ANALYSIS AND MODELLING OF THE EFFECTS
OF THE SOURCE AND MEDIUM ON STRONG MOTION

by

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ANALYSIS AND MODELLING OF THE EFFECTS
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Anthony Frank Shakal
Submitted to the
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ABSTRACT

The effects of the source, medium and site on recorded
strong motion have been studied through the analysis of obser-
vations and through theoretical modelling. The analysis of the
strong motion data from the San Fernando and other earthquakes
indicates the importance of both the source radiation and the
geologic conditions local to the site on the recorded ground
motion.

The effects of a sedimentary site are modelled using a
propagator matrix solution which includes attenuation. This
solution is used in the analysis and interpretation of the site
effects in strong motion data. The observed effects of a sedi-
mentary site depend on the frequency band being considered and
the variation of the medium parameters with depth. For a deep
sedimentary site the amplifying effects of decreased seismic
velocities and internal reflections may be offset, sometimes
severely, by the amplitude reduction effects of attenuation. A
striking example of this is shown in the data from the Puget
Sound earthquakes of 1949 and 1965. Sites with sedimentary
structures of intermediate depth may show amplification at low
frequencies and attenuation at higher frequencies.

The strong motion from the San Fernando earthquake shows significant source radiation effects. Records from stations to the north and east of the fault, on the up-thrust block, are of shortened duration and less complexity than those to the south and west, on the down-thrust block. This is a modellable effect of rupture on a dipping thrust fault, and may be expected to occur for future earthquakes of this geometry. The modelling also shows that the parameters of the initial rupture event determined teleseismically are consistent with those required to model the initial part of the local records.

The early, body wave portions of the velocity records at SL and GPK, bedrock stations in Pasadena and Los Angeles respectively, are successfully modelled. The modelling of these velocity records requires the presence of localized areas of high slip or high strength on the fault. These areas are important in their implications for the generation of high frequencies in strong motion. The predictive modelling of strong motion may be best approached through the deterministic modelling of rupture on a postulated fault, with an overlying stochastic specification of the sub-areas of high slip or pre-stress. Deterministic post-earthquake modelling studies remain to be important for the determination of the dimensions and the magnitude of slip or pre-stress on these localized rupture areas for input to the stochastic modelling.

Thesis Supervisor: M. Nafi Toksoz
Professor of Geophysics
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CHAPTER 1

INTRODUCTION

The last few decades have seen major advances in the understanding of seismic observations at teleseismic distances. The understanding and modelling of ground motion in the vicinity of a damaging earthquake has remained a more difficult problem.

The analysis of strong motion observations began with the studies of peak ground motion by Gutenberg and Richter (1942; 1956), following the earlier development of the strong motion accelerograph by Wenner (1932). More recently, strong motion data have been studied through theoretical waveform modelling of the strong motion record. The first attempts involved modelling the data from the 1966 Parkfield earthquake (Aki, 1968; Haskell, 1969). Since then research on strong motion has proceeded on the dual fronts of peak ground motion analysis and theoretical waveform modelling.

With the occurrence of the 1971 San Fernando earthquake, the set of strong motion observations became large enough to allow quite extensive statistical regression studies on the dependence of peak ground motion on earthquake magnitude, epicentral distance and other variables (e.g., Trifunac, 1976a). Theoretical methods of waveform modelling have also progressed to include more complex aspects of the faulting and the
departure of the medium from a homogeneous full space (e.g., Heaton and Helmberger, 1977; Bouchon, 1979). The current state of the strong motion problem is that the regression analyses show large unexplained variability of peak ground motion parameters, and only a small number of strong motion records have been able to be deterministically modelled by the waveform modelling methods. Page et al. (1975) very aptly describe strong motion observations as 'highly variable from place to place and earthquake to earthquake'. It is the object of this study to improve our understanding of the factors contributing to the wide variability of strong motion observations through detailed seismological analysis of the observations and through the theoretical modelling of individual components of the problem.

The first part of this study is concerned with the analysis of the available strong motion data. Analysis here is used in its strict sense, that is, the separation of a whole into its constituent parts for individual study. The major constituents of the strong motion problem are the earthquake source, the propagation path through the medium, and the local geologic conditions at the site. Chapter 2 is concerned with the study of these components in the strong motion observations from the San Fernando and other earthquakes. Given the realities of station distribution, complex geology and extended faulting, these components can rarely be fully isolated. Nonetheless, understanding strong motion variability requires that as much knowledge as possible be obtained from
the existing data. The analysis of this chapter suggests that, in terms of the Page et al. (1975) description cited above, certain place to place variations in the San Fernando data and earthquake to earthquake variations in the Puget Sound data are due to predictable effects of the source and site. The modelling in later chapters returns to the consideration of these data.

Methods of modelling the site effects are discussed in Chapter 3, and the modification of the propagator matrix method for the case of attenuating layers is discussed. This solution is applied to the analysis of frequency-dependent local variations in the San Fernando data, and to the modelling of the site effects in the Puget Sound data.

The theoretical modelling of the source and propagation effects are considered in Chapter 4. The source radiation is modelled using the analytic solution of Madariaga (1978) for the Haskell dislocation source. Our approach to combining the theoretical solutions for the source radiation and the path and site effects to obtain the solution for an elementary fault segment in a layered halfspace is discussed, as well as the approximations involved.

In Chapter 5 this hybrid modelling method is applied to the modelling and interpretation of the San Fernando data. Particular areas of application include the modelling of the azimuthal variations in the San Fernando data, and the modelling of the initial rupture event which has been previously investigated teleseismically. Finally the velocity records at
SL and GPK are modelled, and the implications of the localized areas of high slip on the fault are discussed. The principal results and conclusions of this study are presented in Chapter 6.
CHAPTER 2

ANALYSIS OF STRONG MOTION OBSERVATIONS

The character of a strong motion record is in general a sum of the contributions of the effects of the source, the propagation path, and the local site conditions. Although recordings of strong motion have been obtained since 1933, only the more recent events were recorded at a sufficient number of stations to allow analyses of the effects of the propagation path or radiation pattern.

In this chapter we consider several aspects of the analysis of strong motion observations from recent earthquakes. In the first section (2.1), we investigate effects of the source and medium in the strong motion data from the Borrego Mountain and San Fernando earthquakes. In the second section (2.2), we investigate the effects of the site structure in the strong motion data from the San Fernando and the Puget Sound earthquakes.

Strong ground motion may in general be comprised of both body wave and surface wave contributions. In section 2.3 we investigate, within the constraints imposed by the triggered nature of the records, the relative importance of the body and surface wave contributions in observational data.

Strong motion levels are often parameterized in earthquake engineering by the peak acceleration or velocity. In
section 2.4 we review aspects of the peak ground motion data and discuss the possible impact of an improved understanding of strong motion generation and propagation on the parameterization of strong motion levels.

The strong motion data analyzed in this study have been made available through the C.I.T. strong motion data processing program (Hudson et al., 1972). This program has now been taken over by the USGS Seismic Engineering Branch. The data used here are the instrument corrected (Vol. II) acceleration data. Studies of the long and short period errors associated with the processing program suggest minimal processing error in the band extending from about .125hz (8 sec) to 10 or 15 hz (Hanks, 1973, 1975; Berrill and Hanks, 1974; Trifunac et al., 1973).
2.1 Source and Medium Effects in Strong Motion Data

Strong motion records are often characterized by large variations over relatively small distances. In this section we consider the source and medium aspects of the strong motion variations in data from the Borrego Mountain and San Fernando earthquakes. The amount of analysis possible increases with the number of records obtained from an earthquake. Thus, although strong motion records have been obtained since the 1933 Long Beach earthquake, only the most recent events have a sufficient number of records to allow detailed studies. The most extensively recorded earthquake is the San Fernando earthquake of 1971, and we study this data in the most detail. We consider first the data from the 1968 Borrego Mountain earthquake.

2.1.1 Borrego Mountain Earthquake

The records from the Borrego Mountain earthquake of 9 April 1968 (33.19N, 116.13W, 02:02:59 GMT, 6.4 $M_L$; Allen and Nordquist, 1972) provide a good introductory example of the regional and local variations in strong motion (Fig. 2-1). The faulting at the source extended for about 30 km, with displacements occurring on a complex of fault planes (Clark, 1972; Allen and Nordquist, 1972). The nearest accelerograph station was at El Centro (ELC), about 70 km to the southeast (approximately down the strike of the fault). The record there was modelled by Heaton and Helmberger (1977), using a set of point shear dislocations in a layer over halfspace.
The geologic structure in the Imperial Valley is well approximated as a flat layered structure (Biehler, 1964), and their generalized ray method was quite successful in modelling this record.

The records shown in Fig. 2-1 are radial components, after rotation to the epicenter, the approximate center of the bilateral faulting. The stations are identified in Table 2-1. Fig. 2-1 also shows contours of the basement depth in the Los Angeles Basin area, after Smith (1964). The depth to basement ranges from exposure down to about 30,000 ft, or about 9 km, at the deepest part of the Los Angeles Basin.

It is interesting to compare the records at SON, SAA and COL. These stations are all at similar distances from the epicenter (135 – 170 km, 280 – 310° azimuth), yet they exhibit quite marked differences. SON has the largest amplitudes except for ELC, though it is more distant than SDG (135 vs 110 km). The record at SAA has reduced amplitude and frequency content relative to that at COL. This may be attributable to the depth of the sedimentary structure at SAA. The depth to basement at SAA is about 10,000 ft, while at COL it is less than 1000 ft. The record at VER, on the other hand, over 20,000 ft, has higher amplitudes compared to neighboring stations.

Detailed analysis of the contributions of the source and local medium effects require a larger set of stations, more widely distributed in azimuth. The San Fernando earthquake of 1971 yielded a much larger set of records and these are
considered in the next section. Records were obtained from the San Fernando earthquake at most of the same stations considered here. Making comparisons to that data, discussed further in the next section, VER was again large, which suggests a local site effect. SON, on the other hand, is quite small, which suggests that its high amplitude here is not a site effect. Its high amplitude relative to SDG may be a source radiation effect, since SDG is nearly normal to the fault strike, an azimuthal range which has low amplitudes for strike-slip faulting.

2.1.2 San Fernando Earthquake

The San Fernando earthquake of 9 February 1971 (34.45N, 118.40W, 14:00:42 GMT; Allen et al., 1973; Hanks, 1974), though of moderate magnitude (6.4 ML, Allen et al., 1973), is a very important event for the study of earthquake strong motion. It provided an unprecedented amount of strong motion data, occurring virtually at the center of the Southern California strong motion instrumentation network. As a result, 241 accelerograms were recorded during the earthquake.

Malley and Cloud (1971, 1973) note that the distribution of the recording stations was relatively inequitable from an engineering point of view. It was also inequitable from a seismological point of view. Of the 241 records obtained, more than 80% were obtained in the Los Angeles area from various levels of high-rise structures (Malley and Cloud, 1971). In total, less than 100 records were obtained at free-field
stations or in building basements, and the majority of these were obtained in the Los Angeles area. One station, serendipitously located over the downward extension of the thrust fault (PAC), recorded the highest accelerations (~1.25g) observed to date. This record has been extensively studied (e.g., Bolt, 1972; Hanks, 1974) and most strong-motion modelling of the earthquake, reviewed in Chapter 5, has been concerned with the modelling of this record.

In studying the San Fernando strong motion data in this section, we consider first the strong motion recorded at regional distances (r < 200 km), and then that more local to the event (r ≤ 40 km).

Records from the San Fernando earthquake at selected regional stations are shown in Fig. 2-2. Where possible, these are records from free-field stations. Otherwise, records were chosen which were as representative as possible of the records at neighboring stations. Finally, stations which had also recorded the 1968 Borrego Mountain earthquake were included. The stations are identified in Table 2-1, along with details of the recording site. The purpose of Fig. 2-2 is the study of the variations in the levels of strong ground motion throughout region. Since all three components could not be shown in the figure, a single component is plotted, but with all three components plotted for reference in Appendix A. The records in Fig. 2-2 are the radial components, except at COS, LBI, SJC and WRI where the transverse is larger. Radial and transverse here are relative to the
 approximate center of the fault, shown on the figure. (As defined here, radial is positive away from the source, transverse is positive 90° counter-clockwise of radial.)

Several effects of the source and medium are striking in the records of Fig. 2-2. The study of these acceleration records leads to conclusions supporting and extending those of Hanks (1975), who studied the long period displacements (i.e., twice-integrated accelerations).

A profile of stations extending to the south-southeast over the Los Angeles Basin shows a quite rapid decay of the acceleration coinciding with the basin. For example, over the 10 km from HSL to WL4 and USC, the acceleration amplitude has been rapidly reduced. Farther to the southwest, AIR is of quite low amplitude, similar to PVE and LBI, another 15 km to the south.

A station which is an exception to this decay is VER. In the records from the Borrego Mountain earthquake VER is also of high amplitude compared to the neighboring stations (see Fig. 2-1). This suggests that the increased amplitudes at VER are a local site effect, although the site geology, as well as it is known (Lastrico at al., 1972), does not indicate that it would be.

Farther to the south and east (FUL, SAA, COS) the records continue with reduced amplitudes and longer periods. SJC has slightly increased amplitude again. Hanks (1975) notes that this distance (120 km) is the critical distance for the Moho, which might explain higher amplitudes at SJC than at COS.
However SON, about 20 km farther from SJC has very low amplitudes again. High attenuation does not characterize the SON site however, as shown by the Borrego Mountain records (Fig. 2-1).

Another aspect apparent in the regional ground motion includes anomalously low amplitude to the northwest, an azimuthal variation in the source radiation. Comparison of the records at FTJ, EDM and WHL shows them to be substantially lower than stations at similar and greater distances to the southeast (WRI, PUD, FUL, COL). In fact, farther to the southeast (HMT, ANZ), the amplitudes are larger even at as much as twice the distance of the northwest stations. Hanks (1975) also observed this variation in the long-period displacements.

The apparent effect of the source radiation is clearer in the records from stations more local to the event. Fig. 2-3 shows records from stations in the epicentral region, at distances of 40 km and less. The principal effect seen in these local records is a tendency toward less energetic, shorter duration records to the north and east of the source than to the south and west. For example, compare the northern stations LH4, LH1 and FAR, to HSL, GPK and GLN, to the south. At LH9, near LH4, the initial amplitudes were apparently low enough so that the instrument did not trigger until later in the wave train. Note that LH9 is approximately along the same direction as the low amplitude stations farther to the northwest, discussed previously (WHL, EDM, FTJ). It is
interesting to speculate that this might be due to a nodal plane in the radiation. LH12, near LH9, is of high amplitude, but the analysis in Chapter 3 suggests that this is due, at least in part, to the effects of the shallow sediments at the LH12 site. Similarly, the later part of the records at PLM and PRL, to the northeast, appear to be dominated by local site effects.

The records in Fig. 2-3 are again all radial components, except at stations to the south where the transverse component is quite pronounced. These stations (LNK, HSL, GPK, GLN and SL) are indicated on the figure. The similarity of the first portion of the record at some of these stations is remarkable. For example, the first 1 or 2 seconds of the S arrival at HSL and GPK are almost identical though the stations are almost 5 km apart and have quite different local site conditions. The initial transverse (i.e., to the east) pulse, though modified, can also be seen at GLN and SL, and at LNK. This common pattern in the initial ground motion could only be recognized after component misidentifications and polarity reversals at GPK and GLN were detected and confirmed (A.G. Brady, USGS, personal communication). The common initial transverse pulse at these stations is a valuable constraint in modelling the initial rupture process on the fault in Chapter 5.

The three-component velocity records at the stations of Fig. 2-3 are shown in Fig. 2-4. In these velocity records, which are not as sensitive to the high frequency effects of the site, the azimuthal differences are much clearer. Records
from stations to the north and east, on the up-thrust block, are of shorter duration and less complexity than those to the south and west, on the down-thrust block. This effect of the thrust fault geometry is a predictable effect of upward rupture on dipping thrust faults, and is considered in greater detail in the modelling of Chapter 5.

As discussed previously, the PAC record has received extensive study because of its proximity to the fault and its high accelerations. Another interesting record is that at ORI, in the San Fernando Valley, approximately 10 km SW of the surface faulting. The three-component velocity record is shown in Fig. 2-5, along with the particle motion of the later part of the record. The particle motion indicates well developed, strong retrograde Rayleigh wave motion of about 3 seconds period. Hanks (1975) also found evidence for Rayleigh wave motion in the displacement records from a set of stations in Los Angeles, but the motion was of longer period and lower amplitude. He observed the motion by stacking the displacement records. At ORI, the peak velocity value occurs during the Rayleigh phase.

The record at ORI is of particular interest as it relates to the theoretical work of Mal (1972). He obtained the solution for the Rayleigh wave radiation from a two-dimensional (infinite horizontal extent) propagating thrust fault in an elastic halfspace. He found that the Rayleigh wave amplitude ahead of the fault break is considerably larger than behind the initial epicenter, and that only a small portion of the
fault near the surface is significant in producing this motion.

It is interesting to compare the motion at ORI with these theoretical results, bearing in mind the departures due to the effects of sedimentary cover and finite fault length. The Rayleigh motion at ORI is very likely due to faulting near the surface, as it occurs quite late in the record, about 15 secs after the arrival at about 2 secs, which is presumably the direct S from the rupture initiation. Secondly, the 'radial' direction (i.e., vertical plane) of the Rayleigh motion points not to the epicenter, or to the approximate center of the fault, but toward the central portion of the surface fault break. The approximate direction of the Rayleigh vertical plane is indicated on the map of Fig. 2-4.

The effects of finite length would include increased interference effects in the Rayleigh motion, though this interference would be minimized along a line normal to the strike of the fault and passing through the mid-length. Although some of the neighboring stations in Fig. 2-4 show similar long period motion on the horizontal components (e.g., VN1, HSL), none show as clearly the 3-component Rayleigh motion exhibited by ORI. On the opposite side of the fault, PLM also shows Rayleigh-like motion, though it is not as clear as that at ORI.

The extensive data from the San Fernando earthquake also allows the comparative study of strong motion from very close stations. Through these comparisons the contributions of the
source and medium, as well as the local site effects, can be investigated.

Several local areas in Los Angeles were quite densely instrumented. In the following, we consider records from the two clusters of stations shown on the map of Fig. 2-6. Hanks (1975) called these clusters Area 1 and Area 2 in his study of the long-period displacements, and we will use that nomenclature here.

The records from six stations within several hundred feet of each other in Area 1 are shown in Fig. 2-7. The records have been shifted to line up the first arrival. These Area 1 stations are in the immediate vicinity of WL4 (see Fig. 2-2). Records from the Area 2 stations are shown in Fig. 2-8. These stations are about 4 km SE of WL4. The vertical components for both sets of stations are shown in Fig. 2-9.

The Los Angeles area acceleration records were first studied by Crouse (1973, 1976) in a study of soil-structure interaction and building response. Hanks (1975, 1976) studied the displacement records at many of the Los Angeles stations. He stacked the displacement records from the Area 1 stations to obtain the Rayleigh wave motion discussed above.

The similarity of the accelerograms recorded within each of the two areas (Figs. 2-7 and 2-8) is quite striking. The Area 2 records in particular (Fig. 2-8) show the arrivals of several separate phases which can usually be correlated from record to record. Lines connecting the 3 or 4 possible phases have been drawn in the figures. The common existence of the
phases in the Area 2 records shows that these phases are not generated by local site effects but arise either from the source, or from medium effects distant from the stations relative to their separation, about 0.6 km (FRE to OL1). The records show that the local site effects cause significant modification of the signal amplitudes and high frequency modulation, however.

Area 1, about 4 km to the NW, is an area of younger surface sediments and greater depth to basement. In these records (Fig. 2-7) the arrivals of the separate phases are not as distinct. However, the phase occurring at about 6 seconds after the first arrival is recognizable in both the Area 1 and Area 2 records. Thus, this phase must arise either from the source, or from medium effects distant relative to the 4 km separation of these two areas.

In a statistical study of the distribution of the time of the peak acceleration in the San Fernando records, Dobry et al. (1979) found that a histogram of these times had peaks at about 3 and 6 seconds. It can be seen in the Area 2 records of Fig. 2-8 how these peaks arise — in some records the peak acceleration occurs in the phase at about 3 seconds after the onset, and in other records in the phase at about 6 seconds. Dobry et al. (1979) inferred that the distribution of the peak acceleration times indicated common wave arrivals which occurred over an extensive area.

Dobry (1979, personal communication) suggested that the records from northern stations were different, that they did
not show the same distribution of peak times. In fact, as discussed earlier in this section, the records from stations to the north and east, on the up-thrust block, are of shorter duration and less complexity than those to the south and west. The stations to the north and east do not sample the source radiation as effectively as those to the south and west, i.e., in the rupture propagation direction. We will return to the analysis and modelling of this effect of a dipping thrust fault in the source modelling of Chapter 5.

In this section we have studied the variations of strong ground motion due to the effects of the source and medium. Significant source effects were observed in the San Fernando data, particularly in the azimuthal dependence of amplitudes and duration, which correspond to predictions from theoretical models. There are suggestions of source-dependent variations in the Borrego Mountain data, but the source effect is difficult to establish because of the limited amount of data. In the next sections we turn to the analysis of site effects in observed ground motion.
2.2 Site Effects

In the previous section we studied data from the Borrego Mountain and San Fernando earthquakes in an investigation of the contributions of the source and medium in recorded strong motion. In this section we investigate the effects of the local geologic structure in ground motion data from the San Fernando and other earthquakes.

The site structure can cause modification of the free-surface ground motion in several ways. High impedance contrasts in the geologic column underlying a site can cause strong frequency-dependent amplification in the strong motion recorded at the free surface. The strong motion recorded in Mexico City, for example, exhibits a prominent spectral peak at about 2.5 secs, attributed to the soft sedimentary deposits of the old lake bed underlying the city (e.g., Zeevaert, 1964; Faccioli and Resendiz, 1976). A striking example of the effects of the local geologic structure is shown in the data from the Puget Sound earthquakes, considered in section 2.2.1.

The site structure can also modify the observed ground motion through the effects of lateral variation in the velocity structure and scattering due to localized inhomogeneities. To investigate these effects in observational data we analyze the inter-station coherency for records from nearby stations in the Pasadena and Los Angeles areas (section 2.2.2).

The effect that the structure in which the instrument is housed may have in modifying the motion recorded may also be
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considered a site effect. The structure may be just an instrument shelter, or as is more typical for present strong motion stations, a multi-story building. An indication of the effects of a structure which must be borne in mind in the seismological interpretation of basement records is provided by a pair of stations in Los Angeles. Data from these stations, one in the basement of a large structure, the other in a neighboring lot, are also considered in the coherency analyses of section 2.2.2.

2.2.1 Puget Sound Earthquakes of 1949 and 1965

The Puget sound region in the western part of the state of Washington experienced damaging earthquakes in 1949 and 1965 (magnitudes 7.1 and 6.5 respectively). Strong motion records were fortunately obtained from both events, at nearly the same sites.

The Puget Sound region differs from the California region in both the regional geology and type of earthquake faulting. The gross tectonic environment is characterized by the present or recent subduction of the Juan de Fuca plate (Atwater, 1970). The regional near-surface geology is characterized by a distribution of tertiary sediments, volcanics and extensive glacial deposits. A simplified geologic map of the Puget Sound region is shown in Fig. 2-10. The epicenters of the earthquakes and the station locations are also shown. The earthquakes were quite deep (60-70 km), with dip-slip mechan-
isms (Nuttli, 1952; Algermissen and Harding, 1965).

Strong motion records were obtained from both the 1949 and 1965 earthquakes at Olympia (OLY) and at Seattle (SAB, SFB). The records from the 1949 event at OLY and SAB are shown in Fig. 2-11. The records from the 1965 event at OLY and SFB are shown in Fig. 2-12. These stations and records are identified in Table 2-1. SFB, the Seattle station at which the 1965 event was recorded, is approximately 5.5 km north of SAB.

The immediate, most striking feature of the records in Figs. 2-11 and 2-12 is the high frequency content of the Olympia records relative to the Seattle records, for both events. It is difficult to ascribe this difference to source or propagation effects.

Because of the depth of the events, the difference in path lengths to the stations is quite small. If we consider, for example, the 1965 event (nearer Seattle), the path length to SFB is less than that to OLY (focal distances of 63 versus 85 km, for 59 km depth). Thus OLY should show a somewhat greater loss of high frequency energy due to path attenuation than SFB, but the opposite is observed (Fig. 2-12). In fact OLY, though more distant than SFB, shows substantially larger amplitudes and higher frequencies. Considering the possible effects of the source mechanism, it is unlikely that the radiation pattern would cause the same high frequencies and amplitudes at Olympia, but low amplitudes and frequencies at Seattle, for both the 1949 and the 1965 event. This suggests that
the differences in the Olympia and Seattle recorded ground motion arise from effects of the local geologic conditions.

The thickness of unconsolidated sediments at OLY, near the southern end of the Puget Lowland, is less than 400 ft (Hall and Othberg, 1974). The Seattle stations, on the other hand, are apparently underlain by an extensive sedimentary formation.

Crosson (1972, 1976) observed a large positive delay (> 1 sec) in P wave arrival times at Seattle, for both teleseismic and local events, which he attributed to an anomalous region of low velocity crustal material in the Seattle vicinity. Langston and Blum (1977) found, in modelling the 1965 event, that the shapes of the pP and sP phases implied thick sediments near the surface in the epicentral area (i.e., near Seattle). Independent of seismic data, Danes et al. (1965) estimate a low-density sedimentary thickness of 10 km or more to explain the large negative Bouguer gravity anomaly in the Seattle vicinity. Data from well logs and seismic profiling indicate that the thickness of the unconsolidated sediments under the Seattle stations is about 3500 feet (Hall and Othberg, 1974).

The spectra of the records at the Olympia and Seattle stations for the two events are shown in Fig. 2-13a,b. The OLY spectra show substantially more energy at all frequencies beyond 1 or 2 hz. The OLY spectra also show a spectral peak at about 7 hz, particularly prominent for the higher frequency 1965 event. These features of the spectra are interpretable
in terms of the local geologic conditions at the sites. The records at the Seattle stations, a region underlain by deep unconsolidated sediments over deep tertiary sediments, are strongly attenuated at higher frequencies. The records at Olympia, which is underlain by shallow sediments, are rich in high frequencies. The pronounced spectral peak at about 7 hz in the 1965 record also suggests the effects of a low velocity layer at shallow depth within the unconsolidated sediments. Both these amplification and attenuation aspects of the effects of attenuating layered structures on the free surface motion are discussed further in the site modelling of Chapter 3.

