Detection, Classification and Localization of Seabed Objects with a Virtual Time Reversal Mirror

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Abstract

The work presented in this thesis addresses the problem of the detection, classification and localization of seabed objects in shallow water environments using a time reversal approach in a bistatic configuration. The waveguide is insonified at low frequency (~kHz) with an omnidirectional source and the resulting scattered field is sampled by a receiving array towed behind an Autonomous Underwater Vehicle (AUV). The recorded signals are then processed to simulate onboard the AUV, the time reversed transmissions which serve to localize the origin of the scattered field on the seabed and estimate the position of the targets present. The clutter rejection based upon the analysis of the singular values of the Time Reversal operator is investigated with simulated data and field measurements collected off the coast of Palmaria (Italy) in January 2008.

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Chapter 1

Introduction

1.1 Background and Motivations

Recent developments in Autonomous Underwater Vehicles (AUVs) technology have brought new perspectives in various fields of ocean engineering such as oil exploration, fishery, marine archeology and mine countermeasures (MCMs). The deployment of autonomous vehicles requires significantly less manpower than surface vessels and is often better adapted to exploring areas with limited access. For economic reasons, the choice of inexpensive platforms to collect data also presents competitive advantages for many applications. The costs involved in mine hunting missions conducted from military ships are reduced by using AUVs which also allow for safer, cooperative and inexpensive operations. However, the shift toward smaller platforms for MCMs as well as for many other fields also introduces new challenges, each of which calls for innovative solutions.

Power limitations onboard AUVs impose reduced power-budgets allowable for the processes running on the payload and call for efficient algorithms. The efficiency of active Detection Localization and Classification (DCL) techniques in the field of MCM can be measured in terms of coverage rates and false alarm rates. Coverage rates are typically constrained by the frequency band of the detection signals. High frequencies (\(\sim\)MHz) used by side-scan sonars provide high-resolution images of the seafloor at the cost of limited coverage rates due to the high attenuation associated with short wavelengths in seawa-
In addition, the poor bottom penetration of high frequency signals also prevents their use for the detection of mines buried in the seabed. In comparison, low-frequency signals (~kHz) suffer little transmission losses, penetrate deeper into the seabed and can reveal the structural response of mine-like elastic objects [1]. The scattering from elastic spherical shells at low-frequency has been investigated during past experiments [2] [3] along with the development of high-fidelity numerical models that can now treat a broad class of scattering problems [4] [5]. The sampling of low-frequency scattered fields using AUVs has long been problematic due to the lack of control techniques adapted to the towing of long receiving antennas by a small platform. A behavior-based control approach [6] recently tested at sea with a 100m long array towed behind a 21 inch AUV demonstrated robust control maneuvers and brings promising sensing capabilities for AUVs.

The presence of multipath and reverberation in shallow waveguides greatly complicates the detection of target echoes and constitute a limiting factor for most DCL techniques. Therefore, the focusing of acoustic energy on potential targets is desirable in order to achieve higher signal to noise ratio. In homogeneous and unbounded media, focusing monochromatic signals in space can be achieved with a set of acoustic sources by properly choosing their amplitude and relative phase delays. Beamforming techniques are commonly used to estimate this weighting from geometrical approximations and produce with an array of sources, a set of pressure wave-fronts that interfere constructively in the focusing region. In ocean waveguides, however, the task of estimating appropriate amplitude and phase delays that account for boundary reflections becomes intractable without the knowledge of the environment. In contrast, the Time Reversal (TR) process automatically determines the response of the waveguide from the scattered field measurements to compensate for multipath effects. The ability of the TR acoustics methods to achieve focusing in time and space without prior knowledge of the environment has attracted a lot of attention among acousticians [7] [8] [9] [10]. Focusing techniques based upon the TR process have been developed to handle multiple scatterers and extract simultaneously the location and classification information about each target with minimal computation effort [11].
1.2 Thesis objectives

The main objective of this research is to investigate the concurrent detection classification and localization of seafloor objects using a TR approach. The DCL technique involves a single acoustic source and a set of receivers in a bistatic configuration. The source insonifies the target field at low frequency while the scattered field is sampled with a receiving array towed behind an AUV. The detection of the scatterers requires the analysis of invariants of the TR process determined from the singular value decomposition of the TR operator. The localization of the scatterers is then achieved with a Time Reversal imaging of the seabed, best described as a "virtual" TR mirror since it does not imply actual acoustic transmissions. Due to the limited amount of calculations involved in the detection and localization process, this approach is well suited to AUV operations. In order to meet our objective, we consider the following intermediary steps:

- Modeling of realistic operational scenarios to test the feasibility of the virtual TR mirror approach and understand its limitations
- Implementation of the TR imaging used to localize the seabed targets
- Testing of the virtual TR mirror approach with experimental data
1.3 Thesis Outline

The thesis report will be organized as follows:

The second chapter introduces the theoretical concepts upon which the proposed approach is based. In particular, the derivation of several focusing methods is presented to formulate the relations between the eigenstates of the TR process and the scatterers present. The theory on scattering from elastic spherical shells is also reviewed. The chapter concludes with the review of past research work in the field of Time Reversal acoustics.

The third chapter describes the modules involved in the modeling of the broadband insonification of a shallow water waveguide in presence of targets located on the seabed and the measurement of the resulting scattered field at the receiving array. It also provides justifications for the modeling simplifications made to reduce computation time while maintaining modeling accuracy. Finally, the principle of the virtual Time Reversal mirror used localize the scatterers on the seabed is presented along with the analysis of simulated results.

The fourth chapter presents the setup of the sea trial that took place near the island of Palmaria (Italy) in January 2008 and the steps taken to process our experimental data. In particular the time of arrival of echoes from the scatterers are determined from the analysis of the singular value of the TR operator and compared to the time of arrival estimated using geometrical consideration. The localization of the scatterers based upon the virtual time reversal mirror approach is also compared to known positions of the scatterers present.

The last chapter concludes the thesis and summarizes the simulated and experimental results. In light of observations made, several limitations of the proposed approach are presented along with possible future developments.
Chapter 2

Theoretical Background

This chapter introduces the theoretical concepts underlying the virtual TR mirror approach presented in the third chapter of the thesis and reviews past research work in the field of Time Reversal Acoustics.

2.1 Time Reversal Acoustics

2.1.1 Background

The wave equation governing the propagation of pressure waves in lossless inhomogeneous media is given by:

\[
\rho(r) \nabla \cdot \left( \frac{1}{\rho(r)} \nabla p \right) - \frac{1}{c^2(r)} \frac{\partial^2 p}{\partial t^2} = 0
\] (2.1)

where \( p, c, \) and \( \rho \) refer respectively to the pressure, sound speed and density in the medium.

Since the order of the time derivative in Eq. (2.1) is even, if a function \( p(r, t) \) satisfies the wave equation, its time reversed form, \( p(r, -t) \) is also a solution of the wave equation. From a practical aspect, the invariance of the wave equation to time inversion implies that the field emitted from a source can be recorded, time reversed and focused back at the source location as if time was running backwards. Achieving time reversed focusing
requires that time shift invariance, linearity and reciprocity hold in the environment.

Time Reversal mirrors (TR mirrors) are physical implementations of the time reversal procedure. The TR mirror typically consists of a set of receivers connected to a storage device that records the pressure field incident on the mirror. A set of sources co-located with the receivers of the TR mirror retransmits the recorded signals in a time reversed manner (First In Last Out). During the backpropagation, the acoustic field produced by each source of the TR mirror interferes constructively where the pressure field initially measured by the TR mirror was emitted and destructively elsewhere. TR mirrors are well suited for active detection purpose since they allow to focus energy on reflective scatterers playing the role of sources after insonification of the environment.

2.1.2 Time Reversal process

We describe in more details the steps involved in a focusing cycle in presence of a single scatterer in a free field environment. The sources of the TR mirror first insonify the environment with a set of probing signals \( e_m(t) \) (\( m = 1, \ldots, N \) assuming \( N \) sources) (Fig. 2-1).

\[
E_m(t) = \sum_{m=1}^{N} k_{lm}(t) \otimes e_m(t) 
\]

Figure 2-1: Target Insonification

The resulting scattered field measured at the \( l^{th} \) receiver of the TR mirror (\( l = 1, \ldots, N \)) (Fig. 2.2(a)) is expressed as follows:

\[
r_l(t) = \sum_{m=1}^{N} k_{lm}(t) \otimes e_m(t) 
\]

(2.2)
where \( k_{lm}(t) \) is the impulse response between element \( m \) and element \( l \) of the TR mirror. "\( \otimes \)" denotes the time domain convolution. In the frequency domain, Eq. (2.2) becomes:

\[
R_i^0(\omega) = \sum_{m=1}^{N} K_{lm}(\omega) E_m^0(\omega)
\]  

(2.3)

where the 0 index refers to the initial cycle.

![Figure 2-2: Backpropagation in presence of a single scatterer](image)

The field recorded at the receivers is then time reversed or equivalently phase conjugated in the frequency domain. The new set of signals to be transmitted (Fig. 2.2(b)) in order to focus on the target is expressed as:

\[
E_l^1(\omega) = \sum_{m=1}^{N} K_{*lm}(\omega) E_m^{0*}(\omega)
\]  

(2.4)

or in matrix notation:

\[
E^1(\omega) = K^*(\omega) E^{0*}(\omega)
\]  

(2.5)

where \( K(\omega) \) refers to the interelement matrix evaluated at the frequency \( \omega \).

In presence of a single scatterer, repeating the focusing cycles progressively reduces the extent of the focusing region and converges to an optimized set of amplitudes and phase delays associated with the scatterer. The analysis of convergence of successive focusing
cycles in presence of multiple scatterers yields the iterative focusing approach.

2.1.3 Iterative Focusing Approach

In presence of multiple reflectors, the procedure described in section 2.1.2 is repeated in order to focus on the most reflective scatterer by iteratively filtering the scattered field of weaker reflectors. This self-converging process initially formulated by C.Prada [12] yields without calculations the amplitudes and phase delays that are to be applied to the sources of the TR mirror in order to achieve focusing on the dominant scatterer. We proceed with an analysis of convergence of the iterative focusing approach in presence of a set of $d$ point-like scatterers (where $d < N$) with distinct reflectivities.

The focusing procedure introduced in section 2.1.2 yields the signals to be transmitted by the TR mirror in order to focus on the scatterer present. The same approach applies in presence of multiple well-resolved\(^1\) scatterers and results in a transmitted field that initially focuses on each of them. Starting from Eq. (2.5), the scattered field resulting from the transmission of $E^1(\omega)$ by the TR mirror is given by:

$$R^1(\omega) = K(\omega)E^1(\omega) \quad (2.6)$$

and yields the transmitted signal of the next cycle:

$$E^2(\omega) = R^{1*}(\omega) \quad (2.7)$$

$$= [K^*(\omega)K(\omega)]E^0(\omega) \quad (2.8)$$

$K^*(\omega)K(\omega)$ in Eq. (2.8) is referred to as the Time Reversal operator (TR operator). Since reciprocity holds in the environment, the interelement matrix $K$ is symmetric and consequently the TR operator matrix is hermitian, has real positive eigenvalues and orthogonal eigenvectors. In presence of point-like well-resolved isotropic scatterers, the rank of

\(^1\)Scatterers are said to be well-resolved when the field focused on one of them does not insonify the other scatterers present
the TR operator is equal to the number of reflectors present [13] and each eigenvalue \( \lambda_i(\omega) \) 
\((i=1, \ldots, d)\) (and corresponding eigenspace) is associated with one of the reflectors present. Therefore, any vector can be expressed as a combination of \( d \) non-null vectors from these eigenspaces. For example, the first transmission vector \( E^0(\omega) \) can be expressed as:

\[
E^0(\omega) = F_1(\omega) + F_2(\omega) + \ldots + F_d(\omega)
\]  

(2.9)  

where \( F_i(\omega) \) is a vector from the \( i^{th} \) eigenspace. Using this form of \( E^0(\omega) \), the transmitted signals corresponding to the \( 2n \) iteration and given by:

\[
E^{2n}(\omega) = [K^*(\omega)K(\omega)]^n E^0(\omega)
\]  

(2.10)  

can be rewritten as:

\[
E^{2n}(\omega) = \lambda_1^n(\omega)F_1(\omega) + \lambda_2^n(\omega)F_2(\omega) + \ldots + \lambda_d^n(\omega)F_d(\omega)
\]  

(2.11)  

Similarly the transmitted signals corresponding to the \( 2n + 1 \) iteration expressed as:

\[
E^{2n+1}(\omega) = [K^*(\omega)K(\omega)]^n K^*(\omega) E^{0*}(\omega)
\]  

(2.12)  

can be rewritten as:

\[
E^{2n+1}(\omega) = \lambda_1^n(\omega)K^*(\omega)F_1^*(\omega) + \ldots + \lambda_d^n(\omega)K^*(\omega)F_d^*(\omega)
\]  

(2.13)  

For a large number of iterations, Eq. (2.11) and Eq. (2.13) converge respectively as follow:

\[
E^{2n}(\omega) \approx \lambda_j^n(\omega)F_j(\omega) \quad E^{2n+1}(\omega) \approx \lambda_j^n(\omega)K^*(\omega)F_j^*(\omega)
\]  

(2.14)  

where \( j \) is the index associated with the largest eigenvalue of the TR operator. Eq. 2.14 shows two distinct limits of convergence of the iterative approach both associated with the dominant scatterer. The DORT method described in the section 2.1.4 extends this analysis.
to the remaining eigenvectors of the TR operator to achieve selective focusing on the each
scatterer present.

