Mutual Fund Trading and Liquidity

by

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Abstract

This thesis uses equities holdings snapshots of mutual funds to study their trading patterns. Using quarter and semi-annual holdings of mutual funds, I am able to extract a main trading component with the application of the asymptotic principle component method. This component demonstrates short term predictability of returns over three months, suggesting overall mutual fund trades contain a liquidity trading component that temporarily pushes up stock prices that reverse over the next few months. I also demonstrates that this particular type of liquidity risk is related to other measures of liquidity risk. Therefore, this trading component can be a useful building block in creating a comprehensive measure of liquidity.

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Chapter 1

Introduction

1.1 Background and motivations for research

There is extensive literature on liquidity. Broadly defined as the ability to trade quickly and at low cost, episodes such as the breakdown of Long Term Capital Management and the recent mortgage meltdown highlighted the downside of holding illiquid assets, and question whether aggregate liquidity should be a risk factor in asset pricing. In light of the unfavorable nature of holding illiquid assets, it appears that aggregate liquidity risk should be a priced risk factor in the economy. This is further supported by the recent credit crisis, where the cost of liquidity drives several investment banks out of existence, and a market crash that is not seen since the Great Depression.

One branch of the empirical literature aim to identify stocks with different level of liquidity through characteristics and compare their returns. Examples of these includes Amihud(2002), Amihud and Mendelson (1986), Brennan and Subrahmanym (1996), Brennan, Chordia and Subrahmanym (1998), Datar, Naik, Radcliffe (1998), etc. Another branch sought to identify liquidity-based trading directly and study its effects. These includes Campbell, Grossman and Wang (1993), Lo and Wang(2000,2006), Pastor and Stambaugh (2003) etc., usually by stock volume. The difficulty of these papers is
that volume is not signed, and one can only observe total trades of a stock, which is the sum of liquidity-based trades and information-based trades.

On the other hand, there is research making use mutual fund holdings. In a series of papers Daniel, Grinblatt, Titman, and Wermers showed that mutual fund trading is related to momentum, and momentum could possibly explain most of the alphas of funds. Lamont and Farazzini (2006) finds that stocks that are bought up by mutual funds have low return in the long run.

This paper aim at bridging the gap between these two literatures, using mutual fund trading as a better proxy for liquidity trades and study its asset pricing implications. I consider mutual fund trading as a candidate for this study for a number of reasons. Mutual funds made up a substantial part of the US stock market. Equities trading mutual funds are all long in the market and use very little leverage or short selling. Also, it is known that mutual funds are forced to engage in liquidity trades. If a fund’s focus is technology stocks and the fund experience large inflow, then the manger is forced to buy stocks in the technology sector even if he believes that the sector is overpriced overall. The fact that many funds index their performance to benchmark such as the S&P 500 also encourage funds to rebalance their portfolio when the benchmark changes, or in the case of index funds, essentially forcing them to engage in liquidity trades that creates short term price fluctuations. (Petajisto 2008)

I find that a liquidity component can be extracted from quarterly change of portfolio holding of mutual funds. This factor is able to predict market return, share the properties of many commonly used proxies for liquidity. In addition, in cross sectional test I found some evidence of the sensitivity of liquidity risk is priced in the cross section.

In this paper I do not investigate the question whether (individual) mutual fund outperforms the market. Instead I investigate if aggregated mutual fund trades contain component(s) that are related to market-wide liquidity. The paper is organized as follows: Chapter 2 discuss the data, a model that relate liquidity trades to stock returns and the asymptotic principle component method I employ to obtain the liquidity factor. Chapter
3 presents the time-series and cross-section results I obtained.

1.2 Related Literature

My research is related to the papers on the relation between returns and volumes. Some early paper focuses on relations between daily market returns and volume (For example, Duffee(1992), Gallant, Rossi and Tauchen(1992) and Campbell, Grossman and Wang (1993)). These paper finds that returns on high-volume days have a tendency to reverse in the next few days. Llorente, Michaely, Saar and Wang (2002) finds that returns accompanied by high volume tends to reverse itself for firms with low level of information asymmetry and that the opposite takes place for firms with high information asymmetry. These paper focuses on daily frequency of returns. My paper uses return of longer horizon and also use the actual trading of the mutual fund industry. This has the benefit of distinguishing buys from sells, whereas in volume both buys and sells contribute positively to volume.

This paper is also related to the literature on mutual fund performance and flows. A large body of the literature is about whether it is possible to pick mutual funds that consistently outperform it peers. Grinblatt and Titman (1989a,1993) finds that some mutual fund managers are able to consistently earn positive abnormal returns before fees and expenses. A collection of paper using a large database of mutual fund holdings [Grinblatt, Titman and Wermers (1995), Wermers (1997), Chen, Jegadeesh and Wermers(2000)] finds that majority of mutual funds to pursue an active momentum strategy and that the best-performing funds use their inflow to buy winner stocks that produces superior returns\(^1\). Consistent with this, Cahart(1997) finds that after controlling for momentum, there is no predictable abnormal returns in the best funds. To the opposite

\(^1\)Other research has reached a similar conclusion using a different data set which contains the trading of all institutions. For reference, see Burch and Swaminathan (2001), Badrinath and Wahal (2002) and Sias (2004, 2007)
end, the paper of Cahart, and the papers by Brown and Goetzmann(1995), Berk and Tonks (2007) also documents that the worst performing fund seems to consistently underperform in the future. However, that may be a topic of lesser interest because there is no way to short these bad mutual funds. Recently there are papers that argues that it is possible to pick fund managers more sophisticated methods. Theses includes: Mamaysky, Spiegel and Zhang (2004), which correct for systematic bias in estimating risk-adjusted fund performance; Aramov and Wermers (2004), which uses a trading strategy that incorporate predictable manager skills related to industry effect; Kacperczyk, Sialm and Zheng (2005) found that funds with higher industry concentration holdings have higher returns both in a 4-factor model and with the DGTW performance measure (in stock selectability); and Cohen, Coval and Pastor (2005), where they distinguish funds that outperform due to commonality of the stocks held from manager who outperform due to stocks traded in the manager’s portfolio.

Another class of the literature is focused on the relation between fund returns and fund flows, as defined as new money invested in a mutual fund (inflow) minus the net withdrawal from a fund (outflow). Coval and Stafford (2007) found that stocks being sold by distressed funds underperform in the short term (with reversal), confirming an asset fire-sale story. My paper also address to this area of research. On one hand, while manager picks stocks within their fund objective, a large part of their trades are flow driven (for example, if a technology stock fund manager receive a large investment, he has to invest mainly in technology stocks no matter what he thinks of the relative value of technology stocks to stocks in other sectors). Therefore, even if managers are able to pick stocks, that does not mean all their trades are information based. My paper studies the effect of the overall trading and try to access whether the majority of trades are based on superior information.

This paper is also related to the literature on liquidity. Liquidity is a vague concept related to cost related to execution of trading, and much of the literature focuses on discovering measures of liquidity and whether liquidity risk is priced. For example,
Chordia, Sarkar and Subrahmanyam (2005) explores cross-market liquidity dynamics by estimating a vector autoregressive model for liquidity (bid-ask spread and depth, returns, volatility, and order flow in the stock and Treasury bond markets). Pastor and Stambaugh (2006) used price reversal caused by volume to measure stock liquidity, and Lo and Wang (2006) studies daily returns to identify hedging portfolios and test if they can predict future returns, and if they are priced in the cross-section. My paper contribute to the literature by arguing that useful information about aggregate market liquidity can be gathered from trades of mutual funds, supplying additional information to identify all sources of liquidity risk.
Chapter 2

Data and research design

2.1 Data

2.1.1 Data description

I obtain mutual fund holdings from the Thompson financial CDA/Spectrum Mutual Fund Holding database. This database covers almost all historical domestic mutual funds plus about 3,000 global funds that hold a fraction of assets in stocks traded in U.S. exchanges as well as Canadian stock market. It collects holdings data for each fund from their SEC N-30D filings, starting from 1980. Thompson financial also use fund prospectus and if necessary, contact funds for updates or additional data.

The SEC filings include the name of the stock, the number of shares held and the market value as of the reporting date. Short positions are not reported. Thompson financial merge this with their stock database to include fund information such as investment objective, total net assets, management company name and location. They also include stock information such as CUSIP, ticker, exchange code, share class, quarter end price and share outstanding. However, it has been found the stock information from Thompson Financial contain errors (Boldin and Ding 2008), therefore I only use their CUSIP field to match the data with the Center for Research in Security Prices (CRSP) monthly
stock file, and use stock characteristics supplied by CRSP instead for more accurate information.

The companies are only required to file their holdings semi-annually\(^1\), but many funds report their holdings in the other two fiscal quarters as well. These are collected in the database as well, therefore quarterly portfolio snapshots for a large proportion of funds are available.

