THE STRESS DISTRIBUTION BENEATH ISLAND ARCS

by

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SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
DEGREE OF MASTER OF SCIENCE
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
September, 1971

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ABSTRACT

In this thesis we develop a numerical simulation of the descending lithosphere at island arcs in order to describe the regional stress distribution derived from earthquakes in terms of the descent conditions for the slab. Two elements comprise the problem: First, the lithospheric slab and the adjacent mantle are simulated as an elastostatic problem with moduli corresponding to their effective rheologies. Assuming similar composition for the lithosphere and mantle, and given the temperature regime for the mantle and its assumptions, we include gravitational body forces generated by thermal volume contraction of the colder slab and by elevation of phase boundaries. In addition, simulated convection is applied to the slab in order to distinguish its role as a driving mechanism. The regional applicability of gravitational sinking is then clarified using a simple numerical integration of the equilibrium equation and comparing this to available information for select island arcs. These analogues suggest a viable hypothesis for the dynamics of the subducting lithosphere.

First, gravitational sinking induced by thermal density anomalies can explain the directions of the principle stress within the descending slab at island
arcs without recourse to convective effects in the adjacent mantle. The rheologies of the mantle and slab represent instead the dominant effect. Second, under these assumptions the convergence rate of the island-arc system constitutes a major factor influencing the stress distribution by means of the gravitational body forces and the mantle's resistance or support. The models indicate that a fine balance exists between these forces. Furthermore, if large deviatoric stresses are necessary for intermediate and deep earthquakes, the elevation of phase-transformation boundaries are a necessary feature of the simulation for a mantle and slab with similar composition. The post-spinel phase change at 650 km depth suggests itself as a possible factor in this mechanism. Finally, a low-strength channel corresponding to the seismic low-velocity zone and a stronger but constant-strength mantle yields reasonable agreement between the simulation and the regional stress patterns derived from earthquakes.

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Acknowledgements

To M. Nafi Toksoz goes my deep appreciation for inspiring this research and for providing a critical commentary. Special thanks go to Norman Sleep and Jorge Mendiguren for many fruitful discussions. And finally all my friends, particularly Jessie, I thank for their understanding and love.

The formative aspects of this problem were performed while I was supported by an MIT fellowship and research assistantship, and completed with the assistance of a Hertz Foundation fellowship. The research has been supported by the Advanced Research Projects Agency monitored by the Air Force Office of Scientific Research through contract F44620-71-C-0049 and by NASA with contract NGL 22-009-187.
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New global tectonics implies both spreading of the sea floor and consumption of crustal plates into the mantle. Various geological and geophysical observations confirm the motion of these plates: the distribution of volcanoes, heat flow, paleomagnetics, and seismicity are among the most important indications (McKenzie, 1969; Sclater and Francheteau, 1970). Each of these imposes some constraints upon the general motion, both past and present. Yet the mechanism producing these motions is a topic of considerable debate. Gravitational sinking of the lithosphere, elevation of the mid-ocean ridges, and viscous drag have each been suggested as the mechanism coupling thermal convection to the lithosphere. All these depend on thermal convection within the mantle as their source of energy, while each limits the possible configuration of mantle convection. One means of limiting the possible mechanisms causing the motion is to compute models of the stresses within a downgoing slab, incorporating various driving mechanisms and relevant body forces. Comparing the stress patterns, seismicity, energy, and focal mechanisms of earthquakes yields constraints upon the type of mechanism producing the plate motion. This technique has been used for just
Deep and intermediate focus earthquakes provide initial evidence for the state of stress within the descending lithosphere. Focal plane solutions and locations for Tonga-Fiji-Kermadec and Japan reveal that the deep earthquakes occur within the descending lithosphere rather than at the mantle-slab boundary (Isacks and Oliver, 1968). The fault planes do not follow the mantle-slab boundary; instead, the axis of principle stress aligns itself with the dipping slab (Isacks and Molnar, 1969). These and succeeding solutions confirm that the earthquakes appear to reflect stresses in the relatively strong slab of the descending lithosphere.

McKenzie (1969) has quantitatively approached the problem of stresses within the descending lithosphere and surrounding mantle. Using an analytical solution of the slab's thermal regime, the resulting force pulling upon the lithosphere is estimated from the negative bouyancy of the cold slab. Elsasser (1967) has suggested this as maintaining the observed surface motion. Focal mechanism solutions indicate that other processes must be operating; not all shallow or intermediate depth earthquakes have tensional fault plane solutions along the axis of the slab. Still the sinking slab qualitatively explains a variety of earthquake solutions (Isacks and Molnar, 1971). The problem lies
with determining the role that gravitational sinking plays as a driving mechanism.

One solution is modeling the descending slab and the surrounding mantle as an elastostatic problem, incorporating body forces due to thermal density differences between the colder slab and surrounding mantle. Symmetry of the ideal slab geometry allows a plane strain formulation; consequently, the simulation reduces to an elliptic boundary value problem allowing numerical solution. Knowing the temperature regime within the slab (Toksoz, et al., 1971), the induced stresses are then computed for elastic moduli selected to simulate the dynamic and static support provided by the surrounding mantle. In addition, the effects of convection are superimposed to resolve its role as a driving mechanism. These models can provide insight and restrictions upon the probable driving mechanism, and in conjunction with other data, improve our understanding of the motions of lithospheric plates.

In the following chapters the theoretical assumptions and practical considerations required for these stress models are developed, and the computational technique is described. The final chapters include models which indicate the dependence of stresses upon the slab and mantle's rheology and suggest their relationship to stresses and source mechanisms of earthquakes.
Chapter 2
SIMULATION OF LITHOSPHERIC SLABS

Section 2.1. Introduction

The rheology of the earth represents a special problem for modeling the downgoing slab. The lithosphere and the mantle each exhibit different behavior to long term stresses: the former acts nearly elastically (Walcott, 1970), while the asthenosphere and probably the upper mantle approximate a fluid with a low yielding stress (Crittenden, 1967). Consequently, a model of the descending lithosphere must incorporate the effects of these regions and the dynamic interaction of the system composed of the descending slab and asthenosphere. Limiting our solution to the descending lithosphere and assuming an elastic model reduces this to a tractable problem. We review the assumptions required for this formulation in the following sections.

Section 2.2. Properties of Lithosphere and Descending Slab

Numerous data indicates that the lithosphere behaves elastically. It possesses a definite flexure rigidity (Walcott, 1970) and displays phenomena such as bending at island arcs (Hanks, 1970) which are attributed to its elasticity. Utsu (1967) and others (Davies and McKenzie, 1969) have observed strong seismic-velocity
anomalies at island arcs corresponding to a high velocity channel in the mantle. This appears to be a direct consequence of the colder lithosphere descending into the mantle (Minear and Toksoz, 1970; Jacobs, 1970). Furthermore, the descending lithosphere attenuates seismic waves much less than the surrounding mantle (Utsu, 1967; Molnar and Oliver, 1969; Kanamori, 1970), again an indication of its elasticity. These and the observation of earthquakes support the assertion of an elastic lithosphere and its simulation as an elastic solid.

Yet it is not justified to assume constant elastic moduli throughout the slab, for it is not at a constant temperature. The thermal gradient separating the core of the slab and the mantle must correspond to a change in the elastic moduli. Discussion of the temperature dependence will be postponed until section 2.4 in order to first assess the behavior of the mantle.

Section 2.3. Simulation of Mantle Properties

To treat the effects of the surrounding mantle with an elastic model, the mantle must be modeled as a 'soft' elastic solid capable of partially supporting the down-going slab. The low yielding strength and the viscosity of the asthenosphere and the upper mantle have been deduced primarily from uplift data after glacial loading.
A low-viscosity channel of $10^{20+1}$ poise is generally agreed upon, while viscosities of $10^{23}$ poise are indicated for depths of 600 to 1200 km. There exist large differences between investigators and possible problems of nonlinearity, so these should be considered as provisional. Both experimental and theoretical results support the low creep strength of the upper mantle (Carter and Ave'Lallemant, 1970; Weertman, 1970; Goetz, 1971). The low strength may even extend to include the lower mantle depending upon the nature of the non-hydrostatic bulge (Wang, 1966; Goldreich and Toomre, 1969; Cathles, 1970). Seismic attenuation (Anderson, et al., 1967; Kanamori, 1968), electrical conductivity (Everett and Hyndman, 1968), and geothermal and melting information (Clark and Ringwood, 1964) substantiate the marked differences between the lithosphere and the asthenosphere which has been attributed to partial melting (Anderson and Sammis, 1970). The upper mantle must then be approximated as a low strength zone whose effect upon the lithosphere arises from both dynamic stresses and static support. The simulation must incorporate these two effects consistent with the geophysical observations.

