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REDUNDANCY RESOLUTION OF MANIPULATORS THROUGH TORQUE OPTIMIZATION

John M. Hollerbach
Ki C. Suh

Abstract. Methods for resolving kinematic redundancies of manipulators by the effect on joint torque are examined. When the generalized inverse is formulated in terms of accelerations and incorporated into the dynamics, the effect of redundancy resolution on joint torque can be directly reflected. One method chooses the joint acceleration null-space vector to minimize joint torque in a least squares sense; when the least squares is weighted by allowable torque range, the joint torques tend to be kept within their limits. Contrasting methods employing only the pseudoinverse with and without weighting by the inertia matrix are presented. The results show an unexpected stability problem during long trajectories for the null-space methods and for the inertia-weighted pseudoinverse method, but rarely for the unweighted pseudoinverse method. Evidently a whiplash action develops over time that thrusts the endpoint off the intended path, and extremely high torques are required to overcome these natural movement dynamics.

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1 Introduction

Increasing interest in manipulator redundancy is a direct consequence of perceived limitations of current 6 degree of freedom robots. On the one hand, singularity regions of 6 degree of freedom rotary manipulators occupy such a significant portion of the workspace as to render them functionally only 5 degree of freedom manipulators (Paul and Stevenson, 1983). On the other hand, workspace obstacles may sufficiently constrain movement as to effectively reduce the degrees of freedom.

Most analysis of redundant arms has proceeded independent of consideration of any particular mechanism. Hollerbach (1984b, 1985) proposed a 7 degree of freedom kinematic design with a spherical shoulder joint to eliminate singularities and to improve workspace. Yoshikawa (1985a) proposed a 4 degree of freedom wrist to overcome the usual problem of wrist singularity. In practice very few redundant manipulators have been constructed. At research laboratories, 7 degree of freedom arms of anthropomorphic geometry include a tendon-driven, torque-controlled robot (Takase, Inoue, and Sato, 1974) and the UJIBOT, driven by DC servo motors (Nakamura, 1985). An 8 degree of freedom redundant sheep-shearing robot has been discussed in (Trevelyan, Kovesi, and Ong, 1983). Commercially, 7 degree of freedom robots have been produced by the Robotics Research Corp., also of anthropomorphic geometry, and by Cybotech, whose P-15 robot has a yaw axis in the forearm (Litvin et al., 1985). In addition, a number of 6 degree of freedom robots have been mounted on linear tracks, but it is hard to count this 7th degree of freedom into fine motion control of the endpoint.

Thus, while the incidence of redundant arms in research labs and industry is gradually increasing, present research in redundant arms has progressed primarily at a theoretical and simulation level, in advance and expectation of available mechanical hardware.

1.1 Kinematic Resolution of Redundancy

The vast majority of research into the control of redundant arms has involved the instantaneous resolution of the redundancy at the velocity level through use of the pseudoinverse $\mathbf{J}^\dagger$ of the Jacobian matrix $\mathbf{J}$. If $\dot{\underline{x}}$ is the 6-dimensional velocity vector of the hand and $\underline{\theta}$ is the $n > 6$ dimensional vector of joint angles, then

$$\dot{\underline{x}} = \mathbf{J}\dot{\underline{\theta}} \tag{1}$$

$$\dot{\underline{\theta}} = \mathbf{J}^\dagger \dot{\underline{x}} + (\mathbf{I} - \mathbf{J}^\dagger \mathbf{J})\dot{\underline{\phi}} \tag{2}$$

$$\mathbf{J}^\dagger = \mathbf{J}^T(\mathbf{J}\mathbf{J}^T)^{-1} \tag{3}$$

where $\dot{\underline{\phi}}$ is an arbitrary joint velocity vector and $(\mathbf{I} - \mathbf{J}^\dagger \mathbf{J})\dot{\underline{\phi}}$ is its projection into the null space of $\mathbf{J}$, corresponding to a self-motion of the linkage that does not move the end effector. The pseudoinverse is also known as the Moore-Penrose generalized inverse. The attractiveness of this approach is twofold. First, the pseudoinverse is one of the types of generalized inverse that has a least squares property (Ben-Israel and Greville, 1980), in the present case minimizing $\dot{\underline{\theta}}^T \dot{\underline{\theta}}$. Presumably any joint is prevented from moving too fast, leading to a more controllable motion (Whitney, 1969). It is also presumed that

squared velocities are approximately related to kinetic energy, which would then also be approximately minimized (Whitney, 1969, 1972).

Second, the redundancy available beyond that required for the tip motion is succinctly characterized by the null space of the Jacobian, which may be freely utilized to assist in the realization of some chosen objective. Liegeois (1977) developed a general formulation for instantaneously minimizing a position-dependent scalar performance criterion $p = g(\underline{\theta})$. The null space vector that minimizes p is $\dot{\underline{\phi}} = k\partial g/\partial \underline{\theta}$, where k is an arbitrary constant, and yields the solution

$$\dot{\underline{\theta}} = \mathbf{J}^{\dagger}\dot{\underline{x}} + (\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J})k\frac{\partial g}{\partial \underline{\theta}} \tag{4}$$

Liegeois demonstrated a way of avoiding joint limits, by minimizing the scalar function

$$p = \sum_{i=1}^{n}\left(\frac{\theta_i - \theta_i^{mid}}{\theta_i^{mid} - \theta_i^{max}}\right)^2 \tag{5}$$

where θ_i^{max} and θ_i^{min} are the upper and lower joint angle limits for joint i and $\theta_i^{mid} = (\theta_i^{min} + \theta_i^{max})/2$.

