Abstract

We put forward a new class of mechanisms. In this extended abstract, we exemplify our approach only for single-good auctions in what we call a conservative-Bayesian setting. (Essentially, no common-knowledge about the underlying distribution of the players’ valuations is required.) We prove that our mechanism is optimal in this challenging and realistic setting.

1 Preliminaries

Single-Good Auctions. In a single-good auction, a seller has one good. He wants to sell the good to players. Individual players, denoted by $1,\ldots,n$, have valuations $v_1,\ldots,v_n \in \mathbb{R}$ for the good, and player $i$ gets utility $v_i - p$ if he wins the good and pays price $p$. A mechanism consists of a message space $M_i$ for each player $i$, together with functions $x(m_1,\ldots,m_n) \in \{1,\ldots,n\}$ and $p(m_1,\ldots,m_n) \in \mathbb{R}$ that determine, respectively, who gets the good and how much the winner pays for the good. The revenue of the mechanism is $p(m_1,\ldots,m_n)$.

Notation We define the star player to be the player with the highest valuation, and denote him by $\star$.

Internal and External Information For player $i$, we define the internal information about player $i$ to be what player $i$ knows about himself. In this particular case, player $i$’s internal information is his own valuation $v_i$. We define the external information about player $i$ as what can be deduced from player $i$’s type by looking exclusively at the information held by others. The key concept is that the external information about player $i$ is out of player $i$’s control, and thus does not affect his incentives to report the truth. As a concrete example, take the Vickrey auction where player $i$ is the winner. Given that player $i$ wins, all other players know that $v_i$ is higher than them. In particular, let player $j$ have the second highest valuation $v_j$. Since player $j$ didn’t win, he knows that $v_i \geq v_j$. The mechanism can extract this knowledge from player $j$ without changing player $i$’s incentives. By doing this, the auctioneer knows that if he charges a price of $v_j$ to player $i$ then it is still dominant strategy truthful for player $i$ to report his own valuation.

2 The Conservative Bayesian Setting

In auctions of a single good, the traditional Bayesian setting assumes that

1. The valuation profile $v$ is drawn from a distribution $D$.
2. $D$ is common knowledge to the players and to the mechanism designer.

\footnote{\textsuperscript{1}In a more detailed note, it should be proved that a mechanism that charges money to people who are not the winner cannot be dominant strategy truthful.}
3. Each player $i$ privately knows his own $v_i$ and thus his conditional distribution $D_i = D|v_i$ over the other players’ valuation subprofile.

4. The only information available to each player $i$ is that of items 2 and 3, together with common knowledge of rationality.

(The last point is crucial. Should a player $i$ gather additional information about $v_{-i}$, then he may want to deviate from a Bayesian Nash equilibrium.)

In a conservative Bayesian setting, the valuation profile $v$ is drawn from an underlying distribution $D$. But $D$ is possibly unknown to the players and the designer. Instead, each player $i$ privately knows his own valuation $v_i$ and his own knowledge $D_i$, which is a distribution over the other players’ valuation subprofiles, obtained from $D$ conditioned on $v_i$ and any other true signal observed by $i$.

**True Types** For each player $i$, the pair $(v_i, D_i)$ is called player $i$’s true type, denoted by $\theta_i$. A mechanism $M$ is revealing if the message space for each player $i$, $M_i$, coincides with $i$’s type space, that is, $\mathbb{R} \times \Delta(\mathbb{R}^{n-1})$. Notice that $\theta_i$ includes information both about $i$ himself and others. A single-good auction context is uniquely identified by the profile of the players’ true types in that context.

### 3 Our revenue bound

Our upper bound on revenue works for all (mechanism, solution concept) pairs $(M, S)$ that satisfy the following weak requirements:

0. For any context with the profile of true types $\theta$, $S(\theta) = S_1(\theta_1) \times \cdots \times S_n(\theta_n)$.

1. For any context, the profile of the true types $\theta$ belongs to $S(\theta)$.

2. For any player $i$, no strategy in $S_i(\theta_i)$ is strictly dominated over $S(\theta)$.

3. safe-truthfulness. $\mathbb{E}[u_i(M(\theta_i, m_{-i}))] \geq 0$ for all $m_{-i} \in S_{-i}(\theta_{-i})$.

**Definition 1.** We say a pair $(M, S)$ is safe if it satisfies the above four requirements.

Intuitively, safe-truthfulness guarantees that “If players announce their true information, then they cannot get negative utility.” In particular, it is safe for players to announce their true beliefs about others. This property is implied, for example, by dominant strategy truthfulness, which specifies that it should be a dominant strategy to announce one’s true information.\(^2\)

We prove a limit on the revenue obtained by any safe-truthful mechanisms for single-good auctions. We show that this revenue cannot be higher than that obtained by only using the external information about the winning player. A priori, one could imagine using the player’s own internal information as well to extract some extra revenue. We show that if the winning player has enough information about the other players’ strategies, then this is impossible.