2.1.4 DORT Method

The DORT\(^2\) method enables selective focusing in a multiple-scatterer environment. The
 technique consists of the following steps:

1. Construction of the interelement matrix \(K(\omega)\),

2. Extraction of the eigenvalues and eigenvectors of the TR operator \((= K^*(\omega)K(\omega))\),

3. Focusing on a selected scatterer by transmission of its associated eigenvector.

The formulation of the DORT method is a direct consequence of the convergence analy-
sis of the iterative focusing approach. It was demonstrated in section 2.1.3 that the iterative
approach yields the focusing on the strongest scatterer present. The following derivations
show that in the presence of point-like well-resolved isotropic scatterers of different reflec-
tivity, the transmission of each eigenvector of the TR operator allows for the focusing on
its associated scatterer.

Considering the transmission of an impulse signal \(\delta(t)\) from the location of the \(i^{th}\)
scatterer present, the field measured at the TR mirror by the \(l^{th}\) receiver is expressed as
\(a_r(t) \otimes h_{il}(t)\) where \(a_r(t)\) is the acoustoelectrical response in reception of the receivers
of the TR mirror. For simplicity the receivers of the array are assumed to have identical
response in reception. \(h_{il}(t)\) refers to the diffraction impulse response between the \(i^{th}\)
scatterer and the \(l^{th}\) receiver of the TR mirror. The field to be emitted by the \(l^{th}\) source of the
TR mirror in order to focus at the location of the \(i^{th}\) scatterer is given by \(a_r(-t) \otimes h_{il}(-t)\)
and the pressure field at the location of the \(j^{th}\) scatterer after the time reversed transmission
is expressed as:

\[
P_j(\omega) = A_r^*(\omega)A_e(\omega) \sum_{l=1}^{N} H_{jl}(\omega)H_{il}^*(\omega)\]

\[(2.15)\]

\(^2\)French acronym for "Décomposition de l'Opérateur de Retournement Temporel"
The term $A_e(w)$ in Eq. (2.15) refers to the Fourier transform of the response in emission of the sources of the TR mirror. For simplicity the sources of the array are assumed to have identical response in emission.

In a vector form Eq. (2.15) becomes:

$$P(w) = A_e^*(w)A_e(w)^T H_j(w)H_i^*(w)$$  \hspace{1cm} (2.16)

The condition of well-resolved scatterers implies that the vectors $H_i(w)$ are orthogonal and that the field produced at the $j^{th}$ reflector and given by equation (2.16) is null unless $i = j$. The preceding analysis demonstrates that propagating $H_i^*(w)$ from the TR mirror allows to focus on the $i^{th}$ reflector present. We now show that $H_i^*(w)$ is the eigenvector of the TR operator associated with the eigenvalue $\lambda_i(w)$. Transmitting $H_i^*(w)$ from the TR mirror produces a scattered field measured at the TR mirror and given by $K(w)H_i^*(w)$. The component of the scattered field measured at the receiver $l$ is expressed as:

$$R_l(w) = \sum_{m=1}^{N} H_{im}^*(w)K_{ml}(w)$$  \hspace{1cm} (2.17)

Introducing the reflectivity of each scatterer $C_1(w), C_2(w), ..., C_d(w)$, the interelement frequency response $K_{ml}(w)$ can be related to the diffraction response as follows:

$$K_{ml}(w) = \sum_{k=1}^{d} H_{mk}(w)C_k(w)H_{kl}(w)$$  \hspace{1cm} (2.18)

Using the fact that the diffraction responses are orthogonal, Eq. (2.17) becomes:

$$R_l(w) = H_{il}(w)C_i(w) \sum_{m=1}^{N} |H_{im}(w)|^2$$  \hspace{1cm} (2.19)

or in a vector form:

$$K(w)H_i^*(w) = C_i(w) \sum_{m=1}^{N} |H_{im}(w)|^2 H_i(w)$$  \hspace{1cm} (2.20)
Finally multiplying Eq. (2.20) by \( K^*(\omega) \) and replacing \( K(\omega)^*H_i(\omega) \) by its form in Eq. (2.20) yields:

\[
K^*(\omega)K(\omega)H_i^*(\omega) = |C_i(\omega)|^2 \left( \sum_{m=1}^{N} |H_{im}(\omega)|^2 \right)^2 H_i^*(\omega) \tag{2.21}
\]

From Eq. (2.21), it is clear that the eigenvector \( H_i^*(\omega) \) of the TR operator is associated with the eigenvalue \( \lambda_i(\omega) \) defined as:

\[
\lambda_i(\omega) = |C_i(\omega)|^2 \left( \sum_{m=1}^{N} |H_{im}(\omega)|^2 \right)^2 \tag{2.22}
\]

Thus the magnitude of each eigenvalue is equal to the square of the effective reflectivity of the corresponding reflector. It is important at this point to make the distinction between the effective reflectivity of the scatterer and its structural reflectivity. The form of \( \lambda_i(\omega) \) shows that the effective reflectivity of a scatterer depends on the coefficient of reflection but also on the transmission between the scatterer and the receivers of the TR mirror.

The iterative focusing approach generates a vector field that converges to the eigenvector produced by the DORT method for the dominant scatterer. However in presence of scatterers with similar effective reflectivities, the convergence process requires a large number of iterations. Furthermore, it does not provide any of the eigenvectors associated to the weaker scatterers present. In these regards, the DORT method overcomes the limitations of the iterative focusing approach but does so at the cost of the time consuming construction of the TR operator which is not needed by the iterative approach. The interelement response \( k_{lm}(t) \) between element \( l \) and element \( m \) is measured by emitting an impulse from the source \( l \) of the TR mirror and recording the scattered field at the receiver \( m \). Due to the reciprocity of the transmission, \( k_{lm}(t) = k_{ml}(t) \) and for a TR mirror of \( N \) element the construction of the TR operator requires \( N^2/2 \) measurements. An alternative approach well suited to reverberating waveguides has been introduced by Lingevitch [14] and results in higher signal to noise ratios during waveguide insonification. A set of beams defined by orthogonal weighting vectors at the transmitting array is used to insonify the
waveguide in place of the conventional insonification from individual sources. The process results in a "beam space" interelement response matrix from which the "element space" interelement response is recovered with a simple matrix manipulation.

2.1.5 Covariance matrix representation of the TR operator

In section 2.1.3 and 2.1.4, the construction of the TR operator is formulated for a set of stationary co-located sources and receivers. The virtual TR mirror approach presented in this thesis however involves a moving source that insonifies the waveguide at regular time intervals and a receiving array towed behind an AUV (Fig 4-1).

Figure 2-3: Bistatic configuration envisioned for the virtual TR mirror approach

Here, the condition of a stationary array is re-created by restricting our processing to a set of receivers that overlap at the time of emissions of the source. In the configuration described here, Doppler shifts are introduced due to the relative motion between the source and the targets and due to the motion of the receivers relative to the target. However, since the waveguide is insonified at low frequency and since the speed of the source of the receivers is small Doppler effects are assumed to be negligible. Therefore the source is considered stationary at each time of emission and it re-creates from its successive positions a stationary transmitting array (Fig 2-4).
For configurations that involve separated array of sources and receivers, invariants of transmissions of the TR process are identified with a similar approach as described in section 2.1.3: the interelement matrix $K$ is constructed by measuring the field incident on a stationary receiving array subsequent to successive emissions of probing signals from each source of the transmitting array [8]. The Singular Value Decomposition of the interelement matrix (or equivalent eigenvalue decomposition of the $TRO = K^*K^T$) provides each eigenstate of the TR process. This form of the TR operator that differs from the previously introduced $K^*K$, allows to obtain a Hermitian matrix from non-symmetric interelement matrix [15] which is particularly useful in channels where the short time coherence affects the reciprocity. Our approach to construct the TR operator (in the form $K^*K^T$) involves a passive source detection technique based on the decomposition of the covariance matrix. We present below some of the steps of the construction of the covariance matrix described in [10] [16] [17].

Assuming a linear time-invariant environment where $d$ sources emit the signals $s_1(t),...,s_d(t)$ measured by $L$ receivers. The signal measured at receiver $l$ is given by:

$$r_l(t) = \sum_{i=1}^{d} h_{li}(t) \otimes s_i(t) + b(t) \quad (2.23)$$

where $h_{li}(t)$ refers to the impulse response from the source $i$ to the receiver $l$. $b(t)$ refers to the noise signal measured at the receiver. In the frequency domain, Eq. (2.23) yields the following matrix formulation:
\[ R(\omega) = H(\omega)S(\omega) + B(\omega) \quad (2.24) \]

where \( H(\omega) \) is the transfer matrix from the sources to the receivers and \( B(\omega) \) is the noise vector. Each of the \( M \) realizations is associated with the column vector \( R_m(\omega) \) of the received signal. An estimate of the covariance matrix is obtained from:

\[ C(\omega) = \sum_m R_m(\omega)^T R_m(\omega)^* \quad (2.25) \]

Assuming that the background noise and the sources are mutually decorrelated, the covariance matrix is expressed as:

\[ \langle C(\omega) \rangle = H(\omega)^T S(\omega)^* H(\omega)^* + \langle B(\omega)^T B(\omega)^* \rangle \quad (2.26) \]

The assumption of uncorrelated sources yields:

\[ H(\omega)^T S(\omega)^* H(\omega)^* = \langle |S(\omega)|^2 \rangle H(\omega)^T H(\omega)^* \quad (2.27) \]

Also assuming an incoherent background noise yields:

\[ \langle B(\omega)^T B(\omega)^* \rangle = \sigma^2 I \quad (2.28) \]

Combining Eq. (2.26), Eq. (2.29) and Eq. (2.28), the diagonalization of the covariance matrix is given by:

\[ \lambda_i = \langle |S_i(\omega)|^2 \rangle \sum_{l=1}^{L} |H_{il}(\omega)|^2 + \sigma^2 \quad (2.29) \]

For the implementation of the virtual TR mirror approach, the TR operator is constructed from the estimated covariance matrix given in Eq. (2.25). The sum over several source realizations allows to average out the noise contribution in the measured signals.
2.2 Scattering from elastic targets

The classification of seabed objects as "elastic target" or "clutter" relies on the comparison of their response subsequent to an incident acoustic field. In the context of the DORT method, the singular value decomposition of the TR operator constructed from successive scattered field measurements provides classification information for each target present. However, understanding the characteristics of the scattered fields associated with objects of different nature is essential to implement a reliable classification procedure based on the resulting singular values. In this section, the scattering from elastic spherical shells subsequent to a plane wave insonification is introduced with an emphasis on the differences between the scattering from rigid (i.e. high density contrast) and elastic objects.

The Resonance Scattering Theory originally introduced in quantum mechanics and first applied to classical physics by Flax [18] has been used to investigate various acoustic scattering problems involving elastic objects. The formulation of scattering problems with the Resonance Scattering Theory underlines the physical meaning of each component of the scattered field and their angular dependence. The scattering from cylinders and spheres has been studied based upon this approach. We examine in this section the scattering in free field resulting from a plane wave incident on a spherical shell. A complete analysis of the scattered field from elastic spherical shells based upon the resonance scattering theory has previously been reported [19] and [20]. Here, we present several of the steps of the derivation presented in [20] along with properties of the elastic waves given in [1].