The null space vector has also been used in singularity avoidance. Yoshikawa (1984) proposed minimizing a dexterity measure w, called by him the manipulatability measure, given by

$$w = \sqrt{\det(\mathbf{J}\mathbf{J}^T)} \tag{6}$$

Since at a singularity $w = 0$, then the scalar function $p = -w(\underline{\theta})$ instantaneously maximizes w and tends to keep the arm away from singularities. Klein (1985) compared various dexterity measures including w and decided instead on the minimum singular value.

At a singularity a nonredundant manipulator can actually execute a self motion, since for example a 6 degree of freedom robot has an excess freedom with regard to the 5 degree of freedom endpoint motion possible at the singularity. The null space vector corresponding to this self motion is useful if the endpoint variables are partitioned into high-priority variables $\dot{\underline{x}}_h$, which must be realized, and low-priority variables $\dot{\underline{x}}_l$, which are sacrificed to avoid the singularity but which should be realized insofar as is possible (Hanafusa, Yoshikawa, and Nakamura, 1981). For example, in spray painting the control of rotation about the spray direction is not as important as the control of the other positioning variables, and could be sacrificed at a singularity. From the relations

$$\dot{\underline{x}}_h = \mathbf{J}_h\dot{\underline{\theta}} \tag{7}$$

$$\dot{\underline{x}}_l = \mathbf{J}_l\dot{\underline{\theta}} \tag{8}$$

$$\dot{\underline{\theta}} = \mathbf{J}_h^{\dagger}\dot{\underline{x}}_h + (\mathbf{I} - \mathbf{J}_h^{\dagger}\mathbf{J}_h)\dot{\underline{\phi}} \tag{9}$$

the null space vector left over after realizing the high priority variables is found by substituting (9) into (8):

$$\dot{\underline{\phi}} = [\mathbf{J}_l(\mathbf{I} - \mathbf{J}_h^{\dagger}\mathbf{J}_h)]^{\dagger}(\dot{\underline{x}}_l - \mathbf{J}_l\mathbf{J}_h^{\dagger}\dot{\underline{x}}_h) \tag{10}$$

Substituting into (9) yields

$$\dot{\underline{\theta}} = \mathbf{J}_h^{\dagger}\dot{\underline{x}}_h + [\mathbf{J}_l(\mathbf{I} - \mathbf{J}_h^{\dagger}\mathbf{J}_h)]^{\dagger}(\dot{\underline{x}}_l - \mathbf{J}_l\mathbf{J}_h^{\dagger}\dot{\underline{x}}_h) \tag{11}$$

where use has been made of the identity $B[CB]^{\dagger} = [CB]^{\dagger}$, with $B = (\mathbf{I} - \mathbf{J}_h^{\dagger}\mathbf{J}_h)$ a hermetian and idempotent matrix (Maciejewski and Klein, 1985).

The null space vector can aid obstacle avoidance (Maciejewski and Klein, 1985). Suppose $\underline{x}_o$ is some point on the arm closest to an obstacle, $\mathbf{J}_o$ is the Jacobian such that $\dot{\underline{x}}_o = \mathbf{J}_o\dot{\underline{\theta}}$, and $\dot{\underline{x}}_o$ is a movement of this point away from the obstacle that would be desirable. In a procedure like that above, $\dot{\underline{\phi}}$ is found by substituting (1) into this relation:

$$\dot{\underline{\phi}} = [\mathbf{J}_o(\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J})]^{\dagger}(\dot{\underline{x}}_o - \mathbf{J}_o\mathbf{J}^{\dagger}\dot{\underline{x}}) \tag{12}$$

After substitution and simplification, the joint rates that avoid the obstacle are:

$$\dot{\underline{\theta}} = \mathbf{J}^{\dagger}\dot{\underline{x}} + [\mathbf{J}_o(\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J})]^{\dagger}(\dot{\underline{x}}_o - \mathbf{J}_o\mathbf{J}^{\dagger}\dot{\underline{x}}) \tag{13}$$

One problem with the Moore-Penrose inverse is that its application is nonconservative (Klein and Huang, 1983). Repetitive motions planned with the pseudoinverse alone do not return at a given tip point to the same joint configuration. Baillieul (1985) has proposed an extended Jacobian method, which may be used to make a repetitive motion conservative. If $\underline{n}_J$ is a vector from the one-dimensional null space of $\mathbf{J}$, obtained by taking cross products of columns of $\mathbf{J}$, and $g(\underline{\theta})$ is a scalar function to be minimized, then define

$$\mathbf{G}(\underline{\theta}) = \frac{\partial g(\underline{\theta})}{\partial \underline{\theta}} \cdot \underline{n}_J \tag{14}$$

The extended Jacobian $\mathbf{J}_{ex}$ is then defined by

$$\mathbf{J}_{ex}\dot{\underline{\theta}} = \begin{bmatrix} \dot{\underline{x}} \\ 0 \end{bmatrix}, \qquad \mathbf{J}_{ex} = \begin{bmatrix} \mathbf{J} \\ \frac{\partial \mathbf{G}}{\partial \underline{\theta}} \end{bmatrix} \tag{15}$$

Assuming the manipulator is nonsingular, this relation may be inverted directly to find $\dot{\underline{\theta}}$. It is possible to recast this method in terms of a generalized inverse with null space vector.

An entirely different mechanism from use of a null space vector to realize desired performance characteristics is the weighted pseudoinverse. The generalized inverse $\mathbf{J}^{\dagger}{}_{\mathbf{w}}$ that instantaneously minimizes the cost $\dot{\underline{\theta}}^T\mathbf{W}\dot{\underline{\theta}}$ is (Whitney, 1969, 1972)

$$\mathbf{J}^{\dagger}{}_{\mathbf{w}} = \mathbf{W}^{-1}\mathbf{J}^T(\mathbf{J}\mathbf{W}^{-1}\mathbf{J}^T)^{-1} \tag{16}$$

where $\mathbf{W}$ is a matrix of weights. Whitney (1969) proposed an alternative method for enforcing high and low priority of hand variables through appropriate selection of weights. Konstantinov, Markov, and Nenchev (1981) used weights to avoid joint limits.

