*(P,K,NRY)

IMPLICIT REAL*(A-H,O-Z)

COMMON/PAYPAP,KUD(100),KRSR(100),LTP(100),LTS(100),LTP(100),LRSQ(100)
COMMON/STUFF/C(100),S(100),D(100),TH(100),XRCSQ(100),RSSQ(100)

PSQ = P**2

PTIM = PSQ

RETURN

END

J = DSQRT(DABS(RCSQ(J)-PSQ))

= PTIM

E = DSQRT(DABS(RSSQ(J)-PSQ))

= PTIM + P**2*PTIM

RETURN
SUBROUTINE PAYDEF
COMMON/RAYPAR/KUD(100),KRSP(100),LTS(100),LTP(100),LREF(100,4)
COMMON/PLACE/THIC,H,KSSP
COMMON/LPRINT/PRNT,PRNTS
LOGICAL PRNT,PRNTS
REAL*3 THIC,H
WRITE (6,800)
800 FORMAT (10X,'WHOOPIE, WE MADE IT TO RAYDEF')
READ (5,400) THIC,H,KSSP
400 FORMAT (2F10.0,Il0)
WRITE (6,700) THIC,H
700 FORMAT (/10X,'LAYER THICKNESS ',F6.3/10X,'SOURCE DEPTH',F6.3)
IF (KSSP .EQ. 0) WRITE (6,1300)
1300 FORMAT (15X,'SHEAR SOURCE')
IF (KSSP .EQ. 1) WRITE (6,1400)
1400 FORMAT (15X,'COMPRESSIONAL SOURCE')
READ(5,1200) (KUD(J),LTS(J),LTP(J),KRSP(J),J=19,64)
1200 FORMAT (4I2)
KUD(1)=1
KUD(2)=0
KUD(3)=1
KUD(4)=0
KUD(5)=1
KUD(6)=0
KUD(7)=1
KUD(8)=0
KUD(9)=1
KUD(10)=0
KUD(11)=1
KUD(12)=0
KUD(13)=1
KUD(14)=0
KUD(15)=1
KUD(16)=0
KUD(17)=1
KUD(18)=0
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<th>LTS(10)</th>
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<td>00012810</td>
<td>00012820</td>
<td>00012830</td>
<td>00012840</td>
</tr>
</tbody>
</table>
KRSP(1) = 0
KRSP(2) = 0
KRSP(3) = 0
KRSP(4) = 0
KRSP(5) = 0
KRSP(6) = 0
KRSP(7) = 0
KRSP(8) = 0
KRSP(9) = 0
KRSP(10) = 0
KRSP(11) = 0
KRSP(12) = 0
KRSP(13) = 0
KRSP(14) = 0
KRSP(15) = 0
KRSP(16) = 0
KRSP(17) = 0
KRSP(18) = 0
KR(18) = 1
DO 200 NR = 1, KR
DO 100 K = 1, 2
100 LREF(NR, K) = 0
200 CONTINUE
SH SOURCE
LREF(2, 2) = 1
LREF(3, 1) = 1
LREF(3, 2) = 1
LREF(4, 1) = 1
LREF(4, 2) = 2
LREF(5, 1) = 2
LREF(5, 2) = 2
LREF(6, 1) = 2
LREF(6, 2) = 3
LREF(7, 1) = 3
LREF(7, 2) = 3
LREF(8, 1) = 3
LREF(3,2)=4
LREF(7,1)=4
LREF(9,2)=4
LREF(10,1)=4
LREF(10,2)=5
LREF(11,1)=5
LREF(11,2)=5
LREF(12,1)=5
LREF(12,2)=6
LREF(13,1)=6
LREF(13,2)=6
LREF(14,1)=7
LREF(14,2)=6
LREF(15,1)=7
LREF(15,2)=7
LREF(16,1)=7
LREF(16,2)=8
LREF(17,1)=8
LREF(17,2)=8
LREF(18,1)=8
LREF(18,2)=9
READ(5,1000) IB,IE
1000 FORMAT(215)
READ(5,1100) ((LREF(II, JJ), JJ=1,2), II=IB,IE)
1100 FORMAT(212)
IF(PRNT) WRITE(6,600) ((LREF(J, L), L=1,2), J=1,6)
IF(PRNT) WRITE(6,600) ((LREF(J, L), L=1,2), J=7,12)
600 FORMAT(* R2, *6(212,2X))
500 FORMAT(* RAYDEF, *4(1212,2X))
RETURN
END
SUBROUTINE FECVR(B1,P,KR,DZ,DR,KSSP,RSS)
C    TO COMPUTE SURFACE RECEIVER AND REFLECTION FUNCTIONS
C
IMPLICIT COMPLEX*16 (A-H,O-Z)
REAL*8 B1,RSS
DZ=-2./CDSQRT(1/B1**2-P*P)
DR = DZ
RSS=1.
RETURN
END
SUBROUTINE REFFT(P,B1,B2,D,DRDS)