Given a plane wave insonification of amplitude $p_0$ expressed as:

$$p_i = p_0 \exp[i(kr - \omega t)]$$

The scattered field from an elastic spherical shell takes the form:

$$p_s = p_0 \exp[-i\omega t] \sum_{n=0}^{\infty} i^n(2n + 1) R_n h_n^{(1)}(kr) P_n(\cos(\theta))$$
where $R_n$ is a function of the wavenumber $k$ of the incident plane wave and $a$ the outer radius of the spherical shell which involves the density and phase speed of the shell material and surrounding medium. $r$ is the radial distance from the center of the shell, $n$ is the modal order, $h^{(1)}_n$ is the spherical Hankel function of the $n$th order and $P_n$ is the Legendre polynomial of the $n$th order. The partial-wave scattering function $S_n$ and the scattering phase shift $\delta_n$ are related as follows:

$$S_n = 2R_n + 1 \equiv \exp(2i\delta_n) \quad (2.32)$$

Introducing the form function $f(\theta)$ expressed as a sum of partial waves functions

$$f(\theta) = \sum_{n=0}^{\infty} f_n(\theta) \quad (2.33)$$

$$f_n(\theta) = \frac{2}{ka} (2n + 1) S_{n}^{1/2} \sin \delta_n P_n(\cos(\theta)) \quad (2.34)$$

and the asymptotic representation of the spherical Hankel function

$$h^{(1)}_n(kr) \sim \frac{1}{kr} i^{-(n+1)} \exp(ikr) \quad (2.35)$$

The scattered field in the limit $r \to \infty$ takes the form

$$p_s \sim p_0 \frac{a}{2r} \exp[i(kr - \omega t)] f(\theta) \quad (2.36)$$

The scattering from elastic objects can be represented as the superposition of a rigid body contribution and a radiation contribution. For thin spherical void elastic shells, the radiation contribution at low-frequency (i.e for small values of $ka$) is limited to the lowest order compressional and flexural modes and their respective circumferential Lamb-type waves: the symmetric wave $S_0$ and antisymmetric wave $A_0$. Upon fluid loading, the antisymmetric wave $A_0$ bifurcates into two dispersive waves $A_0-$ and $A_0+$. The symmetric $S_0$ wave is supersonic and reradiates periodically out of the spherical shell at the coupling angle defined as follow:
\[ \theta_c = \arcsin \left( \frac{c_{\text{ext}}}{c_{\text{shell}}} \right) \]  \hspace{1cm} (2.37)

where \( c_{\text{ext}} \) refers to the sound speed of the surrounding medium and \( c_{\text{shell}} \) refers to the phase speed of the \( S_0 \) wave in the shell. Fig. 2-5 depicts the travel paths of the \( A_0 \)- and \( S_0 \) Lamb-type elastic waves backscattered by a spherical shell after insonification around the coincidence frequency. The radiation angles are outlined for the two wave types.

At higher frequencies, the amplitude of the response of the elastic target increases within the mid-frequency enhancement region where the phase speed of the \( A_0 \)- wave approaches that of the surrounding medium and reaches a maximum at the coincidence frequency \( f_c \) defined as:

\[ f_c \approx \frac{c_{\text{ext}}}{2\pi d} \]  \hspace{1cm} (2.38)

where \( d \) defines the shell thickness. From the measurement of the time of arrival of the \( S_0 \) reradiated waves, Eq. (2.38) can be used to estimate the shell thickness. As the frequency increases above the mid-frequency enhancement region the number of modes excited increases.

Figure 2-5: Scattering from a spherical shell subsequent to a plane wave insonification
The dissipation of the circumferential waves determines the number of reradiation detectable and consequently the duration of the elastic response. The scattering from rigid objects defined by large density contrast with the surrounding medium, differs from that of elastic objects in that the radiation contribution is negligible and the scattered field reduces to the specular echo of the incident field. From the perspective of the TR operator, the presence of the elastic response in the scattered field introduces new invariants of transmission and therefore additional eigenstates of the TR process associated with the modes of the target. As a result, for similar configuration and target size, the number of significant singular values associated with an elastic target is expected to be larger. In a waveguide, the number of singular values associated with an object also depends on the presence of boundaries that introduce its images and therefore virtually extends its size.

2.3 Literature review

2.3.1 Iterative Focusing Approach

In 1989, Fink extended the phase conjugation method - used in optics to correct the wavefront distortion affecting monochromatic signals - to broadband signals with the formulation of the Time Reversal Mirror [21]. The time reversal process was first applied to the problem of focusing through aberrating media in pulse echo mode and to selective focusing in presence of multiple scatterers. Numerical simulations of the iterative focusing approach were presented in 1991 [22] to examine the effect of the array aperture and of the difference of reflectivities between scatterers. In 1995, the convergence of the iterative focusing method was investigated for well and poorly resolved scatterers [12]. The theoretical analysis showed distinct limits of convergence for odd and even number of iterations. The number of iterations needed to achieve convergence on the dominant scatterer was found to depend on the differences of reflectivity between the scatterers. The iterative focusing approach was explored at sea in 1999 [23] with a TR mirror spanning the water column and a receiving array located at the focusing range. A probe source was used in place of a scatterer to insure higher signal to noise ratios and the field transmitted by the TR mirror
after each iteration was amplified. A spread in time of the signal measured at the retrofocus location was attributed to the bandpass filtering introduced by the transducers of the TR mirror and the waveguide. In 2004, Montaldo [24] proposed a solution to the problem of time spreading. The method based on the iterative focusing approach resolves the wavefronts associated with each scatterers from the signals measured at the TR mirror. Each set of wavefronts detected is then recompressed in time prior to its retransmission to limit the bandpass filtering introduced by the transducers of the TR mirror and the waveguide. The same authors [25] introduced a new method based on the iterative focusing approach that involves two Source Receivers Arrays (SRA). A control array measures the field produced by the emitting array and compares it to a objective/desired field at the control array. The difference between the desired field and the measured field at the control array is then used to adjust iteratively the field transmitted from the emitting array and successively achieve optimum focusing. This technique overcomes the time spreading effects mentioned earlier and allows to achieve optimum focusing faster and with little computation efforts.

2.3.2 DORT method

In order to provide the reader with an organized literature review of the DORT method, the following section is divided into categories of problems to which the DORT method has been applied. Within each category, the publications are presented in chronological order.

Selective focusing on point-like and extended scatterers

In 1993, the theoretical formulation of the DORT method was introduced from the derivation of the iterative focusing approach [26] and the predictions made with the method were confirmed by simulated results [11]. The selective focusing based on the DORT method was investigated theoretically for well-resolved wires and the focusing ability of the DORT method was compared to that of the iterative approach for the most reflective scatterer of a set. Experimental results showed that both methods have the same ability to focus in space however the DORT method achieves focusing with only one iteration. Prada extended the DORT method to a finite size hollow cylinder and found that each eigenstate of the TR
operator is associated with a circumferential elastic wave and two points of emission on the cylinder [27]. The experimental backpropagation of each eigenvector confirmed this analysis and allowed an accurate estimation of the phase speed associated with each type of wave. In 1996, the special case of selective focusing on two targets with same apparent reflectivities was investigated theoretically and experimentally [7]. In such configuration, the DORT method produces two eigenvectors with unique characteristics. The magnitude of the first eigenvector shows in-phase contributions of the two targets while the second eigenvector shows their out-of-phase contributions. Selective focusing in this case requires the backpropagation of a linear combination of the two eigenvectors. In light of these results, the scattering analysis from the hollow cylinder conducted in 1994 was revisited [28]. The two secondary sources (associated with the radiation of elastic waves) have identical "reflectivities" and as observed for the two well resolved wires, the DORT method results in two eigenvectors whose magnitude exhibit interferences of these secondary sources radiating in-phase and out-of-phase. In 2001, Chambers carried out the theoretical analysis of the time reversal process associated with a homogeneous point-like scatterers (i.e. subwavelength and spherical) and demonstrated that one eigenstate of the TR operator is associated with the compressibility contrast between the scatterer and the medium while three other eigenstates are associated with the density contrast [13]. The analysis of the TR operator conducted on subwavelength isotropic and anisotropic cylinders results in observations validated by ultrasonic experiments [29]. Chambers applied the theoretical analysis of the DORT method to finite objects of simple geometry such as spheres and finite objects of arbitrary geometry in far-field configurations and confirmed theoretically the relation between the number of eigenvalues associated with an extended scatterer and its size [30]. Minonzio investigated the relation between the modes of vibrations of an elastic scatterers (caused by the density and compressibility contrasts), the projected harmonics associated with each mode and the singular values of the TR operator [31]. In particular, the maximum number of projected harmonics resolvable by the receiving array yields the possible reduction of the dimension of the TR operator.
Environment effects on selective focusing

In 1995, the selective focusing method was tested with a layer of inhomogeneous aberrating media separating the TR mirror from the scatterers [32]. It was shown experimentally that selective focusing on well-resolved scatterers can be achieved under the condition that the phase variations introduced by the layer of aberrating medium are smooth. In 1999, Mordant showed significant improvements in resolution obtained when focusing in a waveguide. Each image of the TR mirror relative to the waveguide boundaries virtually increases its extent and allows for higher resolution focusing compared to predicted resolution in free field. The effects of time variance of the environment on the selective focusing were investigated experimentally in presence of waves of varying amplitude [9]. It was observed that increasing the wave amplitudes increases the amplitude of the noise eigenvalues and decreases the amplitude of the eigenvalues associated with the targets. Finally, Mordant formulated the condition for selective focusing in time and space. The time domain DORT method requires that the reflectivity of each scatterers present in the waveguide allows to attribute to each of them the corresponding eigenvector over the frequency band of a short duration pulse. A similar study from Roux [33] also provided an analysis of the temporal focusing in the waveguide and general formulations for the size of the focal spot and sidelobe levels around the focal spot. In particular, it was observed that the size of the retrofocus increases when the insonifying source is located near the interface. Sabra and Dowling reported on the effect of background noise [34], array motion [35], array deformation [36] and ocean currents [37] on retrofocusing in shallow water environments providing theoretical background for realistic time reversal scenarios in ocean waveguides. The effect of bottom absorption and reverberation on the resolution of the focusing have been investigated [38] and a method was presented to compensate for transmission losses affecting each multipath between the TR mirror and the scatterer and improve signal to noise ratio at the scatterer but requires prior knowledge of the environment. In the light of earlier investigations showing degraded retrofocus in reverberating environments, several solution were proposed that attempt to reduce the amount of backpropagated field reaching the seabed at the focusing range. Kim [39] discussed the potential benefits of using a prob-
ing source located at mid-depth to determine the field to be transmitted by the TR mirror in order to produce a null of transmission near the bottom at the range of the probing source. A similar approach consisted in insonifying a mid-depth target with a probing source and showed improvements in the backpropagated field reaching the target. The use of a probing source however limited the potential of these approaches. Instead, simulated results from Song [40] explored the possibility of determining the eigenvector associated with the seabed reverberation at a range of desired focusing to generate a nulling vector by linearly combining the remaining orthogonal eigenvectors of the TRO. [41]

**Time Reversal Imaging**

The localization and shape estimation of extended targets based on TR imaging has been investigated in publications related to inverse problems. In 2004, Hou layed out an approach based upon the coupling of the target location estimation with a shape estimation algorithm based upon a level set method [42]. In 2007, Hou examined the construction of imaging functions for several types of scatterer boundary conditions (i.e. scatterer properties) [43] which were used to determine the location of the target and its boundaries using a prior estimation of the number of singular values associated with the target subspace. The number of target associated eigenstates was determined by comparing ratios of singular values to a predetermined threshold. It was observed during numerical experiments that the resolution of the focusing is found to improve when estimated boundary conditions considered for the imaging functions match actual boundary conditions at the scatterer. Carin [44] presented similar observations in presence of a strong scatterer, the waveguide characteristics which are unknown and supposed for the TR imaging of the scattered field can be adjusted until an optimum focusing is reached.
Chapter 3

Modeling of the virtual TR mirror approach

3.1 Introduction

In order to predict the performance of the concurrent detection classification and localization method based on Time Reversal, a numerical model of the virtual TR mirror approach is implemented. The model allows the localization of the seabed targets by transmitting the singular vectors of the TR operator (see Chapter 2) and also provides valuable insights into the method’s limitations. Since the backpropagation does not involve actual transmissions underwater, the process is referred to as a "virtual Time Reversal mirror". In order to meet the limited power constraints imposed by autonomous operations, the amount of computations involved is minimized by making appropriate simplifications on the modeling. The steps taken to simulate the target insonification, target scattering, and scattered field measurements are described in the first section of this chapter with an emphasis on the underlying theory. The second section explains the implementation of the virtual TR mirror, presents results and summarizes observations made from the simulations.
3.2 Scattered field modeling

An overview of the modules involved in the modeling of the virtual TR mirror approach is shown on Fig. 3-1.

