MESOSCALE ANALYSIS OF A COMPLEX COLD FRONT

BASED ON SURFACE AND TOWER DATA

by

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ABSTRACT

Mesoscale analyses are performed on a complex cold front which
passes through the NSSL Beta Network. Mesos-network surface,
meteorological tower and rawinsonde data is employed. Historical
and theoretical backgrounds are presented.

Hourly mesoscale surface analyses are performed. We find the
initial front dissipates in the network and a new front forms. We
calculate vorticity, divergence, resultant deformation and fronto-
genetical values for three hours of data. We find that the position
of the zone of maximum confluence of the wind field, in relation to
the position of the leading edge of cold air, plays a dominant role
in frontal intensification and weakening.

Tower data is throughly analyzed in the vicinity of the front.
Conclusions are drawn about frontal shape in the lowest 1500 feet.
No prominent nose structure is found. Second-by-second analyses
of wind components normal and parallel to the front are performed.
Conclusions are drawn on the time scale of the frontal process,
turbulent effects in the vicinity of the front, vertical varia-
bility in the vicinity of the front and relative strengths of
normal and parallel flows in the vicinity of the front.

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I. INTRODUCTION

Meteorology is, in the broad view, the sum of a group of semi-organized attempts to better understand, explain and forecast atmospheric phenomena which affect man. Tolerable temperatures and sufficient water sources are the necessary conditions man requires to inhabit a given location on the earth's surface. Once man is established in an area, he and his endeavors are affected by high and low values of temperature, wind and precipitation and by rapid changes of atmospheric variables such as temperature. In general, it may be said that the better rapid changes and high and low anomalies are predicted, the less likely they are to cause a detrimental outcome for man. We choose to define, as do most people, atmospheric fronts as zones of rapid temperature change (see Bergh, 1967). The investigation presented herein is an attempt to better understand and explain these frontal zones.

A. Historical Aspects

M. Margules in 1906 derived a formula which expressed the equilibrium slope of the line of separation between a warm air mass and a cold mass, assuming geostrophic and hydrostatic balance. Acceptance of the concept that there did in fact exist, in the real atmosphere, lines which delineated sharp contrasts in temperature between warm and cold air masses and along which there tended to be discontinuous wind shears, is due to the work of Vilhelm Bjerknes, Jacob Bjerknes, Solberg and others at the Bergen (Norwegian) School of Meteorology.
V. Bjerknes called these lines "lines of convergence" and instigated an investigation of them while in Leipzig from 1913 to 1917 (according to Eliassen, 1962). It is interesting that Bjerknes considered convergence patterns in the horizontal wind field to be the predominant causative factor in the formation and sustenance of these lines which in subsequent terminology were called "steering surfaces" and finally fronts. We say it is interesting because V. Bjerknes' work was so definitive that only recently (Bergh, 1967) has it been shown that deformation patterns in the horizontal wind field appear to have a stronger effect on frontal formation than do convergence patterns. After moving to Bergen in 1917, V. Bjerknes organized a close observational network along Norway's western coast in order to carry out a more detailed study of the presence of lines of convergence and the atmospheric processes involved with them. From these studies evolved the contributions to the science by the Norwegian school. We wish that the reader will keep in mind the dependence of the work done by V. Bjerknes and his collaborators on the establishment of a proper network for data collection.

Observations showed that the flow around storm cyclones was not symmetrical. One line of convergence was generally found ahead of the cyclone's path and a second extended from the center of the storm to the right of its path of motion. Respectively, the lines were cold and warm fronts. The air enclosed between the fronts was warmer than the air excluded by them. Further development showed the lines to be intersections of the earth's surface and sloping
surfaces of discontinuity in the atmosphere. Vertical motions, cloud and precipitation patterns and various other phenomena were described in terms of the warm and cold fronts.

Meteorologists built on the Bergen school's foundation until it was realized that fronts were not prime movers of the atmosphere and did not provide all, or possibly any, of the answers they sought. Like the atmosphere itself where troughs are seen to appear where a ridge had before existed, the theory and study of fronts fell into a trough of disuse, and absence of a good understanding of them is the result.

Today, fronts are drawn on weather maps, their positions more or less determined by the vagaries of the analyst. The public is aware of them because the television weatherman finds them to be a graphic method of explaining the weather. It often appears that an inverse proportion exists between his meteorological knowledge and his dependence on fronts to explain the forecast. We do not believe that the study of fronts should be resurrected to the position it held in its golden youth, nor do we feel it should be swept into a corner and forgotten. Fronts are a real and important feature of our environment, and an effort should be made to better understand them. We hope that this investigation is a contribution in that direction.

B. Data Reportage

Availability of data coverage with proper spacial and temporal
scales is essential for the study of any physical phenomena. The synoptic scale, where observing stations are spaced at intervals on the order of 100 miles and the regular measurement interval is one hour, is the generally available data network for meteorological purposes. Although the word synoptic applies not properly to a scale but rather to a method, the word is used for want of a better one and with the feeling that it presents no difficulty of expression. Frontal structures are usually present in synoptic analyses; they may be seen to form and dissipate, speed up and slow down and change their nature. Familiarity with these analyses soon convinces one that the frontal process must be studied in networks of smaller space and time increments than the synoptic. Microscale data collection, where time is on the order of a few seconds or less and space on the order of a few feet, has so far proved to be too small for the study of fronts. Clearly what is needed is a data reporting network intermediate in scale between the micro and synoptic, a mesoscale network.

In recent years the National Severe Storms Laboratory of ESSA has developed such a mesoscale reporting network. Located in southern Oklahoma is NSSL's mesoscale Beta Network. The network consists of a grid of 56 surface stations, a 1600' meteorological tower, 10 rawinsonde stations, radar installations, a rain gauge network and an aircraft program. Our interest lies in the first three mentioned. The surface station network lies between 34° and 36° north latitude and 97° and 99° west longitude, the 56 stations are spaced at
approximately 10 mile intervals. Temperature, wind speed, relative humidity, station pressure and rainfall are recorded continuously and displayed by analogue formats. Wind direction is recorded at one-minute intervals with respect to eight channels. Wind direction is given to 16 points of the compass by 2 channels combined print out or single channel print out. Wind instruments are mounted at a height of 20 feet.\(^1\)

The meteorological tower is jointly operated by WKT-TV and NSSL and is located 20 miles north of Norman, Oklahoma. Terrain in the area is flat and virtually featureless. The tower is instrumented at six levels and at a surface station with Bendix Aerovanes (fig. 1). Analogue recording of wind speed and wind direction at a chart speed of 6"/hour is done routinely. Fast run data at a chart speed of 6"/minute is collected when warranted by interesting events. Commencing in December 1966, temperature data was continuously recorded at various tower levels; unfortunately, this type of data was not available in time for this paper. Only when strong convective activity is anticipated does the rawinsonde network become fully operative. When this event occurs the 10 stations release rawinsondes hourly for the duration of the activity, resulting in a high resolution (approximately 100 contact points per sounding) soundings (fig. 2).

The NSSL Beta Network exists to study severe weather. Data

\(^1\) For more specific details of instrumentation and data display consult NSSL publications.
for frontal passage is an illegitimate offspring. As the bastard child, frontal passages, for which fast-run tower data and rawinsonde are available, are the occasional ones which are concurrent with severe weather. We should consider ourselves fortunate that this type of data exists at all.

C. Objectives of this Investigation

This investigation is intended to be a self-contained study of certain aspects of the frontal process and a preliminary study on which to base possible future work on other important aspects. We wish to look into the applicability and usefulness of mesoscale data and analysis techniques in studying a complex cold-frontal passage. We will try to provide some sort of temporal and spacial mesoscale dimensions to the vertical structure of the front. We will attempt to provide a preliminary survey of the usefulness of high-resolutions, fast-run, analogue-recorded tower data and dense rawinsonde data in the study of the frontal process. We believe that these are desirable areas of endeavors.
II. THEORETICAL BACKGROUND

The proximity of two air masses differing in one or more of their measurable characteristic parameters necessitates some type of divider if the two are expected to maintain their individuality for any period of time. Dividers may vary from sharp discontinuities which approach a linear configuration in two dimensional space to broad, diffuse zones of transition. In most instances the nature of the divider is as much a function of the scale of inquiry as it is a function of the real process. Fronts may be assigned as dividers of various atmospheric quantities, with resulting orders of approximation to orders of discontinuity.

