CORE STRUCTURE CONSTRAINTS
DERIVED FROM SKS AND SKKS OBSERVATIONS

by

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ABSTRACT

We have made an extensive study of SmKS arrivals in order to 1) examine the transmission and reflection response of the core-mantle boundary and 2) constrain the velocity structure of core regions in which PmKP waves do not bottom. The SmKS data set included measurements of SKS/SKKS amplitude ratios, SKKS phase lags (relative to SKS), travel times and $dT/dA$ values. In addition to long period data, we have measured over 500 short period travel times corresponding to SKS, SKKS, and SKKKS. SKKKS-SKKS and SKKS-SKS travel time differences have also been employed in this study.

We have isolated an SKS/SKKS amplitude ratio minimum and a region of phase shifts in our data for epicentral distances of 109°-110°. We find that these observations are associated
with the transmission of SKS through the core-mantle boundary and that as a consequence, the SKS p-Δ curve is constrained to pass through the mean value of wave slowness for diffracted P (≈ 4.55 sec/deg) between distances of 109° and 110°. Once the SKKS internal caustic is taken into consideration, we find that theoretical transfer functions for a sharp core-mantle boundary model similar to the B2 model of Bullen and Haddon (1967) provide a reasonable fit to our amplitude ratio and phase lag data. We also find that the SKS/SKKS amplitude ratios exhibit a negligible dependence on frequency, and that if present, core-mantle boundary layering is probably less than about 10 to 15 kilometers in extent.

The second part of this thesis delineates the results from our study of the outer core velocity structure. We derive a velocity model (SKORl) for the outer core and discuss the fit of this model to our SmKS travel times and dT/dΔ values as well as the 1968 Herrin times for PKP. The outer 600 kilometers of SKORl were constrained by our SKKS dT/dΔ values as well as SKKS, SKKKS, SKKKS-SKKS and SKKS-SKS travel time data. We find that the P wave velocity at the core mantle boundary is about 8.015 km/sec and that it increases at a nearly constant rate of about -0.00155 sec⁻¹ throughout the outer 250 kilometers of the core. However, the velocity gradient changes to a value of about -0.0022 sec⁻¹ between radii of 3050 and 3250 kilometers, and this change is reflected in our amplitude and dT/dΔ data.
The remaining portions of SKOR1 were constrained by SKKS-SKS time differences and by the absolute times of the SKS first arrivals (usually referred to as the AC branch). We find that SKOR1 possesses a positive perturbation in velocity (relative to other recent core models) centered at a radius of about 2500 kilometers. This feature is very similar to the results given in the B1 model of Jordan (1973). The improved resolution of the SmKS arrivals provided by our short period data has resulted in initial evidence for additional SKS travel time branches. We are currently undertaking an analysis of a combined PmKP and SmKS data set in order to obtain a consistent explanation for these "secondary" arrivals.
Acknowledgements

The results presented in this thesis would not have been possible without the help of many friends. To name them all seems like a formidable task. I am particularly grateful to Professor Keiiti Aki for originally suggesting study of SmKS arrivals, to Dr. Clint Frasier for the many hours which he spent helping me with the core-mantle boundary portion of this thesis, and to Professor M. Nafi Toksoz for his guidance of my efforts to determine the velocity structure of the outer core. I would also like to thank Dr. Bruce Julian, Dr. Anton Dainty, Ken Anderson and Robert M. Sheppard for making their computer programs available to me, Russ Needham and Larry Lande for showing me how to interpret seismograms and to Dr. David Davies, Dr. John Filson and Raymon Brown for their suggestions throughout the course of this study. The presentation of this thesis was greatly expedited because of the help that I received from Dorothy Frank and Carol Van Etten in typing the manuscript. The long hours spent by these two people is deeply appreciated.

The time spent in preparing a thesis can sometimes produce a significant burden on the human spirit. For this reason, I feel a need to express gratitude for the patience, encouragement, understanding, and above all, friendship of Professor M. Nafi Toksoz, Dr. Clint Frasier, Raymon Brown, Sara Brydges, Ben Powell, Mary Laulis and my psychiatrist.
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1. **Introduction**

   Improvements in instrumentation and technique developed in the past decade have been largely responsible for an increasing number of seismic wave studies aimed at refining the early core models of Jeffreys (1938, 1939a, 1939b), Gutenberg and Richter (1934, 1935, 1946, 1939) and Gutenberg (1958). Most of these refinements have been based on studies of short period core arrivals which propagate through the mantle in the longitudinal mode. Seismic waves used for this purpose include PmKP (m = 1 through 9), PKIKP, PKiKP and PKJKP (Adams and Randall, 1964; Bolt, 1964, 1970; Ergin, 1967; Shurbet, 1967; Engdahl, 1968; Shahidi, 1968; Bolt and Qamar, 1970; Engdahl et al., 1970; Buchbinder, 1971; Adams, 1972; Yanovskaya, 1972; Jordan, 1973; Qamar, 1973; Toksoz et al., 1973). However, all of these studies are biased towards constraining core models at depths greater than about 3900 kilometers, because PmKP waves do not bottom in the upper regions of the outer core. This bias has resulted in a deficiency of the high quality data prerequisite to a better understanding of the core at depths less than 3900 kilometers. Indeed, complete characterization of the outer core is particularly germane to the questions of core formation and generation of the earth's magnetic field.

   This thesis presents original data and preliminary interpretations of SmKS waves (multiply reflected core
arrivals which propagate through the mantle in the transverse mode) which have been examined in order to: 1) determine their usefulness for constraining core-mantle boundary (CMB) models; 2) constrain core regions in which the PmKP waves do not bottom; and 3) test models derived to explain PmKP observations (this is possible, because PmKP and SmKS waves sample overlapping regions of the core). Typical SKS and SKKS ray paths shown in Figure 1.1 (Toksoz et al.; 1973) indicate that study of SKKS arrivals is well suited for constraining the upper 600 kilometers of the core (since S interferes with SKS at distances less than 85°), while study of SKS arrivals provides us with the desired constraints for outer core regions at radii less than approximately 3000 kilometers.

Recently, several core studies employing SKS and SKKS observations have appeared in seismological literature. The data employed in these studies included travel times (Hales and Roberts, 1970, 1971a, 1971b; Randall, 1970) and dT/dA values (Toksoz et al., 1973; Wiggins et al., 1973) measured on seismograms from long period horizontal component instruments. However, we believe that the long period nature of these waves (typically 15 to 30 seconds) has limited their usefulness for detecting fine structure in the earth's outer core. The approach taken in this investigation presents two main advantages over those previously used for SmKS study: 1) the data set includes many short period SKS and SKKS observations which provide improved resolution of travel
times at closer distances and of additional SKS branches and; 2) travel times have been combined with dT/dΔ measurements, SKS/SKKS amplitude ratios and SKS-SKKS phase lags in order to obtain a consistent understanding of model constraints.

The second chapter of this thesis describes the techniques which we have employed for data collection and analysis, while the remaining chapters delineate our interpretations of the SmKS data set. Results from the core-mantle boundary study are presented in chapter 3. Chapter 4 describes our base model for the outer core, and the fit of this model to SmKS and PKP travel times. The fourth chapter also contains a description of some initial evidence for SKS "secondary" arrivals which may imply the necessity for future perturbations to our base model at radii less than about 2700 kilometers.
TABLE 1.1
DEEP EVENTS STUDIED

<table>
<thead>
<tr>
<th>DATE</th>
<th>ORIGIN TIME</th>
<th>DEPTH(KM)</th>
<th>MAG</th>
<th>REGION</th>
</tr>
</thead>
<tbody>
<tr>
<td>7/06/65</td>
<td>18:36:47.3</td>
<td>510</td>
<td>6.5</td>
<td>Solomon Islands</td>
</tr>
<tr>
<td>11/03/65</td>
<td>1:39:02.5</td>
<td>583</td>
<td>6.2</td>
<td>Peru-Brazil Border</td>
</tr>
<tr>
<td>3/17/66</td>
<td>15:50:32.2</td>
<td>626</td>
<td>6.2</td>
<td>Fiji Islands</td>
</tr>
<tr>
<td>6/22/66</td>
<td>20:29:03.6</td>
<td>507</td>
<td>6.1</td>
<td>Banda Sea</td>
</tr>
<tr>
<td>2/15/67</td>
<td>16:11:11.8</td>
<td>597</td>
<td>6.2</td>
<td>Peru-Brazil Border</td>
</tr>
<tr>
<td>10/07/68</td>
<td>19:20:20.3</td>
<td>516</td>
<td>6.1</td>
<td>Bonin Islands</td>
</tr>
<tr>
<td>8/04/69</td>
<td>17:19:19.6</td>
<td>521</td>
<td>6.2</td>
<td>Banda Sea</td>
</tr>
<tr>
<td>7/31/70</td>
<td>17:08:05.4</td>
<td>651</td>
<td>7.1</td>
<td>Colombia</td>
</tr>
<tr>
<td>8/30/70</td>
<td>17:46:09.0</td>
<td>645</td>
<td>6.6</td>
<td>Sea of Okhotsk</td>
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<td>1/29/71</td>
<td>21:58:05.4</td>
<td>544</td>
<td>6.1</td>
<td>Sea of Okhotsk</td>
</tr>
<tr>
<td>11/20/71</td>
<td>7:23:01.0</td>
<td>551</td>
<td>6.0</td>
<td>South of Fiji Islands</td>
</tr>
</tbody>
</table>
FIGURE CAPTIONS

Figure 1.1  SKS and SKKS ray plots for the core model of Toksöz et al. (1973) showing the SKKS internal caustic and SKS focusing.
2. Sources and Analysis of Data

2.1 Data Source

The results presented in this study are based on original data extracted from SmKS waveforms and travel times. The SKS and SKKS observations utilized in constructing this data base came from three sources: 1) long period WWSSN seismograms for 1970 and 1971 $M_b > 6.0$ events; 2) LASA and NORSAR data tapes from 1965 to 1973; and 3) long and short period WWSSN seismograms for the 11 deep events listed in Table 1.1. Although thousands of seismograms were examined during this study, we choose to limit our data base to approximately 700 high quality SKS, SKKS and SKKKS arrivals. Examples of these arrivals are shown in Figures 2.1 (long period) and 2.2 (short period).

