THE STATISTICAL STRUCTURE OF TURBULENT CONVECTION:

AN EXPERIMENTAL STUDY

by

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The Statistical Structure of Turbulent Convection:
An Experimental Study

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Submitted to the Department of Meteorology on August 12, 1966 in partial fulfillment of the requirements for the degree of Master of Science.

ABSTRACT

The purpose of this work is to obtain the vertical variation of statistical properties of the temperature fluctuations in a turbulent convection field between parallel plates, heated from below. The height of the apparatus is designed such that the Rayleigh number is large enough to give turbulence, and the horizontal extent of the apparatus is such that the convective cells are not constrained by the vertical boundaries to remain fixed spatially. The convective cells being free to wander, the statistical properties at a point in the vertical should be equivalent to the spatial average of the plane if a sufficiently long record is taken.

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Introduction

It is observed when a fluid is heated from below that beyond a certain thickness and temperature of the layer motion will occur. This was first studied by Benard who noted that the motion took on a cellular pattern with rising in the center and descending at the cell boundaries. Rayleigh modeled the problem mathematically finding under what conditions the layer would become unstable for the condition of no lateral forces at two rigid boundaries. The result was a stability criterion in terms of a dimensionless ratio bearing his name.

The Rayleigh number is defined as:

\[
Ra = \frac{8 \alpha \Delta T h^3}{K \nu}
\]

\(T_m\) = mean temperature  
\(\alpha\) = the thermal expansion coefficient = \(1/T_m\)  
\(\Delta T\) = temperature difference = 48°C  
\(h\) = height = 4.7 cm  
\(K\) = thermometric diffusivity of heat  
\(\nu\) = kinematic viscosity = \(\mu/\rho\)

Essentially the Rayleigh number is the ratio of the buoyancy forces to the viscous forces. This can be seen if we define a dimensionless vertical velocity \(\omega' = \frac{\omega}{k/h}\) where \(k/h\) is a lower bound on the integrated vertical velocity scale since at speeds below this cutoff the temperature difference of the parcel would be erased.
The viscous shear force tending to stabilize the parcel can be expressed as

\[ \frac{\mu \omega}{h^2} = \frac{\mu K}{h^3} \quad \text{per unit area} \]

The buoyancy destabilizing force can be expressed as:

\[ \beta \rho \delta \Delta T \]

The Rayleigh number of the apparatus used in these experiments was \(3.73 \times 10^5\). The Rayleigh number for the onset of convection according to stability analysis for two rigid plates is 1708 (Pellow & Southwell Ref. 14).

The heat flux through the chamber is expressed by the heat diffusion equation, which if:

\[ T = \overline{T} + T' \]

\[ \overline{T} = \text{average temperature and a function of } Z \text{ only} \]

\[ T' = \text{temperature fluctuations} \]

yields

\[ \dot{H} = \rho c_p K \frac{\partial \overline{T}}{\partial z} + \rho c_p \overline{T'\omega} \]

\( \overline{T'\omega} \) is the heat transport by the convection

In the regions adjacent to the plates the vertical velocity must go to zero and thus conduction must be the mode of heat transport.

The nusselt number is defined as the ratio of the total heat transport
by conduction and convection to the heat transport which would occur by conduction only.

\[ Nu = \frac{\Phi_T}{kA\frac{\partial T}{\partial Z}} \]

The Nusselt number has been determined experimentally as a function of Rayleigh number (Silverton Ref. 1). The Nusselt number may be used to estimate the conductive boundary layer thickness (δ).

\[ \delta = \frac{\delta_{1/2}}{Nu} \]

The Prandtl number is defined as:

\[ Pr = \frac{\nu}{k} \]

and with the Rayleigh number specifies the state of the fluid.

Turbulent convection in air has been studied previously by Townsend and more recently by Deardorff and Willis. The results of both of their works were used as a basis for comparison with findings obtained in these experiments.