Figure 3-1: Overview of the modeling steps

3.2.1 Target insonification - OAST

The waveguide characteristics considered in this section are chosen to reflect the sea trial during which the virtual TR mirror approach was tested. The sea trial was conducted during the winter of 2008 in a shallow waveguide (30m deep). For modeling purposes, we assume an isospeed water column of 1500m/s. In absence of bottom properties measurements, we make the assumption of an isospeed bottom and use table values corresponding to a sandy bottom [45]. Considering the previous assumptions and the fact that the bathymetry in the area of the sea trial is roughly constant, the propagation environment is conveniently modeled as a Pekeris waveguide.
In a range independent waveguide, the Helmholtz equation satisfied by the total field in each layer takes the following form:

$$[\nabla^2 + k^2(z)]\psi(r) = f(r)$$  \hspace{1cm} (3.1)

where $k$ is the wavenumber, $\psi$ the field parameter of interest and $r$ is the position vector.

A common approach to solving the Helmholtz equation (3.1) is the use of integral transforms: the total field is decomposed into plane waves defined by their individual horizontal wavenumbers. This approach simplifies the treatment of the boundary conditions at each layer interface of the waveguide. The Hankel transform pairs are given by:

$$f(r, z) = \int_0^\infty f(k_r, z)J_0(k_r r)dk_r$$ \hspace{1cm} (3.2)

$$f(k_r, z) = \int_0^\infty f(r, z)J_0(k_r r)dr$$ \hspace{1cm} (3.3)

The Hankel transform applied to the Helmholtz equation reduces the three dimensional Helmholtz equation 3.1 to the following depth-separated wave equation:

$$\left[ \frac{d^2}{dz^2} + (k^2 - k_{r}^2) \right] \psi(k_r, z) = S_{\omega} \frac{\delta(z - z_s)}{2\pi}$$ \hspace{1cm} (3.4)

where $S_{\omega}$ is the source strength at the frequency $\omega$, $k_r$ is the wavenumber in the radial direction, and $z_s$ is the depth of the source.

The solution of the depth-separated wave equation is obtained using the Direct Global Matrix (DGM) approach. The field within each layer of the waveguide is represented in terms of source contributions and unknown scalar fields amplitudes of up and down going conical waves [46] [47] [5]. The unknown scalar fields which are superposed to the fields resulting from the source contributions are integrated into local sets of equations which are then mapped into a global set of equations solved simultaneously for the amplitude of the

\textsuperscript{1}we provide the reader with several of the key equations given in [45]
unknown scalar fields within all the layers. The implementation of this method involves some complexity but this approach is unconditionally stable.

Once the depth-separated wave equation has been solved, the field parameters of interest are determined in the waveguide at any given range and depth by evaluating the inverse Hankel transform (Eq. 3.2) using the Fast Fourier approximation. For practical purposes, the integration domain of Eq. 3.2 is truncated to the largest wavenumber providing significant kernel contribution. The reader can refer to [45] for the detailed derivations.

The waveguide insonification from an omnidirectional source is modeled with OAST, the transmission loss module of the OASES package (Ocean Acoustics and Seismic Exploration Synthesis) developed by H.Schmidt [48]. OASES is a propagation model based on wavenumber integration and the Direct Global Matrix approach. It supports a large variety of environmental models including isovelocity fluids, fluids with speed gradient, isotropic elastic media. The transmission loss module takes as inputs the depth of the source and the characteristics of each homogeneous layer constituting the waveguide (depth of the layer boundaries, fluid density, compressional/shear sound speeds, attenuation coefficients). Since we are concerned with the modeling of a Pekeris waveguide, only three layers are necessary: the upper half space, the water column layer, the sea bottom layer. Solutions are provided in the frequency domain and time domain analysis is obtained by Fourier synthesis. In order to compute the field in the vicinity of a scatterer, OAST accepts as an input the range from the source to the scatterer and the layer where the target is located.

### 3.2.2 Scattered field - SCATT

The scattering module of OASES called SCATT uses the incident field computed in the vicinity of the scatterer to determine the resulting scattered field using the virtual sources approach. The virtual sources approach consists in replacing the scatterer by a set of sources distributed within its volume. The strength of each source is determined so that
the field produced by the virtual sources when superimposed to the incident field, satisfies the boundary conditions imposed by the type of scatterer. In the case of elastic targets, the dynamic stiffness matrix relates the pressure on the surface of the scatterer to the normal displacement of its surface. We follow several of the step of [3] to describe the computation of the virtual source strengths.

The total pressure $p$ and total displacement $u$ on the surface of the scatterer are first expressed as:

$$p = p_i + p_s \quad u = u_i + u_s$$  \hspace{1cm} (3.5)

where $p_i$ and $u_i$ refer to the incident field contribution. $p_s$ and $u_s$ refer to the scattered field contribution.

Considering a set of $N$ virtual sources of strengths $s$, the scattered field contribution is expressed as:

$$p_s = Ps \quad u_s = Us$$  \hspace{1cm} (3.6)

where $P$ and $U$ are $N \times N$ matrices containing the pressure and normal displacement Green functions.

The total pressure is then related to the total displacement through the dynamic stiffness matrix $K$ as follows:

$$p = Ku$$  \hspace{1cm} (3.7)

It is important to note that the stiffness matrix of the target is computed independently of the medium that surrounds it and can be used to treat scattering problems for different burial depth and target orientation. The stiffness matrix can be computed with an exact spherical harmonics representation in the case of spherical shells or with a finite element method for objects of arbitrary shapes. Combining Eq. 3.5, 3.6 and 3.7, the source strengths are computed using the following equation:
$$s = [P - Ku]^{-1}[Ku_i - p_i]$$ (3.8)

The scattering module of OASES, SCATT, is used to compute the virtual source strengths associated with various scatterer geometries and properties. In the special case of spherical targets, the command sphcvs3d takes the characteristics of the scatterer as an input to compute the virtual source strengths and the scattered field in the surrounding medium can be obtained from the spectral Green functions.

### 3.2.3 Transmission of the scattered field to the receiving array

Given the distribution of virtual sources and their respective strength, the Green functions from the virtual sources to the surrounding medium are computed with the command OAST3D. Since the virtual sources are distributed in a volume generally much smaller than the region of interest, the scattered field is conveniently derived in a cylindrical coordinate system [49]. Within each layer of the stratified surrounding medium, the scattered field is expressed as the superposition of the field produced by the virtual sources present (if any) and the unknown field required to satisfy the boundary conditions at the layers interfaces. It is governed by the homogeneous wave equation. In a horizontal fluid layer, the resulting displacement potential is expressed with an azimuthal Fourier series as follows:

$$\phi(r, \theta, z) = \sum_{m=0}^{\infty} \left[ \phi_S^m(r, z) + \phi_H^m(r, z) \right] \left\{ \begin{array}{l} \cos m\theta \\ \sin m\theta \end{array} \right\}$$ (3.9)

where $\phi_S^m(r, z)$ is the contribution from the virtual sources and $\phi_H^m(r, z)$ the contribution that satisfies the layers boundary conditions. Each contribution can be expressed in terms of horizontal wavenumber integrals as follows:

$$\phi_S^m(r, \theta, z) = \frac{e^{im\theta}}{4\pi} \int_0^\infty \left[ \sum_{j=1}^N s_j \left\{ \begin{array}{l} \cos m\theta_j \\ \sin m\theta_j \end{array} \right\} \times J_m(k_r r_j) \frac{e^{jkrz}}{jk_z} \right] k_r J_m(k_r r) dk_r$$ (3.10)
\[
\phi_H^m(r, \theta, z) = \int_0^\infty \left[ A_+^m(k_r) e^{jk_z z} + A_-^m(k_r) e^{-jk_z z} \right] k_r J_m(k_r r) dk_r
\]  

(3.11)

where \( k_r \) and \( k_z \) are respectively the horizontal and vertical wavenumber, \( s_j \) is the \( j^{th} \) virtual source strength to be determined and \( A_+^m \) and \( A_-^m \) are the azimuthal Fourier coefficients of the up and down-going planes waves satisfying the boundary conditions in each layer. The factor \( \epsilon_m \) equals 1 for \( m = 0 \) and 2 otherwise.

For the case where both virtual sources and receivers are located in the water column, the Green function becomes:

\[
G(\mathbf{r}_i, \mathbf{r}_j) = s_j \frac{e^{ik_r |\mathbf{r}_i - \mathbf{r}_j|}}{4\pi |\mathbf{r}_i - \mathbf{r}_j|} + s_j \sum_{m=0}^\infty \left\{ \begin{array}{c}
\cos m\theta_i \cos m\theta_j \\
\sin m\theta_i \sin m\theta_j 
\end{array} \right\} 
\times \frac{\epsilon_m}{4\pi} \int_0^\infty J_m(k_r r) R_{11}(k_r) e^{jk_z(z_1 - z_j - r_i)}
\times k_r J_m(k_r r_i) dk_r 
\]  

(3.12)

### 3.3 Modeling implementation

The commands involved in the modeling described in sections 3.2.1, 3.2.2 and 3.2.3 are executed from a Matlab code. The position of the source and receivers are expressed in a Cartesian coordinate system centered on the elastic target. Using this representation, the computation of distances between the elements of the setup reduces to simple vector calculus operations. This coordinate system also simplifies the grid points representation of the seabed described in the next section. The simulations involve a set of stationary receivers and a set of stationary sources all located at the same depth. The target modeled here is the GOATS sphere and we consider for the clutter a rigid spherical shell of dimensions identical to the GOATS sphere. Each insonification from one of the source location yields a set of complex pressures at the receivers which are stored to construct the interelement matrix. For each of the frequencies of the insonifying signal, the interelement matrix is determined in order to produce the singular values and singular vectors associated with each scatterer over the frequency band of the insonifying signal.
3.4 Virtual Time Reversal Mirror

The previous section of this chapter describes the steps taken to model the waveguide insonification and the scattering from targets present in the waveguide. The complex pressures at the receivers obtained from this numerical model or from actual measurements are used to virtually transmit the time reversed scattered field from the receivers to the seabed and localize the scatterers present.

In the modeling framework introduced above, the seabed is represented as a grid (Fig. 3-2) that extends over the operating area of the AUV. The pressure amplitudes resulting from each backpropagations on the seabed are computed at every point of the grid. The Green functions required to calculate the pressure field on the seabed from the complex pressure at the receivers are precomputed using the oasp command for predefined source depths, frequencies and waveguide characteristics. At run time, the real and imaginary part of the Green functions are interpolated at the radial distances of each grid point. Since the waveguide is assumed to be range independent and the receiving array assumed to be horizontal, the same set of Green functions is used to compute the field produced on the seabed by all elements of the array. The resolution of the grid can be adjusted to reduce the time of computation or improve the resolution.

The backpropagation of the singular vectors can also be computed in free field as it will be discussed in the next chapter. The pressure at each grid point of the seabed is then determined by the cross product of the singular vector with the propagation vector $M$ defined by:

$$M = \left( \frac{e^{-jk|r-r_1|}}{|r-r_1|}, \frac{e^{-jk|r-r_2|}}{|r-r_2|}, \cdots, \frac{e^{-jk|r-r_n|}}{|r-r_n|} \right)$$

(3.13)

where $r$ refers to the position vector of the grid point and $r_j$ refers to the position vector of the $j^{th}$ receiver.
3.5 Results and observations

The modeling described in sections 3.2 and 3.4 is used to test the ability of our TR based approach to reject clutter and localize seabed scatterers in configurations similar to the set up of the sea trial presented in chapter 4. For the following, we consider a set of stationary sources and a set of stationary receivers located at the same depth in a shallow water waveguide. The targets to be detected and localized are an elastic spherical shell and a rigid spherical shell. Multiple scattering effects between targets are neglected.

3.5.1 Clutter rejection

It was discussed in the second chapter that the response of elastic targets exhibits resonances which are not present in the response of rigid objects. In order to illustrate this point, the TR operator is constructed using the modeling of our approach in presence of an elastic spherical shell and in presence of a rigid spherical shell of identical dimensions. Fig. 3-3 depicts the amplitude of the first singular values for both targets as a function of frequency. The singular value associated with the elastic sphere clearly exhibits a resonance near 900Hz. In contrast the singular value associated with the rigid sphere does not show significant variations over the same frequency band. For elastic targets, the time of arrival of the Lamb-type waves re-radiated in the direction of the receivers varies depending on the array position and orientation. Therefore, the interference of these waves results at the receivers in time signals that differ significantly depending on the configuration source target receivers. As a consequence, the frequencies at which the singular values have maxima.
can not directly be used to classify the target; instead the variability of a singular value with respect to frequency can be used to classify a target as rigid or elastic.