Elastic constants are selected appropriate to the degree of support by the surrounding mantle. A rough guide can be had from the effective viscosities deduced
by McConnell (1968) and others for the upper mantle. Suppose one observes at an instant in time the stress distribution within the mantle and assumes that the stresses all originate dynamically from viscous drag. As the slab is moving at a constant velocity, a very rough assumption of constant strain rate, \( \dot{\varepsilon} \), can be used with the effective viscosity \( \eta \) to define the resulting stress:

\[
\sigma = \eta \cdot \dot{\varepsilon}
\]

This represents the supporting stress imposed upon the descending lithosphere within the limitations of a linear theory. The elastic moduli of the mantle region can then be adjusted to fit these approximate stresses to yield the corresponding support of the downgoing slab.

A similar and more rigorous justification relies upon modeling the real earth as a quasi-static, linear, viscoelastic problem. Although this ignores nonlinearity of the real earth, the model can incorporate both the short term elastic and the long term fluid behavior of the mantle. Using viscoelastic theory, the differential operators corresponding to the time dependent behavior of the medium replace the elastic moduli of the constitutive equations. Ignoring inertial terms for mantle processes in the equation of motion, a simpler elastic problem having specific body forces, boundary tractions, and elastic moduli is solved yielding the corresponding
viscoelastic solution. This is known as the elastic-viscoelastic correspondence principle or elastic-viscoelastic analogy (Christensen, 1971). These elastic constants incorporate both the static and time-dependent elements of the viscoelastic medium. The only parameter of consequence is then the supporting stress of the mantle, thereby partially avoiding non-linear flow problems and temperature dependent parameters. Thus, selection of effective elastic moduli yielding the desired support of the slab provides an approximate model for mantle behavior with resistance.

Figures 4 to 10 display Young's modulus for the computed models as a function of depth. These models are selected to illustrate a range of mantle conditions: from constant elastic moduli throughout the mantle and slab to a very low-strength mantle relative to the slab. Intermediate cases having a 'soft' asthenosphere and 'hard' mantle offer closer approximations to the viscous models deduced by McConnell (1968). The relative importance of these factors can be judged from the resulting models.

Yet one problem remains unsolved: the mantle's dynamic resistance and static support are indistinguishable in these models. Suppose a viscoelastic, Maxwellian fluid is assumed for the mantle. Replacing Young's modulus by a corresponding differential operator (Flugge, 1967), and applying a stress $\sigma$ for time $\Delta t$, \"
an effective modulus $E$ results:

$$E = \frac{E_0}{(1 + \Delta t/t_o)}$$

for $t_o = \eta/E_0$  \hspace{1cm} (2.3.1)

and

$$\sigma = E\Delta \varepsilon$$

and where $\Delta \varepsilon$ is the total deformation, $t_o$ is the relaxation time, $\eta$ is the viscosity of the Maxwellian fluid, and $E_0$ is the asymptotic Young's modulus for high frequencies. $\Delta \varepsilon$ and $\Delta t$ define a strain rate, so decreasing $\Delta t$ is equivalent to increasing the strain rate. For a given strain rate, $\Delta \varepsilon/\Delta t$, any value of $E_0$ and $\eta$ is possible so long as equation 2.3.1 holds true. A similar effect can occur between the slab and mantle, and unlike the Maxwellian model, it is nonlinear due to temperature and stress dependence. Still the relationship that the mantle's resistance increases with strain rate holds so long as the energy is minimized.

Section 2.4. Temperature Dependence of Elastic Moduli

One crucial factor remains for realistic simulation of the slab: the elastic constants are temperature dependent. Chung (1971) and others have estimated the thermal effects upon elastic moduli for seismic frequencies and found it to be $\frac{1}{E} \frac{\partial E}{\partial T} = -1.5 \times 10^{-4} \degree C$ for olivine.
But for long term stresses this dependence is not realistic; a more appropriate relation uses the rate of diffusion creep at high temperatures. For this form of creep an exponential dependence of strain rate upon temperature is implied by theoretical and experimental evidence (Weertman, 1970; Carter and Ave'Lallemant, 1970):

$$\dot{\varepsilon} = f(\sigma) \exp(-Q/RT) \quad (2.4.1)$$

where $Q$ is the activation energy for diffusion, $R$ is the gas constant, $T$ is the temperature measured in degrees absolute, and $f(\sigma)$ is a function that depends primarily on the stress but that may contain other variables. Substituting this in equation 2.3.1 for constant load time $\Delta t$ and stress $\sigma$, it follows that

$$E = \frac{\sigma}{\Delta t} \cdot f(\sigma) \exp(Q/RT) = E_0 \exp(Q/RT) \quad (2.4.2)$$

or to first order the dependence is

$$\frac{\Delta E}{E_0} = -\frac{Q}{RT^2} \cdot \Delta T \quad (2.4.3)$$

Using approximate values for the parameters, a very rough upper bound results for the descending slab (Weertman, 1970):
\[ \frac{1}{E} \frac{\Delta E}{\Delta T} \approx 2.2 \times 10^{-2} \text{C}^{-1} \quad (2.4.4) \]

when \( Q = 100 \text{ kcal.}, \ T = 1500 \text{ C}, \) and \( E \) is the value of
Young's modulus. Figures 5 to 7 give models incorporating
a temperature dependence for the descending slab lying
within the bounds of 2.4.4 and the high-frequency dependence.
These models provide a means of illustrating the marked
effect that the slab's material strength has upon the
resulting stress distribution; one expects the real earth
to lie within this range.
Chapter 3
COMPUTATION OF STRESSES

Simplifying the rheologies of the slab and mantle to effective elastic moduli allows solving an elasto-static problem instead of a difficult viscoelastic problem. Formulation as a viscous flow problem would not incorporate the desired elastic behavior of the slab. With this in mind the representation of the descending lithosphere reduces to a plane strain, elastostatic problem adopting moduli corresponding to the mantle's' support and using body forces based upon the slab's density anomalies. Assuming similar composition, the body forces originate from buoyancy due to the temperature difference between the slab and the adjacent mantle. A region within the colder slab is denser relative to the surrounding mantle; thus, it has a tendency to sink, thereby inducing stresses within the slab and mantle. These lateral density variations are computed using the temperature regime obtained by Toksoz, et al (1971) for a slab with a spreading rate of 8 cm/yr. The value of volume expansion, $\alpha$, is derived from an average of Birch and Verhoogen's (Toksoz, et al, 1971):

$$\alpha = \exp(3.58 - 0.0072z) \times 10^{-6} C^{-1}$$

where $z$ is depth in km. (3.1)
For average density \( \rho_0 \) of the slab, the vertical body force \( F_y \) due to negative bouyancy becomes

\[
F_y = g\rho_0\alpha\Delta T
\]  

(3.2)

when the lateral density variation \( \Delta \rho \) is

\[
\Delta \rho = \rho_0\alpha\Delta T
\]  

(3.3)

where \( g \) is the acceleration of gravity, and \( \Delta T \) is the horizontal temperature contrast between the mantle and the slab. Two separate values of the heat conduction have been used for the temperature calculations illustrated in Figures 1 and 2. One set of models uses Mac-Donald's formulation of the radiative term (Toksoz, et al, 1971), while the second set has a reduced radiative term derived from Schatz's (1971) experimental investigations for olivine. In the latter case the radiative conductivity is given by

\[
K_R = 0 \text{ for } T \leq 500^\circ K, \text{ and } \\
K_R = 5.5 \times 10^{-6} (T-500) \text{ for } T>500^\circ K
\]  

(3.4)

In addition, the effect of the phase change is included in the body force as a separate problem. The depth and density changes of the olivine-spinel transition are estimated using Ringwood and Major's data (1970), and
the elevation of the boundary is derived from the thermal regime in accordance with Toksoz's calculations. With the inclusion of these parameters, the problem reduces to an elliptic boundary value problem with variable coefficients.

