Measurement That Transcends Time:
A Lebesgue Integral Approach to Existential Sentences

by

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ABSTRACT

In the study of natural language semantics, sentences that assert the existence of entities predicated of by noun phrases have traditionally been analyzed with simple usage of the existential quantifier. In this thesis, I challenge this standard approach through discussion of sentences whose semantics cannot be correctly captured in this manner, and develop an alternative, novel approach that employs Lebesgue integration.

The first chapter is centered around Musan's (1995) generalization that states that the temporal (situational) interpretation of a non-presuppositional (i.e. existential) noun phrase is obligatorily dependent on that of the main predicate. It defends the view that the situational dependence is obtained by virtue of being in the scope of some operator and argues that in order to obtain the correct interpretation of plural non-presuppositional noun phrases, the numeral part of non-presuppositional noun phrases must be separated and interpreted above the said operator.

The second, and last chapter incorporates the result of the former chapter into Krifka's (1990) analysis of the readings of existential sentences which Krifka terms event-related readings. After we observe that Musan's generalization is extended to the situational interpretation of units of measurement, it becomes evident that a proper semantic analysis of sentences that describe continuous production or consumption of mass entities requires the capability of treating infinitesimally small time intervals. This leads to a new theory where the truth conditions of an existential sentence are expressed as a condition on the value of the Lebesgue integral of an appropriate function defined on situations calculated over the set of all situations whose projections onto the time axis are contained in a context time interval. The theory makes a fundamental connection between temporal (situational) interpretation and existential assertion. Furthermore, our natural intuition of a dichotomy between discrete (telic) events and continuous (atelic) events are captured by the decomposition of the measure used in natural language semantics into modified versions of the counting measure and the Lebesgue measure.

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Prelude

Many sentences in natural language make an existential assertion. For instance, (1) asserts the existence of an individual predicated truly of by the noun *20-year-old*.

(1) Mary kissed a 20-year-old.

On the traditional, standard approach to the semantics of such sentences is make simple use of the existential quantifier ($\exists$). Thus, (1) is usually analyzed as follows:

(2) $\exists x[\text{20-yr-old}(x) \wedge \text{kiss}(x)(\text{Mary})]$

The main thesis that I put forward is that existential sentences like (1) should not actually be analyzed with such simple-minded use of the existential quantifier. In the dissertation, I propose a new semantic theory to replace the traditional approach where the truth conditions of an existential sentence are expressed as a condition of the value of a Lebesgue integral of an appropriate function from situations into nonnegaitve reals.

One empirical problem that has not been explained before and that is treated here is the precise semantics of sentences that describe continuous production or consumption of mass entities.

Imagine the following scenario. Our factory has a machine called Machine P which is capable of producing liquid XYZ. Machine P is connected to a tank into which the XYZ produced by Machine P is poured. The tank is further connected to a machined called Machine C that converts XYZ into another liquid called ZYX. At the beginning of yesterday, the tank had exactly 1 liter of XYZ, and this was all the XYZ in the world at that point. At 9 a.m. of yesterday, we ran Machine P, which started producing 0.3 liters of XYZ per second. At the same time, we also ran Machine C, which started converting 0.3 liters of XYZ into 0.2 liters of ZYX per second. We stopped both machines at 5 a.m. of yesterday. Then, since Machine P was running for $60 \times 60 \times 8 = 28800$ seconds in total, we are confident to report our factory's achievement yesterday with the following sentence:

(3) Machine P produced 8640 liters of XYZ yesterday.

Of course, this figure has been calculated as follows:

(4) $0.3 \times 28800 = 8640$

Consequently, we gained $8640 \times (0.2/0.3) = 5760$ liters of ZYX in the end yesterday by the conversion of Machine C.

What are the truth conditions of sentence (3), then? The standard approach would propose something like the following as its truth conditions:

(5) $\exists x[\mathsf{XYZ}(x) \wedge \mathsf{liter}(x) = 8640 \wedge \mathsf{produce}(x)(\mathsf{Machine\ P})]$

Is this really the correct analysis of (3)? That is very unlikely to be true. Since the volume of XYZ that Machine P produced per unit time and the volume of XYZ that Machine C consumed per unit time were exactly the same, the volume of the XYZ kept in the tank must have been exactly 1 liter throughout yesterday. Since this was assumed to be all the XYZ in the world, there was no time point in yesterday when 8640 liters of XYZ existed in the world. How, then, can (3) be truthfully asserted in this situation?

It turns out that the semantics of sentences like (3) cannot be captured without recourse to integration, especially when we think of the possibility that the size of an individual may change as time goes by. The present dissertation develops this theory step by step, starting from discussion of Musan's (1995) generalization. In Chapter 1, we will first see that the numeral part of non-presuppositional noun phrases must be raised at LF for the correct interpretation of plural non-presuppositional noun phrases. In Chapter 2, the results of Chapter 1 is incorporated into Krifka's (1990) analysis of event-related readings. By observing that Musan's generalization is extended to units of measurement, the necessity for integration becomes evident at last. The rest of the dissertation then introduces the theory of Lebesgue integration and builds a general theory that encompasses the analyses of measure phrases like *three kilograms* and temporal measuring expressions like *three times* and *for three hours* that works for discrete (telic) and continuous (atelic) events uniformly.

0.1 Preliminaries

I assume the following semantic types:

(6) Semantic types

e	individuals
w	possible worlds
i	intervals
s	situations
v	events
n	real numbers

Since n is the type of real numbers, $D_n = \mathbb{R}$.

Time intervals are nonempty convex sets of time points. We assume that the time line is isomorphic to the real line $\mathbb{R}$. Thus,

(7) $$\begin{aligned} D_i = {} & \{(a,b) \mid a,b \in \mathbb{R} \wedge a < b\} \cup \{(a,b] \mid a,b \in \mathbb{R} \wedge a < b\} \\ & \cup \{[a,b) \mid a,b \in \mathbb{R} \wedge a < b\} \cup \{[a,b] \mid a,b \in \mathbb{R} \wedge a \leq b\}, \end{aligned}$$

where $(a,b) = \{p \mid p \in \mathbb{R} \wedge a < p < b\}$, $(a,b] = \{p \mid p \in \mathbb{R} \wedge a < p \leq b\}$, $[a,b) = \{p \mid p \in \mathbb{R} \wedge a \leq p < b\}$ and $[a,b] = \{p \mid p \in \mathbb{R} \wedge a \leq p \leq b\}$,

For each semantic type, we can think of the corresponding lattice obtained as the closure under the join and meet operations of entities of the semantic type.

For a semantic type σ, $\sqcup_\sigma$ denotes the join operation. Thus, if $\mathsf{John} \in D_e$ and $\mathsf{Mary} \in D_e$, then $\mathsf{John} \sqcup_e \mathsf{Mary}$ is also an individual that belongs to D_e. Similarly, $\sqcap_\sigma$ denotes the meet operation for semantic type σ.

Note that given a time interval $I = [a,b]$ and $J = [b,c]$, since time intervals are sets of time points, $I \cup J = [a,b] \cup [b,c] = [a,c]$. On the other hand, $I \sqcup_i J = [a,b] \sqcup_i [b,c]$, and thus $I \cup J \neq I \sqcup_i J$. The reader should not confuse these two.

The order given to the lattice for semantic type σ is denoted by $\sqsubseteq_\sigma$. For example, $\mathsf{John} \sqsubseteq_\sigma \mathsf{John} \sqcup_e \mathsf{Mary}$.

The bottom of the lattice for semantic type σ is denoted by $\perp_\sigma$. $\perp_\sigma$ is defined as the element such that for every $x \in D_\sigma$, $\perp_\sigma \sqsubseteq_\sigma x$.

If $x \in D_\sigma$ and if $y \sqsubsetneq \sigma x$ entails $y = \perp_\sigma$, then x is called atomic. The set of all atomic elements in the lattice for semantic type σ is denoted by AT_σ. Clearly, $AT_\sigma \in D_\sigma$.

For any $x, y \in D_\sigma$, we say x and y overlap and write $x \circ_\sigma y$, if and only if there is a $z \in D_\sigma$ such that $z \neq \perp_\sigma \wedge z \sqsubseteq_\sigma x \wedge z \sqsubseteq_\sigma y$. Obviously, if $x \in AT_\sigma \wedge x \circ_\sigma y$, then $x = y$.

An atomic situation is a pair of an atomic world and an atomic time interval. Thus, $AT_s = AT_w \times AT_i$. The lattice of situations are the closure of AT_s under the join and meet operations. π_w and π_i are the projection maps to the world and the time interval, respectively. Thus, for a given atomic situation $s = \langle w, I \rangle$, $\pi_{\mathrm{w}}(s) = w$ and $\pi_{\mathrm{i}}(s) = I$. Hence, for any atomic situation s, $s = \langle \pi_{\mathrm{w}}(s), \pi_{\mathrm{i}}(s) \rangle$ holds.

Although I have introduced lattices for all semantic types, not all of them are needed in the end. Particularly, assuming the situation lattice, I sometimes talk about non-atomic situations, but this is in order to discuss analyses that pluralizes predicates with the * operator. However, I eventually argue for a theory that gets rid of the * operator, and thus it turns out unnecessary to assume non-atomic situations.

$A \overset{\mathrm{def}}{=} B$ means that A is defined as B.

In semantic computation, $\overset{\mathrm{FA}}{=}$ indicates that the equality is obtained by virtue of Functional Application (Heim and Kratzer, 1998) and $\overset{\beta}{=}$ indicates that the equality is obtained by β-reduction.

Chapter 1

A Scope Theory of Non-presuppositional Noun Phrases

This chapter presents a scope theory of non-presuppositional noun phrases to account for Musan's (1995) generalization about the obligatory situationally dependent reading of non-presuppositional noun phrases, and defends this view by providing evidence from bi-clausal sentences in favor of the scope theory. Nevertheless, it turns out that this scope approach runs into an apparent scope puzzle when we attempt to analyze plural non-presuppositional noun phrases which are associated with plural times (events). To solve this puzzle, it is concluded that the numeral part of a non-presuppositional noun phrase gets separated at LF and enters into computation after the noun and the main predicate are combined to from a relation that expresses situational dependence.

1.1 Musan's generalization

1.1.1 Temporal dependence

Since Enç's (1981; 1986) work, it has been known that the evaluation time of a noun phrase may be different from the evaluation time of the main predicate. For example, in Enç's sentence in (1), if the evaluation time of *fugitive* should be the same as the evaluation time of the main predicate *be in jail*, the sentence should be asserting that some individuals are both fugitives and in jail simultaneously, a contradiction.

(1) Every fugitive is now in jail.

However, (1) clearly has a non-contradictory reading which can be paraphrased as "every individual who *was* a fugitive is now in jail." This means that *fugitive* can be evaluated with respect to some past time, while the main predicate is evaluated with respect to the present time.

Following the above empirical observation by Enç, Musan (1995) notices that it is not always possible for a noun phrase to have a different evaluation time from the main predicate. Consider the German setnences in (2) (= Musan's (5) on p.79), where the words in capital letters are stressed. Sentences (2a, b, d, f) can easily be about individuals who are professors now but who were not professors in the 60's. With Musan I refer to this reading as a **temporally independent reading** in the sense that the evaluation time of the noun phrase is independent of that of the main predicate. By contrast, sentences (2c, e) may only be about individuals who were professors and happy simultaneously in the 60's. This reading is referred to as a **temporally dependent reading**.

(2) a. Die meisten Professoren waren in den sechziger Jahren glücklich.
the most professors were in the sixites happy
'Most professors were happy in the sixties.'

b. EINIGE Professoren waren in den sechziger Jahren glücklich.
some professors were in the sixites happy
'Sóme professors were happy in the sixties.'

c. Einige PROFESSOREN waren in den sechziger Jahren glücklich.
some professors were in the sixites happy
'Sm professors were happy in the sixties.'

d. Einige von den Professoren waren in den sechziger Jahren glücklich.
some of the professors were in the sixites happy
'Some of the professors were happy in the sixties.'

e. In den sechziger Jahren waren ja doch Professoren glücklich.
in the sixites were (indeed) professors happy
'In the sixties, [some] professors were (indeed) happy.'

f. Professoren waren ja doch in den sechziger Jahren glücklich.
professors were (indeed) in the sixites happy
'Professors [in general] were (indeed) happy in the sixties.'

Based on data like (2), Musan observes the following generalization (putting aside sentences with what she calls existence-independent predicates):

(3) Musan's Generalization (temporal version, Musan 1995, p. 81)
A noun phrase can be temporally independent if and only if it is presuppositional.

A note is in order on the meaning of **presuppositionality** here. Essentially, a presuppositional noun phrase presupposes the existence of individuals predicated truly of by the noun. It has been commonly assumed that noun phrase with a strong determiner in the sense of Milsark (1974) such as *every*, *most*, etc. and partitive noun phrases (e.g. *some of the rabbits*) are presuppositional. Thus, usage of such noun phrase as *most rabbits* and *some of the rabbits* signals that the discourse of the conversation contains a nonempty set of rabbit individuals.

On the other hand, Diesing (1992) argues that noun phrases with a weak determiner such as *some*, *many*, etc. are ambiguous as to their presuppositionality. According to Diesing, noun phrases with a weak determiner are presuppositional when they are in a VP-external position at LF, which can be indicated by being to the left of the particle *ja doch* in case of German, whereas noun phrases with a weak determiner which are in a VP-internal position at LF are non-presuppositional (see Subsection 1.2.1 below). Furthermore, Musan (1995) notes that stress plays a role in determining whether or not a noun phrase is presuppositional. When the determiner is stressed, the noun phrase is preferredly presuppositional, but when the noun is stressed, the noun phrase tends to be non-presuppositional.

If there is no clear indicator such as particle *ja doch* that can tell the presuppositionality or non-presuppositionality of a noun phrase with a weak determiner, and if the sentence containing the noun phrase is uttered "out of the blue", that is, without a context that has established the existence of the sort of individuals predicated truly of the noun, I assume that the noun phrase is interpreted non-presuppositionally by default. For example, if a sentence containing the noun phrase *some rabbit* is uttered in a discourse where no rabbits have been talked about or assumed to exist, then the interpretation of *a rabbit* shall be non-presuppositional. The presuppositionality of noun phrases can be checked with von Fintel's (1999) test as we see in Section 1.4, but until then, I will go with this intuitively natural assumption just described.

Now that the concept of presuppositionality has been clarified, one can see that Musan's Generalization accounts for the distribution of temporally independent and dependent readings in (2). The subject noun phrase is presuppositional in (2a, b, d, e), but not in (2c, e). Therefore, sentences (2a, b, d, f) can exhibit temporally independent readings, while sentence (2c, e) have only temporally dependent readings. The distinction between (2b) and (2c) comes from whether the determiner or the noun is stressed. Essentially, (2b) can be used only when the context has established a set of some professors, whereas (2c) may be used without such a context. The distinction between (2e) and (2f) is due to the relative position of the noun phrase with respect to *ja doch*, which indicates whether the noun phrase is interpreted in a VP-internal or VP-external position.

Tonhauser (2002) claims that Musan's generalization is not entirely right. According to Tonhauser, at a reunion of the survivors of the Titanic disaster, the following sentence may be uttered:

(4) Look, there are even some crew members here.

As the people referred to here are no longer crew members at the reunion, Tonhauser contends that (4) shows that when an appropriate context is provided, even noun phrases that are syntactically classified by Musan as "non-presupposiitonal" can be interpreted with respect to a time distinct from the main predicate. While I agree with Tonhauser that (4) may be uttered in the context considered, I do not think that this example is a convincing piece of evidence for her conclusion.

As David Pesetsky (p.c.) points out to me, if a person held a particular title in some community or society, he or she might still be referred to with that title even after he or she has resigned from that position. For example, someone who was the president of the USA in the past may still be called "president" even if he is not the current president, and if you were a student representative in your college, your ex-classmates might refer to you as a "student rep" even though you graduated many years ago. I do not know whether in such cases people pretend that some propositions that were only true at some past time hold true at the time of utterance, or the lexical meaning of the relevant noun is somehow altered and extended. In any case, it is clear to me that when an ex-president of the USA is called "president", the speaker is "pretending" that he is a president at the time of utterance and when an ex-student-rep is called "student rep", the speaker is "pretending" that he or she is a student rep at the time of utterance. Similarly, so long as the speaker is committed to the assertion in (4), he or she is also committed to the view that the people referred to there are in some sense regarded as crew members at the time of utterance. After all, it is not very clear when a crew member of a ship ceases to be a crew member. If we hypothesized that a crew member ceases to be one when he disembarks the ship, it would have to be the case that he stops being a crew member and resumes to be one again every time the ship stops at a port on its way and he gets off the ship and on board again. This is not an usual interpretation of what it means to be a crew member.

In any case, even if Diesing's (and Musan's) syntactic classification of presuppositional and non-presuppositional noun phrases is not entirely correct, Tonhauser's observation will not undermine Musan's generalization *per se*. The main point of Tonhauser (2002) is that a specific context must be provided in order to shift the evaluation time of a noun phrase from that of the main predicate. This means that the existence of entities truly predicated of by the noun phrase is established by the context, and so the noun phrase becomes presuppositional. Therefore, even if some syntactic positions that Musan thinks invariably indicate the non-presuppositionality of noun phrases that are located there turn out to be also able to host a presuppositional noun phrase, Musan's generalization (3) will still hold.

1.1.2 World dependence

In Chapter V of Musan (1995), Musan discusses the possibility of extending her generalization to world interpretation. Musan considers the following pair:[1]

(5) a. Things would be different if a senator had grown up to be a rancher instead.
b. Things would be different if there was a senator having grown up to be a rancher instead.

Musan points out that while both (6a) and (6b) are possible interpretations of (5a), (6b) is the only reading of (5b):

(6) a. For all worlds w such that there is a senator in the actual world that grew up to be a rancher in w, things are different in w.
(*de re* reading of *a senator*)
b. For all worlds w such that there is a senator in w that grew up to be a rancher in w, things are different in w.
(*de dicto* reading *a senator*)

In (5b), due to the existential *there* construction, *a senator* is non-presuppositional. On the other hand, in (5a), *a senator* is ambiguous as to its presuppotiaionality. Based on this observation, Musan considers extending her generalization on temporal interpretation of noun phrases to world interpretation. That is, the world with respect to which a non-presuppositional noun phrase is interpreted is dependent on the evaluation world of the main predicate, but a presuppositional noun phrase can be interpreted with respect to a

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[1] Musan cites (5a) from Abusch (1994 p. 104) and attributes (5b) to Kai von Fintel (pc).

different world. The data in (5) can then be explained. In (5), it is not clear whether the main predicate of the embedded clause is *was* or *having grown up ...*, but in either case, the main predicate is evaluated in a hypothetical world introduced by the *if* clause. Since *a senator* is non-presuppositional here, its world interpretation depends on the main predicate of the embedded clause, and as a result it only has a *de re* reading. On the other hand, in (5a), *a senator* can be presuppositional (which requires some context involving some established senators), thus can be evaluated with respect to a different world than the hypothetical worlds introduced by the *if* clause. It is thus possible for *a senator* to be evaluated with respect to the actual world, resulting in a *de re* reading.

The two generalizations made by Musan are about intensional interpretation of noun phrases, and one concerns temporal interpretation and the other world interpretation. In order to integrate these two, I postulate pairs of an atomic possible world and an atomic time interval, and I refer to them as **situations**. Given an atomic possible world $w \in \mathrm{D_w}$ and an atomic time interval $I \in \mathrm{D_i}$, $s = \langle w, I \rangle$ is an atomic situation. Conversely, given an atomic situation, we can extract the time-interval and the possible world that constitute the situation by means of projection maps π_w and π_i. For the situation s just given, $\pi_\mathrm{w}(s) = w$ and $\pi_\mathrm{i}(s) = I$. In general, for any atomic situation s, $s = \langle \pi_\mathrm{w}(s), \pi_\mathrm{i}(s) \rangle$. In Montague's (1973) PTQ, world-time pairs are treated as objects of type s. I follow Montague and use s as the semantic type of situations, except that I consider time intervals while Montague thought of time points. The intensional interpretation of a given predicate is then determined when an evaluation situation is provided. In this sense, intensional interpretation becomes a synonym of **situational interpretation**, although the latter terminology clarifies that both the temporal and world interpretations are taken into account. With this terminology, Musan's two generalizations can be stated in the following single condition:

(7) Musan's Generalization (situational version)
A noun phrase can be situationally independent if and only if it is presuppositional.

In the rest of this chapter, a scopal theory is introduced to account for this generalization and is discussed.

1.2 A scopal account of situationally dependent readings

In this section, I sketch a theory which associates the availability of situationally independent readings with Diesing's (1992) Mapping Hypothesis. The idea is that non-presuppositional noun phrases have situationally dependent readings because they are in the scope of some intensional operator located within or at the edge of the VP domain, but presuppositional noun phrases move out of the VP and thus escapes the scope of this operator. Musan (1995) in fact considers this scope approach as a possible explanation of her generalization, but eventually rejects it because she thinks that it is not capable of accounting for certain data. Instead, Musan develops an alternative approach which utilizes stages of individuals, which I discuss in Section 1.4.

1.2.1 Diesing's Mappying Hypothesis

To begin with, let us reveiw Diesing's (1992) Mapping Hypothesis, which is cited below:

(8) Mapping Hypothesis (Diesing 1992, p. 10)
Material from VP is mapped into the nuclear scope.
Material from IP is mapped into a restrictive clause.

First, regarding the second sentence of (8), note that if something is in the VP, it is automatically inside the IP above the VP. This sentence should therefore be interpreted as "material from IP *which is outside VP* is mapped into a restrictive clause." Second, let us change Diesing's terminology so as to fit my framework. Since I am adopting TP instead of IP, let us interpret IP in (8) as TP. Furthermore, it is becoming increasingly common to decompose VP into a VP and a *v*P above it, which introduces the agent argument respectively (Kratzer, 1996, to appear). Also, with the recent expansion of functional heads (Cinque, 1999), we might still have other fucntional heads between TP and *v*P. Let us then assume that what Diesing refers to as VP in (8) is whatever is the complement of the T head, be it *v*P or AspP, and let us refer to this as the VP domain. Then, Diesing's Mapping Hypothesis is restated as follows:

(9) Mapping Hypothesis
Material from the VP domain is mapped into the nuclear scope.
Material from outside the VP domain is mapped into a restrictive clause.[2]

The terms **nuclear scope** and **restrictive clause** are due to Heim (1982). According to Heim, indefinites, which had usually been treated as existential quantifiers, do not have

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[2] It is possible for a noun phrase outside the VP domain to be even outside TP if it is in Spec, CP for example. Therefore, the second sentence of (9) is, strictly speaking, not identical to the second sentence of (8). I am therefore ignoring CP or any functional projection above TP.

quantificational force on their own. Instead, they merely introduce variables into the domain called the nuclear scope, which is then closed off by existential closure that binds these variables in its scope. On the other hand, genuine quantifying noun phrases such as *every rabbit* form a restrictive clause, which serves as the domain of the quantification. Unlike Heim, however, Diesing argues that not all indefinites are mapped into the nuclear scope. Indefinite noun phrases are either bare plurals or noun phrases with weak determiners such as *a*, *some*, *many*, *three*, etc. The interpretation of bare plurals is ambiguous between generic interpretation and existential interpretation, and according to Diesing, bare plurals with generic interpretation are mapped into a restrictive clause, while bare plurals with existential interpretation are mapped into the nuclear scope. Similarly, noun phrases with a weak determiner are ambiguous as to whether they are presuppositional or not, and according to Diesing, presuppositional ones are mapped into a restrictive clause while non-presuppositional ones are mapped into the nuclear scope. The behavior of such presuppositional indefinites is the same as that of noun phrases with a strong determiner, which are all presuppositional and mapped into a restrictive. To sum up, what Diesing's Mapping Hypothesis says can be restated as follows:

(10) Non-presuppositional noun phrases and existential bare plurals are interpreted in the VP domain (hence mapped into the nuclear scope).
Presuppositional noun phrases and generic bare plurals are interpreted outside the VP domain (hence mapped into a restrictive clause).

The Mapping Hypothesis thus determines what kind of interpretation a noun phrase receives depending on where in the syntactic tree it is interpreted.

1.2.2 Basic idea

To sketch the scope approach, we will focus on the following sentence throughout this section:

(11) Mary kissed a 20-year-old.

When (11) is uttered out of the blue without context, it claims the existence of an individual who got kissed by Mary when he or she was of age 20, as predicted by Musan's generalization.

If we do not adopt Musan's (1995) stage semantics approach, the only way to capture the situational interpretation of noun phrases and main predicates, as far as I can see, is to assume that noun phrases and main predicates are interpreted with respect to evaluation situations. Technically, there are two different systems for having evaluation situations. In one system, the denotations of nouns and main-predicates take an evaluation situation as their argument. In the other system, the denotations of nouns and main-predicates do not themselves have a situation argument, but instead, the interpretation function $[\![\quad]\!]$ is evaluated with respect to a situation. However, the choice of the system between these two does not affect the validity of the argumentation made in this chapter, and I will adopt the first system. Assuming that nouns denote predicates, the noun *20-year-old* shall have the lexical entry in (12a). Likewise, the verb *kiss* shall have the lexical entry in (12b) assuming that the agent argument is introduced later by the *v* head.

(12) a. $[\![\texttt{20-year-old}]\!] = \lambda s \in \mathrm{D_s}.\, \lambda x \in \mathrm{D_e}.\, \textsf{20-yr-old}(s)(x)$

b. $[\![\texttt{kiss}]\!] = \lambda s \in \mathrm{D_s}.\, \lambda x \in \mathrm{D_e}.\, \textsf{kiss}(s)(x)$

Given such lexical entires, the temporally dependent reading of a non-presuppositional noun phrase should be accounted for by feeding the non-presuppositional noun phrase and the main predicate that it depends on with the same situation variable. Essentially, there are two conceivable ways to do this, depending on whether this feeding shall be done in the denotation of some lexical item or in syntax.

The first hypothesis is that some lexical item in a non-presuppositional noun phrase, say the determiner, takes the denotations of the non-presuppositional noun phrase and of the main predicate and feeds them with one and the same situation. More concretely, one can assume a lexical entry like the following for the indefinite article *a*:

(13) $[\![\mathrm{a}]\!] = \lambda f \in \mathrm{D}_{\langle \mathrm{s},\langle \mathrm{e,t}\rangle\rangle}.\, \lambda g \in \mathrm{D}_{\langle \mathrm{s},\langle \mathrm{e,t}\rangle\rangle}.\, \lambda s \in \mathrm{D_s}.\, \exists x \in \mathrm{D_e}[f(s)(x) \wedge g(s)(x)]$

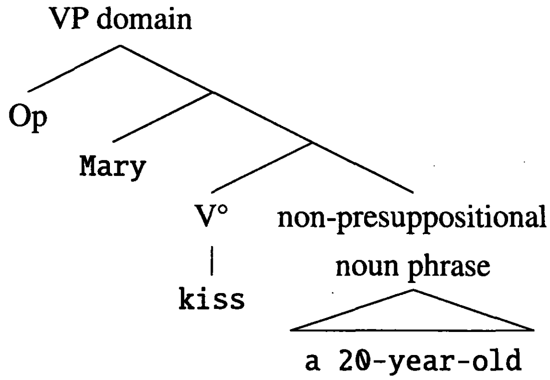
The problem of this approach becomes apparent when one considers sentences with more than one non-presuppositional plural noun phrase like the following:

(14) Three 30-year-olds kissed four 20-year olds.

As observed by Scha (1981), sentences like this have a cumulative reading where neither of the two noun phrases takes scope over the other. That is, (14) can mean that each of the five kissers kissed at least one of the three kissees, and each of the three kissees was kissed by at least one of the five kissers. When (14) is uttered out of the blue, Musan's generalization takes effect so that what is asserted is each kisser was of age 30 at the time of the kissing(s) and each kissee was of age 20 at the time of the kissing(s). If the situational dependence of each noun phrase on the main predicate is obtained as the result of the application of the denotation of the determiner, each of the two determiners should individually be applied to create the situational dependence of the nonpresuppositional noun phrase it is part of in the course of the semantic computation, but then, the noun phrase that enters the semantic computation later would have to take scope over the other one that had already been processed. This would make it impossible to yield the scope-less, cumulative reading.

Given this difficulty of the lexical approach, I would like to adopt the other, syntactic hypothesis, and this is what I refer to as the scopal theory. On this approach, the situation interpretation of a non-presuppositional noun phrase is determined by virtue of being in the scope of some operator. To begin with, let us suppose that there is an operator at the edge of the VP domain which manipulates the evaluation situation of the main predicate. Now, according to Diesing's Mapping Hypothesis, non-presuppositional noun phrases are always interpreted in the VP domain, and thus are interpreted in the scope of the operator postulated at the edge of the VP domain. This is illustrated by the following, where Op is the operator that gives the situational interpretation of the main predicate *kiss*.

(15)



Therefore, if it is the case that the situational interpretation of a non-presuppositional noun phrase is obligatorily determined by the nearest operator that c-commands it, it follows that the evaluation situations of the main predicate and the non-presuppositional noun phrase are simultaneously given by the same operator, and this would yield a situationally dependent reading, deriving Musan's generalization. This account is very appealing since it very naturally connects Musan's generalization with Diesing's independently motivated theory.

In the next two subsections, two versions of the scope theory are presented, first with syntactically overt situation variables, and then without such variables in syntax.

1.2.3 An implementation with situation variables in syntax

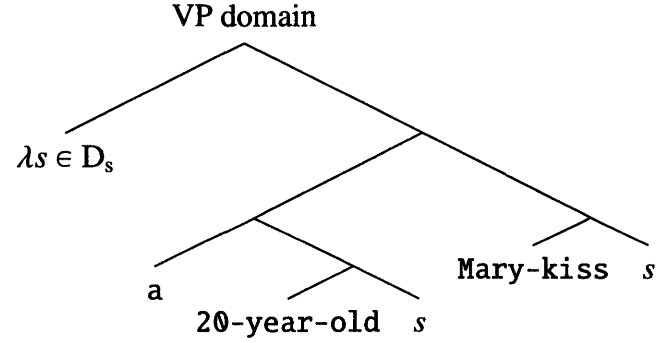
The first implementation of the scope theory assumes with Percus (2000) and Kusumoto (2005) that the situation arguments of predicates are always present as variables in syntax. Percus (2000) argues that once we assume that the situation argument of the main predicate is present in syntax, it is constrained to be bound by the nearest λ abstractor that can bind it (Generalization X). Therefore, once we assume that this λ abstractor that binds the situation argument of the main predicate is at the edge of VP domain, the operator postulated in (15) should be taken to be this λ abstractor. The situationally dependent reading of a non-presuppositional noun phrase is then obtained when this λ abstractor also binds the situation argument of the non-presuppositional noun phrase.

To illustrate this, let us assume the following composite lexical entry:

(16) $[\![\texttt{Mary-kiss}]\!] = \lambda s \in D_s . \lambda x \in D_e . \mathsf{kiss}(s)(x)(\mathsf{Mary})$

Although I actually hold the view that the agent of the verb *kiss* is introduced by the *v* head above *kiss*, for the sake of expository simplicity and clarity, I employ the above composite lexical entry where the agent Mary has already been introduced. Since where *Mary* enters the semantic computation does not in the end have any consequences in the interpretation of the whole sentence unlike in the case of quantifiers such as *everyone*, utilizing the above complex lexical entry does no harm. With this lexical entry, the LF of (11) will look like the following:

(17)



Here, the variable *s* that is the sister of *20-year-old* saturates the situation argument of *20-year-old*, and similarly, the variable *s* that is the sister of *Mary-kiss* saturates the situation argument of *Mary-kiss*. These two situation variables are then bound by the same λ abstractor. With the lexical entry in (18a) of *a*, (17) is interpreted as in (18b):

(18) a. $[\![\mathsf{a}]\!] = \lambda f \in D_{\langle e,t\rangle} . \lambda g \in D_{\langle e,t\rangle} . \exists x \in D_e[f(x) \wedge g(x)]$

b. $[\![(17)]\!] = \lambda s \in D_s . \exists x \in D_e[\mathsf{20\text{-}yr\text{-}old}(s)(x) \wedge \mathsf{kiss}(s)(x)(\mathsf{Mary})]$

The complete truth conditions of sentence (11) can then be obtained by closing (18b) with an existential quantifier over situations in the past. The truth conditions thus obtained can then represent a situationally dependent reading, since the nominal predicate and the main predicate have the same evaluation situation.

As explained in the previous subsection, Musan's generalization will then follow from Diesing's Mapping Hypothesis by ensuring that the situation argument of the non-presuppositional noun phrase obligatorily gets bound by the λ abstractor that binds the situation argument of the main predicate. This is possible, if we postulate the following:

(19) The variable that saturates the situation argument of a non-presuppositional noun phrase is obligatorily bound by the nearest λ abstractor.

I will return to this in Subsection 1.4.3.

1.2.4 An alternative implementation with situation-variable-free syntax

In the above implementation, the situation argument of a predicate was assumed to be always present as a variable in syntax. It should be realized, however, that this assumption is not crucial for the scope theory *per se*. A scope account is possible without syntactically overt situation variables.

Keshet (2008) proposes that predicates are in general of type $\langle s, \langle e, t\rangle\rangle$ just as we have above, but that they can combine by means of a special version of Predicate Modification (Heim and Kratzer, 1998) given in (20) while keeping their situation argument unsaturated.

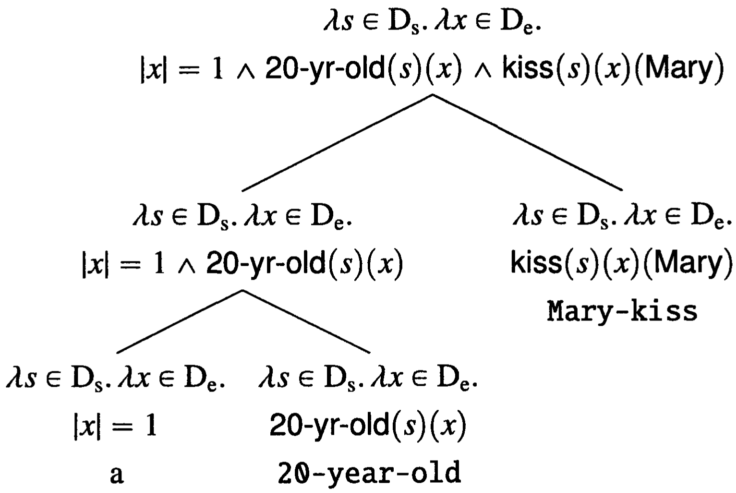
(20) Intensional Predicate Modification
Let α and β be sisters such that $[\![\alpha]\!]$ and $[\![\beta]\!]$ are of type $\langle s, \langle e, t\rangle\rangle$. Then,
$[\![\alpha \frown \beta]\!] = \lambda s \in D_s. \lambda x \in D_e. [\![\alpha]\!](s)(x) \wedge [\![\beta]\!](s)(x)$

Following Landman (2004), Keshet assumes that indefinite numeral determiners are adjectives that denote properties which are true of individuals that consist of a certain number of atomic members. For example, the indefinite article *a* has the following lexical entry, where $|\ |$ gives the number of atomic members comprising a given individual:

(21) $[\![\text{a}]\!] = \lambda s \in D_s. \lambda x \in D_e. |x| = 1$

Then, with the special Predicate Modification rule, the semantics of [[*a 20-year-old*] *Mary-kiss*], which does not contain situation variables, can be computed as follows:

(22)



The top node here will combine with an operator which gives the value of the open situation variable *s* at the top node by existential closure. The value of the open individual variable *x* will also be given by some existential closure eventually.

In this implementation, the non-presuppositional noun phrase receives its situationally dependent interpretation by virtue of being interpreted as the sister of the main predicate. Since the value of the situation variable is given by existential closure at the top node of (22), it is important that the non-presuppsotional noun phrase stay in the scope of the operator responsible for this existential closure. Thus, this is also a version of the scope theory.

In the implementation discussed in the previous subsection, we have situation variables in syntax, and therefore we have to constrain them by stipulating that the situation variable of non-presuppositional noun phrases must be bound locally. By contrast, in the implementation considered here, it is not necessary or possible to make such a stipulation since we do not have situation variables in syntax to begin with, and thus, Musan's generalization automatically follows from the structure. Apart from this technical difference, the two versions of the scope theory make the same predictions as to the interpretation of non-presuppostional noun phrases.

On the other hand, regarding presuppositional noun phrases, we have seen in Section 1.1 that their evaluation time or world can be different from that of the main predicate. Percus (2000), among others, argues for the view that in order to represent such interpretations of noun phrases, the evaluation situation of presuppositional noun phrases must be present in syntax. If Percus is right, and if we also adopt the situation-variable-free approach just presented, then we would need to postualte completely different syntactic representations for presuppositional and non-presuppositional noun phrases. To avoid such a complication that is not readily justified, I adopt the first version of the scope theory, where every evaluation situation is present as a variable in syntax.

The reader should be aware, however, that the conclusions we reach in this dissertation do not depend on a specific implementation of the scope theory, but only on the thesis that the interpretation of both noun phrases and main predicates is evaluated with respect to a situation.

1.2.5 Are the temporal arguments of predicate the exact times?

I have explained above that in the theory where predicates are interpreted with respect to evaluation situations, the situationally dependent reading of (11) is analyzed as in the following (= (18b)):

(23) $\lambda s \in \mathrm{D_s}. \exists x \in \mathrm{D_e}[\text{20-yr-old}(s)(x) \wedge \text{kiss}(s)(x)(\text{Mary})]$

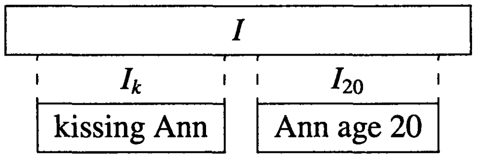
However, close consideration of (23) reveals that (23) *per se* is not sufficient to guarantee situational dependence unless we stipulate an appropriate temporal interpretation of the situation argument of predicates 20-yr-old and kiss. Recall that a situation is defined to be a pair of a possible world and a time-interval. When a predicate takes a situation argument, it turns out to be important whether or not the relevant property holds only in some subinterval of the whole time interval of the given situation argument.

To see this, suppose the following interpretations of 20-yr-old and kiss:

(24) a. $\text{20-yr-old}(s)(x)$ is true iff x is of age 20 in some subinterval of $\pi_{\mathrm{i}}(s)$ in $\pi_{\mathrm{w}}(s)$.
b. $\text{kiss}(s)(x)(y)$ is true iff there is an event of y's kissing x in $\pi_{\mathrm{w}}(s)$ and its running time-interval is a subinterval of $\pi_{\mathrm{i}}(s)$.

If we assume (24), (23) will not guarantee temporal dependence even though the two predicates take the same situation argument. Consider the following scenario. Mary kissed only one individual Ann only once, and its running time-interval is I_k. Ann was of age 20 throughout time interval I_{20}, and before I_{20}, and before or after I_{20}, Ann's age it never 20. Furthermore, I_k precedes I_{20}. Now, let I be a big time interval that contains both I_k and I_{20}. (25) below depicts this scenario:

(25)



Then, letting $s = \langle @, I \rangle$ (where @ is the actual world), both 20-yr-old(s)(Ann) and kiss$(s)(a)$(Mary). Therefore, (23) yields 1 when variable s in (23) is saturated with $\langle @, I \rangle$. This means that the analysis in (23) predicts that (11) is true under this scenario. However, the kissing took place before the Ann turned 20 in this scenario, and thus, there is no temporal dependence and the sentence is clearly not true.

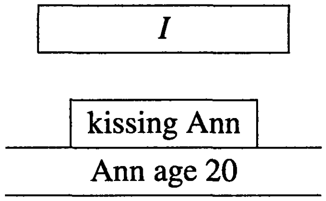
The above discussion shows that we cannot maintain both of (24a) and (24b). In order to guarantee situational (temporal) dependence, we should have either the interpretation of 20-yr-old in (26a) or the interpretation of kiss in (26b), or both:

(26) a. 20-yr-old$(s)(x)$ is true iff x is of age 20 throughout $\pi_{\mathrm{i}}(s)$ in $\pi_{\mathrm{w}}(s)$.

b. kiss$(s)(x)(y)$ is true iff there is an event of y's kissing x in $\pi_{\mathrm{w}}(s)$ and its running time is exactly $\pi_{\mathrm{i}}(s)$.

For example, suppose that we have (26a) and we do not have (22b) but (20b), and that 20-yr-old(s)(Ann) $\wedge$ kiss(s)(Mary)(Mary) holds for $s = \langle @, I \rangle$. According to (26a), 20-yr-old(s)(Ann) is true iff Ann is of age 20 throughout I. According to (24b), kiss(s)(Ann)(Mary) is true iff there is an event of Mary's kissing Ann and its running time interval is a subinterval of I. Therefore, whenever the kissing might have taken place, as long as ti was within I, Ann must have been of age 20 throughout the time interval of kissing, as illustrated by (27):

(27)

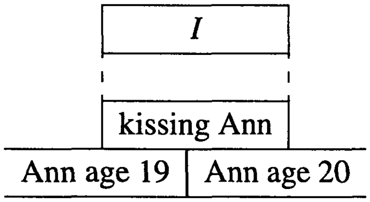


In Section 2.2 of Chapter 2, I show that in order to ensure correct counting of events, the time interval of the situation argument of the verb must be the exact time interval of an event that the verb describes. Thus, (26b) should indeed be the required interpretation.

Now, let us take (26b) for granted. Let us furthermore suppose that we do not have (26a) but (24a), and that 20-yr-old(s)(Ann) $\wedge$ kiss(s)(Ann)(Mary) holds for $s = \langle @, I \rangle$.

According to (26b), $\mathsf{kiss}(s)(\mathsf{Ann})(\mathsf{Mary})$ is true iff there is an event of Mary's kissing Ann and its running time is exactly I. Now according to (24a), $\mathsf{20\text{-}yr\text{-}old}(s)(\mathsf{Ann})$ is true iff Ann is of age 20 in some subinterval of I. Then, (28) should be a possible scenario for this case:

(28)



Here, the event of kissing Ann begins when she is 19, but during the kissing, Ann's birthday comes and she turns 20. Therefore, it is true that Ann was of age 20 in some subinterval of the whole kissing time interval. In this case, however, it does not seem right to describe this situation with sentence (11). I therefore propose that (26a) is needed after all.

To sum up, I think that the correct interpretations of $\mathsf{20\text{-}yr\text{-}old}$ and kiss must eventually be the ones in (26). However, the important point to bear in mind, which will becomes crucial in the next section, is that at least one of (26a) and (26b) needs to be accepted in order to account for situationally dependent readings.

1.3 A scope puzzle with plural noun phrases

Suppose that (29) is uttered out of the blue.[3] [4] The object noun phrase *three 20-year-olds* will then be interpreted non-presuppositionally.

(29) In the last five years, Mary kissed three 20-year-olds.

---

[3] It seems that sentences with the perfect *have* such as (i) below sound more natural than sentences with the simple past tense such as (29) in describing the meaning which is discussed shortly.

(i) In the last five years, Mary has kissed three 20-year-olds.

However, my informants report that sentences with the simple past tense are also fine. Since I do not want to complicate the discussion by introducing the perfect *have*, I stick with sentences with the simple past tense.

[4] Szabó (2006) discusses sentences of the same sort in refutation of presentism. He cites Diodorus Cronus, who says of the following sentence that it is true because Helen was consecutively married to Menelaus, Paris, and Deïphobus:

(i) Helen had three husbands.

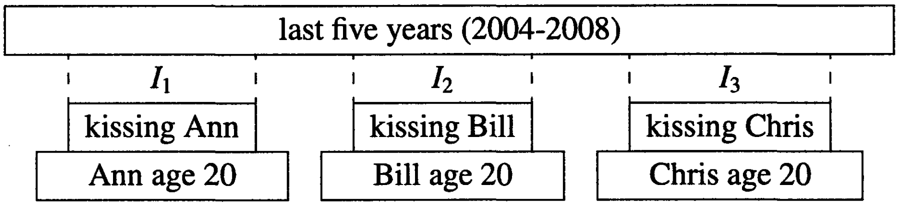
Importantly, the sentence is true even though there was no single past moment where Helen had three husbands simultaneously. In the analysis of (i) that Szabó proposes, *three* takes scope over the past operator, which in turn takes scope over the noun *husband.* Thus, the conclusion I draw below is in this sense the same as Szabó's.

(29) is different from (11) in that the non-presuppositional noun phrase is plural. What are the truth conditions of (29), then? Now, imagine the following scenario:

(30) Ann was born in 1984, and Mary kissed Ann only once and its was in 2004 after her birthday.
Bill was born in 1986, and Mary kissed Bill once and its was in 2006 after his birthday.
Chris was born in 1988, and Mary kissed Chris once and its was in 2008 after his birthday.
Mary kissed nobody else in the last five years.

Suppose that the running time intervals of the events of Mary's kissing Ann, Bill and Chris were I_1, I_2 and I_3 respectively. The above scenario is then visually illustrated by the following:

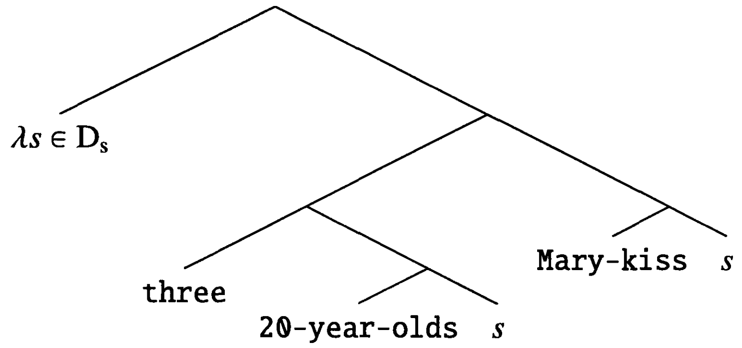
(31)



If uttered at the end of the year 2008, (29) is clearly a true statement about this scenario. On the other hand, if Mary kissed exactly three people in the last five years, but they were not each of age 20 when they were kissed by Mary, (29) should be false. Therefore, for (29) to be true, each of the three people must have been of age 20 when Mary kissed him/her. Thus, the temporal dependence of the non-presuppositional noun phrase on the main predicate obtains for (29), too, as predicted by Musan's generalization. What is interesting about this case is that there is no single time-interval throughout which all the three people were simultaneously of age 20 or in which all the three people simultaneously got kissed by Mary.

The question is whether the scope theory presented in the previous section can predict this meaning of (29). Ignoring the temporal adverbial, the scope theory will give the following LF for (29):

(32)



We should now ask ourselves how this LF may be interpreted. So far, it has been assumed that each predicate takes a situation argument and individual arguments, where the situation argument expresses a situation in which an atomic event of the relevant sort occurs. Assuming Neo-Davidsonian event semantics (Parsons, 1990, among others), each individual argument of a predicate bears a thematic relation to this atomic event that the predicate talks about. Following Landman (1996), I assume that each atomic event is thematically related to a unique individual or collection of individuals by Landman's collectivity criterion. Now, for the above scenario, we know that we have the following true formulae:

(33) $\text{20-yr-old}(\langle @, I_1\rangle)(\text{Ann})$
$\text{20-yr-old}(\langle @, I_2\rangle)(\text{Bill})$
$\text{20-yr-old}(\langle @, I_3\rangle)(\text{Chris})$
$\text{kiss}(\langle @, I_1\rangle)(\text{Ann})(\text{Mary})$
$\text{kiss}(\langle @, I_2\rangle)(\text{Bill})(\text{Mary})$
$\text{kiss}(\langle @, I_3\rangle)(\text{Chris})(\text{Mary})$

However, since 20-yr-old only talks about atomic states of being a 20-year-old, and an atomic state of being a 20-year-old involves only one person, there is no situation s such that $\text{20-yr-old}(s)(\text{Ann} \sqcup_e \text{Bill} \sqcup_e \text{Chris})$ holds. Similarly, kiss only talks about atomic events of kissing, and as long as we assume that one can kiss only one person in one kissing, there is no situation s such that $\text{kiss}(s)(\text{Ann} \sqcup_e \text{Bill} \sqcup_e \text{Chris})(\text{Mary})$ holds. Therefore, in order to deal with plural individuals, we need to "pluralize" these predicates. One strategy is to apply the * operator (Link, 1983) to the inidividual arguments so that these predicates take joins of individuals as their arguments. The other strategy is to pluralize the predicates for the individual argument and for the situation argument simultaneously, i.e., to turn them into predicates that talk about cumulative relations between plural individuals and plural situations. In what follows, I will reveal that (32) cannot be the correct LF by showing that neither of these attempts at interpreting it can capture the correct semantics of (29).

1.3.1 Attempt I: simple pluralization

In this first attempt, we pluralize the predicates with Link's (1983) * operator. On this approach, individuals are elements on a lattice, and the join of any collection of individuals is again on the lattice. Given a property P of individuals, $*P$ is defined as the closure of P under the join. Thus, for any $x, y \in \mathrm{D_e}$,

(34) $P(x) \rightarrow *P(x)$
$*P(x) \wedge *P(y) \rightarrow *P(x \sqcup_e y)$

Using the * operator, LF (32) will be interpreted as follows:[5]

(35) $\lambda s \in \mathrm{D_s}. \exists X \in \mathrm{D_e}[|X| = 3 \wedge [*\lambda x \in \mathrm{D_e}. \textsf{20-yr-old}(s)(x)](X)$
$\wedge\ [*\lambda x \in \mathrm{D_e}. \textsf{kiss}(s)(x)(\textsf{Mary})](X)]$

The value of s is given by existential closure. (35) can be compositionally derived, for instance, by giving the following lexical entry to *three*:[6]

(36) $[\![\texttt{three}]\!] = \lambda p \in \mathrm{D_{\langle e,t\rangle}}. \lambda q \in \mathrm{D_{\langle e,t\rangle}}. \exists X \in \mathrm{D_e}[|X| = 3 \wedge *p(X) \wedge *q(X)]$

Now, suppose that for some situation s_0, (35) yields 1 as a truth-value, where X_1 instantiates the existential quantification. Given the assumption made above that the unstarred predicates 20-yr-old and kiss are true only of atomic individuals, the starred predicates with s_0 applied to X_1 have the following interpretations:

(37) a. $[*\lambda x \in \mathrm{D_e}. \textsf{20-yr-old}(s_0)(x)](X_1) \leftrightarrow \forall x[x \sqsubseteq_e X_1 \wedge x \in \mathrm{AT_e} \rightarrow \textsf{20-yr-old}(s_0)(x)]$
b. $[*\lambda x \in \mathrm{D_e}. \textsf{kiss}(s_0)(x)(\textsf{Mary})](X_1)]$
$\leftrightarrow \forall x[x \sqsubseteq_e X_1 \wedge x \in \mathrm{AT_e} \rightarrow \textsf{kiss}(s_0)(x)(\textsf{Mary})]$

Thus, there ought to be a plural individual consisting of three people such that each person x of these three, $\textsf{20-yr-old}(s_0)(x)$ and $\textsf{kiss}(s_0)(x)(\textsf{Mary})$ hold. The reader should now recall the discussion in Subsection 1.2.3 that either There, we saw that we should either accept the interpretation of 20-yr-old in (26a) or accept the interpretation of Mary-kiss in (26b). Now, if we have the interpretation of 20-yr-old in (26a), all these three people must have been

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[5] It does not matter whether pluralization is done before or after the intersection of the two predicates. (i) below is thus equivalent to (29):

(i) $\lambda s \in \mathrm{D_s}. \exists X \in \mathrm{D_e}[|X| = 3 \wedge [*\lambda x \in \mathrm{D_e}. \textsf{20-yr-old}(s)(x) \wedge \textsf{kiss}(s)(x)(m)](X)]$

[6] Alternatively, we can assume that the denotations of *20-year-olds* and *Mary-kiss* come with *. In that case, we do not need * in the denotation of *three*.

simultaneously of age 20 throughout the time-interval $\pi_i(s_0)$. On the other hand, if we have the interpretation of Mary-kiss in (26b), it must be the case that the these three people got kissed simultaneously by Mary in $\pi_i(s_0)$. However, as we saw with scenario given by (30) and (31) above, in order for (29) to be true, there does not have to be a single time interval throughout which all the three people were simultaneously of age 20 or in which all the three people simultaneously got kissed by Mary. The analysis in (35) is therefore incorrect, as it yields too strong truth conditions even though it predicts temporal dependence.

1.3.2 Attempt II: cumulative relation formation

In the second attempt, we convert predicates into cumulative relations between individuals and situations. This can be done with the double * operator (Krifka, 1986; Sternefeld, 1998). Let P be a predicate that takes an individual argument and a situation argument. Then, for any $x, y \in \mathrm{D_e}$ and $s, t \in \mathrm{D_s}$, $**P$ satisfies the following conditions:

(38) $P(x)(s) \rightarrow **P(x)(s)$
$**P(x)(s) \wedge **P(y)(t) \rightarrow **P(x \sqcup_{\mathrm{e}} y)(s \sqcup_{\mathrm{s}} t)$

With the double * operator, LF (32) will be interpreted as follows:[7]

(39) $\lambda s \in \mathrm{D_s}.\, \exists X \in \mathrm{D_e}[|X| = 3 \wedge [**\lambda x \in \mathrm{D_e}.\, \lambda t \in \mathrm{D_s}.\, \textsf{20-yr-old}(t)(x)](X)(s)$
$\wedge\ [**\lambda x \in \mathrm{D_e}.\, \lambda t \in \mathrm{D_s}.\, \textsf{kiss}(t)(x)(\textsf{Mary})](X)(s)]]$

The value of s is given by existential closure just as before except that s is now a plural situation.

