Essays on Corporate Bonds

by

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Abstract

This thesis consists of three empirical essays on corporate bonds, examining the role of both credit risk and liquidity. In the first chapter, I test the ability of structural models of default to price corporate bonds in the cross-section. I find that the Black-Cox model can explain 45% of the cross-sectional variation in yield spreads. The unexplained portion is correlated with proxies for credit risk and thus, cannot be attributed solely to non-credit components such as liquidity. I then calibrate a jump diffusion model and a stochastic volatility model, finding that the jump diffusion model weakly improves cross-sectional explanatory power while the stochastic volatility model does not. However, much of the cross-sectional variation in yield spreads remains unexplained by structural models.

In the second chapter (co-authored with Jun Pan), we examine the connection between corporate bonds, equities, and Treasury bonds through a Merton model with stochastic interest rates. We construct empirical measures of bond volatility using bond returns over daily, weekly, and monthly horizons. The empirical bond volatility is significantly larger than model-implied volatility, particularly when daily returns are used, suggesting liquidity as an explanation. Indeed, we find that variables known to be linked to bond liquidity are related to excess volatility in the cross-section. Finally, controlling for equity and Treasury exposures, we find a systematic component in bond residuals that gives rise to the excess volatility.

In the third chapter (co-authored with Jun Pan and Jiang Wang), we examine the liquidity of corporate bonds and its asset-pricing implications. Our measure of illiquidity is based on the magnitude of transitory price movements. Using transaction-level data, we find the illiquidity in corporate bonds to be significant, substantially greater than what can be explained by the bid-ask bounce, and closely related to bond characteristics. We also find a strong commonality in the time variation of bond illiquidity, which rises sharply during market crises. Monthly changes in aggregate bond illiquidity are strongly related to changes in the CBOE VIX index. Finally, we find a relation between our measure of bond illiquidity and the cross-sectional variation in bond yield spreads.

Thesis Supervisor: Jun Pan
Title: Associate Professor of Finance

Acknowledgments

Since I started working on my thesis, I have looked forward to the day that I could write the Acknowledgments because it would signal that I had finally completed my thesis. I never realized how hard it would be. I have been lucky enough in my life to be blessed with wonderful family and friends who have helped me at every step along the way. There are no words that can fully describe the gratitude that I feel to everyone. Nevertheless, the following few paragraphs will be my attempt to thank the people who have made all of this possible.

My committee has been crucial during the process of doing my research. Jun Pan, my committee chair, has taught me a lot about doing research in Empirical Asset Pricing. Co-authoring two papers (Chapters 2 and 3) with her, I have learned a lot from the way she thinks about problems. She has also provided a lot of encouragement over the years, from assuring me before Generals that I would be ok to helping me with the job market process. In co-authoring a paper with Jiang Wang (Chapter 3), I have been able to learn from his perspective as a theorist, an important contrast to how I think as an Empiricist. His suggestions for my research have been extremely helpful. Gustavo Manso has provided numerous useful comments for my research and his upbeat attitude was very helpful when I was on the job market.

A number of other professors (past and present) at MIT have also contributed greatly to my learning process. Nittai Bergman spent a lot of time helping me on the introduction to my job market paper. Tim Johnson visited MIT during my second year and taught the Empirical Asset Pricing course. It was from that course that I realized that I was interested mainly in empirical work. Dirk Jenter was always very willing to read papers and paper proposals. In addition, I have received numerous comments from Hui Chen, Serdar Dinc, Carola Frydman, Scott Joslin, Leonid Kogan, Jon Lewellen, Stewart Myers, and Antoinette Schoar.

My PhD classmates have been essential in both helping me to learn Finance and also in providing moral support. Their numerous comments about my research have improved my understanding and my research. I have also greatly enjoyed the time spent with my classmates. Their support has been essential to my life at MIT. The moral support that they have given was particularly essential in the last year while I was on the job market. A number of my classmates have become great friends and they deserve special thanks.

For the last three years, Jason Abaluck has been my roommate and my Econometrics consultant at all hours of the day. We were randomly assigned as roommates by MIT graduate housing and we have had numerous interesting conversations that have involved relating all sorts of mundane things in life to Economics. In fact, Jason told me that I should thank him in my acknowledgments for his messy room because it would increase my utility if I had a Keeping-up-with-the-Joneses habit utility function.[1]

Manuel Adelino has been my officemate for the last two years and I have greatly enjoyed the time that we have spent discussing research. Despite the fact that his research focuses on Corporate Finance and mine on Asset Pricing, I found our conversations to be extremely intellectually stimulating. I often turned to Manuel for opinions on different things related

[1] I countered that his messes were a negative externality.

to my research. Though one of the many things that PhD candidates look forward to is having their own office, I will miss tossing around a miniature football with Manuel while discussing research.[2]

I have learned a lot from the critical way that Tara Bhandari thinks about research. She has never been afraid to challenge me intellectually and it has forced me to think more deeply about research. Tara has also been a very willing participant in the zany games that may entertain only the two of us[3] and a willing teller of cheesy jokes to entertain her fellow students.[4] However, what I really want to thank Tara for is how supportive she has been. This was particularly evident when I was on the job market. She constantly checked up on me when I came back from job talks and messaged me to wish me luck when I was on the road. Perhaps the best example of Tara using her sense of humor to be supportive was an e-mail she sent out on the day before my Finance Seminar at MIT mimicking the e-mails that are sent out when we have external speakers come for seminars. That was about the only time that I actually smiled in the week leading up to my seminar.[5]

Alex Edmans was the only other student in my Finance PhD entering class. We took numerous classes together, played on the same intramural sports teams, studied for Generals together, and worked on research together.[6] Alex is probably as responsible for my understanding of fundamental Finance as anyone. Working with Alex on problem sets and outside the classroom as co-captains of our intramural softball team greatly improved my early years at MIT. I really had a lot of fun teaching him about North American sports like hockey, football, and softball.

Adam Kolasinski was one of my first officemates and was the upper-year classmate most responsible for explaining how the department functioned. He patiently answered all sorts of questions that ranged from choosing courses to living at MIT.

In many ways, Jiro Kondo taught all of us about the dedication needed to be an academic. We watched as he spent years collecting data without guarantee of success. He was able to eventually write a paper that both won him the Lehman Brothers Fellowship and get him a job at Northwestern. We were both genuinely happy about his success, but also selfishly relieved that our own hard work might actually pay off. Despite how busy he was with his own research, Jiro always took time to see how I was doing and to encourage me. Jiro and I were also teammates in intramural hockey and one of my fondest memories at MIT is of assisting on many of Jiro's goals.[7]

Dimitris Papanikolaou has given me a lot of advice and provided his extremely good Economic insights. Not only did he teach me a lot about Finance, but Dimitris constantly encouraged me both when it came to research and non-academic things. Since he graduated two years ago, he has been sure to call me every couple of weeks to see how I have been doing

[2]Manuel has also taught me some interesting Portuguese words.

[3]This may be due to both of us having grown up in New Jersey, but I am sure that Tara will come up with an endogeneity story for that explanation.

[4]I do not know anyone else who would congratulate someone on his new spin-off in a card congratulating the birth of a new child.

[5]Tara also arranged to have most of the Finance PhDs over to her apartment for ice cream after the seminar.

[6]Alex also tried for a couple of years to throw me a surprise birthday dinner.

[7]I also want to mention that seeing Jiro's reaction on the day that his daughter, Isabelle, was born was one of those beautiful moments in life that showed all of us what was truly important.

and to ask about my research progress. This was particularly helpful when I was on the job market. Dimitris was able to give me advice about upcoming talks, console me about the talks that had gone badly, and to provide encouragement that was particularly necessary towards the end of the job market process.

Weiyang Qiu has been my consultant for everything Mathematical. Despite the fact that many of my questions are probably very basic for him, he has always been extremely patient in answering my questions.

Mary Tian has basically been the best officemate that I could possibly ask for. She has be a trusted source for honest opinions since she started at MIT. For the last couple of years, Mary has had to endure all of the guys in the office as our only female officemate. I think it is fair to say that Mary's personality has made it very easy for all of us to embrace her as our voice of reason.[8] Mary attended the AFAs on a student travel grant this year and patiently endured my freaking out before interviews. She deserves a lot of credit for my ability to function at the AFAs.

Jialan Wang, whom I have dubbed the department Soccer Mom, has always shown her concern for her fellow PhDs, myself included. I would like to thank her for always caring and taking the time to make sure that all of us are coping with the typical problems that come with doing research and being doctoral students.

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I am fortunate enough to still be very close with many of my childhood friends, some of whom I have known for over twenty years. Over the years, they have been extremely supportive of my endeavors. Despite the fact that almost all of us have moved away from Edison, NJ, we are still a close group that regularly checks to see how everyone is doing. I want to especially thank Brad Zerlanko. His unique perspective as a childhood friend and a PhD candidate has allowed him to be particularly sympathetic when I was down or stressed out.[9]

My extended family has been incredibly supportive throughout my life and in particular, a number of my cousins have really tried to encourage me. Dennis has lived in Cambridge for the last four years and endured a number of Sunday brunches in which I spent 90% of the time talking about how stressed out I was about my research. He has always been willing to listen and provide encouragement. Sharon has always provided a willing ear and a certain quirkiness that has often made me smile.[10] Patti has always been great with words and able to bring a smile to my face even on bad days. On my 22nd birthday (and my first spent at MIT) she wrote, "I have this feeling that you'll make it through brilliantly at MIT, but I hope that you reach out and make the most of all those people there too. Otherwise, you're not so much depriving yourself as the world!" Patti may have had higher than realistic expectations for me, but it was nice that she had such faith in me.

Finally, I would like to thank my first teachers, my parents. They have always been there for me and tried to give me every opportunity to succeed. My Mom took the teacher bit

[8]One thing that I will always remember from my time at MIT is Mary's incredible shooting display when we were playing basketball at the Finance Retreat.

[9]Neither of us was lucky enough to have siblings, but Brad has been like a brother to me.

[10]As an example, she once drew an In-N-Out burger on a birthday card that she sent me.

very seriously. I remember doing math problems at the age of 4. I could not read yet, but I could do addition and multiplication. My Dad made me his star pupil, patiently explaining things in a way that a child could understand.[11] At the time, I really did not understand how lucky I was.

My parents have taught me by example and I greatly admire many of their traits. My Mom has always shown unbelievable toughness. She was already a Chemistry professor in Taiwan when she came to the United States to be with my Dad. Eventually, she would finish a Master's degree in Computer Science by taking courses in the evening while taking care of me during the day. She accomplished all of this despite the fact that English was not her native tongue and she had no prior training in Engineering of any sort, much less in Computer Science. My Mom has always been the type of person who could endure a lot in the pursuit of her goals.

I admire my Dad for his work ethic and his willingness to sacrifice for the people he loves. He still works on research at 6am on weekends despite the fact that he is a tenured faculty member. Working hard is really the only way that my Dad understands how to live. When I was 13, my Dad took a new job that was 80 miles from where we lived. Instead of doing the natural thing of moving our family closer to his work, my Dad instead opted to make a one and a half hour drive each way because he wanted my Mom to keep her 10-minute commute and he did not want to uproot me from a life that I was comfortable with.[12] My Dad's main goal in life for the last twenty-something years has been to do everything to protect my Mom and me.

Thus, I dedicate my thesis to my parents for spending the last 27+ years putting my hopes and dreams ahead of their own.

[11] I would also like to thank my Dad, a third generation Accounting Professor, for not being disappointed that I decided not to become a fourth generation Accounting Professor.

[12] This decision was extremely important in allowing me to maintain the strong friendships with my childhood friends described above.

Contents

List of Figures

List of Tables

Chapter 1

Structural Models of Default and the Cross-Section of Corporate Bond Yield Spreads

1.1 Introduction

The literature on structural models of default, beginning with Merton (1974), has sought to price corporate debt by modeling firm fundamentals and combining equity and debt information to capture the default processes of firms. Since Merton's work, numerous structural models of default incorporating different assumptions have been developed.[1] Though structural models are appealing from an economic perspective, Huang and Huang (2003) argue that a broad group of structural models underpredict the levels of empirically observed yield spreads.[2] Proposed solutions have included both models that generate larger yield spreads and also non-credit solutions such as liquidity and the choice of benchmark security. It remains unclear how important each component is.

Structural models of default provide predictions about the level of yield spreads as well as the relative yield spreads of different bonds. While there is a large literature on the level of yield spreads, the implications of structural models in *relative* pricing have largely been ignored. This chapter focuses on the cross-section to better understand the disconnect between observed and model yield spreads. Cross-sectional tests provide an important metric for evaluating structural models beyond whether they are able to generate larger model yield spreads than previous models. While some structural models cannot generate large levels

[1]See for example Black and Cox (1976), Leland (1994), Longstaff and Schwartz (1995), Leland and Toft (1996), Anderson and Sundaresan (1996), Collin-Dufresne and Goldstein (2001), and Goldstein, Ju., and Leland (2001).

[2]See also studies by Eom, Helwege, and Huang (2004), Ericsson, Reneby, and Wang (2005), Cremers, Driessen, and Maenhout (2006), and Huang and Zhou (2007)

of spreads, it is possible that they contain mechanisms explaining the cross-section of yield spreads. Furthermore, by examining relations between unexplained yield spreads and credit risk proxies, cross-sectional tests can determine whether a structural model's unexplained yield spreads are solely due to non-credit components in bond yields or are due to the model failing to fully capture credit risk. Correlations between unexplained yield spreads and credit risk proxies may suggest structural models of default that contain dynamics which can further explain the cross-section of yield spreads.

My cross-sectional tests begin with the Black and Cox (1976) model as the base case. The Black-Cox model captures two fundamental determinants of the likelihood of default, leverage and asset volatility. To calibrate the Black-Cox model, I focus on matching firm-level parameters such as market leverage, equity volatility, and payout ratio. From this balance sheet and equity information, I am able to construct a panel of model yield spreads which can be compared to observed yield spreads. As an initial examination, I consider the cross-sectional explanatory power of the Black-Cox model spread, finding a within-group R^2 of 44.9% when regressing observed yield spreads on Black-Cox yield spreads. Thus, I find that over half of the cross-sectional variation in yield spreads remains unexplained by the Black-Cox model. This could be for two reasons. First, the remaining unexplained yield spread could be due to elements that structural models are not designed to capture such as liquidity, transitory price movements, and other non-credit components. Alternatively, the unexplained portion of yield spreads could reflect the Black-Cox model's inability to fully capture credit risk in the cross-section.[3]

To better understand the cross-sectional variation of observed yield spreads, and particularly, the variation that is not explained by the Black-Cox model, I examine the relation between unexplained yield spreads and credit risk proxies such as recent equity volatility and ratings. After controlling for Black-Cox yield spreads, observed yield spreads are related to these proxies of credit risk. The cross-sectional effects of these credit risk variables are economically large. For example, a move from the 25th to 75th percentile in recent equity volatility accounts for a 48 basis point difference in unexplained yield spreads. There is also evidence that unexplained yield spreads are related to option expensiveness, a proxy for credit risk premia. These cross-sectional relations suggest different structural models which may potentially improve our understanding of the cross-section of yield spreads. I explore this in two directions, through a jump diffusion model and also through a stochastic volatility model.

First, the relation between unexplained yield spreads and option expensiveness suggests that there may be a common risk premium priced in the equity option and corporate bond

[3]Of course, these two explanations are not mutually exclusive.

markets beyond a standard diffusion risk premium. I calibrate a jump diffusion model to address this relation. I use individual equity option expensiveness to infer jump risk premia firm-by-firm and then use this information to construct model yield spreads.[4] My results indicate that there is weak evidence that this model helps to explain the cross-section of yield spreads above and beyond the Black-Cox model. Constructing a residual from a regression of jump model spreads on Black-Cox spreads, I find that a one standard deviation change in this residual corresponds to a 16 basis point difference in observed yield spreads. However, the improvement in the within-group R^2 when including jump residuals in a regression of observed yield spreads on Black-Cox yield spreads is only from 43.4% to 45.3%. Despite this limited improvement in cross-sectional explanatory power, it is still interesting that the corporate bond and equity option markets seem to price a common risk.

The limited ability of the jump model to explain the cross-section above and beyond a Black-Cox model presents an important contrast to its ability to explain the level of yield spreads over the Black-Cox model. In a calibration using ratings-level information, Cremers, Driessen, and Maenhout (2006) find that a jump model with an equity index option-implied jump risk premium greatly increases the level of yield spreads relative to a diffusion-only model. Using individual equity options, I also find that the jump model helps to explain the level of yield spreads, though the improvement is somewhat smaller in my study.

The relation between unexplained yield spreads and recent equity volatility suggests a second potential solution to unexplained yield spreads, a stochastic volatility (Heston (1993)) model. Such a model has richer volatility dynamics that incorporate both recent and long-run volatility. This contrasts with the Black-Cox model which uses only a constant long-run volatility. In addition, the stochastic volatility model can generate further cross-sectional variation in yield spreads through across-firm differences in the correlation between asset return and variance shocks. This correlation is typically thought to be negative, meaning that asset variance is high exactly when asset returns are low. Thus, larger shocks to asset value are more likely to occur when asset value is low, increasing the probability of left-tail events. For firms sufficiently far from default, this will generate a greater yield spread through a greater probability of default.

I find that the stochastic volatility model cannot solve the puzzle of strong correlation between yield spreads and recent equity volatility. The mean estimate of the mean-reversion parameter of asset variance is 11.7, indicating a half-life of 0.71 months. Since bond maturities are typically multiple years, it will be the long-run asset variance that has a strong effect on bond pricing rather than short-run variance. In addition, the correlation between asset

[4] Cremers, Driessen, Maenhout, and Weinbaum (2006) document a relation between equity option implied volatilities and skews and yield spreads.

returns and variance, ρ, has little explanatory power for the cross-section of observed yield spreads. Estimates of ρ are generally negative with a mean of -0.15, a magnitude that is significantly smaller in economic terms than for equity indices.[5] Controlling for Merton model spreads[6], firms with more negative values of ρ have bonds with larger stochastic volatility model spreads, but these bonds do not have higher observed yield spreads. I further explore time-varying volatility by pricing bonds with a slower variance mean-reversion parameter and also by calibrating a Black-Cox model using only recent equity volatility. The former calibration does not help explain the cross-section of observed yield spreads while the latter does provide some additional explanatory power.

I also consider the cross-sectional pricing of credit default swaps (CDS). CDS have similar credit risk exposures as corporate bonds, but are thought to have different non-credit risk exposures. Whereas the primary non-credit component in the corporate bond market is liquidity, the primary non-(firm)credit component in the CDS market is counterparty risk. The results for CDS are similar to corporate bonds: unexplained CDS spreads are related to recent equity volatility, ratings, and option expensiveness. In addition, I find that unexplained CDS spreads are significantly related to unexplained corporate bond yield spreads. The within-group R^2 of a regression of unexplained CDS spreads on unexplained corporate bond yield spreads is 49.6%, smaller than the R^2 of 70.2% for a regression of CDS spreads on corporate bond yield spreads, but still quite economically significant. Since the most important common exposure of credit default swaps and corporate bonds is credit risk, this commonality provides further evidence that the Black-Cox model does not fully capture credit risk in the cross-section. Though part of this commonality could be due to commonality in liquidity, the magnitude of the commonality suggests that it is at least in part due to common unexplained credit risk.

My focus on the cross-section of yield spreads is related to the reduced-form regressions framework used by Collin-Dufresne and Goldstein (2001) and Campbell and Taksler (2003). CDGM find a common component in yield spread changes that they are unable to relate to standard macroeconomic and financial variables.[7] Campbell and Taksler (2003) document a relation between observed yield spreads and equity volatility and argue that the relation is too strong to be explained by a structural model such as Merton (1974). A reduced-form regression framework is parsimonious and allows researchers to determine what yield spreads

[5]Pan (2002) estimates this correlation to be -0.57 for the S&P 500 and a stochastic volatility model without a volatility premium.

[6]For the stochastic volatility model, the Merton model is the correct benchmark as the Heston model does not capture a first passage time default.

[7]See Avramov, Jostova, and Philipov (2007) for a follow-up which argues that yield spread changes are indeed related to changes in firm characteristics.

are related to. However, my focus is on testing whether a theoretically-founded measure of risk – a structural model of default – can explain the cross-section of empirically observed yield spreads.[8]

Another related strand of literature is on the relation between yield spreads and liquidity. Since structural models of default are not designed to capture liquidity, it is likely that unexplained yield spreads are related to liquidity proxies. Multiple studies have shown that corporate bond yield spreads are negatively related to proxies for liquidity.[9] In particular, most studies have focused on age and issued amount as proxies for liquidity. Consistent with this literature, I find that observed yield spreads are negatively related to liquidity in the cross-section even after controlling for Black-Cox yield spreads. In particular, moving from the 25th to the 75th percentile of bond age corresponds to a 10 to 15 basis point difference in unexplained yield spreads. Recently, Longstaff, Mithal, and Neis (2005) and Nashikkar, Subrahmanyam, and Mahanti (2007) have studied the relation between yield spreads and liquidity by first using CDS to control for credit risk and find evidence that the non-default component of yield spreads is related to liquidity. Their motivation for using CDS to control for credit risk is based on the fact that CDS are considered to be very liquid and is similar to my motivation for considering commonality in unexplained CDS and corporate bond spreads above.

The remainder of the chapter is organized as follows. Section 1.2 describes the data and provides summary statistics. Section 1.3 details the base case calibration and the relation between unexplained yield spreads and firm- and bond-level characteristics. Section 1.4 presents calibrations for a jump diffusion model. Section 1.5 presents calibrations for a stochastic volatility model. Section 1.6 briefly discusses endogenous default models. Section 1.7 presents results on credit default swaps and commonality between CDS and corporate bonds. Section 1.8 concludes.

1.2 Data and Summary Statistics

1.2.1 Data Description

The bond pricing data for this chapter is obtained from FINRA's TRACE (Transaction Reporting and Compliance Engine). This data set is a result of recent regulatory initiatives to increase the price transparency in the secondary corporate bond markets. FINRA, formerly

[8]Bharath and Shumway (2008) examine the default prediction of Moody's KMV model, a model based on the Merton model. Though this is a theoretically-founded measure of risk, it only reflects one-year P-measure default probabilities and is not designed to price corporate bonds.

[9]See Houweling, Mentink, and Vorst (2005) and references therein.

NASD,[10] is responsible for operating the reporting and dissemination facility for over-the-counter corporate trades. Trade reports are time-stamped and include information on the clean price and par value traded, although the par value traded is truncated at \$1 million for speculative grade bonds and at \$5 million for investment grade bonds.

On July 1, 2002, the NASD began Phase I of bond transaction reporting, requiring that transaction information be disseminated for investment grade securities with an initial issue of \$1 billion or greater. At the end of 2002, the NASD was disseminating information on approximately 520 bonds. Phase II, implemented on April 14, 2003, expanded reporting requirements, bringing the number of bonds to approximately 4,650. Phase III, implemented on February 7, 2005, required reporting on approximately 99% of all public transactions.

From TRACE, I am able to construct quarterly observed bond yields for 2003 to 2007. For my sample, I filter out canceled, corrected, and special trades and also drop cases where prices are obviously misreported. In addition, I only use pricing data from trades that are at least \$100k in face value as both Edwards, Harris, and Piwowar (2007) and Chapter 3 of this thesis find that smaller trades are subject to larger transitory price movements. From the Fixed Income Securities Database (FISD), I obtain bond characteristics that include flags for callability, putability, and convertibility along with various other characteristics such as issuance, offering date, and coupons. I drop callable, putable, and convertible bonds and also bonds with variable coupons from my sample. From MarketAxcess, I obtain the characteristics of benchmark treasuries and from Datastream, the prices of these treasuries. From the TRACE, FISD, MarketAxcess, and Datastream data, I am able to compute observed yield spreads. For each issuer, I keep one bond that is close to four years maturity and one bond that is close to ten years maturity. This is done so that the results below are not due to a handful of firms that have an extremely large number of issues.

The remaining data used consists mostly of firm-level characteristics used to construct model yield spreads and other firm and equity characteristics.[11] These are obtained from standard sources such as CRSP and Compustat.

1.2.2 Variable Construction: Model Inputs

In calibrating a structural model, the basic inputs are the same as that of options, $\frac{K}{V}$, T, r, δ, and σ_v. As will be described below, $\frac{K}{V}$, the default boundary over the total value of the firm and σ_v, the asset volatility, will be calibrated to match market leverage and equity volatility,

[10]In July 2007, the NASD merged with the regulation, enforcement, and arbitration branches of the New York Stock Exchange to form the Financial Industry Regulatory Authority (FINRA).

[11]Following Fama and French (2001), I drop utilities (SIC codes 4900-4949) and financial firms (SIC codes 6000-6999).

respectively. Market leverage is constructed as the market value of debt divided by the sum of the market value of debt and the market value of equity. The market value of equity is constructed as the product of share price and shares outstanding from CRSP. It should be noted from Table 1.1 that my sample contains firms with larger than average equity market capitalization. The median of $13.27 billion is larger than the 80th percentile cutoff of $9 billion for NYSE firms as of December 2007. Since over 70% of the firms in my sample are S&P 500 firms, the large mean market capitalization is unsurprising. Market value of debt is constructed as the sum of long-term debt and debt in current liabilities multiplied by a firm's average market value of debt to face value of debt ratio calculated from TRACE data. Equity volatility is constructed using the volatility of daily log equity returns. The maturity of a firm, T, is constructed as the average duration of a firm's outstanding bond issues where the appropriate Lehman rating index is used as the interest rate for discounting purposes. The mean firm maturity in my sample is slightly more than six years. The interest rate, r, is calculated as the average interest rate over the last ten years. Finally, the firm payout ratio, δ, is calculated as a firm's annual dividends plus annual coupon payments divided by total firm value.

1.2.3 Firm and Bond Characteristics

Table 1.1 also contains summary statistics for a number of additional firm and bond characteristics. The average implied volatility of a short-term out-of-the-money (OTM) put option and a short-term at-the-money (ATM) put option minus recent realized equity volatility, IV - equity volatility, is meant to capture option expensiveness. The average implied volatility tends to be greater than recent realized volatility which is unsurprising given results that options, and in particular, OTM puts tend to be overpriced (relative to realized equity volatility and Black-Scholes). Firm age is calculated as the number of years since the firm's BEGDT obtained from CRSP's msfhdr file. Firms in my sample are mostly mature firms with an average age over 40 years. Return on assets for a quarter are calculated as income before extraordinary items divided by the mean of quarter-start and quarter-end total assets. The firms in my sample tend to be profitable with a mean quarterly ROA of 1.35% as compared to an average of -0.9% for the full Compustat universe during the time period I study. Equity beta is calculated using a standard 60-month rolling window and the estimates for my sample do not exhibit unusual properties. Asset tangibility is constructed using the estimates from Berger, Ofek, and Swary (1996) as 0.715 Receivables + 0.547 Inventory + 0.535 Capital + Cash Holdings, scaled by total book assets. Interest coverage is calculated as earnings before interest and taxes divided by the interest expense. Finally, deviations from historical leverage are calculated as the difference between a firm's mean leverage over the 10 years prior to my sample minus its current leverage.

Most of the bonds in my sample have an A or Baa rating, with the average rating being Baa1. The average amount outstanding is approximately \$300 million in face value, which is larger than the average bond in FISD which has an average face value of approximately \$170 million.[12] Mean trade sizes are over half a million dollars in face value and trades on average occur on less than half the days in which the bonds are in the sample.

1.3 Base Case and Cross-Sectional Tests

As a base case, I calibrate a Black-Cox model. This model has the advantage of capturing both leverage and asset volatility while still having closed-form solutions for derivative prices and probabilities of default. It also improves on the Merton model in that default occurs the first time that asset value falls below a boundary rather than only if firm value is below the face value of debt at maturity. Such a framework is perhaps more intuitive for pricing coupon bonds as a firm's value can be below K at time t and above K at time s where s > t in the Merton model. This would allow a firm to be in default at one point in time and solvent at a future point in time.

1.3.1 Asset Value Process and Claims on Cashflow

The asset value process is a standard Geometric Brownian Motion,

$$\frac{dV_t}{V_t} = (\pi^v + r - \delta)dt + \sigma_v dW_t^v, \tag{1.1}$$

where π^v is the asset risk premium, r is the risk-free rate, δ is the payout ratio, and σ_v is the constant asset volatility. Under the risk-neutral measure, the asset value process is a Geometric Brownian Motion with $\pi^v = 0$. If the total firm value falls below K, the face value of debt, the firm defaults and $(1 - R_{firm})$ is lost in bankruptcy. Firm-level recovery is set at 80% to be consistent with Andrade and Kaplan (1998) finding that the cost of financial distress is approximately 15 to 20% of firm value. For each firm, the issuance-weighted average duration of its outstanding debt is taken to be the firm's "maturity", T. If the firm is solvent at T, debtholders receive K and equityholders receive the remaining firm value. Given these claims to cashflows, modeling the firm involves pricing three components, (1) equity, (2) debt, and (3) bankruptcy costs.

[12] See Chapter 2.

1. Equity

   At maturity, equity is the residual claimant if the firm is still solvent. With a first-passage time model and the face value of debt used as the default boundary, equity is always in the money if the firm is solvent and is paid $V_T - K$. This piece of equity is equivalent to a down-and-out call option. Equity is also a claim on dividends that the firm pays out.

2. Debt

   At maturity, debt receives K if the firm is solvent and $R_{firm}K$ if the firm has defaulted. Additionally, debt is also a claim on coupon payments.

3. Bankruptcy costs

   If the firm defaults, there is a sunk bankruptcy cost of $(1 - R_{firm})K$.

Equity and debt at maturity and bankruptcy costs have closed form solutions.[13] Dividends and coupons are claims on the remaining asset value. To divide the remaining asset value between equity and debt, I construct the total payout ratio, δ, and the equity and debt payout ratios, δ_e and δ_b, respectively. δ_e is the value of dividends paid divided by firm value while δ_b is equal to coupon payments divided by firm value. $\frac{\delta_e}{\delta}$ is then the proportion of the remaining firm value attributed to equity.

1.3.2 Calibration Methodology

For each firm, the values of σ_v and $\left(\frac{K}{V}\right)_t$ are determined by matching model-implied values of equity volatility and market leverage to observed values:

$$\sigma_E^2 = \left(\frac{\partial \log E}{\partial \log V}\right)^2 \sigma_v^2 \tag{1.2}$$

$$\text{Market Leverage}_{empirical} = \frac{\text{Model Debt}(\frac{K}{V}, \sigma_v, T, r, \delta)}{\text{Model Debt}(\frac{K}{V}, \sigma_v, T, r, \delta) + \text{Model Equity}(\frac{K}{V}, \sigma_v, T, r, \delta)}$$

In the Black-Cox model, σ_v is constant while $\frac{K}{V}$ is time-varying due to its denominator. This, along with the time-varying nature of $\frac{\partial \log E}{\partial \log V}$, leads to two important modeling choices. First, equity volatility is time-varying in the model even though asset volatility is constant. Thus, it is necessary to calculate equity volatility using short enough time periods so that $\frac{\partial \log E}{\partial \log V}$ does not change much. Changes in $\frac{\partial \log E}{\partial \log V}$ are largely driven by changes in the firm's market leverage, which typically does not change much at short horizons. Thus, I calculate

[13] See Appendix 1.9.3 and also Bjork (2004)).

equity volatility quarterly using daily data. An alternative strategy would be to estimate equity volatility at even shorter horizons such as by using minute-by-minute data. However, as documented in Appendix 1.9.2, even small bid-ask bounces can generate large equity volatility when such high frequency data is used.

The second important modeling choice I make is to aggregate the volatility equation over time. In the simplest scheme, the two equations could be simultaneously solved period-by-period.[14] However, this would give a time-varying asset volatility, which would be an inconsistent application of the Black-Cox model. In addition, period-by-period estimates may be imprecise due to noisy estimates of equity volatility. Thus, I also extend the volatility data back to 1993 for 15 years of total data, so that estimates of σ_v will be more precise. I solve for asset volatility so that the volatility equation holds on average:

$$\sigma_v^2 = \frac{\sum_t \sigma_{E,t}^2}{\sum_t \left(\frac{\partial \log E}{\partial \log V}\right)_t^2}$$

Since $\frac{K}{V}$ is time-varying in the model, I allow for one market leverage equation per quarter. I then simultaneously solve for σ_v and a time-series of $\left(\frac{K}{V}\right)_t$ through a single volatility equation and a market leverage equation for each quarter.

Important distinctions between my calibration methodology and that of Huang and Huang (2003) are that I calibrate firm-by-firm and that I calibrate to match equity volatilities rather than historical probabilities of default.[15] Because they are concerned with the level of the yield spread for typical firms, Huang and Huang calibrate by rating to the average firm balance sheet information within a rating and the historical probability of default for the rating. I substitute firm-level balance sheet information and equity volatility. Huang and Huang's choice to match historical default probabilities is based on generating model-implied credit spreads with empirically reasonable parameters. Matching equity volatilities is consistent with the spirit of this goal. In addition, there are two important reasons to match equity volatility rather than the historical probability of default. First, it is not necessarily true that historical default probabilities reflect forward-looking default probabilities. In fact, Huang and Huang indicate that they would ideally calibrate their models at each time period separately to reflect each period's default probability, but are constrained by the limited data on defaults. Second, from a practical perspective, default probabilities, by definition, are not available on a firm-by-firm basis. Since Huang and Huang calibrate by rating, they are able to match the historical default probability for each rating. By matching each individual firm's default probability to the historical probability of default

[14]In Section 1.5.3, I explore such an estimation strategy.

[15]In Appendix 1.9.4, I consider the default probabilities implied by the model in my calibrations.

for its rating, I would prevent structural models from explaining cross-sectional variation in yield spreads within a rating. Matching historical default probabilities would also hard-code model yield spreads to be sorted cross-sectionally by rating. One possible method to estimate firm-by-firm default probabilities would be to use the logit method in Campbell, Hilscher, and Szilagyi (2007). However, they already use equity volatility and leverage as inputs and thus, do not provide a theoretical improvement over simply using a structural model's predicted defaults.

After firm-level parameters are calculated, pricing a coupon bond is an application of risk-neutral pricing. Specifically, a T-year bond with semi-annual coupons c and face value of \$1 has a model price of:

$$B_t = \sum_{i=1}^{2T} \frac{c}{2} e^{-ri/2} P^{i/2} + e^{-rT} P^T + \sum_{i=1}^{2T} R_{bond} e^{-ri/2} (P^{(i-1)/2} - P^{i/2}) \tag{1.3}$$

where P^t is the risk-neutral probability that a firm is still solvent at t and R_{bond} is the recovery rate of the bond.[16] The first term in the bond pricing equation is the value of coupon payments and the second term is the value of the face value. The third term is the value of the recovery given default. $P^{(i-1)/2} - P^{i/2}$ represents the risk-neutral probability of default between times $\frac{i-1}{2}$ and $\frac{i}{2}$. The cumulative risk-neutral probability of default is:

$$1 - P^t = N\left(\frac{-\log\left(\frac{V}{K}\right) - \left(r - \delta - \frac{\sigma_v^2}{2}\right)t}{\sigma_v\sqrt{t}}\right) \tag{1.4}$$
$$+ \exp\left(\frac{-2\log\left(\frac{V}{K}\right)\left(r - \delta - \frac{\sigma_v^2}{2}\right)}{\sigma_v^2}\right) N\left(\frac{-\log\left(\frac{V}{K}\right) + \left(r - \delta - \frac{\sigma_v^2}{2}\right)t}{\sigma_v\sqrt{t}}\right)$$

1.3.3 Calibration Results and Cross-Sectional Tests

I assess the performance of the Black-Cox model by first testing whether the credit spread puzzle on levels established by Huang and Huang (2003) holds in my sample of bonds. Because I construct a panel of model-implied yield spreads, my exercise is an empirical exercise in which I am able to provide t-stats.[17] In Table 1.9.1, I confirm that observed yield

[16] Note that the bond recovery rate and the firm recovery rate do not have to be the same. In fact, Andrade and Kaplan (1998) find that the cost of financial distress is approximately 15-20% of firm value at bankruptcy. Carey and Gordy (2007) find that recovery for senior unsecured debt is slightly above 50%. The disparity between overall firm recovery and senior debt recovery is largely due to bank debt having priority over public debt. Here, I set the bond-level recovery equal to 50%, but consider setting recovery rates by industry in Section 1.3.6.

[17] Huang and Huang (2003) calibrate by ratings using the mean firm fundamentals for a rating. Thus, their conclusion is about the economic significance of the difference between model and observed yield spreads.

spreads are too low to be explained solely by the Black-Cox model. The mean difference between observed and model spreads is 90 basis points for four-year bonds and 88 basis points for ten-year bonds. Compared to mean observed yield spreads of 160 basis points and 176 basis points, respectively, the results are very economically significant. In addition, both differences have t-statistics that are significant at the 1% level when standard errors are clustered by firm and time. Differences are also significant for each rating with the exception of four-year poorer-rated bonds for which the sample size is small.

The results in Table 1.9.1 suggest that either (1) the model does not properly account for the credit risk inherent in corporate bonds or (2) part of the level of yield spreads is due to liquidity or some other non-credit risk factor. Simply comparing the levels of observed and model yield spreads does not allow one to distinguish between these two competing, but not mutually exclusive hypotheses. However, focusing on the cross-section, I can assess the ability of the Black-Cox model to capture credit risk in the cross-section through a regression framework. To test the model in the cross-section, I use the framework:

$$\text{observed yield spread}_{it} = \alpha_t + \alpha_1 \text{model yield spread}_{it} + \varepsilon_{it} \tag{1.5}$$

From this regression, I examine how much of the cross-section of observed yield spreads is explained by the model yield spread and also construct the unexplained yield spread, $\hat{\varepsilon}_{it}$. This residual is orthogonal to the model yield spread by construction and can be thought of as the portion of the observed yield spread unexplained by the model. I examine the relation between the unexplained yield spread and various credit risk proxies, liquidity proxies, and firm-level characteristics in this and the following sections. This methodology is similar in spirit to the cross-sectional anomalies literature for equity returns.[18] In the literature on equity returns, stocks are typically sorted on a characteristic into portfolios. If there is a spread in the return based on the characteristic and the portfolios have similar risk (typically measured by β), an anomaly has been discovered (and is potentially an indication that the underlying risk model does not fully capture risk in the cross-section). Here, I explicitly control for credit risk by first partialing it out before considering relations with firm-level and bond-level characteristics. An alternative methodology would be to include characteristics in the left-hand side of equation (1.5). However, given the large correlations of the model yield spread with credit risk proxies, I instead first partial out the model yield spread to allow it the best chance to succeed.

Running the regression from equation (1.5), I find that the coefficient of the model yield spread is 0.58 with a robust t-stat of 8.00. It is statistically different from both 0 (the case

[18]See Lakonishok, Shleifer, and Vishny (1994) for a typical example.

where the model contains no information) and 1 (the case in which the model yield spread and observed yield spread move one-for-one). I find a within-group R^2 of 44.9%, indicating that the model yield spread explains almost half of the cross-sectional variation in the observed yield spread. Though this R^2 does provide a sense of the ability of the model to explain the observed yield spread in the cross-section, it does not on its own allow a determination of whether the model successfully explains the cross-section. The remaining unexplained observed yield spread could potentially be due to liquidity, transitory price movements, and other non-credit components that structural models are not designed to capture.

To examine whether the model can fully capture credit risk in the cross-section, I regress the unexplained yield spread on proxies for credit risk and credit risk premia in the final four columns of Table 1.3, Panel A. The residuals from a regression of the observed yield spread on the model yield spread are, by construction, orthogonal to the model yield spread. These unexplained yield spreads should be unrelated to risk proxies if the model has sufficiently controlled for credit risk in the cross-section. The unexplained spreads are significantly related to leverage, recent equity volatility, and ratings, though the relation to leverage is statistically insignificant once recent equity volatility is included. Using the first partial derivative of the Merton model with respect to equity volatility and average firm parameters, Campbell and Taksler (2003) find that the theoretical sensitivity of yield spreads to equity volatility is smaller than suggested by their regression framework. The results in Table 1.3 formalize this finding for firm-by-firm model-implied yield spreads. It is important to note that this relation holds even after controlling for market leverage and the model estimate of asset volatility, the two major theoretical determinants of credit risk in the Black-Cox model. In unreported results, I also find that unexplained spreads are related to the difference between recent equity volatility and the mean of historical equity volatility. A move from the 25th to 75th percentile in recent equity volatility in the sample corresponds to a 48 basis point difference in the unexplained yield spread. A move of three points in rating (which would be equivalent to moving from A1 to Baa1) corresponds to a 50 basis point difference in the unexplained yield spread. Both of these quantities are very significant economically compared to the average observed yield spread of approximately 165 basis points. Finally, once recent equity volatility is controlled for, the unexplained yield spread is significantly related to option expensiveness as proxied for by option-implied volatility minus equity volatility.