Most research involving the pseudoinverse deals with the instantaneous kinematics of motion, that is to say, motion that is locally optimized by incremental movement

from the current arm state. The Jacobian has also been used in dexterity measures to aid global trajectory planning and the avoidance of singularities. Uchiyama, Shimizu, and Hakomori (1985) parameterized joint angles by polynomials and optimized manipulatability w integrated across the whole trajectory by gradient adjustment of polynomial coefficents. Nakamura and Hanafusa (1985) used Pontryagin's Maximum Principle to minimize a cost function combining manipulatability w, and $\underline{\dot{\theta}}^T \underline{\dot{\theta}}$, interpreted by them as energy, for global determination of trajectory.

1.2 Kinetic Resolution of Redundancy

The previous redundancy resolution schemes are purely kinematic, despite the presumption that since the sum of squares of joint velocities is minimized by the pseudoinverse, then the kinetic energy is approximately minimized. True minimization of kinetic energy would only be realized if the generalized inverse were weighted with the inertia matrix, so that the relation to kinetic energy of an unweighted pseudoinverse is not clear. The kinetic energy is

$$T = \frac{1}{2} \underline{\dot{\theta}}^T \mathbf{H} \underline{\dot{\theta}} \tag{17}$$

where $\mathbf{H}$ is the inertia matrix. Minimizing the kinetic energy T is just Whitney's criterion with the weighting matrix $\mathbf{W} = \mathbf{H}$ in (16).

To incorporate the generalized inverse into dynamics, the pseudoinverse must be formulated in terms of accelerations. Khatib (1980, 1983) was one of the first researchers to do this, in his case using the inertia-weighted pseudoinverse:

$$\underline{\ddot{x}} = \mathbf{J} \underline{\ddot{\theta}} + \dot{\mathbf{J}} \underline{\dot{\theta}} \tag{18}$$

$$\underline{\ddot{\theta}} = \mathbf{J}^{\dagger}{}_{\mathbf{H}} (\underline{\ddot{x}} - \dot{\mathbf{J}} \underline{\dot{\theta}}) \tag{19}$$

The dynamic equations in closed form are (Hollerbach, 1984)

$$\underline{\tau} = \mathbf{H} \underline{\ddot{\theta}} + \underline{\dot{\theta}} \cdot \mathbf{C} \cdot \underline{\dot{\theta}} + \underline{g}. \tag{20}$$

where previously undefined terms are $\mathbf{C}$, the matrix of Coriolis and centrifugal coefficients, and $\underline{g}$, the gravity vector. Hence the joint torques are given by

$$\underline{\tau} = \mathbf{H} \mathbf{J}^{\dagger}{}_{\mathbf{H}} (\underline{\ddot{x}} - \dot{\mathbf{J}} \underline{\dot{\theta}}) + \underline{\dot{\theta}} \cdot \mathbf{C} \cdot \underline{\dot{\theta}} + \underline{g}. \tag{21}$$

Vukobratovic and Kircanski (1985) broadened the method of Khatib to include energetic models of hydraulic and electromagnetic motors, and applied the resultant generalized inverse to velocities.

While kinetic resolution of redundancy incorporates dynamics, the resulting formulation is only indirectly related to torque production at the joints. A major reason for attempting to reflect directly the effect of redundancy on joint torque is to avoid exceeding torque limits. Past work has been concerned with characterizing the endpoint accelerations that depend on the utilization of the redundancy and on torque limits,

rather than on an actual application to trajectory planning. Yoshikawa (1985b,c) defined a dynamic manipulability ellipsoid, derived from a pseudo-endpoint acceleration $\ddot{\tilde{x}} = \ddot{x} - \dot{\mathbf{J}}\dot{\theta}$ related to a pseudo-torque $\tilde{\tau} = \tau - \dot{\theta} \cdot \mathbf{C} \cdot \dot{\theta} - g$ by

$$\ddot{\tilde{x}} = \mathbf{J}\mathbf{H}^{-1}\tau \tag{22}$$

The derivation proceeds at zero velocity by a kind of normalization of the torques according to range. The upper τ^+ and lower τ^- joint torque bounds are presumed equal, and each pseudo-joint torque is normalized as $\hat{\tau}_i = \tilde{\tau}_i/(\tau_i^+ - |g_i|)$. Ignoring bounds on acceleration, then

$$\ddot{\tilde{x}} = \mathbf{J}\tilde{\mathbf{H}}^{-1}\hat{\tau} \tag{23}$$

where $\tilde{\mathbf{H}} = \mathbf{W}\mathbf{H}$ and $\mathbf{W} = diag(1/(\tau_i^+ - |g_i|))$ is a diagonal weighting matrix. The dynamic manipulability ellipsoid consists of the set of $\ddot{\tilde{x}}$ that are derivable from a unit input $\hat{\tau} \leq 1$, obtained from the dot product

$$\ddot{\tilde{x}}^T(\tilde{\mathbf{H}}\mathbf{J}^\dagger)^T\tilde{\mathbf{H}}\mathbf{J}^\dagger\ddot{\tilde{x}} \leq 1 \tag{24}$$

The dynamic manipulability index w is derived from this relation, and is an extension of (6) through the inclusion of inertia:

$$w = \sqrt{det(\mathbf{J}(\tilde{\mathbf{H}}^T\tilde{\mathbf{H}})^{-1}\mathbf{J}^T)} \tag{25}$$

This index has not yet been applied to redundancy resolution.

Khatib (1985) presented a different normalization for torque range in a procedure similar to Yoshikawa's. Defining

$$\hat{\tau}_i = min(|\tau_i^- - g_i|, |\tau_i^+ - g_i|)$$

the weighting matrix is $\mathbf{W} = diag(1/\hat{\tau}_i)$. Khatib applied this procedure to adjust a two-link manipulator's link lengths and masses to optimize the isotropic nature of endpoint acceleration throughout the workspace.