Deardorff performed experiments varying the horizontal dimensions of his convection chamber. He had the capability to probe spatially as well as obtain the statistics at a point. He obtained plots of a "degree of turbulence" parameter (R) defined as

\[ R = \frac{\overline{T'^2}}{\langle T'^2 \rangle} \]
where \( (\cdot) \) is the time average at a point and \( \langle \cdot \rangle \) refers to a spatial average. The degree of turbulence is plotted versus the aspect ratio defined as the length/height of the tank for various Rayleigh numbers (see p. 349 Ref. 5). The "degree of turbulence" is found to increase for larger aspect ratios and smaller Rayleigh number in the range of \( Ra = 10^5 \) to \( 10^7 \).

Townsend's apparatus had an aspect ratio of 5.7 and he used a 6 minute averaging time. He noted influences in his mean temperature profiles which he attributed to a larger scale circulation which wasn't averaged over.

L.N. Howard sought to explain the sharp thermal spikes occurring in Townsend's temperature records. He interpreted the spikes as "blobs" of warm fluid rising vertically past the probe. He mathematically modelled the thermal boundary layer assuming at \( t=0 \) the fluid temperature at the warm plate was that outside the boundary layer and allowed the temperature to increase until a time \( T^* \) (blob formation time) when the layer was gravitationally unstable and separated. Howard found \( T^* \) to be:

\[
T^* = \frac{\sigma^2}{\pi k}
\]
Description of Apparatus and Techniques

A. Apparatus

The convection chamber has a cylindrical geometry having a height of 4.70 cm and a diameter of 70 cm. It is bounded at the top by a .635 cm thick plexiglass plate and the bottom by a 1.27 cm thick ground aluminum plate. The side wall of the chamber consisted of 6.35 cm of closed cell styrofoam. The plexiglass and aluminum plates are parallel to each other. The aluminum plate floats in a reservoir of molten cerrobend having a melting point of 73°C. The mass of the cylindrical convection chamber, ice reservoir, and aluminum plate system is uniformly distributed about the axis of revolution. This insures that the constant temperature aluminum and plexiglass plates are horizontal. Failing to satisfy this requirement would result in a large-scale convective circulation of the scale of the tank as a gravitational instability would result from isotherms tilted with respect to geopotential surfaces. The aspect ratio of the tank, length divided by height, is 15. This large an aspect ratio has been found adequate by previous investigators to provide the convective cells with freedom to wander.

Constant temperature reservoirs are formed by jet mixed ice water bath on top of the plexiglass and fifty-five pound of cerrobend, a eutectic alloy principally composed of lead and bismuth. Cerrobend has a latent heat of fusion of 14 BTU/lb, and was kept at its melting temperature. The cerrobend was heated by circulating water through a 1500 watt heater into a circular tank below the cerrobend. The time required to bring the convection chamber up to operating temperature is approximately one hour. At steady state the heat input is controlled by a 15 ampere variac, adjusted
Convection Apparatus

Fig. 1
to maintain a heat input equal to the heat flux through the chamber plus extraneous losses of the system. The output voltage of the variac is read on AC volt meter.

Vertical Traverse

A vertical traverse was mounted on a frame above the ice bath. The traverse barrel was driven by a thread gear and its scale with .001 inches per division with 50 divisions per revolution. A voltage divider potentiometer (1% linearity) was geared to the traverse and found to yield .075 volts per revolution. Both methods were used to obtain the height of the probe as the voltage measure was recorded as a check.

The probes were fastened to the traverse by pin vise mounts. The probes were passed through .25 cm diameter holes bored through 1.27 cm plexiglass stock of 7.6 cm lengths into the convection chamber. The plexiglass rods acted as guides for the probe stems and also sealed the holes from the ice bath.