Because the double * operator must be applied before the plural situation s is provided, we will need to assume that the lexical entries of *20-year-olds* and *kiss* come with the double * operator. We should thus assume the following lexical entries:

(40) a. $[\![\texttt{20-year-old}]\!] = \lambda s \in \mathrm{D_s}.\, \lambda X \in \mathrm{D_e}.$
$[**\lambda x \in \mathrm{D_e}.\, \lambda t \in \mathrm{D_s}.\, \textsf{20-yr-old}(t)(x)](X)(s)$

b. $[\![\texttt{Mary-kiss}]\!] = \lambda s \in \mathrm{D_s}.\, \lambda X \in \mathrm{D_e}.$
$[**\lambda x \in \mathrm{D_e}.\, \lambda t \in \mathrm{D_s}.\, \textsf{kiss}(t)(x)(\textsf{Mary})](X)(s)$

c. $[\![\texttt{three}]\!] = \lambda p \in \mathrm{D}_{\langle e,t\rangle}.\, \lambda q \in \mathrm{D}_{\langle e,t\rangle}.\, \exists X \in \mathrm{D_e}[|X| = 3 \wedge p(X) \wedge q(X)]$

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[7] In the resultant cumulative predicates, the order of the individual argument and the situation argument is reversed. This is merely to make them look similar to what I have in the theory developed later, and it has no significance here in and of itself.

With the above lexical entires, (39) is compositionally computed from LF (32).

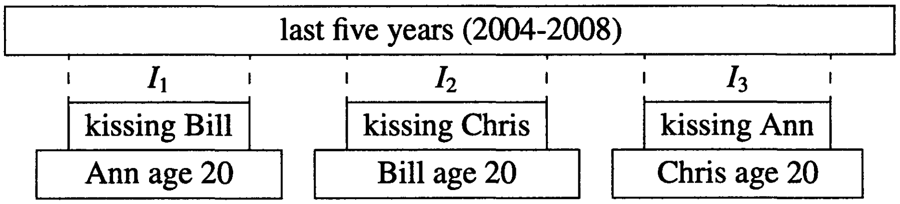
Given the assumption that the unstarred predicates $\mathsf{20\text{-}yr\text{-}old}$ and kiss are true only of atomic individuals, the double-starred predicates have the following interpretations:

(41) a. $[{**}\lambda x \in \mathrm{D_e}.\lambda t \in \mathrm{D_s}.\mathsf{20\text{-}yr\text{-}old}(t)(x)](X)(s)$
$\leftrightarrow \forall x[x \sqsubseteq_e X \wedge x \in \mathrm{AT_e} \rightarrow \exists t[t \sqsubseteq_s s \wedge \mathsf{20\text{-}yr\text{-}old}(t)(x)]]$
$\wedge \forall t[t \sqsubseteq_s s \wedge t \in \mathrm{AT_s} \rightarrow \exists x[x \sqsubseteq_e X \wedge \mathsf{20\text{-}yr\text{-}old}(t)(x)]]$

b. $[{**}\lambda x \in \mathrm{D_e}.\lambda t \in \mathrm{D_s}.\mathsf{kiss}(t)(x)(\mathsf{Mary})](X)(s)$
$\leftrightarrow \forall x[x \sqsubseteq_e X \wedge x \in \mathrm{AT_e} \rightarrow \exists t[t \sqsubseteq_s s \wedge \mathsf{kiss}(t)(x)(\mathsf{Mary})]]$
$\wedge \forall t[t \sqsubseteq_s s \wedge t \in \mathrm{AT_s} \rightarrow \exists x[x \sqsubseteq_e X \wedge \mathsf{kiss}(t)(x)(\mathsf{Mary})]]$

Now, consider the scenario depicted by the following:

(42)



Here, Mary kisses Bill when he is a teenager, and the running time of this event is I_1. Mary also kisses Chris when he is a teenager, and the running time of this event is I_2. Finally, Mary kisses Ann when she is no longer of age 20, and the running time of this event is I_3. No other kissing takes place. Under this scenario, we have the following true formulae:

(43) $\mathsf{20\text{-}yr\text{-}old}(\langle @, I_1\rangle)(\mathsf{Bill})$
$\mathsf{20\text{-}yr\text{-}old}(\langle @, I_2\rangle)(\mathsf{Ann})$
$\mathsf{20\text{-}yr\text{-}old}(\langle @, I_3\rangle)(\mathsf{Chris})$

Therefore, the double-starred predicate ${**}\lambda x \in \mathrm{D_e}.\lambda t \in \mathrm{D_s}.\mathsf{20\text{-}yr\text{-}old}(t)(x)$ should hold of the join of Ann, Bill and Chris, and the join of the three situations $\langle @, I_1\rangle$, $\langle @, I_2\rangle$ and $\langle @, I_3\rangle$. That is, we have the following true formula:

(44) $[{**}\lambda x \in \mathrm{D_e}.\lambda t \in \mathrm{D_s}.\mathsf{20\text{-}yr\text{-}old}(t)(x)]$
$(\mathsf{Ann} \sqcup_e \mathsf{Bill} \sqcup_e \mathsf{Chris})\big(\langle @, I_1\rangle \sqcup_s \langle @, I_2\rangle \sqcup_s \langle @, I_3\rangle\big)$

Similarly, the following is also a true formula:

(45) $[{**}\lambda x \in \mathrm{D_e}.\lambda t \in \mathrm{D_s}.\mathsf{kiss}(t)(x)(\mathsf{Mary})]$
$(\mathsf{Ann} \sqcup_e \mathsf{Bill} \sqcup_e \mathsf{Chris})\big(\langle @, I_1\rangle \sqcup_s \langle @, I_2\rangle \sqcup_s \langle @, I_3\rangle\big)$

holds. Then, $s = \langle @, I_1 \rangle \sqcup_s \langle @, I_2 \rangle \sqcup_s \langle @, I_3 \rangle$ satisfies (39). The analysis thus predicts that (29) can be truthfully uttered in the scenario depicted in (42). However, this is plainly wrong as nobody that Mary kissed was of age 20 at the time of the kissing and thus no temporal dependence holds.

1.3.3 What goes wrong

In order to capture the correct interpretation of sentence (29), we want to be able to describe the scenario given in (30) and (31). The point is that situational dependence holds for each of the three people involved, but for each person, a different situation ought to be considered. The correct truth conditions we seek should then look like the following:

(46) $\exists X \in \mathrm{D_e}[|X| = 3 \wedge [{*}\lambda x \in \mathrm{D_e}.\exists s \in \mathrm{D_s}[\pi_{\mathrm{i}}(s) \text{ is before now} \wedge$
$\pi_{\mathrm{i}}(s) \text{ is in the last 5 years} \wedge \textsf{20-yr-old}(s)(x) \wedge \textsf{kiss}(s)(x)(\textsf{Mary})]](X)]$

Here, existential closure of the situation variable has already been applied. Using the double * operator, we can keep a situation variable open at the top as follows:

(47) $\lambda S \in \mathrm{D_s}.\exists X \in \mathrm{D_e}[|X| = 3 \wedge [{*}{*}\lambda x \in \mathrm{D_e}.\lambda s \in \mathrm{D_s}.\pi_{\mathrm{i}}(s) \text{ is before now} \wedge$
$\pi_{\mathrm{i}}(s) \text{ is in the last 5 years} \wedge \textsf{20-yr-old}(s)(x) \wedge \textsf{kiss}(s)(x)(\textsf{Mary})](X)(S)]$

Variable S in (47) represents a plural situation. Both in (46) and in (47), variable s represents an atomic situation (i.e., a situation in which an atomic event occurs).

It should then be obvious why no attempt at interpreting LF (32) succeeds in capturing the correct interpretation of sentence (29). In order to obtain the correct analysis (46) or (47), before the meaning of *three* (the part that says $|X| = 3$) enters into computation, we first have to state a relation holding between an individual (i.e., x in (46) and (47)) and an *atomic* situation (i.e., s in (46) and (47)). This relation is composed of *20-year-old* and *Mary-kiss*, and in order to obtain situational dependence between these two predicates, it is necessary to simultaneously bind their situation arguments by an operator. Returning to LF (32), however, *three* is located in the scope of the λ abstractor that binds the situation variables. Therefore, one cannot create the sought relation to be combined with the denotation of *three*. As long as we assume (32), *three* is destined to combine with the two predicates before the situation variables get bound. This is why (32) cannot be the correct LF.

We then seem to have a case of scope paradox. We know that the numeral *three* ought to be interpreted above the λ abstractor. At the same time, the λ abstractor ought o bind the situation variable taken by the non-presuppositional noun phrase. It then looks as if the non-presuppositional noun phrase must be simultaneously in two different positions at LF, above and below the λ abstractor. In Section 1.5, I will resolve this paradox by proposing that the numeral is separated off from the rest of the non-presuppositional noun phrase and raised at LF.

1.4 Evidence for the scope theory

In the previous section, we have discovered that once the scope approach to Musan's generalization is adopted, we run into a scope puzzle when analyzing plural non-presupoositional noun phrases. However, if an alternative account of Musan's generalization without recourse to scope is feasible, the puzzle will become only a theory-internal problem which does not have to be of interest of all linguists. As far as I can see, Musan's original approach is the only alternative to the scope theory. In this section, I discuss Musan's theory and argue in favor of the scope theory by presenting data that Musan's theory has difficulty in accounting for.

1.4.1 Musan's argument against the scope approach

Musan claims that the scope approach is not tenable, arguing that it cannot account for sentences like (48) (= Musan' s (24) on p. 91). Musan focuses on the temporally independent reading of *Professoren* of (48a) described in (48b).

(48) a. In den sechziger Jahren spielten meistens alle Professoren, deren Eltern
In the sixties played mostly all professors whose parents
gerade in Urlaub waren, Federball.
just in vacation were badminton
'In the sixties, most often all professors whose parents were just then on vacation played badminton.'

b. For most times t, such that t is in the past and in the sixties, all x such that x is *now* a professor and x's parents were on vacation at t, x played badminton at t.

Musan identifies the operator that binds the time variables (= situation variables in my version) of the noun and the main predicate in the scope approach as a temporal adverb of quantification (TAQ). In (48a), *meistens* is the TAQ and the Tense and the temporal adverbial are restrictors to this TAQ. In reading (48b), the presuppositional, quantificational noun phrase *alle Professoren* ... is in the scope of the TAQ *meistens*, since the time of the main predicate of the relative clause is apparently bound by *meistens*. We thus have the following LF for (48a), where RC and NC stands for the restrictive clause and the nuclear scope respectively:

(49) meistens $[_{\text{RC(meistens)}}$ PAST & in-den-sechziger-Jahren]
$[_{\text{NS(meistens)}}$ alle $[_{\text{RC(alle)}}$ Professoren, deren ...] $[_{\text{NS(alle)}}$ Federball spielten]]

In the version of the scope approach that Musan assumes, a TAQ obligatorily binds every time variable in its scope. In (49), because the noun phrase *Professoren* is in the scope of the TAQ, its time argument should then be obligatorily bound by the TAQ. This should result in a temporally dependent reading 'professors in the sixties', but *Professoren* in (48a) actually has a temporally independent reading 'current professors'. To recapitulate, *meistens* takes scope over the quantificational noun phrase, but at the same time, this noun phrase should not be in the scope of *meistens* so as to have a temporally independent interpretation. This is a scope paradox, and Musan concludes that the scope approach is therefore not tenable.

I do not think, however, that Musan's argumentation is sufficient to reject the scope approach. Musan's refutation of the scope theory relies on two assumptions. The first assumption is that the TAQ is the only operator that binds time variables and the second one is that the TAQ must bind every time variable in its scope. If we do not have these assumptions, no scope paradox arises as discussed below.

To begin with, let us reject Musan's first assumption and assume that (48a) has another temporal/situational operator than the TAQ *meistens*, and this operator is located within or at the edge of the VP domain. According to Diesing's Mapping Hypothesis, the presuppositional noun phrase *alle Professoren ...* should move out of the VP domain to Spec, TP. Let us furthermore assume that the TAQ *meistens* comes above the presuppositional noun phrase. Then, abstracting away from the details, the LF for (48a) looks roughly like (50),[8] where Op represents the postulated operator:

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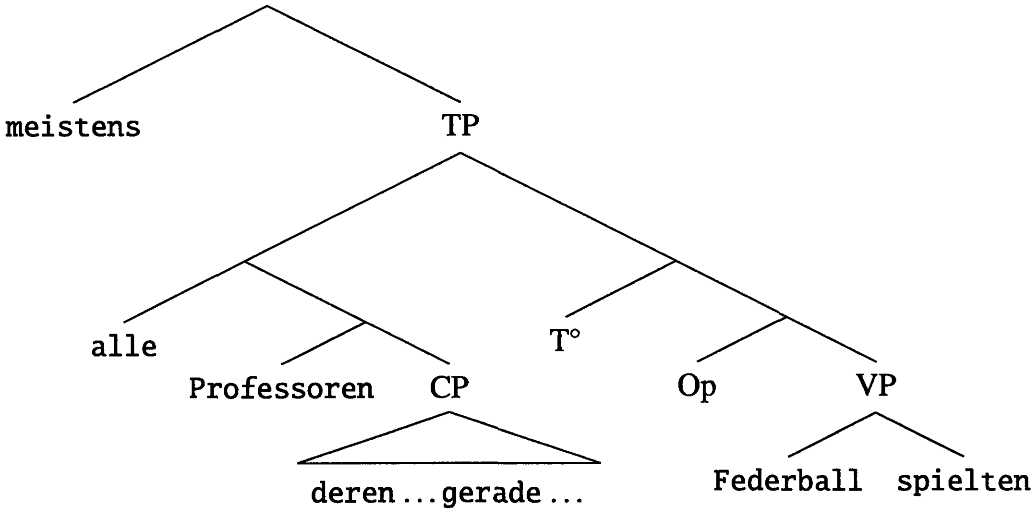
[8] For simplicity's sake, situation variables are not overtly indicated here, but each predicate is actually accompanied with a syntactically overt situation variable as its sister.

(50)



Now, let us modify Musan's second assumption as follows:

(51) A situational operator obligatorily binds every situation variable in its scope if and only if the operator is located in the VP domain.

In (50), since Op is in the VP domain, the situation argument of the verb *spielten*, which is in its scope, gets automatically bound by Op. The situation argument of the noun *Professoren* cannot of course be bound by Op, because it has moved out of its scope. The TAQ *meistens* binds a situation variable in the relative clause (which could be identified with *gerade*), but unlike Op, being located outside the VP domain, *meistens* does *not* obligatorily bind every variable in its scope. The situation argument of the noun *Professoren* may thus remain free, and when a variable assignment function assigns $\langle @, \mathsf{now} \rangle$ (where now represents a present time interval) to this free situation variable, the temporally independent reading 'current (and actual) professors' is obtained.

In the above proposal, then, we were able to obtain the situationally independent reading without inducing a scope paradox. Yet, this is still a version of the scope approach. Non-presuppositional noun phrases only receive situationally dependent readings, as they stay in the VP domain according to Diesing's Mapping Hypothesis and get trapped in the scope of an operator in the VP domain. On the other hand, presuppositional noun phrases move out of the VP domain, and there, their situation variables do not have to be bound locally. Thus, (48) is not evidence against the scope theory as Musan claims.

1.4.2 Musan's stage semantics approach

In this subsection, we first review Musans's stage semantics account of Musan's generalization. I then present data that cannot be dealt with by Musan's theory, but requires the scope approach.

On Musan's theory, noun phrases quantify over stages of individuals, and a temporally dependent reading is obtained when one and the same individual-stage is simultaneously predicated of by the nominal and main predicates. Musan construes stages of individuals as pairs of an individual and a time interval. Since I want to deal with the extended, situational version of Musan's generalization, let us assume that stages of individuals are pairs of an individual and a situation. On this approach, (29) should then be analyzed as follows:

(52) There are 3 maximal[9] stages of individuals x_{st} situated in the last five years such that [20-yr-old(x_{st}) $\wedge$ kiss(x_{st})(m)]

(52) *per se*, however, does not guarantee temporal dependence without ensuring an appropriate interpretation of the predicates. To guarantee temporal dependence, it is necessary to postulate the following principle:

(53) For every predicate P and for every stage of an individual $x_{st} = \langle x, s \rangle$, where $x \in D_e$ and $s \in D_s$, if x_{st} is an argument of P, then the event described by P occurs in situation s.

With (53), for 20-yr-old(x_{st}) to be true, where $x_{st} = \langle x, s \rangle$, there must be a (stative) event of x being of age 20, and this event occurs in situation s. Similarly, for Mary-kiss(x_{st}) to be true, there must be an event of Mary kissing x, and this event occurs in situation s. Therefore, when both of these hold, x must be of age 20 when Mary kissed him / her. Thus, Musan's stage semantics approach can naturally account for (29)'s reading.

It might then appear as if Musan's approach would work better than the scope theory I advocate, because as I discussed in Section 1.3, the scope approach seems to run into a scope puzzle in analyzing (29). However, as I present below, Musan's approach has fundamental difficulty in accounting for some bi-clausal sentences which the scope approach can deal with naturally as I discuss shortly.

Imagine that the sentences in (54) are uttered when no 20-year-olds have been contextually established. *three 20-year-olds* in (54) will then be non-presuppositional.

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[9] For a given individual, we count only the "maximal" stage of that individual that satisfy relevant properties. This way, we can avoid counting different stages of one and the same individual.

(54) a. When he was young, John promised to marry three 20-year-olds on their 30th birthday.

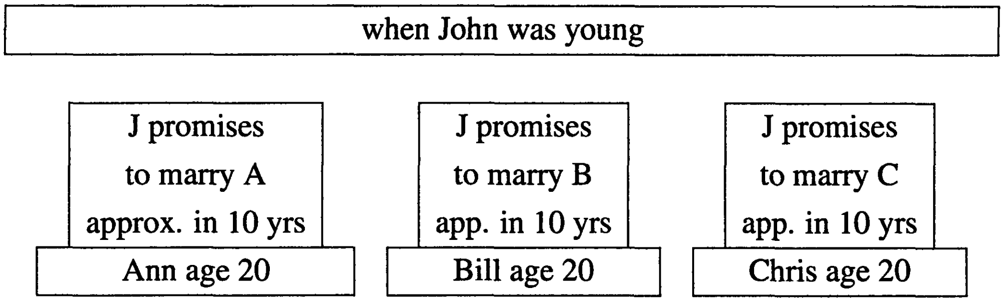
b. When he was young, John wanted to marry three 20-year-olds on their 30th birthday.

What are the truth conditions of the sentences in (54)? Now, consider the following scenario:

(55) When John was young, he met Ann, and when Ann was of age 20, John promised Ann that he would marry her on Ann's 30th birthday.
After a while, John got tired of Ann and met Bill. When Bill was of age 20, John promised Bill that he would marry him on Bill's 30th birthday.
After a while, John got tired of Bill and met Chris. When Chris was of age 20, John promised Chris that he would marry him on Chris's 30th birthday.
All of this happened decades ago when John was young.
Incidentally, Ann is 5 years younger than Bill, who is in turn 5 years younger than Chris. Thus, none of them have been of age 20 at the same time.

(56) illustrates the scenario:

(56)



Such a story is probable, if for example, John gets attracted to people just about 20 but gets tired of them when they become a bit older. Under this scenario, (54a) can clearly be uttered truthfully. Now, imagine a similar scenario, where John made the same promise to Ann, Bill and Chris respectively, but he made the promise to Ann when she was 21, to Bill when Bill was 22, and to Chris when Chris was 19, and John made such a promise only in these three occasions. In this case, it is wrong to describe the story with (54a).

In order for (54a) to be true, then, it must be the case that for each person that John made such a promise, that person was of age 20 at the time of the promise. This means that the temporal interpretation of *three 20-year-olds* depends on that of the matrix predicate *promise*. It must be noted, at the same time, that the temporal interpretation of *three 20-year-olds* is necessarily independent of that of the embedded predicate *marry*. In each promise, the marriage was supposed to take place when John's boyfriend/girlfriend turns 30. Obviously, he or she will no longer be of age 20 at the time of the envisaged marriage.

By altering the above scenario by replacing events of making a promise with events of John's having a relevant desire, one can easily see that in (54b), too, the temporal interpretation of *three 20-year-olds* depends on the matrix predicate (viz., *want*) while being independent of the embedded predicate *marry*.

We can then see that Musan's generalization is at work again in (54). Although the temporal interpretation of *three 20-year-olds* is not dependent on the main predicate of the embedded clause, it is still a temporally dependent reading as it depends on the main predicate of the matrix clause. Apparently, this is because *three 20-year-olds* is non-presuppositional.

Can Musan's stage semantics approach account for the temporally dependent readings in (54), then? In (54), *three 20-year-olds* is an argument of the embedded predicate *marry*. Therefore, on Musan's approach, sentences in (54) assert the existence of three stages of individuals x_{st} such that $\mathsf{marry}(x_{\text{st}})$ holds. Then, the principle in (53) predicts that the temporal interpretation of *three 20-year-olds* should depend on the embedded predicate *marry*. On the other hand, *three 20-year-olds* is not an argument of the matrix predicate *promise* or *want* in (54), and hence, Musan's theory cannot express its temporal dependence on the matrix predicate. However, as we have just seen above, the temporal interpretation of *three 20-year-olds* in (54) is independent of the embedded predicate, and depends on the matrix predicate instead. The predictions that Musan's theory makes is then exactly the opposite of the case with (54).

To recapitulate, Musan's theory accounts for the temporally dependent reading of a non-presuppostional noun phrase by virtue of the fact that the non-presuppostional noun phrase receives a thematic role from the main predicate it temporally depends on. However, this association was a mere coincidence which is due to considering only mono-clausal examples.

To fortify my argument, let us see a few more examples. In (54), we looked at cases where the temporal interpretation of a non-presuppostional noun phrase originated in an embedded clause depends on the matrix predicate. This time, we see cases where the world interpretation of a non-presuppostional noun phrase from an embedded clause depends on the matrix predicate.

Imagine that the sentences in (57) are uttered when no ugly 20-year-olds have been contextually established. *three ugly 20-year-olds* will then be non-presuppositional.

(57) a. In the last five years, three ugly 20-year-olds appeared to Mary to be beautiful.
b. In the last five years, Mary believed three ugly 20-year-olds to be beautiful.

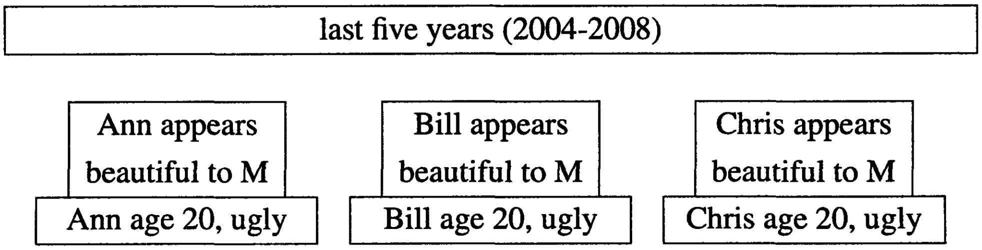
When uttered at the end of the year 2008, the sentences in (57) are true statements given the following scenario:

(58) Ann, Bill and Chris was born in 1984, 1986 and 1988 respectively.
At some point in 2004 after Ann's birthday, Ann was ugly as a matter of fact, but Mary saw Ann then and thought that Ann was beautiful.
At some point in 2006 after Bill's birthday, Bill was ugly as a matter of fact, but Mary saw Bill then and thought that Bill was beautiful.
At some point in 2008 after Chris's birthday, Chris was ugly as a matter of fact, but Mary saw Chris then and thought that Chris was beautiful.

Such a scenario is conceivable if we assume that beauty and ugliness are somehow absolute concepts in reality and that Mary has the wrong taste. Now, suppose that Ann, Bill and Chris were the only people that Mary have had contact with in the last five years and that Mary found each of them beautiful. Suppose further that none of these three people was ugly or of age 20 at the time Mary gave her aesthetic judgment to him/her unlike the scenario above. In this case, the sentences in (57) will be false.

Let us now focus on (57a), the model of which is depicted in (59):

(59)



Similarly to the case of sentences in (54), there are three events taking place in the actual world. These are events of someone appearing beautiful to Mary, and for each event, the person who appeared so was actually an ugly 20-year-old at the time of the event. It is then clear that *ugly 20-year-olds* has a temporally dependent reading here.[10] However,

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[10] *Ugly 20-year-olds* actually consists of the adjective *ugly* and the noun *20-year-olds*. When a non-presuppositional noun phrase is modified with an adjective, the situational interpretation of the adjective seems to coincide with that of the noun. Thus, both the adjective and the noun have a situational interpretation dependent on that of the main predicate. See Keshet's (2008) generalization.

unlike in (54), the evaluation times of the matrix clause and of the embedded clause seem to coincide here when the matrix predicate is the raising predicate *appear*. We therefore cannot conclude without careful examination whether the temporal interpretation of *ugly 20-year-olds* is dependent on the main predicate of the matrix clause or on that of the embedded clause.

Let us then focus on the world interpretation of *ugly 20-year-olds* instead. Suppose that *ugly 20-year-olds* and the embedded clause's main predicate *beautiful* would be evaluated with respect to the same world (i.e., *ugly 20-year-olds* is interpreted *de dicto*). As I have just pointed out above, (57a) asserts the existence of three people each of whom is predicated of by *ugly 20-year-olds* and *beautiful* with some identical reference time-interval. It would then follow that (57a) asserts the existence of three people each of whom is simultaneously ugly and beautiful in some world, which is, however, a contradiction so long as we assume that *ugly* and *beautiful* are antonyms. The evaluation world of *ugly 20-year-olds* must be the actual world instead (i.e., *ugly 20-year-olds* is interpreted *de re*), since (57a) talks about three people each of whom was actually ugly and of age 20 at some point. Then, *ugly 20-year-olds*'s situational interpretation, the combination of its temporal and world interpretations, is not dependent on the situational interpretation of the embedded predicate. Rather, we can conclude that the situational interpretation of *three ugly 20-year-olds* depends on that of the matrix predicate.

On Musan's approach, (57a) would be analyzed as asserting the existence of three (maximal) stages of individuals. Since the matrix predicate in (57a) is a raising predicate, *three ugly 20-year-olds* is not an argument of the matrix predicate *appear*, but an argument of the embedded predicate *beautiful*. Therefore, these stages of individuals only saturate an argument of *beautiful*. With (53), it is then predicted that the situational interpretation of *ugly 20-year-olds* should depend on the embedded predicate and nothing else. In fact, Musan's theory predicts that a non-presuppostional noun phrase can never have a *de re* interpretation (be it world-wise or temporal). Musan's theory thus makes the wrong prediction here. An analogous argument can be made for (57b), except that the matrix predicate is an ECM verb there.

Before moving on to the next subsection, let me provide some argument that the readings we have considered above are in fact non-presuppostional readings. To that end, I employ von Fintel's (1998) test to detect presuppositional indefinites. In this test, indefinites to be investigated are embedded in the antecedent of a conditional, which is preceded by a context to make it clear that the speaker is agnostic as to whether or not individuals of the relevant sort exist. Because presuppositions from the antecedent of a conditional are generally projected, if an indefinite in this position is presuppositional, it will clash with the preceding context. Consider (60) (= von Fintel's (9) on pp. 9-10):

(60) I'm not sure yet whether there are any mistakes at all in this book manuscript, but we can definitely not publish it

  a. if there turn out to be {some / more than a few / a significant number of} major mistakes in there.

  b. if {some / more than a few / a significant number of} major mistakes are found.

  c. # if there turn out to be {some / more than a few / a significant number of} mistakes are major.

(60a) and (60b) are fine because the indefinite *{some / more than a few / a significant number of} major mistakes* is non-presuppositional here. As for (60a), this is due to the existential *there* construction. As for (60b), the indefinite can be either presuppositional or non-presuppositional in principle. However, the presuppositional reading would clash with the preceding text and thus we interpret it non-presuppositionally. On the other hand, (60c) sounds strange. According to Diesing (1992), the subject of an individual-level predicate is always interpreted in a VP-external position and presuppositional. Since the adjective *major* is an individual-level predicate, the indefinite in (60c) should be presuppositional. More concretely, it presupposes that some mistakes exist. However, the preceding text says that the speaker does not know whether there are mistakes at all, which contradicts the presupposition.

Let us now embed our sentences in (54) and (57) (in a slightly modified form) in the schema of von Fintel's test. The examples all start with *I'm not sure if 20-year-old people are ever ugly ...* to make it clear that the speaker is agnostic as to the existence of ugly 20-year-olds. Accordingly, I have changed *20-year-olds* to *ugly 20-year-olds* in (54).

(61) a. I'm not sure if 20-year-old people are ever ugly, but I will certainly conclude that John has a bad taste, if when he was young, he promised to marry three ugly 20-year-olds on their 30th birthday.

  b. I'm not sure if 20-year-old people are ever ugly, but I will certainly conclude that John has a bad taste, if when he was young, he wanted to marry three ugly 20-year-olds on their 30th birthday.

(62) a. I'm not sure if 20-year-old people are ever ugly, but I will certainly conclude that Mary has a bad taste, if in the last five years, three ugly 20-year-olds appeared to her to be beautiful.

b. I'm not sure if 20-year-old people are ever ugly, but I will certainly conclude that Mary has a bad taste, if in the last five years, she believed three ugly 20-year-olds to be beautiful.

For instance, if I believe that young people are generally beautiful, I am not sure if there are any ugly 20-year-olds. Even so, it is clear that I can felicitously utter the sentences in (61) and (62), which indicates that *three ugly 20-year-olds* is non-presuppositional here. This point can be made clearer by comparing the above examples with (63) and (64) below:

(63) a. I'm not sure if 20-year-old people are ever ugly, but I will certainly conclude that John has a bad taste, if when he was young, he promised to marry three of the ugly 20-year-olds on their 30th birthday.

b. I'm not sure if 20-year-old people are ever ugly, but I will certainly conclude that John has a bad taste, if when he was young, he wanted to marry three of the ugly 20-year-olds on their 30th birthday.

(64) a. I'm not sure if 20-year-old people are ever ugly, but I will certainly conclude that Mary has a bad taste, if in the last five years, three of the ugly 20-year-olds appeared to her to be beautiful.

b. I'm not sure if 20-year-old people are ever ugly, but I will certainly conclude that Mary has a bad taste, if in the last five years, she believed three of the ugly 20-year-olds to be beautiful.

Here, the noun phrases at issue have been replaced with presuppositional ones (viz. partitives), and thus the sentences sound infelicitous.

For (61) and (62), where *three ugly 20-year-olds* is non-presuppositional, the reader may confirm the situational dependence of *three ugly 20-year-olds* on that of the matrix predicate in each of the above examples. In order to conclude that John has a bad taste, it is required that there be three people for each of whom, John had a desire to marry him/her, and furthermore, he/she was *actually* ugly (and of age 20) *at the time John had such a desire*. Whether they might have become beautiful with the help of plastic surgery by ten years later when marriage was planned to take place would not matter in deciding my opinion of John's aesthetics. Likewise, in order to conclude that Mary has a bad taste, it is required that there be three people for each of whom, John thought of him/her as beautiful, and furthermore, he/she was *actually* ugly (and of age 20) *at the time of the thought.* If ugliness were not these people's real property, I would not be able to be so judgmental on John's aesthetics.

1.4.3 The meaning of (non-)presuppositionality and binding theory of situation variables

We have seen above that Musan's stage semantics approach has difficulty in accounting for bi-clausal exmamplese like (54) and (57), which are repeated below, where a non-presuppositional noun phrase originated in the embedded clause has a situational interpretation dependent on the main predicate:

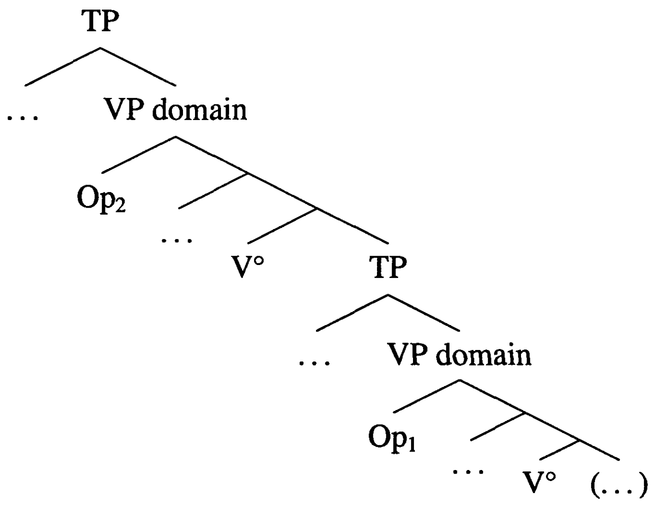
(54) a. When he was young, John promised to marry three 20-year-olds on their 30th birthday.
  b. When he was young, John wanted to marry three 20-year-olds on their 30th birthday.

(57) a. In the last five years, three ugly 20-year-olds appeared to Mary to be beautiful.
  b. In the last five years, Mary believed three ugly 20-year-olds to be beautiful.

The question is then whether the scope approach I advocate is able to account for such sentences. If the answer is yes, then an obvious question to ask is where (i.e. in which scope) the non-presuppositional noun phrase *three 20-year-olds* is interpreted.

Assuming that the situational interpretation of every main predicate is subject to an operator (i.e. λ abstractor in the implementation with syntactically overt situation variables) located at the edge of the local VP domain containing the main predicate, bi-clausal sentences as in (54) and (57) have two operators for the main and embedded main predicates as illustrated below:

(65)



Here, Op$_1$ is responsible for the situational interpretation of the embedded main predicate, and Op$_2$ is for the main predicate. The non-presuppositional noun phrase *three 20-year-olds* is originated inside the VP domain of the embedded clause. Should it then stay there and interpreted in the scope of Op$_1$, or should move out and escape this scope?

Let us first see what we would have to say if the noun phrase *three 20-year-olds* should stay inside the VP domain of the embedded clause at LF and be interpreted there. Because the interpretation of *20-year-olds* in (54) and (57) is situationally dependent not on the embedded verb but on the matrix verb, this must be given by Op$_2$ and not by Op$_1$. If *three 20-year-olds* were to stay inside the VP domain of the embedded clause at LF, it should involve a long-distance dependency between Op$_2$ and the noun phrase. However, it is not clear what mechanism would ensure this long-distance dependency correctly. Unfortunately, the hypothesis in (51) says nothing about this case, because it does not take such bi-clausal cases into account. Let us now furthermore consider the scopal interpretation of the numeral olxthree. The main predicate of the matrix clause in the sentences in (54) and (57) are attitude verbs that express a relation between events (or states) and propositions. If *three* were interpreted in the embedded clause in these sentences, they should be asserting the existence of an event which is related by the matrix attitude verb to propositions that concern three 20-year-olds. However, in the readings that we considered in the previous subsection, these sentences each assert the existence of *three* events which are each related by the matrix attitude verb to propositions that concern *one* 20-year-old. Therefore, *three* has to be interpreted outside the embedded clause. Again, if *three 20-year-olds* were to stay inside the VP domain of the embedded clause at LF, it would be a mystery how the desired meaning could be derived.

On the other hand, once we entertain the possibility that *three 20-year-olds* moves out of the scope of Op_1, but remains in the scope of Op_2, it turns out to be possible to account for its interpretation in the same way as for the situational dependent reading of a non-presuppositional noun phrase in mono-clausal examples.

In Subsection 1.4.1, I reviewed Musan's argument against the scope approach and concluded that her argument was not convincing because once we assume multiple temporal (situational) operators in a single clause and the hypothesis in (51), which is repeated below, the scope approach does not lead to a problem contrary to Musan's claim.

(51) A situational operator obligatorily binds every situation variable in its scope if and only if the operator is located in the VP domain.

Now, instead of this ad hoc hypothesis in (51), let us take (19) seriously and say that the situation variable that saturates the situation argument of a non-presuppositional noun phrase is syntactically an anaphor. Being an anaphor, it should obey something like Principle A of binding theory. I thus propose the following principle:

(66) Principle A of Binding Theory of Situations
A situational anaphor must be bound within the smallest VP domain that contains it.

Thus, the "binding domain" of a situation variable is the smallest VP domain that contains it. With this, the part of Diesing's Mapping Hypothesis about non-presuppositional noun phrases automatically follows. Since the situation argument of a non-presupposinal noun phrase is a situation anaphor, a non-presupposinal noun phrase always has to be in a VP domain so that its situation argument may be bound as required by Principle A of Binding Theory of Situations (66). Essentially, the operator that binds the situation anaphor is the operator that is responsible for the situation interpretation of the main predicate in the VP domain, so a situationally dependent reading results as Musan's generalization predicts.

On the above proposal, *20-year-olds* ought to move out of the VP domain of the embedded clause and end up in the VP domain of the matrix clause for the non-contradictory readings of the sentences in (54) and (57), as illustrated by the following:[11]

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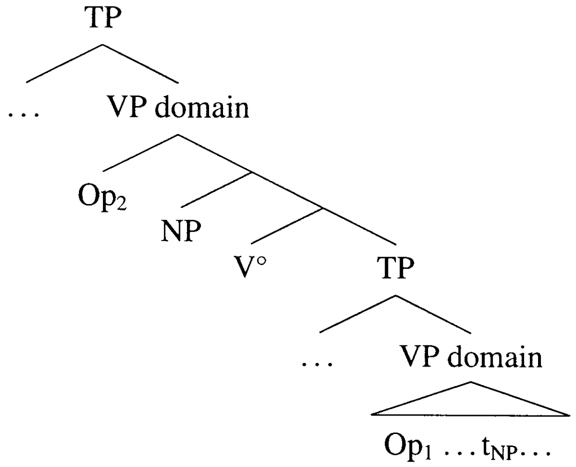
[11] I have assumed here that the noun phrase has moved out of the embedded TP. However, it is also possible to assume that the noun phrase stays inside the embedded TP after moving out of the embedded VP domain.

(67)



If *20-year-olds* stayed in the VP domain of the embedded clause, Principle A of binding theory of situations would dictate that its situation argument should be bound by Op_1, which would result in the contradictory reading of *20-year-olds* that is situationally dependent on the main predicate of the embedded clause. Thus, *20-year-olds* has to move out of the VP domain of the embedded clause. However, if *20-year-olds* moves all the way upstairs to even escape the VP domain of the matrix clause, its situation argument cannot satisfy Principle A of binding theory of situations, as it is no longer in a binding domain. Hence, *20-year-olds* ends up in the VP domain of the matrix clause and its situation argument gets bound by Op_2, and receives a reading that is situationally dependent on the main predicate of the matrix clause. As a consequence, *three* is also interpreted outside the embedded clause, but a scope puzzle of the sort discussed in Section 1.3 obtaines. This is treated together with the original example presented in Section 1.3 by postulating movement of the numeral in the next section.

The assumption that the situation variable that saturates the situation argument of a non-presuppositional noun phrase is an anaphor is not an ad hoc stipulation, but it is actually motivated by semantic/pragmatic reasons. A non-presuppositional noun phrase is a noun phrase used to assert the existence of individuals that are predicated of by the noun. Nevertheless, a noun phrase itself is not the main predicate of a sentence.[12] Instead, a noun phrase appears as an argument of a main predicate or as the complement of the preposition in a PP which directly or indirectly modifies a main predicate. Since a declarative sentence

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[12] In the sentences like the following, the noun phrase *a 20-year-old* may appear to be the main predicate:

(i) There is a 20-year-old.

(ii) John is a 20-year-old.

For such sentences, however, the verb *be* can be regarded as the main predicate.

asserts the existence of an event of a particular sort, the existential assertion by a non-presuppositional noun phrase is made indirectly by means of modifying an event whose existence is asserted by the sentence in which it appears. If some event is asserted to occur which involves an individual predicated truly of by the noun, then it should amount to asserting the existence of an individual predicated truly of by the noun at the same time and in the same world as the event occurs. For example, when the sentence "Mary kissed a 20-year-old" is uttered out of the blue, it asserts the existence of a kissing event that occurred at some time in the actual world. As the non-presuppositional noun phrase *a 20-year-old* modifies this event, there must have been some 20-year-old who was involved in this event. It is then natural that he or she was of age 20 at the time of the event in the actual world. Thinking this way, it makes sense to assume that a non-presuppositional noun phrase comes with a situation anaphor, so that it situational interpretation coincides with that of the main predicate.

By contrast, a presuppositional noun phrase requires that the discourse have established the existence of an individual or individuals predicated truly of by the noun. This in turn means that the discourse has already made reference to some situation (time-world pair) with respect to which that individual is (or those individuals are) predicated truly of by the noun. Therefore, the situation variable that saturates the situation argument of a presuppositional noun phrase should not be co-bound with the situation argument of the main predicate, and its value must instead be given independently of the evaluation situation of the main predicate. In other words, the situation argument of a presuppositional noun phrase must be a pronoun rather than an anaphor, and its value must be given by a variable assignment function. The value assigned to the situation argument of a presuppositional noun phrase ought to be a situation which has been made salient in the discourse and in which individuals that satisfy the noun predicate are already known to exist. The presupposition of the noun phrase is thus satisfied. If it happens that no individual satisfies the noun predicate in whatever situation assigned to the situation argument of the presuppositional noun phrase, a presupposition failure obtains. Just as situation anaphors are subject to something like Principle A of binding theory, situation pronouns should be subject to something like Principle B of binding theory, which should look like the following:

(68) Principle B of Binding Theory of Situations

A situational pronoun must be free within the smallest VP domain that contains it.

This actually turns out to be problematic as I discuss shortly.

To recapitulate, I propose the following:

(69) The type of situation argument of (non-)presuppositional noun phrases

a. The situation argument of a non-presuppositional noun phrase is saturated by a situational anaphor.

b. The situation argument of a presuppositional noun phrase is saturated by a situational pronoun and not by a situational anaphor.

Ordinary pronouns (i.e. pronouns that range over individuals) obey Principle B of binding theory, so they must be free in their binding domain, but they can still be bound within a sentence. Similarly, a situation pronoun may be bound within a sentence. This should be possible if a presuppositional noun phrase is c-commanded by an operator that is located outside the binding domain of the noun phrase. Such a configuration will give rise to a reading of a presuppositional noun phrase that bears no global presupposition. A relevant example would be the interpretation of *the 20-year-old* in the following sentence:

(70) If Mary met a 20-year-old, she kissed the 20-year-old when he was 21.

Here, the *if* clause would bind the situation argument of the presuppositional noun phrase *the 20-year-old*, but this clause is located outside the VP domain of the main clause. Notice that the temporal interpretation of *the 20-year-old* is independent of the main predicate *kissed*, as the kissing would occur when the kissee was no longer 20. We can then propose the following:

(71) Presupposition of noun phrases

A noun phrase bears presupposition in a given domain if and only if its situation argument is free in that domain.

Let us now turn to Diesing's Mapping Hypothesis before we can dicuss the problem of Principle B of binding theory of situations. Below, I repeat the restatement of Diesing's (1992) Mapping Hypothesis in (10) from Subsection 1.2.1:

(10) Non-presuppositional noun phrases and existential bare plurals are interpreted in the VP domain (hence mapped into the nuclear scope).

Presuppositional noun phrases and generic bare plurals are interpreted outside the VP domain (hence mapped into a restrictive clause).

Disregarding bare plurals, which I do not discuss here, this can be restated as the following:

(72) Mapping Hypothesis

If a noun phrase is interpreted in the VP domain, it must be non-presuppositional.

If a noun phrase is interpreted outside the VP domain, it must be presuppositional.

(72) makes clear predictions for mono-clausal sentences, but it does not necessarily do so for bi-clausal sentences. What shall the Mapping Hypothesis say about LF like (67) that are postulated for sentences as in (54) and (57)? The noun phrase in (67) (denoted as NP) has moved out of the VP domain of the embedded clause. Therefore, the second statement in (72) seems to predict it to be presuppositional. On the other hand, since the noun phrase remains in the VP domain of the matrix clause, the first statement in (72) seems to predict it to be non-presuppositional. Thus, the Mapping Hypothesis is not well-defined to deal with cases where there is more than one VP domain in a sentence.[13] This seems to indicate that the Mapping Hypothesis may not be a principle but an observation based on a limited set of data. The reader should be aware that even though the Mapping Hypothesis makes a very interesting connection between noun phrases' syntactic positions and their interpretations, it does not explain why there has to be such a connection.

Now, recall that our original account of Musan's generalization resorts to the Mapping Hypothesis and the hypothesis in (51). However, if we accept that the Mapping Hypothesis is not a principle, our theory should not ultimately be based on the Mapping Hypothesis. Rather, our desideratum would be to derive the observation called Diesing's Mapping Hypothesis from some deeper principles. We have already seen above that Principle A of binding theory of situations derives the part of Diesing's Mapping Hypothesis on non-presuppositional noun phrases. Also, the proposal in (69a) renders hypothesis (51) unnecessary for non-presuppositional noun phrases. However, unlike that case, Principle B of binding theory of situations as posited in (68) cannot derive the other half of Diesing's Mapping Hypothesis regarding presuppositional noun phrases, i.e., the claim that presuppositional noun phrases move out of the VP domain. According to (68), there would be no problem even if a presuppositional noun phrase stayed in a VP domain, as long as its situation argument remained free there. Therefore, being a presuppositional noun phrase does not require that it move out of some VP domain.

Moreover, consideration of the Principle B posited in (68) reveals a number of problems. Once we accept the Mapping Hypothesis, (68) is rather meaningless for mono-clausal sentences. Since there is only one VP domain in a mono-clausal sentence, once a presuppositional noun phrase in a mono-clausal sentence has moved out of the single VP domain

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[13] It seems that Diesing does not think of the possibility that a noun phrase moves out of an embedded clause and stops within the VP of the matrix clause. For one thing, she considers examples with raising predicates, and claims that when the raised subject is interpreted outside the embedded clause, it has to be interpreted at Spec, IP (= TP) of the matrix clause (Diesing, 1992, p. 25). My sentences in (54) and (57) constitute counterexamples to this claim.

of the sentence as dictated by the Mapping Hypothesis, its situation argument can no longer be in a binding domain, so Principle B is vacuously satisfied. In addition, we might have a problem with bi-clausal sentences whose embedded sentence is like (70) or (48a) in the sense that a noun phrase containing a situation pronoun moves out of the VP domain of an embedded clause but this situation pronoun gets bound by some operator within the embedded clause (the *if* clause in (70) and the TAQ in (48a)). Then, being in the VP domain of the matrix clause, this situation pronoun would violate (68), because the situation is bound within the smallest VP domain containing it. To remedy this, it appears necessary to seek an appropriate definition of binding domain for binding theory of situations.

Given the above concerns, it is clear that we have not yet captured the real nature of presuppositional noun phrases. Maybe we need to postulate a principle radically different from what was posited in (68), which might be able to derive the other half of the mapping hypothesis. Or, hypothesis (51) might turn out to be necessary after all. Because then, a non-presuppotiona noun phrase would be forced to move out of a VP domain in order to avoid getting bound there. For now, I would like to leave these issues for the future research.

Finally, let us return to Musan's argument against the scope approach again. Recall that Musan claimes that the scope theory runs into a scope paradox for sentences like (48a). However, Musan herself actually notes that the scope paradox can be avoided if one assumes that the time argument of a presuppositional noun phrase does not have to be bound by the TAQ, unlike the time argument of a non-presuppostional noun phrase. Musan says that such a stipulation would amount to attributing the availability of temporally dependent readings to some internal property of non-presuppositional and presuppositional noun phrases, and regards this as the defeat of the scope approach. Musan then goes on to develop a stage-semantics approach which entirely relies on internal properties of noun phrases (i.e., different denotations of non-presuppositional and presuppositional determiners). However, I think that this reasoning is misguided.

It may be true that an account of Musan's generalization purely in terms of scope is not possible. The version of the scope approach I propose here resorts to some intrinsic difference between non-presuppositional and presuppositional noun phrases. More concretely, I have proposed above that only non-presuppositional noun phrases takes a situational anaphor. Be that as it may, the scope approach still has theoretical advantages over Musan's stage semantics approach, as it has a robust association with the independently motivated Mapping Hypothesis by Diesing, while Musan's stage semantics approach has nothing to say on this count. We still have not answered why presuppositional noun phrases move out of the VP domain as the mysterious half of Diesing's Mapping Hypothesis states, but this cannot be a reason to reject the scope approach and adopt Musan's theory, since neither does Musan's theory answer this question.

To sum up this section, I conclude that on both empirical and theoretical grounds, the scope approach fares better than Musan's stage semantics approach.

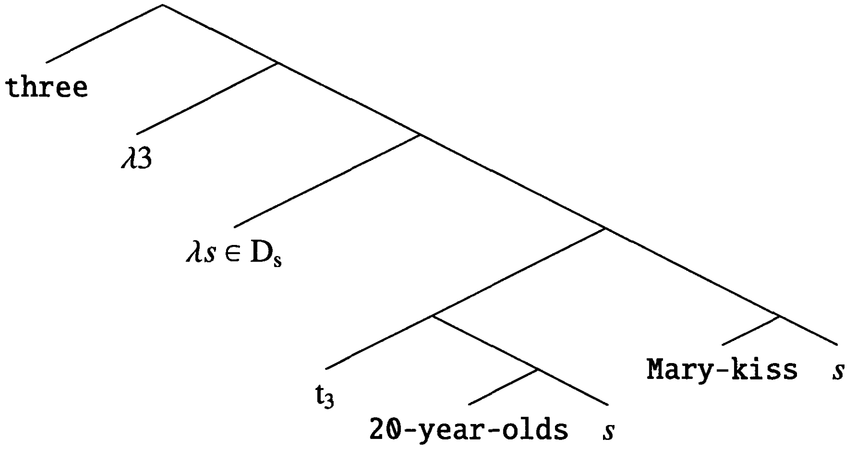
1.5 Solution to the scope puzzle: Movement of the numeral

I have argued thus far that the scope theory is the most reasonable approach to the situational dependence of non-presuppositional noun phrases. However, if my analysis is on the right track, then the scope puzzle revealed in Section 1.3 cannot be marginalized as a merely theory-internal problem. The problem was that the numeral part of a a non-presuppositional noun phrase such as *three* need to take scope over this λ abstractor that is supposed to co-bind the situation arguments of the non-presuppositional noun phrase and the main predicate, but this appears to require the non-presuppositional noun phrase to be above and below the λ abstractor simultaneously.

1.5.1 Basic account

In order to solve the scope puzzle, I propose that the non-presuppositional noun phrase is interpreted in the scope of the λ abstractor, but the numeral separates off and moves to the position right above the λ abstractor. Thus, ignoring the temporal adverbial, we have the following LF for the sentence in (29):

(73)



This LF cannot be interpreted without modifying some lexical entries and/or interpretation principles, however. This is explained and fixed in what follows.

First, remember that the denotation of the numeral *three* has been assumed to be of type $\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$ earlier. This was because the numeral was assumed to be interpreted in the sister position of the noun predicate. If we kept assuming this type for the numeral, (73) would be uninterpretable unless we assume a higher-type trace and lambda abstraction over variables of type $\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$. However, if we did this, the moved element would be semantically reconstructed, and hence the very idea of raising the numeral would be voided. We should therefore write a new denotation for *three* which is of some other semantic type so that it is semantically interpreted in the top position.

Now, recall that in Subsection 1.3.3, we saw that the correct interpretation of (29) should be given by (47), which is repeated below:

(47) $\lambda S \in D_s.\exists X \in D_e[|X| = 3 \wedge [{**}\lambda x \in D_e.\lambda s \in D_s.\pi_i(s)$ is before now $\wedge$
$\pi_i(s)$ is in the last 5 years $\wedge$ $\mathsf{20\text{-}yr\text{-}old}(s)(x) \wedge \mathsf{kiss}(s)(x)(m)](X)(S)]$

As I discussed there, in order to obtain this interpretation, we first have to have a relation holding between an individual and an atomic situation before the meaning of *three* enters into computation. Ignoring the details that restricts situations to those located in the past and in the last five years, this relation we are after is the following:

(74) $\lambda x \in D_e.\lambda s \in D_s.\mathsf{20\text{-}yr\text{-}old}(s)(x) \wedge \mathsf{kiss}(s)(x)(m)$

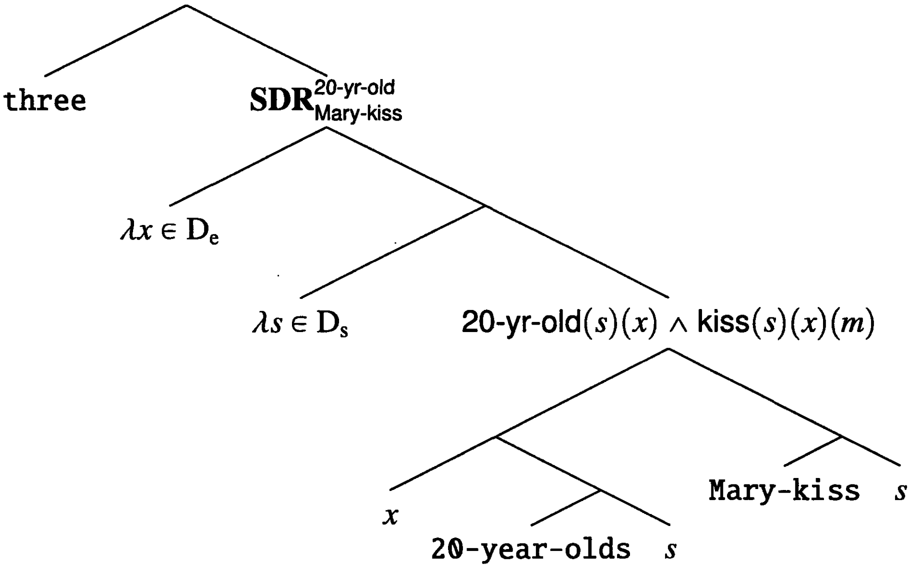
Since this relation describes situational dependence, let us refer to such a relation as a **situationally dependent relation**. For convenience, given a non-presuppositional noun phrase whose head noun denotes predicate φ and a main predicate ψ, let us notate the relevant situationally dependent relation with $\mathbf{SDR}^{\varphi}_{\psi}$. Thus,

(75) $\mathbf{SDR}^{\varphi}_{\psi} \overset{\text{def}}{=} \lambda x \in \mathrm{D_e}.\, \lambda s \in \mathrm{D_s}.\, \varphi(s)(x) \wedge \psi(s)(x)$

With this notation, the situationally dependent relation given in (74) is written as $\mathbf{SDR}^{\textsf{20-yr-old}}_{\textsf{Mary-kiss}}$, where Mary-kiss is short for $\lambda s \in \mathrm{D_s}.\, \lambda x \in \mathrm{D_e}.\, \textsf{kiss}(s)(x)(\textsf{Mary})$. The crucial assumption is that situationally dependent relations hold true only of atomic situations. Therefore, even if $\mathbf{SDR}^{\textsf{20-yr-old}}_{\textsf{Mary-kiss}}(a)(s)$ and $\mathbf{SDR}^{\textsf{20-yr-old}}_{\textsf{Mary-kiss}}(b)(t)$ hold true for some individuals a and b and some atomic situations s and t, $\mathbf{SDR}^{\textsf{20-yr-old}}_{\textsf{Mary-kiss}}(a \sqcup_{\mathrm{e}} b)(s \sqcup_{\mathrm{s}} t)$ never holds. That $\mathbf{SDR}^{\textsf{20-yr-old}}_{\textsf{Mary-kiss}}$ holds only of atomic situations can be derived by assuming that kiss holds only of atomic situations. Note that situationally dependent relations may be true of non-atomic individuals.

