

XXIV. SPONTANEOUS RADIOFREQUENCY EMISSION
FROM HOT-ELECTRON PLASMAS*

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RESEARCH OBJECTIVES

We plan to continue our studies in order to identify the instability observed in the afterglow of our pulsed electron-cyclotron resonance discharge. Theoretical and experimental studies during the past year have indicated that two types of instability may explain our observations. One of these is an electrostatic mode in which the cold plasma background interacts with the hot anisotropic plasma. The other possible mode is an electromagnetic instability of the hot-plasma component alone. Diagnostic equipment and measurements are being prepared for distinguishing between these two possible instabilities.

A. Bers

A. DIAMAGNETISM OF A RADially INHOMOGENEOUS PLASMA
IN A UNIFORM MAGNETIC FIELD

It is well known that a plasma of finite radial extent confined by a static magnetic field will have an internal magnetic field that is less than it would be if the plasma were absent. This may be seen by noting that the gyration of each charged particle in the static magnetic field constitutes a current loop whose effective magnetic moment opposes the applied field. If the plasma is taken to be of uniform density n_0 within a radius r_p and zero outside, it has been shown¹ that the value of the reaction magnetic field which is due to all of the particles is given by

$$B_1 = -\bar{i}_z \frac{\mu_0}{B_0} \left(\frac{1}{2} m v_{\perp}^2 n_0 \right), \quad (1)$$

where v_{\perp} is the magnitude of the particle's velocity perpendicular to the static magnetic field, $B_0 \bar{i}_z$. The plasma is assumed to be of infinite extent along the field lines. Thus the reaction field is proportional to the product of the plasma density and temperature. Of itself, the measurement of this reaction field has become a useful diagnostic.²

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As a generalization of this model we have considered the effects of a radial density gradient upon the value of the diamagnetic field. The plasma is taken to be uniform along the direction of the static magnetic field $\bar{B} = B_0 \bar{i}_z$, and to have a radially symmetric density profile. For simplicity, all particles are assumed to have the same magnitude of velocity across the magnetic field.

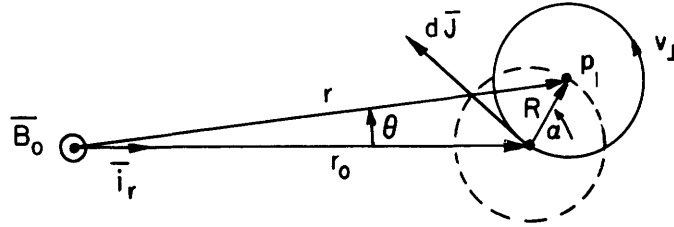


Fig. XXIV-1. Geometry for determining the contribution of the charge density of point p_1 to the current density at r_0 . Note that the charge density associated with point p_1 is assumed to be gyrating about the point p_1 .

From Fig. XXIV-1 it can be seen that the contributions to the macroscopic current density in the plasma at a point r_0 can come only from those charge-density elements dp associated with points located one Larmor radius R from the point in question. This contribution is given by

$$d\bar{J} = \frac{env_{\perp}}{2\pi} (-\bar{i}_r \sin \alpha + \bar{i}_{\theta} \cos \alpha) d\alpha, \quad (2)$$

where e is the electronic charge, n is the radially dependent electron density, and α is an angle that locates the charge-density increment in question. In order to perform the integration over all allowed contributions, we assume that $n(r) \cong n(r_0) + \frac{\partial n}{\partial r} \Delta r$. That is, we assume that the density varies slowly over distances of the order of a Larmor radius. From Fig. XXIV-1, we find

$$\Delta r = r - r_0 = (R^2 + r_0^2)^{1/2} \left[1 + \frac{2Rr_0}{R^2 + r_0^2} \cos \alpha \right]^{1/2}. \quad (3)$$

Noting that \bar{i}_r and \bar{i}_{θ} are not functions of α , we find

$$\bar{J}(r_0) = \bar{i}_{\theta} \frac{ev}{\pi} \left(\frac{\partial n}{\partial r} \right)_0 (R^2 + r_0^2)^{1/2} \int_0^{\pi} d\alpha (1 + \lambda \cos \alpha)^{1/2} \cos \alpha, \quad (4)$$

where $\lambda \equiv \frac{2Rr_o}{R^2 + r_o^2}$. The success of the analysis depends upon the calculation of the non-elementary integral

$$I = \int_0^\pi da \cos a (1 + \lambda \cos a)^{1/2}. \quad (5)$$

For small λ this can be expanded to yield

$$I \cong \int_0^\pi da \cos a \left(1 + \frac{\lambda}{2} \cos a\right) = \frac{\pi}{4} \lambda. \quad (6)$$

For $\frac{r_o}{R} \gg 1$,

$$\lambda = \frac{2 \frac{R}{r_o}}{1 + \left(\frac{R}{r_o}\right)^2} \cong 2 \frac{R}{r_o} \ll 1. \quad (7)$$

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$$\lambda = \frac{2 \frac{r_o}{R}}{1 + \left(\frac{r_o}{R}\right)^2} \cong 2 \frac{r_o}{R} \ll 1. \quad (8)$$

Thus we find

$$J_\theta(r) = \begin{cases} \frac{ev}{2} R \frac{\partial n}{\partial r} & r \gg R \\ \frac{ev}{2} R \frac{\partial n}{\partial r} \left(\frac{r}{R}\right) & r \ll R \end{cases} \quad (9)$$

where the subscript has been removed from r_o . For $\lambda = 1$ corresponding to $r = R$, the integral I can be evaluated exactly to yielding $I = \frac{2\sqrt{2}}{3}$. Thus the current density must be as pictured in Fig. XXIV-2. Note that far from the origin, the current density depends only upon the local radial density gradient. In this region the expression is the same as that obtained in rectangular geometry with a density gradient along one coordinate axis perpendicular to the magnetic field. Close to the origin, however, the current density vanishes, because of the effects of the cylindrical geometry.