C************************************************************************************C
C REFFT CALCULATES THE REFLECTION FUNCTION FOR RAY INTERACTION WITH AN
C SOLID INTERFACE. NOTE THAT THIS SUBROUTINE IS WRITTEN ESPECIALLY FOR
C A SH SOURCE
C
C THE PROGRAM CR IS CALLED. CR CALCULATES THE RADIAL ETA=SQRT(1/
C VELOCITY)**2 - P**2)
C
C CALL LIST: P, THE INTEGRATION PARAMETER, B1 IS VELOCITY IN THE LAYER, B2
C IS VELOCITY IN THE HALF-SPACE, D RATIO OF THEIR DENSITIES, RDR, THE
C REFLECTION FUNCTION FOR THE LOWER(SOLID-SOLID) INTERFACE

C
C IF THE SEVERAL KINDS OF SOURCES ARE INTEGRATED IN ONE PROGRAM THIS
C SUBROUTINE MUST BE GIVEN A DISTINCT NAME
C************************************************************************************C

IMPLICIT COMPLEX*16 (A-H,O-Z)
REAL*8 B1,B2,D,DS
EA = CR(P,B1)
EB = CR(P,B2)
CS = D*B1**2/B2**2
EC = DS*EA
RDS = (EC-EB)/(EC+EB)
RETURN
END
SUBROUTINE SETUP(K,MM,NS,NJ,MO,MPLOT,MPUNCH)

C ***********w*********************************
C THE PROGRAM IS EXECUTED THROUGH THIS SUBROUTINE, WHICH CALLS THE
C FOLLOWING SUBPROGRAMS DIRECTLY; RAYDEF, TS, HIGH, ADJUST, INTERP, PRINTED
C AND PUNCHED OUTPUT OF THE TIME SERIES OF SURFACE RESPONSE ARE PRODUCED
C THE VARIABLE LIST: K IS ALWAYS 1, MM IS APPARENTLY NO LONGER USED
C NS DESIGNATES THE FIRST RAY, NJ, THE LAST RAY; MO=0 INDICATES THAT THIS
C IS THE FIRST TIME SETUP CALLED; IF NOT THE FIRST TIME USE ANY OTHER
C NUMBER .GT.1, MPLOT IS NOT USED; MPUNCH .LT.1 PRODUCES PUNCHED OUTPUT