Provided that the denotation of the sister node of λs in (73) is $\textsf{20-yr-old}(s)(x) \wedge \textsf{kiss}(s)(x)(m)$, we expect to obtain $\mathbf{SDR}^{\textsf{20-yr-old}}_{\textsf{Mary-kiss}}$ if the λ abstractor created by the movement of the numeral ($\lambda 3$ in (73)) abstracts over individuals:

(76)



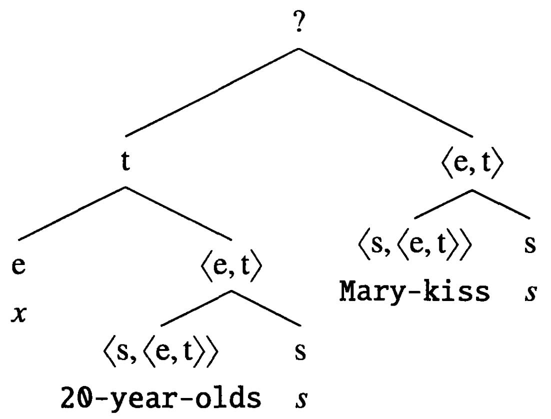
Then, since $\mathbf{SDR}^{\textsf{20-yr-old}}_{\textsf{Mary-kiss}}$ is the denotation of the sister of *three*, what we need to do is to write a lexical entry for *three* which takes $\mathbf{SDR}^{\textsf{20-yr-old}}_{\textsf{Mary-kiss}}$ and then yields (47). The lexical entry of *three* should then be as follows:

(77) $[\![\texttt{three}]\!] = \lambda R \in \mathrm{D}_{\langle \mathrm{e}, \langle \mathrm{s}, \mathrm{t} \rangle \rangle}.\, \lambda S \in \mathrm{D_s}.\, \exists X \in \mathrm{D_e}[|X| = 3 \wedge {}^{**}R(X)(S)]$

The semantic type of the new denotation of numerals is thus $\langle \langle \mathrm{e}, \langle \mathrm{s}, \mathrm{t} \rangle \rangle, \langle \mathrm{s}, \mathrm{t} \rangle \rangle$. With (77), we can compositionally compute the desired interpretaion in (47).

What is now necessary is to arrive at the intersection of the noun predicate and the main predicate, viz., $\textsf{20-yr-old}(s)(x) \wedge \textsf{kiss}(s)(x)(m)$. It turns out, however, that without further adjustment, not only can we not arrive at this, there is a semantic type mismatch when one attempts to combine the noun and the main predicate, as illustrated by the following:

(78)

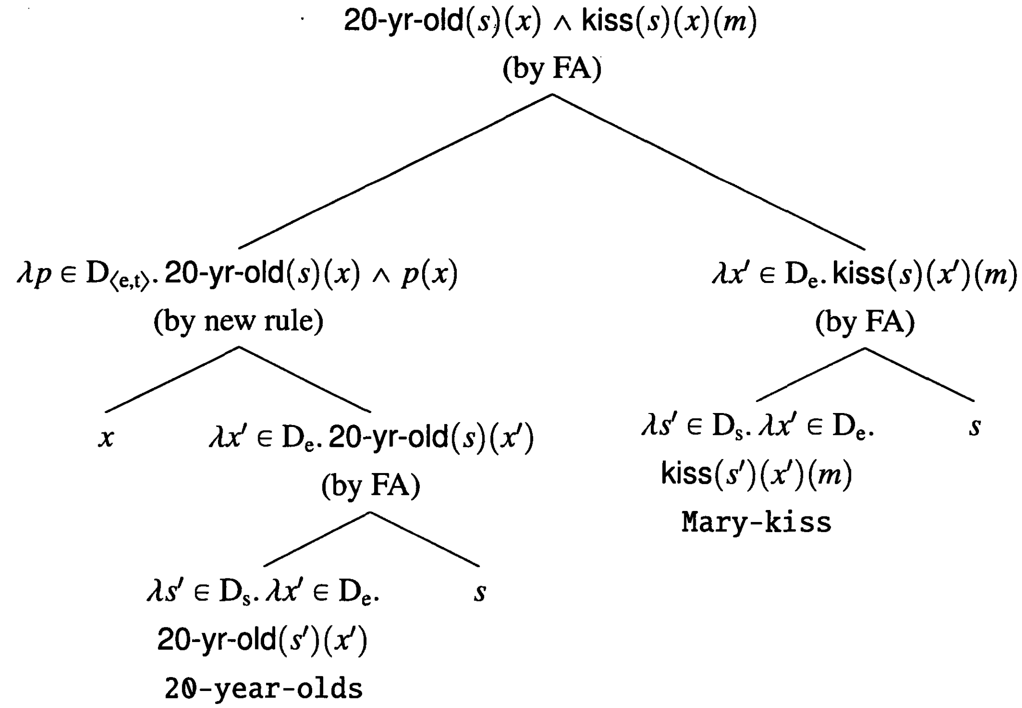


When the denotation of numerals was assumed be of type $\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$, the formation of the intersection of the noun predicate and the main predicate was encoded in their denotation (see (18a), (36) and (40c)). However, now that numerals do not take two predicates, the requisite intersection must be performed independently. A semantic type mismatch arose since nothing in (78) computes this intersection. In order to remedy this, I propose a new principle of interpretation as follows:

(79) Let α and β be sisters such that $[\![\alpha]\!]$ is of type e and $[\![\beta]\!]$ is of type $\langle e,t\rangle$. Then,
$$\left[\!\!\left[\alpha\ \widehat{\ }\ \beta\right]\!\!\right] = \lambda p \in D_{\langle e,t\rangle}.\ [\![\beta]\!]([\![\alpha]\!]) \wedge p([\![\alpha]\!])$$

With this new principle, the denotation of tree (78) can now be computed as follows:

(80)



We can then see that the correct semantics of sentence (29) can now be computed compositionally from LF (76).

Instead of adopting a new interpretation principle, one could achieve the same results only by means of Functional Application if one postulate some functional head which takes the noun and the main predicate to form their intersection and applies it to the variable represented by the trace of *three*. However, this strategy also has to stipulate when one needs this functional head and when one should not have it. For example, when analyzing noun phrases such as *the three 20-year-olds*, one should not have such a functional head because one need not take the intersection of two predicates, but stipulating the relevant conditions as to when to have this functional head seems to be an undesirable complication. The above proposal with the new interpretation principle, on the other hand, does not face such a problem.

I would now like to dispel a possible worry concerning the solution proposed above. According to this solution, when a non-presuppositional noun phrase is a subject (as in (57a)), the numeral will have to be extracted out of a subject. However, extraction out of a subject is often thought to be impossible (Huang's (1982) CED).

Diesing (1992, pp. 31-41) has already made a point regarding this. She argues that while extraction out of a subject in Spec, TP[14] is not possible, extraction of a subject in Spec, VP is possible.[15] In (81) and (82) (from Diesing's (26) p. 32 and (27) on p. 33 respectively), the particles *denn* and *ja* mark the edge of VP respectively. When the subject is to the right of the particle, i.e., in Spec, VP, *was* or *Ameisten* can be extracted out of it. By contrast, when the subject is to the left of the particle, i.e., in Spec, TP, such extraction is not possible.

(81) a. [$_{CP}$ Was$_i$ haben [$_{TP}$ denn [$_{VP}$ t$_i$ für Ameisten einen Postbeamten
what have 'indeed' for ants a postman
gebissen]]]?
bitten
'What kind of ants have bitten a postman?'

---

[14] This is Diesing's IP. I have changed the terminology to make it consistent with the assumption here. The label TP in (81) was orginially IP.

[15] Diesing attributes this observation to Angelika Kratzer in her class lectures in 1988.

b. * [$_{CP}$ Was$_i$ haben [$_{TP}$ t$_i$ für Ameisten denn [$_{VP}$ einen Postbeamten
what have for ants 'indeed' a postman
gebissen]]]?
bitten

(82) a. Ameisten$_i$ haben ja einen Postbeamten [$_{NP}$ viele t$_i$] gebissen.
ants have PRT a postman many bitten
'As for ants, many have bitten a postman.'

b. * Ameisten$_i$ haben [$_{NP}$ viele t$_i$] ja einen Postbeamten gebissen.
ants have many PRT a postman bitten

The reader is referred to Diesing (1992) for more examples of this kind. We can find more supportive evidence from German in Haider (2000):

(83) a. [Welches Buch]$_i$ hat [t$_i$ zu lesen] dir mehr Spaß gemacht?
which book has to read you more fun made

b. * [Welches Buch]$_i$ sagte sie [$_{CP}$ [t$_i$ zu lesen] [habe [ihr Spaß gemacht]]]?
which book said she to read has her fun made

According to Haider, in (83a), the subject is in a VP-internal position and extraction out of is possible, whereas in (83b), the subject is in a VP-external position and such extraction is not possible. Finally, consider the following English sentence:

(84) [Which student]$_i$ are there pictures of t$_i$ on the table?

Here, *wh*-movement takes place out of the subject, which is located in the VP-domain due to the existential *there* construction.

Given the above examples, I conclude that extraction out of the subject becomes impossible when the subject has moved out of the VP-domain, but such extraction is possible when the subject is in the VP-domain. Then, the extraction of the numeral out of non-presuppositional noun phrases that I have proposed above should be no problem, since non-presuppositional noun phrases stay in the VP domain at LF as discussed so far.

1.5.2 Bi-clausal cases

We can now account for bi-clausal sentences like (54) and (57), which are repeated below:

(54) a. When he was young, John promised to marry three 20-year-olds on their 30th birthday.

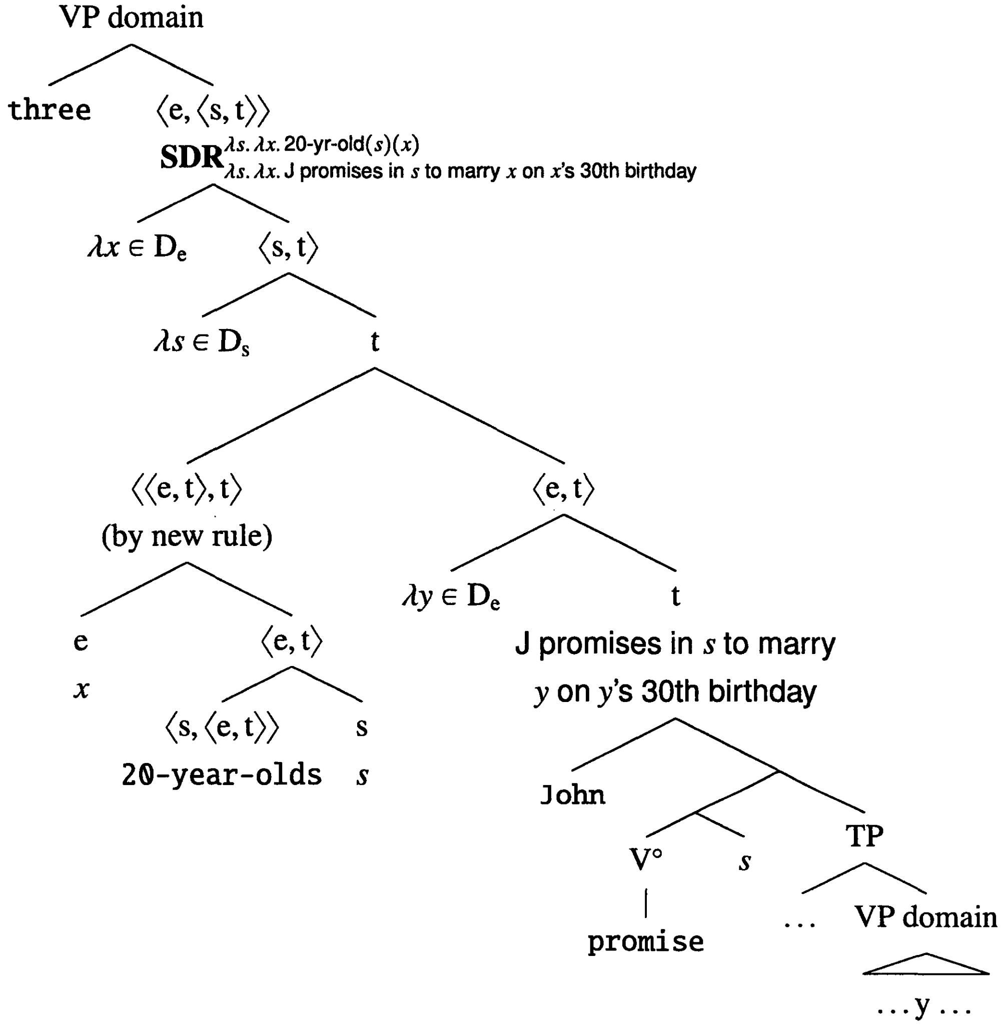
b. When he was young, John wanted to marry three 20-year-olds on their 30th birthday.

(57) a. In the last five years, three ugly 20-year-olds appeared to Mary to be beautiful.

b. In the last five years, Mary believed three ugly 20-year-olds to be beautiful.

The truth conditions of (54a), for instance, can be derived from the following LF:

(85)



Here, *three 20-year-olds* first moves out of the VP domain of the embedded clause and lands inside the VP domain of the matrix clause as proposed in (67), creating a λ abstraction (λy). In order to obtain the reading where events predicated of by the matrix verb are distributed over three people, the numeral *three* gets separated and moves further albeit locally as in (76). The new principle of interpretation is utilized in interpreting the non-presuppositional noun phrase whose numeral has moved out in the same manner as in (80). Thus, the truth conditions of the sentence are computed compositionally without addtional assumptions.

Chapter 2

How to Measure Size

This chapter analyzes what Krifka (1990) terms as event-related readings. We will first see that a novel "pluralization" method such as what Krifka proposes should be adopted to account for event-related readings. Our result from Chapter 1 is then incorporated into this theory. It is then argued that units of measurement must be sensitive to a situation, which discussion reveals the necessity to employ integration to describe the semantics of measure phrases correctly. The chapter then introduces the theory of Lebesgue integration, under which the intuition and concepts behind the analysis will be articulated. The theory offers a unified analysis of discrete and continuous events where existential quantifications over times are replaced by conditions on integrals.

2.1 Four thousand ships passed through the lock

This section introduces the notion of event-related readings (Krifka 1990). In Chapter 1, in order to account for plural non-presuppositional noun phrases, I proposed that a relation between atomic time intervals and individuals is first obtained and is then pluralized using the (double) * operator. I show that this way of pluralization is inadequate to capture event-related readings. I will then review Krifka's own analysis of event-related readings.

2.1.1 What is an event-related reading?

Krifka (1990) observes that sentence (1) has two readings which are paraphrased as in (2):

(1) Four thousand ships passed through the lock last year.

(2) a. There are four thousand different ships that passed through the lock last year.
b. There were four thousand events of passing through the lock by a ship last year.

Krifka terms reading (2a) as the **object-related reading** of (1), as it counts the number of relevant objects (viz. ships), and reading (2b) as the **event-related reading** of (1), as it counts the number of relevant events. These two readings are distinct. Even if there were less than four thousand ships in the world, if they each passed through the lock many times last year so that the total number of the lock traversals has reached 4000, (1) should be true in the event-related reading. However, on the very same scenario, (1) should be false in the object-related reading, as it requires the existence of four thousand distinct ships (that passed through the lock). The aim of this chapter it to develop a theory of how to account for event-related readings. Unfortunately, I have not been able to include my analysis of object-related readings in this dissertation, but I will talk about those in future writing.

The event-related reading of a sentence concerns the interpretation of a particular noun phrase in the sentence. In the case of (1), it is the interpretation of noun phrase *four thousand ships* which is at issue. In particular, I will focus only on the non-presuppositional reading of such event-related-reading-inducing noun phrases. As Musan's generalization predicts, when (1) is uttered out of the blue (i.e. without presuppositions), the situational interpretation of *four thousand ships* depends on that of the main predicate *passed through the lock.* To see this clearly, let us modify the noun with the stage-level adjective *damaged* and embed the whole sentence in a belief context:

(3) Mary believes that four thousand damaged ships passed through the lock last year.

Clearly, this sentence can be uttered felicitously without presupposing the existence of some damaged ships. Furthermore, the interpretation of *four thousand damaged ships* is situationally dependent on the embedded predicate *passed through the lock.* Let us focus on the event-related reading now. For (3) to be true, it is necessary that in Mary's belief world, four thousand events have taken place last year, each of which was an event of passing though the lock by an entity which was damaged and a ship *at the time of the event* and *in Mary's belief world.* If in Mary's belief world, a ship passed though the lock last year, but this ship was not damaged at the time of passing (in Mary's belief world), then, this event will obviously not be included in the four thousand.

Interestingly, in event-related readings, the responsible noun phrase seems to have a situationally dependent interpretation even when it is presuppositional. Krifka (1990) observes that event-related readings are possible with quantified noun phrases as in the following (p. 509):

(4) a. Most ships passed through the lock at night.
    b. Every ship passed through the lock at night.

The event-related reading of (4a), for instance, asserts that most of the lock traversals of a ship occurred at night. Due to *most* and *every*, which are strong quantifying determiners and hence presuppositional, (4a) and (4b) presuppose that there have been some lock traversals of a ship. Now, consider the sentences in (5), where the noun is modified by *damaged*:

(5) a. Most damaged ships passed through the lock at night.
    b. Every damaged ship passed through the lock at night.

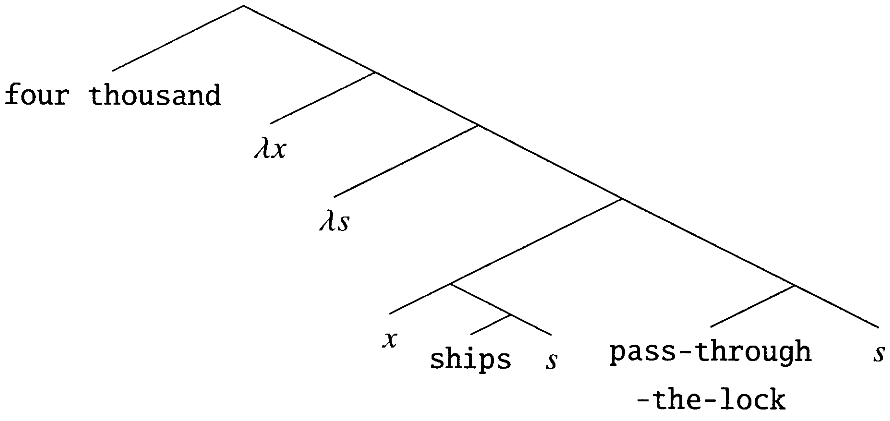
In the event-related reading, (5a) seems to mean that most of the events of passing through the lock by a damaged ship occurred at night. Similarly, (5b) in the event-related reading seems to mean that every event of passing through the lock by a damaged ship occurred at night. Then, it seems that situational interpretation of these noun phrases depends on the main predicate even though they are presuppositional. This contrasts with object-related readings. The sentences in (5) also have object-related readings. (5a) in the object-related reading simply means that most of some damaged ships in a given discourse passed through the lock. Similarly, (5b) in the object-related reading means that every one of some damaged ships in a given discourse passed through the lock. Thus, the sentences in (5) in the object-related readings may well be talking about some ships that are currently damaged, and therefore they do not require that ships have be damaged at the time of their passing through the lock. This is an interesting phenomenon, but I will not discuss presuppositional noun phrases in event-related readings here, and leave it for the future research. In what follows, we focus on non-presuppositional noun phrases in event-related readings.

2.1.2 Inadequacy of the (double) $*$ operator

In this subsection, I would like to apply the analysis of plural non-presuppositional noun phrases developed in Chapter 1 to Krifka's sentence (1), and to show that it does not capture the event-related reading of the sentence.

In Section 1.3 of Chapter 1, we saw that in order to capture the situational dependence of a plural non-presuppositional noun phrase, it is necessary to obtain a situationally dependent relation between an individual and an atomic situation before some "pluralization" for the situation argument is operated. In Section 1.5 of Chapter 1, I proposed that a situationally dependent relation can be formed by (the λ abstraction created by) LF movement of the numeral of the non-presuppositional noun phrase. In the case of (1), the numeral *four thousand* should be split and raised to a position right above the λ abstractor that simultaneously binds the situation argument of the noun phrase and the main predicate. To abstract away from unnecessary details, let us ignore the temporal adverbial *last year* and also assume that *pass through the lock* is one lexical item. The LF of (1) should then look like the following:

(6)

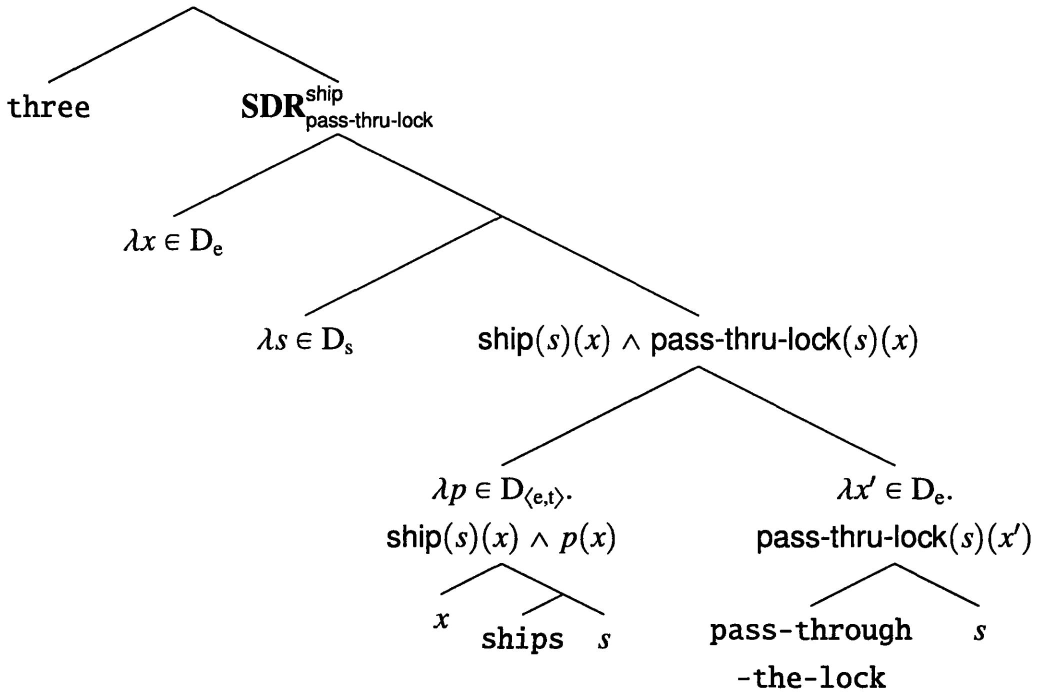


With the lexical entries in (7) and the new interpretation principle proposed in Chapter 1, the sister node of the numeral *four thousand* is shown to denote the desired relation as in (8):

(7) a. $[\![\texttt{ships}]\!] = \lambda s \in D_s. \lambda x \in D_e. \mathsf{ship}(s)(x)$

b. $[\![\texttt{pass-through-the-lock}]\!] = \lambda s \in D_s. \lambda x \in D_e. \mathsf{pass\text{-}thru\text{-}lock}(s)(x)$

(8)



$\mathbf{SDR}^{\text{ship}}_{\text{pass-thru-lock}}$ represents the situationally dependent relation obtained with the nominal predicate ship and the main predicate pass-thru-lock:

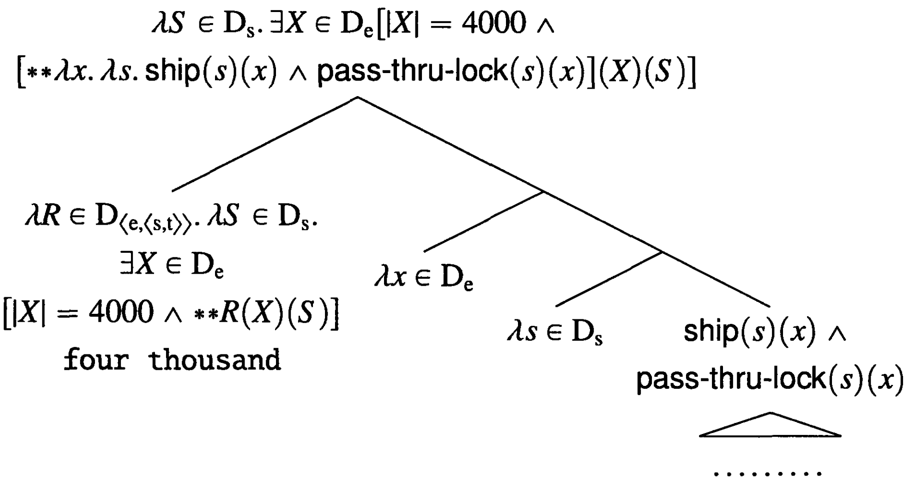
(9) $\mathbf{SDR}^{\text{ship}}_{\text{pass-thru-lock}} = \lambda x \in \mathrm{D_e}. \lambda s \in \mathrm{D_s}. \mathsf{ship}(s)(x) \wedge \mathsf{pass\text{-}thru\text{-}lock}(s)(x)$

Let us return to the computation. In Section 1.5 of Chapter 1, I "pluralized" the situationally dependent relation obtained this way by means of the (double) $*$ operator so that the pluralized version would take a plural (i.e., non-atomic) situation as its situation argument. I encoded this in the semantics of the numeral. By replacing 3 with 4000 in the lexical entry of *three* given in Section 1.5, we obtain the following lexical entry for *four thousand*:

(10) $[\![\texttt{four thousand}]\!] = \lambda R \in \mathrm{D}_{\langle e,\langle s,t\rangle\rangle}. \lambda S \in \mathrm{D_s}. \exists X \in \mathrm{D_e}[|X| = 4000 \wedge {**}R(X)(S)]$

The denotation of the top node of LF (8) can then be computed as follows:

(11)



After event closure, the following truth conditions are obtained:

(12) $\exists S \in \mathrm{D_s}\, \exists X \in \mathrm{D_e}[|X| = 4000 \wedge$
$[{**}\lambda x \in \mathrm{D_e}.\, \lambda s \in \mathrm{D_s}.\, \mathsf{ship}(s)(x) \wedge \mathsf{pass\text{-}thru\text{-}lock}(s)(x)](X)(S)]$

Assuming that never has more than one ship passed through the lock simultaneously, this becomes equivalent to the following:

(13) $\exists X \in \mathrm{D_e}[|X| = 4000 \wedge \forall x \in \mathrm{D_e}[x \sqsubseteq X \wedge x \in \mathrm{AT_e} \rightarrow \exists s \in \mathrm{D_s}[\mathsf{ship}(s)(x) \wedge$ $\mathsf{pass\text{-}thru\text{-}lock}(s)(x)]]]$

According to these truth conditions, (1) asserts the existence of four thousand distinct individuals that are ships. One can then see that we have computed the truth-conditions of the object-related reading of (1) (with the non-presuppositional interpretation of the noun phrase). Thus, the theory developed in Chapter 1 fails to derive the event-related reading of (1).

The reason for the failure to capture the event-realted reading resides in the simple-minded use of the (double) $*$ operator for pluralization. This strategy worked fine in Chapter 1, where we considered sentences like the following:

(14) Mary kissed three 20-year-olds.

The success of the stratey has now turned out to be thanks to the fact that the interpretations of the sentences we considered were their object-related readings indeed, as they happen to be their most natural readings. However, careful consideration of sentences like (14) reveals that they also have event-related readings just as does Krifka's sentence (1). Imagine a scenario where Mary is a professional kisser so she makes her living by kissing her customers. With that in mind, consider the following sentence:

(15) Mary kissed four thousand 20-year-olds last year.

It is obvious that (15) has both an object-related reading and an event-related reading. On the object-related reading, (15) asserts that there were four thousand different individuals who came (at least once) to get kissed by Mary (when they were of age 20) last year. This reading will be suitable when one wants to brag about how many different 20-year-old customers Mary had last year. On the event-related reading, on the other hand, (15) asserts that Mary had four thousand kissing sessions with a 20-year-old customer last year. On this reading, there may have been less than four thousand 20-year-old individuals who came to get kissed by Mary. For instance, even if there were only two hundred different 20-year-old individuals who came for Mary's kissing service, if they each came twenty times during the time that he or she was of age 20, the sentence will be true. This reading may thus be suitable when one wants to calculate Mary's last year's earning (from 20-year-olds), which depends on how many kissing sessions Mary had.

I think that there are a couple of pragmatic factors that affect the matter as to which of the two interpretations is preferred. Essentially, object-related readings talk about how many different individuals have been involved, while event-related readings focus on how many times an event of the relevant kind involving an individual of the relevant sort has occured. Therefore, when we care about the identity of the involved individuals, we interpret such sentences in their object related readings. Since we tend to care about the identity of people more than that of ships, it is more natural to interpret Krifka's sentence (1) with an event-related reading than to do so with sentences like (14) and (15). On the other hand, when we focus on the number of the relevant events (such as when we want to calculate Mary's earnings as in the above example), we interpret these sentences in their event-related readings. It seems that when the number of the relevant events becomes very large, we tend to care less about the identity of the involved individuals and to pay more attention to the number of the events. Thus, it is much easier to interpret (15) with an event-related reading than to do so with (14). However, an event-related reading is also possible with a small number. After uttering (15), (16) would be suitable to express the disappointment that Mary's earning (from 20-year-olds) has significantly reduced this year:

(16) This year, however, Mary has kissed only three 20-year-olds so far.

This sentence concerns the fact that Mary has had only three kissing sessions with a 20-year-old, regardless of whether some 20-year-old has come more than once.

It is then fair to conclude that the interpretation of sentences with a non-presuppositional noun phrase are in general ambiguous between an object-related reading and an event-related reading. Although both readings are logically possible, pragmatic factors such as the ones discussed above would prefer one or the other, but the details of this mechanism is not the interest or within the scope of the present dissertation. The important point is that as long as these sentences can have both object-related and event-related readings, we want our theory to be able to account for both. As I showed above, however, the theory developed in Chapter 1, which utilizes the (double) $*$ operator, only derives object-related readings. This means that we have to revise our "plurization" method in oder to account for event-related readings.

2.1.3 Krifka's analysis

In order to account for event-related readings, Krifka (1990) developed a new way of counting individuals which is more than simply adopting the $*$ operator. In this subsection, we review Krifka's analysis of object-related readings and event-related readings. To begin with, Krifka defines **measure functions compatible with a lattice sort** as follows (p. 494):[1]

(17) μ is a measure function compatible with lattice sort Σ iff

a. $\mu(x) = n \rightarrow \Sigma(x) \wedge n \in \mathbb{R}$
(μ's domain is a subset of Σ, μ's range is a subset of the reals)

b. $\mu(x) = n \rightarrow \mu(x) > 0$ (positivity)

c. $\mu(x) = n \wedge y \sqsubseteq_{\Sigma} x \rightarrow \exists n'[\mu(y) = n']$ (extendability to parts)

d. $\neg x \circ_{\Sigma} y \wedge \mu(x) = n \wedge \mu(y) = n' \rightarrow \mu(x \sqcup_{\Sigma} y) = n + n'$ (additivity)

Krifka then assumes that count nouns denote measure relations. For instance, the denotation of *ship* is the following measure relation:

(18) $[\![\texttt{ship}]\!] = \lambda n \in \mathrm{D_n}. \lambda x \in \mathrm{D_e}. \mathbf{ship}'(x) = n$

Here, $\mathbf{ship}'$ is a measure function compatible with the individual lattice sort. Now, suppose that Candida and Eleonore are atomic individuals such that the following formulae hold true:

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[1] Krifka's original notations have been altered to be compatible with the symbols in the present dissertation. Krifka uses round symbols such as $\cup$, $\cap$, $\subseteq$, etc., and these have been replaced with their square counterparts such as $\sqcup$, $\sqcap$, $\sqsubseteq$, etc. Also, Krifka writes $\cup_E$ for the join operator of events and $\cup_O$ for the join operator of objects (individuals). My notations for these are $\sqcup_v$ and $\sqcup_e$ respectively.

(19) a. $\mathbf{ship}'(\mathsf{Candida}) = 1$

b. $\mathbf{ship}'(\mathsf{Eleonore}) = 1$

Then, because there is no overlap between these two individuals, that is, there is no (non-bottom-element) individual that is both part of Candida and part of Eleonore (i.e. $\neg\mathsf{Candida}\ \circ_{\mathsf{e}}\ \mathsf{Eleonore}$), it follows from the additivity axiom of measure functions (17d) that:

(20) $\mathbf{ship}'(\mathsf{Candida} \sqcup_{\mathsf{e}} \mathsf{Eleonore}) = \mathbf{ship}'(\mathsf{Candida}) + \mathbf{ship}'(\mathsf{Eleonore}) = 1 + 1 = 2$

Therefore, assuming that Candida and Eleonore are ships, one can see that **ship**′ can be understood as a function that yields the number of the ships in that individual.

Let us now look at Krifka's analysis of object-related readings. Krifka gives the following truth conditions to the object-realted reading of sentence (1):

(21) $\exists e \exists u[\mathbf{pass\text{-}through\text{-}the\text{-}lock}'(e, u) \wedge \mathbf{ship}'(u) = 4000]$

pass-through-the-lock′ is a relation between an event on the event lattice and an individual on the individual lattice. It is a **summative** relation in the sense defined as follows:

(22) A relation R is summative iff $R(e, u) \wedge R(e', u') \rightarrow R(e \sqcup_{\mathsf{v}} e', u \sqcup_{\mathsf{e}} u')$.

Given an arbitrary relation between an atomic event and an individual, the relation obtained by applying the double $*$ operator to that given relation will be summative. In other words, Krifka's event relation **pass-through-the-lock**′ could be understood as the result of the application of the double $*$ operator to a relation holding between an atomic lock-traversal event and an individual. To see how Krifka's analysis works, instead of sentece (1), let us consider Krifka's simpler example with the following little scenario (p. 497):

(23) Ship Candida passes through the lock once (called event e_1), and Ship Eleonore passes through the lock twice (called events e_2 and e_3).

Under this scenario, the following sentence is true on its object-related reading:

(24) Two ships passed through the lock.

Krifka's analysis gives the following truth conditions to the object-related reading of (24):

(25) $\exists e \exists u[\mathbf{pass\text{-}through\text{-}the\text{-}lock}'(e, u) \wedge \mathbf{ship}'(u) = 2]$

Let us confirm that (25) is indeed the correct truth conditions. First, under the scenario in (23), we have the following true formulae:

(26) a. $\textbf{pass-through-the-lock}'(e_1, \mathsf{Candida})$

b. $\textbf{pass-through-the-lock}'(e_2, \mathsf{Eleonore})$

c. $\textbf{pass-through-the-lock}'(e_3, \mathsf{Eleonore})$

Because of the summativity of **pass-through-the-lock**′, the following is obtained:

(27) $\textbf{pass-through-the-lock}'(e_1 \sqcup_\mathrm{v} e_2 \sqcup_\mathrm{v} e_3, \mathsf{Candida} \sqcup_\mathrm{e} \mathsf{Eleonore} \sqcup_\mathrm{e} \mathsf{Eleonore})$
$= \textbf{pass-through-the-lock}'(e_1 \sqcup_\mathrm{v} e_2 \sqcup_\mathrm{v} e_3, \mathsf{Candida} \sqcup_\mathrm{e} \mathsf{Eleonore})$

Then, we can see that $e = e_1 \sqcup_\mathrm{v} e_2 \sqcup_\mathrm{v} e_3$ and $u = \mathsf{Candida} \sqcup_\mathrm{e} \mathsf{Eleonore}$ will instantiate (25) (c.f. (20)). Krifka's analysis thus gives the correct truth conditions to the object-related readings of sentences like (1) and (24).

Let us now move on to event-related readings. Krifka's analysis of event-related readings is based on the notion of **iterativity** which is formulated with the the meta-predicate **ITER** as follows:

(28) $\textbf{ITER}(e, R) \leftrightarrow \exists u, e', e''[e' \sqsubseteq_\mathrm{v} e \wedge e'' \sqsubseteq_\mathrm{v} e \wedge e' \neq e'' \wedge R(e', u) \wedge R(e'', u)]$

In words, **ITER** is a relation that holds between an event and an event relation (i.e. relation which holds between an event and an individual) such that $\textbf{ITER}(e, R)$ is true if and only if there is some individual u and there are distinct sub-events e' and e'' of e such that $R(e', u)$ and $R(e'', u)$ both hold. Note that because Kfifka's event lattice does not have a bottom element, e' and e'' in the above definition are assumed to be non-bottom elements. Take Krifka's little scenario in (23) for example. We can see that $\textbf{ITER}(e_1 \sqcup_\mathrm{v} e_2 \sqcup_\mathrm{v} e_3, \textbf{pass-through-the-lock}')$ holds. This is because $e_1 \sqcup_\mathrm{v} e_2 \sqcup_\mathrm{v} e_3$ has e_2 and e_3 as its distinct sub-events, and there is an individual u such that $\textbf{pass-through-the-lock}'(e_2, u)$ and $\textbf{pass-through-the-lock}'(e_3, u)$ both hold, namely, Eleonore. The event $e = e_1 \sqcup_\mathrm{v} e_2 \sqcup_\mathrm{v} e_3$ is thus an iterative event with respect to relation **pass-through-the-lock**′.

Krifka presents two approaches to event-related readings that differ in details regarding the implementation. However, the essential idea behind the two approaches are common. Suppose we are given an event e and we want to count how many ships passed through the lock in e in the sense of the event-related reading. If e is not an iterative lock-traversal event (i.e., $\textbf{ITER}(e, \textbf{pass-through-the-lock}')$ is false), all we need to do is to simply count the number of the distinct ships that passed through the lock in e, as each passed through the lock only once in any case. On the other hand, if e is an iterative lock-traversal event (i.e., $\textbf{ITER}(e, \textbf{pass-through-the-lock}')$ is true), it means that some ship passed through the lock more than once. If that ship passed through the lock n times, for each of these n events, we need to add 1 to the counter. Then, the correct counting can be done by dividing e into non-iterative subevents and adding the number of the ships that passed through the lock in each of those subevents.

To capture this intuition, Krifka's first approach utilizes what he calls **Object-induced Event Measure** (OEM) functions, which are defined with the operator **OEM**, whose definition is given as follows (p. 500):

(29) Let δ be a measure relation and α an event relation. Then,
$\mathbf{OEM}(\delta, \alpha)$ = the smallest measure function μ such that:
i. (Standardization): $\neg\mathbf{ITER}(e, \alpha) \rightarrow [\mu(e) = n \leftrightarrow \exists u[\delta(n, u) \wedge \alpha(e, u)]]$
ii. (Generalization): $\neg e \circ_v e' \wedge \mu(e) = n \wedge \mu(e') = n' \rightarrow \mu(e \sqcup_v e') = n + n'$

Let us now see how this works with Krifka's little scenario (23). Under the scenario in (23), the following sentence is true on its event-related reading:

(30) Three ships passed through the lock.

To capture the truth conditions of (30), one first defines the following OEM function by substituting the denotation of *ships*, viz. $\lambda n.\, \lambda u.\, [\mathbf{ship}'(u) = n]$, for δ, and **pass-through-the-lock′** for α in (29):

(31) $\mathbf{OEM}\big(\lambda n.\, \lambda u.\, [\mathbf{ship}'(u) = n], \mathbf{pass\text{-}through\text{-}the\text{-}lock}'\big)$ is the smallest measure function μ such that:
i. (Standardization): $\neg\mathbf{ITER}(e, \mathbf{pass\text{-}through\text{-}the\text{-}lock}')$
$\rightarrow [\mu(e) = n \leftrightarrow \exists u[\mathbf{ship}'(u) = n \wedge \mathbf{pass\text{-}through\text{-}the\text{-}lock}'(e, u)]]$
ii. (Generalization): $\neg e \circ_v e' \wedge \mu(e) = n \wedge \mu(e') = n' \rightarrow \mu(e \sqcup_v e') = n + n'$

With this OEM function, (30) is analyzed as follows:

(32) $\exists e\, \big[\mathbf{OEM}\big(\lambda n.\, \lambda u.\, [\mathbf{ship}'(u) = n], \mathbf{pass\text{-}through\text{-}the\text{-}lock}'\big)(e) = 3\big]$

It turns out that $e = e_1 \sqcup_v e_2 \sqcup_v e_3$ instantiates (32). To see this, we want to compute the value of $\mathbf{OEM}\big(\lambda n.\, \lambda u.\, [\mathbf{ship}'(u) = n], \mathbf{pass\text{-}through\text{-}the\text{-}lock}')(e_1 \sqcup_v e_2 \sqcup_v e_3\big)$. First of all, as we saw above, e is an iterative event (i.e. $\mathbf{ITER}(e, \mathbf{pass\text{-}through\text{-}the\text{-}lock}')$ holds). Therefore, we cannot use (31i) and are forced to apply (31ii). Let us then do so and split e into $e_1 \sqcup_v e_2$ and e_e. As $\neg(e_1 \sqcup_v e_2) \circ_v e_3$, it follows that

(33) $\mathbf{OEM}(\lambda n.\lambda u.\,[\mathbf{ship}'(u) = n], \mathbf{pass\text{-}through\text{-}the\text{-}lock}')(e)$
$= \mathbf{OEM}(\lambda n.\lambda u.\,[\mathbf{ship}'(u) = n], \mathbf{pass\text{-}through\text{-}the\text{-}lock}')(e_1 \sqcup_v e_2 \sqcup_v e_3)$
$= \mathbf{OEM}(\lambda n.\lambda u.\,[\mathbf{ship}'(u) = n], \mathbf{pass\text{-}through\text{-}the\text{-}lock}')(e_1 \sqcup_v e_2)$
$+ \mathbf{OEM}(\lambda n.\lambda u.\,[\mathbf{ship}'(u) = n], \mathbf{pass\text{-}through\text{-}the\text{-}lock}')(e_3)$

Now, notice that $e_1 \sqcup_v e_2$ is not an iterative event (i.e. $\neg\mathbf{ITER}(e_1 \sqcup_v e_2, \mathbf{pass\text{-}through\text{-}the\text{-}lock}')$ holds), since e_1 is an event of lock-traversal of Candida and e_2 an event of lock-traversal of Eleonore. Therefore, we can use (31i) to obtain the value of $\mathbf{OEM}(\lambda n.\lambda u.\,[\mathbf{ship}'(u) = n], \mathbf{pass\text{-}through\text{-}the\text{-}lock}')(e_1 \sqcup_v e_2)$. Because of the summativity of **pass-through-the-lock**, we have:

(34) $\mathbf{pass\text{-}through\text{-}the\text{-}lock}'(e_1 \sqcup_v e_2, \text{Candida} \sqcup_e \text{Eleonore})$

Since $\mathbf{ship}'(\text{Candida} \sqcup_e \text{Eleonore}) = 2$, we have:

(35) $\mathbf{ship}'(\text{Candida} \sqcup_e \text{Eleonore}) = 2 \wedge \mathbf{pass\text{-}through\text{-}the\text{-}lock}'(e_1 \sqcup_v e_2, \text{Candida} \sqcup_e \text{Eleonore})$

According to (31i), this means that we have the following:

(36) $\mathbf{OEM}(\lambda n.\lambda u.\,[\mathbf{ship}'(u) = n], \mathbf{pass\text{-}through\text{-}the\text{-}lock}')(e_1 \sqcup_v e_2) = 2$

Now, e_3 is obviously not iterative. In an analogous reasoning to the above case, it follows from (31i) that:

(37) $\mathbf{OEM}(\lambda n.\lambda u.\,[\mathbf{ship}'(u) = n], \mathbf{pass\text{-}through\text{-}the\text{-}lock}')(e_3) = 1$

Then, it follows from (33) and (36) and (37) that

(38) $\mathbf{OEM}(\lambda n.\lambda u.\,[\mathbf{ship}'(u) = n], \mathbf{pass\text{-}through\text{-}the\text{-}lock}')(e)$
$= \mathbf{OEM}(\lambda n.\lambda u.\,[\mathbf{ship}'(u) = n], \mathbf{pass\text{-}through\text{-}the\text{-}lock}')(e_1 \sqcup_v e_2)$
$+ \mathbf{OEM}(\lambda n.\lambda u.\,[\mathbf{ship}'(u) = n], \mathbf{pass\text{-}through\text{-}the\text{-}lock}')(e_3)$
$= 2 + 1 = 3$

Hence, (32) is indeed the correct truth conditions of the event-related reading of (30).

Krifka's second approach employs what he calls Object-induced Event Measure Relations (OEMR), which are defined with the operator **OEMR**.[2] For sentence (30), **OEMR (pass-through-the-lock′)** is defined as follows:

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[2] For the definition of **OEMR** and the details of the second approach, the reader is referred to Krifka's (1990) original paper.

(39) $\textbf{OEMR}(\textbf{pass-through-the-lock}')$ is the smallest measure relation σ such that

i. (Standardization): $\neg\textbf{ITER}(e, \textbf{pass-through-the-lock}') \rightarrow$ $[\sigma(e,\beta) \leftrightarrow \exists u[\textbf{pass-through-the-lock}'(e,u) \wedge \beta(u)]]$

ii. (Generalization): $\neg e \circ_{\mathrm{v}} e' \wedge \sigma(e,\beta) \wedge \sigma(e',\beta') \rightarrow \sigma(e \sqcup_{\mathrm{v}} e', \beta +_{\mathrm{e}} \beta')$

Here, β and β' are quantized predicates, which means that they are functions such as $\lambda u.\,[\textbf{ship}'(u) = 3]$, $\lambda u.\,[\textbf{ship}'(u) = 4000]$, etc., where a concrete number is already given. The addition operator $+_{\mathrm{e}}$ is defined so that for any numbers n and n', the following holds:

(40) $\lambda u.\,[\textbf{ship}'(u) = n] +_{\mathrm{e}} \lambda u.\,[\textbf{ship}'(u) = n'] = \lambda u.\,[\textbf{ship}'(u) = n + n']$

Now, with $\textbf{OEMR}(\textbf{pass-through-the-lock}')$, Krifka analyzes (30) as follows:

(41) $\exists e\,[\textbf{OEMR}(\textbf{pass-through-the-lock}')(e, \lambda u.\,[\textbf{ship}'(u) = 3])]$

A consideration analogous to the one made in the first approach reveals that this indeed gives the correct truth conditions of (30). The reader should be aware that the differences between Krifka's two approaches are only superficial and that they merely implement the same idea in slightly different fashions.

As reviewed above, Krifka's theory nicely captures the meaning of event-related readings. This was made possible because Krifka did not simply utilize the $*$ operator for pluralizing event relations, and developed a new way of counting individuals by defining OEM functions or OEMRs. However, Krifka's analysis still leaves the following problems. First, as mentioned in the beginning of this chapter, we want to think of the situationally dependent reading of event-related-reading-inducing noun phrases. However, Krifka's theory simply does not take the situational interpretation of noun phrases into account, and is hence unable to address the question of how or whether the situationally dependence follows. The next section remedies this problem. Second, when considering the meaning of measure phrases with a unit such as *three liters*, *forty-nine kilograms*, it turns out that measuring should be sensitive to the time of events. This is discussed in Section 2.3. Finally, I show in Section 2.4 that in analyzing event-relating reading where events are continuous, OEM functions or OEMRs or their modified versions are insufficient and that integration becomes necessary.

2.2 Basic proposal

In Chapter 1, I argued that in order to account for the obligatory situational dependence of non-presuppositional noun phrases, a situationally-dependent relation should be obtained. However, since the analysis utilizes the (double) * operator in the end, it fails to capture event-related readings, as shown in Subsection 2.1.2. On the other hand, although Krifka's theory accounts for event-related readings nicely, it does not talk about the situational interpretation of noun phrases at all. Therefore, in the present section, I attempt to incorporate these two. After that, I will rewrite the obtained truth conditions in a more refined form and prepare for the discussion to follow.

2.2.1 Situational dependence in event-related readings

In my analysis, in order to explain the situationally dependent interpretation of the noun *ships* in sentences such as (1) or (30), situationally dependent relation $\mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}$ between an individual and an atomic situation, which was given in (9) and is repeated below, ought to be formed:

(9) $\mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}} = \lambda x \in \mathrm{D_e}.\, \lambda s \in \mathrm{D_s}.\, \mathsf{ship}(s)(x) \wedge \mathsf{pass\text{-}thru\text{-}lock}(s)(x)$

Since we have learned that something like Krifka's OEM function or OEMR needs to be employed, let us apply this "pluralization" method of Krifka's to our relation $\mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}$. For this purpose, I will employ Krifka's first approach with OEM functions.

In Krifka's first approach, an event relation **pass-through-the-lock′** was used in order to obtain the OEM function in (31), which is repeated below:

(31) $\mathbf{OEM}(\lambda n.\, \lambda u.\, [\mathbf{ship}'(u) = n], \mathbf{pass\text{-}through\text{-}the\text{-}lock}')$ is the smallest measure function μ such that:

  i. (Standardization): $\neg\mathbf{ITER}(e, \mathbf{pass\text{-}through\text{-}the\text{-}lock}')$
  $\rightarrow [\mu(e) = n \leftrightarrow \exists u[\mathbf{ship}'(u) = n \wedge \mathbf{pass\text{-}through\text{-}the\text{-}lock}'(e, u)]]$

  ii. (Generalization): $\neg e \circ_{\mathrm{v}} e' \wedge \mu(e) = n \wedge \mu(e') = n' \rightarrow \mu(e \sqcup_{\mathrm{v}} e') = n + n'$

Here, what is "pluralized" is relation **pass-through-the-lock′**. The problem is that neither **ship′** nor **pass-through-the-lock′** is evaluated with respect to a situation, and hence the situational dependence of the noun *ships* on the main predicate *passed through the lock* cannot be expressed here. Now that we have constructed relation $\mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}$ to account for the situational dependence, instead of Krifka's **pass-through-the-lock′**, we should use

$\mathbf{SDR}^{\text{ship}}_{\text{pass-thru-lock}}$ to define an appropriate object-induced measure function. Notice, however, that while **pass-through-the-lock′** is a relation between an event and an individual (i.e. of type $\langle \mathrm{v}, \langle \mathrm{e}, \mathrm{t}\rangle\rangle$), $\mathbf{SDR}^{\text{ship}}_{\text{pass-thru-lock}}$ is a relation between an individual and a situation (i.e. of type $\langle \mathrm{e}, \langle \mathrm{s}, \mathrm{t}\rangle\rangle$). Recall that in my terminology, situations are pairs of a world and a time interval, and are thus a different kind of entities than events are. Therefore, we cannot obtain the desired measure function by directly substituting $\mathbf{SDR}^{\text{ship}}_{\text{pass-thru-lock}}$ for **pass-through-the-lock′** in (31). Rather than an Object-induced Event Measure function, what we should need is now an **Object-induced Situation Measure** (OSM) function.

Let us then define a new operator **OSM** which will define OSM functions by adapting the definition of operator **OEM** in (29). First, the relevant notion of "iterativity" to be used to define OSM functions should now concern situations instead of events. So, let us adapt **ITER** to define $\mathbf{ITER}^{(\mathrm{s})}$ as follows:[3]

(42) $\mathbf{ITER}^{(\mathrm{s})}(s, R) \leftrightarrow \exists x, s', s''[s' \sqsubseteq_{\mathrm{s}} s \wedge s'' \sqsubseteq_{\mathrm{s}} s \wedge s' \neq s'' \wedge R(x)(s') \wedge R(x)(s'')]$

Remember that in the definition of Krifka's **ITER**, e' and e'' were assumed to be non-bottom elements of the event lattice. Therefore, being a direct translation of **ITER** into situation talk, s' and s'' in the above definition are assumed to be non-bottom elements of the situation lattice. Similarly, the second clause of the definition of operator **OEM** in (29) should be translated in terms of the situation lattice. The following would then be obtained as a possible candidate for the definition of operator **OSM** by means of direct translation from event talk into situation talk:

(43) Let δ be a measure relation and R a relation between an individual and a situation. Then, $\mathbf{OSM}(\delta, R)$ is the smallest measure function μ such that:

i. (Standardization): $\neg\mathbf{ITER}^{(\mathrm{s})}(s, R) \rightarrow [\mu(s) = n \leftrightarrow \exists u[\delta(n, x) \wedge R(x)(s)]]$

ii. (Generalization): $\neg s \circ_{\mathrm{s}} s' \wedge \mu(s) = n \wedge \mu(s') = n' \rightarrow \mu(s \sqcup_{\mathrm{s}} s') = n + n'$

In the current context, we are only interested in cases where the relation R to be fed to **OSM** is a situationally dependent relation of the form $\mathbf{SDR}^{\varphi}_{\psi}$, where φ is a nominal predicate and ψ is a main predicate. We should thus focus on $\mathbf{OSM}(\delta, \mathbf{SDR}^{\varphi}_{\psi})$:

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[3] Krifka used u as the variable for individual arguments. Here and henceforth, variable x is used instead so that it will be consistent with my use of variables in the rest of this dissertation.