B. Measurement Techniques

The probes: The temperature in the convection chamber is sensed by a resistance thermometer made of a 3 mm long Pt-10% Rd wire of $1.6 \times 10^{-4}$ inch in diameter. The Pt wire is mounted on the filed tips of two ordinary sewing needles which are fastened at one end of a 12 inch ceramic support tube made of common thermocouple wire insulation having two small holes suitable for passing number 28 copper wire. The copper wire is soldered to the eye of the needle which, if the joint is made with sufficient care and after some filing, can be pulled into the thermocouple holes.
Resistance Thermometers

Fig. 2
For conductors, resistance is an increasing function of temperature, following to a linear approximation.

\[ R = R_0 (1 + \alpha \Delta T) \]

- \( R_0 \) is a reference wire resistance at a temperature \( T_0 \).
- \( R \) is the resistance at temperature \( T \).
- \( \Delta T \) is the temperature change \((T - T_0)\).
- \( \alpha \) is the resistance-temperature coefficient.

The resistance-temperature coefficient is determined experimentally by plotting a resistance versus temperature calibration curve for the resistance thermometer. \( \alpha \) is the slope of the best fit straight line. A typical value of \( R_0 = 21.93 \Omega \) at \( 21^\circ C \) for \( \alpha = 0.00166 \Omega/\mu^\circ C \) yields \( 0.03 \Omega \) change in wire resistance per \( ^\circ C \). In order to sense these changes in resistance, a current is passed through the thermometer. The current in the thermometer must be kept at a low level in order to reduce the resistance dependences of the wire on the velocity of the flow.

The first law of thermodynamics reveals the energy balance of a resistance thermometer with current flowing through it.

\[ \frac{dE}{dt} = \frac{dQ}{dt} - \frac{dw}{dt} \]

\( \frac{dE}{dt} \) is the change of internal energy of the wire.

\[ \frac{dE}{dt} = \frac{d^2}{4} C \frac{dT}{dt} \]

\[ \frac{dq}{dt} \text{ is the power heating the wire } = i^2R \]

\[ \frac{dw}{dt} = h\overline{d} (T_w - T_a) \text{ is the conductive and convective heat transfer from the wire} \]

\[ T_a \text{ is the ambient temperature} \]

\[ h \text{ is the film coefficient of the air adjacent to the wire} \]

\[ T_w \text{ is the wire temperature} \]

This film coefficient has a velocity dependence which is sensitive to the heating. Assuming a steady state condition \((t \to \infty)\) for a fixed \(T_a\),

\[ i^2R = h\overline{d} (T_w - T_a) \]

\[ h \approx \frac{k}{d} (A + \sqrt{\frac{d}{\nu}} B \sqrt{\nu}) \text{ where } A \text{ and } B \text{ are constants dependent on the Reynolds number} \]

The quantity \( \frac{dv}{\nu} \) is the local Reynolds in the region of the platinum wire. For a value of \(\nu\) of 100 cm/sec this yields

\[ Re = \frac{dv}{\nu} \]

\[ d = 0.0004 \text{ cm} \]

\[ \nu = 0.19 \text{ cm}^2/\text{sec} \]

\[ v = 100 \text{ cm/sec (Deardorff)} \]

\[ Re = 0.2 \]

For which

\[ A = 0.24 \text{ and } B = 0.56. \]

For the above magnitude Reynolds number conductive and advective effects weigh approximately equally in the film coefficient. As the current is decreased, the wire temperature approaches that of the
surrounding air. In order to minimize the error in temperature fluctuations due to velocity fluctuations, the resistance thermometer was run at the lowest current level necessary to yield temperature changes of 1°C observable in the background noise. The wire length represents a spatial averaging scale. The resistance of the wire increases with length and larger resistance fluctuations are obtained for a given temperature change. To retain the same temperature resolution and reduce velocity sensitivity, longer resistance thermometers were run at lower current (.5 ma) for the latter run.

\[ T_w - T_a = .2°C \text{ at } .5 \text{ ma} \quad v = 0 \]

\[ T_w - T_a = 3°C \text{ at } 2 \text{ ma} \quad v = 0 \]

A velocity of 100 cm/sec will double the film coefficient or introduce 1.5°C fluctuations for a time varying velocity.