Table 1.4 contains a further examination of the relation between observed yield spreads and recent equity volatility. Bonds are first sorted by their Black-Cox yield spread into quintiles and quintiles 1 and 2 are combined as both groups have small model yield spreads. Within each group, bonds are then sorted by past three months' equity volatility. Thus, within each model yield group, I can examine the difference in observed yield spread with the model yield spread reasonably well controlled for. For model yield spread quintiles 1 & 2, 3, and 4, there is a monotonic trend of increasing observed yield spread with increasing recent equity volatility while the model yield spread remains relatively flat. The magnitudes of the differences between quintile 5 and 1 of equity volatility are economically large as the observed yield spread for quintile 5 is close to twice as large as the observed spread for quintile 1.

The overall conclusion from tests of the Black-Cox model is that while the model does capture a significant amount of cross-sectional variation in yield spreads, it does not fully capture cross-sectional differences in credit risk. Even after controlling for the model, observed yield spreads are related to recent equity volatility, equity option expensiveness, and ratings. The relation to recent equity volatility suggests that the use of a stochastic volatility model might be useful while the relation to option expensiveness suggests the use of a model with an additional, non-diffusion, risk premium.

1.3.4 Additional Firm Characteristics

I examine the relation between the unexplained yield spread and additional firm characteristics in Panel B of Table 1.3. My focus is on seven firm characteristics in addition to the credit risk proxies considered previously: firm size (measured by equity market capitalization or total firm value), firm age, return on assets, equity beta, asset tangibility, interest coverage, and deviations from historical leverage. The relations between these variables and unexplained yield spreads may suggest further modeling assumptions needed in a structural model to explain the cross-section of yield spreads. In addition, the relations between these variables and ratings may also shed light on the relation between unexplained yield spreads and ratings.

Firm size potentially affects yield spreads through multiple channels. First, there may be less asymmetric information about accounting information for large firms, leading to lower yield spreads. This would be consistent with the incomplete accounting information model of Duffie and Lando (2001). Second, larger firms may have better reputations in the debt market, decreasing their cost of borrowing. Diamond (1989) formalizes the relation between reputation and borrowing costs in a model of project selection. Finally it is possible that the debt market simply exhibits a size effect similar to the equity market.

Firm age is also a potential proxy for reputation as older firms that continue to borrow have presumably established a positive reputation. Profitability, as measured by ROA, is included as Moody's explicitly acknowledges[19] using profitability in assigning ratings. Equity beta is included as a proxy for how much the firm moves with the market and would be expected to be positively related to yield spreads. Chen (2008) constructs a model in which the (endogenous) default boundary is higher in bad times. This is driven by higher risk premia and lower expected growth rates in bad times. Equity beta is used to capture the former effect as firms which move more closely with the market have higher risk premia exactly when the aggregate risk premium is higher. In addition, I also construct a downside beta, β^-, as in Ang, Chen, and Xing (2006). Downside beta reflects the co-movement of a firm's equity with the market, conditional on a below average market return. Asset tangibility is included as an explanatory variable as I assume 50% recovery in my calibrations as a simplification. If there is in fact cross-sectional variation in expected recovery rates, yield spreads should be negatively related to tangibility. Interest coverage is included as a proxy for corporate liquidity. While firms can theoretically pay interest expenses by liquidating assets, frictions make the ability for firms to pay interest expenses from earnings important. Kim, Ramaswamy, and Sundaresan (1993) present a model in which bankruptcy occurs when a firm's net cashflow cannot cover interest expenses. Finally, the deviation from historical leverage is included as firms may have mean-reverting leverage. A firm with leverage lower than its historical average might be expected to increase its leverage. Thus, just using its current leverage in pricing would generate a model yield spread that is too low. Collin-Dufresne and Goldstein (2001) present a model with a mean-reverting leverage ratio.

When ratings are not controlled for (columns 1 to 4 of Table 1.3, Panel B), unexplained yield spreads are negatively related to firm size and asset tangibility. Both relations are economically significant as moving from the 25th percentile in equity market capitalization to the 75th percentile results in a 32 basis point decrease in unexplained yield spreads and a similar move for asset tangibility corresponds to an 11 basis point decrease in unexplained yield spreads. The result for asset tangibility indicates that there is some cross-sectional variation in recovery rates that my analysis does not capture Firm age, return on assets, equity beta, interest coverage, and deviations from historical leverage are statistically insignificant. In unreported results, I also find that substituting downside beta for beta does not change these findings.

In columns 5 and 6 of Panel B, I include ratings as a control and find that unexplained yield spreads are no longer related to firm size and asset tangibility. As shown in the final two columns of Panel B, ratings are related to both firm size and asset tangibility. This relation is particularly strong for firm size as a move from the 25th to 75th percentile in equity market capitalization corresponds to a two point improvement in rating (i.e. A3 to

[19] See Fons, Cantor, and Mahoney (2002).

A1). These results suggest that at least in part, the relation between unexplained yield spreads and ratings can be explained by firm size and the ability of ratings to capture expected recoveries in the case of default. Interestingly, when I regress unexplained yield spreads on the firm characteristics considered in this section and the credit risk proxies considered previously, the only statistically significant variables are return on assets (at the 10% level with the wrong sign), ratings, recent equity volatility, and option expensiveness. This further affirms the results in Section 1.3.3 that the credit risk proxies considered are important in understanding unexplained yield spreads.

1.3.5 Liquidity

In Panel C of Table 1.3, I consider the cross-sectional variation in the unexplained yield spread due to liquidity variables.[20] As structural models of default are designed to capture credit risk and not liquidity, unexplained yield spreads should be positively related to illiquidity. Older bonds are thought to be less liquid because a larger fraction of their issuance is likely to have been acquired by buy-and-hold investors. The results indicate that older bonds indeed have larger yield spreads. A move from the 25th percentile to the 75th percentile of age corresponds to approximately a 10 to 15 basis point difference in the unexplained yield spread. Larger issues are thought to be more liquid and this is consistent with the finding that larger issues have lower yield spreads. The difference in spreads between a bond at the 25th percentile and a bond at the 75th percentile of amount outstanding is approximately 10 basis points.

In addition to characteristic-based liquidity variables, I also consider trading-based variables. Total volume traded is positively related to yield spreads, but is insignificant. The number of trades is positively related to yield spreads, a surprising result if one believes that more liquid bonds trade less often. A move from the 25th percentile to 75th percentile in number of trades is associated with an unexplained yield spread that is 12 basis points higher. However, traders are more likely to break-up trades for less liquid issues, resulting in smaller trading sizes and a larger number of trades. This is consistent with the negative (albeit insignificant) sign on the average trade size. Finally, turnover and percent of days traded are positively related to yield spreads. Overall, it seems that part of the unexplained cross-sectional variation in observed yield spreads is due to liquidity effects, particularly those effects measured by bond age and amount outstanding.

[20]See Houweling, Mentink, and Vorst (2005) for an examination of different liquidity proxies. Issued amount and age are the most popular liquidity proxies in the literature that the authors survey and also the most applicable here.

1.3.6 Recovery Rates

My calibration of the Black-Cox model is able to capture cross-sectional variation in leverage and asset volatility, but does not capture cross-sectional variation in recovery rates. I have set firm-level recovery rates to 80% to be consistent with Andrade and Kaplan (1998) estimate of the cost of financial distress being between 15 and 20% and bond-level recovery at 50% to be roughly consistent with Carey and Gordy (2007). The assumption about firm-level recovery has little effect in the calibration as it enters only in the first stage when firm-level parameters are inferred from equity and balance sheet information. Since this part of the calibration hinges largely on equity information and equity theoretically has no recovery in default, the impact of firm-level recovery is minimal. However, the latter assumption may potentially be important in bond pricing if there is significant cross-sectional variation in expected recovery rates.

Schuermann (2004) provides a survey of the literature on loss given default (= 1 - recovery rate). He finds three main factors that drive differences in loss given default: (1) seniority and collateral, (2) the business cycle, and (3) the industry of the firm. Priority is important as bank loans have higher recovery rates than corporate bonds and senior bonds have higher recovery rates than subordinated and junior bonds. In addition, senior secured bonds have higher recovery rates than senior unsecured bonds. The vast majority of my sample ($\approx$ 97%) is classified as senior and unsecured by FISD, indicating that it is unlikely that the cross-sectional results presented above are driven solely by differences in seniority or collateral. A caveat is that without full information about the debt structure of companies in my sample, I cannot distinguish between companies with high or low levels of bank loans. Companies with high levels of bank loans would potentially have lower recovery rates on corporate bonds as their corporate bonds are junior to a larger fraction of the firm's debt.

The second determinant of recovery rates, the business cycle, is a variable that predicts time-variation in recovery rates. Altman, Resti, and Sirnoi (2004) find that recovery rates are lower in bad times. Acharya, Bharath, and Srinivasan (2007) find a fire-sales effect. Specifically, the recovery rate for a firm is lower if its industry is in distress. These effects suggest that in bond pricing equation (1.3), the risk-neutral recovery rate should in fact be lower than the P-measure recovery rate, further complicating the treatment of the recovery rate. Calculating a risk-neutral expected recovery rate requires the assumption of a model and also the use of a credit-sensitive security for calibration purposes. This implicitly chooses a risk-neutral expected recovery rate for which the model's pricing is exactly correct, effectively imposing that all remaining variation in yield spreads is due to differences in recovery rates. Since I am examining the ability of these models to price corporate bonds in the cross-section, I do not adopt this methodology.

Altman and Kishore (1996) find that the recovery rate of corporate bonds is related to the industry of the underlying firm. Using the mean industry-level recovery rates from Altman and Kishore's paper, I re-calculate model yield spreads for the Black-Cox model. As shown in Panel A of Table 1.5, the cross-sectional results for risk and risk premia proxies are largely unchanged. In unreported results, I find that the results for liquidity proxies and firm characteristics are also similar.

As described above, an ideal calibration of a structural model would involve bond-by-bond, risk-neutral expected recovery rates. However, without more detailed information about firm-level debt structure and the assumption of an underlying model to infer risk-neutral recovery rates, this cannot be done. Thus, I instead consider the possible effect of the recovery rate by varying recovery rates based on observed yield spreads. The main concern when using the same recovery rate for all bonds is that for two bonds with similar model-implied yield spreads, the bond with the higher observed yield spread is exactly the bond with a lower recovery rate and that this explains exactly what the model is missing in the cross-section. To explore this possibility, I first sort issuers into deciles by the average model yield spread of their bonds. Within each decile, I then sort into terciles by the average observed yield spread of each issuer. I assign a 15.44% recovery to the high observed yield spread tercile, 41% to the middle tercile, and 66.56% to the low tercile[21] and then re-estimate Black-Cox yield spreads. This effectively assumes that bonds with high observed yield spreads compared to the population of bonds with similar model yield spreads have higher yield spreads at least in part because they have lower recovery rates. Mechanically, this improves the cross-sectional explanatory power of the model. However, as shown in Panel B of Table 1.5, this improvement is limited as the within-group R^2 only improves to 50.49%. More importantly, the unexplained yield spread remains related to recent equity volatility, ratings, and option expensiveness. Therefore, it is unlikely that cross-sectional variation in recovery rates can explain the cross-sectional variation in observed yield spreads that the Black-Cox model does not capture.

1.4 Jump Diffusion Model

Making use of the results in Kou and Wang (2003) and the calibration in Huang and Huang (2003), I now calibrate a double-exponential jump diffusion model (henceforth referred to as the jump model). Such a model has the potential to explain the levels of yield spreads as well as the cross-section through changes in the distribution of firm value and an additional

[21]These recovery rates are chose to match the mean recovery rate reported by Altman and Kishore $\pm$ one standard deviation.

source of risk premia, the jump risk premia. Cremers, Driessen, and Maenhout (2006) calibrate a similar model in the aggregate by matching equity index options and find that the incorporation of a jump risk premia greatly reduces the levels puzzle. Here, I calibrate firm-by-firm to individual equity options to examine if a jump model can help to explain the cross-section of yield spreads above and beyond the Black-Cox model. Under the P-measure, asset value is assumed to follow the process:

$$\frac{dV_t}{V_{t-}} = (\pi^v + r - \delta)dt + \sigma_v dW_t^v + d\left[\sum_{i=1}^{N_t}(Z_i - 1)\right] - \lambda\xi dt \tag{1.6}$$

where $Y \equiv \log(Z)$ and $f_Y(y) = p_u\eta_u e^{-\eta_u y}1_{\{y\geq 0\}} + p_d\eta_d e^{-\eta_d y}1_{\{y<0\}}$
The mean percentage jump size is:

$$\xi = E(e^Y - 1) = \tfrac{p_u\eta_u}{\eta_u - 1} + \tfrac{p_d\eta_d}{\eta_d + 1} - 1$$

The probability of up and down jumps are p_u and p_d, respectively and $\frac{1}{\eta_u}$ and $\frac{1}{\eta_d}$ are the mean up and down jump sizes, respectively.

I follow Kou (2002) and Huang and Huang (2003) in defining the transformation from P to Q, the risk-neutral measure, via a single parameter, γ. Under Q, the asset value process is:

$$\frac{dV_t}{V_{t-}} = (r - \delta)dt + \sigma_v dW_t^{vQ} + d\left[\sum_{i=1}^{N_t^Q}(Z_i^Q - 1)\right] - \lambda^Q\xi^Q dt \tag{1.7}$$

where $Y^Q \equiv \log(Z^Q)$ and $f_{Y^Q}(y) = p_u^Q\eta_u^Q e^{-\eta_u^Q y}1_{\{y\geq 0\}} + p_d^Q\eta_d^Q e^{-\eta_d^Q y}1_{\{y<0\}}$,
$\xi^Q = E(e^{Y^Q} - 1) = \frac{p_u^Q\eta_u^Q}{\eta_u^Q - 1} + \frac{p_d^Q\eta_d^Q}{\eta_d^Q + 1} - 1$,
and $\eta_u^Q = \eta_u + \gamma; \eta_d^Q = \eta_d + \gamma; p_u^Q = \frac{\frac{p_u\eta_u}{\eta_u^Q}}{\frac{p_u\eta_u}{\eta_u^Q} + \frac{p_d\eta_d}{\eta_d^Q}}; \lambda^Q = \frac{p_u\eta_u}{\eta_u + \gamma} + \frac{p_d\eta_d}{\eta_d - \gamma}$.
The jump risk premium is $\lambda\xi - \lambda^Q\xi^Q$.

1.4.1 Calibration Methodology

In calibrating the model, I make a number of simplifications for tractability. First, I start with the asset volatility calculated for the Black-Cox model. This assumption is equivalent to accepting that the mapping from equity variance to asset variance given by the Black-Cox model is reasonable.[22] When incorporating jumps, the total asset variance is no longer

[22]Previous papers have considered different methods of mapping equity variance to asset variance. Eom, Helwege, and Huang (2004) use the delta of a call option from the Merton model. In Chapter 2, I follow a similar procedure, but allow for stochastic interest rates. Schaefer and Strebulaev (2008) use a leverage-weighted average of equity and debt return variance.

just variance due to the diffusion component in the asset value process as the jumps also contribute to the total asset variance. The total asset variance is:

$$\sigma_{v,total}^2 = \sigma_v^2 + \lambda \left[\frac{2p_u}{\eta_u^2} + \frac{2p_d}{\eta_d^2} \right] \tag{1.8}$$

The second assumption that I make is regarding the choice of jump parameters. Huang and Huang (2003) choose λ, the jump frequency, to equal 3, and η_u and η_d, the inverse of the mean up and down-jump sizes, respectively, to equal 30. They argue that these parameter values are roughly consistent with the results from Anderson, Benzoni, and Lund (2002). Cremers, Driessen, and Maenhout (2006) find λ to be much smaller and jump sizes to be much larger, though their jump sizes seem much larger than what is empirically observed. I choose $\lambda = 1$, an average of one jump per year, and $\eta_u = \eta_d \equiv \eta$. The mean absolute jump size, $\frac{1}{\eta}$, is then calibrated by matching the fourth moment of equity returns in a similar manner to how asset volatility is calculated for the base case. As a simplification, I use partial derivatives from the Black-Cox model and the equation,[23]

$$\frac{1}{n} \sum \text{fourth moment}_t = \frac{1}{n} \sum \left(\frac{\partial \log E}{\partial \log V} \right)^4 E[(\sigma_v \sqrt{dt} Z + d \sum Y_i)^4] \tag{1.9}$$

To be precise, the transformation from a function of asset value to a function of equity value requires an application of Ito's Lemma with jumps and not a simple application of first partials. The disparity when using the above simplification is particularly severe if jump sizes are large. For my sample, estimated jump sizes turn out to be relatively small (with an average η around 25). In addition, sensitivity analysis shows that yield spreads are not very sensitive to the size of the jump[24] and are instead much more sensitive to the transformation from P to Q-dynamics.

Finally, I estimate γ, the parameter to transform the P-measure jump parameters to Q-measure jump parameters. To estimate γ, I use equity option prices with the intuition that if there is indeed a jump risk premium (or any other non-diffusion risk premium), it should be reflected in the prices of both corporate bonds and equity options. Since equity options are now compound options on asset value with an underlying process that is a double-exponential jump diffusion, I calculate approximate option prices by calculating the risk-neutral probability of asset value falling between discreet levels at option maturity, given γ. Equity options are then priced by inferring the value of equity from assets, calculating

[23] In some cases, fourth moments are too small to back out any jump size. In these cases, η is set equal to 100, a small jump size that has a trivial effect on default probabilities.

[24] Note that the estimate of jump size alone has little effect on yield spreads because large jump sizes will cause a large downward adjustment in σ_v while small jump sizes will cause a small downward adjustment in σ_v.

option payoffs, and using payoffs along with risk-neutral probabilities. The parameter γ is estimated by minimizing the sum of percentage squared pricing errors of a group of options:

$$SSPE = \min_{\gamma} \sum \left(\frac{\text{actual price} - \text{model price}}{\text{actual price}} \right)^2 \tag{1.10}$$

I use four options in calculating γ: (1) a short-horizon (< 3 month) OTM put, (2) a short-horizon ATM put, (3) a short-horizon OTM call, and (4) a longer-horizon (> 6 month) OTM put. Because individual equity options are American while pricing is European, I choose options for which the pricing difference between American and European options is likely to be small. The longer-horizon OTM put is included due to possible differences in the volatility surface across maturities. Among each subgroup of options, I choose the option with the greatest trading volume during the most recent month. Bakshi, Cao, and Chen (1997) minimize the sum of squared dollar pricing errors rather than percentage pricing errors. However, this underweights OTM options which should provide important information about jump risk premia.

One important consideration in calculating the distribution of asset value is the choice of asset volatility. Under jump model dynamics, asset volatility is constant. This is, of course, a simplification of reality. Using the above procedure with a constant asset volatility may generate positive results that are due to the strong relation between yield spreads and recent realized equity volatility rather than a risk premium. Suppose that implied volatility exactly equals recent equity volatility. Some firms have a higher recent equity volatility than their asset volatility (calculated for the full time period) and current leverage can explain. If the full sample asset volatility is used to calculate the distribution of asset value in the next couple of months, it will be exactly these high recent equity volatility firms that will be deemed to have expensive equity options and large jump risk premia. Thus, these firms will have a larger model yield spread. Since high recent equity volatility firms have higher observed yield spreads, the model would be deemed an improvement despite the fact that all equity options are actually of equal expensiveness. To prevent my results from being driven solely by the recent equity volatility effect, I use recent equity volatility to calibrate a short-term asset volatility and use this asset volatility to determine the distribution of asset value at option maturity. Using the full sample asset volatility generates results that suggest that observed yield spreads are strongly related to jump spreads even when Black-Cox spreads are controlled for.

For the firm-level calibration above to be entirely precise would require joint estimation of σ_v, η, and γ as the calibration of σ_v and η requires functions of equity value. Equity value contains a call option on firm value and is, thus, dependent on the risk-neutral distribution of asset value. The risk-neutral distribution of asset value depends on γ which cannot be calculated without σ_v and η. An alternative to simultaneously solving variance, fourth moment, and option valuation equations would be to follow a procedure like Huang and Zhou (2007). They use the difference between empirical and model CDS prices as moment restrictions in a GMM framework to back-out parameters.

In my calibration exercise, I avoid using a series of defaultable securities to estimate parameters or using a simultaneous calibration and instead try to calibrate to equity and equity options data through the sequential process described above. My calibration does capture two essential elements: (1) Asset variance should be related to leverage and the equity variance and (2) jump model spreads should be larger for firms with expensive equity options, holding asset volatility and leverage constant. A sanity check for whether my calibration methodology has reasonably captured that firms with expensive equity options should have higher jump risk premia and thus higher yield spreads is a regression of the jump model spread on the Black-Cox model spread and option expensiveness as measured by the difference between implied and recent equity volatility. I find that the jump model spread is indeed strongly positively related to both the Black-Cox spread and option expensiveness.

Probabilities of default are calculated using the results in Kou and Wang (2003) and previously applied by Huang and Huang (2003). Kou and Wang (2003) find an analytical solution for the Laplace transform of survival probabilities in the jump diffusion model. This transform can be inverted using the Gaver-Stehfest algorithm. Interested readers are referred to Kou and Wang's original work for details. With risk-neutral default probabilities, bonds can be priced using equation (1.3).

1.4.2 Calibration Results

As expected, incorporating jump risk premia decreases the difference between observed and model yield spreads as compared to the Black-Cox model. As shown in Table 1.9.1, the difference between observed and model yield spreads decreases for all ratings and for both the four-year and ten-year bonds. The mean difference drops from 54 to 32 basis points for four-year bonds and from 79 to 57 basis points for ten-year bonds.[25] These drops are unsurprising given that the jump diffusion model incorporates an additional risk premium which increases Q-measure default probabilities. These results using individual equity option-implied jump risk premia are consistent with the finding in Cremers, Driessen, and Maenhout (2006) that index equity option-implied jump risk premia can generate higher model-implied yield

[25]The reader may notice that the reported difference for the Black-Cox model here does not correspond to the numbers reported in Table 1.9.1. This is due to the fact that the sample for the jump model is a subset of the sample for the Black-Cox model.

spreads in ratings-level calibrations than for diffusion-only models.[26]

Since I calibrate at a firm-by-firm level rather than at the ratings level, I can further test whether the jump model can help to explain the cross-section of yield spreads in addition to the level of yield spreads. In Table 1.7, I examine whether the same firms that have high jump risk premia (as inferred from equity options) also have higher observed spreads, with Black-Cox spreads held constant. I find weak evidence that this is indeed the case. In Panel A, I first sort on Black-Cox yield spreads and then jump risk premia within each Black-Cox group. It appears that observed spreads do increase with jump risk premia for jump risk premia quintiles 2 to 5. However, differences between quintile 5 and quintile 1 observed spreads tend to be statistically insignificant, largely due to the high observed spreads of the low jump risk premia quintile. Many of the firms in this quintile are in fact high recent equity volatility firms. As shown in Section 1.3, such firms tend to have high unexplained yield spreads. Thus, the recent equity volatility effect tends to blunt the results for the jump model.

In Panel B, I use a regression framework to study the ability of the jump model to explain the cross-section of yield spreads above and beyond a Black-Cox model. For the full sample and for four-year, ten-year, and investment grade subsamples, the portion of the jump yield spread orthogonal to the Black-Cox yield spread (labeled the jump residual) is positively related to observed yield spreads. The economic significance of the jump residual, however, is relatively small. Moving from the 25th to 75th percentile of the jump residual is approximately a 15 basis point move which then translates to a less than 4 basis point move in observed yield spreads. A move from the 10th to 90th percentile equates to approximately a 12 basis point move in observed yield spreads. A two standard deviation move in the jump residual is much larger at 140 basis points and corresponds to a 33 basis point move in observed yield spreads. Economically, this is still a much smaller move than the 255 basis point difference in observed yield spreads for a two standard deviation difference. Thus, it appears that incorporating a jump risk premium calibrated from equity options does improve the cross-sectional explanatory power over a Black-Cox model, but the economic significance of this improvement is limited. In particular, the jump model is a much greater success in explaining the level of yield spreads than the cross-section.

[26]They argue that once jumps are incorporated and yield spreads are tax-adjusted to reflect the fact that treasuries are exempt from state taxes, the level credit spread puzzle disappears.

1.5 Stochastic Volatility

As a potential solution to the fact that observed yield spreads are strongly related to recent equity volatility, I calibrate a Heston (1993) model[27] where the underlying firm value process has mean-reverting variance. The modeling of claims on firm cashflow is similar to the base case with one important difference. Equity at maturity is a Heston call option rather than a down-and-out call option. The major distinction between the two options besides the way volatility is modeled, is that the Heston option does not capture a first-passage time default. Thus, the proper baseline model to compare the Heston model to is the Merton model and not the Black-Cox model. Numerically, the major difference is that the Δ of the barrier option is much higher when K, the face value of debt, is near V, the total value of the firm.

As in the Heston (1993) stochastic volatility model, the processes of the underlying and of the variance are:

$$\begin{aligned} \frac{dV_t}{V_t} &= (\pi^v + r - \delta)dt + \sqrt{H_t}(\rho dW_t^1 + \sqrt{1-\rho^2}dW_t^2) \\ dH_t &= \kappa_H(\theta_H - H_t) + \sigma_H\sqrt{H_t}dW_t^1 \end{aligned} \tag{1.11}$$

where V_t is asset value and H_t is asset variance. κ_H is the mean-reversion parameter for asset variance, θ_H is the long-run average asset variance, and σ_H is the volatility of variance term. Besides incorporating mean-reverting dynamics for asset variance, the above specification also allows for correlation between firm value and variance shocks, ρ.[28]

Allowing for stochastic variance, the relevant asset variance for pricing bonds is some weighting of the current asset variance and the long-run average asset variance, θ_H. If κ_H, which measures the speed of the mean-reversion in asset variance, is sufficiently small, current asset variance is important in pricing corporate bonds and this could explain the relation between unexplained yield spreads and recent equity volatility. In addition, the correlation parameter, ρ, could potentially further explain the cross-sectional dispersion of yield spreads. If firm value shocks are negatively correlated to asset variance shocks, asset variance tends to be high exactly when firm value is low. For most firms, this increases the probability of default and drives up yield spreads. Thus, cross-sectional variation in ρ would lead to cross-sectional variation in model yield spreads.

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[27] In estimating call option prices for the Heston (1993) model, I follow the Duffie, Pan, and Singleton (2000) formulation. Lord and Kahl (2008) find that the formulation in Heston (1993) is sometimes inaccurate because of a discontinuous characteristic function when only the principal branch of logarithms are used. See Appendix 1.9.5 for the option pricing formula used.

[28] The model calibrated in this section does not have a volatility risk premium.

1.5.1 Calibration Methodology

Given the above processes for asset value and variance, I can calculate model-implied relations between equity variance and asset variance and also the model-implied correlation between asset return shocks and variance shocks. First, denote $E_t = f(t, V_t, H_t)$, where E_t is the value of equity. Then, the aforementioned relations are:

$$\sigma_{E,t}^2 = f_v^2 \left(\frac{V_t}{E_t}\right) H_t + \frac{f_H^2}{E_t^2}\sigma_H^2 H_t + 2\frac{V_t f_v f_H}{E_t^2}\sigma_H H_t \rho \tag{1.12}$$

$$E_t[(d\log V_t - (\pi^v + r - \delta - \frac{1}{2}H_t)dt)(dH_t - \kappa_H(\theta_H - H_t)dt)] = \sigma_H H_t \rho dt \tag{1.13}$$

Since asset value is not observed, this requires backing out asset returns from equity returns. I estimate $d\log V_t$ from $d\log E_t$ (log equity returns without dividends). The model provides the estimate, $d\log V_t \approx \frac{d\log E_t}{f_v}\frac{E_t}{V_t}$, where $d\log E_t$ is estimated as log equity returns minus $\delta_e \frac{V_t}{E_t}dt$ and the other terms are estimated through the model.

To estimate $[\kappa_H, \theta_H, \sigma_H, \rho]$, I adopt the following iterative procedure:

1. Start with observed equity volatility and an initial guess of $[\kappa_H, \theta_H, \sigma_H, \rho]$ and use equation (1.12) to estimate a time-series of H_t.

2. Using the H_t estimated in step 1, estimate $[\kappa_H, \theta_H, \sigma_H]$ using maximum likelihood.

3. Using the H_t and $[\kappa_H, \theta_H, \sigma_H]$ calculated in the previous two steps along with equation (1.13), estimate ρ.

4. Return to step 1 using the $[\kappa_H, \theta_H, \sigma_H, \rho]$ estimated from the previous two steps as the initial guess. Repeat until convergence.

In the Merton model, the risk-neutral probability that asset value is above the face value of debt at time t is $N(d_2)$, where $d_2 = \frac{\ln(\frac{V}{K})+(r-\delta-\frac{\sigma_v^2}{2})t}{\sigma_v\sqrt{t}}$. For a Heston model, it is similarly,

$$e^{rt}G_{0,-1} = e^{rt}\left(\frac{\psi(0, H_0, t)}{2} - \frac{1}{\pi}\int_0^\infty \frac{Im\left[\psi(-iu, H_0, t)e^{iu(\log k)}\right]}{u}du\right) \tag{1.14}$$

where ψ is defined as in Duffie, Pan, and Singleton (2000), but is scaled by e^{-uy} in my application.[29] With the risk-neutral probability of survival, bonds can be priced using equation (1.3).

[29] A stochastic volatility model is a special case of the example in Section 4 of Duffie, Pan, and Singleton (2000). See also Appendix B in Pan (2002) and Section 8F of Duffie (2001) for details of pricing options via transform analysis.

1.5.2 Calibration Results

In Table 1.8, I present calibration results for the stochastic volatility model. One important difference that arises between these results and the Black-Cox results is that the stochastic volatility model is able to generate larger yield spreads for poorer-rated bonds. This is due to the modeling convention that is used. In the stochastic volatility model, the sensitivity of equity to underlying asset value is smaller than in a Black-Cox model for firms that are near the default boundary.[30] It is exactly firms with higher leverage (typically firms with junk ratings) for which this is true. Since asset volatility is inversely related to this sensitivity, the stochastic volatility model infers larger asset volatilities which then lead to larger yield spreads. Thus, the Merton model actually overestimates yield spreads for short-maturity junk debt. However, the results for investment grade and longer maturity bonds remain unchanged as the stochastic volatility model cannot generate sufficiently high yield spreads to match observed yield spreads for these bonds.

To test the cross-sectional explanatory power of the stochastic volatility model, I compare the ability of the model to improve on yield spread estimates from the Merton model. Comparing the model to a Black-Cox model would potentially lead to results that are driven by the difference between barrier options and vanilla options. In Panel A of Table 1.9, I sort first by the Merton model yield spread as a control and then sort within each group by the difference between stochastic volatility and Merton yield spreads. If the additional elements in the stochastic volatility model indeed help to explain yield spreads, the observed yield spread should increase across quintiles while the Merton yield spreads remain relatively flat. The stochastic volatility yield spread should also increase across quintiles. Empirically, observed spreads do not increase across quintiles, suggesting that the stochastic volatility model does not help to explain the cross-section of yield spreads.

Using regressions with time fixed-effects in Panel B, I confirm that the stochastic volatility model has little explanatory power above and beyond the Merton model. The first two columns of Panel B show that regressing observed yield spreads on stochastic volatility yield spreads actually results in a slightly lower within-group R^2 than regressing on Merton model spreads. In the final four columns of Panel B, I first orthogonalize the stochastic volatility model to the Merton model. Then, I regress the observed yield spread on the Merton spread and the residual stochastic volatility spread. For the full sample and for four-year, ten-year, and investment grade subsamples, the stochastic volatility model residual adds little additional explanatory power as the coefficient on the residual stochastic volatility spread is statistically and economically insignificant.

[30]This is due to the fact that equity in a Black-Cox model is based on a down-and-out call option which has a larger Δ near the boundary than a vanilla call option.

The inability of the stochastic volatility model to improve cross-sectional predictions of yield spreads stems from two sources. First, estimated values of κ_H,[31] the mean-reversion parameter, are large (averaging 11.7) and thus, the effect of recent volatility is muted. Second, ceteris paribus, firms with a more negative correlation between return shocks and variance shocks do not seem to have larger observed yield spreads even though they generally have larger stochastic volatility model yield spreads.

1.5.3 Alternative Specifications

In this subsection, I further address the relation between unexplained yield spreads and recent equity volatility. The stochastic volatility model is unable to address the relation between unexplained yield spreads and recent equity volatility largely because of the large estimated mean-reversion parameter for asset variance. Here, I examine whether the relation between the unexplained yield spread and recent equity volatility can be explained if a slower mean-reversion parameter is imposed. I also examine whether allowing asset volatility to be inferred solely from recent equity volatility improves the cross-sectional explanatory power of the Black-Cox model.

Stochastic Volatility with Slower Mean-Reversion

In this section, my calibration methodology follows that of Section 1.5.1, except that I price bonds as if the mean-reversion parameter for asset variance, κ_H, is one. The mean-reversion parameter used here is much smaller than the average P-measure estimates in Section 1.5.1. In a model with a volatility risk premium, the Q-measure mean-reversion parameter is typically much smaller than the P-measure parameter. In such a model[32], θ_H^Q, the Q-measure long-run mean variance, is also greater than θ_H^P, but I do not impose this restriction here as my goal is solely to determine the effect of slower mean reversion on model yield spreads.

I examine the cross-sectional explanatory power of this model through the same regression framework as in Section 1.5.2. The results in Table 1.10 indicate that the stochastic volatility model does not help to explain the cross-section of yield spreads above and beyond the Merton model. Even for four-year bonds where a smaller mean-reversion parameter should have the largest effect, the SV residual is insignificant. Surprisingly, the SV residual is statistically significant and *negative* for ten-year bonds at the 10% level.

[31] See Panel C of Table 1.8 for summary statistics for parameter estimates.
[32] See Pan (2002) for an example.

At first glance, these results seem surprising given the intuition that a slower mean-reversion weights recent volatility more and yield spreads are related to recent volatility more than constant volatility models can explain. However, the volatility of variance parameter, σ_H, complicates this interpretation. With deterministic changes in variance, having a slower mean-reversion parameter when the current variance is lower than the mean historical variance will decrease yield spreads. However, when σ_H is important, this is not necessarily the case. For firms that are unlikely to default, a larger than average asset variance is needed to increase default probabilities while lower than average asset variances have little effect on already low default probabilities. A slower mean-reversion parameter allows for positive shocks in variance to be more persistent and hence, have a greater effect on yield spreads. It seems unlikely that a stochastic volatility model with mean-reverting variance can explain the relation between yield spreads and recent equity volatility.

Black-Cox with Recent Equity Volatility

A method of purely examining whether bonds are priced with recent volatility being more important than P-measure estimates would suggest is to calibrate the Black-Cox model using only recent equity volatility and calibrating equation (1.2) period-by-period. While such a scheme is inconsistent with the Black-Cox modeling assumption of constant asset volatility, it does capture a scenario in which bonds are priced using current volatility as an input for the probability of default.

In Panel B of Table 1.10, I find evidence that observed yield spreads are indeed related to these period-by-period estimates above and beyond the Black-Cox estimates from Section 1.3. A move from the 25th to 75th percentile in the period yield spread residual corresponds to a change in observed yield spread of 16 basis points. A move from the 10th to 90th percentile corresponds to a 54 basis point change in the observed yield spread. Thus, it appears that recent volatility is an important determinant of yield spreads. Interestingly, I find that even when this period-by-period yield spread is partialed out from observed yield spreads, unexplained observed yield spreads are related to recent equity volatility as shown by the statistically significant coefficient for the last column of Panel B.

1.6 Endogenous Default Models

Throughout this chapter, I have focused on exogenous default models, largely because the default barrier is unobservable. This ignores an important literature on endogenous default models typified by the optimal capital structure and endogenous boundary model of Leland

(1994) and the Anderson and Sundaresan (1996) strategic debt service model. Here, I discuss these models and present some preliminary evidence.

In Leland's model, firms optimally choose the level of debt (effectively through the coupon rate) and the default boundary. These endogenous choices generate two comparative statics that can be examined. First, the relation between coupons and asset volatility is U-shaped. Second, the relation between yield spreads and asset volatility is positive for investment grade bonds, but negative for junk grade bonds. A standard exogenous default boundary model makes no predictions about the relation between coupons and asset volatility and predicts a positive relation between yield spreads and asset volatility, regardless of rating.

Examining the two aforementioned implications of the Leland model requires estimates of the asset volatility. While the asset volatility has been estimated earlier for the Black-Cox model, I do not use these estimates here and instead use a model-free estimate. In particular, I first estimate period-by-period asset variance using:

$$\sigma_v^2 = (1 - L)^2 \sigma_E^2 \tag{1.15}$$

where L is a firm's leverage.

Then, I take the time series mean of σ_v^2 for each firm as an estimate of the firm's (constant) asset variance. With estimates of the asset variance, I can now examine the implications of the Leland model.

To examine the relation between coupons and asset volatility, I run a regression of the average coupon rate of a firm on asset volatility and asset variance. I plot the results of this regression in Figure 1 along with the theoretical relation derived by Leland. The empirical relation between coupons and asset volatility is indeed U-shaped, but is less convex than the Leland model suggests.

To further examine the implications of the Leland model, I regress yield spreads on market leverage, asset volatility, and an interaction between asset volatility and an investment grade dummy. The implication from the Leland model is that the coefficient on asset volatility should be negative, reflecting higher prices (lower yield spreads) for junk debt when the asset volatility is higher, and the sum of the asset volatility and interaction coefficients should be positive. However, I find that the coefficient on asset volatility is positive and significant, indicating that yield spreads are positively related to asset volatility, even for junk debt. Overall, it seems unlikely that a Leland-type model can fully explain the cross-section of yield spreads.

The second major type of endogenous default model, the strategic debt service model of Anderson and Sundaresan (1996), hinges largely on the bargaining power of equityholders. If recovery rates are lower, equityholders know that debtholders will receive little in default and can strategically refuse to pay debtholders the full promised amount. This predicts that yield spreads are negatively related to recovery rates. In Section 1.3, it is established that unexplained yield spreads from the Black-Cox model are negatively related to asset tangibility (a proxy for recovery rate), but this relation is weakly significant and becomes statistically insignificant when ratings are controlled for. In addition, it cannot be determined whether this negative relation is due to strategic debt service or simply pricing a lower payoff in case of default.

From the evidence presented in this section, it seems unlikely that endogenous default models can fully explain the cross-section of observed yield spreads as their predicted comparative statics are not strongly supported in the data. However, to truly determine if some of these elements marginally add to explaining the cross-section of yield spreads requires a full calibration that is beyond the scope of this chapter.

1.7 Credit Default Swaps

Credit default swaps are insurance contracts in which the buyer pays a (typically) quarterly payment, the CDS spread, to a seller until the maturity of the contract or when the underlying firm (also known as the reference entity) defaults. In the case of default by the underlying, the seller pays the difference between the notional amount and the recovery to the buyer. Conceptually, CDS are very similar to corporate bonds. The seller of CDS contracts is long credit risk, much like the buyer of corporate bonds while the buyer of CDS contracts is short credit risk, like an investor who is short corporate bonds. The pricing of credit default swaps using structural models also follows the pricing of corporate bonds very closely. The CDS spread is determined by calculating the spread such that the present value of the risk-neutral expected value of the series of spread payments is equal to the present value of the risk-neutral expected value of the credit-event payment. A discretized version of this pricing formula is:

$$\sum_{i=1}^{4T}\left(1-q\left(\frac{i}{4}\right)\right)\frac{s}{4}e^{-r\frac{i}{4}}+\sum_{i=1}^{48T}\left(q\left(\frac{i}{48}\right)-q\left(\frac{i-1}{48}\right)\right)\frac{mod(i-1,12)+1}{12}\frac{s}{4}e^{-r\frac{i}{48}} \tag{1.16}$$
$$=\sum_{i=1}^{48T}\left(q\left(\frac{i}{48}\right)-q\left(\frac{i-1}{48}\right)\right)(1-R)e^{-r\frac{i}{48}}$$

where $q(t)$ is the cumulative risk-neutral probability of default, s is the CDS spread, and R is the recovery rate. The maturity of the CDS contract, T, is five years in my sample.