Most of the data employed in this study fall within the distance range $85^\circ < \Delta < 135^\circ$. The lower limit is imposed by the condition that S, SKS and SKKS travel times intersect near this distance making signal separation difficult for $\Delta < 85^\circ$. We found that signal polarization provided a useful tool for separating S and SmKS arrivals. Continuity of particle velocity at the core-mantle boundary requires SV-polarization of SmKS arrivals (assuming zero core rigidity). Although observed SKS and SKKS arrivals are not of pure SV-mode, they are very nearly so as shown in the examples of Figure 2.3.
Consequently, when possible we compared SKS/SKKS amplitude ratios for both NS and EW components in order to avoid the mistake of incorporating S waves into our data. This approach is particularly effective at distances greater than about 100°, because diffracted S waves are primarily polarized in the SH-mode (Cleary, 1969). The upper distance limit has been imposed by our desire to concentrate primarily on the outer core and core-mantle interface. Consequently, only a limited number of travel times have been measured for distances greater than 135°.

2.2 Signal Travel Time and $\text{dT/dA}$ Analysis

2.2.1 SKS and SKKS travel times.

Examination of seismograms for the deep events listed in Table 1.1 provided approximately 500 short period SKS and SKKS signals for which accurate arrival times could be obtained. These data were supplemented with arrival times from selected long period WWSSN seismograms which exhibited clear signal onsets. We found that it was particularly difficult to determine SKKS arrival times, because their onsets are not sharp as in the case of SKS. This problem is due to the fact that SKKS is not a minimum time path arrival (note the SKKS internal caustic in Figure 1.1; Shimamuro and Sato, 1965; Jeffreys and Lapwood, 1957). All arrival times were converted to travel times using NOAA source parameters.
and recorded to the nearest 0.1 second. Distances and azimuths were computed for each event-station pair using a program provided by Mr. R.M. Sheppard. The travel time versus distance data set was next corrected for depth of focus (correction to 33 kilometers) using the Jeffreys-Bullen Tables (Travis, 1965) and for earth ellipticity using the ellipticity tables of Bullen (1939).

Errors present in our final travel time versus distance table come from several sources: 1) origin time errors; 2) event location errors; 3) onset measurement errors; and 4) errors inherent in the models used for depth of focus and ellipticity corrections. We estimate that SKS travel times have an accuracy of ± 1.0 second and ± 0.2 seconds for the long and short period arrivals respectively, while the error limits for SKKS arrivals are approximately twice these values.

2.2.2 SKS and SKKS dT/dA values.

SKS and SKKS dT/dA values were measured using the Data Analysis facilities of the Lincoln Laboratory Seismic Discrimination Group. LASA and NORSAR long period data tapes provided about 40 events for which SKS and/or SKKS signals were easily identified. Time delays of these signals at each array sensor location were converted to horizontal phase velocities using the Data Analysis
subroutines. The reciprocals of these phase velocities were then computed to obtain $dT/d\Delta$ values. We have computed 95% confidence limits for these measurements using the formula given in equation 2.1 (Kelley, 1964):

$$\sigma = (6/N)^{1/2} \frac{TC}{L}$$

where $\sigma = \text{standard deviation}$, $N = \text{number of sensors used}$, $T = \text{RMS time error of plane wave fit}$, $L = \text{array aperture}$ and $C = \text{expected phase velocity}$.

2.3 Signal Amplitude Analysis

Except for LASA and NORSAR arrivals, all amplitudes used in this study were measured on 15X enlargements of 70 mm film chips for the WWSSN seismograms. Only those long period seismograms which showed SKS and SKKS arrivals with signal to noise ratios greater than about 5 were subjected to amplitude analysis. We also chose to use only a limited number of short period SKS and SKKS arrivals for amplitude study, because these waves have periods which fall on steep portions of the WWSSN instrument magnification curves making corrections difficult.

Peak to peak amplitudes of the SKS and SKKS arrivals were measured with an accuracy of $\pm0.2$ mm. Since the long period arrivals used had paper record amplitudes greater
than 20 mm, measurement errors are estimated to be less than 2%. Short period signal amplitudes could be in error by as much as 10%. Measured amplitudes were corrected for instrument magnification using WWSSN magnification curves and the signal frequency of peak power obtained from spectral analysis. Corrections for azimuthal direction of the incoming waves were applied under the assumption of SV-polarization. Finally, we chose to normalize all amplitudes to a NOAA magnitude of 6.5 $M_b$ by multiplying them by the factor $10^{\Delta M_b}$, where $\Delta M_b$ equals 6.5 minus the NOAA event magnitude.

2.4 Signal Phase Analysis.

In order to avoid the complication of calculating source radiation patterns for all of the events studied, we chose to examine the phase difference of SKS and SKKS. At first, this phase difference was determined using the imaginary and real parts of the SKS/SKKS spectral ratio. We found, however, that phase spectra obtained in this way were oscillatory and difficult to interpret. Consequently, we decided to employ a technique of phase shift filtering and signal correlation. This technique (performed on the computing facilities of the Lincoln Laboratory Seismic Discrimination Group) consisted of several steps: 1) The record section containing SKS and SKKS signals for a given event was displayed on a CRT facility with sampling at 1.0
second intervals; 2) A time window defined by the signal frequency of peak power for the broadest of the two waves was visually placed around each signal; 3) The SKS time window was placed in a reference buffer, while the SKKS time window was iteratively passed through a Hilbert transform phase shift filter at 10 degree intervals. The correlation coefficient of the SKS and SKKS time windows was computed after each iteration; 4) This iteration was repeated in 1 degree intervals over the 20 degree range of best correlation, and the phase shift providing the highest degree of correlation determined; 5) The SKS and shifted SKKS signals were compared on the CRT facility after the analysis to visually check our results.

The phase shift and correlation technique utilized in this study was made under the assumption that all frequencies of the SKS and SKKS signals have been shifted through a constant phase angle. Although this assumption is not strictly true, we feel that the simple time domain shapes of these waves combined with their rapid amplitude decay away from the peak frequency of their power spectra indicates that favoring this dominant frequency should provide a good first approximation to the SKS-SKKS phase difference. Figure 2.4 illustrates typical SKS and SKKS record sections and the corresponding power spectra, while Figure 2.5 shows several examples of the results from our phase shift
and correlation technique. The results shown in Figure 2.5 indicate that SKKS phase lags (relative to SKS) obtained in this study have an accuracy of about ±15 degrees.
FIGURE CAPTIONS

Figure 2.1 Examples of long period SmKS arrivals:

Figure 2.2 Examples of short period SmKS arrivals:

Figure 2.3 Seismograms showing SV-polarization of SKS and SKKS. A: 2/21/71 NUR; B: 11/20/70 KBL.

Figure 2.4 Power spectra for typical long period SKS and SKKS arrivals.

Figure 2.5 Determination of the SKKS phase lag (relative to SKS) using the phase shift and correlation method.
FIGURE 2.2B
\[ \Delta = 111.30^\circ \]
AZIMUTH = 259.31°

\[ \Delta = 119.70^\circ \]
AZIMUTH = 99.21°
FIGURE 2.4

3/27/70 TOL-E
\[ \Delta = 114.82^\circ \]

11/21/69 LF3-N
\[ \Delta = 128.62^\circ \]

FREQUENCY (HZ.)

POWER (DB)

SKS
SKKS
FIGURE 2.5

- SKS
- SKKS

11/21/69 LF3-N

SKKS ADVANCED 30° IN PHASE

3/27/70 TOL-E

SKKS ADVANCED 171° IN PHASE
3. **Study of the Core-Mantle Boundary**

Recent studies of the core-mantle boundary (CMB) have been based on PcP and ScS arrivals (Balakina et al., 1966; Kanamori, 1967; Buchbinder, 1968; Vinnik and Dashkov, 1970; Ibrahim, 1971, 1973; Frasier, 1972; Kogan, 1972; Berzon and Pasechnik, 1972; Bolt, 1972; Anderson and Jordan, 1972), diffracted arrivals (Alexander and Phinney, 1966; Sacks, 1966, 1967) and free oscillations (Dorman et al., 1965; Derr, 1969). Since the nature and extent of any CMB layering is one of the issues still subject to considerable debate, we examined the SmKS data set to determine what information can be obtained from waves which sample the transmission and reflection response of this interface.

