SPECTRUM ANALYSIS IN SEISMOLOGY

by

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A. B. Boston College (1950)

SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE

DEGREE OF DOCTOR OF

PHILOSOPHY

at the

MASSACHUSETTS INSTITUTE OF

TECHNOLOGY

August, 1954

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Department of Geology and Geophysics, August, 16, 1954

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ABSTRACT

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William P. Walsh

Submitted to the Department of Geology and Geophysics on August 16, 1954 in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

We first undertake to reacquaint the reader with the mathematical tools available for determining the spectrum of a function of time and also to establish the problem involved when one attempts to utilize these tools in estimating the spectrum. Adopting the method of estimation devised by Tukey (1949), a technique of computing spectra of a seismogram is suggested and actually applied to both earthquake and prospecting records. The procedure consists of computing spectra from hundreds of successive overlapping intervals and displaying them in the fashion of a contour map — called, here, traveling spectra. The whole process is accomplished at high speed on the Whirlwind digital computer.

Application of the aforementioned process to several earthquake records obtained from the observatory in Weston, Mass. revealed some interesting dispersion curves for both Rayleigh- and Love- waves over Atlantic and continental paths. Those curves for surface waves which traveled the Atlantic path were strikingly similar whereas those curves for surface waves traveling over the continent were most dissimilar. This probably indicates a fair degree of lateral inhomogeneity in the continental portion of the earth's crust by way of comparison to the oceanic portion.

Similar techniques were also applied to some microseisms (seismograms recorded at Weston) due to a storm which existed over the Atlantic near the coast of New England. Our analysis revealed the existence of a resonance phenomenon suspected by Haq (1954) in addition to further corroborating the theory of the origin of microseisms proposed by Longvet-Higgins (1950) and recently studied by Haq.

Since the analysis of earthquake seismograms suggested the use of traveling spectra as a means of "picking" reflections, several trials were made on three prospecting records where reflections were both visible and invisible. The results were quite encouraging, for all types of reflections were put into evidence rather well.

Thesis Supervisor: Dr. S.M. Simpson
Title: Assistant Professor of Geophysics
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ACKNOWLEDGMENTS

The author would like to express his gratitude to Dr. S. M. Simpson for the assistance he has rendered in the supervision of this thesis, and to Dr. N. A. Haskell for his helpful suggestions and comments. We also owe a measure of thanks to Mr. D. T. Ross of the Servomechanisms Laboratory of M. I. T. for answering our questions concerning the computation of Fourier transforms. To Dr. K. E. Haq we are indebted for suggesting the study we have made on microseisms and his comments on the results. We are also pleased to acknowledge the staff of M. I. T.'s Whirlwind computer and Revs. D. Linehan and J. Donohoe of Weston Observatory who gratefully assisted in choosing earthquake seismograms which were loaned to us for our analysis. Finally the author wishes to thank his mother and father for typing this thesis.
A STATEMENT OF THE PROBLEM

Throughout the existence of Seismology surprisingly little has been done to ascertain by means of exact analysis the frequency content of recorded seismic motion, in comparison to theoretical work which has taken place in this regard. The U. S. Coast & Geodetic Survey, we understand, has recently accomplished a good deal of work in this direction, as well as Housner, et al. (1953) who have devoted their attention to analysis of strong motion earthquakes.

Preceding their work, however, only a few papers have been written (Klotz (1918); Caloi (1939)) concerning the application of Fourier analyzing machines to portions of earthquake records. Jakosky, et al. (1952) have been engaged in frequency analysis of reproducible prospecting records (magnetic recordings), and as far as we know, their work and possibly that of a few companies in the petroleum industry (yet unpublished) is all the experimental research that has been projected in this direction in the field of exploration seismology.

It is not difficult to understand the dearth of work in analysis of this sort when one considers the enormous amount of labor that such investigation calls for. Only the advent of high speed computing machinery has made the undertaking of such analysis feasible.

As a matter of fact, it has only been in these recent years which have seen the perfection of these electronic
computors that serious consideration has been given to the problem of calculating spectra. Possibly this is coincidence, but there is reason to believe that such study was motivated by the development of such machines. We believe that there are only two reports which concern this problem of spectrum calculation -- the original (and probably the major) work by Dr. J. W. Tukey (1948-9) and the recent work of D. T. Ross (1954).

In this thesis we have endeavored to apply Tukey's discoveries to the spectrum analysis of seismograms and to exploit as much as possible the high speed of M.I.T.'s electronic computer to that end.

Our purpose was not only to devise a means of seismogram spectrum analysis, but also to apply such analysis in several cases in order to obtain useful information concerning surface wave dispersion; microseisms; and direct, refracted, and reflected energy of earthquakes and explosions. It was also our hope that our choice of subject matter and manner of presentation would be such as to encourage future investigation along these and similar lines.
CHAPTER I

INTRODUCTION AND BACKGROUND

A. Introduction

In the following sections, an attempt is made to reacquaint the reader with the well known mathematical tools utilized in spectrum analysis; and in so doing an effort is made to categorize them — in a fashion similar to that employed by Y. W. Lee (1950) — according to assumptions necessarily attendant on the function to be analyzed.

We have also tried to put the problem of the actual process of computation of spectra into proper light in a section entitled "Concepts of the Spectral Window". It is hoped that, in consideration of these facts, the techniques we have employed in the analysis of seismograms will be brought into better perspective.

A method of analyzing seismogram spectra by determining frequency amplitudes and phase magnitudes of successive overlapping intervals is introduced. We also make known a novel way of displaying these spectra in three dimensional form and a method of accomplishing this at high speed on a digital computer.
B. General Harmonic Analysis

The problem of harmonic analysis is an old one and has developed after passing through many stages, including the use of periodograms, correlograms, and various computational techniques. Many of these methods had little foundation as far as mathematical rigor was concerned when they were first introduced, yet each constituted an advance over previous techniques. This is also true of the so-called auto correlation methods which began to exert some influence over twenty years ago but which have, in recent years, gained a new impetus from Prof. N. Wiener's work on time series. It was a result of the firm mathematical basis which Prof. Wiener has laid down that many engineering and scientific fields have recognized the excellence and versatility of this method and have consequently given more emphasis to it.

In virtue of the above remarks, we will attempt in the following discussion to bring forward the "old" and "new" formulae involved in spectral analysis. The purpose of this is to refamiliarize the reader with the mathematical detail which will be referred to in later sections, and to acquaint him with the contrast and similarity between the methods. We have then, depending on the nature of the functions involved, three general modes of spectral analysis.
Only two modes will be discussed here since we have applied only two of them in forthcoming sections. An excellent treatment of the third -- that which concerns analysis of random functions -- is given by Y. W. Lee (1950).

The first of these methods to be discussed is the analysis of periodic functions; the second that of aperiodic functions.

a. Periodic Functions

According to Fourier, representation of any continuous periodic time function of period $T_1$ is given by

$$ f(t) = \sum_{n=-\infty}^{n=+\infty} F(n) e^{in\omega_0 t} $$

where $\omega_0$ is the fundamental angular frequency $\frac{2\pi}{T_1}$, $i = -1$ and $F(n)$ is the complex line spectrum given by the expression

$$ F(n) = \frac{1}{T_1} \int_{0}^{T_1} f(t) e^{-in\omega_0 t} dt $$

$F(n)$ may also be looked at in the following way:

$$ F(n) = |F(n)| e^{i\phi_n} $$

where

$$ |F(n)| = \left( \frac{\text{Re}[F(n)]^2 + \text{Im}[F(n)]^2}{2} \right)^{\frac{1}{2}} $$

and

$$ \phi_n = \tan^{-1} \frac{\text{Im}[F(n)]}{\text{Re}[F(n)]} $$

where

$$ \text{Re}[F(n)] = \frac{1}{T_1} \int_{0}^{T_1} f(t) \cos n\omega_0 t \, dt $$
and

\[ \text{Im}[F(n)] = -\frac{1}{T_1} \int_0^{T_1} f(t) \sin n\omega_0 t \, dt \quad (7) \]

We can see the relation of the auto correlation function and its transform, the power density spectrum, to the harmonic analysis of periodic functions if we recall the definition of the auto correlation function \( \phi_{11}(\tau) \)

\[ \phi_{11}(\tau) = \frac{1}{T_1} \int_0^{T_1} f_1(t) f_1(t + \tau) \, dt \quad (8) \]

and expand \( f_1(t + \tau) \) in a Fourier Series and resubstitute into \( \phi_{11}(\tau) \) for it can be easily shown that

\[ \phi_{11}(\tau) = \sum_{n=-\infty}^{+\infty} |F(n)|^2 e^{in\omega_0 \tau} \quad (9) \]

We can see then that the spectrum of the auto correlation function is tantamount to the power spectrum of the function and further that \( \phi_{11}(\tau) \) retains all harmonics of the given function but drops all phase angles.

If we specify \( |F(n)|^2 \) (the power spectrum) by \( \overline{F}_{11}(n) \)

we may further state

\[ \overline{F}_{11}(n) = \frac{1}{T_1} \int_0^{T_1} \phi_{11}(\tau) e^{-in\omega_0 \tau} \, d\tau \quad (10) \]

In other words, harmonic analysis of the auto correlation function of a periodic function \( f(t) \) yields the power spectrum of \( f(t) \).

If we let

\[ \phi_{12}(\tau) = \frac{1}{T_1} \int_0^{T_1} f_1(t) f_2(t + \tau) \, dt \quad (11) \]
be the cross-correlation function between \( f_1(t) \) and \( f_2(t) \) and it can be shown by a somewhat similar extension of the aforementioned formulae, that the cross power spectrum is

\[
\bar{F}_{12} = \frac{1}{T_1} \int_0^{T_1} \phi_{12}(\tau) e^{-i\omega \tau} d\tau
\]  

(12)

In passing we should note that the cross power spectrum is in general a complex function, and consequently the cross-correlation function retains relative phase information, in contrast with the auto correlation which discards it. In addition it is also true that the cross-correlation retains only those harmonics which are common to both \( f_1(t) \) and \( f_2(t) \).

b. Aperiodic Functions

If we should consider aperiodic functions as being defined by

\[
\int_{-\infty}^{+\infty} f^2(t) dt \quad \text{Finite}
\]

we may regard \( f(t) \) as being capable of being represented over all time in the manner of the Fourier Integral

\[
f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{i\omega t} d\omega
\]

(13)

where

\[
F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-i\omega t} dt
\]

(14)

Such a manner of representation is, of course, valid in the case where

\[ f(t) = 0 \quad \text{for} \quad -\infty < t < T_1 \]

and for \( T_2 < t < +\infty \)
$F(w)$ is the complex amplitude spectrum of $f(t)$.

There is considerable similarity between the expression for this function and the complex line spectrum of the periodic function, eq. (2). Except for the lack of the multiplicative constant $\frac{1}{T_0}$ expressions for the magnitude, $F(w)$; the phase $\phi(w)$; $\text{Re}[F(w)]$; and $\text{Im}[F(w)]$ may be obtained by substituting the variable $w$ for $n$ in equations 4 through 7 respectively.

It may be shown in a fashion somewhat similar to that of eq. (9) that,

$$
2\pi \int_{-\infty}^{+\infty} F(w) \overline{F}(w) e^{i\omega \tau} \, dw = \int_{-\infty}^{+\infty} f(t) f(t + \tau) \, dt \tag{15}
$$

$$
2\pi \int_{-\infty}^{+\infty} \vert F(w) \vert^2 e^{i\omega \tau} \, dw = \int_{-\infty}^{+\infty} f(t) f(t + \tau) \, dt = \phi_{ll}(\tau) \tag{16}
$$

where $\phi_{ll}(\tau)$ unlike $\phi_{ll}(\tau)$ of periodic function may be expressed

$$
\phi_{ll}(\tau) = \int_{-\infty}^{+\infty} f(t) f(t + \tau) \, dt \tag{17}
$$

and where $F(w)$ is defined by equation (14).

If we let

$$
|\overline{F}_{ll}(\omega)| = 2\pi \vert F(w) \vert^2 \tag{18}
$$

then we have

$$
\phi_{ll}(\tau) = \int_{-\infty}^{+\infty} \overline{F}_{ll}(w) e^{i\omega \tau} \, dw \tag{19}
$$
and by Fourier Transform theory

$$\tilde{\phi}_{11}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \phi_{11}(\tau) e^{-i\omega \tau} d\tau$$  \hspace{1cm} (20)

Since $\phi_{11}(\tau)$ and $\tilde{\phi}(\omega)$ are even functions the foregoing are simplified to read

$$\phi_{11}(\tau) = \int_{-\infty}^{+\infty} \tilde{\phi}(\omega) \cos \omega \tau d\omega$$ \hspace{1cm} (21)

$$\tilde{\phi}_{11}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \phi_{11}(\tau) \cos \omega \tau d\tau$$ \hspace{1cm} (22)

The last equations express the fact that the auto-correlation function and the energy density spectrum of finite total energy are Fourier cosine transforms of one another.

Similar relations exist for cross-correlation of two functions of finite total energy.
C. Computation of Spectra

The time-honored process of Fourier Analysis has in recent years become of great importance. It has always been recognized as a method of expressing in a most elegant manner many natural phenomena, and with the advent of high-speed computing devices it has quite naturally received increasing attention.

Although at one time the Fourier Series approach saw great use, more recent applications have demanded Fourier Transform techniques. However well the Fourier transform with its attendant continuous frequency variable fits the actual case there have nevertheless been additional complications introduced with the transfer.

The following discussion is an attempt to give the reader an insight as to the nature of these complications as well as a resume of what has transpired in recent years in efforts to alleviate them.

a. The concept of "spectral windows"

There are usually three minor types of errors involved in the process of the Fourier transformation

\[ F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-i\omega t} \, dt \]

They are encountered in reading \( f(t) \) and \( \cos \omega t \), in forming their multiplication, and in integration. Their effects on the resultant answer are nevertheless quite small
and may be stated in probabilistic form depending on the methods employed.