A finite-difference scheme with a 20 km grid spacing has been used to solve this problem for an 840 by 680 km region enclosing the slab and adjacent mantle. The digital computer program has been developed for online solution of generalized elliptic boundary value problems by C. Tillman (1971) using a nine-point, integral formulation of the difference equations and successive point over-relaxation to obtain a solution. Utilization of this program has allowed solution of the equilibrium and constitutive equation for an inhomogeneous, isotropic body with infinitesimal deformation (Nowinski and Turski, 1953):

\[
\begin{align*}
\sigma_{\alpha\beta} &= -F_\alpha(x_1, x_2) \\
\sigma_{\alpha\beta} &= \frac{\nu E(x_1, x_2)}{(1 + \nu)(1 - 2\nu)} \delta_{\alpha\beta} \Theta + \frac{E(x_1, x_2)}{2(1 + \nu)} (u_{\alpha, \beta} + u_{\alpha, \beta}) \\
\Theta &= u_{\alpha, \alpha} \\
u &= \text{Poisson ratio} \quad E(x_1, x_2) = \text{Young's modulus}
\end{align*}
\]

Reducing these to the divergence form in terms of displacements, one has a set of coupled equations:
\[ \frac{\partial}{\partial x} \left[ \frac{(1 - \nu)E(x,y)}{(1 + \nu)(1 - 2\nu)} u_{x,x} \right] + \frac{\nu E(x,y)}{(1 + \nu)(1 - 2\nu)} u_{y,y} + \frac{\partial}{\partial y} \left[ \frac{E(x,y)}{2(1 + \nu)} (u_{x,y} + u_{y,x}) \right] = -F_x = 0 \]

\[ \frac{\partial}{\partial x} \left[ \frac{E(x,y)}{2(1 + \nu)} (u_{x,y} + u_{y,x}) \right] + \frac{\partial}{\partial y} \left[ \frac{E(x,y)}{(1 + \nu)(1 - 2\nu)} u_{x,x} \right] + \frac{(1 - \nu)E(x,y)}{(1 + \nu)(1 - 2\nu)} u_{y,y} = -F_y (x,y) \]

(3.6)

where \( F_a \) = a component of body force,

\[ \frac{\partial u_i}{\partial x_j} = u_{i,j} \text{, or } \frac{\partial u_x}{\partial y} = u_{x,y} \text{, etc.} \]

The spatial variations of body forces and elastic parameters are introduced and a solution follows from the difference scheme with the appropriate boundary conditions.

If the boundaries surrounding the slice of mantle are far enough from the slab, their effect upon the descending lithosphere will be minimal. This corresponds to St. Vinent's principle in elastostatics. The two extreme cases are a rigid boundary (no displacements) and a free surface (no tractions). For the side boundaries, the computations indicate minimal effect of the boundaries upon the stress distribution within the descending slab. Our previous arguments for a relatively soft mantle suggest small tractions at the side boundaries; hence, these have been selected for our
computations. The top face, corresponding to the earth's surface, is also specified as free, whereas the bottom face is made rigid. This lower boundary is necessary to satisfy the requirements for a static problem: the sum of all couples and forces must be zero for the region. Again the boundary does not seriously effect the stresses within the slab. In addition, some models have the edges of the lithosphere rigid to simulate the junction of the descending slab and the crustal plate. Yet due to the limitations of these boundary conditions, the lack of information for the lithosphere as it bends into the mantle, and the characteristics of the plate boundaries, these models cannot hope to indicate the stresses for depths less than about 100 km, and that is not their intent. Rather, the intermediate and deep regions of the downgoing slab are within the realistic domain of the models.

Stability and convergence of the solution is ensured by both test problems and on-line inspection of convergence. The program has successfully solved a variety of problems including potential flow, bending of plates, and others (Tillman, 1971). This series has confirmed the utility and stability of the difference scheme. In addition, inhomogeneous, plane strain elastic solutions confirm the stability for problems such as the descending slab (appendix). The 20 km grid spacing used for the slab is adequate for the broad
features encountered in the thermal regime and for the desired resolution of the stress distribution. Finally, on-line monitoring and selection of the relaxation parameters provides for optimum convergence and continual inspection of stability (Mitchell, 1969).
Chapter 4
RESULTS OF SIMULATION

Section 4.1. Introduction

A set of eight models simulating different rheologies and conditions are summarized in Table I together with the important parameters and results. Figures 3 through 10 display these models including the mantle and slab's elastic moduli, each figure depicting the variation of a particular parameter. The differences between the two thermal regimes in Figure 1 and 2 are not the controlling factor in these models; the reduced radiative conductivity increases the gravitational body force and density gradient, yet this is a minute effect compared to the changes in mantle support and slab rheology. The mantle and slab properties control the behavior of the stress pattern: the maximum shear stress concentrates within the colder, harder core for a slab with temperature dependent elastic parameters. Decreasing the mantle's total and distributed support alters the principle stress from compression downgoing to tension along the trend of the slab. These relationships will be clarified in the following discussion of the individual models.

Section 4.2. Comparison of Models

A simulation is first necessary for a control case,
specifically, a model having identical elastic moduli within the mantle and slab. Model 1 in Figure 3 represents such a mode. For this case the maximum shear stress occurs at the lower edge of the slab and within the adjacent mantle, whose stresses have not been contoured. The orientation and the maximum stress reflect the distribution of body forces; at the lower edge and within the adjacent mantle, the compressional axis is vertical as one expects from simple statics. The distribution of body forces also introduces downdip tension for shallow depths. Given this example, the succeeding models illustrate the marked effects produced by different effective rheologies within the slab and mantle.

Models 2 and 3 in Figure 4 represent slabs with constant properties and different surrounding mantle conditions. The slab can be thought of as a bending plate with different degrees of support along its length. In model 2 the mantle differs little from the slab, and only a small reduction in strength occurs in the asthenosphere. The slab is relatively immobile in the deep mantle, whereas the asthenosphere allows 'sagging' analogous to a plate held at its edges: compression along the top face and tension along the bottom. If the asthenosphere is 'softer' as in model 3, the slab almost hangs as though suspended by the lithosphere and only partially supported at its base. Compression then
occurs at the bottom face of the slab and tension along its upper surface. Decreasing the mantle's support also increases the stress guided through the slab. Yet the shear stresses are concentrated at the edges of the slab as one expects for a bending plate but not necessarily for the descending lithosphere.

An alternate set of models is desired which concentrates the shear stress towards the slab's center; models 4 and 5 in Figure 5 show such a transition. The slab's strength is strongly dependent upon temperature according to 2.4.2 with a pressure term included:

\[ E = \exp\left(\frac{Q + pV_{ac}}{RT}\right), \quad \text{or for } V_{ac} = 0, \]

\[ E_{\text{slab}} = 5.0 \times 10^{-10} \exp(-0.36 \times 10^{-2} \Delta T) \]

\[ + E_{\text{mantle}} \]

where \( p \) is the pressure in kilobars, and \( V_{ac} \) is the activation volume for diffusion creep (Weertman, 1970). The stresses are now strongly focused within the colder regions of the slab as expected from diffusion creep. In addition, two extreme cases of mantle support are illustrated: one having the tensional or minimal principle axis following the trend of the slab along its full length, the other with the compressional axis paralleling the slab. The downgoing slab in model 5 hangs from the lithosphere. With increasing strength and support by the mantle, the slab can be placed under compression as in model 4. A balance then exists between the
mantle's support, the slab's strength, and the distribution of density anomalies within the slab. The next set of models clarifies this balance between the mantle's and the slab's rheology.

Suppose the strength of the slab is only weakly dependent upon temperature as given by:

\[ E_{\text{slab}} = E_{\text{mantle}} + 5.0 \times 10^{10} [1 - \frac{(100 + 0.895p)AT}{2(T_{\text{mantle}})^2 \times 10^{-3}}] \]

Model 6 in Figure 6 simulates such a condition for a mantle with a low-strength channel similar to model 4. Its stress distribution is an intermediate case between the downdip compression of model 4 and the bending stresses that are characteristic of model 2; however, the temperature dependence of the elastic moduli is insufficient to focus the stress towards the center of the slab. As in model 4 the narrow asthenosphere prevents tension along the slab's full length. Model 7 is similar to model 6 but has no temperature dependence for the descending lithosphere. As expected, its maximum shear stress concentrates at the edges of the slab; however, the soft asthenosphere introduces tensional features at intermediate depths. The models suggest that a critical value for the temperature dependence may be necessary in order to focus the stress within the slab.

On the other hand, increasing the contrast of the
elastic moduli within the slab strongly focuses the stress and can induce tension at intermediate depths. Figure 7 illustrates this case with model 8 for a temperature dependence given by

\[ \bar{E}_{\text{slab}} = E_{\text{mantle}} \exp \left[ -(1.2 \times 10^{-2} + 5.5 \times 10^{-5} p) \Delta T \right] \]  

(4.2.3)

A very soft asthenosphere combined with a hard central core for the slab produce high shear stresses with the tensional axis following the slab at intermediate depths and with downdip compression for greater depths.

Three parameters are resolved that best described these categories. The first measures the ratio of mantle strength at two depths, 140 and 440 km, thereby resolving the differential support of the asthenosphere and the deeper mantle. Yet this is incomplete without a measure of the contrast between the mantle and the slab, as given by the ratio of their elastic constants near the slab's base. The final parameter measures the temperature dependence of the slab's elastic moduli. It is the ratio of moduli at its edge compared to its center at 160 km depth. Figure 8 gives a three dimensional perspective of these models in terms of the parameters. The categories are representative of the model space spanned by this series of computations. Thus, regions are blocked off depending upon the models locations and characteristics. The boundaries are merely
to illustrate the regional dependence and should only be interpreted as guidelines.

