PLASMA DYNAMICS
VIII. PLASMAS AND CONTROLLED NUCLEAR FUSION

E. High-Temperature Toroidal Plasmas

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1. MODEL FOR THE TURBULENT ION HEATING IN TOKAMAK TM-3

Recently reported experiments on the Tokamak TM-3 machine have shown that the ion energy distribution contains a low-energy part at a temperature of approximately 50 eV and a high-energy tail. The striking feature of this tail is that it continuously disappears as the applied magnetic field is increased from 10 kG to 26 kG. We propose that the enhanced heating of the tail is due to an electrostatic ion-cyclotron instability driven by the electron drift current parallel to the applied magnetic field. We summarize the pertinent results of linear and quasi-linear theories in support of this model.

Linear Instability Theory

The instability under consideration has been studied in the past by several authors. The regime of plasma parameters pertinent to TM-3, where \( T_e \gg T_i \) and, for \( k_{||}a_i \sim 1, \ k_fD_i \gg T_i/T_e \) (\( a_i \) = ion Larmor radius, \( D_i \) = ion Debye length), was not covered by these previous studies, however, and the instability conditions must be rederived. The dispersion relation for electrostatic waves propagating almost across \( B_0 (k_{||} \gg k_{\perp}) \) and with phase velocities \( v_i \ll (\omega/k_{||}) \ll v_e \), where \( v_i \) and \( v_e \) are the ion and electron thermal velocities, has been solved. The growth rate is given by (\( \gamma \equiv \text{imaginary part of } \omega \))

\[
\gamma = -\frac{T_i}{T_e} \frac{\sqrt{\pi} (k_{||}v_e - \omega)}{k_{||}v_i} - \frac{\sqrt{\pi} \omega}{k_{||}v_i} e^{-k_{||}^2a_i^2} \sum_{n=-\infty}^{\infty} \frac{\frac{\partial}{\partial \omega} \left( \int n(k_{||}^2a_i^2) \exp \left[ -\frac{(\omega - n\omega \omega)}{k_{||}v_i} \right] \right)}{\partial \omega},
\]

where \( f(\omega) \) is given in Eq. 4. It is positive as long as

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\[
\frac{u_e}{v_i} > \frac{(k_1/k_\parallel)(\omega/\omega_{ci})}{\sqrt{2} k_1 a_1} \left[ 1 + \sqrt{\frac{m_i T_i^3}{m_e T_e^3}} \frac{e^{-k_1^2 a_1^2}}{k_1 a_1} \sum_{n=-\infty}^{\infty} I_n(k_1^2 a_1^2) e^{-\zeta_n^2} \right].
\]  \hspace{1cm} (2)

where

\[
\zeta_n = \frac{\omega - n\omega_{ci}}{k_\parallel v_i},
\]  \hspace{1cm} (3)

and \(\omega, k_1, \) and \(k_\parallel\) satisfy the real part of the dispersion relation:

\[
1 + \frac{T_i}{T_e} + (kf_{Di})^2 = e^{-k_1^2 a_1^2} \left[ I_0(k_1^2 a_1^2) + \sum_{n=1}^{\infty} \frac{2\omega_n^2}{\omega_n^2 - n^2 \omega_{ci}^2} \right] = f\left(\frac{\omega}{\omega_{ci}}, k_1 a_1\right).
\]  \hspace{1cm} (4)

Equation 4 may be written

\[
1 + \frac{T_i}{T_e} + (k_1 a_1)^2 \left(\frac{\omega_{ci}}{\omega_{pi}}\right)^2 = f.
\]  \hspace{1cm} (5)

Consider first the limit \((kf_{Di})^2 \ll T_i/T_e\), which has been solved numerically. In this limit we note that Eqs. 1, 2, and 4 depend only on \(\omega/\omega_{ci}, k_1 a_1,\) and \(k_1/k_\parallel\). Hence, for a constant ratio of \(T_i/T_e\), as \(B_o\) is changed, both the growth rate \(\gamma/\omega_{ci}\), and the instability onset condition, Eq. 2, remain unchanged if \(k_1\) and \(k_\parallel\) are changed proportionately to \(B_o\). Also, in this limit, the calculation by Lominadze and Stepanov shows that for the onset of instability, \(k_1 a_1\) remains close to unity for a wide range of \(T_i/T_e\) (see Fig. VIII-1 and Table VIII-1). For the case of interest in TM-3, where \(k_1 a_1 \geq T_i/T_e\), it can be seen from Fig. VIII-1 and Eq. 5 that increasing \(B_o\) moves \(\omega\) closer to \(n\omega_{ci}\) and thus cyclotron damping will decrease the growth rate \(\gamma/\omega_{ci}\).