The major error in this process of transforming empirical functions arises because these functions are known only over some finite interval. Such is the case when it is desired to obtain the spectrum of an interval of a time series, for here the function must be regarded as known over the interval in question and zero elsewhere. The effect on the computed transform is to superimpose relatively high amplitude oscillations on the correct transform. (This is the so-called Gibbs phenomenon. cf. Guillman, Carslaw)

The resultant spectrum, needless to say, is the incorrect one. Let us regard this incorrect spectrum (transform) of \( f(t) \) as the correct spectrum of say \( g(t) \). Coalescing the aforementioned errors into a weighting function \( D(t) \) we may further regard \( g(t) = D(t) * f(t) \). Denoting the Fourier transformation process by which we obtained the incorrect spectrum by the operator \( F'' \) and the process by which we obtain the correct spectrum by the operator \( F' \) we obtain the equality,

\[
F'[ D(t) f(t) ] = F''[ f(t) ]
\]  \hspace{1cm} (23)

Letting

\[
F'[ D(t) ] = D(\omega)
\]

and

\[
F'[ f(t) ] = F(\omega) \quad \text{(the correct spectrum)}
\]
and using the well-known fact that the transform of a product is the convolution of the factors involved we have

\[ \int_{-\infty}^{+\infty} D(w-\xi) F(\xi) \, d\xi = \mathcal{F}^{-1}[f(t)] \]  \hspace{1cm} (24) 

where \( \xi \) has the same dimensions as \( w \).

Hence, the value of the spectrum at \( w_n \) which is estimated by the procedure of calculation decided upon, is obtained by taking the inverted transform of the weighting function, centering \( D(t) \) about \( w_n \), and using this to weight the values of \( F(\xi) \) over the entire range of \( \xi \).

Henceforth, \( D(w) \) will be referred to as the - "spectral window". Furthermore it is a function which is entirely independent of \( f(t) \) and dependent only on the method of approximating the transform; and in turn it may be regarded as the measure of goodness of the method utilized.

We have drawn the \( D(w) \) of \( D(t)=1 \) corresponding to the most elementary transform process possible, in fig. (1) (referred to there is the "spectral window of \( L_0 \)" and it may be seen from this that due to the "ripple" on either side of the \( w=0 \) in the fig.) that a poor estimate will be obtained for the spectrum. Ideally \( D(w) \) should equal the impulse function, for then the estimate would be the correct value. The best that can be done is to "smooth"
SPECTRAL WINDOW OF $L$

(The Transform of the Weighting Function $D(t) = 1$)

SPECTRAL WINDOW OF $U_0$

(The Smoothed Transform of the Weighting Function $D(t) = 1$)

Fig. (1)
out the ripple of the $D(\omega)$ so that the estimate may be obtained by averaging the correct spectrum over the narrowest band of frequencies possible. We have shown Tukey's success (1949) in this respect in the same fig. (1) by the configuration entitled "spectral window of $U_0"$ (we will report on this later). Tukey accomplishes this smoothing by the use of an averaging process in forming the transform.

In the frequency plane the spectral window may also be looked upon as a weighting function. -- $W_j(\omega)$. To establish this concept, which will be referred to later, the following development arises.

We may regard the spectrum estimate $S''$ as related to the correct spectrum $S$ by the equality,

$$S''(\omega) = \int_{-\infty}^{+\infty} W_j(\omega) S(\omega) \, d\omega$$  \hspace{1cm} (25)

where $\omega$ and $\omega$ are of the same dimensions and $\omega$ indicates the scanning frequency. We have already seen that for even functions

$$S''(\omega) = \int_{-\infty}^{+\infty} D(t) f(t) \cos \omega t \, dt$$  \hspace{1cm} (26)

If we let

$$f(t) = \int_{-\infty}^{+\infty} S(\omega) \cos \omega t \, d\omega$$ \hspace{1cm} (27)

then

$$S''(\omega) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} S(\omega) \cos \omega t \, d\omega \, D(t) \cos \omega t \, dt$$  \hspace{1cm} (28)
Reversing the order of integration eq. 28 becomes

\[ S''(\omega) = \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} D(t) \cos \phi t \cos \omega t \, dt \right] S(\phi) \, d\phi \]  

(29)

Hence, the spectral window regarded as a weighting function is for cosine transformation

\[ W_{\phi}(\omega) = \int_{-\infty}^{\infty} D(t) \cos \phi t \cos \omega t \, dt \]  

(30)

For sine transformation

\[ W_{\phi}(\omega) = \int_{-\infty}^{\infty} D(t) \sin \phi t \sin \omega t \, dt \]  

(31)

Remembering that \( S'' \) depends on the interval length \( T \)

we can now consider the error involved in calculating

\[ S''(\omega, T) = 2 \int_{0}^{T} f(t) \cos \phi t \, dt \]  

(32)

instead of

\[ S''(\omega, \infty) = 2 \int_{0}^{\infty} f(t) \cos \phi t \, dt \]  

(33)

In order to formulate a comparison let us consider

the case where \( f(t) = \cos \phi t \) and \( \phi = \omega \), for here the answer is known and a measure of the effect of integration over a finite interval may be arrived at. \( D(t) \) here equals 1.

Here then

\[ T = \int_{0}^{\infty} \cos^2 \omega t \, dt = T + \frac{\sin 2\omega T}{2\omega} \]  

(34)
and which when normalized equals
\[
1 + \frac{\sin 2\omega T}{2\omega T}
\]  \hspace{1cm} (35)

Graphically the comparison is

![](image)

Diagram (1)

and we see that as the interval is lengthened the estimated value approaches closer and closer the correct value.

The above process where \( \omega \) is kept constant and the error associated with the method is examined as \( T \) is varied, is, from examination of eq.30\' and 31\', equivalent to measuring the goodness of the process in the frequency plane with the length of interval held constant. In other words if we took eq.32 and did not vary the length of the interval as in diagram (1) we would have an expression equivalent to eq.30', which expresses the value of our estimate of the spectrum in the frequency plane.

In the process of smoothing it is necessary to keep in mind both the window in the frequency domain and the corresponding weighting function in the time domain. Ross (1954) recognizes this in the development of a series of weighting functions \( D_n(t) \), (reproduced in fig. (2) where \( n \) indicates the stage of smoothing, i.e. the greater \( n \) the more is the "ripple" of the spectral window diminished. He cautions
ROSS'S NORMALIZED WEIGHTING FUNCTIONS

INTERVAL LENGTH

n = 0
n = 1
n = 2
n = 3
n = 4
n = 5
n = 6
n = 7

Fig. (2)
that -- if we take \( D_n(t) \) for larger and larger values of \( n \) we, at each stage, essentially reject more and more the finite record of \( f(t) \) for large \( t \), and more and more emphasis is placed on values of \( f(t) \) for small \( t \). The estimate of the spectrum may then be further from the truth than that obtained by using a smaller value of \( n \).

It is clear that a balance must be achieved and this balance will be largely dependent on \( f(t) \). For instance

![Diagram](image)

it is obvious that \( f_2(t) \) can be smoothed more than \( f_1(t) \) with little loss of information at large values of \( t \) and as a result a better spectrum is attained. In general we may say that the greater the degree of smoothing the better the spectrum estimate -- provided that the weighting function in the time domain associated with the process of smoothing in the frequency domain, does not reject a representative portion of the function being transformed.

The auto-correlation function is, generally speaking, somewhat similar in configuration to \( f_2(t) \) in diagram (2). If we should further consider \( f_2(t) = \phi \) where \( \phi \) is the auto-correlation of \( f_1(t) \), we can easily see from our
above discussion that transforming the auto-correlation of a function \( f_1(t) \) will -- for the stage of smoothing involved -- give rise to a better spectrum than the process which entails transforming the function \( f_1(t) \) itself.

(b) **Estimation of the Power Spectrum**

We should now like to outline the method of estimating the power spectrum of a discrete time series \( x_1, x_2, \ldots, x_n = f_1(t) \) utilized in our analysis of seismograms. This method consists of finding numerical approximations of the expressions for the auto-correlation function

\[
\phi(\tau) = \int_{-\infty}^{+\infty} f(t)f(t+\tau)\,dt
\]

(36)

and the spectrum

\[
\overline{S}(\omega) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \phi(\tau) \cos \omega \tau \, d\tau
\]

(37)

which hold for stationary time series. The following discussion of this method is quoted from Report #5 of the Geophysical Analysis Group of M.I.T.

"The auto-correlation coefficients of a time series are given by the sample serial products

\[
R_p = \frac{1}{n-p} \sum_{i=1}^{n-p} x_i x_{i+p}
\]

(38)

The basic problem is to obtain an approximation to the spectrum from the serial products \( R_p \) (\( p = 0, 1, \ldots m \)) for a given number \( m \) which is less than \( n \)."
The observations $x_i$ have the spacing $h$, which may be defined as one unit. Since periods less than two units will not be observable themselves, the effect on the estimation of the spectrum is to fold over the last part of the frequency scale where $\frac{2\pi}{w} < 2$ into that portion of the scale where $\frac{2\pi}{w} > 2$. This means of course that on $w$ scale the distribution of frequencies will now run from $-\pi$ to $+\pi$, and we shall confine our attention to this region. The reduced spectrum is thus defined to be a spectrum which involves frequencies which in magnitude are no greater than $\pi$. Thus the frequencies $w, 2\pi - w, 2\pi + w \ldots$ are treated as aliases of each other. Moreover, to use both positive and negative frequencies, equal power is required at $-w$ and $+w$.

"By choosing discrete values of angular frequency

\[ w = \frac{s\pi}{m} \quad (s = 0, 1, \ldots, m) \]

we may perform numerical integration of equation by the trapezoidal rule and write

\[ \tilde{f}(w_s) = \tilde{f} \left( \frac{s\pi}{m} \right) \approx L_s \]

where

\[ L_s = \frac{1}{\pi} \left[ \frac{1}{2} R_0 \cos 0 + \sum_{j=1}^{m-1} R_j \cos \frac{s\pi j}{m} + \frac{1}{2} R_m \cos s\pi \right] \]

By letting

\[ r_j = \frac{R_j}{R_0} \quad (j = 0, 1, 2, \ldots, m) \]

we have

\[ L_s = \frac{R_0}{2\pi} \left[ 1 + 2 \sum_{j} r_j \cos \frac{s\pi j}{m} + R_m \cos s\pi \right] \]
An approximate integration of $\bar{T}(w)$ from $-\pi$ to $+\pi$ by the trapezoidal rule yields,

$$\int_{-\pi}^{+\pi} \bar{T}(w) \, dw = 2 \int_{0}^{+\pi} \bar{T}(w) \, dw \div \frac{\pi}{m} \left[ L_0 + 2 \sum_{l=1}^{m-1} L_l + L_m \right] \quad (42)$$

Now by standard summation formulae, we find that

$$L_0 + 2 \sum_{l=1}^{m-1} L_l + L_m = \frac{m}{\pi} R_0 \quad (43)$$

Thus, for all values of $m$, the area given by the trapezoidal rule is $R_0$, the serial product of lag zero. Hence, we see, once again that in the estimated spectrum the total power has been compressed into the interval $(-\pi, \pi)$.

(c). Estimation of the Amplitude Spectrum

In a fashion somewhat parallel to that outlined in the previous section, we desire now to outline our method for approximating the Fourier transform of

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(w) e^{iwt} \, dw \quad (44)$$

In particular we wish to approximate $|F(w)|$ and $\phi(w)$ where

$$F(w) = |F(w)| e^{i\phi(w)} \quad (45)$$

and where in turn

$$|F(w)| = \left( \text{Re}[F(w)]^2 + \text{Im}[F(w)]^2 \right)^{1/2} \quad (46)$$

and

$$\phi(w) = \tan^{-1} \frac{\text{Im}[F(w)]}{\text{Re}[F(w)]} \quad (47)$$

From section B we can see that calculation of the
above relies on estimating the integral representations
of $\text{Re}[F(w)]$ and $\text{Im}[F(w)]$ which are

$$\text{Re}[F(w)] = + \int_{-\infty}^{+\infty} f(t) \cos wt \, dt$$

$$\text{Im}[F(w)] = \int_{-\infty}^{+\infty} f(t) \sin wt \, dt$$

(48)

Here we considered $f(t)$ a discrete time series of spacing $h$.

$$f_i(t) \approx f(t_0), f(t_0 + h), \ldots, f(t_0 + ih)$$

$(i = 0, 1, 2, \ldots, m)$

and we also considered

$$\text{Re}[F(w)] = \text{Re}[F(\frac{sn}{m})] \approx M_S$$

(49a)

and

$$\text{Im}[F(w)] = \text{Im}[F(\frac{sn}{m})] \approx -N_S$$

(49b)

where $s = 0, 1, 2, \ldots, m$

As before we numerically integrate (49a) and (49b) by the trapezoidal rule in the familiar fashion.

$$M_S = + \left[ \frac{1}{2} f_0 \cos 0 + \sum_{j=1}^{m-1} f_j \cos \frac{sn_j}{m} + \frac{1}{2} f_m \cos sn \right]$$

$$-N_S = + \left[ \frac{1}{2} f_0 \sin 0 + \sum_{j=1}^{m-1} f_j \sin \frac{sn_j}{m} + \frac{1}{2} f_m \sin sn \right]$$

We point out at this time that the weighting function, $D(t)$, employed in the estimation of the $L_s, M_s,$ and $N_s$ is 1, and as a consequence the estimate is quite far from the truth, which fact is at once evident from the corresponding spectral window, depicted in fig. (1). To overcome this drawback we employed a method of smoothing which we
describe in the subsequent section.

d.. A Method of Smoothing

Dr. T. W. Tukey of Bell Telephone Laboratories and Princeton University (1948-1949) has made what is probably the major contribution to the study of spectrum estimates in recent years. His work, which has undergone several revisions and additions, consists mainly of two studies.

The first part of his work (1949), which was accomplished with the assistance of R. W. Hamming, consists in choosing and applying an improved weighting function which uses the familiar property of Fourier transforms: that the transform of a trigonometric polynomial is a function consisting of equally spaced impulses of unequal strength. The process of convolving the transform of such a polynomial (e.g. a cosine arch) becomes, for digital calculations, the application of a running weighted mean or smoothing formula.

Tukey's work, besides being located in the above cited reference may be found beautifully summarized in Report #5 of the Geophysical Analysis Group, M.I.T. His formulae, which we have used in smoothing our estimated spectra of seismograms, calculated in the second portion of this thesis, are listed below for the reader's convenience.

The Tukey-Hamming smoothed estimate, $U_s$, of the power spectrum density and the cosine and sine transforms of
$f(t)$ is given by,

$$U_s = .23X_{s-1} + .54X_s + .23X_{s+1}$$

where $X$ is equal to the $L$, $M$, and $N$ of sections $b$ and $c$.

