XXII. PROCESSING AND TRANSMISSION OF INFORMATION*

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RESEARCH OBJECTIVES AND SUMMARY OF RESEARCH

The major goals of this research are to generate a deep understanding of communication channels and sources and to use this understanding in the development of reliable, efficient communication techniques.

1. Optical Communication

The fundamental limitations and efficient utilization of optical channels are the central concern of these investigations. Our interests now include the atmospheric channel, the cloud channel, quantum-limited channels, and scatter channels; the investigations range from fundamental coding theorems, through near-optimum digital systems, to "adaptive" systems that measure and compensate for channel fluctuations.

Partial statistical models for the turbulent atmosphere, clouds, and scattering media are now available. The performance that can be realized with these channels by digital communication, in the absence of quantum effects, is being investigated. The quantum aspects of optical communication have also been investigated. In the future these investigations will be unified to obtain a better understanding of the performance limitations of optical channels. Greater emphasis will also be placed upon systems that attempt to measure and "adapt" to the instantaneous state of the channel.

The performance limitations imposed by atmospheric turbulence has been determined for several situations of interest.[1-3] The optimum receiver for this channel differs from those normally employed, in that it utilizes the spatial diversity inherent in the fluctuations of the field across the receiving aperture. Since a receiver that utilizes this diversity in the most straightforward way is often extremely complicated, the development of relatively simple near-optimum receivers is being pursued.

An investigation at the doctoral level of the limitations imposed by the turbulent atmosphere upon high-resolution astronomy, or surveillance, has also been completed.[4] The principal conclusion is that significant gains in performance can be realized for "bright" objects through the use of data-processing techniques suggested by statistical estimation theory. These techniques involve, however, a substantial amount of data processing.

In other areas, three investigations at the doctoral level have been completed. One of these is concerned with the structure and performance of optimum quantum receivers

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for random phase and for fading channels used in conjunction with orthogonal waveform alphabets.[5] Analytical descriptions of the receivers have been developed and exponentially correct bounds to the error probability have been determined.

The limitations upon the transmission of information by combined temporal and spatial modulation, for example, by a sequence of "images," is the subject of another completed doctoral thesis.[6] The fundamental relationships between time, bandwidth, aperture size, and background noise were of particular concern in this investigation. A principal result has been a more complete understanding of the number of degrees of freedom contained within a specified region of the time-frequency-space domain.

A model for communication through clouds has been developed in a third doctoral thesis.[7] The principal result was that the received process is Gaussian and can be described by a scattering function. A partial characterization of this scattering function was obtained. Specific results include the spatial distribution of the power below the cloud and the expected time and frequency spreading.

R. S. Kennedy, E. V. Hoversten

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2. Coding for Noisy Channels and for Sources

The investigation of the properties of convolutional codes and sequential decoding algorithms will continue in the coming year. A doctoral thesis on the difference between systematic and nonsystematic codes has been completed by E. A. Bucher.[1] He shows that for codes with m check digits per one information digit, the exponential decay of error probability with constraint length for maximum-likelihood decoding is degraded by a factor $m/(m+1)$ in going from nonsystematic to systematic codes. The degradation is even greater with sequential decoding. Thus nonsystematic codes have an important advantage over systematic codes for sequential decoding applications.

Source coding is another major area of research. D. L. Cohn, in a doctoral research project, has recently found upper and lower bounds on the minimum mean-square distortion that can be achieved by independently processing successive samples from a discrete time Gaussian source for transmission over a white Gaussian noise channel. At high signal-to-noise ratios the bounds are 6-8 dB above the minimum achievable with joint processing.

A textbook on information theory and coding has also been completed.[2] It presents many new research results, together with providing a simple and precise treatment of the important previously known results.

R. G. Gallager

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3. Quantization

A quantizer converts the value of a random variable into the name of an interval in which it lies. Using the size of the interval as a measure of quantizing error, and the r^{th} mean of the interval size as a measure of performance, bounds have been found for the performance of optimum quantizers for bounded random variables. Further work remains to be done in the unbounded case, and in exploring quantization schemes that permit the receiver to answer a variety of different questions about the input process with moderate efficiency, rather than to answer only one question, albeit with high efficiency.