The current supply

The resistance thermometer was run at a low and constant current. The current supply consisted of a 4.5 volt battery, a milliamp meter, one fixed and one variable resistor. The resistors were two orders of magnitude larger than the resistance thermometer and four orders of magnitude larger than the fluctuations in resistances. Fluctuations in current introduce errors of less than .5% into the temperature readings.

Integrating the current supply with the bridge and potentiometer circuits presented a problem and a diagram is included to show how it was accomplished. Numbers at the nodes refer to pin numbers on the supply connector and also those on Transmetric Inc. circuit diagrams.
Current Supply Bridge Circuit
Balance of D.C. level

The resistance fluctuations from the thermometer are relatively small compared to the mean resistance level of the platinum wire. The tape recorder on which the temperature fluctuations are stored places two voltage constraints on the signal. 1) Maximum voltage level ± 1.4 v, 2) minimum clearly resolvable voltage 40 mv. Using a 50 Ω wire at .5 ma yields a d.c. level of 25 mv with fluctuations of 40 μv/°C. D.C. amplifiers, gain 1000, were necessary to increase signal levels of the fluctuations to tape recorder requirements. The d.c. level of the signal was eliminated by applying an adjustable bucking voltage, by means of a voltage divider. The signal was taken off directly across the resistance thermometer and bucked after being uncoupled through an amplifier from the current supply. A second method of achieving a d.c. balance involves taking the signal off across the bridge circuit in parallel with the galvanometer. The d.c. level was removed by balancing the bridge (Ry is diagram 3). The resistance was adjusted and read at each station and any residue d.c. level remaining in the records was added or subtracted to from the mean temperature at each station. The a.c. bridge attenuates the output signal and the attenuation increases with wire resistance and thus limits the frequency resolution of the fluctuations.
Calibration Procedure

The resistance thermometers were calibrated in water over a temperature range of 0° to 70°C. The water was contained in a thermos and an insulating lid was placed on the top. Temperature was measured using a calibrated thermistor (and a bulb thermometer). The resistance of the thermometer was measured with a Wheatstone bridge scale divisions to .01 Ω. The water was mixed thoroughly after each temperature increment, then the probe was submerged and a reading taken after a time lapse to allow the thermistor to come to equilibrium.

The signal path was gain calibrated using a signal generator connected in place of the resistance thermometer. Amplitudes of a sine wave input were compared directly and at the output of the tape recorder. Similar calibration techniques were used for data reduction electronics.
Resistance vs temp.
Probe l = 3.5 mm
July 20, 1966

\[ \lambda = -0.000166 \frac{\Omega}{\text{m} \cdot \text{oc}} \]

Temp. \text{oc}
Data Collection Procedure

Final data was obtained from stations of one hour duration taken in the vertical. The input of the tape recorder was shorted between stations which allowed recording of tape zero. These zeros were made quite long, 7-10 min, to facilitate identification when the signal was replayed later at higher speeds. It also allowed time for the floating convection chamber to be damped of any oscillations which may have been introduced while adding ice to the cold reservoir.

Straight probes allowed measurement of the temperature of the two plates directly. When probes offset at right angles to the stem were used, it was impossible to measure the upper plate and consequently thermo-couples were mounted against both plates, 3 cm from the edge of the tank. The current was read and noted at the change of stations. Runs on which the d.c. level was compensated on the bridge required adjustment of the balance resistance with height to an eye ball zero mean. The resistance level at each station was noted. On the run using the bucking voltage to balance the mean voltage, the bucking voltage was set constant at the average voltage level between the first station and the center of the tank, thus retaining all mean temperature information on the recorder.

C. Data Reduction

The mean, the variance, and the power spectra of the temperature field were obtained from the records as a function of height.
Extracting the Mean

The mean temperature was the first information extracted from the records. This was accomplished by speeding up the original record by a factor of 100 and smoothing the signal with a (a cps) low-pass filter formed from an operational amplifier with an R-C feedback loop. Discrete points were sampled from a smoothed Sanborn recorder output at intervals no greater than half the time period of the cut-off filter to avoid sampling error. Later this same method was accomplished more simply using a digital voltmeter. The printer output of the digital voltmeter yields numerical voltages directly and eliminated the need for reading voltages from plotted output.