Figure 3.1 illustrates theoretical amplitude and phase curves for the CMB transmission and reflection response. We have calculated these curves using the B2 model of Bullen and Haddon (1967). CMB incidence angles for SKS and SKKS (assuming the B2 model and the Jeffreys-Bullen Tables -- Travis 1965) are shown in Figure 3.2. This diagram indicates that SKKS transmits through the CMB past the P-wave critical angle (2132.5°) for the distance range 85°≤Δ≤135°. On the other hand, SKS transmits through the CMB at angles less than critical for distances greater than about 106°. As shown in
Figure 3.1, the passage of SKS through the CMB at distances where this critical angle is reached should produce a narrow amplitude minimum and region of phase shift. Consequently, we isolate this feature in section 3.1 and then interpret our observations in terms of CMB model studies in section 3.2.

3.1 Data Base for the CMB Study

The observed long period SKS and SKKS amplitudes (corrected for instrument magnification, azimuth and event magnitude as described in section 2.3) are shown in Figure 3.3. The scatter inherent in the present form of these data prohibits their use for interpreting CMB properties. However, valuable information can be extracted from the SmKS data by using amplitude ratios (Kanamori, 1967), since most of this scatter probably results from near source and receiver effects. General expressions for observed SKS and SKKS amplitudes are of the form:

\[ \Lambda(f, \Delta) = S(f, i, \theta) I(f, e, \theta) G(\Delta) B(f, \Delta) D(f, \Delta) \]  

where \( S(f, i, \theta) \) is the source spectrum as a function of frequency, take-off angle and azimuth; \( I(f, e, \theta) \) is the instrument response spectrum as a function of frequency, emergence angle and azimuth; \( G(\Delta) \) is the geometrical
spreading factor; \( B(f,\Delta) \) is the CMB response spectrum; and \( D(f,\Delta) \) is the attenuation spectrum given by

\[
D(f,\Delta) = \exp(-t^*f)
\]

where

\[
t^* = \pi \int_0^s \frac{ds}{v(s) Q(f,s)}
\]

SKS and SKKS take-off angles are shown in Figure 3.4. This diagram indicates that the upper mantle paths for both wave types should be similar. Consequently, formation of the amplitude ratio SKS/SKKS results in the approximate cancellation of source and receiver uncertainties, and hence a reduction in data scatter as shown in Figure 3.5. We also show short period SKS/SKKS amplitude ratios in this diagram for comparison, but these short period ratios have not been corrected for any differences in frequency content between SKS and SKKS.

Figure 3.6 illustrates the correlation between amplitude ratio and phase information derived from the data set. The amplitude ratio portion of this diagram was obtained by taking the median value of Figure 3.5 in 1.0 degree intervals for \( 100^\circ \leq \Delta \leq 135^\circ \). We found no evidence for an amplitude ratio minimum and phase shift at \( \Delta = 106^\circ \), but have isolated these features near distances of \( 109^\circ - 110^\circ \) (region A of Figure 3.6). Although
Figure 3.6 illustrates other regions (B through D) with anomalous amplitude and phase values, we found that these features occurred at distances which precluded the possibility of their being an expression of CMB transfer function effects. Consequently, we concentrate on explaining region A in section 3.2 and then examine the origin of features B through D in chapter 4.

3.2 Interpretation of the CMB Data Base

The results shown in Figure 3.1 as well as the CMB model studies discussed in this section have been calculated using the propagator matrix technique for homogeneous plane layers (Thomson, 1950; Haskell, 1953, 1962; Gilbert and Backus, 1966). These results are subject to our basic assumption of plane waves and layers. In order to evaluate the validity of assuming a plane CMB, we invoked the Sagittal Approximation for estimating the effect of curvature (Born and Wolf, 1964). We find that CMB curvature implies $a/\lambda$ ratios of 0.2% and 4% for waves with periods of 1 and 35 seconds respectively (see Figure 3.7 for nomenclature). Since the effect of curvature is small for $a/\lambda$ ratios less than about 10%, we conclude that the plane layer approximation should not affect the basic results of this study.

Theoretical SKS/SKKS amplitude ratios and phase
differences calculated for the B2 model of Bullen and Haddon (1967) are shown in Figure 3.8. We have calculated these curves using the Jeffreys-Bullen travel times for SKS and SKKS (Travis, 1965). All CMB models discussed in this chapter are listed in Table 3.1. As noted earlier, Jeffreys-Bullen p-Δ values reproduce the amplitude ratio minimum and phase shift at Δ = 106° rather than the observed distance of 109°-110°. Figure 3.8 also shows that the computed amplitude ratios decay more rapidly than our data beyond distances of about 120°.

The coefficient describing the partition of energy for a mantle S wave which transmits through a non-layered CMB and converts to a P wave in the core is given by:

\[ T_{SP} = \frac{4\bar{p}^2\beta_m^3(1-\bar{p}^2\beta_m^2)^{1/2}}{\alpha_c^2 + \frac{\alpha_m^2(1-\bar{p}^2\alpha_m^2)^{1/2}}{(1-4\bar{p}^2\beta_m^2 + 4\bar{p}^2\beta_m^4)^{1/2}} + \frac{4\bar{p}^2\beta_m^3(1-\bar{p}^2\alpha_m^2)^{1/2}(1-\bar{p}^2\beta_m^2)^{1/2}}{\alpha_c^2} } \]

where \( \bar{p} \) = ray parameter divided by the CMB radius ratio and subscripts m and c refer to parameters for the mantle and core respectively. Equation 3.3 shows that the amplitude ratio minimum and phase shift are associated with the zero location of the term \((1-\bar{p}^2\alpha_m^2)^{1/2}\). Bolt (1972) has found a mean wave slowness of about 4.55 ± 0.05 sec./deg. for P waves diffracted at the core-mantle boundary. Consequently, isolation of the minimum and phase shift
associated with $T_{SP}$ near region A implies that the SKS p-Δ curve goes through the value $4.55 \pm 0.05$ sec./deg. at distances of about $109^\circ$-$110^\circ$. We find that the theoretical SKS p-Δ curve corresponding to our velocity model SKOR1 (chapter 4) passes through this critical value at distances of about $107.5^\circ$. This distance is in reasonable agreement with the observed location for feature A. Since a preliminary SKS p-Δ curve was used in constructing model SKOR1, we anticipate an improved fit to the location of feature A once this curve has been constrained to fit all of the SKS observations. Consequently, to aid comparison with our data at this time, we have set $\alpha_m = 13.86$ km./sec. so that theoretical CMB transfer functions computed using the present form of our p-Δ curves would pass through the $T_{SP}$ minimum between $109^\circ$ and $110^\circ$.

Substituting $13.86$ km./sec. for $\alpha_m$, we have recalculated the CMB transfer function for the B2 model of Bullen and Haddon using the p-Δ curves corresponding to model SKOR1. The results of this calculation are compared to our median SKS/SKKS amplitude ratio curve and SKS-SKKS phase lag data in Figure 3.6. The theoretical phase lag curve shown in this diagram has been given a $90^\circ$ correction to compensate for the SKKS internal caustic. The agreement between theoretical and observed
data shown in Figure 3.6 is reasonably good. The improved fit to observed amplitude ratio data beyond distances of about 120° is due to the incorporation of our SKKS p-Δ values in these calculations (primarily because of the high values of SKKS d²T/dΔ² for Δ = 115°-120°).

We next examined our ability to constrain a sharp CMB model. We found that perturbations in α_c and β_m by more than about ±0.3 km./sec. (relative to the values for model B2) were required to produce transfer function changes which exceeded the present scatter in our data. We also calculated CMB transfer functions for several recently published models (Balakina et al., 1966; Buchbinder, 1968). The results of these calculations are shown in Figure 3.9. This diagram indicates that the addition of small amounts of outer core ridigity produces only minor perturbations to models with μ_c = 0 (and hence β_c = 0). Consequently, this study does not allow us to place any constraints on this parameter. On the other hand, theoretical amplitude ratios for the CMB model proposed by Buchbinder (1968) do not fit our observations. Buchbinder proposed this model (characterized by a unity CMB density ratio and 7.5 km./sec. value for α_c) to explain a PcP/P amplitude ratio minimum and phase reversal near Δ = 32°. Our rejection of this model is in agreement with the results of Frasier
(1972), Berzon and Pasechnik (1972), Kogan (1972) and Chowdhury and Frasier (1973) which indicate that the PcP/P amplitude ratio minimum and phase reversal do not exist, but that the first PcP extremum is often buried in noise near $\Delta = 32^\circ$ making phase determination difficult.

Having completed our study of sharp CMB models, we examined theoretical transfer functions for models incorporating layered transition zones in order to:
1) test our ability to resolve the extent of CMB layering; and 2) see if layered model results could provide an improved fit to our data.

We first considered a transition zone model consisting of a homogeneous layer of 100 kilometer thickness (model #81 of Alexander and Phinney, 1966). Theoretical amplitude ratio and phase lag results for this model are shown in Figures 3.10 and 3.11. Although observed phase lag data were only obtained from long period signals, the amplitude ratios shown in Figure 3.5 represent data from signals with periods ranging from 1 to 45 seconds. We find no evidence for the large variation in amplitude ratio with period at distances less than about $110^\circ$ shown in Figure 3.10. Consequently, we next considered a model in which the transition zone thickness was reduced to 30 kilometers (model #94 of Alexander and Phinney, 1966). The theoretical response curves for this model are shown in Figures 3.12
and 3.13. These results also exhibit a dependence on
frequency which exceeded the extent of our data scatter.