Since $U_o$ and $U_m$ respectively involve $X_{-1}$ and $X_m + 1'$ which have not been defined, we set $X_{-1} = X_1$ and $X_m + 1 = X_{m-1}$

This amounts to

$$U_o = .54X_o + .46X_1$$

$$U_m = .54X_m + .46X_{m-1}$$

When $X = L$ we see that because of the identity

$$U_o + \frac{m-1}{1} \sum U_s + U_m = L_0 + 2 \sum_{1}^{m-1} L_s + L_m$$

the smoothing process is area preserving, and hence the total area in the estimated spectrum is given by $R_o$.

See equation (38). This conclusion is Parseval's Theorem.

The frequency in c.p.s. for which the various line spectra $U_s$ have been calculated are

$$\sum_{s=0,1,2,...m} \frac{3}{m+h}$$

where $m$ is the total number of lags or points, and $h$ is the time spacing between points of the discrete time series.
D... Some Considerations of Numerical Fourier Analysis

There are two distinct ways of regarding the numerical method in general -- first as an approximation to the integrals for each line spectrum, and secondly as a process of curve-fitting. It is wise to keep both in mind when applying or interpreting the results.

As may have already been noticed, our approach to calculating the spectrum of an interval on a seismogram has been to choose a fundamental frequency, determined by the length of the interval $T$, and the time spacing between consecutive points, $h$ and then to compute the line spectra of this frequency as well as its odd and even harmonics. The approximation of the intervening frequencies involved in the transform was accomplished by the smooth curve connecting these calculated points. Essentially then, our calculated values were derived in the manner which would have been employed if the interval $T$ were considered representable by a Fourier series.

Riemann's theorem states that the Fourier representation is unique. The theorem, however, applies only when no limit is imposed upon the number of terms which may be included; if any arbitrary limitation is imposed, the result of the theorem is not necessarily valid.

In the following discussion we propose to ascertain the restrictions of and the reliability of our analysis
when a limitation is imposed on the quantity of data. Since the remarks made will pertain to our calculated values, we may for the purposes of this discussion regard \( f(t) \) capable of being expressed in the manner of a Fourier series

\[
f(t) = C + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t
\]  

(54)

where

\[
a_n \sim \frac{1}{N} \sum_{r=1}^{N} f_r(t) \cos(n\omega_0 t) \tag{55}
\]

\[
b_n \sim \frac{1}{N} \sum_{r=1}^{N} f_r(t) \sin(n\omega_0 t) \tag{55}
\]

Let us for the moment consider an example of a function periodic in \( \frac{3\pi}{2} \) radians, for which we have values at \( t = 0, \frac{\pi}{2}, \pi \) and \( \frac{3\pi}{2} \), denoting these values by \( y_0, y_1, y_2, y_3 \)

\[
y_0 = f(0) = C + \sum a_k \cos \frac{kn}{2} + \sum b_k \sin \frac{kn}{2}
\]

\[
y_1 = f(\frac{\pi}{2}) = C + \sum a_k \cos \frac{kn}{2} + \sum b_k \sin \frac{kn}{2}
\]

\[
y_2 = f(\pi) = C + \sum a_k \cos \frac{kn}{2} + \sum b_k \sin \frac{kn}{2}
\]

\[
y_3 = f(\frac{3\pi}{2}) = C + \sum a_k \cos \frac{k3\pi}{2} + \sum b_k \sin \frac{k3\pi}{2}
\]

(56)

Consider the effect of summing these four values

\[
A_0 = \sum_{n=1}^{3} y_n = 4C + \sum a_k D_k + \sum b_k E_k
\]

where

\[
D_k = 1 + \cos \frac{kn}{2} + \cos \frac{kn}{2} + \cos \frac{k3\pi}{2}
\]

\[
E_k = \sin \frac{kn}{2} + \sin \frac{k3\pi}{2}
\]

(57)

\( A_0 \) depends on the value of \( D_k \) and \( E_k \). Now if \( k \) is a multiple of 4, say \( 4X \) where \( X \) is an integer, we find

\[
D_{4X} = 4 \quad E_{4X} = 0
\]

\[
D_{4X-1} = 0 \quad E_{4X-1} = 0
\]
and similarly \( D_{4X-2} = E_{4X-2} = D_{4X-3} = E_{4X-3} = 0 \)

Substituting into the above expression for \( A_0 \)
\[
A_0 = 4\left[ C + \sum_{X} \frac{a_X}{X} \right]
\]  
(58)

Consider the effect of performing the summation
\[
A_1 = y_0 \cos 0 + y_1 \cos \frac{\pi}{2} + y_2 \cos \pi + y_3 \cos \frac{3\pi}{2}
\]
\[
= y_0 - y_2
\]
\[
= \sum a_k \left[ 1 - \cos k\pi \right]
\]
\[
= 2\sum a_{2X-1}
\]

Similarly the summation:
\[
B_1 = y_0 \sin 0 + y_1 \sin \frac{\pi}{2} + \ldots
\]
\[
= y_1 - y_3
\]
\[
= \sum a_k D_k' + \sum b_k E_k'
\]

where
\[
D_k' = \cos \frac{k\pi}{2} - \cos \frac{3k\pi}{2}
\]
\[
E_k' = \sin \frac{k\pi}{2} - \sin \frac{3k\pi}{2}
\]

If \( k = 4X \)
\[
D_{4X} = \cos 2\pi - \cos 6\pi = 0 \quad D_{4X-2} = 0
\]
\[
E_{4X} = 0 \quad E_{4X-2} = 0
\]

If \( k = 4X-1 \)
\[
D_{4X-1} = \cos \frac{\pi}{2} - \cos \frac{3\pi}{2} = 0
\]
\[
E_{4X-1} = - \quad + \quad = -2
\]
We note that if the variation contains harmonics higher than the 2nd, no information concerning the coefficients of any harmonics is afforded by the equations (60).

From the above example and others one can see that the following rule must be observed in numerical Fourier Analysis.

The number of ordinates must exceed twice the reference number of the highest harmonic present in the unlimited and

This restriction has also been determined by Goldman (’53) by a different procedure. The above discussion, however, points out what occurs when the restriction is not met.

Let us for example consider a periodic function which is made up of forty harmonics of a given fundamental frequency. The above result states that at least eighty (80)
discretely spaced points of the function must be used if one wishes to ascertain the line amplitude spectrum of all forty harmonics. It does not state that although one may only be interested in the first twenty (20) it is sufficient to use forty points. Eighty must be used for otherwise, as already pointed out in our analysis, no correct information will be achieved.

In practice one assumes the function he is dealing with as made up of a finite number of harmonics, which for practical purposes is certainly valid. But since the exact number is usually unknown, the only alternative the investigator has is to experiment with sets of points, which have been read at various spacings, until two calculated spectra are similar. At this point it can be assumed that the above criterion is met.
E... Method of Displaying the Estimated Spectra

In the subsequent analysis of seismograms, whether they be prospecting or earthquake records, it was decided to break up the entire trace into intervals $T$, which overlap each other by 75%. The length of $T$ is unspecified, but is assumed to comprise significant frequency information and to be subject to Fourier integral representation. In the following investigation the length of $T$ found to be most practical was $40h$, where $h$ is the time spacing of the discrete time series.

In each interval an analysis is performed yielding either a smoothed estimate of the amplitude and phase spectrum or a power spectrum to be associated with that interval. Instead of plotting and evaluating each curve separately the information derived is presented in the fashion of contour maps, where the amplitude phase and power density is to be represented in relief on a map on which is plotted frequency vs time according to the diagram below.

Diagram (3)
Inasmuch as many spectra are presented in this way, the time dimension, though "blurred" because of interval overlap, has in a loose sense been preserved. The time which we have associated with the spectrum of each interval has been that time associated with \( f_m(t) \), where \( f_m(t) \) is the midpoint reading of the interval in question.

\[
\begin{align*}
\text{center time} = \frac{t_1 + t_2}{2}
\end{align*}
\]

This time will henceforth be referred to as "center time", and the contour representation of spectra of overlapping intervals as "travelling spectra".

The computation and plotting which such an undertaking involves is enormous and the time which would necessarily be expended in such an operation would be such as to discourage it at the outset. However, the entire job of computing and contouring has been accomplished at high speed in the Whirlwind Electronic Computer. The fruits of this computation constitute the second part of this report.

The mechanics of analyzing the spectra have already been described. The actual process of contouring has been accomplished by means of a density plot routine devised by Dr. S. M. Simpson of M.I.T. In this instance the dimension of "height" is brought about by the relative brilliance of a given square area on an oscilloscope screen.

In the case of amplitude or power density spectra all values in twenty-four consecutive spectra are normalized
to the maximum value occurring in all these twenty-four.
Each value is then associated with a number of spots to be focussed on a predetermined square area on the oscillioscope.

The greater the magnitude of the amplitude or power the more densely plotted are the spots.

These predetermined areas are arrayed in a "checkerboard" fashion in such a manner that "center time" is plotted horizontallly (along the abscissa) and frequency is plotted vertically (along the ordinate). For a permanent record a photograph is taken of the density plot as it occurs.

The clarify the language, the following diagram constitutes a negative of a fictitious density plot.

```
\[ \text{Interval - Center-time} \]

Diagram (4)
```

The density plots of the phase spectra are very similar to those described above for amplitude and power spectra. In this instance the only difference lies in that all phase angles are confined to the range of 0 to 360°, and the density of the spots are respectively less to great.
Each photograph of twenty-four overlapping intervals are connected in a consecutive fashion to give rise to a density plot of the entire seismogram trace. Not only does such an approach give rise to a neat, concise presentation of hundreds of seismogram spectra, but it also affords to the seismologist an accurate method of determining frequency and phase changes with time. In addition such a method enables easy comparison of frequency and phase changes with distance and with different component seismographs.

At present we will not be specific in their use or evaluation but will defer such comment to that time when actual cases are investigated.
The Phase Correction

In the calculation of the component frequencies of each overlapping interval \( \nabla \), we found at the outset that, for the sake of ease and speed in digital computation, it would be best to scan each interval with harmonic frequencies which had a zero degree phase shift with respect to the beginning of each interval. Thus for the determination of the phase of the first harmonic we would scan the intervals in the following fashion for determination of \( \Re \left[ F(\omega) \right] \)

![Diagram](image)

For determination of the \( \Im \left[ F(\omega) \right] \)

![Diagram](image)

Diagram (5)

It may be seen that the phase \( \tan^{-1} \frac{\Im}{\Re} \) calculated for such a harmonic will on a traveling spectrum basis be rather arbitrary depending on the interval and degree of overlap employed.

Regardless then of the interval we would like, however, to scan with a frequency which is essentially stationary in time, in order that the calculated phase would have some
relationship with those calculated for other intervals and other frequencies. We would have then for computation of

A similar diagram may be drawn for the intended calculation of

To accomplish this, one merely has to calculate the phase for each harmonic frequency employed (assuming 0° phase shift for the interval in question), and then to subtract the phase angle by which the interval under examination shifts the scanning frequency from the same frequency stationary in time. For instance, suppose we wish to determine the phase relationship between the fundamental scanning frequency curve III and the same component frequency of $f(t)$ in interval $A$ in diagram (7). We would first scan with the cosine arch, curve I, to determine $\text{Re } F(\omega)$ and with the sine arch, curve II, to determine $\text{Im } F(\omega)$. Both curves exhibit 0° phase with respect to interval $A$. The phase relationship between the component frequency of $f(t)$ and curve I is given by eq. (20). The phase relation we desire is then determined by subtracting from this calculation.
It may be easily shown that the phase shift $S$ which must be subtracted is dependent on the scanning frequency and the position of the interval. For intervals which overlap by 75%—the procedure that predominates in our investigation—it may be further shown that this necessary phase shift $S$ is:

$$S = 45^\circ rs$$

where $r = 0, 1, 2, \ldots, m$ and denotes the harmonic.

and where $s = 0, 1, 2, \ldots, m$ and denotes the interval.

From the nature of the overlap utilized it becomes evident that this $S$, if looked at in the phase range $0^\circ$ to $360^\circ$ is repetitive for every eight harmonics and for every eight intervals, i.e. for $r = 0, 1, \ldots, 7$ and for $s = 0, 1, \ldots, 7$. The same is true for $r = 8, \ldots, 15$ and $s = 8, 15$ etc.

This phase shift is shown graphically for $r = 0, 1, \ldots, 7$ and for $s = 0, 1, \ldots, 7$ in figs. (3) and (4). This phase shift which is then superimposed on the traveling phase spectrum, which is determined from scanning frequencies shifted zero degrees for every interval is displayed in fig. (5). The actual densities displayed for the various phases are here, not the true ones, but are nevertheless exemplary of the pattern.
Fig. (4)
A Display of the Phase Correction Superimposed on the Traveling Phase Spectra Calculated from Scanning Frequencies Shifted 0 Degrees.

Fig. (5)
CHAPTER II

APPLICATION OF SPECTRUM ANALYSIS IN SEISMOLOGY

A. Introduction

In this section we have presented, in what we believe to be a rather unique manner, the results of our spectrum analysis of some seismic records. Unfortunately, we could cover only a few aspects in earthquake and exploration seismology. Factors of time and money and the fact that our undertaking was partially a test prevented further computation. It is our hope that the few cases considered here are both representative and interesting.

In passing we would like to point out --- at least for the sake of general interest --- that the calculation and plotting of spectra presented in the following sections would, if undertaken with the ordinary means available, occupy a man working full time for approximately ten years.
A Study of Surface Wave Dispersion

Determination of the earth's crustal structure have been greatly assisted by studies of the reflected and refracted waves of depth charges, quarry blasts, and near earthquakes; by studies of surface wave dispersion; and by considerations of amplitude ratios of direct and surface reflected longitudinal waves. Application of surface wave dispersion to this end has received, it seems, renewed impetus in recent years. Both theoretical study, which has been devoted to the betterment of this technique, and improvements in observation have been the motivating influence.

