

Chapter 3. Superconducting and Quantum-Effect Electronics

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3.1 Nonlinear Dynamics of Discrete Josephson Arrays

Sponsor

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Discrete arrays of nonlinear Josephson oscillators can exhibit diverse spatiotemporal patterns. Although such oscillator arrays are difficult to analyze completely, one can often use the symmetries of the system to construct simple patterns composed of spatially repeated "unit cells." Experiments, simulations, and analysis on a broad class of discrete arrays of Josephson-junction oscillators indicate novel phase-locked states that, due to their special symmetry, reduce the governing equations of the full array to a much smaller set of equations of a unit cell. Networks ranging from single square

and triangular plaquettes to one- and two-dimensional arrays have been studied.

Figure 1 shows the measured IV curves (I is the current per vertical junction normalized by I_C , and V is the voltage per row) for three different array geometries when fully frustrated. The signature of all these IVs is the appearance of jumps at two resonant voltages, V_+ and V_- . The upper step, which ends at V_+ , is independent of temperature suggesting that local geometrical properties determine the voltage. In this state, all the rows of the array act coherently and phase-lock at a voltage that depends on the geometric loop inductance and junction capacitance. The lower voltage V_- , on the other hand, is temperature-dependent, suggesting a dependence on the Josephson inductance and the geometric loop inductance. By taking advantage of the symmetry of the network it is possible to describe the solution as a dynamical checkerboard state and mathematically analyze its resonant behavior in a reduced system of governing equations. However, the conditions for the stability and the temporal periodicity of the checkerboard state, and the dynamics associated with other possible states, are challenging problems for future investigation.

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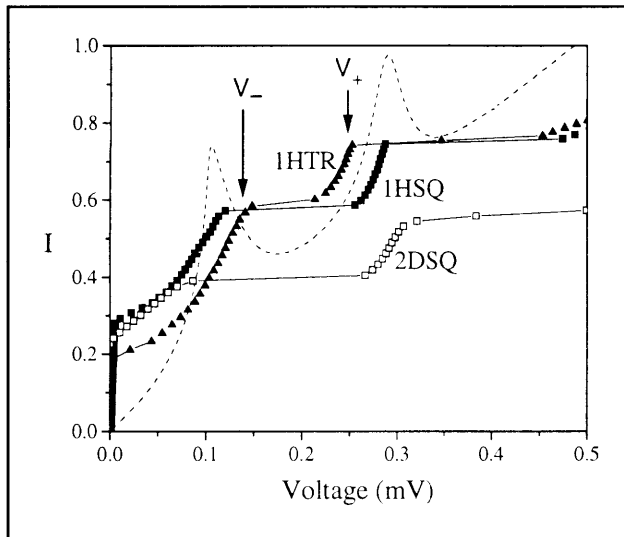


Figure 1. Experimental IV curves for three arrays: triangular array (1 X 9 plaquettes) with $\beta_c = 11$ and $\lambda = 0.64$; square array (1 X 7) with $\beta_c = 11$ and $\lambda = 0.76$; and a square array (7 X 7) with $\beta_c = 20$ and $\lambda = 0.92$. Dashed line, IV from harmonic balance for the squarer array (1 X 7) array with the same β_c and an effective $\lambda_{\text{eff}} = 0.61$ which accounts for mutual inductance effects. V_+ and V_- are indicated.

We have also studied the spatiotemporal dynamics of circular one-dimensional arrays of underdamped Josephson junctions connected in parallel. In these Josephson rings, a traveling wave solution consisting of a single kink can be trapped and studied experimentally without the complications caused by reflections off boundaries. We find that a propagating kink can become phase-locked to linear waves excited in its wake. In the IV curve, resonant steps are observed indicative of this phase-locking. Resonant steps also occur in the IV curves for higher voltages in the return path of the subgap region. These resonant steps have a completely different origin and occur at voltages where the periodic whirling solution undergoes an instability parametrically amplified by the linear modes in the system.