No true spatial discontinuities of any atmospheric properties are present in the real world when one observes above the microscale. Narrow zones across which various properties change rapidly do exist. Degree of approximation to a true discontinuity depends on the narrowness of the zone, the rapidity of the change and the scale of inquiry. The question that arises with atmospheric fronts is to what scale of observation may we reduce our inquiry and still have our approximation to a discontinuity validly hold. Again, various orders of discontinuities are available (fig. 3). Order of discontinuity is equivalent to the order of the derivative of the property at which it is discontinuous. If the property itself is discontinuous, there is a discontinuity of zero order. If its first derivative is discontinuous, there is a first order discontinuity.
present and so on for higher order derivatives. Conceptually a line or surface of discontinuity is considered as a boundary, at which certain boundary conditions hold. Nature will not permit the infinite pressure gradient force that would result from a finite pressure difference across an infinitely small distance. The dynamic boundary condition therefore requires pressure to be continuous through an internal boundary. If fluid motion components normal to the discontinuity surface are not the same on both sides either a vacuum or mixing of the two fluid masses would occur. The kinematic boundary condition restricts the motions so vacuums or mixing do not occur. This implies that the ideal discontinuity moves with the component of motion normal to it. Finally, the no-slip boundary condition states that in a viscous fluid, velocity components tangent to an internal boundary must be equal. We define a front in terms of the temperature field by making the approximation that a zero order discontinuity in temperature may be said to exist to a satisfactory degree of accuracy. Most writers indicate that two first-order temperature discontinuities demarcate a frontal zone.

For ease of description and clearness of thought, we will confine our work and discussion to temperature fronts and mesoscale inquiry whenever possible. We must warn the reader that in the analysis and discussion of tower results that the absence of temperature records forces us to use the wind records as our vehicle of study. Although the highly desirable prospect of continuity is lost, the inaccuracy introduced is not debilitatingly great and is
in any case unavoidable.

Fronts do not just suddenly exist. They form, go through intensifications and weakenings, become diffuse and finally indistinguishable. Rapid transition of temperature as seen in the propinquity of isotherms is our requisite for the existence of a front. Fortunately (planned rather than fortuitous) this in turn leads us to a clear definition of frontal formation. If the isotherms are becoming more closely spaced, frontal formation or intensification is taking place. If the isotherms are becoming less closely spaced, frontal weakening is taking place. Packing of isotherms is termed frontogenesis, and the reverse effect is termed frontolysis. Confluence in the wind field leads to frontogenesis and diffluence to frontolysis. Petterssen evolved an expression for surface frontogenesis in terms of a planer coordinate system oriented along and perpendicular to the front and involving the temperature field and kinematic aspects of the wind field. In Appendix II we present a derivation of the frontogenesis equation in terms of a right-hand planer coordinate system oriented to the four cardinal points of a compass. We feel the necessity to rederive the equation with a north-south orientation for several reasons. By doing so, we dispense with the identification of frontal orientation which in any event is not clearly defined at a reasonable distance from the front. Our resulting equation is

\[ F = D + \frac{1}{2} \text{Grad} T \left[ \cos \phi \text{def}_h \vec{C} - \text{div}_h \vec{C} \right] \]
\[ F = D + \frac{1}{2} \left| \nabla T \right| \left( \cos 2\phi \sec \alpha \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) - \left( \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} \right) \right) \]  

where  

\( D \) = the diabatic term  
\( \nabla T \) = gradient of temperature  
\( \phi \) = angle between isotherms and axis of dilatation of the wind field  
\( \alpha \) = angle between axis of dilatation and principle axis  
\( \alpha \) is determined through the equation:

\[ \tan \alpha = \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \left/ \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) \right. \]  

where the numerator is the shearing deformation and the denominator the stretching deformation. A positive value of \( F \) indicates frontogenesis and a negative value frontolysis. Frontogenesis is seen to be the result of the interaction of the temperature field, deformation and divergence patterns and whatever diabatic effects may be present. Inspection of the equation indicates that the angle \( \phi \) and the divergence determine the sign of the final value (neglecting the diabatic term for the present). Qualitatively convergence helps produce frontogenesis and divergence frontolysis. If the angle between the isotherms and the axis of dilatation is between \(-45^\circ\) and \(+45^\circ\) the deformation field aids frontogenesis. At \(45^\circ\) the term becomes zero and for angles greater than \(\pm 45^\circ\) and less than \(\pm 135^\circ\) frontolysis is aided. This equation applies to
an air parcel. Appearance of a front at a particular location may be due to frontogenesis on parcels at that point, advection of parcels with frontal properties or the two acting in combination.

Surface level fronts represent the intersection of three-dimensional frontal surfaces and the quasi-horizontal earth's surface. Intersections between frontal surfaces and any given plane surface will have similar configurations. The frontal zone is narrowest at the surface, increasing in width with height. In all cases the cold air is seen to underlie the warm air, the two masses being separated by the frontal surface which has a given slope. In the case of a cold front, where cold air is replacing warm air at the point in question, the upper air frontal slope is between 1/50 to 1/150. Warm frontal surfaces have slopes between 1/100 and 1/300. Geophysical texts are apt to wean the student on diagrams with great vertical exaggerations often not stressing the distortions present. Fig. 4 shows what the slopes really look like when the horizontal and vertical scales are equal. In the vertical the frontal zone is a layer of high stability showing as a characteristic "frontal inversion" on an adiabatic chart (fig. 1). As the transition across the frontal zone when ascending is from cold, dry air to warm, moist air, one should ideally anticipate both a temperature and a dewpoint inversion. Shapes of frontal surfaces near the surface of the earth are a point of conjecture. About all that can be definitely said is that a vertical slope is approached. Popular among some meteorologists is the idea that cold fronts
present a "nose" type structure, that is to say that cold air actually lies above warm air and that the front may extend as much as 100 miles ahead of its surface position at some height not far above the surface (Namias, 1940). Potential energy is at a minimum when the warm air overlies the cold air with the slope of the frontal surface at zero. Maximum potential energy is available when cold air lies over the warm air again with zero frontal slope.

Before any satisfactory explanation of the energetics of the frontal process can be given the structure of the lower levels of fronts must be explicitly known. What may be said now is that the baroclinic mode set up by the temperature configuration makes potential energy available to be transformed into the kinetic energy of the winds, which in turn is dissipated in the main by surface friction. Mechanisms for the transformations are not known and in many instances not even hypothesized.

Time series recorded by Eulerian sensors are the type of records we have. Theory involving this type of record is especially important in respect to the tower recordings. Involved in these time series is a turbulent type of flow embedded in a more organized field of fluid motion. Turbulent theory is a major field in itself so the reader will appreciate our hesitancy in going into a discussion of it. Certain basic ideas and concepts are however needed for future discussion and must be briefly stated here. The reader is directed to the literature for more detailed expositions of the material (see bibliography).
According to Hinze (Hinze, 1959) turbulence may be verbally defined as follows: "Turbulent fluid flow is an irregular condition of flow in which the various quantities show a random variation with time and space coordinates, so that statistically distinct average values can be discerned." Turbulent flow can ideally be generated in two ways. The first is called "wall" turbulence; in this manner turbulence is generated by friction forces at fixed walls. When fluid layers flow with different velocities past or over one another, "free" turbulence is generated. Viscous qualities of fluid tend to dissipate turbulence, make it more homogeneous and less direction dependent. Where no outside factors are disturbing the air flow in the atmosphere, the wind field shows greatest turbulence (i.e. gustiness) in the late afternoon of a clear day, when insolation has resulted in a variable vertical temperature regime. The maintainance of the turbulent state implies that masses of air are being continually moved in the vertical. Treatment of turbulent data usually supposes flow to consist of mean motion on which is superimposed an extremely complicated secondary, or eddy, motion of an oscillatory but not obviously periodic nature. Any attempt at treatment must therefore be based on a firm foundation of knowledge of analysis of random data by statistical methods. Most geophysical time series show some degree of smoothness but generally the techniques of random analysis must be applied to them in order to get meaningful results. Again, measurements and analysis of random data is a study in itself and
the reader is directed to the literature (see Bibliography).

