Localization instability and the origin of regularly-spaced faults in planetary lithospheres

by

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Abstract

Brittle deformation is not distributed uniformly in planetary lithospheres but is instead localized on faults and ductile shear zones. In some regions such as the Central Indian Basin or martian ridged plains, localized shear zones display a characteristic spacing. This pattern can constrain the mechanical structure of the lithosphere if a model that includes the development of localized shear zones and their interaction with the non-localizing levels of the lithosphere is available. I construct such a model by modifying the buckling analysis of a mechanically-stratified lithosphere idealization, by allowing for rheologies that have a tendency to localize.

The stability of a rheological system against localization is indicated by its effective stress exponent, $n_e$. That quantity must be negative for the material to have a tendency to localize. I show that a material deforming brittle or by frictional sliding has $n_e < 0$. Localization by shear heating or grain size feedback in the ductile field requires significant deviations from non-localized deformation conditions.

The buckling analysis idealizes the lithosphere as a series of horizontal layers of different mechanical properties. When this model is subjected to horizontal extension or compression, infinitesimal perturbation of its interfaces grow at a rate that depends on their wavelength. Two superposed instabilities develop if $n_e < 0$ in a layer overlying a non-localizing substratum. One is the classical buckling/necking instability. The other gives rise to regularly-spaced localized shear zones, with a spacing proportional to the thickness of the localizing layer, and dependent on $n_e$. I call that second instability the localization instability. Using the localization instability, the depth to which fault penetrate in the Indian Ocean and in martian ridged plains can be constrained from the ridge spacing. The result are consistent with earthquake data in the Indian Ocean and radiogenic heat production on Mars. It is therefore possible that the localization instability exerts a certain control on the formation of fault patterns in planetary lithospheres.
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5-11 a) Growth spectra of the lithosphere models for Solis Planum (thick line, 60 km thick crust) and the northern lowlands (thin line, 30 km thick crust). Strength profiles for the models of b) Solis Planum and c) the northern lowlands. The geotherm is 12 K.km$^{-1}$ and $n_e = -16$ in the brittle regime in both cases. The shaded regions in a) show the range of ridge spacing in Solis Planum (30 to 50 km), and the northern lowlands (80 to 100 km). The spikes in the growth spectra indicate the localization instability.
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Chapter 1

Introduction

1.1 The importance of planetary lithospheres

The surface of the Earth and other solar system bodies is shaped by three main geological processes: the distortion of rocks, or tectonism, the generation, transport, and eruption of melts, or magmatism and the erosion, transportation, and deposition of mineral grains, or sedimentation. The tectonic and magmatic activity of the Earth, Venus, Mars, and the early Moon and Mercury is driven by the need to evacuate the heat produced by radioactive decay in their interior. An alternative source of heat in the icy satellites Io, Europa, Ganymede, and possibly Enceladus, Triton, and Miranda, is the tidal distortion of their interior as they orbit their central body. Variations of the spinning rate or the obliquity of a planet or satellite can also deform the surface. Impacts by asteroids and comets is another important cause of tectonic and magmatic activity in all of these objects, as well as smaller satellites and asteroids. Impacts can be the dominant geological process even in sizeable objects like the satellite of Jupiter, Callisto. Sedimentation, which is fluid-driven, has been important only on the Earth and the early Mars.

The ability of these solar system objects to retain surface structures over long timescales indicates the presence of a rigid lithosphere (sphere of stone in Greek). The “stony” material may be ice in the satellites of giant gas planets, which, under the temperature conditions of the outer solar systems, behaves much like terrestrial rocks.
The lithosphere may be contrasted with the atmosphere (sphere of vapor) prominent in the giant gas planets, and on Earth with the asthenosphere (weak sphere), which occupies the deeper levels of the Earth interior, beneath the lithosphere. These layers flow at shorter time scales than the lithosphere does. However, the lithosphere will also relax eventually, as no deformation is truly permanent. For instance, impact basins eventually relax and leave only a surface scar on icy satellites. On Earth, erosion and sedimentation eventually erase mountain belts and volcanoes and fill rift valleys. It remains that the lithosphere is the layer in each solar system object that remains rigid over the longest geologically significant periods. Hence, it provides us with crucial information about the internal and external history of a solar system object, from which we may deduce the phenomena that control its evolution.