The left-hand side is the value of the quarterly payments by the buyer plus the accrued swap spread if the reference entity defaults between two swap spread payment dates while the right-hand side is the value of the payment given default.[33]

Though corporate bonds and credit default swaps are similar in their exposures to credit risk, their non-credit components are thought to be very different. As of the fourth quarter of 2007, there were $5.8 trillion of corporate bonds outstanding according to the Securities Industry and Financial Markets Association. In contrast, Christopher Cox, the SEC Chairman, cited the size of the CDS market as $58 trillion in testimony to the United States Senate in 2008. Also, it is generally believed that the CDS market is more liquid than the corporate bond market, particularly for five-year CDS contracts. Thus, Longstaff, Mithal, and Neis (2005) and Nashikkar, Subrahmanyam, and Mahanti (2007) use CDS as proxies for credit risk and generate CDS-implied corporate bond yield spreads. In contrast to the corporate bond market, a major source of risk in the CDS market is counterparty risk, the likelihood that the counterparty in a CDS contract will be unable to pay its side of the contract. Since corporate bonds and CDS of the same underlying entity directly share credit risk, but not necessarily liquidity and counterparty risk, it is interesting to examine the performance of structural models of default in the CDS market. In particular, if structural models fail to fully capture credit risk in the cross-section, it is likely that unexplained spreads should be correlated for corporate bonds and CDS of the same company.

1.7.1 Calibration and Cross-Sectional Results

In Panel A of Table 1.9.1, I examine the magnitudes of observed and model CDS spreads for the Black-Cox model, finding that the mean difference between observed and model spreads for five-year CDS contracts is 42 basis points in my sample and statistically significant. The magnitude of this difference is smaller than for both four-year corporate bonds (90 basis points) and for ten-year corporate bonds (88 basis points).[34] In addition, the difference between observed and model CDS spreads is significant for all ratings groups, though it is larger for firms with poorer ratings.

The cross-sectional performance of structural models in the CDS market is similar to that of the bond market. Following the same procedure as for corporate bonds, I first regress the observed CDS spread on the model CDS spread, finding a coefficient of 0.59 and a within-group R^2 of 36.58%. Interestingly, this R^2 is *lower* than the R^2 from an analogous regression

[33] I have made the simplification that the parties determine whether or not the reference entity has defaulted every $\frac{1}{48}$ years and that CDS premia also accrue at this horizon. In contrast, if the parties continuously monitor whether the reference entity has defaulted, the summations in the equation should be replaced by integrals.

[34] This is consistent with findings by Hull, Predescu, and White (2004) that the benchmark risk-free rate in the CDS market is close to the swap rate rather than the Treasury rate. During the 2003 to 2007 period, the mean difference between swap rates and Treasury rates was approximately 45 basis points.

for corporate bonds.[35] I then construct the residual from this regression as the unexplained CDS spread and test whether the unexplained spread is related to credit risk proxies and firm-level characteristics. In Panel B, I find that the unexplained CDS spread is strongly related to recent equity volatility and ratings, much like corporate bonds. Unexplained CDS spreads are also related to firm size as in the corporate bond market. Thus, the cross-sectional performance of the Black-Cox model is similar for both the corporate bond and CDS market.

1.7.2 Commonality Between CDS and Corporate Bonds

The common credit risk component in corporate bonds and CDS suggests a natural test of whether unexplained corporate bond yield spreads are related to unexplained CDS spreads. For each firm-quarter, I choose the bond for a firm that is closest to five years to maturity to compare to my sample of five-year CDS contracts. As a benchmark regression, I first regress the observed CDS spread on the observed corporate bond yield spread with time fixed-effects. As shown in Panel C of Table 1.9.1, CDS spreads and corporate bond yield spreads are strongly related with a coefficient of 1.05 and a robust t-stat of 23.61. The within-group R^2 is large at 70.16%. I then run a similar regression for the unexplained CDS spread and unexplained corporate bond yield spread. finding a coefficient of 0.98 and a robust t-stat of 14.78. The within-group R^2 drops to 49.62%, indicating that the Black-Cox model has captured some common component between corporate bonds and CDS, but that there is also a remaining common component. Since unexplained corporate bond yield spreads and CDS spreads are related to recent equity volatility and ratings, I further purge out recent equity volatility and ratings from the unexplained corporate bond and CDS spreads. The still unexplained spreads are related, though the within-group R^2 declines further to 37.67%.

The results above indicate that a Black-Cox model is able to capture some component of credit risk as the relation between corporate bonds and CDS weaken once the model is controlled for. However, there remains an important relation between unexplained corporate bond yield spreads and CDS spreads. An important caveat is that liquidity has thus far been treated as security-specific (and only for corporate bonds) rather than firm-specific and it is possible that a common liquidity component across markets could account for a portion of this commonality. Empirical evidence about CDS liquidity and in particular, the relation between CDS and corporate bond liquidity, is limited. Nashikkar, Subrahmanyam, and Mahanti (2007) find that less liquid CDS contracts (ones with greater bid-ask spreads) have more expensive corporate bonds (bonds with lower yield spreads). Tang and Yan

[35] However, this can potentially be attributed to the different data sources used for corporate bond prices (TRACE) and CDS spreads (Bloomberg).

(2007) find that less liquid CDS contracts have larger CDS spreads. Together, these results suggest that companies with less liquid CDS contracts should have higher CDS spreads and *lower* corporate bond yield spreads. This actually works against my finding of a positive commonality between unexplained corporate bond and CDS spreads, though the current understanding of liquidity commonality between corporate bond markets and CDS markets is limited and I cannot completely dismiss the possibility of a positive common liquidity component. However, the magnitude of commonality that remains and the previous results on the corporate bond and CDS cross-sections suggest that there is some component of credit risk that the Black-Cox model cannot capture in the cross-section.

1.8 Conclusion

In this chapter, I test the ability of structural models of default to explain the cross-section of corporate bond yield spreads. Though structural models present predictions about both the levels of yield spreads and also the relative yield spreads of different bonds, the literature has thus far focused on levels. Huang and Huang (2003) find that for a broad group of structural models matched to historical default probabilities, yield spreads are too high to be explained solely by credit risk. Much of the literature that has followed involves examining model mechanisms that can generate model yield spreads that are closer to the levels of historically observed yield spreads. Instead, I focus on an alternative test of structural models, their cross-sectional explanatory power, rather than solely asking if they can generate large model yield spreads. This allows me to directly assess the determinants of the disconnect between observed and model yield spreads.

My base case is a Black-Cox model. Regressing observed yield spreads on Black-Cox yield spreads, I find that the Black-Cox model is able to explain 44.9% of the variation in observed yield spreads. I then construct unexplained yield spreads as the residuals from this regression. As expected, unexplained yield spreads are related to proxies for liquidity, as structural models are designed to capture credit risk and not illiquidity. More importantly, I find a significant cross-sectional relation between unexplained yield spreads and proxies for credit risk such as recent equity volatility and ratings. This indicates a failure of the Black-Cox model to fully capture credit risk in the cross-section. In addition, unexplained yield spreads are related to equity option expensiveness, suggesting an additional risk premium.

The relations between unexplained yield spreads and recent equity volatility and option expensiveness suggest that models with stochastic volatility and a non-diffusion risk premium may help to explain cross-sectional yield spreads. Calibrations based on a Heston (1993) model do not improve cross-sectional explanatory power as estimates of the asset variance mean-reversion parameter are high. A double-exponential jump diffusion model with jump risk premia inferred from individual equity options does improve cross-sectional explanatory power, suggesting that there is a risk premium that is priced in both the corporate bond and equity option markets. However, the economic significance of this relation is limited as compared to the Cremers, Driessen, and Maenhout (2006) finding that a model with index equity option-implied jump risk premia can explain much of the levels puzzle.

In addition to examining the cross-sectional explanatory power of structural models for corporate bonds, I examine whether the Black-Cox model can explain the cross-section of CDS spreads. Much like the results for corporate bonds, unexplained CDS spreads are related to proxies for credit risk in the cross-section. I also find that unexplained CDS spreads are related to unexplained corporate bond yield spreads of the same firm, consistent with structural models being unable to fully capture credit risk in the cross-section. Though I cannot completely rule out that this commonality is a liquidity commonality, the magnitude of the relation suggests that it is at least in part due to credit risk that the model has not captured.

In this chapter, I do not address the relation between unexplained spreads and ratings through a structural model of default. I do find that ratings are significantly related to the size of a firm and that firm size is related to unexplained spreads when ratings are not controlled for. A further examination of what ratings truly capture would be useful as they are intended to represent forward-looking predictions of credit-worthiness. Current studies of ratings have generally focused on market reactions to rating announcements. Hand, Hothausen, and Leftwich (1992) find significant bond price reactions to unexpected bond ratings changes. There are potentially two interpretations for this result. First, ratings agencies receive private information from the firms that they rate and bond price reactions could reflect this information. Second, it is possible that changes in ratings elicit changes in bond prices simply because market participants price bonds based on ratings, regardless of how much information ratings capture. A more recent study by Hull, Predescu, and White (2004) finds that the credit default swap market anticipates ratings announcements, casting doubt on the second explanation. This is consistent with ratings reflecting something fundamental that markets use to price bonds rather than ratings leading bond prices. Though ratings may be slower than the market to incorporate information, a potential avenue for future research would be to examine the information that ratings agencies use and to see how this information is related to bond pricing above and beyond what structural models of default suggest. These inputs may then be a guide for future work in structural models.

The tests in this chapter present an additional challenge to researchers in credit risk modeling. Most of the previous work on structural models of default has been focused on constructing a model to match the level of yield spreads. I argue that explaining the cross-section of observed yield spreads through a theoretically-founded model is an equally important and difficult task that should be the focus of future research.

1.9 Appendix

1.9.1 Tables and Figures

Table 1.1: Summary Statistics

Firm Summary Statistics	Obs	Mean	Std Dev	25th	50th	75th
Market Leverage	3,250	30.61	23.46	13.83	23.59	39.98
Equity Volatility	3,250	25.60	12.11	17.86	22.71	30.26
IV - Equity Volatility	2,364	6.73	6.88	2.95	6.33	9.94
Equity Market Cap	3,250	31.22	53.90	4.37	13.27	33.04
Firm Age	3,250	40.73	25.37	17.42	36.66	62.91
Return on Assets	2,874	1.35	1.19	0.65	1.29	1.97
Equity Beta	3,227	0.95	0.75	0.49	0.81	1.24
Avg Bond Duration	3,250	6.07	2.66	4.14	5.81	7.60
Asset Tangibility	3,220	40.57	12.01	33.09	41.97	49.87
Interest Coverage	3,249	8.87	9.90	2.95	5.89	11.37
Hist - Current Lev	3,249	0.89	13.02	-5.79	0.92	7.89
S&P	3,250	71.23				

4yr Bonds	Obs	Mean	Std Dev	25th	50th	75th
Maturity	2,540	3.36	1.36	2.24	3.44	4.19
Rating	2,493	8.16	3.66	6.00	7.00	10.00
Age	2,540	6.94	3.99	4.33	6.79	8.56
Amount Outstanding	2,540	297.45	377.21	100.00	200.00	325.00
Volume	2,540	47.10	132.98	3.50	12.60	37.72
Trades	2,540	146.51	436.43	16.00	45.00	129.50
Turnover	2,532	13.36	19.42	3.15	7.87	15.83
Avg Trade Size	2,540	520.24	763.91	118.90	271.14	585.61
% Days Traded	2,540	41.36	31.77	14.06	32.26	67.74

10yr Bonds	Obs	Mean	Std Dev	25th	50th	75th
Maturity	1,687	12.83	8.12	8.90	10.70	15.29
Rating	1,650	8.30	3.71	6.00	8.00	10.00
Age	1,687	8.69	4.90	4.18	9.49	11.97
Amount Outstanding	1,687	358.39	478.95	150.00	250.00	350.00
Volume	1,687	124.00	593.67	4.95	15.15	45.11
Trades	1,687	203.96	774.65	15.00	41.00	109.00
Turnover	1,680	15.08	22.32	3.08	7.71	16.98
Avg Trade Size	1,687	644.04	791.17	154.57	390.61	803.34
% Days Traded	1,687	40.15	31.25	12.90	32.26	61.90

Observations are quarterly. Market leverage is the ratio of market value of debt to the sum of market value of debt plus market value of equity. Equity volatility is the annualized volatility of daily log equity returns for a quarter. IV - equity volatility is the mean of the implied volatilities of a short-term OTM put and a short-term ATM put minus recent equity volatility. Equity market capitalization is the product of share price and shares outstanding in $B. Firm age is the number of years since a firm's BEGDT in CRSP. Return on Assets is a firm's income before extraordinary items divided by the mean of total assets at the start and end of the quarter. Equity beta is a firm's CAPM beta using a 60-month rolling window. Avg Bond Duration is the issuance-weighted average of the duration of a firm's outstanding bonds. Asset tangibility is calculated using the estimates in Berger, Ofek, and Swary (1996). Interest coverage is EBIT divided by the interest expense. Hist - Current Lev is the difference between a firm's mean leverage from 1993 to 2002 and its current leverage. S&P equals 1 if a firm is an S&P 500 firm. Maturity is the number of years to a bond's maturity and age is the number of years since a bond's issuance. Rating is coded as 1 for Aaa and 21 for C with intermediate ratings coded. Amount Outstanding is the face value outstanding in $mm. Volume is the quarterly trading volume in $mm face value. Trades is the number of trades in a quarter. Turnover is the volume scaled by the amount outstanding. Avg Trade Size is the average trade size in $k. %Days Traded equals the number of days a bond was traded divided by the number of trading days in the quarter.

Table 1.2: Level of Yield Spreads, Black-Cox Model

Panel A: 4yr bonds

Rating	Obs	Lev	Asset Vol	Obs Yield		BC Yield	
				mean	med	mean	med
Aaa	83	21.21	22.42	3.88	4.17	5.12	4.94
Aa	206	16.63	22.69	4.08	4.13	4.91	4.89
A	973	25.80	22.45	4.22	4.37	5.19	4.89
Baa	778	32.75	22.98	5.06	4.72	5.23	4.93
Ba	214	42.36	21.05	6.49	6.59	5.58	5.47
B	154	55.64	20.52	7.55	7.09	7.75	6.93
C	91	70.12	18.36	20.06	10.74	9.91	9.77
Full	2,546	31.90	22.24	5.46	4.52	5.56	4.89

Rating	Observed Spread		BC Spread		Difference		
	mean	med	mean	med	mean	med	t-stat
Aaa	35.03	33.70	12.85	5.42	22.17	19.15	2.94
Aa	36.19	38.32	0.63	0.01	35.56	37.61	9.53
A	61.99	57.89	24.62	0.15	37.37	51.12	4.88
Baa	100.72	99.72	44.07	4.43	56.65	79.01	4.85
Ba	223.76	289.59	85.37	60.72	138.40	213.37	5.56
B	329.25	318.72	302.21	207.75	27.05	162.88	0.35
C	1,591.15	698.28	515.54	490.67	1,075.61	235.56	1.32
Full	159.95	73.26	69.73	0.38	90.21	59.24	3.13

Panel B: 10yr bonds

Rating	Obs	Lev	Asset Vol	Obs Yield		BC Yield	
				mean	med	mean	med
Aaa	90	16.84	21.53	5.03	5.06	5.14	4.89
Aa	117	13.58	23.25	5.19	5.12	4.97	4.92
A	515	29.33	21.78	5.39	5.31	5.48	5.08
Baa	562	29.65	24.59	6.17	6.08	5.58	5.35
Ba	203	40.68	21.52	7.33	7.02	6.05	5.60
B	128	51.16	19.79	8.56	8.15	6.93	6.43
C	40	72.67	12.57	12.02	11.37	8.09	7.96
Full	1,692	31.70	22.48	6.28	5.74	5.71	5.19

Rating	Observed Spread		BC Spread		Difference		
	mean	med	mean	med	mean	med	t-stat
Aaa	53.62	55.68	20.04	0.57	33.57	41.86	2.60
Aa	71.34	68.60	8.09	3.47	63.25	62.22	10.86
A	96.04	85.71	55.30	19.55	40.74	61.12	3.26
Baa	160.09	147.02	80.03	45.97	80.06	77.53	5.32
Ba	271.87	247.23	132.96	83.28	138.91	142.34	6.35
B	394.47	360.82	222.72	158.56	171.75	207.53	4.73
C	748.28	684.33	340.16	325.02	408.12	350.87	2.66
Full	175.72	122.39	88.19	30.04	87.54	74.30	7.07

Observations are at the bond-quarter level from 2003 Q1 to 2007 Q4. Leverage, asset volatility, and yields are reported in %. Spreads and the difference in spreads are reported in basis points. T-statistics for the difference use standard errors that are clustered by firm and time.

Table 1.3: Cross-Sectional Tests

Panel A: Risk Variables

	Obs Spd	Unexplained Spread			
BC Spd	0.58				
	[8.00]				
Rating					16.77
					[9.39]
Mktlev		1.08	-0.21		
		[2.23]	[-0.52]		
Asset Vol		1.22	-1.18		
		[1.00]	[-1.23]		
Eq Vol			3.89	4.03	
			[6.16]	[4.41]	
IV - Eq Vol				2.57	
				[3.01]	
R-sqd	44.86	3.20	15.50	13.55	29.71
Obs	4,227	4,227	4,227	3,145	4,143

Panel B: Additional Characteristics

	Unexplained Spread						Rating	
Mktlev	-0.16	0.28	-0.26	0.16	-0.33	-0.28	0.01	0.03
	[-0.30]	[0.57]	[0.48]	[0.32]	[-0.67]	[-0.65]	[1.24]	[3.70]
Rating					15.71	15.05		
					[6.21]	[5.63]		
Eq Vol	3.10	3.09	3.35	3.33	1.97	1.93	0.07	0.08
	[5.72]	[5.84]	[4.05]	[3.90]	[2.66]	[2.59]	[5.76]	[5.89]
IV - Eq Vol			2.50	2.53	1.28	1.29	0.05	0.05
			[3.87]	[3.84]	[2.43]	[2.44]	[4.15]	[4.30]
ln(Eq Mkt Cap)	-16.18		-16.00		0.75		-1.01	
	[-3.84]		[-4.12]		[0.18]		[-7.99]	
ln(Firm Value)		-17.08		-17.39		-1.96		-0.96
		[-3.93]		[-4.03]		[-0.41]		[-7.96]
Firm Age	0.30	0.31	0.19	0.21	0.32	0.34	-0.00	-0.00
	[1.22]	[1.27]	[0.76]	[0.85]	[1.50]	[1.59]	[-0.70]	[-0.70]
ROA	-7.88	-7.50	-1.86	-1.64	9.98	9.67	-0.63	-0.62
	[-1.21]	[-1.16]	[-0.30]	[-0.27]	[1.67]	[1.63]	[-4.15]	[-4.09]
Eq Beta	1.02	1.72	4.40	4.59	-2.60	-1.87	0.43	0.42
	[0.07]	[0.12]	[0.30]	[0.31]	[-0.21]	[-0.15]	[1.68]	[1.65]
Asset Tangibility	-0.75	-0.73	-0.72	-0.70	-0.15	-0.18	-0.02	-0.02
	[-1.71]	[-1.66]	[-1.76]	[-1.70]	[-0.46]	[-0.57]	[-1.77]	[-1.52]
Interest Coverage	-0.38	-0.21	-0.39	-0.22	0.07	0.14	-0.04	-0.04
	[-0.83]	[-0.48]	[-0.86]	[-0.52]	[0.17]	[0.34]	[-3.26]	[-2.95]
Hist - Current Lev	0.44	0.42	0.41	0.37	0.30	0.29	0.01	0.00
	[0.90]	[0.87]	[0.77]	[0.70]	[0.65]	[0.63]	[0.47]	[0.39]
R-sqd	21.43	21.96	19.01	19.75	29.35	29.38	66.73	66.32
Obs	3,692	3,692	3,021	3,021	2,975	2,975	2,237	2,237

Panel C: Liquidity Variables

	Unexplained Spread					
Rating	14.25	14.20	14.22	14.27	14.29	14.17
	[8.02]	[8.06]	[8.18]	[8.10]	[8.07]	[8.20]
Eq Vol	1.42	1.38	1.36	1.30	1.43	1.38
	[2.83]	[2.77]	[2.72]	[2.60]	[2.85]	[2.76]
Age	2.80	3.14	3.22	3.20	2.67	3.24
	[3.07]	[3.22]	[3.42]	[3.35]	[2.91]	[3.38]
ln(Amt)	-8.79	-11.27	-12.03	-7.91	-7.96	-11.98
	[-3.70]	[-3.74]	[-4.04]	[-3.10]	[-3.15]	[-3.99]
ln(Volume)		3.20				
		[1.58]				
ln(Trades)			5.89			
			[2.00]			
Turnover				0.32		
				[1.84]		
ln(Trd Size)					-3.66	
					[-1.53]	
% Days Trd						0.28
						[2.05]
R-sqd	34.08	34.24	34.60	34.41	34.23	34.55
Obs	4,143	4,143	4,143	4,129	4,143	4,143

The data is at the bond-quarter level from 2003 Q1 to 2007 Q4. For each firm, one bond close to four years to maturity and one bond close to ten years to maturity are used. The only exception is in Panel B when ratings are the dependent variable. For those regressions, the data is at the firm-quarter level. The unexplained spread is the predicted residual from a regression of observed yield spreads on BC yield spreads with time fixed-effects. Yield spreads are reported in basis points and are winsorized at 1% of each tail. The dependent variable is the unexplained yield spread unless otherwise labeled. All regressions include time fixed-effects. Ratings are coded as 1 for Aaa and 21 for C with intermediate ratings also coded. Mktlev is the market leverage of a firm, reported in %. Asset volatility is the model-implied asset volatility in %. Equity volatility is the past three months' equity volatility in %. IV - equity volatility is the difference between equity option implied volatility and recently realized equity volatility. Eq Mkt Cap and Firm Value are reported in $mm. ROA is the mean quarterly return on assets over the last ten years reported in %. Equity Beta is a firm's equity CAPM beta. Asset tangibility is measured using the estimates in Berger, Ofek, and Swary (1996) and is reported in %. Interest coverage is EBIT divided by interest expense. Hist - Current Lev is the difference between a firm's mean leverage from 1993 to 2002 and its current leverage. Age (of a bond) and firm age are reported in years. ln(Amt) is the log face value amount outstanding for a bond in ln($mm). ln(Volume) is the volume of trading for a bond reported in ln($mm face value). ln(Trd Size) is the log of the average trade size in ln($k face value). Turnover is reported in %. % Days Traded is the percentage of days in which a bond was traded at least once. Reported t-stats use standard errors clustered by firm. Reported R^2 values are within-group R^2s.

Table 1.4: The Effect of Recent Equity Volatility

BC Quint		Eq Vol Quintile 1	2	3	4	5	Q5-Q1	t-stat	std
	Eq Vol	13.32	17.08	20.36	24.95	36.25			
1 & 2	Obs Spd	52.88	55.44	67.71	75.78	114.07	61.19	3.30	68.77
	BC Spd	0.11	0.12	0.12	0.12	0.08	-0.03	-1.56	0.21
	Eq Vol	14.90	18.72	22.26	26.59	36.73			
3	Obs Spd	89.84	98.95	94.38	115.73	140.98	51.14	3.41	73.28
	BC Spd	5.13	4.59	5.31	5.32	6.01	0.87	1.59	3.52
	Eq Vol	14.81	19.36	23.13	28.54	41.37			
4	Obs Spd	116.13	122.41	130.25	150.20	226.00	109.87	4.80	105.39
	BC Spd	39.10	36.92	38.16	39.41	40.34	1.25	0.52	18.69
	Eq Vol	17.04	23.69	29.76	38.30	59.61			
5	Obs Spd	173.09	181.00	262.62	338.32	1,181.89	1,008.80	2.51	2,354.65
	BC Spd	278.44	305.76	229.43	288.36	597.81	319.38	2.76	344.87

The sample is the same as in Table 1.3. Bonds are sorted into quintiles by Black-Cox yield spreads. Quintiles 1 and 2 are combined and within each group, bonds are then sorted by past three months' equity volatility. Equity volatility is reported in % and yield spreads in basis points. Reported t-stats use standard errors clustered by firm and time. Reported standard deviations are standard deviations within a Black-Cox quintile.

Table 1.5: Alternative Specifications of Recovery Rates

Panel A: Recovery Rates by Industry

	Obs Spd	Unexplained Spread			
BC Spd	0.46				
	[7.62]				
Rating					17.15
					[9.22]
Mktlev		1.24	-0.12		
		[2.37]	[-0.27]		
Asset Vol		1.44	-1.06		
		[1.02]	[-0.93]		
Eq Vol			4.04	4.22	
			[6.03]	[4.51]	
IV - Eq Vol				2.46	
				[2.77]	
R-sqd	44.57	4.04	16.84	14.57	30.08
Obs	4,034	4,034	4,034	3,009	3,996

Panel B: Recovery Rates by Observed Yield Spread

	Obs Spd	Unexplained Spread			
BC Spd	0.41				
	[8.46]				
Rating					15.66
					[9.32]
Mktlev		1.32	0.35		
		[3.32]	[1.07]		
Asset Vol		1.88	0.08		
		[1.96]	[0.10]		
Eq Vol			2.92	3.61	
			[4.72]	[4.13]	
IV - Eq Vol				2.38	
				[3.10]	
R-sqd	50.49	4.85	12.54	12.00	28.80
Obs	4,227	4,227	4,227	3,145	4,143

Variables are defined as in Table 1.3. The model-implied yield spreads in Panel A are calculated using recovery rates from Altman and Kishore (1996). In Panel B, firms with high average observed yield spreads for their base case average model yield spread decile are assigned a recovery rate of 15.44%, medium average observed yield spreads are assigned a recovery rate of 41%, and low average observed yield spreads are assigned a recovery rate of 66.56%. All regressions include time fixed-effects. Reported t-stats use standard errors clustered by firm. Reported R^2 values are within-group R^2s.

Table 1.6: Level of Yield Spreads, Jump Model

Panel A: 4yr bonds

Rating	Obs	Lev	Asset Vol	Obs Yield		Jump Yield	
				mean	med	mean	med
Aaa	77	20.21	21.84	3.82	4.12	5.26	5.03
Aa	182	15.70	22.04	4.01	4.06	5.09	4.94
A	866	24.78	22.16	4.18	4.30	5.31	4.96
Baa	677	30.05	23.08	5.01	4.64	5.32	5.20
Ba	185	42.11	20.54	6.37	6.54	6.12	5.93
B	125	54.86	20.01	7.42	7.19	8.48	7.66
C	57	67.00	18.99	10.93	10.06	10.60	10.56
Full	2,209	29.79	22.06	4.99	4.41	5.70	4.96

Rating	Observed Spread		Jump Spread		Difference		
	mean	med	mean	med	mean	med	t-stat
Aaa	32.63	31.31	24.71	9.77	7.92	16.01	0.64
Aa	31.94	32.94	14.41	0.26	17.52	29.46	1.55
A	57.38	50.97	33.38	2.13	24.00	40.38	2.67
Baa	94.60	93.01	50.72	26.74	43.88	53.98	4.34
Ba	210.94	285.63	135.98	101.71	74.96	173.17	2.91
B	307.85	326.21	372.72	277.84	-64.87	90.74	-0.89
C	662.88	628.18	584.54	564.46	78.35	127.39	0.54
Full	113.20	63.22	81.16	2.77	32.04	45.11	3.24

Panel B: 10yr bonds

Rating	Obs	Lev	Asset Vol	Obs Yield		Jump Yield	
				mean	med	mean	med
Aaa	84	16.48	21.00	5.00	5.02	5.23	4.96
Aa	105	13.77	22.42	5.16	5.09	5.08	5.02
A	458	29.00	21.25	5.34	5.25	5.69	5.28
Baa	478	29.13	23.95	6.09	6.06	5.78	5.69
Ba	173	40.42	20.98	7.33	6.92	6.39	6.01
B	98	51.46	18.70	8.39	7.99	7.24	6.63
C	25	73.43	11.39	10.49	11.06	8.60	8.90
Full	1,443	30.97	21.83	6.12	5.63	5.89	5.41

Rating	Observed Spread		Jump Spread		Difference		
	mean	med	mean	med	mean	med	t-stat
Aaa	51.29	51.25	25.99	2.46	25.30	38.37	1.55
Aa	69.28	65.61	14.80	8.24	54.48	54.27	7.97
A	92.22	80.35	72.59	34.01	19.63	42.46	1.34
Baa	152.87	142.92	96.52	75.71	56.35	51.99	3.73
Ba	270.36	240.21	165.09	119.76	105.27	105.93	4.24
B	375.46	344.54	249.14	173.15	126.32	163.80	3.44
C	585.83	652.88	389.53	414.94	196.29	234.01	2.12
Full	160.01	113.44	102.57	47.15	57.44	54.12	4.66

Observations are at the bond-quarter level from 2003 Q1 to 2007 Q4. Leverage, asset volatility, and yields are reported in %. Spreads and the difference in spreads are reported in basis points. T-statistics for the difference use standard errors that are clustered by firm and time.

Table 1.7: Cross-Section, Jump Model

Panel A: Sorts

BC Quint		Jump Risk Premium Quintile 1	2	3	4	5	Q5-Q1	t-stat	std
	Jump RP (%)	0.49	2.92	5.24	7.67	15.15			
1 & 2	Obs Spd	71.79	64.00	62.65	58.80	66.77	-5.02	-0.56	56.87
	BC Spd	0.10	0.09	0.10	0.12	0.09	-0.01	-0.26	0.19
	Jump Spd	0.19	0.34	2.02	4.97	32.22	32.04	2.44	92.05
	Jump RP (%)	0.56	3.02	5.43	8.02	13.63			
3	Obs Spd	101.70	92.58	91.35	95.92	109.91	8.21	0.56	66.53
	BC Spd	4.47	5.08	4.48	4.57	4.88	0.42	1.24	3.13
	Jump Spd	5.91	7.92	10.93	16.58	46.65	40.74	5.90	41.87
	Jump RP (%)	0.32	2.32	4.84	7.32	14.08			
4	Obs Spd	140.43	111.24	139.52	128.97	166.47	26.03	1.99	86.68
	BC Spd	35.52	33.77	37.22	34.27	33.43	-2.09	-1.56	16.73
	Jump Spd	35.85	39.61	53.50	60.44	114.38	78.53	6.83	71.11
	Jump RP (%)	0.04	0.92	2.91	6.07	19.39			
5	Obs Spd	300.17	252.44	250.32	268.59	396.26	96.09	1.34	329.81
	BC Spd	308.05	318.61	345.79	223.15	304.93	-3.12	-0.05	304.18
	Jump Spd	306.17	326.65	378.77	271.42	488.91	182.74	2.24	334.53

Panel B: Regressions

	Full	Full	Full	4yr	10yr	IG
BC Spread	0.54		0.54	0.52	0.55	0.36
	[8.29]		[8.59]	[8.00]	[6.85]	[7.99]
Jump Spread		0.45				
		[10.02]				
Jump Residual			0.24	0.22	0.31	0.16
			[4.77]	[3.87]	[3.71]	[2.37]
R-sqd	43.44	43.64	45.29	48.14	41.17	33.73
Obs	3,652	3,652	3,652	2,209	1,443	2,927

The data is at the bond-quarter level from 2003 Q1 to 2007 Q2. The bonds in this table are the subset of bonds from Table 1.3 3 where a jump model spread could be calculated. In Panel A, bonds are sorted into quintiles by Black-Cox yield spreads. Quintiles 1 and 2 are combined and within each group, bonds are then sorted by jump risk premia. The jump risk premia are reported in %. Spreads are reported in basis points. Reported t-stats use standard errors clustered by firm and time. Reported standard deviations are standard deviations within a Black-Cox quintile. In Panel B, the Jump residual is constructed as the residual from a regression of the jump model yield spread on the Black-Cox yield spread with time fixed-effects. The dependent variable in the reported regressions is the observed yield spread and all regressions contain time fixed-effects. Yield spreads in this panel are winsorized at 1% of each tail. Reported t-stats use standard errors clustered by firm. Reported R^2 values are within-group R^2s.

Table 1.8: Level of Yield Spreads, Stochastic Volatility Model

Panel A: 4yr bonds

Rating	Obs	Lev	Asset Vol	Obs Yield		SV Yield	
				mean	med	mean	med
Aaa	83	21.21	23.37	3.88	4.17	5.10	4.98
Aa	200	14.51	23.91	4.06	4.13	4.92	4.89
A	952	25.35	23.27	4.24	4.37	5.23	4.89
Baa	778	32.75	25.16	5.06	4.72	5.72	4.98
Ba	214	42.36	24.52	6.49	6.59	7.03	5.77
B	154	55.64	26.41	7.55	7.09	10.15	9.26
C	90	69.94	35.90	12.78	10.71	13.87	13.04
Full	2,518	31.62	24.72	5.21	4.52	6.15	4.91

Rating	Observed Spread		SV Spread		Difference		
	mean	med	mean	med	mean	med	t-stat
Aaa	35.03	33.70	11.73	8.62	23.30	19.08	4.13
Aa	36.09	38.08	0.80	0.14	35.30	37.25	9.27
A	62.33	57.43	28.96	0.55	33.37	50.13	4.09
Baa	100.72	99.72	93.61	9.37	7.11	68.80	0.32
Ba	223.76	289.59	229.87	90.24	-6.11	188.81	-0.06
B	329.25	318.72	541.89	441.38	-212.63	-71.48	-1.68
C	860.67	694.97	912.21	817.43	-51.53	-6.08	-0.25
Full	134.50	73.00	129.33	1.78	5.17	52.75	0.30

Panel B: 10yr bonds

Rating	Obs	Lev	Asset Vol	Obs Yield		SV Yield	
				mean	med	mean	med
Aaa	90	16.84	21.99	5.03	5.06	5.07	4.89
Aa	117	13.58	23.87	5.19	5.12	4.95	4.92
A	515	29.33	22.96	5.39	5.31	5.43	5.08
Baa	562	29.65	26.31	6.17	6.08	5.58	5.35
Ba	203	40.68	27.20	7.33	7.02	6.37	5.75
B	128	51.16	25.56	8.56	8.15	7.49	7.35
C	40	72.67	25.67	12.02	11.37	9.49	9.67
Full	1,692	31.70	24.95	6.28	5.74	5.80	5.20

Rating	Observed Spread		SV Spread		Difference		
	mean	med	mean	med	mean	med	t-stat
Aaa	53.62	55.68	13.05	0.40	40.56	42.94	4.87
Aa	71.34	68.60	6.07	3.50	65.27	63.98	11.53
A	96.04	85.71	49.53	18.95	46.51	63.68	4.58
Baa	160.09	147.02	80.36	46.15	79.73	84.97	5.58
Ba	271.87	247.23	165.22	97.99	106.65	132.60	4.09
B	394.47	360.82	278.38	250.26	116.09	131.81	2.80
C	748.28	684.33	480.11	495.77	268.17	212.13	1.27
Full	175.72	122.39	97.16	31.08	78.56	74.46	7.30

Panel C: Parameter Estimates, Stochastic Volatility Model

Parameter	Mean	Std Dev	25th	50th	75th
κ_H	11.67	6.11	7.53	10.68	14.20
θ_H	0.0727	0.0518	0.0443	0.0606	0.0868
σ_H	0.9436	0.4409	0.6614	0.8464	1.1796
ρ	-0.1490	0.2256	-0.2164	-0.1138	-0.0193

In Panels A and B, observations are at the bond-quarter level. Leverage, asset volatility, and yields are reported in %. The reported asset volatility is the square root of the average long-run asset variance, θ_H. Spreads and the difference in spreads are reported in basis points. T-statistics for the difference use standard errors that are clustered by firm and time. In Panel C, firm-level parameter estimates are reported for the 286 firms in the sample.

Table 1.9: Cross-Section, Stochastic Volatility Model

Panel A: Sorts

		SV - Mer Quintile							
Mer Quint		1	2	3	4	5	Q5-Q1	t-stat	std
	Rho	-0.05	-0.11	-0.11	-0.14	-0.20			
1 & 2	Obs Spd	60.43	52.86	65.86	71.00	76.77	16.33	1.99	49.44
	Mer Spd	0.13	0.01	0.08	0.34	0.49	0.36	5.67	0.37
	SV Spd	0.11	0.02	0.16	0.67	8.64	8.53	3.15	18.69
	Rho	0.02	-0.08	-0.12	-0.24	-0.23			
3	Obs Spd	143.01	97.63	97.20	95.37	92.19	-50.82	-2.90	67.05
	Mer Spd	8.39	6.24	6.99	7.75	8.97	0.59	0.92	4.79
	SV Spd	7.43	6.67	8.32	10.55	43.02	35.59	3.46	36.74
	Rho	-0.04	-0.08	-0.17	-0.20	-0.27			
4	Obs Spd	188.05	129.93	149.09	154.94	151.78	-36.27	-1.33	100.02
	Mer Spd	52.14	37.88	42.25	47.02	54.45	2.31	0.41	22.88
	SV Spd	46.85	38.47	45.06	52.94	100.24	53.39	4.50	40.46
	Rho	-0.40	-0.18	-0.18	-0.24	-0.21			
5	Obs Spd	508.40	439.38	363.30	320.52	196.34	-312.05	-3.50	482.40
	Mer Spd	1,058.01	446.09	321.34	318.61	376.22	-681.79	-3.24	564.72
	SV Spd	910.44	430.94	323.75	336.33	475.67	-434.77	-2.66	465.34

Panel B: Regressions

	Full	Full	Full	4yr	10yr	IG
Merton Spread	0.38		0.38	0.35	0.61	0.24
	[7.00]		[7.00]	[6.54]	[8.14]	[4.26]
SV Spread		0.39				
		[7.05]				
SV Residual			0.04	0.05	-0.05	-0.11
			[0.27]	[0.42]	[-0.13]	[-1.40]
R-sqd	46.01	45.11	46.02	50.04	53.12	23.64
Obs	4,186	4,186	4,186	2,499	1,687	3,273

The data is at the bond-quarter level from 2003 Q1 to 2007 Q4. For each firm, one bond close to four years to maturity and one bond close to ten years to maturity are used. In Panel A, bonds are sorted into quintiles by Merton yield spreads. Quintiles 1 and 2 are combined and within each group, bonds are then sorted by the difference between stochastic volatility yield spreads and Merton yield spreads. Rho is the estimated correlation between asset return and asset variance shocks. Spreads are reported in basis points. Reported t-stats use standard errors clustered by firm and time. Reported standard deviations are standard deviations within a Merton quintile. In Panel B, the SV residual is constructed as the residual from a regression of the stochastic volatility yield spread on the Merton yield spread with time fixed-effects. The dependent variable in the reported regressions is the observed yield spread and all regressions contain time fixed-effects. Yield spreads are winsorized at 1% of each tail. Reported t-stats use standard errors clustered by firm. Reported R^2 values are within-group R^2s.

Table 1.10: Alternative Specifications

Panel A: Stochastic Volatility with Slower Mean-Reversion

	Full	Full	Full	4yr	10yr	IG
Merton Spread	0.38		0.38	0.35	0.61	0.24
	[7.00]		[7.13]	[6.63]	[8.85]	[4.11]
SV Spread		0.46				
		[6.09]				
SV Residual			-0.12	-0.07	-0.41	0.17
			[-0.88]	[-0.54]	[-1.87]	[1.49]
R-sqd	46.01	41.80	46.18	50.10	54.24	24.11
Obs	4,186	4,186	4,186	2,499	1,687	3,273

Panel B: Black-Cox with Recent Equity Volatility

	Full	Full	Full	4yr	10yr	IG	Unexp Spd
BC Spread	0.58		0.58	0.58	0.58	0.32	
	[8.02]		[7.98]	[6.90]	[6.66]	[8.10]	
BC Spread (period)		0.70					
		[8.84]					
BC Spread (period) Residual			0.44	0.45	0.42	0.25	
			[5.90]	[4.83]	[4.72]	[4.23]	
Eq Vol							1.71
							[3.23]
R-sqd	44.92	48.12	52.95	55.05	49.50	32.97	3.32
Obs	4,238	4,238	4,238	2,546	1,692	3,324	4,238

In Panel A, the SV residual is constructed as the residual from a regression of the (alternative) stochastic volatility model yield spread on the Merton yield spread with time fixed-effects. In Panel B, the BC (period) residual is constructed as the residual from a regression of the BC (period) yield spread on the Black-Cox yield spread with time fixed-effects. In both panels, the dependent variable is the observed yield spread with the exception of the last column of Panel B where the dependent variable is the residual from a regression of the observed yield spread on BC (period) yield spread. All yield spreads are winsorized at 1% of each tail and all regressions contain time fixed-effects. Reported t-stats use standard errors clustered by firm. Reported R^2 values are within-group R^2s.