In summary, the differences in the ground motion recorded at Seattle and at Olympia appear to be primarily due to the effects of the local geologic structure at the two sites. There seems to be no reason not to expect similar effects in records from future deep Puget Sound earthquakes. These spectral characteristics of the Olympia and Seattle ground motion are important in evaluation of the local earthquake hazard. They also provide insight into the mechanisms giving rise to the wide scatter in peak ground motion data, reviewed in section 2.4.
2.2.2 Coherence of Strong Motion at Nearby Sites

In this section the variations of ground motion at nearby sites are investigated by calculating the spectra, spectral ratio and coherency for records from the San Fernando and Kern County earthquakes. The results show that the ground motion at nearby stations is coherent at low frequencies, but that the coherency drops off at higher frequencies—between 1 and 10 hertz, depending on station spacing and site characteristics.

As an example of the local variations in ground motion, the San Fernando records form the four stations in the Pasadena basin area, which is about 35 km southeast of the San Fernando epicenter, are shown in Fig. 2-14. The records from this area have been discussed by Hudson (1972a) in comparing the strong motion data to the small earthquake studies of Gutenberg (1952). We will return to a detailed analysis of these records and the comparison of strong motion and small earthquake response in Chapter 3. In this section we study the coherence of the strong motion recorded at the nearby ATH and MLK sites, and other sites in Los Angeles.

The coherency is an estimate of the correlation between two records as a function of frequency (Jenkins and Watts, 1968). It is defined as the smoothed cross power spectrum normalized by the product of the smoothed individual amplitude spectra

\[
C_{xy}(f) = \frac{\overline{P_{xy}(f)}}{[\overline{P_{xx}(f)} \cdot \overline{P_{yy}(f)}]^{1/2}} \tag{2.1}
\]
The spectral ratio relates the amplitude of a frequency component in one record to that in the other, without regard to phasing. Thus the spectral ratio and the coherency, sensitive to phasing, are complementary. In soil-structure interaction studies the spectral ratio is often called the empirical transfer function (e.g., Crouse and Jennings, 1975).

As an example, Fig. 2-15a shows the estimate of the coherency between the San Fernando records at the MLK and ATH stations in Pasadena, less than .5 km apart (see Fig. 2-14). The spectra have been smoothed by applying a cosine taper to the covariance functions in the time domain (e.g., Jenkins and Watts, 1968). The bandwidth of that smoothing operator in the frequency domain is indicated in the figure. The coherency spectrum for these two records drops rapidly around 3 hz. In general, the width of this band of coherence narrows as the intersite distance increases, but it is quite variable.

As an example at two much closer stations, a similar analysis for the Hollywood Storage stations in Los Angeles is shown in Fig. 2-15b. HSB is in the basement of the Hollywood Storage building, and HSL is about 100 ft from the building in a neighboring lot. Fortunately, records from both the San Fernando and Kern County earthquakes were obtained at these two stations, and so the between-station comparison can be made for two very different records.

The San Fernando records show a net loss at high frequencies in the building record, apparent both in the records and the amplitude spectra. The coherency is nearly unity out to 8
or 10 hz. Crouse and Jennings (1975) studied these records in terms of soil-structure interaction. The lower coherency values (i.e., reduced correlation) at about 2 and 5 hz are at the frequencies that estimates of the theoretical building transfer function are minimum (Crouse and Jennings, 1975; Hradilek et al., 1973). Thus these lows are consistent with their conclusion that soil-structure interaction influenced the basement record.

The Kern County records, also shown in Fig. 2-15b, have much less high frequency content than the San Fernando records, being recorded about 120 km from the source, as opposed to about 35 km. The coherency is high out to about 5 hz. The spectra show that the building record once again has suffered a loss at the higher frequencies, and the difference is again roughly a factor of two. This is better shown by the ratio of the amplitude spectra, also shown in Fig. 2-15b.

The spectral ratios for both the San Fernando and Kern County earthquakes are quite similar despite the different source mechanisms and direction and distance of the two earthquakes. This appears to be a real site-to-site repeatable variation. The spectral ratios are near unity at low frequency, and near .5 for frequencies above about 5 hz. Crouse and Jennings (1975) found that the predictions from theoretical structural models were in general agreement with the observed spectra up to about 5 hz, beyond which the theoretical models were unable to predict satisfactorily the filtering out of the higher frequencies by the building. At these
higher frequencies scattering may be important.

Considering a building resting on (or embedded in) the surface of a half space, for wavelengths much longer than the building dimension, \( \lambda \gg L \), the entire surface of the half-space moves as one, and the waves recorded at the building should be little different from those recorded at any other point on the surface. For wavelengths near the dimension of the building, \( \lambda \sim L \), complex interaction and coupling between \( P \) and \( S \) waves occurs (e.g., Landers and Claerbout, 1972). For wavelengths much shorter than the building dimension, \( \lambda \ll L \), the wavefield at the building will be independent of that at the free-field station many wavelengths away. At these high frequencies, even if both energy spectra were the same, the ratio of the two amplitude spectra would be expected to be less than one, being controlled by the ratio of the effective elastic moduli of the building near the recording device to that at the free-field station.

The coherency analysis examples considered here, and further analysis using other station pairs (e.g., within Areas 1 and 2 in Los Angeles), lead to several conclusions. The coherency of strong motion at nearby stations is high at low frequencies. This is, of course, equivalent to Hank's (1975) conclusion that the long period displacements are similar at nearby stations. At higher frequencies, the coherency drops off, and there is some correlation between the frequency at which it drops off and station spacing. This is not a strong correlation however. Other effects in addition to station
spacing must be important in determining the local variations. These may range from scattering due to localized medium inhomogeneities to the effects of surface topography (e.g., Boore, 1973; Bouchon, 1973) or interaction effects of the structure housing the strong motion instrument.

Although local effects can cause significant differences in the amplitudes and polarization of individual phases, and thus cause low values of the estimated coherence, specific phases are often recognizable from record to record. This is shown, for example, by the Area 1 and Area 2 records of Figs. 2-7 and 2-8. The Pasadena area records also show two separate phases, about 2 secs apart, recognizable in the records at SL, ATH and MLK (Fig. 2-14). In sum, strong motion records from nearby stations show more coherency in the times of individual arrivals than in the amplitudes of the arrivals.

Analyses of the strong motion data from the Hollywood Storage stations (HSL, HSB) suggests that, for these stations, the principal effect of the building appears to be a significant amplitude reduction at high frequencies (\( > 5 \) hz) in the basement record. This is consistent with the results of an experiment by Borcherdt (1970), who recorded a distant explosion at a similar pair of stations in San Francisco. His results do not indicate a major effect of the building on the basement record, at least below 2.5 hz.
2.3 Body and Surface Waves in Strong Motion Observations

Strong motion records are usually taken to be comprised primarily of body wave energy, particularly the S wave, though there is a continuing question as to the importance of the surface wave contribution. In this section we investigate the contribution of body and surface waves in strong motion data, and consider some examples of clear P, S and surface wave arrivals.

As discussed in the section on the San Fernando earthquake (2.1.2), Hanks (1975) found evidence of Rayleigh wave motion in the long period (~5 sec) displacements at some Los Angeles stations. Other recent work on surface waves in strong motion includes that of Hartzell et al. (1978). They studied a record from a 1974 Acapulco earthquake which could be interpreted as being comprised of direct S waves from a deep source, or as high frequency surface waves from a shallow source, their preferred interpretation.

Resolution of the importance of surface waves in strong motion is made difficult by the triggered nature of most strong motion instruments. Some portion of the incoming wavetrain is lost in starting up the recorder. Until recently, the standard instrument used in this country employed a vertical pendulum, which triggered the recorder when its maximum excursion reached some preset level. The recorder was thus triggered by ground motion in the horizontal plane, and the P wave arrival, and often even its coda, is
absent on nearly all the resulting records. Recently a vertical electromagnetic starter has been introduced (Rihn and Beckmann, 1977), sensitive to the primarily vertical motion associated with the P wave arrival. Instruments with this starter often trigger within the P wave coda. The most recent developments in strong motion instrumentation include an internal or WWV time code (e.g., Deilman et al., 1975) and a digital buffer, allowing a record of the entire wavetrain to be obtained, with absolute rather than relative time. However, nearly all currently available strong motion data was recorded by the traditionally triggered analog instrument.

The uncontrolled triggering is an important characteristic of the available strong motion data. Some of the records from the 1968 Borrego Mountain earthquake provide a good example of the effect of this triggering in observed records. The records from SON, COL and SAA, at epicentral distances of approximately 135, 145 and 175 km, respectively, are shown in Fig. 2-16. They have been time shifted according to typical Southern California S-P times corresponding to the station distance, so the P onset would occur at approximately time zero. It is clear that only the SON record was triggered during the early part of the P wave train. Just to underscore the rarity of a record like the SON record, showing both the P and S wave trains, early analysis ascribed the features of this record to two separate events (Housner et al., 1970). Boucher (1971) later pointed out that the time separation is appropriate for the Pg and Sg phases for a distance of 135 km.
Of the remaining stations in Fig. 2-16, SAA apparently triggered late in the P coda, while COL triggered midway between the P and S.

Another example is provided by some of the Los Angeles area records. The records from HSL, W5A and SUB are shown in Fig. 2-17. The inferred S phases (discussed further below), have simply been lined up, as the stations are all at about the same distance (approximately 220 km) from the epicenter.

The importance of forming, when possible, suites of records such as shown in Figs. 2-16 and 2-17 is that they allow the identification of the seismic phases on the record. The SON record (Fig. 2-16), while showing distinct P and S body wave arrivals, does not show a distinct surface wave arrival, and neither do COL or SAA. In the Los Angeles records (Fig. 2-17) correlation of the very similar phases on the transverse components shows that the same phase that triggered SUB occurred about 23 secs into the record at HSL. The HSL record was apparently triggered in the P wave coda, as indicated by the remnant signal on the vertical component, and the fact that the expected S-P time for 220 km is about 25 secs. Thus SUB triggered on the S wave, rather than the surface waves, a fact which could not otherwise be known. These records, then, similar to those of Fig. 2-16, show evidence of the P wave arrival, a clear S wave arrival, but no distinct surface wave arrivals.

These Borrego Mountain records show that in some common cases the principal contribution to the strong motion is due
to the body waves, with the surface wave contribution being of less importance. However, that this is not always the case is shown by the records from the 1957 San Francisco earthquake.

The magnitude 5.3 San Francisco earthquake of 22 March 1957 (37.67N, 122.48W, 19:44:21 GMT; Tocher, 1959) was recorded at several stations in the San Francisco vicinity. Complete recordings of an aftershock which occurred before the instruments turned off were also obtained at these stations. The records from the main shock at the San Francisco stations, about 12 to 18 km from the center of the faulting (Hudson and Housner, 1958), are shown in Fig. 2-18. The aftershock was recorded some 25 secs after the instruments triggered. The records from the aftershock are shown in Fig. 2-19.

The aftershock records in Fig. 2-19 show clear P and S arrivals, marked on the records, and clear, dispersed surface waves. In fact both the Love and Rayleigh waves can be recognized in the particle motion of the surface waves, shown in Fig. 2-20. Note that the Rayleigh wave shows prograde motion, which is unusual. This is particularly interesting because the local geology here, shallow older bay sediments (~100 ft) over bedrock (Borcherdt, 1970), corresponds to one of the layer-over-halfspace models considered in the theoretical computations of Mooney and Bolt (1966). For their 'alluvium' example (which had a high velocity and density contrast) their solutions indicated prograde motion in the fundamental mode of the Rayleigh wave. The agreement of the observed ground
motion with this theoretical result is remarkable.

In contrast to the distinguishable body and surface wave arrivals in the aftershock records, the main shock records (Fig. 2-18) show complicated motion not easily interpreted. Because of the effects of uncontrolled triggering, and without absolute time, we can not establish whether the main packet of energy is the direct S, implying triggering on the P coda, or whether it is surface waves showing strong interference effects, implying triggering on the S coda.

Tocher (1959) suggested that the focus of the main event was in the 7-11 km depth range of the larger aftershocks. Nearly all the aftershocks he located were between 4 and 14 km depth, and confined to within about 2 km of the San Andreas. Thus the paths to the stations were not greatly different for the mainshock and aftershock, and the sites were of course identical.

The appearance of surface waves at a station depends on the ratio of the station distance to the source depth. Pekeris and Lifson (1957) showed that Rayleigh wave amplitudes become significant at distances of five or ten times the depth of a point source in a halfspace. Anderson (1976) and Israel and Kovach (1977) showed that this also holds for finite sources in a halfspace. Levy and Mal (1977) considered the dependence of surface wave generation on the type of faulting. Their solutions show a predominance of body wave motion for vertical strike-slip faulting and a predominance of Rayleigh wave motion for dip-slip faulting. These solutions are of
course for smoothly propagating coherent sources. For realistic faulting, the complexity of the faulting process may also be an important factor, similar to the rupture incoherence effects for body waves discussed by Boore and Joyner (1978).

Returning to the San Francisco data, the presence of the clear surface waves in the aftershock records probably indicates a shallow focus for the aftershock. However, a large distance/depth ratio is apparently not a sufficient requirement for surface wave generation from realistic sources. The Borrego Mountain records considered earlier (Figs. 2-16 and 2-17) were all recorded at large distance/depth ratios (> 10). The complex rupture process, which extended over some 30 km, and possibly for as long as 15 sec (Burdick and Mellman, 1976) probably represented a very incoherent source, except at very long periods. Thus, the presence of clear surface waves in the San Francisco aftershock records may be due as much to its shallow depth as to a very simple source, as indicated by the simple P and S pulses in Fig. 2-19. Of course, another factor making it difficult for high frequency surface waves to be observed at extended distances is the high attenuation and scattering they undergo in the shallow structure along the path. Body waves, propagating through the deeper, higher Q crust, will not be as rapidly attenuated.

In summary, this observational study of body and surface waves in strong motion suggests several conclusions. First, the triggered nature of most currently available strong motion records, with relative rather than absolute timing, makes the
identification of seismic phases in strong motion records difficult. Only in cases where common phases can be recognized at neighboring stations, as in the Borrego Mountain examples, can a late-triggered record be interpreted unambiguously.

Secondly, this study suggests that the presence of a significant surface wave contribution to high frequency ground motion requires the usual large distance/depth ratio but also requires a simple, small dimensioned source, so that the faulting process appears coherent at these frequencies. The few records that do show clear surface waves are interpretable in terms of these aspects. The surface waves in the unusual ORI record from San Fernando (Sec. 2.1.2 and Fig. 2-5) apparently arise from the near-surface thrust faulting, corresponding to the work of Mal (1972). The presence of the clear surface waves in the San Francisco aftershock records are apparently due to a simple source, at shallow depth. Of course, at longer periods, which average over a larger portion of the fault, and suffer less attenuation and scattering along the path, surface waves may be more readily seen (e.g., Hanks, 1975).
2.4 Parameterization of Strong Motion Levels

This thesis is primarily concerned with the analysis and modelling of the complete time series (or spectrum) of recorded strong motion. However, this study is motivated by the wide variability observed in statistical correlation studies of strong motion parameters.

The strong motion level at a site is most often parameterized by the maximum value in the recorded ground motion. Empirical relationships involving peak ground motion were first used by Gutenberg and Richter (1942, 1956). Although it is becoming generally recognized that the peak value may not characterize the most important features of a record (Seed et al., 1976; Crouse, 1976), it continues to be used because it is a simple, easily obtained parameter.

In statistical analyses of strong motion, the primary factors controlling the level of strong ground motion at a site are usually taken to be the earthquake magnitude and its distance from the site. The general functional relationship usually assumed is of the form

\[ y = Ae^{bM(r+d)-c} \]

where \( y \) is the peak ground motion (acceleration, velocity or displacement) at a distance \( r \) from an earthquake of magnitude \( M \). The constant \( d \) is often taken to be 25 or 40, and the parameters \( A, b \) and \( c \) are obtained by least squares regression (e.g., Kanai, 1961; Esteva, 1970; McGuire, 1974). Strong
motion observations exhibit large unexplained variations for a given magnitude and distance however (e.g., Cornell et al., 1979; McGuire, 1974). Extensive statistical regression studies of peak ground motion values (e.g., Trifunac, 1976a, 1976b; Trifunac and Brady, 1975) still leave large residual scatter. The influence of factors such as the type of faulting, the geology at the site and the geology along the propagation path is presently not well understood (Page et al., 1975). The strong motion analysis and modelling in this thesis is directed toward a better understanding of these effects.

As an example of the observed dependence of maximum acceleration on distance Fig. 2-21a shows data obtained from 20 earthquakes between 1933 and 1969 (Cloud and Perez, 1971). In general, the maximum acceleration is found to decay as \( r^{-1.5} \), for distances in the range 15 < \( r < 300 \) km (Page et al., 1975). Peak acceleration values generally increase with magnitude in this same distance range, although the data in Fig. 2-21a, for magnitudes 4.8 to 8.3, do not show a strong magnitude dependence. For distances less than 15 km and greater than 300 km the number of observations has been too limited to allow general conclusions.

A central feature of collected peak acceleration data sets is their wide variability. It is reasonable to expect that this variation is at least in part attributable to dependences not included in a simple distance plot, such as station azimuth relative to the slip and rupture directions, the
source depth, and many other factors, including the local geologic conditions at the site. Another likely source of significant variation is that the peak acceleration samples only the high frequency end of the spectrum, as pointed out by Hudson (1972b). As shown in Chapter 3, the high frequency part of the spectrum is very dependent on shallow layering effects. The examples considered in the previous sections also showed that the peak amplitudes vary significantly over short distances, although wave arrivals are correlatable between records (e.g., Figs. 2-7, 2-8, 2-14).

Another characteristic of plots showing total data sets, like Fig. 2-21a, is that events are plotted for earthquakes and sites throughout the world. Any dependence that might exist on faulting type or regional geologic conditions may be masked. As an example of strong motion data from a single earthquake, the peak accelerations from the well-recorded 1971 San Fernando earthquake are shown in Fig. 2-21b (after Malley and Cloud, 1971). It is immediately clear that this earthquake exhibited a more rapid decay with distance than most of the events of Fig. 2-21a. It is hoped that studies such as this can begin to show whether this is due to the faulting type, or to the occurrence of the earthquake in a sedimentary area.

A plot of maximum accelerations for a single earthquake may still mask the dependence on local geologic conditions. Malley and Cloud (1971) note that all the San Fernando peak accelerations in Fig. 2-21b fall below the average line of
Fig. 2-21a, except for the value at Pacoima Dam and those at 25 and 29 km, which are the stations LH12 and CAS, respectively. It is interesting to note that the analysis of Chapter 3 suggests that the high peak acceleration value at the Lake Hughes station is due at least in part to high frequency sediment amplification at that site. Thus, at least some of the variations in peak ground statistics may be deterministically attributed to local site effects.

In summary, it is hoped that studies like that undertaken here will lead to a better understanding of the variability of peak ground motion statistics. The resolution of this problem will require a better understanding of the physical processes involved, but may also entail improved methods of parameterizing the ground motion.
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* Instrument Location, Structure: I.S. - Instrument shelter. B(10) indicates instrument is on basement level of a 10 story building. G indicates ground level.

** Earthquake, indicated in parentheses:
- blank - San Fernando, 9 Feb. 1971, 14:00:42GMT
- B - Kern County, 21 July 1952, 11:52:14GMT
- C - San Francisco, 22 March 1957, 19:44:21GMT
- D - Puget Sound, 29 April 1965, 15:28:44GMT
- E - Borrego Mountain, 9 April 1968, 02:28:59GMT
Figure Captions - Chapter 2

Figure 2-1 Acceleration records from the Borrego Mountain earthquake of 9 April 1968, radial components. The ELC record is plotted at 1/4 gain of the other records, and the south component is plotted. The stations are identified in Table 2-1.

Figure 2-2 Acceleration records from the San Fernando earthquake at selected regional stations. Radial components, relative to the approximate center of the fault (34.40N, 118.40W), except transverse at COS, LBI, SJC and WRI. All three components are plotted for reference in Appendix A. The circles indicate distances of 40, 80, 120 and 160 km from the center of the fault. (The CAS instrument malfunctioned ~1 sec after triggering, so that 1-2 secs of the record were lost just prior to the large negative pulse in the record (Trifunac, 1974; Brady and Hudson, 1971))

Figure 2-3 Acceleration records from the San Fernando earthquake at selected local stations. Radial components except transverse at stations to the south, indicated by the diamond (LNK, HSL, GPK, GLN and SL). See Appendix A for all 3 components. The approximate fault trace and aftershock zone (after Allen et al., 1973) and the epicenter of Hanks (1974) are shown. The record at PAC, the
station directly over the fault, is plotted at 1/4 the gain of the other records (N54E component). (A suspected polarity reversal at LH1 has been recently confirmed (A.G. Brady, 1979, personal communication); the correction is not reflected here or in Fig. 2-4. The LH1 record plotted in Appendix A includes the corrected polarity.)

Figure 2-4 The three-component (radial, transverse, up) velocity records from the San Fernando earthquake at the stations of Fig. 2-3. The PAC components are oriented N54E and S36E (after Boore and Zoback, 1974b).

Figure 2-5 The three-component velocity record at ORI and the particle motion for the segment within the cursors. The particle motion is plotted in the horizontal and vertical planes. The horizontal components, and thus the vertical plane, are rotated to the approximate orientation of the Rayleigh wave motion.

Figure 2-6 Central Los Angeles accelerograph station map (from Malley and Cloud, 1971). The two groups of stations studied are indicated. The locations of HSL and WL4 are also indicated to allow reference to the maps of Figs. 2-2 and 2-4.

Figure 2-7 San Fernando acceleration records from the stations in Area 1, North and East components. These sta-
tions are in the vicinity of WL4 (see Fig. 2-6).

Figure 2-8 Acceleration records from the stations in Area 2, about 4 km SE of WL4.

Figure 2-9 Vertical (up) components of the acceleration records of Area 1 (upper) and Area 2 (lower). The records are shifted as in Figs. 2-7 and 2-8.

Figure 2-10 Simplified geologic map of the Puget Sound region. The epicentral locations of the 1949 and 1965 events are shown, as well as the locations of the stations at Seattle (SAB, SFB) and Olympia (OLY).

Figure 2-11 Acceleration records at SAB and OLY from the 13 April 1949 Puget Sound earthquake, 7.1 M (Murphy and Ulrich, 1951), 70 km focal depth (Nuttli, 1952). The epicentral distances to SAB and OLY are approximately 47 and 26 km, respectively, and the hypocentral distances are approximately 85 and 75 km.

Figure 2-12 Acceleration records at SFB and OLY from the 29 April 1965 Puget Sound earthquake, 6.5M$_s$, 59 km depth (Algermissen and Harding, 1965). Epicentral distances to SFB and OLY are approximately 22 and 61 km, respectively, the hypocentral distances are approximately 63 and 85 km.

Figure 2-13 A) Amplitude spectra of the records at the OLY and SAB stations from the 1949 Puget Sound earthquake, north components. Spectra are of the approximately 17
second segment containing the principal energy, smoothed by tapering the covariance function, .5 hz bandwidth.

B) Amplitude spectra at OLY and SFB from the 1965 event, north components. Spectra are of segment containing the principal energy (approx. 13 sec).

Figure 2-14 Acceleration records form the San Fernando earthquake recorded in the Pasadena basin area. Components are radial, transverse and up. Contours show approximate depth (in feet) to basement (after Gutenberg, 1957). The San Fernando epicenter is approximately 35 km to the northwest.

Figure 2-15 A) Normalized amplitude spectra and coherency for the EW San Fernando records at MLK and ATH. Spectra and coherency are of the high amplitude portion of the records within the cursors.

B) Normalized amplitude spectra, coherency and spectral ratio for the San Fernando and Kern County records at the Hollywood Storage stations, EW components.

Figure 2-16 Acceleration records from the 1968 Borrego Mountain earthquake at SON, COL and SAA showing the effect of uncontrolled triggering. The records have been shifted by their expected S-P times so that the onset of P, if recorded, would occur at approximately zero time.

Figure 2-17 Borrego Mountain acceleration records at the stations HSL, W5A and SUB in Los Angeles, radial, transverse
and vertical components. The records have been shifted to line up the inferred S phase on the transverse components.

Figure 2-18 Acceleration records from the 5.3 M San Francisco earthquake of 22 March 1957 at four stations in San Francisco, approximately 12-18 km from the center of faulting.

Figure 2-19 Acceleration records from the aftershock of the 1957 San Francisco earthquake. Note the expanded vertical scale relative to the main shock records in Fig. 2-18. The P and S arrivals are indicated. They are clearer when plotted at larger scale. However, the P wave arrival is unclear at GGP, a bedrock station.

Figure 2-20 Records from the San Francisco aftershock at ALX, and the surface wave particle motion in the horizontal and vertical planes. (The epicenter is to the left, in the negative radial direction.) The particle motion in segment A shows clear purely-transverse Love wave motion. Segment B, after the Love motion has decayed, shows prograde Rayleigh wave motion.

Figure 2-21 A) Collected maximum acceleration values for 20 earthquakes between 1933 and 1969, with magnitudes from 4.8 to 8.3 (after Cloud and Perez, 1971). Lines indicating an $r^{-1}$ through $r^{-2}$ decay with distance are also shown for reference. The acceleration values plotted are the
peak value recorded on any of the components. With few exceptions, the peak value occurs on one of the horizontal components.