Finally a word on the often used practice of transference of time domain records to the space domain is in order. Tacitly, and often without explanation or even realization, the researcher by doing so is assuming certain pertinent physical characteristics of the process under examination. What he is doing is applying Taylor's "Frozen Wave" hypothesis (Taylor, 1935) which allows for space-time interchanges by assuming that for a given distance fluctuations of the parameter being measured travel as an invariant quantity so that a time record may be laid off as a space record. Mathematically this may be expressed as \[ \frac{\partial}{\partial t} = U \frac{\partial}{\partial x} \], a change in the time record is equivalent to a change in the space record propagating at a mean velocity. We do not wish to imply that what has been done is incorrect but only wish to point out that limitations do exist and should be considered. Absence of a properly close-knit spacial observation network of the scale needed for the particular study necessitates that temporal records be used as indicative of spacial records.
III. SYNOPTIC SITUATION 8-9 JUNE 1966

Maximum beta-network data reportage and an interesting complex case were coincident on 8-9 June 1966.

The surface system formed to the west-northwest of the beta network and first appeared on the U.S. Weather Bureau's 12 Z 8 June 1966 surface analysis as a short cold front extending from a low pressure center in the vicinity of the Oklahoma-Kansas-Colorado borders to central New Mexico. Formation of the front occurred in the wake of a varying frontal system which had persisted in the Central and Southern Plains States for several days. By 00 Z on 9 June (1800 CST 8 June, fig. 5a) the low was centered in eastern Kansas and the cold front extended from it to southeastern New Mexico, lying just west and north of the network. At 12 Z on 9 June (0600 CST 9 June, fig. 5b) the low had moved to central Illinois and the southwest-northeast oriented cold front had just left the network. By the 00Z 10 June surface analysis the front extended from southern Texas to the low centered in the central Appalachian.

A 500-mb short-wave trough was in evidence progressing eastward from the eastern side of the Rocky Mountains across the North and Central Plains during the period 00 Z 8 June through 00 Z 10 June. At 850 mb the surface pattern was repeated with the usual northwestward height lag. A 15°C (27°F) temperature gradient existed across the most intense section of the 850 mb front.
IV. ANALYSIS

A. Data Available

During the period that the front was passing through the beta network the surface, tower and rawinsonde phases of the network were in full operation. NSSL supplied us with temperature, humidity, wind direction, wind speed, rainfall and station pressure records from the 56 surface stations, fast run tower data from all six levels and the surface station for the time that the front was progressing through the WKY area, and 67 rawinsonde records reduced to digital data printout. Observations from surrounding Weather Bureau and Air Force stations were available.

All surface and tower data received was of generally excellent quality and all was usable. Rawinsonde data was too late in arriving to be given anything but a precursory look. Major use made of the temperature, wind direction and wind speed records from the surface stations and the wind direction and speed records from the tower. All data was on microfilm and data reduction was done from that. Temperature was readable to the nearest degree Fahrenheit, surface wind direction to the nearest 22.5° and surface wind speed to the nearest knot. Relative time accuracy for the surface data was on the order of one minute but absolute accuracy in time was dependable to two or three minutes. Tower wind records were readable to one second, 5 degrees and one knot. Absolute timing of the tower records was excellent, never being worse than two or three seconds.
Best indications garnered from reports on investigations of tower mounted wind instruments are that for the type of tower, the method of instrument mounting and direction of winds measured; wind direction and wind speed are definitely accurate to $\pm 10\%$ and probably accurate to $\pm 5\%$ (Gill et al, 1966). This accuracy rating refers to the effects of the tower on the flow. Errors introduced by the instruments themselves, in this case Bendix Aerovanes, are another matter. If conclusions are to be drawn based on a required accuracy on the order of one or two seconds, strong proof must be provided, preferably by wind tunnel tests, of the high frequency performance of the wind instruments. The resonance frequency of the Bendix Aerovane approaches one second and the possibility of reinforced buffeting must not be overlooked. Examination of the records, especially the direction traces, indicates no severe instrument bias in the time ranges we wish to examine and comment on. It appears even that the instruments performed admirably in the critical high frequency regions.

We characterize frontal passage as the beginning of a sharp step-like drop in the temperature trace (fig. 6a). Tests of the temperature sensors and recorders have shown that the response is in the form of a 95% decay curve, or in other words the actual temperature drop is sharper than the recorded trace indicates.

Because of a problem in the program to reduce raw rawinsonde data to a usable form, the 67 rawinsonde soundings were delayed four
months and time did not allow any major examination of them. Also only a small percentage of the soundings ascended through the frontal zone, most telemetering information in the warm air only. Temperature, wind vectors, humidity and pressure identified at about 100 points whose position is pinpointed by azimuth, range and elevation angle are given for each sounding. NSSL was also kind enough to provide computations of stability and various other nuggets of potentially useful information.

B. Analysis Techniques

Making use of the surface station temperature traces, and establishing the beginning of a step-like break in the traces (fig. 6a) as our criterion for cold front arrival, we were able to ascertain the chronology of the frontal passage through the network. Some stations did not show a clear temperature break, but the majority caused no problems in this way. Temperature, wind direction and wind speed were then read by the minute for a period from 4 minutes before the break to 12 minutes after the break. At stations where more than one break was evident, corresponding data reduction was done for each break. Isochromes of the breaktime were drawn (fig. 7) and from them frontal orientations were established. Rainfall records were checked for places and times of precipitation in order to establish if the temperature records were influenced by convective activity. These initial steps indicated that an hour-by-hour full network analysis would be useful. Thirteen hours of readings were plotted for each of the 56 surface stations. Microfilm charts were read at 5 minutes
to the hour because it was felt that this timing would be closest to the hourly observations taken at surrounding Weather Bureau and Air Force stations. Wind direction, wind speed, temperature, relative humidity, station pressure and rainfall were plotted and analyzed from 1800 CST 8 June 1966 through 0600 CST 9 June 1966 (figs. 8a-m). Three hours, 1900, 2200 and 0200 were chosen for frontogenesis calculations. Wind fields were separated into north-south and east-west components and analyzed. A finite-difference scheme was set up for a grid interval of 10 nautical miles. Since station spacing is about every 10 miles the grid interval should not be less. \( \frac{\delta u}{\delta x}, \frac{\delta u}{\delta y}, \frac{\delta v}{\delta x} \) and \( \frac{\delta v}{\delta y} \) increments were read and the orientation of the axis of dilatation was calculated by use of equation (2) for each station. Temperature gradients and angles between isotherms and axis of dilatation were then read for each station and the frontogenesis quantity, vorticity, divergence, and deformations were calculated.

We then began to analyze the tower data. Our first step was to take five-second eyeball averages of the wind speed and wind direction at the seven levels, at 30-second intervals, for a five-minute thirty-second period which encompassed the major wind activity. These results were then displayed as vectors on a polar plot. We then decided to read the wind direction and speed at one-second intervals, even if questionable instrumentation would not allow us to draw conclusions on that fine a time scale. Since we knew the orientation of the front in the tower vicinity quite well, we were in
fact interested in frontal structure and had to use the wind data
to study that structure we resolved that one-second values into
vector components parallel and normal to the front. A ten-second
averaging was then performed so that any instrument bias present at
the high one-second frequency would be smoothed out. Another inspection
of the analogue data was made for further assurance of the data's
respectability.