3.2 Resonance Splitting in Inductively Coupled Arrays

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A row of Josephson junctions connected in parallel traps magnetic vortices in its interconnecting loops. Measurements of long single rows show that an applied bias current shuttles the vortices through the system, inducing both an average dc voltage across the row and ac voltages in a traveling-wave pattern. When sufficient bias current is applied, the vortices move at the maximum speed for electromagnetic waves in the system. Further increases in current do not increase the vortex velocity or the average dc voltage across the array. At this point, a large step appears in the dc current-voltage (IV) characteristic, where a range of bias currents correspond to a single voltage. This so-called Eck step also corresponds to voltage oscillations which are nearly a single harmonic in the junctions.

When two rows are connected in series, as in figure 2, they are also inductively coupled. Thus, vortices tend to travel through the two rows in either a checkerboard pattern or side-by-side. This results in traveling wave patterns with the two rows either exactly in-phase or shifted by π . These two stable states manifest themselves in the dc IV by splitting the Eck step. A typical IV is shown in figure 3. The higher voltage state corresponds to the in-phase waves, whereas the lower voltage state corresponds to anti-phase waves in the two rows. When the system is biased in the in-phase state, the ac voltages add, increasing the power output. Since the frequency, amplitude, and dispersion of these oscillations are well-defined, the system is desirable for oscillator applications.

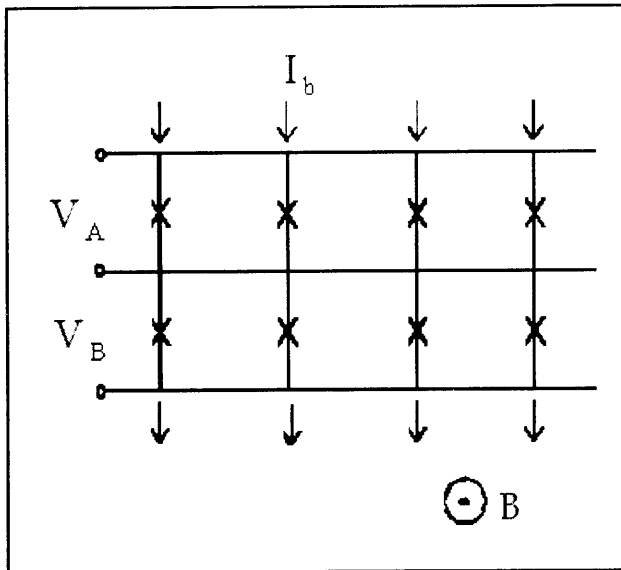


Figure 2. Schematic of inductively coupled array. Fifty-four junctions per row.

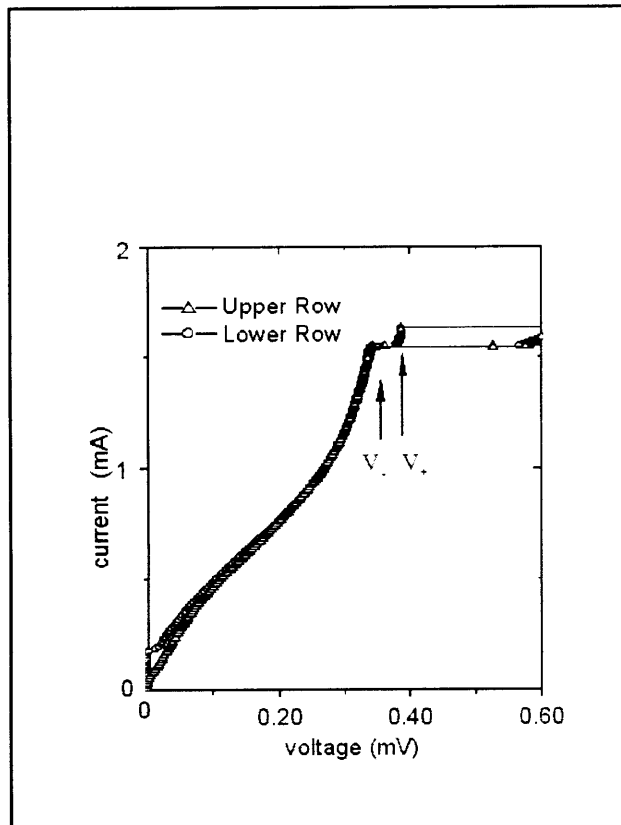


Figure 3. Measurements of an inductively coupled array, $N=2 \times 54$, on a ground plane. The triangles represent the voltage values measured across the top row, while the circles are voltage values across the bottom row. The measurements were taken at 8.2 K.