For my analysis I imposed a few data screens:
- I only use observations at the end of calendar quarters (March, June, September and December).
- I only keep domestic funds with investment objective that is Aggressive Growth, Growth, Growth and Income, or Balanced.
- Only common stocks traded in NYSE, AMEX or NASDAQ, that can be matched to the CRSP equity data base (stock code 10 or 11). There are 15,304 such stocks in my sample.
- Data is available from June 1980 up to March 2007 for a total of 108 quarterly observations.

Overall these funds covers 46% of common stocks covered by CRSP and covers about 3% of the market cap of these stocks in 1980, and the numbers steadily grows to 90% coverage and 13% market value in 2006. See Graph 1.1 for a description of the stock coverage and market share (as a percentage to the market) of mutual funds in my sample.

Stock return, price, share outstanding are from the CRSP, book-to-market ratios from COMPUSTAT annual files, and the stock market excess returns, small-minus-big, high-minus-low and winner-minus-loser returns are from Ken French’s website.

\(^1\)Quarterly reports are required before 1985
2.1.2 Stock-level trading activity

After collecting all valid observations, I have a collection of all quarter-to-quarter change in stock position on each stock, by each mutual fund. But the data is very sparse as very few funds survive the whole sample period. They also do not hold all the stocks every quarter. To alleviate this problem, I aggregate the trading of all funds by stock in order to make the data more densely populated.

For each stock and each quarter, I aggregate the trading of all funds in the stock. Each quarter I define stock $i$’s net-trading activity ($M_{it}$) as

$$M_{it} = \frac{\sum_j(n_{ij,t} - n_{ij,\tau})}{O_{it}}$$

(2.1)

Where $n_{ij,t}$ is the number of shares (split-adjusted) held in stock $i$ by fund $j$, for all funds that reports holding data for quarter $t$, and $O_{it}$ is the split-adjusted number of share outstanding for stock $i$. $n_{ij,\tau}$ is the holding of either 1 or 2 quarters ago, using the more recent holding report available for each fund. Since the database may not report zero holding in a stock, I set all missing stock holdings to zero. Therefore, if there is a gap of 6 months between reporting, I assume the change in portfolio in the first quarter is 0 and the second quarter is the overall change. Portfolio changes with a gap more than 6 months is not used in this analysis.

To avoid errors caused by a surviving fund entering or exiting the sample, if time $t$ represent the time that a fund appears in the dataset for the first time or if $\tau$ is the last time a fund reports its holdings, then these funds are excluded in the calculation of $M_{it}$ as their comparing their reported holdings with zero has a high likelihood of creating outliers in my sample, and do not reflect the actual trading of the fund.

2.1.3 Data filters and scaling
To remove errors, outliers and to drop stocks that are not frequently traded by mutual funds, I apply these additional data screens:

- I delete all observations where fund $j$’s holding in stock $i$ is larger than its shares outstanding. This removes any obvious error.

- Stocks that are owned by very few funds are dropped. Define $OWN_{i,t}$ as the number of funds that has a non-zero holding in stock $i$, and $PCT_{10,t}$ as the 10-percentile number of $OWN_{i,t}$ at time $t$. Then, I only keep $m_{it}$ if it satisfies.

\[
OWN_{i,t} \geq \min(\max(3, PCT_{10,t}), 5) \tag{2.2}
\]

And drops stocks that are owned by very few funds. I utilize a cap and a floor because the number of funds is small in the early 1980s. I want to remove any stock with very little coverage and keep stock with sufficient coverage, while keeping the sample more balanced by adjusting the cutoff as the average stock coverage is higher.

- After filtering out stocks with low ownership, I winsorize the data by dropping the cross-sectional top and bottom 0.25% stocks (sorted by $M_{it}$) for each quarter to eliminates any remaining outlier.

Interestingly the equal- and value-weighted average $M_{it}$ have relatively low correlation (49.9%) with each other, reflecting the difference in trading between large and small stocks. Since the total size of the mutual fund industry is growing during my sample period, there are more quarters that funds are buying overall than selling. The time series averages of the value- and equal-weighted average $M_{it}$ are 0.1% and 0.2% respectively, reflecting a steady increase in mutual funds’ overall market share. The largest downward spikes are the last quarter of 1999 and 2002, and are both stronger when I consider the value-weighted average.
Additionally, the cross-section variance of $M_{it}$ increase over time, reflecting the increasing trading of the mutual fund industry. Since my measure of trading is closely related to a fund’s trading volume, I scale $M_{it}$ by dividing the it by the value-weighted average turnover of all NYSE stocks from the previous twelve months to normalize variance. Define the variance-scaled $M_{it}$ as $m_{it}$:

$$m_{it} = \frac{M_{it}}{TO_{NYSEt-12,t}}$$ (2.3)

I will be working with $m_{it}$ for the remainder of the article. Graphs 1.2 and 1.3 plots the value- and equal-weighted average $m_{it}$. The correlation between $m_{it}$ and $M_{it}$ is very high (95.7% for value-weight average and 92.4% for equal-weight averages), showing that the scaled variable $m_{it}$ still retains much of the information in $M_{it}$.

2.2 Research Design and theory

2.2.1 Theory

The analysis is motivation by Merton’s ICAPM. The key insights of the ICAPM are

1) Asset returns changes over time if the investment opportunity set is not constant, and returns follow a factor structure.

For example, in a discrete time model with one state-variable $X_t$, it can be shown that excess returns follow the relation:

$$E(r_i) - r_f = \beta_i (r_m - r_f) + \beta_i^X (r_X - r_f)$$

The intuition is that asset returns should only be determined by its covariance with aggregate wealth and shocks to state variables.

2) Investor’s stock portfolio follows two-fund separation.

They invest in the market portfolio and a hedging portfolio, therefore, investor $i$’s stock holding at time $t$ is equal to:
\[ S^i_t = h^i_{Mt} S_M + h^i_{S_t} S_H \]

where \( h^i_{Mt} \) and \( h^i_{S_t} \) contains information about investor \( i \)'s (possibility stochastic) preference for market risk and hedging demand.

If the investment set opportunity set is constant, then the model reduce to CAPM where everyone invest only in the risk-free asset and the market portfolio. In addition, if the change in \( h^i_{S_t} \) is small compared to \( h^i_{Mt} \), then I can identify \( S_H \) by considering the change of positions of the investor over time.

3) The hedging portfolio \( S_H \) must be the portfolio with the highest predictability of \( X_t \).

Merton(1973) proved that if one of the asset replicates \( X_t \) (interest rate), then the hedging portfolio consist of that asset (long term bond) only, and clearly these two assets have a correlation of 1. If the state variable cannot be replicated by an asset, then the hedging portfolio is the traded portfolio with the highest predictability of \( X_t \) (For example, see Wang and Lo (2006). See Appendix for proof).

This is also intuitive because the hedging portfolio is picked to minimize fluctuation in investor's utility. The hedging portfolio helps the investor smooth out shocks in the economy that is attributed to state variables. Suppose investor, instead of investing in the hedging portfolio, invest in another portfolio with the same betas to the market and to the state variable. This means that the new portfolio is essentially the hedging portfolio plus noise (that is not correlated to the market and \( X_t \)). Then from (1), this asset will not earn excess returns. Additionally, this investor will have higher fluctuations in his wealth, attributed to the deviation from the hedging portfolio. This is a contradiction and thus the hedging portfolio must have the highest predictability to \( X_t \).

These three predictions support my methodology, which is to identify a main trading component explain the trading activities of a large set of investors. Using this component I can analyze its time-series and cross-section predictability and implications.
2.3 Asymptotic Principal Factor

2.3.1 Extraction of Common component

After obtaining stock-level trading activities \( m_{it} \), I need to find a method to extract the most important information contained in the data. The most commonly used method is a principle-component analysis, but for my sample I have a cross-section that is much bigger than the time series. In this case the covariance matrix is singular and I cannot apply the principle component analysis directly.

A common method used is to group the cross-section into smaller groups (than the time series) to reduce the size of the cross-section, then apply principle component analysis to the sample with reduced cross-section. However information is lost in the process and there is not clear-cut way to determine how the data should be summarized.

In this paper I apply an alternate methodology developed by Connor and Korajczyk (1986, 1988). It is called the Asymptotic Principle Factor and is used for data that has a large cross-section. This method enables me to extract the principle components without reducing the size of the cross section. The next section describe the theory and application of this method.

2.3.2 Theory

In Connor and Korajczyk (1986), they derived a form of APT resulting in a k-factor model:

\[
\tilde{r}_i = r_f + \mu_i + \sum_j \beta_{ij} \tilde{f}_j + \tilde{\varepsilon}_i
\]

where \( \tilde{r}_i \) is the (realized) return for stock \( i \), \( r_f \) is the risk-free rate and \( \mu_i \) the risk premium for stock \( i \) and \( \tilde{f}_j \) are the realizations of the \( k \) factors. Since I do not observe the \( k \) factors directly, I need to estimate them from the data. The problem with financial data is that the size cross section much bigger than the number of time periods, and
therefore the covariance matrix is singular and I cannot use a direct principle factor extraction. Connor and Korajczyk suggest using the "asymptotic principle component" method:

Define $B^n$ as the set of $\beta_{ij}$’s, $\gamma = \{\gamma_i\}_{i=1}^n$ be the vector of factor risk premiums such that $\mu = B^n \gamma$, and $F$ to be the set of $\gamma + \hat{f}$, the realizations of the factor.