IMPLICIT REAL*8 (A-H,O-Z)
COMMON/CONFIX/DEL,NN,NDP,THM,XDI,YDIM,DP,KO
COMMON/FOURCT/MF,NMF,KMF,KNMF
COMMON/THZ/TT(1000),PPZ(1000),PPR(1000)
COMMON/EXACT/PHIZ(1000),PHIR(1000),TD(1000),NEND,NM
COMMON/PLACE/THIC,H,KSSP
COMMON/LPRINT/PRNT,PRNTS,KST,KEND,PRNTC
DIMENSION PPC(800)
REAL*4 ZZX(890),ZZY(890),ZXDIM,ZYDIM
LOGICAL PRNT,PRNTS,PRNTC
IF(MO.EQ.0) CALL RAYDEF
C IF MO.EQ.0 CALL RAYDEF TO ESTABLISH RAY PARAMETERS (SEE PREVIOUS
C COMMENTS ON READ IN OF RAY PARAMETERS)
NF = 1
IF(MO.GT.1) GO TO 11
C IF MO.NE.0 DO NOT INITIALIZED THE ARRAYS WHICH CONTAIN THE OUTPUT
I = K
TT(1) = TS(I)
C TS IS A SUBPROGRAM WHICH ESTABLISHES THE TIME OF FIRST SIGNAL ARRIVAL
PPZ(1) = 0.
PPR(1) = 0.
C PPZ AND PPR WILL CONTAIN THE VERTICAL AND HORIZONTAL SURFACE RESPONSE
C RESPECTIVELY
C TT IS AN ARRAY OF SAMPLING TIMES FOR THE TIME SERIES
DJ 10 J=2,NN


```plaintext
TT(J) = TT(J-1) + DEL
C NOTE THAT DEL = DP SET IN MAIN
PPZ(J) = 0.
PPR(J) = 0.
10 CONTINUE
11 CONTINUE
C ITERATE OVER THE RAYS FROM NS TO NO
DJ 32 N=NS,NO
WRITE (6,102) N
102 FORMAT (5X,'RAY NUMBER',13)
CALL HIGH(NDP,TMX,K,KI,N)
C THE SUBPROGRAMME HIGH CALCULATES THE SURFACE RESPONSE FOR EACH RAY
CALL ADJUST(NN,NFIX)
N2 = 1
M = NM+1
IF(NFIX.LT.1) GO TO 37
N2 = NFIX+1
N1 = NFIX-1
IF(N1.LE.2) GO TO 42
DO 31 J=1,N1
   CALL INTERP(TDPHI,NENDTT(J),Y)
   PPZ(J) = PPZ(J)+Y*NF
   CALL INTERP(TDPHI,NENDTT(J),Y)
   PPR(J) = PPR(J)+Y*NF
31 CONTINUE
32 CONTINUE
   PPZ(NFIX) = PPZ(NFIX)+NF*PHIZ(M)
   PPR(NFIX) = PPR(NFIX)+NF*PHIR(M)
37 CONTINUE
   DO 33 J=N2,NN
   CALL INTERP(TD,PHIZ,NENDTT(J),Y)
   PPZ(J) = PPZ(J)+Y*NF
   CALL INTERP(TD,PHIR,NENDTT(J),Y)
   PPR(J) = PPR(J)+Y*NF
33 CONTINUE
L=0
```

!CONTINUE
!IF(.NOT..PRNT) GJ TO 12
PRINT 13,(TD(J),PHIZ(J),PHIR(J),J=1,NEND)
PRINT 14,(TT(J),PPZ(J),PPR(J),J=1,NN)
!FORMAT (1HO,15X,'TD',15X,'PH',/3(E18.6))
!FORMAT (1HO,15X,'TT',15X,'PP',/3(E18.6))
!CONTINUE
!AQ = NN/10
!NP = NQ + 1
!NR = NN + NQ
DO 20 I = 1, NQ
ZZX(I) = TT(1) - (NQ-I+1)*DP
20 ZZY(I) = 0.0
!NDIM=NN
IER=0
!CALL DDGT3(TT,PPZ,PPR,NDIM,IAR)
DO 300 IZ = NP, NR
ZZX(IZ) = TT(IZ-NP+1)
300 ZZY(IZ) = PPC(IZ-NP+1)
WRITE(6,202)(ZZX(IZ),ZZY(IZ),IZ=1,NR)
!FORMAT(' DEBUG 202 ',2E20.10)
!CONTINUE
!IF(MPUNCH.LT.1) GJ TO 2
WRITE(7,200) TT(1),DP,NN
WRITE(7,100) (PPC(J),J=1,NN)
!FORMAT (5E15.6)
!CONTINUE
200 FORMAT (2E15.6,I10)
RETURN
END
REAL FUNCTION SF2*8 (P,K,NRY,DP)
IMPLICIT REAL*8 (A-H,O-Z)
COMMON/STUFF/C(100),S(100),D(100),TH(100),X,RCSQ(100),RSSQ(100)
COMMON/RAYPAR/KUD(100),KRSP(100),LTS(100),LTP(100),LREF(100,4)
PSQ = P ** 2
J = 2
ESQ = DABS (RCSQ(J)-PSQ)
E = SQRT(ESQ)
TE = (TH(4)+TH(2)*LTP(NRY))*RCSQ(2)/(ESQ*E)
ESQ = DABS (RSSQ(J)-PSQ)
E = SQRT(ESQ)
TE = (TH(3)+TH(2)*LTS(NRY))*RSSQ(2)/(ESQ*E) + TE
TE = TE*2.
SR = 1.
SF2 = SR/DSQRT(TE)
RETURN
END
SUBROUTINE TIME2(PR, PI, DL, Q, DPT, T, KN, N, PIL)