Data Speed Up

The frequency of the temperature fluctuations can be seen, see sample records, to have very low frequency components associated with the unsteadiness of the motion causing the "cells" to shift spatially. The period of the shifting, defined by the average time between equal intensity regions, is on the order of 150 seconds. It was desired to have the power spectra representative of the frequency content of the temperature fluctuations within the cells. The lower bound on the frequency content was dictated by the limitations of the H-P wave from analyzer, range 20 cps to 50 kc, and the transfer of data between tape recorders while preserving high frequency information to 10 cps (real time). A schematic (Fig. 5) shows the transfer loop with low-pass cut-off filter limitations. The transfer loop allows speeding up of the signal by a factor of 1000 as the P.I. 6100 has speed ranges .375 ips, 3.75 ips and 37.5 ips.

An adjustable bucking voltage is included in the transfer loop to eliminate the d.c. level which exists on the original record. The mean voltage information
Signal Path including Speed up Loop
was used to adjust the d.c. level at each station to the tape zero. The bucking voltage was set to zero between stations. This eliminates the square wells on which the fluctuations ride. It was necessary to eliminate these square waves as they contained higher order harmonics of 0.3 cps which would yield pseudo energy in the variance and spectral analysis.

Obtaining the Variance

The variance of the temperature fluctuations was obtained using a hotwire ampmeter coupled to an amplifier with high and low pass filters. The hotwire ampmeter is a power resistor with a thermocouple attached. The rectified A.C. signal is fed through the resistor which dissipates energy proportional to voltage squared ($V^2$). If the electrical energy is converted to heat energy then this will cause a temperature rise of the resistor.

$$\Delta T = \frac{V^2/R}{\rho C_p}$$

$\rho$ = density of resistor

$C_p$ = heat capacity of resistor

The time constant of the hotwire ampmeter was 8 seconds. The speeded up temperature record was played into the hotwire ampmeter with the low-pass filter set at 0.01 cps and the high pass filter set at 50 cps real time. The output was sampled every 2 seconds and an average formed from forward and reverse sweeps.

There is a frequency weighting factor introduced into the variance due to the response character of the high and low pass filters. Calibration of the meter was done at a frequency of 0.1 cps which is about the centroid of the power spectra.
The Power Spectra

The average energy level in discrete frequency bands was obtained from the transferred data speeded up by a factor of 1000. The integrating digital voltmeter was used with a preset and externally controlled integrating time to average the RMS output of the wave from analyser at each station. The duration of each station (x1000) was 3.6 seconds. An integration time of 3 seconds was found to allow sufficient to reset the voltmeter between stations. A bucking voltage was used to cancel a 5.5 volt d.c. at the recorder output of the wave from analyser. An integrated zero level reading was taken before and after each sweep of the record and any residue zero level was subtracted from the station readings.

A sharp spike of energy was found at 1.5 cps. Tape level zero was played through the transfer loop and the noise level of the PI 2070 A record was found to be the source.
Data Reduction with Speeded up Record
Convection Apparatus

Left to Right Waveform Analyser, Digital Voltmeter and PI Tape Recorders

Fig. 7
Current Supply, Bridge and Potentiometer Circuits

Hotwire Ammeter

Fig. 8
Discussion of Results

The mean temperature profiles are in Figs. 9 and 10 obtained from 20 minute and one hour stations respectively. The ordinate is the vertical scale normalized against total height. The conductive boundary layer is seen between 0 and .1 h. In the region immediately adjacent to the hot reservoir the probe is influenced by radiation (see Appendix). A distance of .2 mm was found adequate to specify conductive temperature to .5°C.