Suppose I observe returns for $n$ stocks over $T$ time periods where $n > T$. I can write the observed stock returns in matrix form:

$$R^n = \bar{r} - \bar{f} = B^n(\gamma + \hat{f}) + \bar{\varepsilon} = B^n F + \varepsilon^n$$

Define theses matrix products:

$$\Omega^n = (1/n)R^n'R^n$$
$$A^n = (1/n)F'B^nB^nF$$
$$Y^n = (1/n)(F'B^n\varepsilon^n + \varepsilon^n'B^nF)$$
$$Z^n = (1/n)(\varepsilon'^n\varepsilon^n)$$

And also assume: (existence of average cross-sectional idiosyncratic variance)

$$\sigma^2 = \lim_{n \to \infty} (1/n)\varepsilon'^n\varepsilon^n$$

Notice that $\Omega^n$ is the matrix of cross-product of asset excess returns, and $\Omega^n = A^n + Y^n + Z^n$ while $A^n$ contains information about the true factors. My goal here is to show that if I extract the principle components from $\Omega^n$, it will be approximately a linear transformation of the true factors $F$. Let $H^n$ and $\Lambda^n_H$ be the eigenvector matrix and the eigenvalues of $A^n$. That is, $H'^nA^n = \Lambda^n_H H^n$. Then it can be proved that $H^n = L^n F$ where $L^n$ is a nonsingular matrix. This is the same as saying that an eigenvalue decomposition only unique up to a linear transformation. Similarly define Let $G^n$ and $\Lambda^n_G$ be the eigenvector matrix and the eigenvalues of $\Omega^n$, then I would want to show that $G^n \to H^n$ when $n$ is large./The actual proof is technical but a sketch of the proof is provided in Connor and Korajczyk (1986).
Recall $\Omega^n = A^n + Y^n + Z^n$. Since $\epsilon$ is idiosyncratic noise, it is clear that $Y^n$ converges to zero as they are uncorrelated with the factors. Based on equation 2.4, they also show that $Z^n$ converge to a scalar matrix where all off-diagonal terms are zero. The eigenvectors of a matrix are unaffected by the addition of a scalar matrix., therefore for large $n$ $G^n$ will be approximately equal to $H^n$, and therefore $G^n$ is a approximately a linear transformation of $F$.

Connor and Korajczyk (1988) also shows that this technique can be extended to returns with time-varying average.

### 2.3.3 Application

After showing the validity of the asymptotic principle component method, I discuss how I adopt this method below:

Denote by $m_t$ the $k \times T$ matrix of scaled net-trading activity for all stocks. Assume $m_t$ follows an approximate factor model with time-varying average:

$$m_t = B(\gamma_t + f_t) + \epsilon_t$$ (2.5)

In essence, this is the same as ordinary factor analysis except that the role of the time-series and cross-section are reversed. Given the matrix of trading of all stocks over time $M = [m_{it}]$, I first compute the cross-product matrix $\Omega$, where

$$\Omega_{i1,i2} = \frac{\sum_j m_{j,t1} * m_{j,t2}}{\sum_j 1_{j,t1} * 1_{j,t2}}$$ (2.6)

Over all $(m_{j,t1}, m_{j,t2})$ pairs. If an observation in the pair is missing (for example, for a stock that did not survive the whole sample period), then the product is set to zero.
$1_{j,t}$ is the indicator function which is set to 0 if $m_{j,t}$ is missing and 1 otherwise. Therefore each item in the above matrix is the simple average of all $(m_{j,t1} \times m_{j,t2})$ where both items are not missing.

Then the asymptotic principle factors of the is simply obtained by the eigenvectors of $\Omega$. I use this method to extract the principle factor, and the properties of these factors will be discussed below.

At this point it is worth discussing the effect of missing observations and outliers. Suppose that stock $i$ only have $k$ valid observations $(t_1, t_2, ... t_k)$. Then stock $i$ will enter to the sum for $\Omega_{r_1,r_2}$ only if both $\tau_1$ and $\tau_2$ and in the set $(t_1, t_2, ... t_k)$. Therefore there can only be $k^2$ terms involving stock $i$. This means that the covariance matrix naturally gives more weight to stocks with more observation, therefore stocks that only appear in the panel for very few periods is unlikely to affect the covariance matrix.

On the other hand, the same cannot be said for outliers. If $m_{j,r'}$ is an outlier, then it will affect all the terms $\Omega_{r,r'}$ and $\Omega_{r',r'}$, which will be $2t - 1$ terms out of $t^2$ terms in the covariance matrix, especially $\Omega_{r',r'}$ which includes $(m_{j,r'})^2$.

Therefore, I need to be very careful about eliminating outliers, and this justifies the winsorization of the data, and the removal of stocks that are only traded by very few funds, where they are small stocks and often contain big changes in $m_{it}$.

Then under technical conditions I can identify the realized factor $\tilde{\gamma}_t + \tilde{f}_t$, which is equal to the eigenvectors of the matrix $m_t' \times m_t$. Connor and Korajczyk proved that the eigenvectors converge to the true factors as the number of assets grow to infinity.

### 2.3.4 Properties of principle factor

I employed the asymptotic principal factor technique to $m_t$. To test the power of my extracted factor, I regress the value-weighted $m_t$ and equal weighted $m_t$ against the first 5 factors. The results are summarized in Table 1.4.

In both regressions, the first factor is significantly stronger than all other factors,
in both magnitude and significance. The first factor also explains a large part of the variance explained by the first 5 factors. For the value-weighted average, the first factor explains 29.00% (59.06%) of the variance for the value(equal)-weighted case and while 5 factors explains 33.01% (71.71%), showing that the first factor explains much more of the commonality in trades than any other factor. Moreover the significance of the other factors are unstable and can change if I slightly alter out screening procedure, while the first factor does not change by much.

Therefore, I only use the first factor for the remainder of the paper. Figure 2 plots the first factor and Table 3a and 3b shows the correlation matrix of the factor with the Fama-French factors and average net-trading. It appears that the factor is persistent at least over two quarters and do not decay very quickly. The factor is also not strongly related to the any Fama-French factors, even if I consider possible lead-lag relations. The largest correlation is the HML factor and lagged factor, with a correlation of 0.332.

This factor I extracted does not appear to be driven by any of the commonly used risk-factor, so I move on to explore its time-series and cross-section explanatory power of stock returns. From here I will name the extracted factor as $Z_t$. Graph 1.5 plots the factor with the standard deviation scaled to 1.
Chapter 3

Results

In this section, I want to explore the time-series predictability of the liquidity measure.

The factor $Z_t$ captures commonality in trading of mutual funds. It is extracted form the net change in stock holdings by mutual funds, so my results is very relevant to the field of research between trading and stock returns.

There is ample research on the relationship between stock returns and trading. In a classical framework relating trades to prices (for example, Kyle(1985)), trades are information-based and stock returns correlates positively with informational traders as information is revealed over time through trades. In reality, trades can be motivated also by rebalancing or shocks to one’s non-tradeable portfolio (for example, income). Wang (1993,1994) demonstrated that in an environment with information asymmetry, while trades and contemporaneous stock returns will be positively correlated, future stock returns could either be positively or negatively correlated with current trades depending on the true motive behind the trade. In analyzing the properties of $Z_t$ I can explore what is the most important factor affecting mutual fund trading decisions.

My results also add to existing research on the relationship between stock returns and trading volume. One limitation of current research on return/volume relationship is that trading volume is not signed. Therefore, one cannot identify if volume is contributed from an active buy or sell. This paper takes a more direct approach and takes the actual
(signed) trading activity and investigate the relationship with current and future stocks
return. Therefore my results will not only able to determine if mutual fund trading
relates to a source of risk, the results will also show if mutual fund is a provider or risk
insurance or demand risk protection.

3.1 Time Series Regression

3.1.1 Description of Time Series Results

I explore the time-series property of $Z_t$ by employing the regression model:

$$R_t = a + b_Z * Z_{t-1} + b_R * R_{t-1} + X_{t-1} + \epsilon_t \quad (3.1)$$

Where $R_t$ is the return of the (value- or equal-weighted) market portfolio, $Z_{t-1}$ is my
liquidity factor, and $X_{t-1}$ is a set of control variables. They include dividend yield, default
spread, term spread, return volatility (VIX, or RVOL: daily return standard deviation of
the value-weighted index over the past 3 months) and the risk-free rate. Table 1.8 and 1.9
shows the result for whole sample (for value and equal weighted returns, respectively),
and Table 1.10 is the subsample 1986-2006, where the VIX index is available.