Recently, several authors (Dorman et al., 1965; Ibrahim,
1971, 1973) have proposed CMB models consisting of a number
of interbedded layers 10 to 15 kilometers thick. Figures
3.14 and 3.15 show theoretical CMB transfer function results
for the model of Dorman et al. (1965). As anticipated,
the results for this model are very similar to those for
a sharp CMB. In particular, the addition of a thin layer
transition zone does not improve the fit to our data.

3.3 Conclusions from the CMB Study

Theoretical transfer functions for layered and
non-layered CMB models have been compared to observed
SKS/SKKS amplitude ratios and SKS-SKKS phase lags. We
have isolated an SKS/SKKS amplitude ratio minimum and region
of SKS-SKKS phase shift near $\Delta = 109.5^\circ$ due to the CMB
transfer function, and find that our observations can be
explained reasonably well by a non-layered interface model
similar to the B2 model of Bullen and Haddon (1967), but
with $\alpha_m = 13.86$ km./sec. We do not purport that 13.86
km./sec. represents a derived value for the compressional wave
velocity at the CMB. Rather, we indicate that this value
for $\alpha_m$ is required to compensate for the inaccuracy of our
SKS $p-\Delta$ curve near 109°-110°. At present, our data does
not allow us to resolve variations in $\alpha_c$ or $\beta_c$ as suggested by Balakina et al. (1966). On the other hand, we conclude that the model of Buchbinder (1968) which incorporates a CMB density ratio of unity does not imply transfer functions which fit our data.

We also find that theoretical transfer functions for models containing homogeneous transition zones 30 kilometers or more in thickness exhibit a dependence on frequency which exceeds the scatter of our data. However, this observation does not preclude the existence of lateral variations in the mantle's D" region as suggested by Alexander and Phinney (1966), Toksöz et al. (1967), Husebye et al. (1971), Davies and Sheppard (1972), Kanasewich et al. (1972), Vinnik et al. (1972), Julian and Sengupta (1973) and Jordan and Lynn (1973). Rather, it indicates that a single homogeneous layer is an inadequate model for this region. If present, we conclude that a CMB transition zone consists of homogeneous layers less than about 10 to 15 kilometers in thickness. The "sharpness" of this interface is also suggested by the remarkable similarity of P, PcP and PKKKP for a given event-station pair (Davies, 1971; Frasier, 1972).
### TABLE 3.1

**CORE - MANTLE BOUNDARY MODELS**

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<tr>
<th></th>
<th>$\alpha$ (km/sec)</th>
<th>$\beta$ (km/sec)</th>
<th>$\rho$ (g/sec)</th>
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<td></td>
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<td>#94 (Alexander and Phinney, 1966)</td>
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FIGURE CAPTIONS

Figure 3.1 Theoretical amplitude and phase curves for CMB transmission and reflection coefficients. Calculations were made assuming the B2 model of Bullen and Haddon (1967).

Figure 3.2 SKS and SKKS CMB incidence angles calculated using the B2 model of Bullen and Haddon (1967) and the Jeffreys-Bullen Tables for SKS and SKKS (Travis 1965).

Figure 3.3 Observed long period SKS and SKKS amplitudes (corrected for instrument magnification, azimuth and event magnitude as described in section 2.3).

Figure 3.4 SKS and SKKS take-off angles assuming the ray parameters implied by equations 4.1 and 4.2 and $\beta_0 = 3.40 \text{ km./sec.}, \beta_{500} = 5.28 \text{ km./sec.}$

Figure 3.5 SKS/SKKS amplitude ratios. Short period ratios (solid circles) have not been corrected for any differences in the frequency content of SKS and SKKS.

Figure 3.6 Correlation between amplitude ratio and phase lag data. Amplitude ratio curve (solid line) was obtained by taking the median value of Figure 3.5 in 1 degree intervals. Dashed curves
represent theoretical values computed for the B2 model of Bullen and Haddon (1967), (but with \( \alpha_m = 13.86 \) km./sec.) and the SmKS ray parameters corresponding to model SKOR1.

Figure 3.7  Nomenclature for the Sagittal approximation to curvature effect.

Figure 3.8  Theoretical SKS/SKKS amplitude ratios and SKKS phase lags (relative to SKS) for the B2 model of Bullen and Haddon (1967) and the Jeffreys-Bullen travel times for SKS and SKKS (Travis 1965).

Figure 3.9  Theoretical SKS/SKKS amplitude ratios and SKKS phase lags (relative to SKS) for the CMB models of Balakina et al. (1966) and Buchbinder (1968).

Figure 3.10  Theoretical SKS/SKKS amplitude ratios as a function of period for CMB model #81 of Alexander and Phinney (1966).

Figure 3.11  Theoretical SKKS phase lags (relative to SKS) as a function of period for CMB model #81 of Alexander and Phinney (1966).

Figure 3.12  Theoretical SKS/SKKS amplitude ratios as a function of period for CMB model #94 of Alexander and Phinney (1966).
Figure 3.13  Theoretical SKKS phase lags (relative to SKS) as a function of period for CMB model #94 of Alexander and Phinney (1966).

Figure 3.14  Theoretical SKS/SKKS amplitude ratios as a function of period for the CMB model of Dorman et al. (1965).

Figure 3.15  Theoretical SKKS phase lags (relative to SKS) as a function of period for the CMB model of Dorman et al. (1965).
FIGURE 3.1
Figure 3.2

CMB S-Wave Incidence Angle (Deg)

DELTA (Deg)

SKS

SKKS

0 100 110 120 130

0 30 60 90
FIGURE 3.3
Figure 3.4: Graph showing the relationship between take-off angle (deg) and delta (deg) for different altitudes (H=0 km, H=500 km). The graph compares SKS and SKKS scenarios.
FIGURE 3.6

SKS/SKKS AMPLITUDE RATIO

- MEDIAN CURVE
- CMB TRANSFER FUNCTION

SKKS PHASE LAG (RELATIVE TO SKS)

DELTA (DEG)
\[(X-a)^2 + Y^2 = R^2\]
\[x^2 - 2aX + a^2 + Y^2 = R^2\]
\[a \text{ SMALL}\]
\[a^2 \approx 0\]
\[X \approx R\]
\[R^2 - 2aR + Y^2 \approx R^2\]
\[Y^2 = \lambda^2 \approx 2aR\]
\[\frac{a}{\lambda} \approx \frac{1}{2R}\]

**SAGGITAL APPROXIMATION**
Figure 3.8

SKS/SKKS AMPLITUDE RATIO

SKKS PHASE LAG (RELATIVE TO SKS)

DELTA (DEG)

PHASE LAG (DEG)
FIGURE 3.9

BALAKINA ET AL. (1966)

BUCHBINDER (1968)
FIGURE 3.11

SKKS PHASE LAG (RELATIVE TO SKS)

DELTA (DEG)

PERIOD (SEC)

1
2
10
30
Figure 3.14

SKS/SKKS Amplitude Ratio

DELTA (DEG)

Period (sec) 1, 2, 10, 30
4. **Inversion of the Outer Core Velocity Structure**

This chapter delineates our preliminary model for the outer core velocity structure which has been inverted to fit SmKS travel time and dT/dΔ observations. Since our study indicates that the true nature of the SKS travel time curve may be more complicated than previously thought, we have decided not to invert specific details of the velocity structure below a radius of about 2900 kilometers until a more sophisticated inversion incorporating PmKP data and an analysis for uniqueness has been completed. Instead, we present a detailed analysis of velocities in the core's outer 600 kilometers based on dT/dΔ observations and travel times corresponding to SKKS, SKKKS, SKKKS-SKKS and SKKS-SKS. The remaining portions of the core are constrained only by our SKKS-SKS time differences and by the absolute times of the first SKS arrivals. We should point out that the results presented in this chapter are subject to any errors inherent in the Jeffreys-Bullen ScS travel times used for "stripping" the mantle contributions from our observations.

4.1 **Constraints on the Velocity Structure for the Outer 600 Kilometers of the Core**

Figure 4.1 shows the total SmKS travel time data set. Those travel times corresponding to SKKS, SKKKS and the initial SKS onsets (usually referred to as the
AC branch) are shown in Figure 4.2. The SKS and SKKS data from Figure 4.2 were least squares fit with a second degree polynomial in delta and the derivative taken in order to obtain a ray parameter versus epicentral distance table for SKS and SKKS. These polynomials (valid for $90^\circ < \Delta < 135^\circ$) are given in equations 4.1 and 4.2

$$T_{SKS} = 1481.28 + 4.5926 (\Delta - 105^\circ) - 0.0337 (\Delta - 105^\circ)^2 \quad 4.1$$

$$T_{SKKS} = 1523.99 + 7.0314 (\Delta - 105^\circ) - 0.0125 (\Delta - 105^\circ)^2 \quad 4.2$$

where the RMS deviations are 1.84 and 2.28 seconds respectively for SKS and SKKS. We compare SKS and SKKS travel times corresponding to equations 4.1 and 4.2 to those of Hales and Roberts (1971) and the Jeffreys-Bullen Tables in Table 4.1. Since we have mainly used short period data, the fact that our travel times are generally shorter than those of Hales and Roberts (1971) may reflect the effect of instrument group delay.