Rawinsonde data was copious when it arrived, but its late arrival
allowed only for plotting of wind direction, wind speed, temperature
and dewpoint on thermodynamic charts.
V. DISCUSSION OF RESULTS

A. Surface

The cold front enters the northwest sector of the network between 1800 and 1900 CST on 8 June 1966. Grady, the most southeasterly station in the network, does not experience a distinct temperature break until 0600 CST 9 June 1966. Isochrones of temperature break (fig. 7) show a general southwest to northeast orientation throughout the network. Intensity of the temperature breaks at the various stations, as indicated by 12-minute temperature decreases (fig. 12), is marked by a strong decay between the front's entrance into the network and its departure from the network. Station 1D Waterloo has a temperature drop of 16°F in 12 minutes, from 90°F to 74°F. Station 8H, Grady, has a 3°F temperature drop in 12 minutes, from 71°F to 68°F. Neither station's temperature trace is affected in the vicinity of the break by precipitation or any other convective activity. Station 1D shows a distinct, strong temperature break; station 8H shows a distinct, weak temperature break. A strong cold front enters the network with distinct and large temperature breaks. By the time (4 1/2 hours) it is two-thirds of the way through the network it has been emasculated to the extent that it no longer produces any clear temperature break. Then some mechanism revives the systematic temperature breaks and passes through the remainder of the network. The substantial surface that is associated with the temperature breaks in the upper two-thirds of
the network is not the substantial surface associated with the
temperature breaks in the southern third of the network. This
case therefore differs radically from cases previously studied
(Sanders, 1966 and Bergy, 1967).

Twenty-eight station traces show one distinct temperature
break, seventeen show two distinct breaks, two show three distinct
breaks, one shows four distinct breaks, three show traces where the
break is strongly affected by convective activity, five show no
distinct break and one station is unavailable. We are convinced,
after much data inspection, that the characteristic step-like
break can be caused only by frontal passage. This is important
because the presence of a distinct break in a temperature trace,
even if the resulting temperature decrease is only a few degrees, is
indicative of a frontal passage at that location. The isochrones in
fig. 7 are in all cases drawn for the first indicated break. From
1800 when the front is approaching the network to the 2300 isochrone,
found lying just southeast of a line running roughly 8D-7E-6F-5G,
the movement averages about 18 knots and the isochrone spacing is
fairly even. Some wave structure is seen in the western section of
the network, this is probably due to the Wichita Mountains and other
high ground in that area. Between 2300 CST on the eighth and 0300
on the ninth the isochrones are packed and show little movement.
The 0000, 0100 and 0200 isochrones are broken lines because they
are not supported by actual observational but rather are drawn to fill in the space between 2300 and 0300. The isochrones of temperature break, and thereby the front, are not continuous through the network.

For the stations with usable data the network-wide 12-minute decrease in temperature associated with the initial break is 5.7°F. If we divide the network into a section having an initial temperature break by 2300 and a section having an initial temperature break after 2300 we get average temperature drops of 6.8°F and 2.9°F respectively. Dual and multiple breaks occur only at stations north of the 2300 isochrone. Those in the upper network are separated in time by 1/2 hour to 1-1/2 hours, those in the vicinity of the 2300 isochrone by two or more hours.

Plotkin (1965), Sanders (1966) and Bergh (1967) have investigated the relationship between the wind shift and the temperature break in the vicinity of cold fronts. Change of wind direction is referred to as the direction shift and change of wind speed as the speed surge, the dual terminology necessitated by their lack of coincidence. Analysis of the relationships indicates where diffluence and confluence are occurring in regard to the position of the temperature break. Because timing is of the essence we did not use the occasional record where either the wind or temperature could not be adequately referred to absolute time. In this case we find that indeed the direction shift and speed surge are a dual phenomena, the wind shift preceding the vector shift by one or more minutes in almost all instances.
Maximum confluence occurs at or slightly after the temperature break for all stations analyzed which experienced the temperature break by 2000. Huge amounts of frontogenesis by confluence are experienced by those stations in the northernmost section of the network. This shows good correlation with the intensities of temperature drops felt. After the 2000 isochrone the direction shifts and speed surges get increasingly ahead of the temperature break until by the 2300 isochrone they occur substantially ahead of the temperature break. Thus maximum confluence is found further in the warm air as time goes on and diffluence is occurring in the cold air giving frontolyses. In the southeastern corner of the network, the area experiencing temperature breaks after 0300, we find that the direction shifts are occurring slightly before the temperature breaks and the speed surges at or slightly after the breaks. Maximum confluence is found at or slightly behind the front due to the speed surge and resulting in systematic, if not strong, frontogenesis. The results seem to substantiate the thoughts of Plotkin, Sanders and Bergh.

Analyses of 16-minute time slices about the temperature breaks would not be sufficient to give a complete understanding of the surface characteristics of the complex case. The wind field does not maintain a consistent northerly flow component after the frontal passage, nor, as we have said, does the same substantial surface produce the temperature breaks in the northwest and southeast sections. Fig. 8 a through m are hourly beta-network surface charts with the mesoscale wind and temperature fields indicated. The isotherms are not smoothed
so that both real temperature field irregularities and irregularities produced by station peculiarities are present.

At 1800 CST 8 June 1966 the entire network is in the warm air with most stations reporting southerly components. Strong convective activity occurs during the hour at 3B and 4A. Thunderstorms at these two stations obscure the temperature break. A small tornado is reported over open ground two miles north of Alfalfa, 4A. By 1900 the front has begun to move into the network causing sharp temperature drops and wind shifts to the northwest at 1D and 2C. Low temperatures at 3B and 4A are due to convective activity. Station 3A, which never receives precipitation, remains in the warm air. Intense temperature drops are limited to a relatively short segment of the front. At this time the 85°F isotherm appears to be the best identifier of the leading edge of the front. Diabatic effects are beginning to decrease the temperatures in the warm air resulting in a lessening of the gradient between the warm air and the cold air. The 85°F isotherm is well through a third of the network by 2000, but diabatic effects have lowered the temperature in the warm air to the extent that the temperatures in the warm air are not significantly different from 85°F at most stations. Notice that the 85°F isotherm has begun to outdistance the strongest isotherm packing and that the winds at the northern and northwestern stations have switched back to southerly components. Thus there is very strong influence ten to twenty miles behind the packed isotherms and no strong temperature gradient exists behind the 70°F isotherm. At 2100 the 85°F isotherm has become innocuous. Isotherms are starting
to spread and now the 80°F isotherm best defines the beginning of what has become a more diffused frontal zone. Winds to the north maintain southerly components for the most part and the overall wind field is acting to destroy the previous isotherm concentration. Processes of frontolysis continue at 2200. Synoptic scale analysis would still indicate a fairly well defined front, but here at the mesoscale we can see that what was a strong, concentrated frontal zone two hours earlier has now degenerated to nothing more than a wide zone of temperature decrease. One would certainly be hard put to call it a good approximation to a zero order temperature discontinuity. Only 6G reports a temperature break between 2300 and 0200, this a 2° drop commencing at 2325 and associated with the 80° isotherm in its last appearance as the leading edge of a frontal zone. The temperature field at 2300 shows some organization, but not of a degree seen previously. Notice that the winds at 2C and 1D have reverted to northerly components - they will continue to have northerly components through 0600. The reader is asked to pay particular attention to the relative spacing and movements of the 70°F and 75°F isotherm in the period 2300 through 0300. During the next three hours, 0000, 0100 and 0200, rebuilding of a substantial surface is taking place. Northerly wind components, noted as returning to 1D and 2C at 2300, return throughout the air colder than 75° until at 0200 confluence, and resultant isotherm packing, is taking place behind the 75° isotherm. This isotherm begins to act as the leading edged of a new frontal zone. Its position in the network varies little during the hours that the windfield is
reorganizing into predominately northerly flow in the cold air and packing the other isotherms behind it. From 0300 through 0600 the new front, which is weaker by almost an order of magnitude than the original, moves through the remainder of the network producing clearly defined, weak temperature breaks. In view of the fact that temperature gradient between the warm and the cold air has been considerably lessened by diabatic effects and by the vestiges of the original front we can not expect too vigorous a front to be set up. As soon as the new front starts to move it in fact begins to decay, as can be seen by the spreading of the 60°F and 65°F isotherms during the last four hours. What is important is that even in the absence of an intense temperature gradient across the network the wind field becomes sufficiently organized to condense what temperature gradient there is into an effective frontal zone. Temporal and spacial scales involved are sub-synoptic.