As shown in Table 1.8, my factor has strong predictability for value-weighted returns.
A one standard deviation change in my factor predicts the market return to be -0.17
standard deviation lower in the subsequent quarter, and is significant at 5%. With
controls, the parameter is between -0.16 and -.22, and is significant at 10% except for
the last case. In general, adding controls do not change my results, except when the
3-month interest rate and dividend yields are both included. However, in my sample
these two has high correlation (77%), and this may have introduced extra noise in the
errors. Moreover, my point estimate of the regression coefficient for $Z_{t-1}$ is still very
comparable to the estimate in other regressions. Overall the evidence supports that $Z_t$
can predict future market returns.

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Results are weaker for equal weighted returns. Change of one standard deviation above average in $Z_{t-1}$ predicts that the return will be lower by 0.11 standard deviation. The regression coefficient is not significant but still has the same sign. Adding controls largely leave my result unchanged, except one case where I regress $R_t$ with $Z_{t-1}$ and and the term spread I have a coefficient of -0.17 which is significant at 5%. In all other situations the coefficient changes between -0.06 and -0.013 and are not significant at 10%.

I also present the results of one particular subperiod here. I use the period from 1986-2006, which is the period where the VIX index is available and I use VIX as a substitute of RVOL. VIX has the desirable feature of being a forward looking measure of volatility where RVOL represents volatility in the previous year. Because of a shorter sample, there may be a decrease in the power of the test. That is confirmed in Table 1.10, where the results are similar with weaker significance. Using only $Z_{t-1}$, the regression coefficient is -0.15 and is significant at 10%. VIX (and RVOL) weakens my result to -0.11(-0.16) and is not significant at 10%, and so is the case where I use both the dividend yield and the 3-month risk free rate (-0.07, not significant). The other controls do nothing, the estimates ranges between -0.16 and -0.18 and are significant at 10%

As an additional test, I also divide the time series into two subperiod of equal length and repeat my regressions. In each subperiod the regression coefficients remain stable, and the results are presented in the Appendix.

3.1.2 Discussing Time Series Results

The most important result from this section is that a high level of $Z_t$ indicates that the market return will be lower in the future. In the context of liquidity, this is consistent with mutual fund demands liquidity, as demand in stocks drive up prices temporarily and drive down future returns. This result is also consistent with results by Wermers (2000) where purchase of mutual funds caused by capital inflow drives up stock prices temporarily, and also the results by Lamont and Farazzini (2006) where stock purchases made by mutual fund has low returns in the long run. This also suggests that the majority
of trades made by mutual funds are for liquidity purpose rather than information, which
is not very surprising given the difficulty in consistently identify funds that trades with
information and outperform the market.

In addition, $Z_t$ has stronger predictability on value-weighted returns than equal-
weighted returns in both magnitude and significance. This is likely because large stocks
have higher correlation with each other, but it is also possible that $Z_t$ contain more in-
formation about large stocks as they tend to stay in the sample for a longer time period.
This demonstrates the desirability of using the asymptotic principle factor technique as
it picks out the optimal weight for trading in each stock to contribute to the factor with
the most information.

Since the time-series results suggest that mutual fund demands liquidity, I will move
on to a cross-section test to confirm the results of this section. I will use the methods of
Fama-Macbeth to test if the risk correlated to $Z_t$ is priced in the cross-section.

3.2 Cross-section tests

In this section I perform cross-section regressions to determine whether a stock’s expected
return is related to its sensitivity to my innovations in the trade factor $Z_t$.

3.2.1 Identifying innovations in liquidity

The first step for cross section test is to remove the predictable component of $Z_t$: To
achieve this, I run the AR(1) regression

$$Z_t = \alpha + \rho Z_{t-1} + \zeta_t$$  \hspace{1cm} (3.2)

Using current and lagged values of $Z_t$. The residuals, $\zeta_t$, are the unexpected change
in liquidity and is used to determine if liquidity risk is priced. It is possible to add other
factors to equation 3.2 (for example, the lagged market return or Fama-French factors)
but the results would not be materially affected. The goal of the cross-section test is to
determine if risk premium in each stock is related to its sensitivity to \( \zeta_t \), controlling for
other source of risk premium. The sensitivity of stock \( i \) to \( \zeta_t \) is simply the slope coefficient
on \( \zeta_t \) in a multiple regression with stock \( i \)'s return where other asset pricing factors are
included. To investigate whether the stock's expected return is related to \( \beta^L_i \), I follow
the methods of Fama and French (1993). At the end of each year, I sort stock based
on their predicted values of \( \beta^L_i \) and form 10 portfolios where their betas are sufficiently
dispersed. The postranking returns on these portfolios during the next 12 months are
linked across years to form a single return series for each decile portfolio. The excess
returns on those portfolios are then regressed on return-based factors that are commonly
used in empirical asset pricing studies. The extent that the regression intercepts (alphas)
are different from zero explains the component of returns that is explained by \( \beta^L_i \) but not
by exposures to other risk factors.

### 3.2.2 Pre-ranking betas

For the purpose of portfolio formation, define \( \beta^L_i \) as the coefficient on \( \zeta_t \) in a multiple
regression that also includes the three factors of Fama and French (1993).

\[
rt_i = a_i + \beta^L_i \zeta_t + \beta^M_i MKT_t + \beta^S_i SMB_t + \beta^H_i HML_t + \beta^U_i UMD_t + e_{i,t}
\]

where \( r^e_{i,t} \) denotes asset \( i \)'s excess return, MKT denotes the excess return on a broad
market index, and the other two factors, SMB and HML, are payoffs of long-short spreads
constructed by sorting stocks according to their market capitalization and book-to-market
ratio\(^1\). This definition captures stock excess return's co-movement with \( \zeta_t \) that is distinct
from its co-movement from other commonly used factors.

\(^1\)These factors are directly downloaded from Ken French's website
For each quarter $t$, I estimate $\beta^L_i$ for all common stocks traded in AMEX, NASDAQ or NYSE, using on data available up to and including $t$, if they satisfy the following commonly applied data screens:

- Price and return data is available for stock $i$ at time $t$ from the CRSP database, and stock price at time $t$ is higher than $5$ and lower than $1000$.
- Quarterly stock return is available for the stock over the past 5 years.

These estimates will be very noisy, as I use a short time horizon to estimates the betas, and the estimates can be affected by shocks to individual stock returns. My hope is that despite that the estimate will still allow us to sort stocks by their true betas (to some extent), and I will re-estimates the loadings once portfolios are formed to obtain more accurate estimates for my other tests.

3.2.3 Postranking portfolio betas

At the end of each year, stocks are sorted by their pre-ranking liquidity betas and assigned to 10 portfolios. The value-weighted average of each portfolio is calculated. The postranking returns for each portfolio are linked across each year, forming a single return series for each decile covering the period from January 1984 to December 2006. Then the post-ranking beta of each decile portfolio is re-estimated using equation 3.2 over the whole sample period. Panel A of table 1.11 reports the postranking liquidity betas. Comparing portfolios 1 and 10, I created a spread in liquidity betas, but the difference is not statistically significant and the betas do not line up in an increasing fashion from portfolio 1 to 10. Particularly the loadings on portfolios 5 and 6 are low, and the loading of portfolio 10 is lower than that of portfolios 8 and 9. The largest estimate is 4.06 with a $t$-statistic of 1.59, and the smallest value is -3.33 with a $t$-statistic of -1.95. The difference between the betas of the top and bottom decile is 5.21, with a $t$-statistic of 0.9. Overall, there is positive correlation between the ranking of pre-ranking betas to post-ranking
betas, but there is certainly concern that the pre-ranking betas estimates are not very precise and do not sort stocks precisely on their true liquidity betas.

Panel B reports the postranking betas of each decile portfolio for Fama-French factor and the momentum factor. The market risk shows a U-shape pattern, with the extreme portfolios having higher market risk. Portfolios 1 and 10 have betas of 1.10 and 1.13 respectively, and portfolios 5 and 6 have betas of 0.95 and 0.97. There is a similar pattern for the SMB and HML factor: Portfolios 1 and 10 have smaller stocks (loadings of 0.09 and 0.63) and low book-to-market ratios (-0.55 and -0.07) while portfolio 5 and 6 have larger stocks (loadings of -0.10 and -0.31) and value stocks (HML loadings of 0.17 and 0.01). There is very little spread and no pattern for the UMD betas.

3.2.4 Alphas

Panel C reports the alphas of regression of the decile portfolios using the Fama-French factors (MKT, SMB, HML) only, and in the second line shows the alphas when I regress with the Fama-French factors plus the momentum factor. If the alphas shows an upward(downward) trend over the portfolios, that will be an indication that there is a premium(discount) to the risk loading to my liquidity factor. Same as the case of the postranking betas, the alphas do not line up perfectly with the deciles. However, they are still positively correlated.