We next "stripped" the mantle contributions from calculated epicentral distances using the relations presented in equations 4.3 and 4.4 and the Jeffreys-Bullen Tables for ScS:
This process provided us with ray parameter information for core distances between 15° and 120°. Since core-mantle boundary studies did not permit determination of the outer core velocity at this interface, a technique given by Randall (1970) was employed for this purpose. This technique assumes that \( \xi = 2\ln r/\ln a \) is constant throughout the region of interest, and that as a consequence, velocity follows the simple power law

\[
v = v_{cmb}(r/r_{cmb})^\alpha\ 
\]

(\text{where } \eta = r/v \text{ and } \alpha = \xi = r\Delta v/vdr = \text{constant}). Equations 4.5 and 4.6 (Randall, 1970) are derived under this assumption and solved simultaneously to provide estimates for \( v_{cmb} \).

\[
t_K = \left[2r_{cmb}/(1-\alpha)v_{cmb}\right] \sin(\Delta_K(1-\alpha)/2) \quad 4.5
\]

\[
p = dt_K/d\Delta_K = (\pi r_{cmb}/180 v_{cmb}) \cos(\Delta_K(1-\alpha)/2) \quad 4.6
\]

This approach was applied to SKKS(90°-130°) and SKS(90°) data on \( p, t_K, \text{ and } \Delta_K \). Values for \( v_{cmb} \) obtained by this method (assuming \( r_{cmb} = 3485 \) kilometers) were grouped between 8.006 and 8.011 km./sec. using data from SKKS.
(90°-100°), but then increased steadily as the ray bottoming depth increased. We show this trend in the results of Table 4.2.

The values of \( v_{\text{cmb}} \) which we have obtained are slightly higher than the values of 7.893 to 7.909 km./sec. obtained by Hales and Roberts (1971), but in good agreement with the recently derived velocities of 8.02 km./sec. (Jordan, 1973), 8.05 km./sec. (Toksoz et al., 1973) and 8.056 km./sec. (Qamar, 1973). It is important to note that similar to our results, Randall (1970) obtained a relatively high value for \( v_{\text{cmb}} \) (8.256 km./sec.) using SKS data. This observation suggests that the simple velocity power law assumption may only be valid in the outer few hundred kilometers of the core.

The \( p-\Delta_K \) curves obtained from equations 4.1 and 4.2 are compared to observed SKKS \( dT/d\Delta \) values from LASA and NORSAR in Figure 4.3. This diagram indicates that the \( p-\Delta_K \) curve relevant to the determination of compressional wave velocities in the upper 600 kilometers of the core possesses structure not contained in the limited information provided by travel times alone. In particular, our observations indicate the existence of a significant increase in the slope of the \( p-\Delta_K \) curve between core distances of about 35° to 40°. This region of increased \( p-\Delta_K \) slopes corresponds to SKKS arrivals between epicentral
distances of about 110° to 120°. Since the geometrical spreading effect is proportional to \( |d^2T/d\Delta^2| \) (Bullen, 1965), we anticipate that this slope change should be reflected in our amplitude data. Figure 3.3 shows that this is indeed the case. SKKS amplitudes increase rapidly with increasing epicentral distance, reach a maximum between distances of about 110° and 120° and then decrease. We conclude that this rapid slope increase in the p-\( \Delta_K \) curve is responsible for the amplitude ratio minimum in region C of Figure 3.6. Although this feature is localized between distances of about 116° to 120° in Figure 3.6, we believe that region C encompasses the approximate distance range 112°<\( \Delta \)<120° (the minimum being located near 117.5°), but that SKS focusing (discussed in section 4.3) is superimposed on this feature in region B (112°<\( \Delta \)<115°). As an independent check, we examined SKKKS/SKKS amplitude ratios. Preliminary results from this study shown in Figure 4.4 indicate that the SKKKS/SKKS amplitude ratio gradually increases with increasing delta until it reaches a maximum at distances of about 150° to 165°. This maximum location is consistent with the results for SKKS.

Using 8.01 km./sec. as a starting value for \( v_{cmb'} \), we next examined a suite of p-\( \Delta_K \) models in order to determine which ones would simultaneously fit our \( dT/d\Delta \) data and produce velocity structures consistent with observed
travel times. We found that the $p-\Delta_K$ curve shown in Figure 4.3 provided the best fit to both sets of data. The resulting velocity structure (shown in Figure 4.3) is characterized by a CMB velocity of $8.015 \text{ km./sec.}$ and a change in $dv/dr$ from about $-0.00155 \text{ sec}^{-1}$ for the outer 250 kilometers to a maximum of about $-0.0022 \text{ sec}^{-1}$ near a radius of 3175 kilometers. These results are consistent with those from the $v_{\text{cmb}}$ extrapolation, and the breakdown of the Randall technique for rays which bottom more than a few hundred kilometers below the CMB. We should point out that $p-\Delta_K$ models which provided a better fit to observed SKKS ray parameters less than about 6.2 sec./deg. produced core velocities too high to satisfy our travel time data for SKS and SKKS ($\Delta > 125^\circ$).

Although a detailed analysis similar to the study by Wiggins et al. (1973) should be made, a preliminary study indicates that the scatter in our SKS, SKKS, SKKKS, SKKKS-SKKS and SKKS-SKS times constrain the model of Figure 4.3 within about $\pm 0.02 \text{ km./sec}$ (subject to uncertainties in the Jeffreys-Bullen ScS times). We discuss the fit to these travel times in section 4.2.

4.2 Analysis of Core Model SKORl

Having obtained a velocity model for the outer 600 kilometers of the core, we combined our $p-\Delta_K$ data from section 4.1 with the SKS ray parameter data implied by
equation 4.1. The resulting $p-\Delta K$ curve was inverted using the Herglotz-Wiechert technique to obtain our base model for the outer core velocity structure. This velocity model (SKORl) is compared to other core models in Figure 4.5 and listed in Table 4.3. Except for the change in $dv/dr$ between radii of 3050 and 3250 kilometers derived in section 4.1, SKORl is similar to most of the other recently derived core models (particularly the B1 model of Jordan, 1973). As we discuss in section 4.3, however, SKORl may require revision at radii less than about 2700 kilometers in order to satisfy our total SKS data set.

We first considered the fit of SKORl to our absolute travel times. Figure 4.2 shows the comparison between theoretical SKS, SKKS and SKKKS travel times computed for SKORl and those observed in this study. The theoretical travel times shown in this diagram all fall within the limits of our data scatter, with SKS showing a slight tendency to be earlier than our observed times at distances greater than about 100° (bottoming radius less than about 2700 kilometers). Next, we compared theoretical SKKKS-SKKS and SKKS-SKS time differences computed for SKORl to our observed data. Figure 4.6 shows that these theoretical time differences fall within the scatter of our data as in the case of the absolute travel times.

Figure 4.7 shows the smoothed SKS/SKKS geometrical
spreading ratio computed for SKORl. We find that SKORl reproduces the amplitude ratio minimum at distances of about 117.5°. It should also be noted that the decrease in slope of our p-$\Delta K$ curve between 37°-$\Delta K$-42° produces a region of high SKS/SKKS amplitude ratios between 120°-$\Delta$-123° similar to our observations in region D of Figure 3.6. Computed slopes of $\ln(A_{SKS}/A_{SKKS})$ versus frequency plots are shown in Figure 4.8. This diagram indicates that the effect of attenuation may differ for SKS and SKKS. Consequently, differences in SKS and SKKS attenuation, and possibly tunnelling effects (Richards, 1973) will have to be considered before theoretical amplitude ratios such as those given in Figure 4.7 can be placed in proper perspective to observed ratios.

As an independent check, we computed theoretical PKP travel times for model SKORl and the mantle P wave velocities from the Bl model of Jordan (1973). These theoretical travel times are compared to the PKP times of Herrin (1968) in Table 4.4. We find that theoretical and observed PKP$_{AB}$ times agree within 0.5 seconds for epicentral distances of 170° to 150° (bottoming radius greater than about 2250 kilometers), but that our times are increasingly too long for rays bottoming below this radius. We also find that our PKP focus location of 145.6° is slightly larger than the value of 144.2°
determined by Shahidi (1968). However, we do not purport to have constrained velocities in the lower portions of the outer core at this time.

4.3 Evidence for Additional SKS Travel Time Branches

The data presented in this section represent preliminary observations indicative of additional SKS travel time branches. As such, they are presented at this time in order to provide insight into areas requiring future study.

Figure 4.9 shows a comparison between the p-\(\Delta\) curve implied by SKOR1 and our observed SKS ray parameters. The data shown in this diagram indicates that the SKS travel time curve may possess "secondary" branches in addition to the AC-branch. Figure 4.1 shows an alignment of arrivals between SKS and SKKS. Similar observations were made by Gutenberg and Richter (1936). These travel times may correlate with our \(dT/d\Delta\) observations between 5.0 and 5.5 sec./deg. Figure 4.10 shows a detailed section of our travel time data for \(85^\circ < \Delta < 105^\circ\). It is important to note that the triangles shown in this diagram represent travel times for arrivals observed after the initial SKS onset for a given seismogram. This diagram shows that secondary SKS arrivals form a cusp at distances of about 92°.
Figure 4.11 shows a detailed section of our data at distances where other "secondary" SKS arrivals have been observed. This diagram illustrates one alignment of arrivals which have a slope of about 4.0 sec./deg. These data may correlate with the p-Δ structure observed at this value of wave slowness in Figure 4.9. Figures 3.1 and 3.6 illustrate the presence of high SKS amplitudes and SKS-SKKS phase shifts indicative of SKS focusing between distances of 112° and 115°. The \( \frac{dT}{dA} \) observations near 4.0 sec./deg. shown in Figure 4.9 suggest that this focusing may be associated with the cusp of the proposed SKS branch.