Kinematic analysis of the wind field, temperature field and temperature break for this case indicates both a mechanism for frontal decay and a mechanism for frontal generation. It appears that isotherm spreading in frontolysis is not uniform but rather that the leading edge of the colder air initiates the decay by advancing rapidly into the warm air. Advance by this leading component is such that it separates from the frontal zone at a rate faster than the zone is spreading. Thus frontolysis is enhanced both by the greater homogeneity of temperature between the warm and cold air, caused by the mixing of this lead component, and diabatic effects, and by the spreading
of the remainder of the frontal zone. The mesoscale wind field, rather than microscale (frictional) effects, seems to be the main perpetrator of frontal decay in this case. Generation of the secondary front also appears to be the result of a systematic organization of the wind field. The 75° isotherm remains virtually immobile while the wind in the cold air shifts to northerly components of flow and packs the temperature gradient into a new frontal zone. Relationships between the position of maximum confluence in the wind field and the temperature break give good indications of whether frontogenesis or frontolysis is taking place.

B. Frontogenesis

Frontogenesis was calculated by equation (1a), using techniques described in Section IV B, for the hours 1900, 2200 and 0000. In the process of calculation of the frontogenetical value for each station, values for the vorticity, divergence and resultant deformation were found. Results are given in figs. 9, 10 and 11.

Very large rates of frontogenesis are computed at 1900. The maximum of $+2.5^\circ F/nm/hr$ concentrated between 1D and 2C is one unit greater than Bergh found in any of his three cases. The 1900 temperature-break isochrone lies slightly ahead of the frontogenesis maxima, the highest concentrations of frontogenesis thus being in the cold air. This coincides well with the position of maximum confluence at this hour as discussed above. Concentration of frontogenesis, in contrast to even distribution along the front, holds with previous
results. Some frontolysis is found at 3A, but the wind field at this station and at 3B is questionable for this hour, so the negative value should not be taken in any manner as proof of frontolysis behind the frontal zone. Resultant deformation is transposed southwest along the front in relation to the divergence and the frontogenesis. Convergence is of slightly less magnitude than the resultant deformation, maximum convergence being about \(25 \times 10^{-4}\) units sec\(^{-1}\) and resultant deformation having a maximum of \(30 \times 10^{-4}\) units sec\(^{-1}\). Vorticity is poorly defined in comparison to the other three parameters. Maxima of cyclonic vorticity of \(12 \times 10^{-4}\) units sec\(^{-1}\) found at 4A and 4B coincide with strong precipitation at the time of observation. Anticyclonic vorticity indicated at 3C is a result of the troublesome wind field in the 3B area and may or may not be a true feature.

Strongest temperature decreases are correlated well with frontogenesis and convergence maxima (at 1D for example). Dependence of the frontogenetical value on the temperature gradient is evident from equation (1a); we find that the wind field makes an equally weighted contribution to the computed values as one should expect from physical reasoning. Zero values of the frontogenetical function are found at most stations in the southeastern half of the network because of the homogeneous temperatures in the warm air.

The 2200 temperature-break isochrone lies through the 7C and 4G-5G frontogenesis centers. Divergence, deformation and frontogenesis fields show less organization than they showed three hours earlier. The principle zone of convergence is you found to lie mainly ahead of the
temperature-break isochrone. Magnitude of convergence is somewhat decreased with maximum about $18 \times 10^{-4}$ units sec$^{-1}$. One strong center and one medium center of divergence have appeared in the cold air. Absolute resultant deformation has become more diffuse and maximum values are now about $20 \times 10^{-4}$ units sec$^{-1}$. Deformation and convergence centers do not coincide, the result being that the three frontogenesis centers split the difference between the two kinematic fields in determining their location. What is significant here is that the frontogenesis values are approximately 20% of those found at 1900 although convergence and resultant deformation are still approximately 70% of their earlier values. This underscores the necessary interplay between these two aspects of the wind field and the temperature field that is needed to produce and maintain fronts. Now compare the slight off-set of convergence and deformation with the relation between the divergence maxima and deformation. In the first mentioned, the two fields show fair balance, with the deformation somewhat larger in magnitude. Where divergence is large, deformation is small - here deformation is subservient, not a stronger partner. Frontolysis is found in most of the cold air, the exact zone being between the zero lines. Waviness in the frontogenetical field is not seen in the isotherm field (fig. 8e), indicating that the waviness is caused by the wind field. The vorticity field shows more character than at 1900 and more organization about the front. Cyclonic vorticity at 7C is associated with a wave in the 2200 isochrone and precipitation commences at 7C at 2246. Station 6G never has precipitation and no wave is apparent near it, yet it has cyclinic vorticity
equal or greater than that at 7°C.

Discussion in VA suggests that frontal formation should be taking place at 0200. Values computed for the frontogenetical function are lower than at either previous hour. Deformation, divergence and vorticity are about half what they were at 2200. We can only suggest that during the four hours between 2200 and 0200 values were even lower and that at 0200 they are just beginning to reassert themselves. A well organized, if weak, band of frontogenesis is seen in fig. 11d. Significantly this band lies entirely within the cold air. We are dealing with the development of a weak cold front in a not too conducive temperature field, and we see that development takes place within the cold air. It produces a front which propagates through the remainder of the network producing distinct temperature breaks. The new front features the bend indicated by the frontogenesis field.

In the occurrence of strong convergence $\frac{\partial v}{\partial y}$ is usually large and negative, $\frac{\partial u}{\partial x}$ is generally smaller in magnitude than $\frac{\partial v}{\partial y}$ and either positive or negative. Where strong divergence is present $\frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial x}$ are usually both positive and more nearly equal in value than the convergent case. Resultant deformation magnitude is complicated by the orientation of the axis of dilatation. Vorticity is found to have some fairly large values but does not show distribution of the areal extent of the convergence and deformation and is not as well organized in regard to frontal position. Frontogenetical values are high when the deformation and convergence
approximately coincide, but fall off rapidly when the fields are displaced relative to each other. Further, the placement of the maxima of frontogenesis in relation to the leading edge of the cold air is important and follows the results of placement of maximum confluence on frontal growth and decay. Analysis at 0200 shows that organized frontogenesis, although weak, in the cold air produces a front capable of causing distinct temperature breaks.

Temperature decrease in the warm air definitely reduces the effectiveness of frontal forming processes. Diabatic cooling under scattered to broken cloud cover, is a contributor to the decrease. Advective cooling caused by the remnants of the initial front is the other major contributor. What percentage of the total cooling is contributed by each factor is hard to discern; it appears to us that they participate quite equally.

C. Tower

Tower data of the high quality of that supplied by NSSL for 8 June 1966 has never been used before to study a frontal passage.

Frontal orientation at the tower is 230°-050° and frontal speed is approximately 20 knots. The surface station lies 250 feet to the west northwest of the tower and for the given frontal orientation the travel distance between the tower and the surface station is 222 feet. For frontal speeds of 15, 20 and 30 knots, travel times are respectively 8.8, 6.6 and 4.4 seconds. A temperature drop of 10°F in 12 minutes is felt at the surface station, commencing with
a temperature break at 1909. A second break is seen at 1933 resulting in an 11 degree drop in 8 minutes (fig. 6). Precipitation begins at 2003 and totals 1.41" in 40 minutes. Absolute timing of events at the surface station is not of the accuracy we would desire. For example, the ordinary surface wind readable to the minute indicates that the wind shift occurs at 1907 and the vector shift at 1909, while the fast run, tower type record at the surface station indicates a wind shift about 1906'45" and vector shift about 1908'08". Position of the temperature break, as an indicator of the leading edge of cold air, in relation to the zone of maximum confluence is important, as has been discussed. An order of magnitude difference in absolute timing between the temperature and wind records is therefore a drawback.