B) Peak acceleration values from the San Fernando earthquake of 1971 (from Malley and Cloud, 1971). The solid lines correspond to those which had been fit to the data in part A. Note that both the distance and acceleration scales are expanded relative to the plot in part A. The acceleration values plotted are the peak value recorded on either of the horizontal components, without rotation.
SAN FERNANDO
ACCELERATION RECORDS

Figure 2-3
1949 7.1M

E

Up

OLY

---

0  5  10  15  20  25  30 sec

---

150.0 --

cm/sec2  N SAB

---
Figure 2-13
SAN FERNANDO AT PASADENA STATIONS

Figure 2-14
Figure 2-16

A set of seismograms showing different angles and directions, such as N-S, E-W, and Up, with time axes ranging from 0 to 30 seconds and acceleration values ranging from 0 to 40.0 cm/sec².
Figure 2-20
ATTENUATION OF MAXIMUM ACCELERATION

EARTHQUAKES DURING WHICH THE PLOTTED ACCELERATIONS WERE RECORDED

1 = PARXFIELD, CALIF. 27 JUNE 1966 (M=5.3)
2 = EL CENTRO, CALIF. 18 MAY 1940 (M=7.1)
3 = PUGET SOUND, WASH. 13 APR 1949 (M=7.1)
4 = EUREKA, CALIF. 31 DEC 1956 (M=6.6)
5 = LONG BEACH, CALIF. 10 MAR 1933 (M=6.2)
6 = KERN COUNTY, CALIF. 21 JULY 1952 (M=7.5)
7 = EL CENTRO, CALIF. 30 DEC 1933 (M=6.5)
8 = LOGAN, UTAH 30 AUG 1962 (M=6.7)
9 = PORTLAND, OREGON 6 NOV 1962 (M=6.8)
10 = SANTIAGO, CHILE 13 SEPT 1945 (M=7.1)
11 = HEBGEN LAKE, MONTANA 17 AUG 1959 (M=7.1)
12 = SAN JOSE, COSTA RICA 30 DEC 1960 (M=7.7)
13 = MEXICO CITY, MEXICO 11 MAY 1980 (M=7.2)
14 = GUATEMALA 23 OCT 1950 (M=7.3)
15 = SAN JOSE, COSTA RICA 15 NOV 1949 (M=7.0)
16 = BISHOP, CALIF. 10 APR 1947 (M=6.4)
17 = LIMA, PERU 17 OCT 1968 (M=7.5)
18 = LITUYA BAY, ALASKA 10 JULY 1958 (M=7.8)
19 = PRINCE WILLIAM SOUND, ALASKA 27 MAR 1964 (M=8.3)
20 = LIMA, PERU 31 MAY 1970 (M=7.6)

DISTANCE IN MILES TO FAULT TO EPICENTER

MAXIMUM SINGLE COMPONENT ACCELERATION (gal/sec^2)

DISTANCE FROM THE EPICENTER OR FAULT

MAXIMUM ACCELERATION VALUES
- Circles: Stations Nearest the Episenter
- Square: Distance from the epicenter
- Diamond: Distance from surface faulting
- Cross: Other Stations
- Circle: Distance from the epicenter

Figure 2-21
CHAPTER 3

ANALYSIS AND MODELLING OF SITE EFFECTS

As discussed in Chapter 2, the geologic structure in the vicinity of a site can have a significant effect on the recorded strong motion. In this chapter we focus on methods of modelling the effects of the geologic structure at a site, and on the application of these methods in the analysis of site effects in strong motion observations.

3.1 Theoretical Modelling of the Site Response

The importance of the site geology on recorded strong motion has spurred substantial work on modelling the response of the site structure to a wavefield incident from the underlying basement. In the following sections the range of existing models are reviewed (3.1.1), and then the development of the method we will use is summarized (3.1.2). This is an extension of the propagator matrix method for attenuating layers, and is discussed in greater detail in Appendix B.

3.1.1 Methods of Modelling the Site Response

The effect of the site geology on recorded strong motion has been modelled with varying degrees of complexity and generality. The earliest work involved one-dimensional wave propagation or lumped-mass (one-dimensional finite element)
approximations to model S waves vertically incident to flat layers (e.g., Kanai, 1952; 1957; 1961). Much of the early work is covered in the bibliography by Duke (1958).

The general problem of plane wave propagation in a flat layered elastic medium was solved by Thomson (1950), with modification and elaboration by Haskell (1953, 1960, 1962). The approach of Thomson was used by Wuenschel (1960) to investigate vertical propagation in elastic layers. Roesset and Whitman (1969) and Roesset (1970) review solution methods for one-dimensional wave propagation in elastic and viscoelastic layers.

The first application of the work of Thomson (1950) and Haskell (1953, 1962) for general oblique body wave incidence was by Hannon (1964). He considered the effect of low-velocity surface layers on the free surface motion due to obliquely incident P waves. Roesset (1969) and Jones and Roesset (1970) studied the free-surface spectra when the layers are anelastic. A more recent work on the Thomson-Haskell propagator matrix approach is that of Silva (1976) who considers the extension of the Thomson-Haskell approach for the case of linear viscoelastic layers, in which attenuation is vectorial in nature (Buchen, 1971; Borcherdt, 1973). Except for the stability aspects discussed below, our extension of the Thomson-Haskell approach for attenuating layers shares aspects with the work of Silva and of Jones and Roesset (1970).

A characteristic of the propagator matrix approach is a
numerical instability at high frequencies. Growing exponentials which cancel analytically do not cancel on finite word length machines (e.g., Schwab and Knopoff, 1970). Knopoff (1964) avoided this difficulty by reformulation of the matrix method. Dunkin (1965) and Thrower (1965) showed that the problem could be circumvented by expanding the layer matrices into second-order compound matrices (e.g., Pestel and Leckie, 1963). This transformation was used by Harkrider (1970) and Watson (1970) and will also be used here. The most recent approaches to the numerical instability problem include Abo-Zena (1979), who reposed the matrix manipulation, and Kennett and Kerry (1979) who reformulated the problem using what they term reflection matrices rather than propagator matrices. These new approaches hold promise in that they appear to solve, rather than circumvent, the problems at high frequency.

More general approaches to the theoretical calculation of the site response include allowance for non-planar boundaries (e.g., Bouchon and Aki, 1977) or allowance for the nonlinear behavior of the medium under large strains. The most general methods are finite-difference or finite-element schemes allowing complex geometries (e.g., Joyner and Chen, 1975; Joyner, 1975). A widely used (Schnabel et al., 1972a) method of approximating nonlinear response in a plane-layered medium is the 'equivalent linear method' (Idriss and Seed, 1968; Schnabel et al., 1972b). This method involves one-dimensional wave propagation (i.e., vertical incidence) through attenuating layers which are linear in stress and strain. The solution is
iterative, with the medium velocity (slope of the stress-strain curve) and the attenuation being determined from the average strain developed in the iteration and the assumed strain-dependent non-linear characteristics of the medium (e.g., Hardin and Drnevich, 1970; Dobry, 1970). Despite the simplicity of this approach, it has been found to be a reasonable approximation to more exact nonlinear methods (Constantopoulos et al., 1973). The solution discussed below could be used in the same iterative manner in approximating nonlinear response in the general case of non-vertical incidence.

3.1.2 Propagator Matrix Solution for Attenuating Layers

The inclusion of attenuation can cause significant modification in the theoretical modelling of the response of near-surface sedimentary layers. In this section we outline the method we will use in modelling the site response in the presence of attenuation. The details of the development are presented in Appendix B.

The type of departure from perfect elasticity we assume is that of solid friction (Knopoff, 1956, 1959) which predicts attenuation proportional to frequency (Q independent of frequency). This proportionality has been observed in studies of attenuation in rocks (e.g., White, 1965) and also in sediments (e.g., Tullos and Reid, 1969; Kudo and Shima, 1970). Dobry (1970) and Dobry et al. (1971) suggest that the solid friction model, referred to as the 'linear-hysteretic solid', best approximates the hysteretic loss mechanism in soils. When the
strain levels exceed the range of linear response, strain-dependent medium properties must be used (e.g., Schnabel et al., 1972b). Although the solid friction model is noncausal, the travel time in typical site structures is so short (order of 1 sec.) that the noncausal effect is negligible. Much new data are becoming available in this area (Johnston and Toksoz, 1979a,b; Toksoz et al., 1979; Winkler and Nur, 1979) substantiating that for most dry rocks, \( Q \) may be independent of frequency and that \( Q \) decreases for strain amplitudes above about \( 10^{-5} \).

The equation of motion for the solid friction medium can be written (Knopoff, 1959)

\[
\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu)\left\{1 + \frac{1}{|\omega|Q_\alpha} \frac{\partial}{\partial t}\right\} \text{grad div} u - \mu \left\{1 + \frac{1}{|\omega|Q_\beta} \frac{\partial}{\partial t}\right\} \text{curl curl} u
\]

where \( Q_\alpha, Q_\beta \) are the quality factors for P and S waves. The (harmonic) dilatational and rotational potentials satisfying this equation of motion can be written in the form

\[
\phi = e^{i\omega t - ik\xi_\alpha(x + \nu_\alpha z)}
\]

\[
\psi = e^{i\omega t - ik\xi_\beta(x + \nu_\beta z)}
\]

where

\[
\xi_\alpha = 1/[1 + i \text{sgn}(\omega)/Q_\alpha]^{1/2} = 1 - i \text{sgn}(\omega)/2Q_\alpha + O(1/Q_\alpha^2)
\]

and

\[
\nu_\alpha = [c^2/\alpha^2 - 1]^{1/2}, \quad c > \alpha
\]

\[
\nu_\alpha = -i[1 - c^2/\alpha^2]^{1/2}, \quad c < \alpha
\]
and similarly for $\xi_\beta, v_\beta$. As discussed in Appendix B, this choice of variables is made to reduce the notational complexity and also to make as clear as possible the effect of departure from perfect elasticity in the solution (the attenuation factors $\xi_\alpha, \beta$ go to unity as $Q_\alpha, \beta$ go to infinity).

The derivation in Appendix B parallels that first posed by Thomson (1950). The boundary conditions are matched at each of the layer interfaces, up to the stress-free interface at the free surface. The compound matrix transformation (e.g., Dunkin, 1965) is made to control the numerical instability at high frequencies and incidence angles. The final matrices of the solution are similar to those obtained if the layers are perfectly elastic, but are more complex as most of the cancellations that occur in the infinite $Q$ case cannot be made. Finally the crustal transfer functions $U_P, SV, V_{SH}, W_P, SV$ (Haskell, 1962) are formed, relating each component of the free surface motion to that of the incident wave. Using these crustal transfer functions, the free surface motion due to a specified incident motion can be obtained by spectral multiplication. That is, the incident motion is convolved with the site response, yielding the free surface motion. As noted in the Appendix, the crustal transfer functions defined in this manner are similar to the 'elastic rock' amplification functions used in soil amplification studies (e.g., Roesset abd Whitman, 1969).

An example of the effect of $Q$ on computed free surface seismograms for a particular sedimentary structure is shown in
Fig. 3-1. The velocity structure approximates the crustal model of Kanamori and Hadley (1975) overlain by a low velocity layer. The example shown is for an SH wave incident at 30°. For the non-attenuating case the seismogram exhibits strong ringing because of the energy trapped by the velocity contrast at the sediment-rock interface. For the case with a shear Q of 20 in the sediment layer, the ringing is rapidly damped. Although the velocity contrast in this example makes the difference quite striking, the comparison underscores the importance of including the effect of attenuation when modeling the effects of low velocity sediments on computed ground motion.

It is also quite informative to study the effect of attenuation in the frequency domain. Various aspects of this problem have been considered in the literature, particularly in one-dimensional soil amplification studies (e.g., Kanai, 1961; Roesset and Whitman, 1969). As an example, consider the spectral response of a single layer over a halfspace under SH excitation (Fig. 3-2). For no attenuation, the modes, or peaks in the spectral response, occur at frequencies

$$f_n = n \delta / 4h \cos i, \ n = 1, 3, \ldots$$

where $\delta$ and $h$ are the shear velocity and thickness of the layer, and $i$ is the incidence angle in the layer. Thus, one effect of non-vertical incidence is to shift the modes to slightly higher frequencies, but the most significant effect is a reduction in the amplitudes of the modes (e.g., Roesset, 1969). This is caused by the reduction in the transmission coefficient for waves incident to the
layer at higher angles of incidence.

With the introduction of attenuation the modes shift to lower frequencies, but the shift is negligible. The principal effect is the reduction in the amplitude of the modes with increasing frequencies. From the work of Roesset and Whitman (1969) it can be shown that for $Q \gg 1$ the amplitude of the peaks of the crustal transfer function are given by

$$|V_{SH}(f_n)| = 2 \frac{\rho_r \beta_r}{\rho_h} \frac{1}{1 + \frac{\rho_r \beta_r}{\rho_h} \left(\frac{h}{2\beta Q}\right) f_n}$$

where $\rho_r, \beta_r$ are the density and shear velocity in the half-space. Thus the peaks of the crustal transfer function decay as $f^{-1}$.

In summary, the spectral response is maximum for vertical incidence and no attenuation. Non-vertical incidence reduces the response equally over all frequencies, while attenuation introduces a $f^{-1}$ decay in the peaks of the response. These effects are shown in the spectra of Fig. 3-2.
3.2 Amplification and Attenuation Effects of Site Structure in Strong Motion Observations

In the following sections of this chapter we apply the site modelling method described in the previous section to the interpretation of site effects in several sets of strong motion data. In particular, the frequency dependent aspects of both amplification and attenuation are considered.

The extensively recorded San Fernando earthquake provided the opportunity for detailed study of the local variations of strong ground motion. Subsequent analysis of the Pasadena area data suggested that the sediment amplification effects on strong motion are apparently different from the amplification effects observed for small earthquake response. In section 3.2.1 these data are re-examined to investigate this apparent discrepancy. The strong motion data from the Lake Hughes array are also considered. It is found that at the low frequencies in the passband of the Wood-Anderson instrument, the strong motion data shows effects similar to those previously observed for small earthquake response, though the effects of nonlinear response to large amplitude motion may be significant. At the higher frequencies to which the acceleration is most sensitive, the shallow sedimentary layering has the greatest effect.

The strong site effects in the Puget Sound strong motion data are considered in terms of the amplification and attenuation effects of the site structure in section 3.2.2.
3.2.1 Frequency-Dependent Local Variations in Strong Motion, San Fernando Earthquake

The first detailed study of the local variations in ground motion was made by Gutenberg (1957). He studied seismograms from small earthquakes as recorded at various locations in the Pasadena area and found a correlation of amplification with sediment thickness. He observed strong amplification and increased duration in seismograms obtained over deep sediment as compared to those obtained at the CIT Seismological Laboratory (SL), on basement.

The occurrence of the well-recorded San Fernando earthquake in 1971 provided the opportunity to determine whether the same effects seen for small earthquakes were observed for strong motion. Hudson (1972a) analyzed the strong motion records from the same area studied by Gutenberg. He found that the strong motion records do not show a strong effect of the sediment under the stations on the amplitude of the acceleration (e.g., see Fig. 2-14). Hudson concluded that the local distribution of ground shaking predicted on the basis of small earthquake studies may not correspond very well with that during a damaging earthquake. Because of the importance of this apparent discrepancy to understanding the variations in strong motion on a local scale, we have re-examined this problem considering the frequency dependence of the sediment layering effects and the frequency response of the recording
instruments.

A major difference between the experiment of Gutenberg and the strong motion data from the San Fernando earthquake is the passband of the recording instruments. Gutenberg recorded with the low frequency Wood-Anderson instrument, the San Fernando data is recorded on accelerometers and seismoscopes. The responses of these instruments are shown in Fig. 3-3. The accelerometer responds most strongly to the higher frequencies (10-15 hz), where the Wood-Anderson is down by a factor of ten from its maximum at about 1 hz. (Except for phasing, the difference between the accelerometer response and the acceleration is negligible below 15 hz.)

The effect of the low frequency response of the Wood-Anderson on the resulting record can be seen in Fig. 3-4, where the Pasadena area acceleration records of Fig. 2-14 have been passed through the Wood-Anderson instrument response for comparison. The Pasadena acceleration records in Fig. 3-4 are all roughly similar except that the SL record is higher frequency. In the Wood-Anderson records, on the other hand, the SL record has been considerably reduced because its higher frequencies are outside the preferred band of the instrument.

The amplitudes and durations of these computed Wood-Anderson records are in better agreement with the results of Gutenberg. The duration at SL is less than that at ATH or MLK by a factor of 3 or 4. In amplitude, one component at SL is similar to those at ATH and MLK, the other is lower by a factor of perhaps 2 or 3. All of Gutenberg's results are based
on single-component records. If only one component of the San Fernando strong motion had been recorded, the resulting amplification ratio would lie within these extremes, depending on orientation. Gutenberg found a duration factor of about 4, and an amplification factor about 4 (Gutenberg and Richter, 1956), though varying widely, for ATH relative to SL. Thus, the computed Wood-Anderson seismograms show a duration increase at ATH roughly the same as Gutenberg's, and an amplification similar, but somewhat less than that seen by Gutenberg.

Seismoscope recordings were obtained during the San Fernando earthquake (Morrill, 1971) from several of the same sites at which Gutenberg made his recordings, including SL and ATH. In discussing the amplification ratios, it is important to note that the strongly polarized (roughly transverse) motion at SL is anomalously large compared to that indicated by the seismoscope records at MtW and WHS, the other bedrock stations considered by Gutenberg. (The WHS station is located on the small bedrock exposure in the middle of Pasadena basin, shown in Fig. 3-4.). Amplification ratios formed relative to SL are very sensitive to the SL record. Of course, because of the very narrow frequency response of seismoscopes (Fig. 3-3), peak amplitudes on seismoscope records would not in general be expected to correlate well with Wood-Anderson amplitudes. In this case, however, the predominant period of the motion as shown by the Wood-Anderson records is about 1 sec (Fig. 3-4),
which is close to the seismoscope response peak.

In sum, it is difficult to establish at this point whether the lower amplification factors relative to SL are caused by strong motion being less amplified by sediment, due for example, to nonlinear response, or whether they are due to an anomalous record at SL. It is interesting to note however that this reduction in amplification factor, without a similar reduction in the duration factor, is an effect to be expected from nonlinear response.

As another example of the frequency-dependent differences in amplification as seen on acceleration records and Wood-Anderson records, we consider the records at the Lake Hughes array, shown in Fig. 3-5. These records were all obtained in instrument shelters or 1-story buildings and so should be free of building effects. The Lake Hughes acceleration records, on the right, show roughly similar amplitude and duration except for LH12, which is large. LH9 apparently triggered late. According to the USGS (1976) station list, LH1 is on about 300m of alluvium over granitic basement, LH4 is on weathered granite, LH9 is on gneiss, and LH12 is on thin alluvium over conglomerate. The Wood-Anderson seismograms computed from the acceleration records are shown for comparison on the left. They show quite clearly the sediment layer amplification at LH1 where the amplitude is 2 or 3 times that at LH4 or LH9. Note that though the LH1 record shows strong amplification at low frequencies (~1 hz), the higher frequencies show the attenuation effects of the sediments and the low Q material in
the vicinity of the fault zone, where $Q_s$ may be as low as 20 (Kurita, 1975). The thin-alluvium LH12 site has a high amplitude acceleration record, but does not have a proportionately high amplitude Wood-Anderson record because the peak amplitudes in the acceleration are at about 4-6 Hz, where the Wood-Anderson response is down by a factor of 3.

To consider in greater detail what is occurring in the amplification, Fig. 3-6 shows theoretical seismograms as would be recorded by the Wood-Anderson and accelerometer over the layered media shown. We consider sediment layers 15 and 90 meters thick (50 and 300 ft) overlying basement corresponding to the upper crust of Kanamori and Hadley (1975). The sediment layer has a shear velocity of .5 km/sec (1600 ft/sec), and a shear $Q$ of 20, as typically observed for shallow sedimentary deposits (e.g., Kudo and Shima, 1970). For simplicity, we consider SH waves, incident from the halfspace at $30^\circ$. The two input time functions are shown in the figure, their spectra peak at 2 and 7 Hz, respectively.

For the low frequency input, the Wood-Anderson record has increased amplitude and duration over the thick layer because of the instrument passband and the low frequency response of the layer. The acceleration is not amplified because the high frequencies to which it is sensitive are attenuated by the $Q$ in the layer. The layer is only a fifth of a wavelength thick at 1 Hz, while its two wavelengths thick at 10 Hz. In contrast, for the high frequency input, the acceleration record
is quite strongly amplified over the shallow layer.

To summarize the discussions of this section, the amplification effects due to the local geologic conditions are strongly dependent on frequency. Thus such effects when measured by a Wood-Anderson seismograph or by a strong motion accelerometer appear to be different. The Wood-Anderson passband tends to reflect the deeper sediment layering, as seen by Gutenberg, while acceleration records are influenced to a large extent by the shallow layering and do not show the deep layering well.

This explains some of the differences cited by Hudson (1972a), in comparing strong motion and small earthquake results. It may be that non-linear response plays a significant role in the remaining difference (i.e., lower amplification ratio) between the San Fernando data and Gutenberg's average results.

An important point which must be borne in mind is that significant local variability is not at all unique to the ground motion from damaging earthquakes. Gutenberg's small earthquake observations also show station differences which vary significantly from earthquake to earthquake. As an example, Fig. 3-7a shows records obtained by Gutenberg at SL, ATH and other Pasadena area stations from a small earthquake at about the same distance as San Fernando, but to the NE. The San Fernando strong motion records from this area (Fig. 3-4) actually show less variation than this example. As a further example, Fig. 3-7b shows the set of ATH/SL amplification
values observed by Gutenberg. Though the average may be about 4, there is a wide variation, ranging from less than one to nearly ten. The San Fernando earthquake would only represent a single sample in these data. If the San Fernando amplification value lies between 1 and 3, as suggested above, it would hardly represent a significant departure from this observed population.

In conclusion, there are significant local variations in recorded ground motion, whether that ground motion arises from a large or small earthquake. Understanding the local variations in strong ground motion from large earthquakes, for which the additional complexities of an extended source and nonlinear response are important, probably can not precede the understanding of the local variations of ground motion from small earthquakes. Intensive local studies of ground motion (e.g., Borchert and Gibbs, 1976; Tucker et al., 1978) will be important in achieving this understanding.

3.2.2 Puget Sound Site Effects

The strong motion data from the 1949 and 1965 Puget Sound earthquakes shows striking differences at the Olympia and Seattle stations, and in section 2.2 these differences were interpreted in terms of the geologic structure at the sites. In this section we model the response of inferred velocity structures appropriate for the two sites. Attenuation in the deep sedimentary structure under Seattle appears to be the principal cause of the spectral differences in the Olympia and
Seattle strong motion.

The velocity models used in this modelling are given in Table 3-1. The results of Langston and Blum (1977), Hall and Othberg (1974), Crosson (1972, 1976) and Danes et al. (1965) were utilized in arriving at these models. The Seattle model is a modification of the PS-9 model of Langston and Blum (1977). There is approximately 1 km of unconsolidated sediments overlying the tertiary sediments near Seattle (Hall and Othberg, 1974), and this is reflected in the Seattle model in Table 3-1. The Olympia model is similar to the Seattle model except that the sediments are shallower. The unconsolidated sediments are approximately .1 km thick at Olympia (Hall and Othberg, 1974), and the underlying tertiary sediments are taken to be approximately 1 km thick. Both Seattle and Olympia are taken to have a shallow (~10 m) surficial sediment layer with a nominal shear velocity of .25 km/sec (800 ft/sec) (e.g., Duke et al., 1973). The work of Howell (1963), Kurita (1975) and the recent work of Johnston and Toksoz (1979a,b) were drawn upon to arrive at the estimated Q structures in Table 3-1.

To consider the response of these models for earthquake-like excitation, rather than the response to an impulse or a wavelet, we consider as input an observed strong motion record obtained at an exposed-bedrock site. The broad-band GPK record from the San Fernando earthquake (see Fig. 2-3) was chosen for this purpose. The GPK (Griffith Park, Los Angeles) instrument is located in an instrument shelter on exposed
granite, though it is near a three-story building (USGS Station Lists).

The free surface seismograms computed for the Seattle and Olympia models, for this input, are shown in Fig. 3-8. For reference, the observed records at Seattle and Olympia from the 1949 event are also shown. The computed seismograms show the marked differences in amplitude and frequency content which are seen in the observed records, discussed in Chapter 2 (Figs. 2-11 and 2-12). These differences are primarily due to the attenuation in the deep sediments of the Seattle model, though the shallow surficial layer causes some high frequency amplification at Olympia. Note that though both models have this surficial layer, the Seattle motion does not have the high frequency amplification because too much of the high frequency energy has been lost due to attenuation in the deep sediment structure.

3.2.3 Summary

The observed effects of a sedimentary site structure on the recorded ground motion depend on the frequency band being considered and the variation of the medium parameters with depth. For a site with shallow sediments overlying a high-velocity basement, the amplification effects can be severe at high frequencies. For a deep sedimentary site, the amplifying effects of decreased seismic velocities and internal reflections can be offset by the amplitude reduction effects of attenuation, as shown by the Puget Sound data. For sites of
intermediate depth, amplification effects may be observed at low frequencies and attenuation effects at high frequencies, as shown for example by the Lake Hughes LH1 data.

Some of the observed local variations in strong motion can be understood, and modelled, in terms of these effects of the site structure. However, variations not predictable with simple plane layered site models remain, particularly at high frequencies.

Significant local variability is not unique to strong motion, but also occurs for small earthquake response. Perhaps a reduction in site amplification factors is the principal difference between strong motion and small earthquake response.

While the site effects can be of principal importance, as shown by the Puget Sound data, in other data the local variations are clearly not attributable to site effects. For example the amplitude differences at the nearby exposed-bedrock sites LH4 and LH9 (Fig 3-5) are probably not due to site effects, and in fact the source modelling of Chapter 5 shows that the difference can be explained in terms of the source radiation. Thus a final conclusion of the investigations of this Chapter and of Chapter 2 is that, as Hudson and Udwadia (1973) first suggested, though the effects of the source, medium and site are all important in understanding the local variations in strong motion, no single factor dominates.
# TABLE 3-1

Puget Sound Velocity Models

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<tr>
<th>Layer</th>
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<th>$V_p$ (km/sec)</th>
<th>$V_s$ (km/sec)</th>
<th>$\rho$ (gm/cm$^3$)</th>
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Seattle Model

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<th>$V_s$ (km/sec)</th>
<th>$\rho$ (gm/cm$^3$)</th>
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Olympia Model
Figure Captions - Chapter 3

Figure 3-1 Free surface seismograms for SH wave incidence to the velocity structure shown, for a non-attenuating and attenuating ($Q_\beta = 20$) low velocity layer. The lower seismograms are the computed motions at each of the layer interfaces, showing the incident and all reflected arrivals (for the non-attenuating case).

Figure 3-2 SH crustal transfer function for the layer-over-halfspace velocity model shown. Solid line for vertical incidence and infinite $Q$, shows the spectral response peaks at $f_n = n\beta/4h$, $n = 1,3...$. Dashed line shows effect of non-vertical incidence for the infinite $Q$ case ($i = 45^\circ$ in the halfspace). Dotted line shows effect of attenuation, for the vertical incidence case ($Q_\beta = 30$ in the layer).