The heat flux was computed from the

\[ q = kA \frac{dT}{dz} \]

temperature gradient of the thermal boundary layer and found to be 47 watts. This compares with 52 watts, based on the Nusselt number obtained from Silverton's data on Nu versus Ra number (Ref. 1).

Before the temperature fluctuations are reduced to statistical properties of the temperature field, the records composing the one hour stations in the vertical are presented. The first set of figures 11, 13 and 15 cover 5/6's of the station duration on a one cm = 200 second time base; and indicate the "cells"—defined as long period cyclic structure—which form the sample. The cells are seen to vary in intensity which is also the case for other data collected and spatial sampling of Deardorff [(Ref. (5)]. In order to see the fine structure of the temperature fluctuations which are to compressed and also filtered above .7 cps in the first records, 5 minute sections, covering
the middle of each station, are presented on a one centimeter equals 20 second time base. These records are shown in Figs. 12, 14 and 16. Notice the difference in the direction of the thermal spikes between stations at 3.80 mm and 5.80 mm, just inside and outside conductive influence region. The spikes are superimposed on a low frequency oscillation associated with the cells shifting spatially. The boundary layer spikes oscillate approximately an equal amplitude hot and cold but the 5.30 mm station are predominantly hot spikes growing in amplitude as temperature increases.

The variance of the temperature fluctuations are plotted in Fig. 17 for the one hour stations. The graph indicates a maximum in the variance just outside the thermal boundary layer. This maximum of the variance is also observed by Townsend (Ref. 12) and Deardorff (Ref. 6). The abscissa of these graphs are scaled to $\Delta T$. The graph of variance does not indicate a monotonic decay with height from the maximum to the center of the tank as other investigators have found. This could be due to the frequency weighting factor in the technique for obtaining the variance but the power spectra indicate the same trend in the low frequency component.

Malkus develops an expression for the spatially averaged variance for the region.

$$\frac{\langle \sigma^2 \rangle}{\langle T^2 \rangle} = \frac{2}{3\pi^2} \left( \frac{\log \frac{T_h}{T_c}}{\phi h} \right)^3 \frac{kT_m}{\phi h} \frac{\log \phi h}{kT_m}$$
\( Q = \) heat flux

\( K = \) conductivity

\( T_m = \) mean temperature

\( \frac{T_h}{T_c} = \) ratio of hot to cold plate temperature

\( \Upsilon = 5.0^\circ C \) from above

The spatially averaged variance from Fig. 17 is \( 4.2^\circ C \).

The power spectra of each station are plotted in Figs. 18, 19, 20 and 21. The power density is scaled with \( \frac{Q c_p}{(T_h - T_m)^2} \). The power spectra exhibit an increase in energy with increasing frequency between .8 and 1 cps in all but the station immediately adjacent to the plate. This energy bump may be indicative of a separation between the time scale of the unsteady shifting of the "cells" and the time scale of the fine structure forming the "cells".

It was thought that the energy bump could perhaps be interpreted as L.N. Howard's "blob" frequency but a calculation yields a formation time of .2 secs or 5 cps.

The fine structure time scale could be defined by the time to transport a parcel between boundary layers. A balance of forces for a parcel in the interior is written using a rough estimate of the fine structure temperature spikes from the "cell" structure as \( \Delta T \) in the buoyancy term.

\( f_b = \) buoyancy force per unit mass

\( f_b = g \alpha \Delta T = \ddot{z} \)

\( h_x = \frac{1}{2} \ddot{z} t^2 \)

\( h_x = \) height of the interior region \( 4 \) cm.
\[ \Delta T = 5^\circ C \] obtained from the temperature records of Figs. 12, 14 and 16.

\[
t_*^2 = \frac{2 \rho}{\rho_0 \Delta T}
\]

\[
t_* = .8 \text{ sec.}
\]

\[
f_* = 1.2/\text{sec.}
\]

This leads one to suspect that something on the transport time scale is generating the bump in the power spectra. To speculate, imagine chimneys draining fluid from the boundary layer. If these chimneys oscillate spatially, they could produce the observed thermal spikes.