The three- and four-factors alpha are quite similar, except for portfolio 2. The two alphas have a correlation of 89%. And they both exhibit an upward trend, but the alpha for portfolio 0 is quite high in both cases. The correlation of the alphas with the liquidity betas I estimated in the previous part is low (4% for three factor alpha, -7% for four factor alpha), and this suggests that my asset pricing test in the next part may not yield significant results, despite the results this and the previous part are pointing to a positive direction.
3.2.5 Estimating Risk Premium using All 10 Portfolios

In this part I use the methodology of Fama and Macbeth to estimate the risk premium associated with the risk to $\zeta_t$. Using the beta estimates from the ten postrank portfolios, I estimate the regression

$$r_{it}^e = a_i + \beta_i r_t^L + \beta_i r_t^M + \beta_i r_t^S + \beta_i r_t^H + \beta_i r_t^U + e_{it}$$

(3.4)

to obtain estimates of $\hat{r}_t^L, \hat{r}_t^M, \hat{r}_t^S, \hat{r}_t^H$ and $\hat{r}_t^U$ for each quarter. My estimates of the risk premium for each factor is then the time-series average of these slope coefficients, and the standard error is the time-series standard deviation divided by the square-root of the number of observations.

My results show that the estimate of the risk premium is insignificant. When I include liquidity, Fama-French and momentum factors, I find that economically the difference of returns between the top and bottom decile portfolio due to liquidity is 0.33% per quarter, or 1.31% per year. But the t-statistic is only 1.15 and not significant. This may imply that there is no risk premium associated with liquidity, but this could also simply be caused by the imprecise ranking of stocks when I sort by historical liquidity betas. The result is also too weak to say much about the sign of the risk premium. Therefore in the next section I use a different sorting methodology as an attempt to achieve better sorting.

3.2.6 Sorting by liquidity measures

Sorting by historical liquidity betas yield weak results. This may be due to the fact that individual stocks returns are too volatile and stocks with extreme betas estimates are often the stocks with the highest estimation errors. As a result, historical betas may not be the best estimator of true betas of the portfolios. Papers such as Pastor and Stambaugh (2006) and Mamaysky, Spiegel and Zhang (2004) use other factors or back-
testing methods to improve the accuracy of the beta estimates. Therefore in this part I sort stock based on stock characteristics that are more stable over time, and investigate if this will give us stronger results.

From the literature on liquidity, commonly used proxies for liquidity includes:

- Firm size
- Number of shares outstanding
- Stock price
- dollar volume.
- turnover

- Amihud(2002)’s liquidity measure. I use a 1-year average (daily average for each month, then a 12-month average) for each stock

\[
ILLQ_i = \frac{1}{M} \sum_{\tau=1}^{M} \frac{|R_{i,\tau}|}{V_{i,\tau}}
\] (3.5)

- Past 12-month volatility (using monthly returns)

Therefore I sort stocks based on these characteristics to determine if they can create spread in \( \beta_i^L \).

Sorting using dollar volume, size, illiquidity ratio and shares outstanding gives us further insight into the role of my liquidity beta. Panel A of Table 1.12 shows the post-ranked liquidity betas when sorted by these stock characteristics. When I consider portfolios sorted by dollar volume, the post-ranking betas are not strictly lined up, but still demonstrate a clear trend. The spread between the top and bottom portfolios is 5.64 with a t-statistic of 2.34. Therefore, although the size of the spread is similar, the other portfolios line up better and also the betas are measured with less noise. A similar
patter can be seen if you consider sorting by size, illiquidity ratio and shares outstanding, creating a spread in liquidity betas between 4.07 and 4.74, with t-statistic between 1.64 and 1.9. In each of these cases the betas line up better and the top and bottom decile’ betas are measured with higher precision than using historical liquidity betas. Stocks with higher dollar volume, bigger size and lower illiquidity ratio are associated with higher liquidity, as these stocks have a deeper market and their prices are not easily moved by trades. And as shown in Table 1.12, Stocks with high dollar volume, big size and low illiquidity ratio both have low post-ranking liquidity beta, indicating a high loading in my liquidity beta measure is correlated with stocks of low liquidity. This is also sensible in terms of equation 3.2, as stocks that experience high return when there is a positive surprise in $Z_t$ should be stocks that can be moved easily, and are thus less liquid stocks.

In Panel B of Table 1.12 is the post-ranking (3-factor) alpha of each portfolio. Again, I am able to obtain better bigger spreads than the previous section. If I focus on dollar volume, size and the illiquidity ratio, we can see that portfolios that is generally considered to have low liquidity have higher returns, confirming my intuition that there should be a risk premium associated with low liquidity (the 10-1 spread in alphas are 0.5% if I sort by dollar volume and illiquidity ratio, 0.2% if I sort by size). I obtain similar results if I include the momentum factor.

Similar results in both spread of betas and alphas can be found when sorting by share outstanding. Sorting by price, volatility, turnover or share volume does not produce a clear pattern in post-ranking betas and alphas.

Table 1.13 shows the risk-premium when I repeat the Fama-Macbeth procedure from the previous section. Portfolio sorted by dollar volume and illiquidity ratio confirms my results, with the risk premium being negative and significant at 10%. However, when I sort by size the risk premium is insignificant, and when I sort by shares outstanding the risk premium is positive and significant at 10%. Sorting by the other variables seem to suggest that the risk premium is negative, but the results are not strong.

I do not claim that sorting by any particular stock characteristic is the best way
to achieve maximum spread in post-ranking betas, as no single variable is perfectly correlated with liquidity. But the results in Table 1.13 seems encouraging, as I find some evidence of a negative risk premium, and its sign confirms with my intuition. In the next section I attempt to combine the predicting power of different estimates to achieve even strong ranking of stocks.

3.2.7 Sorting by predicted liquidity betas

Since the liquidity characteristics from the previous part seems to capture variation in risk in my liquidity factor, in this section I adopt the methods by Pastor and Stambaugh (2006) to combine these stock characteristics to predict each stock’s liquidity beta with more precision. The intuition is that I want to find the optimal linear combination of the stock characteristics to best predict liquidity, but with a short sample, there is some concerns for not estimating the weights accurately, in the same line as the criticism with using preranking betas. I use the same factor that I used in the previous section, but I dropped stock price, turnover and firm size to avoid multi-collinearity. The Pastor-Stambaugh procedure is briefly described below.

First, I compute 3-factor excess returns for each stock by:

\[ R_{i,t} = \hat{\beta}_i^M MKT_t + \hat{\beta}_i^S SMB_t + \hat{\beta}_i^H HML_t + e_{i,t} \quad (3.6) \]

Where the each beta is estimated in a 4 factor model with the three Fama-French factors and my liquidity factor \( \zeta_t \). I then pool all stocks and time periods together and regress

\[ e_{i,t} = \beta_1 \zeta_t + \beta_2 Y_{t-1} \zeta_t + \nu_{i,t} \quad (3.7) \]

Where \( Y_t \) is the vector of stock characteristics that includes the illiquidity ratio, log dollar volume, shares outstanding, share volume and standard deviation of returns.
(Unlike Pastor and Stambaugh, I did not use historical beta because the estimates have showed to be unstable). This procedure is repeated every quarter with only data before the formation date is used (That includes re-estimation of $\zeta_t$ and betas). The liquidity ratio, turnover and volatility is the most stable over time.

After obtaining the estimates $\hat{\beta}_1$ and $\hat{\beta}_2$, I can estimate the predicted liquidity beta for each stock by

$$\beta^L_{i,t} = \hat{\beta}_1 + \hat{\beta}_2 Y_t$$

(3.8)

I then rank stocks into 10 portfolios based on the predicted beta and apply the Fama-Macbeth procedure to these portfolios.

Since my goal here is to achieve sorting of portfolios based on their true liquidity betas, here I present the result that achieves the best sorting. That is obtained when I only consider dollar volume and illiquidity ratio. The post-ranking portfolio has liquidity betas from 0.21 (portfolio 1) to 4.27(portfolio 10), the difference being 4.06 and significant at 95%. The betas line up pretty well, with only a tiny drop from portfolio 2 to 3 and a bigger drop from portfolio 6 to 7.

Fama-Macbeth regressions gives us a risk premium of -0.15% per quarter for liquidity, which is the sign I expect, but the t-statistic is only 1.64 and not significant. The result is almost unchanged when I include the momentum factor.