C*******************************************************************************
C FOR A PARTICULAR VALUE OF THE COMPLEX INTEGRATION PARAMETER P, TIME2
C FINDS A VALUE FOR THE COMPLEX PARAMETER TIME SUCH THAT THE IMAGINARY
C PART OF TIME IS MINIMIZED
C
C NO SUBPROGRAMS ARE CALLED
C
C CALL LIST: PR, THE SPECIFIED REAL PART OF P; PI, A GUESS AT THE IMAGINARY
C PART; DL, A SPECIFIED FRACTION OF PI USED AS AN ITERATION STEP; Q IS THE
C FINAL VALUE OF THE COMPLEX INTEGRATION PARAMETER P; DPT IS DP/DT FOR
C THIS VALUE; T IS THE COMPLEX INTEGRATION PARAMETER T FOR THIS VALUE OF
C P, KN HAS NO APPARENT USE; N IS THE RAY NUMBER, AND PL IS A MINIMUM VALUE
C FOR PI
C*******************************************************************************

IMPLICIT REAL*8 (A-H,O-Z)
COMMON/RAYPAR/KUD(100), KRSP(100), LTS(100), LTP(100), LREF(100, 4)
COMMON/L PRINT/PRNT, PRNTS, KST, KEND, PRNTC, NDC, DET
COMMON/STUFF/C(100), S(100), D(100), TH(100), R
DIMENSION E(100)
COMPLEX*16 PE, BL, T, PC, DPT, Q
DIMENSION Y1(100), Y4(100), X1(100), X4(100)
LOGICAL PRNT, PRNTS, PRNTC
RNY = N
X1M = 1.0E+4
X4M = 0.0
NNN = 0
I = 0

P = PR*(1.0, 0.0) + PI*(0.0, 1.0)
T = P*R
J = 2
BL = 1.0/(C(J)**2) - P*P
E(J) = SQRT(BL)
T = T*E(J)*(TH(4) + TH(2)*LTP(NRY))
J = 3
BL = 1./(S(2)**2)-P*P
E(J) = CDSQR(BL)
T = T*E(J)*(TH(3)+TH(2)*LTS(NRY))
IF(PRNTC) WRITE(6,110) P,E(2),T
IF(PRNTC) WRITE(6,111) E(3),I,NNN
111 FORMAT('OE(3)=',2G18.6,' I=',I5,' NNN=',I5)
TI = T*(0.,-1.)
IF (DABS(TI) .LE. DET) GO T3
IF(I.GT.90) GJ TO 2
1 = I+1
X1(I) = 100.*
X4(I) = 0.*0
IF(TI.GT.0.) Y1(I)=TI
IF(TI.GT.0.) X1(I)=PI
IF(TI.LT.0.) Y4(I)=TI
IF(TI.LT.0.) X4(I)=PI
IF(I.EQ.1) GO TO 43
IF(NNN.GT.1) GO TO 44
IF(TI*TL .LE. 0.) GO TO 44
43 IF(TI.GT.0.) PI=PI-DL
IF(TI.LE.0.) PI= PI+DL
IF(P1.LE.1.E-5) PI=PK/2.*
NNN = 1
PK = PI
TL = TI
GO TO 6
44 DO 52 J=1,I
IF(X1(J).GT.X1M) GO TC 53
X1M = X1(J)
NJ = J
53 IF(X4(J).LE.X4M) GO TC 54
X4M = X4(J)
NJ = J
54 CONTINUE
52 CONTINUE
Y1M = Y1(NJ)
Y4M = Y4(MJ)
DPI = (X1M-X4M)/(Y1M-Y4M)
DPM = Y1M*DPI
PI = X1M-DPM
NNN = 2
IF(PI.LE.PIL) PI=PIL
GO TO 6
2 CONTINUE