Like any recording medium, the lithosphere modulates whatever signal is imposed on it. Impact craters may form a central peak or rings depending on the force of the impact relative to the mechanical behavior of the lithosphere [Melosh, 1989]. The shape of riverbeds expresses the strength of the different strata that they cut. Tectonic structures, being generated by the deformation of the lithosphere, depend critically on its mechanical properties. A mountain belt reaches a maximum height controlled by the strength of the lithosphere [Molnar and Lyon-Caen, 1988]. The flexural rigidity of the lithosphere influences the width of sedimentary basins [Allen and Allen, 1990]. Even the thermal evolution of a terrestrial body is influenced by the behavior of the lithosphere. A strong lithosphere reduces the efficiency of convection in a planetary interior [Solomatov, 1995; Conrad and Hager, 1993], leading the interior to retain more heat than if the lithosphere was mobile, thereby causing the planet to heat up. This may result in more vigorous convection and melt production, although their expression at the surface may be reduced. A mobile lithosphere as on Earth, might result in a relatively cool interior. It has been proposed that the Martian interior has been cooled by an early occurrence of plate tectonics [Nimmo and Stevenson, 2000].

To understand how the lithosphere influences geological processes and planetary evolution, we must know its mechanical properties. On Earth, several techniques are available that indicate what the lithosphere is made of. They include seismic
reflection and refraction, topography analysis, electromagnetic sounding, heat flow measurements, structural geology, and chemical analyses of sediments or xenoliths (rock carried in volcanic eruptions). Laboratory experiments characterize the behavior of rocks, giving a possible picture of the manner that the lithosphere behaves. This understanding of the lithosphere's behavior must be verified by comparing the patterns of tectonism and volcanism that it predicts with actual geological features. On other planets, very little information about the composition of the lithosphere is available. Hence, theoretical models based on our experience with the geology of the Earth must be adapted for other planets so that the geological structures observed from robotic exploration can be used to constrain the structure of the lithosphere.

1.2 Mechanical properties of the lithosphere and non-localized tectonic patterns

The rheology of a rock determines its response to a loading. A load may be quantified by a deviatoric stress, and the response by the resulting strain or strain rate [Turcotte and Schubert, 1982; Guéguen and Palciauskas, 1994; Ranalli, 1995]. If the loading is of relatively small intensity, the rock behaves elastically; the load is supported by recoverable strains. If the ambient temperature is high enough, crystalline defects—vacancies and dislocations—are mobilized. Then, the rock flows like a fluid, although the strain rate, which is controlled by the applied stress, the temperature, the grain size, and chemical variables such as oxygen and water fugacity, is rather slow [Evans and Kohlstedt, 1995; Kohlstedt et al., 1995]. If the deviatoric stress is high enough compared to the pressure, brittle failure may occur, which is indicated by the opening and sliding of cracks [Paterson, 1978; Lockner, 1995]. Upon failure, the rock accumulates a permanent, or plastic, deformation.

Brittle failure and ductile flow are introduced here as microscopic deformation mode of rocks. The behavior of the lithosphere is also characterized by the type of deformation patterns produced by these deformation mechanisms. The large-scale
deformation field may be localized, meaning that deformation is enhanced in a particular location. Ductile flow is sometimes used in contrast to localized deformation [Evans and Kohlstedt, 1995]. However, it is important to differentiate between the rheology of the rock, which can be described only by local properties, and the tendency to form localized deformation patterns. These patterns develop at a scale that is larger than that at which the deformation mechanism is active. Therefore, one may reserve the appellation of ductile for the fluid-like behavior of rock and characterize the large-scale deformation patterns as localized or distributed. Localized deformation is often associated with brittle failure processes, although localized ductile shear zones are observed as well [Ramsay, 1980].

Theoretical models are needed to link the laboratory-determined rock behavior to observed tectonic patterns. As mentioned above, these models are the only way to constrain the mechanical structure of the lithosphere of other planets. Understandably, most of these models are inspired by continuum mechanics and structural engineering theories. They necessarily involve a simplification of the lithosphere’s mechanical properties of one sort or another to constitute a well-posed problem.

Foremost among these geodynamics models is the flexural analysis that approximates the lithosphere as a thin elastic plate [Love, 1927; Turcotte and Schubert, 1982]. The flexural rigidity, $D$, of that plate, is proportional to $H_E^3$, with $H_E$ the thickness of the equivalent elastic plate, which corresponds roughly to the depth range of the lithosphere that does not flow ductily and does not fail brittly under the imposed load [Goetze and Evans, 1979; McNutt, 1984; McAdoo and Sandwell, 1985; Solomon and Head, 1990]. The load, usually a volcanic edifice, a mountain belt, a subducted slab, or a mantle plume, produces deflections of that plate, with a horizontal scale that is related to the flexural rigidity. The deflections of the plate may be constrained using the topography of the solid surface, the gravity field, and possible sedimentation and drainage patterns. Elastic flexure analysis usually ignores that the actual lithosphere has a finite thickness and that ductile flow and plastic deformation can produce their own structures. Nevertheless, flexural analysis is widely used in many settings and provides a useful first look at the structure of the lithosphere, provided
that the lithosphere is loaded from the top or from the bottom.