Table 1.11: Credit Default Swaps

Panel A: Levels of CDS Premia

Rating	Obs	Observed CDS Spread	Model CDS Spread	Difference		t-stat
				Mean	Med	
Aaa & Aa	195	12.42	4.66	7.76	9.97	2.50
A	538	26.51	10.23	16.28	18.98	3.48
Baa	741	63.91	31.98	31.92	32.65	3.76
Junk	407	337.48	229.23	108.25	118.90	2.01
Full	1,905	107.61	65.23	42.38	25.64	3.24

Panel B: Cross-Sectional Regressions

	Obs Spd	Unexplained Spread					
BC Spread	0.59						
	[3.94]						
Rating					17.80		
					[6.66]		
Mktlev		2.15	0.40			0.70	0.95
		[2.51]	[0.46]			[0.74]	[1.01]
Asset Vol		1.08	-1.50				
		[0.74]	[-1.28]				
Eq Vol			5.60	6.41		5.68	5.67
			[5.27]	[5.06]		[5.30]	[5.45]
IV - Eq Vol				4.57		3.71	3.72
				[4.09]		[5.13]	[5.08]
ln(Eq Mkt Cap)						-10.88	
						[-1.77]	
ln(Firm Value)							-12.40
							[-2.28]
Firm Age						0.34	0.37
						[1.47]	[1.55]
ROA						6.35	6.61
						[0.69]	[0.72]
Eq Beta						18.68	18.88
						[1.10]	[1.10]
Asset Tangibility						-1.57	-1.58
						[-3.30]	[-3.27]
Interest Coverage						0.08	0.19
						[0.12]	[0.34]
Hist - Current Lev						0.28	0.27
						[0.52]	[0.50]
R-sqd	36.58	8.91	22.66	22.42	23.27	29.59	29.93
Obs	1,905	1,905	1,905	1,405	1,881	1,358	1,358

Panel C: Commonality

	CDS Spread	$\hat{\varepsilon}_{CDS,1}$	$\hat{\varepsilon}_{CDS,2}$
Bond Spread	1.05		
	[23.61]		
$\hat{\varepsilon}_{Bond,1}$		0.98	
		[14.78]	
$\hat{\varepsilon}_{Bond,2}$			0.87
			[8.33]
R-sqd	70.16	49.62	37.67
Obs	1,914	1,914	1,876

The sample is a panel of quarterly five-year CDS with a matching panel of bonds that represent the bond closest to five years to maturity for each issuer. Data is from 2004 Q4 to 2007 Q4. CDS spreads are reported in basis points. Ratings are the average of the ratings of the corporate bonds for the issuer. In Panel A, t-stats use standard deviations clustered by time and firm. In Panel B, the dependent variable is the residual from a regression of observed CDS spreads on model CDS spreads with the exception of the first column for which the dependent variable is the observed CDS spread. In Panel B, CDS spreads and model CDS spreads are winsorized at 1% of each tail and all regressions contain time fixed-effects and reported t-stats use standard errors clustered by firm. Dependent variables are as defined in Tables 1.1 and 1.3. In Panel C, $\hat{\varepsilon}_{Bond,1}$ and $\hat{\varepsilon}_{CDS,1}$ have the Black-Cox model spreads partialed out while $\hat{\varepsilon}_{Bond,2}$ and $\hat{\varepsilon}_{CDS,2}$ also have recent equity volatility and ratings partialed out. All regressions contain time fixed-effects. T-statistics in Panels B and C are clustered by firm. Reported R^2 values are within-group R^2s.

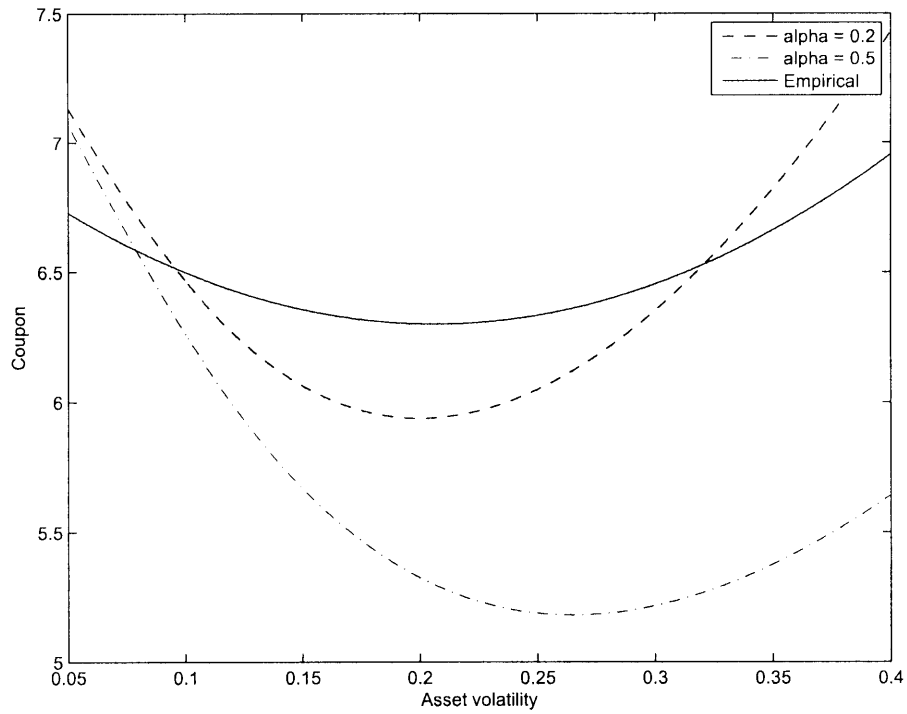

Figure 1-1: Relation Between Coupons and Asset Volatility

1.9.2 Volatility Due to Bid-Ask Spreads

In equation (1.2), $\frac{\partial \log E}{\partial \log V}$ and σ_E are not constant over time, but I only update these values quarterly due to the fact that Compustat data is updated quarterly. An alternative specification is to update $\frac{\partial \log E}{\partial \log V}$ more frequently by using a linear interpolation of Compustat values between quarterly reports and use higher frequency updates of σ_E. This, however, creates problems in volatility estimates due to bid-ask spread. Using a framework similar to Roll (1984), I examine the amount of annualized equity volatility that can be generated solely through bid-ask bounce depending on the sampling horizon. Suppose that all changes in equity price are due to whether the transaction is at the bid or ask price, denoted b and a, respectively. Also, denote the bid-ask spread as $s = a - b$. If transactions at bid and ask are equally likely, the distribution of log returns is:

$$\log(R) = \begin{cases} 0 & \text{w.p. } 0.5, \\ log(1 + \frac{s}{b}) & \text{w.p. } 0.25, \\ log(1 - \frac{s}{a}) & \text{w.p. } 0.25 \end{cases}$$

Suppose $\frac{b+a}{2} = 100$, then the annualized volatility generated solely by bid-ask spreads for 5-minute, 30-minute, and daily sampling (in %) are:

Table 1.12: Bid-Ask Spread Induced Volatility

Bid-Ask Spread	Vol (5 min)	Vol (30 min)	Vol (daily)
0.1	9.91	4.05	1.12
0.2	19.83	8.09	2.24
0.3	29.74	12.14	3.37
0.4	39.65	16.19	4.49
0.5	49.57	20.24	5.61

As can be seen above, sampling at too high a frequency can generate large equity volatility even when there are no changes in firm fundamentals. As a baseline for comparison, the median spreads as a percentage of stock value for IBM, GE, GM, JNJ, and WMT in January 2003 were 0.18, 0.16, 0.36, 0.30, and 0.29, respectively.[36]

1.9.3 Claims on the Firm in the Black-Cox Model

Following Bjork (2004), the value of a down-and-out call option where the barrier equals the strike price is:

$$Call = Ve^{-\delta T}N(d_1) - Ke^{-rT}N(d_2) - (\frac{K}{V})^{2(r-\delta-\frac{\sigma_v^2}{2})/\sigma_v^2}[\frac{K^2}{V}e^{-\delta T}N(\tilde{d}_1) - Ke^{-rT}N(\tilde{d}_2)] \tag{1.17}$$

$$d_1 = \frac{\ln(\frac{V}{K}) + (r - \delta + \frac{\sigma_v^2}{2})T}{\sigma_v\sqrt{T}}, d_2 = d_1 - \sigma_v\sqrt{T}$$
$$\tilde{d}_1 = \frac{\ln(\frac{K}{V}) + (r - \delta + \frac{\sigma_v^2}{2})T}{\sigma_v\sqrt{T}}, \tilde{d}_2 = \tilde{d}_1 - \sigma_v\sqrt{T}$$

The value of equity at maturity is the value of this call option.

Defining Q as the risk-neutral probability of default (see equation (1.4), the value of debt at maturity and the value of bankruptcy costs are:

$$\text{Debt at Maturity} = Ke^{-rT} - Ke^{-rT}Q(1 - R_{firm}) \tag{1.18}$$
$$\text{Bankruptcy Costs} = Ke^{-rT}Q(1 - R_{firm})$$

[36]Quote-by-quote spreads from the NYSE TAQ data are used.

Thus, the remaining value of the firm, the value of payouts, is equal to $V - Call - Ke^{-rT}$, and is attributed to equity and debt in a proportion equal to $\frac{\delta_e}{\delta}$ and $\frac{(\delta-\delta_e)}{\delta}$, respectively.

1.9.4 Model-Implied Default Probabilities

In contrast to Huang and Huang (2003), I match equity volatilities rather than probabilities of default when calculating firm parameters. This is done both because historical default probabilities do not necessarily reflect forward-looking default probabilities and because there is no data on firm-by-firm default probabilities. Here, I examine model-implied default probabilities for the Black-Cox model and compare them to historical default probabilities.

Huang and Huang use average firm parameters within a rating and use the Bhandari (1988) estimates to generate an equity risk premium to match along with historical probabilities of default. From this, they generate implied asset volatilities and implied asset risk premia. As my calculations in Section 1.3 infer an asset volatility from equity volatility, I only need to infer an asset risk premium to be able to calculate probabilities of default. I follow the Huang and Huang procedure in using the Bhandari estimates (columns labeled with Lev) and also use equity risk premia calculated from the CAPM. In the third and fourth columns of Panels A and B, I report the model-implied default probabilities for four-year and ten-year horizons, respectively. For four-year horizons, the model tends to underestimate default probabilities for all ratings. At the ten-year horizon, the model actually does a reasonable job for investment grade bonds, but vastly underestimates default probabilities for junk debt.

In addition to using average firm parameters, I also calculate model-implied default probabilities firm-by-firm for comparison with historical default probabilities. I find that the mean model-implied default probabilities tend to be higher than historical default probabilities for investment grade firms at both the four-year and ten-year horizon. However, median default probabilities are much lower, indicating that these results are driven by cases where a few firms have very high model-implied default probabilities.

Table 1.13: Model-Implied Default Probabilities

Panel A: 4yr Default Probabilities

		Average Firm		Firm-by-Firm CAPM			Firm-by-Firm Lev		
Rating	Historical	CAPM Used	Lev Used	Mean	Std	Med	Mean	Std	Med
Aaa	0.04	0.01	0.01	0.47	0.92	0.00	0.45	0.88	0.00
Aa	0.23	0.00	0.00	0.18	0.85	0.00	0.14	0.64	0.00
A	0.35	0.07	0.12	1.12	4.59	0.01	0.83	3.25	0.01
Baa	1.24	0.38	0.62	1.97	7.30	0.04	1.49	4.90	0.04
Ba	8.51	0.89	1.41	5.07	14.49	0.38	2.96	6.50	0.50
B	23.32	6.43	8.14	9.78	15.81	1.98	6.97	11.20	2.02
C		20.80	19.90	21.76	21.12	18.20	17.70	19.41	10.02

Panel B: 10yr Default Probabilities

		Average Firm		Firm-by-Firm CAPM			Firm-by-Firm Lev		
Rating	Historical	CAPM Used	Lev Used	Mean	Std	Med	Mean	Std	Med
Aaa	0.77	0.88	0.91	2.45	3.89	0.01	2.40	3.85	0.01
Aa	0.99	0.70	0.97	1.87	4.73	0.32	1.60	3.61	0.34
A	1.55	1.78	3.10	3.98	8.70	0.69	3.23	6.27	0.82
Baa	4.39	3.77	6.40	6.80	12.76	1.60	5.56	9.52	1.73
Ba	20.63	5.02	8.36	12.01	21.19	3.71	10.33	14.63	5.25
B	43.91	17.83	23.04	24.49	28.98	11.01	20.32	22.90	12.44
C		36.93	35.19	36.74	28.19	38.35	28.20	25.78	27.27

1.9.5 Option Pricing Formula for the Stochastic Volatility Model

In this section, I present call pricing formulas for a stochastic volatility model. This formulation is a special case of Duffie, Pan, and Singleton (2000) and also of Pan (2002).

$$\frac{Call}{V_t} = G_{1,-1}(-\log k, H_0, T) - kG_{0,-1}(-\log k, H_0, T) \tag{1.19}$$

$$G_{1,-1} = \frac{\psi(1, H_0, T)}{2} - \frac{1}{\pi}\int_0^\infty \frac{Im[\psi(1-iu, H_0, T)e^{iu(\log k)}]}{u}du$$

$$G_{0,-1} = \frac{\psi(0, H_0, T)}{2} - \frac{1}{\pi}\int_0^\infty \frac{Im[\psi(-iu, H_0, T)e^{iu(\log k)}]}{u}du$$

$$\psi(s, H_0, T) = \exp(\alpha(T,s) + \beta(T,s)H_0)$$

$$\alpha(T,s) = -rT + (r-\delta)sT - \kappa_H\theta_H(\frac{\gamma+b}{\sigma_H^2}T + \frac{2}{\sigma_H^2}\log[1 - \frac{\gamma+b}{2\gamma}(1-e^{-\gamma T})])$$

$$\beta(T,s) = -\frac{a(1-e^{-\gamma T})}{2\gamma - (\gamma+b)(1-e^{-\gamma T})}$$

$$a = s(1-s)$$

$$b = \sigma_H \rho s$$

$$\gamma = \sqrt{b^2 + a\sigma_H^2}$$

Chapter 2

Excess Volatility of Corporate Bonds

2.1 Introduction

We examine the connection between corporate bonds and stocks under the structural model of Merton with stochastic interest rates. In particular, we focus on the volatility of corporate bonds and its connection to the equity volatility of the same firm, as well as Treasury volatility. Using daily returns on bonds and stocks, we find an overwhelming amount of excess volatility in corporate bonds. In annualized terms, the difference between the empirical volatility $\hat{\sigma}_D$ and its model-implied counterpart $\hat{\sigma}_D^{\text{Merton}}$ is on average 12.64% with a robust t-stat of 35. Moving from daily returns to weekly and monthly returns, this excess volatility tapers off quite dramatically, suggesting a liquidity component in corporate bonds that is more pronounced at short horizons.

The motivation for this empirical study is two-fold. First, while the structural models pioneered by Merton (1974) have a direct impact on our conceptual understanding of the connection between bonds and stocks of the same firm, their empirical reach remains somewhat limited. Limited access to quality bond data in the past certainly is an important factor. The recently available TRACE data, however, greatly improves the situation by offering transaction-level data with both price and volume information. The second, and perhaps more important motivation for this study is the severe liquidity issues in the corporate bond market. While the corporate bond market is as large as the Treasury bond market, the difference in liquidity between these two markets can offer a stark contrast. By the first quarter of 2007, the total amount outstanding is \$4.45 trillion in Treasuries and \$5.45 trillion in corporate debt. By comparison, in January 2007, the average daily trading volume is around \$492 billion in Treasuries and \$16.7 billion in corporate debt. Thus, there is active trading in Treasuries and hardly any trading in corporates. Liquidity issues of this magnitude are bound to find its way to corporate bond pricing.

In this chapter, we tackle these two issues by introducing and comparing two measures of corporate bond volatility: the empirically estimated versus the model implied. We use the Merton model to take into account the two main drivers of corporate bond volatility: the firm's asset volatility and the Treasury volatility. Measuring Treasury volatility using Treasury bond returns and inferring asset volatility from the firm's equity volatility through the Merton model, we feed these these estimates collected from equity and Treasury markets back to the Merton model with stochastic interest rates to obtain model-implied return volatilities for the respective corporate bonds. We then compare this model-implied bond volatility $\hat{\sigma}_D^{\text{Merton}}$ with the empirically observed bond return volatility $\hat{\sigma}_D$. The discrepancy between the two volatility measures sets the stage for both an empirical evaluation of the Merton model and an empirical investigation on the nature of illiquidity in the corporate bond market.

For a broad cross-section of corporate bonds from July 2002 through December 2006, we find that the annualized bond volatility $\hat{\sigma}_D$ is on average 18.06% using daily bond returns, 9.62% using weekly returns, and 7.18% using monthly returns. This pattern of decreasing annualized volatility with increasing measurement horizon is found to be unique only in corporate bonds. In particular, when daily, weekly, and monthly equity returns are used for the same pool of bond issuers in our sample, the estimated annualized equity volatilities are similar in magnitudes across measurement horizons. The same is true when daily, weekly, and monthly Treasury bond returns are used to estimate the Treasury bond volatility. Absent a structural model, this result along the measurement horizon by itself has an immediate implication on the liquidity of corporate bonds. It quantifies and contrasts the illiquidity of corporate bonds in relation to that of the equity and Treasury markets.

To connect the diverging information in the three markets – corporate bond, equity and Treasury bond — into one unified framework, we adopt the Merton model with stochastic interest rates. In the model, the corporate bond volatility has two contributions: random fluctuations in firm value and in the risk-free interest rate. While the Treasury bond volatility can be directly estimated using Treasury bond returns, we can only infer, via the Merton model, the firm's asset volatility from estimates of the equity volatility of the firm. Consequently, the two important inputs of the model come from the volatility estimates of equity and of Treasury bond returns. In addition, we also rely on the firm's balance sheet information to estimate the other firm-level characteristics that are important in the model. We find that, for the same pool of bonds and for the same time periods, the annualized bond volatility is on average 5.42% using daily equity and Treasury returns, 5.14% using weekly returns, and 5.35% using monthly returns. Comparing this set of model-implied volatilities against the ones estimated empirically, we see a clear pattern of excess volatility in corporate bonds that is most severe at the short horizon, but remains significant, both statistically and economically, even at longer measurement horizons.

To further shed light on the economic origin of the excess bond volatility, we examine its cross-sectional determinants. Our results paint a general picture that links the degree of excess bond volatility to the illiquidity of a bond. For example, our results show that excess volatility is higher for smaller bonds, which are typically less liquid. More interestingly, our results also show that, after controlling for bond characteristics including maturity, rating and size, older bonds have higher excess volatility. As newly issued bonds are typically more liquid, while the old bonds are more likely to be have larger fractions held by buy-and-hold investors, our result is consistent with a liquidity explanation. Finally, we also find interesting connections to trading related variables. For example, we find more excess volatility in corporate bonds whose average trade size is small. This is consistent with the possibility that, after controlling for bond size, the bonds with smaller average trade size are more likely to be traded in less liquid bond trading platforms and therefore have a larger liquidity component. In addition to this cross-sectional explanation, the monthly time-series variation in excess volatility and the bond trading variable could also contribute to the result. In particular, it is consistent with the assumption that the month during which the average trade size of a particular bond is small is a less liquid month, resulting in higher excess volatility for that bond during that month.

Compared with the existing literature that examines the empirical performance of the Merton model from the angle of the first moment, our particular focus on the second moment of corporate bond returns sets us apart and provides us with a new angle. Our motivation, however, is very much aligned with this literature. In particular, we would like to be able to add to the debate that is central to this literature and has been raised, among others, by Huang and Huang (2003): How much of the market-observed corporate bond yield spreads is due to the firm's credit risk? Our empirical results point in the direction of an illiquidity component in corporate bond returns. One might argue that with the reliance of a model, the conclusion is always a joint hypothesis of the empirical performance, or, in this case, the lack of empirical performance, of the Merton model. Nevertheless, it would be highly implausible for any default-based model to generate the observed pattern of increasing bond volatility with decreasing measuring horizon. A pattern like this is more reminiscent of a microstructure model with bid-ask bounce playing a more important role at a short horizon. The fact that we find this pattern only in corporate bonds but not in equities and Treasury bonds is indicative of a liquidity problem beyond simple bid-ask bounce. Indeed, using the quoted bid-ask spreads of corporate bonds, we find the effect of bid-ask bounce to be rather minor in addressing the excess volatility puzzle. In summary, the empirical pattern of bond volatility documented in this chapter stands on its own to provide a clear and unambiguous support of the importance of liquidity in corporate bond prices.

To the extent that the structural model of Merton is important in our analysis, it provides a set of benchmark numbers of corporate bond volatility, incorporating the firm's balance sheet information as well as information from the equity and Treasury bond markets. A formal and extensive empirical evaluation of the Merton model constitutes another important motivation of this chapter. In relation to the work of Eom, Helwege, and Huang (2004), who examine the empirical performance of structural models of default including the Merton model using only 182 data points, our contribution is to perform an empirical analysis of the Merton model on a much larger scale of corporate bonds. More importantly, by contrasting the model-implied bond volatility against the empirical volatility measures, we are able to shed light on the empirical performance of the Merton model from a perspective that has not been looked at before.

Our result indicates that while at the daily and weekly measurement horizons, the Merton model cannot even begin to generate the kind of bond volatility observed in the data due to the liquidity problems in corporate bonds, at the monthly return horizon, the model is able to generate an average volatility of 5.35% that is relatively close to the empirically observed bond volatility of 7.18%. This excess volatility of 1.83%, however, is still statistically significant with a robust t-stat of 2.29, and, perhaps more importantly, still accounts for a quarter of the observed empirical bond volatility. Whether or not this is due to liquidity or model mis-specification remains an interesting question. We find that even at the monthly measurement horizon, the cross-sectional determinants of the excess volatility are still closely related to liquidity variables such as the age of a bond and its average trade size.

On the other hand, we can look for potential model mis-specifications by examining the model-implied bond volatility more closely. In constructing the model-implied volatility, the corporate bond's sensitivities to its firm's asset and the Treasury bond are the two basic building blocks. As such, whether or not the model-implied sensitivities match their empirical counterparts provides a more detailed test of the model. In fact, Schaefer and Strebulaev (2008) show the Merton model provides quite accurate predictions of the sensitivity of corporate bond returns to changes in the value of equity. However, the empirical sensitivity to Treasury bonds is significantly lower than those prescribed by the model.[1] This result indicates that the excess volatility puzzle documented in this chapter is somewhat understated. Given the importance of Treasury bond volatility in generating the model-implied corporate bond volatility, the magnitude of the excess volatility puzzle would have been more severe had we not used the model-implied sensitivity measures.

[1] We find that this empirical result holds for our sample.

Indeed, this is confirmed in our finding of an even more exacerbated volatility puzzle when excess bond returns are used to avoid explicitly modeling the risk-free interest rate. Given the importance of interest rate risk in corporate bonds, the stochastic interest rate component of the model should perhaps be subject to the most severe scrutiny. In particular, the bonds in our sample have a median maturity close to 7 years. To properly account for the risk-free bond's volatility in our main results, we employ the Vasicek (1977) model for its simplicity, but calibrate the volatility coefficient of the model so that the model generates the empirically observed level of volatility for a 7-year Treasury coupon bond. Missing in this simple one-factor term structure model is the potentially rich term-structure of interest rate volatility. As a robustness check, we work with excess bond returns to avoid relying on a term structure model. Specifically, we calculate excess bond returns by subtracting, from the corporate bond returns, the contemporaneous Treasury bond returns of a similar maturity. Comparing the volatility measured by bond excess returns to the model-implied excess bond volatility, we find that, in annualized terms, the excess volatility is 17.21% using daily returns, 8.02% using weekly returns, and 4.76% using monthly returns. In other words, the excess volatility puzzle is more severe in this treatment.

This chapter is related to Collin-Dufresne, Goldstein, and Martin (2001), who regress monthly changes in corporate bond yields on variables that should in theory determine credit spread changes, and find very low R^2's in their regressions. We are able to put a horizon dimension to this illuminating and intuitive result. Specifically, regressing daily corporate bond returns on daily equity returns on its firm equity and on a Treasury bond of a similar maturity, we find a cross-sectional average R^2 of 18.28%, which increases to 46.38% when monthly returns are employed. More importantly, by working in a structural setting, we are able to contrast the data and the model more closely. Instead of simply comparing the magnitudes of R^2, we are able to construct formal empirical tests linking the empirical volatility to the model-implied volatility. In the liquidity dimension, this chapter is related to the papers by Houweling, Mentink, and Vorst (2005), Downing, Underwood, and Xing (2005), Mahanti, Nashikkar, and Subrahmanyam (2008), deJong and Driessen (2005), and Chen, Lesmond, and Wei (2007), which examine the liquidity impact in corporate through a liquidity premium component in bond yields. Our empirical evaluation of the Merton model is closely related to the papers by Crosbie and Bohn (2003), Leland (2004) and Bharath and Shumway (2008), which use structural models of default to forecast default probability. Finally, also related are the papers by Vassalou and Xing (2004) and Campbell, Hilscher, and Szilagyi (2007), which use default probability to examine the expected equity returns of the same firm.

The rest of the chapter is organized as follows. Section 2.2 outlines the empirical specification. Section 2.3 summarizes the data and the empirical volatility estimates. Section 2.4 details the model implied volatility. Section 2.5 summarizes the main empirical results of this chapter. Section 2.6 reports the cross-sectional determinants of excess volatility. Section 2.7 supplements with a time-series analysis of corporate bond returns. Section 2.8 concludes.

2.2 Empirical Specification

2.2.1 The Merton Model

We use the Merton (1974) model to connect the equity and corporate bonds of the same firm. Let V be the total firm value, whose risk-neutral dynamics are assumed to be

$$\frac{dV_t}{V_t} = (r_t - \delta)\, dt + \sigma_v\, dW_t^Q\,, \tag{2.1}$$

where W is a standard Brownian motion, and where the payout rate δ and the asset volatility σ_v are assumed to be constant.

We adopt a simple extension of the Merton model to allow for a stochastic interest rate.[2] This is important for our purposes because a large component of the corporate bond volatility comes from the Treasury market. Specifically, we model the risk-free rate using the Vasicek (1977) model:

$$dr_t = \kappa\,(\theta - r_t)\; dt + \sigma_r\, dZ_t^Q\,, \tag{2.2}$$

where Z is a standard Brownian motion independent of W, and where the mean-reversion rate κ, long-run mean θ and the diffusion coefficient σ_r are assumed to be constant.

Following Merton (1974), let us assume for the moment that the firm has, in addition to its equity, a single homogeneous class of debt, and promises to pay a total of K dollars to the bondholders on the pre-specified date T. Equity then becomes a call option on V:

$$S_t = V_t\, e^{-\delta T}\, N(d_1) - K\, e^{a(T)+b(T)\, r_t}\, N(d_2), \tag{2.3}$$

where $N(\cdot)$ is the cumulative distribution function for a standard normal, $d_1 = d_2 + \sqrt{\Sigma}$,

$$d_2 = \frac{\ln(V/K) - a(T) - b(T)r_t - \delta T - \frac{1}{2}\Sigma}{\sqrt{\Sigma}}, \tag{2.4}$$

$$\Sigma = T(\sigma_v^2 + \frac{\sigma_r^2}{\kappa^2}) + \frac{2\sigma_r^2}{\kappa^3}(e^{-\kappa T} - 1) - \frac{\sigma_r^2}{2\kappa^3}(e^{-2\kappa T} - 1), \tag{2.5}$$

[2]See Shimko, Tejima, and vanDeventer (1993).

and where $a(T)$ and $b(T)$ are the exponents of the discount function of the Vasicek model:

$$b(T) = \frac{e^{-\kappa T} - 1}{\kappa}; \quad a(T) = \theta\left(\frac{1 - e^{-\kappa T}}{\kappa} - T\right) + \frac{\sigma^2}{2\kappa^2}\left(\frac{1 - e^{-2\kappa T}}{2\kappa} - 2\frac{1 - e^{-\kappa T}}{\kappa} + T\right). \tag{2.6}$$

Note that a Merton model extended to have Vasicek interest rates simply has e^{-rT} replaced by $e^{a(T)+b(T)r_t}$ and σ_v^2 T replaced by Σ.

2.2.2 From Equity Volatility to Asset Volatility

We first use the Merton model to link the firm's asset volatility to its equity volatility. Let σ_E be the volatility of instantaneous equity returns. In our model, the equity volatility is affected by two sources of random fluctuations:

$$\sigma_E^2 = \left(\frac{\partial \ln S_t}{\partial \ln V_t}\right)^2 \sigma_v^2 + \left(\frac{\partial \ln S_t}{\partial r_t}\right)^2 \sigma_r^2. \tag{2.7}$$

Using equation (2.3), we can calculate the sensitivities of equity returns to the random shocks in asset returns and risk-free rates:

$$\frac{\partial \ln S_t}{\partial \ln V_t} = \frac{1}{1 - \mathcal{L}} \quad \text{and} \quad \frac{\partial \ln S_t}{\partial r_t} = \frac{b(T)\,\mathcal{L}}{1 - \mathcal{L}},$$

where

$$\mathcal{L} = \frac{K}{V}\frac{N(d_2)}{N(d_1)} \exp\left(\delta\, T + a(T) + b(T)\, r_t\right).$$

Combining the above equations, we have

$$\sigma_E^2 = \left(\frac{1}{1 - \mathcal{L}}\right)^2 \sigma_v^2 + \left(\frac{\mathcal{L}}{1 - \mathcal{L}}\right)^2 b(T)^2\, \sigma_r^2. \tag{2.8}$$

As expected, the firm's equity volatility σ_E is closely related to its asset volatility σ_v. In addition, it is also affected by the Treasury volatility σ_r through the firm's borrowing activity in the bond market. This is reflected in the second term of equation (2.8), with $b(T)\,\sigma_r$ being the volatility of instantaneous returns on a zero-coupon risk-free bond of the same maturity T. The actual impact of these two random shocks is further amplified through $\mathcal{L}$, which, for lack of a better expression, we refer to as the "modified leverage." Specifically, for a firm with a higher $\mathcal{L}$, a one unit shock to its asset return is translated to a larger shock to its equity return. Of course, this is the standard leverage effect. Moreover, as shown in the second term of equation (2.8), for such a highly "levered" firm, its equity return also bears more interest rate risk. Conversely, for an all-equity firm, $\mathcal{L} = 0$, and the interest-rate component diminishes to zero.

As it is true in many empirical studies before us, a structural model such as the Merton model plays a crucial role in connecting the asset value of a firm to its equity value. Ours is not the first empirical exercise to back out asset volatility using observations from the equity market.[3] In the existing literature, there are at least two alternative ways to approximate K/V. For example, in the approach pioneered and popularized by Moody's KMV, the Merton model is used to calculate $\partial S/\partial V$ as well as to infer the firm value V through equation (2.3). By contrast, we use the Merton model to derive the entire piece of the sensitivity or elasticity function $\partial \ln S/\partial \ln V$, as opposed to using only $\partial S/\partial V$ from the model and then plugging in the market observed equity value S for the scaling component. At a conceptual level, we believe that taking the entire piece of the sensitivity function from the Merton model is a more consistent approach. At a practical level, while the Merton model might have its limitations in the exact valuation of bond and equity, it is still valuable in providing insights on how a percentage change in asset value propagates to percentage changes in equity value for a levered firm.

In this respect, our reliance on the Merton model centers on the sensitivity measure. To the extent the Merton model is important in our empirical implementation, it is in deriving the analytical expressions that enter equation (2.8). In particular, we rely on the Merton model to tell us how the sensitivities or elasticities vary as functions of the key parameters of the model including leverage K/V, asset volatility σ_v, payout rate δ, and debt maturity T. When it comes to the actual calculations of these key parameters, we deviate from the Merton model as follows.

The key parameter that enters equation (2.8) is the ratio K/V, where K is the book value of debt and V is the market value of the firm. We calculate the book debt K using Compustat data, and approximate the firm value V by its definition $V = S + D$, where S is the market value of equity and D is the market value of debt. To estimate the market value of debt D, we start with the book value of debt K. To further improve on this approximation, we collect, for each firm, all of its bonds in TRACE, calculate an issuance weighted market-to-book ratio, and multiply K by this ratio.

Implicit in our estimation of the firm value V is the acknowledgment that firms do not issue discount bonds as prescribed by the Merton model. In particular, we deviate from the zero-coupon structure of the Merton model in order to take into account of the fact that firms typically issue bonds at par. By adopting this empirical implementation, however, we do have to live with one internal inconsistency with respect to the relation between K

[3]See, for example, Crosbie and Bohn (2003), Eom, Helwege, and Huang (2004), Bharath and Shumway (2008), and Vassalou and Xing (2004).

and D, and central to this inconsistency is the problem of applying a model designed for zero-coupon bonds to coupon bonds.

The main implication of our choice of V is on the ratio of K/V, which in turn, affects the firm's actual leverage. We can therefore gauge the impact of our implementation strategy by comparing the market leverage implied by the Merton model with the empirically estimated market leverage. In unreported results, we find that with our choice of K/V, the two market leverage numbers, model implied vs. empirically estimated, are actually very close for the sample of firms considered in this chapter. Closely related to this comparison is the alternative estimation strategy that infers K/V by matching the two market leverage ratios: model-implied and empirically estimated.[4] From our analysis, we expect this approach to yield K/V ratios that are close to ours.[5]

Finally, two other parameters that enter equation (2.8) are the firm-level debt maturity T and the firm's payout ratio δ. Taking into account the actual maturity structure of the firm, we collect, for each firm, all of its bonds in FISD and calculate the respective durations. We let the firm-level T be the issuance-weighted duration of all the bonds in our sample. Effectively, we acknowledge the fact that firm's maturity structure is more complex than the zero-coupon structure assumed in the Merton model, and our issuance-weighted duration is an attempt to map the collection of coupon bonds to the maturity of a zero-coupon bond. In calculating the payout ratio δ, we aggregate the firm's equity dividends, repurchases, and issuances and the debt coupon payments and scale the total payout by firm value V, with the details of calculating V summarized above.

2.2.3 Model-Implied Bond Volatility

The second step of our empirical implementation is to calculate, bond-by-bond, the volatility of its instantaneous returns, taking the inferred asset volatility $\hat{\sigma}_v$ from the first step as a key input. Again, we have to take make a simplification to the Merton model to accommodate the bonds of varying maturities issued by the same firm. Specifically, we rely on the Merton model to tell us, for any given time τ, the risk-neutral survival probability up to time τ: $P^\tau = N(d_2)$, where d_2 is as defined in equation (2.4) with T replaced by τ. Instead of taking the Merton model literally, which would imply no default between time 0 and the maturity date T. We find this to be a more realistic adoption of the model.[6]

Equipped with the term structure of default probabilities implied by the Merton model,

[4]We thank Hayne Leland for pointing this out and for extensive discussions on this issue.

[5]See the previous chapter for an example of how K/V can be inferred.

[6]A more self-consistent approach is to use the Black and Cox (1976) model, which generates a term structure of default probability that is the complementary first passage time distribution.

we can now price defaultable bonds issued by each firm. Consider a τ-year bond paying semi-annual coupons with an annual rate of c. Assuming a face value of \$1, the time-t price of the bond is

$$B_t = \sum_{i=1}^{2\tau} \frac{c}{2}\, e^{a(i/2)+b(i/2)\, r_t} P^{i/2} + e^{a(\tau)+b(\tau)\, r_t} P^{\tau} + \sum_{i=1}^{2\tau} \mathcal{R}\, e^{a(i/2)+b(i/2)\, r_t} \left(P^{(i-1)/2} - P^{i/2}\right), \quad (2.9)$$

where $\mathcal{R}$ is the risk-neutral expected recovery rate of the bond upon default. The first two terms in equation (2.9) collect the coupon and the principal payments taking into account the probabilities of survival up to each payment. The third term collects the recovery of the bond taking into account the probability of default happening exactly within each six-month period, which is $P^{(i-1)/2} - P^{i/2}$ for the i-th six-month period. The terms involving a and b are the respective risk-free discount functions implied by the Vasicek model as defined in equation (2.6).

Let σ_D^{Merton} be the volatility of the instantaneous returns of the defaultable bond. The model-implied bond volatility can be calculated as

$$\left(\sigma_D^{\text{Merton}}\right)^2 = \left(\frac{\partial \ln B_t}{\partial \ln V_t}\right)^2 \sigma_v^2 + \left(\frac{\partial \ln B_t}{\partial r_t}\right)^2 \sigma_r^2 . \quad (2.10)$$

Using the bond priced in equation (2.9) as an example, we can see that its asset-sensitivity, $\partial \ln B_t / \partial \ln V_t$, arises from the sequence of risk-neutral default probabilities, $P^{i/2}$ for $i = 1, \ldots, 2\tau$, while the Treasury-sensitivity, $\partial \ln B_t / \partial r_t$ arises both explicitly from the sequence of Vasicek discount functions and implicitly from the sequence of risk-neutral default probabilities.

It might be instructive to consider a τ-year zero-coupon bond, since its calculation can be further simplified to

$$\frac{\partial \ln B_t}{\partial \ln V_t} = \frac{n(d_2)\,(1-\mathcal{R})}{N(d_2) + (1-N(d_2))\,\mathcal{R}} \frac{1}{\sqrt{\Sigma}} \quad \text{and} \quad \frac{\partial \ln B_t}{\partial r_t} = b(\tau)\left(1 - \frac{\partial \ln B_t}{\partial \ln V_t}\right),$$

where $n(\cdot)$ is the probability distribution function of a standard normal. As expected, with full recovery upon default, $\mathcal{R} = 1$, the bond is equivalent to a treasury bond and its asset-sensitivity is zero and its Treasury-sensitivity becomes $b(\tau)$. The asset-sensitivity becomes more important with increasing loss given default, $1 - \mathcal{R}$, as well as with increasing firm leverage K/V. From this example, we can also see the importance of allowing for a stochastic risk-free rate, as the Treasury volatility is an important component in the defaultable bond volatility.

In calculating the model-implied bond volatility, we take advantage of the model-implied term structure of survival probabilities but avoid treating the defaultable bond as one large piece of zero-coupon bond with face value of K and maturity of T. This calculation is similar to the reduced-form approach of Duffie and Singleton (1999), except for the fact that our term structure of survival probabilities come from a structural model while theirs derives from a stochastic default intensity.

2.3 Data and Construction of Volatility Estimates

The main dataset used in this chapter is the TRACE dataset, a transaction-level dataset on corporate bonds distributed by FINRA. TRACE is describe in more detail in Chapter 1.

2.3.1 The Bond Sample

We use the transaction-level data from TRACE to construct bond return volatility for non-financial firms. First, we construct daily bond returns as follows. For any day t, we keep the last observation of the day for the bond and calculate the log return on day t as:

$$R_t = \ln\left(\frac{P_t + AI_t + C_t}{P_{t-1} + AI_{t-1}}\right),$$

where P_t is the clean price as reported in TRACE, AI_t is the accrued interest, and C_t is the coupon paid at t if day t is a scheduled coupon payment day. We use FISD to get bond-level information on coupon rates and payment dates. Accrued interest is calculated using the standard 30/360 convention. Returns are only calculated for day t if there is a price available for both t and $t-1$. Given the daily return data, we next construct time-series of monthly bond volatilities by taking the standard deviation of the daily bond returns (if there are at least 10 bond returns in a month) and annualizing. The sample of bonds that survive this calculation form the basis of our bond sample. In addition, to exclude very infrequently traded bonds, we include bonds for which we can construct monthly volatilities for at least 75% of its presence in TRACE.

Table 2.1 summarizes our bond sample. The number of bonds increases throughout our sample period largely due to the coverage expansion of the TRACE. Compared with the universe of U.S. corporate bonds documented in FISD, our sample contains only a small number of bonds. In terms of size, however, these bonds are orders of magnitude larger than the median size bond in FISD. For example, at the beginning of our sample in 2002, the median bond size is \$1,368 million in our sample, compared with \$68 million in FISD. In the early sample, this is largely due to the limited coverage of TRACE, but overall, our sample construction biases toward picking more frequently traded bonds, which are typically larger.

The average maturity of the bonds in our sample is about 8.5 years, similar in magnitude but slightly higher than the average maturity of 7.3 years for the bond universe in FISD. While the cross-sectional median maturity is close to 7 years in our sample, it is only around 4.5 years in the FISD sample. These observations are consistent with a relatively higher degree of cross-sectional dispersion of bond maturity in FISD. In the early sample period, the bonds in our sample are noticeably younger than those in the FISD sample, although this difference diminishes toward the later sample period. The representative bonds in our sample are investment grade, with a median rating of roughly 7 (Moody's A3) during the early sample and 9 (Moody's Baa2) during the later sample. By contrast, the median rating in the FISD sample remains stable.

