HYDRAULIC CONTROL BY A

WIDE WEIR IN A ROTATING FLUID

by

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HYDRAULIC CONTROL BY A WIDE WEIR IN A ROTATING FLUID

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Abstract

The basic concepts of steady, inviscid, free-surface, open-channel fluid flow are first reviewed and then extended to include effects of the rotation of the reference frame. Specifically, flow control by a very wide, very deep weir in a rotating fluid is examined. Theoretical calculations are based on conservation of the Bernoulli function along streamlines whose curvature is determined by the vorticity constraint, together with standard hydraulic flow-control assumptions. For fixed volume flux Q, it is claimed that the "potential head" of the upstream basin, or depth of the fluid layer above the weir crest, must increase as Ω^2 for a steady-state solution to exist (Ω = angular frame rotation rate).

Experiments that tend to buttress the theoretical predictions are described. Upstream heights as a function of Ω are measured with a micrometer in a recirculating tank system fitted with a large flow barrier and mounted on a variable-speed rotating turntable. For low rotation rates, the upstream height can be fairly well fitted by an Ω^2 relation. There is also an indication that a new regime of hydraulic control comes into play when experimental parameters are such that the "wide-weir" assumptions no longer hold.

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I. On the use of open-channel, free-surface hydraulics

A considerable literature has been built up over the past two hundred years about flow control in open channels with free surfaces. One of the major concerns of the hydraulic engineer is to design weirs, sluice gates, locks, etc. so that, for given fluid and channel parameters, the free-surface flow is "controlled" in the sense that "topographic" barriers act to determine the possible flow regimes. The hydraulicist's concept of flow control has been applied to problems of geophysical interest, such as airflow over mountains (Long (1954 and Houghton and Kasahara (1968)), but the formulations generally ignore effects of the earth's rotation. The development presented here includes frame rotation and momentum advection in the equations for flow over a wide, deep weir. The introduction of hydraulic assumptions in a rotating fluid cannot at this time be rigorously defended theoretically, but using them, we can make a first attempt at a viable solution for the rotating weir, and the predicted results can be tested experimentally.

To introduce the hydraulic concepts used in the formulation of the rotating-weir problem, we review a standard problem in nonrotating flow over an obstacle, following Long (1954) and Rouse (1961).

Figure 1 is a sketch of the geometry cross-section and flow pattern to be considered. A symmetric obstacle of maximum (crest) height b_c and half-width x_b extends infinitely in the plane perpendicular to the page. (The solution reduces to an effectively one-dimensional flow pattern derivation.) The bottom of the water column is at $z = z_B$, where

$$z_B = \begin{cases} b(x) & , \quad |x| \leq |x_b| \\ \\ 0 & , \quad |x| > |x_b| \end{cases}$$

(The coordinate origin is taken with $x = 0$ at the crest.) Far upstream, a fluid column of uniform depth h_o and uniform velocity $\underset{\sim}{u} = (u_o, 0, 0)$ approaches the barrier. $h(x)$ defines the height of the fluid column above the bottom; $h(x) + b(x)$ defines the free-surface profile. All dependent variables are functions of x alone.

We want to find a set of values (u_o, h_o, h_c) (where h_c = fluid-layer thickness at the crest) such that the steady-state, inviscid Navier-Stokes equations for a homogeneous fluid are satisfied, *given the volume flux* $Q = uh$. Assume that the vertical acceleration equation reduces to a hydrostatic balance (valid if the radius of curvature of the obstacle is large with respect to h_o). Then the horizontal Navier-Stokes equation reads simply
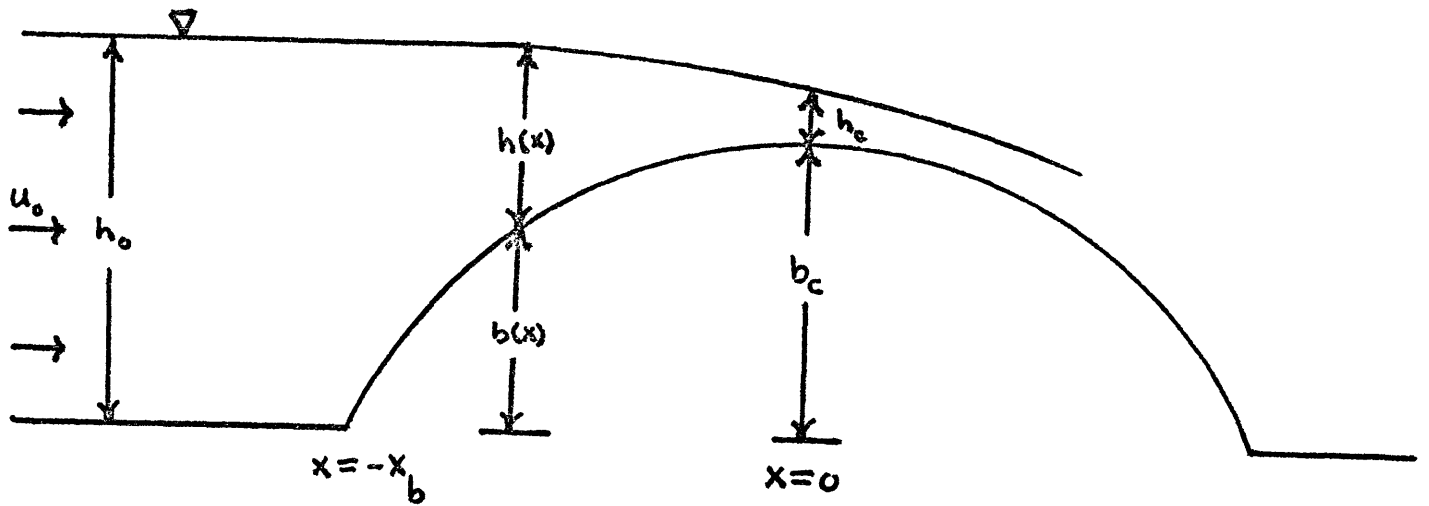
Figure 1. A sketch of the obstacle geometry and flow pattern.