Elastic flexure has more difficulty explaining the behavior of the lithosphere under horizontal end-loads. The lithosphere displays regular undulations in several locations where it is thought to be under horizontal compression [Weissel et al., 1980; Burov et al., 1993; Jin et al., 1994]. An elastic plate under horizontal compression may become unstable and fold if the load exceeds a critical value [Turcotte and Schubert, 1982]. As the wavelength of these folds is proportional to $D^{1/4}$ or $H_{E}^{3/4}$, it can be used to estimate the elastic thickness of the lithosphere. However, the minimum end-load that can produce undulations at the observed wavelength exceeds reasonable estimates of the strength of the lithosphere [Turcotte and Schubert, 1982]. Hence, the lithosphere should lose its elastic behavior before the elastic plate buckles. In addition, buckling of an elastic plate requires horizontal compression, whereas there are examples of undulations in extensive environments, most notably the Basin-and-Range [Smith, 1978; Fletcher and Hallet, 1983; Froideveau, 1986; Zuber et al., 1986].

Fortunately, buckling is not restricted to elastic plates. Biot [1961] pioneered the use of viscous and elasto-viscous rheologies in the instability analysis of thin plates. Fletcher [1974] and Smith [1977] derived a buckling theory using a non-linear viscous rheology, which is a good description of the ductile flow of rocks. Plastic flow has been approximated by highly non-Newtonian creep [Smith, 1979; Fletcher and Hallet, 1983; Zuber et al., 1986]. In addition, the thin plate formalism was abandoned in favor of a thick plate theory that allowed smaller wavelengths to be considered, and also periodic undulations of the lithosphere to develop in extension (necking) as well as in compression (buckling) [Smith, 1975, 1977; Fletcher and Hallet, 1983; Zuber and Parmentier, 1986; Zuber et al., 1986]. The wavelengths of the buckling and necking instabilities are proportional to $H_{S}$, the thickness of a strong layer that may correspond to the depth range in the lithosphere where rocks are predominantly brittle [Fletcher and Hallet, 1983]. This theory has been applied in many geological examples on Earth [Fletcher and Hallet, 1983; Zuber et al., 1986; Ricard and Froideveau, 1986; Zuber, 1987a; Bassi and Bonnin, 1988a; Martinod and Davy, 1992; Burov et al., 1993], Mars [Zuber and Aist, 1990], Venus [Zuber, 1987b; Zuber and Parmentier, 1990], and
Ganymede [Fink and Fletcher, 1981; Dombard and McKinnon, 1996], two of which we will revisit in the Chapters 4 (Central Indian Ocean) and 5 (martian wrinkle ridges).

In addition to allowing buckling and necking of the lithosphere, ductile flow plays an important role in a wide variety of tectonic environments [England and McKenzie, 1982; Sonder and England, 1986; England and Molnar, 1997; Deng et al., 1998; Freed and Lin, 2001]. For instance, a zone of lithospheric convergence may produce flat plateaus as the result of ductile flow [Bird, 1991; Royden, 1996]. The slope of the Tibetan plateau may reflect the viscosity of the lower crust [Clark and Royden, 2000]. Wide strike-slip plate boundaries may indicate ductile flow in the lithosphere [Bourne et al., 1998]. Ancient orogens collapse through ductile flow under the effect of gravity if the horizontal convergence that created them ceases [Artyushkov, 1973; Molnar and Tapponnier, 1978; Molnar and Lyon-Caen, 1988; Sonder and Jones, 1999]. Impact craters may relax over time as the lithosphere and the underlying asthenosphere flow into the depression [Thomas and Schubert, 1986; Hillgren and Melosh, 1989; Grimm and Solomon, 1988; Dombard and McKinnon, 2000]. All these phenomena can be modeled and their recognition at the surface of solid planets constitute so many possible probes into their lithosphere.

1.3 Patterns of localized structures

All the models mentioned above predict distributed deformation in the lithosphere. Unfortunately, distributed deformation patterns are sometimes hard to recognize from satellite data. Folds are easily recognized on Earth from satellite images because erosion exposes different rock types at the core of folds. This is of little help on other planets where erosion and sedimentation are minor. Folds can be expressed in the gravity and magnetic fields, but the resolution of the data available for planets other than the Earth is seldom sufficient, and their interpretation is non-unique. Analyses of the topography of other planets are also difficult because the configuration of the surface before a particular tectonic event is not known.

On the other hand, localized deformation is easy to recognize. The intensity of
the deformation is high within a narrow zone, and vanishes out of it, so that faults can be recognized in satellite images at visible as well as radar wavelengths. If the displacement on a localized shear zone is high enough, it may juxtapose crustal blocks of different origins, so that the fault can be identified from multispectral images or from gravity, topography or magnetic data. Sharp features like faults scars may be quickly eroded on Earth, However, the fault zone and the surrounding blocks may have different erodability, so that the fault may have leave a specific signatures in the configuration of drainage networks or the profile of incised riverbeds.