P. Elias

A. IMPROVED OPTICAL COMMUNICATION THROUGH ATMOSPHERIC TURBULENCE USING STATE KNOWLEDGE*

Optical communication systems in which the clear turbulent atmosphere comprises part of the transmission medium are characterized by reduced system performance when compared with free-space systems. From a communications viewpoint, the loss of spatial coherence caused by the turbulence limits the performance of optical systems in two ways:

1. a maximum transmitting aperture diameter beyond which the far-field beamwidth is turbulence-limited and independent of aperture size; and

2. a receiving aperture diameter (for a heterodyne receiver) beyond which the signal-to-noise ratio is not enhanced by increased aperture size.

Each of these effects may be taken to define a coherence length for the turbulence, so that performance saturates when the related aperture diameter is made larger than this length.

Much work has been devoted to finding receiver structures capable of using more receiving aperture than a coherence area,[1,2] and indeed for a certain reasonable model of the atmospheric communication problem the optimal receiver has been derived.[2] In this report the turbulent channel is attacked from the standpoint of transmitter design. We assume that there exists a feedback channel from which the transmitter may obtain sufficient state knowledge of the atmosphere to combat the loss of spatial coherence through spatial modulation of the antenna pattern. A ray optics analysis is used to define conditions under which the channel state may be determined, and to show how it may be used (in an abstract sense) to maximize the received energy-to-noise ratio for Earth to deep space transmission. We show that this communication scheme leads to performance that does not saturate with increasing transmitter aperture diameter. An experiment is proposed to test the state-knowledge hypothesis, and connections to a rigorous, linear system analysis are indicated.

1. Apodization through Turbulence

Before introducing the known state concept, we shall develop the framework of a general apodization problem that we wish to solve. We have a transmitter located on the Earth, and we wish to transmit (optically) to a spacecraft, through the atmosphere. We assume that the transmitter uses linear polarization, and that all information is transmitted by means of temporal modulation. Also, we assume that the transmitter is capable of generating any spatial field pattern over its aperture, and we seek to adjust this spatial modulation to deliver maximum carrier energy to the spacecraft.

For the deep-space problem, the spacecraft will be sufficiently into the far field of any transmitting aperture that we might build that the original problem is equivalent to the following statement. We define a "window" at the top of the atmosphere as being a region, R_2, in the plane tangent to the top of the atmosphere, with origin along the line connecting the spacecraft with the center of the transmitting aperture (see Fig. XXII-1). In terms of this window, the problem that we wish to solve is to maximize the carrier energy in the normally incident plane wave component of the field in the region R_2, by adjusting the spatial modulation in the region R_1 (transmitter aperture).

At this point a rigorous approach to the problem would be to find the impulse response of the transmission from R_1 to R_2, and solve the apodization problem in a manner similar to the free-space development of Greenspan.[3] The solution is straightforward in terms of the eigenfunctions of a Hermitian kernel obtained from the impulse response. Unfortunately, this direct approach is not particularly fruitful; the impulse response for a given state of the atmosphere cannot (usually) be found, much less the eigenfunctions and eigenvalues of the derived kernel. It must be remembered, also, that the impulse response changes in time as the state of the atmosphere changes, and thus we would have to imagine solving for the eigenfunctions and eigenvalues of a

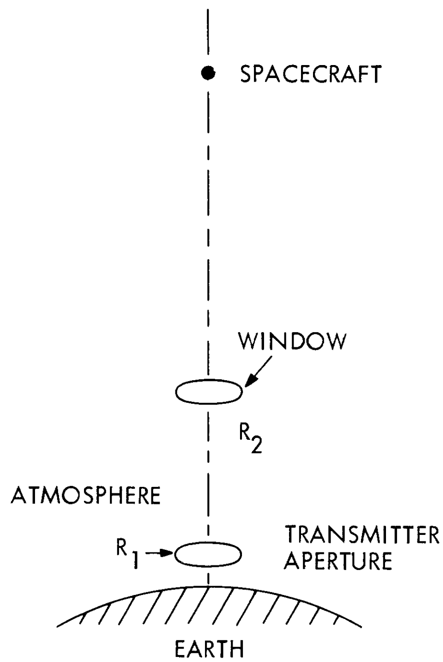

Fig. XXII-1. Apodization problem.

different kernel every few milliseconds.