$$\frac{Du}{Dt} = - g \frac{d}{dx} (h+b).$$

For steady, irrotational conditions, this equation reduces further to

$$\frac{d}{dx} [1/2 \ u^2 + g(h+b)] = 0$$

or

$$\frac{u^2}{2g} + h + b = F = \text{const.}$$

The mass balance equation is simply

$$uh = u_o h_o = Q = \text{const.}$$

F is the Bernoulli head function:

$$F = \frac{u_o^2}{2g} + h_o \ .$$

Then

$$\frac{u^2}{2g} + h + b = \frac{u_o^2}{2g} + h_o.$$

We now define the specific energy e:

$$e = \frac{u^2}{2g} + h$$

or

$$e + b = e_o = \frac{u_o^2}{2g} + h_o,$$

and, using the relation $Q = uh$, we obtain

$$e = \frac{Q^2}{2gh^2} + h.$$

Differentiating this last equation for constant Q,

$$\frac{de}{dx} = (1 - \frac{Q^2}{gh^3}) \frac{dh}{dx} = - \frac{db}{dx} .$$

At the crest of the obstacle, $\frac{db}{dx} = 0$, so that *at that point*, we require that either $Q^2/gh_c^3 = 1$ (or, equivalently, that $u_c^2 = gh_c$) or $\frac{dh}{dx}\Big|_{x=o} = 0$. In the latter case h is either at a minimum or a maximum at the crest; the former possibility represents truly "subcritical" flow, with the magnitude of the fluid velocity everywhere smaller than the corresponding shallow-water wave phase speed. The free surface dips symmetrically over the obstacle as the fluid draws on its potential energy to mount the barrier. Conversely, if h is at a maximum over the crest, the flow is truly "super-critical." All shallow-water waves are swept downstream be-cause the magnitude of the fluid velocity exceeds the wave phase speed. In both cases, the barrier is unimportant to

upstream or downstream flow patterns: the obstacle has a
purely localized influence and cannot be considered a flow
control. In other words, given Q, we do not know whether
a steady-state flow solution exists, and even if one does,
we do not have enough information to deduce one or the
other of u_o and h_o. Experimentally, the upstream Froude
number $u_o/\sqrt{gh_o}$ must be specified. The Froude number and
volume-flux rate then determine the kind of flow over the
barrier and the values of u_o and h_o.

When $u_c^2 = gh_c$, the flow is termed "critical." Here the
obstacle acts to control upstream conditions, since, with
given Q, h_o and u_o are both given when b_c is known. Waves
cannot propogate upstream through the crest point because
the magnitude of the fluid velocity matches the phase speed.
That the flow be critical at the crest is a necessary condi-
tion for the steady-state solution if $\left.\dfrac{dh}{dx}\right|_{x=o} \neq 0$.

The critical condition can be cast in other forms (see
Rouse (1961)). For our purposes, it is useful to note that
manipulation of the Bernoulli-head and continuity equations
gives

$$\frac{1}{2} Q^2 \left[1 - \left(\frac{h_o}{h(x)}\right)\right]^2 = gh_o^2 [h(x) + b(x) - h_o].$$

Differentiating implicitly with respect to h and setting
$\left.\dfrac{dQ}{dh}\right|_{h_*} = 0$ gives $h_* = h_c$, the critical depth, so that maximiza-
tion of Q for a given fluid-column thickness at the control

point is an alternate statement of the critical condition.

One interesting flow-control device familiar to hydraulic engineers is the deep "broad-crested" weir. If we take the obstacle height to be almost as high as the (large) upstream depth, then $u_o^2/2g \ll h_o$ (the velocity head can be made vanishingly small for given Q), and

$$u_c^2/2g + h_c + b_c = h_o \; .$$

Since $u_c^2 = gh_c$, we have $h_c = \frac{2}{3}(h_o - b_c)$. Also, $Q = uh = u_c h_c = g^{1/2}[2/3(h_o - b_c)]^{3/2}$. These are the well-known formulas for the broad-crested weir, which is an ideal flow-control device that cannot exist (see the comments of Rouse (1961), pp. 319 ff), but in practice conditions close to ideal can be obtained, where the flow in the region of the critical point is very nearly rectilinear. (Note, however, that we must always have $\frac{dh}{dx} \neq 0$, since the term represents the loss of "potential head" that is used by the fluid to gain kinetic energy and to overcome resistance to flow.)

In what follows, these simple principles are extended to flow over a barrier in a rotating reference frame. The Bernoulli-head function is conserved along streamlines whose curvature is determined by the vorticity constraint. If the barrier is very wide, mass conservation in the x-direction must hold. Hydraulic assumptions are used to close the set of equations. These theoretical predictions are then tested experimentally.

II. Theoretical development

Calculations of the effect of frame rotation on a weir flow depend on assertions that cannot be theoretically verified. These assertions will be clearly denoted in the following development. All the uncertainties and imprecisions of nonrotating hydraulics (i.e., free-surface effects, the "broad-crested" assumption, etc.) are injected, along with some problems in simplifying the vorticity equations. These inadequacies are tested in laboratory experiments, which in this case lend credence to the theoretical ideas and the assumptions that lie behind them.

To develop the equations for a rotating weir, we retain the geometry and notation of Figure 1, and, after the theory has been worked out, the limit where $u_o^2/2g \ll h_o$ and $b_c \sim O(h_o)$ (b_c, h_o both very large) is deduced. These limiting conditions are sufficient to define a weir. Furthermore, the weir extends infinitely in the y-direction, making all dependent variables independent of y: it is in this sense that we speak of a wide weir.