![Graph showing Amplitude of the first singular value as a function of frequency for a void elastic sphere and a rigid sphere.](image)

Figure 3-3: Amplitude of the first singular value as a function of frequency for a void elastic sphere and a rigid sphere

It is also important to observe from Fig. 3-3 that the dominant singular value is not necessarily associated with the same scatterer throughout the whole frequency band. The backpropagation of the first singular vector therefore results in a field that focuses on the dominant scatterer at the frequency of interest.
3.5.2 Selective focusing

We investigate in this section the ability of the virtual TR mirror approach to localize targets on the seabed. The scattered field computed at the receivers in presence of an elastic target and a rigid target is used to construct the TR operator. The backpropagation of the eigenvectors associated with each targets is achieved using the virtual Time Reversal mirror approach described in section 3.4. The receiving array considered here is composed of 20 elements spaced every 0.75m for a total length of 14.25m. In presence of well resolved scatterers (i.e scatterers separated by a distance larger than the resolution cell), the singular vectors of the TR operator provide the phase and amplitude information to focus selectively on each scatterer present. Fig. 3.4(a) and 3.4(b) show respectively the field backpropagated on the seabed for the transmission of the singular vector associated with the elastic target located at (0,0) and the singular vector associated with the rigid target located at (-20,0). We observe that the transmission of each singular vector shows a maximum at the location of each corresponding scatterer. The size of the retrofocus is related to the resolution of the array. For the smaller array extent considered during the sea trial, we expect a decrease in resolution.

Figure 3-4: Backpropagation of singular vectors associated with an elastic sphere located at (0,0) and a rigid sphere located at (-20,0) compensated for spherical spreading
Chapter 4

Experimental validation

4.1 Introduction

The virtual TR mirror approach is tested with a set of experimental data collected near the island of Palmaria (Italy) on the 25th of January 2008. The first section of this chapter provides a description of the setup of the CCLNet08 trial. The steps taken to process the data collected are explained in the second section. The last part of the chapter concludes with the results of the processing.

4.2 Experimental configuration

4.2.1 Description of the sea trial

One of several objectives of the CCLNet08 sea trial described in this chapter was to collect scattered field measurements in presence of seabed objects. The waveguide was insonified with an omnidirectional source towed behind a research vessel (Leonardo) and the acoustic field scattered by the seabed objects was sampled with an array of hydrophones towed behind an Autonomous Underwater Vehicle (Ocean EXplorer or OEX version C) (Fig. 4-1).
The choice of a bistatic configuration is best suited to sample efficiently the anisotropic scattered field from the seabed targets. Fig. 4-2 shows the path of the Leonardo and of the OEX during the run as well as the positions of the scatterers in local grid coordinates. Both vehicles started their mission at the north east of the scatterers then moved west, south and east. Over the course of the mission, the scatterers were successively located at endfire and broadside relative to the receiving array.

The targets to be detected and localized are an elastic sphere and a rock. Prior to the experiment, the targets were lowered on the seabed and their GPS location logged. The elastic target referred to as the GOATS sphere is hollow, made of steel (density 7700
kg/m$^3$, compressional wave speed 5950 m/s shear wave speed 3240 m/s). The shell outer radius and wall thickness are respectively 0.53 m and 0.03 m. The scattering strength of the GOATS sphere is known from past numerical modeling investigations and experimental measurements (Fig. 4-3). The rock which played the role of clutter is of comparable size and its scattering strength is also known. The two targets were separated by a distance of 50 m.

![Figure 4-3: Experimental scattering strength of the GOATS sphere](image)

4.2.2 Waveguide insonification

The waveguide is insonified with a Lubell source (Ref. LL1424HP) which transmits a 1-ms Ricker pulse$^1$ centered at 2 kHz with a ping rate of 4 Hz. Fig. 4.4(a) and Fig. 4.4(b) show respectively the Ricker pulse applied to the Lubell source in the time domain and in the frequency domain. The short duration of the pulse provides the spatial resolution suitable for localization purposes and the low frequency band allows for a higher coverage rate.

Before the beginning of the run, the source is lowered at a depth of 10 m and calibrated by measuring its sound pressure level 13 m away from its center for applied driving voltage of frequency varying from 0.1 kHz to 5 kHz. Fig. 4-5 and Fig. 4-6 show respectively the

$^1$The Ricker pulse is defined as the second derivative of a Gaussian
4.2.3 Data acquisition

The SLITA (Slim LIne Towed Array) receiving array consists of 48 hydrophones distributed in 2 octave spacings: 0.211m and 0.422m. The processing described in this chapter is concerned with the measurements from the 32 hydrophones spaced every 0.211m. The hydrophones (Benthos AQ-4) have a sensitivity of 201dB ref 1 Volt per μPa and are de-
Figure 4-6: Transmitting voltage response of the Lubell source measured during calibration signed to compensate for acceleration noise [50]. During the mission, the acoustic field is sampled with an A/D board (PCI-24DSI32) at an acquisition rate of 51kHz and with a 24 bits resolution. Every 4 seconds, the data acquisition system produces a binary file that contains the time series of each of the 32 channels. Continuous measurements were conducted during the mission of the OEX which started at 11.34am and finished at 1.14pm local time on the 25th of January 2008. Our processing is limited to the part of the run during which the distance between the source the target and the receivers ensures sufficient signal to noise ratios.

4.2.4 Environmental data

As explained in section 3.2.1, the bathymetry data available in the region of the sea trial indicates that the depth of the waveguide is approximately constant and equal to 30m. The sound speed profile in the winter time in a very shallow waveguide is assumed to be constant over the whole water column and was estimated to be 1500m/s. The sound speed of the sandy bottom was not measured and the value of the compressional sound speed was assumed to be 1800m/s.
4.3 Processing

4.3.1 Estimation of the source and receivers locations

In order to compute the time of arrival of the target and rock reflected echoes, the location of the receivers and of the source during the mission must be estimated. This section describes the steps taken to estimate precisely their location during the run.

The location of the hydrophones is determined from the logs of the OEX and the specifications of the SLITA array. Before the dive, the Global Positioning System (GPS) of the OEX provides its location. During the mission, the navigation system of the OEX uses its initial GPS fix as a reference to estimate its future positions from dead-reckoning and DVL measurements. The GPS fix taken after the vehicle surfaces at the end of the mission, is used to correct for navigation errors that occurred during the run. In the local grid coordinate system, the cumulated navigation error was found to be of 2.74m in the \( x \) direction and 34m in the \( y \) direction. Since the SLITA files processed were produced toward the end of the run, the corresponding vehicle locations are assumed to be affected by the same navigation error and the same correction is applied.

The location of the receivers relative to the AUV are estimated by replaying the logs of the OEX in a simulation environment [51] and running in parallel a dynamic array model. The model takes as inputs the speed of the OEX logged during its missions and the characteristics of each section of the array (number of subsections, weight in water, diameter, tangential and normal drag coefficient, modulus of elasticity). The SLITA array is composed of 3 main sections: the extension module, the acoustic module and the tail and the number of subsections is chosen to provide a fine element spacing in the extension and the acoustic module. Given the previous (or initial) shape estimate and the current tow point velocities, the model determines the new array shape by applying the Newton-Raphson iteration to solve the set of nonlinear algebraic equations found in [52]. During the run, the depth of the hydrophones is assumed to be equal to the depth of the vehicle provided by its depth sensor.
The location of the source towed behind the research vessel is determined during the run from the GPS measurements of the ship knowing the position of the towing point relative to the center point of the ship. The length of the ship is 28.6m and the distance between the GPS positioned on the mast and the towing point was estimated to be 17m. Due to the short length of the towing cable, it is assumed to remain straight and to point in the direction of motion of the ship. Before the beginning of the mission, the source is lowered at a depth of 10m while the ship has zero forward speed and during the mission a sensor mounted on the source records its depth (Fig. 4-7).

![Figure 4-7: Depth of the Lubell source during the run](image)

Finally, the timestamps of the logs of the Leonardo take as a reference the GPS absolute time while the timestamps of the logs of the OEX and of the SLITA files take as a reference the clock of its payload. At the end of the run, the payload clock was found to be 28 secs out of synchronization with the GPS time. This additional correction was applied to the timestamp of each AUV location and each SLITA file timestamp.
4.3.2 Validation of the source and receivers location estimations

The processing of multipath echoes is irrelevant to our analysis since they do not interact with the target or the rock. However, their time of arrival can be used to verify that the source and receiver locations are correctly estimated after the navigation and synchronization errors have been accounted for. Since the wavelength of the insonifying signals are much shorter than the characteristics lengths of the waveguide, the time of arrivals of each multipath echo is predicted from ray theory and using the source and receivers locations estimate during the run. The experimental time of arrival are determined from the signals measured at each hydrophone. The binary SLITA files produced by the data acquisition system of the OEX during the mission are converted to voltage time series of each channel using a Matlab script provided by M. Mazzi. The following analysis is concerned with the processing of the measured signals in the frequency band of the Ricker pulse (0-6kHz) used for insonification. Prior to the processing, the signals are low pass filtered with Butterworth filter of order 10 and a cut-off frequency of 12kHz.

The strongest arrivals present in the time signals measured at the hydrophones are associated with the direct path between the source and the receivers. Each time series is matched filtered with the first direct arrival to determine precisely the following direct arrival times. Once the later have been detected, the time series are divided into time windows of 250 milliseconds and staked to reveal the common features of the pings and facilitate the post processing.

Fig. 4-8 shows a small set of pings stacked together. We can see from the figure that the direct arrival is followed by the surface reflected arrival which is an inversed and slightly attenuated replica of the direct arrival (due to the phase inversion of the signal at the pressure release surface). The following multipath echoes are associated with paths depicted in Fig. 4-9: bottom reflected, bottom-surface reflected, surface-bottom reflected etc.

The experimental arrival times of the multipath echoes are compared to predicted ar-
rival times inferred from geometrical considerations for the first receiver. Fig. 4-10 shows the predicted time of arrival of the direct arrival followed by the multipath arrivals for the pings of the run. Fig. 4-11 shows for the same set of pings, the amplitude of the experimental signals stacked as in Fig.4-8. The good match between predicted and experimental arrival times validates the set of corrections applied in section 4.3.1 and our assumptions on the waveguide characteristics. Since the location of the source and receivers are correctly estimated, the predicted arrival time of the target and rock reflected echo (Fig. 4-10) are also assumed to be correct.
Figure 4-10: Predicted time of arrivals of the multipath echoes

Figure 4-11: Experimental time of arrivals of the multipath echoes
### 4.3.3 Construction of the TR operator

The construction of the TR operator is based upon the covariance matrix approach introduced in Section 2.1.5. Given the vectors of complex pressures $P_n(\omega)$ measured at the receivers for each of the $N$ source positions, the TR operator is expressed as follows:

$$ TRO(\omega) = \sum_{n=1}^{N} P_n(\omega)^t P_n(\omega)^* $$

In order to recreate a stationary receiving array from the moving receivers, the largest number of receivers that can be considered is 16. The construction of the TR operator therefore involves 16 successive insonifications that occur over a time period of 4 seconds (which in calm water can be assumed to be smaller than the coherence time of the channel). In practice, the requirement of overlapping receivers at the emission time of the source is not strictly satisfied, but the assumption of a range independent environment allows to report small discrepancy from overlapping positions as a delay in the time of emission. Insonifying the waveguide from an omnidirectional source as in the sea trial results in significant transmission losses caused by the geometrical spreading and the attenuation of the portion of the transmitted field incident on the seabed at supercritical angles. The presence of sources of noise during the mission also reduces the signal to noise ratios at the receivers. It was reported that the cavitation at the propeller of the Leonardo introduces a strong permanent noise in the frequency band of the insonifying signal [50]. The self noise of the array and the noise introduced by the propeller of the AUV are both assumed to be dominated by the noise caused by the Leonardo in the frequency band of interest. As a result, the weak echoes from both targets can not be detected visually when stacking pings of the run together. Instead, the arrival times of the targets is determined from the analysis of the singular values of the TR operator constructed for a sliding time window. In order to limit the impact of background noise, the duration of the sliding time window over which the TR is computed needs to be reduced to 5ms with an overlap of 80%. This window duration is sufficiently long to include both the specular echo of the target and its elastic response if present [1] while the overlap of 80% allows to determine more precisely the arrival time of each echo.
4.4 Results

4.4.1 Detection of the target and rock echoes

As explained in the second chapter, each singular value and corresponding eigenvector is associated with an invariant of the TR process. With respect to the experimental setup, we expect that the propeller of the Leonardo which constitutes a strong permanent source of noise will introduce at least one singular value which may dominate the singular values associated with the scatterers to be detected. Since the measured signals were observed to have very low amplitude above 3kHz, the following figures only show the frequency band [0-3kHz].