**Definition 2.** In single-good auctions, we say that

- a function $r$ is a revenue benchmark if for any context with true-type profile $\theta$, $r$ maps $\theta$ to a real number;
- a (mechanism, solution concept) pair $(M, S)$ achieves a revenue benchmark $r$, if for any single-good auction context with true-type profile $\theta$ and any strategy profile $\sigma \in S(\theta)$, $\text{rev}(M(\sigma)) \geq r(\theta)$;
- $(M, S)$ achieves $r$ with $\epsilon$ bonus, if for any context with true-type profile $\theta$ and any $\sigma \in S(\theta)$, $\text{rev}(M(\sigma)) \geq r(\theta) + \epsilon$ whenever $\max_i v_i \geq r(\theta) + \epsilon$, and $\text{rev}(M(\sigma)) \geq r(\theta)$ otherwise; and

\(^2\)This applies for mechanisms where a player can choose not to participate.
Intuitively, any auction that wants to generate a large amount of revenue should sell the good to this player. Furthermore, any safe mechanism should not charge money to any of the players that do not get the good. Thus, the revenue generated by the mechanism is exactly the price charged to the \(*\) player. A natural benchmark would be the maximum revenue we can obtain by using only external knowledge about the winning player. In particular, we can define this benchmark as

\[ \text{rev}_{\text{ext}}(\theta_\bullet) = \max_p p \cdot \Prob[v_1 > p | \theta_\bullet], \text{ Player } \star \text{ won the auction }. \]

The corresponding price that achieves this expected revenue would be

\[ p_{\text{ext}}(\theta_\bullet) = \arg\max_p p \cdot \Prob[v_1 > p | \theta_\bullet], \text{ Player } \star \text{ won the auction }. \]

We now show that we cannot achieve a revenue higher than this benchmark.

**Theorem 1.** For single-good auctions, no safe \((M,S)\) pair can achieve \(\text{rev}_{\text{ext}}\) with \(\epsilon\) bonus, for any arbitrarily small constant \(\epsilon > 0\).

**Remark:** This proof also works for the case where the mechanism designer has some information \(\theta_0\). This information can also be used to extract money from player \(*\).

**Intuition.** Assume without loss of generality (after relabeling the players) that the players are numbered according to their valuations. So \(v_1 \geq v_2 \geq \ldots \geq v_n\), and \(\star = 1\). We call player 1 the potential winner of the auction. For each play of the auction, we denote by \(\star\) the winner of the play, that is, the player who actually gets the good. Note that player \(\star\) needs not coincide with player \(*\). But if the mechanism is going to generate a revenue higher than the second price, then it is necessary that \(\star = \star\).

Now we show the mechanism cannot achieve a revenue higher than \(\text{rev}_{\text{ext}}\). Proceed by contradiction. Suppose that the mechanism can guarantee revenue \(R > \text{rev}_{\text{ext}}(\theta_2,...,\theta_n) + \epsilon\). The definition of \(\text{rev}_{\text{ext}}\) was the largest expected revenue that achievable using only external information about player 1. Thus, to achieve a revenue of \(R\), the mechanism must use some internal information about player 1. That is, player 1 must make an announcement \(m_1\) such that \(\mathcal{M}(m_1,...,m_n)\) produces a revenue of \(R\). Player 1’s utility will be \(v_1 - R\).

In some cases, it is not rational for player 1 to do this. In particular, assume player 1 knows what \(\theta_2,...,\theta_n\) are. Then player 1 does not maximize his utility by announcing \(m_1\). Instead he should announce the message \(\tilde{m}_1\):

- “My internal valuation is \(\text{rev}_{\text{ext}}(\theta_2,...,\theta_n) + \epsilon\), and
- I have no information about others.”

Because \(\tilde{m}_1,\theta_2,...,\theta_n\) could be a truthful strategy profile, the mechanism cannot make any player unhappy. In particular, if player 1 wins the auction, he cannot be charged more than his announced valuation \(\text{rev}_{\text{ext}}(\theta_2,...,\theta_n) + \epsilon\). So his utility is larger than or equal to \(v_1 - \text{rev}_{\text{ext}}(\theta_2,...,\theta_n) - \epsilon\), which is larger than or equal to \(v_1 - R\).

The contradiction arises from the assumption that the mechanism can guarantee revenue \(R\) regardless of what player 1 announces. Thus, we have an upper bound on the revenue of \(\text{rev}_{\text{ext}}(\theta_2,...,\theta_n) + \epsilon\), for arbitrary \(\epsilon\).