In vector form, the momentum equations for a frictionless homogeneous fluid in a rotating reference frame with gravity vertical are

$$\frac{D\underset{\sim}{u}}{Dt} + 2\underset{\sim}{\Omega} \times \underset{\sim}{u} = -\frac{1}{\rho}\nabla p - g\underset{\sim}{k}$$

where

$\underset{\sim}{u}$ = velocity vector

and

$\underset{\sim}{\Omega}$ = rotation-rate vector for the frame.

(The centripetal acceleration $\underset{\sim}{\Omega} \times (\underset{\sim}{\Omega} \times \underset{\sim}{r})$ has been absorbed into the pressure-gradient term.)

We are interested in the steady-state equations, since they are much more tractable than the unsteady equations; for steady flow, streamlines and trajectory paths coincide, which will prove to be very useful. Thus, after rearranging terms, the steady momentum equations become

$$(2\underset{\sim}{\Omega} + \underset{\sim}{\omega}) \times \underset{\sim}{u} = - \nabla(p/\rho + gz + 1/2\, \underset{\sim}{u} \cdot \underset{\sim}{u})$$

where $\underset{\sim}{\omega} = \nabla \times \underset{\sim}{u}$ = relative vorticity vector as observed in the rotating reference frame.

It is most convenient and instructive at this point to decompose this vector equation into its components in "natural streamline" coordinates. The sketch below demonstrates the decomposition:

The orthonormal vector triad is defined so that

$$\underset{\sim}{u} = |\underset{\sim}{u}|\underset{\sim}{\sigma} \qquad \text{and} \qquad \underset{\sim}{\sigma} \times \underset{\sim}{\nu}_1 = \underset{\sim}{\nu}_2 \quad .$$

Along a streamline, the steady-state momentum equation is

$$\frac{\partial}{\partial s}(p/\rho + gz + 1/2\ \underset{\sim}{u} \cdot \underset{\sim}{u}) = 0$$

or

$$F = p/\rho + gz + 1/2\left(\underset{\sim}{u} \cdot \underset{\sim}{u}\right) = F(\psi).$$

$F(\psi)$, the Bernoulli potential function, is conserved along streamlines.

$F(\psi)$ will vary across streamlines. Let us assume that $\underset{\sim}{\Omega} = \Omega\underset{\sim}{k}$. In general, then, there will be Corioles accelerations in both n_1 and n_2 directions. We obtain

$$\frac{\partial F}{\partial n_1} = (2\underset{\sim}{\Omega} + \underset{\sim}{\omega})_{n_2} |\underset{\sim}{u}|$$

and

$$\frac{\partial F}{\partial n_2} = -\ (2\underset{\sim}{\Omega} + \underset{\sim}{\omega})_{n_1} |\underset{\sim}{u}| \quad .$$

An important assumption to be made is that
$(2\Omega + \omega)_{n_1} = 0$. This sets the n_2- and z-directions parallel.
We must assert that $\omega_{n_1} \sim 0$ (no horizontal vorticity). In
that event,

$$\frac{\partial F}{\partial n_2} = \frac{\partial F}{\partial z} = 0.$$

Asserting, further, no shear of horizontal currents and
negligible vertical velocity, the n_2-momentum equation reduces
to a vertical hydrostatic balance, and the natural coordinates
s and n_1 can be defined in terms of a continuously changing
coordinate-axis rotation in the xy-plane as we move along a
streamline.

When the hydrostatic balance $p = \rho g(h + b - z)$ is sub-
tracted from F, we have

$$F = g(h + b) + 1/2\left(\underset{\sim}{u} \cdot \underset{\sim}{u}\right)$$

where h + b = profile of the free surface.

The n_1-momentum equation becomes

$$\frac{\partial F}{\partial n_1} = (2\Omega + \omega_z)\left|\underset{\sim}{u}\right|$$

where ω_z = relative vertical vorticity. This expression
will be examined more later.

The basic flow will then be in horizontal planes. Even though there is necessarily a vertical excursion of stream-lines as the fluid is driven up the side of the barrier, the rise is assumed to be sufficiently gentle so that the flow is almost rectilinear, and the flow patterns are effectively two-dimensional.

For the rotating weir we introduce the potential-vorticity equation. Taking the vector momentum equations and using the curl operation, we get

$$\frac{\partial \underset{\sim}{\omega}}{\partial t} + (\underset{\sim}{u} \cdot \nabla)(2\Omega \underset{\sim}{k} + \underset{\sim}{\omega}) - [(2\Omega \underset{\sim}{k} + \underset{\sim}{\omega}) \cdot \nabla]\underset{\sim}{u} = 0$$

or

$$\frac{D}{Dt}(2\Omega \underset{\sim}{k} + \underset{\sim}{\omega}) - [(2\Omega \underset{\sim}{k} + \underset{\sim}{\omega}) \cdot \nabla]\underset{\sim}{u} = 0.$$

The first term of the last equation represents changes in the vorticity of a fluid parcel as it moves along its trajectory, and the second term represents vorticity changes through stretching and tilting of the vortex lines. We *assume,* in consonance with the "two-dimensional" assumption for the momentum equations, that the only significant contribution from the stretching and tilting terms is the vertical vortex stretching we use to derive the potential vorticity equation. Thus we assume negligible horizontal vorticity and no vertical shear of horizontal currents (to prevent horizontal vorticity production brought about by vortex-line tilting). This is

equivalent to assuming that the fluid moves up the side of the barrier in vertically uniform columns.

With the assumption of zero horizontal vorticity, the vorticity equations reduce to the single scalar equation

$$\frac{D}{Dt}(2\Omega + \omega) = (2\Omega + \omega)\frac{\partial w}{\partial z}$$

where ω = relative vertical vorticity. Integrating over the depth of the fluid,

$$h\frac{D}{Dt}(2\Omega + \omega) = (2\Omega + \omega)\frac{Dh}{Dt}$$

or

$$\frac{D}{Dt}\left(\frac{2\Omega + \omega}{h}\right) = 0.$$

Along fluid-parcel trajectories, the quantity $\frac{2\Omega + \omega}{h}$ is conserved. This is the well-known potential vorticity theorem. In steady flow, particle trajectories trace paths coincident with the streamlines, so that we can combine the potential-vorticity and Bernoulli-head conservations as parts of a system of equations to be solved (at least in a limiting case to be described below).