This paper reports on a method for instantaneously minimizing torque loading at the joints in a least squares sense, by specification of a null space vector applied to generalized inverse accelerations and derived from manipulator dynamics. This method is compared to use of a straightforward pseudoinverse and to use of an inertia-weighted pseudoinverse with regard to the effect on joint torques. Portions of this research have been previously reported (Baillieul, Hollerbach, and Brockett, 1984; Hollerbach and Suh, 1985).

2 The Generalized Inverse and Torque Optimization

Consider an n degree of freedom manipulator whose task is described by m hand variables x, where $m < n$. Assume that the upper and the lower torque limits of the joints are τ^+ and τ^- respectively, where for simplicity the limits are assumed motion independent. Then given a desired hand trajectory $x(t)$, we would like to find the set

of joint torques that results in $\underline{x}(t)$ and at the same time reduces actuator demands. One way of reducing the actuator demands is to place the joint torques closest to the midpoint of the joint torque limits $\frac{1}{2}(\underline{\tau}^+ + \underline{\tau}^-)$. In the following, a method is presented for finding this set of joint torques using the null space of the Jacobian matrix.

In order to incorporate dynamics, the redundancy is resolved at the acceleration level rather than at the velocity level. Using the pseudoinverse, the inverse kinematics readily follows from (18).

$$\ddot{\underline{\theta}} = \mathbf{J}^\dagger(\ddot{\underline{x}} - \dot{\mathbf{J}}\dot{\underline{\theta}}) + (\mathbf{I} - \mathbf{J}^\dagger\mathbf{J})\ddot{\underline{\phi}} \tag{26}$$

where the null space vector $(\mathbf{I} - \mathbf{J}^\dagger\mathbf{J})\ddot{\underline{\phi}}$ now appears relative to (19). Substituting (26) into (20),

$$\underline{\tau} = \tilde{\underline{\tau}} + \mathbf{H}(\mathbf{I} - \mathbf{J}^\dagger\mathbf{J})\ddot{\underline{\phi}} \tag{27}$$

where

$$\tilde{\underline{\tau}} = \mathbf{H}\mathbf{J}^\dagger(\ddot{\underline{x}} - \dot{\mathbf{J}}\dot{\underline{\theta}}) + \dot{\underline{\theta}} \cdot \mathbf{C} \cdot \dot{\underline{\theta}} + \underline{g}. \tag{28}$$

The goal is to place $\underline{\tau}$ closest to $\frac{1}{2}(\underline{\tau}^+ + \underline{\tau}^-)$ in a least squares sense, that is to say, to minimize

$$\left\| \underline{\tau} - \frac{\underline{\tau}^+ + \underline{\tau}^-}{2} \right\|^2 \tag{29}$$

Substituting (27) into (29) and defining $\hat{\underline{\tau}}^+ = \underline{\tau}^+ - \tilde{\underline{\tau}}$ and $\hat{\underline{\tau}}^- = \underline{\tau}^- - \tilde{\underline{\tau}}$, the goal is recast as finding the vector $\ddot{\underline{\phi}}$ to minimize

$$\left\| \mathbf{H}(\mathbf{I} - \mathbf{J}^\dagger\mathbf{J})\ddot{\underline{\phi}} - \frac{\hat{\underline{\tau}}^+ + \hat{\underline{\tau}}^-}{2} \right\|^2 \tag{30}$$

This least squares problem can also be solved by the generalized inverse (Ben-Israel and Greville, 1980), yielding

$$\ddot{\underline{\phi}} = \left[\mathbf{H}(\mathbf{I} - \mathbf{J}^\dagger\mathbf{J})\right]^\dagger \frac{\hat{\underline{\tau}}^+ + \hat{\underline{\tau}}^-}{2}. \tag{31}$$

This result can be extended to incorporate the idea that the available torque range is smaller for some joints than for others, through the use of weighted least squares (Ben-Israel and Greville, 1980). Given a weighting matrix $\mathbf{W}$, we now want to minimize

$$\left\| \underline{\tau} - \frac{\underline{\tau}^+ + \underline{\tau}^-}{2} \right\|_{\mathbf{W}}^2 = \left(\underline{\tau} - \frac{\underline{\tau}^+ + \underline{\tau}^-}{2} \right)^T \mathbf{W} \left(\underline{\tau} - \frac{\underline{\tau}^+ + \underline{\tau}^-}{2} \right). \tag{32}$$

The solution to this is given by

$$\ddot{\underline{\phi}} = \left[\mathbf{W}^{\frac{1}{2}}\mathbf{H}(\mathbf{I} - \mathbf{J}^\dagger\mathbf{J})\right]^\dagger \left(\mathbf{W}^{\frac{1}{2}} \frac{\hat{\underline{\tau}}^+ + \hat{\underline{\tau}}^-}{2} \right) \tag{33}$$

where by definition $\mathbf{W}^{\frac{1}{2}}\mathbf{W}^{\frac{1}{2}} = \mathbf{W}$. Torque range can be incorporated into the least squares by a diagonal matrix $\mathbf{W} = diag(1/(\tau_i^+ - \tau_i^-)^2)$. Weighting the least squares by