3.3 Self Field Effects on Flux Flow in Josephson Arrays

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Two-dimensional arrays of Josephson junctions provide controllable model systems for the study of vortex transport in thin film superconductors. A crucial parameter that determines the dynamics of these vortices is the characteristic penetration depth of the applied field. It is this length that also governs the effects of self-induced magnetic fields.

Typical current-voltage IV characteristics vs. applied magnetic field are shown in figure 4. The applied magnetic field B_0 is measured in units of the frustration $f = B_0 p^2 / \Phi_0$. The depinning current I_{dep} indicates the onset of the flux-flow region. There is a curving transition in the IV from the depinning current to a linear region. The flux-flow resistance R_{ff} is defined as the slope of this linear region which is shown by a straight line for each of the IVs. The inset shows that the assigned resistance values are linear in f up to $f \approx 0.3$. For larger values of f up to $f=0.5$, R_{ff} is no longer linear with f due to the increased interaction between vortices.

Other measurements and numerical studies of the effects of self-induced magnetic fields on the flux flow resistance have been performed. It was found that the flux-flow resistance becomes larger as the penetration depth of the array decreases. A phenomenological model, which agrees qualitatively with the experiments and simulations, has been developed to explain the self-field effects on flux flow. Due to the smaller spatial extent of supercurrents around a vortex when self-fields are important, both the mass of the vortex and the array viscosity decrease. The decreased mass and viscosity lead to an increase in flux-flow resistance. The effects of spin-wave damping have also been investigated for underdamped arrays.

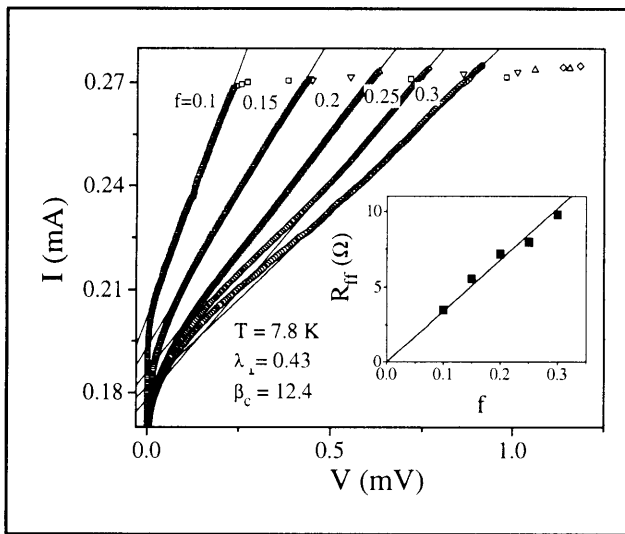


Figure 4. Current-voltage characteristics for various magnetic fields from $f = 0.1$ to 0.3 . The data is from sample H2 taken at 8.6 K with $\lambda_{\perp} = 1.04$ and $\beta_c = 5.1$. The numbers indicate values of f . The solid straight lines denote the linear region of flux-flow. The inset shows that the flux-flow resistance R_H is linear in f .

The flux-flow region appears to be richer in its dynamics than the presented model can account for. In particular, treating the effective linear viscosity of the array as the sum of the Bardeen-Stephen damping and spin-wave damping is probably an over-simplification. Although these deviations do not diminish the useful and intuitive results from the phenomenological model, they point the way for further research on the richness of the dynamics in the flux-flow regime.