Section 4.3. Phase Changes and Convection

Figure 9 considers the elevation effect of the olivine-spinel phase change within the slab (Schubert and Turcotte, 1971). Using Ringwood and Major's (1970) estimation of the slope for the phase transformation boundary, 30 bar/°C, the density anomalies within the slab are computed using Figure 1 when the transition occurs over 50 km (Toksöz, et al, 1971). The problem then lends itself to superposition: calculating the solution independently and summing the stresses for the separate models. The transition induces local stress concentrations directly above and below the perturbed depth. The slab tends to be under tension at 350 km and compression below this depth; consequently, if the slab is originally under compression at 350 km, the transition reduces the maximum shear stress above that region while increasing it below the phase change.

In the same manner the convection problem in Figure 10 is treated as a slab with boundary tractions along its faces. The right-hand problem does not include the mantle's support; instead, the slab acts as a free plate with applied tractions. The figure illustrates an example using 50 bars resistance along the slab's upper
surface and no tractions along its bottom face. This roughly simulates convection along the bottom face and viscous drag on the top. As expected for a plate, bending effects are superimposed upon the dominate horizontal principle stress. The left-hand model with mantle support incorporates a shear couple along the upper, mantle-slab boundary to simulate drag. The stress orientations are now more complicated due to the mantle support. Yet the models and intuition reveal that such convection generally increases compression along the trend of the slab. Gravitational sinking also induces similar stress distributions; accordingly, the two cannot be distinguished a priori. One must rely on geophysical and geological inferences; these are reviewed in the next chapter in view of the preceding models.
Chapter 5
COMPARISON TO ISLAND ARCS

Section 5.1. Introduction

Isacks and Molnar (1971) have summarized the observed stress patterns for island-arc regions and their relationship to the regional tectonics and gravitational sinking of the descending lithosphere. Figure 11 gives their interpretation of the principle stresses derived from focal-mechanism solutions. Three crucial regions are apparent in the stress pattern and the numerical models: at shallow depths the tensional principal stress, \( T \), parallels the slab's orientation; intermediate orientations, \( B \), or down-dip extension occurs at intermediate depths; and down-dip compression, \( P \), dominates for greater depths. The following sections will show the correspondence between the computed models and the regional stress distribution, and in addition, will corroborate the sinking lithosphere hypothesis. Regions have been selected as representative of island-arc systems and for adequate knowledge of the arc, a prerequisite for discriminating the crucial elements of the mechanism. Unless referenced to the contrary, the seismicity and the focal-mechanism solutions are contained in Isacks and Molnar (1971).
Section 5.2. **Honshu**

The dipping slab beneath North Honshu and the Sea of Japan closely resembles model 4: a 30 degree dipping slab under compression for depths greater than 80 km. The seismic activity is continuous and does not extend much beyond 600 km in depth. Figure 12 summarizes the cumulative results for Honshu and Kurile island arc. Concentrating upon the relatively uncontorted central region, one notices the preferred down-dip orientation of the compressional axis and the interchanging of the B and T axes for three solutions. Closer inspection reveals a slight rotation towards down-dip tension of the P axis with decreasing depth, yet nothing that can be termed definitive. Kanamori's (1971) solution of the Saniki earthquake of 1933 adds further evidence to the gradual rotation of the principle axis; it has down-dip extension and is located near the trench axis. Only for shallow depths, then, is the slab under tension; the bulk remains under down-dip compression as attested by the focal-mechanism solutions.

Comparing these orientations to those of model 4 in Figure 5, the principle stress is also down-dip compression for intermediate and greater depths. The simulation implies that a narrow, low-strength asthenosphere can exist if sufficient support is provided by the metasphere. In addition, the model depicts a gradual rotation of the stress orientations for inter-
mediate depths. The nature of these rotations cannot be determined on the basis of a two-dimensional simulation of the real earth; stresses and contortions along the island arc influence the direction of the principle stresses within the slab, such as interchanging the B and T axes. The two-dimensional model is unable to distinguish these effects: it merely indicates rotation of the P and T axes within the slab. Moreover, Kanamori's (1971) tensional solution indicates that the simulation is not fully satisfactory; the model is still compressional for shallow depths. This effect has its origin from either of two related factors: the slab's core is insufficiently 'hard', or its corrolary, the mantle is supporting an excessive fraction of the slab's body force. The major factor, however, is the rigid boundary at the edge of the lithosphere. It, in effect, simulates the lithosphere pushing the descending slab. Consequently, a simulation having parameters intermediate to models 4 and 8 is necessary for extension at shallow depths.

Section 5.3. Tonga

The Tonga arc again represents down-dip compression for all depths except shallow. Figure 13 summarizes the seismicity and focal-mechanism solutions and indicates that the slab can be represented by an inclined plane dipping at approximately 45 degrees. Avoiding the contorted edges, the seismicity is
continuous for depths less than 680 km. Again the focal solutions exhibit downdip compression for all depths except possibly the shallow zone, even for regions near pronounced contortions. These solutions indicate that bending effects within the slab are minimal; the pattern is dominated by stress transmitted through the slab rather than local bending, unless the stresses along the arc are severe. Anomalous solutions include 39 and 49 which do not lend themselves to a simple explanation.

In addition, solution 50 suggests rotation of the principle stresses for shallow depths, and an extensional solution is reported at 80 km depth (Isacks and Molnar, 1969).

Distinguishing the mechanisms as thrusting between lithospheric plates or as tension within the slab is difficult for shallow solutions. Only when the solution is near the trench can we safely claim the latter.

The marked similarity between Tonga and Honshu stress distributions dictate a corresponding interpretation of the models; both involve support by the mesosphere and a soft asthenosphere as suggested by Isacks and Molnar (1969). Utsu (1967) and Kanamori (1968) have given compelling evidence for this interpretation of the asthenosphere beneath Japan. The zone represents a region of high attenuation and strong travel time delays. Yet the form that the mantle's support takes, dynamic or static, is unclear. Both convergence rates are approximately 9 cm per year (Le Pichon,
1968) implying greater density anomalies and depth before equilibrium (Toksoz and Minear, 1970), higher strength within the colder slab, alterations of phase boundaries, and perhaps greater resistance or drag by the mantle. Their interrelationship clarifies itself when additional regions are examined.

5.4. **Kermadec**

The Kermadec arc, lying just south of Tonga, represents a marked change in the stress distribution. The inclined zone parallels a slab dipping 60 degrees west and extends to 500 or 550 km depth (Sykes, 1966). Limited focal plane solutions show down-dip extension at 230 km depth and compression for 350 km. The region corresponds, then, to model 8 in Figure 7, if the mantle support is further reduced to allow extension at 230 km depth. Thus in Figure 8 the regions are evolving towards decreasing mantle support. Moreover, the convergence rate at the Kermadec arc is markedly less than the preceding two cases, 5.4 cm/yr as opposed to 9 cm/yr. (LePichon, 1968, 1970), suggesting a causal relationship.

5.5. **South America**

Peru and Northern Chilean regions carry this deduction one step further: a gap in seismicity exists between intermediate depth, extensional earthquake mechanisms and
compressional mechanisms at 600 km. Briefly, the shallow or intermediate zone is characterized by down-dip tension for a shallow dipping slab; however, no conclusive seismic activity occurs between 200 or 300 km and 500 km for Peru and Chile, respectively. Between 500 and 600 km the seismic activity dramatically increases, and the mechanisms indicate down-dip compression. Whether the slab is continuous through the seismicity gap remains a conjecture. In Chile high-frequency shear waves pass through the gap, while the results are inconclusive for Peru and western Brazil (Molnar and Oliver, 1969; Sacks, 1969). Either the zone of attenuation in the mantle occurs above the gap, or the descending lithosphere is continuous to depths of 650 km.

Assuming the slab is continuous, the seismicity and focal mechanisms follow from the dynamics of gravitational sinking. The North Chilian slab extends 200 km deeper than Kermadec, but the convergence rate is similar, 5.2 cm/yr (Le Pichon, 1968). The additional length of slab has two opposing effects: the total body force is increased by the added segment and by the contribution of any deep phase changes within the slab. Yet deeper penetration of the slab and variations in the mantle's resistance or support opposes the increased body force.
An extrapolation of model8, which has tension at intermediate depths, could produce such a gap given the proper combination of mantle and slab support, body forces, and perhaps elevation of phase-transformation boundaries. The additional length of slab increases the total body force and offsets any mantle support gained by greater penetration. A balance must exist between the body force, mantle resistance, and depth of penetration for the occurrence of this pattern. For Kermadec and Chilean slabs, the resistance or support at each depth is held constant by similar convergence rates; consequently, the effects caused by the additional length of slab are present given that their preceding history is inconsequential. The latter assumption is questionable. Otherwise, if the slab is continuous and if the convergence rate has been reasonably stable over the past 10 million years, it is an adequate proposition.