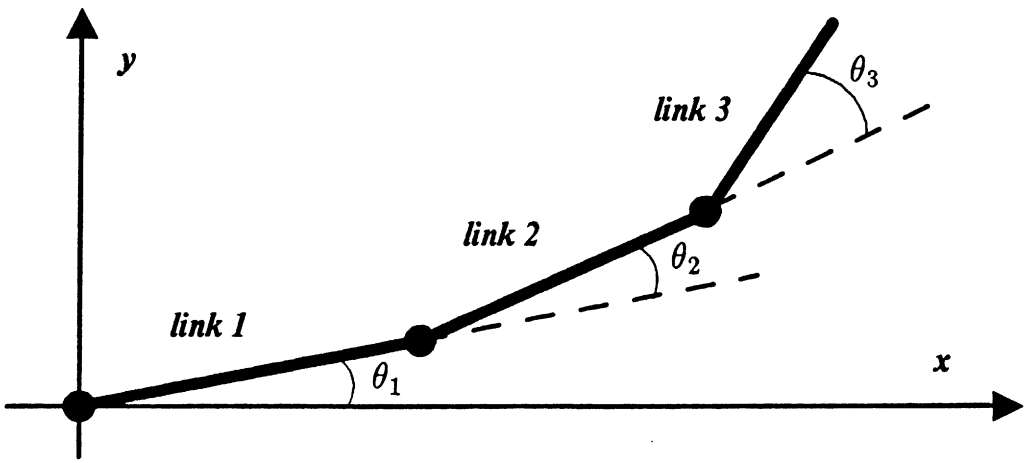

Figure 1: Geometry of the three-link planar manipulator.

the magnitude of the torque range seems a better procedure than the weighting schemes of Khatib and Yoshikawa, which are biased towards zero.

3 Simulation

The various algorithms for redundancy resolution at the acceleration level are demonstrated by simulation.

1. The *unweighted pseudoinverse algorithm* derives the joint torques from (27) and (28).

$$\underline{\tau} = \mathbf{H}\mathbf{J}^{\dagger}(\ddot{\underline{x}} - \dot{\mathbf{J}}\dot{\underline{\theta}}) + \dot{\underline{\theta}} \cdot \mathbf{C} \cdot \dot{\underline{\theta}} + \underline{g}. \tag{34}$$

2. The *inertia-weighted pseudoinverse algorithm* is obtained from (21).

$$\underline{\tau} = \mathbf{H}\mathbf{J}^{\dagger}_{\mathbf{H}}(\ddot{\underline{x}} - \dot{\mathbf{J}}\dot{\underline{\theta}}) + \dot{\underline{\theta}} \cdot \mathbf{C} \cdot \dot{\underline{\theta}} + \underline{g}. \tag{35}$$

3. The *unweighted null-space algorithm* derives the joint torques from (27), (28), and (31).

$$\underline{\tau} = \mathbf{H}\mathbf{J}^{\dagger}(\ddot{\underline{x}} - \dot{\mathbf{J}}\dot{\underline{\theta}}) + \dot{\underline{\theta}} \cdot \mathbf{C} \cdot \dot{\underline{\theta}} + \underline{g} + \mathbf{H}\left[\mathbf{H}(\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J})\right]^{\dagger} \frac{\hat{\underline{\tau}}^{+} + \hat{\underline{\tau}}^{-}}{2}. \tag{36}$$

4. The *weighted null-space algorithm* is derived from (27), (28), and (33).

$$\underline{\tau} = \mathbf{H}\mathbf{J}^{\dagger}(\ddot{\underline{x}} - \dot{\mathbf{J}}\dot{\underline{\theta}}) + \dot{\underline{\theta}} \cdot \mathbf{C} \cdot \dot{\underline{\theta}} + \underline{g} + \mathbf{H}\left[\mathbf{W}^{\frac{1}{2}}\mathbf{H}(\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J})\right]^{\dagger}\left(\mathbf{W}^{\frac{1}{2}}\frac{\hat{\underline{\tau}}^{+} + \hat{\underline{\tau}}^{-}}{2}\right) \tag{37}$$

Methods 3 and 4 again make use of the identity $B[CB]^{\dagger} = [CB]^{\dagger}$, where $B = (\mathbf{I} - \mathbf{J}^{\dagger}\mathbf{J})$ is hermetian and idempotent (Maciejewski and Klein, 1985).

The simulated manipulator is a planar rotary manipulator with three links (Figure 1). The parameters of the arm are link lengths $l_1 = l_2 = l_3 = 1.0$ and masses

$m_1 = m_2 = m_3 = 10.0$; link inertias are modeled by thin uniform rods. The simulated movements are straight-line Cartesian paths starting and ending with zero velocity, with constant tangential acceleration and deceleration of the magnitude $\sqrt{2}$ over first and last half of the movement respectively. Total duration of the movement is varied to give the trajectories of different lengths. Tip orientation is not constrained, giving one degree of freedom redundancy. For simplicity gravity is ignored, and the upper and lower torque limits are set equal in magnitude yielding $\underline{\tau}^+ + \underline{\tau}^- = \underline{0}$. Torque limits for joints 1–3 are set at 54, 24, and 6 respectively, derived from ratios of mass moments at each joint when the arm is perfectly straight to ensure appropriate relative scaling. For the weighted null-space algorithm, the diagonal elements of the weighting matrix $\mathbf{W}$ were set as the inverse squares of these torque limits.

The pseudoinverse was found using the singular value decomposition of a matrix (Strang, 1980), via a routine taken from *Linpak User's Guide* (Dongarra, 1979). After the joint torques were found, a fourth-order Runge-Kutta algorithm was used to find the next joint velocities and angles with an integration time interval of 1 msec. Programs were written in Fortran 77 in single precision and ran on the VAX 11/750 computer running 4.2 BSD UNIX.

4 Results

Performance of the unweighted null-space and pseudoinverse algorithms are compared in Figs. 2-4 for representative trajectories. For the short movement of Fig. 2, the null-space algorithm seems to give a substantial reduction in required joint torques. The most dramatic decrease is in the joint 1 torque, which is much smaller than for the unweighted pseudoinverse algorithm. While the other joint torques in the present example are decreased less, the joint 3 torque is pulled away from the perilously close torque limit of 6. For the intermediate length movement of Fig. 3, which is 2.5 times as long as in Fig. 2, the unweighted null-space algorithm still outperforms the unweighted pseudoinverse algorithm, although near the movement midrange and end the joint 2 torques are somewhat larger though still within the torque range.