Generally speaking there are two ways for evaluating the amount of dispersion of observed surface waves. One is to determine the period and speeds of conspicuous or of first surface wave arrivals from the records of any number of earthquakes and stations, sort them as to type and path, and plot the data points for comparison with theoretical curves computed from some assumed type of crustal layering. The second method is to determine the pertinent data for successive waves in a surface wave group from a single seismogram and plot these data for comparison with theoretical curves. Both have been used extensively although the latter is to be preferred, since it supplies a dispersion curve consistent with respect to path and does not entail the difficulties encumbant with respect to choice of what to measure in the former method.
Whichever of the two methods is employed various notions prevail as to the choice of group, but only one method exists in the actual measurement of the group's period, i.e. an assumption is made that what is to be measured is a sinusoid and then what is assumed to be the period is measured with dividers, and a time scale of some sort. This method may be regarded as sufficient at the initial portions of the Rayleigh or Love train where the record usually exhibits long period sinusoidal motion. However, as time proceeds the sinusoidal character diminishes and at certain times is completely absent. Present techniques of measurement then can hope to be accurate only over a limited range of the surface wave record.

Furthermore, theoretical considerations have shown that it is conceivable for two or more groups to arrive at a station at the same time depending on the layering configuration of the earth's crust through which the waves have passed. Since such observation is beyond the scope of existing methods, comparison with theoretical dispersion curves for assumed layering is greatly hampered.

Our proposed technique of "traveling spectra" analysis, comparatively speaking, affords a more accurate measurement of the period of a particular group and also exhibits change of period in time. The plotting of the spectra of overlapping intervals according to density not only automatically
affords a travel time curve of the dispersing trains of one quake but will also yield a more complete observation than has hitherto been offered. Furthermore such a manner of analysis could also be looked upon as a standard by which to establish actual dispersion, since it appears from the diversity of method and scatter of "observations" of seismologists that some sort of standardization is desirable.

We have in our study chosen five earthquakes recorded at Weston, Massachusetts. In two of these an Atlantic Ocean path predominates, in two others the path is entirely continental (U.S.), and in the fifth approximately half the path is over the Pacific Ocean and half over the United States. The foci of all of these quakes were of normal depth. Only the long period Benioff records were used.

**Case I -- Atlantic Path**

(1) Data Analyzed -- Methods Employed

The pertinent data for the two quakes studied here were derived from the U.S. Coast and Geodetic Survey and Jesuit Seismological Assn. respectively.

Quake (a) Date- May 31, 1953  
Epicenter -- N. Coast Dominican Rep. (20 N 70½ W)  
Time -- 19:58: 35 G.S.T.  
Magnitude -- 7 (Pas); 7½ (Berk)  
Epicentral Distance -- 2500 kms; 22.5°
Quake (b)  Date- Aug. 15, 1941
Epicenter -- "Atlantic Ocean" (19° N  27° W)
Time -- 06: 09; 00 G.S.T.
Magnitude
Epicentral Distance -- 2850 kms; 43.65°

Photostatic reproductions of these quakes may be found on plates (1) and (3).

For quake (a) data points—or trace amplitudes—we’re read from the seismogram every .446 seconds over the interval marked "A" on the traveling spectra of this quake (plate 2) and every 1.78 seconds over the interval marked "B" on this plate. For quake (b) data points we’re read at every 1.78 seconds.

The traveling spectra density plots of the spectra computed for quake (a) may be found on plate (2). In region "A" spectra were computed from 80 point intervals; in region "B" spectra were computed from 40 point intervals. In "A" n-harmonic frequencies of .014 C.P.S. were calculated (n = 0,1,2,3, ----40) per interval. A frequency scale appears at the left margin for this region. For region "B" n-harmonic frequencies of .007 C.P.S. were calculated per interval (n = 0,1,2,3, ----40); a frequency scale appears at the right of plate (2) for this region. The traveling spectra computed for quake (b) are found on plate (4). For this quake harmonic frequencies of .007 C.P.S. were calculated for intervals consisting of 40 points (n = 0,1,2,3, ----40).
In all instances our group velocity curves were obtained by drawing a smooth curve through the spectral density maxima which the traveling spectra depict for the various groups of waves involved in the Love and Rayleigh trains, and reading choosing those times where the curve intersected the frequency coordinate lines. Unfortunately the dispersion of the Love waves was not observed for quake (a) which fact may be due to the short distance involved and to the nature of the path (Wilson and Baykal 1948 have observed this for Love waves in their studies) -- although in retrospect, it may be due to our not overlapping intervals in region "A" by a sufficient percentage. At any rate our observations may be found tabulated in Tables I - III and depicted in figures (6) - (8).

For the purposes of comparison we have included in fig. (6) the observations of Wilson & Baykal (1948) of the dispersion of Rayleigh waves across the Atlantic.

The theoretical curve recently calculated by Jardetzky & press (1953) is also reproduced in fig. (7). In this latter curve the layer of sediment and water were considered. The basement layering for this curve is $H_1 = H_0$, $H_2 = \infty$, $\alpha_0 = 1.52$ kms/sec.; $\alpha_0 = 6.9$ kms/sec.; $\alpha_1 = 8.1$ kms/sec.; $\phi_1 = 3.0 \phi_0$, where $\alpha$ is the compressional wave velocity, $H$ is the thickness, $\phi$ the density, and the subscript the layer in question. Such a curve is according to these authors experimentally indistinguishable from the case where $H_1 = \infty$, $\phi_1 = 3.0 \phi_0$, $\alpha_0 = 1.52$ kms/sec.; $\alpha_1 = 7.90$ kms/sec.
We have also on fig. (8) reproduced two theoretical curves for Pacific Love wave dispersion, which have been recently published by Evernden (1954). The fig. (8) contains the pertinent assumptions involved and refer as usual to the shear wave velocity and density in gms/cm³ respectively).

(2) Discussion
The dispersion observed for Rayleigh waves over the Atlantic is very similar for the two paths studied, although the "Atlantic Ocean" quake gives values of velocity which are somewhat lower for the higher periods (50-70 sec.) and the lower periods (11-18 sec.)

The lower values in the latter range in quake (a) occur on the N-S component. Our value for 10.1 sec. - paralleling the Dominican Republic Quake is questionable and we have indicated this on our plate.

If we assume that the land path in both cases was 450 kms -- the distance from Weston to the 2000 fathom line - and that the speed for Rayleigh waves of the minimum group velocity at (15.6 sec) is 3.4 kms/sec. over this path we see that the velocity over the oceanic portion is 2.26 kms/sec. for the Dominican Republic Quake and 2.23 kms/sec. for the other. The velocity given by Press, Ewing & Jardetzky for this period is 1.6 kms/sec. and their minimum velocity occurs between periods 7-10 sec., for the model already described.
Comparison of our observations for Rayleigh waves in figs. (6) -- (7) with the theoretical curve of Jardetzky & Press (1953) seems to suggest that a decrease in \( v \) to about 4.35 kms/sec would probably give rise to better comparison between the two in the interval 40 - 70 sec. This decrease seems to be suggested by our observations for Love waves of quake (b) fig. (8), for which when extrapolated to higher periods a value of 4.35 kms/sec. is indeed approached.

Consideration of the curves in fig. (8) for dispersing Love waves seems to indicate that the structure proposed by Evernden (1954) for Pacific crustal layering might possibly exist under the Atlantic. For it can be seen that our curve would fit quite well with curve B if the layer of \( \varphi = 3.00 \) were to be increased to 11 or 12 kms and the shear velocity in the semi-infinite medium were decreased to 4.35 kms/sec. as has already been suggested from observed Rayleigh wave dispersion. It may also be thought curve "A" could be made to fit the observations if the layer's thickness were increased and \( \varphi \) were decreased, but the velocity for 13.0 sec. - 3.83 kms/sec. - and the general trend of our curve do not favor such a notion.

The above model of low velocity material 2.5 kms thick - consolidated sediment - overlying a layer of basalt of twice the thickness of that proposed by Jardetzky and Press may serve to explain our observations of Rayleigh waves in the 10 - 16 sec. period range. Comparison of quake (a) dispersion with
the curve derived by Haskell (1951) for a two layered continental structure (fig.11) seems to point more and more in this direction.

This discussion of course does not rule out the Airy phase (corresponding to the minimum of the group velocity curve) since our observations have not been extended into the frequency range (7 - 10 sec. period) of this phenomenon. It does point out, however, that a layer capable of supporting shear overlying a layer of basalt should be seriously considered. We may also conclude from our comparison with Evernden's curve that Atlantic and Pacific layering is similar.

Recently Ewing and his co-workers (see Ewing, et al. (1950,'51, '54); Officer, et al. (1952); Hersey, et al. (1952), have established some interesting results in their seismic refraction studies in the Atlantic. A review of these papers will show discrepancies among the measurements of the investigators as regards layering and velocities, however, they are not major ones. We will regard, then, Hersey's findings (1952) as representative of the results of their work, and quote them here for comparison with the conclusions obtained from the surface wave study.

Hersey interprets his measurements in two ways, one of which is the following:

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<tr>
<th>Thickness</th>
<th>Velocity (Longitudinal Waves)</th>
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</thead>
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<tr>
<td>1.. 5.30 kms (Water)</td>
<td>1.51 kms./sec.</td>
</tr>
<tr>
<td>2.. 0.42</td>
<td>1.69</td>
</tr>
<tr>
<td>3.. 2.26</td>
<td>4.31</td>
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<td>4.. 2.49</td>
<td>6.64</td>
</tr>
<tr>
<td>5..</td>
<td>7.94</td>
</tr>
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</table>
These results do point to the existence of low velocity material overlying a basaltic layer as was inferred from our comparison of the observed Love wave velocities with the curve calculated by Evernden. However, they do not bear out the 11 km. thickness of the basalt (6.64 km/sec.) which we surmised from the same curve (Ewing, et al. 1950, report a thickness of about 5 kms. and a velocity 6.42 km/sec.). It may well have been that if Evernden had regarded a still lower velocity overlying that which overlies the basalt such a thickness (11 kms.) would not have been required to obtain a decent fit with the observed values.

It is difficult, of course, to make statements concerning crustal layering from surface wave data because of the scarcity of theoretical curves which must be available for comparison. For instance, our observed Rayleigh wave dispersion may be accounted for by a structure consisting of three to five layers.

The improvements in observation of dispersion phenomena which we have presented certainly should warrant the calculation of such curves — an undertaking which should not be too difficult with the means which have now been made available in the form of electronic computers.
EPICENTER: ATLANTIC OCEAN (9°N 27°W)
DATE: AUG. 15, 1941
PLATE: 3
### TABLE I

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<th>TRAVEL TIME</th>
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# .098 N-S | 10.1 | 20.06 | 35.32 | 2.27 |

Questionable
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b. Case II  Continental Path

(1) Data Analyzed --- Methods Employed

The data listed below was obtained from the U.S. Coast and Geodetic Survey.

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<th>Time</th>
<th>Magnitude</th>
<th>Epicentral Distance</th>
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<tbody>
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<td>Oct. 13, 1953</td>
<td>N. Gulf Calif. 30 N 113° W</td>
<td>08:53:45 G.S.T.</td>
<td>6½ (Pas.)</td>
<td>3950 kms.</td>
</tr>
<tr>
<td>(d)</td>
<td>Dec. 4, 1953</td>
<td>Coast Vancouver Is. 49¾ N 129 W</td>
<td>14:54:46 G.S.T.</td>
<td>6½ (Pas.)</td>
<td>4450 kms.</td>
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</tbody>
</table>

Photostatic reproductions of the records of these quakes may be found on plates (5) and (7).

Data points were read from all seismograms at approximately every .894 seconds; 40 point intervals were utilized in all spectra calculations. As a result of this choice n-harmonic frequencies of .014 C.P.S. = 0, 1,2, ... 40 were calculated for each interval.

Density plots of the traveling spectra computed for quake (c) and (d) are displayed in plates (6) and (8) respectively.
The methods of obtaining the group velocity curves have already been mentioned under Case I. Our observations are tabulated in Tables IV - VII and plotted in figures (9) - (12). Since there was some question in our mind as to how a smooth curve could be drawn through the density plot (UD component) of the Vancouver Is. quake we have plotted as a result three curves for the Rayleigh wave group velocity. Of these three, curve "B" seems to us to be the most plausible, after consideration of travel time and velocity plots. Curve "A" was drawn in its present form to display possible comparison between the two continental paths analyzed, although it could have been extended as the two plotted points indicate. However, the fact that the excitation of more than one mode might have occurred to give rise to the two curves should not be overlooked.

We have also reproduced for purposes of comparison the theoretical curves for continental Rayleigh curve dispersion derived by Haskell (1951) on fig. (11). Figure (9) contains the curve derived by Wilson and Baykal (1948). There is also included on this same figure the recent observations of Brilliant and Ewing (1954) of Rayleigh wave dispersion across the United States. We have also computed several Love wave group velocity curves of assumed continental structure and have plotted same on figures (10) and (12). The assumptions pertinent to each theoretical curve are contained on the respective graphical plot.
Consideration of our dispersion curves for both Love and Rayleigh waves show marked dissimilarity. Generally speaking, however, they clearly show that corresponding groups exhibit higher velocities over Canada and Northern United States than they do over Southern portions of the United States. This fact is probably attributable to greater thicknesses of sialic rock over the latter route.

The observations of Brilliant and Ewing (1954) of Rayleigh wave dispersion across the United States were made from quakes (Pacific) which took the same continental path to Weston as did our N. Gulf California quake. It is interesting to note the similarity between our values and theirs.

Only in one case, namely, the Vancouver Is. quake, is it possible to envision comparison between a theoretical curve and the observed values. In the curves, fig. (11) for Rayleigh wave velocities we do see some similarity between curve B and curve I-- that which was derived by Haskell (1951) from a two layer case where velocity increases with depth. Due to the complexity of the calculations it is difficult to say in what manner the various parameters involved should be changed to bring about a decent fit. We will, however, venture to speculate that an increase in $q_1$, and $q_2$, a decrease in $\xi_2$, an increase in the first layers thickness, and a decrease in the thickness
of the second layer might serve to accomplish the fit between the two curves.

There seems to be no doubt that the scatter of values obtained for the two paths was effected by a high degree of heterogeneity in the upper crustal layers. Such a fact of necessity rules out the method of using a number of quakes to determine a general continental dispersion curve. It appears then that any study of continental surface wave dispersion must take into consideration direction of approach to the station and the epicentral distance, because various paths will most likely entail different layering.
TRAVELING SPECTRA - N GULF CALIFORNIA QUAKE

PLATE 6
TRAVELING SPECTRA - VANCOUVER IS. QUAKE

U-D

CENTER TIME IN MINUTES MEASURED FROM 15:14:565 (G.S.T.)