Other evidence from South America suggests that the slab behaves as a stress guide for both shallow and deep earthquakes. Carr, et al (1971) reported evidence of lithospheric bending for shallow depths: tensional earthquakes near the surface and down-dip compression along the bottom face. This cannot be attributed to thrusting of one lithospheric plate under another. The Kurile arc also shows compression for shallow depths at the lower face. For greater depths Fukao (1971) has obtained the solution for a deep-focus earthquake (577 km) in western Brazil.
including the fault plane parameters. It verifies shear faulting for deep earthquakes and indicates that the P axis is nearly vertical, corresponding to the trend of the slab. The stress drop, 300 to 460 bars, implies that the compressional principle stress is greater than 600 to 920 bars. Moreover, the faulted area, 510 to 580 km², the average dislocation, 265 to 355 cm, and the location of subsequent earthquakes suggests large-scale faulting over nearly the entire thickness of the descending lithosphere. These observations corroborate gravitational sinking as the mechanism inducing earthquakes within the descending lithosphere.

Section 5.6. Middle America and the Aleutians

The Middle American arc corresponds to a short slab dipping about 40 to 60 degrees at depths greater than 85 km. Focal-mechanism solutions are consistent with down-dip extension along its 200 to 300 km length. Thus, the arc closely resembles model 5 with minimal support from the surrounding mantle.

The Aleutian arc also conforms to a shallow-slab representation with a dip of approximately 60 degrees (Davies and McKenzie, 1969) and with seismicity generally less than 200 to 300 km deep. Unlike the Middle American arc, compressional solutions are evident for intermediate depths, while down-dip tension occurs for shallow depths adjacent to the trench (Stauder, 1968a,b). The arc could
illustrate a truncated version of model 8, similar perhaps to model 7; however, the factors that determine the individuality of the Aleutians and Middle American arcs remain obscure. Conceivably, the distinctive stress distributions result from the increased convergence rate of the Aleutian arc, 5.2 cm/yr (Le Pichon, 1968) compared to roughly 3.5 cm/yr for Middle America (Molnar and Sykes, 1969), thereby augmenting the thermal body forces and the mantle resistance and placing the slab under partial compression. These convergence rates are, however, questionable.

Section 5.7. Kurile-Kamchatka

The Kurile-Kamchatka arc, dipping at 45 degrees, poses a crucial validation of these mechanisms. Contrary to the other island arcs, the seismicity suggests the maximum depth of the zone is probably less than 600 km and perhaps nearer 500 km. Yet the convergence rate is approximately 7.5 cm/yr (Le Pichon, 1968). Based upon the proposed mechanism, one would expect a distribution similar to Kermadec with its 500 km slab but with additional body forces and mantle resistance generated by the faster convergence rate. Indeed, the effect is apparent in the stress distribution: a minimum in activity, and not a aseismic gap, is reported between about 200 and 400 km. The shallow events above the gap exhibit down-dip extension, whereas for events located beneath the gap the stress orientation is down-dip compression. Moreover,
rotation of the principle axis occurs for events near the periphery of each region just as in model 8. These characteristics place the arc as an intermediate case between Kermadec and Tonga, and substantiate the idea that the descent velocity of the lithosphere plays a major role in the dynamics and manifests itself in the seismicity and stress orientations by means of the body force and mantle resistance.
Chapter 6
DISCUSSION AND IMPLICATIONS

Section 6.1. Model of Descending Lithosphere

A simple analogue explains the regional stress patterns observed for these island arcs. If the earthquake characteristics, the limiting depth of down-dip tension and compression, are plotted in terms of the convergence rate, a regular pattern appears between the seismicity, focal mechanisms, and the convergence rate. Figure 14 illustrates this pattern in addition to a theoretical model. Beginning with New Zealand and slow convergence rates, a large seismicity gap occurs between down-dip tensional solutions and deep compressional solutions. For faster convergence rates with a deep slab, the zone narrows and then disappears for the Kurile island arc, yet a minimum in seismicity persists between 200 and 400 km in addition to fluctuations in the stress orientations. Tonga also displays a minimum within this range. The second figure indicates an analogous pattern for steeply-dipping slabs. Indeed, the relationship can be deduced from gravitational sinking of the descending lithosphere if a continuous slab is presupposed.

A simulation of gravitational sinking elucidates the dependence of the stress distribution upon the body force and the mantle resistance via the convergence rate.
upon the convergence rate of the island arc, and assuming the depth dependence for the support and phase changes, a numerical integration of the forces along the slab yields the stress within the slab as a function of convergence rate, length of slab, and dip. The stresses are only those along the direction of the slab and indicate whether tension or compression prevail. Figure 14 portrays a numerical solution given the following assumptions: First, the body force due to thermal contraction is constant along the slab and varies as the square root of the rate. It has been normalized to yield 140 dyne/cm$^3$ at 8 cm/yr; the precise value does not affect the solution. The rate function roughly corresponds to the models of Minear and Toksoz (1970) and equals the rate of conduction for an infinite body with initial temperature distribution (Carslaw and Jaeger, 1958). Alterations of the phase boundaries are also given this rate dependence when normalized to 8 cm/yr boundaries calculated by Toksoz, et al (1971). Viscous drag is now assumed for the support or resistance; thus, resistance is linearly dependent upon the rate. In addition, the resistance at its end is 2 to 5 times greater than at the faces. Increasing tension with decreasing convergence rate as in Figure 14 suggests that resistance must decrease faster than the total body force if the slab is continuous through the aseismic gap. The dependence upon depth is then selected to yield the desired distribution. Finally,
the resistance is normalized at 10 cm/yr to obtain compression throughout the slab. To compute steeply dipping slabs, only the component of body force along the dip is altered, while other factors such as greater temperature contrasts remain constant. Restrictions upon the length of the downgoing slab and the convergence rate have not been imposed and are not necessary in this analogue (Luyendyk, 1970). Table II summarizes the parameters and solution for one successful model.

Certain observations are in order regarding the characteristics of successful models: If stresses greater than 1 kilobar are deemed necessary (Wyss, 1970; Fukao, 1971) and if compositional differences are insignificant, the additional body force provided by elevated phase boundaries is a prerequisite. Alone, body forces from thermal contraction are insufficient to cause the seismicity and the large apparent stresses observed, particularly at great depths in South America. Perhaps the deep seismicity maximum at 600 km reflects a change in the material properties of the slab as could occur in the post-spinel phase change. Only the depth of the phase change must decrease within the slab. A positive slope of dP/dT for the post-spinel phase transformation would imply both higher body forces and a change in physical properties at 600 km. Anderson (1967) has estimated a negative slope, yet it is not conclusive by
any means. The transformation may ultimately be responsible for the seismicity at 600 km.

The analogues also employ a low-strength asthenosphere extending to 220 km; however, little or no increase in resistance is demanded for greater depths. This is in accord with a recent solution of glacial uplift data (Cathles, 1970), but directly contradicts models of gravitational sinking relying upon increasing strength with depth (Isacks and Molnar, 1969, 1971).

Assuming a continuous slab for all seismic zones entails no presuppositions for the mechanism of separation. Instead, the aseismic gap stems from its low stress. Neither hypothesis subjects the deeper portions to stress; both must rely upon forces adjacent to and within the deep slab as the mechanism inducing deep earthquakes in Chile and similar locals.

Figure 15 further corroborates the model's relationship to seismicity. A distinct correlation appears between the regional seismicity and the stress levels within the hypothetical slab. Only the relative shapes of the curves are significant; the actual magnitude of stress within the slab depends upon the precise model. Underthrusting of lithospheric plates also contributes to shallow seismicity, and variations in the material properties of the slab may be important for all depths.

Finally, the models readily explain the pattern for steeply dipping slabs: the greater down-dip component of the
body force pulls the zone of tension further down the slab. Yet constant resistance along the length of the slab and localized body forces from phase transitions allow compression even for short slabs. This emphasizes the importance of these phase transformations upon the proposed slab dynamics.