A more productive, albeit less rigorous, approach to the apodization problem is to use geometrical optics. This approach leads to a natural way of obtaining the spatial waveform needed, without recourse to solving integral equations, and it is possible to estimate (roughly) the resultant system performance.

2. Ray Optics Approach

For convenience we will assume that the apertures R_1 and R_2 are both circular, of diameters D and d respectively. We will assume that R_1 is in the near field of the aperture R_2. By this we mean that in the absence of turbulence if a normally incident uniform plane wave is transmitted from R_2 (towards R_1), then all the rays in this beam fall on R_1. Since in the presence of turbulence little energy is scattered far out of the beam, it is possible that we can make D large enough so that all the rays from R_2 will still fall on R_1. We now assume that there exists D such that for almost all possible states of the atmosphere all of the rays in a normally incident uniform plane wave from R_2 fall on R_1. Furthermore, we assume that this D is of "reasonable" size, in the sense that we could conceive of building such an aperture for sizes of d of interest. If these conditions are satisfied, we say that the atmospheric state is known to the transmitter. There is some experimental evidence to justify our assumption that these conditions are satisfied.[4]

Fig. XXII-2. Ray-by-ray reciprocity.

Having assumed that the transmitter knows the channel state, the rest follows. From Fermat's principle, the atmosphere is reciprocal on a ray-by-ray basis; that is, it follows that if ray 1 will go from A to B along a given path, then ray 2 will follow the same path in going from B to A (see Fig. XXII-2). Thus we see that if we were to transmit a wave from R_1, using the same phase front that is received when a plane wave is sent from R_2 to R_1, then, by using ray-by-ray reciprocity, the field received at R_2 would be a normally incident uniform plane with the same energy as the wave transmitted from R_1.

3. Degrees of Freedom

The state knowledge argument may be restated in a more elegant form, with the aid of the concept of degrees of freedom. We define the number of degrees of freedom in the R_1-R_2 system to be[3]

$$\mathscr{D} = (\pi Dd)^2/16(\lambda z)^2,$$

where z is the center-to-center distance between R_1 and R_2. In the absence of turbulence, there exists a number, N_o, such that essentially all of the rays in a normally incident uniform plane wave from R_2 fall on R_1 if and only if

$$\mathscr{D} \geq N_o.$$

Now we assume a particular state of the (turbulent) atmosphere, and that there exists a number, N, such that

(i) $N = N_o + N_T$.

(ii) If and only if $\mathscr{D} \geq N$, essentially all of the rays in a normally incident uniform plane wave from R_2 fall on R_1.

Thus, with the use of ray-by-ray reciprocity, if $\mathscr{D}$ is such that the second condition is satisfied, then we say that the transmitter knows that particular state of the atmosphere. If we fix d, then, for a given state of the atmosphere, the channel state will be known only if

$$D^2 \geq 16(N_o+N_T)(\lambda z)^2/(\pi d)^2. \tag{1}$$

Let D_T be the minimum of all D satisfying this criterion for the given atmosphere. In order for the transmitter to be able to use state knowledge, for a given d, there must exist a number D such that, given $\epsilon > 0$,

(i) D is a "reasonable" aperture size for d of interest.

(ii) Pr (the atmosphere is s. t. $D_T > D$) $< \epsilon$.

As before, we assume that this condition is satisfied.

Let D_o be the minimum of all D satisfying the state knowledge criterion (1) in the absence of turbulence ($N_T=0$). We define the parameter δ_T to be

$$\delta_T = D_T/D_o$$

for a given state of the atmosphere. Thus δ_T is a random variable that measures how much aperture is needed at the transmitter in the turbulent case, relative to the non-turbulent case, to achieve state knowledge. The statement that D, as defined above, be implementable is equivalent to the requirement that δ_T be a relatively small number (order of magnitude 10 or less) with high probability. Evaluating δ_T, using Eq. 1, we have

$$\delta_T^2 = 1 + N_T/N_o.$$

Therefore, when we assume that the transmitter can use knowledge of the channel state we are assuming that N_T/N_o is "small" with high probability. N_T is the number of degrees of freedom associated with receiving the whole beam at R_1; hence, it can be seen to depend only on so-called gross turbulence effects, that is, beam steering and beam spreading.[5] Thus, if $\mathcal{N}$ is the total number of degrees of freedom of the turbulence (for a particular state) we may express $\mathcal{N}$ as

$$\mathcal{N} = N_T + n_T,$$

where n_T is the number of degrees of freedom associated with small-scale perturbations (scintillation, etc.). Roughly speaking, n_T is equal to the number of coherence areas in R_1, which can be a very large number. We shall see that it is essentially N_T that limits system performance, while n_T affects system complexity.