Figs. 13a through 1 give the results of our initial analysis, with a time increment of 30 seconds. Using an estimate of 20 knots for frontal speed we get a travel time of 6.6 seconds between the surface station and the tower. Therefore the surface station displacement is not likely to be crucial for a 30-second time increment and we will consider the surface station to be at the tower in this initial analysis. Beginning at 1905'00" and ending at 1910'30" a systematic clockwise wind shift of 140° takes place. The diagrams show the surface station to come around first and the shift follows up the tower levels. Even the most skeptical persons must appreciate this as proof that at as small a time interval as one-half minute, determinable, well organized, three-dimensional wind fields exist in
the vicinity of frontal surfaces. So that we may have a reference to tie our tower discussion to, we assert that the frontal zone is that zone at a given level in which flow shifts from predominantly southerly to predominantly northwesterly. We have previously explained the necessity of doing this and that we fully realize that this definition is not necessarily consistent with our earlier temperature break definition.

Wind is seen to back with height through the frontal zone, which is consistent with thermal wind considerations. With time at a level, the wind turns in a clockwise manner. If the front had any substantial nose structure some level would experience a wind shift ahead of the surface shift. This is clearly not the case. It is just as clear that the front slopes backward towards the cold air between the lowest and highest levels.

We begin our discussion of the results we obtained from our second-by-second analysis of the tower wind records by informing the reader that we examined data taken during steady flow in the warm air and found no evidence of bias in either direction or speed for any level. About 400 seconds of data, centered about the front, were analyzed for each level. Total time covered was 9 minutes, 1904'30" to 1913'30". Henceforth, the component normal to the front and positive into the cold air will be designated as the $V$ component and the component parallel to the front and positive to the right of will be called the $u$ component. Figs. 14 and 15 are the one-second $V$ and $u$ components and figs. 16 and 17 are 10-second averaged $V$ and $u$. All seven levels are referenced to real time with no
allowance made for surface station displacement.

Many measures of flow parameters can be garnered from the data. We list here five that are of interest — greatest one-second wind speed, greatest one-second $\dot{V}$, greatest ten-second averaged $V$, greatest ten-second increase in $V$ and greatest ten-second increase in average $V$ — for each level; all values expressed in knots, negative values indicating flow from cold to warm air:

<table>
<thead>
<tr>
<th>Tower Level</th>
<th>$C$</th>
<th>$\dot{V}$</th>
<th>$\bar{V}$</th>
<th>$\Delta_{10s}V$</th>
<th>$\Delta_{10s}\bar{V}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SFC</td>
<td>122</td>
<td>-22</td>
<td>-20</td>
<td>-10</td>
<td>-5</td>
</tr>
<tr>
<td>1</td>
<td>126</td>
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<tr>
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<td>130</td>
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<td>-10</td>
<td>-9</td>
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<tr>
<td>3</td>
<td>129</td>
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</tr>
<tr>
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<td>-15</td>
<td>-8</td>
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<td>-12</td>
<td>-9</td>
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<td>125</td>
<td>-25</td>
<td>-24</td>
<td>-12</td>
<td>-9</td>
</tr>
</tbody>
</table>

Greatest horizontal wind shear and greatest confluence across the front occurs at intermediate levels. Behind the frontal zone the greatest upward increase in $\dot{V}$ is between the surface and level one, levels two and three show the highest sustained $\bar{V}$. Cyclonic vorticity is greatest at levels three and four. Considering the five quantities presented above as indicative, levels two, three, four and five show the most vigorous wind fields. Two thousand feet is the generally quoted top of the friction layer and theory indicates that in average flow wind speed should increase with height through the friction layer in a logarithmic manner. Our results stand in
contradiction because we are dealing with a frontal surface in which the wind field is adjusted so that maximum winds are found at, in this case, a level below 1500 feet. Kinetic energy is directly proportional to the velocity squared, if the wind field about a front shows a certain preferred height for maximum velocities, a preferred energy distribution in the frontal process is indicated.

When we first looked at our results as they are displayed in figs. 13, 14, 15 and 16 we felt that they were a perfect paradigm of a statistically definable, random flow field. We still feel this to be true and hope that time and interest will lead to machine analysis of this data. For the present we will use our eyes and mind to interpret the implications of these time series.

We seek to identify the shape of the front in the first 1500 feet of the atmosphere, with special attention payed to any possible overrunning nose structure. We can do this by getting the time of occurrence at each level of some identifiable aspect of the wind that connotes the front or by correlating the entire records and reading the log times between levels. Our most successful results come from a combination of the two methods, and they are not as good as we would like. Temporal uncertainty ranges from ten to thirty seconds, this is equivalent to a spatial range of 350 to 1000 feet. Diagrams therefore are not justified, but discussion is. Allowing for seven seconds of travel time between the surface station and the tower we find that level one experiences the front about the same time as the surface station or slightly sooner. In the lowest 146 feet the
frontal surface is vertical or slopes slightly towards the warm air. Between levels one and two the slope is nearly vertical but definitely towards the cold air. No extensive nose structure exists on this frontal surface. Between levels two and three the slope towards the cold air becomes more horizontal. The trend continues through level six. Best estimates of frontal slope are - vertical between surface and level one, $30/10$ between one and two, $4/10$ between two and three, $3/10$ between three and four, $2/10$ between four and five and $1/10$ between five and six. Energy in the form of less turbulent, higher speed flow at intermediate levels is somehow transferred down to the surface levels where it takes the form of more turbulent, lower speed flow. Frictional dissipation is facilitated by increased turbulence, thus our results indicate that microscale dissipation of the front takes place at lowest levels with the aid of high frequency, turbulent flow.

Finally let us address ourselves to the question of how much of a flow component parallel to the front is present - the smoothed traces in figure 17 show that once flow is predominately out of the cold air, only levels three and four have any appreciable flow parallel to the front. Ahead of the zone of maximum confluence all levels have parallel components of flow. If the temperature break lies ahead of the zone of maximum confluence there is flow parallel to the leading edge of the cold air, if the break lies in or behind the zone of maximum confluence there is no important parallel flow.
D. Rawinsonde

Little work was done with the rawinsonde data. Thirty temperature soundings from four stations within the surface network grid were plotted on Stuve diagrams. The few whose release times were after the frontal passage at the surface and ascended through the frontal zone, had dewpoint and wind plotted. Fig. 2 shows the sounding of the rawinsonde released at Ft. Sill at 2252. The temperature break for 6°C was at 2123 with a 7°F temperature drop in 12 minutes. Inversions in the temperature and dewpoint soundings at 960 mb represents the classical cold-frontal configuration. Through the inversion the wind shifts from NNE to NE and does not become southerly until 850 mb. Only five releases among the thirty ascended through the frontal zone.
VI. CONCLUSIONS

We have investigated the complex cold front passing through the NSSL Beta-Network 8-9 June 1966. We found that a vigorous cold front entered the network between 1800 CST and 1900 CST on 8 June 1967. In the five hours it took the front to cross two-thirds of the network, dissipation took place to the extent that the front could no longer be called a substantial surface. Later a new substantial surface formed and moved through the remainder of the network. We noted the uniqueness of having a case where both dissipation and regeneration take place within the mesoscale network. The position of the zone of maximum confluence in the wind field, in relation to the leading edge of the cold air, was found to indicate future frontal intensity. If the maximum confluence was found in the cold air, the front would either intensify or continue at the same strength. Maximum confluence in the warm air was found to precede frontal weakening. Progressive frontal weakening was seen to occur as the confluent zone got increasingly ahead of the cold air.