Now to formulate the wide-weir problem: Returning to the geometry and notation of Figure 1, we must have, in the wide limit for any $x > -x_b$ up to the crest, that

$$\frac{2\Omega + \omega}{h(x)} = \frac{2\Omega}{h_o} \quad .$$

We have assumed negligible relative vertical vorticity $k|\underset{\sim}{u}| - \frac{\partial|\underset{\sim}{u}|}{\partial n_1}$ upstream. In rectangular coordinates with $\frac{\partial}{\partial y} \equiv 0$, the vorticity equation becomes

$$\frac{2\Omega + \frac{dv}{dx}}{h(x)} = \frac{2\Omega}{h_o} \quad ,$$

which can be immediately integrated to give

$$v = v(-x_b) + 2\Omega \int_{-x_b}^{x} \left(\frac{h(x')}{h_o} - 1 \right) dx'.$$

Define

$$A(x) = \int_{-x_b}^{x} \left(\frac{h(x')}{h_o} - 1 \right) dx'.$$

In general, $A(x)$ will be difficult to evaluate, since $h(x)$ is one of the unknowns to be solved for. (This complication results from having a free surface.) But in the "deep" limit, A can be simplified. We set

$$h_o = h(x) + b(x) + \delta(x)$$

where $\delta(x)$ = change in the free-surface elevation from $z = h_o$. We assume that $\delta(x) \ll b(x)$; then $h_o \approx h(x) + b(x)$, and

$$A(x) = \int_{-x_b}^{x} \left(\frac{h_o - b(x')}{h_o} - 1 \right) dx'$$

$$= - \frac{1}{h_o} \int_{-x_b}^{x} b(x') dx' \underset{\sim}{} - \frac{1}{b_c} \int_{-x_b}^{x} b(x') dx'$$

A(0) becomes a measure of the centroid of the obstacle, and if $\underset{\sim}{u} = 0$ at $x = -x_b$,

$$v = - \frac{2\Omega}{b_c} \int_{-x_b}^{x} b(x') dx'.$$

The physical interpretation is that the fluid, as it is driven up the side of the obstacle, develops a relative vorticity (or spin) opposite in sign to that of the frame rotation, in the absence of external torques. A(0) will be crucially dependent on the length of the obstacle from base to crest.

The vorticity constraint also determines streamline curvature $\frac{ds}{dx}$:

$$\frac{ds}{dx} = \tan \phi(x) = \frac{v}{u} = \frac{2\Omega}{Q} h(x) A(x)$$

or

$$\phi(x) = \tan^{-1} \left(\frac{2\Omega}{Q} h(x) A(x) \right).$$

Along streamlines, the Bernoulli constant is conserved. With the assumption of vertical hydrostatic balance at every point over the barrier, we have

$$u(x)^2 + v(x)^2 + 2g(h(x) + b(x)) = F_o.$$

In the wide-weir formulation, it is, however, incorrect to assume $F_o = u_o^2 + 2g\, h_o$. To see why we return to the cross-stream Bernoulli equation

$$\frac{\partial F}{\partial n_1} = (2\Omega + \omega)|\underset{\sim}{u}| \; .$$

In slow, steady upstream flow with $2\Omega \gg \omega$, we have

$$2\Omega|\underset{\sim}{u}_o| = g \frac{\partial h_o}{\partial n_1} ,$$

a statement of geostrophic balance. In an infinite plane, though, it is not physically possible to have $\frac{\partial h_o}{\partial n_1} \neq 0$. This difficulty is partially overcome by allowing $|\underset{\sim}{u}_o|$ to become arbitrarily small (through setting h_o large enough for given Q). In other words, a truly motionless upstream in a rotating fluid is impossible, but by making the weir deep enough, we can approach the stagnant limit. Only in this limit can a constant Bernoulli upstream head for all y be realized. Streamlines do not exist upstream; all fluid flowing over the barrier is drawn from a "reservoir." (These arguments are similar in spirit to those reported by Charney (1955) and Whitehead et al. (1974) about necessary relations between "upstream" potential vorticity and Bernoulli head.)

Proceeding with the above cautionary notes in mind, we obtain *along streamlines*

$$u(x)^2 + 4\Omega^2 A(x)^2 + 2g(h(x) + b(x)) = 2gh_o,$$

or

$$u(x) = [2g(h_o - h(x) - b(x)) - 4\Omega^2 A(x)^2]^{1/2}.$$

Then

$$Q = h(x)u(x) = h(x)[2g(h_o - h(x) - b(x)) - 4\Omega^2 A(x)^2]^{1/2}.$$

Now we apply the hydraulic assumption

$$\left.\frac{\partial Q}{\partial h}\right|_{h_{cr}} = 0 \qquad \text{where } h_{cr} = \text{critical-point fluid depth.}$$

Differentiating the expression for Q implicitly with respect to h, and setting $\left.\frac{\partial Q}{\partial h}\right|_{h_{cr}} = 0$, we get

$$h_{cr} = \frac{2}{3}(h_o - b_{cr}) - \frac{4\Omega^2 A_{cr}^2}{3g}$$

where $\quad b_{cr} = b(x_{cr}),$

$$h_{cr} = h(x_{cr}),$$

$$A_{cr} = -\frac{1}{b_c} \int_{-x_b}^{x_{cr}} b(x')dx',$$

$$u_{cr} = \left[\frac{2}{3} g(h_o - b_{cr}) - \frac{4\Omega^2 A_{cr}^2}{3} \right]^{1/2},$$

and

$$Q = h_{cr} u_{cr} = \frac{1}{g} \left[\frac{2}{3} g(h_o - b_{cr}) - \frac{4}{3} \Omega^2 A_{cr}^2 \right]^{3/2}.$$

Next we assume that $x_{cr} = 0$ (the critical control point is at the crest). The whole obstacle from base to crest has a role in the flow control. This certainly seems reasonable, but it will be necessary to test the assumption experimentally. In that case,

$$Q = \frac{1}{g} \left[\frac{2}{3} g(h_o - b_c) - \frac{4}{3} (\frac{\Omega}{b_c})^2 \left(\int_{-x_b}^{o} b(x')dx' \right)^2 \right]^{3/2}.$$

Frame rotation thus acts as a block to steady flow: for a given volume flux Q, as Ω increases, so must h_o if a steady state is to be maintained. If $\Omega = 0$, the result for Q is the classical formula for steady discharge over a wide, broad-crested weir.