### 3.2.8 Discussion on Cross-Section tests

In this section I used different ways to explore if my liquidity factor is priced. The results in this section is a little weak, but it still provides us with important insights. First is that my measure $Z_t$ indeed measures liquidity. When I sort by common proxies of liquidity, I can clearly see that stocks with lower liquidity has a higher loading of $\beta^L_t$. As discussed in the previous section, this confirms with my intuition of how $\beta^L_t$ should reflect liquidity. In addition, the risk premium to liquidity risk appears to be priced at the right sign, unfortunately it does not have very high statistical significance. Overall,
combining both the time-series and cross-section results, I still consider this to be strong evidence to suggest that the factor extracted from the net trading activities of mutual funds is consistent with a measurement of liquidity.
Chapter 4

Conclusion

Market wide liquidity is a state variable that is important for pricing for common stocks. Using quarterly changes in equity holdings of mutual funds, I am able to extract a trading component that can predict changes in market returns, is related to a number of proxies for liquidity, and shocks to this factor also appears to be priced. I find that I can use this factor to predict market return, as well as to explain cross-sectional differences in returns among stocks. Stocks that are more sensitive to the liquidity measure has higher returns, even after I account for exposure to the market return as well as size, value and momentum factors. This paper serves as support to the literature that liquidity is a common risk factor and is priced in the market, and changes in aggregate liquidity is one of the causes to time-varying risk premium.

This paper confirms that liquidity risk should be an important risk factor in asset pricing, and that aggregate liquidity could be extracted from changes in portfolio holding. Currently my sample of mutual fund still represents only a small fraction of market, with a market share of 13% at the end of my sample period. As data becomes more available more research can be done using all trading activities, and possibly at a higher frequency as well. Future research can also study the effect of the latest credit crisis on trading activity. In will be highly interesting to study if this period of financial distress will allow us to identify additional factors that contribute to the elusive concept of liquidity.
Chapter 5

Appendix

5.1  A simple version of ICAPM

In this section I present an ICAPM model used in Lo and Wang (2006) to support statements I made in Chapter 2

5.1.1  Model set-up

Consider a economy with the following characteristics: for simplicity,

Utility: There are I individuals, each maximize his expected utility function of the following form:

\[ E_t[-e - W_{t+1}^i - (\lambda X_t + \lambda Y_t) D_{Mt+1} - \lambda Z(1 + Z_t^i) X_{t+1}] \]  

I use this formulation of utility for simplicity. This utility function captures the dynamic nature of the investment problem without the need to explicitly solve a dynamic optimization problem. For a model that fully solves the dynamic optimization problem, refer to Wang (1994) and Lo and Wang (2003). The resulting value function is very similar to the one I proposed.
Stocks: There are J stocks in the economy. Let $D_{jt}$ denotes the dividend paid by stock at time $t$.

Assumes all shocks to the economy $(D_t, X_t, Y_t^i, Z_t^i)$ are iid over time. I also assume that $D_t$ and $X_t$ are jointly normally distributed:

$$u_t \equiv \begin{pmatrix} D_t \\ X_t \end{pmatrix} \sim N(., \sigma), \text{ where } \sigma = \begin{pmatrix} \sigma_{DD} & \sigma_{DX} \\ \sigma_{XD} & \sigma_{XX} \end{pmatrix} \quad (5.2)$$

where $\sigma_{DD}$ is positive definite. All stocks are normalized to have a share volume of 1. Denote the vector of excess dollar returns on stock as

$$Q_{t+1} = D_{t+1} + P_{t+1} - (1 + r)P_t \quad (5.3)$$

### 5.1.2 Equilibrium

The economy defined above has a unique linear equilibrium where

$$P^i_t = -a^i - (1 + b^i)X_t \quad (5.4)$$

and

$$S^i_t = (\frac{1}{I} - \lambda_Y Y_t^i)S_M - [\lambda_Y (b^i)' Y_t^i + \lambda_Z Z_t^i](\sigma_{QQ})^{-1}\sigma_{QX} \quad (5.5)$$

where

$$\sigma_{QQ} = \sigma_{DD} - (b\sigma_{XD} + \sigma_{XD}b') + \sigma^2_X b'b' \quad (5.6)$$

$$\sigma_{QX} = \sigma_{DX} - \sigma^2_X b \quad (5.7)$$
\[ a = \frac{1}{\bar{r}}(\bar{a}\sigma_{QQ}S_M - \lambda_Z\sigma_{QX}) \]  \tag{5.8}

\[ b = \lambda_X[(1 + r) + (\lambda_Z\sigma_{XD}S_M)]^{-1}\sigma_{DD}S_M \]  \tag{5.9}

where \( \bar{a} = 1/I \) and \( S_M \) is the market portfolio with investment in 1 share of all stocks.

### 5.1.3 Proof

I solve this equilibrium by first deriving investor asset demand under the assumed price function, then solve for coefficient vectors \( a \) and \( b \) to clear the stock market.

From equation 5.4, I have
\[ Q_{t+1} = \bar{Q}_t + \bar{Q}_{t+1} \]
where \( \bar{Q}_t = ra + (1 + r)bX_t \), \( \bar{Q}_{t+1} = (\iota, -b)u_{t+1} \). \( \iota \) is the \((n \times n)\) identity matrix.

I now consider investor \( i \)'s optimal portfolio choice. Denote \( S_t \) the vector of his stock holding in period \( t \). His next-period wealth is
\[ W_{t+1} = W_t + S'_t\bar{Q}_{t+1} + S'_t(\iota, -b)u_{t+1} \tag{5.10} \]

(superscript \( i \) has been suppressed).

Also define
\[ \lambda_{1t} = \lambda_X X_t + \lambda_Y Y_t \]

and
\[ \lambda_{2t} = \lambda_Z(1 + Z_t) \]

Then
\[
E_t[-e - W^t_{t+1} - \lambda_{1t}D_{Mt+1} - \lambda_{2t}X_{t+1}] = E_t[e^{-W_t - S'_t\bar{Q}_t + (S_t + \lambda_{1t}(\iota, -b)'S_t + \lambda_{2t})'u_{t+1}}] \tag{5.11}
\]
\[
= e^{-W_t - S'_t\bar{Q}_t + \frac{1}{2}(S_t + \lambda_{1t}(\iota, -b)'S_t + \lambda_{2t})'\sigma(S_t + \lambda_{1t}(\iota, -b)'S_t + \lambda_{2t})} \tag{5.12}
\]
where \( \sigma \) is the covariance matrix of \( u_t \). The investor’s optimization problem then reduces to

\[
\max_{S_t} S_t' \bar{Q}_t - \frac{1}{2} (S_t + \lambda_{1t}(\mu; -b)'S_t + \lambda_{2t})' \sigma (S_t + \lambda_{1t}(\mu; -b)'S_t + \lambda_{2t})
\]  

(5.13)

The first-order condition is

\[
0 = \bar{Q}_t - (\sigma_{DD} - b\sigma'_{DX} - \sigma_{DX} b' + \sigma_{XX} bb') S_t - \lambda_{1t}(\sigma_{DD} - b\sigma_{DZ})\mu - \lambda_{2t}(\sigma_{DX} - b\sigma_{XX})
\]  

(5.14)

Therefore investor’s stock demand is

\[
S_t = (\sigma_{DD} - b\sigma'_{DX} - \sigma_{DX} b' + \sigma_{XX} bb')^{-1} [\bar{Q}_t - \lambda_{1t}(\sigma_{DD} - b\sigma_{DZ})\mu - \lambda_{2t}(\sigma_{DX} - b\sigma_{XX})]
\]  

(5.15)

Summing equation 5.15 over all investors and imposing the market clearing condition

\[
\sum_i S^i_t = S_M \text{ gives us }
\]

\[
0 = [r a + (1 + r)bX_t] \bar{Q}_t - (1/I)\sigma_{QQ} S_M - \lambda_X \sigma_{QD} \mu - \lambda_Z \sigma_{QX}
\]  

(5.16)

It follows that

\[
r a = (1/I)\sigma_{QQ} S_M - \lambda_Z \sigma_{QX}
\]  

(5.17)

\[
(1 + r)b = \lambda_X \sigma_{QD} S_M
\]  

(5.18)

which uniquely determine the equilibrium \( a \) and \( b \). Substituting equation 5.16 into 5.15, I obtain investor \( i \)'s equilibrium stock holding

\[
S^i_t = (\frac{1}{I} - \lambda_Y Y^i_t) S_M - [\lambda_Y (b'Y^i_t + \lambda_Z Z^i_t)](\sigma_{QQ})^{-1}\sigma_{QX}
\]  

(5.19)

Which is identical with equation 5.5. Q.E.D.
From this model I can observe that the claims about ICAPM is held. 1 and 2 are straightforward, and 3 is true because the hedging portfolio return, \( Q_{t+1}(\sigma_{QQ})^{-1}\sigma_{QX} \), is the regression slope for predicting \( Q_{t+1} \) with \( X_t \). Therefore it must be the portfolio that has the highest predicting power of future returns.

5.2 Further time-series analysis based on subperiod and subsample analysis

There is an influx of funds entering the sample after June of 1993, and the number of stocks and coverage increased dramatically. Therefore in this section I will divide my data to before June 1993 and after to see if there is a change in regime, and to test if my time-series results hold in subperiod analysis.