By using the calculated current density in the Biot-Savart law, the diamagnetic field on the axis of the cylindrical geometry system may be determined. For simplicity, we approximate the current density by its small r/R value for $r < R$ and by its large r/R

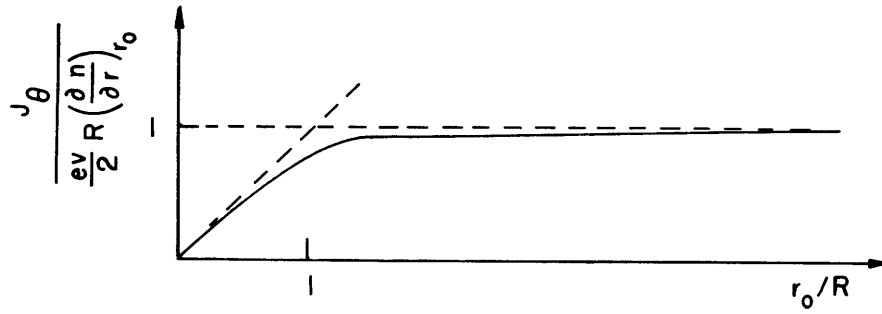


Fig. XXIV-2. Dependence of normalized current density upon radius.

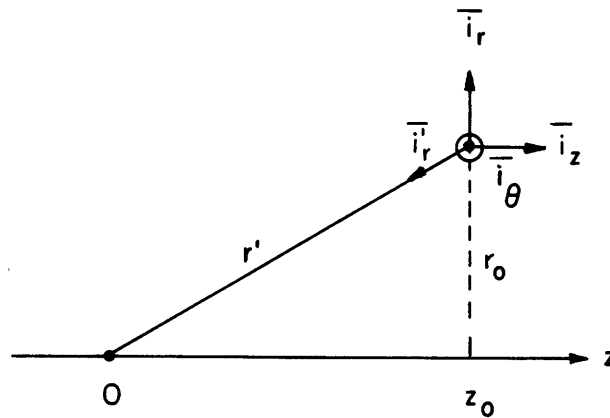


Fig. XXIV-3. Geometry for applying the Biot-Savart law.

value for $r > R$. We note, from Fig. 3 and the Biot-Savart law

$$d\bar{H}_1 = \frac{d^3 r \bar{J} \times \bar{i}'_r}{4\pi r'^2}, \quad (10)$$

that integrating over ϕ from 0 to 2π and over z from $-\infty$ to $+\infty$ yields

$$\bar{H}_1 = \bar{i}_z \left\{ \frac{mv_{\perp}^2}{2B_0} \int_R^{\infty} dr \frac{\partial n}{\partial r} + \frac{ev}{2} \int_0^R dr r \frac{\partial n}{\partial r} \right\} \quad (11)$$

$$\bar{H}_1 = -\bar{i}_z \left\{ \frac{mv_{\perp}^2}{2B_0} n(R) - \frac{ev}{2} \int_0^R dr r \frac{\partial n}{\partial r} \right\}. \quad (12)$$

Normally, the density profile should be radially symmetric. Thus for small r , n is given by $n(r) \cong n_0 \left[1 - k \left(\frac{r}{R} \right)^2 \right]$, where n_0 is the density at the center, and k is a constant. When substituted in the expression for \bar{H}_1 the result is

$$\bar{B}_1 = -\bar{i}_z \frac{n_o m v_{\perp}^2}{2B_o} \mu_o \left\{ 1 - \frac{k}{3} \right\}. \quad (13)$$

If the density gradient were such that the density were decreased by 10% at a radius of one Larmor radius, the field measured on the axis would be

$$\bar{B}_1 = -\bar{i}_z \frac{n_o m v_{\perp}^2}{2B_o} \mu_o (.967) \cong -\bar{i}_z \frac{n_o m v_{\perp}^2}{2B_o} \mu_o. \quad (14)$$

Thus the diamagnetic field on the axis of an infinitely long plasma with a radial density gradient, which is small over a Larmor radius, is of the same form as that derived for a uniform plasma column with the density taken equal to that on the axis.

We are now expanding this analysis to include the effects of finite-length plasmas and of inhomogeneous mirror magnetic fields.

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References

1. S. Chandrasekhar, Plasma Physics (Phoenix Books, University of Chicago Press, Chicago, 1960), pp. 17-18.
2. T. J. Fessenden, "Pulsed Electron-Cyclotron Resonance Discharge Experiment," Technical Report 442, Research Laboratory of Electronics, M.I.T., Cambridge, Mass., March 15, 1966, p. 64.