Given that TRACE is transaction-level data, we can further collect trading information for the bonds in our sample. For example, in 2002, an average bond is traded on average 534 times a month with $245 million of average trading volume and 13.69% turnover.[7] Over time, this set of numbers decrease quite significantly, reflecting the coverage expansion of TRACE to include smaller and less frequently traded bonds. By 2006, an average bond was traded on average 150 times a month with $58 million of average trading volume and 7.31% turnover. Also, the average trade size is $781 thousand in 2002 and $450 thousand in 2006, reflecting the inclusion of smaller trades. Overall, compared with the entire TRACE sample, our sample is biased toward bonds that are more frequently traded. For example, on over 95% of the business days, a median bond in our sample is traded at least once on that day.

Merging our bond sample with CRSP and Compustat by bond issuer, the firm-level summary statistics are reported in Table 2.1. The average number of firms in our sample is 60 in 2002 and grows to 236 in 2006. By equity market capitalization, the firms whose bonds are in our sample are typically large, with an average market capitalization of $35.90 billion in 2002 and $26.96 billion in 2006.

2.3.2 Bond Return Volatility $\hat{\sigma}_D$

The direct outcome of our sample construction is a monthly time-series of bond return volatility, $\hat{\sigma}_D$, for cross-sections of bonds. Building on the same bond sample, we also use weekly bond returns to construct a quarterly time-series of bond volatility, and monthly bond returns to construct a yearly time-series.[8]

[7]Note that transaction-level statistics are biased downwards as TRACE truncates trading volume for individual trades at $5mm for investment grade bonds and $1mm for speculative grade bonds.

[8]Like the case for daily returns, we require at least 10 weekly bond returns in a quarter to form a quarterly estimate of bond volatility, and at least 10 monthly bond returns in a year to form a yearly estimate.

The first panel of Table 2.2 summarizes the empirically estimated bond volatility, $\hat{\sigma}_D$, using daily, weekly, and monthly returns. Moving across the three return horizons, the magnitude of $\hat{\sigma}_D$'s, all annualized, decreases markedly. Specifically, the sample mean of $\hat{\sigma}_D$ is 18.06% when estimated using daily returns, contrasted with 9.62% using weekly returns, and 7.18% using monthly returns. The time-series averages of the cross-sectional median of $\hat{\sigma}_D$ exhibit a similar pattern: 15.78% at daily, 8.48% at weekly, and 6.36% at monthly frequency.

Implicit in this pattern are strong negative auto-covariances of daily and weekly bond returns. Given that we are using transaction prices to construct bond returns, bid-ask bounce could be a natural candidate for such negative autocorrelations.[9] One could use the volatility estimate proposed by French, Schwert, and Stambaugh (1987) to take out this autocorrelation and therefore construct a volatility estimate that is more closely linked to the fundamental movements. For our purposes, however, it is more appropriate to use a simple measure of volatility that includes the fundamental component as well as the potential liquidity component. As we move on next to construct equity return volatility from daily, weekly and monthly stock returns, we will adopt the same treatment. To emphasize the cross-sectional variation of the empirical bond volatility, we sort $\hat{\sigma}_D$, at the appropriate frequencies, by a set of bond- and firm-level variables into quartiles, and report the means for each quartile. As reported in Table 2.3, bonds with smaller issuance, longer maturity, and lower rating are more volatile. Bonds issued by firms with higher equity volatility and higher leverage are also more volatile. The relation of $\hat{\sigma}_D$ to the bond trading variables such as turnover and the frequency of its trading is not as clear, nor is there a clear pattern linking $\hat{\sigma}_D$ to the firm payout ratio. Finally, it is interesting to notice that moving across measurement horizons from daily to monthly returns, the cross-sectional patterns hold quite well, while the overall magnitude decreases in a dramatic fashion.

2.3.3 Equity Return Volatility $\hat{\sigma}_E$

The equity return volatility, from which the asset volatility for the firm can be backed out, is one key input to our structural model. The equity sample used to construct the equity volatility mirrors the bond sample summarized in Table 2.1. For each firm whose bonds enter our bond sample, we use CRSP daily, weekly, and monthly returns to form monthly, quarterly, and yearly estimates of equity volatility.

The second panel of Table 2.2 summarizes the empirical equity return volatility $\hat{\sigma}_E$. When we average across firms and time, the annualized equity volatility of our sample is

[9] See, for example, Niederhoffer and Osborne (1966) and Roll (1984).

26.86% when estimated using daily equity returns, 27.46% using weekly returns, and 24.39% using monthly returns. Compared with the dramatic pattern of decreasing empirical bond volatility with increasing measurement horizon, this is a strong indication of the relative importance of liquidity in the bond and equity markets. The cross-sectional variation of the empirical equity volatility is reported in the second panel of Table 2.3. As expected, smaller firms are more volatile, and so are more leveraged firms.

Overall, our volatility measures are lower than those reported for U.S. equities, in part due to the fact that firms in our sample are typically larger firms. Another important driver is that our sample period, from July 2002 through 2006, is a relatively low volatility period. We maintain a contemporaneous sample of empirical bond and equity volatilities so as to capture the time-variation in asset volatility and its impact on both the bond and equity volatilities. Nevertheless, our model is set in a constant volatility setting. So a lower than average equity volatility would have a more permanent impact on our estimate of the firm asset volatility than it otherwise would in a stochastic volatility setting. We consider the robustness of our results with respect to this limitation of our model in Section 2.5.2.

2.4 Model-Implied Volatility

2.4.1 Parameter Calibration of the Merton Model

The parameters that govern the dynamics of the risk-free rate are calibrated as follows. First, we use the daily time-series of three-month T-bill rates from 1982 through 2006 to calibrate the long-run mean parameter θ and the rate of mean reversion parameter κ. Specifically, we set $\theta = 5.46\%$, so that the long-run mean equals its time-series average; $\kappa = 0.2443$, so that the daily autocorrelation of the model matches the sample autocorrelation.[10] Second, we set the volatility parameter σ_r so that, for a 7-year Treasury coupon bond, the model-implied volatility of its instantaneous returns matches the sample volatility. More specifically, to parallel our treatment of the empirical bond and equity volatilities, we use daily 7-year Treasury coupon-bond returns to form monthly estimates of Treasury bond volatilities, and weekly returns to form quarterly estimates and monthly returns to form yearly estimates. The Treasury volatility estimates are reported in the third panel of Table 2.2. When averaged across time, the annualized volatility estimates remain stable regardless of the measurement horizons, again, contrasting with the pattern in corporate bonds.

The random fluctuations of the Treasury rate are an important component in corporate

[10] To be more precise, we should match the risk-neutral value of κ since we are using the model for pricing purposes.

bonds. Our choice of the risk-free parameters, particularly σ_r, have a big impact on the model-implied bond volatility. The median maturity of the bonds in our sample is close to 7 years. We choose σ_r to match the volatility of a 7-year Treasury coupon bond so that on average, the Treasury component of the bond volatility is matched to its sample counterpart. While σ_r varies over time in our calibrations, its time-series average is around 2.3%. By contrast, the volatility coefficient σ_r estimated from the time-series of three-month T-bill rates is only at 1.3%, which would severely under-estimate the Treasury component of corporate bond volatility. Implicit in this difference in σ_r is the fact that the simple one-factor model of Vasicek cannot match well the term structure of Treasury bond volatility. Our approach is to force the model to match well near the 7-year maturity, which is close to the median maturity of our bond sample.

Apart from the asset volatility σ_v, which is to be inferred from the model, the firm-level parameters to be calibrated are the payout ratio δ, leverage K/V, and maturity T. For each firm, its leverage K/V is calculated as follows. First, we calculate the book value of the firm's debt, K, to be the sum of long-term debt and debt in current liabilities using Compustat data. Second, we set $V = S + D$ to be the firm value with S equaling the market value of the equity and D equaling the market value of the debt. Given that firms typically issue bonds at par, the market value of the debt should be close to the book value. To improve on this approximation, however, we collect, for each firm, all of its bonds covered by TRACE and calculate an issuance weighted market-to-book ratio. We then approximate the market value of the debt by multiplying K by the market-to-book ratio.

In calibrating the firm-level debt maturity T, we take into account the actual maturity structure of the firm and collect all of the firm's bonds in FISD and calculate the respective durations. We let the firm-level T be the issuance-weighted duration of all the bonds in our sample. Effectively, we acknowledge the fact that firm's maturity structure is more complex than the zero-coupon structure assumed in the Merton model, and our issuance-weighted duration is an attempt to map the collection of coupon bonds to the maturity of a zero-coupon bond. Finally, we calculate the firm's payout ratio δ by adding its annual dividends plus repurchases minus issuances and plus annual coupon payments and scale the total dollar payout by the firm value V. With the exception of leverage K/V, the other firm-level parameters are constant in the model. We do, however, take its time-variation into account and update the firm level parameters at the appropriate frequencies.

2.4.2 Model-Implied Asset Return Volatility $\hat{\sigma}_v^{\text{Merton}}$

For each firm in our sample, we back out its asset volatility $\hat{\sigma}_v^{\text{Merton}}$ using the Merton model via equation (2.8). The risk-free parameters as well as the firm-level model parameters including leverage K/V, payout ratio δ and firm T are calibrated as described in Section 2.4.1. The fourth panel of Table 2.2 summarizes the model-implied asset volatility using the firm-level parameters, the estimated equity volatility $\hat{\sigma}_E$, and the bond volatility σ_r as inputs. Among the inputs, however, the key variable is equity volatility, which remains quite stable across measurement horizons. The model-implied asset volatility inherits this pattern. In addition, it also inherits a relatively low asset volatility from the relatively low equity volatility.

The cross-sectional variation of the model-implied asset volatility is reported in the third panel of Table 2.3. It shows that smaller firms have higher asset volatilities. It is interesting that firms with lower leverage have higher asset volatilities, consistent with the possibility of leverage being an endogenous variable. On the other hand, firms whose bonds are speculative grades have markedly higher volatilities than that of the investment grades. Within investment grade, there is some evidence of increasing asset volatility with decreasing credit rating, although this pattern is not robust. Finally, the relation of asset volatility to equity volatility is monotonically increasing, indicating that the cross-sectional variation in leverage, or more precisely the modified leverage $\mathcal{L}$, does not break the link between the two.

Finally, we should mention one bias in our sample regarding the calculation of model-implied asset volatility. Although it is clear from equation (2.8) that, for each firm with a fixed set of parameters, the equity volatility will be the deciding factor in backing out the asset volatility, the interest rate volatility component does plays a role. For firms with high leverage, its equity volatility should have a component tied to the volatility of the risk-free rate. But for some highly levered firms in our sample, their empirical equity volatility is too low to account for the risk-free interest-rate volatility component. In such cases, an asset volatility cannot be backed out from equation (2.8), and we exclude the firm and their bonds from our sample. The frequency of such incidents is not rare, and happens for about 10% of the firms and 20% of the bonds in our sample. Had we used a zero asset volatility in the model, the bond volatility would be identical to the Treasury bond volatility of the same maturity. In practice, however, these bonds' volatilities are higher than their Treasury counterparts because of default risk. By excluding these bonds from our sample, we effective create an downward bias in the difference between empirical and model-implied bond volatility.

2.4.3 Model-Implied Bond Return Volatility $\hat{\sigma}_D^{\text{Merton}}$

For each bond in our sample, we calculate its model-implied volatility $\hat{\sigma}_D^{\text{Merton}}$ using the Merton model via equation (2.10). The risk-free parameters as well as the firm-level model parameters are the same as before, except that we are taking the model-implied asset volatility $\hat{\sigma}_v^{\text{Merton}}$ as a key put. Moreover, we no longer need the firm maturity T. Instead, the respective bond maturity is used in calculating the return volatility for coupon bonds and the loss given default is set at 50%.

The last panel of Table 2.2 summarizes the model-implied bond volatility, $\hat{\sigma}_D^{\text{Merton}}$. It is interesting to note that while the sample mean of $\hat{\sigma}_D$ is 18.06%, 9.62%, and 7.18% when estimated using daily, weekly, and month bond returns, the sample mean of $\hat{\sigma}_D^{\text{Merton}}$ is 5.42%, 5.14%, and 5.35%, respectively. Of course, this lack of variation across horizons is not surprising given that a key input in estimating $\hat{\sigma}_D^{\text{Merton}}$ is the equity return volatility $\hat{\sigma}_E$, which is relatively stable when various horizon returns are used. It does, however, reflect an interesting disconnect between the bond and equity market. We will compare $\hat{\sigma}_D$ and $\hat{\sigma}_D^{\text{Merton}}$ more closely in the next section.

The cross-sectional variation of $\hat{\sigma}_D^{\text{Merton}}$ is summarized in the last panel of Table 2.3. Quite intuitively, bonds with higher firm leverage, longer maturity, and lower rating are more volatile. In fact, among all the variables used in the sorting procedure, bond maturity is among the most effective variables in generating a spread in $\hat{\sigma}_D^{\text{Merton}}$. In other words, the duration risk is an important component in the cross-sectional determinants of $\hat{\sigma}_D^{\text{Merton}}$. The other most effective variable is the bond volatility $\hat{\sigma}_D$ estimated directly from the data. The fact that $\hat{\sigma}_D$ and $\hat{\sigma}_D^{\text{Merton}}$ line up in the expected direction is encouraging for our model. In addition, the fact that the spread is even wider when sorted by the empirical bond volatility $\hat{\sigma}_D^{\text{Merton}}$ estimated using monthly bond returns is even more telling. It indicates that the model-implied bond volatility, which includes no information about the potential liquidity problems in corporate bonds, lines up better cross-sectionally with the empirical bond volatilities that are estimated using longer horizon returns and are less subject to liquidity contaminations. Sorting by equity volatility $\hat{\sigma}_E$ generates a cross-sectional variation in $\hat{\sigma}_D^{\text{Merton}}$, although the effect is somewhat muted.[11] Finally, the relation of $\hat{\sigma}_D^{\text{Merton}}$ to the size of the bond, however, is not as clear as it is the case for $\hat{\sigma}_D$.

[11]It should be noted that $\hat{\sigma}_D^{\text{Merton}}$ is not necessarily monotonic in $\hat{\sigma}_E$. For example, while the first term equation (2.10), which measures the defaultable bond's exposure to firm risk, is clearly increasing in σ_v, the second term, however, is decreasing in σ_v. For long duration bonds, the second term, which captures the risk-free interest rate exposure, could outweigh the first.

2.5 Bond Volatility: Empirical vs. Model

2.5.1 Excess Volatility

Table 2.4 summarizes the main result of this chapter. Specifically, there is a strong discrepancy between the bond volatility $\hat{\sigma}_D$ estimated directly using bond return data and the bond volatility $\hat{\sigma}_D^{\text{Merton}}$ implied by the Merton model. Using daily returns to construct the volatility estimates, the sample means of $\hat{\sigma}_D$ and $\hat{\sigma}_D^{\text{Merton}}$ are 18.06% and 5.42%, respectively. The full sample mean of $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$ is 12.64% with a robust t-stat of 34.55 (clustered by month and bond). The economic magnitude of such a discrepancy is quite large, and it indicates a volatility component in corporate bonds that is disconnected from the equity volatility of the same issuer and the interest rate volatility in Treasury bonds. Figure 2-2 paints a very similar picture by reporting the cross-sectional distribution the time-series averages of $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$, bond by bond. In fact, 618 out of the 623 bonds in our sample have a $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$ that is positive with t-stat greater than 1.96.

To better understand this large excess volatility component, we examine $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$ across various measurement horizons.[12] When the volatility estimates are constructed using weekly returns, the sample mean of $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$ shrinks to 4.47% with a robust t-stat of 15.52. Moving to the monthly horizon, the difference is further reduced to 1.83% with a robust t-stat of 2.29. Putting aside the fact that even at the monthly level the discrepancy is still significant statistically and large economically, the dramatic reduction in $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$ across measurement horizons indicates that the disconnect is most severe at shorter horizons. Indeed, this horizon result is driven almost entirely by the short-term behavior of corporate bond returns. Specifically, the sample means of the empirical bond volatility $\hat{\sigma}_D$ are 18.06%, 9.62%, 7.18%, respectively, when measured using daily, weekly, and monthly bond returns. By contrast, the empirical equity volatility $\hat{\sigma}_E$ remains stable across the different measurement horizons. So does the model-implied bond volatility. Implicit in this unique horizon result is a high degree of negative auto-covariances in short-horizon bond returns, accentuating a severe liquidity component in the corporate bond market.

To exclude the possibility that our results are driven by a few bonds with extreme values of $\hat{\sigma}_D$, we examine the time-series average of the cross-sectional medians of the volatility estimates. Specifically, the medians of $\hat{\sigma}_D$ are 15.78%, 8.48%, and 6.36% for daily, weekly, and monthly measurement horizons respectively; the medians of $\hat{\sigma}_D^{\text{Merton}}$ are 6.30%, 6.16%, and 6.18% respectively; and the medians of $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$ are 9.85%, 2.63%, and 0.47%,

[12]It should be mentioned that the model-implied volatility $\hat{\sigma}_D^{\text{Merton}}$ is derived for instantaneous returns. As such, when we move on to calculate volatility of bond returns over monthly horizons, the approximation error would increase.

respectively. The patterns are similar to those reported for the sample mean results, with the exception that the median discrepancy in volatility estimates is only 47 basis points when measured with monthly returns. In other words, the excess volatility in corporate bond is most severe when measured with short-horizon returns, and then tapers off near monthly returns.

As shown in Table 2.4, at the daily and weekly measurement horizons, the pattern of excess volatility is quite robust, whether the sample is split by bond type, by year, by credit rating, or by bond duration. While our full sample includes only straight bonds and callable bonds, we also report the results for convertible and putable bonds separately. It is expected that, except for straight bonds, the model-implied bond volatility will be off in capturing the real bond volatility. In particular, the model would under-estimate the volatility for a convertible bond. Indeed, we find a much higher level of excess volatility for convertible bonds. The fact that the putable bonds also have higher excess volatility, however, is puzzling, although the sample is quite small. Given that the callability feature is more tied to the random fluctuation of interest rates and effectively shortens the duration of a callable bond, one would expect the model to over-estimate the volatility in a callable bond, and therefore generate a lower degree of excess volatility. Table 2.4, however, shows that the excess volatility is slightly higher for callable bonds than for straight bonds. This comparison, however, fails to factor in bond-level characteristics such as maturity and rating, which could be important in driving the excess volatility results. Indeed, the average maturity of the callable bonds in our sample is about 3.5 years longer than for the straight bonds. Their credit ratings are also on average one or two notches below the straight bonds.

It is not surprising that the model performs the best at the monthly return horizon. Specifically, the model is able to generate an average volatility of 5.35% that is relatively close to the empirically observed bond volatility of 7.18%. This excess volatility of 1.83%, however, is still statistically significant with a robust t-stat of 2.29, and, perhaps more importantly, still accounts for a quarter of the observed empirical bond volatility. The subsample results at the monthly measurement horizon, however, are not as robust as those at the shorter horizons. For example, at the monthly measurement horizon, excess volatility is not significant for A and above rated bonds, but remains to be important for bonds rated Baa and below. The U-shaped pattern of excess volatility by bond duration is also interesting. At the monthly measurement horizon, the average empirical volatility is 3.14% for bonds with duration less than 2 years, and the corresponding excess volatility is 0.91%, which is close to a third of the observed empirical volatility and is statistically significant. For median duration bonds, however, excess volatility becomes less significant. But as we move to the category of bonds with the longest duration, excess volatility becomes significant again. The average empirical volatility for bonds with duration longer than 8 years is 11.19%, and the corresponding excess volatility is 4.98%, which is about 45% of the observed empirical volatility.

Overall, our results indicate that while at the daily and weekly measurement horizons, the Merton model cannot even begin to generate the kind of bond volatility observed in the data due to the liquidity problems in corporate bonds, the model does a much better job at the monthly horizon. Whether or not the remaining amount of excess volatility is due to liquidity or model mis-specification remains an interesting question. We pay close attention to this aspect of our result in the next few sections.

2.5.2 Further Considerations

Stochastic Interest Rate

The simple term-structure model employed in this chapter is an issue of concern. A proper account of the risk-free volatility is important because small fluctuations in the risk-free interest rate will be magnified by the duration of the bond to a sizeable volatility. Because of this, we calibrate the volatility coefficient σ_r in the risk-free rate model so that the model generates the empirically observed level of volatility for a 7-year Treasury coupon bond. Effectively, we force the model to match well near the 7-year maturity, which is close to the median maturity of our bond sample. This, however, still does not fully capture the entire term-structure of interest rate volatility.

To account for this, we work with excess bond returns to avoid relying on a term structure model. We calculate excess bond returns by subtracting contemporaneous Treasury bond returns of a similar maturity from the corporate bond returns.[13] Comparing the volatility measured by bond excess returns to the model-implied excess bond volatility, we find that the results are similar to our main result. For the daily, weekly, and monthly measurement horizons, the sample means of $\hat{\sigma}_D - \hat{\sigma}_D^{\mathrm{Merton}}$ are respectively 17.21%, 8.02%, and 4.76% with robust t-stats of 42.98, 31.37, and 18.85; the time-series averages of the cross-sectional medians are 14.54%, 6.39%, and 3.79%.

Overall, the excess bond volatility puzzle is somewhat deepened here with the adoption of the model-free approach. The main reason is that in working with the Merton model with stochastic interest rates, we inherit the model-implied correlation between the corporate bond and Treasury bond. In the model-free approach, the empirical correlation is used. Implicit in the current result, the empirical link is weaker than that prescribed by the model, consequently making the excess volatility puzzle even larger in magnitude.

[13] The return horizons are matched at daily, weekly, and monthly, respectively. We use 1-, 2-, 5-, 7-, 10-, 20-, and 30-year Treasury returns as the basis of our extrapolation to get the target maturity.

Bid-Ask Bounce

Given the importance of liquidity as a potential explanation of our results, we further consider correcting the bond volatility measure by factoring in the effect of bid-ask bounce. Following Roll (1984) and assuming that transactions at bid and ask are equally likely, we can map an observed bid-ask spread to its impact on return volatility. For all of the bonds in our sample, we collect monthly bid-ask spreads from Bloomberg Terminals, and calculate the associated "bid-ask bounce" contribution to the bond return volatility.[14]

The last panel in Table 2.4 reports $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$, where $\hat{\sigma}_D$ is the square-root of the difference between the empirical bond variance minus the variance generated by the bid-ask bounce. While decreasing somewhat from the previous results, the magnitude of the discrepancy remains similar to our main result. In other words, the excess bond volatility documented here cannot be explained by bid-ask bounce alone.

Firms with Missing Asset Volatility

We use the Merton model to back out asset volatility from equity and Treasury volatilities via equation (2.8). For most firms, the key input of this calculation is equity volatility, with interest rate volatility relegated to playing only a minor role. It is for this reason, many of the existing studies do not include Treasury volatility in the calculation. For firms with higher than usual leverages, however, this Treasury component becomes too large to be ignored. If the high leverage is further coupled with a higher than usual payout ratio δ, it would result in a high level of modified leverage $\mathcal{L}$. And from equation (2.8), we see that for such a firm, its equity volatility would collect a component amplified by $\mathcal{L}/(1-\mathcal{L})$ from the Treasury volatility. If in practice, the equity volatility for such a firm is not large enough to account for this component alone, then we run into the problem of not being able to back out asset volatility from equation (2.8).

Indeed, for 12% of the firm-years in our sample based on monthly returns, we run into this problem of missing asset volatility. This pool of firms has an average leverage of 70%, twice the sample average of 35%. They have an average payout ratio of 11.55%, almost one and a half deviations from the sample average of 4.92%. On the other hand, their firm-T is on average 5.3 years, not that different from and slightly lower than the sample average. Their equity volatility is on average 32.15%, which is indeed higher than the sample average

[14] Bloomberg typically provides bid-ask quotes from various dealers and we use the Bloomberg Generic (BGN) Quote, which reflects consensus market quotes. BGN quotes are available for a larger number of bonds in our subsample and typically have a longer time series than quotes by other dealers. It should also be noted that our adjustment is one-sided, since the bid-ask spread in the equity market might also have an impact as it finds its way to σ_D^{Merton}.

of 25.68%, but not high enough to account for their interest rate exposure.

This problem of missing asset volatility affects 20% of the bonds in our sample that is summarized in Table 2.1. Using monthly bond returns, we construct empirical bond volatility for this sample of bonds and find an average bond volatility of 13.81% and a median volatility of 10.44%, which are markedly higher than the average of 8.96% and median of 6.74% for the sample of bonds without the missing asset volatility problem. The exclusion of such firms and their bonds effectively creates a downward bias in our main result. In other words, the magnitude of excess volatility reported in our main result would have been larger had we included such bonds in our analysis.

Conditional vs. Unconditional Asset Volatility

One limitation of our modeling approach is the tension between conditional versus unconditional volatility. Using conditional volatility sharpens the contemporaneous connection among the equity, Treasury, and corporate bond volatilities, capturing the link both across firms and across time. Indeed, our approach leans heavily toward this conditional approach by constructing monthly estimates of volatility using daily returns, and yearly estimates using monthly returns. The limitation, however, is that when the Merton model is used to calculate the sensitivity coefficients in equations (2.7) and (2.10), the conditional rather than unconditional volatility is plugged into the model. This has the wrong implication that if the current volatility is low, it will stay low for the entire life of the firm or the entire maturity of the bond. Given that our sample of July 2002 through December 2006 falls under a low equity volatility period, this tension could be important. The best resolution of this tension is to have a stochastic volatility model. Given that it will dramatically increase the complexity of the problem without the benefit of additional insights, we examine the robustness of our main result by using the following "hybrid" approach.

We first obtain unconditional estimates of equity and Treasury bond volatilities using monthly equity and Treasury bond returns going as far back into history as possible. We then plug in the unconditional volatilities to equation (2.7) to obtain an unconditional version of the asset volatility. Averaged across all firms, this unconditional asset volatility is 27.84% with a cross-sectional median of 24.18%, which are indeed higher than the the model-implied asset volatilities reported for our sample period. Armed with the unconditional asset volatility, we can now fix the problem with respect to the model-implied sensitivity coefficients in equations (2.7) and (2.10). Given that the horizon of the firm is typically long (firm-T is on average 6 years), and the rate of mean reversion of equity volatility is relatively fast, calculating the sensitivity coefficients using an unconditional approach seems reasonable. Applying this hybrid approach, we find only minor effects on our main result. For the monthly measurement horizon, the excess volatility measure $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$ is on average 1.36% with a t-stat of 3.96. In other words, excess volatility remains significant both statistically and economically even at the monthly horizon.

2.6 Cross-Sectional Determinants of Excess Volatility

To shed light on the discrepancy measure $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$, we examine its cross-sectional determinants. Using the daily measurement horizon, we have monthly time-series of $\hat{\sigma}_D$ and $\hat{\sigma}_D^{\text{Merton}}$ for cross-sections of bonds. Table 2.5 reports the Fama and MacBeth (1973) cross-sectional regressions at monthly frequency with $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$ as the dependent variable.

We find that, after controlling for other bond characteristics such as maturity and rating, excess volatility is more severe in smaller bonds. Given that smaller bonds are typically less liquid, this could potentially be a liquidity explanation. Similarly, the result on age is also quite interesting. In terms of the level of bond volatility, there is no theoretical link between the age of a bond and its volatility, keeping bond maturity fixed. By contrast, we find that $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$ is higher for those bonds that are older. Specifically, controlling for other bond characteristics including maturity, rating and size, a bond that is one-year older has an additional excess volatility of 58 basis points, and the t-stat is 4.55. As newly issued bonds are typically more liquid, while the old bonds are more likely to have a larger fraction held by buy-and-hold investors, our result is consistent with a liquidity explanation.

Table 2.5 shows that both the bond maturity and rating play an important role in explaining $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$. More specifically, longer maturity bonds and lower rated bonds exhibit more excess volatility. This effect, however, could be confounded with the fact that longer maturity bonds and lower rated bonds are more volatile in general. The liquidity connections in terms of rating and maturity are not apparent, although it is probably true that investment grades are more liquid than speculative grades, and certain maturity cohorts are more liquid. Given the importance of these two variables in bond volatility, we add them to serve more as controls.

The results using the firm-level variables are mixed. After controlling for rating, bonds that are issued by firms with higher leverage have higher excess volatility. Specifically, a 10% increase in a firm's leverage increases the excess volatility by 41 basis points. The firm's equity volatility is found to be positively related to the cross-sectional variation of $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$, but the result is not statistically significant. Given the inherent connection between the corporate bonds and equity of the same firm, one would expect a positive link between the empirical bond volatility $\hat{\sigma}_D$ and the equity volatility $\hat{\sigma}_E$. Indeed, Table 2.3 shows that the cross-sectional variation in empirical bond volatility is closely connected to the cross-sectional variation in equity volatility. The fact that the equity volatility can no longer explain, with statistical significance, the cross-sectional variation in $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$ indicates that the model-implied bond volatility $\hat{\sigma}_D^{\text{Merton}}$ is doing a good job in capturing the cross-sectional variation in $\hat{\sigma}_D$ that was previously proxied by $\hat{\sigma}_E$. And whatever is left unexplained is unlikely to be related to the issuer's equity volatility.

The trading related variables also show some interesting results. We find more excess volatility in corporate bonds whose average trade size is small. Controlling for other bond characteristics including bond size, maturity, rating, and age, a bond with an average trade size of \$100,000 face value would have an additional excess volatility of $2.26\ln(10)=5.20\%$ than a bond with an average trade size of \$1,000,000 face value.[15] One could argue that, after controlling for bond size, the bonds with smaller average trade size are more likely to be traded in less liquid bond trading platforms and therefore have a larger liquidity component. It should be mentioned that the monthly time-series variation of the variables could also contribute to the result. That is, for each bond, the month during which the average trade size is small is a less liquid month, therefore resulting in a higher $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$ during that month relative to other bonds. The results from the other trading variables are somewhat mixed. For example, bonds with higher trading volume yield lower excess volatility. The turnover result points in the same direction, but is not statistically significant. On the other hand, bonds with a larger number of trades yield higher excess volatility and is statistically significant. While trading volume and number of trades are positively related, their implications for liquidity might be different, and this observation is consistent with our result for average trade size.[16] Similarly, we find that bonds with higher percentage days of trading exhibit more excess bond volatility.[17]

Finally, using the monthly measurement horizon, we have yearly time-series of the empirical and model-implied bond volatilities, and the Fama-MacBeth cross-sectional regression could be done at yearly frequency. It is plausible that the liquidity problem is less prominent at this measurement horizon. Indeed, we find that the size of the bond is no longer important in explaining the excess volatility. The age of a bond remains important: older bonds have more excess volatility, but the importance diminishes when the average trade size of a bond is used. Overall, the average trade size of a bond remains to be important: bonds trading in smaller average sizes have higher excess volatility. The leverage of the firm, however, is no

[15]It should be mentioned that TRACE truncates the trade size at \$1 million for speculative grade bonds and at \$5 million for investment grade bonds.

[16]A bond with high number of trades from investors trading with small average trade size is different from a bond with low number of trades from investors trading large average trade size.

[17]Overall, it should be mentioned that our bond sample selection biases toward more liquid bonds. So to the extent that we would like to interpret some of the trading variables as proxying for bond liquidity, we are working within the domain of relatively liquid bonds.

longer important in explaining the cross-sectional variation in excess volatility.

2.7 Time-Series Determinants of Bond Returns

In this section, we focus on the random shocks that give rise to the excess volatility puzzle documented in this chapter. In particular, we are interested in knowing if these random shocks exist only at the individual bond level, and whether or not they aggregate to a systematic component.

For this, we apply the method of Campbell, Lettau, Malkiel, and Xu (2001) to decompose the volatility associated with the random shocks into systematic and idiosyncratic components. The results are summarized in Figure 2-1. Given that uneven panels might introduce time variations in the relative magnitude of idiosyncratic and systematic volatility, we perform our analysis for the sample period after April 14, 2003, when Phase II of TRACE was introduced and the coverage was broader. Moreover, we exclude bonds that entered only after Phase III. The stock sample mirrors this bond sample. The bond residuals are the regression residuals from[18]

$$R_t^D = \alpha + \beta^E R_t^E + \beta^T R_t^T + \epsilon_t\,. \tag{2.11}$$

Intuitively, this equation captures the variation of bond returns after taking out the exposure to the equity of the same firm and the exposure to a Treasury bond of a similar maturity.

As shown in the top panel of Figure 2-1, the systematic volatility of corporate bonds is on average 3.82%, which is low compared with the equity market. The bond residuals have an even lower systematic volatility at 2.11%. According to the model, however, this systematic volatility should be zero. In practice, however, over 50% of the systematic volatility in the corporate bond market actually arises from this systematic component of the residuals that should have been zero according to the model. Moreover, as shown in Figure 2-1, the systematic volatility of bonds moves quite closely with the systematic volatility of bond residuals, with a correlation of 85%. Interestingly, they also co-move with the CBOE VIX index: the correlation is 71.62% with bonds and 76.21% with bond residuals.

The bottom panel of Figure 2-1 plots the idiosyncratic volatility of stock, bond, and bond residual. Unsurprisingly, the idiosyncratic volatility of the residuals is disproportionally large. The average idiosyncratic volatility is 18.52% for equities, 20.11% for bonds, and 17.02% for bond residuals. Given that the equity market in aggregate is several times more volatile than the aggregate bond market, the similar magnitudes of their idiosyncratic

[18] For each bond in our sample, this regression is run every month (at least 10 daily observations are required).

volatility indicate a disproportionally large idiosyncratic component in corporate bond returns. Moreover, the fact that the idiosyncratic volatility of the bond residual remains high, at a level of 17.02%, indicates that this large idiosyncratic component cannot be explained by our model. Interestingly, this idiosyncratic volatility has a correlation of 80% with the systematic volatility of the residuals and a correlation of 83.56% with the CBOE VIX index.

2.8 Conclusions

In recent years, we have seen increasing research activities on the empirical performance of structural models of default. Much attention has been focused on the model's ability or inability to match credit spreads. In addition, our knowledge of the empirical performance of structural models, while intriguing and informative, is formed in large part by calibrations at the level of credit ratings or by applying the model to only a handful of bond observations. With the availability of the high frequency data from TRACE, which offers better data quality for a broad cross-section of corporate bonds, it is perhaps an opportune time to examine the structural models of default more closely.

This chapter's contribution is to provide an alternative angle from which the empirical performance of the structural models can be evaluated. In addition, by taking advantage of the high frequency data from TRACE, we can evaluate the empirical performance of the Merton model from varying measurement horizons. Applying the model at the firm level for a relatively broad cross-section of bonds, we are also able to form a better sense of how the Merton model actually performs at the individual firm and bond level and examine the cross-sectional and time-series determinants of the discrepancies between model and data.

For a broad cross-section of corporate bonds that extend from July 2002 through December 2006, we find an overwhelming amount of excess volatility in corporate bonds that cannot be explained by the Merton model of default. In fact, perhaps no structural model of default can explain our results at the daily and weekly measurement horizons: the magnitudes of the discrepancy are too large and the patterns too unique to be contributed by default risk. At these horizons, the issue of liquidity is unambiguously important. Moreover, we find that variables known to be linked to bond liquidity are important in explaining the cross-sectional variations in excess volatility, providing further evidence of a liquidity problem in corporate bonds.

At the monthly measurement horizon, excess volatility becomes less severe, although on average it still accounts for a quarter of the observed empirical bond volatility. At this horizon, it becomes interesting to question whether or not the documented excess volatility is due to liquidity or model mis-specification. Our additional analyses show that it cannot be attributed to the lack of a sophisticated term-structure model or to the bid-ask bounce in the quoted bid-ask spread for corporate bonds. Finally, even at the monthly measure horizon, we still find interesting connections between variables known to be linked to bond volatility and the cross-sectional variations in excess volatility,

Overall, the main result of this chapter is the pattern of excess volatility in corporate bonds and its connection to the liquidity of the corporate bond market. In the following chapter, we link this excess volatility more closely to the micro-structure of corporate bonds so as to understand the economic driver of the illiquidity in corporate bonds.

2.9 Tables and Figures

Table 2.1: Bond Sample Summary Statistics

	2002			2003			2004			2005			2006		
Our Sample	mean	med	std	mean	med	std	mean	med	std	mean	med	std	mean	med	std
#Bonds	184			341			467			773			784		
Maturity	8.51	7.05	7.97	8.34	6.49	7.67	8.26	6.19	7.51	8.67	6.48	7.61	8.55	6.12	7.64
Amt	1,686	1,368	976	1,233	1,082	944	1,033	895	903	782	521	775	799	579	746
Rating	6.58	7.00	2.88	6.31	6.92	2.82	6.89	6.96	3.14	8.79	8.75	4.13	9.27	9.00	4.31
Age	2.02	1.58	1.69	2.74	2.07	2.31	3.28	2.59	2.53	3.82	3.13	2.92	4.04	3.56	3.03
#Trades	534	241	941	326	171	582	213	131	285	208	116	345	150	101	151
Volume	245	169	255	172	101	257	113	57	227	75	36	145	58	33	82
Turnover	13.69	11.12	9.50	11.79	9.02	9.77	9.14	6.68	8.34	8.44	5.73	8.10	7.31	4.81	7.42
%Traded	96.12	98.44	6.17	95.18	98.82	7.02	93.68	98.43	8.00	92.93	96.48	8.34	91.79	95.31	8.84
Trd Size	781	610	606	651	484	571	541	392	519	418	279	462	450	269	521
#Firms	60			92			129			227			236		
Mkt Cap	35.90	17.61	52.78	36.56	19.05	50.76	34.67	20.33	51.65	25.23	10.46	46.65	26.96	11.50	48.22
US Corporates in FISD															
#Bonds	21,465			22,305			24.203			26,590			28,710		
Maturity	7.11	4.33	8.97	7.24	4.43	8.88	7.43	4.55	8.74	7.45	4.65	8.61	7.23	4.39	8.57
Amt	172	68	328	174	57	336	171	48	337	164	31	335	164	25	339
Rating	7.49	6.00	4.26	7.41	6.58	4.25	7.02	6.17	4.10	6.77	6.00	4.03	6.51	5.58	4.11
Age	4.47	3.84	3.82	4.15	3.11	3.86	3.76	2.18	3.89	3.56	2.04	3.85	3.55	2.29	3.79

#Bonds and *#Firm* are the average numbers of bonds and firms per month. *Maturity* is the bond's time to maturity in years. *Amt* is the bond's amount outstanding in millions of dollars. *Rating* is a numerical translation of Moody's rating: 1=Aaa and 21=C. *Age* is the time since issuance in years. *#Trades* is the bond's total number of trades in a month. *Volume* is the bond's total trading volume in a month in millions of dollars of face value. *Turnover* is the bond's monthly trading volume as a percentage of the amount outstanding. *%Traded* is the percentage of business days in a month when the bond is traded. *Trd Size* is the average trade size of the bond in thousands of dollars of face value. *Mkt Cap* is the equity market capitalization in billions of dollars. The reported std and median are the time-series averages of cross-sectional values.