Figure 4.11 also shows an alignment of arrivals with a slope of about 3.1 sec./deg. These data may be the SKS equivalent of the precursors to PKP. Figure 4.2 shows several examples of the observed "secondary" SKS arrivals.

4.4 Conclusions From The Outer Core Velocity Study

We have obtained a preliminary model for the outer core velocity structure based on absolute and differential SmKS travel times. The main emphasis of this study was that of constraining the outer 600 kilometers of the core. We found that compressional wave velocities
start from a value of about 8.015 km/sec. at the CMB and increase with a nearly constant rate in the outer 250 kilometers of the core. The rate of velocity increase with increasing depth is significantly higher between radii of about 3250 and 3050 kilometers, however.

We inverted an outer core velocity model (SKOR1) by using our results for the outer 600 kilometers and constraining the remaining depths with our travel times for initial SKS onsets. SKOR1 possesses a positive perturbation in compressional wave velocity (relative to other recent core models) centered at a radius of about 2500 kilometers. This feature is very similar to the results given in the B1 model of Jordan (1973).

Section 4.3 of this chapter provided initial evidence which indicates the existence of additional SKS travel time branches. We are currently examining a combined PmKP and SmKS data set in an effort to arrive at a consistent explanation for these "secondary" arrivals.
### TABLE 4.1

COMPARISON OF SKS AND SKKS TRAVEL TIMES

(DEPTH OF FOCUS = 33 KM)

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<thead>
<tr>
<th>DISTANCE</th>
<th>EQUATION 4.1</th>
<th>HALES AND ROBERTS</th>
<th>J-B</th>
<th>EQUATION 4.2</th>
<th>HALES AND ROBERTS</th>
<th>J-B</th>
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<td>90°</td>
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<td>1664.0</td>
<td>1657.9</td>
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<td>$r_{\text{bottoming}}$ (km)</td>
<td>$V_{CMB}$ (km/sec)</td>
<td>$\alpha$</td>
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TABLE 4.3
MODEL SKOR1

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<tr>
<th>RADIUS (km)</th>
<th>VELOCITY (km/sec)</th>
<th>RADIUS (km)</th>
<th>VELOCITY (km/sec)</th>
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<td>HERRIN (1968)</td>
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FIGURE CAPTIONS

Figure 4.1 Observed SKS, SKKS and SKKKS Travel Times

Figure 4.2 Travel times for SKKS, SKKKS and first arrivals of SKS. Fit of theoretical travel times for SKOR1 to these data.

Figure 4.3 Observed SKKS dT/dΔ values and p-ΔK curve for outer 600 kilometers of core. Bars on ray parameters indicate 95% confidence limits. Diagram also shows inverted velocity structure for outer 600 kilometers of core.

Figure 4.4 Observed SKKKS/SKKS amplitude ratios.

Figure 4.5 Comparison of velocity model SKOR1 to other recently published models (Hales and Roberts, 1971; Toksoz et al., 1973; Jordan, 1973; Qamar, 1973).

Figure 4.6 Fit of theoretical SKKKS-SKKS and SKKS-SKS time differences computed for SKOR1 to observed data from this study.

Figure 4.7 Theoretical SKS/SKKS geometrical spreading ratios (smoothed) computed for core model SKOR1.

Figure 4.8 Observed slopes from plots of ln (A_{SKS}/A_{SKKS}) versus frequency.
Figure 4.9  Comparison between observed SKS $dT/d\Delta$ values and the SKS $p-\Delta$ curve calculated for core model SKOR1. Bars on ray parameters indicate 95% confidence limits.

Figure 4.10  Detailed section of observed travel times at distances between 85° and 105°. Triangles indicate travel times for onsets observed after the first SKS arrival on a given seismogram.

Figure 4.11  Detailed section of observed travel times showing "secondary" SKS arrivals. Triangles indicate travel times for onsets observed after the first SKS arrival on a given seismogram. Dashed lines indicate a preliminary interpretation of these "secondary" arrivals.
Figure 4.2

OBSERVED TRAVEL TIMES

TRAVEL TIMES COMPUTED FOR SKORI

DELTA (DEG)

TRAVEL TIME (MIN)
FROM OUR DATA OF HALES AND ROBERTS (1971)

**Figure 4.4**

- **SKKKS/SKKS FROM OUR DATA**
- **SKKKS/SKKS FROM DATA OF HALES AND ROBERTS (1971)**
Figure 4.7: SKS/SKKS Geometrical Spreading Ratio vs. Delta (Deg)
SLOPE OF $\ln(\frac{A_{SKS}}{A_{KS}})$ VS. FREQUENCY

DELTA (DEG)

FIGURE 4.8
Figure 4.9

**Ray Parameter Curve from SKORI**

- **OBSERVED RAY PARAMETER**
- **SKS P-Δ CURVE FROM SKORI**

Ray Parameter (sec/deg) vs. Delta (deg)
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from travel times of successive arrivals of PKP
Appendix. Reflection and Transmission Coefficients

The nomenclature used in the following derivation of the CMB reflection and transmission response is provided in Figure A1 and Table A1. Coefficients pertinent to our discussion of SKS and SKKS arrivals include:

1) P-P reflection coefficient for a fluid-solid interface

\[ R_{pp} = \frac{\Delta n'}{\Delta n} \]

2) P-S transmission coefficient for a fluid-solid interface

\[ T_{ps} = \frac{\omega_l''}{\Delta n} \]

3) S-P transmission coefficient for a solid-fluid interface

\[ T_{sp} = \frac{\Delta n'}{\omega_l'} \]

A1 The P-P Reflection Coefficient

Transverse wave potentials vanish in the fluid half space. In addition, there will be no potentials corresponding to incident longitudinal and transverse waves in the solid half space. Consequently,

\[ \Delta l' = \omega_{l'} = \omega_{n'} = \omega_{n''} = 0 \]

Using the notation of Haskell (1953), and letting

\[ A = a_{n-1} \cdot a_{n-2} \cdot \ldots \cdot a_2 \] we have:
\[
\begin{bmatrix}
\dot{\mathbf{u}}_n \mathbf{c} \\
\dot{\mathbf{w}}_n \mathbf{c} \\
\sigma_{n-1} \\
\tau_{n-1}
\end{bmatrix} = \mathbf{A} \cdot 
\begin{bmatrix}
\dot{\mathbf{u}}_1 \mathbf{c} \\
\dot{\mathbf{w}}_1 \mathbf{c} \\
\sigma_1 \\
\tau_1
\end{bmatrix}
\]  

Since \( \tau_{n-1} = 0 \) and \( u_{n-1} \) does not have to be continuous (because the \( n \)th layer is fluid), we have the following relationship:

\[
\begin{bmatrix}
\dot{\mathbf{u}}_n \mathbf{c} \\
\dot{\mathbf{w}}_n \mathbf{c} \\
\sigma_{n-1} \\
0
\end{bmatrix} = \mathbf{E}_n \cdot 
\begin{bmatrix}
\Delta'_n \mathbf{c} + \Delta''_n \mathbf{c} \\
\Delta'_n \mathbf{c} - \Delta''_n \mathbf{c} \\
0 \\
0
\end{bmatrix}
\]  

But now \( E_n \) must be appropriately modified to account for the fluid nature of the \( n \)th layer. First consider the equations for \( \dot{\mathbf{w}}_n \mathbf{c} \) and \( \sigma_{n-1} \).

\[
\begin{bmatrix}
\dot{\mathbf{w}}_n \mathbf{c} \\
\sigma_{n-1}
\end{bmatrix} = 
\begin{bmatrix}
0 & -(\alpha_n \mathbf{c})^2 r_{an} \\
-\rho_n \alpha_n^2 (\gamma_{n-1}) & 0
\end{bmatrix} \cdot 
\begin{bmatrix}
\Delta'_n \mathbf{c} + \Delta''_n \mathbf{c} \\
\Delta'_n \mathbf{c} - \Delta''_n \mathbf{c}
\end{bmatrix}
\]  

\[
\begin{bmatrix}
0 \\
-\rho_n \alpha_n^2 (\gamma_{n-1})(\Delta'_n \mathbf{c} + \Delta''_n \mathbf{c})
\end{bmatrix} = 
\begin{bmatrix}
0 & -(\alpha_n \mathbf{c})^2 r_{an} \\
-\rho_n \alpha_n^2 (\gamma_{n-1}) & 0
\end{bmatrix} \cdot 
\begin{bmatrix}
\Delta'_n \mathbf{c} + \Delta''_n \mathbf{c} \\
\Delta'_n \mathbf{c} - \Delta''_n \mathbf{c}
\end{bmatrix}
\]  