Figure 3-3 Velocity response of typical Wood-Anderson seismograph, strong motion accelerometer and seismoscope. The natural frequency of the seismoscope is approximately the same as that of the Wood-Anderson (about 1.25 hz), but the seismoscope is very lightly damped (typically 10-15% of critical).

Figure 3-4 Pasadena area acceleration records from the San Fernando earthquake and the synthetic Wood-Anderson
records computed from them (radial and transverse components). Geologic map showing exposed basement and contours of sediment depth (in feet) after Gutenberg (1957).

Figure 3-5 Lake Hughes array acceleration records from the San Fernando earthquake and the Wood-Anderson records computed from them, N21E components. LH12 is about 25 km NW of the approximate center of the faulting (see Fig. 2-3). The apparent polarity reversal of the LH1 record has been confirmed (A. G. Brady, 1979, personal communication).

Figure 3-6 Computed Wood-Anderson and accelerometer records for a shallow and deep sedimentary layer over a halfspace for the velocity source functions shown.

Figure 3-7 A) Seismograms (Wood-Anderson) recorded at stations in the Pasadena area by Gutenberg (1957) from an M = 2.5 earthquake at about the same distance as San Fernando, but to the NE.

B) Amplification ratios, ATH/SL, as a function of period, observed by Gutenberg (1957). The solid circles are for S phase from local shocks, and most directly relate to the San Fernando strong motion data.

Figure 3-8 Computed horizontal free surface acceleration for the Seattle and Olympia site models (upper). The GPK acceleration record from San Fernando (transverse component) is input as SH, incident to the site models at 5
km depth, at 30°. Vertical scale arbitrary, indicating free surface amplitudes relative to that of the input record. For reference, in the lower part of the figure are shown the observed records at Seattle (SAB) and Olympia (OLY) from the 1949 event, north components (cf. Fig. 2-11).
Free Surface, with $Q_b = 20$ in top layer

Free Surface $Q_b = \infty$ in top layer

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$30^\circ$

1 sec
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--- $i = 90^\circ$
--- $i = 45^\circ$
--- $i = 90^\circ, q_B = 30$

![Figure 3-2](image-url)
SAN FERNANDO

AT

PASADENA STATIONS

WOOD-ANDERSON

ACCELEROMETER

Figure 3-4
ACCELEROMETER

LOW FREQUENCY WAVELET

HIGH FREQUENCY WAVELET

WOOD-ANDERSON

DEEP LAYER

SHALLOW LAYER

V_a

3.2

15m

90m

Q_a = 20

1 sec

Figure 3-6
Figure 3-7

A

1957 Jan. 8, 2012 P.S.T.  \( \Delta = 40 \pm 2 \text{ km, from NE} \)

- FP
- CIT (ATH)
- WHS
- CFR
- SL
- SRS

10 sec.

B

LOCAL SHOCKS
TELESEISMS
MICROSEISMS

Figure 3-7
The calculation of ground motion at intermediate distances from an earthquake fault in the heterogeneous earth is an important but difficult problem. Relatively simple methods can be used for the calculation of ground motion close to the fault where assumptions such as fault two-dimensionality or medium homogeneity are reasonable. At intermediate distances from the fault the effects of medium layering, free surface and the full dimensions of the fault become significant and cannot be neglected. We approach this problem using a hybrid method combining the effects of the source, propagation path and site characteristics, each being calculated with a suitable available technique. Our strong motion modelling is directed toward the interpretation of the observed strong motion records and the identification of the important parameters that determine the levels and frequency band of strong ground motion at intermediate distances from a fault.

Methods presently exist for calculating the strong motion close to a fault with an assumed source function. These methods are either based on analytic models or are numerical solutions obtained by finite element or finite difference methods. The assumptions in analytically-based methods, such as infinite medium or two-dimensionality, are reasonable at
near field distances where the medium characteristics and the
three dimensional nature of the fault are less important.
Similarly, finite element models can be useful close to the
fault but become more prohibitive at greater distances since
the intervening medium must be specified in detail. In this
chapter we discuss the development of a hybrid method allowing
the calculation of ground motion at points intermediate
between very near-field and teleseismic distances. The
effects of the source kinematics as well as the effects of
medium layering and local site geology are included in these
calculations.

In the first section of this chapter (4.1) we review
present modelling methods and discuss their limitations in
certain practical applications. In the second section (4.2)
we outline the analytic solution of Madariaga (1978) for the
Haskell source, which we will use in the calculation of the
source radiation. A more detailed discussion of the Madariaga
solution is given in Appendix C. In the third section (4.3)
we discuss the approach used to include the effects of complex
faulting, and the approximation of the surrounding medium as a
layered halfspace. The propagation through the local site
geology, a region of typically high velocity contrasts and
significant attenuation, is performed using the propagator
matrix solution for attenuating layers discussed in Chapter 3.
4.1 Methods of Modelling Strong Motion

Present methods for modelling strong motion range in complexity from the analytic representation of an earthquake source as a finite dislocation in an infinite homogeneous medium, to the numeric (finite element) representation of a dynamic fault in a heterogeneous half space. Each has certain limitations, imposed by their assumptions or computational practicalities. We discuss these methods and their application to modelling strong motion observations in this section.

The earliest modelling of near-field strong motion was the modelling of the 1966 Parkfield earthquake as recorded at Station 2 (80 m from the fault) by Aki (1968) and Haskell (1969). Aki applied the dislocation solutions obtained by Maruyama (1963), and Haskell used the equivalent representation theorem approach (Haskell, 1964). These are solutions for the displacements due to a finite fault in an infinite homogeneous medium. The full space assumption is not a severe limitation in modelling the Station 2 record, the station being very close to the strike-slip fault. Attempts to model the records at the more distant Stations 5 through 12 (5 - 15 km from the fault) using this full space formulation have not been as successful (Tsai and Patton, 1973; Anderson, 1974; Trifunac and Udwadia, 1974). The free surface and layering effects become significant at these distances but are not taken into account in this formulation. The 1971 San Fernando earthquake provided a similar situation. The Pacoima Dam station was close to the fault, allowing relatively successful
modelling of the strong motion record using the Haskell (1969) formulation (e.g., Tsai and Patton, 1973; Trifunac, 1974). Modelling the strong motion at some more distant stations was again less successful (Trifunac, 1974). The Pacoima Dam record has also been successfully modelled using a two-dimensional dislocation model (e.g., Boore and Zoback, 1974b).

A central feature of dislocation models is the prescription of the slip function on the fault. This prescription is purely kinematic and involves no consideration of the dynamics of the faulting process. Recent work (e.g., Richards, 1976; Madariaga, 1976, 1977) has been directed toward a more physical solution of the faulting process using shear-crack models. These are more physically realistic in that the slip function is no longer prescribed but results from the solution of a crack propagating in a prescribed pre-stressed medium. The numerical method of Das and Aki (1977a, 1977b) allows solution of the crack problem in the case of variable strength or pre-stress. These dynamic solutions are still full space solutions, but the slip functions which are computed can be used as the prescribed slip functions in the more flexible applications of the kinematic solution methods.

Full space models are of course limited by their inability to include the effects of the surrounding medium. The effect of a free surface on direct body waves is often approximated by the inclusion of an image source, which, for strike-slip faulting, is equivalent to doubling the calculated ground motion amplitudes. This is a good approximation for SH
waves, but can be a poor approximation for P and SV waves, especially at high angles of incidence (Anderson, 1976). The effect of the free surface on surface wave generation cannot be approximated in such a simple manner.

Half space models can, however, include the effects of the free surface on both the direct body waves and on the generation of surface waves. Anderson (1976) used a half space Green's function (Johnson, 1974) with the representation theorem formulation to study the effects of the free surface on calculated motions near a shallow rupturing fault. Israel and Kovach (1977) recently applied the generalized multipolar ray theory of Ben-Menahem and Vered (1973) to calculations of near field motions from a strike slip fault in a homogeneous half space. Levy and Mal (1976), also using a half space formulation, attempted to model the Parkfield records at the stations more distant from the fault than Station 2. Although their results are somewhat better matches to the observations than the results from full space models (Tsai and Patton, 1973; Anderson, 1974; Trifunac and Udwadia, 1974), a significant amount of the records remains unexplained. This is probably due at least in part to the significant departures of the medium from a homogeneous half space.

To allow for the effect of layering in a non-homogeneous half-space, Helmberger and others have modelled several earthquakes in the Imperial Valley using generalized ray theory (Helmberger, 1974; Helmberger and Malone, 1975). This approach allows inclusion of the principal effects of medium
layering, although the source is modelled as a point dislocation. The results using this method are especially good when the source is either small or distant so that it can be modelled as a point, and the medium is simple enough to allow using a computationally practical number of rays. Heaton and Helmberger (1977) were quite successful in modelling the El Centro record from the Borrego Mountain earthquake using this method. Heaton (1978) recently used this approach in modelling the San Fernando earthquake as a distribution of point shear sources in a halfspace. The discrete wave-number representation of Bouchon and Aki (1977b) allows the solution for strong motion from a finite source in a non-homogeneous half space, with the restriction that the finite source be two-dimensional. Recently Bouchon (1979) has extended that solution, allowing the source to be three-dimensional.

Dynamic finite element and finite difference methods have recently been applied in seismology and have potential in modelling the effects of complex faulting and media. McCowan et al. (1977) modelled the San Fernando earthquake using a two-dimensional representation of the fault and medium. Archuleta and Frazier (1978) have discussed three-dimensional modelling using finite element methods, and Cherry has performed extensive computational modelling, including the effects of nonlinear behavior immediately surrounding the fault (e.g., Cherry, 1973). Wiggins et al. (1978) have discussed some of their modelling results, based on a finite element solution of the faulting process (Archuleta and Frazier,
1978) and propagation through a two-dimensional medium using Green's functions convolutions.

These various methods for calculating expected ground motion each have certain limiting characteristics. The fullspace models cannot produce free surface effects such as Rayleigh waves or differential amplification of body waves. The homogeneous halfspace models cannot account for any medium layering effects. Thus although they produce a Rayleigh wave, it is undispersed, and Love waves cannot be accounted for at all. Perhaps most significantly, in a homogeneous half space the energy propagates in a straight line from each fault segment to the station. In a medium with a velocity-depth dependence, the ray path bends, so that the energy leaves the fault segment at a varying take off angle, stations farther from the fault sampling the radiation from a deeper angle.

Considering the strong motion observations in terms of the model assumptions, it is clear that neither a homogeneous fullspace nor halfspace model can explain many of the observed features. Two-dimensional models have not been successful in modelling strong motion records except very close to the fault, where the two-dimensionality is a reasonable assumption (such as San Fernando at Pacoima Dam, Parkfield at Station 2, and similar configurations). In certain situations, the path and site geology can be as important as the source in determining the relative amplitudes and frequency content of the ground motion. This is illustrated by some of the San Fernando data considered in Chapters 2 and 3, and particularly by
the Puget Sound data. The San Fernando data also indicate the importance of a complex rupture process, as opposed to a smooth, uniformly propagating rupture and constant dislocation. In the following sections we discuss a hybrid method allowing the inclusion of complexity in the rupture process and in the propagation. The strong motion at a site is obtained by combining the radiation of seismic energy at the source with the propagation of the energy along the path and finally through the crustal section at the site.
4.2 The Madariaga Solution for the Haskell Source

In this section we summarize the solution of Madariaga (1978) for the Haskell source and discuss some of the properties of that solution. The Madariaga solution will be used in section 4.3 in the modelling of elementary segments of a fault. The radiation from these segments is then propagated through the medium and to the site. Before discussing Madariaga's solution, we will discuss the Haskell source model for which Madariaga obtained an analytic solution.

4.2.1 The Haskell Full-Space Source Model

Haskell (1969) modelled the earthquake faulting process as a finite propagating line of dislocation, leaving a rectangular ruptured area behind the rupture front. This model of earthquake faulting, introduced by Ben-Menahem (1962), is often called the Haskell source model. Haskell (1969) presented many examples of theoretical seismograms from such a fault in a full-space (see also Thomson and Doherty, 1977).

It is of interest to study the general formulation of Haskell (1969) for the insights which it provides. The expression for the displacement at a point $\hat{x}$ due to a dislocation $D(\xi_1, \xi_2, t)$ on the fault plane $\Sigma$ can be written
\[
\begin{align*}
\mathbf{u}_1(\mathbf{x},t) &= \int \int \left\{ \frac{R_1}{r^2} \int_{r/a}^{r/b} D(\xi',t-t') t' dt' \\
&\quad + \frac{1}{r^2} \left[ \frac{R_2}{a^2} D(\xi,t-r/a) - \frac{R_3}{b^2} D(\xi,t-r/b) \right] \\
&\quad + \frac{1}{r} \left[ \frac{R_4}{a^3} D(\xi,t-r/a) - \frac{R_5}{b^3} D(\xi,t-r/b) \right] \right\} d\xi_1 d\xi_2 
 \end{align*}
\] (3.1)

The integration is over elements of the fault \( d\Sigma \), at 
\( \xi = (\xi_1,\xi_2) \), and \( r = |\mathbf{x} - \xi| \) is the distance from the observation point to the element on the fault. The \( R_n \) are radiation pattern terms, dependent on the component and type of slip (longitudinal or transverse). They are given in their entirety in Haskell (1969), but as an example, for longitudinal slip, and for the \( x \)-component,

\[ R_1 = \frac{3\mu}{2\pi\rho} \gamma_3 \left( 5\gamma_1^2 - 1 \right) \]

where \( \gamma_i \) are the direction cosines, \( \gamma_i = (x_i - \xi_i)/r \).

The general expression (3.1) is independent of the particular choice of slip function. Haskell used a ramp time function, propagating with velocity \( v_R \), and reaching a permanent displacement \( D_0 \)

\[
D(\xi_1,t) = \begin{cases} 
0 & t-\xi_1/v_R < 0 \\
(D_0/T)(t-\xi_1/v_R) & 0 \leq t-\xi_1/v_R \leq T \\
D_0 & t-\xi_1/v_R > T 
\end{cases}
\]
where $T$ is the rise time of the ramp. A schematic of this model of the fault kinematics is shown in Fig. 4-1a. Of course, other time functions could be used rather than the ramp, such as a step-function, or an exponential function (e.g., Ben-Menahem and Toksoz, 1963).

Regardless of the particular choice of slip function, however, the general form of the displacement, eq. 3.1, is worth studying. First, the integrand is made up of terms decaying with distance as $r^{-1}$ through $r^{-4}$. The $r^{-1}$ term is the far-field term, the only term used in teleseismic studies. It has terms recognizable as propagating with P and S velocities. The far-field term shows that the far-field displacement time function is the derivative of the displacement time function on the fault, and that its amplitude varies as the inverse cube of the medium velocities. The higher order decay terms are often called the near-field terms. They represent coupled P and S waves traveling with a 'spectrum of velocities' from $\alpha$ to $\beta$ (Boatwright and Boore, 1975). The near-field terms can be decomposed into waves apparently propagating with P and S velocities however (Aki and Richards, 1980). In this case these near-field P and S waves have both radial and transverse components.

The displacement expression (3.1) also shows that, whatever the nature of the slip function $D(\xi,t)$, the highest frequencies in the displacement arise from the far-field term, with the high frequency contribution decreasing for each of the terms down to the $r^{-4}$ near-field term. The near-field
terms are most important at low frequencies. This is of interest because in our modelling of strong motion, the higher frequencies will be of greatest concern.

The application of Haskell's solution entails two basic difficulties circumvented in the analytic solution obtained by Madariaga (1978). First, it involves a two-dimensional spatial integration over the fault, a time-consuming numerical procedure (though with modification the integration can be reduced to one dimension (Boatwright and Boore, 1975; Sato, 1975)). Second, it is a solution in displacement, requiring numerical differentiations to obtain velocity or acceleration, the observed variable. Numerical differentiation has extensive inherent noise problems as evidenced even in the original paper of Haskell (1969). Although the observed data can be twice integrated for comparison to the theoretically computed displacement, as is usually done, most of the energy, and detail, in the strong motion is at higher frequencies, severely attenuated by the integration process.

A compromise approach is to match velocities, the once-differentiated theoretical displacement and the once-integrated observed acceleration. Consideration of velocity is recommended by other aspects also (e.g., Boore and Zoback, 1974b), the velocity being related to the fault pre-stress, and also to the energy in the propagating wave. The analytic solution of Madariaga (1978), discussed in the following section, is a solution in velocity, and no approximate numerical differentiation procedures are required. This, plus the fact
that it is a computationally rapid solution, recommends it highly for application in strong motion modelling studies.

4.2.2 The Madariaga Analytic Solution

Madariaga (1978) obtained a closed-form representation for the integral solution of Haskell (eq. 3.1). He obtained an analytic solution in particle velocity for a step source slip function (a delta function in the slip velocity). Thus, his solution is essentially an impulse response solution, and the corresponding solution for a general slip function is obtainable via convolution of the desired slip velocity function with this impulse solution. We outline general aspects of the Madariaga solution in this section, with a more detailed discussion of the solution and aspects of its application being given in Appendix C.

The finite aspects of a rectangular fault are made manageable in the Madariaga solution by representing a finite rectangular dislocation as the superposition of four infinite quadrantally dislocations suitably delayed in time and space. Referring to Fig. 4-1b, the velocity field due to the rectangular dislocation can be written in terms of those due to the infinite quadrantally dislocations as

\[ U(x,y,z,t) = \hat{u}_Q(x,y,z,t) - \hat{u}_Q(x,y-W,z,t) \]

\[- \hat{u}_Q(x-L,y,z,t-L/v_R) + \hat{u}_Q(x-L,y-W,z,t-L/v_R) \]

The problem thus becomes one of obtaining the solution for an
infinite quadrantal dislocation. Madariaga obtained this solution by using the Laplace transform of the velocity solution for an impulse in slip velocity, and then using the Cagniard-de Hoop technique to invert the transforms. He obtained the solution for a quadrantal dislocation as the sum of a cylindrical and spherical wave contribution. The cylindrical waves arise from the sudden appearance of the dislocation line along the y-axis. The spherical waves are generated by the motion of the tip of the dislocation line along the x-axis.

When the solutions for the four dislocations are superposed, the cylindrical waves cancel except in the strip \(0 < y < W\). In the region outside this strip, i.e., above and below the fault in \(y\), only the spherical waves are present. In this total solution, the cylindrical waves are radiated by the sudden initialization of rupture along the line of length \(W\) at \(x = 0\), at \(t = 0\), and from the sudden termination of rupture at \(x = L\), at \(t = L/v_R\). The spherical waves are radiated by the corners of the dislocation.

The solution for the cylindrical wave has a square root singularity in the velocity at the time of the P and S arrivals. However, the singularities are integrable, and the convolved velocity for more general slip velocity functions than an impulse is not singular at the arrival times. The singular behavior does lead to certain computational difficulties in obtaining the velocity convolution however, and these aspects are discussed in Appendix C. Boore and Zoback (1974a)
also discuss the computational aspects of this problem. Their solution exhibits the same singular behavior since, as Madariaga (1978) points out, their two-dimensional solution corresponds to the cylindrical wave part of the complete solution. The relationship between Madariaga's solution and that of Boore and Zoback (1974a) is discussed in greater detail in the appendix.

The complete solution for the velocity or displacement, obtained by superposition of the four dislocations, can be used to study the distribution of ground motion around a rectangular fault, in a manner similar to Haskell (1969). As an example, Fig. 4-2 shows velocity seismograms computed for stations distributed in azimuth around a vertical strike-slip fault. The stations are equispaced and located a constant distance from the fault. The fault is of dimensions $L \times W = 4 \times 2 \text{ km}$, and the top of the fault is $1 \text{ km}$ below the plane of the stations. The rupture velocity and rise time of the (ramp) source slip function correspond to those used in modelling the Parkfield earthquake (e.g., Tsai and Patton, 1973). These are full-space seismograms, but the incidence angles are in a range where, for strike slip faulting, the usual doubling should reasonably approximate the effect of the free surface (Anderson, 1976).

The effect of the propagating rupture on the focusing of energy in front of the rupture is clear in the seismograms of Fig. 4-2. The stations toward which the rupture propagates have higher amplitudes and shorter durations compared to those
behind the rupture. The magnitude of this focusing effect depends strongly on the ratio of the rupture velocity to the medium velocity, or the Mach number (e.g., Boore et al., 1971; Boore and Zoback, 1974a). In terms of cylindrical and spherical waves, note that since the stations of Fig. 4-2 are outside of the strip \(0 < y < W\), there are no cylindrical waves, and only the spherical waves contribute to the computed motion. An illustrative example showing the contributions of the individual dislocations to the total solution, and the contributions of the cylindrical and spherical waves for an individual dislocation is included in Appendix C (see Figs. C-2 through C-4).

The Haskell source model, for which Madariaga obtained the analytic solution discussed here, is based on a purely kinematic description of the faulting process. The most serious aspect is probably the prescription of a constant dislocation over the fault. Dynamic shear-crack solutions show the dislocation to vary from zero at the fault edges to a maximum near the center of the fault (e.g., Richards, 1976; Madariaga, 1976). Solutions allowing variable pre-stress or rupture strength over the fault show an even more complex distribution of slip (Das and Aki, 1977a). Post-earthquake observations also often indicate varying slip along the fault (e.g., Aki et al., 1979). As Madariaga (1978) shows, the uniform dislocation of the Haskell model implies singularities in the stress at the fault edges, regardless of the choice of source slip.
functions.

Thus, we can not expect straightforward modeling with a Haskell-type source to realistically match observations, except at very long periods, or at stations so close to the fault as to be insensitive to the effects of the fault edges (such as Parkfield at Sta. 2). However, a realistic source model, with dislocation varying over the fault as in crack solutions, can be approximated by dividing the fault into elementary segments of constant dislocation. With decreasing segment size, the method can closely approximate any given dislocation variation over the fault. The impact of Madariaga's solution in this approach is that it is computationally rapid enough to make feasible the approximation of a complex faulting process using many elementary segments. In the next section we discuss the methods used in propagating the radiation from each of these elementary segments through the surrounding medium and then through the site geology.
4.3 Modelling the Radiation from Elementary Fault Segments in a Layered Halfspace

In Chapters 2 and 3 we considered the observed strong motion data and studied the effects of the source, medium and site in these observations. In this section we discuss the method used to model strong motion, which includes the ability to account for these effects.

Considering the results of the analyses in Chapters 2 and 3, the study of the site effects showed that the site structure can cause significant amplification and that attenuation effects can be severe for deep sedimentary structures. The analysis of the San Fernando data in chapter 2 showed a significant azimuthal dependence in the data and suggested a complex rupture process. The study of the arrivals in records from several earthquakes showed that the recorded ground motion is often primarily S waves. Surface waves, which can be observed in some records, are usually of longer period and arrive later in the record. Thus, modelling the early, energetic portion of strong motion records entails modelling the S wave propagation from a possibly complex rupture process on a finite, three-dimensional fault, with the inclusion of the effects of the geology at the site. We discuss in the following the modelling method which we will use. Certain approximations are of course required, and these will be discussed following the description of the method.
In our approach the fault is subdivided into elementary segments. We consider several paths by which the radiation from each elementary segment may reach the site. These are shown schematically in Fig. 4-3a. They include propagation directly to the top of the basement, reflection off the free surface, and reflection at depth. We shall call these the direct, surface-reflected and depth-reflected arrivals. There may be any number of surface-reflected or depth reflected arrivals, depending on the complexity of the layered medium, but only a single direct arrival. Typically only those surface and depth reflected arrivals which arrive during a specified time window and have a minimum amplitude are included. Note that P-S conversions along the path are not included.

The raypaths for each of the possible arrivals are obtained by raytracing. Our principal concern being with the S radiation, we solve for the S-wave raypath using the method of false position (Acton, 1970) to obtain the ray parameters of the minimum time paths from the center of the fault segment to the site. We also evaluate the net product of the P, SV and SH transmission and reflection coefficients to the top of the basement along the path determined. These coefficients are obtained by the solution of the Zoeppritz-Knott equations at each interface. Finally, from the ray path, we obtain the image point A' of the point A at the base of the site. Thus, A and A' are in the vertical plane containing the site and the center of the fault segment. These are demonstrated for a depth reflection in Fig. 4-3b. We also obtain the net
attenuation coefficient along the path to the base of the site for P and S waves.

The source radiation from the elementary fault segment is then obtained at the image point A' using the Madariaga (1978) solution as discussed in the previous section. With the application (in the frequency domain) of the product of the reflection and transmission coefficients and the net attenuation coefficients, we obtain the effective source radiation at the point A at the base of the site. This is then propagated up through the site layering using the layering transfer functions \( U_{P,SV}, V_{SH} \) and \( W_{P,SV} \) obtained using the propagator matrix solution for attenuating layers discussed in Chapter 3 and Appendix B. Thus, we include all multiple reflections and the complete effect of Q in the region of the propagation path having the greatest velocity contrasts and attenuation. The entire procedure is presented in flowchart format in Fig. 4-4.

The severity of the approximations which must be made in this approach depend on the segment sizes and path lengths. As was clear in the discussion of the radiation from a finite dislocation, the near-field P and S waves have both radial and transverse components, and the radiation is, in general, comprised of spherical waves (and cylindrical waves, for certain orientations). Yet the reflection and transmission coefficients, and the propagation through the site layering all assume plane P and S waves. This involves two assumptions - that the radiation can be approximated as P and S waves with far-field like behavior (having only radial and transverse
components, respectively), and that the curvatures of the wavefronts are small enough to allow their approximation as planes. The first requires that we be at a sufficient distance in terms of fault dimensions, the second that we be at a sufficient distance in terms of wavelengths. These approximations are applied at the first interface encountered along the path. If we let this distance be $r_0$ (Fig. 4-3), then these distance requirements can be expressed as $r_0/w \gg 1$ and $r_0/\lambda \gg 1$ ($w$ being the segment dimension). In terms of frequency, the second implies a lower frequency constraint, $f \gg v_s/r_0$, below which the plane-wave approximation will be poor.

Thus the first approximation is a small-source approximation, the second is a high-frequency approximation. In terms of the source modelling, small elementary segments are already required in order that a variable dislocation over the fault can be approximated by segments of constant dislocation, as discussed in the previous section. Also, the high-frequency approximation is not inappropriate if our greatest concern is with modelling the early portion of strong motion records, which is usually of high frequency, with the longer period energy arriving later.