E-W

CENTER TIME IN MINUTES MEASURED FROM 15:12:965 (P.S.T.)

PLATE 8
RAYLEIGH WAVE GROUP VELOCITY VS. PERIOD

N. GULF CALIFORNIA EARTHQUAKE

\[ c = \frac{3.7 \text{ sec}}{\text{sec}} \]

\[ c = \frac{4.85 \text{ sec}}{\text{sec}} \]

Theoretical curve

Observations of Brilliant (1894)

Theoretical spectra observations
Love wave group velocity vs. period

Vancouver 1° quake

Theoretical curve

A
q = 3.14 sec
v = 4.4

q = 2.58 sec
v = 3.66

q = 2.36 sec
v = 3.69

B
q = 3.57 sec
v = 3.77

q = 3.30 sec
v = 3.83

Theoretical curve
### TABLE IV

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<th>Rayleigh Wave</th>
<th>Group Velocity</th>
<th>N. Gulf California Quake</th>
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<td>CENTER TIME</td>
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<td>10.1</td>
<td>4.86</td>
</tr>
<tr>
<td># .112</td>
<td>8.9</td>
<td>4.426</td>
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</tbody>
</table>

# Uncertain
### TABLE VI

<table>
<thead>
<tr>
<th>FREQUENCY</th>
<th>CENTER TIME</th>
<th>TRAVEL TIME</th>
<th>VELOCITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.028 c.p.s.</td>
<td>1.47 Min.</td>
<td>21.66 Min.</td>
<td>3.52</td>
</tr>
<tr>
<td>0.042</td>
<td>2.95</td>
<td>23.14</td>
<td>3.2</td>
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<td>0.056</td>
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<td>3.04</td>
</tr>
<tr>
<td>0.126</td>
<td>3.25</td>
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<td>0.140</td>
<td>2.36</td>
<td>22.55</td>
<td>3.28</td>
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</table>

(Curve B)

<table>
<thead>
<tr>
<th>FREQUENCY</th>
<th>CENTER TIME</th>
<th>TRAVEL TIME</th>
<th>VELOCITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.028</td>
<td>1.47</td>
<td>21.66</td>
<td>3.52</td>
</tr>
<tr>
<td>0.042</td>
<td>2.22</td>
<td>22.41</td>
<td>3.31</td>
</tr>
<tr>
<td>0.056</td>
<td>2.95</td>
<td>22.14</td>
<td>3.21</td>
</tr>
<tr>
<td>0.070</td>
<td>2.95</td>
<td>23.14</td>
<td>3.21</td>
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<tr>
<td>0.082</td>
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<td>22.84</td>
<td>3.25</td>
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<td>0.112</td>
<td>3.25</td>
<td>23.44</td>
<td>3.15</td>
</tr>
<tr>
<td>0.126</td>
<td>3.25</td>
<td>23.44</td>
<td>3.15</td>
</tr>
<tr>
<td>0.140</td>
<td>2.94</td>
<td>23.13</td>
<td>3.20</td>
</tr>
<tr>
<td>0.154</td>
<td>2.36</td>
<td>22.55</td>
<td>3.28</td>
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</table>

### TABLE VII

<table>
<thead>
<tr>
<th>FREQUENCY</th>
<th>CENTER TIME</th>
<th>TRAVEL TIME</th>
<th>VELOCITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.014 c.p.s.</td>
<td>.89 Min.</td>
<td>19.37 Min</td>
<td>3.84</td>
</tr>
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<td>0.028</td>
<td>2.22</td>
<td>20.41</td>
<td>3.63</td>
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<td>0.042</td>
<td>2.95</td>
<td>21.14</td>
<td>3.51</td>
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<tr>
<td>0.056</td>
<td>3.25</td>
<td>21.60</td>
<td>3.44</td>
</tr>
<tr>
<td>0.070</td>
<td>3.39</td>
<td>21.72</td>
<td>3.40</td>
</tr>
</tbody>
</table>
(C) Case III -- Pacific and Continental Path

(1) Data Analyzed -- Methods Employed

The data given below was obtained from the United States Coast and Geodetic Survey.

Quake (e)  Date - February 26, 1953
           Epicenter - Santa Cruz Isles  11° S  164°E
           Time --  11:42:26  G.S.T.
           Magnitude - 7½ (Pas.)
           Epicentral Distance - 13,650 kms.

Photostatic reproduction of the component seismograms of this quake may be found on plate (9).

Readings were taken from the record every 1.78 sec., and n-harmonic frequencies of .007 c.p.s. (n = 0,1,2,--40) were computed from intervals consisting of 40 points. The density plot of the calculated traveling spectra are photographed on plate (10).

The same procedure as mentioned previously in drawing our dispersion curves was employed here in plotting the Rayleigh wave group velocity curve. Since there was some evidence of generation of a shearing motion transverse to the direction of propagation (Love waves) we corrected our observed values of Rayleigh and Love motion for continental dispersion effects, in order to ascertain which groups -- if any -- were generated at the western coast of the continent -- (Our findings in this regard will be presented in a following section). The curves used for continental correction were our Rayleigh
(curve B) and Love wave observations for the Vancouver Is. quake, since the continental path of the two were practically the same. The groups which were found to be generated at the source were utilized in plotting our Love wave curve for the continental path - Fig. (14).

For comparison purposes we have subtracted the continental Rayleigh wave dispersion (curve B, Fig. (11) from that of the combined path, and have plotted the results in Fig. (13). We assumed that the continental path was approximately 4650 kms (the distance from the 2000 fathom line off Vancouver to Weston, Mass.). A similar curve was plotted for the Love waves on Fig. (14).
Fig. (14)
### TABLE VIII

<table>
<thead>
<tr>
<th>FREQUENCY</th>
<th>PERIOD</th>
<th>CENTER TIME</th>
<th>TRAVEL TIME</th>
<th>VELOCITY KMS/SEC</th>
</tr>
</thead>
<tbody>
<tr>
<td># .014 c.p.s.</td>
<td>71.5 Sec</td>
<td>42.480 Min</td>
<td>58.310 Min</td>
<td>3.90</td>
</tr>
<tr>
<td>.028</td>
<td>35.7</td>
<td>43.955</td>
<td>59.785</td>
<td>3.80</td>
</tr>
<tr>
<td>.035</td>
<td>28.6</td>
<td>45.135</td>
<td>60.965</td>
<td>3.74</td>
</tr>
<tr>
<td>#.042</td>
<td>23.8</td>
<td></td>
<td>62.735</td>
<td>3.62</td>
</tr>
<tr>
<td>.049</td>
<td>20.4</td>
<td>49.560</td>
<td>65.390</td>
<td>3.49</td>
</tr>
<tr>
<td># .056</td>
<td>17.9</td>
<td>53.690</td>
<td>69.520</td>
<td>3.27</td>
</tr>
<tr>
<td>#.063</td>
<td>15.9</td>
<td>63.72</td>
<td>79.550</td>
<td>2.86</td>
</tr>
<tr>
<td>.020</td>
<td>14.3</td>
<td>61.42</td>
<td>97.25</td>
<td>2.33</td>
</tr>
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</table>

# Questionable

### TABLE IX

<table>
<thead>
<tr>
<th>FREQUENCY</th>
<th>PERIOD</th>
<th>CENTER TIME</th>
<th>TRAVEL TIME</th>
<th>VELOCITY KMS/SEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>.014 c.p.s.</td>
<td>71.5 Sec</td>
<td>35.99 Min</td>
<td>51.82 Min</td>
<td>4.42</td>
</tr>
<tr>
<td>.021</td>
<td>47.6</td>
<td>37.17</td>
<td>53.10</td>
<td>4.29</td>
</tr>
<tr>
<td>.028</td>
<td>35.7</td>
<td>38.94</td>
<td>54.77</td>
<td>4.15</td>
</tr>
<tr>
<td># .035</td>
<td>28.6</td>
<td>40.12</td>
<td>55.95</td>
<td>4.06</td>
</tr>
<tr>
<td>#.042</td>
<td>23.8</td>
<td>42.48</td>
<td>58.33</td>
<td>3.91</td>
</tr>
<tr>
<td># .049</td>
<td>20.4</td>
<td>44.84</td>
<td>60.67</td>
<td>3.75</td>
</tr>
<tr>
<td>.056</td>
<td>17.9</td>
<td>52.51</td>
<td>68.34</td>
<td>3.31</td>
</tr>
<tr>
<td>.063</td>
<td>15.9</td>
<td>59.00</td>
<td>74.83</td>
<td>3.04</td>
</tr>
<tr>
<td>.070</td>
<td>14.3</td>
<td>73.75</td>
<td>89.58</td>
<td>2.55</td>
</tr>
</tbody>
</table>

# Questionable
C... A Study of Reflected, Refracted and Direct Body Waves.

(a) The Reflected and Refracted Phases.

In the foregoing sections it was noted on those seismograms which were analyzed over times prior to the arrival of the dispersing surface wave groups, that the traveling spectra gave evidence of what appeared to be pulses of energy arriving at discrete intervals of time. Suspecting that these occurrences might well be correlated with the times of arrival of the various P and S phases we undertook to compare the center times of these pulses with the travel times given by Gutenberg and Richter (1939) and now circulated by the United States Coast & Geodetic Survey. The results were very encouraging, displaying in each instance remarkable correlation in arrival time.

We have arbitrarily chosen our maximum error as being the time from the center-time of the interval to the extremities of the interval in question.

Hence we have for

Dominican Republic Quake -- ±.295 Min. --Region A
±.590 " -- " B
N. Gulf California Quake -- ±.295 "
Atlantic Ocean Quake -- ±.295 "
Santa Cruz Isles Quake - ±.590 "

In Tables X - XIII below we have listed the results of this investigation for four of the quakes analyzed in this report.
**TABLE X**

**Dominican Republic Quake**

<table>
<thead>
<tr>
<th>Phase</th>
<th>Dominant Frequencies</th>
<th>Travel Time (Gutenberg)</th>
<th>Travel Time (Trav.Spec.)</th>
<th>Center Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>.000 - .070 c.p.s.</td>
<td>5.00 Min.</td>
<td>5.23 Min.</td>
<td>.885 Min.</td>
</tr>
<tr>
<td>PP</td>
<td>.000 - .014</td>
<td>5.50</td>
<td>5.28</td>
<td>1.63</td>
</tr>
<tr>
<td>S</td>
<td>.070 - .560</td>
<td>8.70</td>
<td>8.18</td>
<td>3.83</td>
</tr>
<tr>
<td>Love</td>
<td>.126 - .560</td>
<td>9.15</td>
<td>9.22</td>
<td>4.87</td>
</tr>
</tbody>
</table>

**TABLE XI**

**Atlantic Ocean Quake**

<table>
<thead>
<tr>
<th>Phase</th>
<th>Dominant Frequencies</th>
<th>Travel Time (Gutenberg)</th>
<th>Travel Time (Trav.Spec.)</th>
<th>Center Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>.075 - .063 c.p.s.</td>
<td>14.90 Min.</td>
<td>15.85 Min.</td>
<td>.59 Min (E-W)</td>
</tr>
</tbody>
</table>

**TABLE XII**

**N.Gulf California Quake**

<table>
<thead>
<tr>
<th>Phase</th>
<th>Dominant Frequencies</th>
<th>Travel Time (Gutenberg)</th>
<th>Travel Time (Trav.Spec.)</th>
<th>Center Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>ScS (V)</td>
<td>.070 - .560 c.p.s.</td>
<td>17.25 Min.</td>
<td>18.19 Min.</td>
<td>2.07 Min. (U-D)</td>
</tr>
<tr>
<td>ScS (H)</td>
<td>.000 - .560</td>
<td>-----</td>
<td>18.62</td>
<td>2.50</td>
</tr>
<tr>
<td>?</td>
<td>.000 - .560</td>
<td>-----</td>
<td>29.10</td>
<td>12.98</td>
</tr>
</tbody>
</table>

**TABLE XIII**

**Santa Cruz Islands Quake**

<table>
<thead>
<tr>
<th>Phase</th>
<th>Dominant Frequencies</th>
<th>Travel Time (Gutenberg)</th>
<th>Travel Time (Trav.Spec.)</th>
<th>Center Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP</td>
<td>.063 - .084</td>
<td>(21.65) Min.</td>
<td>21.54 Min.</td>
<td>5.71 Min.</td>
</tr>
<tr>
<td>PKS</td>
<td>.063 - .070</td>
<td>23.00</td>
<td>22.91</td>
<td>7.08</td>
</tr>
<tr>
<td>PPP</td>
<td>.035 - .077</td>
<td>30.85</td>
<td>30.89</td>
<td>15.06</td>
</tr>
<tr>
<td>PS</td>
<td>.028 - .063</td>
<td>32.03</td>
<td>31.77</td>
<td>15.94</td>
</tr>
<tr>
<td>PPS</td>
<td>.000 - .028</td>
<td>33.05</td>
<td>33.53</td>
<td>17.70</td>
</tr>
<tr>
<td>PPPS</td>
<td>.021 - .049</td>
<td>37.95</td>
<td>38.25</td>
<td>22.42</td>
</tr>
<tr>
<td>PSPS</td>
<td>.000 - .021</td>
<td>40.60</td>
<td>40.61</td>
<td>24.78</td>
</tr>
<tr>
<td>ScSScS</td>
<td>.021 - .042</td>
<td>41.85</td>
<td>42.09</td>
<td>26.26</td>
</tr>
</tbody>
</table>
With regard to the Dominican Republic Quake we found it difficult to ascertain the nature of a prominent wide frequency band energy pulse which occurred on both U-D and E-W component records at 20:15:03 G.S.T. (Center Time 11.46 Min.). At first we thought that might be some reflected S phase caused by the P but the frequency spectrum of this pulse contains energy outside that of the P for this quake. Comparison of its spectrum with that of the S showed striking similarity between the two and the likeness seemed to suggest a reflection of the S, however, the travel time charts indicate nothing of such a nature arriving at the time in question. It seems that all that one has left to explain the occurrence of this phase is the Love wave, when one takes into account the band of frequencies involved. Further consideration of the difference in time between the arrival time of this phase and the Love wave strongly suggests the generation of a "Rayleigh" wave by the Love wave as it strikes the continent. The nature of the components of the motion indicate retrograde elliptical motion in a plane, perpendicular to the direction of travel—a screw-like motion—and seemingly detracts from this notion. Definite statements, however, must necessarily await further mathematical analysis and observation.