For the long movement of Fig. 4, which is four times as long as in Fig. 2, the unweighted null space algorithm unexpectedly encounters stability problems near the movement end. The torques required to keep the manipulator endpoint on the planned trajectory become extremely high, occasioned by the joint alignment of links 2 and 3 coupled with large velocities and accelerations in all the joints (Fig. 5). Evidently the unweighted null-space algorithm needs a long enough path for this situation to develop. Indeed, nearly any long path led to similar instability problems, since joint alignment and high velocities inevitably followed. This rarely happened in the case of the unweighted pseudoinverse algorithm. Although the unweighted null-space algorithm instantaneously gives smaller torques than the unweighted pseudoinverse algorithm, globally the unweighted null-space algorithm degrades unacceptably in performance with longer trajectories.

Ordinarily one would expect a proximal joint such as joint 1 to have a larger actuator and a wider torque range than the distal joints, since it is required to exert more torque.

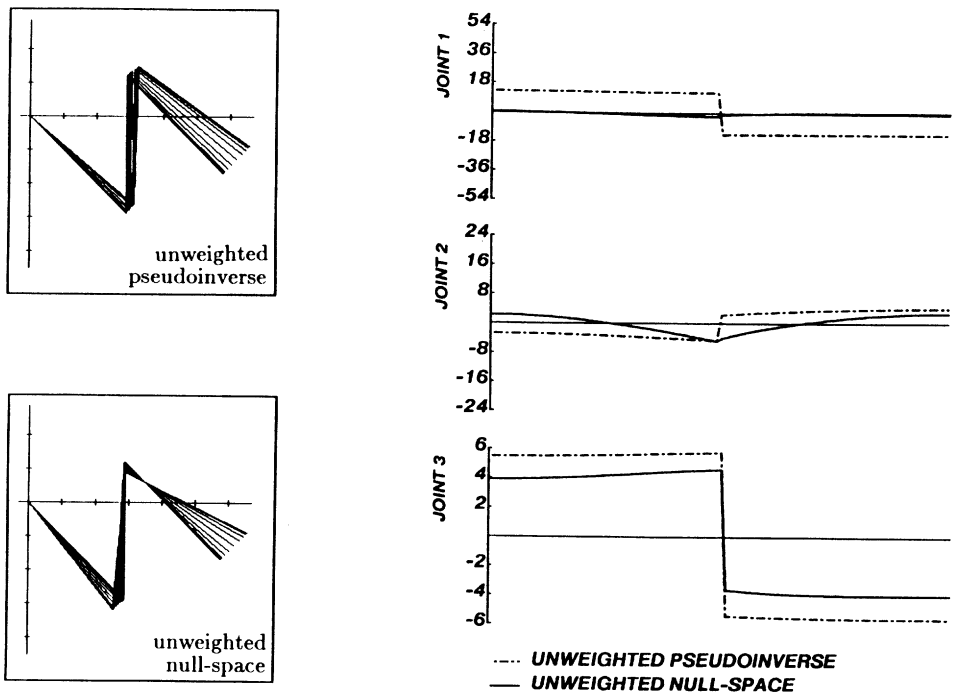

Figure 2: Torque profiles for a short movement for the pseudoinverse and null-space algorithms.

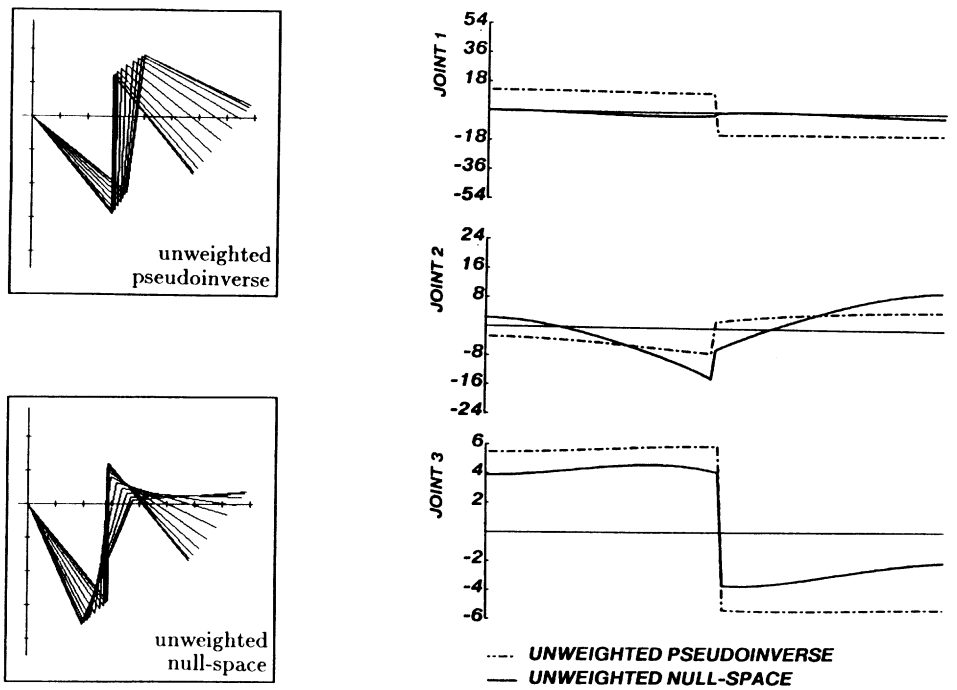

Figure 3: Medium-length movement.