(44) $\mathbf{OSM}(\delta, \mathbf{SDR}^{\varphi}_{\psi})$ is the smallest measure function μ such that:

i. (Standardization): $\neg\mathbf{ITER}^{(s)}(s, \mathbf{SDR}^{\varphi}_{\psi})$
$\rightarrow [\mu(s) = n \leftrightarrow \exists u[\delta(n, x) \wedge \mathbf{SDR}^{\varphi}_{\psi}(x)(s)]]$

ii. (Generalization): $\neg s \circ_{s} s' \wedge \mu(s) = n \wedge \mu(s') = n' \rightarrow \mu(s \sqcup_{s} s') = n + n'$

As I have established in Chapter 1, situationally dependent relations hold true only of atomic situations. For a given atomic situation s, s itself is the only non-bottom situation that is part of s, and thus it is not possible to find non-bottom situations s' and s'' such that $s' \sqsubseteq_{s} s \wedge s'' \sqsubseteq_{s} s \wedge s' \neq s''$. Therefore, for any situationally dependent relation $\mathbf{SDR}^{\varphi}_{\psi}$ and for any atomic situation s, $\neg\mathbf{ITER}^{(s)}(s, \mathbf{SDR}^{\varphi}_{\psi})$ trivially holds. Then, from (44i), we can deduce the following:

(45) $s \in \mathrm{AT}_{s} \rightarrow [\mathbf{OSM}(\delta, \mathbf{SDR}^{\varphi}_{\psi})(s) = n \leftrightarrow \exists u[\delta(n, x) \wedge \mathbf{SDR}^{\varphi}_{\psi}(x)(s)]]$

Now, turning to (44ii), one can see that for a non-atomic situation s, the value of **OSM** $(\delta, \mathbf{SDR}^{\varphi}_{\psi})(s)$ will be computed by dividing s into its atomic subsituations and then taking the sum of the value of $\mathbf{OSM}(\delta, \mathbf{SDR}^{\varphi}_{\psi})$ for each of them, which is expected to be given by (45).

It turns out that (45) is not well-defined, unfortunately. This in turn means that (43) and (44) are not well-defined. To see what the problem is, let us compare **OSM** with **OEM**. For any relation α between an event and an individual, if e is an atomic event, $\neg\mathbf{ITER}(e, \alpha)$ automatically holds. Therefore, according to (29), the value of $\mathbf{OEM}(\delta, \alpha)$ for e will be given by the following:

(46) $\mathbf{OEM}(\delta, \alpha)(e) = n \leftrightarrow \exists u[\delta(n, u) \wedge \alpha(e, u)]$

For a given atomic event e, if $\alpha(e, u)$ holds for some individual u, then u is the unique such individual, since α in fact expresses a thematic relation. If $\mathbf{pass\text{-}through\text{-}the\text{-}lock}'(e)(u)$ holds, u is the unique individual which bears the theme role of event e, namely, the unique ship that passes through the lock in event e.[4] Now, recall that δ is a measure function represented as $\lambda n. \lambda u. [M(u) = n]$, where M is a measure function compatible with the individual lattice sort. Thus, for an atomic event e, if there is some individual u such that $\alpha(e, u)$ holds, $\mathbf{OEM}(\delta, \alpha)(e)$ is uniquely determined as $M(u)$. On the other hand, a similar situation does not obtain for (45). Now, let us remember Krifka's little scenario in (23) and assume that this scenario is about what happened in the last week in the actual world.

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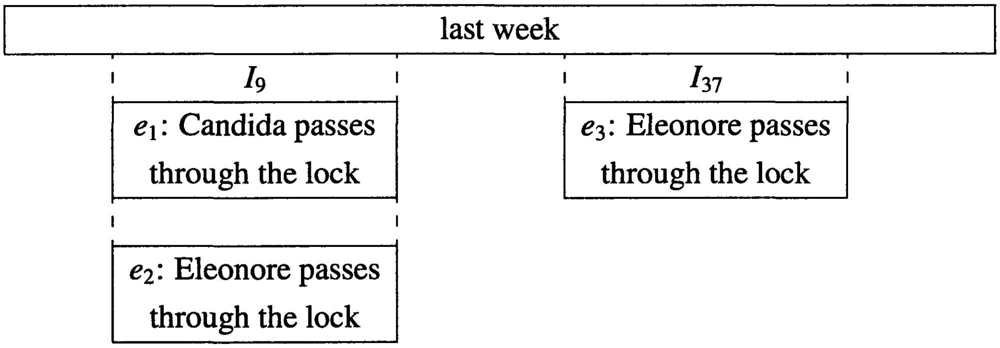
[4] Or, the unique ships that pass through the lock in event e, if multiple ships collectively receive the theme role of atomic event e.

Suppose further that e_1 and e_2 took place simultaneously, i.e., the running time intervals of e_1 and e_2 happen to be exactly the same. Let I_9 be this time interval, and I_{37} be the running time interval of e_3. Suppose that no other lock-traversal events of a ship occurred in the last week. The following depicts this scenario:

(47)



For $s = \langle @, I_9 \rangle$, there are two individuals x such that $\mathbf{SDR}^{\text{ship}}_{\text{pass-thru-lock}}(s)(x)$ holds, namely, Candida and Eleonore. Therefore, even though $\langle @, I_9 \rangle$ is an atomic situation, we cannot determine the value of $\mathbf{OSM}(\delta, \mathbf{SDR}^{\text{ship}}_{\text{pass-thru-lock}})\,(\langle @, I_9 \rangle)$ according to (45), because we do not know whether to measure Candida or Eleonore.

Obviously, we want to count both Candida and Eleonore, for situation $\langle @, I_9 \rangle$. The solution I propose is to take the join of Candida and Eleonore here. In general, for a given situationally dependent relation $\mathbf{SDR}^{\varphi}_{\psi}$, let us define a function $\mathbf{SDIC}^{\varphi}_{\psi}$, from (atomic) situations into individuals as follows:

(48) $\mathbf{SDIC}^{\varphi}_{\psi} \overset{\text{def}}{=} \lambda s \in \mathrm{D_s}. \bigsqcup_{\mathrm{e}} \{x \mid \mathbf{SDR}^{\varphi}_{\psi}(x)(s)\}$

We call functions obtained this way **situationally dependent individual concepts**. For a given atomic situation s, $\mathbf{SDIC}^{\varphi}_{\psi}(s)$ gives the join of every individual x such that $\mathbf{SDR}^{\phi}_{\psi}$ $(x)(s)$ holds. In order for $\mathbf{OSM}(\delta, \mathbf{SDR}^{\varphi}_{\psi})$ to yield the desired value for an atomic situation s, all we need to do is measure $\mathbf{SDIC}^{\varphi}_{\psi}(s)$ by the measure function given by δ. Thus, for $s \in \mathrm{AT_s}$ and a measure relation $\delta = \lambda n. \lambda x. [M(x) = n]$, let us define $\mathbf{OSM}(\delta, \mathbf{SDR}^{\varphi}_{\psi})\,(s)$ as follows:

(49) $\mathbf{OSM}(\delta, \mathbf{SDR}^{\varphi}_{\psi})(s) \overset{\text{def}}{=} M(\mathbf{SDIC}^{\varphi}_{\psi}(s)) = M\left(\bigsqcup_{\mathrm{e}} \{x \mid \mathbf{SDR}^{\varphi}_{\psi}(x)(s)\}\right)$

Let us now consider non-atmoic situations. For any atomic situation $s \neq \bot_{\mathrm{s}}$, if $s \circ_{\mathrm{s}} t$ and $s \neq t$, then $t = \bot_{\mathrm{s}}$. Therefore, for any atomic situations $s_1, s_2 \neq \bot_{\mathrm{s}}$, if $s_1 \neq s_2$ then $\neg s_1 \circ_{\mathrm{s}} s_2$. It then follows from (44ii) that for any atomic situations $s_1, s_2 \neq \bot_{\mathrm{s}}$, we have:

(50) $\mathbf{OSM}(\delta, \mathbf{SDR}^{\varphi}_{\psi})\,(s_1 \sqcup_{\mathrm{s}} s_2) = \mathbf{OSM}(\delta, \mathbf{SDR}^{\varphi}_{\psi})(s_1) + \mathbf{OSM}(\delta, \mathbf{SDR}^{\varphi}_{\psi})(s_2)$

Let us generalize this to define the value of $\mathbf{OSM}(\delta, \mathbf{SDR}^{\varphi}_{\psi})$ for $s \notin \mathrm{AT_s}$. Given a situation $s \notin \mathrm{AT_s}$, $\{t \mid t \sqsubseteq s \wedge t \in \mathrm{AT_s}\}$ is the set of all atomic situations that are part of s. Now, let $\{s_i\}_{i \in \gamma}$ be a family of atomic situations indexed by γ such that $\{s_i\}_{i\in\gamma} = \{t \mid t \sqsubseteq s \wedge t \in \mathrm{AT_s}\}$ and $s_i \neq s_j \rightarrow i \neq j$. Then, let us generalize (50) and define $\mathbf{OSM}\left(\delta, \mathbf{SDR}^{\varphi}_{\psi}\right)(s)$ as follows:

(51) $\mathbf{OSM}(\delta, \mathbf{SDR}^{\varphi}_{\psi})(s) \stackrel{\text{def}}{=} \sum\limits_{i\in\gamma} \mathbf{OSM}(\delta, \mathbf{SDR}^{\varphi}_{\psi})(s_i)$

Since s_i is atomic for any $i \in \gamma$, we can use (49) and finally obtain the following definition of **OSM**:

(52) Let $\delta = \lambda n. \lambda x. [M(x) = n]$ be a measure relation and $\mathbf{SDR}^{\varphi}_{\psi}$ a situationally dependent relation between an individual and an atomic situation. Then,
$\mathbf{OSM}(\delta, \mathbf{SDR}^{\varphi}_{\psi})(s) \stackrel{\text{def}}{=} \sum\limits_{i\in\gamma} M(\mathbf{SDIC}^{\varphi}_{\psi}(s_i)) = \sum\limits_{i\in\gamma} M\left(\bigsqcup_{\mathrm{e}}\{x \mid \mathbf{SDR}^{\varphi}_{\psi}(x)(s_i)\}\right)$,
where $\{s_i\}_{i\in\gamma}$ is a family of atomic situations indexed by γ such that
$\{s_i\}_{i\in\gamma} = \{t \mid t \sqsubseteq s \wedge t \in \mathrm{AT_s}\}$ and $s_i \neq s_j \rightarrow i \neq j$.

Notice that this definition works even if s is an atomic situation, in which case $\{t \mid t \sqsubseteq s \wedge t \in \mathrm{AT_s}\} = \{s\}$ and thus $\mathbf{OSM}(\delta, \mathbf{SDR}^{\varphi}_{\psi})(s) = M\left(\bigsqcup_{\mathrm{e}}\{x \mid \mathbf{SDR}^{\varphi}_{\psi}(x)(s)\}\right)$.

Let us now discuss the treatment of the noun. In Krifka's analysis, count nouns such as *ship* are treated as denoting measure relations (see (18)). In the present analysis, however, the noun in a non-presuppositional noun phrase must denote a predicate of type $\langle \mathrm{s}, \langle \mathrm{e}, \mathrm{t}\rangle\rangle$ and together with a main predicate constitute a situationally dependent relation. In the case of sentence (1) or (30), we consider situationally dependent relation $\mathbf{SDR}^{\text{ship}}_{\text{pass-thru-lock}}$. If the measure relation with which the relevant OSM function should be defined (i.e., the measure relation to substitute δ for in (52)) were $\lambda n. \lambda x. [\mathbf{ship}'(x) = n]$ as Krifka claims, it would mean that the noun *ship* should be contributing to the semantic composition of the sentence twice, and what is more, once as a predicate and the other time as a measure relation. This would pose a serious problem in semantic compositionality. It should be realized that now that the semantics of *ship* has been incorporated to the relation to be pluralized, we need no longer have *ship* as a measure function. For count nouns such as *ship*, I therefore postulate a measure function compatible with the individual lattice sort ATOM. ATOM is the function that yields the number of atomic elements in the given individual. For example, $\mathsf{ATOM}(\mathsf{Candida}) = 1$ and $\mathsf{ATOM}(\mathsf{Candida} \sqcup_{\mathrm{e}} \mathsf{Eleonore}) = 2$. Instead of $\lambda n. \lambda x. [\mathbf{ship}'(x) = n]$, the measure relation $\lambda n. \lambda x. [\mathsf{ATOM}(x) = n]$ should be used for count nouns in general. Then, the relevant OSM function for sentences like (1) and (30) is $\mathbf{OSM}\big(\lambda n.\,\lambda x.\,[\mathsf{ATOM}(x) = n], \mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}\big)$.

Finally, let us return to Krifka's truth conditions of event-related readings. According to Krifka, sentence (30) has the truth conditions in (32), which are repeated below:

(32) $\exists e\,\big[\mathbf{OEM}\big(\lambda n.\,\lambda u.\,[\mathbf{ship}'(u) = n], \mathbf{pass\text{-}through\text{-}the\text{-}lock}'\big)(e) = 3\big]$

Now, if we replace the OEM function here with the above OSM function, we obtain the following:

(53) $\exists s\big[\mathbf{OSM}\big(\lambda n.\,\lambda u.\,[\mathsf{ATOM}(x) = n], \mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}\big)(s) = 3\big]$

Let us confirm that (53) is indeed the correct truth conditions. Think of the model (47) and the scenario considered there. We need only show that $s = \langle @, I_9\rangle \sqcup_{\mathsf{s}} \langle @, I_{37}\rangle$ instantiates the existential quantification in (53). Since $\langle @, I_9\rangle$ and $\langle @, I_{37}\rangle$ are both atomic situations, it follows form (52) that:

(54)
$$\begin{aligned}
&\mathbf{OSM}\big(\lambda n.\,\lambda x.\,[\mathsf{ATOM}(x) = n], \mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}\big)\big(\langle @, I_9\rangle \sqcup_{\mathsf{s}} \langle @, I_{37}\rangle\big)\\
&= \mathsf{ATOM}\Big(\bigsqcup\nolimits_{\mathsf{e}}\{x \mid \mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(x)\big(\langle @, I_9\rangle\big)\}\Big)\\
&\qquad + \mathsf{ATOM}\Big(\bigsqcup\nolimits_{\mathsf{e}}\{x \mid \mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(x)\big(\langle @, I_{37}\rangle\big)\}\Big)
\end{aligned}$$

We know that $\mathbf{SDR}^{\varphi}_{\psi}(\mathsf{Candida})\big(\langle @, I_9\rangle\big)$ and $\mathbf{SDR}^{\varphi}_{\psi}(\mathsf{Eleonore})\big(\langle @, I_9\rangle\big)$ hold and that there is no other individual x such that $\mathbf{SDR}^{\varphi}_{\psi}(x)\big(\langle @, I_9\rangle\big)$ holds. Therefore,

(55)
$$\begin{aligned}
&\mathsf{ATOM}\Big(\bigsqcup\nolimits_{\mathsf{e}}\{x \mid \mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(x)\big(\langle @, I_9\rangle\big)\}\Big)\\
&= \mathsf{ATOM}\Big(\bigsqcup\nolimits_{\mathsf{e}}\{\mathsf{Candida, Eleonore}\}\Big) = \mathsf{ATOM}\,(\mathsf{Candida} \sqcup_{\mathsf{e}} \mathsf{Eleonore}) = 2
\end{aligned}$$

Similarly, we obtain:

(56)
$$\begin{aligned}
&\mathsf{ATOM}\Big(\bigsqcup\nolimits_{\mathsf{e}}\{x \mid \mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(x)\big(\langle @, I_{37}\rangle\big)\}\Big)\\
&= \mathsf{ATOM}\Big(\bigsqcup\nolimits_{\mathsf{e}}\{\mathsf{Eleonore}\}\Big) = \mathsf{ATOM}\,(\mathsf{Eleonore}) = 1
\end{aligned}$$

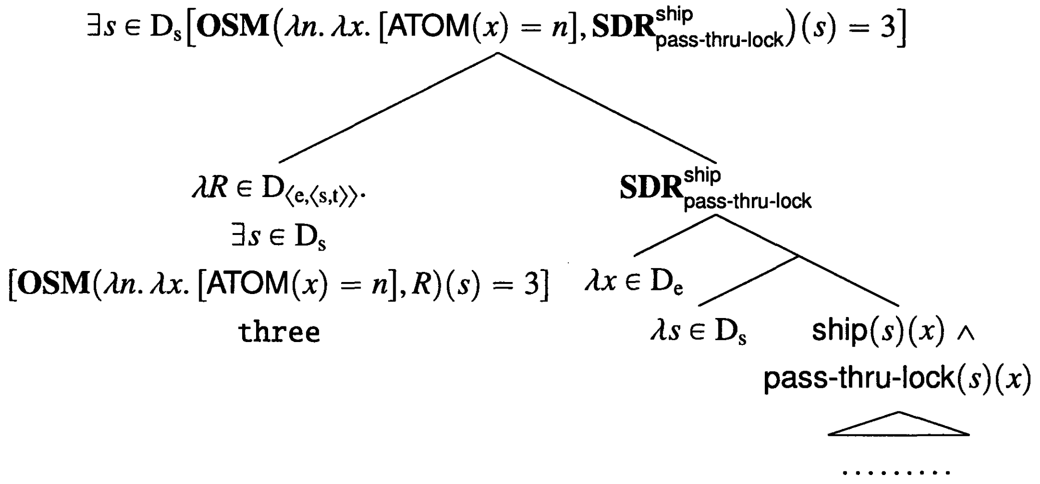
From (54), (55) and (56), we obtain:

(57) $\mathbf{OSM}\big(\lambda n.\,\lambda x.\,[\mathsf{ATOM}(x) = n], \mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}\big)\big(\langle @, I_9\rangle \sqcup_{\mathsf{s}} \langle @, I_{37}\rangle\big) = 2 + 1 = 3$

which proves (53). Returning to our LF in (8), we would like to derive the truth conditions in (53) compositionally. This is indeed possible by assuming the lexical entry for *three* in (58), as shown in (59):

(58) $[\![\mathtt{three}]\!] = \lambda R \in \mathrm{D}_{\langle \mathrm{e},\langle \mathrm{s},\mathrm{t}\rangle\rangle}.\,\exists s \in \mathrm{D}_{\mathrm{s}}[\mathbf{OSM}(\lambda n.\,\lambda x.\,[\mathsf{ATOM}(x) = n], R)(s) = 3]$

(59)



This completes the exposition of a sketch of the theory obtained by applying Krifka's pluralization to our result in Chapter 1.

2.2.2 Refinement

By now, the reader may have been overwhelmed by the apparent complexity of the theory. Indeed, the obtained truth conditions in (53) appear to be somewhat hard to grasp intuitively. This point may become even clearer when one considers sentences like the following:

(60) a. Exactly three ships passed through the lock last week.

b. At most three ships passed through the lock last week.

Under the current analysis, in order to capture the truth conditions of the above sentences, one has to know the maximum number m such that $\exists s[\mathbf{OSM}(\lambda n.\lambda x.[\mathsf{ATOM}(x) = n], \mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}})(s) = m]$, where s ranges over the set of all atomic time intervals in the last week and their joins. Let k be this number:

(61) $$k = \sup\Big\{m \;\Big|\; \exists s\Big[s \sqsubseteq \bigsqcup\nolimits_{\mathrm{e}} \{t \mid t \in \mathrm{AT_s} \wedge t \text{ is in last week}\} \wedge \mathbf{OSM}(\lambda n.\lambda x.[\mathsf{ATOM}(x) = n], \mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}})(s) = m\Big]\Big\}$$

With k, (60a) and (60b) will have the truth conditions in (62a) and (62b) respectively:

(62) a. $k = 3$

b. $k \leq 3$

Now, these truth conditions look very hard to comprehend.

Fortunately, the complexity of these truth conditions can be simplified greatly once we notice that k in (61) can be directly computed as the sum of the values of OSM function $\mathbf{OSM}(\lambda n.\, \lambda x.\, [\mathsf{ATOM}(x) = n], \mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}})$ for *all* atomic situations in the last week. Let A be the set of all atomic situations in the last week:

(63) $A = \{s \mid s \in \mathrm{AT_s} \wedge s \text{ is in last week}\}$

Then, k must be given by the following:

(64) $k = \mathbf{OSM}(\lambda n.\, \lambda x.\, [\mathsf{ATOM}(x) = n], \mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}) \left(\bigsqcup_{\mathsf{e}} A\right)$
$= \sum_{i \in \gamma} \mathsf{ATOM}(\mathbf{SDIC}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(s_i)) = \sum_{i \in \gamma} \mathsf{ATOM}\left(\bigsqcup_{\mathsf{e}} \{x \mid \mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(x)(s_i)\}\right),$
where $\{s_i\}_{i \in \gamma}$ is a family of atomic situations indexed by γ such that
$\{s_i\}_{i \in \gamma} = A$ and $s_i \neq s_j \rightarrow i \neq j$.

This strategy encounters a problem, however. When we look at model (47), we see that for all $s \in A$ such that $s \neq \langle @, I_9 \rangle \langle @, I_{37} \rangle$ in the last week, there is no individual that is a ship and passes through the lock in s. Therefore, for those situations s, we have:

(65) $\mathbf{SDIC}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(s) = \bigsqcup_{\mathsf{e}} \{x \mid \mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(x)(s)\} = \bigsqcup_{\mathsf{e}} \varnothing = \bot_{\mathsf{e}}$

This means that in order to calculate $\mathsf{ATOM}(\mathbf{SDIC}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(s))$ for all $s \in A$, $\mathsf{ATOM}(\bot_{\mathsf{e}})$ has to be defined, and its value is furthermore expected to be 0. According to Krifka's definition of measure function, however, a measure function is not defined for the bottom element, and always yields a positive value as stated in (17b). This is because Krifka assumes that lattices to be considered should have no bottom element. Now that we should add the bottom element, the definition of measure functions must be revised accordingly. Because I will introduce measures defined on σ-algebras later, in order to avoid confusion, I will refer to the functions defined below as **lattice-measure functions**. Also, I reserve the greek letter symbol μ for measures on σ-algebras. Below is the definition of lattice-measure functions, but this is actually soon revised in the next section so that they will be modified to be sensitive to a situation:

(66) $M \in D_{\langle e,n\rangle}$ is a lattice-measure function iff

i. $x \in \mathrm{dom}(M) \rightarrow M(x) \geq 0$ (non-negativity)

ii. $M(\perp_e) = 0$ (the bottom element is mapped to 0)

iii. $x \in \mathrm{dom}(M) \wedge y \sqsubseteq_e x \rightarrow y \in \mathrm{dom}(M)$ (extendability to parts)

iv. If $x_n \in \mathrm{dom}(M)$ for all $n \in \mathbb{N}$ and if $\{x_n\}$ is a pairwise non-overlapping set, i.e., for any $i, j \in \mathbb{N}, i \neq j \rightarrow x_i \sqcap_e x_j = \perp_e$, then $M\left(\bigsqcup_{e\,n=1}^{\infty} x_n\right) = \sum_{n=1}^{\infty} M(x_n)$ (countable additivity)

The last clause might strike as significantly different from the additivity axiom in (17). Here, I am proposing that the additivity of measure functions is countable additivity, which means that the additivity holds of any countable set of non-oeverlapping individuals. Accordingly, the individual lattice is assumed to be complete, so that any collection of elements on the individual lattice has a unique join (supremum). The countable additivily proves to be useful in Subsection 2.6.2. Since that the bottom element is added to the lattice, the condition $x_i \sqcap_e x_j = \perp_e$ says that x_i and x_j do not overlap, so it expresses the same condition as $\neg x_i \circ_e x_j$.

ATOM is a lattice-measure function. (64) will now compute the correct value 3 in the case of our model (47). It follows from (62) and (64) that we have (67a) and (67b) as the truth conditions for (60a) and (60b) respectively, given $\{s_i\}_{i \in \gamma}$ such that $\{s_i\}_{i \in \gamma} = A$ and $s_i \neq s_j \rightarrow i \neq j$:

(67) a. $\sum_{i \in \gamma} \mathsf{ATOM}(\mathbf{SDIC}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(s_i)) = 3$

b. $\sum_{i \in \gamma} \mathsf{ATOM}(\mathbf{SDIC}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(s_i)) \leq 3$

Furthermore, sentence (30), or rather, sentence *at least three ships passed through the lock last week*, will have the following analogous truth conditions:

(68) $\sum_{i \in \gamma} \mathsf{ATOM}(\mathbf{SDIC}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(s_i)) \geq 3$

These truth conditions do not differ empirically from the ones in (53) and (62), but in this fashion, we can treat sentences with *exactly n*, *at most n* and *at least n* all uniformly and understand their truth conditions much more intuitively.

Here, a note on the ontology of situations is in order. In (53) and (61), the existentially bound situation variable ranges over both atomic and non-atomic situations. However, now that our truth conditions are expressed as in (67) and (68), we need no longer think of non-atomic situations (i.e., joins of situations). Therefore, from now on, we will not be dealing with lattice of situations, and $\mathrm{D_s}$ can simply be assumed the set of pairs of a world and a time interval.

Although (67) and (68) are the truth conditions we want, it turns out that the correct counting is not guaranteed unless we give a specific temporal interpretation of the verbal predicates. This is a problem I have neglected so far. In Section 1.3 of Chapter 1, we briefly debated whether $\mathsf{kiss}(s)(x)(y)$ should mean "an event of y's kissing x occurs in $\pi_\mathrm{w}(s)$ and its running time interval is *within* $\pi_\mathrm{i}(s)$ (i.e., a subinterval of $\pi_\mathrm{i}(s)$)" or "an event of y's kissing x occurs in $\pi_\mathrm{w}(s)$ and its running time interval is *exactly* $\pi_\mathrm{i}(s)$" . There, how this question should be answered *per se* did not affect the discussion and the conclusion drawn there. However, in the current discussion, it becomes important. Suppose that $\mathsf{pass\text{-}thru\text{-}lock}(s)(x)$ means "an event of x's passing through the lock occurs in $\pi_\mathrm{w}(s)$ and its running time interval is *within* $\pi_\mathrm{i}(s)$". In model (47), $\mathsf{pass\text{-}thru\text{-}lock}(\langle @, I_{37}\rangle)(\mathsf{Eleonore})$ obviously holds. However, in addition to this, for any time interval I in the last week such that $I_{37} \subset I$, $\mathsf{pass\text{-}thru\text{-}lock}(\langle @, I\rangle)(\mathsf{Eleonore})$ will also be true, and in turn, $\mathsf{ATOM}\ (\mathbf{SDIC}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(\langle @, I\rangle)) = \mathsf{ATOM}(\mathsf{Eleonore}) = 1$.[5] Since there are infinitely many such time intervals in the last week, $\sum_{i\in\gamma} \mathsf{ATOM}(\mathbf{SDIC}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(s_i))$ would be infinite. One should then be able to truthfully utter "One million ships passed through the lock last week", etc. for the scenario depicted in (47), which is clearly the wrong prediction. This means that the correct interpretation of $\mathsf{pass\text{-}thru\text{-}lock}(s)(x)$ must be "an event of x's passing through the lock occurs in $\pi_\mathrm{w}(s)$ and its running time interval is *exactly* $\pi_\mathrm{i}(s)$". Then, for any time interval I in the last week such that $I_{37} \subset I$, there is no individual x such that $\mathsf{pass\text{-}thru\text{-}lock}(\langle @, I\rangle)(x)$ holds, and thus $\mathsf{ATOM}(\mathbf{SDIC}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(\langle @, I\rangle)) = \mathsf{ATOM}(\bot_\mathrm{e}) = 0$. To conclude, in order to ensure the correct counting, the situation argument of a verbal predicate always provides the exact time interval of the event described by the predicate.

To end this section, let me hint at how the theory developed so far leads to integration. Above, A was defined to be the set of all atomic time intervals in the last week. We assume that the time line is isomorphic to the real line. Thus, the time line is continuous and any given time point corresponds to a unique real. Now, let p_0 and p_1 be the beginning time point and the ending time point of the last week respectively. Then, A can be represented

---

[5] This holds only if I does not include I_9. If $I_9 \subset I$, $(\mathbf{SDIC}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(\langle @, I\rangle)) = \mathsf{ATOM}(\mathsf{Candida} \sqcup_\mathrm{e} \mathsf{Eleonore}) = 2$.

as follows:

$$(69)\quad A = \{(p,p') \mid p_0 \le p < p' \le p_1\} \cup \{(p,p'] \mid p_0 \le p < p' \le p_1\} \\ \cup \{[p,p') \mid p_0 \le p < p' \le p_1\} \cup \{[p,p'] \mid p_0 \le p \le p' \le p_1\}$$

Since there are uncountably many points in $[p_0, p_1]$, set $\{[p,p'] \mid p_0 \le p \le p' \le p_1\}$, for instance, is clearly an uncountable set. It is then immediate that A is an uncountable set. At this point, one might argue that the above assumption that the time line is isomorphic to the real line is wrong. Now, suppose, for instance, that there are only a finite number of time points in the last week. Then, assuming that time points are totally ordered, for any given time point t_1 in the last week which is not the final time point in the last week, one will always be able to find the succeeding time point t_2 in the last week. Recall now that we are committed in looking at exact time intervals in which events of the relevant sort occur, as discussed right above. Because events may begin at (or "right after") any random time point and end at (or "right before") another random time point,[6] we can imagine events beginning at t_1 and at t_2. However, since there is no time point between t_1 and at t_2, we should not be able to think of events beginning between t_1 and t_2, but this is extremely odd intuitively. The order of time points should thus be dense at least. Given this, it seems natural enough to assume that the time line is isomorphic to the real line. In any case, it will become clear in Section 2.4 that the time line should actually be continuous so that we may perform integration on it.

Given that A is an uncountable set, the index set γ that appeared in (67) and (68) must also be an uncountable set, and that the summation in (67) and (68) must be taken over uncountably many values. It should now be recalled that integration was devised to take summation over uncountable sets rigorously. To see the connection somewhat heuristically, let us think of Riemann integration of a continuous function $f(x)$ defined on the closed interval $[a, b]$. Let $x_0, x_1, x_2, \ldots, x_n$ be points in $[a, b]$ such that $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$. For $i = 1, 2, \ldots, n$, let $\Delta x_i = x_i - x_{i-1}$ and let $\tilde{x}_i$ be an arbitrary point such that $x_{i-1} < \tilde{x}_i < x_i$. If $\lim\limits_{n\to\infty} \max \Delta x_i = 0$, then the definite integral of $f(x)$ over the closed interval $[a, b]$ is given as follows:

$$(70)\quad \lim_{n\to\infty} \sum_{i=1}^{n} f(\tilde{x}_i)\Delta x_i = \int_a^b f(x)\,dx$$

---

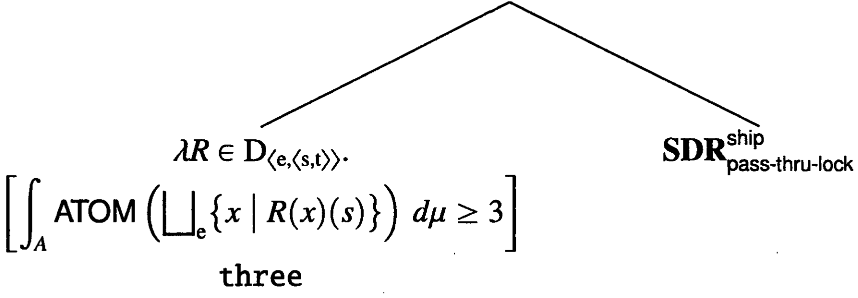
[6] If the exact running time interval of an event is an open interval (a, b), then, the event has not begun at a yet, but it is to begin "right after" a, and similarly, it does not end at b, but "right before" b. Whether or not such events should indeed be considered is an empirical matter.

The reader may recall that Gottfried Leibniz created the integral symbol $\int$ from the elongated initial letter of the word "sum". Now, comparing (67) and (68) with (70), one might wonder if the summation in (67) and (68) might also be expressed as some kind of integral. In fact, as will become clear in Section 2.6, it is equal to a Lebesgue integral of function $\mathsf{ATOM}\left(\mathbf{SDIC}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(s)\right)$ over A:[7]

(71) $$\sum_{i\in\gamma} \mathsf{ATOM}\left(\mathbf{SDIC}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(s_i)\right) = \int_A (\lambda s \in \mathrm{D_s}.)\ \mathsf{ATOM}\left(\mathbf{SDIC}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(s)\right) d\mu$$

Here, μ is the counting measure defined on a σ-algebra over A. It is then expected that the event-related reading of sentence (30) would be computed as follows:

(72) $$\left[\left[\int_A \mathsf{ATOM}\left(\mathbf{SDIC}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(s)\right) d\mu \geq 3\right]\right]$$
$$= \left[\left[\int_A \mathsf{ATOM}\left(\bigsqcup_{\mathsf{e}} \{x \mid \mathbf{SDR}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(s)(x)\}\right) d\mu \geq 3\right]\right]$$



In Section 2.5, the theory of Lebesgue integration is formally introduced, where the present analysis's insight will be illuminated. Before that, I discuss a couple of remaining problems in Krifka's analysis in the next two sections.

2.3 Units of measurement and its situation-sensitivity

In addition to sentences like (1) with a count noun such as *ship*, Krifka (1990) also discusses sentences involving mass nouns like the following:

(73) Sixty tons of radioactive waste {passed / were transported} through the lock.

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[7] The "$\lambda s \in \mathrm{D_s}$." part in the parentheses on the right hand side of the equation is there to make it explicit that the function being integrated is a function of s. This is usually omitted, and in the case of Riemann integration, having this would in fact be superfluous as the dx part makes it clear that x is the relevant variable. One may note, however, that $\int_a^b f(x)\,dx$ is equivalent to $\int_a^b \lambda x.\, f(x)\,dx$.

Here, the size of the entity denoted by the mass noun *radioactive waste* is described by the phrase *sixty tons*. I refer to phrases like *sixty tons* as **measure phrases**, which consist of a numeral[8] and a **unit of measurement** like *ton*. This section discusses the event-related reading of sentences where the event-realted-reading-inducing non-presuppositional noun phrase contains a unit of measurement as in (73), and concludes that the lattice-measure function denoted by a unit of measurement must be sensitive to time (situation).

2.3.1 Units of measurement

In the previous section, we reached the conclusion that the event-related reading of sentence (30) (on the "at least" reading) is analyzed as in (68), which is repeated below:

(68) $\sum_{i \in \gamma} \mathsf{ATOM}(\mathbf{SDIC}^{\mathsf{ship}}_{\mathsf{pass\text{-}thru\text{-}lock}}(s_i)) \geq 3$

Here, $\{s_i\}_{i \in \gamma}$ is a family of atomic situations indexed by γ such that $\{s_i\}_{i \in \gamma} = A$ and $s_i \neq s_j \rightarrow i \neq j$, where A is the set of all atomic situations in a given context time interval such as the last week, the last five years, etc. Let us assume for the moment that A is properly supplied by the context. To analyze (73), let us assume the following lexical entry for *radioactive waste*:

(74) $[\![\texttt{radioactive waste}]\!] = \lambda s \in \mathrm{D_s}.\, \lambda x \in \mathrm{D_e}.\, \mathsf{RA\text{-}waste}(s)(x)$

Krifka assumes that a unit of measurement such as *ton* represents a measure function compatible with the individual lattice. By analogy with (68), (73) on the "at least" reading is then expected to be analyzed as follows:

(75) $\sum_{i \in \gamma} \mathsf{ton}\,(\mathbf{SDIC}^{\mathsf{RA\text{-}waste}}_{\mathsf{pass\text{-}thru\text{-}lock}}(s_i)) \geq 60$

To derive this compositionally, let us also consider the following sentences:

(76) a. Exactly sixty tons of radioactive waste passed through the lock.

b. At most sixty tons of radioactive waste passed through the lock.

(76a) and (76b) will be analyzed as in (77a) and (77b) respectively:

(77) a. $\sum_{i \in \gamma} \mathsf{ton}\,(\mathbf{SDIC}^{\mathsf{RA\text{-}waste}}_{\mathsf{pass\text{-}thru\text{-}lock}}(s_i)) = 60$

---

[8] It could be phrasal such as *at most three*. Worlds such as *some*, *many*, etc. may also find themselves in this position.

b. $\sum_{i\in\gamma} \text{ton}\left(\mathbf{SDIC}^{\text{RA-waste}}_{\text{pass-thru-lock}}(s_i)\right) \leq 60$

By the definition of **SDIC** in (48), we have:

(78) $$\sum_{i\in\gamma} \text{ton}\left(\mathbf{SDIC}^{\text{RA-waste}}_{\text{pass-thru-lock}}(s_i)\right) = \sum_{i\in\gamma} \text{ton}\left(\bigsqcup\nolimits_{\text{e}}\{x \mid \mathbf{SDR}^{\text{RA-waste}}_{\text{pass-thru-lock}}(x)(s_i)\}\right)$$

With this, from (75) and (77), we can extract the denotation of the measure phrases as follows:

(79) a. $[\![\texttt{at least sixty tons}]\!] = \lambda R \in \text{D}_{\langle\text{e},\langle\text{s,t}\rangle\rangle}.\ \sum_{i\in\gamma} \text{ton}\left(\bigsqcup\nolimits_{\text{e}}\{x \mid R(x)(s_i)\}\right) \geq 60$

b. $[\![\texttt{exactly sixty tons}]\!] = \lambda R \in \text{D}_{\langle\text{e},\langle\text{s,t}\rangle\rangle}.\ \sum_{i\in\gamma} \text{ton}\left(\bigsqcup\nolimits_{\text{e}}\{x \mid R(x)(s_i)\}\right) = 60$

c. $[\![\texttt{at most sixty tons}]\!] = \lambda R \in \text{D}_{\langle\text{e},\langle\text{s,t}\rangle\rangle}.\ \sum_{i\in\gamma} \text{ton}\left(\bigsqcup\nolimits_{\text{e}}\{x \mid R(x)(s_i)\}\right) \leq 60$

To derive (79), I posit the following lexical entry for unit of measurement *ton*:

(80) $$[\![\texttt{ton}]\!] = \lambda v \in \text{D}_{\langle\text{n,t}\rangle}.\ \lambda R \in \text{D}_{\langle\text{e},\langle\text{s,t}\rangle\rangle}.$$
$$\exists n \in \text{D}_{\text{n}}\left[v(n) \wedge \sum_{i\in\gamma} \text{ton}\left(\bigsqcup\nolimits_{\text{e}}\{x \mid R(x)(s_i)\}\right) = n\right]$$

The first argument of *ton*, *p*, is the characteristic function of a set of numbers in a certain range, and this is saturated by the denotation of the numeral phrase such as *at most sixty*. The numeral phrases in (79) thus have the following denotations:

(81) a. $[\![\texttt{at least sixty}]\!] = \lambda n \in \text{D}_{\text{n}}.\ [n \geq 60]$

b. $[\![\texttt{exactly sixty}]\!] = \lambda n \in \text{D}_{\text{n}}.\ [n = 60]$

c. $[\![\texttt{at most sixty}]\!] = \lambda n \in \text{D}_{\text{n}}.\ [n \leq 60]$

Let us see, for example, how (79a) can be derived from (80) and (81a):

(82) $$\left[\!\!\left[\overbrace{\text{at least sixty}\quad\text{tons}}\right]\!\!\right] \overset{\text{FA}}{=} [\![\texttt{ton}]\!]([\![\texttt{at least sixty}]\!])$$

$$= \left[\lambda v \in \text{D}_{\langle\text{n,t}\rangle}.\ \lambda R \in \text{D}_{\langle\text{e},\langle\text{s,t}\rangle\rangle}.\ \exists n \in \text{D}_{\text{n}}\left[v(n) \wedge \sum_{i\in\gamma} \text{ton}\left(\bigsqcup\nolimits_{\text{e}}\{x \mid R(x)(s_i)\}\right) = n\right]\right]$$
$$(\lambda n \in \text{D}_{\text{n}}.\ [n \geq 60])$$

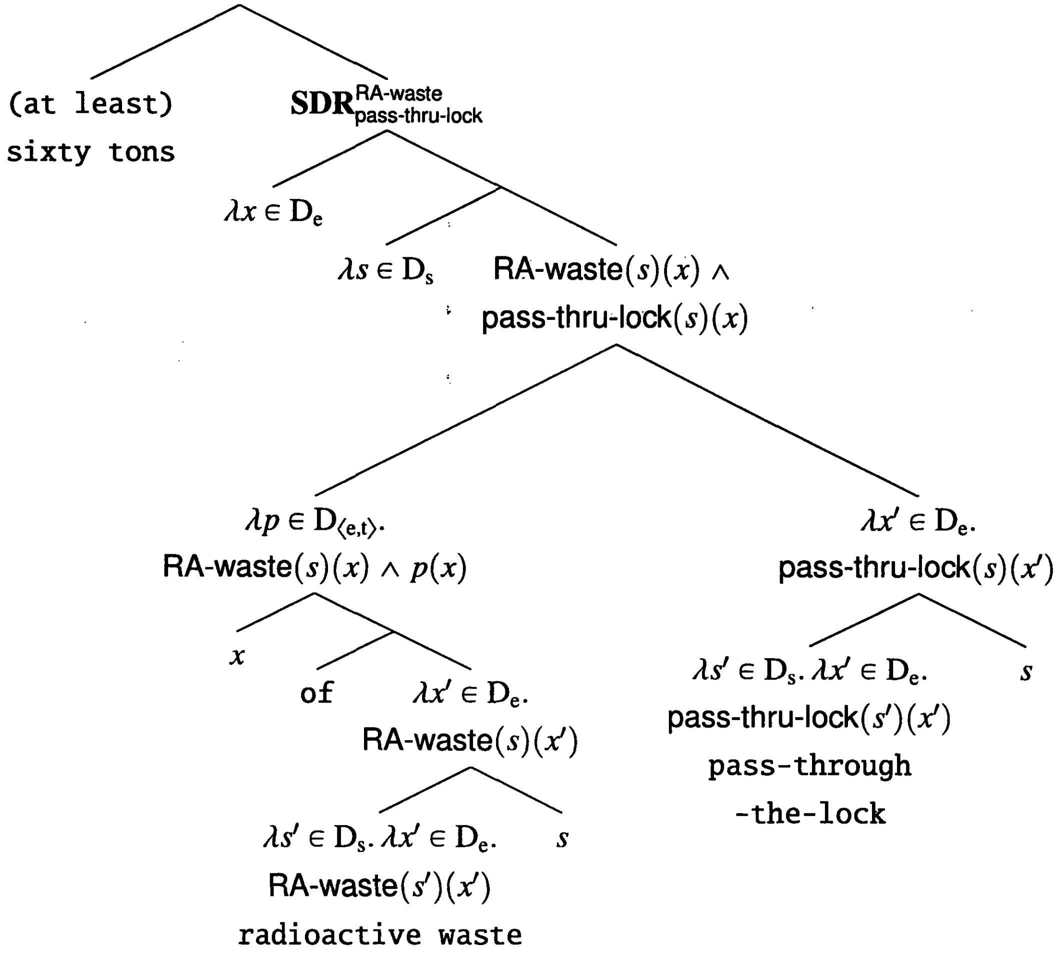
$$\overset{\beta}{=} \lambda R \in \text{D}_{\langle\text{e},\langle\text{s,t}\rangle\rangle}.$$
$$\exists n \in \text{D}_{\text{n}}\left[[\lambda n \in \text{D}_{\text{n}}.\ [n \geq 60]]\,(n) \wedge \sum_{i\in\gamma} \text{ton}\left(\bigsqcup\nolimits_{\text{e}}\{x \mid R(x)(s_i)\}\right) = n\right]$$

$$\overset{\beta}{=} \lambda R \in \mathrm{D}_{\langle e,\langle s,t\rangle\rangle}. \exists n \in \mathrm{D}_{\mathrm{n}} \left[n \geq 60 \wedge \sum_{i\in\gamma} \text{ton}\left(\bigsqcup\nolimits_{\mathrm{e}} \{x \mid R(x)(s_i)\}\right) = n \right]$$

$$= \lambda R \in \mathrm{D}_{\langle e,\langle s,t\rangle\rangle}. \left[\sum_{i\in\gamma} \text{ton}\left(\bigsqcup\nolimits_{\mathrm{e}} \{x \mid R(x)(s_i)\}\right) \geq 60 \right]$$

The measure phrases in (79), whose semantics is thus computed, take situationally dependent relation $\mathbf{SDR}^{\text{RA-waste}}_{\text{pass-thru-lock}}$ to yield the truth conditions in (75) and (77). $\mathbf{SDR}^{\text{RA-waste}}_{\text{pass-thru-lock}}$ is obtained in almost the same way as $\mathbf{SDR}^{\text{ship}}_{\text{pass-thru-lock}}$. Unlike the case of count nouns such as *three ships*, however, not only the numeral, but the whole measure phrase containing the unit of measurement gets raised at LF to a position right above the situation λ abstractor. This movement strands *of* + Noun. The partitive *of* is then treated as semantically vacuous. The following illustrates the LF and the semantic computation:

(83)



Thus, syntactically speaking, in sentences where the event-realted-reading-inducing non-presuppositional noun phrase contains a unit of measurement, the whole measure phrase such as *sixty tons* corresponds to the simple numeral such as *four thousand* in sentences like (1).

2.3.2 Measurement of individuals of unstable size

Let us consider the following sentence:

(84) Sixty tons of 20-year-olds {passed / were transported} through the lock last year.

If (73) is to be analyzed as in (75), this sentence should be analyzed analogously as follows:

(85) $$\sum_{i \in \gamma} \mathsf{ton}\left(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{pass-thru-lock}}(s_i)\right) \geq 60$$

However, there is a problem. Unlike the weight of radioactive waste, people's weight naturally fluctuates quite a bit even within a single year. For example, even if Mary weighed 70 kg[9] on her 20th birthday, it is possible that half a year later, she only weighed 50 kg as a result of her successful diet plan. Then, if a measurement of an individual is to be given in some unit of measurement, it should matter when the measurement was taken. Reflecting the meaning of (84), when uttered out of the blue, or embedded in an appropriate context as in von Fintel's test (see Section 1.4) to ensure the non-presuppositional reading of *sixty tons of 20-year-olds*), what it asserts is clearly that the sum of the weight of every 20-year-old person who passed through the lock last year which is measured *at the time of his/her lock-traversal* is 60 tons (or more). If someone passed through the lock twice last year while he was of age 20, his weights at both times (which may be different) should each be added to the sum. Furthermore, notice that if (84) is a statement about the actual world, the relevant weight to be considered is people's weight in the actual world, and not in any other hypothetical world or belief world where they could be thiner or fatter than they actually are.

To see this point more clearly, let us think of a concrete and small scenario, which goes as follows: Last year, Ann passed through the lock twice while she was of age 20, and the exact running time interval of each of the two events was I_{12} and I_{55} respectively; Bill and Chris each passed through the lock only once while being of age 20 last year, at I_{20} and I_{31} respectively; There was no other lock-traversal of a 20-year-old last year. (86) illustrates this scenario:

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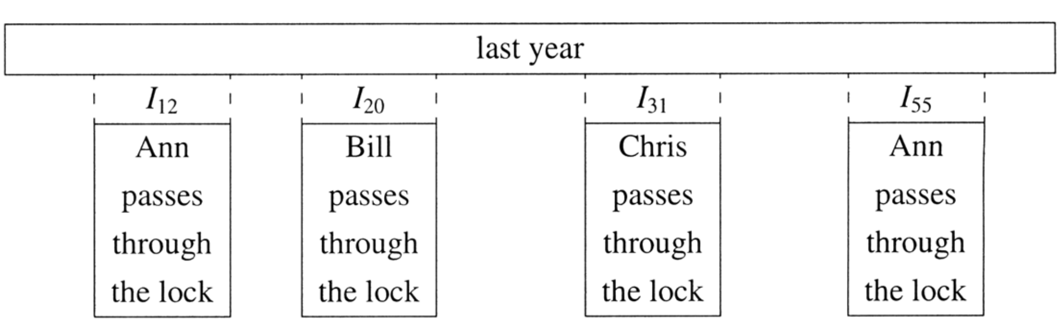
[9] Here, I am using kg to mean kilograms-force, following the usual practice of our daily language. Mass and weight are distinguished in the subsequent discussion of the story of Mary as an astronaut, however.

(86)



The weights of Ann, Bill and Chris at various situations are given by the following table:

(87)

	$\langle @, I_{12} \rangle$	$\langle @, I_{20} \rangle$	$\langle @, I_{31} \rangle$	$\langle @, I_{55} \rangle$	$\langle @, \text{now} \rangle$
Ann	49 kg	52 kg	55 kg	51 kg	55 kg
Bill	94 kg	92 kg	95 kg	98 kg	97 kg
Chris	77 kg	75 kg	72 kg	74 kg	75 kg

A shaded cell indicates that a person's weight given there is measured in the very situation in which an event of a lock-traversal of that person occurred. Now, imagine that the following sentences are uttered our of the blue:

(88) a. Two hundred and sixty-four kilograms of 20-year-olds passed through the lock last year.

b. Two hundred and eighty-two kilograms of 20-year-olds passed through the lock last year.

As the sum of the weights in the shaded cells in (87) is $49 + 92 + 72 + 51 = 264$ kg, (88a) correctly describes the above scenario. If, on the other hand, for each of the four events of lock-traversals of a 20-year-old in the last year, one adds up the *current* weight of the person who was of age 20 and passed through the lock in the event, $55+97+75+55 = 282$ kg will be obtained. However, when (88b) is uttered out of the blue, one cannot imagine the above scenario as a possible story compatible with what is asserted. Thus, what matters is how much a 20-year-old weighed at the time he/she passed though the lock, and his/her weight measured at other times do not count.

The measurement of individuals does not always change due to their intrinsic changes as in the above scenario where individuals get fatter or thinner as time goes by. Even when an individual's intrinsic properties remain constant, its measurement could vary if it is measured in different situations. Suppose the following scenario:

(89) Mary is an astronaut, and she has two boxes of apples. Box #1 contains apples whose mass is 12 kg, and Box #2 contains apples whose mass is 30 kg. While on the Earth surface, Mary threw the apples in Box #1. She collected the thrown apples in Box #1, and then flew to the moon as an astronaut with her two boxes of apples. On the moon surface, Mary threw the apples in Box #2. In no other occasion did Mary throw apples.

Let I_{201} be the exact time interval in which Mary threw the apples in Box #1 and I_{245} the exact time interval in which Mary threw the apples in Box #2. Assuming that the mass of an object never changes, the mass of the apples in Box #1 is always 12 kg. Likewise, the mass of the apples in Box #2 is always 30 kg. On the other hand, the weight of an object is the gravitational force acting upon the object, which is determined by its mass and the gravitational field in which it is located. In situation $\langle @, I_{201} \rangle$, the apples in Box #1 are on the Earth surface, and thus their weight is 12 kgf (= $12g$ N, where g is the gravitational acceleration on the Earth surface). In situation $\langle @, I_{245} \rangle$, the apples in Box #1 are on the moon surface, and as the moon's surface gravity is $1/6$ of that of the Earth, their weight is $12 \times (1/6)g \text{ N} = 2g \text{ N} = 2$ kgf. Likewise, the apples in Box #2 weighted 30 kgf in $\langle @, I_{201} \rangle$ and 5 kgf in $\langle @, I_{245} \rangle$. This is summarized in the following table:

(90)

	$\langle @, I_{201} \rangle$	$\langle @, I_{245} \rangle$
apples in Box #1	mass: 12 kg weight: 12 kgf	mass: 12 kg weight: 2 kgf
apples in Box #2	mass: 30 kg weight: 30 kgf	mass: 30 kg weight: 5 kgf

Now, imagine that the following sentences are uttered out of the blue:

(91) a. On the earth and on the moon, (in total,) Mary threw forty-two kilograms of apples.

b. On the earth and on the moon, (in total,) Mary threw seventeen kilograms-force of apples.

c. On the earth and on the moon, (in total,) Mary threw forty-two kilograms-force of apples.

It is clear that (91a) can be used to correctly describe he above scenario. As the apples Mary threw on the Earth weighed 12 kgf at that time, and the apples she threw on the moon weighed 5 kgf at that time, (91b) can also be used to correctly describe the scenario as she threw $12 + 5 = 17$ kgf of apples in total. On the other hand, if (91c) is uttered out of the blue, the hearer cannot imagine (89) as a possible scenario compatible with what is asserted, even though the apples she threw together weigh 42 kgf if measured on the Earth surface. Thus, what matters here is only how much the apples weighed at the time Mary threw them.

Given the discussion of the above examples, it is clear that when a noun phrase with a measure phrase involving a unit of measurement is interpreted non-presuppositionally, the relevant measurement of an individual that matters must be the one that is measured in the situation in which an event of the sort described by the main predicate that involves that individual occurrs. Put shortly, the situational interpretation of the unit of measurement in a non-presuppositional noun phrase is dependent on the situational interpretation of the main predicate. This makes a contrast with the interpretation of units of measurement in presuppositional noun phrases. Consider the following sentence, where the object noun phrase with a unit of measurement is headed by the definite article:

(92) On the earth and on the moon, Mary threw the forty-two kilograms-force of apples in the two boxes.

Imagine that after the scenario in (89), Mary collected all the apples she threw on the moon back in Box #2 and returned to the Earth with both boxes of apples. Looking at the two boxes in front of him, a speaker may easily utter (92) truthfully. In this case, 42 kgf will be the weight of the apples in the two boxes measured at the time of the utteracne. What we have observed here reminds us of Musan's generalization, according to which, the situational interpretation of the descriptive part of a non-presuppositional noun phrase (e.g., *20-year-olds*), depends on that of the main predicate. In fact, one can regard this observation as an extension of Musan's generalization to the unit of measurement in the measure phrase of a non-presuppositional noun phrase. This generalization is stated as follows:

(93) Extension of Musan's generalization to units of measurement
The situational interpretation of the unit of measurement in a noun phrase can be independent of the main predicate if and only if the noun phrase is presuppositional.