3.4 Triangular Arrays of Josephson Junctions

Sponsors

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Project Staff

Amy E. Duwel, S. Yukon,⁴ Professor Terry P. Orlando

Superconducting arrays of Josephson junctions are potential sources of millimeter and sub-millimeter radiation. However, the low power generated by

each junction, its low impedance, and its broad line-width necessitates the coupling of superconducting junctions into arrays to overcome these three limitations. In arrays based on triangular rather than square cells, numerical simulations and calculations show novel and technologically important dynamical states.

Measurements of underdamped single cell and single row arrays reveal two steps in the current-voltage (IV) characteristic, corresponding to L_sC and L_jC resonances. These steps are characteristics of single cells, and their position does not change significantly with array size. Measurements of two different cell sizes showed that the upper step voltage depends strongly on the cell geometry, while the lower step is only slightly affected. In figure 5, we compare the IV of a triangular cell to that of an 8-cell array with the same parameters. The steps appear only in the presence of a magnetic field, when the average applied flux per cell (called frustration, f) is approximately one-half. They are stable for a range of $f = 0.3$ to 0.7 .

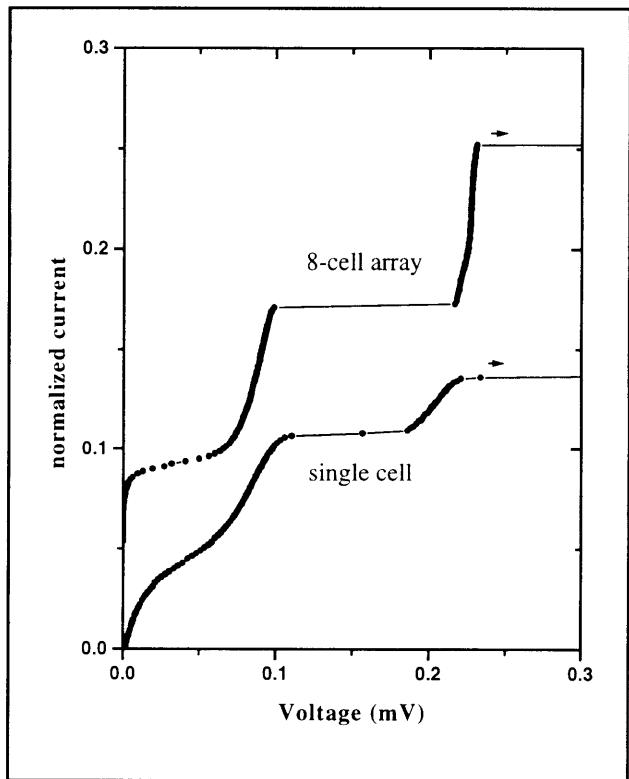


Figure 5. Current-voltage characteristics of a single triangular cell and of an 8-cell array. The parameters are $\beta_c =$ and $\Lambda_j^2 = 2.4$, and $f = 0.5$.

⁴ U.S. Air Force, Rome Laboratory, Rome, New York.

At the L_sC resonance, triangular arrays produce large-amplitude single-harmonic oscillations in the horizontal junctions. According to dc measurements, oscillators based on this resonance operate at frequencies ranging from 70-170 GHz, with bandwidths of 10-20 percent. In addition, Yukon and Lin have suggested methods for making mode-locked oscillators with triangular Josephson junction arrays. Using the available niobium technology, calculations indicate that time-domain pulses can be produced with repetition frequencies ranging from 1-10 GHz. We are currently studying the dc properties of such systems and developing methods for microwave measurements on these and other types of Josephson arrays.