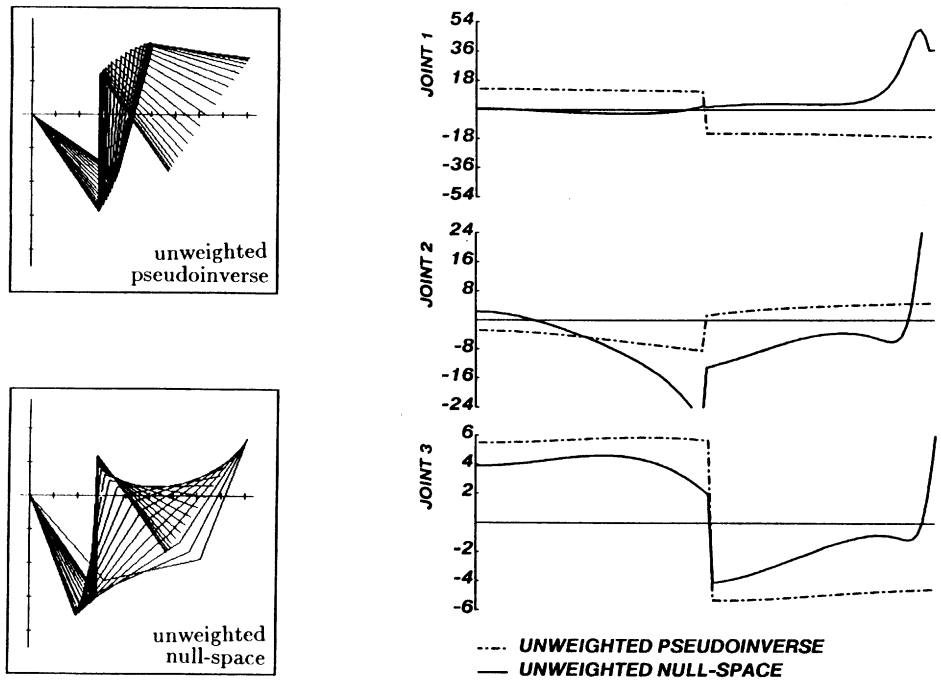

Figure 4: Long movement showing instability.

Yet in the unweighted torque optimization all joints are treated equal; the sum of squares of joint torques is minimized by reducing the largest torque which is the joint 1 torque by generating high accelerations at all the joints. If the torque optimization is done relative to the allowable torque range at each joint, the instability problem may be overcome.

To test this idea, performance of the weighted null-space algorithm is examined for the same trajectory as in Fig. 2. It can be seen that the joint 3 torque is now much smaller at the expense of more torque at joints 1 and 2, but all remain well within their torque ranges (Fig. 6). Unfortunately, the weighted null-space algorithm also yields stability problems for the long movement of Fig. 4, so that the hoped-for elimination of instability with weights is not realized. There were even movements where the instability was shown only by the weighted case, although the reverse was not found. The observed characteristics of the instability in the weighted cases were identical to those of the non-weighted cases.

Performance of the inertia-weighted pseudoinverse algorithm is also shown in Fig. 6. Here and elsewhere the joint torques fell somewhere in between the unweighted pseudoinverse algorithm and the unweighted null-space algorithm. The inertia-weighted pseudoinverse algorithm also suffered from instabilities in longer movements. In most of the trajectories studied, whenever the null-space algorithm was unstable, the inertia-weighted pseudoinverse algorithm was unstable as well.

5 Discussion

The goal of this paper was to develop a local algorithm for reducing actuator require-

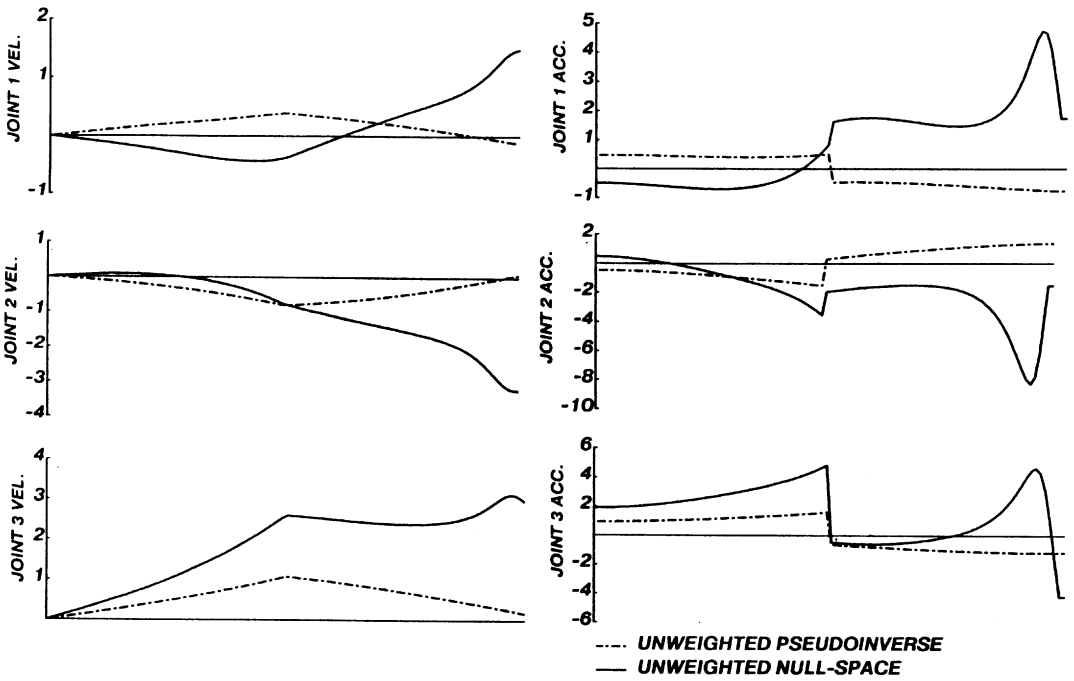

Figure 5: Velocity and acceleration profile for the movement in Figure 4.