(v) The P-Wave

A generally accepted definition of earthquake magnitudes proposed by Richter (see Bullen) is the following: magnitude
is the logarithm (to the base ten) of the maximum amplitude (measured in microns) traced on a seismogram by a standard short-period seismograph (free period 0.8 sec.; statical magnification 2800; damping coef. 0.8), distant 100 kms. from the epicenter. Empirical tables for quakes of normal depth have been set up to enable reduction of amplitudes measured at various distances to the expected amplitudes at the standard 100 kms. These tables have been built on the assumption that the ratio of the maximum amplitudes at two given distances is the same for all earthquakes. Since these assumptions are not strictly true many observers have advocated the use of P-wave period in establishing the magnitude of an earthquake.

For instance, Kanai, et al. (1953) claim that the amplitude of earthquake motion is considerably influenced by direction, damping, construction of earth crust and property of the ground near the observation point. Then it is easily seen that the relationship between the amplitudes at earthquake origin and at the observation point is not so simple. Therefore, in general, there will be a considerable inaccuracy in determining the energy or magnitude of earthquake by using the amplitude of earthquake motion even if we adopt the initial motion of P-waves which is considered to communicate the characteristics of seismic waves generated at the earthquake origin comparatively well.
"On the contrary, the period of seismic waves, particularly that of the initial motion of P-waves, can be considered to keep the characteristics of the waves unchanged from the earthquake origin to the observation point. Consequently, if the relation between the period and the energy can be clarified, the energy or the magnitude of earthquake will be given more accurately by using the period of seismograms than by using the amplitude."

More recently Kanai, Osada and Yoshizawa (1953) have inaugurated and extended studies of the relation between the period and amplitude of the P-wave. The results of their studies indicate among other things that the amplitude is roughly proportional to the square of the period of the motion of this wave. It may be reasoned then, that for quakes of similar magnitude, there should be a noticeable decrease in period with increase in epicentral distance.

It would seem, then, that if any determinations were to be made from the period of the P-wave exact analysis of same for frequency content should be preferred over any rough determinations which might be made by measurement with dividers and time scale. As we have shown previously the frequency estimates of a function of time are obtained by determining the auto-correlation of this function and performing a cosine transformation on this to obtain the power density spectrum.
In an effort, therefore, to test and display the usefulness of spectrum analysis in this regard we have determined the power spectra of four P waves from quakes of various epicentral distance. The data pertaining to these quakes and listed below were obtained from the United States Coast & Geodetic Survey. The results of our calculations are depicted in figures (15) - (18).

<table>
<thead>
<tr>
<th>Date</th>
<th>Epicenter</th>
<th>Time</th>
<th>Magnitude</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>May 31, 1953</td>
<td>19:58:35 G.S.T.</td>
<td>7 (Pas.)</td>
<td>2,500 kms</td>
</tr>
<tr>
<td></td>
<td>Dominican Republic</td>
<td>20°N - 70°W</td>
<td>7 ½ (Berk)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date</th>
<th>Epicenter</th>
<th>Time</th>
<th>Magnitude</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>Dec. 12, 1953</td>
<td>17:31:22 G.S.T.</td>
<td>7 ¾ (Pas.)</td>
<td>5,150 kms</td>
</tr>
<tr>
<td></td>
<td>Peru</td>
<td>3°8'S, 81°W</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date</th>
<th>Epicenter</th>
<th>Time</th>
<th>Magnitude</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>III</td>
<td>Dec. 7, 1953</td>
<td>02:05:37 G.S.T.</td>
<td>7 ½ Pas.</td>
<td>7,220 kms</td>
</tr>
<tr>
<td></td>
<td>N. Chile</td>
<td>22°S, 68°W</td>
<td>100 kms</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date</th>
<th>Epicenter</th>
<th>Time</th>
<th>Magnitude</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV</td>
<td>Sept. 23, 1953</td>
<td>02:14:36 G.S.T.</td>
<td>7 Pas 6 ½ (Berk)</td>
<td>8,650 kms</td>
</tr>
<tr>
<td></td>
<td>N. Kurile Is.</td>
<td>50° N, 156° E</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comparison of the magnitudes of quakes I - III and the spectra of the corresponding P-wave shows that the expected decrease in period with distance for quakes of approximately the same magnitude is a fact for the predominant period involved, although quake IV seemingly indicates the opposite effect. The period of quake IV should have been the least. See Kanai (1953). It would seem that the only way to account
for Quake III and IV having the same prevailing frequency would be to increase the magnitude of quake IV given by the Coast & Geodetic Survey. It may be, however, that effects other than energy play a more dominating role in this instance. For example Kanai, et al. have established the relation

\[ \alpha = \frac{T}{2.6 \sqrt{\frac{V_s}{Q}}} \]

where \( \alpha \) is the radius of the origin (assumed spherical), \( T \) is the P-wave period measured at a station, and \( \sqrt{\frac{V_s}{Q}} \) is the shear wave velocity in the vicinity of the focus. It can be easily seen, then, that if the radius is assumed to be constant in all quakes, deeper foci will give rise to shorter P-wave periods.
POWER DENSITY VS. FREQUENCY
P-WAVE
FROM DOMINICAN REPUBLIC — 2500 MWS.
POWER DENSITY VS. FREQUENCY
C-MODE
FROM N. KURKIN Tables \& 8650 EMS.
D. The Possibility of Surface Wave Generation at the Edge of a Continent

In a previous section dealing with Rayleigh and Love wave dispersion from a quake occurring in the Santa Cruz Isles, we had occasion to mention the procedure we used in determination of groups which we believed to have been generated at the source. We repeat, that the continental dispersion for both types of surface waves determined from the Vancouver Is. quake was subtracted from all groups which evidenced themselves on the traveling spectra of the U-D and E-W components of the Santa Cruz Isles quake. Our method was to determine the continental travel time from our observed Love wave and Rayleigh wave (curve B) curves for a 4,650 km. path and to subtract these times from the center-times of the respective groups or pulses which we observed for the Santa Cruz quake, plate (10). This correction should then have given the "center-time" arrival time of the Rayleigh and Love groups at the Pacific edge of the continent. Our results for this computation are listed in Table XIII.

Such a listing of groups enabled us to determine fairly accurately which groups were from the source, for in most instances the Love center-time at this point would differ from that of the corresponding Rayleigh group by approximately three to four minutes, which roughly should be the case if the two traveled across the Pacific to this point.
It was noticed in the tabulation of these results that several Love and Rayleigh groups exhibited approximately the same center-time arrival time at this point. If we should arbitrarily choose our maximum error as being the time from the center-time of the interval to the extremities of the interval in question, namely, \( \pm 0.590 \) min. we then see that fairly good agreement is had in these instances. It may be possible to discredit this by better observations of surface wave dispersion across the continent, by considerations of chance, by identification of the groups in question as being something other than surface waves, or by determining the direction of approach of these waves as being inconsistent with a direction which is direct from the source to Weston. However, we do claim that the evidence clearly suggests the possibility of generation of Love waves by Rayleigh waves (or visa versa) as they strike the continent.

A firm basis for such a claim, of course, can only be established by further observations and theoretical considerations.
### TABLE XIII A

Continental Travel Times (4,650 kms.)

<table>
<thead>
<tr>
<th>GROUP (FREQUENCY)</th>
<th>LOVE WAVE</th>
<th>RAYLEIGH WAVE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.014 c.p.s.</td>
<td>20.1 min.</td>
<td>----- min.</td>
</tr>
<tr>
<td>.021</td>
<td>20.6</td>
<td>-----</td>
</tr>
<tr>
<td>.028</td>
<td>21.3</td>
<td>22.0</td>
</tr>
<tr>
<td>.035</td>
<td>21.8</td>
<td>22.9</td>
</tr>
<tr>
<td>.042</td>
<td>22.0</td>
<td>23.4</td>
</tr>
<tr>
<td>.049</td>
<td>22.3</td>
<td>23.7</td>
</tr>
<tr>
<td>.056</td>
<td>22.5</td>
<td>24.1</td>
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<td>22.6</td>
<td>24.2</td>
</tr>
<tr>
<td>.070</td>
<td>22.8</td>
<td>-----</td>
</tr>
</tbody>
</table>

### TABLE XIII B

Time of Surface Waves at the Pacific Edge of the Continent

<table>
<thead>
<tr>
<th>GROUP (FREQUENCY)</th>
<th>LOVE WAVE CENTER-TIME</th>
<th>RAYLEIGH WAVE CENTER-TIME</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>.028 c.p.s.</td>
<td>38.9 min.</td>
<td>43.95 min.</td>
<td>21.95 min.</td>
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</tr>
<tr>
<td></td>
<td>42.88</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.035</td>
<td>45.73</td>
<td>45.13</td>
<td>22.23</td>
<td></td>
</tr>
<tr>
<td></td>
<td>43.36</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>40.12?</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>.042</td>
<td>46.32</td>
<td>24.32</td>
<td>24.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>42.48?</td>
<td>20.48</td>
<td></td>
<td></td>
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<tr>
<td>.049</td>
<td>49.56</td>
<td>49.56</td>
<td>25.86</td>
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</tr>
<tr>
<td></td>
<td>44.84?</td>
<td>22.30</td>
<td>50.74</td>
<td></td>
</tr>
<tr>
<td></td>
<td>40.12?</td>
<td>18.32</td>
<td>27.04</td>
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<tr>
<td>.056</td>
<td>52.51</td>
<td>53.69</td>
<td>29.57</td>
<td></td>
</tr>
<tr>
<td></td>
<td>48.38</td>
<td>55.46</td>
<td>31.36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>42.77</td>
<td>20.27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.063</td>
<td>59.00</td>
<td>63.72</td>
<td>39.52</td>
<td></td>
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<td></td>
<td>48.09</td>
<td>25.49</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>42.77</td>
<td>20.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.056-.063</td>
<td>64.31</td>
<td>41.76</td>
<td>66.08</td>
<td></td>
</tr>
<tr>
<td></td>
<td>65.49</td>
<td>42.74</td>
<td>69.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td>69.03</td>
<td>46.48</td>
<td>71.78</td>
<td></td>
</tr>
<tr>
<td></td>
<td>74.34</td>
<td>51.79</td>
<td>76.11</td>
<td></td>
</tr>
<tr>
<td>.070</td>
<td>42.48</td>
<td>19.68</td>
<td>74.34</td>
<td></td>
</tr>
<tr>
<td></td>
<td>47.20</td>
<td>24.40</td>
<td>81.42</td>
<td></td>
</tr>
<tr>
<td></td>
<td>73.75</td>
<td>50.25</td>
<td>57.12</td>
<td></td>
</tr>
</tbody>
</table>

A-- Love center-time minus continental travel time.
B-- Rayleigh " " " " " " " ".
E... A Study of Microseisms

Recently, Dr. K. E. Haq (1954) in a doctor's thesis at M.I.T. has clearly shown that the microseisms are generated by standing ocean waves which are formed by a system of traveling waves of approximately the same period coming from opposite directions, as originally proposed by Longuet-Higgins (1950). His investigations and arguments, in addition, completely negate microseisms being generated by the pounding of surf against a rocky coast, which method has been vigorously defended by Gutenberg and others.

Michel (1944) has shown from a theoretical study of wave motion that the ocean bottom beneath a train of standing waves of the form

$$\xi = a \cos kx \cos \beta t + O(a^2)$$

where

$$\beta^2 = \frac{gk \tan kh}{2\pi}$$

$$\lambda = \frac{2\pi}{k}$$

$$Z = z \quad \ldots \quad \text{the ocean surface}$$

$$Z = -h \quad \ldots \quad \text{the ocean bottom}$$

will be subjected to a fluctuating mean pressure,

$$\frac{P - P_0}{\rho} = gh - \frac{1}{2} a^2 \beta^2 \cos 2\beta t + O(a^3)$$

whose frequency is twice the frequency of the waves on the surface and whose amplitude is proportional to the square of the amplitude of the stationary waves on the surface. Haq (1954) has already shown this frequency
relationship to be a matter of fact in his investigations of the power density spectra calculated from microseisms recorded at Weston Observatory, Weston, Mass. and from swell records obtained from Woods Hole, Mass. and Gilgo, L. I.; and also by comparing periods of the microseisms with those of the ocean waves reported from the weather ship 4YH, located 36°N 70°W.

Theoretical considerations show that if the depth of water

$$h = \left( \frac{n^2}{2} + \frac{1}{4} \right)$$

where

- $h$ .... depth of water
- $\lambda$ .... wave length of compression al waves in water
- $n$ .... 0, 1, 2, ..........

the amplitude of microseisms may increase by a factor of 4 - 5 due to resonance effects. We can see, then, that if standing waves are situated over an ocean floor which is variable in depth, many frequencies will give rise to resonance and the generated microseisms will contain wide band of frequencies. Should the standing waves occur in those regions of the ocean whose depth is constant the microseisms arising therefrom will exhibit narrow band frequency spectra. Further speculation seems to indicate that if the area of origin is near to the recording station those frequencies which are not those of resonance and which are relatively of minor importance will also contribute to the
broadness of the microseisms frequency spectra. As this generating area becomes further removed from the station those frequencies, and especially the higher ones, which are not favorable to resonance, will become of increasingly less importance due to effects of decay and absorption.

In view of the aforementioned considerations we would in actuality expect to find for a storm proceeding from the land but over the continental shelf into deeper and deeper water, microseisms frequency spectra displaying in time increasing narrowness, and increasing predominance of the lower frequencies. It has been Haq's observation "that at the beginning of a storm, microseisms are usually low in amplitude and very irregular in character, at which times the fronts are usually on the shelf or at the edge of it. As the fronts move into the deep water regions, where the depth is more uniform, the microseisms increase in amplitude and look like regular pulses. This shows that the depth of water in the generating area has a significant effect on the nature of microseisms." (Above is evident on plate 1.)

We have in an effort to confirm the veracity of these speculations, calculated the traveling amplitude spectra -- of the two minute intervals -- and power density spectra -- of thirty second intervals -- of four samples (separated by approximately twelve hours) of microseisms. The storm
in question was recorded at Weston December 11th through December 13th, 1953, and the seismograms from which the samples were taken were the long period verticals. Photostatic reproductions of those portions of the microseism record utilized may be found on plate (11); the actual two minute samples employed in this study occur between the superimposed brackets. The progress of the storm's intensity as far as microseisms and wave heights at 4 Y H are concerned are depicted in fig. (19) (Courtesy of K.E. Haq).