**Target echo**

Fig. 4-12 and 4-13 show the amplitude of the four largest singular values in a time window that contains the echo from the target. For the set of pings used to construct the TR operator, the echo from the target is predicted to occur at sample 2900. This predicted time of arrival coincides with the time at which the amplitude of the third singular value exceeds that of the background noise. We suspect that the first singular value is associated with the Leonardo while the fourth and remaining singular values are associated with the background noise.

![Figure 4-12: Elastic target echo - First and second singular value](image)
Figure 4-13: Elastic target echo - Third and fourth singular value

Rock echo

Fig. 4-14 and 4-15 show the amplitude of the four largest singular values in a time window that contains the echo from the rock. For the set of pings used to construct the TR operator, the echo from the rock is predicted to occur at sample 3100. This predicted time of arrival coincides with the time at which the amplitude of the third singular value exceeds that of the background noise 4.15(a). It is unclear from the observation of the first and second singular values if they are associated with the echo from the rock or with the propeller of the Leonardo. The remaining singular values are associated with the background noise.

Figure 4-14: Rock echo - First and second singular value
4.4.2 Target localization

In order to determine to which scatterer corresponds each of the singular value depicted in section 4.4.1, the virtual TR mirror approach is used to backpropagate the corresponding singular vector. It was explained in section 4.3.3 that the presence of background noise can be limited by reducing the duration of the time window used to construct the TR operator. This comes at the cost of removing the multipath between the source and the receivers. As a consequence, the resolution during the backpropagation process is reduced and consequently the size of the retrofocus increased. The singular vectors obtained from the decomposition of the TR operator computed for a short time window provides the amplitude and phase to apply to each transducer of the array to focus in free space at a given frequency (neglecting the target bottom reflected path). The resolution cell of the array is defined in free field as follows [29]:

$$ R = \frac{\lambda_0 F}{D} \quad (4.2) $$

where $\lambda_0$ is the wavelength of the signal transmitted by the TRM, $F$ is to the distance between the target and the array and $D$ is the length of the stationary section of the array (equal 3.165m). For the frequency band of interest, $\lambda_0$ varies from 0.25m at 6kHz to 1.5m at 1kHz. At the time when the signals used to construct the TR operator in section 4.4.1 are recorded, the distance $F$ between the target and receivers of the array is approximately
45m and corresponds to a resolution cell of $R = 21m$ at 1kHz and 3.5m at 6kHz.

**Elastic target echo**

![Graph showing singular values](image)

**Figure 4-16: Singular Values associated with the echo from the elastic target**

The decomposition of the TR operator at the time of arrival of the elastic target echo yields the singular value distribution depicted in Fig 4-16.

Fig 4.17(a) shows the location of the propeller of the Leonardo and the receivers at the time when the TR operators of Fig. 4-12 and Fig. 4-13 are constructed. Fig. 4.17(b) and 4-18 depict a top view of the backpropagated field on the seabed associated with the first three singular vectors. The backpropagated field is compensated for spherical spreading and is depicted for an array pointing in the $-x$ direction and located at the origin (matching the orientation of the array of the experimental setup). The field associated with the transmission of the first singular vector points in the direction of the Leonardo which is consistent with the assumption that the dominant singular value is associated with the noise produced by its propeller. It is unclear if the backpropagated field associated with the second singular vectors points in the direction of the target or in the direction of the propeller of the Leonardo. The backpropagated field associated with the third singular vector does point in the direction of the elastic target which is also consistent with the fact that the time at which the singular value emerges coincides with the predicted time of arrival.
Rock echo

The decomposition of the TR operator at the time of arrival of the elastic target echo yields the singular value distribution depicted in Fig 4-19.

Fig 4.20(a) shows the location of the propeller of the Leonardo and receivers corresponding to the pings from which the TR operators depicted in Fig. 4-14 and Fig. 4-15 are constructed. We observe from Fig. 4.15(a) that the third singular value which is assumed to be associated with the rock, emerges from the background noise at a frequency of 1kHz. The resolution provided by the configuration at this frequency does not allow to separate
the contribution of the propeller from that of the rock. The backpropagated field corresponding to the first (Fig. 4.20(b)) and third singular vectors (Fig. 4.21(b)) are pointing in the direction of the rock. However the backpropagated field associated with the second singular vector (Fig. 4.21(a)) points in a direction that can not clearly be associated to the rock or the Leonardo.

Figure 4-19: Singular Values associated with the echo from the rock

Figure 4-20: Free field backpropagation of the first singular vector
Figure 4-21: Free field backpropagation of the second and third singular vector

It was demonstrated in [31] that the field scattered by an elastic spherical target can be represented in terms of monopole and multipole contributions. In order to be able to resolve the singular values associated with the multipole contributions, the extent of the receiving array needs to be sufficient to capture the angular variations of the corresponding projected harmonics. The configurations source target receivers depicted in Fig. 4.17(a) and Fig. 4.20(a) allow to resolve the isotropic component of the scattered field from both target but do not allow to detect the anistropic field associated with the elastic target scattering. Therefore, the rock and the elastic target appear identical in terms of their singular values and can not be classified. The localization process requires the combination of bearing estimates obtained from the transmission of singular vectors associated with successive sensing points-of-view. The benefit of a bistatic configuration becomes apparent here as it allows for adaptative resolution adjustments needed to classify the targets previously localized.
Chapter 5

Conclusion

5.1 Summary

A virtual TR mirror approach for the detection, localization and classification of seabed objects has been modeled and tested with experimental data. A numerical code was implemented to simulate the broadband insonification (0-6kHz) of elastic and rigid targets from a set of source locations and to determine the resulting pressure fields at a stationary receiving array. The TR operator was constructed using the covariance matrix approach, and it was shown that the analysis of the singular values allows the detection of elastic scatterers by observing the presence of resonance frequencies. The virtual TR mirror approach was modeled and used to localize seabed objects by simulating the backpropagation of their corresponding singular vectors.

The second part of this work involved the processing of scattered field measurements collected at sea in the presence of seabed targets. The location of the source and receivers during the sea trial once determined accurately was used to predict the time of arrival of the echoes from the targets. A permanent source of noise was present during the sea trial measurements. In order to limit its effect, the TR operator was constructed over short overlapping time windows. The analysis of its singular values allowed the determination of the target echoes arrival time which were found to be in good agreement with the predictions based upon geometric considerations. Reducing the duration of the time window when
constructing the TR operator came at the cost of removing contributions from the waveguide boundaries and resulted in a degraded resolution during the backpropagation of the singular vectors. Due to the limited extent of the stationary section of the array, the backpropagation of the singular vectors only allowed an estimate of the bearing to the targets but fails at providing an estimate of their range. Localizing the scatterers with the same array aperture would therefore require a sufficient signal to noise ratio to combine several bearing estimations.

5.2 Future work

One major challenge associated with the virtual TR mirror approach consists in matching each singular value with the corresponding scatterer. It was observed that the presence of permanent sources of noise results in irrelevant eigenstates that need to be identified as such. The classification of targets based upon the singular value analysis also requires to take into account the changes of effective reflectivity associated with each target as the mobile sensing platforms operates. The observation of the singular values with respect to time will have to be investigated further as it may provide ways of tracking these changes and sort the singular values in a systematic manner.

Recreating a stationary array from a set of moving receivers reduces its exploitable aperture which affects the localization process. In that regard, the use of a bi-static configuration provides the flexibility needed for adaptative resolution adjustments and facilitates the classification and localization of seabed targets. However, translating a distribution of singular values into target structural characteristics remains a complex problem that requires to account for varying sensing point-of-views and unknown possible scattered fields. For the virtual TR mirror approach to be exploitable by autonomous platforms, the processing of the singular values will need to be addressed in a systematic manner.
The computational efforts involved in the transmission of the singular vectors have been minimized by considering a range independent waveguide and limiting our analysis to seabed objects. One can anticipate, however, that the backpropagated field on the seabed associated with mid-depth scatterers would yield erroneous location estimations. Extending the domain over which the field is backpropagated would require an adaptive resolution technique in order to limit the amount of computation while allowing the localization of scatterers or sources at different depths.
Appendix A

Matlab User Guide

A.1 Experimental setup - Estimation of echoes arrival time

The following code is used to read the logs produced by the OEX and by the Leonardo and plot their location during the sea trial described in Chapter 4 (see Fig. 4-2). The location of the source and receivers (respectively towed behind the Leonardo and the OEX) are estimated using the ship characteristics and the specifications of the SLITA array. All the locations provided in the logs as latitude and longitude were converted to local grid coordinates to facilitate the computation of distances between the elements of the setup. Using the location of the sources and the first receiver, the code generates the estimated time of arrival of multipath and target reflected echoes (see Fig. 4-10). Several corrections are introduced to compensate for the lack of synchronization between the clock of the payload on the OEX and the GPS time (line 79) and to account for the navigation errors which were evaluated during the post processing (line 302).
% defines origin of local grid coordinate system
LatOrigin = 44.016;
LongOrigin = 9.82;
geo_datum = [LatOrigin LongOrigin];
zone = floor((geo_datum(2)+180)/6) + 1;
prime_meridian = zone*6-183;
utm_datum = geo2utm(geo_datum, prime_meridian);

% loads logs of Leonardo
log_leo = load('log_leo.mat');
lst = log_leo.lat;
lon = log_leo.long;

utc_time = log_leo.utc_time;
local_time = log_leo.normal_time;

time ms(:,1) = ((str2num(local_time(:,13:14))+5)*60*60*1000)+((str2num(local_time(:,16:17))*60*1000)+(str2num(local_time(:,19:20)))*1000);

% LOCATION AND HEADING OF LEONARDO BETWEEN 12:53:36 and 13:03:36 (01/25/08)
A = load('head_leo.mat');

% Note 2441 -> 125336 and 2931 -> 130336
leotime_ms = time ms(2441:2931,1);
leo_pos_latlong = [lst(2441:2931,1) lon(2441:2931,1)];
leo_head = A.head(2441:2931,1);

% converts location from lat/long to localgrid
for k1 = 1:length(leo_pos_latlong)
    pos_leo(k1,:) = geo2utm(leo_pos_latlong(k1,:), prime_meridian) * utm_datum;
end
xy_leo = [pos_leo(:,2) pos_leo(:,1)];

% LOCATION, HEADING AND DEPTH OF OEX BETWEEN 12:53:36 and 13:03:36 (01/25/08)
start_run = '125336';
end_run = '130336';

start_runms = (str2num(start_run(1:2))*60*60*1000)+((str2num(start_run(3:4))*60*1000)+(str2num(start_run(5:6)))*1000);
end_runms = (str2num(end_run(1:2))*60*60*1000)+((str2num(end_run(3:4))*60*1000)+(str2num(end_run(5:6)))*1000);
% AuvPosition_data.m -> OEX location and corresponding milliseconds timestamps
AuvPos = AuvPosition_data;

% delay in sec between OEX and GPS clock
clock_dif = 28; % time difference between AUV clock and Leonardo (GPS) clock

clock = -clock_dif; % corresponding correction in ms

delay = -clock * 1000;

% correct AUV clock time to account for delay with GPS time of Leonardo
AuvPosTime = AuvPos.TimeStamp.time_t + delay;

templ = abs(AuvPosTime - start runms);
[val indl] = min(templ);
temp2 = abs(AuvPosTime - end run ms);
[val ind2] = min(temp2);

% extract OEX timestamps - converts lat/long to local grid coordinates
oextime_ms = AuvPosTime(ind1:ind2,1);
oex_pos_latlong = [AuvPos.AuvPosition.latm.min(ind1:ind2,1) ...] AuvPos.AuvPosition.longm.min(ind1:ind2,1)]/60;

for kl = 1:length(oexpos_latlong)
pos_oex(kl,:) = geo2utm(oex_pos_latlong(kl,:),prime meridian)-utm datum;
end