As an example of theoretical seismograms obtained using this method, Fig. 4-5 shows velocity seismograms from a single fault element, computed at points distributed in azimuth around the source. The layered velocity structure corresponds to an upper crustal structure (Kanamori and Hadley, 1975).
overlain by a sedimentary layer. These seismograms include the direct arrival and the reflection off the bottom of the first crustal layer. The surface-reflected arrival has negligible amplitude in this case because of the very low downward transmission coefficient at the sediment-bedrock interface. Considering the small-source and high-frequency approximations, for this example we have $\frac{r_0}{w} \approx 7$ and $f \gg \frac{v_s}{r_0} \approx 0.2 \text{Hz}$. These are typical values in our applications. Note that at close stations, the first approximation can be satisfied if the element size is made sufficiently small. The second cannot be circumvented in this manner — for small $r_0$ the approximation will only be valid at high frequencies, regardless of element size.

The effects of both the source and the layered structure are apparent in the seismograms of Fig. 4-5. The internal reflections in the sedimentary layer increases the duration, though the attenuation rapidly damps the oscillations. The depth-reflected arrival also increases the complexity of the seismograms. Its arrival, about 1/2 sec after the direct arrival, can be clearly seen in the high frequency records at 5 and 30° azimuth. The seismograms show a pronounced source directivity effect, with high frequencies and amplitudes in the rupture propagation direction. The directivity is more severe than in the previous example (Fig. 4-2), since the Mach number, or ratio of rupture velocity to shear velocity, is $2.8/3.2 \approx 0.88$, while it is $\approx 0.65$ in the example of Fig. 4-2.
It is of interest to compare our modelling approach to a segmented-fault approach like that used by Trifunac (1974) and Trifunac and Udwadia (1974). Trifunac (1974) modelled the San Fernando records at PAC and three other stations using the Haskell (1969) fullspace dislocation solution. The fault was divided into segments with dimensions of several km. To approximate the effect of the free surface, he used the image-source doubling as is often done. Of course, for thrust faulting, the presence of an image thrust fault is physically impossible, as Trifunac (1974) notes. By including the effects of both the site layering and the free surface, the approach discussed here should allow more successful modelling of the ground motion at intermediate distances from the source. Of course the plane wave, high frequency approximation will limit the application of this approach to the modelling of higher frequencies than the long periods considered by Trifunac (1974).

Sato (1977) used an approach similar in some respects to that discussed here. Being primarily concerned with long period ground motions of a few seconds and greater, he did not segment the fault. However, he propagated the radiation through the near-surface layering in a manner similar to that discussed here. He also attempted to include the effects of the wavefront curvature by an approximate integration over wavenumber. Mikumo (1973) also investigated a similar approach, including the use of plane wave corrections at the free surface. He found the plane wave corrections to be
unsatisfactory at the long periods involved in modelling the displacements. This is a reflection of the limitation to high frequencies and short wavelengths involved in the plane wave approximation, as discussed above. The validity of the plane wave assumption for the long wavelengths associated with the long period displacements would require that the station be quite distant from the source.

Our method is applied to modelling some of the San Fernando strong motion records in the next chapter. Though the method involves approximations, these are not severe for stations at intermediate distances from the source, and for sufficiently small elementary fault segments. The method is of course restricted to the modelling of the body wave portion of the observed strong motion. However, as shown in the next chapter, it can be very useful in modelling, and understanding, the source-dependent azimuthal variations in strong motion around a complex fault. It is also useful in studying the modification of the strong motion due to the effects of geologic conditions local to the site.
Figure Captions - Chapter 4

Figure 4-1  A) Haskell source model of a rectangular fault of length L by width W. Rupture initiates instantaneously along the width at \( t = 0 \), and terminates at \( x = L \), at \( t = L/v \). Slip may be longitudinal or transverse (parallel or perpendicular to the rupture direction, respectively). The ramp source slip function used by Haskell (1969) is also shown.

B) Representation in the x-y plane of the four infinite quadrantal dislocations used by Madariaga (1978) in obtaining the solution for the finite rectangular Haskell model of (A). The velocity field due to the dislocation of (A) can be obtained by the superposition of those of the four quadrantal dislocations in (B), as I - II - III + IV.

Figure 4-2  Computed velocity seismograms for points distributed in azimuth about a strike-slip fault. Fault is 4 by 2 km, 1 km below the plane of the stations. \( V_p, V_s, V_{rupt} \) are 5.5, 3.4 and 2.2 km/sec, respectively. Rise time is 0.9 sec.

Figure 4-3  A) Schematic diagram showing direct, surface-reflected and depth-reflected paths from an elementary fault segment to a site. Depth-reflected paths have
take-off angles less than $90^\circ$, direct and surface-
reflected paths, greater than $90^\circ$. The near-surface
structure seen by the surface-reflected arrival in the
vicinity of the fault is in general assumed to be dif-
ferent from that near the site.

B) The image point $A'$ of the point $A$ at the base
of the site, for a depth-reflected path. $A'$ is deter-
mined by the take-off angle $i_0$ and the total path length
from the center of the segment to the point $A$. For the
direct arrival, $A'$ and $A$ may be identical if there are no
layer interfaces between the segment center and the base
of the site (as in this example).

Figure 4-4 Flowchart representation of the procedure used in
modelling the ground motion at a site. For the $i$-th ele-
mental fault segment, the velocity $\dot{u}(t)$ is obtained at
the image point of each $j$-th arrival, convolved with
$\delta_i(t)$, and delayed by $t_{o_1}^i$, the time of rupture initiation
on the $i$-th segment. Convolution with the path
reflection/transmission coefficients and the path
attenuation, and then the site layering transfer func-
tions, yields the free-surface velocity at the site.
Finally, the total ground motion is obtained by summation
over each arrival, and each element.

Figure 4-5 Velocity seismograms (horizontal components) com-
puted at points distributed in azimuth around a $1 \times 1$ km
fault element embedded in a crustal layer overlain by a
sedimentary ($Q_s = 20$) layer. Strike-slip faulting, $v_R = 2.8$ km/sec, 10 cm slip, ramp source time function with 1 sec rise time.
Figure 4-1
For i-th segment

For j-th arrival

Obtain S raypath and Image Pt. by ray tracing

Obtain wRT along path to base of site by Zoeppritz-Knott

Eval. \( \hat{u}(t) \) at Image Pt., convolved with \( \hat{s}_i(t) \), delayed \( t_{o_i} \)

Rotate to P, SV, SH along ray

Apply wRT, path attenuation, obtaining \( \hat{u}(t) \) at base of site

Propagate through site layering using P, SV, SH transfer functions

Rotate \( \hat{u}(t) \) at surface to a common orientation

Sum over j, i

Figure 4-4
Figure 4-5

Az = 175°

Az = 150°

Az = 120°

Az = 90°

Az = 60°

Az = 30°

Az = 5°
CHAPTER 5

COMBINED EFFECTS OF SOURCE, MEDIUM AND SITE - MODELLING STRONG MOTION FROM SAN FERNANDO

In this chapter we apply the methods discussed in the previous chapters for modelling strong motion at intermediate distances from a fault. We consider the strong motion observations from the San Fernando earthquake, which was recorded with a better distribution of stations than any other earthquake. In the first section (5.1) we review the results of earlier studies, both teleseismic and near-field, of the San Fernando source mechanism. The results of these studies indicate that the thrust faulting initiated at depth on a steeply dipping plane, continuing onto a shallower plane as it neared the surface. We begin our source modelling studies in section 5.2 by considering the effects of a general dipping-thrust source geometry of this type on the azimuthal variations of local ground motion.

Following this general study, we turn to consideration of the San Fernando event in particular. The results of several teleseismic and local studies indicate that the faulting initiated with a massive localized rupture. In section 5.3 we model this initial rupture event, attempting to match the first part of the record at several stations distributed in azimuth around the source. In section 5.4 we extend the
modelling beyond this localized initial event, attempting to model the records at the two bedrock stations SL (Seismological Laboratory) and GPK (Griffith Park Observatory). A summary of the results of the various aspects of strong motion modelling considered in this chapter is included in section 5.5.

In general our modelling in this chapter is concerned with the ground motion at intermediate distances, rather than at the close-in PAC station. This is partly a reflection of the approximations inherent in our method, as discussed in the last chapter. Additionally, our greatest interest is in the modelling and interpretation of the wealth of strong motion observations obtained at intermediate distances from the source.
5.1 Source studies of the San Fernando Earthquake

A large number of studies have been made on various aspects of the San Fernando earthquake. In this section we review the principal results of the teleseismic and local studies of the source mechanism.

The teleseismic first motion studies indicate that the faulting initiated on a plane striking about N70W and dipping approximately 50°NE, with both thrust and strike slip (left lateral) components (Whitcomb, 1971; Dillinger and Espinosa, 1971; Canitez and Toksoz, 1972; Whitcomb, et al., 1973; Dillinger, 1973; Langston, 1978). Typical values, and their associated uncertainties, are given by the recent study of Langston (1978): strike N70±8W, dip 53±2NE, rake 76±14.

Although earlier studies (Allen et al., 1972) estimated the hypocentral depth to be about 8 km, most studies now place the hypocenter at 13-14 km depth (Canitez and Toksoz, 1972; Hanks, 1974; Langston, 1978). Displacement near the surface of approximately 2 m occurred on a plane with a shallower dip than that indicated by the fault plane solutions (Kamb et al., 1971). The aftershock distribution also indicates a fault surface which steepens with depth (Whitcomb et al., 1973). Thus, most modelling studies include a fault plane which dips at about 30° near the surface, steepening to about 50° below 5-8 km (e.g., Boore and Zoback, 1974b; Niazy, 1975; Langston, 1978; Heaton and Helmberger, 1979)

The results of more detailed studies of the source
faulting are also of interest. Hanks (1974) studied the Pacoima Dam (PAC) strong motion record and concluded that rupture initiated with massive, but localized faulting in the hypocentral region. The fault model obtained by Trifunac (1974) also includes a local area of high slip at the initiation of rupture. Bouchon (1978a), studying the teleseismic P waveforms, concluded that the high amplitude of the first pulse required a massive localized initial rupture, in agreement with the work of Hanks (1974) and Trifunac (1974). Langston (1978) included a short downward rupturing portion with a high rupture velocity in his fault model to generate the high amplitude teleseismic pulse. Hanks estimated the dimensions of the initial rupture to be 3-6 km in radius. Bouchon estimated the initial event to be near 1.5 km in radius, and Langston modelled the initialization as a 2 km strip along the bottom of the fault.

In summarizing these results of the San Fernando source studies, two aspects are of particular importance. First, there is general agreement that the faulting initiated at depth and propagated toward the surface on a steeply dipping fault plane, continuing onto a plane with shallower dip as it neared the surface. Second, both local and teleseismic studies suggest that the faulting initiated with a massive but localized rupture. These two aspects are considered individually in the next two sections of this chapter.
5.2 Effects of a Dipping Thrust Mechanism on Local Strong Motion

In this section we consider the theoretical distribution of ground motion from a thrust geometry in which the faulting initiates at depth on a steeply dipping plane, similar to that of the San Fernando earthquake. The variations with station azimuth are shown to be quite significant. The San Fernando strong motion data considered in Chapter 2, and shown in Fig. 5-1, are discussed in terms of this result.

Boore and Zoback (1974a) considered the theoretical ground motion at points in the vicinity of a dipping thrust fault. Constrained by the two-dimensional nature of their solution, discussed in Chapter 4, their study was limited to points along a profile normal to the fault strike. With the relaxation of the two-dimensionality constraint, we study the azimuthal dependence of local ground motion, considering station distances similar to those of the stations in Fig. 5-1.

For this study we consider a simple dipping fault model, 6 km in length (down dip) by 1 km in width. Midway down the fault, at 7 km depth, the dip changes from $50^\circ$ to $30^\circ$. The fault strikes N70W. This fault model is shown schematically in Fig. 5-2a. The theoretical velocity records are computed at eight stations equally distributed in azimuth and 25 km from the fault center, as indicated in Fig. 5-2b. These stations and the strike and dip of the fault are chosen for qualitative comparison with the San Fernando data (Fig. 5-1).
though our concern here is also with implications of a general
dipping thrust fault on local ground motion.

The theoretical velocity seismograms at the stations of
Fig. 5-2b are shown in Fig. 5-3. A simple exposed-bedrock
site model (Table 5-1) has been used for all stations. The
seismograms are based on the solutions of 1 x 1 km elementary
segments, with rupture initiating at depth and propagating up
the dip at 2.7 km/sec. A slip of one meter, constant over the
fault, and a ramp source time function with a .25 sec rise
time are used. We consider pure thrust faulting, or 90° rake.
(For clarification in the following, the rake measures the
angle the slip vector makes with the horizontal, in the plane
of the fault. Thus pure thrust is 90° rake, left lateral
strike slip is 0° rake. We will use the same convention to
discuss the direction of rupture propagation.)

The theoretical records in Fig. 5-3 show a significant
variation with azimuth. The stations on the up thrust block,
to the north and east, are of low amplitude compared to those
on the down thrust block, and this aspect is considered in
greater detail in the following. This effect was noted in the
analyses of the San Fernando data in Chapter 2, and is
apparent in the data shown in Fig. 5-1.

It is particularly interesting to consider the separate
contributions of the lower and upper portions of the fault to
the total seismograms. Fig. 5-4 shows the contributions from
the lower, 50° dip segment. The amplitudes at the northern
and southern stations are quite similar. In contrast, the
contributions from the upper, 30° dip segment show a marked
difference between the northern stations and those to the
south (Fig. 5-5). The southern stations have several times
larger amplitude than those to the north. Thus in the seismo-
grams for the whole fault (Fig. 5-3), the amplitudes at the
northern stations are almost entirely due to the 50° dip por-
tion of the fault, while the 30° dip portion adds almost noth-
ing. The southern stations, on the other hand, have signifi-
cant contributions from both portions, but the greatest con-
tribution is due to the segment of the fault dipping at 30°.

The differences seen at stations to the north and south
of the dipping fault are due to the effect of the rupture
velocity on the radiation pattern from the source. This
directivity effect was discussed in Chapter 4 and was apparent
in the examples considered there. To clarify this effect, the
S wave first-motion radiation pattern for a propagating dislo-
cation is shown in Fig. 5-2c, after Savage (1965). The direc-
tion of rupture is a preferred radiation direction, and the
magnitude of this effect depends on the ratio of the rupture
velocity to the shear velocity, or Mach number (e.g., Boore et
al., 1971). The case considered here is that of longitudinal
slip, i.e., slip in the direction of rupture propagation.
This reasonably approximates the San Fernando event, which was
primarily thrust faulting with rupture propagating upward from
its initiation at depth.

The question may arise as to how well the directivity
effect predicted for a theoretical smoothly propagating
dislocation corresponds to the effects of more realistic models of faulting. Boore and Joyner (1978) investigated the effects of variations of rupture velocity and fault slip on the directivity. They found that the predicted directivity effects were as strong or stronger than for the corresponding smoothly propagating dislocation, provided that the average rupture velocity was the same. Madariaga (1977a) showed that the theoretical directivity effects for a shear crack are similar to those for a dislocation, differing mainly in the backward direction. The numerical crack propagation solutions of Das (1976) also show significant directivity effects for both smooth and complex strength distributions on the crack plane. In terms of observations, Bakun et al. (1978) related the azimuth differences in the high frequency radiation from two small, nearby earthquakes with the same fault-plane solutions to rupture directivity effects. In laboratory fracture experiments Vinogradov (1978) found that non-uniform fracture yielded directivity effects similar to those for smooth fracture, except for wavelengths short relative to the sample dimension. Significant directivity effects have also been observed in foam rubber faulting experiments (Hartzell and Archuleta, 1979).

In sum, the directivity effects for a propagating dislocation imply a significant azimuthal variation in the radiation. This theoretical effect predicted for a simple dislocation is also predicted by more realistic models of faulting and has received verification in some field and laboratory
studies. The importance of this effect for local strong motion is in its implications for the distribution of strong motion levels around a dipping thrust fault. It suggests that the strong azimuthal dependence shown by the San Fernando data (Fig. 5-1) may be expected to occur in general for this type of faulting.
5.3 Modelling the Initial Rupture Event at Intermediate-Distance Stations

The results of both local and teleseismic studies of the San Fernando source mechanism indicate that the faulting initiated with a massive, localized rupture event, as discussed in section 5.1. In this section we model this initial rupture event, and attempt to match the early part of the record at several of the stations distributed around the source (Fig. 5-1). Being able to match at least the initiation of the rupture is of interest since the source geometry of the initiation determined from the teleseismic studies provides a valuable constraint on the modelling. Secondly, the three-dimensional effects of the medium and the complexities of rupture on an extended rupture front should be less important in the early part of the records.

Utilizing the teleseismically determined values of strike, dip and rake, we start with a relatively small source area (e.g., Bouchon, 1978a). The parameters which remain to be specified include the rupture velocity, the rupture direction, the source time function and the magnitude of slip. We will consider the theoretical seismograms for a particular set of parameters and then discuss the effects of perturbations in the various parameters.

The comparison between the theoretical velocity seismograms and the first part of the observed records are shown in
Fig. 5-6a,b for seven stations ranging from HSL, to the south of the fault, to LH9, to the northwest. For this solution, the initial rupture occurs on a plane with strike, dip and rake of N60W, 52°NE and 76°, respectively. The fault ruptures with \( v_R = 2.7 \) km/sec, at 110° (measured like the rake), and the slip is 10 m on the fault of dimensions 1 x 1 km. The source time function is a ramp, with a rise time of .25 sec. The theoretical records in Fig. 5-6 are all computed for the simple exposed bedrock site model considered previously (Table 5-1). For certain of the stations this site model is inappropriate, but it allows the study of the variations with the site conditions held constant. The effects of a sedimentary site will be considered below.

The correspondence between the theoretical records and the initial part of the observed records in Fig. 5-6a is quite good. In multi-station modelling, the fit to individual records as well as the fit to observed variations from station to station need to be considered. The fit to the first, almost purely transverse pulse in the HSL and GPK records is quite good, both in amplitude and period (~.7 sec). The theoretical amplitude of the pulse reduces as we move in azimuth to GLN and then to SL. At SL the amplitude of this pulse is less than half that at GPK, both in the observed and theoretical records.

At GLN, a sedimentary site, the observed amplitude is significantly greater than the theoretical. The large amplitudes in the GLN record, nearly double those at GPK (Fig. 5-6a
and 5-1), suggests a significant site effect at GLN. Duke et al. (1973) give a velocity model for this site, with a sedimentary structure extending down to basement at about 1.5 km depth. Using an approximation to this model, we will recompute the theoretical record at GLN. It should be noted however that the velocity models of Duke et al. (1973) are estimates except at very shallow (~100 ft) depths. The approximated Glendale model (14 layers simplified to 10), overlying the crustal structure of the simple rock site of Table 5-1, is given in Table 5-2. Reasonable values of Q were assumed to complete the specification of the site model.

The comparison between the GLN record and the theoretical record obtained using this site model is shown in Fig. 5-7a. The effect of the sedimentary site structure has been to approximately double the amplitude of the theoretical pulse. The theoretical and observed amplitudes are now in quite good agreement. There is a significant difference in the theoretical arrival times using these two models. The arrival time for the sedimentary model solution is more than a second later than for the rock site solution (Fig. 5-7a, 5-6a). We note in passing that, in addition to the advantages cited in Chapter 2, instruments with an absolute time base could provide confirmation of the magnitude of the site effect at GLN.

The sedimentary effects at HSL are apparently not as significant as those at GLN. The station is located at the northern edge of the Los Angeles basin, where the depth of the basin is still shallow (Fig. 5-1, 2-2). Both the amplitudes
and frequencies at HSL are similar to those at GPK (Fig. 5-1). The GLN record has approximately doubled amplitudes and also shows attenuation of the higher frequencies present in the GPK and HSL records. The HSL record does show large amplitude, long period motion later in the record however, possibly due to surface waves.

The fit between the theoretical and the initial part of the observed records is not as good at the northern azimuths (Fig. 5-6b). In terms of the station to station patterns however, the observed record at LH9 is of low amplitude, and triggered the instrument late. This low in the observed radiation pattern is also present in the theoretical solution, the theoretical LH9 record being of very low amplitude, except for the vertical component, compared to that at LH4, only about 6 km away.

At LH4, although the first motion is correct on the radial, the theoretical records are of too long period, and the transverse and vertical amplitudes are too large. These effects may be due to our simple model of the rupture initiation, in which the rupture nucleates instantaneously along the starting edge of the dislocation. Also, at the incidence angles appropriate for the Lake Hughes stations, the converted S to surface P phase, not included in our solution, may be important (e.g., Anderson, 1976; Bouchon, 1978b; Heaton, 1978).

At PLM both the theoretical and observed amplitudes are low, though the actual fit of the records is again poor.
Since PLM is a sedimentary site, though the site structure is unavailable, we recompute the theoretical record using the same sedimentary structure used for GLN (Table 5-2). The record obtained using this structure (Fig. 5-7b) shows increased amplitudes and coda on the radial, due to the internal SV reflections. The long duration of the observed PLM record (Fig. 5-1) may be due in part to this effect, although surface waves are probably the principal cause.

Considering the effects of changes in the source parameters on the theoretical records of Figs. 5-6 and 5-7, several aspects are noteworthy. Increasing the size of the source area increases the period of the pulse at the southern stations, unless the rupture velocity is simultaneously increased. This, in turn, increases the directivity effects. However, if the source area is increased by the addition of a downward rupturing segment, the period at the southern stations is only slightly broadened. Thus, a model assuming initially bilateral rupture (Bouchon, 1978a; Langston, 1978) is consistent with the observed records. At the northern stations, increased amplitudes result from the downward rupturing portion, suggesting that the slip on that portion is not as large as that on the upward rupturing portion. However the effects of instantaneous line nucleation in our model may also be contributing to the northern amplitudes, as discussed above.

The theoretical records are quite sensitive to changes in strike. Significantly different strike from the teleseismic
estimate degrades the fit between the theoretical and observed amplitudes, particularly in matching the low amplitudes at SL and LH9. The strike used here (N60W) differs little from the N70±8W estimate of Langston (1978). In contrast, the fit between the theoretical and observed amplitudes is relatively insensitive to changes in dip. Theoretical records based on dips of 5° from the 52° value show only slight differences.

The magnitude of slip (10 m) required here to match the observed amplitudes, given the assumed source time function, is consistent with other estimates from local studies. Hanks (1974) suggested an initial slip of 5-9 m, and the model of Trifunac (1974) as well as one of the models of Heaton and Helmberger (1979) includes an initial slip of 12 m.

In summary the modelling of the initial San Fernando rupture event as seen in the velocity records at intermediate-distance stations is generally quite consistent with previous local and teleseismic studies of the rupture initiation. The fit between the theoretical and observed records demonstrates that, at least at stations to the south and east of the event, the model of the faulting and the theoretical modelling method used are sufficiently accurate to allow successful modelling of the body-wave portion of the velocity records at these intermediate distances. In the next section we turn to the modelling of a larger part of the fault, beyond just the initial rupture event.
5.4 Modelling the San Fernando Velocity Records at SL and GPK

Since the initial part of the fault was modelled relatively successfully in the previous section, at least at southern azimuths, we now turn to consideration of larger parts of the fault and later parts of the observed records. Limiting ourselves to exposed-bedrock stations, since the site structure at most non-bedrock stations is poorly known, we will attempt to model the records at the Seismological Laboratory and Griffith Park Observatory stations (SL and GPK). These two stations are separated by about 20° in azimuth, relative to the fault center. They are about 10 km from each other, and about 25 km from the center of the fault.

Many parameters must be specified to proceed with the modelling, some largely by guesswork, as Heaton (1978) notes. Heaton (1978) and Heaton and Helmberger (1979) based their starting model on the results of the teleseismic modelling by Langston (1978). We in turn will base our starting model on the results of Heaton and Helmberger.

A schematic of the fault geometry is shown in Fig. 5-8a. The fault model consists of a lower segment dipping at 53°, which intersects at 5 km depth with the upper segment, dipping at 29°. The entire fault system strikes N75W. Contours of the slip on the fault plane for Heaton's preferred model, Norma 163, are shown in Fig. 5-8b.

Heaton and Helmberger (1979) modelled the long period displacements using the generalized ray approach. Our study
differs in two aspects relative to their modelling. First, since we will attempt to match the velocity records, the frequencies of interest are higher in our modelling. Second, we will only attempt to match the body-wave portion of the record, before the surface waves and the effects of the near-surface faulting become important.

As a first example, we compute the theoretical velocity for the smoothly varying dislocation function of the Norma 163 model (Fig. 5-8b). The computed seismogram at SL for this fault model is shown, with the SL record, in Fig. 5-9. In this solution, the total record is based on the solutions of (185) 1 x 1 km elementary segments, each with a constant slip (at 76° rake) corresponding to the contours of the Norma 163 model. The near-surface faulting has not been included in the solution. A ramp source time function with a rise time of .5 sec, and the simple rock site model considered previously (Table 5-1) are used. We have chosen a rupture velocity of 2.7 km/sec, and rupture for each elementary segment propagates directly up-dip. Radial rupture propagation is approximated by delaying the initiation of rupture for each element according to its distance from the origin. For comparison, Heaton (1978) used a rupture velocity of 2.8 km/sec on the lower segment and 1.8 km/sec on the upper segment, and a triangular slip velocity function of .8 sec duration. He also approximated radial rupture propagation by delaying each point shear source according to its distance from the origin.
It is immediately apparent that this smooth dislocation function does not yield sufficient amplitudes or high frequency content in the computed velocity record at SL (Fig. 5-9). The effect of convolution with a long duration source time function is to remove much of the higher frequencies. This is shown by the amplitude spectra of the slip velocity functions, \( \dot{\delta}(t) \), in Fig. 5-10. For either a box or triangular \( \dot{\delta}(t) \) (ramp or quadratic slip time function), the spectra drops to low values by \( f \approx 1/T \), where \( T \) is the duration of \( \dot{\delta}(t) \). Thus, for \( T \approx 1 \) sec, there would be little energy beyond 1 hz in the convolved velocity even if the unconvolved velocity spectrum were nearly flat. In the following we will use a ramp source time function with a rise time of .25 sec, so \( 1/T = 4 \) hz, and we will attempt to model the observed records up to frequencies of 2-3 hz.