Table 2.2: Volatility Estimates

	Daily Returns			Weekly Returns			Monthly Returns		
	mean	med	sd	mean	med	sd	mean	med	sd
	Empirical Bond Volatility $\hat{\sigma}_D$								
2003	21.14	18.32	13.11	11.45	10.03	7.09	8.79	8.42	4.50
2004	17.37	14.04	12.30	8.82	7.50	6.26	6.21	6.07	3.14
2005	17.50	13.88	13.56	9.30	7.40	8.10	6.50	5.28	4.85
2006	16.06	13.31	10.99	8.63	6.99	8.99	7.50	5.67	5.85
Full	18.06	15.78	13.51	9.62	8.48	8.43	7.18	6.36	4.58
	Empirical Equity Volatility $\hat{\sigma}_E$								
2003	27.17	25.37	11.54	26.07	25.09	11.27	25.46	23.27	10.73
2004	22.05	18.63	12.40	23.18	19.57	13.27	18.73	16.43	8.20
2005	26.09	21.09	17.36	26.79	21.92	17.88	26.51	21.91	20.30
2006	26.21	21.51	17.70	27.06	21.58	20.01	24.87	19.61	18.50
Full	26.86	24.50	16.06	27.46	25.56	16.74	24.39	20.31	14.43
	Empirical 7-Year Treasury Bond Volatility								
2003	6.83	7.11	1.15	6.98	7.06	1.20	7.96	NA	NA
2004	5.77	5.61	1.16	5.36	5.33	1.73	6.15	NA	NA
2005	4.65	4.81	0.62	4.18	4.01	0.72	5.00	NA	NA
2006	3.93	4.12	0.54	3.43	3.41	0.50	3.35	NA	NA
Full	5.51	5.20	1.55	5.30	5.19	1.93	5.62	5.58	1.94
	Model-Implied Asset Volatility $\hat{\sigma}_V^{\text{Merton}}$								
2003	18.19	17.47	9.99	17.35	16.54	10.06	17.30	16.46	8.28
2004	15.14	13.36	10.58	15.92	13.32	11.16	13.32	12.54	6.29
2005	16.78	13.50	13.12	17.24	13.93	13.15	17.24	14.26	16.69
2006	16.98	13.98	13.50	17.42	14.21	14.66	16.41	13.34	13.00
Full	17.81	16.52	13.53	18.15	16.92	13.85	16.26	14.15	11.07
	Model-Implied Bond Volatility $\hat{\sigma}_D^{\text{Merton}}$								
2003	7.42	8.00	2.51	7.31	8.03	2.50	8.10	9.08	2.54
2004	5.79	6.38	2.16	5.41	5.88	2.16	5.97	6.68	2.15
2005	4.85	5.00	2.86	4.42	4.48	2.62	5.00	5.27	2.73
2006	4.03	4.08	2.37	3.76	3.63	2.69	4.06	3.70	2.82
Full	5.42	6.30	2.72	5.14	6.16	2.87	5.35	6.18	2.56

All volatility estimates are annualized and expressed in percentages. The empirical bond and equity return volatilities are constructed using daily, weekly, and monthly bond and equity returns, respectively. The model-implied asset and bond return volatilities are backed out from equations (2.8) and (2.10), respectively, using the equity return volatility $\hat{\sigma}_E$ as inputs. The reported med and std are the time-series averages of cross-sectional medians and standard deviations. For empirical Treasury bond volatility, the reported numbers are time-series medians and standard deviations. The full sample includes data from July 2002 through 2006.

Table 2.3: Volatility Estimates by Firm or Bond Characteristics

	Daily Returns				Weekly Returns				Monthly Returns			
	low	Q2	Q3	high	low	Q2	Q3	high	low	Q2	Q3	high
Empirical Bond Volatility $\hat{\sigma}_D$												
Bond Amt	36.58	21.60	18.10	17.79	20.70	11.90	9.80	9.88	12.77	7.99	7.34	7.30
Bond Maturity	13.83	19.04	24.96	36.15	7.09	10.32	14.33	20.50	4.82	7.12	9.78	14.27
Rating	15.75	17.02	24.67	33.85	7.82	8.82	13.65	19.76	5.32	6.26	7.79	14.63
Equity Volatility	16.85	17.43	23.81	35.93	8.73	9.07	16.57	18.66	5.78	6.32	8.61	15.29
Firm Leverage	17.47	20.16	19.76	37.03	9.52	10.40	11.45	21.87	6.61	7.14	7.46	15.05
Firm Payout	20.05	17.84	29.53	25.59	10.64	9.76	17.66	13.56	7.40	7.22	11.83	8.91
Bond Turnover	25.84	21.45	20.40	25.83	14.16	11.53	10.95	15.31	9.76	7.17	7.53	11.43
%Days Traded	23.61	24.92	23.81	20.71	13.24	13.68	13.16	11.44	10.93	8.23	8.36	8.24
Empirical Equity Volatility $\hat{\sigma}_E$												
Equity Mkt Cap	36.58	24.27	23.05	21.33	40.37	25.23	23.32	21.35	38.59	24.37	20.68	18.81
Firm Leverage	23.80	24.33	23.89	39.54	23.90	24.43	24.78	41.04	20.19	23.86	21.06	37.81
Model-Implied Asset Volatility $\hat{\sigma}_V^{\text{Merton}}$												
Equity Mkt Cap	21.21	17.96	17.52	18.43	22.66	18.87	17.21	18.07	19.92	17.27	15.90	15.55
Firm Leverage	21.67	19.19	15.78	17.48	21.84	19.68	15.76	18.37	19.02	17.91	15.41	16.33
Equity Volatility	11.04	15.92	19.67	28.70	10.72	15.38	20.50	31.65	8.82	13.51	17.80	28.73
Rating	15.66	17.19	17.61	21.67	15.68	16.89	17.91	22.43	14.25	14.87	14.43	22.74
Model-Implied Bond Volatility $\hat{\sigma}_D^{\text{Merton}}$												
Bond Amt	5.91	5.22	5.38	5.53	5.74	4.81	5.05	5.28	6.08	5.34	4.83	5.36
Firm Leverage	5.03	5.59	5.26	6.64	4.60	5.05	5.06	6.42	4.85	5.15	5.33	6.34
Bond Maturity	3.19	5.50	6.57	6.88	3.00	5.15	6.30	6.50	2.76	5.34	6.53	7.04
Equity Volatility	4.92	5.17	5.58	6.57	4.61	4.70	5.01	6.99	4.90	4.92	5.03	6.84
Firm Payout	5.64	5.24	5.59	5.53	5.18	5.11	5.28	5.26	5.49	5.03	6.10	5.00
Bond Volatility $\hat{\sigma}_D$	3.68	5.34	6.13	7.20	3.41	4.86	5.82	7.04	2.96	5.04	6.05	7.61
Rating	5.08	5.30	5.76	6.00	4.74	4.96	5.42	5.81	4.99	5.26	5.34	6.08

All volatility estimates are annualized and expressed in percentages. The empirical bond and equity return volatilities are constructed using daily, weekly, and monthly bond and equity returns, respectively. The model-implied asset and bond return volatilities are backed out from equations (2.8) and (2.10), respectively, using equity return volatility $\hat{\sigma}_E$ as inputs. The sample is sorted by the respective variables from low to high into quartiles: Q1 (low), Q2, Q3, and Q4 (high). For bond ratings: Q1=Aaa&Aa, Q2=A, Q3=Baa and Q4=Junk.

Table 2.4: Data Estimated vs. Model Implied Bond Volatility

$\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$

	Daily Returns			Weekly Returns			Monthly Returns		
	#obs	mean	t-stat	#obs	mean	t-stat	#obs	mean	t-stat
Full Sample*	20,486	12.64	34.55	7,057	4.47	15.52	1,692	1.83	2.29
Straight	7,067	11.60	21.46	2,428	3.79	11.81	577	1.26	2.14
Callable Only	13,419	13.19	29.46	4,629	4.83	13.12	1,115	2.13	2.43
Convertible	1,848	27.36	26.45	624	13.84	13.31	133	8.59	7.97
Putable Not Conv.	117	23.78	6.02	41	12.76	3.64	10	4.60	3.50
By Year									
2003	3,199	13.73	23.81	1,034	4.14	8.09	277	0.69	*3.38*
2004	4,147	11.58	23.59	1,441	3.41	6.64	319	0.23	*1.86*
2005	5,957	12.66	26.25	2,022	4.88	9.08	482	1.50	*9.98*
2006	6,340	12.04	31.88	2,274	4.86	12.09	614	3.45	*19.02*
By Rating									
Aaa	2,090	10.74	12.05	685	3.24	7.37	154	0.44	1.04
Aa	2,258	10.23	12.75	751	2.90	7.86	166	0.21	0.48
A	7,088	10.41	19.82	2,387	3.13	10.42	571	0.59	1.51
Baa	5,274	13.66	19.78	1,828	4.98	10.55	419	1.86	3.50
Ba	1,665	17.39	15.38	649	7.79	8.47	202	6.10	5.28
B	1,230	17.10	17.28	444	6.84	8.80	96	3.16	5.66
By Duration									
≤ 2	3,402	7.41	18.99	1,208	2.46	9.10	317	0.91	2.55
2 - 4	4,442	9.01	22.73	1,554	2.43	8.24	366	0.34	0.62
4 - 6	5,126	11.43	28.33	1,753	3.93	8.60	442	1.22	1.59
6 - 8	3,917	14.50	27.52	1,312	5.23	12.82	267	2.95	1.94
> 8	3,150	22.86	21.66	1,090	9.59	19.04	270	4.98	4.47
Using Excess (Bond - Treasury) Returns									
	19,079	17.21	42.98	6,665	8.02	31.37	1,668	4.76	18.85
Factor in Bid/Ask Spreads									
	17,734	11.64	30.75	6,121	3.97	14.69	1,487	1.74	2.12

The full sample includes straight and callable bonds, excluding convertibles and putables. Convertibles and putables are excluded from all tests except those reported under Covertible, and Putable Not Covertible. $\hat{\sigma}_D$ is estimated using daily, weekly, and monthly bond returns, respectively. σ_D^{Merton} is the model-implied bond volatility. #obs is bond month for daily, bond quarter for weekly, and bond year for monthly. The t-stat's are calculated using robust standard errors, clustered by time and by bond, with the by-year results at the monthly horizon being the only exception.

Table 2.5: Cross-sectional Determinants of $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$

Constant	16.99	14.85	12.01	3.16	23.88	23.65	19.88	8.22
	[7.13]	[7.27]	[5.79]	[1.80]	[6.82]	[10.61]	[8.58]	[2.80]
ln(Amt)	-2.48	-1.53	-3.38	-1.83	-3.54	-1.08	-1.83	-2.81
	[-7.31]	[-4.98]	[-9.08]	[-5.45]	[-7.57]	[-3.61]	[-5.45]	[-7.84]
Maturity	0.61	0.64	0.61	0.65	0.63	0.66	0.65	0.61
	[16.86]	[19.59]	[16.25]	[19.58]	[19.54]	[19.92]	[19.58]	[17.41]
Age	0.58	0.44	0.62	0.33	0.57	0.25	0.32	0.59
	[4.55]	[3.33]	[4.54]	[2.72]	[4.75]	[2.29]	[2.72]	[4.51]
Rating	0.34	0.36	0.43	0.55	0.27	0.52	0.55	0.39
	[4.29]	[5.24]	[5.06]	[7.67]	[4.40]	[7.76]	[7.67]	[4.57]
Leverage	4.05	4.30	3.50	3.27	5.07	3.52	3.27	3.92
	[5.80]	[6.59]	[5.53]	[6.30]	[7.07]	[6.19]	[6.30]	[5.94]
Equity Vol	0.050	0.069	0.021	0.037	0.060	0.056	0.037	0.045
	[1.57]	[2.37]	[0.67]	[1.28]	[1.89]	[1.87]	[1.28]	[1.40]
Volume		-1.18		-2.42				
		[-4.71]		[-10.46]				
ln(#Trd)			2.26	3.80			1.38	
			[11.09]	[14.44]			[6.77]	
Turnover					-0.009			
					[-1.10]			
ln(Trd Sz)						-2.78	-2.42	
						[-15.38]	[-10.46]	
%Days Trd								0.12
								[5.79]
Callable	-0.49	-0.48	-0.40	-0.39	-0.27	-0.45	-0.39	-0.51
	[-1.23]	[-1.28]	[-1.01]	[-1.13]	[-0.80]	[-1.28]	[-1.13]	[-1.29]
R-sqd	30.17	31.51	31.71	34.82	31.72	34.20	34.82	30.92

Reported are monthly Fama-MacBeth cross-sectional regressions with $\hat{\sigma}_D - \hat{\sigma}_D^{\text{Merton}}$ as the dependent variable, where $\hat{\sigma}_D$ and $\hat{\sigma}_D^{\text{Merton}}$, both in %, are estimated using daily returns. The Fama-MacBeth t-stats are corrected for autocorrelation using Newey-West. The reported R-sqd's are the time-series averages of cross-sectional R^2's. Convertible and putable bonds are excluded from the regression, and Callable is one for a callable bond and zero otherwise. Age is year since issuance, Amt is in \$m, Ratings are coded as 1 for Aaa and 21 for C, Leverage is in decimals, Volume is monthly bond trading volume in \$m, Trd Sz is average trade size in \$thousand, and Turnover, %Day Trd and Equity Vol are all in %. See Table 2.1 for the summary statistics of the independent variables.

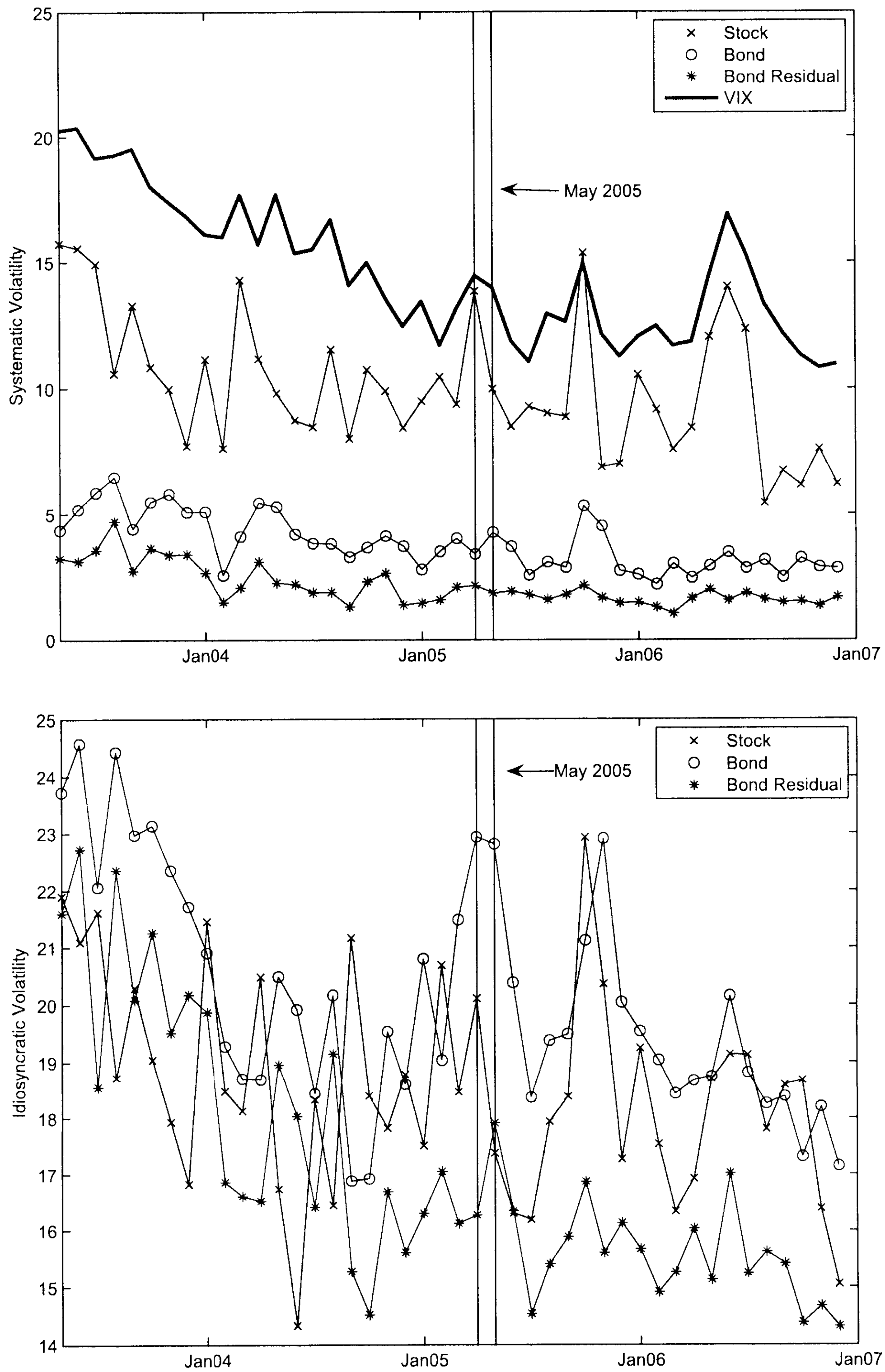

Figure 2-1: Systematic (top panel) and idiosyncratic (bottom panel) volatility of daily returns on stocks, bonds, and bond residuals.

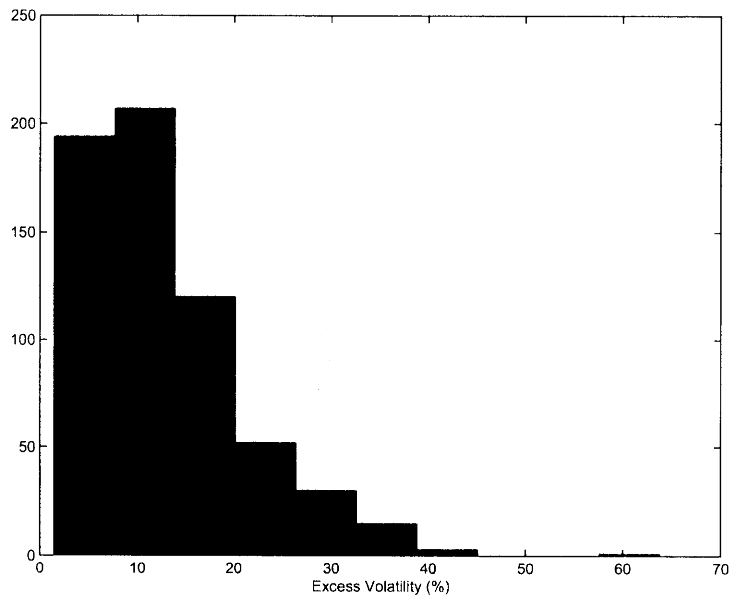

Figure 2-2: The cross-sectional distribution of the time-series means (bond by bond) of $\hat{\sigma}_D - \hat{\sigma}_D^{\mathrm{Merton}}$

Chapter 3

Liquidity of Corporate Bonds

3.1 Introduction

The liquidity of the corporate bond market has been of interest for researchers, practitioners and policy makers. Many studies have attributed deviations in corporate bond prices from their "theoretical values" to the influence of illiquidity in the market.[1] Yet, our understanding of how to quantify illiquidity remains limited. Without a credible measure of illiquidity, it is difficult to have a direct and serious examination of the asset-pricing influence of illiquidity and its implications on market efficiency.

Several measures of illiquidity have been considered in the literature for corporate bonds. A simple measure is the effective bid-ask spread, which is analyzed in detail by Edwards, Harris, and Piwowar (2007).[2] Although the bid-ask spread is a direct and potentially important indicator of illiquidity, it does not fully capture many important aspects of liquidity such as market depth and resilience. Alternatively, relying on theoretical pricing models to gauge the impact of illiquidity allows for direct estimation of its influence on prices, but suffers from potential mis-specifications of the pricing model.

In this chapter, we rely on a salient feature of illiquidity to measure its significance. It has been well recognized that the lack of liquidity in an asset gives rise to transitory components in its prices.[3] The magnitude of such transitory price movements reflects the

[1] For example, Huang and Huang (2003) find that yield spreads for corporate bonds are too high to be explained by credit risk and question the economic content of the unexplained portion of yield spreads (see also Collin-Dufresne, Goldstein, and Martin (2001) and Longstaff, Mithal, and Neis (2005)). Chapter 2 documents a significant amount of transitory excess volatility in corporate bond returns and attributes this excess volatility to the illiquidity of corporate bonds.

[2] See also Bessembinder, Maxwell, and Venkataraman (2006) and Goldstein, Hotchkiss, and Sirri (2007).

[3] Niederhoffer and Osborne (1966) are among the first to recognize the relation between negative serial covariation and illiquidity. More recent theoretical work in establishing this link include Grossman and Miller (1988) and Huang and Wang (2007), among others.

degree of illiquidity in the market. Since transitory price movements lead to negatively serially correlated price changes, the negative of the autocovariance in price changes, which we denote by γ, provides a simple, yet robust measure of illiquidity. In the simplest case when the transitory price movements arise purely from bid-ask bounce, as considered by Roll (1984), $2\sqrt{\gamma}$ equals the bid-ask spread. But in more general cases, γ captures the broader impact of illiquidity on prices, above and beyond the effect of bid-ask spread. Moreover, it does so without relying on specific bond pricing models.

Indeed, our results show that the lack of liquidity in the corporate bond market is substantial, significantly more severe than what can be explained by bid-ask bounce, and closely related to bond characteristics that are known to be linked to liquidity. More importantly, taking advantage of this measure of illiquidity, we are able to analyze the time variation of the aggregate illiquidity in corporate bonds and its asset-pricing implications. The main results of this chapter can be further detailed as follows.

First, we uncover a level of illiquidity in corporate bonds that is important both economically and statistically. Using TRACE, a transaction-level dataset, we estimate γ for a broad cross-section of the most liquid corporate bonds in the U.S. market. Our results show that, using trade-by-trade data, the median estimate of γ is 0.41, and the mean estimate is 0.60; using daily data, the median γ is 0.67, and the mean γ is 1.04. Both means are highly statistically significant. To judge the economic significance of such magnitudes, we can use the quoted bid-ask spreads to calculate a bid-ask implied γ. For the same sample of bonds and for the same sample period, we find that the median and mean γ implied by the quoted bid-ask spreads are respectively 0.031 and 0.045, which are tiny fractions of our estimated γ. An alternative comparison is to use Roll's model to calculate the γ-implied bid-ask spread, which is $2\sqrt{\gamma}$, and compare it with the quoted bid-ask spread.[4] Using our median estimates of γ, the γ-implied bid-ask spread is \$1.28 using trade-by-trade data and \$1.64 using daily data, significantly larger than the median quoted bid-ask spread of \$0.31 or the estimated bid-ask spread reported by Edwards, Harris, and Piwowar (2007) (see Section 3.5 for more details). Such comparisons suggest that our illiquidity measure γ captures the price impact of illiquidity above and beyond the effect of simple bid-ask bounce.

Second, we establish a robust connection between our illiquidity measure γ and bond characteristics known to be relevant for liquidity. Regressing our illiquidity measure γ on a spectrum of bond characteristics, we find a strong positive relation between γ and bond age

[4]Roll's model assumes that directions of trades are serially independent. For a given bid-ask spread, positive serial correlation in trade directions, which could be the case when liquidity is lacking and traders break up their trades, tends to increase the implied bid-ask spreads for a given γ. This could potentially increase the magnitude of the γ implied bid-ask spreads, further deepening its difference from the quoted bid-ask spreads.

— a variable widely used in the fixed-income market as a proxy of illiquidity; and a strong negative relation between γ and the size of the bond issuance — another variable potentially linked to bond liquidity. Moreover, we find that the measure of illiquidity captured by γ is related to but goes beyond the information contained in the quoted bid-ask spreads. Specifically, adding the bid-ask implied γ as an additional explanatory variable, we find that it has a positive cross-sectional relation with our γ measure, but it does not alter the established cross-sectional relation between γ and bond characteristics, including age and issuance size. Controlling for bond characteristics including age and issuance size, we also find that bonds with smaller average trade sizes typically have higher illiquidity measure γ.

Third, focusing on the systematic component of bond illiquidity, we construct a time-series of aggregate γ. Examining its variation over time and its connection with broader financial markets, we find some rather interesting patterns in the aggregate γ. Before the onset of the subprime crisis in 2007, there is a general trend of decreasing γ, indicating an overall improvement of liquidity in the corporate bond market before the summer of 2007. Against this backdrop of an overall time trend, however, we find substantial monthly movements in the aggregate measure of illiquidity. In particular, the aggregate γ rises sharply during market crises, including the periods that lead to the downgrade of Ford and GM bonds to junk status, the sub-prime mortgage crisis that started in August 2007, and the credit market turmoil following the collapse of Lehman Brothers. For example, before August 2007, our aggregate γ hovered around a level near 0.38 with a monthly standard deviation of 0.096. In August 2007, it jumped by over 60% to a level near 0.70. During the year after that, it stabilized around 0.81 with a monthly standard deviation of 0.12. Then rose to 1.59 in September 2008 and an all-time high of 2.85 in October 2008.[5]

More interestingly, this common illiquidity component uncovered by our analysis is closely connected with the changing conditions of broader financial markets. Regressing monthly changes in aggregate γ on changes in CBOE VIX, we find a positive and strongly significant relation with an R-squared of 64%. We also consider a number of other variables that may capture changing conditions in financial markets such as the volatility of aggregate bond returns, CDS spread, term spread, default spread and lagged returns on the aggregate stock and bond markets. While there is some evidence that these variables are related to our aggregate liquidity measure during some periods, by far, the most robust relation lies with VIX. Moreover, this connection between VIX is not simply a 2008 phenomena. For the subperiod that excludes 2008, monthly changes in our aggregate γ remain closely related to monthly changes in VIX.

[5] To be conservative, we use the cross-sectional median of the bond-level γ as our aggregate illiquidity measure. The numbers are even more dramatic using the cross-sectional mean.

The fact that the VIX index, measured from index options, is the main variable in explaining changes in aggregate illiquidity of corporate bonds is rather intriguing. Indeed, from an aggregate perspective, this implies that the sources of our estimated bond market illiquidity are not contained just in the bond market. This raises the possibility of illiquidity being an additional source of systemic risk, as examined by Chordia, Roll, and Subrahmanyam (2000) and Pastor and Stambaugh (2003) for the equity market.

Fourth, we examine the asset-pricing implications of bond illiquidity. We find that our illiquidity measure γ explains the cross-sectional variation of average bond yield spreads with large economic significance. Controlling for bond rating categories, we perform monthly cross-sectional regressions of bond yield spreads on bond γ. We find a coefficient of 0.21 with a t-stat of 7.08 using Fama and MacBeth (1973) standard errors. Given that the cross-sectional standard deviation of γ is 1.79, our result implies that for two bonds in the same rating category, a two standard deviation difference in their γ leads to a difference in their yield spreads as large as 75 bps. This is comparable to the difference in yield spreads between Baa and Aaa/Aa bonds, which is 113 bps in our sample. In contrast, quoted bid-ask spreads have rather limited, if any, economic significance in explaining the cross-sectional average yield spreads. Moreover, the economic significance of our illiquidity measure remains robust in its magnitude and statistical significance after we control for a spectrum of variables related to the bond's fundamental information as well as bond characteristics. In particular, liquidity related variables such as bond age, issuance size, quoted bid-ask spread, and average trade size do not change this result in a significant way.

In addition to the main results summarized above, we provide detailed analyses of our illiquidity measure to further shed light on the nature of illiquidity in corporate bonds. We explore the dynamic properties of illiquidity by estimating the magnitude of price reversals after skipping one or several trades. We find significant price reversals even after skipping a trade, indicating a mean-reversion in price changes that lasts for more than one trade.[6] We also find that negative price changes, likely caused by excess selling pressure, are followed by stronger reversals than positive price changes, resulting in an asymmetry in γ.[7] We find that price changes associated with large trades exhibit weaker reversals than those associated with small trades, and this effect is robust after controlling for the overall bond liquidity. Although this result suggests a strong link between liquidity and trade sizes, it is, however, difficult to interpret this negative relation between γ and trade sizes simply as more liquidity

[6]This is consistent with the fact that our γ measured at the daily level, capturing this persistent transaction-level mean-reversion cumulatively, yields a higher magnitude than its counterpart at the transaction level.

[7]Such an asymmetry was described as a characteristic of the impact of illiquidity on prices by Huang and Wang (2007). Our results provide an interesting empirical test of this proposition.

for larger trades, since both trade sizes and prices are endogenous.

This chapter is related to the growing literature on the impact of liquidity on corporate bond yields. Using illiquidity proxies that include quoted bid-ask spreads and the percentage of zero returns, Chen, Lesmond, and Wei (2007) find that more illiquid bonds earn higher yield spreads. Using nine liquidity proxies including issuance size, age, missing prices, and yield volatility, Houweling, Mentink, and Vorst (2005) reach similar conclusions for euro corporate bonds. deJong and Driessen (2005) find that systematic liquidity risk factors for the Treasury bond and equity markets are priced in corporate bonds, and Downing, Underwood, and Xing (2005) address a similar question. Using a proprietary dataset on institutional holdings of corporate bonds, Nashikkar, Mahanti, Subrahmanyam, Chacko, and Mallik (2008) and Mahanti, Nashikkar, and Subrahmanyam (2008) propose a measure of latent liquidity and examine its connection with the pricing of corporate bonds and credit default swaps.

We contribute to this growing body of literature by proposing a measure of illiquidity that is theoretically motivated and empirically more direct. Moreover, the degree of illiquidity captured by our illiquidity measure is significantly higher in magnitude than that implied by the quoted or estimated bid-ask spreads. We are able to establish a connection between our measure of illiquidity and the commonly used liquidity proxies such as age, issuance and trading activities. But more importantly, our illiquidity measure contains information above and beyond these proxies in explaining, for example, the average bond yield spreads across a broad cross-section of bonds. Finally, the close connection between our aggregate illiquidity measure and overall market conditions is a clear indication that our measure indeed extracts useful information about illiquidity from the transaction-level data. We hope that the properties we uncover in this chapter about the illiquidity of corporate bonds can provide a basis to further analyze its importance to the efficiency of the bond market.

It should be noted that the estimation of our illiquidity measures relies on transactions prices. For relatively liquid bonds, transactions are fairly frequent and our estimates are more reliable. This constrains our analysis to more liquid bonds. Since the goal of this chapter is to demonstrate the potential importance of illiquidity for corporate bonds, doing so for the more liquid bonds actually strengthens our case. However, a large fraction of corporate bonds are not traded often. For those bonds, one may need to use the methods proposed by Edwards, Harris, and Piwowar (2007), which rely on more detailed trade information in addition to prices.

The chapter is organized as follows. Section 3.2 describes the data used in our analysis and provides summary statistics. The main results of this chapter are reported in Section 3.3, and Section 3.4 provides further analyses of our illiquidity measure. Section 3.5 compares our illiquidity measure with the effect of bid-ask spreads. Section 3.6 concludes.

3.2 Data Description and Summary

The main data set used for this chapter is FINRA's TRACE (Transaction Reporting and Compliance Engine). Coverage of bond trades was gradually implemented in three phases by FINRA. See chapter 1 for a more complete description of the dataset and the different phases.

In our study, we drop the early sample period with only Phase I coverage. We also drop all of the Phase III only bonds. We sacrifice in these two dimensions in order to maintain a balanced sample of Phase I and II bonds from April 14, 2003 to December 31, 2008. Of course, new issuances and retired bonds generate some time variation in the cross-section of bonds in our sample. After cleaning up the data, we also take out the repeated inter-dealer trades by deleting trades with the same bond, date, time, price, and volume as the previous trade.[8] We further require the bonds in our sample to have frequent enough trading so that the illiquidity measure can be constructed from the trading data. Specifically, during its existence in the TRACE data, a bond must trade on at least 75% of its relevant business days in order to be included in our sample. Finally, to avoid bonds that show up just for several months and then disappear from TRACE, we require the bonds in our sample to be in existence in the TRACE data for at least one full year.

Tables 3.1 and 3.2 summarize our sample, which consists of frequently traded Phase I and II bonds from April 2003 to December 2008. There are 1,205 bonds in our full sample, although the total number of bonds does vary from year to year. The increase in the number of bonds from 2003 to 2004 could be a result of how NASD starts its coverage of Phase III bonds, while the gradual reduction of number of bonds from 2004 through 2008 is a result of matured or retired bonds.

The bonds in our sample are typically large, with a median issuance size of $717 million, and the representative bonds in our sample are investment grade, with a median rating of 6, which translates to Moody's A2. The average maturity is close to 6 years and the average age is about 4 years. Over time, we see a gradual reduction in maturity and increase in age. This can be attributed to our sample selection which excludes bonds issued after February 7, 2005, the beginning of Phase III.[9]

[8]This includes cleaning up withdrawn or corrected trades, dropping trades with special sale conditions or special prices, and correcting for obviously mis-reported prices.

[9]We will discuss later the effect, if any, of this sample selection on our results. An alternative treatment is to include in our sample those newly issued bonds that meet the Phase II criteria, but this is difficult to implement since the Phase II criteria are not precisely specified by NASD.

Given our selection criteria, the bonds in our sample are more frequently traded than a typical bond. The average monthly turnover — the bond's monthly trading volume as a percentage of its issuance size — is 7.51%, the average number of trades in a month is 181. The median trade size is $338,000. While for the the whole sample in TRACE, the average monthly turnover is 4.07%, the average number of trades in a month is 26 and the median trade size is $61,000. Thus, the bonds in our sample are also relatively more liquid. Given that our focus to study the significance of illiquidity for corporate bonds, such a bias in our sample towards more liquid bonds, although not ideal, will only help to strengthen our results if they show up for the most liquid bonds.

In addition to the TRACE data, we use CRSP to obtain stock returns for the market and the respective bond issuers. We use FISD to obtain bond-level information such as issue date, issuance size, coupon rate, and credit rating, as well as to identify callable, convertible and putable bonds. We use Bloomberg to collect the quoted bid-ask spreads for the bonds in our sample, from which we have data for 1,170 out of the 1,205 bonds in our sample.[10] We use Datastream to collect Lehman Bond indices to calculate the default spread and returns on the aggregate corporate bond market. To calculate yield spreads for individual corporate bonds, we obtain Treasury bond yields from the Federal Reserve, which publishes constant maturity Treasury rates for a range of maturities. Finally, we obtain the VIX index from CBOE.

3.3 Main Results

3.3.1 Measure of Illiquidity

Although a precise definition of illiquidity and its quantification will depend on a specific model, two properties are clear. First, illiquidity arises from market frictions, such as costs and constraints for trading and capital flows; second, its impact to the market is transitory.[11] Our empirical measure of illiquidity is motivated by these two properties.

Let P_t denote the clean price of a bond - the full value minus the accrued interest since

[10]We follow Chen, Lesmond, and Wei (2007) in using the Bloomberg Generic (BGN) bid-ask spread. This spread is calculated using a proprietary formula which uses quotes provided to Bloomberg by a proprietary list of contributors. These quotes are indicative rather than binding.

[11]In a recent paper, Vayanos and Wang (2008) provide a unified theoretical model for liquidity, which relates illiquidity with different forms of market frictions. Huang and Wang (2007) consider a model in which trading costs give rise to illiquidity in the market endogenously and show that it leads to transitory deviations in prices from fundamentals.

the last coupon date – at time t. We start by assuming that P_t consists of two components:

$$P_t = F_t + u_t\,. \tag{3.1}$$

The first component F_t is its fundamental value — the price in the absence of frictions, which follows a random walk; the second component u_t comes from the impact of illiquidity, which is transitory (and uncorrelated with the fundamental value).[12] In such a framework, the magnitude of the transitory price component u_t characterizes the level of illiquidity in the market. Our measure of illiquidity is aimed at extracting the transitory component in the observed price P_t. Specifically, let $\Delta P_t = P_t - P_{t-1}$ be the price change from $t-1$ to t. We define the measure of illiquidity γ by

$$\gamma = -\text{Cov}\left(\Delta P_t, \Delta P_{t+1}\right). \tag{3.2}$$

With the assumption that the fundamental component F_t follows a random walk, γ depends only on the transitory component u_t, and it increases with the magnitude of u_t.

Several comments are in order before our proceed with our analysis of γ. First, it should be noted that we use this framework merely to facilitate our empirical estimation. Indeed, our contribution to the literature lies in our empirical results. Second, other than being transitory, we know little about the dynamics of u_t. For example, when u_t follows an AR(1) process, we have $\gamma = (1-\rho)^2\sigma^2/(1+\rho)$, where σ is the instantaneous volatility of u_t, and ρ is its persistence coefficient.[13] In this case, while γ does provide a simple gauge of the magnitude of u_t, it combines various aspects of u_t. Third, in terms of measuring illiquidity, other aspects of u_t that are not fully captured by γ may also matter. In other words, γ itself gives only a partial measure of illiquidity. Finally, given the potential richness in the dynamics of u_t, γ will in general depend on the horizon over which we measure price changes. This horizon effect is important because γ measured over different horizons may capture different aspects of u_t or illiquidity. For most of our analysis, we will use either trade-by-trade prices or end of the day prices in estimating γ. Consequently, our γ estimate captures more of the high frequency components in transitory price movements.

[12]Such a separation assumes that the fundamental value F_t carries no time-varying risk premium. This is a reasonable assumption over short horizons. It is equivalent to assuming that high frequency variations in expected returns are ultimately related to market frictions — otherwise, arbitrage forces would have driven them away. To the extent that illiquidity can be viewed a manifestation of these frictions, price movements giving rise to high frequency variations in expected returns should be included in u_t. Admittedly, a more precise separation of F_t and u_t must rely on a pricing theory incorporating frictions or illiquidity. See, for example, Vayanos and Wang (2008).

[13]The persistent coefficient ρ is less than 1 given that u_t is transitory.

Table 3.3 summarizes the illiquidity measure γ for the bonds in our sample.[14] Focusing first on Panel A, in which γ is estimated bond-by-bond using either trade-by-trade or daily data, we see an illiquidity measure of γ that is important both economically and statistically.[15] For the full sample period from 2003 through 2008, our illiquidity measure γ has a cross-sectional average of 0.60 with a robust t-stat of 22.43 when estimated using trade-by-trade data, and an average of 1.04 with a robust t-stat of 28.35 using daily data.[16] More importantly, the significant mean estimate of γ is not generated by just a few highly illiquid bonds. Using trade-by-trade data, the cross-sectional median of γ is 0.41, and 99.83% of the bonds have a statistically significant γ (the t-stat of γ greater than or equal to 1.96); using daily data, the cross-sectional median of γ is 0.67 and over 98% of the bonds have a statistically significant γ. Moreover, breaking our full sample by year shows that the illiquidity measure γ is important and stable across years.[17]

For each bond, we can further breakdown its overall illiquidity measure γ to gauge the relative contribution from trades of various sizes.Specifically, for each bond, we sort its trades by size into the smallest 30%, middle 40%, and largest 30% and then estimate γ^{small}, γ^{medium} and γ^{large} using prices associated with the corresponding trade sizes. The results are summarized in Table 3.15 in the Appendix. We find that our overall illiquidity measure is not driven only by small trades. In particular, we find significant illiquidity across all trade sizes. For example, using daily data, the cross-sectional means of γ^{small}, γ^{medium} and γ^{large} are 1.44, 0.91, and 0.47, respectively, each with very high statistical significance.

As a comparison to the level of illiquidity for individual bonds, Panel B of Table 3.3 reports γ measured using equal- or issuance-weighted portfolios constructed from the same cross-section of bonds and for the same sample period. In contrast to its counterpart at the individual bond level, γ at the portfolio level is slightly negative, rather small in magnitude, and statistically insignificant. This implies that the transitory component extracted by our

[14]To be included in our sample, the bond must trade on at least 75% of business days and at least 10 observations of the paired price changes, $(\Delta P_t, \Delta P_{t-1})$, are required to calculate γ.

[15]In calculating γ using daily data, price changes may be between prices over multiple days if a bond does not trade during a day. We limit the difference in days to one week though this criteria rarely binds due to our sample selection criteria.

[16]The robust t-stats are calculated using standard errors that are corrected for cross-sectional and time-series correlations. Specifically, the moment condition for estimating γ is $\hat{\gamma} + \Delta P_t^i \Delta P_{t-1}^i = 0$ for all bond i and time t, where ΔP is demeaned. We can then correct for cross-sectional and time-series correlations in $\Delta P_t^i \Delta P_{t-1}^i$ using standard errors clustered by bond and day.

[17]Our γ measure could be affected by the presence of persistent small trades, which could be a result of how dealers deal bonds to retail traders. We thank the referee for raising this point. Such persistent small trades will bias our illiquidity measure downward. In other words, our γ measures would have been larger in the absence of such persistent small trades. Moreover, it will have a larger impact on γ measured using prices associated small trade sizes. As we show in the next paragraph, we find significant illiquidity across all trade sizes.

γ measure is idiosyncratic in nature and gets diversified away at the portfolio level. It does not imply, however, that the illiquidity in corporate bonds lacks a systematic component, which we will examine later in Section 3.3.3.

Panel C of Table 3.3 provides another and perhaps more important gauge of the magnitude of our estimated γ for individual bonds. Using quoted bid-ask spreads for the same cross-section of bonds and for the same sample period, we estimate a bid-ask implied γ for each bond by computing the magnitude of negative autocovariance that would have been generated by bid-ask bounce. For the full sample period, the cross-sectional mean of the implied γ is 0.045 and the median is 0.031, which are more than one order of magnitude smaller than the empirically observed γ for individual bonds. As shown later in the chapter, not only does the quoted bid-ask spread fail to capture the overall level of illiquidity, but it also fails to explain the cross-sectional variation in bond illiquidity and its asset pricing implications.