Let us now return to the analysis of (84). The truth conditions we gave to (84) in the beginning of this subsection were (85), which is repeated below:

(85) $$\sum_{i \in \gamma} \mathsf{ton}\left(\mathbf{SDIC}^{\mathsf{20\text{-}yr\text{-}old}}_{\mathsf{pass\text{-}thru\text{-}lock}}(s_i)\right) \geq 60$$

The problems is that the measure function ton is not sensitive to a situation, and thus the obligatory situational dependence of the unit of measurement on the main predicate discussed above cannot be expressed. In Chapter 1, I postulated a situation argument for predicates in general in order to account for the situational dependence of non-presuppositional noun phrases. Likewise, I propose here that the lattice-measure functions such as ton in general also take a situation argument, which specifies in which situation the measurement is taken.

Before we modify the definition of lattice-measure functions, let us discuss one more complicating factor. Imagine that the following sentence is uttered out of the blue:

(94) Exactly sixty-six tons of 20-year-olds {passed/were transported} through the tunnel.

Suppose that a train runs through the tunnel and transports people through it every day. So this sentence is about what amount of people the train transported last year. Suppose further that the tunnel at issue is a really long one, so it takes two hours for the train to run through it. According to the generalization in (93), the situation in which the weights of the 20-year-old passengers are measured must be dependent on the situations in which they were transported through the tunnel. However, now that each event of a train ride is two hours long, the question arises at which point of the riding their weight should be measured, as it is possible for a passenger to increase his/her weight significantly during the two hours' ride if he/she kept eating all the while. Now, let $r_1, r_2, \ldots, r_n$ be the events of train rides through the tunnel that transported at least one 20-year-old in the last year. Let w_i and W_i be the collective weight of the 20-year-old passengers in r_i at the beginning of r_i and at the end of r_i respectively. Suppose that all 20-year-old passengers had a very big lunch in the train and they each gained a couple of kilograms during the ride. As a consequence, it turns out that $\sum_{i=1}^{n} W_i - \sum_{i=1}^{n} w_i = 2$ tons. We should then ask ourselves the following questions: If $\sum_{i=1}^{n} w_i = 66$ (and thus $\sum_{i=1}^{n} W_i = 68$), can one utter (94) truthfully? Or, if $\sum_{i=1}^{n} W_i = 66$ (and thus $\sum_{i=1}^{n} w_i = 64$), can one utter (94) truthfully? I think that in such cases, (94) would simply be an inappropriate utterance. The absurdity of making a statement like (94) in such cases would become perhaps most clear if one imagined that the size of individuals get doubled or tripled during the relevant events. In order to be able to utter (94) felicitously, it seems to be required that for every i such that $1 \leq i \leq n$, the collective weight of the 20-year-old passengers in r_i was constant during r_i. I thus propose that a sentence with a non-presuppositional noun phrase containing a unit of measurement in general has such a presupposition of regarding the obligatory constancy in size of relevant individuals during relevant events.

I would now like to translate the discussion so far on the interpretation of units of measurement into our semantic formulae. To that end, I will first modify lattice-measure functions so that they will be situation-sensitive. However, they are actually defined only when the time interval of their situation argument is momentary. To analyze a sentence with a non-presuppositional noun phrase containing a unit of measurement, by using the lattice-measure function given by the unit of measurement, a relevant function from situations into reals is defined. Unlike the original lattice-measure function, this function may yield values for situations whose time interval has a positive length.

To begin with, let us define **situation-sensitive lattice measure functions** as follows:

(95) $M \in D_{\langle s, \langle e, n \rangle \rangle}$ is a situation-sensitive lattice-measure function iff

  i. $\langle s, x \rangle \in \mathrm{dom}(M) \rightarrow \exists p[p \in \mathbb{R} \wedge \pi_{\mathrm{i}}(s) = [p, p]]$

  ii. $\langle s, x \rangle \in \mathrm{dom}(M) \rightarrow M(s)(x) \geq 0$ (non-negativity)

  iii. $M(s)(\perp_{\mathrm{e}}) = 0$

  iv. $\langle s, x \rangle \in \mathrm{dom}(M) \wedge y \sqsubseteq_{\mathrm{e}} x \rightarrow \langle s, y \rangle \in \mathrm{dom}(M)$ (extendability to parts)

  v. If $\langle s, x_n \rangle \in \mathrm{dom}(M)$ for all $n \in \mathbb{N}$ and if $\{x_n\}$ is a pairwise non-overlapping set, i.e., for any $i, j \in \mathbb{N}, i \neq j \rightarrow x_i \sqcap_{\mathrm{e}} x_j = \perp_{\mathrm{e}}$, then,

$$M(s)\left(\bigsqcup_{n=1}^{\infty}{}_{\mathrm{e}}\, x_n \right) = \sum_{n=1}^{\infty} M(s)(x_n)$$

(countable additivity)

$[p, p]$ is a closed time interval which contains only one time point p: $[p, p] = \{p\}$. The first clause thus says that a situation-sensitive lattice measure function is defined only on situations whose time interval is actually a moment. For example, ton is a situation-sensitive lattice-measure function, and $\mathsf{ton}(\langle w, [p, p] \rangle)(x)$ gives the weight of individual x in tons as measured at time point p in world w. Now, given a situationally dependent individual concept $\mathbf{SDIC}^{\varphi}_{\psi}$, we want to define a function which, if the size of $\mathbf{SDIC}^{\varphi}_{\psi}(s)$ is constant during $\pi_{\mathrm{i}}(s)$ in $\pi_{\mathrm{w}}(s)$, yields the constant measurement in s, and would otherwise be undefined. For this, I define operator ϱ, which yields the relevant such function from situations into reals when provided a situation-sensitive lattice-measure function and a function from situations into individuals as follows:

(96) Given a situation-sensitive lattice-measure function $M \in \mathrm{D}_{\langle \mathrm{s},\langle \mathrm{e},\mathrm{n}\rangle\rangle}$ and a function from situations into individuals $\xi \in \mathrm{D}_{\langle \mathrm{s},\mathrm{e}\rangle}$, $\varrho_M(\xi) \in \mathrm{D}_{\langle \mathrm{s},\mathrm{n}\rangle}$ is a function from situations into reals such that

$$\varrho_M(\xi) \overset{\mathrm{def}}{=} \lambda s \in \mathrm{D_s} : \inf_{p \in \pi_{\mathrm{i}}(s)} M\big(\langle \pi_{\mathrm{w}}(s), [p,p]\rangle\big)(\xi(s)) = \sup_{p \in \pi_{\mathrm{i}}(s)} M\big(\langle \pi_{\mathrm{w}}(s), [p,p]\rangle\big)(\xi(s)).$$
$$\inf_{p \in \pi_{\mathrm{i}}(s)} M\big(\langle \pi_{\mathrm{w}}(s), [p,p]\rangle\big)(\xi(s))$$

For a given situation s, $\varrho_M(\xi)(s)$ is defined only when $\inf_{p \in \pi_{\mathrm{i}}(s)} M\big(\langle \pi_{\mathrm{w}}(s), [p,p]\rangle\big)(\xi(s)) = \sup_{p \in \pi_{\mathrm{i}}(s)} M\big(\langle \pi_{\mathrm{w}}(s), [p,p]\rangle\big)(\xi(s))$, that is, only when the size of the relevant kind of individual $\xi(s)$ is constant during time interval $\pi_{\mathrm{i}}(s)$. In that case, $\varrho_M(\xi)(s)$ gives the infimum of the size during $\pi_{\mathrm{i}}(s)$, but of course the significance of using infimum (as opposed to supremum, etc.) is trivialized here. If $\xi(s)$'s size ever changes during $\pi_{\mathrm{i}}(s)$, $\varrho_M(\xi)(s)$ is undefined, and the associated sentence ends up failing to have a truth value. The analysis thus captures the obligatory constancy presupposition discussed above.

A brief note is in order on what "constant" is supposed to mean. Strictly speaking, in actuality, almost nothing has a stable size during a time interval. Even if people do not eat while in a train ride, if the air is dry inside the train, for instance, water might keep evaporating from their body surface, which would keep reducing their mass/weight during the ride. However, the amount of the water which evaporates from people's body surface would be so minute that it would be practically negligible. Consequently, if this is the only change of their mass/weight during the train ride, their mass/weight will be normally regarded as constant all the while. We should be aware that the numeral in a measure phrases is a rounded number in general, and that it is only meant to show the significant figures. The numeral in measure phrase "49 kg", for instance, has only two significant figures, and the weight expressed by it actually ranges between 48.5 kg and 49.5 kg. Thus, even if Ann's weight fluctuated between 48.5 kg and 49.5 kg during some time interval, it is correct to say that Ann's weight was "constant" and 49 kg during the time interval. Similarly, depending on the usage, the numeral in measure phrase "50" kg might have only one significant figure, in which case the weight it expresses would range between 45 kg and 55 kg, a much wider range than in the previous case. Thus, the concept of "constancy" depends very much on the precision with which the associated lattice-measure function assigns values to individuals. Contextual dependency of the granularity of measurement has been discussed in the literature, and thus I will not persue this issue further here.

Let us now see how the proposed theory works with the scenario we considered earlier with (86) and (87). In situation $\langle @, I_{12}\rangle$, Ann's weight was constant and it was 49 kg. In this case, the relevant situation-sensitive lattice-measure function is $\mathsf{kilogram}$, and the relevant function from situations into individuals is $\mathbf{SDIC}^{\text{20-yr-old}}_{\text{pass-thru-lock}}$. Following the definition in (91), we obtain the function $\varrho_{\mathsf{kilogram}}\left(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{pass-thru-lock}}\right)$ as follows:

(97) $$\begin{aligned}&\varrho_{\mathsf{kilogram}}\left(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{pass-thru-lock}}\right)\\ &= \lambda s \in \mathrm{D_s} : \inf_{p \in \pi_{\mathrm{i}}(s)} \mathsf{kilogram}\left(\langle \pi_{\mathrm{w}}(s), [p,p]\rangle\right)\left(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{pass-thru-lock}}(s)\right)\\ &\qquad\qquad = \sup_{p \in \pi_{\mathrm{i}}(s)} \mathsf{kilogram}\left(\langle \pi_{\mathrm{w}}(s), [p,p]\rangle\right)\left(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{pass-thru-lock}}(s)\right).\\ &\quad\inf_{p \in \pi_{\mathrm{i}}(s)} \mathsf{kilogram}\left(\langle \pi_{\mathrm{w}}(s), [p,p]\rangle\right)\left(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{pass-thru-lock}}(s)\right)\end{aligned}$$

Let us compute the value that this function yields for situation $\langle @, I_{12}\rangle$. For $s = \langle @, I_{12}\rangle$, the presupposition of (97) becomes the following:

(98) $$\begin{aligned}&\inf_{p \in \pi_{\mathrm{i}}(\langle @, I_{12}\rangle)} \mathsf{kilogram}\left(\langle \pi_{\mathrm{w}}(\langle @, I_{12}\rangle), [p,p]\rangle\right)\left(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{pass-thru-lock}}(\langle @, I_{12}\rangle)\right)\\ &= \sup_{p \in \pi_{\mathrm{i}}(\langle @, I_{12}\rangle)} \mathsf{kilogram}\left(\langle \pi_{\mathrm{w}}(\langle @, I_{12}\rangle), [p,p]\rangle\right)\left(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{pass-thru-lock}}(\langle @, I_{12}\rangle)\right)\end{aligned}$$

Here, $\pi_{\mathrm{i}}\left(\langle @, I_{12}\rangle\right) = I_{12}$ and $\pi_{\mathrm{w}}\left(\langle @, I_{12}\rangle\right) = @$. Also,

(99) $$\begin{aligned}&\mathbf{SDIC}^{\text{20-yr-old}}_{\text{pass-thru-lock}}\left(\langle @, I_{12}\rangle\right) = \bigsqcup_{\mathrm{e}}\left\{x \mid \mathbf{SDR}^{\text{20-yr-old}}_{\text{pass-thru-lock}}(x)\left(\langle @, I_{12}\rangle\right)\right\}\\ &= \bigsqcup_{\mathrm{e}}\{\mathsf{Ann}\} = \mathsf{Ann}\end{aligned}$$

(98) is thus equivalent to the following condition:

(100) $$\inf_{p \in I_{12}} \mathsf{kilogram}\left(\langle @, [p,p]\rangle\right)(\mathsf{Ann}) = \sup_{p \in I_{12}} \mathsf{kilogram}\left(\langle @, [p,p]\rangle\right)(\mathsf{Ann})$$

As Ann's weight was constant in $\langle @, I_{12}\rangle$ (viz., 49 kg), both sides of this equation is equal to 49 and hence this equation actually holds. This means that $\langle @, I_{12}\rangle$ is in the domain of $\varrho_{\mathsf{kilogram}}\left(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{pass-thru-lock}}\right)$. The value it yields for $\langle @, I_{12}\rangle$ is 49:

(101) $$\varrho_{\mathsf{kilogram}}\left(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{pass-thru-lock}}\right)\left(\langle @, I_{12}\rangle\right) = \inf_{p \in I_{12}} \mathsf{kilogram}\left(\langle @, [p,p]\rangle\right)(\mathsf{Ann}) = 49$$

Now that ϱ has been defined, we can finally give the correct truth conditions to (84). What was expressed by $\mathsf{ton}\left(\mathbf{SDIC}^{\varphi}_{\psi}(s)\right)$ in the old analysis, where ton was not situation-sensitive, should be replaced by $\varrho_{\mathsf{ton}}\left(\mathbf{SDIC}^{\varphi}_{\psi}\right)(s)$ in the new analysis, where ton is situation-sensitive. Hence, the correct truth conditions of (84) can be obtained by rewriting (85) accordingly as follows:

(102) $$\sum_{i \in \gamma} \varrho_{\mathsf{ton}}\left(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{pass-thru-lock}}\right)(s_i) \geq 60$$

By the definition of **SDIC** in (48), this is equivalent to the following:

(103) $$\sum_{i\in\gamma}\varrho_{\text{ton}}\left(\lambda s\in \mathrm{D}_{\mathrm{s}}.\bigsqcup_{\mathrm{e}}\{x\mid \mathbf{SDR}^{\text{20-yr-old}}_{\text{pass-thru-lock}}(x)(s)\}\right)(s_i)\geq 60$$

This can be compositionally computed in the same way and with the same syntax as we computed (75) earlier. All we need to do is to rewrite the lexical entry of ton given in (80) in the way explained just above as follows:

(104) $$[\![\texttt{ton}]\!]=\lambda v\in \mathrm{D}_{\langle \mathrm{n,t}\rangle}.\,\lambda R\in \mathrm{D}_{\langle \mathrm{e},\langle \mathrm{s,t}\rangle\rangle}.$$
$$\exists n\in \mathrm{D}_{\mathrm{n}}\left[v(n)\wedge\sum_{i\in\gamma}\varrho_{\text{ton}}\left(\lambda s\in \mathrm{D}_{\mathrm{s}}.\bigsqcup_{\mathrm{e}}\{x\mid R(x)(s)\}\right)(s_i)=n\right]$$

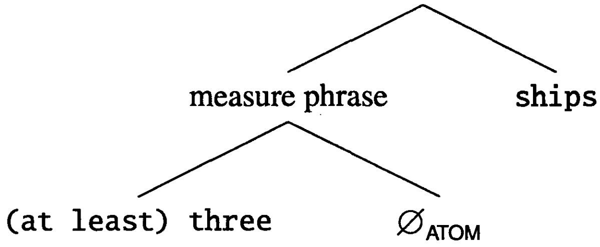
2.3.3 Count nouns and classifiers

In the previous section, following Krifka's **OEM** operator, we defined the **OSM** operator, utilizing a sum operator. There, in analyzing count nouns, this **OSM** operator was encoded in the lexical entry of numerals as in (58). In the analysis developed in the present section, however, the corresponding sum operator is now in the semantics of units of measurement as in (104), and numerals simply denote a property of numbers. For example, *at least three* has the following lexical entry:

(105) $[\![\texttt{at least three}]\!]=\lambda n\in \mathrm{D}_{\mathrm{n}}.\,[n\geq 3]$

It then appears most reasonable to assume that the numeral that precedes a count noun is actually a measure phrase consisting of a numeral and a silent unit of measurement. The relevant lattice-measure function for count nouns is ATOM. I will thus notate this silent unit of measurement for count nouns as $\varnothing_{\text{ATOM}}$. The structure of a non-presuppositional noun phrase with a count noun should then look like the following:

(106)



The whole measure phrase containing $\varnothing_{\text{ATOM}}$ here move to a position right above the relevant situation λ abstractor just as in (83). Unlike noun phrases with an overt unit of measurement, the noun phrase does not have the partitive *of*, which is semantically vacuous in any case.

Now, if we are to follow the theory proposed in previous subsection, ATOM should be modified so that it will be situation-sensitive, and ϱ_{ATOM} should be utilized accordingly to account for the semantics of count nouns. (30), which is repeated below, will then be analyzed as in (107), similarly to (102) above.

(30) Three ships passed through the lock.

(107) $$\sum_{i\in\gamma}\varrho_{\mathsf{ATOM}}\left(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{pass-thru-lock}}\right)(s_i) \geq 3$$

The question arises here as to whether ATOM should really be sensitive to a situation. An individual's weight, height, etc., may naturally change as time progresses, but it is not clear whether an individual's atomicity may change similarly. One could think of some obscure examples. For instance, suppose that individual x is a big stone. If this stone is broken into five smaller pieces, should we think that the same individual x counts as 5 afterwards? Or, suppose that individual y is a planarian. Planaria are known for their extraordinary ability to regenerate. If Mary cut planarian y into three pieces, and each piece has regenerated into a full planarian, should we think that the same individual y now counts as 3? I do not know if these questions should be answered with a yes or no. As Irene Heim (p.c.) suggests, perhaps it is most reasonable to assume that the identity of the individuals is not retained in such cases. On this view, after the cut pieces of the original planarian have regenerated, none of the new planaria is identified as y, and each of them is a new individual with its own identity. I will not go into this philosophical discussion further here, but in any way assume that ATOM is situation-senstitive and maintain the analysis in (107), just so that all units of measurement will be analyzed in a uniform fashion. If one thinks that an individual's atomicity should never change, one can simply assume that for any individual x, the value of $\mathsf{ATOM}(s)(x)$ never varies with s, or in other words, $\lambda s \in \mathrm{D_s}.\,\mathsf{ATOM}(s)(x)$ is a constant function. In order to be able to compute (107), we need only posit the following lexical entry for $\varnothing_{\mathsf{ATOM}}$.

(108) $$[\![\varnothing_{\mathsf{ATOM}}]\!] = \lambda v \in \mathrm{D}_{\langle \mathrm{n,t}\rangle}.\,\lambda R \in \mathrm{D}_{\langle \mathrm{e},\langle \mathrm{s,t}\rangle\rangle}.$$
$$\exists n \in \mathrm{D_n}\left[v(n) \wedge \sum_{i\in\gamma}\varrho_{\mathsf{ATOM}}\left(\lambda s \in \mathrm{D_s}.\bigsqcup\nolimits_{\mathrm{e}}\{x \mid R(x)(s)\}\right)(s_i) = n\right]$$

This is identical to the lexical entry of *ton* except that ton is replaced with ATOM.

The assumption that count nouns have a silent unit of measurement is not an implausible idea. In languages like Japanese, the numeral in count nouns is always followed by a classifier:

(109) a. [san-nin]-no gakusei
three-CL -GEN student
'three students'

b. [yon-tô]-no tora
four-CL -GEN tiger
'four tigers'

c. [go-hiki]-no neko
five-CL -GEN cat
'five cats'

Nin, *tô* and *hiki* seen here are classifiers. Which classifier should follow the numeral is determined by the noun. *Nin* is used to count human beings, *tô* is for large animals or beasts such as tigers or cows, and *hiki* is usually used for small animals such as cats or mice.[10] What is important is that classifiers have the same distribution as units of measurement in Japanese. Compare (109) with the following noun phrase with a unit of measurement:

(110) [ni-rittoru]-no mizu
two-liter -GEN water

Given this parallel, it is natural to assume that classifiers in Japanese are units of measurement. It is then not a strange idea to propose that English count nouns also have a unit of measurement, but it is simply silent unlike Japanese classifiers. What is different between Japanese classifiers and its silent counterpart in English is that Japanese classifiers are more finely grained in the sense that they specify what kind of atomic element they count.

How shall we analyze classifiers semantically, then? The semantics of a classifier should essentially be the same as that of $\varnothing_{\mathsf{ATOM}}$ in (108), but it restricts what kind of objects it counts. We see that $\varnothing_{\mathsf{ATOM}}$ takes the sum of the size of $\bigsqcup_{\mathsf{e}} \{x \mid R(x)(s)\}$ for all atomic situations s in the context, but *nin*, for instance, will pick only the human individuals from $\bigsqcup_{\mathsf{e}} \{x \mid R(x)(s)\}$, and take the sum of their size for all s. To do this, let us define NIN to be the individual concept (i.e., function from situations into individuals) which, given a situation, yields the individual obtained by taking the join of every individual that is human in that situation:

(111) $\mathsf{NIN}(s) \stackrel{\text{def}}{=} \bigsqcup_{\mathsf{e}} \{x \mid x \text{ is a human(-like) being in situation } s\}$

---

[10] Such a classification is not as clear-cut as stated here. For example, if we are to count extraterrestrial intelligent beings, we might as well use *nin* rather than *hiki*, even though they are not human beings. Dogs may be counted with either *tô* or *hiki*. The correct usage of classifiers is often idiosyncratically prescribed. For example, butterflies are supposed be counted with *tô*, although many people might rather use *hiki*.

Then, the desired restriction can be achieved by taking the meet (greatest lower-bound) of $\mathsf{NIN}(s)$ and $\bigsqcup_{\mathrm{e}}\{x \mid R(x)(s)\}$. The lexical entry of *nin* should thus be given by the following:

(112) $$[\![\texttt{nin}]\!] = \lambda \nu \in \mathrm{D}_{\langle \mathrm{n,t}\rangle}.\, \lambda R \in \mathrm{D}_{\langle \mathrm{e},\langle \mathrm{s,t}\rangle\rangle}.\, \exists n \in \mathrm{D}_{\mathrm{n}} \left[\nu(n) \wedge \sum_{i \in \gamma} \varrho_{\mathsf{ATOM}} \left(\lambda s \in \mathrm{D}_{\mathrm{s}}.\, \mathsf{NIN}(s) \sqcap_{\mathrm{e}} \bigsqcup_{\mathrm{e}} \{x \mid R(x)(s)\}\right)(s_i) = n\right]$$

The lexical entries of *tô* and *hiki* can be given by defining appropriate individual concepts TÔ and HIKI similarly.

I would like to end this section by analyzing an interesting example involving classifiers in Japanese. Imagine that Taro was playing a video game in which the player is supposed to defeat enemy characters, which are comprised of human soldiers, large animals and small animals. The following sentence can be used to describe what Taro did in such a scenario:

(113) Taro-wa [[[[san-nin]-to yon-tô]-to go-hiki]-no teki]-wo taosi-ta.
Taro-TOP three-CL -and four-CL -and five-CL -GEN enemy -ACC defeat-PAST
'Taro defeated three human enemies, four large-animal enemies and five small-animal enemies.'

What is interesting here is that measure phrases are conjoined to make up a complex measure phrase. This can be accounted for in the current system once an appropriate lexical entry for *to* 'and' is given. Notice that the denotation of measure phrase *san-nin* is computed from (105) and (112) to be the following, which is of type $\langle\langle \mathrm{e}, \langle \mathrm{s}, \mathrm{t}\rangle\rangle, \mathrm{t}\rangle$:

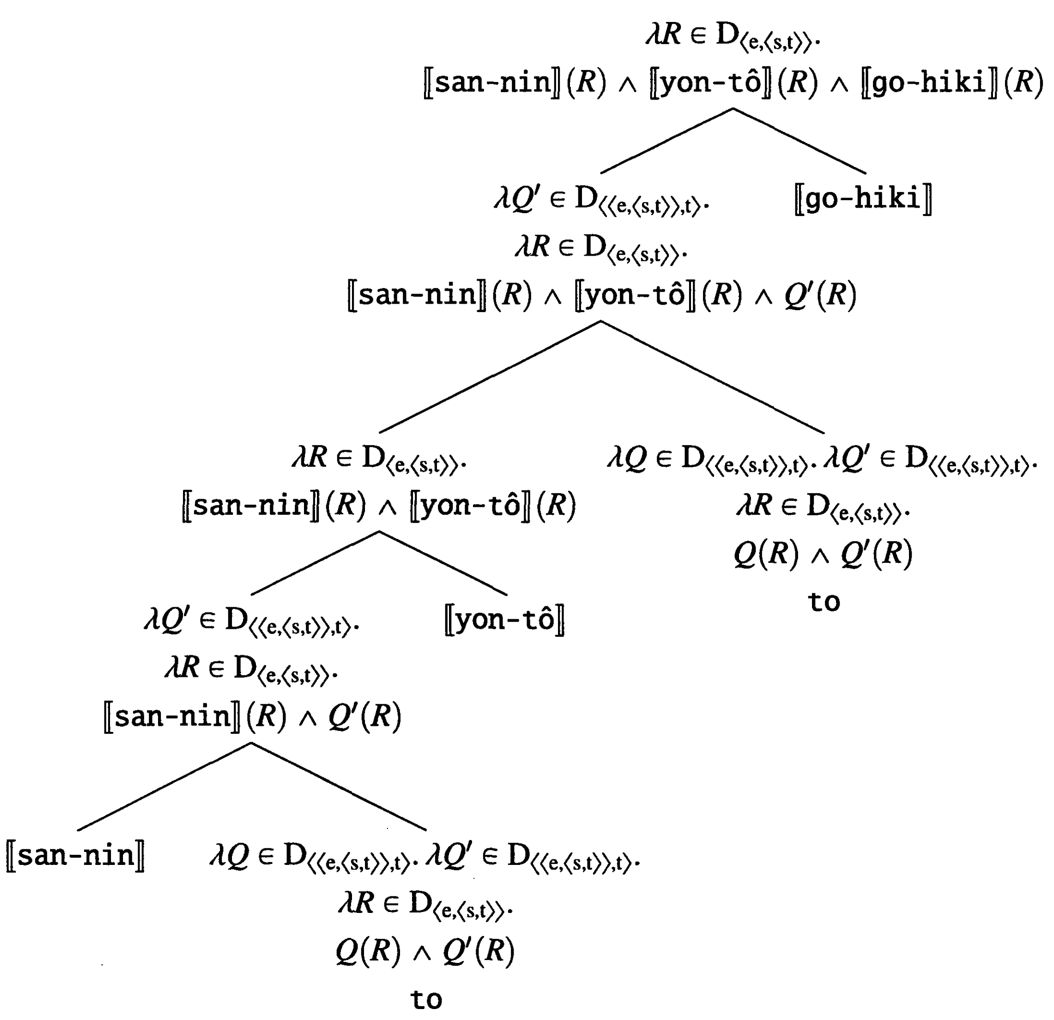
(114) $$[\![\texttt{san-nin}]\!] = \lambda R \in \mathrm{D}_{\langle \mathrm{e},\langle \mathrm{s,t}\rangle\rangle}.\, \left[\sum_{i \in \gamma} \varrho_{\mathsf{ATOM}} \left(\lambda s \in \mathrm{D}_{\mathrm{s}}.\, \mathsf{NIN}(s) \sqcap_{\mathrm{e}} \bigsqcup_{\mathrm{e}} \{x \mid R(x)(s)\}\right)(s_i) \geq 3\right]$$

Thus, the denotation of *to* must be a higher-type conjunction Partee and Rooth (1983) of phrases of type $\langle\langle \mathrm{e}, \langle \mathrm{s}, \mathrm{t}\rangle\rangle, \mathrm{t}\rangle$, which is given by the following:

(115) $$[\![\texttt{to}]\!] = \lambda Q \in \mathrm{D}_{\langle\langle \mathrm{e},\langle \mathrm{s,t}\rangle\rangle,\mathrm{t}\rangle}.\, \lambda Q' \in \mathrm{D}_{\langle\langle \mathrm{e},\langle \mathrm{s,t}\rangle\rangle,\mathrm{t}\rangle}.\, \lambda R \in \mathrm{D}_{\langle \mathrm{e},\langle \mathrm{s,t}\rangle\rangle}.\, Q(R) \wedge Q'(R)$$

The semantics of the whole complex measure phrase *[[[san-nin]-to yon-tô]-to go-hiki]* is then computed as follows:

(116)



This complex measure phrase as a whole undergoes movement to a position right above the relevant situation λ abstractor as usual, and then takes the situation dependent relation made up of the nominal predicate and the main predicate. For (113), the appropriate situation dependent relation is $\mathbf{SDR}^{\text{enemy}}_{\text{Taro-defeat}}$.[11] The truth conditions of (113) are obtained by plugging $\mathbf{SDR}^{\text{enemy}}_{\text{Taro-defeat}}$ into the top node of (116):

(117) $[\![\texttt{san-nin}]\!]\left(\mathbf{SDR}^{\text{enemy}}_{\text{Taro-defeat}}\right) \wedge [\![\texttt{yon-tô}]\!]\left(\mathbf{SDR}^{\text{enemy}}_{\text{Taro-defeat}}\right) \wedge [\![\texttt{go-hiki}]\!]\left(\mathbf{SDR}^{\text{enemy}}_{\text{Taro-defeat}}\right)$

Here, we can for instance compute $[\![\texttt{san-nin}]\!]\left(\mathbf{SDR}^{\text{enemy}}_{\text{defeat}}\right)$, using (114), as follows:

---

[11] Taro-defeat $= \lambda s \in D_s. \lambda x \in D_e.$ [there is an event of Taro defeating x in $\pi_w(s)$ whose running time interval is exactly $\pi_i(s)$]

(118) $$[\![\texttt{san-nin}]\!]\left(\mathbf{SDR}^{\mathsf{enemy}}_{\mathsf{Taro\text{-}defeat}}\right)$$
$$= \left[\sum_{i \in \gamma} \varrho_{\mathsf{ATOM}} \left(\lambda s \in \mathrm{D_s}.\, \mathsf{NIN}(s) \sqcap_{\mathsf{e}} \bigsqcup_{\mathsf{e}} \{x \mid \mathbf{SDR}^{\mathsf{enemy}}_{\mathsf{Taro\text{-}defeat}}(x)(s)\}\right)(s_i) \geq 3\right]$$
$$= \left[\sum_{i \in \gamma} \varrho_{\mathsf{ATOM}} \left(\lambda s \in \mathrm{D_s}.\, \mathsf{NIN}(s) \sqcap_{\mathsf{e}} \mathbf{SDIC}^{\mathsf{enemy}}_{\mathsf{Taro\text{-}defeat}}(s)\right)(s_i) \geq 3\right]$$

$\mathbf{SDIC}^{\mathsf{enemy}}_{\mathsf{Taro\text{-}defeat}}(s)$ is the collection of the individuals that are enemies and are defeated by Taro in s, and $\mathsf{NIN}(s) \sqcap_{\mathsf{e}} \mathbf{SDIC}^{\mathsf{enemy}}_{\mathsf{Taro\text{-}defeat}}(s)$ picks only the humans in it. $[\![\texttt{san-nin}]\!]$ thus counts only humans as desired.

2.4 Continuous production and consumption of mass entities

2.4.1 Limitations of the approach based solely on summation

Based on the definition of the **OSM** operator (52), the truth conditions of event-related readings have so far been translated into formulae with a sum operator. In this section, it is finally shown that the current theory does not work for sentences that describe events of continuous production or consumption of mass entities and that integration must be employed.

To begin with, consider the following sentence with a production verb:

(119) Machine P produced (exactly) 2.7 liters of XYZ yesterday.

Imagine that Machine P is a special machine in a laboratory designed to produce a mysterious liquid called XYZ, and that the XYZ produced by Machine P enters a long pipe (which is itself not part of Machine P) connected to Machine P and pours into a tank in which XYZ (and only XYZ) is kept. If (119) is uttered out of the blue, what truth conditions shall it have?

Initially, one might think that one only needs to check the volume of the XYZ in the tank at 12 a.m. of yesterday and at 12 a.m. of today, and that the sentence is correct if the increase in volume is 2.7 liters. This, however, works only if the produced XYZ never expanded or contracted in volume after the production. As a matter of fact, most liquids expand when heated and contract when cooled. Suppose that the long pipe connected to Machine P is refrigerated all along, and thus the temperature of the XYZ in it goes down as it runs through it towards the tank. Suppose, for example, that Machine P produces XYZ at

90 °C, but the XYZ gets cooled down to around 10 °C by the time it comes out of the pipe. We can imagine that this mysterious liquid changes its volume in an unusually large scale and that its volume is reduced to exactly a third when its temperature falls from 90 °C to 10 °C. If the temperature of the XYZ always becomes exactly 10 °C at the end of the pipe, and if the XYZ in the tank is always kept at exactly 10 °C, all we need to verify (119) is to check whether the volumetric increase of the XYZ in the tank is 0.9 ($= 2.7 \times 1/3$) liters. However, XYZ actually only comes out of the pipe around 10 °C, sometimes below it, and sometimes above it. Worse yet, the power of the refrigeration might fluctuate a lot so that the terminal temperature might range so widely as between 0 °C and 30 °C. It should then be realized that the idea of attempting to know how many liters of XYZ Machine P produced from the volume of the XYZ in the tank is fundamentally flawed. To see this more clearly, imagine that John was drinking XYZ from the tank while Machine P was running. At the end of yesterday, it may be the case that no XYZ was left in the tank if John had drunk it dry. If in that case, there was 1.5 liters of XYZ at the beginning of yesterday, should we then say that Machine P produced minus 1.5 liters of XYZ yesterday? I think not. Regardless of whether or not John was drinking XYZ from the tank, it is possible that Machine P produced 2.7 liters of XYZ. How, then, would one know whether (119) is a correct assertion?

The point of the above discussion might perhaps be more intuitively understood if one considers a case of consumption. Suppose that there is a tank which was filled with 2.7 liter of XYZ at 90 °C at the beginning of yesterday. A long, refrigerated pipe is connected to this tank, and John drank up all the XYZ in the tank through this pipe yesterday. Imagine that the temperature of the XYZ always came down to exactly 10 °C at the end of the pipe, where it entered John's mouth. Then, the total amount of XYZ that entered John's mouth ought to be $2.7 \times 1/3 = 0.9$ liters. Then, consider the following sentences:

(120) a. John drank (exactly) 0.9 liters of XYZ yesterday.
b. John drank (exactly) 2.7 liters of XYZ yesterday.

Clearly, (120a) may be uttered out of the blue to describe this scenario, but intuitively, (120b) may not. One might object to this and claim that (120b) is also possible. However, I have the impression that (120b) can make sense only when the pipe is regarded as part of John, in which case, the moment XYZ enters the pipe, it is considered to have been drunk by John. Still, if you ask John whether he drank 2.7 liters of XYZ yesterday, I think that he would most certainly say no. Things would be different, of course, if the noun phrase with the measure phrase is headed by *the*:

(121) John drank the 2.7 liters of XYZ yesterday.

This is perfectly fine to describe the above scenario. When (121) is uttered felicitously, the context must provide a salient time, which, in this case, should be the beginning of the yesterday, and thus the measurement is taken at that time of a contextually salient individual which is the XYZ in the tank at that time.

Obviously, what is necessary to account for (119) and (120a) is to measure the volume of the XYZ at the very moment it is produced by Machine P or drunk by John, and to calculate its total amount. This is exactly what was discussed concerning the interpretation of the unit of measurement in an event-related-reading-inducing non-presuppositional noun phrase in the previous section. Let us then apply the theory developed there to sentence (119). (119)'s truth conditions are then expected to look like the following:

(122) $\sum_{i \in \gamma} \varrho_{\mathsf{liter}}\left(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}\right)(s_i) = 2.7$

$\{s_i\}_{i \in \gamma}$ is the set of atomic situations in yesterday in this case. XYZ is a predicate such that $\mathsf{XYZ}(s)(x)$ is true if and only if x is XYZ during $\pi_{\mathrm{i}}(s)$ in $\pi_{\mathrm{w}}(s)$. The problem is the meaning of the main predicate which is written as $\mathsf{P\text{-}produce}$ here. Supposedly, $\mathsf{P\text{-}produce}$ should be a predicate such that $\mathsf{P\text{-}produce}(s)(x)$ is true if and only if there is an event of Machine P producing x in $\pi_{\mathrm{w}}(s)$ and its running time is $\pi_{\mathrm{i}}(s)$. Now, suppose that Machine P was running during time interval I_7, which was 10 seconds long, yesterday. Let x_{44} be the individual which was the XYZ that Machine P produced during I_7 in total. One might then expect $\mathsf{P\text{-}produce}(\langle @, I_7\rangle)(x_{44})$ to hold true. This naïve expectation turns out to be problematic, however. Assume that $\mathsf{P\text{-}produce}(\langle @, I_7\rangle)(x_{44})$ holds true. Then, assuming also that $\mathsf{XYZ}(\langle @, I_7\rangle)(x_{44})$ is true, $\mathbf{SDR}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}(x_{44})(\langle @, I_7\rangle)$ holds. As x_{44} is all the XYZ produced by Machine P during I_7, we have

(123) $\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}(\langle @, I_7\rangle) = \bigsqcup_{\mathsf{e}}\{x \mid \mathbf{SDR}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}(x)(\langle @, I_7\rangle)\} = x_{44}$

Let us now think what the value of $\varrho_{\mathsf{liter}}\left(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}\right)(\langle @, I_7\rangle)$ should be. According to the definition (96), $\varrho_{\mathsf{liter}}\left(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}\right)(\langle @, I_7\rangle)$ will be defined only if

(124) $\mathsf{liter}(\langle @, [p,p]\rangle)\left(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}(\langle @, I_7\rangle)\right) = \mathsf{liter}(\langle @, [p,p]\rangle)(x_{44})$

is constant when p ranges within I_7. However, $\mathsf{liter}(\langle @, [p,p]\rangle)(x_{44})$, i.e., the volume of x_{44} in liters at time point p, is not constant obviously. First of all, one should be aware that in the beginning of the 10 seconds of I_7, no part of x_{44} with a non-zero volume exists as XYZ yet, as it has not been produced yet. The whole of x_{44} as XYZ exists only at the end of I_7. Furthermore, no part of x_{44} which is produced as XYZ within the 10 seconds has a constant non-zero volume, because its temperature decreases as it travels down the refrigerated pipe, and as a result, it keeps contracting till the end of I_7. Generalizing this, one can conclude that for any time interval I, as long as I has some positive length, $\varrho_{\mathsf{liter}}(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}})(\langle @, I\rangle)$ is not generally defined. This means, in turn, that the left hand side of the suggested truth conditions in (122) is not generally defined, either.

What the above discussion has revealed is that we should not be considering time intervals with a positive length in analyzing sentences as in (119) and (120). Indeed, what these sentences describe is continuous events, or put differently, (uncountable) collections of momentary events. By contrast, the earlier sentences we considered in the previous sections describe discrete events each of which take some length of time to occur. Therefore, I will refer to the type of event-related readings that describe continuous production or consumption of some mass entity as in (119) and (120) as **continuous event-related readings**, and distinguish these from **discrete event-related readings**, which are the type of event-related readings that describe discrete events as considered earlier.

Given the distinction between discrete and continuous events, in order to analyze (119), which describes a continuous event, its main predicate $\mathsf{P\text{-}produce}$ must be defined so that it is true only of momentary time intervals as follows:

(125) $\mathsf{P\text{-}produce} = \lambda s \in \mathrm{D_s}.\, \lambda x \in \mathrm{D_e}.$ [there is a time point p such that $\pi_{\mathrm{i}}(s) = [p, p]$ and there is an event of Machine P producing x in $\pi_{\mathrm{w}}(s)$ whose running time is $\pi_{\mathrm{i}}(s) = [p, p]$]

Given this, for no time interval I with a positive length, does $\mathsf{P\text{-}produce}(\langle @, I\rangle)(x)$ hold for any individual x. Given a time interval I with a positive length, we have

(126) $\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}(\langle @, I\rangle) = \bigsqcup_{\mathsf{e}}\{x \mid \mathbf{SDR}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}(x)(\langle @, I\rangle)\} = \bigsqcup_{\mathsf{e}}\varnothing = \bot_{\mathsf{e}}$

$\varrho_{\mathsf{liter}}(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}})(\langle @, I\rangle)$ will thus be defined, since for any $p \in I$,

(127) $\mathsf{liter}(\langle @, [p, p]\rangle)(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}(\langle @, I\rangle)) = \mathsf{liter}(\langle @, [p, p]\rangle)(\bot_e) = 0$

We thus have $\varrho_{\mathsf{liter}}(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}})(\langle @, I\rangle) = 0$. Now, let α and β be index sets such that $\alpha \cup \beta = \gamma$ and $\alpha \cap \beta = \varnothing$, and $\{s_i\}_{i \in \alpha}$ is the set of all situations whose time interval is a moment, i.e., of the form $[p, p]$ for some time point p, and $\{s_i\}_{i \in \beta}$ is the set of all situations whose time interval is not a moment but has some positive length. What I have shown just above is that for any $i \in \beta$, $\varrho_{\mathsf{liter}}(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}})(s_i) = 0$. Then,

(128) $$\sum_{i\in\gamma} \varrho_{\mathsf{liter}}\left(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}\right)(s_i) = \sum_{i\in\alpha\cup\beta} \varrho_{\mathsf{liter}}\left(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}\right)(s_i)$$
$$= \sum_{i\in\alpha} \varrho_{\mathsf{liter}}\left(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}\right)(s_i) + \sum_{i\in\beta} \varrho_{\mathsf{liter}}\left(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}\right)(s_i)$$
$$= \sum_{i\in\alpha} \varrho_{\mathsf{liter}}\left(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}\right)(s_i) + \sum_{i\in\beta} 0$$
$$= \sum_{i\in\alpha} \varrho_{\mathsf{liter}}\left(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}\right)(s_i)$$

Thus, (122) becomes equivalent to the following:

(129) $$\sum_{i\in\alpha} \varrho_{\mathsf{liter}}\left(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}\right)(s_i) = 2.7$$

For $i \in \alpha$, what is the value of $\varrho_{\mathsf{liter}}\left(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}\right)(s_i)$, then? If $i \in \alpha$, then with some time point p in yesterday, s_i is written as $s_i = \langle @, [p, p] \rangle$. Suppose that Machine P was in fact running and producing XYZ at p. Then, $\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}(s_i) \neq \bot_{\mathsf{e}}$. So, let $x_{107} = \mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}(s_i) \neq \bot_{\mathsf{e}}$. Because s_i is momentary, $\varrho_{\mathsf{liter}}\left(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}\right)(s_i)$ is obviously defined, and its value is given by $\mathsf{liter}(\langle @, [p, p] \rangle)(x_{107})$. However, $\mathsf{liter}(\langle @, [p, p] \rangle)(x_{107})$ is necessarily 0, because if $\mathsf{liter}(\langle @, [p, p] \rangle)(x_{107}) > 0$, then it means that Machine P can produce some positive amount of XYZ literally in no time. This is obviously not correct, and furthermore, if Machine P can do this at any moment, since there are uncountably many moments in yesterday, the summation in (129) will diverge to the infinity. This means that $\varrho_{\mathsf{liter}}\left(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}\right)(s_i) = 0$ for any $i \in \alpha$, and hence we obtain

(130) $$\sum_{i\in\alpha} \varrho_{\mathsf{liter}}\left(\mathbf{SDIC}^{\mathsf{XYZ}}_{\mathsf{P\text{-}produce}}\right)(s_i) = \sum_{i\in\alpha} 0 = 0 \neq 2.7$$

It should then be concluded that (119) is necessarily wrong. This is clearly not the desired concludtion. One can then see that in general, the theory developed so far faces difficulty in dealing with continuous event-related readings.

2.4.2 Prelude to integration: Eating of volume-changing bread

What, then, should the correct truth conditions of (119) look like? Suppose that $f(p)$ gives the rate of the volume of the XYZ that Machine P produces per unit time at time point p, where the volume in question must be measured at the very same time point p. Then, the relevant volume of the XYZ we seek to evaluate shall be given by the definite integral of f from the beginning to the end of yesterday (i.e., from 12 a.m. of yesterday to 12 a.m. of today). Letting a and b be the beginning and the end of yesterday respectively, the truth conditions of (119) should then be given by the following:

(131) $$\int_a^b f\,dp = 2.7$$

It should now be clear that Krifka's analysis or its variants is insufficient for continuous event-related readings. First of all, Krifka's original analysis cannot deal with rates of any sort, since it does not talk about times to begin with. This inadequacy has been fixed in the previous sections, but a problem has still remained. That is, in spite of the fact that integrals are defined by taking limits of sums (c.f. (70)), Krifka's approach is simply based on summation without taking limits.

Let us now see yet another, more concrete example, with which I demonstrate how an integral can calculate a relevant volume for a continuous event-related reading, albeit by means of Riemann integration. The scenario considered here will be used in Subsection 2.6.2 again so as to demonstrate how the Lebesgue integral approach proposed there may be applied to a concrete example. In the earlier examples, what expands or shrinks is some liquid, and hence the reader might simply think that in such cases, it is actually not the case that the same individual that is XYZ is expanding or shrinking in volume, but rather, new individuals that are XYZ come into being when the liquid appears to expand, and some individuals that are XYZ disappear from the world when the liquid appears to shrink. I will therefore consider consumption of a solid entity.

Suppose that there is a piece of magic bread. Because of a magic cast on it, this piece of bread continually keeps on expanding and shrinking. It has the shape of a long cylinder with a fixed length of l cm. Its cross section is thus a circle, and it is its radius that keeps changing. Assume that for a given time point p, the radius is given by $r(p)$ cm. Then, the volume of the whole bread piece is given by $\pi\{r(p)\}^2\, l\, \text{cm}^3$, which is a function of p. If one is to give a name to this piece of bread, say Bill, one can see that this scenario is parallel to the one where Ann's body volume fluctuates over time as she gets fatter and thinner. Thus, even though the piece of bread gets bigger and smaller, its identity as Bill shall remain the same. The same goes for any parts of the piece of bread. If one is to focus on some proper part of the whole piece, one can give it a name, say Clinton. Then, Clinton shall be keeping on expanding and shrinking as well, but that does not alter the identity of Clinton.

Now, imagine that John begins eating this piece of bread at time point p_0, starting from one end of the cylindrical shape. After a while, he reaches the other end and finishes eating the whole piece of bread. Then, how many cm^3 of bread has John eaten after all, should we say? Or rather, if one hears a sentence like the following out of the blue, how can one know whether or not the sentence is true under this scenario?

(132) John ate (exactly) 666 cubic centimeters of bread.

$\pi\{r(p_0)\}^2\, l$ cm^3 is the volume of the whole piece measured just when John's meal is about to commence, but this is not the volume in question we are after. Clearly, the size of the piece never matters while it is expanding and shrinking outside John's mouth; it only becomes important how big it is the moment it enters the mouth. Hence, we must look at every infinitesimal time span during John's meal, and measure how big the piece John eats during each infinitesimal time span is at that very moment, and collect those amounts.

Let us see how the volume in question can be calculated by means of Riemann integration. First, assume that the position of the bread is fixed, so only John's mouth moves forward to continue the meal, and its speed at time point p is given by $v(p)$ cm/s. Let p_1 be the time point at which John finishes eating the whole bread. p_1 is then the time point that satisfies the following:

(133) $$l = \int_{p_0}^{p_1} v(p)\, dp$$

Now, given a time point p such that $p_0 < p < p_1$, consider a tiny time span $[p, p + \Delta p]$. The distance that John's mouth travels during this time span is given by $v(p)\Delta p$ cm. As a consequence, the part of the bread that enters John's mouth in this time span has the shape of a cylinder with a length of $v(p)\Delta p$ cm. Since the radius of this cylinder at this moment is given by $r(p)$ cm, the volume of the part of the bread that John eats in $[p, p + \Delta p]$ is given by $\pi\{r(p)\}^2\, v(p)\Delta p$ cm^3. Hence, the rate of the volume of the bread that John eats per unit time at p is obtained by taking the quotient of said volume by Δp s, which is $\pi\{r(p)\}^2\, v(p)$ cm^3/s.[12] Then, the sought volume V of the bread that John has eaten in the end can be calculated as the definite integral of $\pi\{r(p)\}^2\, v(p)$ from p_0 to p_1:

(134) $$V = \int_{p_0}^{p_1} \pi\{r(p)\}^2\, v(p)\, dp$$

Notice that the calculation has succeeded because the relevant rate of a size per unit time has been provided. By contrast, Krifka's approach only looks at sizes *for* different times (or events, originially), but just as seen in the discusstion of the earlier examples,

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[12] In the present discussion, I have assumed that $r(p)$ and $v(p)$ are continuous as functions of p. This is a natural assumption, given the physical truth that an object may expand or shrink only gradually and that the speed of an object cannot just leap in no time.

the size for each infinitesimal time is always 0 (the volume of bread that John can eat in an instance is of course 0). Therefore, collecting this amount does not give rise to any positive value. This is why it is necessary to employ integration in order to capture the truth conditions of sentences like (133).

The integration used in (134) is Riemann integration, but in the next section, I introduce a more general sort of integration called Lebesgue integration, which enables us to treat discrete and continuous event-related readings in a unified manner. As briefly mentioned in Section 2.2, once the theory of Lebesgue integration is introduced, the sum operator is replaced by an integral. It is thus expected that in the end, the correct truth conditions of (132) are given be the following:

(135) $\int_A \varrho_{\text{cubic-centimeter}}(\mathbf{SDIC}^{\text{bread}}_{\text{John-eat}})\, d\mu = 666$

Here, $A = \{s_i\}_{i \in \alpha}$. However, we have already seen that this integrand is equal to 0 for all situations, and hence the above truth conditions do not work. We would thus like to make $\varrho_{\text{cubic-centimeter}}(\mathbf{SDIC}^{\text{bread}}_{\text{John-eat}})$ yield the relevant rate for situations that are momentary. Thus, we would like to have the following:

(136) $\varrho_{\text{cubic-centimeter}}(\mathbf{SDIC}^{\text{bread}}_{\text{John-eat}})(\langle @, [p, p] \rangle) = \pi \{r(p)\}^2 v(p)$

In Section 2.6, ϱ is redefined for momentary situations so that for a situation-sensitive lattice-measure function M and an individual concept ξ, $\varrho_M(\xi)$ defines an appropriate rate for situations whose time interval is a moment.

2.5 Measure theory and Lebesgue integration

This section is a very cursory introduction of measure theory and Lebesgue integration. I cannot give a detailed and rigorous discussion of this subject here, as the goal of this section is to provide tools for the analysis of event-related readings and temporal expressions and provide the insight behind the analysis. The reader is referred to such textbooks as Halmos (1974), Rudin (1987) and Wheeden and Zygmund (1977) for full and detailed exposition of the subject.

2.5.1 Limitations of the Riemann integral

Integration is a method of finding the "area" under the graph of a given function. Given a non-negative continuous function $f(x)$ from reals into real defined on the closed interval

$[a, b]$, the area between the graph of $f(x)$ and the x axis is given by its Riemann integral $\int_a^b f\,dx$. Riemann integration is known to have some limitations, which can in fact be overcome by Lebesgue integration. Therefore, let us briefly see those limitations of Riemann integration here.

To begin with, let us review the definition of the Riemann integral. Let f be a function defined on the real line $\mathbb{R}$ with bounded, nonnegative values. We want to find the area under the graph of f over a closed interval $[a, b]$. Think of a partition of $[a, b]$ into n subintervals, and let $x_0, x_1, x_2, \ldots, x_n$ be points in $[a, b]$ such that $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$. For $1 \leq i \leq n$, let

$$(137) \quad M_i = \sup_{x_{i-1} \leq x \leq x_i} f(x), \quad m_i = \inf_{x_{i-1} \leq x \leq x_i} f(x).$$

Then, for each interval $[x_{i-1}, x_i]$, think of a rectangle whose base is $[x_{i-1}, x_i]$ with a height of M_i. Then, it is the smallest rectangle that contains the graph of $f(x)$ over $[x_{i-1}, x_i]$. Similarly, if we think think of a rectangle whose base is $[x_{i-1}, x_i]$ with a height of m_i for each interval $[x_{i-1}, x_i]$, this is the biggest rectangle that is under the graph of $f(x)$ over that interval. Now let

$$(138) \quad S_n = \sum_{i=1}^{n} M_i(x_i - x_{i-1}), \quad s_n = \sum_{i=1}^{n} m_i(x_i - x_{i-1}).$$

Then, one can see that if the graph of $f(x)$ over $[a, b]$ has an area, it is sandwiched by s_n and S_n. Let us assume that the graph of $f(x)$ over $[a, b]$ has an area of S. Then, $s_n \leq S \leq S_n$. Now, if we think of finer and finer partitions of $[a, b]$ so that $\lim_{n\to\infty} \max_{1\leq i\leq n}(x_i - x_{i-1}) = 0$, the limits of s_n and S_n are expected to be equal to S. If these two have a single limit, the Riemann integral of $f(x)$ over $[a, b]$ is defined to be that unique value. Thus,

$$(139) \quad \int_a^b f\,dx = \lim_{n\to\infty} S_n = \lim_{n\to\infty} s_n.$$

The Riemann integral is capable of determining the area under the graph of many "normal" functions. However, it turns out to be unable to deal with a greater variety of functions. A representative example is the Dirichlet function $\chi_{\mathbb{Q}}$, which is defined as the characteristic function of rationals:

$$(140) \quad \chi_{\mathbb{Q}}(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$$

The Dirichlet function is peculiar in the sense that it is nowhere continuous. The question is whether we can find the area under the graph of $\chi_{\mathbb{Q}}$ over $[a, b]$. Since the rational numbers are dense on the real line, no matter how small a subinterval $I = [\alpha, \beta], \alpha < \beta$ of $[a, b]$ is taken, it contains many rationals, and thus $\sup_{x \in I} \chi_{\mathbb{Q}}(x) = 1$. Similarly, for any subinterval $I = [\alpha, \beta], \alpha < \beta$ of $[a, b]$, it contains irrationals and thus $\inf_{x \in I} \chi_{\mathbb{Q}}(x) = 0$. Therefore, if we consider the corresponding S_n and s_n defined for $\chi_{\mathbb{Q}}$, we see that for any n,

$$(141) \quad S_n = \sum_{i=1}^{n} M_i(x_i - x_{i-1}) = \sum_{i=1}^{n} 1(x_i - x_{i-1}) = \sum_{i=1}^{n} (x_i - x_{i-1}) = x_n - x_0 = b - a,$$

and

$$(142) \quad s_n = \sum_{i=1}^{n} m_i(x_i - x_{i-1}) = \sum_{i=1}^{n} 0(x_i - x_{i-1}) = \sum_{i=1}^{n} 0 = 0.$$

Thus, $\lim_{n \to \infty} S_n = b - a$ and $\lim_{n \to \infty} s_n = 0$ and the Riemann integral of $\chi_{\mathbb{Q}}$ over $[0, 1]$ is not defined.