Extracted factor for both subsample

First, I recalculate my factor \( Z_t \) using only data from each subsample. Define \( Z^1_t \) as the factor extracted using only data from 1980- June 1993 and \( Z^2_t \) as the factor extracted using only data from September 1993- December 2006. Table 1.15 shows there is still a high correlation between the whole-sample factor and the sub-sample factor. From 1980-1993, the correlation is between \( Z_t \) and \( Z^1_t \) is 80%. From 1993-2006 the correlation between \( Z_t \) and \( Z^2_t \) is 96.2%.

Time series regression:

I repeat the time series regression with factors extracted from each subsample. For value-weighted returns, I see the same pattern for factors extracted from each subperiod. For equal-weighted returns, the results is actually stronger if I divide my sample into two halves.
5.3 Graphs and Tables

Graph 1.1: Fund ownership of the overall equities market.

Graph 1.2: Value-weighted $m_{it}$
Graph 1.3: Equal-weighted $m_{it}$

![Graph showing equal-weighted $m_{it}$]

<table>
<thead>
<tr>
<th>Table 1.4</th>
<th>value-weighted $m_t$</th>
<th>equal-weighted $m_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor 1</td>
<td>0.5431***</td>
<td>0.7916***</td>
</tr>
<tr>
<td></td>
<td>(4.61)</td>
<td>(9.23)</td>
</tr>
<tr>
<td>Factor 2</td>
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<td>0.2478***</td>
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<td></td>
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<td>(4.93)</td>
</tr>
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<td>Factor 3</td>
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<td>R-square (5 factors)</td>
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<td>R-square(1st factor only)</td>
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Regression of average $M_{it}$ against the first five principle components. All variables are scaled to have the same volatility.

T-statistics are Newey-West errors with four lags. * significant at 90% ** significant at 95% *** significant at 99%
Graph 1.5: Plot of $Z_t$ over time

Table 1.6
Autocorrelation of the first principle component

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Table 1.7
Correlation Matrix of $Z_t$ with Fama-French factors

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<td>VW $m_t$</td>
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### Table 1.8: Regression of value-weighted stock returns on (lagged) liquidity factor and controls

Each column reports the regression coefficients of a time series regression of the value-weighted stock returns against \( Z_{t-1} \) and controls. T-statistics are Newey-West standard errors with lag of 4 quarters.

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<td>( R^2 )</td>
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Standardized beta coefficients; t statistics in parentheses

Note: Put your notes here.

* p < .1, ** p < .05, *** p < .01
Table 1.9: Regression of equal-weighted stock returns on (lagged) liquidity factor and controls

Each column reports the regression coefficients of a time series regression of the equal-weighted stock returns against $Z_{t-1}$ and controls. T-statistics are Newey-West standard errors with lag of 4 quarters.

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Standardized beta coefficients; t statistics in parentheses
* p<0.1, ** p<0.05, *** p<0.01
Table 1.10: Regression of value-weighted stock returns on (lagged) liquidity factor and controls
Subperiod 1986-2006

Each column reports the regression coefficients of a time series regression of the value-weighted stock returns against $Z_{t-1}$ and controls against $Z_{t-1}$ and controls. T-statistics are Newey-West standard errors with lag of 4 quarters.

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$R^2$: 0.013, 0.024, 0.027, 0.055, 0.042, 0.028, 0.080, 0.115, 0.180

Standardized beta coefficients; t statistics in parentheses
* p<.1, ** p<.05, *** p<.01
### Properties of Portfolios Sorted on Historical Liquidity Betas

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### Risk premium of each factor

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<th>UMD</th>
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<td></td>
<td>(1.15)</td>
<td>(-1.87)</td>
<td>(1.71)</td>
<td>(-0.85)</td>
<td>(0.07)</td>
</tr>
</tbody>
</table>

At each year end between 1980 and 2006, eligible stocks are sorted into 10 portfolio according to predicted liquidity betas. The betas are constructed by a linear regression of stock returns over surprises in $Z_t$ and Fama-French factor returns. The estimation and sorting procedure at each year end use only data available at that time. Eligible stocks are defined as ordinary common shares traded on the NYSE, AMEX, or NASDAQ with at least five year of quarterly returns, with year-end stock price between $5 and $1,000. The portfolio returns for the 4 post-ranking quarters are linked across years to form one series of post-ranking returns for each decile. Panel A reports the decile portfolios’ post-ranking liquidity betas, estimated by regressing the value-weighted portfolio excess returns on innovation on $Z_t$ and the Fama-French factors. Panel B reports the Fama-French factor loading estimates. Panel C is the three- and four-factor alphas of each portfolio, estimated by regressing each post-ranking portfolio return against Fama-French factors. The last panel report the estimated risk-premium of the liquidity factor and each Fama-French factor using the Fama-Macbeth methodology. t-statistics are in parentheses.
Table 1.12: Properties of post-ranking portfolio sorted by stock characteristics

<table>
<thead>
<tr>
<th>Properties of Portfolios Sorted on Historical Liquidity Betas</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>10-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dollar Volume</td>
<td>5.59</td>
<td>3.07</td>
<td>4.15</td>
<td>1.09</td>
<td>1.97</td>
<td>1.52</td>
<td>1.42</td>
<td>0.79</td>
<td>1.36</td>
<td>-0.05</td>
<td>-5.64</td>
</tr>
<tr>
<td>(2.49)</td>
<td>(1.40)</td>
<td>(2.29)</td>
<td>(0.73)</td>
<td>(1.38)</td>
<td>(1.07)</td>
<td>(1.01)</td>
<td>(0.54)</td>
<td>(0.98)</td>
<td>(0.10)</td>
<td>(2.34)</td>
<td></td>
</tr>
<tr>
<td>Price</td>
<td>1.80</td>
<td>-1.31</td>
<td>-2.08</td>
<td>-0.95</td>
<td>2.59</td>
<td>-0.98</td>
<td>-0.04</td>
<td>3.07</td>
<td>1.79</td>
<td>-0.07</td>
<td>-1.87</td>
</tr>
<tr>
<td>(0.51)</td>
<td>-0.045</td>
<td>-0.78</td>
<td>-0.44</td>
<td>(1.30)</td>
<td>-0.49</td>
<td>(0.02)</td>
<td>(2.12)</td>
<td>(1.26)</td>
<td>(0.09)</td>
<td>(0.50)</td>
<td></td>
</tr>
<tr>
<td>Illiquidity Ratio</td>
<td>0.04</td>
<td>0.10</td>
<td>0.32</td>
<td>0.08</td>
<td>2.02</td>
<td>3.11</td>
<td>1.61</td>
<td>2.39</td>
<td>5.40</td>
<td>4.11</td>
<td>4.07</td>
</tr>
<tr>
<td>(0.06)</td>
<td>(0.08)</td>
<td>(0.27)</td>
<td>(0.06)</td>
<td>(1.63)</td>
<td>(1.20)</td>
<td>(1.04)</td>
<td>(1.42)</td>
<td>(2.46)</td>
<td>(1.76)</td>
<td>(1.64)</td>
<td></td>
</tr>
<tr>
<td>Share Outstanding</td>
<td>4.31</td>
<td>3.46</td>
<td>2.90</td>
<td>1.80</td>
<td>0.87</td>
<td>1.74</td>
<td>-0.04</td>
<td>0.57</td>
<td>-0.29</td>
<td>0.23</td>
<td>-4.08</td>
</tr>
<tr>
<td>(1.91)</td>
<td>(1.72)</td>
<td>(1.65)</td>
<td>(1.21)</td>
<td>(0.66)</td>
<td>(1.29)</td>
<td>(0.03)</td>
<td>(0.41)</td>
<td>-0.24</td>
<td>(0.42)</td>
<td>(1.73)</td>
<td></td>
</tr>
<tr>
<td>Size</td>
<td>4.93</td>
<td>4.67</td>
<td>2.00</td>
<td>2.14</td>
<td>0.86</td>
<td>2.12</td>
<td>0.52</td>
<td>1.58</td>
<td>0.03</td>
<td>0.19</td>
<td>-4.74</td>
</tr>
<tr>
<td>(2.07)</td>
<td>(2.33)</td>
<td>(1.16)</td>
<td>(1.32)</td>
<td>(0.58)</td>
<td>(1.47)</td>
<td>(0.41)</td>
<td>(1.11)</td>
<td>(0.03)</td>
<td>(0.37)</td>
<td>(1.90)</td>
<td></td>
</tr>
<tr>
<td>Volatility</td>
<td>-1.29</td>
<td>0.11</td>
<td>3.95</td>
<td>2.90</td>
<td>2.41</td>
<td>-1.11</td>
<td>0.60</td>
<td>-3.18</td>
<td>-6.00</td>
<td>-7.27</td>
<td>-5.98</td>
</tr>
<tr>
<td>(0.61)</td>
<td>(0.07)</td>
<td>(2.33)</td>
<td>(1.61)</td>
<td>(1.32)</td>
<td>(0.64)</td>
<td>(0.28)</td>
<td>(1.26)</td>
<td>(1.69)</td>
<td>(1.47)</td>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>Turnover</td>
<td>0.85</td>
<td>0.02</td>
<td>-1.01</td>
<td>3.23</td>
<td>2.15</td>
<td>2.12</td>
<td>2.58</td>
<td>2.19</td>
<td>-3.05</td>
<td>-6.60</td>
<td>-7.45</td>
</tr>
<tr>
<td>(0.45)</td>
<td>(0.01)</td>
<td>(0.55)</td>
<td>(1.79)</td>
<td>(1.18)</td>
<td>(1.07)</td>
<td>(1.06)</td>
<td>(1.43)</td>
<td>(1.89)</td>
<td>(1.60)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share Volume</td>
<td>1.57</td>
<td>5.63</td>
<td>2.55</td>
<td>2.50</td>
<td>2.43</td>
<td>1.34</td>
<td>2.15</td>
<td>0.94</td>
<td>1.55</td>
<td>-0.27</td>
<td>-1.84</td>
</tr>
<tr>
<td>(0.71)</td>
<td>(1.77)</td>
<td>(1.38)</td>
<td>(1.33)</td>
<td>(1.42)</td>
<td>(0.91)</td>
<td>(1.53)</td>
<td>(0.66)</td>
<td>(1.09)</td>
<td>(0.45)</td>
<td>(0.76)</td>
<td></td>
</tr>
<tr>
<td>A. Post-ranking Liquidity Betas</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B. Four Factor Alphas</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