III. Experimental evidence

A series of experiments using water as the single fluid layer were run in order to test the theoretical predictions from the last chapter on upstream heights in a rotating system. We have that

$$Q = \frac{1}{g}\left[\frac{2}{3}g(h_o - b_c) - \frac{4\Omega^2 A_{cr}^2}{3}\right]^{3/2}$$

in the limit of a stagnant upstream basin with centripetal effects removed. If Q is fixed and Ω is varied,

$$\left(\frac{Q^2}{g}\right)^{1/3} = \frac{2}{3}(h_o - b_c) - \frac{4\Omega^2 A_{cr}^2}{3g} \ .$$

But

$$\left(\frac{Q^2}{g}\right)^{1/3} = \frac{2}{3}(h_{o,nr} - b_c)$$

where $h_{o,nr}$ = nonrotating upstream basin height, so that

$$h_o = h_{o,nr} + \frac{2\Omega^2 A_{cr}^2}{g} \ .$$

This is the relation to be tested. It is the upstream effect of frame rotation on the steady-state weir solution.

Figure 2 is a photograph of the experimental apparatus. The tank measures 90 cm x 25.7 cm x 50 cm. Built onto the bottom is a large paraboloid barrier, 60 cm long with an

Figure 2. A photograph of the experimental apparatus.

apex 16 cm above the tank floor. If the coordinate origin
is taken as in Figure 1, the equation of the bottom in
centimeters is

$$b(x) = \begin{cases} 16[1 - (\frac{x}{30})^2] & |x| \leq 30 \\ \\ 0 & |x| > 30. \end{cases}$$

A pump downstream recirculates the water to a diffusion
system of horsehair fiber, pea gravel, and a sprinkler. The
entire system is carefully centered on a variable-speed
rotating turntable.

In the experiments for free-surface elevations, a
micrometer was mounted upstream. The micrometer was operated
both manually and mechanically; despite greater precision in
the mechanical micrometer drive, the accuracy of the readings
was not materially better than when hand-operated, because
the major source of error was pump fluctuation and free-
surface oscillations in resonance with machine vibrations.

Since the tank is not very wide, the wide-weir formula-
tion should be valid only for sufficiently small rotation
rates. A crude idea for the upper bound on the formulation
can be obtained by the following argument:

Calculation of the trajectory of a fluid parcel as it is
driven up the barrier is relatively straightforward if tedious.
Consider the sketch below:
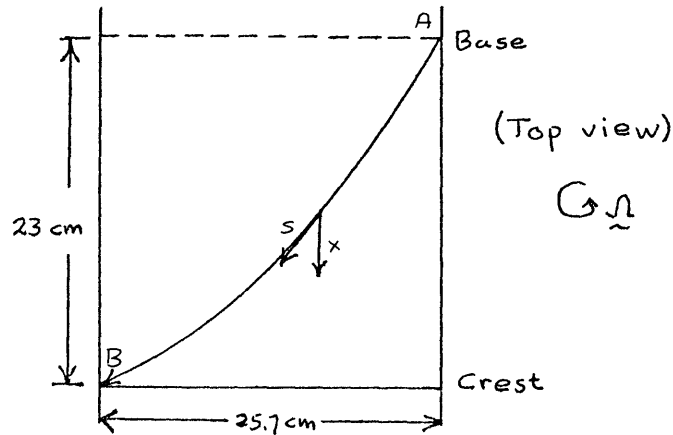
We want to calculate the rotation rate for which a fluid parcel that begins its trajectory at A is driven up to B at the crest. This we can consider the upper bound on the wide-weir applicability.

$$\frac{ds}{dx} = \frac{v}{u} = \frac{fA}{Q/h} = \frac{f}{Q} \, hA.$$

After integration for experimental parameters, it is found that to trace a trajectory from A at (0,0) cm to B at (23,25.7) cm, we need $\Omega \sim 0.6$ sec^{-1}. This value of Ω we take as the upper bound. For higher rotation rates, the water is driven into a recirculating asymmetric upstream gyre. (There is an apparent β-effect due to centripetal curving of the free surface.) A narrow boundary current transports water over the barrier. For low rotation rates the fluid separates from the rotation-leading side of the tank and piles up in a small boundary current on the rotation-lagging

side. For whatever value of the rotation rate, there is always a pronounced rotational effect on the steady-state upstream height, even after corrections have been made for centripetal accelerations.

As was mentioned before, the pump variability was the largest source of experimental error. Also, for low rotation rates, the upstream surface was often disturbed by standing-wave patterns in vibrational resonance with the turntable driveshaft. As a result, all readings have an error bar as large as $\pm$ 0.005 cm (less for higher rotation rates).

For each rotation rate, at least five micrometer readings were taken; the average reading was then corrected for centripetal distortion of the free surface ($\Delta h_{centripetal} = \frac{\Omega^2 r^2}{2g}$, where r = distance from turntable center), and from each average reading, the corresponding nonrotating micrometer reading was subtracted. The final figure is therefore the upstream height difference Δh_o between the given rotation rate and zero rotation (Q fixed), corrected to eliminate centripetal effects.