Another limitation of the Riemann integral lies in the fact that it can be defined only for functions whose domain is a subset of the real line. Removing such a restriction enables application of integration to many cases. A prime example is probability theory. When there are only a finite number of possible events to be considered, one may be able to discuss probabilities etc. without resorting to something like integration. However, we sometimes want to consider cases with infinitely many possible outcomes, such as when a die gets thrown infinitely many times. By considering functions whose domains are events, relevant probabilities, etc. can be calculated by means of Lebesgue integration. In the context of the present dissertation, Lebesgue integration is applied to functions whose domains are situations.

2.5.2 Basic ideas

How does Lebesgue integration overcome the above limitations of Riemann integration? When one looks at the definition of the Riemann integral, one can notice that it relies on two factors.

The first is that calculating the Riemann integral begins with partitioning the domain of the integrand function into subintervals by the order intrinsically given to the real numbers. However, if one is to consider functions whose domains are not a subset of the real line such as functions defined on events, the domain of functions are in general not ordered, and hence this strategy is insufficient. When we deal with a set of situations whose world is fixed to one world, because time intervals are pairs of time points, the reader may think that it is possible to define an order to the set of situations by using the beginning and ending time points of time intervals. This is true. However, there is no linguistic motivation to impose such an order to begin with, and even if such an order is provided, we still need to give an appropriate function that measures the size of sets of situation as discussed right below.

The second factor is that how to measure the length of intervals on the real line has been granted in Riemann integration. After having partitioned the domain into subintervals, the area of rectangles that have these subintervals as their base is considered. The heights are given by the supremum and infimum of the function on each interval, but how does one get the length of the bases? In Riemann integration, it has naturally been assumed that the length of an interval of the from $[a, b]$ is given by $b - a$. Since an interval is actually a set of elements on the domain of the integrand function, what is done here is actually assigning values to subsets of the domain. For intervals, the above assumption may be fair and sufficient, but we would now also like to treat functions defined on a set that is not a subset of the reals. Even so, we should somehow still be able to assign appropriate values to subsets of the domain of the integrand function.

Lebesgue integration solves these problems as follows. To begin with, we consider partitions of the domain of the integrand function not by the order given to the domain, but according to the range of the function. Let f be a function defined on a set X with nonnegative values. Given an interval $[\alpha,\beta)$, one can consider the set of all elements in X that is mapped by f to an element in $[\alpha,\beta)$. This is called the preimage of $[\alpha,\beta)$ by f, and written as $f^{-1}\big([\alpha,\beta)\big)$. Thus,

(143) $f^{-1}\big([\alpha,\beta)\big) = \{x \mid \alpha \leq f(x) < \beta\}.$

If we think of a "rectangle" whose base is $f^{-1}\big([\alpha,\beta)\big)$ with a height of α, this is a "rectangle" that is certain to be under the graph of f on $f^{-1}\big([\alpha,\beta)\big)$.[13] When f is defined on reals, this "rectangle" (although disconnected) looks like the following (in this case, the "rectangle" actually consists of three rectanbles).

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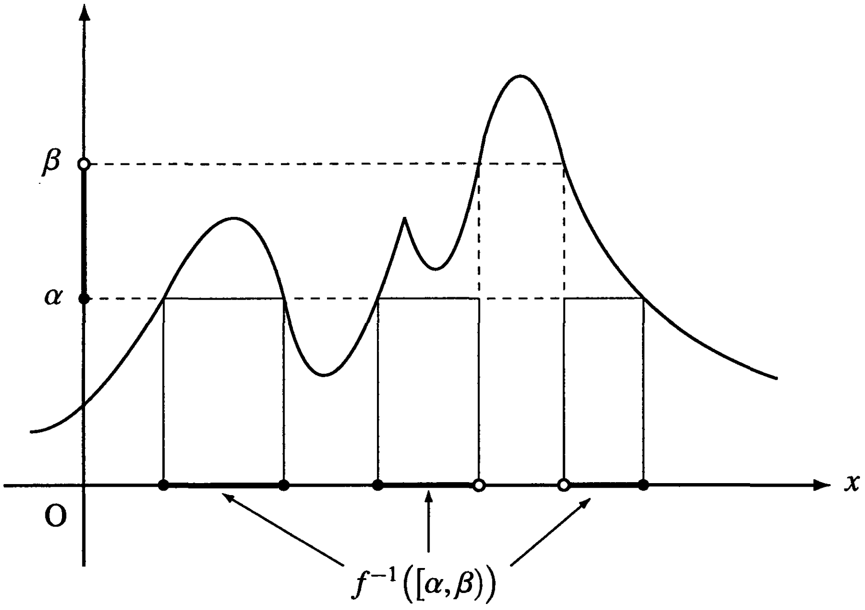
[13] The reason that "rectangle" is in quotation marks is that the domain of the integrand function is in general not on the real line, and therefore it is in general inappropriate to imagine a certain geometric figure. Even when the domain is on the real line, $f^{-1}\big([\alpha,\beta)\big)$ could be a collection of disconnected intervals as in the case in (144).

(144)



Now, if it is possible to measure the size of the set $f^{-1}([\alpha,\beta))$, it will give the length of the base of this rectangle. Let us suppose that $\mu(f^{-1}([\alpha,\beta)))$ gives this size. The "area" of the rectangle is then calculated as $\alpha \cdot \mu(f^{-1}([\alpha,\beta)))$. By partitioning $[0,\infty]$ into mutually disjoint subintervals, we can consider the preimage by f of each interval and the area of the biggest "rectangle" on it under f. Then, by taking the sum of the area of these rectangles, we can have an approximation of the whole "area" under the graph of f. By taking the limit of this sum as the partition gets finer and finer, we can expect to approach the exact "area" under the graph of f. This is essentially the main idea of Lebesgue integration.

In order to be able to find the Lebesgue integral of a function f, it is thus crucial to have some function μ that assigns nonnegative values to subsets of the domain of f, and furthermore, it is important that the preimage by f of any subinterval of the range of f be in the domain of the function μ. Put conversely, if these conditions are met, Lebesgue integraion can enable us to calculate the integral of various functions. In what follows, we will be first introduced to some notions necessary to define the Lebesgue integral.

2.5.3 σ-algebra and measure

For a function f with domain X, we would like to think of a collection of subsets of X on which a function μ called measure may be defined. Such a collection is called a σ-algebra.

(145) Definition.

A collection $\mathfrak{F}$ of subsets of X is called a **σ-algebra** if and only if the following conditions are met:

i. $X \in \mathfrak{F}$.

ii. If $E \in \mathfrak{F}$, $E^{\mathrm{c}} \in \mathfrak{F}$, where E^{c} is the complement of E i.e., $E^{\mathrm{c}} = X - E$.
(closed under complementation)

iii. If $E_k \in \mathfrak{F}$ for all $k \in \mathbb{N}$, then $\bigcup\limits_{k=1}^{\infty} E_k \in \mathfrak{F}$.
(closed under countable unions)

It is immediate that by virtue of de Morgan's laws (viz., $A \cap B = (A^{\mathrm{c}} \cup b^{\mathrm{c}})^{\mathrm{c}}$), σ-algebras are closed under countable intersections as well.

Example.

For any set X, its power set $\wp(X)$ of X is a σ-algebra.

Example.

Let DAY $= \{\mathsf{Mon}, \mathsf{Tue}, \mathsf{Wed}\}$. Then,

$$\mathfrak{W} = \{\varnothing, \{\mathsf{Mon}\}, \{\mathsf{Tue}, \mathsf{Wed}\}, \{\mathsf{Mon}, \mathsf{Tue}, \mathsf{Wed}\}\}$$

is a σ-algebra. Note that $\mathfrak{W} \neq \wp(\mathrm{DAY})$.

(146) Theorem.

Given a collection $\mathscr{A}$ of subsets of X, there is a smallest σ-algebra $\mathfrak{S}$ over X such that $\mathscr{A} \subseteq \mathfrak{S}$.

Proof.

Since $\mathscr{A} \subseteq \wp(X)$, there is at least one σ-algebra containing $\mathscr{A}$ (namely, $\wp(X)$). Now, let $\mathfrak{S}$ be the intersection of all σ-algebras containing $\mathscr{A}$. It is easy to verify that the intersection of σ-algebras is itself a σ-algebra. Therefore, $\mathfrak{S}$ is a σ-algebra, and since any σ-algebra that contains $\mathscr{A}$ contains $\mathfrak{S}$, $\mathfrak{S}$ is the smallest σ-algebra containing $\mathscr{A}$. Q.E.D.

The smallest σ-algebra $\mathfrak{S}$ containing $\mathscr{A}$ is called the **σ-algebra generated by** $\mathscr{A}$.

(147) Definition.

For a topological space X, let $\mathfrak{B}(X)$ denote the σ-algebra generated by the collection of all open sets in X. The elements of $\mathfrak{B}(X)$ are called **Borel sets** of X.

By definition, all open sets belong to $\mathfrak{B}(X)$ and are thus Borel sets. Besides open sets, all closed sets are Borel sets of X as well, since every closed set is the complements of some open set. There are many more Borel sets other than open and closed sets. For example, $[a, b)$ is neither open or closed, but it can be represented as a union of two Borel sets: $[a, b) = [a, a] \cup (a, b)$. Thus, $[a, b)$ is also a Borel set.

(148) Definition.

If $\mathfrak{F}$ is a σ-algebra over X, $(X, \mathfrak{F})$ is called a **measurable space**, and the elements of $\mathfrak{F}$ are called **$\mathfrak{F}$-measurable sets**.

For example, $(\mathrm{DAY}, \mathfrak{W})$ and $(\mathbb{R}^n, \mathfrak{B}(\mathbb{R}^n))$ are measurable spaces. The singleton set $\{\mathsf{Mon}\}$ is a $\mathfrak{W}$-measurable set, but $\{\mathsf{Tue}\}$ is not. The term "measurable" suggests that for a σ-algebra, one can think of a set function (i.e., function whose domain is a collection of sets) defined on the σ-algebra called measure, which assigns nonnegative values to the elements of the σ-algebra.

(149) Definition.

A function μ with the domain $\mathfrak{F}$ is called a **measure** if and only if the following conditions are met:

i. $0 \leq \mu(E) \leq +\infty$ for all $E \in \mathfrak{F}$.

ii. $\mu(\varnothing) = 0$.

iii. If $\{E_k\} \in \mathfrak{F}$ for all $k \in \mathbb{N}$ and $\{E_k\}$ is a pairwise disjoint collection, i.e., for any $i, j \in \mathbb{N}$, if $i \neq j \rightarrow E_i \cap E_j = \varnothing$, then $\mu\left(\bigcup_{k=1}^{\infty} E_k\right) = \sum_{k=1}^{\infty} \mu(E_k)$.

Note that a measure is allowed to take ∞ as its value. The third property is called **countable additivity** or **σ-additivity**.

Example.

The **counting measure**, which I notate as μ_{CT}, is the set function that yields the number of the elements in a given set. Therefore,

$$\mu_{\mathrm{CT}}(\{\mathsf{Mon}\}) = 1$$

$$\mu_{\mathrm{CT}}(\{\mathsf{Mon, Tue, Wed}\}) = 3$$

$$\mu_{\mathrm{CT}}(\varnothing) = 0$$

If the given set is not finite, its value is ∞. Hence,

$$\mu_{\mathrm{CT}}(\{n \mid n \in \mathbb{N}\}) = \infty$$

$$\mu_{\mathrm{CT}}\big([0,1)\big) = \mu_{\mathrm{CT}}(\{x \mid 0 \leq x < 1\}) = \infty$$

Example.
The **Lebesgue measure**, which I notate as μ_{LBG}, assigns a volume (length, area) to subsets of Euclidean space $\mathbb{R}^n$. For a "half open" interval I in $\mathbb{R}^n$ of the following form

$$\begin{aligned} I &= [a_1, b_1) \times [a_2, b_2) \times \cdots \times [a_n, b_n) \\ &= \{(x_1, x_2, \ldots, x_n) \mid a_1 \leq x_1 < b_1 \wedge a_2 \leq x_2 < b_2 \wedge \cdots \wedge a_n \leq x_n < b_n\}, \end{aligned}$$

let us define its "volume", written as $|I|$, as follows:

$$|I| = (b_1 - a_1) \times (b_2 - a_2) \times \cdots \times (b_n - a_n).$$

For any subset E of $\mathbb{R}^n$, define

$$\mu_{\mathrm{LBG}}(E) = \inf \sum_{n=1}^{\infty} |I_n|,$$

where the inf is taken over all coverings of E by a countable collection of half open intervals I_n, i.e., $E \subseteq \bigcup_{n=1}^{\infty} I_n$. It can then be proved that for any half open interval I, $\mu_{\mathrm{LBG}}(I)$ is actually equal to $|I|$. Furthermore, it can be shown that $\mu_{\mathrm{LBG}}(E)$ is indeed a measure on $(\mathbb{R}^n, \mathfrak{B}(\mathbb{R}^n))$ (see the aforementioned textboos for details).

Let us now focus on the 1-dimentional case, that is $\mathfrak{B}(\mathbb{R}^1)$ and the Lebesgue measure defined on it. An important fact is that for any set consisting of only one point $\{a\}$, $\mu_{\mathrm{LBG}}(\{a\}) = 0$. To see this, observe that $\{a\} \subset [a, a + \varepsilon)$ for any positive number ε.

Therefore, $\mu_{\text{LBG}}(\{a\}) \leq \inf\left|[a, a+\varepsilon)\right| = \inf \varepsilon$, and because ε is an arbitrarily small positive number, the right hand side becomes 0.

As mentioned above, $\mu_{\text{LBG}}\big([a, b)\big) = \big|[a, b)\big| = b - a$. Since $[a, b) = \{a\} \cup (a, b)$ and $\mu_{\text{LBG}}(\{a\}) = 0$, one can see that $\mu_{\text{LBG}}\big((a, b)\big) = b - a$. Similarly, we have $\mu_{\text{LBG}}\big((a, b]\big) = \mu_{\text{LBG}}\big([a, b]\big) = b - a$.

With the Lebesgue measure, we can now measure the set of all rationals contained in the interval $[a, b]$, i.e., $\mathbb{Q} \cap [a, b]$. Since the rationals are countable, we can enumerate all the rationals in $[a, b]$ as $r_1, r_2, r_3, \ldots$. Then, $\mathbb{Q} \cap [a, b] = \bigcup_{k=1}^{\infty}\{r_k\}$. It then follows from the countable additivity of the Lebesgue measure that

$$\mu_{\text{LBG}}(\mathbb{Q} \cap [a, b]) = \mu_{\text{LBG}}\left(\bigcup_{k=1}^{\infty}\{r_k\}\right) = \sum_{k=1}^{\infty}\mu_{\text{LBG}}(\{r_k\}) = \sum_{k=1}^{\infty} 0 = 0.$$

Likewise, the Lebesgue measure of any countable set is 0.

(150) Definition.

When μ is a measure on $(X, \mathfrak{F})$, $(X, \mathfrak{F}, \mu)$ is called a **measure space**.

Example.

$(\text{DAY}, \mathfrak{W}, \mu_{\text{CT}})$ is a measure space. This is easy to verify.

Example.

$(\mathbb{R}^n, \mathfrak{B}(\mathbb{R}^n), \mu_{\text{LBG}})$ is a measure space.

2.5.4 Measurable functions and the Lebesgue integral

A function's Lebesgue integral is most straightforwardly defined when the function only takes nonnegative values. Therefore, in what follows, we first consider functions with nonnegative values For the sake of generality, we would not like to confine ourselves to bounded functions, and thus, we allow functions to take the value ∞. We thus think of functions from X into $[0, \infty]$. The topology of $[0, \infty]$ is such that its open sets are $[0, a)$, (a, b), $(a, \infty]$, and their unions.

The functions of which we can think of a Lebesgue integral must be measurable functions, which notion is defined as follows:

(151) Definition.

Let f be a function from X into $[0, \infty]$, and let $\mathfrak{F}$ be a σ-algebra on X. If for every $V \in \mathfrak{B}([0, \infty])$, its pre-image by f is in $\mathfrak{F}$, i.e.,

$$f^{-1}(V) = \{x \mid f(x) \in V\} \in \mathfrak{F},$$

then, f is said to be a **$\mathfrak{F}$-measurable function**.

An important fact is that the preimage is closed under complementation and countable union and intersection. That is, for any set V and any collection of sets $\{V_k\}$,

$$f^{-1}(V^c) = (f^{-1}(V))^c$$

$$f^{-1}\left(\bigcup_{k=1}^{\infty} V_k\right) = \bigcup_{k=1}^{\infty} f^{-1}(V_k)$$

$$f^{-1}\left(\bigcap_{k=1}^{\infty} V_k\right) = \bigcap_{k=1}^{\infty} f^{-1}(V_k)$$

Therefore, if $f^{-1}(V) \in \mathfrak{F}$ holds for every open set V in $[0, \infty]$, since by definition any Borel set of $[0, \infty]$ can be obtained through applications of the above operations to open sets in $[0, \infty]$, the preimage by f of any Borel set of $[0, \infty]$ is in $\mathfrak{F}$, and thus f shall be $\mathfrak{F}$-measurable.

I would like to note a couple of properties of measurable functions without proof.

(152) Theorem.

If f and g are $\mathfrak{F}$-measurable functions on X, then so are $f + g$ and fg.

(153) Theorem.

If $\{f_n\}$ is a sequence of $\mathfrak{F}$-measurable functions on X, then $\sup f_n$, $\inf f_n$, $\limsup_{n \to \infty} f_n$ and $\liminf_{n \to \infty} f_n$ are all $\mathfrak{F}$-measurable.

Here, $\sup f_n(x)$ is a function that returns $\sup\{f_1(x), f_2(x), f_3(x), \dots\}$ for each x. Similarly for $\inf f_n(x)$. $\limsup_{n \to \infty} f_n$ is a function that returns $\lim_{n \to \infty} \sup\{f_n(x), f_{n+1}(x), f_{n+2}(x), \dots\}$ for each x. Similarly, for each x, $(\liminf_{n \to \infty} f_n)(x) = \lim_{n \to \infty} \inf\{f_n(x), f_{n+1}(x), f_{n+2}(x), \dots\}$. Notice that if f is a function that is obtained by the limit of a sequence $\{f_n\}$ of $\mathfrak{F}$-measurable functions, then since $f = \lim_{n \to \infty} f_n = \limsup_{n \to \infty} f_n = \liminf_{n \to \infty} f_n$, f is $\mathfrak{F}$-measurable by the above theorem.

For a given measurable function f, if one can find a suitable sequence of measurable functions that converges to f such that the "area" under the graph of each of the functions in the sequence can be calculated, one can expect that the "area" under the graph of f may be obtained as the limit of these areas. To that end, we consider functions that are called simple, which are constructed with characteristic functions.

(154) Definition.

For any subset E of X, the **characteristic function** of E, written as χ_E, is a function from X into $\{0, 1\}$ such that

$$\chi_E(x) = \begin{cases} 1 & \text{if } x \in E \\ 0 & \text{if } x \notin E \end{cases}$$

Given a measure space $(X, \mathfrak{F}, \mu)$, it is obvious that the characteritic function of E is a $\mathfrak{F}$-measurable function if and only if E is a $\mathfrak{F}$-measurable set. With a characteristic function, the graph of the "rectangle" in (144), which has $f^{-1}([\alpha, \beta))$ as its base and whose height is α, can be represented as $\alpha \chi_{f^{-1}([\alpha,\beta))}$. Assuming that f is $\mathfrak{F}$-measurable, $f^{-1}([\alpha, \beta)) \in \mathfrak{F}$ since $[\alpha, \beta) \in \mathfrak{B}([0, \infty])$, and thus $f^{-1}([\alpha, \beta))$ is in the domain of μ. Interpreting that $\mu(E)$ gives the "length" of any subset E of X, the "area" of this "rectangle" shall be $\alpha \mu(f^{-1}([\alpha, \beta)))$.

(155) Definition.

A function s from X into $[0, \infty)$ is called a **simple function** if and only if for a finite number of disjoint disjoint measurable subets $E_1, E_2, \ldots, E_n$ of X, and a finite number of distinct nonnegative values $\alpha_1, \alpha_2, \ldots, \alpha_n$,

$$s(x) = \sum_{k=1}^{n} \alpha_k \chi_{E_k}(x).$$

That is, $s(x) = \alpha_k$ when $x \in E_k$ for each k, and $s(x) = 0$ when $x \notin \bigcup_{k=1}^{n} E_k$.

Note that ∞ is excluded as a value of simple functions. Since each of the characteristic functions used to define a simple function is measurable, one can easily see that any simple function is measurable. The "area" of the graph under a simple function is the sum of the "area" of the "rectangles" each of which has E_k as its base and a height of α_k for some k.

Thus, for simple function $s(x) = \sum_{k=1}^{n} \alpha_k \chi_{E_k}(x)$, its area is given by $\sum_{k=1}^{n} \alpha_k \mu(E_k)$. Based on this, the definition of the Lebesgue integral of simple functions is given as follows:

(156) Definition.

Given a simple function $s(x) = \sum_{k=1}^{n} \alpha_k \chi_{E_k}(x)$, for any $E \in \mathfrak{F}$, the Lebesgue integral of s over E is defined by

$$\int_E s(x)\, d\mu = \sum_{k=1}^{n} \alpha_k \mu(E_k \cap E).$$

The Lebesgue integral of arbitrary nonnegative functions is defined as follows:

(157) Definition.

Let f be a measurable function from X into $[0, \infty]$. Then, for any $E \in \mathfrak{F}$, the Lebesgue integral of f over E is defined by

$$\int_E f(x)\, d\mu = \sup \int_E s(x)\, d\mu,$$

where the supremum is taken over all simple functions s such that $0 \leq s \leq f$.

The reader might wonder how such a supremum may be obtained. It is known that for a given measurable function f, one can find a sequence of monotone increasing sequence of simple functions, and that the sequence constructed by the Lebesgue integral of each simple function in the sequence then converges to this supremum. Roughly speaking, this can be done by partitioning the range of f and thinking of the simple function whose graph consists of the biggest rectangles under that graph of f whose bases are given by the preimage by f of the partitioned intervals, and then making the partition finer and finer.

We would now like to talk about functions that can take negative values as well.

(158) Definition.

Given a measure space $(X, \mathfrak{F}, \mu)$, for any $\mathfrak{F}$-measurable function f from X into $[-\infty, \infty]$, the **norm** of f, written as $||f||$, is defined as follows:

$$||f|| = \int_X |f|\, d\mu$$

Note that the notion of measurable function here is based on the preimage of Borel sets of $[-\infty, \infty]$.

(159) Definition.

For a measure space $(X, \mathfrak{F}, \mu)$, the space $L^1(X, \mathfrak{F}, \mu)$ is the collection of functions f such that $\|f\| < \infty$. If $f \in L^1(X, \mathfrak{F}, \mu)$, then f is called an **integrable function**.

For a function f from X into $[-\infty, \infty]$, let

(160) $f^+(x) = \max\{f(x), 0\}$ and let $f^-(x) = \min\{-f(x), 0\}$.

f^+ and f^- can be understood as the nonnegative part of f and the mirror image of the negative par of f, respectively. Then, f^+ and f^- are both nonnegative measurable functions and

(161) $f = f^+ - f^-, \quad |f| = f^+ + f^-.$

Therefore, if $f \in L^1(X, \mathfrak{F}, \mu)$,

(162) $\|f\| = \int_X |f| \, d\mu = \int_X f^+ + f^- \, d\mu = \int_X f^+ \, d\mu + \int_X f^- \, d\mu < \infty,$

and thus, both $\int_X f^+ \, d\mu$ and $\int_X f^- \, d\mu$ are finite. For any subset E of X that belongs to $\mathfrak{F}$, clearly,

(163) $\int_E f^+ \, d\mu \leq \int_X f^+ \, d\mu < \infty, \quad \int_E f^- \, d\mu \leq \int_X f^- \, d\mu < \infty.$

The Lebesgue integral of f over E is then defined to be the difference of these two.

(164) Definition.

For a $f \in L^1(X, \mathfrak{F}, \mu)$, the Lebesgue integral of f over $E \in \mathfrak{F}$ is defined as follows:

$$\int_E f \, d\mu = \int_E f^+ \, d\mu - \int_E f^- \, d\mu$$

Let me comment briefly on the relation between the Lebesgue integral and the Riemann integral. It is known that if a bounded function defined on a closed interval is Riemann-integrable, it is also Lebesgue-integrable, and furthermore, its Riemann-integral is equal to its Lebesgue-integral with respect to the Lebesgue measure. Moreover, some functions that are not Riemann-integrable may be Lebesgue integrable. The Dirichlet function $\chi_{\mathbb{Q}}$ is such an example. The integral of the Dirichlet function over $[a, b]$ can be calculated by Lebesgue integration with respect to the Lebesgue measure. Since $\chi_{\mathbb{Q}}$ is a characteristic function,

(165) $\int_{[a,b]} \chi_{\mathbb{Q}} \, d\mu_{\mathrm{LBG}} = \mu_{\mathrm{LBG}}(\mathbb{Q} \cap [a, b]).$

In the explanation of the Lebesgue measure in the previous subsection, we have seen that $\mu_{\text{LBG}}(\mathbb{Q} \cap [a,b]) = 0$. Hence,

(166) $$\int_{[a,b]} \chi_{\mathbb{Q}}\, d\mu_{\text{LBG}} = 0.$$

The reader may see, then, that Lebesgue integration is more general and useful than Riemann integration.

We should now familiarize ourselves with the concept of "almost everywhere". For that, let us first see the following theorem.

(167) Theorem.
$\|f\| = 0$ if and only if $f(x) = 0$ except on a set of measure 0.

Proof.
(⇒): For any $\alpha > 0$, the "area" of the rectangle whose base is $\{x \mid |f(x)| > \alpha\}$ with a height of α is fit under the graph of $|f|$. Since the "area" under the graph of $|f|$ over $\{x \mid |f(x)| > \alpha\}$ is given by its integral over $\{x \mid |f(x)| > \alpha\}$,

$$\alpha\mu(\{x \mid |f(x)| > \alpha\}) \leq \int_{\{x \mid |f(x)| > \alpha\}} |f|\, d\mu \leq \int_X |f|\, d\mu = \|f\| = 0.$$

Then, since the set of all x such that $f(x) \neq 0$ is given by $\bigcup_{k=1}^{\infty}\left\{x \,\middle|\, |f(x)| > \frac{1}{k}\right\}$, which is a countable union of sets of measure 0, its measure is 0.
(⇐): Let $A = \{x \mid |f(x)| > 0\}$. A is the set of all x such that $f(x) \neq 0$. We have to show that $\mu(A) = 0$ entails $\|f\| = 0$. Since $|f(x)| = 0$ when $x \in X - A$,

$$\|f\| = \int_X |f|\, d\mu = \int_{X-A} |f|\, d\mu + \int_A |f|\, d\mu = \int_{X-A} 0\, d\mu + \int_A |f|\, d\mu = \int_A |f|\, d\mu$$

Because $\mu(A) = 0$, for any simple function s such that $0 \leq s < |f|$, $\int_A s\, d\mu = 0$. By the definition of the Lebesgue integral, $\int_A |f|\, d\mu$ is the supremum of $\int_A s\, d\mu$ where $0 \leq s < |f|$. Hence $\|f\| = 0$. Q.E.D.

When some property holds on the entire domain except on some subset of it that measures 0, we say that the property holds **almost everywhere**. When we want to make the relevant measure μ explicit, we write almost everywhere $[\mu]$. "Almost everywhere" is often abbreviated as **a.e.** In this way of talking, the above theorem can be stated as follows: $\|f\| = 0$ if and only if $f(x) = 0$ a.e.

Now, let f and g be functions in $L^1(X, \mathfrak{F}, \mu)$ such that $f = g$ a.e. Then, since $f - g = 0$ a.e., it follows from the above theorem that $\|f - g\| = 0$. Then,

$$(168) \quad \left| \int_X f \, d\mu - \int_X g \, d\mu \right| = \left| \int_X f - g \, d\mu \right| \leq \int_X |f - g| \, d\mu = \|f - g\| = 0,$$

and thus, $\int_X f \, d\mu = \int_X g \, d\mu$. Therefore, Lebegue integration absorbs any differences in the value of integrands as long as the differences are seen only on a set of measure 0. Conversely, even if the integrals of two functions f and g coincide over every set, that only tells that $f = g$ a.e. and thus f and g may be taking different values on a set of measure 0.

2.5.5 Differentiation

Suppose that f is a function from $\mathbb{R}^n$ into $\mathbb{R}$ and that $f \in L^1(\mathbb{R}^n, \mathfrak{B}(\mathbb{R}^n), \mu_{\mathrm{LBG}})$, where μ_{LBG} is the Lebesgue measure for $\mathbb{R}^n$. Let F be the indefinite integral F of f:

$$(169) \quad F(E) = \int_E f \, d\mu_{\mathrm{LBG}}$$

Let $C(x, h)$ denote a cube centered at x whose edges are parallel to the coordinate axes and have a length of $2h$. If the quotient of $F(C(x, h))$ by the volume of the cube $C(x, h)$,

$$(170) \quad \frac{1}{\mu_{\mathrm{LBG}}(C(x, h))} F(C(x, h))$$

has a unique limit as the cube shrinks towards x, that is, as h tends to 0, we call the limit the derivative of F at x and write it as $DF(x)$:

$$(171) \quad DF(x) = \lim_{h \to 0} \frac{1}{\mu_{\mathrm{LBG}}(C(x, h))} F(C(x, h))$$

Lebesgue's differentiation theorem is the generalized version of the fundamental theorem of calculus in Lebesgue theory.

(172) Lebesgue's differentiation theorem
If $f \in L^1(\mathbb{R}^n, \mathfrak{B}(\mathbb{R}^n), \mu_{\mathrm{LBG}})$, its indefinite integral is differentialbe with derivative $f(x)$ a.e. $[\mu_{\mathrm{LBG}}]$. That is, for almost every $x \in \mathbb{R}$,

$$\lim_{h \to 0} \frac{1}{\mu_{\mathrm{LBG}}(C(x, h))} \int_{C(x, h)} f \, d\mu_{\mathrm{LBG}} = f(x).$$

(173) Definition

If $f \in L^1(\mathbb{R}^n, \mathfrak{B}(\mathbb{R}^n), \mu_{\text{LBG}})$, any point x at which the following holds is called a **Lebesgue point** of f:

$$\lim_{h \to 0} \frac{1}{\mu_{\text{LBG}}(C(x,h))} \int_{C(x,h)} \lambda x'. \left| f(x') - f(x) \right| d\mu_{\text{LBG}} = 0$$

For example, every continuous point of f is a Lebesgue point of f. It is known that the equality in Lebesgue's differentiation theorem holds at every Lebesgue point of f, and thus almost every $x \in \mathbb{R}$ is a Lebesgue point of f.

2.5.6 The Radon-Nikodým theorem

The Radon-Nikodým theorem is a fundamental, important result of measure theory, and this theorem will be made use of in the next section when continuous event-related are analyzed.

Let $(X, \mathfrak{F}, \mu)$ be a measure space. Given a nonnegative-valued function f defined on X, let $\Phi(E)$ be the integral over $E \in \mathfrak{F}$.

(174) $\Phi(E) = \int_E f \, d\mu$

It is easy to see that Φ is a measure on $(X, \mathfrak{F})$ by confirming that it satisfies the conditions of measure as follows. First, since $f \geq 0$, for any $E \in \mathfrak{F}$, $\Phi(E) = \int_E f \, d\mu \geq \int_E 0 \, d\mu = 0$. Second, $\Phi(\varnothing) = \int_{\varnothing} f \, d\mu = 0$. Finally, if $\{E_k\}$ is a mutually disjoint collection of elements in $\mathfrak{F}$, then, $\Phi\left(\bigcup_{k=1}^{\infty} E_k\right) = \int_{\bigcup_{k=1}^{\infty} E_k} f \, d\mu = \sum_{k=1}^{\infty} \int_{E_k} f \, d\mu = \sum_{k=1}^{\infty} \Phi(E_k)$. A natural question that arises is this: when a measure Φ on $(X, \mathfrak{F})$ is given first, can Φ be represented as the integral of some function on X? The Radon-Nikodým theorem answers this question positively, provided that Φ satisfies certain properties, namely, σ-finiteness and absolute continuity.

(175) Definition

Let μ be a measure defined on a σ-algebra $\mathfrak{F}$. A set $E \in \mathfrak{F}$ is said to have a **σ-finite** measure if and only if E is a union of a countable collection of sets $\{E_k\}_{k \in \mathbb{N}}$ such that $\mu(E_k)$ is finite for all $k \in \mathbb{N}$.

Example.

The Lebesgue measure μ_{LBG} is a σ-finite measure on $\mathfrak{B}(\mathbb{R})$, since $\mathbb{R}$ can be represented as a countable union of intervals of the from $[Z, Z+1)$ with Z being a whole number, each of which is of measure 0 with respect to μ_{LBG}.

Example.

The counting measure μ_{CT} is not a σ-finite measure on $\mathfrak{B}(\mathbb{R})$. To see this, assume to the contrary that there is a countable collection of sebsets $\{E_k\}_{k \in \mathbb{N}}$ of $\mathbb{R}$ such that $\bigcup_{k=1}^{\infty} E_k = \mathbb{R}$ and $\mu_{\mathrm{CT}}(E_k)$ is finite for each k. Since $\mu_{\mathrm{CT}}(E_k)$ is finite, E_k has only a finite number of elements. Then, being a countable union of finite sets, $\bigcup_{k=1}^{\infty} E_k$ must be countable. However, since $\bigcup_{k=1}^{\infty} E_k = \mathbb{R}$ and $\mathbb{R}$ is uncountable, a contradiction obtains.

(176) Definition

Let μ and ν be measures on $(X, \mathfrak{F})$. ν is a **absolutely continuous** with respect to μ if and only if for all $E \in \mathfrak{F}$, $\mu(E) = 0 \rightarrow \nu(E) = 0$. In this case, we write $\nu \ll \mu$.

It is easy to see that the function Φ in (174) which is given the integral of f is absolutely continuous with respect to μ. Now, the Radon-Nikodým theorem is given below:

(177) The Radon-Nikodým theorem

Let μ be a σ-finite measure on a measurable space $(X, \mathfrak{F})$, and let ν be a σ-finite measure on $(X, \mathfrak{F})$ and $\nu \ll \mu$. Then, there is a function f from X into $[0, \infty)$ such that for any $E \in \mathfrak{F}$,

$$\nu(E) = \int_E f \, d\mu$$

For the proof, the reader is referred to the aforementioned textbooks.

Finally, I would like to end this section by introducing the concept of singular measure.

(178) Definition

Two measures μ_0 and μ_1 on a measurable space $(X, \mathfrak{F})$ are **mutually singular** if and only if there are disjoint sets E_0 and E_1 in $\mathfrak{F}$ such that $E_0 \cup E_1 = X$ and $\mu_0(E_1) = \mu_1(E_0) = 0$. In this case, we write $\mu_0 \perp \mu_1$.

2.6 Application of Lebesgue integration to measure phrases

In Sections 2.2 and 2.3, we analyzed discrete event-reatled readings with truth conditions involving the summation of the following form:

(179) $$\sum_{i \in \gamma} \varrho_M(\mathbf{SDIC}_{\psi}^{\varphi})(s_i)$$

In Section 2.4, I showed that this does not work for continuous event-realted readings, and that an integral method is necessitated. In this section, I will thus analyze continuous event-realted readings with integration, but I will also adopt this approach to discrete event-reatled readings. There are a couple of reasons for this move. First, we must realize that index set γ in (179) is not something prescribed by the sentence. What the earlier analysis says is merely that when A is the set of all atomic situation contained the sentence's context (like yesterday), if there is an index set γ such that $A = \{s_i\}_{i \in \gamma}$ and $s_i \neq s_j \rightarrow i \neq j$, then the sentence can be analyzed with (179). At the end of the day, however, the context time interval (such as yesterday) that generates A must be abstracted over, especially because the abstracted context time variable can be quantified over as observed by Pratt and Francez (2001). Therefore, γ cannot be predetermined before the context time variable is provided, and thus the precise formulation of the analysis must have γ existentially bound locally. The correct analysis of discrete event-related readings should then in general look like the following:

(180) $$\lambda I.\, \exists \gamma \Bigg[\{s_i\}_{i \in \gamma} \text{ is the set of all the situations within the context time interval } I \\ \wedge [s_i \neq s_j \rightarrow i \neq j] \wedge \sum_{i \in \gamma} \varrho_M(\mathbf{SDIC}_{\psi}^{\varphi})(s_i) \geq 4000 \Bigg]$$

Despite the fact that the value of the sum of (179) is not affected by the choice of γ, in this formulation, the value of the sum appears to depend on γ. Furthermore, γ is not rendered any meaningful linguistic interpretation. Thus, (180) is a very awkward way of formulating the analysis and is hence undesired. When integration is employed, however, this undesired variable γ is eliminated from our truth conditions. The second reason is that by analyzing both discrete and continuous event-related readings with integration, we achieve a unified view of event-related readings. By virtue of Lebesgue integration and measure theory on which it is built, the analysis provides an insightful view to our understanding of non-presuppositinoal noun phrases and temporal expressions.

2.6.1 Discrete (telic) events

Let us begin our analysis with discrete event-related readings. What we would like to do is to translate the summation of the form (179) into a Lebesgue integral. The function to be integrated here is $\varrho_M(\mathbf{SDIC}^{\varphi}_{\psi})$, a function from situations into (non-negative) reals. Therefore, we have to consider the measure space $(A, \mathfrak{F}, \mu)$, where μ is an appropriate measure of situations. What becomes crucial is what kind of measure μ should be. In the present discussion, the world of situations is always fixed (to the actual world @). and as a consequence, what we need to consider in effect is to provide an appropriate measure to time intervals. This means that in order to develop a correct analysis of discrete event-related readings, we must first develop a correct theory of the temporal interpretation of the main predicate that describes discrete events.

Let us thus start with the following simple sentence that describes discrete events but has no event-related-reading-inducing noun phrase:

(181) John jumped exactly three times last week.

Suppose that John actually jumped exactly three times during the time period of the last week, and the running time interval of these three jumping events are I_{32}, I_{50} and I_{48}. Under this scenario, (181) is a correct statement. Therefore, to capture its truth conditions, the set of all time intervals that are the running time intervals of John's jumping in the last week, namely $\{I_{32}, I_{50}, I_{48}\}$, should be measured as 3. Thus, the counting measure μ_{CT} is the measure we need:

(182) $\mu_{\mathrm{CT}}(\{I_{32}, I_{50}, I_{48}\}) = 3$

Now, let $f(I)$ be a function from time intervals into $\{1, 0\}$ such that $f(I) = 1$ iff I is the exact running time of an event of John's jumping:

(183) $f = \lambda I \in \mathrm{D_i}$. [there is an event of John jumping in @ whose running time is I]

Under the above scenario, $f(I)$ is represented as follows:

(184) $f(I) = \begin{cases} 1 & \text{if } I = I_{32}, I_{50}, I_{48} \\ 0 & \text{otherwise} \end{cases}$

The characteristic set of f is thus $\{I_{32}, I_{50}, I_{48}\}$. Then, according to the definition of Lebesgue integration, (182) is written as a Lebesgue integral of f with the counting measure over the set of all atomic time intervals in the last week:

(185) $$\int_{\{I | I \in \mathrm{AT}_{\mathrm{i}} \wedge I \subset \text{the-last-week}\}} f \, d\mu_{\mathrm{CT}} = \mu_{\mathrm{CT}}\big(\{I_{32}, I_{50}, I_{48}\}\big) = 3$$

Essentially, this is the truth conditions of (181). However, our assumption is that predicates have a situation argument, rather than a time interval argument. We should therefore translate the above truth conditions in terms of situations. In the case of (181), the VP will provide the property of situations $\lambda s \in \mathrm{D_s}.\,\mathsf{jump}(s)(\mathsf{John})$, whose characteristic set is given by the following:

(186) $$\{s \mid \mathsf{jump}(s)(\mathsf{John})(s)\} = \big\{\langle @, I_{32}\rangle, \langle @, I_{50}\rangle, \langle @, I_{48}\rangle\big\}$$

Then, we can actually obtain the set of time intervals $\{I_{32}, I_{50}, I_{48}\}$ by applying the projection map π_{i} to the members of the above set of situations:

(187) $$\{I_{32}, I_{50}, I_{48}\} = \big\{\pi_{\mathrm{i}}\big(\langle @, I_{32}\rangle\big), \pi_{\mathrm{i}}\big(\langle @, I_{50}\rangle\big), \pi_{\mathrm{i}}\big(\langle @, I_{48}\rangle\big)\big\}$$

Now, given a function φ from domain D and into codomain C, for any subset X of D, let us adopt the writing convention:

(188) $$\varphi(X) = \{\varphi(x) \mid x \in X\}$$

In this notation, φ can then simultaneously be regarded as a map from subset of D into subsets of C. With this notation, (187) can be written as follows:

(189) $$\{I_{32}, I_{50}, I_{48}\} = \pi_{\mathrm{i}}\big(\{\langle @, I_{32}\rangle, \langle @, I_{50}\rangle, \langle @, I_{48}\rangle\}\big)$$

Then,

(190) $$\begin{aligned} \mu_{\mathrm{CT}}\big(\{I_{32}, I_{50}, I_{48}\}\big) &= \mu_{\mathrm{CT}}\big(\pi_{\mathrm{i}}\big(\{\langle @, I_{32}\rangle, \langle @, I_{50}\rangle, \langle @, I_{48}\rangle\}\big)\big) \\ &= (\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}})\big(\{\langle @, I_{32}\rangle, \langle @, I_{50}\rangle, \langle @, I_{48}\rangle\}\big) \end{aligned}$$

Regarding π_{i} as a map from sets of situation into set of time intervals, we can see that the composite function $\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}}$ gives the sought measure for situations.[14] We thus obtain the following equation:

(191) $$\int_{\{I | I \in \mathrm{AT}_{\mathrm{i}} \wedge I \subset \text{the-last-week}\}} f \, d\mu_{\mathrm{CT}} = \int_A \lambda s \in \mathrm{D_s}.\,\mathsf{jump}(s)(\mathsf{John}) \, d(\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}})$$

Here, A is the set of all situations whose time interval is an atomic time interval within the last week and whose world is the actual world @. This in fact yields the correct result:

---

[14] Actually, we could adopt μ_{CT} instead of $\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}}$ and obtain the same results. The reason we adopt $\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}}$ is to make is clear that we are concerned with time intervals, and also to make the analysis consistent with the analysis of continuous events, which is presented shortly.

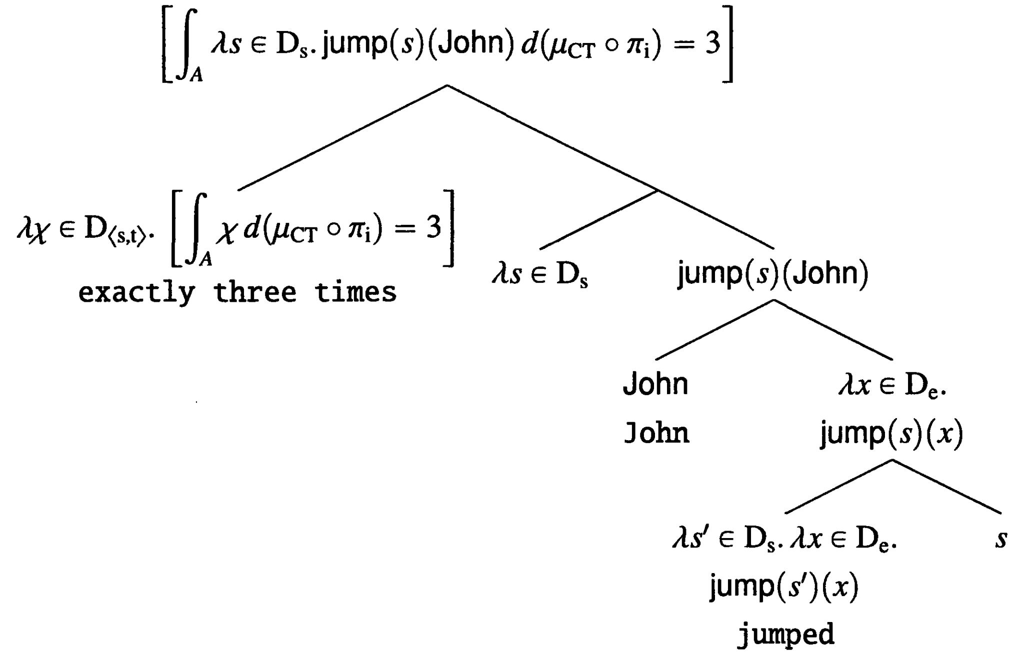
(192) $$\int_A \lambda s \in \mathrm{D_s}.\mathsf{jump}(s)(\mathsf{John})\, d(\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}}) = (\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}})(\{\langle @, I_{32}\rangle, \langle @, I_{50}\rangle, \langle @, I_{48}\rangle\})$$
$$= \mu_{\mathrm{CT}}(\pi_{\mathrm{i}}(\{\langle @, I_{32}\rangle, \langle @, I_{50}\rangle, \langle @, I_{48}\rangle\})) = \mu_{\mathrm{CT}}(\{I_{32}, I_{50}, I_{48}\}) = 3$$

To derive these truth conditions by compositional semantic computation, we need only assume the following denotation for the adverbial *exactly three times*:

(193) $$[\![\texttt{exactly three times}]\!] = \lambda\chi \in \mathrm{D}_{\langle s,t\rangle}.\left[\int_A \chi\, d(\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}}) = 3\right]$$

The semantic computation is shown below:

(194)



Assuming as before that numerals denote properties of numbers, the lexical entry of *times* is given by the following:

(195) $$[\![\texttt{times}]\!] = \lambda\nu \in \mathrm{D}_{\langle n,t\rangle}.\lambda\chi \in \mathrm{D}_{\langle s,t\rangle}.\exists n \in \mathrm{D_n}\left[\nu(n) \wedge \int_A \chi\, d(\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}}) = n\right]$$

Having provided an appropriate measure for analyzing discrete events, let us return to the discussion of event-related readings. Let us consider the following sentence:

(196) Mary hit exactly eleven 20-year-olds last week.

According to the analysis in Section 2.3, the event-related reading of (196) is analyzed as follows:

(197) $$\sum_{i\in\gamma} \varrho_{\mathrm{ATOM}}(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}})(s_i) = 11$$

Now that we have determined that discrete events are counted by the measure $\mu_{\text{CT}} \circ \pi_{\text{i}}$, this is straightforwardly translated as follows:

(198) $\int_A \varrho_{\text{ATOM}}\big(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}\big)\, d(\mu_{\text{CT}} \circ \pi_{\text{i}}) = 11$

Let us confirm that this in fact gives the correct truth conditions. To that end, consider the scenario described below:

(199) Let I_M, I_T, I_W, I_R, I_F and I_S be a time interval on Monday, on Tuesday, on Wednesday, on Thursday, on Friday and on Saturday of the last week respectively.
There was an event whose running time was I_M, which was an event of Mary hitting Ann.
There was an event whose running time was I_T, which was an event of Mary hitting Bill and Chris simultaneously.
There was an event whose running time was I_W, which was an event of Mary hitting Dave.
There was an event whose running time was I_R, which was an event of Mary hitting Ann.
There was an event whose running time was I_F, which was an event of Mary hitting Ed, Fred, George and Henry simultaneously.
There was an event whose running time was I_S, which was an event of Mary hitting Ann and Bill simultaneously.
Ann, Bill, Chris, Dave, Ed, Fred, George and Henry were all of age 20 throughout the last week.

Thus, the individuals hit by Mary are 8 different people, namely, Ann, Bill, Chris, Dave, Ed, Fred, George and Henry, but Bill was hit twice and Ann was hit three times. Under this scenario, (196) should be true on its event-related reading. We should thus verify whether (198) actually holds. To begin with, note that we have the following true formulae under this scenario:

(200) $\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}(\langle @, I_M\rangle) = \text{Ann}$
$\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}(\langle @, I_T\rangle) = \text{Bill} \sqcup_{\text{e}} \text{Chris}$
$\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}(\langle @, I_W\rangle) = \text{Dave}$
$\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}(\langle @, I_R\rangle) = \text{Ann}$
$\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}(\langle @, I_F\rangle) = \text{Ed} \sqcup_{\text{e}} \text{Fred} \sqcup_{\text{e}} \text{George} \sqcup_{\text{e}} \text{Henry}$
$\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}(\langle @, I_S\rangle) = \text{Ann} \sqcup_{\text{e}} \text{Bill}$
For any other situation $s \in A$, $\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}(s) = \bot_{\text{e}}$

Hence[15]

$$
\begin{aligned}
(201)\quad & \varrho_{\mathsf{ATOM}}\big(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}\big)\big(\langle @, I_M\rangle\big) = \inf\nolimits_{p \in I_M} \mathsf{ATOM}\big(\langle @, [p,p]\rangle\big)(\mathsf{Ann}) = 1 \\
& \varrho_{\mathsf{ATOM}}\big(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}\big)\big(\langle @, I_T\rangle\big) = \inf\nolimits_{p \in I_T} \mathsf{ATOM}\big(\langle @, [p,p]\rangle\big)(\mathsf{Bill} \sqcup_{\mathrm{e}} \mathsf{Chris}) = 2 \\
& \varrho_{\mathsf{ATOM}}\big(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}\big)\big(\langle @, I_W\rangle\big) = \inf\nolimits_{p \in I_W} \mathsf{ATOM}\big(\langle @, [p,p]\rangle\big)(\mathsf{Dave}) = 1 \\
& \varrho_{\mathsf{ATOM}}\big(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}\big)\big(\langle @, I_R\rangle\big) = \inf\nolimits_{p \in I_R} \mathsf{ATOM}\big(\langle @, [p,p]\rangle\big)(\mathsf{Ann}) = 1 \\
& \varrho_{\mathsf{ATOM}}\big(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}\big)\big(\langle @, I_F\rangle\big) = \inf\nolimits_{p \in I_F} \mathsf{ATOM}\big(\langle @, [p,p]\rangle\big) \\
& \qquad\qquad\qquad (\mathsf{Ed} \sqcup_{\mathrm{e}} \mathsf{Fred} \sqcup_{\mathrm{e}} \mathsf{George} \sqcup_{\mathrm{e}} \mathsf{Henry}) = 4 \\
& \varrho_{\mathsf{ATOM}}\big(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}\big)\big(\langle @, I_S\rangle\big) = \inf\nolimits_{p \in I_S} \mathsf{ATOM}\big(\langle @, [p,p]\rangle\big)(\mathsf{Ann} \sqcup_{\mathrm{e}} \mathsf{Bill}) = 2 \\
& \text{For any other situation } s \in A, \\
& \qquad \varrho_{\mathsf{ATOM}}\big(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}\big)(s) = \inf\nolimits_{p \in \pi_{\mathrm{i}}(s)} \mathsf{ATOM}\big(\langle @, [p,p]\rangle\big)(\bot_{\mathrm{e}}) = 0
\end{aligned}
$$

Hence

$$
\begin{aligned}
(202)\quad & \big\{s \in A \mid \varrho_{\mathsf{ATOM}}\big(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}\big)(s) = 1\big\} = \big\{\langle @, I_M\rangle, \langle @, I_W\rangle, \langle @, I_R\rangle\big\} \\
& \big\{s \in A \mid \varrho_{\mathsf{ATOM}}\big(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}\big)(s) = 2\big\} = \big\{\langle @, I_T\rangle, \langle @, I_S\rangle\big\} \\
& \big\{s \in A \mid \varrho_{\mathsf{ATOM}}\big(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}\big)(s) = 4\big\} = \big\{\langle @, I_F\rangle\big\} \\
& \text{For any positive number } n \neq 1,2,4,\ \big\{s \in A \mid \varrho_{\mathsf{ATOM}}\big(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}\big)(s) = n\big\} = \varnothing
\end{aligned}
$$

$\varrho_{\mathsf{ATOM}}\big(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}\big)$ is thus a simple function, taking only three positive values $1, 2$ and 4, and is represented as follows:

$$
(203)\quad \varrho_{\mathsf{ATOM}}\big(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}\big) = 1 \cdot \chi_{\{\langle @, I_M\rangle, \langle @, I_W\rangle, \langle @, I_R\rangle\}} + 2 \cdot \chi_{\{\langle @, I_T\rangle, \langle @, I_S\rangle\}} + 4 \cdot \chi_{\{\langle @, I_F\rangle\}}
$$

Its integral over A is thus calculated as follows and yields the desired value:

$$
\begin{aligned}
(204)\quad & \int_A \varrho_{\mathsf{ATOM}}\big(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}\big)\, d(\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}}) \\
& = \int_A 1 \cdot \chi_{\{\langle @, I_M\rangle, \langle @, I_W\rangle, \langle @, I_R\rangle\}} + 2 \cdot \chi_{\{\langle @, I_T\rangle, \langle @, I_S\rangle\}} + 4 \cdot \chi_{\{\langle @, I_F\rangle\}}\, d(\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}}) \\
& = 1 \cdot (\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}})\big(\{\langle @, I_M\rangle, \langle @, I_W\rangle, \langle @, I_R\rangle\}\big) + 2 \cdot (\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}})\big(\{\langle @, I_T\rangle, \langle @, I_S\rangle\}\big) \\
& \quad + 4 \cdot (\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}})\big(\{\langle @, I_T\rangle\}\big) \\
& = 1 \cdot \mu_{\mathrm{CT}}(\{I_M, I_W, I_R\}) + 2 \cdot \mu_{\mathrm{CT}}(\{I_T, I_S\}) + 4 \cdot \mu_{\mathrm{CT}}(\{I_F\}) \\
& = 1 \cdot 3 + 2 \cdot 2 + 4 \cdot 1 \\
& = 11
\end{aligned}
$$

---

[15] Since the atomicity of these people never changes, the presupposition of $\varrho_{\mathsf{ATOM}}\big(\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}\big)$ is automatically satisfied.