Calculation of the frontogenesis function was done for three hours of data. General agreement was found between these quantitative results and earlier qualitative ones. The necessary coincidence between the fields of divergence and resultant deformation needed for high values of frontogenesis was underscored. No huge rates of frontolysis were found, leading us to believe that diabatic effects and turbulent dissipation contributed heavily to overall frontolysis.
We wish to assert that we feel there is an inherent, systematic relationship between the divergence and deformation fields. It appears to us, from studying the relative positions and intensities of convergence and deformation maxima, that the deformation field acts to distribute along the front, large, concentrated values of frontogenesis produced by convergence. The frontogenesis equation provides a quantitative measure of what may be clear from strictly qualitative analysis. If a quantitative measurement is not needed, we advise against performing the calculations, unless time is not of importance or a machine analysis is available.

Results from the Meteorological Tower proved most promising in terms of future enlightenment. We hope that the opportunities provided by temperature data available beginning late in 1966 will not be overlooked. Using only wind data, we found probably frontal shape in the lowest 1500 feet. No significant nose structure was present. Factors of wind intensity were strongest at intermediate levels. Turbulence appeared to be greatest at the surface. Various aspects of the wind field were noted; all are considered important because of the fine time scale of the data. Cohesive frontal characteristics were shown to exist and be identifiable to at least thirty seconds. Statistical analysis of the data is suggested.

We believe that future investigation of fronts can be worthwhile and significant if proper analytical techniques are used and worthy objectives set. Two worthy objectives are a mathematical model, and a program to understand and explain the energy balances
in the vicinity of frontal surfaces. Both are machine oriented projects. Probability of success in the former endeavor has been increased by the observational studies done in the past few years. We feel that a good way to attack the latter is to use wind and temperature data recorded at the WKY Tower and subject it to various statistical, time-series analyses.
Establishment of the invariance of horizontal divergence, vorticity and resultant deformation with axis rotation.

Expressing the two dimensional, horizontal, Cartesian coordinate, instantaneous velocity components by a Taylor's series expansions about the origin:

\[
U = U_o + \left( \frac{\partial U}{\partial x} \right)_o x + \left( \frac{\partial U}{\partial y} \right)_o y + \frac{1}{2} \left( \frac{\partial^2 U}{\partial x^2} \right)_o x^2 + \frac{1}{2} \left( \frac{\partial^2 U}{\partial y^2} \right)_o y^2 + \cdots \quad (1.1)
\]

\[
V = V_o + \left( \frac{\partial V}{\partial x} \right)_o x + \left( \frac{\partial V}{\partial y} \right)_o y + \frac{1}{2} \left( \frac{\partial^2 V}{\partial x^2} \right)_o x^2 + \frac{1}{2} \left( \frac{\partial^2 V}{\partial y^2} \right)_o y^2 + \cdots \quad (1.2)
\]

The subscript \(o\) designating evaluation at the origin by considering only a sufficiently small area about the origin the expansions may be linearized by dropping higher order terms.

\[
U \approx U_o + \left( \frac{\partial U}{\partial x} \right)_o x + \left( \frac{\partial U}{\partial y} \right)_o y \quad (1.3)
\]

\[
V \approx V_o + \left( \frac{\partial V}{\partial x} \right)_o x + \left( \frac{\partial V}{\partial y} \right)_o y \quad (1.4)
\]

What derivatives or combination of derivatives do not vary when the coordinate system is rotated?

The original \(x, y\) coordinate system is rotated through a fixed arbitrary angle \(\gamma\) forming the rotated system \(x', y'\)
\[
X' = x \cos \theta + y \sin \theta \\
y' = -x \sin \theta + y \cos \theta \\
u' = u \cos \theta + v \sin \theta \\
v' = -u \sin \theta + v \cos \theta \\
u = u' \cos \theta - v' \sin \theta \\
v = u' \sin \theta + v' \cos \theta \\
\]

\[
\frac{\partial A}{\partial x} = \frac{\partial x'}{\partial x} \frac{\partial A}{\partial x'} + \frac{\partial y'}{\partial x} \frac{\partial A}{\partial y'} = \cos \theta \frac{\partial A}{\partial x'} - \sin \theta \frac{\partial A}{\partial y'} \\
\frac{\partial A}{\partial y} = \frac{\partial x'}{\partial y} \frac{\partial A}{\partial x'} + \frac{\partial y'}{\partial y} \frac{\partial A}{\partial y'} = \sin \theta \frac{\partial A}{\partial x'} + \cos \theta \frac{\partial A}{\partial y'} \\
\]

Now

\[
\frac{\partial u}{\partial x} = \cos \theta \left[ \frac{\partial u'}{\partial x} \cos \theta - \frac{\partial v'}{\partial x} \sin \theta \right] - \sin \theta \left[ \frac{\partial u'}{\partial y} \cos \theta - \frac{\partial v'}{\partial y} \sin \theta \right] \quad (1.5)
\]

\[
\frac{\partial u}{\partial y} = \sin \theta \left[ \frac{\partial u'}{\partial x} \cos \theta - \frac{\partial v'}{\partial x} \sin \theta \right] + \cos \theta \left[ \frac{\partial u'}{\partial y} \cos \theta - \frac{\partial v'}{\partial y} \sin \theta \right] \quad (1.6)
\]

\[
\frac{\partial v}{\partial x} = \cos \theta \left[ \frac{\partial u'}{\partial y} \sin \theta + \frac{\partial v'}{\partial y} \cos \theta \right] - \sin \theta \left[ \frac{\partial u'}{\partial y} \sin \theta + \frac{\partial v'}{\partial y} \cos \theta \right] \quad (1.7)
\]

\[
\frac{\partial v}{\partial y} = \sin \theta \left[ \frac{\partial u'}{\partial y} \sin \theta + \frac{\partial v'}{\partial y} \cos \theta \right] + \cos \theta \left[ \frac{\partial u'}{\partial y} \sin \theta + \frac{\partial v'}{\partial y} \cos \theta \right] \quad (1.8)
\]
to get divergence, add (1.8) to (1.5)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'}$$  \hspace{1cm} (1.9)

to get vorticity subtract (1.6) from (1.7)

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial v'}{\partial x'} - \frac{\partial u'}{\partial y'}$$  \hspace{1cm} (1.10)

both are seen to be invariant under rotation of coordinate axis.

Now the shearing deformation is

$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = \sin\theta \left[ \frac{\partial u'}{\partial x'} - \frac{\partial v'}{\partial y'} \right] + \cos\theta \left[ \frac{\partial v'}{\partial x'} + \frac{\partial u'}{\partial y'} \right]$$  \hspace{1cm} (1.11)

and the stretching deformation

$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = \cos\theta \left[ \frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} \right] - \sin\theta \left[ \frac{\partial v'}{\partial x'} + \frac{\partial u'}{\partial y'} \right]$$  \hspace{1cm} (1.12)

neither of which is invariant through rotation, but by summing the squares:

$$\left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right)^2 = \left( \frac{\partial v'}{\partial x'} + \frac{\partial u'}{\partial y'} \right)^2 + \left( \frac{\partial u'}{\partial x'} - \frac{\partial v'}{\partial y'} \right)^2$$

we show invariability.

The magnitude of the resultant deformation may be defined as...
which is itself invariant with rotation.