BL = 0.
BL = BL-(TH(4)+TH(2)*LTP(NRY))/E(2)
BL = BL-TH(2)*LTS(NRY)/E(3)-TH(3)/E(3)
BL = R+P*BL
Q = P
DPT = 1./BL
IF(PRNTC) WRITE(6,110) P,E(2),T,DPT
110 FORMAT(1HO,4X,'P=',2G15.6/5X,'E(1) =',2G17.6/5X,'T = ',2G18.6/
1 5X,'DPT = ',2G18.6)
RETURN
END
REAL FUNCTION TS*8 (K)
IMPLICIT REAL*8 (A-H, O-Z)
COMMON/PLACE/THIC, H, KSSP
COMMON/RAYPAR/KUD(100), KRSP(100), LTS(100), LTP(100), LREF(100, 4)
COMMON/IRSTF/CC(100), SS(100), DD(100), TTH(100), XX
COMMON/STUFF/C(100), S(100), D(100), TH(100), X, RCSQ(100), RSSQ(100)
DIMENSION T(200)
COMMON/LPRINT/PRNT, PRNTS, KST, KEND, PRNTC, NDC, DET
LOGICAL PRNT, PRNTS, PRNTC
DO 10 N=1,2
DEL = 1./SS(2)
IF (KSSP .EQ. 0) GO TO 81
IF (LTS(N) .GT. 0) GO TO 81
DEL = 1./CC(2)
81
P  = -1.E-9
DAT = 1.E+12
CALL FIND2(P, M, DEL, DAT, PO, T), N
P = 1./SS(N+1)
TTP = TO
IF (PO .LE. P) GO TO 6
CALL HELP(K, N, P, TTP, DTP, N)
6
T(N) = DMIN1(T0, TTP)
10 CONTINUE
TS = DMIN1(T(1), T(2))
N = 2
IF (PRNT) WRITE (6,1) (T(J), J=1,N)
1 FORMAT (5X, 'T ', 1, 2, 3', 3(1E18.6))
100 FORMAT(I10, 2E18.6)
RETURN
END
C ***THE PROGRAM GENERATES CALCOMP PLOTS OF THE RESPONSE OF AN ELASTIC LAYER OVER AN ELASTIC HALF-SPACE AS CONVOLVED WITH A SPECIFIED SOURCE FUNCTION.
C
C ***THREE PLOTS ARE PRODUCED: THE UNCONVOLVED OUTPUT, THE CONVOLVED OUTPUT, AND THE SOURCE FUNCTION. ALL PLOTS ARE IN SAME SCALE IN TIME.
C
C ***THE SUBROUTINE CONVLV IS USED.
C
C ***THE SOURCE FUNCTION, SRC, A LINEAR ARRAY, IS SPECIFIED IN A DATA STATEMENT.
C
C ***XDIM, YDIM SPECIFY THE SIZE OF THE OUTPUT
C
C ***DELTA IS HALF WIDTH OF THE SOURCE ASSUMED SYMMETRIC
C