To compositionally derive these truth conditions, we need only modify the lexical entry of $\varnothing_{\text{ATOM}}$ in (108) accordingly as follows:

(205) $$[\![\varnothing_{\text{ATOM}}]\!] = \lambda v \in \mathrm{D}_{\langle \mathrm{n,t}\rangle}.\lambda R \in \mathrm{D}_{\langle \mathrm{e},\langle \mathrm{s,t}\rangle\rangle}.$$
$$\exists n \in \mathrm{D}_{\mathrm{n}}\left[v(n) \wedge \int_A \varrho_{\text{ATOM}}\left(\lambda s \in \mathrm{D}_{\mathrm{s}}.\bigsqcup\nolimits_{\mathrm{e}} \{x \mid R(x)(s)\}\right) d(\mu_{\text{CT}} \circ \pi_{\mathrm{i}}) = n\right]$$

With this, the very same syntactic structure as assumed earlier computes the truth conditions in (198). First, it combines with the numeral to yield the following:

(206) $$[\![\texttt{exactly eleven } \varnothing_{\text{ATOM}}]\!] \overset{\text{FA}}{=} [\![\varnothing_{\text{ATOM}}]\!]([\![\texttt{exactly eleven}]\!])$$
$$= \lambda R \in \mathrm{D}_{\langle \mathrm{e},\langle \mathrm{s,t}\rangle\rangle}.\left[\int_A \varrho_{\text{ATOM}}\left(\lambda s \in \mathrm{D}_{\mathrm{s}}.\bigsqcup\nolimits_{\mathrm{e}} \{x \mid R(x)(s)\}\right) d(\mu_{\text{CT}} \circ \pi_{\mathrm{i}}) = 11\right]$$

This in turn takes the situationally dependent relation $\mathbf{SDR}^{\text{20-yr-old}}_{\text{Mary-hit}}(x)(s)$, which has been created as the result of the movement of the measure phrase to the position right above the situation λ abstractor, and yields the desired truth conditions.[16]

As shown just above, the translation from (197) to (198) works just fine. I would now like to show that this is not a lucky coincidence, but it is because the following equation generally holds:

(207) $$\sum_{i \in \gamma} \varrho_M(\mathbf{SDIC}^{\varphi}_{\psi})(s_i) = \int_A \varrho_M(\mathbf{SDIC}^{\varphi}_{\psi})\, d(\mu_{\text{CT}} \circ \pi_{\mathrm{i}})$$

If this is shown, it means that our earlier analysis with summation of discrete event-related readings can always be translated into the new integral approach. This equation actually follows from the fact that any summation can be translated into a Lebesgue integral with the counting measure, and vice versa. This is more or less intuitively obvious. For each positive value, the integral with the counting measure counts the number of the elements that are mapped to that value and multiplies the value by the number, but this is the same as if this value got added that number of times. I state this as a form of a theorem below, which concludes the present subsection:

(208) Theorem

Let A be a set of atomic situations: $A \subseteq \mathrm{AT}_{\mathrm{s}}$, and $(A, \mathfrak{F}, \mu_{\text{CT}} \circ \pi_{\mathrm{i}})$ be a measure space. Let f be a function from D_{s} into nonnegative reals. If $\int_A f\, d(\mu_{\text{CT}} \circ \pi_{\mathrm{i}}) < +\infty$, and if there is a family of atomic situations indexed by γ such that $A = \{s_i\}_{i \in \gamma}$ and $s_i \neq s_j \rightarrow i \neq j$,

---

[16] Recall that $\mathbf{SDIC}^{\text{20-yr-old}}_{\text{Mary-hit}}(x)(s) = \lambda s \in \mathrm{D}_{\mathrm{s}}.\bigsqcup\nolimits_{\mathrm{e}}\left\{x \mid \mathbf{SDR}^{\text{20-yr-old}}_{\text{Mary-hit}}(x)(s)\right\}$.

$$\int_A f\, d(\mu_{\text{CT}} \circ \pi_{\text{i}}) = \sum_{i \in \gamma} f(s_i).$$

Proof.

Let $S_0 = \{s \in A \mid f(s) \geq 1\}$ and $S_n = \left\{ s \in A \;\middle|\; \frac{1}{n+1} \leq f(s) < \frac{1}{n} \right\}$ for $1 \leq n$. Note that

$$\int_A f\, d(\mu_{\text{CT}} \circ \pi_{\text{i}}) \geq \int_{S_n} f\, d(\mu_{\text{CT}} \circ \pi_{\text{i}}) \geq \int_{S_n} \frac{1}{n+1} d(\mu_{\text{CT}} \circ \pi_{\text{i}}) = \frac{1}{n+1} \times (\mu_{\text{CT}} \circ \pi_{\text{i}})(S_n).$$

If S_n contained infinitely many elements, then $(\mu_{\text{CT}} \circ \pi_{\text{i}})(S_n) = \infty$, and thus we would obtain $\int_A f\, d(\mu_{\text{CT}} \circ \pi_{\text{i}}) \geq \frac{1}{n+1} \times \infty = \infty$, which would contradict the assumption that f is integrable. Hence, for each n, S_n is at most finite. $\bigcup_{n=1}^{\infty} S_n$ gives the support of f, written as $\text{supp}(f)$, viz., $\{s \in A \mid f(s) > 0\}$. $\text{supp}(f)$ is countable because it is a countable union of finite sets. This in turn means that $f(\text{supp}(f))$ is countable, i.e., f yields at most countably many positive values.

If $\text{supp}(f) = \emptyset$, i.e., $f = 0$, then the theorem trivially holds. Therefore, let us assume that $\text{supp}(f) \neq \emptyset$, i.e., for some situation in A, f yields a positive value. Because f is integrable, f is bounded in A, and thus has a maximum value in A. Let $\alpha_1 = \max f(A)$. For $k > 1$, let $\alpha_k = \max(f(A) - \{\alpha_1, \alpha_2, \ldots, \alpha_{k-1}\})$. This way, we obtain the sequence $\{\alpha_k\}$ of the positive values that f yields, such that $\alpha_1 > \alpha_2 > \alpha_3 > \ldots$. For any positive value a that f yields, there must be some number k such that $\alpha_k = a$. This follows from that fact that $\{s \in A \mid f(s) > a\}$ is at most finite for any a for the same reason that S_n is at most finite, and that as a result, $f(\{s \in A \mid f(s) > a\})$, viz., the set of the values grater than a that f yields, is at most finite as well.

Now, suppose f yields only a finite number of positive values $\alpha_1, \alpha_2, \ldots, \alpha_n$. Since $f^{-1}(\alpha_k)$ is finite for each k, let $\sigma(k) < \infty$ be the number of the elements in $f^{-1}(\alpha_k)$. Then, for each k, we have $(\mu_{\text{CT}} \circ \pi_{\text{i}})(f^{-1}(\alpha_k)) = \sigma(k)$. Therefore,

$$\int_A f\, d(\mu_{\text{CT}} \circ \pi_{\text{i}}) = \sum_{k=1}^{n} \alpha_k \cdot (\mu_{\text{CT}} \circ \pi_{\text{i}})(f^{-1}(\alpha_k)) = \sum_{k=1}^{n} \alpha_k \cdot \sigma(k).$$

Let us now construct a subfamily $\{s_{i_j}\}_{1 \leq j \leq \sigma(1)+\sigma(2)+\cdots+\sigma(n)}$ of $\{s_i\}_{i \in \gamma}$ as follows. The first $\sigma(1)$ members are the $\sigma(1)$ members in $f^{-1}(\alpha_1)$: $\{s_{i_1}, s_{i_2}, \ldots, s_{i_{\sigma(1)}}\} = f^{-1}(\alpha_1)$. The next $\sigma(2)$ members are the $\sigma(2)$ members in $f^{-1}(\alpha_2)$: $\{s_{i_{\sigma(1)+1}}, s_{i_{\sigma(1)+2}}, \ldots, s_{i_{\sigma(1)+\sigma(2)}}\} = f^{-1}(\alpha_2)$ And in general, $\{s_{i_{\sigma(1)+\cdots+\sigma(k)+1}}, s_{i_{\sigma(1)+\cdots+\sigma(k)+2}}, \ldots, s_{i_{\sigma(1)+\cdots+\sigma(k)+\sigma(k+1)}}\} = f^{-1}(\alpha_{k+1})$. If $i \notin \{i_j \mid$

$1 \leq j \leq \sigma(1) + \sigma(2) + \cdots + \sigma(n)\}$, then $f(s_{i_j}) = 0$. Therefore,

$$\begin{aligned}
\sum_{i \in \gamma} f(s_i) &= \sum_{j=1}^{\sigma(1)+\sigma(2)+\cdots+\sigma(n)} f(s_{i_j}) + \sum_{i \in \gamma,\, i \notin \{i_j | 1 \leq j \leq \sigma(1)+\sigma(2)+\cdots+\sigma(n)\}} f(s_i) \\
&= \sum_{j=1}^{\sigma(1)+\sigma(2)+\cdots+\sigma(n)} f(s_{i_j}) + \sum_{i \in \gamma,\, i \notin \{i_j | 1 \leq j \leq \sigma(1)+\sigma(2)+\cdots+\sigma(n)\}} 0 \\
&= \sum_{j=1}^{\sigma(1)+\sigma(2)+\cdots+\sigma(n)} f(s_{i_j}) \\
&= \sum_{j=1}^{\sigma(1)} f(s_{i_j}) + \sum_{j=\sigma(1)+1}^{\sigma(2)} f(s_{i_j}) + \cdots + \sum_{j=\sigma(n-1)+1}^{\sigma(n)} f(s_{i_j}) \\
&= \sum_{j=1}^{\sigma(1)} \alpha_1 + \sum_{j=\sigma(1)+1}^{\sigma(2)} \alpha_2 + \cdots + \sum_{j=\sigma(n-1)+1}^{\sigma(n)} \alpha_k \\
&= \alpha_1 \cdot \sigma(1) + \alpha_2 \cdot \sigma(2) + \cdots + \alpha_k \cdot \sigma(k) \\
&= \sum_{k=1}^{n} \alpha_k \cdot \sigma(k).
\end{aligned}$$

Hence, we obtain:

$$\int_A f\, d(\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}}) = \sum_{k=1}^{n} \alpha_k \cdot \sigma(k) = \sum_{i \in \gamma} f(s_i).$$

When f yields infinitely many (but countable) positive values,

$$\int_A f\, d(\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}}) = \lim_{n \to \infty} \sum_{k=1}^{n} \alpha_k \cdot (\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}})(f^{-1}(\alpha_k)) = \lim_{n \to \infty} \sum_{k=1}^{n} \alpha_k \cdot \sigma(k).$$

We can then construct a subfamily $\{s_{i_j}\}_{j \in \mathbb{N}}$ of $\{s_i\}_{i \in \gamma}$ in the obvious way, and by taking the limits, the theorem is proved similarly for this case as well. Q.E.D.

2.6.2 Continuous (atelic) events

Before we tackle continuous event-related readings, let us begin with a simple sentence that describes continuous events but does not contain an event-related-reading-inducing noun phrase, in order to determine the appropriate measure for situations when dealing with continuous events.

Consider the following sentence:

(209) John ran for exactly seventeen minutes yesterday.

Imagine that yesterday, John ran from 6:03:00 a.m. through 6:08:00 a.m and also from 4:11:00 p.m. through 4:23:00 p.m. and did not run at any other time. Then, the set of all the time points at which John was running yesterday is represented as the union of two intervals as follows:[17]

(210) $J = [\text{6:03:00 a.m.}, \text{6:08:00 a.m.}] \cup [\text{4:11:00 p.m.}, \text{4:23:00 p.m.}]$

The time interval [6:03:00 a.m., 6:08:00 a.m.] has a length of $5 \times 60 = 300$ seconds, and the time interval [4:11:00 p.m., 4:23:00 p.m.] has a length of $12 \times 60 = 720$ seconds, and thus, J has a length of $300 + 720 = 1020$ seconds in total. This is actually what the Lebesgue measusre gives to J, provided that the time line is isomorphic to the real line and its unit is the second. The truth conditions of (209) are then expected to be given by the following:

(211) $\mu_{\text{LBG}}(J) = 1020$

In the case of (209), the property of situations given by the VP is $\lambda s \in D_s.\, \mathsf{run}(s)(\mathsf{John})$. Since running is a continuous event, this is true only of momentary situations, i.e., for situations of the form $\langle w, [p, p]\rangle$, where p is some time point. For example, $\mathsf{run}(\langle @, [\text{6:06:06 a.m.}, \text{6:06:06 a.m.}]\rangle)(\mathsf{John})$ is true, because John was running at 6:06:06 a.m., but $\mathsf{run}(\langle @, [\text{6:03:00 a.m.}, \text{6:08:00 a.m.}]\rangle)(\mathsf{John})$ is false. The characteristic set of the function $\lambda s \in D_s.\, \mathsf{run}(s)(\mathsf{John})$ is thus given by the following:

(212) $\{s \mid \mathsf{run}(s)(\mathsf{John})(s)\} = \{\langle @, [p, p]\rangle \mid p \in J\}$

Because this characteristic set is a set of situations while the Lebesgue measure takes measurement of sets of time points, we have to convert sets of situations to corresponding sets of time points, just as we converted sets of situations to sets of time intervals in analyzing discrete events in the previous subsection. For situation $\langle @, [\text{6:06:06 a.m.}, \text{6:06:06 a.m.}]\rangle$, we can first apply the projection map π_i to yield a corresponding time interval:

(213) $\pi_i(\langle @, [\text{6:06:06 a.m.}, \text{6:06:06 a.m.}]\rangle) = [\text{6:06:06 a.m.}, \text{6:06:06 a.m.}]$

Although [6:06:06 a.m., 6:06:06 a.m.] is momentary, it is still an interval (i.e., a singleton set containing only one time point), and not a time point. We thus define the map ι as

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[17] Suppose that the times here refer to the times in yesterday only.

follows:[18]

(214) For any intervals of the form $[p, p]$, where p is a time point, $\iota([p, p]) \stackrel{\text{def}}{=} p$

ι is a bijective map between momentary time intervals and time points. Applying ι to momentary time interval [6:06:06 a.m., 6:06:06 a.m.], we can obtain the corresponding time point as follows:

(215) $\iota\big([6{:}06{:}06\text{ a.m.}, 6{:}06{:}06\text{ a.m.}]\big) = 6{:}06{:}06\text{ a.m.}$

We can then see that the composite map $\iota \circ \pi_{\mathrm{i}}$ gives the sought conversion from situations into time points, and $\mu_{\mathrm{LBG}} \circ \iota \circ \pi_{\mathrm{i}}$ gives the sought measure for situations. The truth conditions of (209) can then be expressed by the following:

(216) $\int_B \lambda s \in \mathrm{D_s}.\, \mathsf{run}(s)(\mathsf{John})\, d(\mu_{\mathrm{LBG}} \circ \iota \circ \pi_{\mathrm{i}}) = 1020$

Here, B is the set of all momentary situations in yesterday, i.e., $\{\langle @, [p, p]\rangle \mid p \in \textsf{yesterdsy}\}$. Since ι is not defined for intervals that are not momentary, $\mu_{\mathrm{LBG}} \circ \iota \circ \pi_{\mathrm{i}}$ is not defined for situations that are not momentary. Accordingly, the domain of the integral must be restricted to sets that contain only momentary situations. The above truth conditions are indeed correct as confirmed by the following:

(217)
$$\begin{aligned}
&\int_B \lambda s \in \mathrm{D_s}.\, \mathsf{run}(s)(\mathsf{John})\, d(\mu_{\mathrm{LBG}} \circ \iota \circ \pi_{\mathrm{i}}) \\
&= (\mu_{\mathrm{LBG}} \circ \iota \circ \pi_{\mathrm{i}})(\{s \mid \mathsf{run}(s)(\mathsf{John})(s)\}) \\
&= (\mu_{\mathrm{LBG}} \circ \iota \circ \pi_{\mathrm{i}})\big(\{\langle @, [p, p]\rangle \mid p \in J\}\big) \\
&= (\mu_{\mathrm{LBG}} \circ \iota)\big(\pi_{\mathrm{i}}(\{\langle @, [p, p]\rangle \mid p \in J\})\big) \\
&= (\mu_{\mathrm{LBG}} \circ \iota)\big(\{[p, p] \mid p \in J\}\big) \\
&= \mu_{\mathrm{LBG}}\big(\iota(\{[p, p] \mid p \in J\})\big) \\
&= \mu_{\mathrm{LBG}}(\{p \mid p \in J\}) \\
&= \mu_{\mathrm{LBG}}(J) \\
&= 1020
\end{aligned}$$

To derive these truth conditions by compositional semantic computation, we can assume the following denotation for the adverbial *for exactly seventeen minutes*:

---

[18] This may be identified with the description operator usually notated with ι, since with it, the function defined here can be represented as $\lambda I.\iota p[p \in I]$.

(218) $[\![\texttt{for exactly seventeen minutes}]\!]$

$$= \lambda\chi \in \mathrm{D}_{\langle s,t\rangle}. \left[\int_B \chi\, d(\mu_{\mathrm{LBG}} \circ \iota \circ \pi_{\mathrm{i}}) = 1020\right]$$

The semantic computation proceeds just in the same fashion as in the discrete case with *three times* in the previous section. Assuming as before that numerals denote properties of numbers, the lexical entry of *minutes* should then be given by the following:

(219) $[\![\texttt{minutes}]\!] = \lambda v \in \mathrm{D}_{\langle n,t\rangle}. \lambda\chi \in \mathrm{D}_{\langle s,t\rangle}. \exists n \in \mathrm{D}_{\mathrm{n}} \left[v(n) \wedge \int_B \chi\, d(\mu_{\mathrm{LBG}} \circ \iota \circ \pi_{\mathrm{i}}) = 60n\right]$

Having established the measure for situations used in analyzing continuous events, let us now discuss continuous event-related readings. We would like to analyze (132) from Section 2.4:

(132) John ate (exactly) 666 cubic centimeters of bread.

In the analysis of discrete event-related readings, we only needed to integrate the relevant function defined with ϱ with the measure used in analyzing simple sentences describing discrete events. If that strategy would carry over into continuous event-related readings, (100) would be expected to be analyzed as follows:

(220) $\int_B \varrho_{\text{cubic-centimeter}}(\mathbf{SDIC}^{\text{bread}}_{\text{John-eat}})\, d(\mu_{\mathrm{LBG}} \circ \iota \circ \pi_{\mathrm{i}}) = 666$

Unfortunately, this does not work. In Section 2.4, we saw that $\varrho_{\text{cubic-centimeter}}(\mathbf{SDIC}^{\text{bread}}_{\text{John-eat}}) = 0$ for all momentary situations, since for a time point p, it is given by $\int_p^p \pi\{r(p')\}^2\, v(p')\, dp'$. The integral then becomes 0, and hence the above truth conditions are not correct. This is merely a restatement of what was discussed in Section 2.4. As suggested there, what needs to be integrated is the rate of the volume of the bread eaten by John. Therefore, in what follows, I construct the relevant function that gives the relevant rate for continuous event-related readings.

In general, for a given situation dependent individual concept $\mathbf{SDIC}^{\varphi}_{\psi}$, where ψ is a predicate for continuous events, how can we find the relevant rate to be integrated? To answer this question, let us consider the individual given by $\bigsqcup_{p \in I}{}_{\mathrm{e}}\, \mathbf{SDIC}^{\varphi}_{\psi}(\langle w, [p,p]\rangle)$. $\mathbf{SDIC}^{\varphi}_{\psi}(\langle w, [p,p]\rangle)$ is the individual which bears a relevant thematic role to an event described by ψ at the momentary time interval $[p,p]$ in world w. For instance, $\mathbf{SDIC}^{\text{bread}}_{\text{John-eat}}$ $(\langle @, [p,p]\rangle)$ gives the individual which is bread and is eaten by John at the moment p in the actual world. Assuming that individuals constitute a complete lattice,

$\underset{p\in I}{\bigsqcup_{\mathrm{e}}} \mathbf{SDIC}^{\varphi}_{\psi}(\langle w, [p,p]\rangle)$ defines an individual, which is the collection of all the individuals that were involved in the momentary events during time interval I in world w. Thus, $\underset{p\in I}{\bigsqcup_{\mathrm{e}}} \mathbf{SDIC}^{\mathsf{bread}}_{\mathsf{John\text{-}eat}}(\langle @, [p,p]\rangle)$ is the individual which is the collection of all the bread that was eaten by John during I in the actual world. Consider the scenario considered in Section 2.4, where John was eating the bread during the time interval $[p_0, p_1]$. Assuming that $I = [a,b] \subseteq [p_0, p_1]$, this is the part of the bread that begins from the point that is $\int_{p_0}^{a} v(p)\,dp$ cm away from the original end of the piece of bread where John's mouth started and ends at the point that is $\int_{p_0}^{b} v(p)\,dp$ cm away in the same sense.[19] In other words, it has the shape of a cylinder of a length of $\int_{a}^{b} v(p)\,dp$ cm. If $I \cap [p_0, p_1] = \varnothing$, then $\underset{p\in I}{\bigsqcup_{\mathrm{e}}} \mathbf{SDIC}^{\mathsf{bread}}_{\mathsf{John\text{-}eat}}(\langle @, [p,p]\rangle) = \bot_{\mathrm{e}}$, as John did not eat any bread in I.

Recall that one important difference between event-related readings and object-related readings is that in event-related readings, one and the same individuals could be counted multiple times in case they get involved in multiple events of the same sort, while object-related readings never count one and the same individuals more than once. Since we are discussing continuous event-related readings, we should in general take into account the possibility that the same individuals get involved in multiple events of the relevant sort.

The type of the events we are considering is production and consumption. While it is very hard to imagine that one and the same individual gets produced more than once, it is possible to think of scenarios where one and the same individual gets consumed multiple times in some fashion. For example, one could imagine that while John is drinking XYZ, some XYZ that John has drunk can somehow get out of John's body (maybe a catheter is inserted to John's stomach and drains its contents out) and enter John's mouth again. In this case, some individual that is XYZ gets drunk by John again, and the volume of this body of XYZ at the time of John's drinking it should be counted for each time, if one is to describe this scenario by the event-related reading of a relevant sentence. Or, imagine that John's work is to spread XYZ over the windows.[20] Suppose that John spread some XYZ over the windows and then somebody collected the XYZ off the windows. If John spread this XYZ over the windows again, the volume of this XYZ at the time of John's spreading

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[19] Recall that p_0 is the time point where John started eating, and $v(p)$ is the speed with which John's mouth travels.

[20] I thank Irene Heim for suggesting this story.

it should be counted for each time, if one is to describe this by the event-related reading of a relevant sentence.

However, if one and the same individual is to be "consumed" (eaten, spread over the windows, etc.) more than once, practically, there will always be some time span between the consumption events. Before being able to drink the same XYZ, John has to wait for it to first go through his oesophagus, his stomach, and the catheter. Similarly, before being able to spread the same XYZ over the windows again, someone has to spend time on collecting it. Therefore, I think it is safe to assume that if one looks at a sufficiently short time interval, no individual ever gets "consumed" more than once. I thus make the following claim about the models of worlds we have in our mind:

(221) Claim

For any time point p, there is an open time interval G containing p such that $\forall p_1, p_2 \in G \left[p_1 \neq p_2 \rightarrow \mathbf{SDIC}^{\varphi}_{\psi}(\langle w, [p_1, p_1] \rangle) \sqcap_{\mathsf{e}} \mathbf{SDIC}^{\varphi}_{\psi}(\langle w, [p_2, p_2] \rangle) = \bot_{\mathsf{e}} \right]$

Then, our goal is to construct the relevant rate function to be integrated for a short enough time span that satisfies the above conditions. Once we obtain a series of such rate functions for a series of short time spans, the rate function for any longer time span can be obtained by "stringing them together".

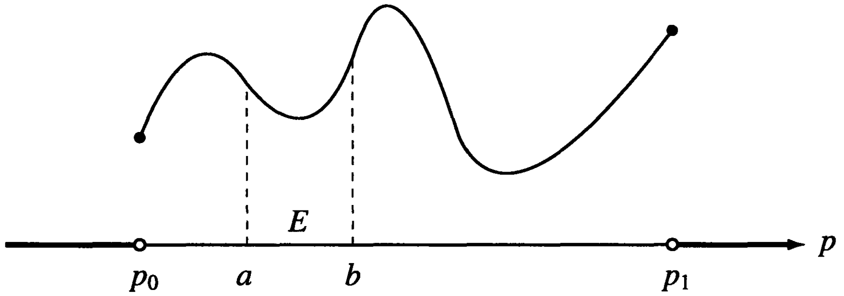
Now, let G be an open time interval that satisfies the condition in Claim (221). We define $\Theta_{w,q}$ from time intervals contained in G into nonnegative reals such that for any $E \subseteq G$,

(222) $$\Theta_{w,q}(E) \overset{\text{def}}{=} M(\langle w, [q, q] \rangle) \left(\bigsqcup_{\mathsf{e}\; p \in E} \mathbf{SDIC}^{\varphi}_{\psi}(\langle w, [p, p] \rangle) \right)$$

$\Theta_{w,q}$ is a function that, given a time interval E, yields the measurement of the individual $\bigsqcup_{\mathsf{e}\; p \in E} \mathbf{SDIC}^{\varphi}_{\psi}(\langle w, [p, p] \rangle)$ at time point q in world w. When $\varphi = \textsf{bread}$, $\psi = \textsf{John-eat}$, $M = \textsf{cubic-centimeter}$, $\Theta_{@,q}(E)$ is the volume in cubic centimeters at time point q of the bread eaten by John during E in the actual world. Under the scenario considered in Section 2.4, no part of the bread was eaten more than once by John. Therefore, for any open time interval $G \supset [p_0, p_1]$, the conditions in Claim (221) is met. $E = [a, b] \subseteq [p_0, p_1]$, $\Theta_{w,q}(E)$ gives the volume of a cylinder of a length of $\int_a^b v(p)\, dp$ cm. Since the radius of this cylinder at time point q is given by $r(q)$ cm, $\Theta_{@,q}(E) = \pi\{r(q)\}^2 \int_a^b v(p)\, dp$. If $E \cap [p_0, p_1] = \varnothing$, then $\Theta_{@,q}(E) = \textsf{cubic-centimeter}(\bot_{\mathsf{e}}) = 0$. For convenience, let us refer to the size of something measured at time point q as q-size. Let $f_q(p)$ denote the rate of q-volume of bread that John eats per unit time at p. From the scenario, we know that $f_q(p)$ is given by (223), and so its graph looks, for instance, like (224):

$$(223)\quad f_q(p) = \begin{cases} \pi\{r(q)\}^2 v(p) & \text{if } p \in [p_0, p_1] \\ 0 & \text{if } p \notin [p_0, p_1] \end{cases}$$

(224)



Obviously, $\Theta_{@,q}(E)$ is given by the area (i.e. integral) of $f_q(p)$ over E:

$$(225)\quad \Theta_{@,q}(E) = \int_E f_q(p)\, d\mu_{\mathrm{LBG}}$$

In the case of the bread-eating scenario, the relevant rate function f_q was provided by the scenario as in (223) above. In the general case, however, the concrete form of the relevant rate function is unknown. Instead, we are only given $\Theta_{w,q}$ by the compositional semantics in (222). Therefore, in order to obtain the relevant rate function, it is necessary to differentiate $\Theta_{w,q}$. According to the definition of derivatives, the derivative $D\Theta_{w,q}$ at time point p is given by the following:

$$(226)\quad D\Theta_{w,q}(p) = \lim_{h\to 0} \frac{1}{\mu_{\mathrm{LBG}}(C(p,h))}\, \Theta_{w,q}(C(p,h))$$

Here $C(p,h)$ is a cube centered at p with an edge length of $2h$. Since G is a subset of $\mathbb{R}^1$, the cube $C(p,h)$ here is 1-dimentional and hence $C(p,h) = [p-h, p+h]$. Therefore,

$$\begin{aligned}(227)\quad D\Theta_{w,q}(p) &= \lim_{h\to 0} \frac{1}{\mu_{\mathrm{LBG}}\big([p-h,p+h]\big)}\, \Theta_{w,q}\big([p-h,p+h]\big) \\ &= \lim_{h\to 0} \frac{1}{2h}\, \Theta_{w,q}\big([p-h,p+h]\big) \\ &= \lim_{h\to 0} \frac{1}{2h}\, M\big(\langle w,[q,q]\rangle\big)\Bigg(\mathop{\bigsqcup\nolimits_{\mathrm{e}}}_{p'\in[p-h,p+h]} \mathbf{SDIC}^{\varphi}_{\psi}\big(\langle w,[p',p']\rangle\big)\Bigg)\end{aligned}$$

If this limit exists, it will give us the rate of q-size of the the individuals which is consumed, etc., per unit time at almost every $p \in G$. Then, by integrating this rate over a given time span, we should be able to calculate the total amount of consumption, etc.

For the bread-eating scenario, $\Theta_{@,q}$ is already expressed as the integral of f_q in (223). Therefore, it follows from Lebesgue's differentiation theorem that $D\Theta_{@,q}(p)$ exists a.e. $[\mu_{\text{LBG}}]$ and that it is equal to f_q a.e. $[\mu_{\text{LBG}}]$. Let us confirm this by actually calculating $D\Theta_{@,q}(p)$. If $p \in (p_0, p_1)$,

(228)
$$\begin{aligned}
&D\Theta_{@,q}(p)\\
&= \lim_{h\to 0}\frac{1}{2h}\,\text{cubic-centimeter}(\langle W, [q,q]\rangle)\Bigg(\bigsqcup_{\substack{\mathrm{e}\\ p'\in[p-h,p+h]}} \mathbf{SDIC}^{\text{bread}}_{\text{John-eat}}(\langle w, [p', p']\rangle)\Bigg)\\
&= \lim_{h\to 0}\frac{1}{2h}\pi\{r(q)\}^2\int_{p-h}^{p+h} v(p')\,dp'\\
&= \pi\{r(q)\}^2\lim_{h\to 0}\frac{1}{2h}\int_{p-h}^{p+h} v(p')\,dp'\\
&= \pi\{r(q)\}^2\,v(p)
\end{aligned}$$

In the last equality, I have used the assumption that $v(p)$ is a continuous function. If $p \notin [p_0, p_1]$, $D\Theta_{@,q}(p) = 0$ because $\Theta_{@,q}(p) = 0$. If $p_0 \in G$,

(229)
$$\begin{aligned}
&D\Theta_{@,q}(p_0)\\
&= \lim_{h\to 0}\frac{1}{2h}\big(\Theta_{@,q}([p_0-h, p_0)) + \Theta_{@,q}([p_0, p_0+h])\big)\\
&= \lim_{h\to 0}\frac{1}{2h}\big(0 + \Theta_{@,q}([p_0, p_0+h])\big)\\
&= \lim_{h\to 0}\frac{1}{2h}\Theta_{@,q}([p_0, p_0+h])\\
&= \lim_{h\to 0}\frac{1}{2h}\int_{p_0}^{p_0+h} v(p')\,dp'\\
&= \frac{1}{2}\pi\{r(q)\}^2\,v(p)
\end{aligned}$$

Similarly, if $p_1 \in G$, $D\Theta_{@,q}(p_1) = \frac{1}{2}\pi\{r(q)\}^2\,v(p_1)$ is obtained. Putting these together, $D\Theta_{@,q}(p)$ is defined for all $p \in G$ as follows:

(230)
$$D\Theta_{@,q}(p) = \begin{cases} \pi\{r(q)\}^2\,v(p) & \text{if } p \in (p_0, p_1)\\ \frac{1}{2}\pi\{r(q)\}^2\,v(p) & \text{if } p = p_0, p_1\\ 0 & \text{if } p \notin [p_0, p_1] \end{cases}$$

Notice that if $\pi\{r(q)\}^2\, v(p_0) \neq 0$, then at $p = p_0$, $D\Theta_{@,q}(p)$ is not equal to $f_q(p)$, even though $\Theta_{@,q}$ is obtained by integrating f_q (see (225)). This is because in that case, f_q is not continuous at p_0, and thus p_0 is not a Lebesgue point of f_q. That p_0 is not a Lebesgue point of f_q when $\pi\{r(q)\}^2\, v(p_0) \neq 0$ can be confirmed as follows:

(231)
$$\begin{aligned}
&\lim_{h\to 0} \frac{1}{\mu_{\mathrm{LBG}}(C(p_0,h))} \int_{C(p_0,h)} \lambda p.\, \left|f_q(p) - f_q(p_0)\right| d\mu_{\mathrm{LBG}} \\
&= \lim_{h\to 0} \frac{1}{\mu_{\mathrm{LBG}}\big([p_0 - h, p_0 + h]\big)} \int_{[p_0-h,\, p_0+h]} \lambda p.\, \left|f_q(p) - f_q(p_0)\right| d\mu_{\mathrm{LBG}} \\
&= \lim_{h\to 0} \frac{1}{2h} \int_{[p_0-h,\, p_0+h]} \lambda p.\, \left|f_q(p) - f_q(p_0)\right| d\mu_{\mathrm{LBG}} \\
&= \lim_{h\to 0} \frac{1}{2h} \Bigg(\int_{[p_0-h,\, p_0)} \lambda p.\, \left|f_q(p) - f_q(p_0)\right| d\mu_{\mathrm{LBG}} \\
&\qquad\qquad + \int_{[p_0,\, p_0+h]} \lambda p.\, \left|f_q(p) - f_q(p_0)\right| d\mu_{\mathrm{LBG}} \Bigg) \\
&= \lim_{h\to 0} \frac{1}{2h} \int_{[p_0-h,\, p_0)} \lambda p.\, \left|f_q(p) - f_q(p_0)\right| d\mu_{\mathrm{LBG}} \\
&\qquad\qquad + \frac{1}{2h} \lim_{h\to 0} \int_{[p_0,\, p_0+h]} \lambda p.\, \left|f_q(p) - f_q(p_0)\right| d\mu_{\mathrm{LBG}}
\end{aligned}$$

The first term is calculated as follows:

(232)
$$\begin{aligned}
&\lim_{h\to 0} \frac{1}{2h} \int_{[p_0-h,\, p_0)} \lambda p.\, \left|f_q(p) - f_q(p_0)\right| d\mu_{\mathrm{LBG}} \\
&= \lim_{h\to 0} \frac{1}{2h} \int_{[p_0-h,\, p_0)} \lambda p.\, \left|0 - \pi\{r(q)\}^2\, v(p_0)\right| d\mu_{\mathrm{LBG}} \\
&= \lim_{h\to 0} \frac{1}{2h} \int_{[p_0-h,\, p_0)} \lambda p.\, \pi\{r(q)\}^2\, v(p_0)\, d\mu_{\mathrm{LBG}} \\
&= \lim_{h\to 0} \frac{1}{2h} \pi\{r(q)\}^2\, v(p_0)\, \mu_{\mathrm{LBG}}\big([p_0 - h, p_0)\big) \\
&= \lim_{h\to 0} \frac{1}{2h}\, \pi\{r(q)\}^2\, v(p_0)\, h \\
&= \lim_{h\to 0} \frac{1}{2}\, \pi\{r(q)\}^2\, v(p_0) \\
&= \frac{1}{2}\, \pi\{r(q)\}^2\, v(p_0)
\end{aligned}$$

Let us now evaluate the second term.

$$
\begin{aligned}
(233)\quad & \lim_{h\to 0}\frac{1}{2h}\int_{[p_0,p_0+h]}\lambda p.\left|f_q(p)-f_q(p_0)\right|d\mu_{\text{LBG}}\\
&=\lim_{h\to 0}\frac{1}{2h}\int_{[p_0,p_0+h]}\lambda p.\left|\pi\{r(q)\}^2 v(p)-\pi\{r(q)\}^2 v(p_0)\right|d\mu_{\text{LBG}}\\
&=\pi\{r(q)\}^2\lim_{h\to 0}\frac{1}{2h}\int_{[p_0,p_0+h]}\lambda p.\left|v(p)-v(p_0)\right|d\mu_{\text{LBG}}
\end{aligned}
$$

Since v is continuous by assumption, for any small positive number ε, there is some δ such that if $|p-p_0|<\delta$, then $\left|v(p)-v(p_0)\right|<\varepsilon$. Then, when $h<\delta$,

$$
\begin{aligned}
(234)\quad & \frac{1}{2h}\int_{[p_0,p_0+h]}\lambda p.\left|v(p)-v(p_0)\right|d\mu_{\text{LBG}}\\
&<\frac{1}{2h}\int_{[p_0,p_0+h]}\lambda p.\,\delta\, d\mu_{\text{LBG}}=\frac{1}{2h}\varepsilon\,\mu_{\text{LBG}}([p_0,p_0+h])=\frac{1}{2h}\varepsilon h=\frac{\varepsilon}{2}
\end{aligned}
$$

Since ε is an arbitrarily small positive number, this means that

$$
(235)\quad \lim_{h\to 0}\frac{1}{2h}\int_{[p_0,p_0+h]}\lambda p.\left|v(p)-v(p_0)\right|d\mu_{\text{LBG}}=0
$$

Therefore,

$$
\begin{aligned}
(236)\quad & \lim_{h\to 0}\frac{1}{2h}\int_{[p_0,p_0+h]}\lambda p.\left|f_q(p)-f_q(p_0)\right|d\mu_{\text{LBG}}\\
&=\pi\{r(q)\}^2\lim_{h\to 0}\frac{1}{2h}\int_{[p_0,p_0+h]}\lambda p.\left|v(p)-v(p_0)\right|d\mu_{\text{LBG}}=0
\end{aligned}
$$

It follows from (231), (232), and (236) that

$$
\begin{aligned}
(237)\quad & \lim_{h\to 0}\frac{1}{\mu_{\text{LBG}}(C(p_0,h))}\int_{C(p_0,h)}\lambda p.\left|f_q(p)-f_q(p_0)\right|d\mu_{\text{LBG}}\\
&=\frac{1}{2}\pi\{r(q)\}^2 v(p_0)+0=\frac{1}{2}\pi\{r(q)\}^2 v(p_0)\neq 0
\end{aligned}
$$

A parallel remark goes for p_1 as well, that is, p_1 is in general not a Lebesgue point of f_q. However, we can see that $D\Theta_{@,q}(p)=f_q(p)$ always holds except at p_0 and p_1 and the set $\{p_0,p_1\}$ is of measure 0, as Lebesgue's differentiation theorem has asserted.

For the bread-eating scenario, we were able to calculate the derivative, since we had given a good, specific model. For the general case, however, we need to check whether the derivative of $\Theta_{@,q}$ is defined. Below, I will show that it is actually defined a.e. with the help of the Radon-Nikodým theorem, but to that end, we should make a couple of natural assumptions about our models of worlds.

To begin with, notice that $\Theta_{w,q}$ is a set function, as its domain consists of intervals, which are sets of time points. We can now observe the following:

(238) Theorem

Let G be an open time interval that satisfies the conditions in Claim (221). $\Theta_{w,q}$ is then a measure on $(G, \mathfrak{B}(G))$.

Proof.

We verify that $\Theta_{w,q}$ satisfies the three conditions of the definition of measure (149).

(i) The nonnegativity of $\Theta_{w,q}$ directly follows from the nonnegativity of situation-sensitive lattice-measure function M.

(ii) $\Theta_{w,q}(\varnothing) = M(\langle w, [q,q]\rangle)\left(\mathop{\bigsqcup\nolimits_{\mathrm{e}}}_{p\in\varnothing} \mathbf{SDIC}^{\varphi}_{\psi}(\langle w, [p,p]\rangle)\right) = M(\langle w, [q,q]\rangle)(\perp_{\mathrm{e}}) = 0.$

(iii) Let $\{E_k\}_{k\in\mathbb{N}}$ be a pairwise disjoint collection of elements in $\mathfrak{B}(G)$. Then,

$$\begin{aligned}\Theta_{w,q}\left(\bigcup_{k=1}^{\infty} E_k\right) &= M(\langle w, [q,q]\rangle)\left(\mathop{\bigsqcup\nolimits_{\mathrm{e}}}_{p\in\bigcup_{k=1}^{\infty} E_k} \mathbf{SDIC}^{\varphi}_{\psi}(\langle w, [p,p]\rangle)\right)\\ &= M(\langle w, [q,q]\rangle)\left(\mathop{\bigsqcup\nolimits_{\mathrm{e}}}_{k=1}^{\infty} \mathop{\bigsqcup\nolimits_{\mathrm{e}}}_{p\in E_k} \mathbf{SDIC}^{\varphi}_{\psi}(\langle w, [p,p]\rangle)\right).\end{aligned}$$

Given that G satisfies the conditions in Claim (221), no two elements of $\left\{\mathop{\bigsqcup\nolimits_{\mathrm{e}}}_{p\in E_k} \mathbf{SDIC}^{\varphi}_{\psi}(\langle w, [p,p]\rangle)\right\}_{k\in\mathbb{N}}$ overlap. Therefore, it follows from the countable additivity of M that

$$\begin{aligned}&M(\langle w, [q,q]\rangle)\left(\mathop{\bigsqcup\nolimits_{\mathrm{e}}}_{k=1}^{\infty} \mathop{\bigsqcup\nolimits_{\mathrm{e}}}_{p\in E_k} \mathbf{SDIC}^{\varphi}_{\psi}(\langle w, [p,p]\rangle)\right)\\ &= \sum_{k=1}^{\infty} M(\langle w, [q,q]\rangle)\left(\mathop{\bigsqcup\nolimits_{\mathrm{e}}}_{p\in E_k} \mathbf{SDIC}^{\varphi}_{\psi}(\langle w, [p,p]\rangle)\right)\\ &= \sum_{k=1}^{\infty} \Theta_{w,q}(E_k).\end{aligned}$$

Hence,

$$\Theta_{w,q}\left(\bigcup_{k=1}^{\infty} E_k\right) = \sum_{k=1}^{\infty} \Theta_{w,q}(E_k).$$

Q.E.D.

I would now like to make a couple of natural claims.

(239) Claim

$\Theta_{w,q}$ is a σ-finite measure on $(G, \mathfrak{B}(G))$ for any w and q.

Given any bounded time interval E, it is normal to assume, for example, that the bread that John eats in w during E cannot have infinite volume regardless of when its volume is to be measured. For instance, under the scenario considered in Section 2.4, for $E = [a, b]$, this volume is given by $\Theta_{@,q}([a, b]) = \pi\{r(q)\}^2 \int_a^b v(p)\, dp$. For this to be infinite, either the diameter must be infinite at q or John must have eaten an infinite length of bread in this bounded time span, but neither is imaginable. Given any open time interval G, one can express G as an at most countable union of bounded intervals.[21] $\Theta_{@,q}$ in this case is then a σ-finite measure on $(G, \mathfrak{B}(G))$. In general, the $\Theta_{w,q}$ that is defined for any nominal predicate φ and a main predicate ψ of production / consumption is expected to be σ-finite since one cannot produce or consume an infinite amount of something in a bounded time span. Therefore, the above claim is a very natural assumption of the models we have in mind.

(240) Claim

$\Theta_{w,q}$ is absolutely continuous with respect to μ_{LBG}.

Let $E \in \mathfrak{B}(G)$ such that $\mu_{\text{LBG}}(E) = 0$. That is, E is a collection of time points in G whose collective length is 0. It is then obvious that one cannot produce or consume any positive amount of something in E and thus $\Theta_{w,q}(E) = 0$. Therefore, this claim as well is a very normal assumption of our models.

Summing up the above two claims, $\Theta_{w,q}$ is a σ-finite measure on $(G, \mathfrak{B}(G))$ and absolutely continuous with respect to the Lebesgue measure μ_{LBG}. Recall now that μ_{LBG} is a also σ-finite measure on $(G, \mathfrak{B}(G))$. Then, by applying the Radon-Nikodým theorem to $\Theta_{w,q}$, we can obtain the following result:

(241) There is a $\mathfrak{B}(G)$-measurable function $\theta_{w,q}$ from G into $[0, +\infty)$ such that for any $E \subseteq G$,

$$\Theta_{w,q}(E) = \int_E \theta_{w,q}(p)\, d\mu_{\text{LBG}}.$$

In the case of the bread-eating scenario, function f_q in (223) qualifies as $\theta_{@,q}$ for the $\Theta_{@,q}$ defined with bread, John-eat and cubic-centimeter. By applying Lebesgue's differentiation theorem to the equation in (241), we can then obtain the following result, just as we have seen for the model of the bread-eating story above:

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[21] If G is itself bounded, we do not even have to consider this.

(242) $\Theta_{w,q}$ is differentiable a.e. $[\mu_{\text{LBG}}]$ on G and its derivative $D\Theta_{w,q}(p)$ is equal to $\theta_{w,q}(p)$ a.e. $[\mu_{\text{LBG}}]$ on G.

By applying the Radon-Nikodým theorem to $\Theta_{w,q}$ for all $q \in G$, we can obtain a collection of functions $\{\theta_{w,q}\}_{q \in G}$ that each satisfy the equation in (214). For each q, we expect $\theta_{w,q}(q)$ to give the rate of the q-size of something consumed or produced per unit time at q. Thus, we can think of the function $\lambda p.\, \theta_{w,p}(p)$ $(= \lambda q.\, \theta_{w,q}(q))$. For the bread-eating scenario, $\lambda p.\, \theta_{@,p}(p) = \lambda p.\, f_p(p)$ gives such a function. The truth conditions of continuous event-related readings of production or consumption are then in general expected to be expressed as a condition on the value of an integral of $\lambda p.\, \theta_{w,p}(p)$, where the corresponding $\Theta_{w,p}(p)$ is defined with appropriate predicates. Unlike in Section 2.4, the integral should now be given as a Lebesgue integral with the Lebesgue measure. Then, the truth conditions of (132) should then look like the following:

(243) $$\int_{[p_0, p_1]} \lambda p.\, \theta_{@,p}(p)\, d\mu_{\text{LBG}} = \int_{[p_0, p_1]} \lambda p.\, f_p(p)\, d\mu_{\text{LBG}} = 666$$

Since q varies within G, $\theta_{w,q}(p)$ can be regarded as a function defined on the open square $G \times G$. Then, calculating the integral of $\lambda p.\, \theta_{w,p}(p)$ can be understood as calculating the integral of $\theta_{w,q}(p)$ on the diagonal $\{(q, p) \mid q = p\}$ of this square.

There is a problem, however. In general, when some $q \in G$ is fixed, what we know is only that $D\Theta_{w,q}$ is defined almost everywhere and that $\theta_{w,q}$ is equal to $D\Theta_{w,q}$ almost everywhere. If $D\Theta_{w,q}(q)$ is not defined for some $q \in G$, $\theta_{w,q}(q)$ will have a random value that does not have to do with the rate. In general, when q is not a Lebesgue point of $\theta_{w,q}$, $\theta_{w,q}(q)$ can be any random value. We know from (242) that the set of non-Lebesgue points of $\theta_{w,q}$ is of measure 0 for any $q \in G$. However, it is in theory possible for the set of points q such that q is not a Lebesgue point of $\theta_{w,q}$ to be of positive measure, that is, it is possible that all such "misbehaving" points on the diagonal constitute a set of positivie measure. If this were the case, we would not be able to obtain the correct total amount of consumption or production by integrating $\lambda p.\, \theta_{w,p}(p)$.

To avoid this problem, I would like to make the following claim.

(244) Claim

Let $\{\theta_{w,q}\}_{q \in G}$ be a collection of functions whose existence is asserted in (241). If p is a Lebesgue point of $\lambda p'.\, \theta_{w,q}(p')$ for some $q \in G$, then, p is in fact a Lebesgue point of function $\lambda p'.\, \theta_{w,q}(p')$ for every $q \in G$, and $\lambda q.\, \theta_{w,q}(p)$, a function of q where p' is fixed to p, is a continuous function of q.

This claim is not an unreasonable one. To begin with, let us consider the first half of the claim, namely, the claim that if p is a Lebesgue point of $\lambda p'.\,\theta_{w,q}(p')$ for some $q \in G$, then, p is a Lebesgue point of function $\lambda p'.\,\theta_{w,q}(p')$ for every $q \in G$. This could be violated if, for instance, for some points $q_0, q_1 \in G$, $\lambda p'.\,\theta_{w,q_0}(p')$ is continuous at p while $\lambda p'.\,\theta_{w,q_1}(p')$ is not continuous at p. However, since $\lambda p'.\,\theta_{w,q}(p')$ is a function that gives the rate of q-size of something consumed or produced per unit time at p' for almost all $p' \in G$, functions $\lambda p'.\,\theta_{w,q_0}(p')$ and $\lambda p'.\,\theta_{w,q_1}(p')$ shall essentially be the same thing (almost everywhere) except that the size on which the rate is based on is measured at different time points. Thus, if $\lambda p'.\,\theta_{w,q_0}(p')$ is continuous at p, $\lambda p'.\,\theta_{w,q_1}(p')$ is also expected to be continuous at p and vice versa. Hence, it does not seem to make sense to think of models where the first half of the claim does not hold. Let us now consider the second half of the claim, namely, the claim that $\lambda q.\,\theta_{w,q}(p)$ is a continuous function of q when p is a Lebesgue point of $\lambda p'.\,\theta_{w,q}(p')$ for some (in fact, every) $q \in G$. This is also a very natural assumption as what it says is just that the rate of q-size of something produced or consumed per unit time at p gradually varies as q varies.[22]

Now, pick some $q_0 \in G$. Since we are assuming that $\Theta_{w,q}(E)$ is finite for any bounded interval E, $\lambda p.\,\theta_{w,q_0}$ is locally integrable on G. Therefore, almost every point of G is a Lebesgue point of $\lambda p.\,\theta_{w,q_0}$. Let $\tilde{G}$ be the set of all Lebesgue points of $\lambda p'.\,\theta_{w,q_0}(p')$. Then, by Claim (244), if $p \in \tilde{G}$, p is a Lebesgue point of $\lambda p'.\,\theta_{w,q_0}(p')$ for every $q \in G$. Thus,

(245) $\tilde{G} = \{p \mid p \in G \wedge \forall q[q \in G \rightarrow p \text{ is a Lebesgue point of } \lambda p'.\,\theta_{w,q}(p')]\}$

By Claim (244), for each $p \in \tilde{G}$, $\lambda q.\,\theta_{w,q}(p)$ is a continuous function of q. Therefore, for any $q_1 \in G$, we can obtain the value of $\theta_{w,q_1}(p)$ by virtue of taking the limit of $\theta_{w,q}(p)$ as q moves from q_0 to q_1. It can then be concluded that for all $p \in \tilde{G}$, i.e., for almost all $p \in G$, $\theta_{w,p}(p)$ gives the relevant rate. By integrating $\lambda p.\,\theta_{w,p}(p)$, we should be able to calculate

---

[22] Let $\xi(p, h_n) = \bigsqcup_{\mathsf{e}\ p' \in [p-h_n, p+h_n]} \mathbf{SDIC}^{\mathsf{bread}}_{\mathsf{John\text{-}eat}}(\langle w, [p', p']\rangle)$. That is, $\xi(p, h_n)$ is the individual which is the bread that John ate during time interval $[p-h_n, p+h_n]$. When p and h_n are fixed, $\xi(p, h_n)$ defines an individual which had different sizes at different time points as it kept expanding and shrinking, but this change ought to be gradual. It is thus natural to assume that the q-size of $\xi(p, h_n)$ is continuous as a function of q for any p and $h_n > 0$. To understand this, it might be helpful to think of people's weight. For instance, in order for Ann's weight to increase, she has to eat a lot of food, which is a time-consuming process. Her weight cannot just rise from 49 kgs to 52 kgs in a moment. Now, assuming that $\lim_{n \to \infty} h_n = 0$, $\theta_{w,q}(p')$ is given as the limit of (q-size of $\xi(p, h_n))/2h_n$ as $n \to \infty$. If p is a Lebesgue point of $\theta_{w,q}$, this is actually the rate of the q-size of the bread John ate per unit time at p. Since (q-size of $\xi(p, h_n))/2h_n$ is continuous for all n, it seems fair to expect that the limit of (q-size of $\xi(p, h_n))/2h_n$ is also continuous as a function of q. This does not follow mathematically. It will, however, provided that (q-size of $\xi(p, h_n))/2h_n$, as a function of q, uniformly converges on G when $n \to \infty$.

the total amount consumed or produced. It should be now recalled, however, that whatever function to be integrated must be measurable to begin with. Therefore, it is important that $\lambda p.\, \theta_{w,p}(p)$ is indeed a measurable function. This can actually be shown to be the case. Here, let us consider bounded open intervals.

(246) Lemma

Let G be a non-empty bounded open time interval that satisfies the condition in Claim (221). Then, there is a $\mathfrak{B}(G)$-measurable function δ defined on G such that $\delta(p) = \theta_{w,p}(p)$ for all $p \in \tilde{G}$, and thus almost everywhere on G.

Proof.