3.5 Quantum Device Simulations

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We carried out numerical simulations of single-electron transport through a quantum dot with superconducting leads, based on an experimental system. We introduce a general phenomenological model of transport through a quantum dot. In this model, we assume that the quantum dot is weakly coupled to the two leads by tunnel barriers. When an appropriate bias voltage V is applied to the leads, an electron can tunnel across one barrier into the dot and subsequently tunnel out through the second barrier. According to general tunneling theory, the tunneling rate across a barrier from side "a" to side "b", can be evaluated using the Fermi's Golden Rule,

$$\Gamma_{a \rightarrow b}(\mu_a, \mu_b) =$$

$$\frac{2\pi}{\hbar} |T_{ab}|^2 N_a(E - \mu_b) f(E - \mu_a) [1 - f(E - \mu_b)] dE$$

where T_{ab} is the phenomenological tunneling matrix element, and $f(x) = 1/[1 + \exp(x/k_B T)]$ is the Fermi function. $N_a(E)$ and $N_b(E)$ are the density of states, and μ_a and μ_b are the chemical potentials, on their corresponding sides. For our system, to compute the tunneling rate from one of the leads to the dot, we take the BCS quasiparticle density of states in

the lead and assume that the dot itself has an evenly spaced (with spacing ϵ) discrete level spectrum.

In figure 6, we show a typical low temperature current-voltage (I-V) characteristic of the system. Here the temperature $k_B T = 0.02E_c$ ($E_c \equiv E_c + \epsilon$ is the spacing between chemical potential levels, and $E_c \equiv e^2/C_x$ is the charging energy), the superconducting energy gap $2\Delta = 0.3E_c$ and the quantum energy level spacing in the dot $\epsilon = 0.2E_c$. When the leads are superconducting (solid curve), the I-V curve consists of a series of sharp peaks spaced ϵ apart. This is in contrast with the I-V curve of the same dot with normal metal leads (dashed curve), which has only gentle steps with the same spacing ϵ . Figure 6 is in good qualitative agreement with the experiment.

In addition to the low temperature transport, our analysis shows that at higher temperatures thermal excitation of quasi-particles in the leads and thermal population of the excited quantum levels within the quantum dot should lead to interesting changes in the I-V curves. We also predict that when RF radiation is coupled to the system, the photon-assisted tunneling phenomena would manifest itself by producing extra periodic structures in the I-V curves, which might be useful in the millimeter wave detector/mixer applications. Due to the presence of many different characteristic energy scales, the rich dynamical properties of this system demands more exploration.

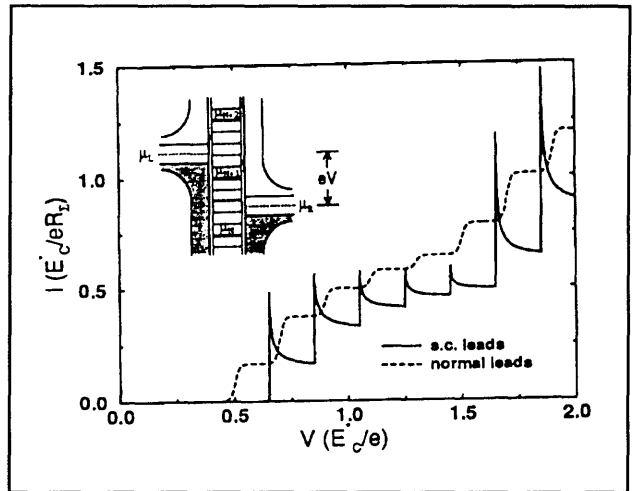


Figure 6. Low temperature I-V characteristics of a quantum dot with superconducting leads (solid curve) and normal metal leads (dashed curve). The temperature is $k_B T = 0.02E_c$, and the superconducting energy gap in the leads is $2\Delta = 0.3E_c$. The quantum level spacing is, $\epsilon = 0.2E_c$. The inset is a sketch of the energy spectra in the leads and the dot. Note the quantum dot energy spectrum includes the excitation spectrum (with spacing ϵ), and addition spectrum (with spacing $E_c = e^2/C + \epsilon$).

3.6 Publications

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- Duwel, A.E., T.P. Orlando, S. Watanabe, and H.S.J. van der Zant. "A Novel Phase-Locked State in Discrete Josephson Oscillators." *IEEE Trans. Appl. Superconduct.* Forthcoming.
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