% Dynamics_data.m -> OEX depth, heading and corresponding milliseconds timestamps
auv_dyn = Dynamics_data;
auvDynTime = auv_dyn.TimeStamp.time_t + delay;

temp3 = abs(AuvDynTime - start runms);
[val ind3] = min(temp3);
temp4 = abs(AuvDynTime - end run ms);
[val ind4] = min(temp4);
auv_dyn_time_ms = AuvDynTime(ind3:ind4,1);
oex_depth = -auv_dyn.AuvDepth.depth.m(ind3:ind4,1);
oex_heading = auv_dyn.auvHeading.deg(ind3:ind4,1);
waveguide_depth = auvdyn.AuvAlt.altitude.m(ind3:ind4,1)+abs(oex_depth);

% DEPTH OF THE LUBELL SOURCE DURING THE RUN

% POSITION OF ROCK AND TARGET (lat/long and localgrid)
targetlatlong = [44+(0.8649/60) 9+(49.2638/60)];
xy_target = [flipr(geo2utm(target_latlong(1,:),prime_meridian)) -utm_datum]*-28;  
rock_latlong = [44+(0.893/60) 9+(49.2646/60)];  
xy_rock = [flipr(geo2utm(rock_latlong(1,:),prime_meridian)) -utm_datum]*-27.9;  

% POSITION OF OTHER FEATURES OF INTEREST (OPTIONAL)  
gps_surface = '131335';  
gps_surface_ms = (str2num(gps_surface(1:2))*60*1000)+(str2num(gps_surface(3:4))*60*1000)+(str2num(gps_surface(5:6))*1000);  
temp8 = abs(AuvPos.TimeStamp.time_t-gps_surface_ms);  
[ val ind8] = min(temp8);  
oex_11_GPS1 = [AuvPos.AuvPosition.latm.min(ind8,1) AuvPos.AuvPosition.longm.min(ind8,1)]/60;  
oex_11_GPS1 = [AuvPos.AuvPosition.latm.min(ind8,1) AuvPos.AuvPosition.longm.min(ind8,1)]/60;  
oex_11_GPS1 = geo2utm(oex_11_GPS1(1,:),prime_meridian) -utm_datum;  
gps_surface = '125117';  
gps_surface_ms = (str2num(gps_surface(1:2))*60*1000)+(str2num(gps_surface(3:4))*60*1000)+(str2num(gps_surface(5:6))*1000);  
temp9 = abs(AuvPos.TimeStamp.time_t-gps_surface_ms);  
[ val ind9] = min(temp9);  
oex_11_125117 = [AuvPos.AuvPosition.latm.min(ind9,1) AuvPos.AuvPosition.longm.min(ind9,1)]/60;  
oex_11_125117 = [AuvPos.AuvPosition.latm.min(ind9,1) AuvPos.AuvPosition.longm.min(ind9,1)]/60;  
oex_11_125117 = geo2utm(oex_11_125117(1,:),prime_meridian) -utm_datum;  

if 1  
figure  
plot(xy_oex(:,1),xy_oex(:,2),'k');  
hold on  
plot(xy_leo(:,1),xy_leo(:,2),'r');  
hold on  
plot(xy_target(1,1),xy_target(1,2),'ko');  
hold on;  
plot(xy_rock(1,1),xy_rock(1,2),'ro');  
hold on  
end  
grid on  
xlabel('X (m)');  
ylabel('Y (m)');  
title('Location of OEX and Leonardo during between 12:53:26 and 13:03:26');  
legend('OEX path','Leonardo path','elastic target','rock');
ARRIVAL TIME OF TARGET AND MULTIPATH ECHOES FOR THE SLITA FILES OF THE RUN

if 1

% timestamp of the slita files
f1 = [125402:4:125458]; f2 = [125502:4:125558]; f3 = [125602:4:125658];
f4 = [125702:4:125758]; f5 = [125802:4:125858]; f6 = [125902:4:125958];
f7 = [130002:4:130058]; f8 = [130102:4:130158]; f9 = [130202:4:130258];
f10 = [130302:4:130358];

files_timestamp = [f1 f2 f3 f4 f5 f6 f7 f8 f9 f10 130402];

% file tag is used to specify the index of a timestamp and extract
% corresponding locations of the OEX and Leonardo

file_tag = 100;

for kl=1:length(files_timestamp)

A = floor(files_timestamp(kl,kl)/10000);
B = floor((files_timestamp(kl,kl) - A*10000)/100);
C = files_timestamp(kl,kl) - A*10000 - B*100;

time_file = A*60*60*1000 + B*60*1000 + C*1000 - clock_dif*1000;

% extracts OEX and Leonardo locations associated with timestamp

temp3 = abs(oextimems - time_file);
[val ind3] = min(temp3);

temp4 = abs(leotimems - time_file);
[val ind4] = min(temp4);

% extracts heading of Leonardo associated with timestamp

xy_leo_f = [xy_leo(ind4,:) 0];
if (ind4 > 10)
    leo_head_f = mean(leo_head(ind4-10:ind4,1));
else
    leo_head_f = leo_head(ind4,1);
end

% depth of Lubell source associated with timestamp

temp5 = abs(time_depth_log - time_file);
[val ind5] = min(temp5);
source_depth_f = depth_lubell(ind5,1);

% position of the Lubell source relative to the center point Leonardo

dist_mast_frame = 15.5;
frame_length = 5.14;
zs0 = 10.8; % depth of the source before the ship moves
dist_Aframe_water = 8;
compens = dist_mast_frame + (frame_length/sqrt(2)) + 
    sqrt((zs0+dist_Aframe_water)^2 - 
        (dist_Aframe_water+abs(source_depth_f))^2);
% location of the source in localgrid coordinates
xy_source_f(kl,:) = [xy_leo_f(1,1) - const*\sin(\textit{leo}_\textit{head}_f*\pi/180) \ xy_leo_f(1,2) - const*\cos(\textit{leo}_\textit{head}_f*\pi/180) \ \textit{source}\_\textit{depth}_f];

L1 = 71.3;

straight_array = 1; % if 1 assumes straight array

%... % location of the 1st receiver of the array in localgrid coord
if (straight_array)
    xy_rec_f(kl,:) = [xy_oex_f(1,1) - L1*\sin(oex\_\textit{head}_f*\pi/180) \ xy_oex_f(1,2) - L1*\cos(oex\_\textit{head}_f*\pi/180) \ oex\_\textit{depth}(\textit{ind6},1)] + [2.74 -34 0];
    % note [2.74 -34 0 ] -> navigation error correction
else
    % estimation based on array dynamics model
    B = load('recl.mat');
    C = load('time_rec.mat');
    temp7 = abs(C.time_rec+delay - time_file);
    [val ind7] = min(temp7);
    temp20 = abs(oex\_time_ms - time_file - 56*1000);
    [val ind20] = min(temp20);
    xy_rec_f(kl,:) = [B.recl(ind7,:) oex\_\textit{depth}(\textit{ind6},1)] + [2.74 -34 0];
    % note [2.74 -34 0 ] -> navigation error correction
end

if kl == file_tag
    xy_rec_tag = xy_rec_f(kl,:);
    xy_src_tag = xy_source_f(kl,:) - [0 10 0];
end

d2_dist_sc_rec(1,kl) = norm(xy_source_f(kl,1:2) - xy_rec_f(kl,1:2),'fro');
d3_dist_sc_rec(1,kl) = norm(xy_source_f(kl,1:3) - xy_rec_f(kl,1:3),'fro');

temp_oex = [xy_oex(~ind3,:) -16];
dist_source_oex_rec = norm(xy_source_f(kl,:) - temp_oex,'fro') + ...  
    norm(temp_oex - xy_rec_f(kl,:),'fro');

% computes multipath distance between source and 1st receivers

% surface reflected path
srf_path(1,kl) = sqrt((abs(xy_rec_f(kl,3)) + abs(source_depth_f))^2 + ...  
    + d2_dist_sc_rec(1,kl)^2);

% bottom reflected path
bot_path(1,kl) = sqrt((abs(xy_rec_f(kl,3)) + 2*H)^2 + ...  
    + d2_dist_sc_rec(1,kl)^2);

% bottom surface reflected path
bot_srf_path(1,kl) = sqrt((abs(xy_rec_f(kl,3)) + 2*H + ...  
    abs(source_depth_f))^2 + d2_dist_sc_rec(1,kl)^2);

% surface bottom reflected path
srf_bot_path(1,kl) = sqrt((abs(xy_rec_f(kl,3)) + 2*H + ...  
    abs(source_depth_f))^2 + d2_dist_sc_rec(1,kl)^2);

% surface bottom surface reflected path
srf_bot_srf_path(1,kl) = sqrt((abs(xy_rec_f(kl,3)) + 2*H + ...  
    abs(source_depth_f))^2 + d2_dist_sc_rec(1,kl)^2);
% bottom surface reflected path
bot_srf_bot_path(l,kl) = (sqrt((abs(xy_rec_f(kl,3)) + 4*H - ...
    abs(source_depth_f))^2 + d2_dist_sc_rec(l,kl)^2));

% computes target reflected path between source and 1st receivers

% computes target surface target path between source and 1st receivers

targetpath(l,kl) = norm(xy_source_f(kl,:)-xy_target,'fro') + ...% target path
    norm(xy_rec_f(kl,:)-xy_target,'fro');

% computes rock path to target
rock_path(l,kl) = norm(xy_source_f(kl,:)-xy_rock,'fro') + ...
    norm(xy_rec_f(kl,:)-xy_rock,'fro');

% computes leo reflected path
leo_rec(l,kl) = norm(xy_leo_f(l,:)-xy_rec_f(kl,:),'fro');

% computes source target path
src_target(l,kl) = norm(xy_source_f(kl,:)-xy_target,'fro');

tgt_rec(l,kl) = norm(xy_rec_f(kl,:)-xy_target,'fro');

end

% plots location of 1st receiver/ Lubell source during the run

if 0
    hold on
    plot(xyrec_f(:,l),xy_rec_f(:,2),'g');
    hold on
    plot(xy_source_f(:,l),xy_source_f(:,2),'m');
    hold on
end

%---------------------------------------

% PLOT OF MULTIPATH AND TARGET REFLECTED ARRIVALS

c0 =1500; % waveguide sound speed
fs = 51000; % sampling rate
dt = 1/fs; % sampling period

% sets the sample at which the direct arrival is aligned on the plot

time_direct_arrival = 360;

figure
pr = linspace(0,2432,151);
plot(time_direct_arrival*ones(1,151),pr,'r');
hold on
plot(((targetpath-dirpath)/(c0*dt))+time_direct_arrival,pr,'k');
hold on
plot(((rock_path-dir_path)/(c0*dt))+time_direct_arrival,pr,'m');
hold on
plot(((srf_path-dir_path)/(c0*dt))+time_direct_arrival,pr);% direct
hold on
plot(((bot_path-dir_path)/(c0*dt))+time_direct_arrival,pr);
hold on
plot(((bot_srf_path-dir_path)/(c0*dt))+time_direct_arrival,pr);% direct
hold on
plot(((bot_srf_path-dir_path)/(c0*dt))+time_direct_arrival,pr);
hold on
plot(((srf_bot_path-dir_path)/(c0*dt))+time_direct_arrival,pr);
hold on
plot(((srf_bot_srf_path-dir_path)/(c0*dt))+time_direct_arrival,pr);
hold on
plot(((bot_srf_bot_path-dir_path)/(c0*dt))+time_direct_arrival,pr);
hold on
axis ij;
axis([0 12000 0 2432]);grid on
legend('direct arrival','target arrival','rock arrival','multipath arrivals');
xlabel('Time (sample)');
ylabel('Ping #');
title('Arrival time of target and multipath echoes during the run');
end
A.2 Construction of the TRO -Singular values analysis