Another aspect controlling the amplitudes and frequencies in the computed velocity is the spatial variation of the rupture process over the fault. A smooth dislocation variation like that of the Norma 163 model will generate long periods satisfactorily but cannot generate the higher frequencies in the SL velocity record. Thus, the generation of higher frequencies will require quite detailed specification of the variations in the rupture process over the fault. To limit the resulting large parameter space, we constrain the fault model to be similar to the Norma 163 model and allow increased variability only in the dislocation function. Thus we hold the geometry fixed, as well as the rupture velocity (2.7
km/sec) and source time function (.25 sec ramp). The spatial variability over the fault is limited by the size of the elementary segments. The solutions discussed in this section are based on 1 x 1 km segments. This 1 km dimension is reasonable for the SL station distance in terms of the tests of Aki (1968). Though smaller elements would allow modeling at higher frequencies, the computational time requirements rapidly increase as the element size is decreased (the present solution requires ~ 10 cpu hours on a PDP11).

To understand the effect of variation of the dislocation amplitude as a function of location on the fault, we study the contribution to the SL record of elementary segments along various profiles on the fault.

We first consider a profile across the width of the fault, given by A - A" in Fig. 5-11. The velocity seismograms at SL due to unit (1 m) slip on each 1 x 1 km element along the profile are shown in Fig. 5-12. (For a manageable number of records, only the transverse components, which dominate at this azimuth, are plotted.) These seismograms show several effects. The earliest arrival is from the element at A', directly up the fault from the hypocenter at H in Fig. 5-11. The elements on either side of A' begin rupture according to their distance from A' and the rupture velocity. The arrivals from elements to the east of A' (toward A") are earlier than corresponding elements to the west because the travel distance to SL decreases as the profile is traversed from west to east (A to A"). The most important aspect of these seismograms is
the increasing amplitudes as the profile is traversed from west to east. This is partly due to the decreasing distance, but is primarily due to the directivity effect in the radiation. SL is closer to the main radiation lobe in the direction of rupture for the eastern elements. Thus, in terms of modelling the SL record, the eastern edge of the fault is of the greatest importance. For example, a slip of 1 m near the eastern edge of the fault yields the same amplitude at SL as a slip of more than 2 m near the western edge. This is an azimuth-dependent aspect. For example, a similar profile evaluated for GPK, 20° CW in azimuth from SL, shows nearly constant amplitudes across the fault.

We next consider a profile of elementary segments up the dip of the fault, profile B - B' in Fig. 5-11. The seismograms computed at SL for this profile are shown in Fig. 5-13. These seismograms show the effect of downward rupture as seen at SL. The downward-rupturing elements below the hypocenter at H are much lower amplitude and longer period than the corresponding upward-rupturing elements above H, again because of the rupture directivity. Langston (1978) modelled the high amplitude, short period teleseismic pulse by a small portion of the fault rupturing downward. These seismograms show that the record at SL, and in fact any of the southern stations, cannot resolve this downward rupture feature. The stations to the north, however, show more sensitivity to a downward rupture component.
The seismograms for this profile up the dip of the fault (Fig. 5-13) also show the effect of the increasing angle of incidence at the base of the site on the free surface record. The seismograms from the shallower elements (toward B') show longer durations due to stronger internal reflections within the site structure as the angle of incidence increases.

Finally, we consider a profile extending over the hinge line of the fault, as shown by the profile C - C' in Fig. 5-11, the seismograms for which are shown in Fig. 5-14. The seismograms from elements on the upper portion of the fault at 29° dip (toward C') yield lower amplitudes at SL than those from the elements at 53° dip. In terms of the record at SL, the modelling will be less sensitive to slip on the upper portion of the fault than on the lower. Of course, the contributions from the upper part of the fault also begin arriving later in the SL record, approximately 3 seconds after the arrival from the hypocenter (Fig. 5-13), for the assumed rupture velocity of 2.7 km/sec.

Following the analysis of amplitudes from various sections of the fault, and a parallel study of arrival times, a trial specification of the slip magnitude as a function of location on the fault can be made. The first attempt was to obtain a slip distribution which would match the SL record, beginning by perturbation of the Norma 163 model. The results of this effort are shown in Fig. 5-15. The fault model giving rise to this record at SL is shown in Fig. 5-16. It is clear that the match to the GPK record, also shown in Fig. 5-15, is
quite poor. (For these comparisons, the observed velocity records are bandpass filtered, .25 to 3.5 hz, using a cascaded (phase free) 3rd order Butterworth filter (e.g., Oppenheim and Schafer, 1975)) However certain aspects of this study merit discussion before proceeding. The first pulse is relatively well matched in both the SL and GPK records, since we are using approximately the initial rupture model discussed in section 5.3. Beyond that the principal alterations to the Norma 163 model are the introduction of localized areas of high slip along the eastern portion of the fault. The larger, approximately 3 x 5 km area contributes the large pulse of approximately 1 sec duration which arrives about 2 secs after the first pulse in the SL record. This pulse is also observable in the JPL record (Fig. 5-1) and in the ATH and MLK records. Marked on the fault in Fig. 5-16 is a band from within which the radiation arrives at SL during the duration of this 1 sec pulse. Thus the SL record implies a local area of high slip somewhere within this band, though it need not be at its location in the Fig. 5-16 model. In fact the poor match at GPK suggests that this location is incorrect.

Our modelling efforts proceeded in this manner, studying the fault time history relative to the observed velocity records at both SL and GPK. Eventually we arrived at our final fault model, for which the comparison between the theoretical and observed velocity records are shown in Fig. 5-17. The fault model yielding these theoretical records is
shown in Fig. 5-18.

The fit of the theoretical to the observed velocity record at SL is seen to be quite good (Fig. 5-17). The observed record departs from the theoretical in the latter part of the record, when the radiation from the topmost part of the fault is arriving. The unfiltered record (Fig. 5-1) also shows a longer period, probably surface wave component arriving at about this time.

The fit to the GPK record is also quite good during the early part of the record. At the azimuth of GPK, the radiation from the top part of the fault is sensed more efficiently, so there is significant energy later in the record not present in the theoretical record. Another departure is the pulse on the radial component beginning about 3 seconds after the first arrival on the transverse, which is not present in the theoretical record. The generation of this pulse appears to require a more complex fault geometry than the simple planar fault model used here. The pulse does not appear to be a site effect since it has a recognizable counterpart on the radial component of both the HSL and GLN records (Fig. 5-1).

The fault model giving rise to the theoretical velocity records of Fig. 5-17 shows strong local variations in the slip magnitude over the fault (Fig. 5-18). These localized areas of high slip, with dimensions of 2-5 km, may represent zones of low strength or high pre-stress. Thus, the effort to model the velocity records directs us to the consideration of some
of the recent theoretical concepts of the details of the rupture process on a fault. Aki and others (Aki, 1979; Aki et al., 1979) have discussed the importance of high strength barriers on the rupture process and modelled their effects on the resulting radiation (Das and Aki, 1977a; 1977b). The results here are quite consistent with the barrier concept, and the dimensions of the high slip areas in Fig. 5-18 are within the range inferred for barrier intervals by Aki et al. (1979). Of course, as we model the faulting process in greater detail we are beginning to see the variations first discussed in terms of a statistical coherence length by Haskell (1966) and Aki (1967).

It is also of interest to consider the stress drop implied by the fault model of Fig. 5-18. For a finite rectangular fault, the stress drop is approximately related to the slip and dimensions as (e.g., Kanamori and Anderson, 1975)

\[ \Delta \sigma = \frac{2}{\pi} \mu \frac{\bar{D}}{w} \]

where \( \mu \) is the rigidity and \( \bar{D} \) is the average slip over the area of width \( w \). As discussed by Madariaga (1977b), this assumes a constant stress drop, which is probably not the case at the scale of the slip areas in Fig. 5-18, and some spatial averaging method should be used. If we estimate the stress drop by forming the spatial average \( \frac{\bar{D}}{w} \) within each slip area, we obtain a stress drop for the initial slip area at the hypocenter of \( \Delta \sigma \approx 600 \) bars (for \( \mu = 3 \times 10^{11} \) dynes/cm\(^2\)). This is consistent with the estimates of Hanks (1974) and Bouchon
(1978a) for the stress drop of the initial rupture event (350-1400, and 400-500 bars, respectively). Our stress drop estimate must be considered poorly resolved, however, since just the source geometry constant varies by a factor of two (e.g., Madariaga, 1977b).

A final point that must be made concerning the fault model of Fig. 5-18 is that it is not within the resolution of velocity modelling to ascertain that the slip is zero between the high slip areas. The theoretical velocity scales as $D_o/T$, rather than $D_o$, as discussed in Chapter 4. For arbitrary $D_o$, as $T$ increases $D_o/T$ decreases, and the convolution operation of width $T$ effectively removes the high frequencies. Thus, slip of low amplitude or long rise time would yield very little difference in the theoretical velocity records at SL and GPK.

A final aspect of this study is the comparison of our fault model with previous, smoother models obtained in displacement modelling studies (e.g., Heaton and Helmberger, 1979; Trifunac, 1974). Since the longer period displacements are sensitive to the local spatial average of the slip on the fault, spatially averaging our fault model should yield a dislocation function similar to those obtained in displacement studies. As an example, Fig. 5-19 shows the dislocation function of Fig. 5-18 averaged with a 3 x 3 km operator (boxcar). Wider averaging operators would of course yield a smoother function. The important point is that the smoothed slip function is similar to the Norma 163 model of Heaton and
Helmberger (1979) and the model of Trifunac (1974). For all three models the principal slip on the lower part of the fault occurs within a zone of 4-6 km width and aligned slightly east of the up-dip direction.

5.5 Summary

Several aspects of the modelling of strong motion have been considered in this chapter. These exercises yield several conclusions. First, there is a significant azimuthal variation in the strong motion from dipping thrust faults. This is shown in the theoretical modelling and corresponds to the variations seen in the San Fernando observations. This is an effect of the upward-propagating rupture and probably should be expected to occur in the strong motion from future thrust earthquakes which initiate at depth. This effect is also of interest in terms of the variations seen in empirical peak ground motion data discussed in Chapter 2. Perhaps with a better understanding of the effects of faulting in thrusting, as well as other tectonic regimes, some of the wide variations in strong motion parameters can be accounted for.

Another result of the modelling in this chapter is that the determination of the initial rupture event obtained teleseismically has been shown to be quite consistent with the initiation of the faulting as seen at the local stations. This underscores the importance of teleseismic studies for bracketing the allowed class of models to begin with in model-
ling local strong motion data.

Finally, successful modelling of the local velocity records requires the existence of localized regions of high slip on the fault. This is an aspect of modelling a higher frequency part of the spectrum than is important for satisfactory models of the displacement records. These local areas are of greater importance than simply being required to fit the velocity records. It is these small-dimensioned areas which determine the high frequency content of strong motion records. For the higher frequencies in the acceleration records the areas involved may be quite small. For example in the Brune (1970) model, the corner frequency in the displacement, and thus the peak frequency in the velocity, is related to the dimension as

$$f_C = \frac{2.34\gamma/2\pi r}{r}.$$  

Thus dimensions of the order of .2 km are necessary to understand the spectrum at 5 hz. Of course, at these high frequencies, the effects of the medium may become quite severe. Nonetheless, given the human and cpu time, it may in principle be possible to successfully model the body waves in strong motion observations at least up to several hertz. However, this cannot be done in a predictive sense. The location of areas on a fault which may act as high strength barriers in the rupture process probably could not be known before hand, even if a likely fault plane could be postulated.
The most fruitful approach in strong motion prediction is probably through deterministic modelling of rupture on a postulated fault, with an overlying stochastic specification of sub-areas of high strength or high slip. This approach shares features with the modelling of random fault patches by Boore and Joyner (1978) and Savy (1979).

This does not diminish the importance of deterministic post-earthquake modelling studies. It is only through these studies that knowledge of the variables to be used in the stochastic modelling can be obtained. The results of the present study should be a first step toward that end.
Table 5-1
Simple Rock Site Model

<table>
<thead>
<tr>
<th>Layer</th>
<th>Vp</th>
<th>Vs</th>
<th>Qp</th>
<th>Qs</th>
<th>rho</th>
<th>h</th>
<th>(Depth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.00</td>
<td>3.00</td>
<td>200</td>
<td>120</td>
<td>2.73</td>
<td>1.50</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>6.10</td>
<td>3.40</td>
<td>240</td>
<td>140</td>
<td>2.80</td>
<td>--</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Table 5-2
GLN Sedimentary Site Model

<table>
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<tr>
<th>Layer</th>
<th>Vp</th>
<th>Vs</th>
<th>Qp</th>
<th>Qs</th>
<th>rho</th>
<th>h</th>
<th>(Depth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.52</td>
<td>0.26</td>
<td>70</td>
<td>40</td>
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<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>3</td>
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<td>0.60</td>
<td>100</td>
<td>50</td>
<td>2.16</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>4</td>
<td>1.52</td>
<td>0.67</td>
<td>100</td>
<td>55</td>
<td>2.37</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>5</td>
<td>1.76</td>
<td>0.82</td>
<td>110</td>
<td>65</td>
<td>2.37</td>
<td>0.18</td>
<td>0.12</td>
</tr>
<tr>
<td>6</td>
<td>2.03</td>
<td>1.01</td>
<td>120</td>
<td>70</td>
<td>2.53</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>7</td>
<td>2.22</td>
<td>1.11</td>
<td>120</td>
<td>75</td>
<td>2.37</td>
<td>0.30</td>
<td>0.60</td>
</tr>
<tr>
<td>8</td>
<td>2.37</td>
<td>1.18</td>
<td>150</td>
<td>80</td>
<td>2.43</td>
<td>0.30</td>
<td>0.90</td>
</tr>
<tr>
<td>9</td>
<td>2.49</td>
<td>1.24</td>
<td>160</td>
<td>80</td>
<td>2.43</td>
<td>0.30</td>
<td>1.20</td>
</tr>
<tr>
<td>10</td>
<td>5.00</td>
<td>3.00</td>
<td>200</td>
<td>120</td>
<td>2.73</td>
<td>0.50</td>
<td>1.50</td>
</tr>
<tr>
<td>11</td>
<td>6.10</td>
<td>3.40</td>
<td>240</td>
<td>140</td>
<td>2.80</td>
<td>--</td>
<td>2.00</td>
</tr>
</tbody>
</table>
Figure Captions - Chapter 5

Figure 5-1 The three-component (radial, transverse, up) velocity records from the San Fernando earthquake at the selected local stations considered in Chapter 2.

Figure 5-2 A) Schematic illustration of simple dipping fault model. Rupture propagates upward from the bottom of the 50° dip segment (at 9.3 km depth). The dip changes to 30° at a depth of 7 km. Sectional view of vertical plane, normal to the N70W fault strike (N20E is to the right).

B) The stations at which the theoretical velocity records from the fault model are computed. The eight stations are equally distributed in azimuth, 25 km from the fault center.

C) S wave first-motion radiation pattern for a unilateral propagating dislocation, after Savage (1965). Radiation pattern is for the case of longitudinal slip, with a rupture velocity of .9 times the shear wave velocity.

Figure 5-3 Velocity seismograms at the stations of Fig. 5-2b for the fault model of Fig. 5-2a. Time is relative to the onset of rupture on the fault.

Figure 5-4 Velocity seismograms showing the contribution from the lower, 50° dip segment of the fault model to the
total seismograms of Fig. 5-3.

Figure 5-5  Seismograms showing the contribution from the upper 30° dip segment of the fault model to the total seismograms of Fig. 5-3.

Figure 5-6  Comparison between the theoretical (dashed) and the initial part of the observed (solid) records for stations at (a) southern azimuths and (b) northern azimuths. See Fig. 5-1 for station locations. All theoretical records are obtained using a simple rock site model (Table 5-1). Time (in secs) is relative to the onset of rupture on the fault. The alignment of the theoretical and observed records is arbitrary for the late-triggered LH9 record.

Figure 5-7  Comparison between theoretical and observed records at the sedimentary sites GLN (A) and PLM (B). The theoretical records were obtained using the sedimentary site model of Table 5-2.

Figure 5-8  A) Schematic of fault geometry used in modelling San Fernando. Geometry is from Heaton (1978), based on the teleseismic modelling results of Langston (1978).

   B) Contours of slip magnitude (meters) on the two-segment, dipping plane shown in (A) for Heaton's preferred model, Norma 163. The rake is 76° on the lower segment, 90° on the upper. The segments intersect at a depth of 5 km. The heavy line indicates the approximate
surface fault trace.

Figure 5-9 Comparison between the SL velocity record and the theoretical record obtained using the smoothly varying dislocation function of the Norma 163 model. The near-surface faulting above 3 km is not included. The source time function is a ramp with .50 sec rise time. (The observed SL record is bandpass filtered, .25 - 3.5 hz.)

Figure 5-10 Source time functions and spectra. The source slip function \( s(t) \) and its derivative \( \dot{s}(t) \) are shown for the case of a box and triangular slip velocity function \( f(t) \). (lower) Amplitude spectra of the \( \dot{s}(t) \) functions. They are given by \( D_0 \text{sinc}(\pi fT) \) and \( D_0 \text{sinc}^2(\pi fT/2) \) for the box and triangular functions, respectively.

Figure 5-11 Profiles on the fault for which the seismograms from individual 1 x 1 km elements are studied in Figs. 5-12 through 5-14. Elements above the hypocenter at \( H \) rupture upward, those below rupture downward. The profile C-C' spans the hinge line at 5 km depth. The .5 m slip contour of the Norma 163 model is shown for reference.

Figure 5-12 Velocity seismograms (transverse) at SL due to unit (1 m) slip on individual 1 x 1 km elements along the profile \( A - A'' \), across the width of the fault (see Fig. 5-11). The element at \( A' \) is directly up the fault from the hypocenter, the elements at \( A \) and \( A'' \) are at the west
and east ends of the profile, respectively. All records are plotted at the same scale, time is relative to the onset of rupture at the hypocenter H.

Figure 5-13  Velocity seismograms (transverse) at SL for elements along the profile B - B', up the dip of the fault (see Fig. 5-11). Rupture initiates at the hypocenter H. The elements above H (toward B') rupture upward, those below rupture downward. At A' this profile intersects that of Fig. 5-12.

Figure 5-14  Velocity seismograms (transverse) at SL for elements along the profile C - C', extending over the hinge line of the fault (see Figure 5-11). The first three seismograms are from the elements on the 53° dip segment, the last three from elements on the 29° dip segment. The latter show that the radiation from the 29° segment is not as strongly recorded at SL.

Figure 5-15  Comparison between theoretical and observed records at SL (upper) for the dislocation function shown in Fig. 5-16. The corresponding comparison for GPK is also shown for reference (lower). The observed velocity records are bandpass filtered (Butterworth, 3rd order) from .25 to 3.5 hz for these comparisons.

Figure 5-16  Contours of slip magnitude for the fault model giving rise to the theoretical records of Fig. 5-15. The slip distribution was obtained through perturbation of
the slip distribution of the Norma 163 model (Fig. 5-8b). The dashed lines mark the band on the fault from within which the radiation arrives at SL during the pulse discussed in the text.

Figure 5-17 Comparison between the theoretical and observed velocity records at SL and GPK for our final fault model of Fig. 5-18.

Figure 5-18 The final fault model, yielding the theoretical records in Fig. 5-17. Slip magnitude contoured in 2 m intervals. The .5 m contour of the Norma 163 model is shown (dashed) for locational reference.

Figure 5-19 Smoothed dislocation function obtained by averaging the dislocation function of Fig. 5-18 with a 3 x 3 km boxcar. The .5 m contour of the Norma 163 model is shown for reference (dashed).
Figure 5-3
50 Dip Segment

Figure 5-4
Figure 5-6a
Figure 5-6b
Figure 5-7
Figure 5-9
Figure 5-10
Figure 5-11
Figure 5-12
Figure 5-13
Figure 5-14
Figure 5-15
Figure 5-18
CHAPTER 6

CONCLUSIONS

The effects of the source, medium and site on recorded strong motion have been studied through the analyses of observations and through theoretical modelling. Each of these parts of the problem are found to be important, while none are uniformly dominant.

The effects of a sedimentary site structure on the amplitude and frequency content of the recorded ground motion may be slight or they may be severe, depending on the frequency band being considered and the variation of the medium parameters with depth. The high frequencies are very dependent on the shallow structure local to the site. At longer periods the properties of the site structure at depth are most important. Because of the frequency dependence of the site effects, conclusions that the site conditions are of minor importance for peak acceleration (Trifunac and Brady, 1976), and that the velocity structure can profoundly affect the displacements (Heaton and Helmberger, 1979) are not incompatible. The peak acceleration is a high frequency measure, while the displacements are sensitive to the site response at low frequencies.

A sedimentary site structure increases the duration of the ground motion, for both strong motion and small earthquake
response. However, the amplification effects for strong motion may be less than for small earthquake response, probably due to nonlinear response.

The application of a propagator matrix solution which includes attenuation underscores the importance of attenuation in modelling ground motion at a sedimentary site. The amplifying effects of decreased seismic velocities and internal reflections can be offset, sometimes severely, by the amplitude reduction effects of attenuation. Also, the long duration, ringing effect caused by a site structure with a high velocity contrast decays rapidly in the presence of typical sedimentary Q values.

The three-dimensional effects of the medium were not found to be as severe as expected in this study. It is not expected that these effects are insignificant but rather that their appreciation awaits better modelling of the source and site effects. For example, the three-dimensional medium effects on station-to-station variations can only be discovered when the component of those variations due to the source radiation and site effects are known.

The effects of the source radiation are significant, and modellable, particularly at intermediate and low frequencies. The azimuthal variations in the San Fernando data show a significant dependence on the rupture propagation on the dipping fault.

The modelling of the velocity records at SL and GPK suggests that common effects of the source radiation can be
observed throughout the region. For example, the distinct, long duration pulse in the velocity records from the Pasadena stations is also present, and modellable, in the GPK record. This same pulse is apparently the common arrival about 3 secs after the first arrivals in the records from the Area 2 stations in Los Angeles. The maximum at about 3 secs in the peak-times histograms of Dobry et al. (1979) is tentatively identified with this same rupture phase.

The modelling of the strong motion records at SL and GPK also shows that the velocity records can be successfully matched up to 1-2 hz. More importantly, it demonstrates the importance of localized areas on the fault for the generation of high frequencies in strong motion. The statistics of peak ground motion parameters are probably strongly related to the statistics of the dimensions and the slip or stress drop on these localized rupture areas.
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APPENDIX A

San Fernando Strong Motion Data

Figures A-1 through A-10. Three component (radial, transverse, up) San Fernando acceleration records for the stations considered in Figs. 2-2 and 2-3. All records plotted at same scale except for PAC, which is plotted at 1/4 gain. Records identified by station code (see Table 2-1) and C.I.T. ID numbers.
150.0 cm/sec²

- T
- Up
- R SJC N195
- T
- Up
- R SL G186
- T
- Up
- R SOM L171
- T
- Up

0 5 10 15 20 sec
APPENDIX B

PROPAGATOR MATRIX SOLUTION FOR ATTENUATING
LAYERS, WITH HIGH FREQUENCY STABILITY

In this appendix we outline the propagator matrix solution used in modelling body wave propagation in the site layering, as discussed in chapter 3 (section 3.1). We first consider the P-SV solution, including the compound-matrix transformation for high frequency stability. The SH solution follows, which is simpler, and also does not require the transformation for stability. We finally consider normalizations of the free-surface motion, contrasting the crustal transfer function (Haskell, 1962) with the amplification functions used in soil amplification studies (e.g., Roesset and Whitman, 1969).

B.1 P-SV Solution

We consider N-1 layers over a halfspace as shown in Fig. B-1, each layer having down and up going dilatational and shear waves (P,SV). We will use the dilatational and shear displacement potentials \( \phi \) and \( \psi \), with \( \phi_n' \) and \( \phi_n'' \) being the down and up going components, respectively, in the n-th layer, and with \( \psi_n' \), \( \psi_n'' \) similarly defined. Except for the inclusion of attenuation-dependent terms, the use of potentials to simplify notation, and the compound-matrix extension for stability, the following parallels the original development of Thomson (1950)
and Haskell (1953).

As discussed in chapter 3, the dilatational and shear potentials for the solid-friction medium can be written, for the n-th layer (with n suppressed),

$$
\phi = \phi' e^{i\omega t - ik\xi_\alpha (x + v_\alpha z)} + \phi'' e^{i\omega t - ik\xi_\alpha (x - v_\alpha z)}
$$

$$
\psi = \psi' e^{i\omega t - ik\xi_\beta (x + v_\beta z)} + \psi'' e^{i\omega t - ik\xi_\beta (x - v_\beta z)}
$$

where

$$
\xi_\alpha = 1/[1 + i \sgn(\omega)/Q_\alpha]^{1/2} = 1 - i \sgn(\omega)/2Q_\alpha + O(1/Q_\alpha^2)
$$

and

$$
\nu_\alpha = \left[ c^2/\sigma^2 - 1 \right]^{1/2}, \quad c > \alpha
$$

$$
= -i[1 - c^2/\sigma^2]^{1/2}, \quad c < \alpha
$$

and similarly for $\xi_\beta$ and $\nu_\beta$. $k$ is the horizontal component of the wavenumber,

$$
k = \frac{\omega}{c} = \frac{\omega}{\sigma} \sin i_\alpha = \frac{\omega}{\beta} \sin i_\beta
$$

where $c$ is the horizontal phase velocity. The approximate expression for $\xi_\alpha$ is not actually used here, but is shown for reference. The form of $\xi_\alpha$ and other attenuation-dependent factors introduced in the following were chosen such that they go to unity in the infinite-$Q$ case, clarifying the role of the attenuation terms in modifying the equations.
The boundary conditions at each internal interface are continuity in displacements and stresses. Supressing $e^{i\omega t}$, taking $x = 0$, and defining

\[
P(z) = k_{x\alpha}v_{\alpha}z, \quad R(z) = k_{y\beta}v_{\beta}z,
\]

the horizontal and vertical particle velocity equations can be written

\[
\begin{align*}
\frac{\dot{u}}{\omega} &= k_{x\alpha}(\phi'e^{-iP(z)} + \phi''e^{iP(z)}) - k_{y\beta}(\psi'e^{-iR(z)} - \psi''e^{iR(z)}) \\
\frac{\dot{v}}{\omega} &= k_{x\alpha}(\phi'e^{-iP(z)} - \phi''e^{iP(z)}) + k_{y\beta}(\psi'e^{-iR(z)} + \psi''e^{iR(z)})
\end{align*}
\]

Defining $q = \xi_{\alpha}/\xi_{\beta}$, which goes to unity for no attenuation, and $\gamma = 2\beta^2/c^2$, the normal and shear stress equations can be written

\[
\begin{align*}
\frac{\sigma_{zz}}{\omega^2} &= \rho(\gamma q^2 - 1)(\phi'e^{-iP(z)} + \phi''e^{iP(z)}) - \rho\gamma \nu_{\beta}(\psi'e^{-iR(z)} - \psi''e^{iR(z)}) \\
\frac{\sigma_{zx}}{\omega^2} &= -\rho\gamma q^2 \nu_{\alpha}(\phi'e^{-iP(z)} - \phi''e^{iP(z)}) - \rho(\gamma - 1)(\psi'e^{-iR(z)} + \psi''e^{iR(z)})
\end{align*}
\]

These four equations can be written in the matrix form

\[
\dot{U}(z) = E(z)\hat{\chi}
\]

where $\dot{U}(z)$ and $\hat{\chi}$ are given by

\[
\begin{align*}
\dot{U}(z) &= \begin{bmatrix}
\frac{\dot{u}}{\omega} & \frac{\dot{v}}{\omega} & \frac{\sigma_{zz}}{\omega^2} & \frac{\sigma_{zx}}{\omega^2}
\end{bmatrix}^T \\
\hat{\chi} &= \begin{bmatrix}
\phi' + \phi'' & \phi' - \phi'' & \psi' - \psi'' & \psi' + \psi''
\end{bmatrix}^T
\end{align*}
\]

and will be referred to as the motion-stress vector and the
potentials vector, respectively.