Al.3
We next note that $\gamma_n = 0$ (since $\gamma = 2(\beta/c)^2$). Therefore,

$$
\begin{bmatrix}
    \cdot \\
    w_{n-1}/c \\
    \sigma_{n-1}
\end{bmatrix}
= 
\begin{bmatrix}
    -\left(\frac{\alpha_n}{c}\right)^2 r_{an} (\Delta_n' - \Delta_n'') \\
    \rho_n \alpha_n^2 (\Delta_n' + \Delta_n'')
\end{bmatrix}
\text{ Al.4}
$$

According to equation 2.12 of Haskell (1953), we have the following:

$$
\begin{bmatrix}
    \cdot \\
    u_1/c \\
    w_1/c \\
    \sigma_1 \\
    \tau_1
\end{bmatrix}
= E_2 \cdot 
\begin{bmatrix}
    \Delta_2' + \Delta_2'' \\
    \Delta_2' - \Delta_2'' \\
    \omega_2' - \omega_2'' \\
    \omega_2' + \omega_2''
\end{bmatrix}
\text{ Al.5}
$$

However, $u, w, \sigma, \text{ and } \tau$ are continuous from layer 1 to layer 2. Figure A2 and the definitions of the D and E matrices given in Haskell (1953) show us that

$$
E_1 = \lim_{d \to 0} D_1.
$$

Consequently,

$$
E_2 \cdot 
\begin{bmatrix}
    \Delta_2' + \Delta_2'' \\
    \Delta_2' - \Delta_2'' \\
    \omega_2' - \omega_2'' \\
    \omega_2' + \omega_2''
\end{bmatrix}
= E_1 
\begin{bmatrix}
    \Delta_1'' \\
    -\Delta_1'' \\
    -\omega_1'' \\
    \omega_1''
\end{bmatrix}
\text{ Al.6}
$$
Therefore,

\[
\begin{pmatrix}
    \dot{u}_1/c \\
    \dot{w}_1/c \\
    \sigma_1 \\
    \tau_1
\end{pmatrix}
= E_1 \cdot
\begin{pmatrix}
    \Delta_1'' \\
    -\Delta_1'' \\
    -\omega_1'' \\
    \omega_1''
\end{pmatrix}
\]

We now let \( B = AE_1 \) and substitute Al.7 into Al.1 to obtain:

\[
\begin{pmatrix}
    \dot{u}_{n-1}/c \\
    \dot{w}_{n-1}/c \\
    \sigma_{n-1} \\
    0
\end{pmatrix}
= B \cdot
\begin{pmatrix}
    \Delta_1'' \\
    -\Delta_1'' \\
    -\omega_1'' \\
    \omega_1''
\end{pmatrix}
\]

We need only three equations to determine \( R_{pp} \). Al.4 and the last three rows of Al.8 give

\[
\begin{pmatrix}
    -(\alpha_n/c)^2 r_{an}(\Delta_n' - \Delta_n'') \\
    \rho_n \alpha_n^2 (\Delta_n' + \Delta_n'') \\
    0
\end{pmatrix}
= \begin{pmatrix}
    B_{21} & B_{22} & B_{23} & B_{24} \\
    B_{31} & B_{32} & B_{33} & B_{34} \\
    B_{41} & B_{42} & B_{43} & B_{44}
\end{pmatrix}
\begin{pmatrix}
    \Delta_1'' \\
    -\Delta_1'' \\
    -\omega_1'' \\
    \omega_1''
\end{pmatrix}
\]

Al.9
Expanding Al.9 provides the three required equations:

\[-(\alpha_n/c)^2 r_{an} (\Delta_n' - \Delta_n'') = \Delta_1'' (B_{21} - B_{22}) + \omega_1'' (B_{24} - B_{23}) \quad \text{Al.10}\]

\[\rho_n \alpha_n^2 (\Delta_n' + \Delta_n'') = \Delta_1'' (B_{31} - B_{32}) + \omega_1'' (B_{34} - B_{33}) \quad \text{Al.11}\]

\[0 = \Delta_1'' (B_{41} - B_{42}) + \omega_1'' (B_{44} - B_{43}) \quad \text{Al.12}\]

Equation Al.12 is easily solved for \(\omega_1''\) to give

\[\omega_1'' = \frac{-\Delta_1'' (B_{41} - B_{42})}{(B_{44} - B_{43})} \quad \text{Al.13}\]

After substituting Al.13 into Al.10 and Al.11 we have

\[-(\alpha_n/c)^2 r_{an} (\Delta_n' - \Delta_n'') = \frac{\Delta_1}{(B_{44} - B_{43})} \left[ (B_{21} - B_{22}) (B_{44} - B_{43}) - (B_{41} - B_{42}) (B_{24} - B_{23}) \right] \quad \text{Al.14}\]

and

\[\rho_n \alpha_n^2 (\Delta_n' + \Delta_n'') = \Delta_1'' \frac{[ (B_{31} - B_{32}) (B_{44} - B_{43}) - (B_{41} - B_{42}) (B_{34} - B_{33}) ]}{(B_{44} - B_{43})} \quad \text{Al.15}\]

Now let

\[F_1 = (B_{21} - B_{22}) (B_{44} - B_{43}) - (B_{41} - B_{42}) (B_{24} - B_{23})\]

\[F_2 = (B_{31} - B_{32}) (B_{44} - B_{43}) - (B_{41} - B_{42}) (B_{34} - B_{33})\]

Upon cancellation of the common term \(B_{44} - B_{43}\), Al.14 and Al.15 allow us to eliminate \(\Delta_1''\) to give:

\[\frac{-(\alpha_n/c)^2 r_{an} (\Delta_n' - \Delta_n'')}{F_1} = \frac{\rho_n \alpha_n^2 (\Delta_n' + \Delta_n'')}{F_2} \quad \text{Al.16}\]
Consequently,
\[
\Delta_n'\cdot [F_2(\alpha_n/c)^2 r_{\alpha n} - F_1 \rho_n \alpha_n^2] = \Delta_n' [F_2(\alpha_n/c)^2 r_{\alpha n} + F_1 \rho_n \alpha_n^2]
\]
A1.17

The desired reflection coefficient is obtained after cancelling \(\alpha_n^2\):
\[
R_{pp} = \frac{\Delta_n'}{\Delta_n''} = \frac{-F_1 \rho_n + F_2 \frac{r_{\alpha n}}{c^2}}{F_1 \rho_n + F_2 \frac{r_{\alpha n}}{c^2}}
\]
A1.18

A2 The P-S Transmission Coefficient

The boundary conditions on the potentials for this case are identical to those for the previous reflection coefficient. Equation A1.12 implies the following identity:
\[
\Delta_1''' = -\frac{\omega_1'''(B_{44}'-B_{43}')}{(B_{41}'-B_{42}')}
\]
A2.1

Equation A2.1 is next substituted into A1.10 and A1.11 to give
\[
-(\alpha_n/c)^2 r_{\alpha n}(\Delta_n' - \Delta_n''') = \omega_1'' \frac{(B_{24}'-B_{23}') (B_{41}'-B_{42}') - (B_{44}'-B_{43}') (B_{21}'-B_{22}')}{(B_{41}'-B_{42}')}
\]
\[
= \frac{-F_1 \omega_1}{(B_{41}'-B_{42}')}
\]
A2.2

And
\[
\rho_n \alpha_n^2(\Delta_n' + \Delta_n''') = \omega_1'' \left[ \frac{(B_{34}'-B_{33}') (B_{41}'-B_{42}') - (B_{44}'-B_{43}') (B_{31}'-B_{32}')}{B_{41}'-B_{42}'} \right]
\]
\[
= \frac{-F_2 \omega_1}{(B_{41}'-B_{42}')}
\]
A2.3
Equation A2.2 implies that

$$\Delta_n' = \Delta_n'' + \frac{F_1 \omega_1''}{(\alpha_n/c)^2 r \alpha_n (B_{41} - B_{42})}$$  \hspace{1cm} A2.4

and

$$\Delta_n' = -\Delta_n'' - \frac{F_2 \omega_1''}{\rho_n \alpha_n^2 (B_{41} - B_{42})}$$  \hspace{1cm} A2.5

We can now eliminate $\Delta_n'$ from A2.4 and A2.5 to obtain

$$2\Delta_n'' = \frac{-\omega_1''}{(B_{41} - B_{42})} \left[ \frac{F_1}{(\alpha_n/c)^2 r \alpha_n} + \frac{F_2}{\rho_n \alpha_n^2} \right]$$  \hspace{1cm} A2.6

Consequently (after cancelling $\alpha_n^2$) we have the desired transmission coefficient:

$$T_{ps} = \frac{\omega_1''}{\Delta_n''} = \frac{-2(B_{41} - B_{42})(r \alpha_n/c^2) \alpha_n^2 \rho_n}{\rho_n F_1 + (r \alpha_n/c^2) F_2}$$  \hspace{1cm} A2.7

A3 The S-P Transmission Coefficient

The wave potential boundary conditions are now different from those of the previous two derivations. As before, transverse wave potentials vanish in the fluid half space. In this case, however, there will be no potentials corresponding to incident longitudinal waves in the fluid or solid half spaces. Consequently,

$$\Delta_1' = \Delta_n'' = \omega_n' = \omega_n'' = 0$$
This modification implies that A1.2 becomes

\[
\begin{bmatrix}
\dot{u}_{n-1/c} \\
\dot{w}_{n-1/c} \\
\sigma_{n-1} \\
0 \\
\end{bmatrix}
= \begin{bmatrix}
\Delta_n' \\
\Delta_n' \\
0 \\
0 \\
\end{bmatrix}
\]