Section 6.2. Speculations upon the Driving Mechanism

The interaction of the descending plate and the rigid lithosphere still remains a problem. Given the tentative hypothesis of subduction, what is the mode of coupling and the forces exerted by the lithosphere? Assorted evidence suggests a hypothesis. First, the models indicate that convection at island-arc regions may not be significant. Rather, resistance to the motion of the slab, gravitational sinking including elevation of phase (transformation) boundaries, and pushing from mid-ocean ridges are perhaps much more important. In particular, a recent focal plane solution by Mendiguren (1971) for an earthquake located in the Nazca plate off Peru indicates that the principle axis of compression is in the direction of the plate's motion. When combined with the overall lack of seismicity under abyssal plains, it suggests that the rigid plate is under uniform compression and that the driving mechanism inducing the compression is in the vicinity of the ridge. Localized convective effects near the ridges, or sliding of the lithosphere off the ridges
(Jacoby, 1970) could account for the mechanism. The convection need not necessarily extend to the downgoing slab; the two phenomena can be quite independent. Instead, the mantle convection may only be effective as a driving mechanism in the vicinity of the ridges, while a very extended and gradual return flow occurs under the oceanic lithosphere. This pattern is consistent with our expectations; it minimizes the energy according to Helmholtz theorem (Batchelor, 1967; Elsasser, 1971). The ascending mantle is significantly warmer than the adjacent mantle, which implies a lower viscosity and higher velocity gradients than the colder return flow (Weertman, 1970). Energy minimization then constrains the higher viscosity and colder return flow to a broad pattern with low velocity gradients. This flow could remain largely decoupled from the lithosphere by the upper asthenosphere.

The coupling is expected, instead, on the flanks of the ridge. Both high velocity gradients and solidification of upwelling mantle occur there (Torrance and Turcotte, 1971; Forsyth and Press, 1971). The coupling could allow tensional earthquakes near the ridge axis and compressional solutions for the bulk of the plate. The mechanism is also consistent with heat flow (Sclater and Francheteau, 1970) and regional seismic attenuation (Ward and Toksoz, 1971). But compressional solutions in the direction of motion are expected near the island-arcs if the stress is transmitted through the crustal plate.
Apparently the lithosphere and the subducting slab are partially decoupled; shallow, downdip tensional earthquakes are observed near the trenches (Stauder, 1968a,b; Isacks and Molnar, 1971). Kanamori's (1971) solution of the 1933 Sanriku earthquake verifies the shallow extensional solutions for major tectonic earthquakes. The faulting involves the slab's total thickness, leading him to surmise that the slab and lithosphere are decoupled near their junction. Other evidence (Lister, 1971) suggest that at some trenches the oceanic lithosphere is failing in shear rather than bending elastically. This could act as a decoupling mechanism between the plate and the downgoing slab. Perhaps the asthenosphere actually resists the plate's motion and prevents any compression near the island arcs. A balance between tension induced by sinking and compression from the ridge could then exist within the rigid lithosphere. This, of course, would require the effect of the return flow from the ridge to be inconsequential. While only speculations, the reasoning suggests possible avenues of exploration.
Section 7.1. **Summary**

Simulation of the descending lithosphere at island arcs has provided an effective means to isolate the important factors that contribute to the subduction. The analogue consists of two elements: The first is the elastostatic problem which shows the characteristics of the stress distribution within the slab given the mantle and slab's simulated rheologies, the thermal regime and its assumptions, and similar composition for the lithosphere and mantle. Then, the regional applicability of gravitational sinking and the factors affecting it are clarified using a simple numerical integration of the equilibrium equation and comparing these to available information for select island arcs. This in turn suggests a viable hypothesis for the dynamics of the subducting lithosphere under these conditions. The deductions that follow are based upon the elastostatic solution with these presuppositions and provide the initial impetus for the hypothesis.

First, gravitational sinking induced by thermal density anomalies can explain the directions of the principle stress within the descending slab at island arcs without recourse to significant convective effects in the adjacent mantle. Rather, the rheologies of the
mantle and slab are the dominant factors influencing the stress pattern. Earthquake source mechanisms together with the models indicate that the temperature dependence of the slab's material properties is significant; bending effects that are characteristic of a plate are only observed for shallow depths. Instead, the stress is transmitted predominately down-dip within the slab. A low-strength asthenosphere or channel below the lithosphere is also consistent with the models and with the regional earthquake mechanisms.

Numerical integration for a simple model containing the relevant forces and their assumptions suggests a hypothesis for the dynamics of subduction. An essential element is the dependence upon the velocity of the converging lithospheric plates; it represents a major influence upon the dynamics of descent. The thermal body forces, boundaries of phase changes, and elastic constants are manifestations of the convergence rate. Yet a crucial dependence results between the mantle's support or resistance and the velocity in order to account for the regional distribution of earthquakes; the resistance increases with faster convergence rates. The viscous end-resistance may also be more important than hitherto acknowledged. Finally, increasing strength with depth in the mantle is not a prerequisite for simulating the regional stress distributions. A low-strength channel corresponding to the seismic low-velocity zone and a
stronger but constant strength mantle yields reasonable
agreement between the models and the regional stress
patterns derived from earthquakes.

In addition, the contribution of phase changes is
predicted as a crucial element of the hypothesis given
the large apparent stresses observed for intermediate and
depth earthquakes and the initial assumptions for the
models. The hypothesis is then consistent with a form of
shear faulting as the dominant source mechanism for these
earthquakes. The simple models suggest further that the
post-spinel phase change at 650 km could provide a
mechanism needed to induce deep earthquakes, given a
positive slope for dP/dT at the phase boundary. Otherwise,
the high stresses proposed for deep earthquakes are
probably inconsistent with the hypothesis.

Section 7.2. Recommendations

The hypothesis suggested within this paper relies
upon a variety of theoretical and experimental inferences
that require further research. Analysis of earthquake
focal-mechanism solutions may shed light upon the
characteristics of the aseismic gap, particularly whether
it is indeed continuous, and on the properties of the slab-
lithospheric junction. Experimental evidence on rock
mechanics is sorely needed to distinguish the source
mechanisms of deep earthquakes: shear-instability,
phase-change instability, or some unrecognized mechanism.
The reliability of fault plane parameters, such as the stress drop, must also be resolved for earthquakes. The hypothesis depends too upon the rheology of the slab and mantle; however, more experimental and observational data, and theoretical inferences may resolve the behavior. If the mantle and slab have significantly different compositions, the simulations are no longer valid. Finally, the hypothesis offers predictions which must satisfy conditions imposed by the mantle: these constraints include the mantle-convection problem, thermodynamics and heat flow, and the location and properties of phase changes. Further analysis and numerical models may resolve the validity of the hypothesis.
Appendix I

FORMULATION OF FINITE-DIFFERENCE EQUATIONS

The finite-difference approximation for equation (3.6) within the digital program (Tillman, 1971) uses the self-adjunct form of the elliptic equations together with an integral formulation for the difference equations (Varga, 1962). A complete development may be found in the references; however, we will review the formulation for the elastostatic problem.

The self-adjunct, elliptic boundary value problem has the form

\[ \frac{d}{dx} \left( A \frac{d u}{dx} + B \frac{d u}{dy} + Cu \right) + \frac{d}{dy} \left( D \frac{d u}{dx} + E \frac{d u}{dy} + Fu \right) + Gu = H \]  

(I.1)

where \( x \) and \( y \) are the independent variables, \( u=(u_1, u_2, \ldots, u_N) \) is the set of unknown functions to be determined, and \( A=(a_{kl}), \ B=(b_{kl}), \ldots, \ G=(g_{kl}) \) and \( H=(h_k), \ k,l=1,1,\ldots,N \), are specified matrices whose components may vary with \( x \) and \( y \). The integral formulation of equation (I.1) reduces to an area integral over any subregion (Varga, 1962; Tillman, 1971):
On application of Gauss's theorem, this becomes

\[
\oint \left[ \frac{d}{dx} \left( A \frac{du}{dx} + B \frac{du}{dy} + Cu \right) + \frac{d}{dy} \left( D \frac{du}{dx} + E \frac{du}{dy} + Fu \right) \right] dx
\]

\[
\oint G u dxdy = \oint H dxdy
\]

\[
\int \left[ - \left( A \frac{du}{dx} + B \frac{du}{dy} + Cu \right) dy + \left( D \frac{du}{dx} + E \frac{du}{dy} + Fu \right) dx \right]
\]

\[
\oint G u dxdy = \oint H dxdy
\]

(I.2)

(I.3)
where the boundary $\Sigma$ of subregion $R$ is traced in the clockwise direction.