Replacing the explicit z-dependence of $\hat{U}(z)$, $E(z)$, $P(z)$ and $R(z)$, let $\hat{U}^+$, $E^+$, $P$ and $R$ be their values at the bottom of the layer ($z = d$), and let $\hat{U}^-$ and $E^-$ be the values of $\hat{U}(z)$ and $E(z)$ at the top. Then at the top, and at the bottom, of the $n$-th layer,

$$
\hat{U}_n^- = E_n^- \hat{x}_n, \quad \text{and} \quad \hat{U}_n^+ = E_n^+ \hat{x}_n, \quad (B.1a,b)
$$

respectively. These relate the motion-stress at the top and bottom of the layer through $\hat{x}_n$ as

$$
\hat{U}_n^+ = H_n \hat{U}_n^- , \quad \text{where} \quad H_n = E_n^+ (E_n^-)^{-1}
$$

is the downward transfer function matrix, or layer propagator matrix. The matrices $E_n^-$, its inverse and $H_n$ are given in an appendix to this section (B.1.1).

By the continuity conditions across each interface,

$$
\hat{U}_{n+1}^- = \hat{U}_n^+ , \quad \text{so} \quad \hat{U}_{n+1}^- = H_n \hat{U}_n^- ,
$$

which relates the motion-stress at the top of layer $n+1$ to that at the top of layer $n$. Continuity across each of the $N-1$ internal interfaces implies

$$
\hat{U}_N^- = H_{N-1} H_{N-2} \cdots H_2 H_1 \hat{U}_1^- = \left\{ \prod_{n=1}^{N-1} H_n \right\} \hat{U}_1^-
$$

which relates the motion-stress at the top of the halfspace to that at the free surface. Using equation (B.1a) and letting $u_0$, $w_0$ be the displacements at the free surface (where the
stresses are zero), then
\[
\hat{\chi}_N = J \begin{bmatrix} \hat{u}_o / \omega, \hat{w}_o / \omega, 0, 0 \end{bmatrix}^T, \quad \text{where} \quad J = (E_n)^{-1} \prod_{n=1}^{N-1} H_n,
\]
relates the free-surface motion to the potentials in the half-space.

Haskell (1960, 1962) first suggested forming a 'crustal transfer function' by normalizing the free-surface displacements to the displacements which would occur in the absence of the layering or free surface (i.e., in a full space). Thus, the free surface displacements are normalized to the displacements in the halfspace due only to the incident potentials \((\phi_n'''' , \psi_n'''' )\). With these ratios denoted as \(U_P, W_P\) for P incidence \((\psi_n''' = 0)\), and \(U_{SV}, W_{SV}\) for SV incidence \((\phi_n''' = 0)\), we obtain
\[
U_P = u_P^D / u_N^D = 2( J_{32} - J_{42} ) / k \xi_{\alpha N} D
\]
\[
W_P = w_P^D / w_N^D = 2( J_{31} - J_{41} ) / k \xi_{\alpha N} \nu_{\alpha N} D
\]
and
\[
U_{SV} = 2( J_{12} - J_{22} ) / k \xi_{\beta N} \nu_{\beta N} D
\]
\[
W_{SV} = 2( J_{21} - J_{11} ) / k \xi_{\beta N} D
\]
where
\[
D = ( J_{11} - J_{21} )( J_{32} - J_{42} ) + ( J_{12} - J_{22} )( J_{41} - J_{31} ).\]

These ratios are of course the same for particle velocities or accelerations. Using these ratios we can obtain the free surface motion for arbitrary incident P or SV displacements by spectral multiplication. They are effectively the impulse
response of the layering at a particular phase velocity (incidence angle).

The numerical instability in the propagator matrix method, discussed in section 3.1, arises in the calculation of $D$, called the secular or dispersion function in surface wave studies. This instability does not always arise in the body wave problem, as it can only occur for $c < \alpha_{\text{max}}$, when $v_{\alpha_n}$ is pure imaginary for some $n$. Then the trigonometric functions of $P$ become hyperbolic, involving exponentials whose argument increases with frequency and layer thickness. (If $c < \beta_{\text{max}}$ this will also apply to $v_\beta$ and $R$.) If the velocity function is increasing with depth, so that $\alpha_{\text{max}} = \alpha_N$, the instability will never arise for $P$ incidence. For $SV$, the problem will not arise for $0 \leq i_s \leq \sin^{-1}(\beta_N/\alpha_{\text{max}})$. For typical $P$ and $S$ velocities in the halfspace, this corresponds to $0 \leq i_s \leq 30^\circ$, which is the range Jones and Roesset (1970) consider. For higher angles of incidence, the instability occurs as in surface wave problems, though typically at larger values of frequency or depth since the phase velocity is higher.

Dunkin (1965) and Thrower (1965) suggested circumventing the numeric stability problem by expansion of the 4th order matrices into compound matrices made up of all possible 2nd order subdeterminants. This expansion is called the delta-matrix in mechanics (Pestel and Leckie, 1963), where the solutions for certain mechanical systems involve instabilities identical to those of $D$. 
The transformation of a matrix $M$ to its compound matrix $M$ may be written as (e.g., Harkrider, 1970)

$$M_{ij} = M^{(kl)}_{mn} = M_{km}M_{ln} - M_{kn}M_{lm}$$

where $i = 1, 2, \ldots, 6$ corresponds to the index pairs $kl = 12, 13, 14, 23, 24, 34$ with an identical correspondence between $j$ and $mn$. Letting $J$, $(E_N^{-1})^{-1}$ and $H_n$ be the compound matrices corresponding to $J$, $(E_N^{-1})^{-1}$ and $H_n$, then by the properties of compound matrices (Gantmacher, 1959), if

$$J = (E_n^{-1})^{-1} \prod_{N-1}^1 H_n$$

then

$$J = (E_n^{-1})^{-1} \prod_{N-1}^1 H_n$$

The elements of $H_n$, which is cross-symmetric in our case, and the required elements of $J$ are given in B.1.1. The crustal transfer functions of equations (B.2a,b) can now be obtained with $D$ of equation (B.2c) given in terms of $J$ as

$$D = J_{21} - J_{31} - J_{41} + J_{51}$$

Note that for the crustal transfer functions, both the $4 \times 4$ and $6 \times 6$ solutions must be carried through, as contrasted with surface wave dispersion studies in which only the zeroes of $D$, the secular function, are usually sought. Certain economies can be realized in the solution, however. Only the first two columns of $J$ are required, so only the first two columns of each $H_n H_{n-1}$ matrix product need be obtained.
Similarly, only the first column of $J$, and thus of each $H_n H_{n-1}$ product, is required. The full matrices $H_n$ and $H_n$ must be obtained for each layer (except the first) however, though the symmetry of $H_n$ reduces the number of distinct elements to be calculated. In contrast to solutions for non-attenuating layers, all elements of these matrices are complex.

B.1.1 Matrix Elements

Suppressing the layer index $n$, $E_n^-$ and its inverse are given by

$$E_n^- = \begin{bmatrix}
  k\xi_\alpha & 0 & -k\xi_\beta v_\beta & 0 \\
  0 & k\xi_\alpha v_\alpha & 0 & k\xi_\beta \\
  \rho(Yq^2-1) & 0 & -\rho v_\beta & 0 \\
  0 & -\rho v_\beta^2 v_\alpha & 0 & -\rho(Y-1)
\end{bmatrix}$$

$$\left(E_n^-ight)^{-1} = \begin{bmatrix}
  \frac{Y}{k\xi_\beta^F} & 0 & -\frac{1}{\rho^F} & 0 \\
  0 & \frac{1-Y}{k\xi_\alpha v_\alpha^G} & 0 & -\frac{1}{\rho q v_\alpha^G} \\
  \frac{Yq^2-1}{k\xi_\beta^F v_\beta^F} & 0 & -\frac{q}{\rho v_\beta^F} & 0 \\
  0 & \frac{Yq^2}{k\xi_\alpha^G} & 0 & \frac{1}{\rho G}
\end{bmatrix}$$

where

$$F = 1 + Yq - Yq^2, \quad G = 1 - Y + Yq$$

which both go to unity in the non-attenuating case.

For brevity in the following expressions, $\cos P$ and $\cos R$ are denoted by $CP$ and $SP$, and $\cos R, \sin R$ by $CR, SR$. 
The elements of $H_n$ (with $n$ suppressed) are given by

$$H_{11} = \frac{1}{F} [YqCP - (Yq^2-1)CR]$$

$$H_{12} = \frac{1}{G} \left[ \frac{1}{\nu_\alpha} (Y-1)SP + Yq_\nu_\beta SR \right]$$

$$H_{13} = -\frac{k_\xi_\alpha}{\rho F} [CP - CR]$$

$$H_{14} = \frac{ik_\xi_\beta}{\rho G} \left[ \frac{1}{\nu_\alpha} SP + \nu_\beta SR \right]$$

$$H_{21} = -\frac{i}{F} \left[ Yq_\nu_\alpha SP + \frac{1}{\nu_\beta} (Yq^2-1)SR \right]$$

$$H_{22} = -\frac{1}{G} \left[ (Y-1)CP - YqCR \right]$$

$$H_{23} = \frac{ik_\xi_\alpha}{\rho F} [\nu_\alpha SP + \frac{1}{\nu_\beta} SR]$$

$$H_{24} = \frac{F}{qG} H_{13}$$

$$H_{31} = \frac{\rho Y(Yq^2-1)}{k_\xi_\beta F} [CP - CR]$$

$$H_{33} = -\frac{1}{F} \left[ (Yq^2-1)CP - YqCR \right]$$

$$H_{32} = \frac{i\rho}{k_\xi_\alpha G} \left[ \frac{1}{\nu_\alpha} (Yq^2-1)(Y-1)SP + Y^2 q^{2}_\nu_\beta SR \right]$$

$$H_{34} = \frac{i}{qG} \left[ \frac{1}{\nu_\alpha} (Yq^2-1)SP + Yq_\nu_\beta SR \right]$$

$$H_{41} = \frac{i\rho}{k_\xi_\alpha F} [\nu_\alpha Y^2 q^{2}_\nu SP + \frac{1}{\nu_\beta} (Yq^2-1)(Y-1)SR]$$

$$H_{42} = \frac{q(Y-1)F}{(Yq^2-1)G} H_{31}$$

$$H_{43} = -\frac{i\rho}{F} \left[ Yq_\nu_\alpha SP + \frac{1}{\nu_\beta} (Y-1)SR \right]$$

$$H_{44} = \frac{1}{G} \left[ YqCP - (Y-1)CR \right]$$

The compound layer matrix $H_n$ is symmetric about the secondary (i.e., lower-left to upper-right) diagonal. The elements in the upper left are given below. The remaining elements in the lower right are given by $H_{ij} = H_{7-j,7-i}$, for $i = 2, \ldots, 6$, $j = 8-i, \ldots, 6$. 
\[ H_{11} = -\frac{1}{FG} \left[ \gamma q (\gamma q^2 + 1) - 2 - \{\gamma q^2 (2\gamma - 1) - \gamma + 1\} \right] CPCR + \left( \frac{(\gamma - 1)(\gamma q^2 - 1)}{\nu_\alpha \nu_\beta} + \gamma q^2 \nu_\alpha \nu_\beta \right) SPSR \]

\[ H_{21} = \frac{i \rho}{k \xi_\alpha FG} \left[ \frac{1}{\nu_\alpha} (\gamma - 1)(\gamma q^2 - 1) CPCR + \gamma q^2 \nu_\alpha SPSR \right] \]

\[ H_{31} = \frac{\rho}{k \xi_\beta FG} \left[ \gamma q (\gamma - 1)(2\gamma q - F) \right] (1 - CPCR) + \left( \frac{(\gamma - 1)(\gamma q^2 - 1)}{\nu_\alpha \nu_\beta} + \gamma q^2 \nu_\alpha \nu_\beta \right) SPSR \]

\[ H_{41} = \frac{\rho}{k \xi_\alpha FG} \left[ \gamma q (\gamma q^2 - 1)(2\gamma q - G) \right] (1 - CPCR) + \left( \frac{(\gamma - 1)(\gamma q^2 - 1)}{\nu_\alpha \nu_\beta} + \gamma q^2 \nu_\alpha \nu_\beta \right) SPSR \]

\[ H_{51} = -\frac{i \rho}{k \xi_\beta F} \left[ \frac{1}{\nu_\beta} (\gamma - 1)(\gamma q^2 - 1) CPCR + \gamma q^2 \nu_\alpha SPSR \right] \]

\[ H_{61} = \frac{\rho^2 q}{k^2 \xi_\alpha^2 FG} \left[ 2\gamma q^2 (\gamma - 1)(\gamma q^2 - 1)(1 - CPCR) + \frac{(\gamma - 1)(\gamma q^2 - 1)}{\nu_\alpha \nu_\beta} + \gamma q^2 \nu_\alpha \nu_\beta \right] SPSR \]

\[ H_{12} = \frac{ik \xi_\alpha}{\rho F} \left[ \frac{1}{\nu_\beta} \right] CPCR + \nu_\alpha SPSR \]

\[ H_{22} = CPCR \]

\[ H_{32} = -\frac{i q}{F} \left[ \frac{\gamma - 1}{\nu_\beta} CPCR + \gamma q \nu_\alpha SPSR \right] \]

\[ H_{52} = \frac{q G v}{F} \nu_\alpha SPSR \]

\[ H_{42} = -\frac{i}{F} \left[ \frac{\gamma q^2 - 1}{\nu_\beta} CPCR + \gamma q \nu_\alpha SPSR \right] \]

\[ H_{13} = -\frac{k \xi_\beta}{\rho FG} \left[ (2\gamma q - F)(1 - CPCR) + \frac{(\gamma q^2 - 1)}{\nu_\alpha \nu_\beta} + \gamma q \nu_\alpha \nu_\beta \right] SPSR \]

\[ H_{23} = \frac{i}{q G} \left[ \frac{1}{\nu_\alpha} (\gamma q^2 - 1) SPCR + \gamma q \nu_\beta SPCR \right] \]

\[ H_{33} = \frac{1}{FG} \left[ 1 + \gamma q (\gamma q^2 + 1) - \gamma q (\gamma q^2 + 1) - 2 \right] CPCR + \left( \frac{(\gamma - 1)(\gamma q^2 - 1)}{\nu_\alpha \nu_\beta} + \gamma q^2 \nu_\alpha \nu_\beta \right) SPSR \]
\[ H_{43} = \frac{1}{qFG} \left[ 2Y(Y-1)(1-CPCR) + \left(\frac{Yq^2-1}{\nu_{\alpha}\nu_{\beta}} \right) + Yq^2\nu_{\alpha}\nu_{\beta} \right] \]

\[ H_{14} = -\frac{k\xi_{\alpha}}{\rho FG} \left[ (2Yq-G)(1-CPCR) + \left(\frac{Y-1}{\nu_{\alpha}\nu_{\beta}} \right) + Yq\nu_{\alpha}\nu_{\beta} \right] \]

\[ H_{24} = \frac{1}{G} \left[ \frac{Y-1}{\nu_{\alpha}} \right] \]

\[ H_{34} = \frac{q}{FG} \left[ 2Yq(Y-1)(1-CPCR) + \left(\frac{Y-1}{\nu_{\alpha}\nu_{\beta}} \right) + Y^2q^2\nu_{\alpha}\nu_{\beta} \right] \]

\[ H_{15} = -\frac{ik\xi_{\beta}}{\rho G} \left[ \frac{1}{\nu_{\alpha}} \right] \]

\[ H_{16} = \frac{k^2\xi_{\alpha}\xi_{\beta}}{\rho^2FG} \left[ 2(1-CPCR) + \left(\frac{1}{\nu_{\alpha}\nu_{\beta}} \right) + \nu_{\alpha}\nu_{\beta} \right] \]

In the compound-matrix equation

\[ J = (E_n^-)^{-1} \sum_{n=1}^{N-1} H_n \]

only four elements of the first column of \( J \) are needed to obtain \( D \). For brevity these elements will be given explicitly, rather than giving the elements of the matrix \((E_n^-)^{-1}\). If, only in the following, we let \( H \) denote the final product of the \( H_n \), so \( H = \prod_{n=1}^{N-1} H_n \) then the four elements of \( J \) are given as

\[ J_{21} = -\frac{1}{\rho k\xi_{\beta}\nu_{\beta}} H_{21}, \quad J_{51} = \frac{1}{\rho k\xi_{\alpha}\xi_{\alpha}} H_{51} \]

\[ J_{31} = \frac{1}{FG} \left[ \frac{Y^2q}{k^2\xi_{\beta}} H_{11} + \frac{Y}{\rho k\xi_{\beta}} (H_{31} + qH_{41}) - \frac{1}{\rho^2} H_{61} \right] \]

\[ J_{41} = \frac{1}{\nu_{\alpha}\nu_{\beta}FG} \left[ \frac{(Y-1)(Yq^2-1)}{k^2\xi_{\alpha}\xi_{\beta}} H_{11} \right. \]

\[ \left. + \frac{1}{k\xi_{\alpha}} \left( (Yq^2-1)H_{31} + q(Y-1)H_{41} \right) - \frac{1}{\rho^2} H_{61} \right] \]

where the layer parameters are those of the halfspace, layer \( N \).
B.2 SH Solution

The solution for SH waves in this section parallels that for the P-SV problem in section B.1, but is much simpler. Working directly with the transverse component of the displacement, in the $n$-th layer,

$$v = v'e^{i\omega t} - ik\xi_\beta(x + v_\beta z) + v''e^{i\omega t} - ik\xi_\beta(x - v_\beta z)$$

where $k$, $\xi_\beta$ and $v_\beta$ are as defined in section B.1. The boundary conditions at each internal interface are continuity in the transverse displacement and the shear stress $\sigma_{zy}$. With normalization as in the previous section, the displacement and shear stress equations can be written

$$\frac{\dot{v}}{\omega} = i(v'e^{-iR(z)} + v''e^{iR(z)})$$

$$\frac{\sigma_{zy}}{\omega^2} = -\frac{i\rho Yv}{2k\xi_\beta}(v'e^{-iR(z)} - v''e^{iR(z)})$$

If $\vec{U}(z)$, $E(z)$ and $\vec{\chi}$ are redefined, these equations can be written as $\vec{U}(z) = E(z)\vec{\chi}$ again, where now $\vec{U}(z)$ and $\vec{\chi}$ are given by

$$\vec{U}(z) = [\dot{v}/\omega, \sigma_{zy}/\omega^2]^T, \quad \vec{\chi} = [v' + v'', v' - v'']^T$$

With $-,+$ superscripts indicating evaluations at the top and bottom of the $n$-th layer, as in the P-SV solution, then

$$\vec{U}_n^+ = E_n^+(E_n^-)^{-1}\vec{U}_n^- = H_n\vec{U}_n^-$$
where now

\[ H_n = \begin{bmatrix} \cos R & \frac{i2k\xi_\beta}{\rho \nu \beta} \sin R \\ \frac{i\rho \nu \beta}{2k\xi_\beta} \sin R & \cos R \end{bmatrix} \quad \text{and} \quad (E_n^{-1})^{-1} = \begin{bmatrix} -i & 0 \\ 0 & \frac{i2k\xi_\beta}{\rho \nu \beta} \end{bmatrix} \]

Continuity at each of the interfaces yields

\[ \vec{U}_n^- = \{ \Pi \}_{n-1} \vec{U}_1^- \]

and with \( \vec{U}_1^- = [\phi_o / \omega, 0]^T \),

\[ \vec{\chi}_n = J \begin{bmatrix} \phi_o / \omega, 0 \end{bmatrix}^T, \quad J = (E_n^-)^{-1} \{ \Pi \}_{n-1} H_n \]

Defining an SH crustal transfer function similar to those of equations (B.2a,b),

\[ V_{SH} = \frac{v_o}{v_N} = \frac{2i(J_{11} - J_{21})}{J_{11} - J_{22}} \]

Given the simple form of \( J \), we can write this directly in terms of the product of the layer matrices. Letting

\[ H = \prod_{n=1}^{N-1} H_n \]

we have

\[ V_{SH} = \frac{2k\xi_\beta}{H_{11} + \frac{2k\xi_\beta}{\rho \nu \beta} H_{21}} \]

(B.3)

where the layer parameters are those of the halfspace, layer \( N \). The numeric instability problems which can occur in the P-SV solution do not arise in the SH problem.
B.3 Normalizations of the Free-Surface Motion

Haskell (1962) defined the crustal transfer function as the ratio of the free surface motion to the motion in the halfspace due to the incident wave. We have used this function because of its utility as an impulse response. The response of a layered structure has also been studied through other spectral ratios however, and we will discuss the connection between these and the crustal transfer function in this section.

Consider normalizing the free surface motion to that of the top of the halfspace if the layering were absent. This yields the spectral amplification relative to the motion at an exposed-bedrock site. Roesset and Whitman (1969) call this amplification function $A_2(\omega)$, and Jones and Roesset (1970) call it the 'elastic rock amplification function'. This function is closely allied to the crustal transfer function. Considering the SH case for simplicity, the free surface motion at the top of a halfspace can be given by

$$\hat{\mathbf{U}}^-(\frac{1}{2}) = \mathbf{E}^{-1} \hat{\mathbf{X}}$$

where $\hat{\mathbf{U}}^- = [\hat{\mathbf{v}}_o \hat{\mathbf{j}}/\omega, 0]^T$, so

$$A_2(\omega) = \frac{v_o}{v_o^{(1/2)}} = 1/i(j_{11} - j_{21}) = 1/[H_{11} + \frac{2k_\xi B}{\rho \gamma v_\beta} H_{21}]$$

where $H$ is the product of the layer matrices as in the previous section. Thus, for the case of SH, the crustal transfer function and the elastic rock amplification function differ.
only by the factor of 2 associated with the doubling of amplitude for SH at a free surface.

Another normalization which might be considered is to form the ratio of the free surface motion to the total motion at the top of the halfspace, incident and reflected, rather than that due just to the incident wave as in the crustal transfer function. Roesset and Whitman (1969) call this ratio $A_1(\omega)$. Thus

$$A_1(\omega) = \frac{v_o}{v_N} = \frac{1}{H_{11}}$$

Comparing this expression to that for $A_2(\omega)$, they would be identical in the case of infinite rigidity in the halfspace. Thus this ratio has been called the 'rigid rock amplification function' (Jones and Roesset, 1970). For a single layer over the halfspace, these two ratios become

$$A_1(\omega) = \frac{1}{\cos R}$$

$$A_2(\omega) = \frac{1}{\cos R + \frac{i\rho_1 \beta_1 \xi_2}{\rho_2 \beta_2 \xi_1} \sin R}$$

where $R = k\xi_1 \beta_1 h$, and $h$ is the thickness of the layer. Roesset and Whitman (1969) review the behavior of these two functions in the case of vertical incidence. The simple form of $A_1(\omega)$ indicates the modes occurring at $\cos R = 0$, or $f_n = n\beta/4h \cos i_1$, $n = 1, 3, \ldots$ for the non-attenuating case, when $A_1(\omega)$ is infinite. In the attenuating case, it can be shown that these frequencies are lowered slightly, but the principal effect of the attenuation is to reduce the amplitudes of the modes, as discussed in chapter 3.
Figure B-1 A) Geometry of layered halfspace and numbering of layers
The $x, y, z$ components of displacements are denoted $u, v, w$.

B) Detail of variables in the $n$-th layer.
APPENDIX C

ASPECTS OF THE MADARIAGA SOLUTION FOR THE HASKELL SOURCE

The closed form solution obtained by Madariaga (1978) for the Haskell source is very useful in the modelling of strong motion since it is a complete solution in the velocity, and is computationally economic. In this appendix the important practical formulae and details of the Madariaga solution are reviewed, partly for the insight they provide about the types of waves radiated in the vicinity of a propagating rupture. Additionally, this will allow certain typographical errors and omissions in Madariaga (1978) to be set right, which were resolved through extended correspondence (Madariaga, 1978, personal communications). Finally, certain numerical aspects of the application of the Madariaga solution will be discussed.

The Madariaga solution may be considered a successor to the analytic solution used by Boore and Zoback (1974a), which draws upon the theoretical solutions of Ang and Williams (1959) and Mitra (1966). These are solutions for infinite half-plane edge and screw dislocations, respectively. With the superposition of two such half-plane dislocations suitably delayed in time and space, Boore and Zoback (1974a) obtained the velocity solution for a strip of finite length and
infinite width. Boore and Zoback (1974b) used this two-dimensional approach in modelling the San Fernando Pacoima Dam record. By obtaining the solution for an infinite quadrantal dislocation, Madariaga (1978) could, by superposition of four such dislocations, obtain the solution for a fault for which both the length and width were finite. Both in this solution, and that used by Boore and Zoback, cylindrical waves are associated with the line of dislocation with which rupture initiates. But while only these cylindrical waves are included in the solutions of Boore and Zoback (1974a), Madariaga's solution also includes the spherical waves generated by each of the dislocation corners.