A3.1

and A1.4 becomes

\[
\begin{bmatrix}
\dot{w}_{n-1/c} \\
\sigma_{n-1} \\
\end{bmatrix}
= \begin{bmatrix}
-(\alpha_n/c)^2 r_{an}(\Delta_n') \\
\rho_n \alpha_n^2 (\Delta_n') \\
\end{bmatrix}
\]

A3.2

Similarly, equations A1.7 and A1.8 are modified to the following form:

\[
\begin{bmatrix}
\dot{u}_1/c \\
\dot{w}_1/c \\
\sigma_1 \\
\tau_1 \\
\end{bmatrix}
= \begin{bmatrix}
\Delta_1'' \\
-\Delta_1'' \\
\omega_1'' - \omega_1'' \\
\omega_1'' + \omega_1'' \\
\end{bmatrix}
\]

A3.3

and

\[
\begin{bmatrix}
\dot{u}_{n-1}/c \\
\dot{w}_{n-1}/c \\
\sigma_{n-1} \\
0 \\
\end{bmatrix}
= \begin{bmatrix}
\Delta_1'' \\
-\Delta_1'' \\
\omega_1'' - \omega_1'' \\
\omega_1'' + \omega_1'' \\
\end{bmatrix}
\]

A3.4
Therefore,

\[
\begin{bmatrix}
-(\alpha_n/c)^2 r_{an} \Delta_n'
\rho_n^2 \Delta_n'
0
\end{bmatrix}
= \begin{bmatrix}
B_{21} & B_{22} & B_{23} & B_{24} \\
B_{31} & B_{32} & B_{33} & B_{34} \\
B_{41} & B_{42} & B_{43} & B_{44}
\end{bmatrix}
\begin{bmatrix}
\Delta_1'' \\
-\Delta_1'' \\
\omega_1'-\omega_1'' \\
\omega_1'+\omega_1''
\end{bmatrix}
\]

Expanding A3.5 produces three equations which can be solved for the desired potential relationships.

\[
\Delta_1''(B_{41}-B_{42}) + \omega_1'(B_{43}+B_{44}) - \omega_1''(B_{43}-B_{44}) = 0 \tag{A3.6}
\]

\[-(\alpha_n/c)^2 r_{an} \Delta_n' = \Delta_1''(B_{21}-B_{22}) + \omega_1'(B_{23}+B_{24}) - \omega_1''(B_{23}-B_{24}) \tag{A3.7}
\]

\[
\rho_n^2 \Delta_n' = \Delta_1''(B_{31}-B_{32}) + \omega_1'(B_{33}+B_{34}) - \omega_1''(B_{33}-B_{34}) \tag{A3.8}
\]

Equation A3.6 produces the following identity for \( \omega_1'' \):

\[
\omega_1'' = \frac{\Delta_1''(B_{41}-B_{42}) + \omega_1'(B_{43}+B_{44})}{(B_{43}-B_{44})} \tag{A3.9}
\]

A3.9 can now be substituted into A3.6 and A3.7 to give

\[
\rho_n^2 \Delta_n' = \frac{-\Delta_1''F_2 + \omega_1'F_3}{(B_{43}-B_{44})} \tag{A3.10}
\]

and

\[-(\alpha_n/c)^2 r_{an} \Delta_n' = \frac{-\Delta_1''F_4 + \omega_1'F_4}{(B_{43}-B_{44})} \tag{A3.11}
\]
where
\[ F_3 = 2(B_{34}B_{43} - B_{33}B_{44}) \]
\[ F_4 = 2(B_{24}B_{43} - B_{23}B_{44}) \]

Elimination of \( \Delta_1'' \) from equations A3.10 and A3.11 provides the following relationship:
\[
\frac{-(B_{43}B_{44})\rho_n\alpha_n^2\Delta_1'' + \omega_1 F_3}{F_2} = \frac{(B_{43}B_{44})(\alpha_n/c)^2 r_{\alpha n} \Delta'_n + \omega_1 F_4}{F_1}
\]
A3.12

Consequently,
\[
T_{sp} = \frac{\Delta_1'}{\omega_1} = \frac{F_1 F_3 - F_2 F_4}{(B_{43}B_{44}) [\rho_n\alpha_n^2 F_1 + (\alpha_n/c)^2 r_{\alpha n} F_2]}
\]
A3.13

A4 Conversion to Energy

Although most calculations are made for single interface core-mantle boundary models, the general case of multilayer transition boundaries has been considered. The work done per unit time per unit area of interface by the incident and reflected waves at the \((n-1)\)th interface of such a model is given by the following equation (Haskell, 1962):
\[
\varepsilon_{n-1} = R_{\varepsilon} \left[ u_{n-1}^* v_{n-1} + w_{n-1}^* \theta_{n-1}^* \right]
\]
A4.1

Following Haskell's derivation (his equations 23-25) based on radiation conditions, we obtain the following equivalent to A4.1:
\[
\varepsilon_{n-1} = (\rho_n\alpha_n^4/c)(|\Delta_n'''|^2 - |\Delta_n''|^2)R_{\varepsilon} r_{\alpha n}
\]
\[
+ 4(\rho_n\beta_n^4/c)(|\omega_n'''|^2 - |\omega_n''|^2)r_{\beta n}
\]
A4.2
The terms in $|A_n'|^2$ and $|\omega_n'|^2$ represent power fluxes due to incident P and SV waves respectively across a unit area of interface. Likewise, the power carried off by the reflected P and SV waves is given by the terms in $|A_n|^2$ and $|\omega_n|^2$.

No mode conversion occurs in the case of P to P reflection in the outer core. Consequently, $A_{1.18}$ also represents the square root of the incident to reflected P-wave energy ratio. Mode conversion takes place during both of the transmissions, however. Equation B2 enables us to obtain the factors prerequisite to converting $T_{PS}$ and $T_{SP}$ to square root of energy ratios.

Consider the case of $n = 2$ in Figure A2. An arbitrary interface (labeled "0" and dashed in this figure) is constructed in the homogeneous solid half space. The energy incident on this imaginary interface is equal to the energy transmitted through the first interface. Therefore, equation A4.2 can be employed to obtain the following relationships for a P-wave incident in the fluid half-space:

$$\varepsilon_0 = \rho_1 \alpha_1^4/c (|\Delta_1'|^2 - |\Delta_1|^2) Re(r_{a1}) + 4(\rho_1 \beta_1^4/c)(|\omega_1'|^2 - |\omega_1|^2) r_{\beta 1}$$

$$\varepsilon_1 = \rho_2 \alpha_2^4/c (|\Delta_2'|^2 - |\Delta_1|^2) Re(r_{a1}) + 4(\rho_2 \beta_2^4/c)(|\omega_2'|^2 - |\omega_2|^2) r_{\beta 2}$$

These two equations provide the necessary relationship between incident and transmitted energy. This relationship is given in equation A4.5:
\[
\frac{\varepsilon_{OS}'''}{\varepsilon_{1P}'''} = \frac{4\rho_1 \beta_1^4 r_{\beta_1} |\omega_1'''|^2}{\rho_2 \alpha_2^4 r_{\alpha_2} |\Delta_2'''|^2}
\]

Consequently, the square root of the energy ratio for the case of P to S transmission is given by

\[
\left[ \varepsilon_{T_{PS}} \right]^{1/2} = \frac{2\beta_1^2}{\alpha_2^2} \left[ \frac{\rho_1}{\rho_2} \frac{r_{\beta_1}}{r_{\alpha_2}} \right]^{1/2} T_{PS}
\]

Similarly, the square root of energy ratio for the case of S to P transmission is given by

\[
\left[ \varepsilon_{T_{SP}} \right]^{1/2} = \frac{\alpha_2^2}{2\beta_1^2} \left[ \frac{\rho_2}{\rho_1} \frac{r_{\alpha_2}}{r_{\beta_1}} \right]^{1/2} T_{SP}
\]

As anticipated (because of reciprocity), \( \varepsilon_{T_{PS}} = \varepsilon_{T_{SP}} \) for a given value of wave slowness.
TABLE A1

\( \rho_n = \) density

\( d_n = \) thickness

\( \gamma_n, \mu_n = \) Lamé elastic constants

\( \alpha_n = \) P-wave velocity

\( \beta_n = \) S-wave velocity

\( K = p/c = \) wave number

\[
  r_{\alpha n} = \begin{cases} 
    +[(c/\alpha_n)^2 - 1]^{1/2} & c > \alpha_n \\
    -i[1 - (c/\alpha_n)^2]^{1/2} & c < \alpha_n 
  \end{cases}
\]

\[
  r_{\beta n} = \begin{cases} 
    +[(c/\beta_n)^2 - 1]^{1/2} & c > \beta_n \\
    -i[1 - (c/\beta_n)^2]^{1/2} & c < \beta_n 
  \end{cases}
\]

\( \gamma_n = 2(\beta_n/c)^2 \)

\( u, w = \) displacement components in x and z directions

\( \sigma = \) normal stress

\( \tau = \) tangential stress
FIGURE CAPTIONS

Figure A1  Notation for wave potentials used in deriving CMB reflection and transmission coefficients

Figure A2  Model notation used in deriving energy relations
FIGURE AI
FIGURE A2