For the elastostatic equations, the formulation reduces to a set of difference equations for a uniform orthogonal lattice with grid lines parallel to the $x,y$ axes and separated by a distance $h$. The interior node, $N_0$, is surrounded by 8 adjacent nodes, each numbered consecutively starting at the $x$ axis and working clockwise. In this case the set of interior difference equations at an interior node $N_0$ becomes:

\[
\begin{align*}
\sum_{k=1}^{8} & \left\{ -[A\delta \left( \frac{du}{dx} \right) + B\delta \left( \frac{du}{dy} \right)] S_k \Delta y_k \\
& + [D\delta \left( \frac{du}{dx} \right) + E\delta \left( \frac{du}{dy} \right)] S_k \Delta x_k \right\} \\
G_1 &= \sum_{k=1}^{8} H_{T_k} \Delta a_k
\end{align*}
\]

where $\delta(z)$ is a difference approximation to $z$. Thus, $\delta \left( \frac{du}{dx} \right)$ is a vector approximation to $\frac{du}{dx}$ in terms of the approximations to the dependent variables $u=(u_1, u_2)$ at node $N_0$ and its neighbors $N_1, N_2, \ldots, N_8$. The summation
is along the boundary $\Sigma$ of the subregion. This subregion is a square centered upon the interior node $N_0$ and with side $h$. The subregion may be further subdivided into 8 equal right triangles, $t_k$, all having a vertex in common at the interior node $N_0$. $s_1, \ldots s_8$ denotes the boundary of subregion starting at the $x$ axis and continuing clockwise, and $y_k, x_k$ is the length of the segment $s_k$ in the $x$ and $y$ directions. $S_k$ indicates that the matrix or difference approximation is evaluated at the midpoint of segment $s_k$, and $T_k$ requires evaluation at the centroid of the triangular subregion $t_k$. The evaluation of the governing system of matrices $A, B, \ldots, H$ and the approximation $\delta \frac{du}{dx}$, etc., are accomplished by a bilinear interpolation from the values at the surrounding nodes. Thus, evaluation for the midpoint of segment $s_1$ requires a linear interpolation using nodes $N_0, N_1, N_2,$ and $N_3$. The formulation results then in a nine-point difference approximation to equation (3.6) for the displacement field. The matrices for equation (1.4) become

$$A = \begin{bmatrix}
\frac{(1 - \nu)E(x,y)}{(1 + \nu)(1 - 2\nu)} & 0 \\
0 & \frac{E(x,y)}{2(1 + \nu)}
\end{bmatrix}$$
E(x,y) represents the spatially varying Young's modulus, and \( F_y(x,y) \) is the gravitational body force. Using the finite-difference approximation to the coupled equations, a solution may be obtained using successive over-or-under-relaxation.

The finite-difference equations must be represented in matrix form

\[ AU = B \]  \hspace{1cm} (I.6)
where $A$ and $B$ are a coefficient matrix and inhomogeneity vector, respectively, produced by means of the integration technique. The vector of approximations to the nodal values of the dependent variables $\mathbf{u}$, is determined by successive over-relaxation. For a problem with $N$ dependent variables, each node has a set of $N$ equations which are ordered successively in (I.6) according to the grid numbering. Correspondingly, the unknown vector $\mathbf{U}$ of the system is formed by stacking sets of nodal approximations $\mathbf{u}_p=(u_{1p}, u_{2p})$ end-on-end in the order of the numbering. Thus for a lattice with $M$ nodes, the vector $\mathbf{U}$ of (I.6) would have $MN$ components while $A$ would be a square, $MN$ by $MN$, coefficient matrix, and $B$ an inhomogeneity vector with $MN$ components. The solution is then obtained using the method of successive point over- or under-relaxation (Varga, 1962).
Appendix II

STABILITY AND CONVERGENCE

The solution of the general plane elastostatic problem given by equation (1.4) and (1.5) requires theorems for stability and convergence of coupled elliptic equations with variable coefficients. Since these are not available, we must depend upon empirical tests of convergence (Mitchell, 1969).

Mild restrictions are, however, imposed by the integration technique (Varga, 1962; Tillman, 1971):

1. dependent variables \( u=(u_1, \ldots, u_n) \) must be continuous functions of \( x \) and \( y \);

2. all coefficients of the equations must be piecewise continuous;

3. there must be flux continuity in any direction and at any point of the region.

Flux continuity is actually a property imposed upon the solution \( u \) by the integration technique and guarantees that proper interface conditions are formulated along lines of material discontinuities without special attention (Varga, 1962).

Convergence towards a true, discrete solution using successive over-relaxation depends upon the properties of the coefficient matrix, which in turn depends on the particular differential system and finite-difference lattice under consideration. Successive relaxation will
converge for any relaxation factor $\omega$ in the range $0 < \omega < 2$.

If matrix $A$ is symmetric, nonsingular, and positive definite, and further, convergence will occur for $\omega = 1$ (Gauss-seidel method) if $A$ is strictly or irreducibly diagonally dominant (Varga, 1962). The last condition is satisfied; however, showing that matrix $A$ possess the other properties is a difficult task, particularly for variable coefficients. One must then rely upon empirical tests to evaluate the convergence.

Tillman (1971) describes several sample problems successfully solved by the program. These include conduction problems, simple plane stress, potential flow, and linear and nonlinear plate bending, including irregular lattice structures. Comparison to exact solutions available for some problems indicates insignificant errors; the final answers are within a few percent of the theoretical.

Figure 16 depicts the numerical solution for a plane strain problem with variable elastic moduli. A square with 11 grid points to the side is taken for the region, and Young's moduli is made to vary along the $y$-axis. A rigid boundary is placed along the left-hand face; free sides are specified; and 25 dynes/cm$^2$ outward traction is imposed by means of body forces upon the right-hand face. Thus the square is under tension. The exact solution along the axis of the square corresponds within 6% to the numerical results for $\omega = 1.5$ and after 150 iterations.
The problem indicates that on-line monitoring can provide an accurate evaluation of the convergence.

For general solutions, empirical evaluation must be relied upon to determine convergence, for it does not necessarily occur for all values of the relaxation parameter. Yet on-line tests for different parameters furnish a reliable appraisal of the convergence; both convergence to a discrete solution and the optimum rate may be evaluated. This technique has proven the surest means of guaranteeing stability and convergence for the problems.
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Table I. Models of Descending Lithosphere using Elastostatic Simulation

<table>
<thead>
<tr>
<th>Model</th>
<th>Thermal Regime (Figure)</th>
<th>Depth of Slab (km)</th>
<th>Plate* Boundary</th>
<th>Poisson Ratio</th>
<th>Log Ratio of Young's Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Slab(center) Slab(edge)</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mantle(440 km) Mantle(140 km)</td>
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<tr>
<td>1</td>
<td>1</td>
<td>560</td>
<td>free</td>
<td>0.25</td>
<td>0.0</td>
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<td>rigid</td>
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<td>rigid</td>
<td>0.30</td>
<td>0.5</td>
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<td>0.0</td>
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<td>rigid</td>
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<td>0.0</td>
</tr>
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<td>560</td>
<td>rigid</td>
<td>0.25</td>
<td>0.0</td>
</tr>
<tr>
<td>11</td>
<td>-</td>
<td>560</td>
<td>rigid</td>
<td>0.25</td>
<td>0.0</td>
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* Boundary condition on edge of lithosphere
† Measured at 100 km from end of slab
Table I. (continued)

<table>
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<tr>
<th>Model</th>
<th>Down-dip Principle Stress in Slab*</th>
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<tr>
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<td>Intermediate (160 km depth)</td>
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<tr>
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<td>Top Face</td>
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<tr>
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<td>P low</td>
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<tr>
<td>2</td>
<td>P high</td>
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<tr>
<td>3</td>
<td>T high</td>
</tr>
<tr>
<td>4</td>
<td>P med</td>
</tr>
<tr>
<td>5</td>
<td>I low</td>
</tr>
<tr>
<td>6</td>
<td>P high</td>
</tr>
<tr>
<td>7</td>
<td>P,I med</td>
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<tr>
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<td>I low</td>
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<tr>
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<td>T low</td>
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<tr>
<td>10</td>
<td>P low</td>
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* | Deep (>250 km) |
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<td></td>
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<td>P high</td>
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<tr>
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<td>T med</td>
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<tr>
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<td>8</td>
<td>P high</td>
</tr>
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<td>9</td>
<td>T,P med</td>
</tr>
<tr>
<td>10</td>
<td>P,I low</td>
</tr>
<tr>
<td>11</td>
<td>I low</td>
</tr>
</tbody>
</table>


Maximum shear stress: low, <150 bar; med, 150-300 bar; high, >300 bar.
Table II. Numerical Integration of Equilibrium Equation for Descending Slab: $45^\circ$ dip; 8 cm/yr; 80 km thick

<table>
<thead>
<tr>
<th>Depth (km)</th>
<th>Body Force (dyne/cm$^3$)</th>
<th>Mantle Resistance (bar)</th>
<th>Down-dip Stress* slab depth: 700</th>
<th>560</th>
<th>280</th>
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<td>495.</td>
<td>-100.</td>
<td>606.</td>
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</table>

End-resistance (bar): -2591 -1520 -1514

* Positive denotes tension (bar)
Figure 1.