C ***NOTE THAT THE BEGINNING OF THE OUTPUT IS PADDED WITH ZEROS TO 10% OF ITS LENGTH
C
REAL*8 P, SRC, PC
DIMENSION P(1000), SRC(1000), PC(2000), PPLCT(1000), TPLCT(1000)
DIMENSION XL(2), YL(4), YM(4)
DIMENSION SRCP(1000)
DIMENSION PPL(1000)
DATA XL/'TIME', 'SEC'/
DATA YL/'CONV', 'OLVE', 'D OU', 'TPUT'/
DATA YM/'UNCO', 'NVOL', 'V OU', 'TPUT'/
C SPECIFY SOURCE FUNCTION
DATA SRC/0., 2.5, 5., 2.5, 0., 995*0./
C SPECIFY PROGRAM PARAMETERS
XDIM=6.
YDIM=2.
DELTA=.4
C READ IN PUNCHED DATA
READ(5,100) TT, DP, NN
PRINT 200, TT, DP, NN
200 FORMAT(1X,TT=F15.6,DP=F15.6,NN=I10)
READ(5,101)(P(I),I=1,NN)
100 FORMAT(2E15.6,I10)
101 FORMAT(5E15.6)
102 FORMAT(3(I10,G15.6))
C PERFORM CONVOLUTION WITH SOURCE FUNCTION
NS = 2*DELTA/DP + 1.5
CALL CONVLV(NN,P,NSRCPC)
C PAD RECORD WITH ZEROES
NQ = NN/10
NA = NQ + 1
NB = NN + NQ
DO 15 I = 1, NQ
SRCP(I) = 0.0
TPLDT(I) = TT-(NQ-I+1)*DP
PPL(I) = 0.0
15 PPL(I) = 0.0
DO 16 I = NA, NB
SRCP(I) = SRC(I-NA+1)
TPLDT(I) = TT + (I-NA)*DP
PPL(I) = P(I-NA+1)
16 PPL(I) = PC(I-NA+1)
C LIST OUTPUT
PRINT 201
201 FORMAT(1X,'OUTPUT AFTER CONVOLUTION-BEFORE CONV-SOURCE FUNC'/)
PRINT 202,(TPLDT(I),PPL(I),SRCP(I),I=1,NB)
202 FORMAT(4G15.6)
C PLOT ON CALCOMP
CALL NEWPLT('M8800','6894','WHITE','BLACK')
CALL PICTUR(XDIM,YDIM,XL,8,YM,16,TPLDT,PPL,NN,0.,0.)
CALL PICTUR(XDIM,YDIM,XL,8,YL,16,TPLDT,PLOT,NN,0.,0.)
DO 17 I = 1, NB
17 TPL(T(I)) = TLOT(I) - TT
    CALL PICTU(XDIM,YDIM,XL,8,'SOURCE FUNCTION ',16,TLOT,SRCP,NB,0.,
    1,0)
    CALL ENDPLT
    CALL EXIT
    STOP
    END
SUBROUTINE CONVLV(LX,XX,LY,YY,CC)
C
TITLE - CONVLV = CONVOLVE
COMPLETE CONVOLUTION OF TWO TRANSIENTS