Although our focus is on extracting the transitory component at the trade-by-trade and daily frequencies, it is nevertheless interesting to provide a general picture of γ over varying horizons. First, our results show that the magnitude of the illiquidity measure γ is stronger at the daily than the trade-by-trade horizon. Given that the autocovariance at the daily level cumulatively captures the mean-reversion at the trade-by-trade level, this implies that the mean-reversion at the trade-by-trade level persists for a few trades before fully dissipating, which we show in Section 3.4.1. Second, moving from the daily to weekly horizon, we find that the magnitude of γ increases from an average level of 1.04 to 1.11, although its statistical significance decreases to a robust t-stat of 11.70, and 74.94% of the bonds in our sample have a positive and statistically significant γ at this horizon. Third, extending to the bi-weekly and monthly horizons, γ starts to decline in both magnitude and statistical significance.[18]

As mentioned earlier in the section, the transitory component u_t might have richer dynamics than what can be offered by a simple AR(1) structure for Δu_t. By extending γ over various horizons, we are able to uncover some of the dynamics. We show in Section 3.4.1 that at the trade-by-trade level Δu_t is by no means a simple AR(1). Likewise, in addition to the mean-reversion at the daily horizon that is captured in this chapter, the transitory component u_t may also have a slow moving mean-reversion component at a longer horizon. To examine this issue more thoroughly is an interesting topic, but requires time-series data for a longer sample period than ours.[19]

[18]In addition to reducing the available data, the higher differencing intervals also decreases the signal to noise ratio as the fundamental volatility starts to build up. See Harris (1990) for the exact small sample moments of the serial covariance estimator and of the standard variance estimator for price changes generated by the Roll spread model.

[19]By using monthly bid prices from 1978 to 1998, Khang and King (2004) report contrarian patterns in

3.3.2 Illiquidity and Bond Characteristics

Our sample includes a broad cross-section of bonds, which allows us to examine the connection between our illiquidity measure γ and various bond characteristics, some of which are known to be linked to bond liquidity. The variation in our illiquidity measure γ and bond characteristics are reported in Table 3.4. We use daily data to construct yearly estimates for γ for each bond and perform pooled regressions on various bond characteristics. Reported in square brackets are the t-stat's calculated using standard errors clustered by year.

We find that older bonds on average have higher γ, and the results are robust regardless of which control variables are used in the regression. On average, a bond that is one-year older is associated with an increase of 0.1 in its γ, which accounts for 10% of the full-sample average of γ. Given that the age of a bond has been widely used in the fixed-income market as a proxy for illiquidity, it is important that we establish this connection between our illiquidity measure γ and age. Similarly, we find that bonds with smaller issuance tend to have larger γ. We also find that bonds with longer time to maturity and lower credit ratings typically have higher γ.

Using weekly bond returns, we also estimate, for each bond, its betas on the aggregate stock- and bond-market returns, using the CRSP value-weighted index as a proxy for the stock market and the Lehman US Bond Index as a proxy for the bond market. We find that γ is positively related to the stock beta and weakly related to bond beta. However, in unreported results, we find that the inclusion of the idiosyncratic volatility (estimated from the residual of the betas regression) drives out the significance of both stock and bond betas.

Given that we have transaction-level data, we can also examine the connection between our illiquidity measure and bond trading activity. We find that, by far, the most interesting variable is the average trade size of a bond. In particular, bonds with smaller trade sizes have higher illiquidity measure γ.

To examine the cross-sectional connection between our illiquidity measure and the quoted bid-ask spreads, we use the quoted bid-ask spreads for each bond in our sample to calculate the bid-ask spread implied autocovariance, or bid-ask implied γ. We find a positive relation between our γ measure and the γ measure implied by the quoted bid-ask spread.[20] The

corporate bond returns over horizons of one to six months. Instead of examining autocovariance in bond returns, their focus is on the cross-sectional effect. Sorting bonds by their past monthly (or bi-monthly up to 6 months) returns, they find that past winners under perform past losers in the next month (or 2-month up to 6 months). Their result, however, is relatively weak and is significant only in the early half of their sample and goes away in the second half of their sample (1988–1998).

[20]As it is possible that the relation between γ and γ implied by quoted bid-ask spreads is mechanical due to spreads being a fixed proportion of prices and price levels being different, we also examine the relation between γ calculated using log price changes and γ implied by bid-ask spreads, finding a significant correlation between the two variables.

regression coefficient is on average around 4 and is statistically significant. The magnitude of the coefficient implies that one unit difference in γ implied by quoted bid-ask spreads gets amplified to four times the difference in our measure of γ. Adding the bid-ask implied γ as an explanatory variable, however, does not alter the relation between our γ measure and liquidity-related bond characteristics such as age and size. Overall, we find that the magnitude of illiquidity captured by our γ measure is related to but goes beyond the information contained in the quoted bid-ask spreads.

We also introduce a CDS dummy, which is one if the bond issuer has credit default swaps traded on it, to examine whether or not there is a difference in our illiquidity measure for bonds with and without CDS traded on their issuers. About 71% of the bonds-years in our sample have traded CDS and our results show that, after controlling for bond age, maturity, issuance size and rating, such bonds on average have insignificantly different γ, although they have slightly lower γ in the sample pre-2008. Finally, we introduce CDS spreads, finding that higher CDS spreads are correlated with greater illiquidity, though this result is largely driven by 2008.

3.3.3 Commonality in Illiquidity and Market Conditions

We next examine the time variation of illiquidity in the bond market. From Table 3.3, we see a steady reduction in the annual γ averaged over all bonds in our sample from 2003 through 2006. For example, the median γ using daily data is 0.71 in 2003, which decreases monotonically to 0.42 in 2006, suggesting an overall improvement of liquidity in the bond market from 2003 through 2006. During 2007, however, the median γ jumped back to 0.59 and in 2008, the median γ dramatically increased to 1.50, reflecting worsening liquidity in the market.[21] Using the cross-sectional mean of γ, we can observe the same and even a somewhat more dramatic pattern.

We now investigate this time variation more closely. For this, we turn our attention to monthly fluctuations in the illiquidity measure γ. Monthly illiquidity measures γ are calculated for each bond using daily data within that month.[22] We then use the median γ

[21] By focusing only on Phase I and II bonds in TRACE to maintain a reasonably balanced sample, we did not include bonds that were included only after Phase III, which was fully implemented on February 7, 2005. Consequently, new bonds issued after that date were excluded from our sample, even though some of them would have been eligible for Phase II had they been issued earlier. As a result, starting from February 7, 2005, we have a population of slowly aging bonds. Since γ is positively related to age, the overall downward trend in γ would have been more pronounced had we been able to maintain a more balanced sample. It should be mentioned that the sudden increases in aggregate γ during crises are too large to be explained by the slow aging process. Finally, to avoid regressing trend on trend, the time-series regression results presented later in this section are based on regressing changes on changes.

[22] In calculating the monthly autocovariance of price changes, we can demean the price change using the sample mean within the month, within the year, or over the entire sample period. It depends on whether

as our aggregate γ measure. Compared with the cross-sectional mean of γ, the median γ is a more conservative measure and is less sensitive to those highly illiquid bonds that were most severely affected by the credit market turmoil.

In Figure 3-1, we plot our aggregate γ along with the CBOE VIX index. It is clear that the aggregate γ exhibits significant time variation, which suggests that there are important commonalities in the illiquidity measure captured by our bond-level γ. In particular, after decreasing markedly but relatively smoothly during 2003 and the first half of 2004, it reversed its trend and started to climb up in late 2004 and then spiked in April/May 2005. This rise in γ coincides with the downgrade of Ford and GM to junk status in early May 2005, which rattled the credit market. The illiquidity measure γ quieted down somewhat through 2006, but rose sharply in August 2007, when the sub-prime mortgage crisis first hit. Its August 2007 value of 0.70 is quite dramatic compared to its late 2006 value of 0.30. Though this rise is fairly sharp compared to changes in γ preceding 2007, it is small compared to changes in 2008. Aggregate γ remained in the 0.8 to 1.1 range for much of 2008 before jumping to 1.59 in September 2008, when Lehman Brothers filed for bankruptcy. In October 2008, γ rose further to 2.85 before slightly declining towards the end of 2008.

The fact that γ increased drastically during periods of credit market turmoil in our sample indicates that not only does bond market illiquidity vary over time, but, more importantly, it also varies together with the changing conditions of the market. In Figure 3-1, we also plot the CBOE VIX index with aggregate γ. The CBOE VIX index is used to capture the overall market condition and is also known as the "fear gauge" of the market. In Figure 3-2, we plot the aggregate γ along with several other variables that are known to be linked to market conditions. To capture the conditions of the credit market, we use default spread, measured as the difference in yields between AAA- and BBB-rated corporate bonds, using the Lehman US Corporate Intermediate Indices. To capture the overall volatility of the corporate bond market, we construct monthly estimates of annualized bond return volatility using daily returns to the Lehman US Investment Grade Corporate Index. We also consider an average CDS index, constructed as the average of five-year CDS spreads covered by CMA Datavision in Datastream.

Comparing the time variation in these variables with that of our aggregate γ, we have several observations. First, the aggregate γ comoves with VIX in a rather significant way. Regressing changes in γ on contemporaneous changes in VIX, we obtain a slope coefficient of 0.0351 with a t-stat of 8.15 (adjusted for serial correlation using Newey-West) as reported in

we view the monthly variation in the mean of price change as noise or as some low-frequency movement related to the fundamental. In practice, however, this time variation is rather small compared with the high-frequency bouncing around the mean. As a result, demeaning using the monthly mean or the sample mean generates very similar results. Here we report the results using the former.

Panel A of Table 3.5. The adjusted R-squared of the OLS regression is 64.12%. Furthermore, this result is robust to only including data through 2007 as reported in Panel B.

Second, there seems to be a link between γ and the volatility of bond returns towards the end of our sample. Indeed as shown in Panel A of Table 3.5, changes in γ and contemporaneous changes in bond return volatility are related with a coefficient of 0.0409 and a t-stat of 2.03 when we use the whole sample. However, as Panel B shows, this result is mostly driven by 2008 and it disappears when the data from 2008 is excluded. Third, the aggregate γ also comoves weakly with CDS, but not the default spread.

We further examine in Table 3.5 the relation between monthly changes of our aggregate γ and the performance of the aggregate stock and bond markets in the previous month. We find that our aggregate γ has statistically insignificant relations with lagged aggregate bond and stock market returns. In the pre-2008 period, however, aggregate γ typically increased following down markets. Towards the end of 2008, our aggregate illiquidity measure flattened out while stock and bond markets continued to decline.

The various market condition variables considered so far are closely inter-connected. To evaluate their relative importance, Table 3.5 also reports the result of the multivariate regression using all variables that are univariately significant in full sample regressions. VIX remains robustly significant and bond volatility remains significant only when 2008 is included while the CDS index does not. Our illiquidity measure is in fact most related to variables that tend to measure aggregate uncertainty and fear.

Our time-series analysis of the aggregate illiquidity reveals two important properties of γ as a measure of illiquidity for corporate bonds. First, there exists commonality in the illiquidity of individual bonds, which is reflected in the significant time variation in aggregate γ. Second, such common movements in bond market illiquidity are closely connected with overall market conditions in an important way.

3.3.4 Bond Yield Spreads and Illiquidity

We now examine the pricing implications of bond illiquidity. For this purpose, we focus on the bond yield spread, which is the difference between the corporate bond yield and the Treasury bond yield of the same maturity. For Treasury yields, we use the constant maturity rate published by the Federal Reserve and use linear interpolation whenever necessary. We perform monthly cross-sectional regressions of the yield spreads on the illiquidity measure γ, along with a set of control variables. We report our results for our full sample of bonds here, including both investment-grade and junk bonds. Results using just investment grade bonds are similar.

The results are reported in Table 3.6, where the t-stat's are calculated using the Fama-MacBeth standard errors with serial correlation corrected using Newey and West (1987). To include callable bonds in our analysis, which constitute a large portion of our sample, we use a callable dummy, which is one if a bond is callable and zero otherwise.[23] We exclude all convertible and putable bonds from our analysis. In addition, we also include three rating dummies for A, Baa, and junk ratings, respectively. The first column in Table 3.6 shows that the average yield spread of the Aaa and Aa bonds in our sample is 113 bps, relative to which the A bonds are 69 bps higher, Baa bonds are 119 bps higher, and junk bonds are 541 bps higher.

As reported in the second column of Table 3.6, adding γ to the regression does not bring much change to the relative yield spreads across ratings. This is to be expected since γ should capture more of a liquidity effect, and less of a fundamental risk effect, which is reflected in the differences in ratings. More importantly, we find that the coefficient on γ is 0.21 with a t-stat of 7.08. This implies that for two bonds in the same rating category, if one bond, presumably less liquid, has a γ that is higher than the other by 1, the yield spread of this bond is on average 21 bps higher than the other. To put an increase of 1 in γ in context, the cross-sectional standard deviation of γ is on average 1.79 in our sample. From this perspective, our illiquidity measure γ is economically important in explaining the cross-sectional variation in average bond yields.

To control for the fundamental risk of a bond above and beyond what is captured by the rating dummies, we use equity volatility estimated using daily equity returns of the bond issuer. Effectively, this variable is a combination of the issuer's asset volatility and leverage. We find this variable to be important in explaining yield spreads. As shown in the third column of Table 3.6, the slope coefficient on equity volatility is 0.06 with a t-stat of 3.82. That is, a ten percentage point increase in the equity volatility of a bond issuer is associated with a 60 bps increase in the bond yield. While adding γ improves the cross-sectional R-squared from a time-series average of 42.79% to 46.55%, adding equity volatility improves the R-squared to 53.07%. Such R-squared's, however, should be interpreted with caution since it is a time-series average of cross-sectional R-squared, and does not take into account the cross-sectional correlations in the regression residuals. By contrast, our reported Fama-MacBeth t-stat's do and γ has a stronger statistical significance. It is also interesting to observe that by adding equity volatility, the magnitudes of the rating dummies decrease significantly. This is to be expected since both equity volatility and rating dummies are designed to control for the bond's fundamental risk.

When used simultaneously to explain the cross-sectional variation in bond yield spreads,

[23]In the Appendix, we also report results with callable bonds excluded.

both γ and equity volatility are significant, with the slope coefficients for both remaining more or less the same as before. This implies a limited interaction between the two variables, which is to be expected since the equity volatility is designed to pick up the fundamental information about a bond while γ is to capture its liquidity information. Moreover, the statistical significance of our illiquidity measure γ is virtually unchanged.

Taking advantage of the fact that a substantial sub-sample of our bonds have CDS traded on their issuers, we use CDS spreads as an additional control for the fundamental risk of a bond. We find a very strong relation between bond yields and CDS yields: the coefficient is 0.53 with a t-stat of 12.20. For the sub-sample of bonds with CDS traded, and controlling for the CDS spread, we still find a strong cross-sectional relation between our illiquidity measure γ and bond yields. The economic significance of the relation is smaller: a difference of γ of 1 translates to a 14 bps difference in bond yields. On the other hand, the statistical significance improves because the sample is less noisy.

Adding three bond characteristics — age, maturity and issuance — to compete with γ, we find that the positive connection between γ and average bond yield spreads remains robust. Both bond age and bond issuance are known to be linked to liquidity.[24] Our results show that bond age remains an important liquidity variable above and beyond our γ measure. In particular, a bond that is one year older is associated with an increase of 6 bps in average yield spreads.[25]

Including the bond trading variables reveals that bonds with higher turnover and a large number of trades have higher average yield spreads. The slope coefficients for both variables are statistically significant. If one believes that more frequently traded bonds are more liquid, then this result would be puzzling. It is, however, arguable whether this variable actually captures the liquidity of a bond. We also find that bonds with higher average trade size have lower yield spreads. This result seems to be consistent with a liquidity explanation. Another possibility is that more frequent trades also reflect the speculative interest in the bonds, which can lead to higher yields. Overall, these variables are important control variables for us, since they are shown in Table 3.4 to be connected with our illiquidity measure γ. Our results show that these variables do not have a strong impact on the positive relation between our illiquidity measure γ and average yield spreads.

Finally, we examine the relative importance of the quoted bid-ask spreads and our illiquidity measure γ. As shown in the last two columns of Table 3.6, the quoted bid-ask spreads are negatively related average yield spreads. Using both the quoted bid-ask spreads and our

[24] See, for example, Houweling, Mentink, and Vorst (2005) and additional references therein.

[25] We find that this relation is robust to both an investment grade subsample and to a subsample of non-callable bonds.

illiquidity measure γ, we find a robust result for γ and a statistically insignificant result for the quoted bid-ask spread. This aspect of our result is curious since Chen, Lesmond, and Wei (2007) report a positive relation between the quoted bid-ask spreads and yield spreads. We find that this discrepancy is due to both the junk bonds in our sample and to 2008 data. This is not surprising given that the Phase I and II bonds in TRACE are predominantly investment grades, and the junk bonds covered by TRACE could be an unrepresentative pool. In unreported results, we consider the subsample of investment grade bonds, finding that γ remains an important explanatory variable for yields in this subsample. Furthermore, this holds even after controlling for quoted bid-ask spreads. In contrast to Chen, Lesmond, and Wei (2007), we do not find a significant positive relation between yield spreads and quoted bid-ask spreads. We do, however, find such a relation in the pre-2008 data.

3.4 Further Analyses of Illiquidity

3.4.1 Dynamic Properties of Illiquidity

To further examine the dynamic properties of the transitory component in corporate bonds, we measure the autocovariance of price changes that are separated by a few trades or a few days:

$$\gamma_\tau = -\mathrm{Cov}\left(\Delta P_t, \Delta P_{t+\tau}\right). \tag{3.3}$$

The illiquidity measure we have used so far is simply γ_1. For $\tau > 1$, γ_τ measures the extent to which the mean-reversion persists after the initial price reversal at $\tau = 1$. In Table 3.7, we report the γ_τ for $\tau = 1, 2, 3$, using trade-by-trade data. Clearly, the initial bounce back is the strongest while the mean-reversion still persists after skipping a trade. In particular, γ_2 is on average 0.11 with a robust t-stat of 15.50. At the individual bond level, 74% of the bonds have a statistically significant γ_2. After skipping two trades, the amount of residual mean-reversion dissipates further in magnitude. The cross-sectional average of γ_3 is only 0.027, although it is still statistically significant with a robust t-stat of 11.60. At the individual bond level, fewer than 14% of the bonds have a statistically significant γ_3.

The fact that the mean-reversion persists for a few trades before fully dissipating implies that autocovariance at the daily level is stronger than at the trade-by-trade level as it captures the effect cumulatively, as shown in Table 3.3. At the daily level, however, the mean-reversion dissipates rather quickly, with an insignificant γ_2 and γ_3. For brevity, we omit these results.

3.4.2 Asymmetry in Price Reversals

One interesting question regarding the mean-reversion captured in our main result is whether or not the magnitude of mean-reversion is symmetric in the sign of the initial price change. Specifically, with ΔP properly demeaned, let $\gamma^- = -Cov(\Delta P_t, \Delta P_{t+1} | \Delta P_t < 0)$ be a measure of mean-reversion conditioning on an initial price change that is negative, and let γ^+ be the counterpart conditioning on a positive price change. In a simple theory of liquidity based on costly market participation, Huang and Wang (2007) show that the bounce-back effect caused by illiquidity is more severe conditioning on an initial price movement that is negative, predicting a positive difference between γ^- and γ^+.

We test this hypothesis in Table 3.8, which shows that indeed there is a positive difference between γ^- and γ^+. Using trade-by-trade data, the cross-sectional average of $\gamma^- - \gamma^+$ is 0.1025 with a robust t-stat of 7.10. Skipping a trade, the asymmetry in γ_2 is on average 0.0488 with a robust t-stat of 8.74. Compared with how γ_τ dissipates across τ, this measure of asymmetry does not exhibit the same dissipating pattern. In fact, in the later sample period, the level of asymmetry for $\tau = 2$ is almost as important for the first-order mean-reversion, with an even higher statistical significance. Using daily data, the asymmetry is stronger, incorporating the cumulative effect from the transaction level. The cross-sectional average of $\gamma^- - \gamma^+$ is 0.19, which is close to 20% of the observed level of mean reversion. Skipping a day, however, produces no evidence of asymmetry, which is expected since there is very little evidence of mean-reversion at this level in the first place.

3.4.3 Trade Size and Illiquidity

Since our illiquidity measure is based on transaction prices, a natural question is how it is related to the sizes of these transactions. In particular, are reversals in price changes stronger for trades of larger or smaller sizes? In order to answer this question, we consider the autocovariance of price changes conditional on different trade sizes.

For a change in price $P_t - P_{t-1}$, let V_t denote the size of the trade associated with price P_t. The autocovariance of price changes conditional on trade size being in a particular range, say, R, is defined as

$$\text{Cov}\left(P_t - P_{t-1}, P_{t+1} - P_t, \,\middle|\, V_t \in R\right) , \tag{3.4}$$

where six brackets of trade sizes are considered in our estimation: (\$0, \$5K], (\$5K, \$15K], (\$15K, \$25K], (\$25K, \$75K], (\$75K, \$500K], and (\$500K, ∞), respectively. Our choice of the number of brackets and their respective cutoffs is influenced by the sample distribution of trade sizes. In particular, to facilitate the estimation of γ conditional on trade size, we need to have enough transactions within each bracket for each bond to obtain a reliable conditional γ.

For the same reason, we construct our conditional γ using trade-by-trade data. Otherwise, the data would be cut too thin at the daily level to provide reliable estimates of conditional γ. For each bond, we categorize transactions by their time-t trade sizes into their respective bracket s, with $s = 1, 2, \ldots, 6$, and collect the corresponding pairs of price changes, $P_t - P_{t-1}$ and $P_{t+1} - P_t$. Grouping such pairs of prices changes for each size bracket s and for each bond, we can estimate the autocovariance of the price changes, the negative of which is our conditional $\gamma(s)$.[26]

Equipped with the conditional γ, we can now explore the link between trade size and illiquidity. In particular, does $\gamma(s)$ vary with s and how? We answer this question by first controlling for the overall liquidity of the bond. This control is important as we find in Section 3.3.2 the average trade size of a bond is an important determinant of the cross-sectional variation of γ. So we first sort all bonds by their unconditional γ into quintiles and then examine the connection between $\gamma(s)$ and s within each quintile.

As shown in Panel A of Table 3.9, for each γ quintile, there is a pattern of decreasing conditional γ with increasing trade size and the relation is monotonic for all γ quintiles. For example, quintile 1 consists of bonds with the highest γ and therefore the least liquid in our sample. The mean γ is 2.13 for trade-size bracket 1 (less than \$5K) but it decreases to 0.69 for trade-size bracket 6 (greater than \$500K). The mean difference in γ between the trade-size bracket 1 and 6 is 1.37 and has a robust t-stat of 9.22. Likewise, for quintile 5, which consists of bonds with the lowest γ measure and therefore are the most liquid, the same pattern emerges. The average value of γ is 0.23 for the smallest trades and then decreases monotonically to 0.02 for the largest trades. The difference between the two is 0.21, with a robust t-stat of 8.20, indicating that the conditional γ between small and large size trades remains significant even for the most liquid bonds. To check the potential impact of outliers, we also report the median γ for different trade sizes. Although the magnitudes are smaller, the general pattern remains the same.

Overall, our results demonstrate a clear negative relation between trade sizes and our illiquidity measure.[27] The interpretation of this result, however, requires caution. It would be simplistic to infer from this pattern that larger trades face less illiquidity or have less impact on prices. It is important to realize that both trades sizes and prices are endogenous variables. Their relation arises from an equilibrium outcome in which traders of different

[26] Specifically, we compute six conditional covariances for each bond, one for each size bracket. The negative of these conditional covariances is our conditional γ.

[27] In the Appendix, we consider an alternative method of examining γ by trade size, simply cutting the data into trade size brackets and calculating γ separately for each bracket. We find a similar negative relation between trade sizes and our illiquidity measure using this methodology.

types optimally choose their trading strategies, taking into account the dynamics of the market including the actions of their own and others. Non-competitive factors such as negotiation power for large trades can also contribute to the relation between trade sizes and γ.

3.5 Illiquidity and Bid-Ask Spread

It is well known that the bid-ask spread can lead to negative autocovariance in price changes. For example, using a simple specification, Roll (1984) shows that when transactions prices bounce between bid and ask prices, depending on whether they are sell or buy orders from customers, their changes exhibit negative autocovariance even when the "underlying value" follows a random walk. Thus, it is important to ask whether or not the negative autocovariances documented in this chapter are simply a reflection of bid-ask bounce. Using quoted bid-ask spreads, we show in Table 3.3 that the associated bid-ask bounce can only generate a tiny fraction of the empirically observed autocovariance in corporate bonds. Quoted spreads, however, are mostly indicative rather than binding. Moreover, the structure of the corporate bond market is mostly over-the-counter, making it even more difficult to estimate the actual bid-ask spreads.[28] Thus, a direct examination of how bid-ask spreads contribute to our illiquidity measure γ is challenging.

We can, however, address this question to certain extent by taking advantage of the results by Edwards, Harris, and Piwowar (2007) (EHP hereafter). Using a more detailed version of the TRACE data that includes the side on which the dealer participated, they provide estimates of effective bid-ask spreads for corporate bonds. To examine the extent to which our illiquidity measure γ can be explained by the estimated bid-ask spread, we use our illiquidity measure γ to compute the implied bid-ask spreads, and compare them with the estimated bid-ask spreads reported by EHP. The actual comparison will not be exact, since our sample of bonds is different from theirs. Later in the section, we will discuss how this could affect our analysis.

It is first instructive to understand the theoretical underpinning of how our estimate of γ relates to the estimate of bid-ask spreads in EHP. In the Roll (1984) model, the transaction price P_t takes the form of equation (3.1), in which P is the sum of the fundamental value and a transitory component. Moreover, the transitory component equals to $\frac{1}{2}\,S\,q_t$ in the Roll model, with S being the bid-ask spread and q_t indicating the direction of trade. Specifically,

[28]The corporate bond market actually involves different trading platforms, which provide liquidity to different clienteles. In such a market, a single bid-ask spread can be too simplistic in capturing the actual spreads in the market.

q is $+1$ if the transaction is buyer initiated and -1 if it is seller initiated, assuming that the dealer takes the other side. More specifically, in the Roll model, we have

$$P_t = F_t + \tfrac{1}{2}\, S\, q_t\,. \tag{3.5}$$

If we further assume that q_t is *i.i.d.* over time, the autocovariance in price change then becomes $-(S/2)^2$, or $\gamma = (S/2)^2$. Conversely, we have

$$S_{\text{Roll}} = 2\,\sqrt{\gamma}\,, \tag{3.6}$$

where we call S_{Roll} the implied bid-ask spread.

EHP use an enriched Roll model, which allows the spreads to depend on trade sizes. In particular, they assume

$$P_t = F_t + \tfrac{1}{2}\, S(V_t)\, q_t\,, \tag{3.7}$$

where V_t is the size of the trade at time t.[29] Since the dataset used by EHP also contains information about q_t, they directly estimate the first difference of equation (3.7), assuming a factor model for the increments of F_t.

Table 3.10 reproduces the results of EHP, who estimate percentage bid-ask spreads for average trade sizes of \$5K, \$10K, \$20K, \$50K, \$100K, \$200K, \$500K and \$1MM. The cross-sectional medians of the percentage bid-ask spreads are 1.20%, 1.12%, 96 bps, 66 bps, 48 bps, 34 bps, 20 bps and 12 bps, respectively. To compare with their results, we form trade size brackets that center around their reported trade sizes. For example, to compare with their trade size \$10K, we calculate our illiquidity measure γ conditional on trade sizes falling between \$7.5K and \$15K, and then calculate the implied bid-ask spread. Using the average price for the respective bond, we further convert the spread to percentage spread so as to compare with the EHP result. The results are reported in Table 3.10, where to correct for the difference in our respective sample periods, we also report our implied bid-ask spreads for the period used by EHP. For the EHP sample period, the cross-sectional medians of our implied percentage bid-ask spreads are 1.82%, 1.80%, 1.59%, 1.23%, 91 bps, 68 bps, 57 bps, and 54 bps, respectively. As we move on to compare our median estimates to those in EHP, it should be mentioned that this is a simple comparison by magnitudes, not a formal statistical test.

Overall, our implied spreads are much higher than those estimated by EHP. For small

[29]The model EHP use has an additional feature. It distinguishes customer-dealer trades from dealer-dealer trades. The spread they estimate is for the customer-dealer trades. Thus, in (3.7), we simply do not identify dealer-dealer trades. This decreases our estimate of γ relative to EHP since we are including inter-dealer trades which have a smaller spread than customer-dealer trades.

trades, our median estimates of implied spreads are over 50% higher than those by EHP. Moving to larger trades, the difference becomes even more substantial. Our median estimates are close to doubling theirs for the average sizes of \$100K and \$200K, close to tripling theirs for the average size of \$500K, and more than quadrupling theirs for the average size of \$1,000K. In fact, our estimates are biased downward for the trade size group around \$1,000K, since our estimated bid-ask spreads include all trade sizes above \$750K, including trade sizes of \$2MM, \$5MM, and \$10MM, whose median bid-ask spreads are estimated by EHP to be 6 bps, 2 bps, and 2 bps, respectively. We have to group such trade sizes because in the publicly available TRACE data, the reported trade size is truncated at \$1MM for speculative grade bonds and at \$5MM for investment grade bonds.

In addition to differing in sample periods, which is easy to correct, our sample is also different from that used in EHP in the composition of the bonds that are used to estimate the bid-ask spreads. In particular, our selection criteria bias our sample towards highly liquid bonds. For example, to be included in our sample, the bond has to trade at least 75% of business days, while the median frequency of days with a trade is only 48% for the bonds used in EHP. The median average trade sizes is \$462K in 2003 and \$401K in 2004 for the bonds used in our sample, compared with \$240K for the bonds used in EHP; the median average number of trades per month is 148 in 2003 and 122 in 2004 for the bonds in our sample, while the median average number of trades per day is 1.1 for the bonds used in EHP. Given that more liquid bonds typically have smaller bid-ask spreads, the difference between our implied bid-ask spreads and EHP's estimates would have been even more drastic had we been able to match our sample of bonds to theirs. It is therefore our conclusion that the negative autocovariance in price changes observed in the bond market is much more substantial than merely the bid-ask effect. And our measure of illiquidity captures more broadly the impact of illiquidity in the market.

Finally, one might be curious as to what is the exact mechanism that drives our estimates apart from those by EHP. Within the Roll model as specified in equation (3.6), our estimates should be identical to theirs. In particular, using equation (3.5) to identify bid-ask spread S implies regressing ΔP_t on Δq_t. But using our model specified in equation (3.1) as a reference, it is possible that the transitory component u_t does not take the simple form of $\frac{1}{2}\,S\,q_t$. More specifically, the residual of this regression of ΔP_t on Δq_t might still exhibit a high degree of negative autocovariance, simply because u_t is not fully captured by $\frac{1}{2}\,S\,q_t$. If that is true, then our measure of illiquidity captures the transitory component more completely: both the bid-ask bounce associated with $\frac{1}{2}\,S\,q_t$ and the additional mean-reversion that is not related to bid-ask bounce. Overall, more analysis is needed, possibly with more detailed data as in EHP, in order to fully reconcile the two sets of results.

3.6 Conclusions

The main objective of this chapter is to gauge the level of illiquidity in the corporate bond market and to examine its key properties and implications. Using a theoretically motivated measure of illiquidity, i.e., the amount of price reversals as captured by the negative of autocovariance of prices changes, we show that this illiquidity measure is both statistically and economically significant for a broad cross-section of corporate bonds examined in this chapter. We demonstrate that the magnitude of the reversals is beyond what can be explained by bid-ask bounce. We also show that the reversals exhibit significant asymmetry: price reversals are on average stronger after a price reduction than a price increase.

We find that a bond's illiquidity is related to several bond characteristics. In particular, illiquidity increases with a bond's age and maturity, but decreases with its rating and issue size. As compared to a bond's idiosyncratic return volatility, a bond's illiquidity shows limited relation with its market risk exposures, as measured by its beta with respect to the stock and bond market indices. We also find that price reversals are inversely related to trade sizes. That is, prices changes accompanied by small trades exhibit stronger reversals than those accompanied by large trades.

Furthermore, the illiquidity of individual bonds fluctuates substantially over time. More interestingly, these time fluctuations display important commonalities. For example, the median illiquidity over all bonds, which represents a market-wide illiquidity, increases sharply during the periods of market turmoil such as the downgrade of Ford and GM to junk status around May of 2005, the sub-prime market crisis starting in August 2007, and in late 2008 when Lehman filed for bankruptcy. Exploring the relation between changes in the market-wide illiquidity and other market variables, we find that changes in illiquidity are positively related to changes in VIX and that this relation is not driven solely by the events in 2008. During pre-2008 periods, we also find that aggregate illiquidity tends to rise following down markets.

We also find important pricing implications associated with bond illiquidity. Our result shows that for two bonds in the same rating category, a one-standard-deviation difference in their illiquidity measure would set their yield spreads apart by almost 40 bps. This result remains robust in magnitude and statistical significance, after controlling for bond fundamental information and bond characteristics including those commonly related to bond liquidity.

Our results raise several questions concerning the liquidity of corporate bonds. First, what are the underlying factors giving rise to the high level of illiquidity? This question is particularly pressing when we contrast the magnitude of our illiquidity measure in the corporate bond market against that in the equity market. Second, what causes the fluctuations in the overall level of illiquidity in the market? Are these fluctuations merely another manifestation of more fundamental risks or a reflection of new sources of risks such as a liquidity risk? Third, does the high level of illiquidity for the corporate bonds indicate any inefficiencies in the market? If so, what would be the policy remedies? We leave these questions for future work.

3.7 Appendix

3.7.1 Tables and Figures

Table 3.1: Summary Statistics: Year-by-Year

	Panel A: Our Sample																	
	2003			2004			2005			2006			2007			2008		
	mean	med	std	mean	med	std	mean	med	std	mean	med	std	mean	med	std	mean	med	std
#Bonds	773			1,150			1,093			970			831			670		
Issuance	1,012	1,000	730	864	700	683	861	700	689	844	681	666	842	650	676	847	688	678
Rating	5.59	5.67	2.55	6.77	6.00	3.84	7.05	6.00	4.07	7.41	6.00	4.52	7.51	6.00	4.73	7.98	7.00	4.71
Maturity	7.33	5.23	6.80	7.88	5.66	7.24	7.36	5.19	7.24	7.02	4.66	7.24	6.92	4.42	7.31	6.70	3.96	7.35
Coupon	5.87	6.00	1.66	5.87	6.13	1.87	5.83	6.00	1.87	5.78	5.88	1.90	5.81	6.00	1.90	5.90	6.00	1.86
Age	2.68	1.94	2.62	3.17	2.38	2.94	3.90	3.17	2.94	4.74	3.96	2.95	5.69	4.73	3.07	6.66	5.74	3.15
Turnover	11.57	8.31	9.52	9.40	7.10	7.51	8.26	6.17	6.90	6.30	5.11	4.78	5.08	4.10	3.90	4.82	4.11	3.33
Trd Size	584	462	488	527	401	477	430	336	385	383	293	344	336	256	313	257	183	247
#Trades	243	148	360	180	122	191	201	121	291	158	109	144	143	102	127	200	126	219
Avg Ret	0.59	0.38	0.85	0.64	0.35	1.57	-0.03	0.17	1.04	0.63	0.39	1.10	0.39	0.45	0.71	-1.18	0.27	3.59
Volatility	2.49	2.25	1.48	1.76	1.61	1.09	2.17	1.44	2.87	1.79	1.22	2.06	1.86	1.33	1.82	6.99	4.03	8.68
Price	108	109	10	106	106	11	103	103	11	100	101	11	102	101	12	96	100	20
	Panel B: TRACE																	
	2003			2004			2005			2006			2007			2008		
	mean	med	std	mean	med	std	mean	med	std	mean	med	std	mean	med	std	mean	med	std
#Bonds	4,161			15,270			23,415			22,627			23,640			23,442		
Issuance	470	260	542	220	85	384	189	50	363	209	56	371	221	42	403	219	30	417
Rating	5.30	5.00	2.60	6.46	6.00	3.26	7.37	7.00	4.00	7.18	6.00	4.26	6.77	6.00	4.19	6.75	6.00	4.29
Maturity	8.51	4.55	10.77	8.34	5.38	8.88	7.86	5.05	8.41	8.01	5.12	8.66	8.08	5.05	8.97	8.00	4.94	8.97
Coupon	6.51	6.75	1.69	5.76	5.85	1.96	5.80	5.70	2.16	5.74	5.63	2.13	5.60	5.55	2.16	5.35	5.50	2.32
Age	4.61	3.75	3.87	3.24	1.82	3.61	3.37	2.00	3.73	3.64	2.44	3.78	3.77	2.83	3.71	4.06	3.33	3.70
Turnover	5.87	3.82	6.36	5.20	2.58	7.07	3.95	2.44	4.52	3.56	2.18	4.18	3.29	1.97	4.02	3.10	1.84	3.69
Trd Size	1,017	532	1,263	534	59	991	477	55	869	509	58	905	487	49	899	386	46	761
#Trades	66	19	184	31	9	86	26	6	90	21	5	56	21	5	71	28	5	97
Avg Ret	0.63	0.38	4.07	0.50	0.28	2.56	0.11	0.21	2.27	0.85	0.54	2.06	0.36	0.45	2.02	-0.90	0.14	6.33
Volatility	2.73	2.37	2.27	1.93	1.68	1.29	2.65	1.94	2.81	2.31	1.75	2.28	2.42	1.96	2.24	9.34	5.79	11.08
Price	109	110	12	104	103	16	100	100	17	99	99	19	100	100	34	92	97	22

Panel A presents summary statistics for our sample and Panel B for bonds traded in TRACE. *#Bonds* is the number of bonds in a given period. *Issuance* is the bond's amount outstanding in millions of dollars. *Rating* is a numerical translation of Moody's rating: 1=Aaa and 21=C. *Maturity* is the bond's time to maturity in years. *Coupon*, reported only for fixed coupon bonds, is the bond's coupon payment in percentage. *Age* is the time since issuance in years. *Turnover* is the bond's monthly trading volume as a percentage of its issuance. *Trd Size* is the average trade size of the bond in thousands of dollars of face value. *#Trades* is the bond's total number of trades in a month. Med and std are the time-series averages of the cross-sectional medians and standard deviations. For each bond, we also calculate the time-series mean and standard deviation of its monthly log returns, whose cross-sectional mean, median and standard deviation are reported under *Avg Ret* and *Volatility*. *Price* is the average market value of the bond in dollars.

Table 3.2: Summary Statistics: Full Sample

	Our Sample						TRACE					
	2003			Full			2003			Full		
	mean	med	std	mean	med	std	mean	med	std	mean	med	std
#Bonds	773			1,205			4,161			38,012		
Issuance	1,012	1,000	730	879	717	685	470	260	542	209	40	407
Rating	5.59	5.67	2.55	7.13	6.00	4.19	5.30	5.00	2.60	6.87	6.00	4.04
Maturity	7.33	5.23	6.80	6.36	3.83	6.90	8.51	4.55	10.77	7.25	4.25	8.42
Coupon	5.87	6.00	1.66	5.87	6.00	1.87	6.51	6.75	1.69	5.51	5.60	2.43
Age	2.68	1.94	2.62	4.44	3.49	2.85	4.61	3.75	3.87	3.31	2.05	3.52
Turnover	11.57	8.31	9.52	7.51	6.38	4.90	5.87	3.82	6.36	4.07	2.49	4.83
Trd Size	584	462	488	417	338	348	1,017	532	1,263	517	61	930
#Trades	243	148	360	181	126	184	66	19	184	26	6	88
Avg Ret	0.59	0.38	0.85	0.20	0.31	0.76	0.63	0.38	4.07	0.05	0.33	3.90
Volatility	2.49	2.25	1.48	3.08	1.81	3.73	2.73	2.37	2.27	4.49	2.73	5.81
Price	108	109	10	103	103	11	109	110	12	98	99	31

#Bonds is the number of bonds in a given period. *Issuance* is the bond's amount outstanding in millions of dollars. *Rating* is a numerical translation of Moody's rating: 1=Aaa and 21=C. *Maturity* is the bond's time to maturity in years. *Coupon*, reported only for fixed coupon bonds, is the bond's coupon payment in percentage. *Age* is the time since issuance in years. *Turnover* is the bond's monthly trading volume as a percentage of its issuance. *Trd Size* is the average trade size of the bond in thousands of dollars of face value. *#Trades* is the bond's total number of trades in a month. Med and std are the time-series averages of the cross-sectional medians and standard deviations. For each bond, we also calculate the time-series mean and standard deviation of its monthly log returns, whose cross-sectional mean, median and standard deviation are reported under *Avg Ret* and *Volatility*. *Price* is the average market value of the bond in dollars.