4. System Performance

To gain some insight into the performance gained by going to the state knowledge system, using spatial modulation of the wavefront at the transmitter, in Fig. XXII-3 we have plotted the energy received at the spacecraft against window diameter, d, for

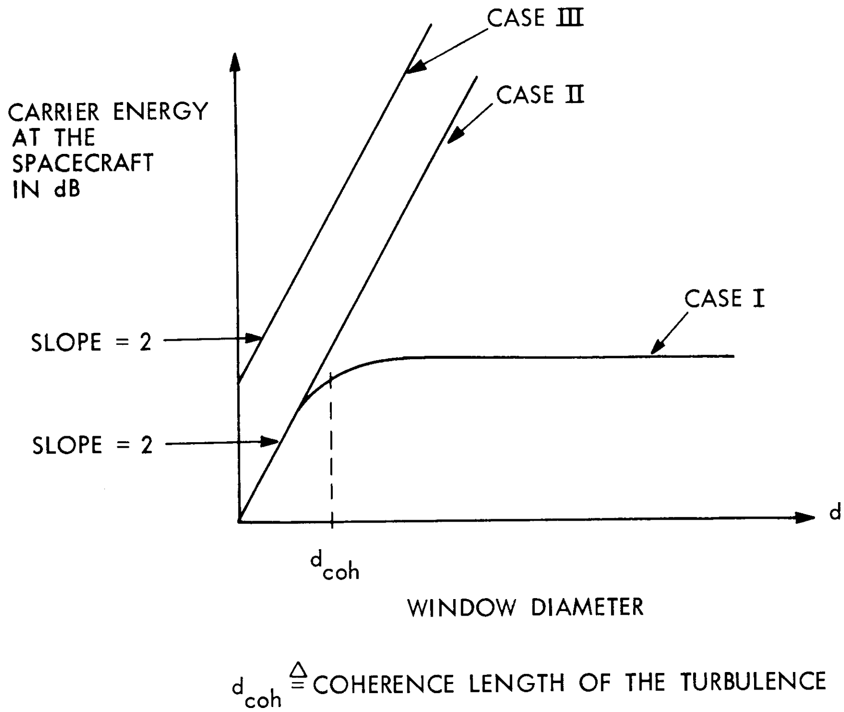

Fig. XXII-3. Performance comparison.

three different cases.

Case I. A normally incident uniform plane wave transmitted over an aperture of diameter d through the turbulence.

Case II. A spatially modulated wavefront transmitted over an aperture of diameter D, with the use of a state knowledge adjusted wavefront.

Case III. A normally incident uniform plane wave transmitted over an aperture of diameter D with no turbulence.

Case I illustrates the saturation effect that has been discussed, but Cases II and III both show performance increasing as d^2, without saturation. The difference, in energy, between Cases II and III is constant for all d, and, in some sense, is a measure of how much performance is lost because of the turbulence. The difference depends upon the statistics of N_T, through δ_T, and order-of-magnitude arguments give a performance difference of approximately 20-25 dB. The most significant aspect of these results, however, is that we have demonstrated a communication scheme for the turbulent channel whose performance does not saturate with increasing aperture diameter.

5. An Experiment

If we had a device that would reverse the direction of propagation of an incident wave without changing its phase front, then the state knowledge criterion could be tested as follows. On the earth we set up two apertures with the geometry of R_1 and R_2, and transmit a uniform plane wave from R_2 to R_1. At R_1 we employ a device of the type described in this report to reverse the direction of the incident wave without changing the phase front and, by examining the field returned to R_2, we may determine whether or not the state knowledge criterion is satisfied.

6. Future Work

A more rigorous approach to the state knowledge problem is now being investigated by utilizing linear systems techniques. It appears that the state knowledge concept may be generalized somewhat, to allow a weakening of the state knowledge criterion and still maintain performance that does not saturate with increasing aperture diameter.

J. H. Shapiro

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