Figures 3, 3a, 4, and 4a present data from two experimental runs where the upstream height was measured. Experimental parameters are given in the figure captions. The data in Figures 3 and 3a represent a wider range in Ω; note the apparent transition in Δh_o.
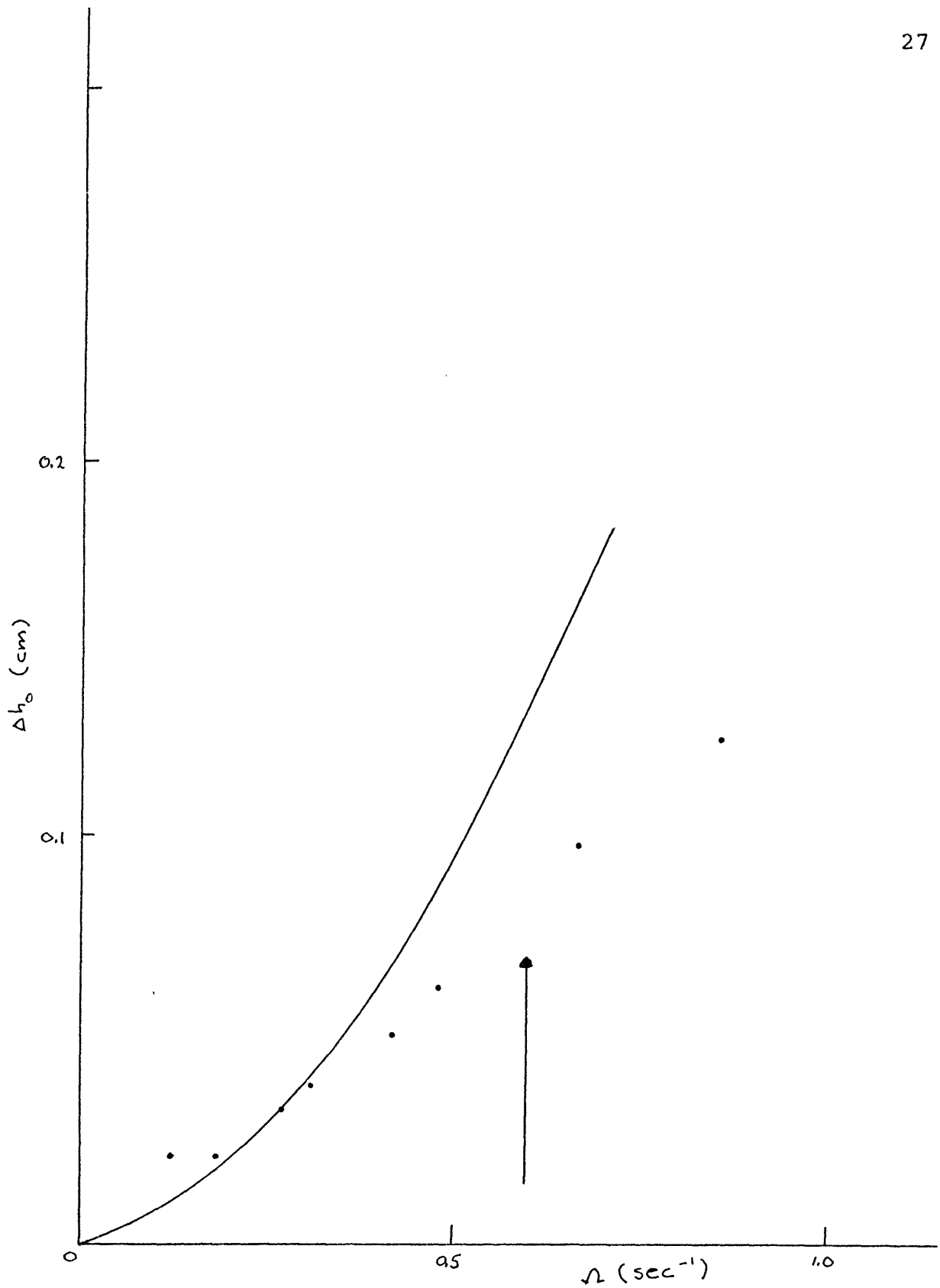
Figure 3. Upstream height as a function of the rotation rate.