Using TM-3 data in the operating regime of strong anomalous resistance \((u_e \approx 60 \nu_i \approx v_{Te}/3, n_e \approx 10^{11} - 10^{12}/cm^3, T_e \approx 1 keV, T_i \approx 50 eV\), we observe the hot-ion tail at \(B_o \approx 10 kG,\) and it disappears at \(B_o \approx 26 kG\). If the density is taken as \(10^{11}\), \((\omega_{ci}/\omega_{pi})^2\) varies from \(1/25\) to \(1/4\) for this variation in \(B_o\). A rough scaling of the computations to these parameters shows that at \(10 kG\) the instability conditions are well satisfied and the growth rate should be large, while for \(26 kG\) a strong
Fig. VIII-1. Dispersion characteristics for the electrostatic ion cyclotron wave (Lominadze and Stepanov). (a) Eq. 4 for \(k_1 a_1\) constant. (b) \(\omega(k_1)\) at \(T_i/T_e = 0.1, 1.0\).

Table VIII-1. Conditions for onset of instability for \(m_i/M_i = 1/3680\).

<table>
<thead>
<tr>
<th>(T_i/T_e)</th>
<th>(u_e/v_i)</th>
<th>(k_{1,i}^2 a_{1,i}^2)</th>
<th>(\cos \theta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>23</td>
<td>1.1</td>
<td>0.05</td>
</tr>
<tr>
<td>0.33</td>
<td>15</td>
<td>0.8</td>
<td>0.12</td>
</tr>
<tr>
<td>0.10</td>
<td>12</td>
<td>0.8</td>
<td>0.15</td>
</tr>
</tbody>
</table>

reduction in growth rate may be expected. Computer calculations for the growth rate have been initiated for obtaining quantitative estimates of the reduction in growth rate with increasing magnetic field.

Quasi-linear Theory

We shall now examine the way in which this instability heats the ions. The quasi-linear diffusion equation for ions in the presence of weak turbulence is well known, and can be written
In order to estimate the ion heating rate, we shall assume that \( f_i \) is isotropic. In spherical coordinates in velocity space, integrating Eq. 6 over the spherical angles \( \theta \) and \( \phi \), we obtain

\[
\frac{\partial f_i}{\partial t} = \nabla_v \cdot D \cdot \nabla_v f_i, \tag{6}
\]

where

\[
\tilde{D}_{vv} = \frac{1}{4\pi} \int D_{vv} \sin \theta \, d\theta d\phi
\]

\[
= \sum_{n,k} \frac{\pi}{2k_{\parallel}v} \left| \frac{eE(k)}{m} \right|^2 \left( \frac{\omega}{k_v} \right)^2 J_n^2 \left( \frac{k_{\parallel}V_n}{\omega_{ci}} \right),
\]

where \( V_n = \sqrt{v^2 - \left( \frac{\omega - n\omega_{ci}}{k_{\parallel}} \right)^2} \), for \( v^2 > \left( \frac{\omega - n\omega_{ci}}{k_{\parallel}} \right)^2 \),

and

\[
0 \quad \text{for} \quad v^2 < \left( \frac{\omega - n\omega_{ci}}{k_{\parallel}} \right)^2. \tag{8}
\]

For TM-3 parameters and the instability conditions discussed above, we find that all waves are linearly stable unless \( (\omega - n\omega_{ci})/k_{\parallel} \gtrsim 2.5 \, v_i \). This, together with Eq. 8, shows that only the tail \( (\epsilon \lesssim 6 \, \epsilon_i) \) of the ion energy distribution function will be heated, which is in agreement with experimental observations.\(^1\) For the high-velocity ions \( (v > 2.5 \, v_i) \), an order-of-magnitude estimate of \( \tilde{D}_{vv} \) from Eq. 8 is

\[
\tilde{D}_{vv} \approx \left| \frac{eE}{m} \right|^2 \frac{v_i^2}{k_{\parallel}v^3}, \tag{9}
\]

where \( |E| \) is an average field strength. A heating rate \( \nu_h \equiv \tilde{D}_{vv}/v_i^2 \) is then given approximately by

\[
\nu_h \approx \omega_{pi} \left( \frac{v_i}{k_{\parallel}v_1} \right) \left( \frac{v_i}{v_1} \right)^2 \left( \frac{\epsilon_F}{\epsilon_i} \right), \tag{10}
\]
where $\epsilon_F$ is the average electric field energy, and $\epsilon_i$ the ion thermal energy. We can compare this heating rate with the electron-ion energy transfer rate, $v e \approx 2(m_e/m_i) \omega_{pe}/n \omega_{De}^3$. Assuming $v \approx 5 v_i$ and $k_\parallel \approx 10 \omega_{ci}/u_e$, we find $v_h/v_e \approx 10^6 (\epsilon_F/\epsilon_i)$. Thus even if the total electric field energy is small compared with the ion thermal energy, turbulent heating may be dominant. Furthermore, if the nonlinear process that stabilizes the wave is such that the final electric field energy scales with the linear growth rate, then $\epsilon_F$ will decrease as the magnetic field increases. Therefore the heating rate of the tail (and hence its temperature) will also decrease with increasing magnetic field strength.

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References