Assume that $G = (a,b)$, $a < b$. For a given $n \in \mathbb{N}$, divide G into 2^n number of intervals $\{E^n_k\}$, $1 \leq k \leq 2^n$ such that

$$E^n_1 = \left(a,\, a + \frac{1}{2^n}(b-a)\right), \quad E^n_2 = \left[a + \frac{1}{2^n}(b-a),\, a + \frac{2}{2^n}(b-a)\right), \quad \ldots,$$

$$E^n_k = \left[a + \frac{k-1}{2^n}(b-a),\, a + \frac{k}{2^n}(b-a)\right), \quad \ldots,$$

$$E^n_{(2^n-1)} = \left[a + \frac{2^n-2}{2^n}(b-a),\, a + \frac{2^n-1}{2^n}(b-a)\right), \quad E^n_{2^n} = \left[a + \frac{2^n-1}{2^n}(b-a),\, b\right).$$

For all n and k such that $1 \leq k \leq 2^n$, let q^n_k be an arbitrary point in E^n_k, $q^n_k \in E^n_k$. For all $q \in G$, let $\theta_{w,q}(p)$ be a $\mathfrak{B}(G)$-measurable function whose existence is guaranteed by the Radon-Nikodým theorem as stated in (241). Since we vary q, let us write $\theta_w(q)(p)$ rather than $\theta_{w,q}(p)$ for the sake of better legibility. Define $\{\delta_n\}$ to be a sequence of functions defined on G such that

$$\delta_n(p) = \theta(q^n_k)(p) \quad \text{if } p \in E^n_k, 1 \leq k \leq 2^n.$$

Put differently,

$$\delta_n(p) = \sum_{k=1}^{2^n} \theta(q^n_k)(p)\chi_{E^n_k}.$$

Being a characteristic function of a measurable set, $\chi_{E^n_k}$ is $\mathfrak{B}(G)$-measurable for any n and k. Then, δ_n is a sum of products of $\mathfrak{B}(G)$-measurable functions, and hence $\mathfrak{B}(G)$-measurable (see). Now define δ as follows:

$$\delta = \liminf_{n \to \infty} \delta_n.$$

δ is then a function whose existence is asserted by the theorem. First, observe that δ is

$\mathfrak{B}(G)$-measurable since it is the limit inferior of a sequence $\{\delta_n\}$ of $\mathfrak{B}(G)$-measurable functions (see). Now, given any $p \in \tilde{G}$, one can find a unique sequence of intervals $\{E^n_{k_n}\}_{n\in\mathbb{N}}$ such that $p \in E^n_{k_n}$ for all $n \in \mathbb{N}$. Clearly, $E^1_{k_1} \supset E^2_{k_2} \supset E^3_{k_3} \supset \ldots$ and $\bigcap_{n=1}^{\infty} E^n_{k_n} = \{p\}$. Then, according to the definition of δ_n, we have

$$\delta_n(p) = \theta_w(q^n_{k_n})(p).$$

Since $q^n_{k_n} \in E^n_{k_n}$ for all n, it follows that $\lim_{n\to\infty} q^n_{k_n} \in \bigcap_{n=1}^{\infty} E^n_{k_n} = \{p\}$, which means that $\{q^n_{k_n}\}$ converges to p. Because $p \in \tilde{G}$, $\lambda q.\, \theta_w(q)(p)$ is a continuous function of q by assumption. Hence,

$$\lim_{n\to\infty} \delta_n(p) = \lim_{n\to\infty} \theta_w(q^n_{k_n})(p) = \theta_w(p)(p).$$

As $\lim_{n\to\infty} \delta_n(p) = \liminf_{n\to\infty} \delta_n(p) = \limsup_{n\to\infty} \delta_n(p)$, this means that $\delta(p) = \theta_{w,p}(p)$. Since p was an arbitrary point in $\tilde{G}$, the theorem has been proved. Q.E.D.

Recall that in the discussion so far, time interval G has been assumed to satisfy the condition in Claim (221). In the case of bread-eating scenario, since no part of the bread was eaten more than once by John, any bounded open interval G that contains $[p_0, p_1]$ satisfies the conditions in Claim (221). Therefore, by integrating a function δ obtained as in the above lemma for this case will correctly calculate the total amount of bread that John has eaten. However, we would now like to deal with cases where the same individuals get consumed more than once, as in the story where John drinks the same XYZ more than once by use of a catheter inserted into his stomach. In this story, if $I_{\text{drink}} = [p_5, p_6]$, where p_5 and p_6 are the time at which John started drinking XYZ and the time at which John stopped drinking XYZ, respectively, then, no open time interval G that contains I_{drink} satisfies the condition in Claim (221). We would thus like a more general version of the above lemma that is stated without the condition on G. Now, let I be a closed time interval which is exactly the running time interval of some consumption. By Claim (221), I can be covered with a collection of small enough open time intervals that each satisfy the condition in Claim (221). For each of these small open time intervals, the above lemma asserts the existence of an appropriate measurable function defined on it. Then, by connecting these measurable functions, we expect to obtain a relevant measurable function defined on the whole of I.

(247) Theorem

For any bounded closed time interval I, there is an open interval G and a function δ such that $I \subset G$ and δ is a $\mathfrak{B}(G)$-measurable function defined on G whose value is given by the following for almost all $p \in G$:

$$\delta(p) = D\Theta_{w,p}(p) = \lim_{h \to 0} \frac{1}{2h} M(\langle w, [p,p] \rangle) \left(\bigsqcup_{\mathrm{e}\ p' \in [p-h, p+h]} \mathbf{SDIC}_{\psi}^{\varphi}(\langle w, [p', p'] \rangle) \right).$$

Proof.

By Claim (221), for each $p \in I$, one can find an open time interval G_p that satisfies the condition in Claim (221). Then, $\bigcup_{p \in I} G_p$ gives an open cover of I, i.e., $I \subset \bigcup_{p \in I} G_p$. Since I is a closed bounded interval, I is compact (the Heine-Borel theorem), and hence, I can be covered by a finite subset $\{G_{p_1}, G_{p_2}, \dots, G_{p_n}\}$ of $\{G_p\}_{p \in I}$. Let $G = \bigcup_{k=1}^{n} G_{p_k}$. Then,

$$I \subset \bigcup_{k=1}^{n} G_{p_k} = G.$$

Now, for each $k = 1, 2, \dots, n$, apply Lemma (246) to G_{p_k} and let δ_k be a function whose existence is asserted by the lemma. Thus, for each k, δ_k is a $\mathfrak{B}(G_{p_k})$-measurable function defined on G_{p_k} such that $\delta_k(p) = \theta_{w,p}(p)$ for almost all $p \in G_{p_k}$. Because δ_k is $\mathfrak{B}(G_{p_k})$-measurable, for any $V \in \mathfrak{B}([0, \infty])$,

$$\delta_k^{-1}(V) = \{p \mid p \in G_{p_k} \wedge \delta_1(p) \in V\} \in \mathfrak{B}(G_{p_k})$$

Since $G_{p_k} \subset G$, $\mathfrak{B}(G_k) \subset \mathfrak{B}(G)$. It then follows that $\delta_k^{-1}(V) \in \mathfrak{B}(G)$ for each k. Let us now define a function δ on G as follows:

$$\delta(p) = \begin{cases} \delta_1(p) & \text{if } p \in G_{p_1} \\ \delta_2(p) & \text{if } p \in G_{p_2} \cap G_{p_1}{}^{\mathrm{c}} \\ \delta_3(p) & \text{if } p \in G_{p_3} \cap (G_{p_1} \cup G_{p_1})^{\mathrm{c}} \\ \vdots & \\ \delta_n(p) & \text{if } p \in G_{p_n} \cap \left(\bigcup_{j=1}^{n-1} G_{p_j} \right)^{\mathrm{c}} \end{cases}$$

Then, for any $V \in \mathfrak{B}([0,\infty])$,

$$\begin{aligned}
\delta^{-1}(V) &= \{p \mid p \in G \wedge \delta(p) \in V\} \\
&= \left\{p \,\middle|\, \bigvee_{k=1}^{n} \left[p \in G_{p_k} \cap \left(\bigcup_{j=1}^{k-1} G_{p_j}\right)^{\mathrm{c}} \wedge \delta_k(p) \in V\right]\right\} \\
&= \bigcup_{k=1}^{n} \left\{p \,\middle|\, p \in G_{p_k} \cap \left(\bigcup_{j=1}^{k-1} G_{p_j}\right)^{\mathrm{c}} \wedge \delta_k(p) \in V\right\} \\
&= \bigcup_{k=1}^{n} \left(\{p \mid p \in G_{p_k} \wedge \delta_k(p) \in V\} \cap \left(\bigcup_{j=1}^{k-1} G_{p_j}\right)^{\mathrm{c}} \right) \\
&= \bigcup_{k=1}^{n} \left({\delta_k}^{-1}(V) \cap \left(\bigcup_{j=1}^{k-1} G_{p_j}\right)^{\mathrm{c}} \right)
\end{aligned}$$

Since $G_{p_j} \in \mathfrak{B}(G)$ for any j, the complements of their unions are also in $\mathfrak{B}(G)$. Because ${\delta_k}^{-1}(V) \in \mathfrak{B}(G)$ for each k as seen above, $\delta^{-1}(V)$ is then a union of intersections of members in $\mathfrak{B}(G)$, and thus it is itself a member of $\mathfrak{B}(G)$. Hence, δ is $\mathfrak{B}(G)$-measurable. Since $\delta_k(p) = \theta_{w,p}(p)$ for almost all $p \in G_{p_k}$, $\delta(p) = \theta_{w,p}(p)$ for almost all $p \in G$. According to (215), whenever $D\Theta_{w,p}(p)$ is defined, it is given by the following:

$$D\Theta_{w,p}(p) = \lim_{h \to 0} \frac{1}{2h} M(\langle w, [p,p] \rangle)\left(\bigsqcup_{p' \in [p-h, p+h]}^{\mathrm{e}} \mathbf{SDIC}_{\psi}^{\varphi}(\langle w, [p', p'] \rangle) \right)$$

By (227), we see that this gives the value of $\theta_{w,p}(p)$, and thus $\delta(p)$ for almost all $p \in G$. Q.E.D.

Thus, we have finally obtained a measurable function that yields the rate of p-size of the relevant stuff produced or consumed per unit time at time point p in w. By integrating $\delta(p)$, the total amount of the relevant stuff produced or consumed will be calculated!

Let us now return to the starting point of our discussion. Originally, we had hoped that the following would give the correct truth conditions for (132):

(220) $\int_B \varrho_{\text{cubic-centimeter}}(\mathbf{SDIC}_{\text{John-eat}}^{\text{bread}})\, d(\mu_{\mathrm{LBG}} \circ \iota \circ \pi_{\mathrm{i}}) = 666$

This did now work since the integrand is equal to 0 for momentary situations. Let us then redefine ϱ so that for momentary situations, it will give a relevant rate function to be integrated for the given continuous event-related reading. In other words, for momentary situations, $\varrho_M(\mathbf{SDIC}^{\varphi}_{\psi})$ is a function δ whose existence is asserted by Theorem (247). For situations that are not momentary, the definition of ϱ remains unaltered. To define ϱ, let **MO** (for MOments) denote the set of all momentary situations, $\mathbf{MO} = \{\langle w, [p, p]\rangle \mid w$ is a world and p is a time point$\}$ and let **SP** (for time SPans) denote the set of all situations whose time interval is not momentary. Here is the final definition of ϱ:

(248) Given a situation-sensitive lattice-measure function $M \in \mathrm{D}_{\langle \mathrm{s},\langle \mathrm{e},\mathrm{n}\rangle\rangle}$ and a function from situations into individuals $\xi \in \mathrm{D}_{\langle \mathrm{s},\mathrm{e}\rangle}$, $\varrho_M(\xi) \in \mathrm{D}_{\langle \mathrm{s},\mathrm{n}\rangle}$ is a function from situations into reals such that

(i) if $s \in \mathbf{MO}$,

$$\varrho_M(\xi)(s) \overset{\text{def}}{=} \lim_{h\to 0} \frac{1}{2h} M(s)\left(\bigsqcup_{p' \in [(\iota\circ\pi_{\mathrm{i}})(s)-h,\, (\iota\circ\pi_{\mathrm{i}})(s)+h]} \xi(\langle \pi_{\mathrm{w}}(s), [p', p']\rangle)\right) \quad \text{a.e.}$$

(ii) if $s \in \mathbf{SP}$,

$$\varrho_M(\xi) \overset{\text{def}}{=} \lambda s \in \mathrm{D_s} : \inf_{p\in\pi_{\mathrm{i}}(s)} M\big(\langle \pi_{\mathrm{w}}(s), [p, p]\rangle\big)(\xi(s)) = \sup_{p\in\pi_{\mathrm{i}}(s)} M\big(\langle \pi_{\mathrm{w}}(s), [p, p]\rangle\big)(\xi(s)).$$
$$\inf_{p\in\pi_{\mathrm{i}}(s)} M\big(\langle \pi_{\mathrm{w}}(s), [p, p]\rangle\big)(\xi(s))$$

The clause (i) does not speak of the values that $\varrho_M(\xi)$ when the limit does not exist. Since the set of those points is of measure 0 by Theorem (247), they have no effect when $\varrho_M(\xi)$ is integrated. $\varrho_M(\xi)$ is thus allowed to take any values at those points.[23] Let us see how this definition works in the case of the bread-eating scenario. By substituting $\mathbf{SDIC}^{\text{bread}}_{\text{John-eat}}$ for ξ, we have

(249) $\varrho_{\text{cubic-centimeter}}(\mathbf{SDIC}^{\text{bread}}_{\text{John-eat}})(\langle @, [p, p]\rangle)$

$$= \lim_{h\to 0} \frac{1}{2h} \text{cubic-centimeter}(\langle @, [p, p]\rangle)\left(\bigsqcup_{p' \in [p-h,\, p+h]} \mathbf{SDIC}^{\text{bread}}_{\text{John-eat}}(\langle @, [p', p']\rangle)\right)$$
$$= D\Theta_{@,p}(p)$$

We have already calculated $D\Theta_{@,q}(p)$ in (230). Since $p = \iota(\pi_{\mathrm{i}}(\langle w, [p, p]\rangle))$, we can obtain the following:

(250) $\varrho_{\text{cubic-centimeter}}(\mathbf{SDIC}^{\text{bread}}_{\text{John-eat}})(s)$

$$= \begin{cases} \pi\{r(\iota(\pi_{\mathrm{i}}(s)))\}^2\, v(\iota(\pi_{\mathrm{i}}(s))) & \text{if } s \in \{\langle @, [p, p]\rangle \mid p \in (p_0, p_1)\} \\ \frac{1}{2}\pi\{r(\iota(\pi_{\mathrm{i}}(s)))\}^2\, v(\iota(\pi_{\mathrm{i}}(s))) & \text{if } s = \langle @, [p_0, p_0]\rangle, \langle @, [p_1, p_1]\rangle \\ 0 & \text{if } s \in \{\langle @, [p, p]\rangle \mid p \notin [p_0, p_1]\} \end{cases}$$

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[23] Assuming that $\lambda s : s \in \mathbf{MO}.\varrho_M(\xi)(s)$ is integrable, (248I) determines a unique equivalence class of functions as an element of L^1.

Therefore, the integral in (220) is calculated as follows:

(251)
$$\int_B \varrho_{\text{cubic-centimeter}}\left(\mathbf{SDIC}^{\text{bread}}_{\text{John-eat}}\right) d(\mu_{\text{LBG}} \circ \iota \circ \pi_{\text{i}})$$

$$= \int_{B \cap \{\langle @,[p,p]\rangle \mid p \in (p_0,p_1)\}} \lambda s \in \text{D}_{\text{s}}.\, \pi\{r(\iota(\pi_{\text{i}}(s)))\}^2\, v(\iota(\pi_{\text{i}}(s)))\, d(\mu_{\text{LBG}} \circ \iota \circ \pi_{\text{i}})$$
$$+ \int_{B \cap \{p_0,p_1\}} \lambda s \in \text{D}_{\text{s}}.\, \frac{1}{2} \pi\{r(\iota(\pi_{\text{i}}(s)))\}^2\, v(\iota(\pi_{\text{i}}(s)))\, d(\mu_{\text{LBG}} \circ \iota \circ \pi_{\text{i}})$$
$$+ \int_{B \cap \{\langle @,[p,p]\rangle \mid p \notin [p_0,p_1]\}} 0\, d(\mu_{\text{LBG}} \circ \iota \circ \pi_{\text{i}})$$
$$= \int_{B \cap \{\langle @,[p,p]\rangle \mid p \in (p_0,p_1)\}} \lambda s \in \text{D}_{\text{s}}.\, \pi\{r(\iota(\pi_{\text{i}}(s)))\}^2\, v(\iota(\pi_{\text{i}}(s)))\, d(\mu_{\text{LBG}} \circ \iota \circ \pi_{\text{i}})$$

If the sentence is a description of what happened yesterday, B is the set of all momentary situations in yesterday, and $[p_0, p_1] \subseteq B$. Therefore, $B \cap \{\langle @, [p,p]\rangle \mid p \in (p_0, p_1)\} = \{\langle @, [p,p]\rangle \mid p \in (p_0, p_1)\}$. Furthermore, $\pi_{\text{i}}(\{\langle @, [p,p]\rangle \mid p \in (p_0, p_1)\}) = \{[p,p] \mid p \in (p_0, p_1)\}$ and $\iota(\{[p,p] \mid p \in (p_0, p_1)\}) = \{p \mid p \in (p_0, p_1)\} = (p_0, p_1)$. Therefore,

(252)
$$\int_{B \cap \{\langle @,[p,p]\rangle \mid p \in (p_0,p_1)\}} \lambda s \in \text{D}_{\text{s}}.\, \pi\{r(\iota(\pi_{\text{i}}(s)))\}^2\, v(\iota(\pi_{\text{i}}(s)))\, d(\mu_{\text{LBG}} \circ \iota \circ \pi_{\text{i}})$$
$$= \int_{\{\langle @,[p,p]\rangle \mid p \in (p_0,p_1)\}} \lambda s \in \text{D}_{\text{s}}.\, \pi\{r(\iota(\pi_{\text{i}}(s)))\}^2\, v(\iota(\pi_{\text{i}}(s)))\, d(\mu_{\text{LBG}} \circ \iota \circ \pi_{\text{i}})$$
$$= \int_{\{[p,p] \mid p \in (p_0,p_1)\}} \lambda I \in \text{D}_{\text{i}}.\, \pi\{r(\iota(I))\}^2\, v(\iota(I))\, d(\mu_{\text{LBG}} \circ \iota)$$
$$= \int_{(p_0,p_1)} \lambda p \in \text{D}_{\text{n}}.\, \pi\{r(p)\}^2\, v(p)\, d\mu_{\text{LBG}}$$

This Lebesgue integral is equal to the Riemann integral $\int_{p_0}^{p_1} \pi\{r(p)\}^2\, v(p)\, dp$, which we had in Section 2.4, provided that $\pi\{r(p)\}^2 v(p)$ is Riemann-integrable.

Since we have derived the truth conditions only by redefining ϱ, the lexical entry for *cubic-centimeter* looks very similar to that given to $\varnothing_{\text{ATOM}}$ in the end of the previous subsection. The differences are only obvious. First, *cubic-centimeter* has cubic-centimeter instead of ATOM as the situation-sensitive lattice-measure function. Second, rather than the set of all situations in a given time interval, the domain of integration is now the set of all momentary situations in a given time interval (i.e. B above and below). Finally, the measure is now $\mu_{\text{LBG}} \circ \iota \circ \pi_{\text{i}}$. Thus, the lexical entry for *cubic-centimeter* is given by the following:

(253) $[\![\texttt{cubic centimeter}]\!] = \lambda v \in \mathrm{D}_{\langle \mathrm{n,t}\rangle}.\, \lambda R \in \mathrm{D}_{\langle \mathrm{e},\langle \mathrm{s,t}\rangle\rangle}.\, \exists n \in \mathrm{D}_{\mathrm{n}}$

$$\left[v(n) \wedge \int_B \varrho_{\text{cubic-centimeter}} \left(\lambda s \in \mathrm{D}_{\mathrm{s}}. \bigsqcup\nolimits_{\mathrm{e}} \{x \mid R(x)(s)\} \right) d(\mu_{\mathrm{LBG}} \circ \iota \circ \pi_{\mathrm{i}}) = n \right]$$

2.6.3 Decomposition of the measure

We have seen right above that when *cubic centimeters* is in the measure phrase of a continuous-event-related reading-inducing noun phrase, it has the lexical entry in (253), where the relevant measure is $(\mu_{\mathrm{LBG}} \circ \iota \circ \pi_{\mathrm{i}})$ and the domain of integration is a set of momentary situations. However, if *cubic centimeters* is in the measure phrase of a discrete-event-related reading-inducing noun phrase, such as (254) below, the measure $\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}}$ should be used, and the domain of integration should be the set of all situations in a context time interval, just as we saw in Subsection 2.6.1:

(254) Mary threw 65535 cubic centimeters of apples.

The lexical entry of *cubic centimeters* should now be the following in this case, where A is the set of all situations in a context time interval:

(255) $[\![\texttt{cubic centimeter}]\!] = \lambda v \in \mathrm{D}_{\langle \mathrm{n,t}\rangle}.\, \lambda R \in \mathrm{D}_{\langle \mathrm{e},\langle \mathrm{s,t}\rangle\rangle}.\, \exists n \in \mathrm{D}_{\mathrm{n}}$

$$\left[v(n) \wedge \int_A \varrho_{\text{cubic-centimeter}} \left(\lambda s \in \mathrm{D}_{\mathrm{s}}. \bigsqcup\nolimits_{\mathrm{e}} \{x \mid R(x)(s)\} \right) d(\mu_{\mathrm{CT}} \circ \pi_{\mathrm{i}}) = n \right]$$

This is problematic, since it is not desirable to conclude that units of measurement in general have an ambiguous lexical entry. Furthermore, it is not clear how the lexical semantics of units of measurement could be sensitive to the type of the main predicate. Ideally, we would like an analysis of a unified form for all cases.

To achieve this goal, first, recall that the problem of the domain of integration has arisen from the fact that $\mu_{\mathrm{LBG}} \circ \iota \circ \pi_{\mathrm{i}}$ is not defined for sets containing situations that are not momentary. So, let us replace ι with a function ι_{MO} defined below, which takes sets of time intervals and returns sets of time points.

(256) ι_{MO} is a function from sets of time intervals into sets of time points such that for any set $\mathscr{T}$ of time intervals,

$$\iota_{\mathrm{MO}}(\mathscr{T}) \overset{\mathrm{def}}{=} \{\text{the unique } p \text{ such that } p \in I \mid I \in \mathscr{T} \cap \pi_{\mathrm{i}}(\mathbf{MO})\}$$

When $S \subseteq \mathbf{MO}$, ι_{MO} does the same job as our old ι did. When $S \subseteq \mathbf{SP}$, on the other hand, $\mu_{\mathrm{LBG}} \circ \iota_{\mathrm{MO}} \circ \pi_{\mathrm{i}}(S) = \mu_{\mathrm{LBG}}(\varnothing) = 0$. Thus, $\mu_{\mathrm{LBG}} \circ \iota_{\mathrm{MO}} \circ \pi_{\mathrm{i}}$ can now take sets that contain situations that are not momentary as well, but it "ignores" non-momentary situations entirely. Therefore, if we use $\mu_{\text{LBG}} \circ \iota_{\text{MO}} \circ \pi_{\text{i}}$ instead of $\mu_{\text{LBG}} \circ \iota \circ \pi_{\text{i}}$ for continuous event-related readings (i.e., if $\mu_{\text{LBG}} \circ \iota_{\text{MO}} \circ \pi_{\text{i}}$ is used in (253)), the domain of integration can now always be the set of all situations in a context time, be they momentary or not. One difference between (253) and (255) can thus be eliminated (i.e., B in (253) can be replaced with A).

Next, notice that any discrete events we think of seem to have a running time of some positive length. For instance, an event of John's jumping begins at the moment John's feet leave the ground and ends at the moment they touch the ground again. Let us therefore assume that discrete events always take a positive length of time to occur. Then, we need no longer count momentary situations with the counting measure when we consider discrete event-related readings. Therefore, let us define ι_{SP} as follows:

(257) ι_{SP} is a function from sets of time intervals into sets of time intervals such that for any set $\mathscr{T}$ of time intervals,

$$\iota_{\text{SP}}(\mathscr{T}) \overset{\text{def}}{=} \mathscr{T} \cap \pi_{\text{i}}(\mathbf{SP})$$

When $S \subseteq \mathbf{MO}$, $\mu_{\text{CT}} \circ \iota_{\text{SP}} \circ \pi_{\text{i}}(S) = \mu_{\text{CT}}(\varnothing) = 0$. When $S \subseteq \mathbf{SP}$, ι_{SP} does nothing, so $\mu_{\text{CT}} \circ \iota_{\text{SP}} \circ \pi_{\text{i}}(S) = \mu_{\text{CT}} \circ \pi_{\text{i}}(S)$. Thus, $\mu_{\text{CT}} \circ \iota_{\text{SP}} \circ \pi_{\text{i}}$ does the same job as $\mu_{\text{CT}} \circ \pi_{\text{i}}(S)$ for non-momentary situations while "ignoring" momentary situations entirely.

To recapitulate, $\mathbf{MO} \cap \mathbf{SP} = \varnothing$ and $\mathbf{MO} \cup \mathbf{SP} = \text{D}_{\text{s}}$, and $\mu_{\text{LBG}} \circ \iota_{\text{MO}} \circ \pi_{\text{i}}(\mathbf{SP}) = \mu_{\text{CT}} \circ \iota_{\text{SP}} \circ \pi_{\text{i}}(\mathbf{MO}) = 0$. Thus, $\mu_{\text{LBG}} \circ \iota_{\text{MO}} \circ \pi_{\text{i}}$ and $\mu_{\text{CT}} \circ \iota_{\text{SP}} \circ \pi_{\text{i}}$ are mutually singular measures.

(258) $\mu_{\text{LBG}} \circ \iota_{\text{MO}} \circ \pi_{\text{i}} \perp \mu_{\text{CT}} \circ \iota_{\text{SP}} \circ \pi_{\text{i}}$

Let us now define a new measure μ as the sum of these two.

(259) $\mu \overset{\text{def}}{=} \mu_{\text{LBG}} \circ \iota_{\text{MO}} \circ \pi_{\text{i}} + \mu_{\text{CT}} \circ \iota_{\text{SP}} \circ \pi_{\text{i}}$

Let $(\text{D}_{\text{s}}, \mathfrak{F}, \mu)$ be a measure space. For any $E \in \mathfrak{F}$, and a non-negative-valued function f defined on E,

(260)
$$\begin{aligned}\int_E f\,d\mu &= \int_E f\,d(\mu_{\text{LBG}} \circ \iota_{\text{MO}} \circ \pi_{\text{i}} + \mu_{\text{CT}} \circ \iota_{\text{SP}} \circ \pi_{\text{i}}) \\ &= \int_E f\,d(\mu_{\text{LBG}} \circ \iota_{\text{MO}} \circ \pi_{\text{i}}) + \int_E f\,d(\mu_{\text{CT}} \circ \iota_{\text{SP}} \circ \pi_{\text{i}})\end{aligned}$$

$$= \int_{(E\cap\mathbf{MO})\cup(E\cap\mathbf{SP})} f\, d(\mu_{\mathrm{LBG}} \circ \iota_{\mathrm{MO}} \circ \pi_{\mathrm{i}}) + \int_{(E\cap\mathbf{MO})\cup(E\cap\mathbf{SP})} f\, d(\mu_{\mathrm{CT}} \circ \iota_{\mathrm{SP}} \circ \pi_{\mathrm{i}})$$

$$= \int_{E\cap\mathbf{MO}} f\, d(\mu_{\mathrm{LBG}} \circ \iota_{\mathrm{MO}} \circ \pi_{\mathrm{i}}) + \int_{E\cap\mathbf{SP}} f\, d(\mu_{\mathrm{LBG}} \circ \iota_{\mathrm{MO}} \circ \pi_{\mathrm{i}})$$

$$+ \int_{E\cap\mathbf{MO}} f\, d(\mu_{\mathrm{CT}} \circ \iota_{\mathrm{SP}} \circ \pi_{\mathrm{i}}) + \int_{E\cap\mathbf{SP}} f\, d(\mu_{\mathrm{CT}} \circ \iota_{\mathrm{SP}} \circ \pi_{\mathrm{i}})$$

$$= \int_{E\cap\mathbf{MO}} f\, d(\mu_{\mathrm{LBG}} \circ \iota_{\mathrm{MO}} \circ \pi_{\mathrm{i}}) + \int_{E\cap\mathbf{SP}} 0\, d(\mu_{\mathrm{LBG}} \circ \iota_{\mathrm{MO}} \circ \pi_{\mathrm{i}})$$

$$+ \int_{E\cap\mathbf{MO}} 0\, d(\mu_{\mathrm{CT}} \circ \iota_{\mathrm{SP}} \circ \pi_{\mathrm{i}}) + \int_{E\cap\mathbf{SP}} f\, d(\mu_{\mathrm{CT}} \circ \iota_{\mathrm{SP}} \circ \pi_{\mathrm{i}})$$

$$= \int_{E\cap\mathbf{MO}} f\, d(\mu_{\mathrm{LBG}} \circ \iota_{\mathrm{MO}} \circ \pi_{\mathrm{i}}) + \int_{E\cap\mathbf{SP}} f\, d(\mu_{\mathrm{CT}} \circ \iota_{\mathrm{SP}} \circ \pi_{\mathrm{i}})$$

If $f = \varrho_M(\mathbf{SDIC}^{\varphi}_{\psi})$, where the main predicate ψ describes discrete events, then for any $s \in \mathbf{MO}$, $f(s) = \varrho_M(\mathbf{SDIC}^{\varphi}_{\psi})(s) = 0$. Therefore,

(261) $$\int_E f\, d\mu = \int_{E\cap\mathbf{MO}} f\, d(\mu_{\mathrm{LBG}} \circ \iota_{\mathrm{MO}} \circ \pi_{\mathrm{i}}) + \int_{E\cap\mathbf{SP}} f\, d(\mu_{\mathrm{CT}} \circ \iota_{\mathrm{SP}} \circ \pi_{\mathrm{i}})$$

$$= \int_{E\cap\mathbf{MO}} 0\, d(\mu_{\mathrm{LBG}} \circ \iota_{\mathrm{MO}} \circ \pi_{\mathrm{i}}) + \int_{E\cap\mathbf{SP}} f\, d(\mu_{\mathrm{CT}} \circ \iota_{\mathrm{SP}} \circ \pi_{\mathrm{i}})$$

$$= \int_{E\cap\mathbf{SP}} f\, d(\mu_{\mathrm{CT}} \circ \iota_{\mathrm{SP}} \circ \pi_{\mathrm{i}})$$

$$= \int_{E\cap\mathbf{SP}} \varrho_M(\mathbf{SDIC}^{\varphi}_{\psi})\, d(\mu_{\mathrm{CT}} \circ \iota_{\mathrm{SP}} \circ \pi_{\mathrm{i}})$$

We have thus obtained the integral of $\varrho_M(\mathbf{SDIC}^{\varphi}_{\psi})$ only over non-momentary situations, and this is as desired since it is for discrete event-related readings. On the other hand, if $f = \varrho_M(\mathbf{SDIC}^{\varphi}_{\psi})$, where the main predicate ψ describes continuous events, then for any $s \in \mathbf{SP}$, $f(s) = \varrho_M(\mathbf{SDIC}^{\varphi}_{\psi})(s) = 0$. Thus,

(262) $$\int_E f\, d\mu = \int_{E\cap\mathbf{MO}} f\, d(\mu_{\mathrm{LBG}} \circ \iota_{\mathrm{MO}} \circ \pi_{\mathrm{i}}) + \int_{E\cap\mathbf{SP}} f\, d(\mu_{\mathrm{CT}} \circ \iota_{\mathrm{SP}} \circ \pi_{\mathrm{i}})$$

$$= \int_{E\cap\mathbf{MO}} f\, d(\mu_{\mathrm{LBG}} \circ \iota_{\mathrm{MO}} \circ \pi_{\mathrm{i}}) + \int_{E\cap\mathbf{SP}} 0\, d(\mu_{\mathrm{CT}} \circ \iota_{\mathrm{SP}} \circ \pi_{\mathrm{i}})$$

$$= \int_{E\cap\mathbf{MO}} f\, d(\mu_{\mathrm{LBG}} \circ \iota_{\mathrm{MO}} \circ \pi_{\mathrm{i}})$$

$$= \int_{E\cap\mathbf{MO}} \varrho_M(\mathbf{SDIC}^{\varphi}_{\psi})\, d(\mu_{\mathrm{LBG}} \circ \iota_{\mathrm{MO}} \circ \pi_{\mathrm{i}})$$

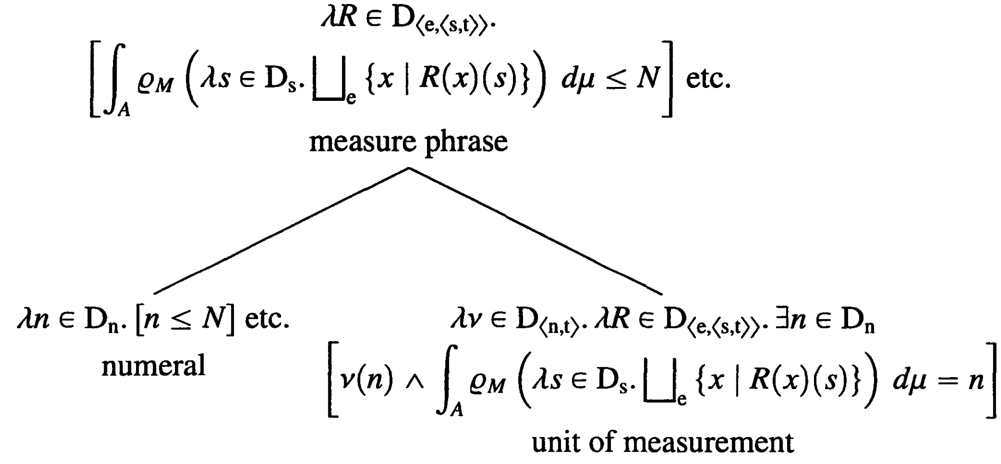
This time, we have obtained the integral of $\varrho_M(\mathbf{SDIC}^{\varphi}_{\psi})$ only over momentary situations. Again, this is as desired since it is for continuous event-related readings. Thus, we need no longer choose an appropriate measure depending on whether the sentence describes discrete or continuous events. Thus, in our lexicon, we need only one measure μ, and the other difference between (253) and (255) is now eliminated. *cubic centimeter* thus has a unique lexical entry given below:

(263) $$[\![\texttt{cubic centimeter}]\!] = \lambda v \in \mathrm{D}_{\langle \mathrm{n,t}\rangle}.\, \lambda R \in \mathrm{D}_{\langle \mathrm{e},\langle \mathrm{s,t}\rangle\rangle}.\, \exists n \in \mathrm{D}_{\mathrm{n}} \left[v(n) \wedge \int_A \varrho_{\text{cubic-centimeter}}\left(\lambda s \in \mathrm{D}_{\mathrm{s}}.\, \bigsqcup\nolimits_{\mathrm{e}} \{x \mid R(x)(s)\}\right) d\mu = n\right]$$

In general, units of measurement (including $\varnothing_{\text{ATOM}}$) have a lexical entry of the above from where cubic-centimeter is replaced by an appropriate situation-sensitive lattice-measure function. The analysis thus neatly captures our intuition that for continuous events, we look at relevant momentary time intervals and thus measure those by considering the length of the interval they constitute together by means of the Lebesgue measure, while for discrete events, which are necessarily have positive length, we simply count the number of relevant time intervals by means of the counting measure.

Let me summarize my analysis of event-related readings in general. A unit of measurement with a situation-sensitive lattice-measure function M combines with a numeral, which denotes a property of numbers, to yield a measure phrase:

(264)



$$\lambda R \in \mathrm{D}_{\langle \mathrm{e},\langle \mathrm{s,t}\rangle\rangle}.\, \left[\int_A \varrho_M\left(\lambda s \in \mathrm{D}_{\mathrm{s}}.\, \bigsqcup\nolimits_{\mathrm{e}} \{x \mid R(x)(s)\}\right) d\mu \leq N\right] \text{ etc.}$$

measure phrase

$$\lambda n \in \mathrm{D}_{\mathrm{n}}.\, [n \leq N] \text{ etc.}$$

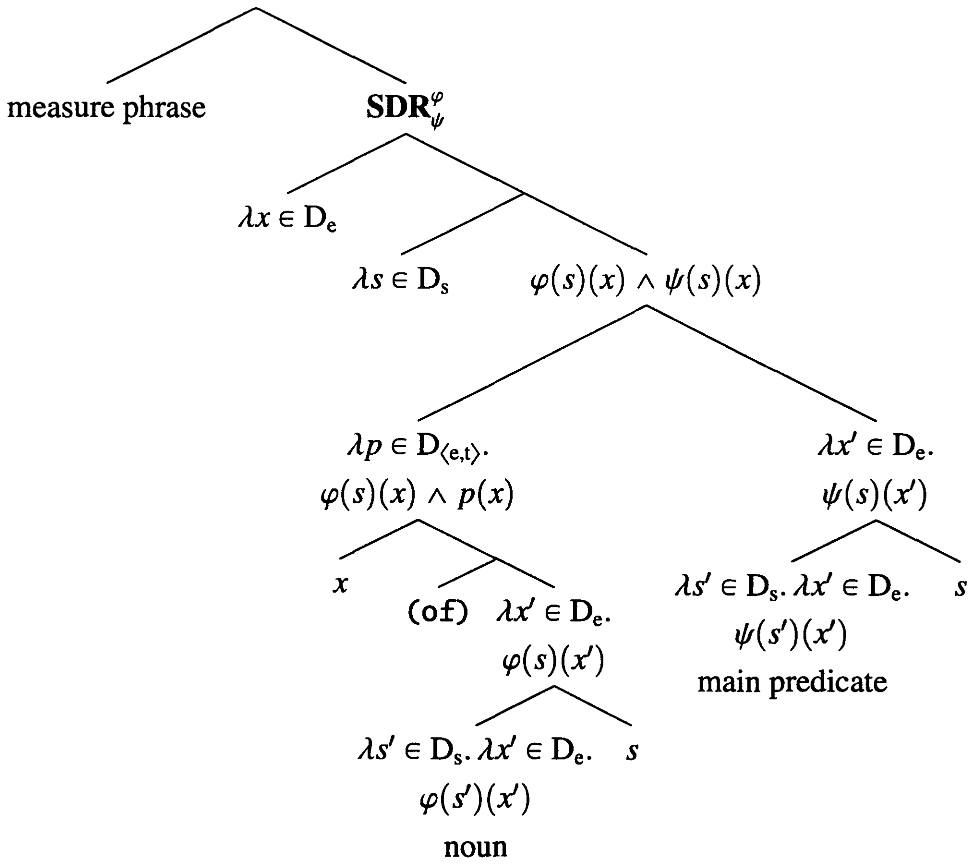
numeral

$$\lambda v \in \mathrm{D}_{\langle \mathrm{n,t}\rangle}.\, \lambda R \in \mathrm{D}_{\langle \mathrm{e},\langle \mathrm{s,t}\rangle\rangle}.\, \exists n \in \mathrm{D}_{\mathrm{n}} \left[v(n) \wedge \int_A \varrho_M\left(\lambda s \in \mathrm{D}_{\mathrm{s}}.\, \bigsqcup\nolimits_{\mathrm{e}} \{x \mid R(x)(s)\}\right) d\mu = n\right]$$

unit of measurement

The measure phrase moves out of the event-related-reading-inducing non-presuppositional noun phrase, and its sister forms a situation-dependent relation from the nominal predicate and the main predicate:

(265)



At the top node, the measure phrase takes the situation-dependent relation as its argument, and as a result, the following is obtained as the truth conditions of the sentence:

(266)
$$\left[\int_A \varrho_M \left(\lambda s \in \mathrm{D_s}. \bigsqcup_{\mathrm{e}} \{x \mid \varphi(s)(x) \wedge \psi(s)(x)\}\right) d\mu \leq N\right]$$
$$= \left[\int_A \varrho_M \left(\lambda s \in \mathrm{D_s}. \bigsqcup_{\mathrm{e}} \{x \mid \mathbf{SDR}^{\varphi}_{\psi}(x)(s)\}\right) d\mu \leq N\right]$$
$$= \left[\int_A \varrho_M (\mathbf{SDIC}^{\varphi}_{\psi})\, d\mu \leq N\right]$$

Here, A is the set of all situations in some world contained in a context time interval.

2.6.4 Tense

So far, I have argued that sentences that assert the existence of entities described by a noun phrase are analyzed not with a simple existential quantification, but with an integral of an appropriate function over situations. In so doing, I have already analyzed how the temporal interpretation of the main verb should be done as well. In the tradition of Prior's (1957;

1967) tense logic, simple declarative sentence like (267) are analyzed with an existential quantification over times. Under this view, (267) will be analyzed as (268).

(267) John jumped once.

(268) $\exists I[\mathsf{jump}(I)(\mathsf{John})]$

In the analysis I developed above, however, (267) is analyzed as follows, where A is the set of all situations in a context time interval:

(269) $\int_A \lambda s.\,\mathsf{jump}(s)(\mathsf{John})\,d\mu \geq 1$

If one is only interested in analyzing (267), (268) seems to suffice, but then, the deep connection between simple sentences describing discrete events without an event-related-reading-inducing noun phrase such as (267) and discrete event-related readings cannot be captured. Krifka's original analysis manages to account for simple cases of discrete event-realted readings, but it did not provide this insight. This point can most clear be seen when we consider continuous cases. A simple existential quantification approach would analyze (270) as (271).

(270) John ran for seventeen minutes yesterday.

(271) $\exists I[I \text{ is 17 minutes long } \wedge \mathsf{run}(I)(\mathsf{John})]$

In my approach, (270) receives an analysis parallel to the analysis of discrete events as in (269):

(272) $\int_A \lambda s.\,\mathsf{run}(s)(\mathsf{John})\,d\mu \geq 60 \times 17$

Again, if one is only interested in (270), (271) may suffice, but it does not give any hint at how continuous event-related readings should be analyzed, where John drinks XYZ that keeps expanding and contracting, of which some part is drunk twice, or where John eats a piece bread that keeps expanding and shrinking. Moreover, the simple existential quantification approach already faces difficulty when we attempt to analyze (270) with *exactly*:

(209) John ran for exactly seventeen minutes yesterday.

(271) does not work, since we look at the total of John's running time. On the other hand, the integral approach can straightforwardly analyze (209) just by replacing the inequality in (272) with an equality:

(273) $\int_A \lambda s.\,\mathsf{run}(s)(\mathsf{John})\,d\mu = 60 \times 17$

For all of these sentences, my analysis provides truth conditions of a unified form, where our intuitions are formally articulated from a view point of measure theory. It is somewhat odd that I am discussing tense in the very end after I have discussed various event-related readings, which are much more complex. However, it must be realized that it is through the discussion of event-related readings that we have reached this new treatment of tense.

Let us now turn to Partee's (1973) referential theory of tense. Partee, who considered the following sentence, objected to the existential analysis of tense as in (268).

(274) I didn't turn off the stove.

Let us say that turn-off-the-stove is an unanalyzed predicate, and the speaker of the sentence is Partee herself. (275) is the positive counterpart of the Partee sentence, and under the approach with the existential quantifier, it will be analyzed as in (276).

(275) I turned off the stove.

(276) $\exists I[\text{turn-off-the-stove}(I)(\text{Partee})]$

Since the Partee sentence (274) can be regarded as being obtained by adding *not* to (275), the following two possible analyses suggest themselves for the Partee sentence.

(277) a. $\neg\exists I[\text{turn-off-the-stove}(I)(\text{Partee})]$

b. $\exists I\neg[\text{turn-off-the-stove}(I)(\text{Partee})]$

As Partee correctly points out, neither captures the correct meaning of her sentence. (277a) entails that Partee has never turned off the stove in her life, while (277b) will almost trivially true, presumably unless Partee keeps turning off the stove forever. Partee then proposes that when the speaker utters sentences like (274), she has a specific time in mind and therefore that the sentence should be evaluated with respect to that time. On this view, the time of the main predicate is like a pronoun like *he*, whose extension is determined by the context, and hence the time of the main predicate is not quantified over by an existential quantifier.

As Ogihara (2003) and von Stechow (to appear) point out, however, Partee's analysis does not work as long as one is committed to the view that $\text{turn-off-the-stove}(I)(\text{Partee})$ is true if and only if *I* is the exact time interval of an event of Paree's turning off the stove, because according to Partee's analysis, the speaker then needs to know the exact time in which the stove might have been turned off. In uttering sentences like (274), however, the speaker would not have such a specific time. She would have a rather longer time span in mind. Suppose that the time span that Partee had in mind when uttering the sentences is between the 1pm and 2 pm on August 27 of 1973. Since turning off the stove does not take an hour to accomplish, the sentence merely asserts that there is no time interval between 1 pm and 2pm on August 27 of 1973 at which the stove is turned off. Ogihara and von Stechow thus both conclude that the analysis with the existential quantifier must be retained, although the domain of the quantification should now be properly restricted.

My answer to this controversy is the following. Under my theory, Ogihara's and von Stechow's conclusion that Partee's anslysis does not work is inevitable, since in my analysis, it is crucial that the time interval of the situation argument of the main predicate be the exact time interval in which an event described by the predicate occurs, in order to ensure correct counting (see Subsection 2.2.2). Therefore, we do not have an option of encoding a temporal inclusion relation in the semantics of the main predicate itself. In my analysis, however, existential quantification is replaced with a condition on the value of a relevant integral. Partee's reference time then actually corresponds to the domain of integration in a sense. In my theory, the positive counterpart of the Partee sentence is analyzed as follows:

(278) $[\![\texttt{I turn off the stove once}]\!]$

$$= \left[\int_A \lambda s \in \mathrm{D_s}.\,\textsf{turn-off-the-stove}(s)(\textsf{Partee})\, d\mu > 0\right]$$

If the sentence is uttered in the context described above, the domain of integration A here is the set of all situations whose time interval is contained in Partee's reference time.

(279) $A = \{\langle @, I\rangle \mid I \subseteq [\text{1pm on Aug 27 of 1973}, \text{2 pm on Aug 27 of 1973}]\}$

In order to obtain (278) compositionally, let us assume with Bäuerle (1979) that when a sentence does not have an adverb such as *once* or *three times* overtly, there is an invisible adverb meaning "'(at least) once". Under my theory, the meaning of this hidden adverb is given as follows:

(280) $[\![\texttt{(at least) once}]\!] = \lambda\chi \in \mathrm{D}_{\langle s,t\rangle}.\left[\int_A \chi\, d\mu > 0\right]$

Assuming the following obvious semantics for *not* in (281), we can calculate the truth conditions of the Partee sentence as in (282).

(281) $[\![\mathsf{not}]\!] = \lambda t \in \mathrm{D_t}.\,\neg t$

(282) $[\![\texttt{I didn't turn off the stove}]\!]$

$$\overset{\text{FA}}{=} [\![\texttt{not}]\!]\left([\![\texttt{I turn off the stove (at least once)}]\!]\right)$$

$$= [\lambda t \in \mathrm{D_t}.\, \neg t]\left([\![\texttt{I turn off the stove (at least once)}]\!]\right)$$

$$\overset{\beta}{=} \neg\, [\![\texttt{turn off the stove (at least once)}]\!]$$

$$= \neg \left[\int_A \lambda s \in \mathrm{D_s}.\, \text{turn-off-the-stove}(s)(\text{Partee})\, d\mu > 0\right]$$

$$= \left[\int_A \lambda s \in \mathrm{D_s}.\, \text{turn-off-the-stove}(s)(\text{Partee})\, d\mu \not> 0\right]$$

$$= \left[\int_A \lambda s \in \mathrm{D_s}.\, \text{turn-off-the-stove}(s)(\text{Partee})\, d\mu = 0\right]$$

Summary and Prospect

I hope to have shown the following main points convincingly through my discussion in the dissertation.

Chapter 1:

- Bi-clausal examples where a noun phrase originated in the embedded clause exhibits obligatory situational dependence on the matrix predicate strongly argues for the scope theory of non-presuppositional noun phrases.
- The scope theory helps develop a binding theory of situation variables under which what is means for a noun phrase to have existential presupposition can be articulated.
- To interpret plural non-presuppostional noun phrases correctly, a situationally dependent relation holding between an individual and an atomic situation must be formed.
- The numeral part of a non-presuppostional noun phrase must be separated and enters into computation after the formation of a situationally dependent relation.

Chapter 2:

- To account for event-related readings, situationally dependent relations should not be "pluralized" with the (double) * operator, and Krifka's (1990) strategy is called for.
- Musan's (1995) generalization is extended to the interpretation of units of measurement.
- Consideration of sentences that describe continuous production or consumption of mass entities reveals the necessity of employing integration.

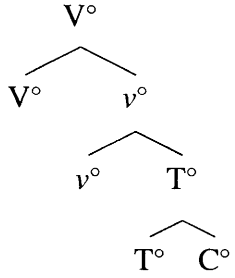
- Using Lebesgue integration, I developed a new semantic theory where an existential statement is expressed as the condition of the value of a Lebesgue integral of an appropriate function from situations into non-negative reals.

- Discrete (telic) events are measured with the counting measure, and continuous (atelic) events are measured with the Lebesgue measure. This is expressed as a decomposition of the measure used in natural language semantics.

I suspect that the reader might feel as if this dissertation ended abruptly in the midst of a story, as I have left many questions and problems unanswered and even unmentioned. In particular, I have not discussed how to deal with object-related readings. For object-related readings, I propose with Musan (1995) and Szabó (2006) that we can count the maximal time intervals in which the state described by the relevant noun holds.

The reason that this dissertation falls short of a more complete and satisfactory story is that I had originally hoped to write about four times of the present amount, but I unfortunately did not have enough time to accomplish this as it started very late. The semantic theory developed in the present dissertation was going to be embedded in the new theory of head movement whose outline was presented in my generals paper (Shimada, 2007). In this theory, the derivation of a sentence begins with the formation of a complex head consisting of the heads that are going to be in the spine of the sentence. For instance, the derivation of a sentence with C, T, *v*, V starts with the formation of following head complex:

(1)



From this, head complexes successively move out and get projected. This is the head movement in this theory. Unlike the standard theory of head movement, this theory does not violate Chomsky's (1995) extension condition. Furthermore, Baker's (1985) mirror principle can be derived by compositional morphological computation.

The crucial proposal of this theory is that head movement creates a relevant λ abstractor. Throughout the dissertation, I have postulated a λ abstractor that binds the situation argument of the main predicate (and the situation argument of a non-presuppositional noun phrase). I was going to argue that this λ abstractor is in fact created by head movement of Asp°. The account of object-related readings was going to be given in the general theory of aspect entertained in this new framework of head movement that accounts for Pratt and Francez's (2001) observation on sentences with nested temporal quantification. This is the reason that I could not include my analysis of object-related reading. The theory has an advantage of deriving Percus's (2000) generalizations on the constraint of situation variables in syntax directly from the well-known locality of head movement.

Despite its evident importance to the theory of natural language semantics, it seems to me that Krifka's (1990) observation and theory of event-related readings have more or less been ignored, as nobody has seriously tried to incorporate it into the "main" theory of natural language semantics as far as I know. I suspect that this was due to two factors. One is the apparent complexity of Krifka's analysis. The other is that Krifka did not explain why two radically different analyses are needed for object-related readings and event-related readings. In Krifka's proposal, a noun phrase simply comes equipped with different kinds of determiners depending on whether the sentence has an object-related reading or an event-related reading. I feel that for these reasons, people have marginalized event-related readings as some "exotic" epiphenomenon that requires special complicated treatment that Krifka devises, while considering object-related readings to be the basic case as they seem to only require simple usage of the existential quantifier.

However, such a view must be abandoned, since we quite normally use sentences with an event-related reading and it is not that only English exhibits event-related readings. Event-related reading should then be at least as central to the analysis of existential sentences as are object-related readings. In the present dissertation, I hope to have developed a theory of simple cases of event-related readings. Once an analysis of this sort is accepted, as alluded to briefly above, the analysis of object-related readings should be given by constructing a more complex integrand function using Asp, and thus, object-related shall actually be derived from a more complex structure than are event-related readings. Hence, it shall not be object-related readings, but the seemingly complex event-related readings that turn out to represent the basic case of existential sentences. If this is the case, we can no longer marginalize event-related readings as an "exotic" epiphenomenon. I am aware that I have not shown any of the above claim in detail yet, and I regret I have not had enough time to accomplish my original goal to include this all in this dissertation. Nevertheless, I intend to work on this project and to keep posting my writing on the web in the near future.

Finally, I would like to mention an expected application of the Lebesgue integral approach to possible worlds. Given that existential claims are in general translated with an integral, it is expected that sentences that have been taken to assert the existence of certain possible worlds like (2) are treated analogously:

(2) It is possible that Mary likes John.

This is typically analyzed roughly as follows, where acc(@) represents the set of possible worlds accessible from the actual world.

(3) $\exists w[w \in \mathsf{acc}(@) \wedge \mathsf{like}(w)(\mathsf{John})(\mathsf{Mary})]$

Under the integral approach, this would be translated into something like the following:

(4) $\int_{\mathsf{acc}(@)} \lambda w.\, \mathsf{like}(w)(\mathsf{John})(\mathsf{Mary})\, d\mu > 0$

Here, the measure μ is a suitable measure defined on sets of possible worlds. Let $\mathfrak{F}$ be a σ-algebra over acc(@). If we succeed in defining an appropriate measure μ so that the measure space $(\mathrm{acc}(@), \mathfrak{F}, \mu)$ becomes a probability space, the integral in (4) is expected to directly give the probability of Mary liking John. This opens up a possibility of capturing the difference between the sentences in (5) by simply comparing the values of the integrals, whereas the standard approach with simple usage of the existential quantifier might translate all these sentences as in (3).

(5) a. It is very probable that Mary likes John.
    b. Perhaps Mary likes John.
    c. It is 40 percent likely that Mary likes John.

To this end, it is probably necessary to define an appropriate topology and metric to the set of possible worlds. This is not an outlandish idea at all. The classic analyses of counterfactual conditionals by Stalnaker (1968) and Lewis (1973) already talk about the nearest possible world(s) to the actual world, and thus this line of approach seems necessary in any case. Since Lebesgue integration has been put in the fundamentals of probability theory, this line of approach seems to be promising.

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