Experimental run of 9/21/74.
Transition marked by arrow. Solid curve denotes
theoretical predictions.   A = 13.3 cm. h_0 = 7.5 cm.
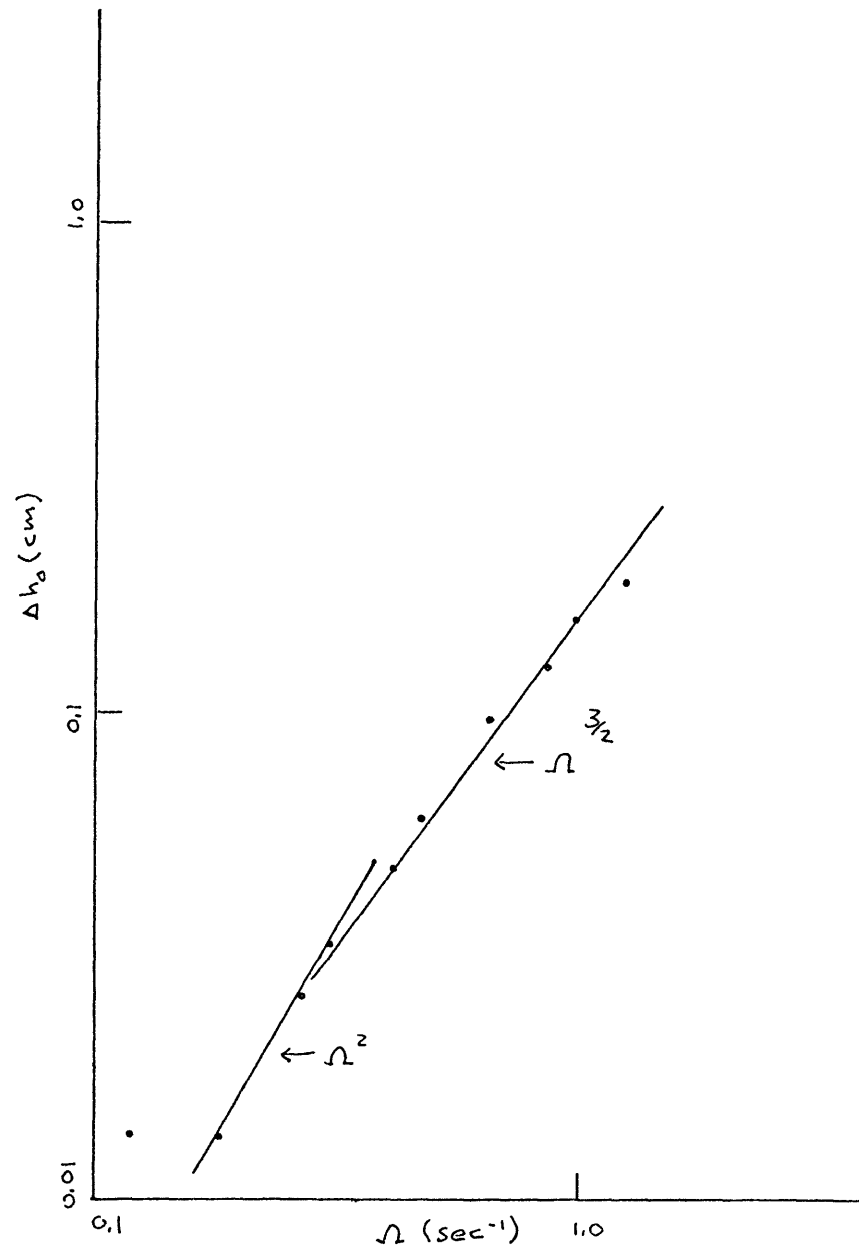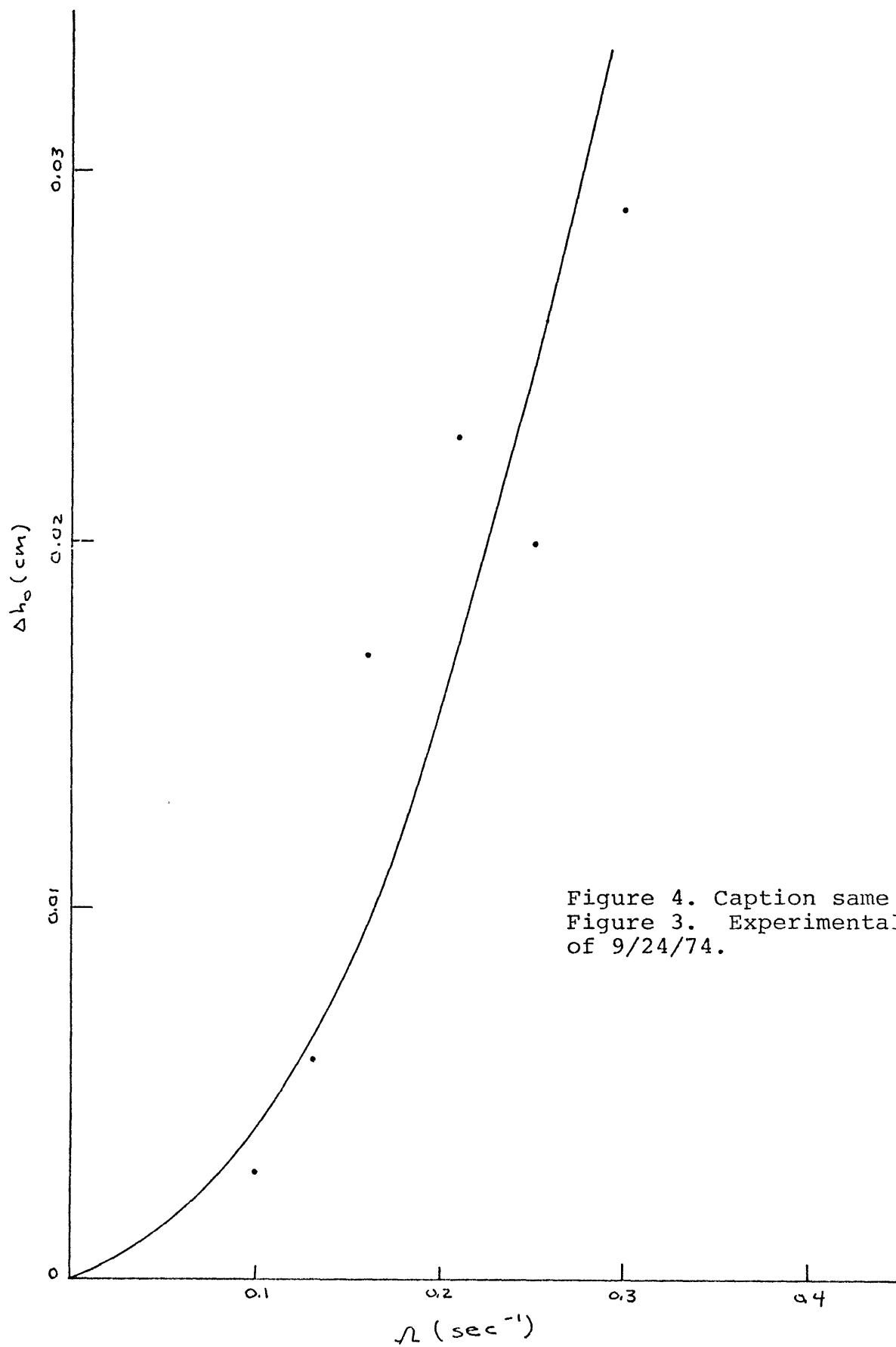
Figure 3a. Same as Figure 3, but plotted in log-log format.
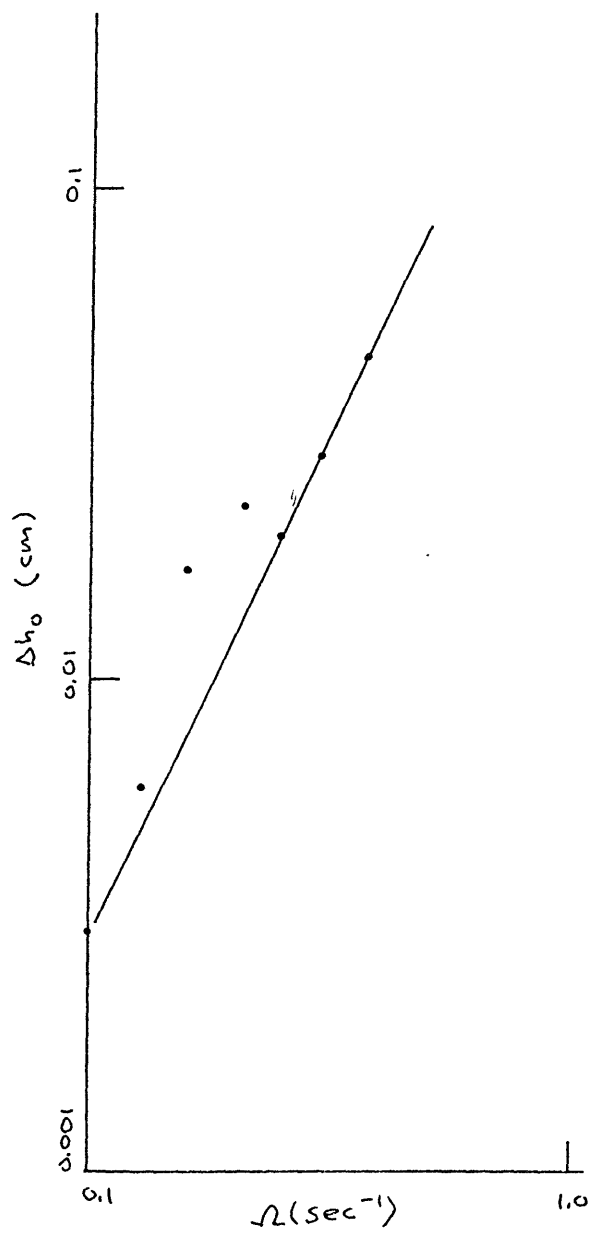
Figure 4. Caption same as Figure 3. Experimental run of 9/24/74.