As discussed above, Madariaga obtained the solution for a finite rectangular fault by the superposition of the solutions for four infinite quadrantal dislocations appropriately delayed in time and space,

\[
\hat{\mathbf{u}}(x,y,z,t) = \hat{\mathbf{u}}_Q(x,y,z,t) - \hat{\mathbf{u}}_Q(x,y-W,z,t) - \hat{\mathbf{u}}_Q(x-L,y,z,t-L/v_R) + \hat{\mathbf{u}}_Q(x-L,y-W,z,t-L/v_R)
\]

where \( \hat{\mathbf{u}}_Q \) is the solution for an infinite quadrantal dislocation, as illustrated in Fig. C-1a. The problem thus becomes one of obtaining the solution for an infinite quadrantal dislocation, rather than a finite rectangular dislocation. Madariaga obtained the solution for each quadrantal dislocation as the sum of a cylindrical and a spherical wave component,
\[ \dot{u}_Q = \dot{u}_{Q}^{\text{cyl}} + \dot{u}_{Q}^{\text{sph}} \]  
(C.2)

The coordinate geometry used in the solution is shown in Fig. C-1b.

The cylindrical waves are radiated from the sudden appearance of the semi-infinite dislocation line along the \( y \)-axis. Since the dislocation is semi-infinite, the cylindrical waves are only radiated into the space on the positive side of the \( x \)-\( z \) plane \( (y > 0) \). They arrive at the time corresponding to the travel distance \( \rho_c \) from the \( y \)-axis, where \( \rho_c = \sqrt{x^2 + z^2} \). The expression for the cylindrical wave is given by (suppressing \( Q \))

\[ \dot{u}_i^{\text{cyl}}(\hat{x}, t) = \frac{D_o \nu_p}{2\pi k' \rho_c} \text{Re} \left[ \frac{f_{\chi}^{\alpha} m_{\chi}}{P_x + \gamma} \right] \frac{1}{(\gamma^2 - c_{\alpha}^2)} H(\gamma - c_{\alpha}) H(y) \]  
(C.3)

where the index \( \alpha \) ranges over \( P \) and \( S \) waves, and \( c_p = 1 \), \( c_s = v_p/v_s = \kappa \), and \( \gamma = v_p/v_R \). \( D_0 \) is the amplitude of the step source slip function, discussed further below. The functions \( F_{i \chi}^{\alpha} \) are given in Table C-1, and

\[ p_x = -\gamma \cos \psi - \text{isin} \sqrt{\gamma^2 - c_{\alpha}^2}, \quad p_y = 0, \]
\[ m_{\alpha} = \gamma \sin \psi - \text{icos} \sqrt{\gamma^2 - c_{\alpha}^2} \]  
(C.4)

The nondimensional cylindrical time \( \gamma \) is \( v_p t/\rho_c \), and \( \rho_c, \gamma \) are the cylindrical coordinates about the \( y \)-axis. The Heaviside function in \( y \) indicates the cylindrical waves are only radiated in the \( y > 0 \) region, and the Heaviside in \( \gamma - c_{\alpha} \) indicates the arrival time of the cylindrical \( P \) and \( S \) waves, at
\( t = \frac{\rho_c}{v_p}, \frac{\rho_c}{v_s} (\gamma_c = 1, k) \). The radical in the denominator gives rise to square root singularities at the arrival times of P and S. Another singularity arises from the \( p_x + \gamma \) term, but this occurs only for \( z \) close to zero, i.e., for points \( \mathbf{\hat{x}} \) very near to the plane of dislocation. For \( z \sim 0, \gamma \sim 0 \), then from (C.4), \( p_x \sim -\gamma_c \), and so the singularity occurs at cylindrical time \( \gamma_c \sim \gamma \), or \( t = x/v_R \), when the rupture front is passing the point \( \mathbf{\hat{x}} \). Being so localized, this singularity is less troublesome than those associated with the arrivals of the P and S waves. The computational aspects of dealing with these singularities are discussed later in this appendix.

\( D_0 \), the amplitude of the step source slip function corresponds to \( D_x \) for longitudinal slip and \( D_y \) for transverse slip. The only other dependence of \( u_c^\text{cyl} \) on the type of slip occurs in the functions \( F_i^\wedge \) (Table C-1). Longitudinal slip (edge dislocation) has slip in the \((\pm)\) direction of rupture propagation. Transverse slip (screw dislocation) has slip normal to the rupture direction. Since \( D_0 \) is defined by \( D_0 = u(+) - u(-) \) as in Haskell (1969), positive \( D_0 \) in the longitudinal case corresponds to right-lateral slip.

The spherical waves are radiated by the motion of the tip of the dislocation line along the x-axis. They arrive at travel times corresponding to the vectorial distance \( r \) of the point \( \mathbf{\hat{x}} \) from the origin. Madariaga obtained the solution for the spherical waves as a sum of three components,

\[
\frac{\mathbf{u}_i^{\text{sph}}(\mathbf{x}, t)}{4\pi} = -\lambda_i \frac{D_v v_p}{4\pi} \left( I_{c_i}^\wedge + I_{s_i}^\wedge + I_{\omega_i}^\wedge \right) H(\gamma - C_a) \tag{C5}
\]
The Heaviside indicates the arrivals of the spherical P and S waves at \( \gamma = 1, k \), where \( \gamma \) is the nondimensional spherical time, \( \gamma = v_p t / r \). Thus the P and S waves arrive at \( t = v_p / r, v_s / r \). The leading constant \( \lambda_i \) is a sign term, dependent on the component and type of slip. It has the value +1 except for \( i = 1, 3 \) for longitudinal slip, and \( i = 2 \) for transverse slip, when \( \lambda_i = \text{sgn}(y) \). Letting \( k = 1, 2 \) for longitudinal and transverse slip, this can be denoted

\[ \lambda_i = [\text{sgn}(y)]^{i+k-1} \quad (C.6) \]

Rather than the square root singularities associated with the P and S arrivals for cylindrical waves, the spherical waves arrive with step function discontinuities, and are not as difficult to handle computationally. An exception is the \( I_c \) component, at points \( \bar{x} \) such that \( y \) is near zero. In fact, for points in the x-z plane (\( y = 0 \)), \( \gamma = \gamma_c \) (since \( \rho_c = r \)), and the cylindrical and spherical waves arrive simultaneously. Then \( I_c \) exhibits the same singular behavior as the cylindrical wave.

Each of the three terms in (C.5) are complex, but their imaginary parts cancel in forming the sum. The expression for the \( I_c \) component is very similar to that of the cylindrical wave,

\[ I_c^{x'} = \frac{F_c^{x'}}{(p_x + k) \sqrt{\frac{\gamma_c^2 - c_x^2 c_y^2}{1 - \nu_y^2}}} \quad (C.7) \]

where \( p_x, p_y \) and \( m_\alpha \) are the same as those given for the
cylindrical wave (eq. C.4). Considering the leading coefficient in (C.5), since \( r\sqrt{1-u_y^2} = \rho_c \), the contribution due to the \( I_c \) term is \(-1/2\) that of the cylindrical wave (eq. C.3), and acts as a stopping phase for the cylindrical wave, as discussed further below.

The expression for the \( I_s \) term is given by

\[
I_s^s = - \frac{F_i^s}{\rho_y \sqrt{\gamma_s^2 + M^2 \sqrt{1 - u_y^2}}} \tag{C.8}
\]

where \( M^2 = \gamma^2 - c^2 \), and

\[
\gamma_s = (\gamma - \gamma u_y) / \sqrt{1 - u_y^2} = (t - x/v_R) \nu_P / \rho_s
\]

Although the \( F_i^m \) are still given by Table C-1, \( \rho_x \), \( \rho_y \), and \( m_m \) are now given by

\[
\rho_y = -\gamma_s |\cos\eta| - \sin\eta \sqrt{\gamma_s^2 + M^2} \cos\eta \sqrt{\gamma_s^2 + M^2}, \quad \rho_x = -\gamma
\]

\[
m = \gamma_s \sin\eta - i |\cos\eta| \sqrt{\gamma_s^2 + M^2}
\]

where \( \rho_s, \eta \) are the cylindrical coordinates about the x-axis (see Fig. C-1b).

The P and S contributions of the final term \( I_{s\infty} \) are equal and of opposite signs. Thus they cancel after the arrival of the S wave, and only \( I_P \) need be considered, contributing only between the P and S arrival times. The expressions for each component are (where \( \nu_i = x_i / r \), but \( \nu_j = |y| / r \)),

\[
I_{s\infty}^P = 2F_\nu \nu_y D - (\gamma + \nu_x) C + \nu_z
\]

\[
I_{s\infty}^P = -2(\nu_x - \nu_y) C \tag{C.10a}
\]
\[ I_{\omega z}^D = 2D[\gamma \nu_y D - \gamma C + 2 \nu_z] \]

for longitudinal slip, and

\[ I_{\omega x}^D = -2(\gamma \nu_z - \gamma C) \]
\[ I_{\omega y}^D = -(2/F)[- \gamma \nu_y D - (\gamma - \gamma \nu_x)C + \nu_z] \] (C.10b)
\[ I_{\omega z}^D = 2C[-(\gamma - \gamma \nu_x)C + 2 \nu_z] \]

for transverse slip. The complex constants \( C, D \) and \( F \) are given by

\[ C = (\nu_x \nu_y + i \nu_y)/(1-\nu_y^2) \]
\[ D = (\nu_y \nu_z - i \nu_x)/(1-\nu_x^2) \] (C.10c)
\[ F = -(\nu_x \nu_y + i \nu_z)/(1-\nu_z^2). \]

With these three components of the spherical wave, the spherical wave contribution can be obtained using (C.5), and the solution for the quadrantal dislocation is then given by the sum of the cylindrical and spherical wave components (eq. C.2). The complete solution for the rectangular fault is then obtained by the superposition of the solutions for the four quadrantal dislocations (eq. C.1). Note that \( x, y, z \) are defined independently within the solution for each dislocation as indicated by the right hand side of (C.1).

This summarizes the pertinent formulae for the application of Madariaga's solution for the rectangular fault. The departures from Madariaga (1978), established through extended
communication (Madariaga, 1978, personal communications), include elements of equations (C.4), (C.5), (C.6), (C.8) and (C.10b).

Before considering computational aspects which arise in the practical application of the solution, we consider an example illustrating the individual contributions to the solution. We take for this example a rectangular fault of dimensions \( L \times W = 2 \times 1 \text{ km} \), with \( v_p = 1 \), \( v_s = v_p / \sqrt{3} \) and \( v_R = 0.9v_s \), and unit transverse slip, so \( D_0 = D_y = 1 \text{ cm} \). (Thus the medium velocities and fault dimensions are similar to those in the examples of Madariaga (1978)). The velocities and displacements at a point a fault-width away from the center of the fault \( (x = 1, 1/2, 1) \) are shown in Fig. C-2. The source slip function is a step (actually a minimum ramp, of length \( T = t \), here .02 secs) so that the individual arrivals can be seen clearly. Convolution with a longer duration slip velocity function makes the individual arrivals less distinct of course. The velocity and displacement waveforms for the total solution, corresponding to the sum of the four quadrantal dislocations, are shown in Fig. C-2. The first arrivals of P and S are indicated in the figure. The sharp onset of the S arrival for this example makes clear the difficulty that can arise in estimating the velocity by the numeric differentiation of a displacement solution. An adequate estimate of the velocity would require the computation of the displacement at very small time increments near the time of the S arrival.

The contributions of each of the four quadrantal
dislocations to the total solution are shown in Fig. C-3. As is clear in the figure, quadrants 1 and 2 are associated with the starting of the rupture at \( t = 0 \), and quadrants 3 and 4 with the rupture termination at \( t = L/v_R \). The cancelling nature of the quadrant contributions can also be observed. The \( x \) and \( z \) components of the total solution are zero for this example (see Fig. C-2), the point \((1,1/2,1)\) being on a plane of symmetry. However, these components are not zero in the contributions from the individual quadrants. For example, the \( x \) and \( z \) components from quadrants 1 and 2 are not zero (Fig. C-3), but are equal and of opposite sign, cancelling each other in the sum.

The individual contributions of the cylindrical wave and the \( I_c, I_s, \) and \( I_\infty \) contributions to the spherical wave for the quadrant 1 solution are shown in Fig. C-4. The square root singularity is apparent at the \( S \) arrival for the cylindrical wave (there is no cylindrical \( P \) wave for transverse slip). The \( I_c \) contribution has the same inverse square root behavior, though negative, and acts as a stopping phase for the cylindrical wave. The effect of \( I_c \) on the sum for quadrant 1 (in Fig. C-3) can be seen as a sudden drop in \( \dot{u}_y \) from its square root decay at the spherical \( S \) arrival time, at \( r/v_s - r/v_c \approx 0.2 \) sec after the pulse at the cylindrical \( S \) arrival time. For this case, the cylindrical wave is the dominant contribution to quadrant 1, and in fact to the total solution.

In general, as Haskell (1969) noted, the displacement is constant (the velocity is zero) after the \( S \) arrival from the
farthest corner of the fault (including the slip function duration, if it is finite). In terms of the quadrant dislocations, the contributions of the four dislocations sum to zero after the (spherical) \( S \) travel time from the most distant of the terminating quadrants. This can be written as

\[
\text{ts}_{\text{max}} = \frac{L}{v_R} + [(x-L)^2 + \max(y,W-y)^2 + z^2]^{1/2} / v_S + T
\]

where \( T \) is the duration of the slip velocity function.

For clarity in the discussion of the example considered above, no mention was made of the computational aspects of the application of the Madariaga solution. These are discussed in the following. Before considering the numerical aspects, certain simplifications and economies are noted.

Since cylindrical waves are only radiated for \( y > 0 \) for each quadrant solution, in the superposed solution, they cancel except in the strip \( 0 \leq y \leq W \). Thus the cylindrical wave contribution need not be considered for any of the quadrants for points \( x \) such that \( y < 0 \) or \( y > W \). Even for points within \( 0 < y < W \), the cylindrical contribution need only be considered for quadrants 1 and 3. Also, since \( p_x \) is zero for the cylindrical wave and for \( I_c \) of the spherical wave (eq. C.4), many terms of \( F_i^x \) of Table C-1 are zero. For longitudinal slip, there is no contribution to the \( y \)-component, while for transverse slip only the \( y \)-component, for the \( S \) wave, contributes. That these terms are zero is understandable in terms of the motion associated with longitudinal and transverse slip. However, in practice this limits
consideration of the $I_c$ contribution, and that of the cylindrical wave when it must be calculated, to only those components for which the exponent of $\chi_i$ in (C.6) is odd.

Although the $I_c$, $I_s$ and $I_\infty$ terms are all complex, their imaginary parts sum to zero, so only their real parts need to be obtained. The $I_\infty$ term can be simplified by noting that all the $I_\infty$ expressions (eqs. C-10a,b,c) can be written as a constant plus a $\gamma$-dependent term,

$$I_{\infty i}^p = C_1 + C_2 \gamma$$  \hspace{1cm} (C.11)

and this linear $\gamma$ (and thus $t$) dependence can be observed in the $I_\infty$ contributions in Fig. C-4. The constant and $\gamma$-dependent terms in the expression for $I_\infty$ (real part) are given in Table C-2.

A modification which should be made for computational purposes is the replacement of terms in $1-\nu_i^2$ by their equivalent expressions. Since $\nu_i$ is a direction cosine, $\sqrt{1-\nu_i^2}$ is really the direction sine, and the problem of obtaining the sine from the cosine for small angles ($\nu_i \approx 1$) is well known. Thus the $1-\nu_i^2$ terms are replaced by their equivalent direction sine forms, denoted $J_i$ in Table C-2. For computational purposes the $\sqrt{1-\nu_i^2}$ should also be replaced by $J_i$ in the expressions for $I_c$ and $I_s$ of the spherical wave (eqs. C.7 and C.8). Problems can still arise when $\nu_i$ becomes very close to 1 however, with numerical instability due to word-length effects similar to that of the propagator matrix method, as discussed in Chapter 3 and Appendix B. Individual components of the
solution can become very large, though because of the cancellations, the total solution can be relatively small. Our interim solution is the computation of all contributions in extended precision when $\nu_i - 1$.

The remaining computational consideration concerns the square root singularities at the arrival times of the cylindrical P and S waves. In general, we convolve the solution with a slip velocity function, since in our application we are concerned with slip velocity functions other than the impulse in slip velocity (step in displacement). The convolution entails the local integral of the velocity over the duration of the slip velocity function. The existing expressions can be used in the numerical integration, but very small time increments or approximate methods are required at the P and S arrival times, as discussed by Boore and Zoback (1974a).

Obtaining the convolution of the velocity, or just the displacement, requires the evaluation of the integral of $\hat{u}(\vec{x},t)$. For the displacement,

$$u(\vec{x},t) = \int_0^t \hat{u}(\vec{x},t')dt'.$$

If we denote by $\hat{u}'$ the result of convolving $\hat{u}$ with a desired slip velocity function $\hat{s}(t)$, of duration $T$, then

$$\hat{u}'(\vec{x},t) = \int_{t-T}^t \hat{u}(\vec{x},t')\hat{s}(t-t')dt'$$

where $\hat{s}(t)$ is normalized such that

$$\int_0^T \hat{s}(t)dt = 1.$$
We will consider the displacement integral for clarity, then discuss the differences for the convolution integral. The displacement integral for the cylindrical wave, from (C.3), is (suppressing \( H(y) \))

\[
\begin{align*}
\mathbf{u}^*_i^{cyl}(\mathbf{x}, t) &= \frac{D_0 y}{\pi \kappa \rho_c} \int_0^t \Re \left[ \frac{\frac{\mathbf{r}_0}{\rho_x + \beta}}{\sqrt{\gamma_c^2 - \beta^2}} \right] H(\gamma_c' - c_\alpha) \, dt \\
&= \frac{D_0}{\pi \kappa^2} \int_{c_\alpha}^{\gamma_c'} \Re \left[ \frac{\frac{\mathbf{r}_0}{\rho_x + \beta}}{\sqrt{\gamma_c'^2 - \beta^2}} \right] \, d\gamma_c'
\end{align*}
\]

The singularity in the integrand is removable (Acton, 1970) using a trigonometric substitution as suggested by Madariaga (1978). Letting \( \gamma_c = c_\alpha \cosh \chi \), the radical becomes \( c_\alpha \sinh \chi \), and the integral becomes

\[
\begin{align*}
\mathbf{u}^*_i^{cyl}(\mathbf{x}, t) &= \frac{D_0}{2 \pi \kappa^2} \int_0^{\cosh^{-1}(\gamma_c'/c_\alpha)} \Re \left[ \frac{\frac{\mathbf{r}_0}{\rho_x + \beta}}{\sqrt{\gamma_c'^2 - \beta^2}} \right] \, d\chi
\end{align*}
\]

(C.12)

where in terms of \( \chi \) (cf. eq. C.4),

\[
\begin{align*}
p_x &= -c_\alpha \cos(\omega - i\chi), & p_y &= 0, \\
m_\alpha &= c_\alpha \sin(\omega - i\chi).
\end{align*}
\]

This integral no longer has the singular behavior at the P and S arrival times. The integral for the contribution of \( I_c \) can be transformed similarly, the only difference being a leading factor of \(-1/2\), and the lower limit being \( \cosh^{-1}(r/\rho_c) \).

The integration of (C.12) can be performed with no difficulty for the cylindrical wave and \( I_c \) contributions. Of the
remaining components, $I_s$ only represents a step function discontinuity. Because $I_\infty$ can be written as a constant plus a $\tau$-dependent term (C.11), it can be integrated analytically.

The numerical integration is performed using an adaptive quadrature algorithm (in which subinterval lengths are chosen dynamically according to the function behavior). Although several adaptive schemes (e.g., Kahaner, 1971) were investigated, greatest accuracy and economy were obtained using the adaptive Romberg scheme of deBoor (1971). This algorithm, also available in the International Mathematical and Statistics Library (I.M.S.L.) routines (as 'cadre'), was also used by Madariaga (personal communication, 1978).

With the evaluation of the integral, the displacement, or with adjusted limits, the convolution of the velocity, can be obtained. Improved computational efficiency can be obtained if the integration is broken into subintervals. Using simplified notation, the displacement can be obtained recursively as

$$u(t) = \int_0^t \dot{u}(t') dt' = u(t-\Delta t) + \int_{t-\Delta t}^t \dot{u}(t') dt'$$

and the convolved velocity as

$$\dot{u}'(t) = \int_{t-T}^t \dot{u}(t') \dot{s}(t-t') dt'$$

$$= \dot{u}'(t-\Delta t) + \dot{s}(t-\Delta t/2) \int_{t-\Delta t}^t \dot{u}(t') dt' - \dot{s}(t-T-\Delta t/2) \int_{t-T-\Delta t}^{t-T} \dot{u}(t') dt'$$

provided $\dot{s}(t)$ does not vary greatly within $t$, and special care is taken with the limits in the initialization. Note that for a ramp source slip function, $\dot{s}(t)$ is a constant. If
$(t)$ does vary rapidly, the velocity convolution can not be done recursively.

As outlined above, the computational aspects of the Madariaga solution require greater care in its application than in the application of the original formulation of Haskell (1969). However, even when all computational aspects have been included, the Madariaga solution is more than an order of magnitude faster.
TABLE C-1

The Functions $F_i^\alpha$*

<table>
<thead>
<tr>
<th>Component</th>
<th>$F_i^p$</th>
<th>$F_i^s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal Slip</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$2p_x^2$</td>
<td>$-2p_x^2 + \kappa^2$</td>
</tr>
<tr>
<td>2</td>
<td>$2p_xp_y$</td>
<td>$-2p_xp_y$</td>
</tr>
<tr>
<td>3</td>
<td>$-2m_p^2p_x$</td>
<td>$(2m_s^2 - \kappa^2)p_x/m_s$</td>
</tr>
<tr>
<td>Transverse Slip</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$2p_y^2p_x$</td>
<td>$-2p_y^2p_x$</td>
</tr>
<tr>
<td>2</td>
<td>$2p_y^2$</td>
<td>$-2p_y^2 + \kappa^2$</td>
</tr>
<tr>
<td>3</td>
<td>$-2m_p^2p_y$</td>
<td>$(2m_s^2 - \kappa^2)p_y/m_s$</td>
</tr>
</tbody>
</table>

*Letting $k = 1, 2$ for longitudinal, transverse slip, and letting $p_3 = -m_\alpha$, these functions can be written

$$F_i^p = 2p_ip_k$$

$$F_i^s = -2p_ip_k$$, unless $i = k$ or 3, when $F_i^s = -2p_ip_k + \kappa^2 p_k/p_i$
TABLE C-2
Terms* of \( I_{\infty}^P = 2\gamma I_{\infty}^1 + 2\eta I_{\infty}^2 \)

<table>
<thead>
<tr>
<th>Component</th>
<th>( I_{\infty}^1 )</th>
<th>( I_{\infty}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal Slip</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>( v_y v_z / \xi^2 )</td>
<td>( v_x v_y v_z (3-v_y^2) / \xi^2 )</td>
</tr>
<tr>
<td>2</td>
<td>( v_x v_z / \xi^2 )</td>
<td>(-v_z )</td>
</tr>
<tr>
<td>3</td>
<td>(-v_x v_y (1+v_y^2) / \xi^2 )</td>
<td>( v_y [v_z^2 (2-v_y^2) - v_x^2] / \xi^2 )</td>
</tr>
<tr>
<td>Transverse Slip</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>( v_x v_z / \xi^2 )</td>
<td>(-v_z )</td>
</tr>
<tr>
<td>2</td>
<td>(- (1+v_x^2) I' )</td>
<td>( v_x (3-v_x^2) I' )</td>
</tr>
<tr>
<td>3</td>
<td>( (v_y^2 - v_x^2 v_z^2) / \xi^2 )</td>
<td>( v_x [v_y^2 (2-v_y^2) - v_z^2] / \xi x )</td>
</tr>
</tbody>
</table>

*Note that \( v_i = x_i / r \), but \( v_y = |y| / r \). \( \xi_i \) are the direction sines corresponding to the direction cosines \( v_i \). They are obtained as
\( \xi_x = \rho_s / r \), \( \xi_y = \rho_c / r \) (\( \xi_z \) does not appear). Also, \( I' = v_y v_z \xi_y / [\xi_x^2 (v_y^2 + v_z^2)] \).
Figure Captions - Appendix C

Figure C-1  A) Model of unilateral rupture on a rectangular fault of length $L$ by width $W$, and Madariaga's decomposition of the rectangular fault into four infinite quadrantal dislocations. The rectangular fault is given by $Q_1 - Q_2 - Q_3 + Q_4$, dislocation starts at $t = L/v_R$ for quadrants 3 and 4, cancelling 1 and 2 which start at $t = 0$.

B) Coordinate geometry used in the solution of an infinite quadrantal dislocation. The coordinates $\rho, \eta$ are cylindrical coordinates about the $y$-axis, associated with the radiation of the cylindrical waves. $\rho, \eta$ are cylindrical coordinates about the $x$-axis. $\nu_x, \nu_y, \nu_z$ are the direction cosines of $\hat{r}$ relative to the coordinate axes.

Figure C-2  Displacement and velocity for unit transverse slip ($D_0 = D_y = 1$ cm) on a fault of dimensions $L \times W = 2 \times 1$ km. The observation point is at $x = (1,1/2,1)$, a fault-width away from the center of the fault. The earliest P and S arrivals are indicated. The first P and S arrival from the terminal end of the fault at $x = L$ are also indicated ($P', S'$). ($v_p, v_s, v_{rupt}$ are $1, 1/\sqrt{3},$ and $0.9/\sqrt{3}$, Trise = $\Delta t = .02$ sec).

Figure C-3  Contributions of the four quadrantal dislocations to the displacements and velocities in Fig. C-2. The earliest P and S arrivals from each dislocation are indi-
cated (cylindrical wave for quadrants 1 and 3, spherical for 2 and 4). Note change in displacement scale from Fig. C-2.

**Figure C-4** Contribution of the cylindrical wave and the contribution of the individual components of the spherical wave to the quadrant 1 displacements and velocities of Fig. C-3. The P and S arrival times are indicated. (There is no cylindrical P wave, nor $I_c$ component of the spherical P wave for transverse slip.)
Figure C-2
Figure C-11

Cyl. Wave

I₀, Sph. Wave

Iₒ, Sph. Wave

I∞, Sph. Wave

<table>
<thead>
<tr>
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<th></th>
<th></th>
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</tr>
</thead>
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<td>cm/Sec X Vel</td>
<td>cm/Sec X Vel</td>
<td>cm/Sec X Vel</td>
</tr>
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<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>5</td>
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