In general, the storm was caused by the eastward passage of a cold front from the coast of the United States into the Atlantic Ocean; the actual details of meteorology, and the method of generation of standing ocean waves in this storm have been adequately described by Haq in his "Case 1". According to Haq, standing waves were generated on the shelf at approximately 16:00 G.S.T. Dec. 11th, over the slope at about 4:00 G.S.T. Dec. 12th, in deeper regions at 16:00 G.S.T. Dec. 12th, and farther into the ocean at approximately 4:00 G.S.T. Dec. 13th.

The traveling spectra figs. (20) through (21) computed in the vicinity of these respective times show that, as time proceeds, the spectra become more sharply peaked. Figs. (22) -- (25) depict this fact in two dimensional form. The two resonant peaks at .225 and .36 C.P.S. at 16:00 Dec. 11th would seem to indicate that the standing waves had proceeded farther out onto the shelf than Haq had expected from
meteorological considerations at that time. At 4:00 Dec. 12th we see that the major contribution of frequencies exists in the band .135 -- .36 C.P.S. with a maximum at .247 C.P.S. At 16:00 G.S.T. Dec. 12th the band of frequencies is .147 -- .36 C.P.S. with the major contribution from .225 C.P.S. In fig. (25) we see that the same band of frequencies, with the same maximum, occurs at 4:00 G.S.T. Dec. 13th, although the contribution from higher frequencies (.63 -- .81 C.P.S.) is decidedly less.

This study shows rather conclusively that in one instance, the resonating phenomena expected from considerations of the standing wave generation of microseism is indeed a fact. In addition to this, if the nature of microseisms and their spectra are studied as the storm proceeds we see that additional evidence is had for the standing wave theory of generation. For at the beginning of the storm the microseisms are irregular and the spectra are of the wide band type as the storm proceeds into deep water the microseisms appear as pulses and the derived spectra are more narrow with major contributions at lower frequencies. Such phenomena could hardly be expected from surf action on a rocky coast. As a matter of fact no change would be evidenced, as already shown by Haq's spectra on this matter.
Microseism amplitude at Weston

Trace amplitude of waves at Woods' Hole

Wave height in feet at 4YH

Graph 1.
Traveling spectra of a two-minute interval in the vicinity of 16:00 (GST) Dec. 11, 1953.

Traveling spectra of a two-minute interval in the vicinity of 16:00 (GST) Dec. 12, 1953.
Traveling Spectra of a Two Minute Interval in the Vicinity of 4:00 (EST) Dec. 12, 1953.

Fig. (21)
POWER DENSITY VS. FREQUENCY
OVER A 30 SEC. INTERVAL IN THE
VICINITY OF 16:00 DEC. 11

Fig. (22)
POWER DENSITY VS. FREQUENCY OVER A 30 SEC. INTERVAL IN THE VICINITY OF 4:00 DEC. 12

Fig. (23)
DENSITY VS. FREQUENCY
OVER A 30 SEC. INTERVAL IN THE
VICINITY OF 16:00 DEC. 12

Fig. (24)
POWER DENSITY VS. FREQUENCY
OVER A 30 SEC. INTERVAL IN THE
VICINITY OF 4:00 DEC. 13

Fig. (25)
The Phase Spectra of Seismograms.

We would, at this time, like to report that the high hopes we entertained for traveling phase spectra, which we have already described in the first chapter, did not in actuality materialize. Possibly improvements of our techniques in this respect or applying it to other branches of geophysics may bear more fruit. For example, studies of the phase relations between variations in telluric current and the earth's magnetic field as suggested by Cagniard (1953) and Wait (1954) may be profitable.

We present, nevertheless, the results obtained for traveling phase spectra in one instance from our analysis of a quake which occurred in the N. Gulf of California (already described) in Plate A solely for the purposes of illustration.
G... A Study of Seismic Prospecting Records

The success achieved in a foregoing section of correlating high amplitude spectra pulses with reflected and refracted energy of P and S phases of earthquakes suggests the possibility of using such means as travelling spectra to ascertain the presence of reflections not visible on prospecting seismograms. It seems doubtful at the present whether such a procedure would be able to pinpoint a reflection to the millisecond, but it would be of use in some cases in determining positions of the record which do not constitute reflected energy. To these regions then it would be possible to fit a linear operator for purposes of filtering the record to better enhance the reflections. Thus far the determination of the regions of the record over which such operators are fitted has been rather arbitrary.

All the records which we have analyzed in this section have had data points read every 2.5 milliseconds. The travelling spectra have been calculated from successive forty point intervals overlapping by 75%. It is possible that such a technique could be enlarged on to bring out better portions of the record consisting of reflected energy, times of arrival and characteristics of each reflection, by taking shorter intervals with more closely read points, by overlapping intervals a greater amount, by investigating high frequency contribution, etc.

(*) The Geophysical Analysis Group of M.I.T. has devoted much attention and research in this direction in recent years. Their reports #1--#5 and the paper by Wadsworth, et al. (1953) explain this research quite thoroughly.
However, we will present our results that have been obtained to date for three such records.

(1) Presentation of the Results.

(a) A Record Containing Many Visible Reflections

Recently the Atlantic Refining Co. has supplied a number of records from Roosevelt County, New Mexico to the Geophysical Analysis Group for analysis. All the records exhibit a large number of reflections. The record chosen here was M.I.T. Record 7.4.

Twenty equally spaced groups of three seismometers each were used in a straight line in each spread, the interval between groups being equal to the group length. The frequency response curve of the amplifier and filter used on these records is given in fig. (26). Mixing in this record consisted of the addition of one half the output of each group of three seismometers connected in parallel to the output of the group next farthest away from the shot point. Each trace represents the recording of the output of such a combination. A reproduction of Record 7.4 may be found in Plate (12).

The results of a velocity survey taken in the locality are given in fig. (27). There is also given in this figure the tops of geologic formations present in this area.

The results of our computation for traces 1, 4, 7, 10, 13, 16 and 19 depicted photographically on plate (13).
We have also actually contoured the frequency time plot of trace 1 for purposes of comparison with the corresponding density plot. This contour is found on fig. (28).

In the following table we have listed the times of arrival of the reflections obtained from the record, together with traces whose traveling spectra seem to put the reflection into evidence by large power contribution at a particular time.

**TABLE XIV**

**TRACES WHOSE TRAVELING SPECTRA EXHIBIT REFLECTIONS BEST**

<table>
<thead>
<tr>
<th>Reflection Time</th>
<th>Trace</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.690 sec.</td>
<td>T - 1, 7, 10, 16</td>
</tr>
<tr>
<td>0.820 - 0.830</td>
<td>T - 1, 4, 7, 10, 13, 19</td>
</tr>
<tr>
<td>0.880</td>
<td>T - 1, 13, 19</td>
</tr>
<tr>
<td>0.970</td>
<td>T - 1, 2, 7, 10, 13, 16, 19</td>
</tr>
<tr>
<td>1.030</td>
<td>T - 13, 16, 19</td>
</tr>
<tr>
<td>1.110 - 1.120</td>
<td>T - 1, 4, 13, 16, 19</td>
</tr>
<tr>
<td>1.180</td>
<td>T - 1, 4, 7, 10, 13</td>
</tr>
<tr>
<td>1.260</td>
<td>T - 7, 10, 13, 16, 19</td>
</tr>
<tr>
<td>1.330 - 1.350</td>
<td>T - 19</td>
</tr>
<tr>
<td>1.360</td>
<td>T - 4, 7</td>
</tr>
<tr>
<td>1.380</td>
<td>T - 1, 4, 7, 10, 13, 16, 19</td>
</tr>
<tr>
<td>1.430</td>
<td>T - 13, 16, 19</td>
</tr>
<tr>
<td>1.800</td>
<td>T - 10, 13, 19</td>
</tr>
</tbody>
</table>

(b) Record Containing Few Visible Reflections

This record was supplied by Magnolia Petroleum Co. to the Geophysical Analysis Group for analysis, the results of which have already been reported by Wadsworth, Robinson, et al. (1953). The only information pertaining to this record which is designated as M.I.T. Record No. 1 or 10.1 may be found on the reproduced record, Plate (14).
The traveling spectra for traces 1 - 6 of this record may be found on plate (15). Our ability to determine reflected energy by means of these analysis is recorded in the following table, which, as in the foregoing section, displays those traces from which spectral analysis was best in this regard. Reflection times were picked by Magnolia and are shown on plate (14).

**TABLE XV**

**TRACES WHOSE TRAVELING SPECTRA EXHIBIT REFLECTIONS BEST**

<table>
<thead>
<tr>
<th>Reflection Time</th>
<th>Traces</th>
</tr>
</thead>
<tbody>
<tr>
<td>.51 - .54 sec.</td>
<td>T - 1 to 6</td>
</tr>
<tr>
<td>1.00 - 1.04</td>
<td>T - 1 to 6</td>
</tr>
<tr>
<td>1.16 - 1.24</td>
<td>T - 1 to 6</td>
</tr>
</tbody>
</table>

(c) Record Containing No Visible Reflection

This record was also furnished to the Geophysical Analysis Group by Magnolia Petroleum Co. from a prospect in Henderson Co., Texas. The charge used was 5 lbs. at 295 ft. Each trace represents the recording of nine geophones. The record is shown in plate (16) and is designated as 10.9.

Reflection times on the top trace T 1 are marked by . The times were determined by Magnolia from a different shooting procedure.

Our traveling spectra for the top four traces are shown in plate (17). A comparison of the traveling spectra and the reflection times on plate (16) shows that all five are displayed fairly well by the spectral analysis of traces.
It is evident from our foregoing analysis that much can still be done to perfect our method, if it is, in the future to be seriously considered as a means to assist in the determination of reflected or refracted energy. What has transpired thus far should be regarded as experimental, the hope being that what has been presented here will stimulate more research in this direction.
FREQUENCY RESPONSE
25A AMPLIFIER
FILTER SETTING: 35-55

Records 7.1 to 7.6

Fig. (26)
This velocity function is based on information obtained in the immediate vicinity of the Elida records, but penetrated only to approximately 7,000 feet.

ELEIDA AREA

Fig. (27)
TRAVELING SPECTRA

G.A.G. RECORD 7.4
TRACE I

TIME IN TENTHS OF A SECOND
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BIOGRAPHICAL NOTE

Graduated from Boston College, June 1950 with an A.B. in Physics. In Sept. 1950, entered the Department of Geology and Geophysics, Massachusetts Institute of Technology. Associated with the Geophysical Analysis Group in the capacity of a research assistant from 1952 to 1954.
Appendix A
APPENDIX A

Description and Use of the Power Spectrum Programs

The following outline separates these programs into two groups - the separation being based on the nature of the data which is to be operated on. In each case, whether the output be a spot on a photograph or in digital form, the information is representative of the square of the amplitude spectrum or the power spectrum.

I. A. Input: Auto correlation function

B. Output: An option exists for the use of either direct or delayed printer. In particular there is for the smoothed spectra --

<table>
<thead>
<tr>
<th>Code</th>
<th>Type</th>
<th>Lags</th>
</tr>
</thead>
<tbody>
<tr>
<td>106-43-05018</td>
<td>Direct</td>
<td>20</td>
</tr>
<tr>
<td>106-43-05019</td>
<td>Delayed</td>
<td></td>
</tr>
<tr>
<td>106-43-05014</td>
<td>Direct</td>
<td>40</td>
</tr>
<tr>
<td>106-43-05015</td>
<td>Delayed</td>
<td></td>
</tr>
<tr>
<td>106-43-05024</td>
<td>Delayed</td>
<td>80</td>
</tr>
<tr>
<td>106-43-05025</td>
<td>Direct</td>
<td></td>
</tr>
<tr>
<td>106-43-05016</td>
<td>Delayed</td>
<td>100</td>
</tr>
<tr>
<td>106-43-05017</td>
<td>Direct</td>
<td></td>
</tr>
</tbody>
</table>

C. General: Should additional spectrum programs be desired it will be necessary to prepare a table of cosines to be stored in registers 1310 - 1372 and also to present the following registers:
A "filing" flexor character if so desired.

where \( M \) = no. of spectra to be calculated

\( \hat{M} \) = no. of autocorrelations to be used in the calculation of each spectrum.

e.g. for 100 lags \( N = 101 \)

Furthermore if an option is desired between a smoothed and unsmoothed spectrum the following registers should be present

263 - sp. 274 --- if just the smoothed spectrum is desired

1062 - sp. 1071 --- if just the unsmoothed spectrum is desired

The following tapes are then attached in the order indicated below

1) 106-43-05008 (Basic Program)
2) 106-43-05010 (Factoring Routine)
3) a. 106-43-05011 - if direct output is desired
   b. 106-43-05003 - if delayed
4) Tape containing the aforementioned cosine table and preset registers.

D. Preparation of Data: to prevent over-flow it is necessary to factor the data to the form \( \pm \cdot 0XXX \)
e.g. $\left( \dfrac{+35789}{+0.0358} \right)$
after the mean has been subtracted. Thereupon the first
and last value of those values to be used in the calculation
of one spectrum are divided by two.

Having accomplished the foregoing these values
\[ \frac{R_0}{2}, R, \frac{R}{2}, \ldots, \frac{R_M}{2} \]
are placed in registers,

\[(1373) \quad \text{----} \quad (1373 \div N)\]

The values of R for the following spectra to be calcu-
lated are placed in succeeding registers.

Registers 1373 - to - 3777 are to be used for data
storage.

E. Error: All values calculated are in error by approxi-
mately $\pm 0.3$ percent.

All values calculated (E's or U's) should be multiplied
by $2x$, where $x$ is the reciprocal of the data scale factor.

F A Typical Performance Request:

1) E Si 1 "on"
2) 106-43-05018
3) 106-(-)-(--)- Data Tape
4) Put (-M) in Reg. 722 (octal) - (this is done if
   722 has not been preset on data tape)
5) S.A. 130

where M is the no. of spectra to be computed.
II. A. Input: Equally spaced trace readings.