See notes to Table 1.11. Panel A shows the post-ranking liquidity beta when the portfolio are sorted by stock characteristics. The betas are estimated as the slopes from the regression of excess portfolio post-ranking returns on Fama French factor, momentum and innovation in liquidity. Panel B reports each decile portfolio's post-ranking alphas. The alphas are estimated as intercepts from the regression of excess portfolio post-ranking returns on Fama-French factors and momentum. The t-statistics are in parentheses.
### Table 1.13: Risk premium of postranking portfolios sorted by stock characteristics

<table>
<thead>
<tr>
<th>Portfolios sorted by:</th>
<th>LIQ</th>
<th>MKT</th>
<th>SMB</th>
<th>HML</th>
<th>UMD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dollar Volume</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Four Traded Factors</td>
<td>-0.17%**</td>
<td>-3.77%**</td>
<td>0.66%</td>
<td>0.86%</td>
<td></td>
</tr>
<tr>
<td>Five Traded Factors</td>
<td>-0.14%**</td>
<td>-2.45%</td>
<td>0.37%</td>
<td>0.86%</td>
<td>5.28%*</td>
</tr>
<tr>
<td><strong>Illiquidity Ratio</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Four Traded Factors</td>
<td>-0.11%*</td>
<td>-1.87%</td>
<td>0.02%</td>
<td>3.33%**</td>
<td></td>
</tr>
<tr>
<td>Five Traded Factors</td>
<td>-0.05%</td>
<td>-1.79%</td>
<td>0.03%</td>
<td>1.82%</td>
<td>2.66%</td>
</tr>
<tr>
<td><strong>Size</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Four Traded Factors</td>
<td>0.04%</td>
<td>-0.66%</td>
<td>0.49%</td>
<td>0.38%</td>
<td></td>
</tr>
<tr>
<td>Five Traded Factors</td>
<td>0.03%</td>
<td>-1.70%</td>
<td>0.52%</td>
<td>0.28%</td>
<td>-1.32%</td>
</tr>
<tr>
<td><strong>Share Outstanding</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Four Traded Factors</td>
<td>0.23%*</td>
<td>0.23%</td>
<td>0.37%</td>
<td>0.57%</td>
<td></td>
</tr>
<tr>
<td>Five Traded Factors</td>
<td>0.24%</td>
<td>0.43%</td>
<td>0.32%</td>
<td>0.53%</td>
<td>1.02%</td>
</tr>
<tr>
<td><strong>Price</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Four Traded Factors</td>
<td>-0.11%*</td>
<td>0.38%</td>
<td>-0.98%</td>
<td>1.83%*</td>
<td></td>
</tr>
<tr>
<td>Five Traded Factors</td>
<td>-0.11%*</td>
<td>1.03%</td>
<td>-0.64%</td>
<td>2.42%**</td>
<td>0.7%</td>
</tr>
<tr>
<td><strong>Volatility</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Four Traded Factors</td>
<td>0.04%</td>
<td>-1.80%</td>
<td>-0.70%</td>
<td>-1.05%</td>
<td></td>
</tr>
<tr>
<td>Five Traded Factors</td>
<td>0.01%</td>
<td>0.89%</td>
<td>2.51%</td>
<td>3.36%</td>
<td>5.39%***</td>
</tr>
<tr>
<td><strong>Turnover</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Four Traded Factors</td>
<td>-0.03%</td>
<td>-0.96%</td>
<td>0.28%</td>
<td>-0.07%</td>
<td></td>
</tr>
<tr>
<td>Five Traded Factors</td>
<td>0.00%</td>
<td>-0.79%</td>
<td>0.41%</td>
<td>-0.15%</td>
<td>2.02%</td>
</tr>
<tr>
<td><strong>Share Volume</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Four Traded Factors</td>
<td>-0.13%*</td>
<td>-1.00%</td>
<td>0.82%</td>
<td>1.65%</td>
<td></td>
</tr>
<tr>
<td>Five Traded Factors</td>
<td>-0.03%</td>
<td>-0.98%</td>
<td>0.88%</td>
<td>1.73%</td>
<td>-5.27%*</td>
</tr>
</tbody>
</table>

See notes to Table 1.11. This table reports the risk premium associated with the liquidity factor, with the portfolios being sorted by stock characteristics. The premium is reported as monthly values. The t-statistics are in parentheses.
Table 1.14: Properties of post-ranking portfolios sorted by predicted liquidity

<table>
<thead>
<tr>
<th>Property of Portfolios Sorted on Predicted Liquidity Beta</th>
<th>Value Weighted Decile Portfolios</th>
<th>Value Weighted Decile Portfolios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>0.21</td>
<td>0.90</td>
</tr>
<tr>
<td>A. Post-ranking Liquidity Betas</td>
<td>(0.44)</td>
<td>(0.65)</td>
</tr>
<tr>
<td>B. Fama-French Factor and Momentum Betas</td>
<td>0.90</td>
<td>0.62</td>
</tr>
<tr>
<td>Fama-French and LIQ Premium</td>
<td>-3.04%</td>
<td>-1.99%</td>
</tr>
<tr>
<td>C. Alpha</td>
<td>0.23%</td>
<td>0.07%</td>
</tr>
<tr>
<td>Risk premium of each factor</td>
<td>0.13%</td>
<td>0.07%</td>
</tr>
</tbody>
</table>

See notes to Table 1.11. Panel A reports the decile portfolios’ post-ranking liquidity betas, estimated by regressing the value-weighted portfolio excess returns on innovation on Z and the Fama-French factors. Panel B reports the Fama-French factor loading estimates. Panel C is the three- and four-factor alphas of each portfolio, estimated by regressing each post-ranking portfolio return against Fama-French factors. The last panel reports the estimated risk-premium of the liquidity factor and each Fama-French factor using the Fama-MacBeth methodology. t-statistics are in parentheses.
Table 1.15: Subperiod time series analysis

Panel A: Time series regression with factor extracted from sub-samples.
Value-weighted returns

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z_{t-1} (Whole Sample)</td>
<td>-0.17**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z_{t-1} (1st half)</td>
<td></td>
<td>-0.17**</td>
<td></td>
</tr>
<tr>
<td>Z_{t-1} (2nd half)</td>
<td></td>
<td></td>
<td>-0.16</td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>107</td>
<td>52</td>
<td>55</td>
</tr>
<tr>
<td>R^2</td>
<td>0.030</td>
<td>0.028</td>
<td>0.027</td>
</tr>
</tbody>
</table>

Standardized beta coefficients; Newey-West standard errors with 4 lags; t statistics in parentheses
*p<.1, **p<.05, ***p<.01

Panel B: Equal-weighted returns

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z_{t-1} (Whole Sample)</td>
<td>-0.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z_{t-1} (1st half)</td>
<td></td>
<td>-0.12</td>
<td></td>
</tr>
<tr>
<td>Z_{t-1} (2nd half)</td>
<td></td>
<td></td>
<td>-0.19**</td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>107</td>
<td>52</td>
<td>55</td>
</tr>
<tr>
<td>R^2</td>
<td>0.015</td>
<td>0.014</td>
<td>0.036</td>
</tr>
</tbody>
</table>

Standardized beta coefficients; Newey-West standard errors with 4 lags; t statistics in parentheses
*p<.1, **p<.05, ***p<.01
Bibliography