Further it may be shown that the angle \( \alpha \) between the axis of dilatation of the deformation field and whatever coordinate axis orientation we may seek to use can be found from \[
\tan 2\alpha = \frac{\partial v/\partial x + \partial u/\partial y}{\partial u/\partial x + \partial v/\partial y}
\]
Using the identity \( \sec 2\alpha = \sqrt{\tan^2 2\alpha + 1} \)
in (1.13)

We get

\[
\left| \text{def}_h \mathbf{C} \right| = \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) \sec 2\alpha = \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \csc 2\alpha
\] (1.14)
APPENDIX II

The frontogenesis equation

The frontogenetical function first introduced by Petterssen:

\[ F = \frac{d}{dt} \left| \nabla T \right| \]  \hspace{1cm} \text{where} \ \nabla T = \frac{\partial T}{\partial x} \ i + \frac{\partial T}{\partial y} \ j + \frac{\partial T}{\partial z} \ k \]  

The operator \( \frac{d}{dt} \) is a process following a specific air parcel, and represents temperature as a scalar quantity. A value of \( F > 0 \) gives frontogenesis, \( F < 0 \) frontolysis.

\[ F = \frac{d}{dt} \left| \nabla T \right| = \frac{1}{2 |\nabla T|} \frac{d(\nabla T \cdot \nabla T)}{dt} = \mathbf{N}_T \cdot \frac{d(\nabla T)}{dt} \]  \hspace{1cm} \text{where} \ (2)

\( \mathbf{N}_T \) is a unit vector in the direction of \( \nabla T \). Now expanding

\[ \frac{d}{dt} (\nabla T) = \frac{\partial (\nabla T)}{\partial t} + \mathbf{u} \cdot \nabla (\nabla T) + \mathbf{v} \cdot \nabla (\nabla T) + \mathbf{w} \cdot \nabla (\nabla T) \]

\[ \frac{d}{dt} (\nabla T) = \nabla \left( \frac{\partial T}{\partial t} \right) + \nabla \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) \]

\[ \frac{d}{dt} (\nabla T) = \nabla \left( \frac{d T}{d t} \right) - \left( \frac{\partial T}{\partial x} \nabla u + \frac{\partial T}{\partial y} \nabla v + \frac{\partial T}{\partial z} \nabla w \right) \]  \hspace{1cm} \text{(3)}

Using (2) and (3)

\[ F = \mathbf{N}_T \cdot \nabla \left( \frac{d T}{d t} \right) - \mathbf{N}_T \cdot \left( \frac{\partial T}{\partial x} \nabla u + \frac{\partial T}{\partial y} \nabla v + \frac{\partial T}{\partial z} \nabla w \right) \]  \hspace{1cm} \text{(4)}
the first term on the right represents frontogenesis through diabatic processes.

Using a Taylor Series expansion and restricting ourselves to a sufficiently small area around the point in question, thereby eliminating higher order terms, we may write the linear horizontal field:

\[
U \approx U_0 - \frac{1}{2} \left( \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \right)_0 y + \frac{1}{2} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)_0 x + \frac{1}{2} \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right)_0 x + \frac{1}{2} \left( \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} \right)_0 y
\]

\[
V \approx V_0 + \frac{1}{2} \left( \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \right)_0 x + \frac{1}{2} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)_0 y - \frac{1}{2} \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right)_0 y + \frac{1}{2} \left( \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} \right)_0 x
\]

Introducing the notation

\begin{align*}
2a &= \text{stretching deformation} = \left( \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \right) \\
2b &= \text{divergence} = \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \\
2c &= \text{vorticity} = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \\
2d &= \text{shearing deformation} = \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)
\end{align*}

we get

\[
U = (a + b)x + (d - c)y
\]

\[
V = (d + c)x + (b - a)y
\]

where \( U_0 \) and \( V_0 \) vanish with a parallel transition.
We now proceed with the derivation of the frontogenetical equation in the horizontal wind field.

We define $\alpha$ as the angle between the x-axis and the axis of dilatation, $\beta$ as the angle between the axis of dilatation and the isotherms and $\gamma$ as the sum of $\alpha$ and $\beta$.

\[
F = D - N_T \cdot (\frac{\partial T}{\partial x} v_x + \frac{\partial T}{\partial y} v_y), \quad D \text{ is the diabatic term}
\]

\[
\frac{\partial T}{\partial x} = |\nabla T| \sin \gamma, \quad \frac{\partial T}{\partial y} = -|\nabla T| \cos \gamma
\]

\[
F = D - (\sin \gamma \xi - \cos \gamma \eta) \cdot |\nabla T| \left\{ \sin \gamma \left[ (a+b) \xi + (d-c) \eta \right] - \cos \gamma \left[ (d+c) \xi + (b-a) \eta \right] \right\}
\]

\[
= D - |\nabla T| \left[ \sin^2 \gamma (a+b) + \cos^2 \gamma (b-a) - \sin \gamma \cos \gamma (d+c) - \sin \gamma \cos \gamma (d-c) \right]
\]

\[
= D - |\nabla T| \left[ b - a (\cos^2 \gamma - \sin^2 \gamma) - 2d \sin \gamma \cos \gamma \right]
\]

\[
= D - |\nabla T| \left[ b - a \cos 2\gamma - 2d \sin \gamma \cos \gamma \right]
\]

\[
= D + |\nabla T| \left[ a \cos 2\gamma + 2d \sin \gamma \cos \gamma - b \right]
\]

\[
\sin \gamma \cos \gamma = \frac{1}{2} \sin 2\gamma
\]

\[
= D + |\nabla T| \left[ a \cos 2\gamma + d \sin 2\gamma - b \right]
\]

\[
= D + |\nabla T| \left[ a \cos 2\gamma + a \tan \alpha \sin 2\gamma - b \right] - |\nabla T| b
\]
\[ F = D + |VT| \left[ \cos 2\beta \sec 2\alpha \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) - \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] \]
Figure 1

Schematic Diagram of NSSF-WKY Tower

Lat. 35°34.2' N
Long. 97°29.4' W
Azimuth and range from NRO AZRAN from NRO = 357°/19.5 nm
Base of tower = 1147 MSL
Surface wind instruments 250 ft WNW of tower, 23 ft above round (40 ft above base of tower, or 1187 MSL

Tower Levels

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<thead>
<tr>
<th>Feet Above Ground</th>
<th>Ft. MSL</th>
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<tr>
<td>1</td>
<td>146</td>
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<tr>
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<td>296</td>
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<tr>
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<td>4</td>
<td>873.5</td>
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<td>5</td>
<td>1166</td>
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<tr>
<td>6</td>
<td>1458.5</td>
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</tbody>
</table>
Figure 2

Temperature, Dewpoint and Wind from Sounding Released from Ft. Sill, Okla. at 2252 CST 8 June 1966.
Figure 3

a. Function $q(s)$

b. First derivative of $q(s)$

c. Second derivative of $q(s)$

A second-order discontinuity in $q$ exists.
Equal Horizontal and Vertical Scale
Depiction of Frontal Slopes.
Figure 5

a. 0000Z 9 June 1966 (1800 CST 8 June 1966)
   Weather Bureau Surface Map

b. 1200Z 9 June 1966 (0600 CST 9 June 1966)
   Weather Bureau Surface Map

Beta Network indicated by hatched area.
Figure 6

Top - Station ID, Waterloo, temperature trace from 1300 to 2300 8 June 1966

Bottom - WKY surface station temperature trace from 1300 to 2300 8 June 1966
Figure 7

Isochrones of Temperature Break. Hourly Times indicated from 1900 CST 8 June 1966 to 0600 CST 9 June 1966.
Wind direction and speed is given for each station and isotherms as drawn.
Figure 9

1900 CST 8 June 1966

a. Vorticity in units $\times 10^{-4}$ sec$^{-1}$

b. Absolute magnitude of the resultant deformation in units $\times 10^{-4}$ sec$^{-1}$

c. Divergence in units $\times 10^{-4}$ sec$^{-1}$

d. Frontogenetical function in units °F/°F/°F
Figure 10

2200 CST 8 June 1966

Same units as fig. 9
Figure 11

0200 CST 9 June 1966

Same units as fig. 9
Figure 12

Isolines of 12-minute temperature decreases associated with temperature breaks.
Figure 13

Polar vector plots of wind at surface and tower levels for every 30 seconds. Time is indicated by hour, minute and second. Levels are identified by numbers.
Figure 14

One-second plots of wind component normal to front, (v) out of cold air, in knots. Negative values indicate flow from cold air towards warm air.
One-second plots of wind component parallel to front (u) in knots. Negative values indicate flow with cold air on right and warm air on left.
Figure 16

Ten-second averaged v-component in knots.
Figure 17

Ten-second averaged u-component in knots.
BIBLIOGRAPHY


SOME REFERENCES ON ANALYSIS OF RANDOM DATA


SOME REFERENCES ON TURBULENCE