Figure 4a. Same as Figure 4, but plotted in log-log format.

Inspection of the graphs indicates that (1) For every Ω, there is a definite increase in the upstream height for steady weir flow when compared to the nonrotating ($\Omega = 0$] case, with Q fixed. (2) For low $\Omega (\leq 0.5 \text{ sec}^{-1})$, there is an approximate $\Delta h_o \sim \Omega^2$ dependence, as predicted by the wide-weir theory. At about $\Omega = 0.5 \text{ sec}^{-1}$, a transition appears, above which an approximate $\Delta h_o \sim \Omega^{3/2}$ dependence holds. This dependence is much less pronounced than what could be expected for the wide-weir case, but is much stronger than calculations of Whitehead et al. (1974) for the "narrow weir" limit, where a cross-stream geostrophic balance is assumed, with the result that $\Delta h_o \sim \Omega^{1/2}$.

Further experimentation is necessary, but it is clear that strong constraints are placed on weir formulations in a rotating fluid.

IV. Concluding overview

An effort has been made to extend the classical results of nonrotating hydraulics to include a strong vorticity constraint in the form of frame rotation. Theory and experiments have combined to demonstrate a definite rotational effect on the steady-state weir solution. Such results are encouraging, but must be viewed with caution.

The use of streamline coordinates brings out many of the problems in the wide, rotating-weir formulation. The

price of a constant upstream Bernoulli velocity head $\frac{u_o^2}{2g}$ in a rotating fluid is a continuously varying potential head $h_o - b_c$, which is physically impossible in a fluid of infinite extent. Cross-stream changes in the Bernoulli head are determined by the (strong) vorticity constraint; the formulation presented here is valid only in the very wide, very deep limit where the velocity head can be set arbitrarily small and h_o becomes a constant everywhere upstream.

The experiments, while showing a definite increasing trend in upstream height with increasing rotation rate, have not been entirely conclusive. This is, I think, more a problem of experimental pitfalls than of theoretical difficulties. The experiments will be rerun soon with better pumps (variability in Q less than 1% over the entire Ω range) and with measures taken to reduce turntable vibrations. The tank may also be widened and the gravity may be reduced by adding a deep layer of kerosene or some other liquid over water (to increase the magnitudes of upstream heights for a given Q).

There are many geophysical situations where the non-linear approach of free-surface hydraulics might be useful. One thinks immediately of airflow over mountain ranges or waterflow through oceanic sills and across seabed ridges. Some such work has already been done, but only recently has the earth's rotation been taken into account. We need careful

investigations of rotating hydraulic flow control - its strengths and its weaknesses, what we can say rigorously and what we must assume - so that geophysical applications can be made with confidence. Such investigations, for the simplified case of a very wide, very deep obstacle, have been the aim of this work.

References

Charney, J., "The Gulf Stream as an inertial boundary layer,"
Proceedings of the National Academy of Sciences 41:
731-740 (1955).

Houghton, D. D. and A. Kasahara, "Nonlinear Shallow Fluid Flow
over an Isolated Ridge," Communications on Pure and
Applied Mathematics, XXI, 1-23 (1968).

Long, R. R., "Some Aspects of the Flow of Stratified Fluids:
II. Experiments with a Two-Fluid System," Tellus 6:2,
98-115 (1954).

Rouse, H., Fluid Mechanics for Hydraulic Engineers, New York,
Dover Publications, Inc. (1961).

Whitehead, J. A., A. Leetma, and R. A. Knox, "Rotating Hydraul-
ics of Strait and Sill Flows," Geophysical Fluid Dynamics
(in press).

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