Table 3.3: Measure of Illiquidity $\gamma = -\mathrm{Cov}\left(P_t - P_{t-1}, P_{t+1} - P_t\right)$

Panel A: Individual Bonds							
	2003	2004	2005	2006	2007	2008	Full
Trade-by-Trade Data							
Mean γ	0.67	0.68	0.57	0.48	0.52	0.89	0.60
Median γ	0.46	0.40	0.32	0.27	0.31	0.63	0.41
Per t $\geq$ 1.96	99.35	97.56	99.63	99.59	99.52	98.06	99.83
Robust t-stat	16.79	16.10	18.61	19.97	19.20	16.21	22.43
Daily Data							
Mean γ	1.05	1.00	0.90	0.77	0.97	2.39	1.04
Median γ	0.71	0.55	0.46	0.42	0.59	1.50	0.67
Per t $\geq$ 1.96	94.55	90.43	96.15	96.27	94.90	93.70	98.76
Robust t-stat	22.29	17.49	26.38	25.10	23.01	16.04	28.35
Panel B: Bond Portfolios							
	2003	2004	2005	2006	2007	2008	Full
Equal-weighted	-0.0021	-0.0044	-0.0024	0.0009	-0.0004	-0.0393	-0.0087
t-stat	-0.38	-1.17	-0.86	0.75	-0.21	-1.08	-1.29
Issuance-weighted	0.0019	-0.0040	-0.0011	0.0008	0.0006	-0.0402	-0.0077
t-stat	0.28	-0.98	-0.35	0.48	0.19	-1.04	-1.09
Panel C: Implied by Quoted Bid-Ask Spreads							
	2003	2004	2005	2006	2007	2008	Full
Mean implied γ	0.045	0.040	0.050	0.050	0.051	0.056	0.045
Median implied γ	0.037	0.030	0.027	0.024	0.027	0.050	0.031

At the individual bond level, γ is calculated using either trade-by-trade or daily data. Per t-stat ≥ 1.96 reports the percentage of bond with statistically significant γ. Robust t-stat is a test on the cross-sectional mean of γ with standard errors corrected for cross-sectional and time-series correlations. At the portfolio level, γ is calculated using daily data and the Newey-West t-stats are reported. Monthly quoted bid-ask spreads, which we have data for 1,170 out of 1,205 bonds in our sample, are used to calculate the implied γ.

Table 3.4: Variation in γ and Bond Characteristics

Cons	1.36	1.59	2.53	1.15	1.35	1.59	1.49
	[4.55]	[4.71]	[5.23]	[2.91]	[6.80]	[7.97]	[3.12]
Age	0.10	0.08	0.05	0.09	0.09	0.09	0.09
	[3.49]	[3.61]	[2.31]	[3.07]	[2.91]	[2.82]	[3.75]
Maturity	0.07	0.08	0.08	0.08	0.07	0.08	0.08
	[11.51]	[9.02]	[10.28]	[9.52]	[9.95]	[8.58]	[8.25]
ln(Issuance)	-0.26	-0.27	-0.06	-0.30	-0.24	-0.28	-0.25
	[-4.20]	[-5.75]	[-3.95]	[-10.05]	[-5.55]	[-6.18]	[-3.78]
Rating	0.04	0.06	0.07	0.05	0.03	0.06	0.01
	[4.10]	[5.41]	[6.13]	[6.24]	[2.99]	[7.53]	[1.59]
beta (stock)	0.58						
	[4.84]						
beta (bond)	0.18						
	[1.75]						
Turnover		-0.01					
		[-1.04]					
ln(Trd Size)			-0.41				
			[-4.72]				
ln(Num Trades)				0.11			
				[1.45]			
Quoted BA Gamma					3.90		
					[4.75]		
CDS Dummy						-0.07	
						[-0.94]	
CDS Spread							0.06
							[5.86]
Obs	4,781	5,323	5,323	5,323	5,076	4,565	3,317
R-sqd	27.49	25.86	29.17	25.85	26.05	24.71	23.33

Panel regression with γ as the dependent variable. T-stats are reported in square brackets using standard errors clustered by year. *Issuance* is the bond's amount outstanding in millions of dollars. *Rating* is a numerical translation of Moody's rating: 1=Aaa and 21=C. *Age* is the time since issuance in years. *Maturity* is the bond's time to maturity in years. *Turnover* is the bond's monthly trading volume as a percentage of its issuance. *Trd Size* is the average trade size of the bond in thousands of dollars of face value. *#Trades* is the bond's total number of trades in a month. *beta(stock) and beta(bond)* are obtained by regressing weekly bond returns on weekly returns on the CRSP value-weighted index and the Lehman US bond index. *Quoted BA* γ is the γ implied by the quoted bid-ask spreads. *CDS Dummy* is 1 if the bond has credit default swaps traded on its issuer. *CDS Spread* is the spread on the five-year CDS of the bond issuer in %. Data is from 2003 to 2008 except for regressions with CDS information which start in 2004.

Table 3.5: Time Variation in γ and Market Variables

				Panel A: 2003-2008				
Cons	0.0224 [1.00]	0.0068 [0.86]	0.0042 [0.46]	0.0261 [1.06]	0.0251 [0.96]	0.0241 [1.15]	0.0205 [1.15]	0.0088 [1.11]
Δ VIX	0.0351 [8.15]							0.0339 [5.70]
Δ Bond Volatility		0.0409 [2.03]						0.0375 [3.26]
Δ CDS Index			0.2288 [1.94]					0.0098 [0.15]
Δ Term Spread				0.3496 [1.45]				
Δ Default Spread					-0.0283 [-0.24]			
Lagged Stock Return						-0.0061 [-0.81]		
Lagged Bond Return							-0.0303 [-1.62]	
Adj R-sqd (%)	64.12	6.50	14.68	10.32	-1.40	-0.10	6.08	69.96
				Panel B: 2003-2007				
Cons	0.0012 [0.13]	0.0004 [0.05]	0.0014 [0.26]	0.0033 [0.35]	0.0003 [0.04]	0.0094 [0.92]	0.0021 [0.24]	0.0069 [1.08]
Δ VIX	0.0187 [3.46]							0.0156 [3.29]
Δ Bond Volatility		-0.0054 [-0.64]						
Δ CDS Index			0.3500 [2.67]					0.0809 [0.89]
Δ Term Spread				0.0868 [1.85]				
Δ Default Spread					0.2705 [1.72]			
Lagged Stock Return						-0.0088 [-2.27]		-0.0047 [-1.63]
Lagged Bond Return							-0.0127 [-3.65]	-0.0058 [-1.10]
Adj R-sqd (%)	40.29	-1.22	32.25	3.20	13.25	11.36	6.17	50.58

Monthly changes in γ regressed on monthly changes in bond index volatility, VIX, CDS index, term spread, default spread, and lagged stock and bond returns. The Newey-West t-stats are reported in square brackets. Panel A includes data through the end of 2008. Panel B includes data through the end of 2007. Regressions with CDS Index do not include 2003 data.

Table 3.6: Bond Yield Spread and Illiquidity Measure γ

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Cons	1.13	0.96	-0.70	-0.70	0.06	-0.36	-0.64	0.02	-1.31	1.66	-0.26	0.20
	[3.46]	[3.31]	[-1.55]	[-1.58]	[0.35]	[-1.20]	[-1.91]	[0.08]	[-2.55]	[2.11]	[-1.17]	[1.76]
γ		0.21		0.21	0.14	0.14	0.15	0.13	0.13		0.21	0.13
		[7.08]		[7.01]	[8.23]	[4.51]	[4.72]	[4.27]	[4.59]		[6.38]	[9.32]
Equity Vol			0.06	0.06	0.02	0.06	0.05	0.06	0.05		0.06	0.02
			[3.82]	[3.40]	[2.99]	[3.41]	[3.36]	[3.43]	[3.32]		[3.55]	[2.96]
CDS Spread					0.53							0.52
					[12.20]							[11.90]
Age						0.06	0.07	0.05	0.07			
						[2.60]	[3.05]	[2.47]	[2.76]			
Maturity						0.00	0.00	0.00	0.00			
						[0.14]	[0.04]	[0.10]	[0.21]			
ln(Issuance)						-0.10	-0.08	-0.05	-0.19			
						[-2.26]	[-1.97]	[-1.49]	[-3.37]			
Turnover							0.03					
							[5.89]					
ln(Trd Size)								-0.13				
								[-2.56]				
ln(#Trades)									0.31			
									[5.00]			
Quoted B/A Spread										-1.34	-0.98	-0.28
										[-1.09]	[-1.12]	[-0.63]
Call Dummy	-0.44	-0.47	0.12	0.07	0.04	0.07	0.09	0.08	0.12	-0.38	0.18	0.08
	[-1.05]	[-1.10]	[2.14]	[1.52]	[1.01]	[1.61]	[2.13]	[1.56]	[2.19]	[-1.11]	[1.48]	[1.04]
A Dummy	0.69	0.66	0.19	0.16	0.21	0.16	0.15	0.18	0.23	0.67	0.09	0.16
	[1.52]	[1.52]	[1.14]	[1.10]	[1.46]	[1.05]	[0.97]	[1.18]	[1.39]	[1.56]	[0.96]	[1.64]
BAA Dummy	1.19	1.08	0.74	0.67	0.50	0.64	0.59	0.71	0.76	1.18	0.63	0.49
	[2.85]	[2.91]	[2.77]	[2.68]	[2.51]	[2.42]	[2.12]	[2.57]	[2.49]	[3.15]	[3.42]	[2.79]
Junk Dummy	5.41	5.00	3.86	3.57	1.28	3.53	3.48	3.59	3.62	5.43	3.58	1.36
	[4.38]	[4.12]	[3.96]	[3.65]	[4.20]	[3.76]	[3.70]	[3.73]	[3.72]	[3.97]	[3.53]	[3.57]
Obs	679	670	679	670	502	670	670	670	670	633	627	472
R-sqd (%)	42.79	46.55	53.07	56.24	75.33	58.74	59.43	59.01	60.22	43.37	57.25	76.40

Monthly Fama-MacBeth cross-sectional regression with the bond yield spread as the dependent variable. The t-stats are reported in square brackets calculated using Fama-MacBeth standard errors with serial correlation corrected using Newey-West. The reported number of observations are the average number of observations per period. The reported R-squareds are the time-series averages of the cross-sectional R-squareds. γ is the monthly estimate of illiquidity measure using daily data. *Equity Vol* is estimated using daily equity returns of the bond issuer. *Age*, *Maturity*, *Issuance*, *Turnover*, *Trd Size*, and *#Trades* are as defined in Table 3.4. *Call Dummy* is one if the bond is callable and zero otherwise. Convertible and putable bonds are excluded from the regression. The sample period is from May 2003 through December 2008.

Table 3.7: Dynamics of Illiquidity: $\gamma_\tau = -\text{Cov}(P_t - P_{t-1}, P_{t+\tau} - P_{t+\tau-1})$

		2003	2004	2005	2006	2007	2008	Full
$\tau = 1$	Mean γ	0.668	0.679	0.575	0.477	0.520	0.887	0.603
	Median γ	0.463	0.400	0.323	0.267	0.307	0.633	0.407
	Per t $\geq$ 1.96	99.35	97.56	99.63	99.59	99.52	98.06	99.83
	Robust t-stat	16.79	16.10	18.61	19.97	19.20	16.21	22.43
$\tau = 2$	Mean γ	0.084	0.068	0.079	0.056	0.105	0.341	0.106
	Median γ	0.038	0.025	0.032	0.027	0.060	0.211	0.061
	Per t $\geq$ 1.96	27.85	20.31	38.06	38.56	54.64	76.38	73.53
	Robust t-stat	10.36	7.61	12.73	10.08	13.47	14.23	15.50
$\tau = 3$	Mean γ	0.011	0.023	0.022	0.030	0.029	0.072	0.027
	Median γ	0.006	0.005	0.005	0.006	0.008	0.020	0.008
	Per t $\geq$ 1.96	4.92	5.75	6.77	8.25	6.76	11.51	13.78
	Robust t-stat	2.98	4.37	8.59	7.54	7.93	7.55	11.60

For each bond, its γ_τ, $\tau = 1, 2, 3$, is calculated using trade-by-trade data. Per t-stat ≥ 1.96 reports the percentage of bond with statistically significant γ. Robust t-stat is a test on the cross-sectional mean of γ with standard errors corrected for cross-sectional and time-series correlations.

Table 3.8: Asymmetry in γ

Panel A: Using trade-by-trade data

Tau		2003	2004	2005	2006	2007	2008	Full
1	Mean	0.1547	0.0739	0.0120	0.0394	0.0679	0.1209	0.1025
	Median	0.1421	0.0183	-0.0064	0.0225	0.0627	0.1080	0.0556
	CS t-stat	8.42	3.90	0.92	3.36	5.62	6.76	7.79
	Robust t-stat	6.86	3.61	0.89	3.22	5.37	6.25	7.10
2	Mean	0.0379	0.0336	0.0428	0.0413	0.0542	0.0732	0.0488
	Median	0.0147	0.0078	0.0096	0.0169	0.0263	0.0519	0.0189
	CS t-stat	5.23	4.24	8.94	8.93	8.26	4.62	9.56
	Robust t-stat	5.21	3.94	7.79	7.59	7.65	4.38	8.74

Panel B: Using daily data

Tau		2003	2004	2005	2006	2007	2008	Full
1	Mean	0.2993	0.1726	0.1155	0.1240	0.1774	0.2046	0.1910
	Median	0.2006	0.0426	0.0171	0.0439	0.1031	0.1763	0.0892
	CS t-stat	10.64	5.47	5.06	5.79	6.30	2.74	9.89
	Robust t-stat	9.46	4.92	4.65	5.00	5.96	2.33	8.57
2	Mean	-0.0028	0.0043	0.0100	0.0003	0.0107	-0.0324	-0.0091
	Median	0.0002	0.0007	0.0011	0.0008	0.0003	0.0037	0.0007
	CS t-stat	-0.25	0.28	1.09	0.03	0.77	-0.59	-0.95
	Robust t-stat	-0.21	0.31	0.94	0.03	0.68	-0.47	-0.71

Asymmetry in γ is measured by the difference between γ^- and γ^+, where $\gamma^- = -E(\Delta P_{t+1}\Delta P_t|\Delta P_t < 0)$, with ΔP properly demeaned, measures the price reversal conditioning on a negative price movement. Likewise, γ^+ measures the price reversal conditioning on a positive price movement. Robust t-stat is a pooled test on the mean of $\gamma^- - \gamma^+$ with standard errors clustered by bond and day. CS t-stat is the cross-sectional t-stat.

Table 3.9: Variation of γ with Trade Size

γ Quint	trade size =	1	2	3	4	5	6	1 - 6
1	Mean	2.13	1.64	1.47	1.29	0.89	0.69	1.37
	Median	1.94	1.53	1.40	1.25	0.84	0.53	1.26
	Robust t-stat	13.14	10.64	9.73	9.56	8.51	6.17	9.22
2	Mean	1.13	0.93	0.83	0.67	0.38	0.24	0.88
	Median	1.05	0.86	0.78	0.62	0.36	0.19	0.82
	Robust t-stat	10.59	10.16	10.60	11.65	13.92	10.15	8.54
3	Mean	0.69	0.56	0.49	0.38	0.22	0.12	0.57
	Median	0.61	0.51	0.45	0.36	0.21	0.10	0.48
	Robust t-stat	8.32	12.26	12.17	12.58	14.19	11.29	7.05
4	Mean	0.43	0.34	0.28	0.20	0.12	0.06	0.37
	Median	0.37	0.30	0.25	0.19	0.11	0.05	0.31
	Robust t-stat	8.69	12.23	12.20	13.31	15.57	11.25	7.64
5	Mean	0.23	0.17	0.14	0.10	0.05	0.02	0.21
	Median	0.21	0.17	0.13	0.09	0.05	0.02	0.18
	Robust t-stat	8.93	13.95	12.41	15.76	18.35	13.23	8.20

Trade size is categorized into 6 groups with cutoffs of \$5K, \$15K, \$25K, \$75K, and \$500K. $\gamma = -\text{Cov}(P_t - P_{t-1}, P_{t+1} - P_t)$. γ is calculated conditioning on the trade size associated with P_t. Bonds are sorted by their "unconditional" γ into quintiles, and the variation of γ by trade size is reported for each quintile group. The trade-by-trade data is used in the calculation. For the daily data, the results are similar but stronger.

Table 3.10: Implied and Estimated Bid-Ask Spreads

	Full Sample Period			EHP Subperiod					
	γ-Implied			γ-Implied			EHP Estimated		
trade size	#bonds	Mean	Med	#bonds	Mean	Med	EHP Size	Mean	Med
$\leq$ 7,500	1,148	2.21	1.90	938	2.07	1.82	5K	1.50	1.20
(7500, 15K]	1,156	1.98	1.73	1,036	1.98	1.80	10K	1.42	1.12
(15K, 35K]	1,160	1.80	1.50	1,043	1.80	1.59	20K	1.24	0.96
(35K, 75K]	1,152	1.56	1.25	906	1.38	1.23	50K	0.92	0.66
(75K, 150K]	1,124	1.28	1.02	817	1.00	0.91	100K	0.68	0.48
(150K, 350K]	1,025	0.94	0.76	678	0.68	0.68	200K	0.48	0.34
(350K, 750K]	1,066	0.82	0.69	786	0.60	0.57	500K	0.28	0.20
> 750K	1,093	0.74	0.61	950	0.52	0.54	1,000K	0.18	0.12

The bid-ask spreads are calculated as a percentage of the market value of the bond and are reported in percentages. The EHP bid-ask spread estimates are from Table 4 of Edwards, Harris, and Piwowar (2007), and the EHP subperiod is Jan. 2003 to Jan. 2005. Our bid-ask spreads are obtained using Roll's measure: $2\sqrt{\gamma}$ divided by the average market value of the bond. The sample of bonds differs from that in EHP, and our selection criteria biases us toward more liquid bonds with smaller bid-ask spreads.

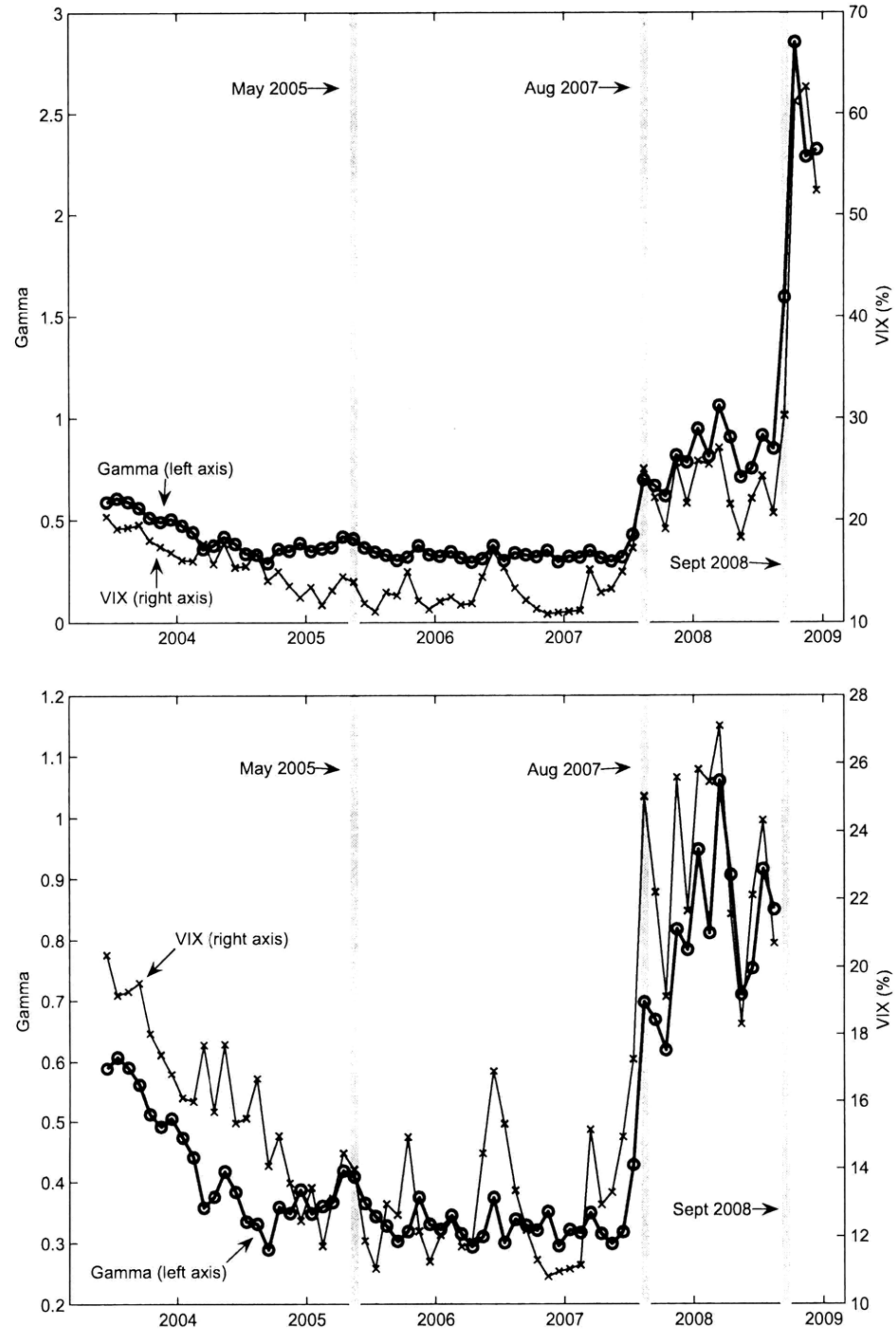

Figure 3-1: Monthly time-series of aggregate γ and CBOE VIX
The bottom panel is for the subperiod before the collapse of Lehman.

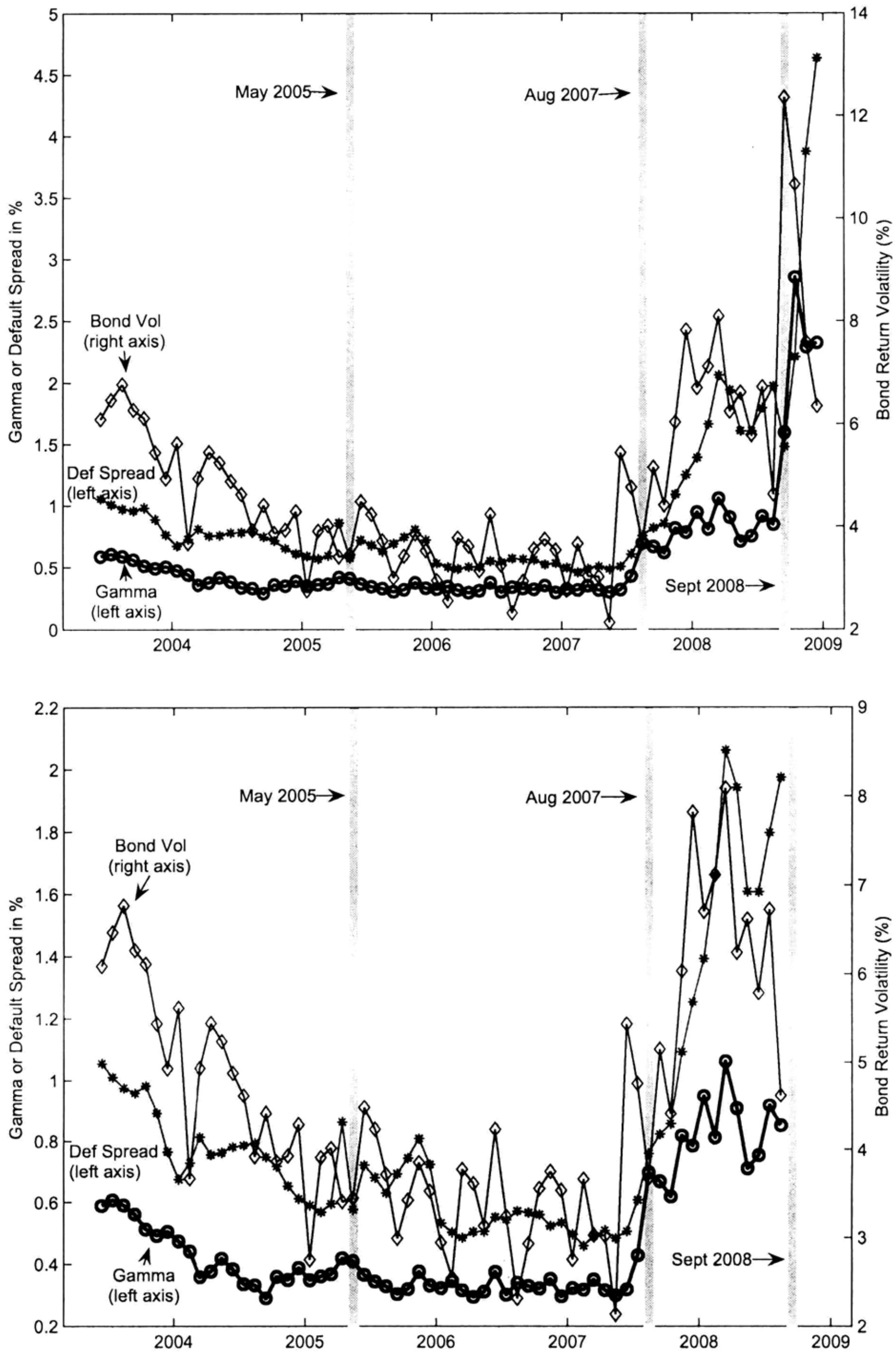

Figure 3-2: Monthly time-series of aggregate γ and macroeconomic variables The bottom panel is for the subperiod before the collapse of Lehman.

3.7.2 Robustness Tables

Measure of Illiquidity, Log Price Changes

In Table 3.11, we reproduce the results in Table 3.3, but use changes in log prices. In particular, we define γ as

$$\gamma = -\text{Cov}\left(Ln(\Delta P_t), Ln(\Delta P_{t+1})\right). \tag{3.8}$$

Note that reported γ's are scaled by 10,000 for easier comparison with γ's calculated using ΔP.

Time Variation in Gamma

In Table 3.12, we present regression results for determinants of the aggregate γ. This table corresponds to Panel A of Table 3.5, but with γ calculated using log price changes rather than price changes.

Cross-Sectional Determinants of Yield Spreads

We report results using log price changes rather than price changes in Table 3.13. Our results remain largely unchanged. In Table 3.14, we consider only the subset of non-callable bonds. As callable bonds of the poorest credit quality are unlikely to be called, bond age may actually be a proxy for credit quality in a sample of callable bonds. We find that in the subsample of non-callable bonds, age remains an important determinant of yield spread.

Gamma by Trade Size

In Table 3.15, we consider γ calculated using only trades of certain sizes. First, we take all trades for a particular bond and sort these trades by into the smallest 30% of trade size, middle 40%, and largest 30%. We then calculate γ using only trades from a given bin to estimate small trade, medium trade, and large trade γ's. These results are supplemental to those presented in Table 3.9, but provide an additional robustness check as these γ's are calculated solely with a subset of trades of a given size rather than conditioning on the trade size at t as in equation (3.4). Furthermore, the size of trades is now grouped relative to a bond's other trades rather than with respect to a fixed cut-off.

Table 3.11: Measure of Illiquidity $\gamma = -\text{Cov}\left(lnP_t - lnP_{t-1}, lnP_{t+1} - lnP_t\right)$

	2003	2004	2005	2006	2007	2008	Full
Panel A: Individual Bonds							
Trade-by-Trade Data							
Mean γ	0.65	0.68	0.63	0.53	0.56	1.45	0.70
Median γ	0.41	0.36	0.30	0.27	0.30	0.70	0.39
Per t $\geq$ 1.96	99.35	97.38	99.63	99.59	99.52	97.91	99.83
Pooled t-stat	15.01	15.07	17.81	19.52	18.22	12.85	20.21
Daily Data							
Mean γ	1.01	1.06	1.01	0.88	1.05	5.05	1.38
Median γ	0.62	0.50	0.44	0.40	0.60	1.74	0.65
Per t $\geq$ 1.96	94.29	90.08	95.33	96.48	94.66	90.85	97.93
Pooled t-stat	17.74	12.84	21.50	21.92	19.36	10.22	18.32
Panel B: Bond Portfolios							
	2003	2004	2005	2006	2007	2008	Full
Equal-weighted	-0.0010	-0.0037	-0.0031	0.0010	-0.0005	-0.1222	-0.0239
t-stat	-0.20	-1.06	-0.93	0.79	-0.21	-1.37	-1.44
Issuance-weighted	0.0021	-0.0035	-0.0012	0.0007	0.0004	-0.1361	-0.0249
t-stat	0.36	-0.97	-0.32	0.47	0.13	-1.77	-1.74
Panel C: Implied by Quoted Bid-Ask Spreads							
	2003	2004	2005	2006	2007	2008	Full
Mean implied γ	0.045	0.041	0.061	0.058	0.058	0.084	0.052
Median implied γ	0.032	0.027	0.026	0.025	0.026	0.052	0.030

At the individual bond level, γ is calculated using either trade-by-trade or daily data. γ is scaled by 10,000. Per t-stat $\geq$ 1.96 reports the percentage of bond with statistically significant γ. Robust t-stat is a test on the cross-sectional mean of γ with standard errors corrected for cross-sectional and time-series correlations. At the portfolio level, γ is calculated using daily data and the Newey-West t-stats are reported. Monthly quoted bid-ask spreads, which we have data for 1,170 out of 1,205 bonds in our sample, are used to calculate the implied γ.

Table 3.12: Time Variation in γ and Market Variables, Ln Prices

Cons	0.0346 [1.05]	0.0084 [0.75]	0.0031 [0.35]	0.0404 [1.06]	0.0401 [1.08]	0.0366 [1.16]	0.0305 [1.29]	0.0148 [1.07]
Δ VIX	0.0578 [5.51]							0.0599 [4.32]
Δ Bond Volatility		0.0482 [1.92]						0.0348 [3.01]
Δ CDS Index			0.3501 [1.63]					-0.0624 [-0.51]
Δ Term Spread				0.6092 [1.35]				
Δ Default Spread					-0.0767 [-0.39]			
Lagged Stock Return						-0.0073 [-0.54]		
Lagged Bond Return							-0.0544 [-1.51]	
Adj R-sqd (%)	63.54	2.53	12.18	11.63	-1.17	-0.80	7.43	65.30

Monthly changes in γ regressed on monthly changes in bond index volatility, VIX, CDS index, term spread, default spread, and lagged stock and bond returns. The Newey-West t-stats are reported in square brackets. Regressions with CDS Index do not include 2003 data. γ is calculated using log price changes and is scaled by 10,000.

Table 3.13: Bond Yield Spread and Illiquidity Measure γ, ln P

Cons	1.13	0.97	-0.70	-0.58	0.14	-0.34	-0.64	-0.05	-1.21	1.66	-0.23	0.26
	[3.46]	[3.14]	[-1.55]	[-1.52]	[0.95]	[-1.30]	[-2.08]	[-0.22]	[-2.72]	[2.11]	[-1.11]	[2.77]
γ		0.22		0.22	0.12	0.17	0.17	0.16	0.16		0.21	0.10
		[4.45]		[5.14]	[3.75]	[3.70]	[3.83]	[3.52]	[3.65]		[4.80]	[3.12]
Equity Vol			0.06	0.05	0.02	0.05	0.05	0.05	0.05		0.05	0.02
			[3.82]	[3.51]	[3.67]	[3.53]	[3.48]	[3.55]	[3.42]		[3.72]	[3.65]
CDS Spread					0.52							0.52
					[11.16]							[10.96]
Age						0.05	0.06	0.04	0.06			
						[2.54]	[3.07]	[2.36]	[2.70]			
Maturity						-0.00	-0.00	-0.00	-0.00			
						[-0.11]	[-0.18]	[-0.16]	[-0.08]			
ln(Issuance)						-0.07	-0.05	-0.03	-0.16			
						[-2.13]	[-1.76]	[-1.24]	[-3.51]			
Turnover							0.03					
							[5.27]					
ln(Trd Size)								-0.10				
								[-2.52]				
ln(#Trades)									0.29			
									[5.12]			
Quoted B/A Spread										-1.34	-0.87	-0.21
										[-1.09]	[-1.14]	[-0.52]
Call Dummy	-0.44	-0.44	0.12	0.07	0.04	0.08	0.10	0.09	0.12	-0.38	0.19	0.08
	[-1.05]	[-1.07]	[2.14]	[1.65]	[1.11]	[1.79]	[2.30]	[1.75]	[2.33]	[-1.11]	[1.57]	[1.22]
A Dummy	0.69	0.66	0.19	0.18	0.24	0.18	0.17	0.19	0.24	0.67	0.11	0.20
	[1.52]	[1.52]	[1.14]	[1.17]	[1.50]	[1.14]	[1.07]	[1.24]	[1.43]	[1.56]	[1.08]	[1.71]
BAA Dummy	1.19	1.07	0.74	0.69	0.56	0.67	0.61	0.72	0.77	1.18	0.67	0.56
	[2.85]	[2.89]	[2.77]	[2.69]	[2.45]	[2.46]	[2.16]	[2.59]	[2.52]	[3.15]	[3.31]	[2.68]
Junk Dummy	5.41	4.65	3.86	3.39	1.28	3.37	3.33	3.42	3.45	5.43	3.43	1.35
	[4.38]	[4.46]	[3.96]	[3.94]	[4.29]	[4.04]	[3.98]	[4.02]	[4.02]	[3.97]	[3.74]	[3.62]
Obs	679	670	679	670	502	670	670	670	670	633	627	472
R-sqd (%)	42.79	49.80	53.07	58.22	76.06	60.66	61.33	60.91	62.02	43.37	59.22	77.21

Monthly Fama-MacBeth cross-sectional regression with the bond yield spread as the dependent variable. The t-stats are reported in square brackets calculated using Fama-MacBeth standard errors with serial correlation corrected using Newey-West. The reported number of observations are the average number of observations per period. The reported R-squareds are the time-series averages of the cross-sectional R-squareds. γ is the monthly estimate of illiquidity measure using daily data and log price changes. *Equity Vol* is estimated using daily equity returns of the bond issuer. *Age*, *Maturity*, *Issuance*, *Turnover*, *Trd Size*, and *#Trades* are as defined in Table 3.4. *Call Dummy* is one if the bond is callable and zero otherwise. Convertible and putable bonds are excluded from the regression. The sample period is from May 2003 through December 2008.

Table 3.14: Bond Yield Spread and Illiquidity Measure γ, Non-Callable Only

Cons	0.98	0.91	-0.41	-0.40	-0.06	-1.59	-2.11	-1.73	-1.92	1.55	-0.03	0.23
	[3.86]	[3.63]	[-0.95]	[-0.96]	[-0.27]	[-2.84]	[-2.72]	[-2.64]	[-2.77]	[1.87]	[-0.15]	[0.83]
γ		0.13		0.14	0.13	0.06	0.07	0.06	0.06		0.12	0.12
		[6.59]		[9.92]	[6.95]	[2.96]	[3.22]	[3.03]	[2.44]		[6.72]	[7.21]
Equity Vol			0.04	0.04	0.03	0.04	0.04	0.04	0.04		0.04	0.03
			[3.31]	[3.05]	[2.01]	[2.99]	[2.92]	[2.98]	[2.92]		[3.05]	[1.95]
CDS Spread					0.54							0.54
					[27.58]							[23.31]
Age						0.08	0.10	0.09	0.09			
						[2.86]	[3.07]	[2.77]	[2.93]			
Maturity						0.00	0.00	-0.00	0.01			
						[0.02]	[0.07]	[-0.02]	[0.28]			
ln(Issuance)						0.11	0.15	0.11	-0.07			
						[4.02]	[4.33]	[3.16]	[-1.25]			
Turnover							0.04					
							[3.02]					
ln(Trd Size)								0.04				
								[0.63]				
ln(#Trades)									0.29			
									[3.72]			
Quoted B/A Spread										-1.29	-0.66	-0.58
										[-0.83]	[-0.74]	[-0.85]
A Dummy	1.16	1.10	0.22	0.17	0.25	0.23	0.25	0.21	0.28	1.12	0.10	0.19
	[1.49]	[1.48]	[1.68]	[1.72]	[1.07]	[1.88]	[1.83]	[1.91]	[2.13]	[1.48]	[1.47]	[1.04]
BAA Dummy	1.44	1.38	0.98	0.89	0.56	0.85	0.78	0.83	0.87	1.42	0.87	0.58
	[3.16]	[2.90]	[3.74]	[3.65]	[2.00]	[2.77]	[2.60]	[2.78]	[2.65]	[3.00]	[3.51]	[1.89]
Junk Dummy	6.24	4.31	5.38	3.53	1.21	3.48	3.45	3.48	3.45	6.33	3.41	1.30
	[3.76]	[3.25]	[3.50]	[3.05]	[3.52]	[3.01]	[2.97]	[3.02]	[2.97]	[3.33]	[2.85]	[3.14]
Obs	373	370	373	370	283	370	370	370	370	357	356	273
R-sqd (%)	44.43	44.62	52.57	52.40	72.22	56.82	57.97	57.11	58.30	46.10	53.61	73.70

Monthly Fama-MacBeth cross-sectional regression with the bond yield spread as the dependent variable. The t-stats are reported in square brackets calculated using Fama-MacBeth standard errors with serial correlation corrected using Newey-West. The reported number of observations are the average number of observations per period. The reported R-squareds are the time-series averages of the cross-sectional R-squareds. γ is the monthly estimate of illiquidity measure using daily data. *Equity Vol* is estimated using daily equity returns of the bond issuer. *Age*, *Maturity*, *Issuance*, *Turnover*, *Trd Size*, and *#Trades* are as defined in Table 3.4. Callable, convertible and putable bonds are excluded from the regression. The sample period is from May 2003 through December 2008.

Table 3.15: γ by Trade Size

Panel A: Using Trade-by-Trade Data								
Size		2003	2004	2005	2006	2007	2008	Full
Small	Mean γ	1.10	1.05	0.83	0.72	0.77	1.18	0.88
	Median γ	0.76	0.62	0.48	0.41	0.45	0.76	0.59
	Per t $\geq$ 1.96	91.32	90.43	95.48	94.62	92.23	88.11	99.09
	Robust t-stat	13.54	14.35	17.47	17.90	17.00	15.34	20.62
Medium	Mean γ	0.69	0.68	0.58	0.47	0.48	0.78	0.57
	Median γ	0.48	0.43	0.33	0.25	0.25	0.53	0.39
	Per t $\geq$ 1.96	95.84	92.40	96.60	96.27	95.16	91.29	97.75
	Robust t-stat	14.91	16.37	17.20	19.20	18.02	15.05	22.11
Large	Mean γ	0.29	0.30	0.27	0.22	0.25	0.47	0.27
	Median γ	0.11	0.08	0.07	0.06	0.08	0.24	0.10
	Per t $\geq$ 1.96	90.13	83.94	90.02	85.42	82.93	79.03	95.35
	Robust t-stat	13.44	12.65	14.20	13.68	13.51	12.82	17.42
Panel B: Using Daily Data								
Size		2003	2004	2005	2006	2007	2008	Full
Small	Mean Gamma	1.55	1.41	1.26	1.04	1.30	2.92	1.44
	Median Gamma	1.05	0.80	0.68	0.59	0.84	2.03	0.98
	Per t $\geq$ 1.96	86.14	83.24	90.22	89.52	89.03	83.59	96.75
	Robust t-stat	21.04	16.29	24.26	24.20	20.76	17.37	25.84
Medium	Mean γ	1.02	0.92	0.83	0.65	0.76	2.05	0.91
	Median γ	0.65	0.54	0.45	0.32	0.40	1.22	0.58
	Per t $\geq$1.96	89.88	86.18	92.51	91.16	89.62	86.33	95.58
	Robust t-stat	23.26	18.72	24.45	22.30	20.34	15.22	27.66
Large	Mean γ	0.50	0.46	0.42	0.35	0.47	1.20	0.47
	Median γ	0.18	0.12	0.10	0.07	0.12	0.48	0.15
	Per t $\geq$1.96	68.59	63.04	70.93	71.18	69.33	63.08	80.33
	Robust t-stat	14.28	13.05	15.30	14.27	9.84	9.53	17.84

γ is calculated using only trades of sizes in the smallest 30%, middle 40%, or largest 30% for each bond. Per t-stat $\geq$ 1.96 reports the percentage of bond with statistically significant γ. Robust t-stat is a test on the cross-sectional mean of γ with standard errors corrected for cross-sectional and time-series correlations.

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