B. Output: There are a number of options in this case as is evidenced below. The means of distinction lies in that S.A. request which is starred in F. of this section.

a) Scope or Density Plot Output

<table>
<thead>
<tr>
<th>OUTPUT TO BE USED WITH</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Four pictures 96 Spectra</td>
</tr>
<tr>
<td>2. Two pictures 48 Spectra</td>
</tr>
<tr>
<td>3. Two pictures 48 Spectra</td>
</tr>
</tbody>
</table>

b) Direct Print Output

1. 40 lag tape  S.A. 3246  Present 3107
2. 80 lag tape  S.A. 3232

If data for each spectrum desired are on separate tapes, then for:

1. 40 lag tape  S.A. 3212  S.A. 3007 for additional calculation
2. 80 lag tape  S.A. 3200  S.A. 3007 for additional calculation

c) Delayed Print Output

1. 40 lag tape  S.A. 3253  Preset 3107
2. 80 lag tape  S.A. 3242

If data for each spectrum desired are on separate tapes, then for:

1. 40 lag tape  S.A. 3226  S.A. 3007 for additional calculations
2. 80 lag tape  S.A. 3223
In each of the above cases the following tapes alone may be used:

- 106-43-05027 40 lag combined tape
- 106-43-05026 80 lag 
- 106-43-05028 100 lag 

C. General: Should additional combined tapes warrant construction, instructions given in (I) of the preceding section should be followed in the development of a fundamental tape.

Thereupon the following registers should be preset:

1401 + M'
1402 + K
1412 - N
1413 - N'
1414 - N'
1415 - N'
1416 - N'
1424 - M
1425 - M

Where:
- \( M \) = no. of spectra to be calculated from each tape
- \( K \) = no. of points to be skipped in each succeeding spectrum calculation
- \( M' \) = lag no.

If scope output slope is desired speedier calculation may be obtained by preseting registers --

733 to (-42)
734 to (-42)

The desired tape is then a combination of the following...
1. fundamental tape
2. 106-43-05021 = Autocorrelation routine
3. 106-43-05023 = Control routine

D. Preparation of Data: All data should be in the form of decimal integers. In general the data tapes should be prepared much in the same way as previous GAG "data" or "trace -reading" tapes, i.e.

\[
\begin{align*}
1054 & \quad \text{Mean} \\
1055 & \quad \text{Data} \\%
S.A. 1033
\end{align*}
\]

In particular, if density plots or printouts of "traveling spectra" be desired, it would be advisable to prepare tapes containing

a. 511 Successive readings for 40 lag spectra
b. 541 " " " 80 " "
c. 676 " " " 100 " "

If this is accomplished it will be possible to calculate

a. 24 -- 40 lag spectra with 10 pt. skips
b. 24 -- 80 " " " 20 pt. "
c. 24 -- 100 " " " 25 pt. "

E. Error: All digital calculations are about ± 0.3 percent in error.

To obtain the true magnitude of each line spectrum a multiplication factor of 20,000 would be in order.
F. A Typical Performance Request:

a. Scope Output or Print-out of More than One Spectrum

1) E, Si 1 "on"

2) 106-43-05027
   106-43  26 RI
   106-43  28

3) Put (106 --------) (data) in petr

4) a. R.S. -- (if density plot is to be used)
   b. S.A. -- (if print-out) -- (if B.)

5) Put (106 --------) (Next data tape) in petr
   immediately.
   Si L "off"

   After computation ceases (s:o 3002)

6) E, Si 1 "off"  First 30 points of each spectrum

7) 3451 P 12 RI, RI

6) E, Si 1 "on"

7) 3451 P 12 RI, RI  First 41 points of each spectrum

8) 3451 P 15 RI, RS

N.B. 6 - 8 are requested only in the case of scope output.

b. Print Out of One Spectrum at a Time

1) E, Si 1 "on"

2) 106-43-05022
   106-43-05026  RI
   106-43-05028

3) Put - (106 --------) (data) in petr.

4) S.A. ------- (of. B.)

   If additional "one-spectrum" data is had request;

5) Put (106 --------) (nest data tape) in petr.

6) After above computation ceases (Si 0 130)
   S.A. 3007
N.B. The above 4 picture output requests assume that the "phase density" routine in the density plot program (3451 P 12 etc.) has been altered to operate on amplitude spectra. If 3451 P 12 is used and picture 3 and 4 are desired it will be necessary to do the following after pictures 1 and 2 have been obtained.

a) E, Si 1 off
b) 106-43-050 RI
c) 3451 P 12 RI, RI
Appendix B
APPENDIX B

Description and Use of the Amplitude and Cross Spectrum Programs

I. Amplitude Spectrum

A. Input: Discretely spaced points or trace readings.

B. Output: At present options exist for delayed printer and scope (density plot) or scope output only. In each case the spectra attained are representative of spectra of overlapping intervals. We have the following:

1) Scope

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>&quot;lags&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>106-43-25</td>
<td>Smoothed spectrum</td>
<td>40</td>
</tr>
<tr>
<td>106-43-06008</td>
<td>Unsmoothed</td>
<td></td>
</tr>
<tr>
<td>106-43-06009</td>
<td>Smoothed</td>
<td></td>
</tr>
<tr>
<td>106-43-06016</td>
<td>Unsmoothed</td>
<td>80</td>
</tr>
<tr>
<td>106-43-06011</td>
<td>Smoothed</td>
<td>100</td>
</tr>
<tr>
<td>106-43-06013</td>
<td>Unsmoothed</td>
<td></td>
</tr>
</tbody>
</table>

The 40 "lag" programs have a 10 point skip between intervals and calculate 48 spectr. The density plot routine displays two amplitude spectra pictures - 24 spectra each - and two phase spectra (corrected) pictures - 24 spectra each.

The 80 "lag" programs, above, have a 20 point skip distance, and calculate 24 spectra. The density plot displays two pictures - one amplitude and one phase spectra (corrected).

The 100 "lag" programs have a 25 point skip between successive intervals, and calculate 24 spectra. The density plot is the same as for the 80 "lag" programs.

We have in addition to the above, for 80 "lags".

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>&quot;lags&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>106-43-06022</td>
<td>smoothed</td>
<td>80</td>
</tr>
<tr>
<td>106-43-06023</td>
<td>unsmoothed</td>
<td></td>
</tr>
</tbody>
</table>
These programs utilize a 10 point skip between intervals and calculate 48 spectra. The density plot in this case is the same as that mentioned for the 40 "lag" programs.

1) Delayed Printer and Scope

- 106-43-06014 - Smoothed 40 "lags"
- 106-43-06015 - Unsmoothed
- 106-43-06017 - Smoothed 80 "lags"
- 106-43-06018 - Unsmoothed
- 106-43-06019 - Smoothed 100 "lags"
- 106-43-06020 - Unsmoothed

The output for this option will be the "digital" spectra the phase and amplitude spectra of the intervals involved. The print-out is in this order.

We have for

a) 40 "lags" a 10 point skip and a total of 48 spectra
b) 80 "20 " " " " 24 "
c) 100 "25 " " " " 24 "

Should, in addition to the above the cosine and / or sine transforms be desired, ca 766 (octal) should be returned to 400 (octal).

In the above description "smoothed" and "unsmoothed" spectrum refers to the cosine and sine transform's being smoothed or not being smoothed by the Tukey method individually, before calculation of the amplitude or magnitude.

Also the term, "lag", is synonymous with "discretely spaced points".
C. General: All of the afore mentioned tapes are combined tapes. The general make-up of each is somewhat as follows:

I Data Factoring Routine

This factors the data and stores same on the magnetic drums, in such a way as to be appropriate for computation of intervals which overlap by 75 percent.

II Phase Amplitude Calculation Routine (3436 M 21 3436 M 17)

This routine is read in and then thrown on the drums. On the scope output tapes before the "sp block" we have the following

606 sp 615 - circumvent amplitude print out
320 -41
321 -41
514 -40
515 -40
516 -40

460 sp 2251 This must occur before the "sp block" in all cases.
473 sp 2067

III Tangent, Angle, Sine, and Sine Sign Table (Tape 3436 M3)

This routine is read in and immediately stored on the drums. Depending on the number of points per interval as new sine table is added on before the "sp block". The values of the sines to be added will be for $0^\circ, \frac{180^\circ}{n}, 2\left(\frac{180^\circ}{n}\right)\ldots$ where $n$ equals the number of points per interval. The number of points per interval is not to exceed 100.
IV Sine and Cosine Transform Calculation Routine

1 (Tape 3436 M20)

2 Data Read in Routine and Phase Correction Routine (Tape 106-43-18)

A cosine table (the reverse of the above sine values) is added. The first value being placed in 1000 (octal).

On the scope output tapes the following modifications occur before the sp block:

733 \(-45\)
734 \(-45\)
2067 sp 2070
2100 sp 2101
2127 sp 2130
2166 sp 2176
2423 sp 473
2110 ad 2472
2137 ad 2472
24.72 \(\dagger 41\) (decimal)

In addition to the above changes we have for the 80 and 100 lag tapes,

2213 sp 2220
2223 -9 After third sp block
2415 -14
627 -9 After first sp block

Registers 560 through 563 contain drum and register of the first factored (halved) value of the overlap in question. In explanation, four drum groups are used for the factored data each having appropriate and different values factored by .5. Diagramatically, the data for each spectrum is stored as follows:

\[ g(o) \] ← Drum Groups

Registers containing halved values.
D. Preparation of Data: Data is prepared much in the same fashion as other GAG data. All points or trace readings are positive decimal integers and appear in registers 1055 etc. The mean appears in 1054.

In particular we have for:

40 lag spectra and a total of 48 spectra - 511 points
80 " " " " " 24 " - 541 "
100 " " " " " 24 " - 676 "

All data is typed for 5-56 basic conversion. A "start at 0" is usual for most data tapes. However, "start at 1033" may appear if this data is to be used with the autocorrelation spectrum programs.

E. Typical Performance Requests

1) Scope Output - (40 lags)

1) E. Si 1 "off"

A 2) fb 106-43 -----Data - RI
3) fb 106-43 -25 RI, RI, RI, RI
   -- After computation classes - (Si 0 2153)
B 4) E, Si 1 "off"
5) 3451 P 12 RI, RI

C 4) E, Si 1 "on"
5) 3451 P 12 RI, RI
6) 3451 P 16 RI, RS

B and C are the density plotting routines

B - is used should the first 31 points of each spectrum be desired

C - is used if the first 41 points of each spectrum are desired.
In this connection, we may reiterate that points of the spectrum are plotted vertically while center time or succeeding spectra are plotted horizontally.

2. Delayed Print Output

This is the same as the foregoing save that B or C are omitted.

II. Cross Spectrum

A. Input: Discrete points of the crosscorrelation function.

B. Output: Cross spectrum and phase difference. At present only delayed print output is possible; the tapes used in calculating amplitude spectra are applicable in this section.

C. Preparation of Data: Both A and B should have the mean subtracted and also be factored by .0010. They both should appear in registers 1055 ---- etc. All data for succeeding spectra should follow in succeeding registers. After all the data has been prepared it will be necessary to make up a small routine. In particular,

a) for S.A. 40

\begin{verbatim}
40 ca 45
41 si 707
42 ca 46
43 po 1055
44 si 0
45 0.34100
46 + N (the quantity of data)
\end{verbatim}

b) for S.A. 40

\begin{verbatim}
40 ca 45
41 si 707
42 ca 46
43 po 1055
44 si 0
45 0.41000
46 + N (the quantity of data)
\end{verbatim}
The above should be attached to each tape.

Instead of the aforementioned it is possible to prepare the cross-correlations as other GAG data tapes - mean in 1054, first number in 1055 etc. in decimal integers.

If the above is the alternative the following presetting of registers must occur, on each tape (one for $\phi_{MN}$ and one for $\phi_{NM}$).

\[
\begin{align*}
74 & = (N-1) \\
77 & + A \\
100 & = (M-1) \\
102 & = M \\
104 & + N \\
105 & 0.34100 \quad \text{for } \phi_{NM} \\
& 0.41000 \quad \text{for } \phi_{NM}
\end{align*}
\]

*WHERE*: \(N\) = no. of data points

\[
M = \frac{N}{A} + 1
\]

\(A = +40; +80; +100 \) (depending on what lag spectra is desired)

These tapes are then to be used in conjunction with the cross-correlation factoring routine - 106-43-06023.

D. Typical Performance Requests

\[
\begin{align*}
\text{E Si 1 "off"} \\
106 - 43 - (\phi_{MN} \text{ data}) \text{ RI} \\
106 - 43 - (\phi_{NM} \text{ data}) \text{ RI} \\
\text{Si 1 "on"}
\end{align*}
\]

-- Starting at second block--

\[
\begin{align*}
106-43-(XXXXX) \text{ RI, RS; RI, RS; RI} \\
106-43-05030 \text{ RI, RS}
\end{align*}
\]
The above supposes that the data has been factored and prepared as mentioned in the foregoing section. If it has not, and it appears as decimal integers then the following request would be appropriate.

E, SI 1 "off"
106-43- ($\Phi_{MN}$ data) RI
106-43- 06023 RI
106-43- ($\Phi_{NM}$ Data) RI
106-43- 06023 RI

si 1 "On"
Starting at second block
106-43-(06xxx) RI, RS; RI, RS; RI
106-43-05030 RI, RS.
Appendix C
APPENDIX C

Weston Observatory -- Pertinent Data

Geodetic coordinates: 42° 23' 04.9" N
71° 19' 19.5" W

Elevation: 60 meters

Lithologic foundation: Metavolcanic

Pendulum mass: 100 kg.


Normal Operating Constants:

<table>
<thead>
<tr>
<th>Instrument</th>
<th>$T_o$ sec.</th>
<th>$T_g$ sec.</th>
<th>Drum Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZSP</td>
<td>1.0</td>
<td>0.5</td>
<td>60 mm/min</td>
</tr>
<tr>
<td>NSP</td>
<td>1.0</td>
<td>0.25</td>
<td>60</td>
</tr>
<tr>
<td>ESP</td>
<td>1.0</td>
<td>0.25</td>
<td>60</td>
</tr>
<tr>
<td>ZLP</td>
<td>1.0</td>
<td>30.0</td>
<td>30</td>
</tr>
<tr>
<td>NLP</td>
<td>1.0</td>
<td>60.0</td>
<td>30</td>
</tr>
<tr>
<td>ELP</td>
<td>1.0</td>
<td>60.0</td>
<td>30</td>
</tr>
</tbody>
</table>

$T_o$ -- Period of pendulum.

$T_g$ -- Period of galvanometer.