## 4 Our Tight Mechanism

We now give a specific mechanism that achieves our revenue benchmark \(\text{rev}_{\text{ext}}\). This shows that this benchmark is a tight bound on the revenue that an auctioneer can generate by selling a single good. In our mechanism, valuations come from a bounded discrete domain. For concreteness we assume this domain is \(D = [0,K]\cap\mathbb{Z}\).
1. Each player $i$ secretly announces his own valuation $v_i$ to the auctioneer.

   a. The auctioneer announces a player, whom we call $\ast$, that has the highest valuation $v_\ast$.

2. Each player $j \neq \ast$ announces his information about $\ast$. This is a distribution $P^j$ whose support is the set of integers between 0 and $K$. The number $P^j_v$ represents the probability (according to $j$’s beliefs) that $v_\ast = v$.

   b. The auctioneer computes
      - $b_1. CP = \text{Second Price} = \max_{j \neq \ast} v_j$.
      - $b_2. KR = \text{Knowledge Revenue} = \max_{j \neq \ast} \max_{p \in [0, K]} \sum_{p \in [0, K]} p \cdot \text{Prob}_{V \leftarrow P^j}[V > p] = \max_{j \neq \ast} KR^j$.
      - $b_3. BIP = \text{Best Informed Player} = \arg\max_{j \neq \ast} KR^j$.
      - $b_4. KP = \text{Best Known Price} = \arg\max_{p \in [0, K]} \sum_{p \in [0, K]} p \cdot \text{Prob}_{V \leftarrow P_{BIP}}[V > p]$.

   c. The auctioneer allocates the item and decides $\ast$’s price with the following rule:
      - $c_1. \text{If } CP \geq KR \text{ then } \ast \text{ wins the item and pays } CP$.
      - $c_2. \text{Else, if } v_\ast \geq KP, \text{ then } \ast \text{ wins the item and pays } KP$.
      - $c_3. \text{In the case that } v_\ast < KP \text{ no one wins the item}$.

   d. Reward each $j \neq \ast$ with $\epsilon(2 - \sum_{k=1}^K (\delta_{v_\ast, k} - P^j_k)^2)$.

5 Analysis

We state some theorems about our mechanism, and give some intuition for their proofs.

**Theorem 2.** When the players are truthful, the mechanism obtains revenue $r_{ext}(\theta_{-\ast})$.

**Intuition.** When players are truthful, the mechanism generates an expected revenue equal to $\max\{CP, KR\}$. $CP$ is the maximum revenue possible if there is no information whatsoever about player $\ast$’s valuation. $KR$ is the maximum revenue that the best informed player can extract from player $\ast$. If we do not aggregate information, then the maximum of these two is the maximum revenue possible.

**Remark:** Our mechanism does not aggregate information. We leave as an extension for future work to replace steps $b_2$ through $b_4$ by a step where the mechanism uses all the reported external information to generate a composite distribution. Notice that doing this, while keeping a proper scoring rule as a reward, does not affect the incentives of the players.

**Theorem 3.** Regardless of the announced external information, it is always dominant strategy for players to announce their true valuations $v_i$ at step 1.

**Intuition.** If a player believes he will win the auction, he gains nothing by overbidding. In particular, announcing a higher $v_i$ does not change either the allocation or the price that the mechanism charges him. Similarly, the player gains nothing by underbidding, since the price does not depend on the particular announcement of $v_i$. If a player believes he will not win the auction, he might be tempted to overbid in order to win the good. However, if he overbids then he will get charged a price higher than $CP$, the second price. Since the player believes he will not win the auction, he must believe the second price is higher than his own valuation. Thus, the player gets negative expected utility from overbidding. His utility does not change if he underbids.

**Theorem 4.** If the winning player is truthful in step 1, then it is strictly dominant for players to truthfully report their beliefs about the winning player in step 2.
Intuition. The players are rewarded for their information at step \( d \) of the mechanism. This reward only depends on their announcement, and on the realization of the (uncertain) variable \( V^* \). Notice that the reward is an increasing linear function of Brier’s quadratic scoring rule. This encourages players to report their information truthfully.

6 Final Remarks

Our mechanism trivially accommodates the case when the designer himself has some distributional knowledge about the star player, or the underlying distribution \( D \) itself. Our mechanism performs at least as well as all previously known mechanisms. In particular, our mechanism gracefully degrades to the Vickrey mechanism when the players only know their own types and have no external knowledge at all. It also matches Myerson’s mechanism when the only information about the star player known to the other players is a prior, common-knowledge to the players and the designer.

7 Extensions

In forthcoming papers we shall deal with generalizing our results to continuous valuations, aggregating the external knowledge of all players, as well as considering other types of external knowledge, and extensions beyond single-good auctions.