Temperature field at time $t=12.96$ by using Mac-Donald's (1959) value of the radiative conductivity. Adiabatic compression, phase changes, shear-heating, and radioactive-heating are included as energy sources in the model. The spreading rate is 8 cm/yr. Shading indicates zones of phase changes. For the elastostatic problem, the slab is truncated when its temperature reaches equilibrium with the surrounding mantle at 560 km (reproduced from Toksoz, et al, 1971).
Figure 2.

Temperature field at time $t=9.45$ my for a 30 degree dipping slab with Schatz's (1971) radiative conductivity. All other parameters are the same. Notice the larger thermal gradients and temperature contrasts. (provided by N. Sleep, 1971).
Figure 3.

Maximum shear stress for Model 1: Contours of maximum shear stress in bars are shown in the slab for stresses less than 500 bars. The arrows indicate the direction of deviatoric compression or tension (principle stresses minus hydrostatic pressure) for select nodes of the finite-difference solution. In this case, Young's modulus, $E$, is constant throughout the region for both the mantle and slab, and it equals $1.5 \times 10^{12}$ dynes/cm$^2$. The gravitational body forces result from thermal contraction using the temperature regime in Figure 1.
Figures 4a and b

Solutions for the maximum shear stress for two different mantle conditions using the temperature regime of Figure 1. The contours on the right are again those of maximum shear stress in bars within the slab. Shear stresses greater than 500 bars are not contoured in these illustrations. The stresses within the mantle are not contoured. On the left, E represents Young's modulus (dynes/cm²) used for the mantle surrounding the slab. Unless otherwise specified, the elastic moduli of the slab are constant within it and equal to the lithosphere's value.

Figure 4a.

Model 2 depicts sagging of the slab at 160 km in the asthenosphere: compression near the top face, tension at the lower surface. Only a slight decrease in the asthenosphere's strength is used for the model.
Figure 4b.

In contrast, model 3 with low strength asthenosphere and increasing strength with depth.
Figure 5a and b

Solution of maximum shear stress for two different mantle supports using a 30 degree dipping slab in Figure 2. Section A-A' depicts the temperature dependence of Young's modulus within the slab for both models.

Figure 5a

Model 4 with a soft asthenosphere and increasing mantle support. Compression results along the full length of the slab.
Figure 5b

Model 5 with little mantle support. The slab hangs under tension.
Figure 6a

Maximum shear stress for Model 6 using the thermal regime in Figure 2 and a slowly varying thermal dependence in the slab given by section A-A'. Sagging effects and stress concentrations again result along the edge of the slab.
Figure 6b

Maximum shear stress for model 7 when the thermal regime of Figure 1 is truncated at 380 km. An extended asthenosphere is used for a slab with constant elastic moduli. Both sagging and intermediate tensional stresses occur within the slab.
Figure 7

Maximum shear stress for model 8 illustrating a slab with strongly varying elastic moduli given by section A-A'. The thermal regime in Figure 1 is used for the computations. Large down-dip tensional stresses predominate at intermediate depths and gradually rotate to compression for greater depths.
Figure 8

Three-dimensional diagram summarizing the dependence of the models upon the elastic moduli. The three coordinates represent increasing mantle strength with depth relative to the asthenosphere (mantle/mantle), increasing temperature dependence of the slab's elastic moduli (slab/slab), and increasing contrast between the slab and mantle's elastic moduli (slab/mantle, incorrectly labeled as mantle/slab in the diagram). The points refer to models 1 through 8 in Table I. The regions blocked off are meant only as a rough guide to the characteristic stress patterns observed for the previous models.
Figure 9

The maximum shear stress for model 9 with olivine-spinel phase change introduced at 400 km depth according to Ringwood and Major's (1970) data and using the thermal regime in Figure 1. A density anomaly of approximately 0.3 gm/cm$^3$ results at 350 km. The elastic moduli in the mantle and slab are the same as model 2.
Maximum shear stress for two models simulating convection. Model 10 on the right is for a free plate with 50 bars traction along its upper face simulating mantle resistance. The arrows again indicate direction of principle stress at various nodes. The maximum shear stress is less than 100 bars throughout the slab. Model 11 on the left uses a shear-couple at the mantle-slab interface to simulate resistance by the mantle. The arrow indicates the component of the shear couple acting on the slab, while a similar component but in the opposite direction acts upon the mantle. The mantle's elastic modulus for this model is given on the left and is the same as Model 2. The maximum shear stress is approximately 70 bars throughout the slab.
Figure 11

Global summary of the distribution of down-dip stresses in inclined seismic zones. The stress axis that is approximately parallel to the dip of the zone is represented by an unfilled (open) circle for compression or P axis, and a filled (solid) circle for the tensional or T axis; an "X" indicates that neither the P nor the T axis is approximately parallel to the dip. For each region the line represents the seismic zone in a vertical section aligned perpendicular to the strike of the zone. The lines show approximately the dips and lengths of the zone and gaps in the seismic activity as a function of depth. (reproduced from Isacks and Molnar, 1971).
Fig 11
Figure 12

The region of North Honshu, the Sea of Japan, and the Kurile Island arc. The large circles are equal-area projections of the lower hemisphere of a focal sphere and are oriented according to the directions of the map projection; in all cases the top and right-hand side corresponds to north and east, respectively. A filled circle represents the axis of tension, T; an unfilled circle represents the axis of compression, P; an X represents the null axis, B. A solid line in the projection is the trace of the plane that best fits the orientation of the seismic zone; a dashed line is the trace of a plane parallel to two of the stress axes (or the average position of groups of axes) and perpendicular to the third axis. If earthquakes occur in thin slab-like stress guides, this plane should give the orientation of the slab. The numbers near the projection refer to the solutions plotted in the projection. On the map the contours of hypocentral depth (in kilometers) are shown by solid lines. The epicenters of the events are shown by filled, upward-pointing triangles for hypocentral depths of 70-229 km; unfilled, upwards-pointing triangles for depths of 300-499 km; and filled, downward-pointing triangles for depths of 500-700 km. The landward side of coastlines are shown by dots. (reproduced from Isacks and Molnar, 1971).
Figure 13

The Tonga island arc. The nomenclature is the same as the preceding figure. (reproduced from Isacks and Molnar, 1971).
Figure 14a and b

Down-dip stresses in the inclined seismic zones as a function of convergence rate and depth. The filled circles indicate the maximum depth of down-dip tension; the open circles show the shallowest down-dip compressional solution (Isacks and Molnar, 1971). If two of the same circles occur for a region, they represent the lower and upper bound for the deepest tensional solution or the shallowest compressional event inferred from the boundaries of the aseismic gap in Figure 11. The Kurile island arc apparently has tensional and compressional solutions scattered throughout the region. The convergence rates are taken from LePichon (1968, 1970). The lines across the diagram depict theoretical limits when the down-dip stress is greater than 1 kilobar. The solid line defines the lower bound for tension; the dashed line is the upper bound for compression; and stresses generally less than 500 bars occur between the limits. The maximum depth of the slab is given on the right for each theoretical model.

Figure 14a

Limits of tension and compression for inclined seismic zones dipping less than 45 degrees. Theoretical models for a 45 degree dipping slab are included for slabs penetrating 560 and 700 km in depth.
Figure 14b

Inclined seismic zones and theoretical models for steeply dipping slabs (60 degrees or more). New Zealand has been included in the previous diagram since the curves merge for slowly converging slabs.
Figure 15a

Annual number of earthquakes per 25-km depth intervals as a function of depth for three select regions: Tonga, Kuril-Kamchatka, and Alaska (data from Sykes, 1966).
Figure 15b

Magnitude of down-dip principle stress as a function of depth for slab models using numerical integration of the equilibrium equation. The gravitational body forces and mantle resistance are a function of convergence rate. The models correspond to the previous three regions: a 700 km depth slab at 9 cm/yr convergence rate for Tonga; 560 km depth at 8 cm/yr for the Kurile island arc; and 300 km depth at 6 cm/yr for Alaska-Aleutian slab. The dips are shown along side the depth. Effects of lithospheric bending and overthrusting are not included for shallow depths. The relative shapes of the curves show a distinct correlation.
Finite-difference solution for a plane-strain elastostatic problem with variable elastic moduli in the y-direction. The boundary conditions for the square region are illustrated at the bottom. The arrows indicate the 25 dyne/cm$^3$ body force imposed at the boundary nodes on the far right. The 20 cm sides consist of 11 grid points. The solution along the y-axis is shown at the top together with the value of Young's modulus. The theoretical solution is 25 dynes/cm$^2$ except near the rigid boundary.
Fig 16

LOG(E)

σ_y (dynes/cm²)

0 10 20 cm

node

0 10 20 cm

axis

y

0 10 20 cm

axis

y