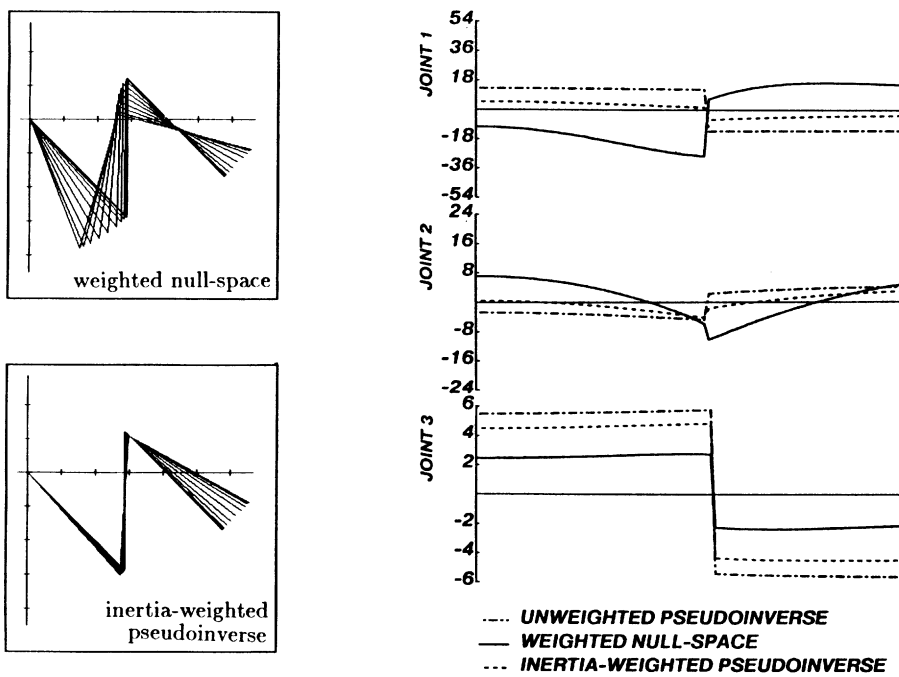

Figure 6: Torque profiles for the short movement for the weighted null-space and inertia-weighted pseudoinverse algorithms.

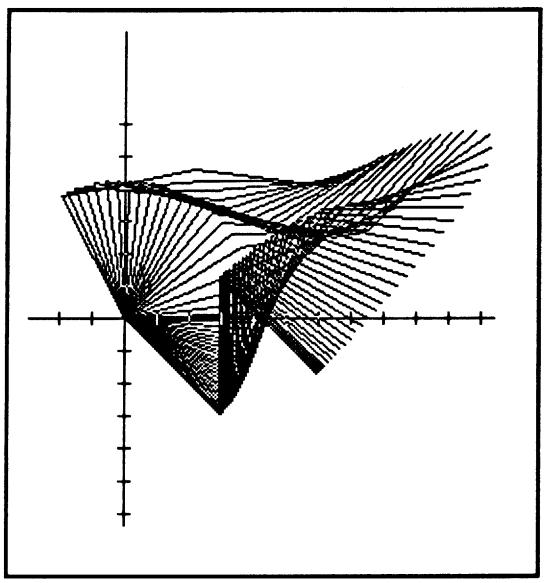

Figure 7: A free-swinging pendulum acting under the influence of gravity directed along the former straight-line motion

ments and avoiding torque limits, and the methods examined include use of a null-space acceleration vector, this vector weighted by torque range, and an inertia-weighted pseudoinverse. These kinetic methods all lead to stability problems, even though locally these methods reduce actuator torques. Globally only the unweighted pseudoinverse algorithm, which by contrast is a kinematic method, is generally well-behaved, although on rare occasions it too shows instabilities.

It seems that local tampering with the energetics of movement has led to global disaster. The torque profiles in Fig. 4 are representative of the problem. High accelerations in the distal joints lead to high joint velocities, and at the point where the third joint is straight these high velocities induce a whipping action that strongly thrusts the endpoint off the path. Substantial torques are required to overcome these natural movement dynamics and to maintain the planned endpoint trajectory.

Insofar as torque optimization tends to reduce the torques towards zero, the situation may be analogous to a free-swinging pendulum. In Fig. 7 the manipulator has been started at the same position as the previous plots, but with gravity now acting along the original straight-line motion with a magnitude that yields an initial acceleration equal to the acceleration step in the simulated trajectories. The manipulator is allowed to fall freely under the influence of this gravity. The free-swinging pendulum exhibits a similar whipping action when the third joint straightens out. Thus the kinetic methods do indeed lead to utilization of the natural movement dynamics, but ultimately to the detriment of the movement goals with which the natural movement dynamics are incompatible.

Whether the kinetic methods may be modified to avoid the instabilities is a subject of continuing research. One possibility is to weight the local optimization criterion with a kinematic term to avoid high velocity buildup. For staying within torque bounds,

linear programming rather than least squares may be more satisfactory. The broader question is whether any local algorithm can ever be completely successful, or whether ultimately only a global resolution of redundancy can be guaranteed problem-free.

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Methods for resolving kinematic redundancies of manipulators by the effect on joint torque are examined. When the generalized inverse is formulated in terms of accelerations and incorporated into the dynamics, the effect of redundancy resolution on joint torque can be directly reflected. One method chooses the joint acceleration null-space vector to minimize joint torque in a least squares sense; OVER

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when the least squares is weighted by allowable torque range, the joint torques tend to be kept within their limits. Contrasting methods employing only the pseudoinverse with and without weighting by the inertia matrix are presented. The results show an unexpected stability problem during long trajectories for the null-space methods and for the inertia-weighted pseudoinverse method, but rarely for the unweighted pseudoinverse method. Evidently a whiplash action develops over time that thrusts the endpoint off the intended path, and extremely high torques are required to overcome these natural movement dynamics.

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