This is far from a conclusive argument, since the time scales discussed are only order of magnitude and the two differ by less than an order of magnitude.

Theories of homogeneous turbulence predict the exchange of energy between various wave number or frequency phenomena. Outside the thermal boundary layer, the decay of energy with frequency can be divided into two regimes. The low frequency component follows a \(-5/3\) power law which is in agreement with Komoloroff's theory for the transfer of energy in the inertial subrange (particle collision exchange). The high frequency energy decay follows a \(-5/2\) power law which is predicted when viscosity is dominant.

For the thermal boundary layer, the low frequency component of the power spectra (.02 to .1 cps) appears to obey a functional relationship for the increase of energy with height of the form:

\[
\varepsilon = \frac{\varepsilon_{1/2}}{\rho \rho \left(T_h - T_m\right)} = C z^2
\]

C is a constant.

Verification of the energy bump at 1 cps should be undertaken. Varying the Rayleigh number, and reproducing the power spectra should show a shift of the bump to lower frequencies with increasing Ra.
20 minute average
1 hour average

Fig. 10
"Cellé" boundary layer

Fig. 11
"Fine" structure boundary layer

Fig. 12
"Cells" outside boundary layer

Fig. 13
"Fine" structure outside boundary layer

Fig. 14
"Fine" structure interior

Fig. 16
1 hour average

\[
\frac{\sigma}{\Delta T}
\]

fig. 17
Fig. 18

power density

frequency cps

- - - - .38 mm
- - - - 1.27 mm
- - - - 1.78 mm
- - - - 2.29 mm

Fig. 18
Fig. 19
Fig. 20
power density

Fig. 21
Appendix

Calculation of Radiational Heat Exchange & Effect on Resistance Thermometer

A body will radiate energy according to the Stefan-Boltzmann law:

\[ Q = A \varepsilon F \sigma T^4 \]

\( \varepsilon \) = emissivity of the material = .24 for AL.

\( \sigma \) is a constant = 5.71x10^{-12} \( \frac{\text{Watts}}{\text{cm}^2 \ \text{o} \ \text{K}^4} \)

\( T \) = temperature of the body in absolute = 346\(^\circ\)K

Using the above numbers yields:

\[ Q_R = 22 \text{ watts} \]

Air is transparent to radiational energy but the resistance thermometer has an absorption coefficient of \( \gamma = .28 \). The radiational energy flux from a point source decays as \( 1/r^2 \). The plate can be thought of as the sum of many such points. The amount of radiation energy exchanged between two bodies will depend on their separation distance and the ratio intercepted radiational energy of a first body to the total radiation energy emitted by the second body is defined as the shape factor \( (F_{2-1}) \).

The problem is determine in what region the probe is influenced to a large degree by the high temperature reservoir. It is valid to approximate the resistance thermometer wire by a rectangular plate provided the region of interest is several wire diameters away from the reservoir.
Roshenow and Choi (Ref. 10) plot shape factor (F) versus plate separation (Z) for square bodies. Using a wire length of 3.5 mm and wire diameter of .004 mm yield a length scale, based on the area of the probe, = .13 mm.

The following shape factors are taken from (Ref. 10).

<table>
<thead>
<tr>
<th>Z (mm)</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1</td>
<td>.27</td>
</tr>
<tr>
<td>.2</td>
<td>.12</td>
</tr>
<tr>
<td>.3</td>
<td>.07</td>
</tr>
<tr>
<td>.4</td>
<td>.04</td>
</tr>
<tr>
<td>.5</td>
<td>.03</td>
</tr>
</tbody>
</table>

The energy balance of the probe neglecting the effect of wire heating yield the error in temperature.

\[
\Delta T = \frac{C F z}{V_w} \left( T_C^4 - T_w^4 \right)
\]

Where the wire temperature is obtained from the mean temperature profile.

A calculation at .2 mm yields:

\[
\Delta T = .4 \, ^\circ C
\]
References