The following code is used to process the SLITA files and construct the TR operator for a sliding time window. It produces a set of figures that depict the evolution in time of the first singular values of the TR operator. The time signals of each hydrophone contained in the SLITA files are first extracted using the function slita_open provided by M.Mazzi (line 34). The time series are stored in the variable data_volts_1. The signal from each hydrophone is then convolved with the impulse response of a Butterworth lowpass filter with cuttoff frequency of 12kHz (line 58). The resulting filtered signals are stored in the variable filt_signal and the time series of the first hydrophone is used as a reference to decompose each signal into pings (line 68). The time window of ping duration is Fourier transformed (line 136) and the complex pressures measured by the receivers are used to construct the TR operator at each frequency (stored in the variable $KKH$ in the code below) using the covariance matrix approach (Eq. 4.1) (line 150). The Singular Value Decomposition of the TR operator (line 164) allows finally to extract its eigenstates. Its singular values and singular vectors are stored at each frequency to be transmitted with the virtual TR mirror (line 177).

```matlab
0001 % ------------------------------------------
0002 % CCLNet08 processing - Singular Value Analysis
0003 %---------------------------------------
0004 0005
0006 % SLITA FILE TIMESTAMPS
0007
0008 clear all;
0009
0010 files_timestamp = [fl f2 f3 f4 f5 f6 f7 f8 f9 f10];
0011 % SLITA files timestamps
0012
0013 f1 = [125402:4:125458];f2 = [125502:4:125558];f3 = [125602:4:125658];
0014 f4 = [125702:4:125758];f5 = [125802:4:125858];f6 = [125902:4:125958];
0015 f7 = [130002:4:130058];f8 = [130102:4:130158];f9 = [130202:4:130258];
0016 f10 = [130302:4:130358];
0017
0018 files_timestamp = [f1 f2 f3 f4 f5 f6 f7 f8 f9 f10 130402];
0019
0020 ww = 100 % index of the file to process
0021
0022 % SLITA files + LOADS HYDROPHONE TIME SERIES
0023 files = ['sliva_files/sliva_2008025' num2str(files_timestamp(1,ww))]; ...
0024 'sliva_files/sliva_2008025' num2str(files_timestamp(1,ww+1))];
0025 [A1 B1]= size(files);
```
0029 data_volts_1 = zeros(32,A1*204000);
0030
0031 for k1=1:A1
0032    filename = [files(k1,:) '.dat']
0033    [data_volts, fs, octave, hydrophone_spacing] = slita_open(filename);
0034    templ1 = length(data_volts);
0035    for k2 = 1:32
0036        data_volts_1(k2,(k1-1)*templ1:k1*templ1) = data_volts(k2,:);
0037    end
0038 end
0039 clear temp;
0040
0041 %========================================================================
0042 % SIGNAL LOW PASS FILTERING
0043 %========================================================================
0044 fh = 12000;  % cutoff frequency of lowpass filter
0045 [B2 A2] = butter(10,fh/(fs/2),’low’);
0046 [H2 T2] = impz(B2,A2,100,fs);
0047
0048 dt = 1/fs;  % time increment between sample (s)
0049 p_rate = 4;  % ping rate (Hz)
0050 ns = (1/p_rate)/dt;  % number of samples per ping
0051 nping = 16;  % number of ping per file
0052
0053 filt_signal = zeros(32,length(data_volts_1));
0054
0055 for k1 = 1:32
0056    templ1 = conv(H2,data_volts_1(k1,:));
0057    filt_signal(k1,:) = templ1(1:length(data_volts_1));
0058 end
0059 clear templ1;
0060
0061 %========================================================================
0062 % DETECTION OF DIRECT ARRIVAL + SPLITTING OF TIME SERIES INTO PINGS
0063 %========================================================================
0064 % match filtering of hydrophone times series with a sample direct arrival
0065
0066 [mm nn] = find((filt_signal(1,1:ns)) == min((filt_signal(1,1:ns))));
0067 pulse = filt_signal(1,nn-min(nn,25)+1:nn+25);
0068 cor_pulse(l,:) = conv(filt_signal(l,:),fliplr(pulse));
0069
0070 for k1 = 1:nping
0071    [templ temp2] = find(cor_pulse(l,(k1-1)*ns+1:k1*ns) == ...)
0072          max(cor_pulse(l,(k1-1)*ns+1:k1*ns)));
0073    max_cor(l,k1) = temp2;
0074    break_beg(l,k1) = (k1-1)*ns+1+temp2-400;
0075    break_end(l,k1) = k1*ns+1+temp2-400;
0076 end
0077 clear templ temp2;
0078
0079 %========================================================================
0080 % DETECTION OF DIRECT ARRIVAL + SPLITTING OF TIME SERIES INTO PINGS
0081 %========================================================================
0082 for k1 = 1:32
0083    for k2 = 1:nping
0084        data(k2,:,k1) = filt_signal(k1,break_beg(1,k2):break_end(1,k2));
0085    end
0086 end
0087
0088 f_l = 500;
0089 [B2 A2] = butter(10,f_l/(fs/2),’high’);
0090 [H2 T2] = impz(B2,A2,100,fs);
0091
0092 for k1 = 1:32
0093    for k2 = 1:nping
0094        templ1 = conv(H2,data(k2,:,k1));
0095        data(k2,:,k1) = templ1(1,1:length(data(k2,:,k1)));
0096    end
0097 end
0097   end
0098 clear temp1;
0099
0100 %=================================================================================
0101 % TIME WINDOWING + CONSTRUCTION OF TIME REVERSAL OPERATOR
0102 %=================================================================================
0103
0104 win_start = 2600;
0105 beg_win = win_start; % time window starting sample
0106 win_duration = 200; % time window duration
0107 win_overlap = 0.75  % overlapping (0.6 - 60%)
0108
0109 for k1=1:15
0110   beg_win = beg_win + win_duration*(1-win_overlap);
0111   end_win = beg_win + win_duration;
0112   han_win = hanning(200);
0113
0114   % hanning window applied to the time windowed signal
0115
0116 for k2 =1:nping
0117   for k3 =1:32
0118     data_f(k2,:,k3) = [data(k2,beg_win-100:beg_win-l,k3).*han_win(1:100)'; ... 
0119       data(k2,beg_win:end_win,k3) ... 
0120       data(k2,end_win+1:end_win+100,k3).*han_win(101:200)';]
0121   end
0122 end
0123
0124 spec_echo = [zeros(nping,1000,32) data_f zeros(nping,10000,32)];
0125 clear ft spec_echo;
0126 clear A;
0127
0128 for k2 =1:nping
0129   for k3 =1:32
0130     temp1 = spec_echo(k2,:,k3);
0131     temp2 = length(temp1);
0132     f = (1/dt)*((0:temp2/2-1)/temp2);
0133     ft_spec_echo(k2,:,k3) = (2/temp2)*fft(temp1,temp2);
0134   end
0135 end
0136 clear temp1 temp2;
0137
0138 for k2 =1:1200
0139   % Contruction of the TRO based covariance matrix approach
0140
0141 if 1
0142   % assuming receivers overlap at each emission
0143   KKH=zeros(16,16);
0144   for k3=1:16
0145     P(:,k3) = ft_spec_echo(k3,k2,k3:k3+15);
0146     KKH = KKH+P(:,k3)*P(:,k3)';
0147   end
0148 else
0149   % assuming receivers overlap at every other emission
0150   KKH=zeros(8,8);
0151   for k3=1:8
0152     P(:,k3) = ft_spec_echo((k3*2)-1,k2,(k3-1)*3+1:(k3-1)*3+1+7);
0153     KKH = KKH+P(:,k3)*P(:,k3)';
0154   end
0155 end
0156
0157 [eig_vec,eig_val] = eig(KKH);
0158 eig_TRO(k2,:) = sum(eig_val);
0159
0160 [U,S,V] = svd(KKH);
sing_val(k2,:) = sum(S);
sum_sing(1,k2) = sum(sum(S));
eig_vec_1(:,k2) = eig_vec(:,1);
eig_vec_2(:,k2) = eig_vec(:,2);
eig_vec_3(:,k2) = eig_vec(:,3);
eig_vec_4(:,k2) = eig_vec(:,4);
eig_vec_5(:,k2) = eig_vec(:,5);

end

store_SV1(:,k1) = sing_val(:,1);
store_SV2(:,k1) = sing_val(:,2);
store_SV3(:,k1) = sing_val(:,3);
store_SV4(:,k1) = sing_val(:,4);

end

XX = [win_start:50:end_win];
YY = fliplr(f(1,1:1200));
figure;imagesc(XX,YY,flipud(store_SV1));
title('SV1');
xlabel('Time (sample)');
ylabel('Frequency (Hz)');

figure;imagesc(XX,YY,flipud(store_SV2));
title('SV2');
xlabel('Time (sample)');
ylabel('Frequency (Hz)');

figure;imagesc(XX,YY,flipud(store_SV3));
title('SV3');
xlabel('Time (sample)');
ylabel('Frequency (Hz)');

figure;imagesc(XX,YY,flipud(store_SV4));
title('SV4');
xlabel('Time (sample)');
ylabel('Frequency (Hz)');

save target_ping1520_eig_vec eig_vec_1 eig_vec_2 eig_vec_3;
A.3 Backpropagation of eigenvectors

The following code is used to backpropagate on the seabed the singular vectors of the TR operator (see section A.2). It produces a set of top view plots of the pressure field on the seabed (see Fig. 4.17(b)). The backpropagation can either be conducted in a free field environment or in a waveguide using precomputed green functions (line 91).

```matlab
% Characteristics of the grid
nX = 60; % number of grid points in x direction
nY = 60; % number of grid points in y direction
nGP = (2*nX+1)*(2*nY+1); % total number of grid points
step_x = 1e-3 % step increment in x direction (km)
step_y = 1e-3 % step increment in y direction (km)

% grid point coordinates
[gridx gridy] = meshgrid(-nX*step_x:stepx:nX*step_x,-nY*stepy:stepy:nY*stepy);

% Characteristics of the array + hydrophone coordinates
freq = 1000; % frequency of backpropagation
nR = 16; % number of receivers considered for the backpropagation
dRR = 0.211; % distance between the receivers of the array (m)
dpthR = 16; % depth of the receivers (m)

% sets the receivers (i.e sources) coordinates
hyd_x = [0:dRR:(nR-1)*dRR]/1e3;
p_r(:,1) = hyd_x';
p_r(:,2) = zeros(1,16)';
p_r(:,3) = dpthR*ones(1,16)/le3';

% Note: defines a horizontal array pointing in the +x direction

% Distance receivers - grid points
dRGP = zeros(nR,nGP);
temp_RGP = NaN*ones(2*nX+1,2*nY+1);

for kl=1:nR
    temp_RGP = sqrt((grid_x - p_r(kl,1)).' * 2 + (grid_y - p_r(kl,2)).' * 2);
dRGP(kl,:) = reshape(temp_RGP',1,nGP);
end
```

83
Load eigenvectors to backpropagate

templ = load('target_ping1450_eig_vec.mat');

def = 4.5532;
vec_tag = floor(freq/df);
T = templ.eig_vec3(:,vec_tag);
temp2 = T ;

Backpropagation in free field or using precomputed green functions

gpmag = NaN*ones(l,nGP);
gp_phase = NaN*ones(l,nGP);
C = NaN*ones(nR,nGP);
dist3D = zeros(nGP,1);
cO = 1500; % sound speed in water
DP = 30; % depth of waveguide
free_field = 1

if (free_field)
    for kl=l:nR
        k1 = 1:nR
        dist3D = sqrt((1000*dRGP(kl,:)').^2+(DP-dpth_R)^2);
        green_func = exp(-j*(2*pi*freq/c0)*dist3D)./dist3D;
        gp_mag = temp2(k1)•abs(green_func);
        gp_phase = phase(green_func);
        C(k1,:) = gp_mag•exp(j•gp_phase);
        dist3D = zeros(nGP,1);
    end
else
    for kl=l:nR
        filename = ['CCLNET08_' num2str(freq) ' 0 0.3_' num2str(dpth_R) '.mat'];
        A=load(filename);
        mag_T = abs(A.T);
        phase_T = phase(A.T);
        gp_mag = temp2(k1)•interplq(A.range', mag_T',dRGP(kl,:)');
        gp_phase = interplq(A.range', phase_T',dRGP(kl,:)');
        C(k1,:) = gp_mag•exp(j•gp_phase);
    end
end

Computes pressure amplitude at each grid point location.
pres_GP = abs(sum(C));
Z = zeros(2•nX+1,2•nY+1,1);
k=0;
for k1=1:2•nY+1
    for k2=1:2•nX+1
        k = k+1;
        Z(k1,k2)=pres_GP(k);
    end
end

Plots pressure field on the seabed
X = [-nX•step_x:step_x:nX•step_x]•le3;
Y = [-nY•step_y:step_y:nY•step_y]•le3;
figure
imagesc(X,Y,Z);
cmap_smooth = jet(500);
colormap(cmap_smooth);
xlabel('X (m)');
ylabel('Y (m)');
mes_title = ['freq = ', num2str(freq)];
title(mes_title);
colorbar;
hold on
drawnow;
Appendix B

SLITA Specifications

The following technical drawing of the SLITA array was provided by Alain Maguer from NURC (NATO Undersea Research Center)
Bibliography