---ABSTRACT---

CONVLV CONVOLVES TWO TRANSIENTS, X(I) I=0,1,...,LX-1
AND Y(I) I=0,1,...,LY-1, TO PRODUCE THE COMPLETE
CONVOLUTION FUNCTION

\[ C(I) = \sum_{J=0}^{LX-1} X(J)Y(I-J) \]

FOR I = 0,1,...,LX+LY-2
WHERE
LX AND LY ARE INPUT PARAMETERS
Y(K) IS ASSUMED = 0.0 FOR K OUTSIDE OF
THE RANGE 0 TO LY-1
NOTE THAT THE CONVOLUTION IS INDEPENDENT OF THE ORDER
OF THE INPUTS X AND Y.

TECHNIQUE USED IS AN ALGORITHM BASED ON ANALOGY TO
MULTIPLICATION OF POLYNOMIALS

---STATISTICS---

LANGUAGE - FORTRAN IV
EQUIPMENT - NO SPECIAL REQUIREMENTS
STORAGE -
AUTHOR - J.F. CLAEBOULT, TRANSLATED FROM FORTRAN II TO FORTRAN
BY R.A. WIGGINS, 6/65

000000020 CONV0001
000000030 CONV0002
000000040 CONV0003
000000050 CONV0004
000000060 CONV0005
000000070 CONV0006
000000080 CONV0007
000000090 CONV0008
00000100 CONV0009
00000110 CONV0010
00000120 CONV0011
00000130 CONV0012
00000140 CONV0013
00000150 CONV0014
00000160 CONV0015
00000170 CONV0016
00000180 CONV0017
00000190 CONV0018
00000200 CONV0019
00000210 CONV0020
00000220 CONV0021
00000230 CONV0022
00000240 CONV0023
00000250 CONV0024
00000260 CONV0025
00000270 CONV0026
00000280 CONV0027
00000290 CONV0028
00000300 CONV0029
00000310 CONV0030
00000320 CONV0031
00000330 CONV0032
00000340 CONV0033
00000350 CONV0034
00000360 CONV0035
00000370 CONV0036
C LIBRARY ROUTINES REQUIRED - NONE

---- USAGE ----

SAMPLE CALL
CALL CONVLV(LX, XX, LY, YY, CC)

INPUTS
LX IS NO. OF TERMS IN X VECTOR
XX(I) I=1,...,LX CONTAINS X(0),...,X(LX-1) RESPECTIVELY
LY IS NO. OF TERMS IN Y VECTOR
YY(I) I=1,...,LY CONTAINS Y(0),...,Y(LY-1) RESPECTIVELY
EQUIVALENCE (XX,YY) IS PERMITTED

OUTPUTS
CC(I) I=1,...,LX+LY-1 CONTAINS C(0),...,C(LX+LY-2) RESPECTIVELY
EQUIVALENCE (XX,CC) ALLOWED IF XX NOT EQUIVALENT TO YY.

EXAMPLES
1. SHOWING REVERSIBILITY CF X AND Y
   INPUTS - LX = 3 XX(1...3) = 1,2,3
             LY = 2 YY(1...2) = 10,1
   USAGE - CALL CONVLV(LX, XX, LY, YY, CC1)
            CALL CONVLV(LY, YY, LX, XX, CC2)
   OUTPUTS - CC1(1...4) = CC2(1...4) = 10,21,32,3
**C 2. ILLEGAL INPUT CASES (NO OUTPUT)**

**INPUTS** - SAME AS EXAMPLE 1, EXCEPT START WITH OUTPUT VECTORS
CLEANED, I.E. \( C_{C1}(1...4) = C_{C2}(1...4) = 0,0,0,0, \)

**USAGE** -
- CALL \( \text{CCNVLV}(-2,XX,LY,YY,CC1) \)
- CALL \( \text{CCNVLV}(LX,XX,0,YY,CC2) \)

**OUTPUTS** -
- \( C_{C1}(1...4) = 0,0,0,0 \) (ILLEGAL LX)
- \( C_{C2}(1...4) = 0,0,0,0 \) (ILLEGAL LY)

**PROGRAM Follows Below**

```fortran
SUBROUTINE CCNVLV(LX,XX,LY,YY,CC)
IMPLICIT REAL*8 (A-H,O-Z)

DIMENSION XX(2),YY(2),CC(2)

CHECK LEGALITIES
IF (LX.LE.0) GO TO 9999
IF (LY.LE.0) GO TO 9999

CLEAR PORTION OF OUTPUT AREA
LC = LX+LY-1
IB = LX+1
DO 10 I = IB, LC
10 CC(I) = 0.

DO 30 J = 1, LX
20 CONTINUE
30 IX = IX-1
```

**CONVOLUTION**

<table>
<thead>
<tr>
<th>IX</th>
<th>DO 30</th>
<th>1</th>
<th>DO 20</th>
<th>K</th>
<th>CC(K-1) = CC(K-1) + X*YY(J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DO 30</td>
<td>J = 1, LY</td>
<td>K = IX+J</td>
<td>CC(K-1) = CC(K-1) + X*YY(J)</td>
<td>20 CONTINUE</td>
<td>IX = IX-1</td>
</tr>
</tbody>
</table>
C
C EXIT
C
9999 RETURN
END

00001090 CONVO109
00001100 CONVO110
00001110 CONVO111
00001120 CONVO112
00001130 CONVO113