Localized features can form patterns of their own, observed in a variety of tectonic environments. The grooved terrain on Ganymede displays relatively long-wavelength undulation indicative of necking, but superposed on them are fault blocks with a characteristic width, the grooves themselves [Collins et al., 1998, Fig. 1.1b]. Sub-parallel strike-slip faults accommodate plate motions in California (Fig. 1.1b) and in New Zealand [Bourne et al., 1998; Molnar et al., 1999]. Horizontal shortening is expressed by wrinkle ridges and fault scars on the Moon, Mercury, Mars and Venus [Watters, 1992]. Wrinkle ridges are probably underlain by a brittle fault [Plescia and Golombek, 1986; Golombek et al., 1991; Schultz, 2000] and form arrays of sub-parallel faults [Watters, 1988; Zuber and Aist, 1990, Fig. 1.1c]. Large orogens on Earth are composed of a succession of sub-parallel slivers, like in the North Canadian Asiak Chain [Hoffman et al., 1988, Fig. 1.1d] or the European Pyrenees [De Paor, 1992; Muñoz, 1992]. In these examples and in others [Bonnet et al., 2001; Ackermann et al., 2001], the faults form sub-parallel arrays, and have associated with them a characteristic spacing. It might be expected that this spacing is related to the depth range where rocks undergo localization.
1.4 Modeling the origin of regularly-spaced localized structures

Unfortunately, the formation of localized features is seldom incorporated in geodynamics models. As we will see in Chapter 2, the viscous and elastic rheologies often used to model lithosphere dynamics inherently form non-localized features. Faults in models are employ these rheologies are either defined \textit{a priori} in the geometry of the lithosphere [Beaumont and Quinlan, 1994], or computed \textit{a posteriori} from the geometry of the non-localized stress field [Anderson, 1951]. As these models ignore the interaction between developing localized features and the stress fields, it may be misleading to use them to explain tectonic patterns expressed by localized features. The interaction between localization and the stress field leads to a rather complex behavior. Deformation mechanisms active at the microscopic scale interact with feedback processes that may be at a much larger scale. In this thesis, I present a simple characterization of the behavior of a material undergoing localization. Then, I modify the buckling/necking analysis of non-Newtonian viscous lithosphere to include materials with these properties and apply this new analysis to two examples of tectonics with regularly-spaced faults, the Central Indian Ocean, and martian ridged plains.

In Chapter 2, I review several processes that produce localized deformation. If localization arises from small perturbations of an initial deformation state, the apparent strength of a rock, $\sigma$ must decrease with deformation. The deformation state

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Figure 1-1 (facinging page): a) Galileo image of Uruk Sulcus on Ganymede overlain on a topographic model from stereo images (courtesy of R. Pappalardo). The topographic undulations have a wavelength of $\sim 6$ km whereas the fault-bounded grooves have a wide of $\sim 0.2$ m; b) Earthquake locations in California, showing the sub-parallel faults that are the San Andreas fault system (after [Hill et al., 1990]; c) Radar map of Zhibek Planitia, Venus, showing wrinkle ridges (map centered on 33°S, 171°E, made from Magellan images using PDS MAP-IMAGE at http://pdsmaps.wr.usgs.gov/maps.html, resolution 176 Pixel/degree); d) Geological cross-section of the Asiak Mountain belt, (Wopmay orogen, at the border of the Slave craton, Northern Canada), showing a succession of fault-bounded crustal slivers (from Hoffman et al. [1988]).
is generally quantified by a variable called $\chi_0$ that may be the strain or the strain rate. The effective stress exponent, defined as

\[
\frac{1}{n_e} = \frac{\chi_0}{\sigma} \frac{d\sigma}{d\chi_0},
\]

expresses the rate of weakening scaled by the current $\sigma$ and $\chi_0$. Thus, $n_e$ provides a measure of the efficiency of localization. Localization requires negative $n_e$ and is more efficient for more negative $1/n_e$. It is important to include in Eq. 1.1 the contributions of all the variables coupled to $\chi_0$ that influence the rheology. Together, they form the rheological system. The effective stress exponent is defined for any rheological system, and provides a unified measure to compare the efficiency of different systems. I present in Chapter 2 the mathematical expression of $n_e$ for a broad range of localization processes: loss of cohesion upon failure; non-associated elastic-plastic flow; rate-and-state dependent friction; shear heating; dynamic grain size reduction. The effective stress exponent for brittle processes is of order -10 to -50 and between -50 and -300 for frictional sliding. Localization in the ductile regime needs to be triggered by a pre-existing heterogeneity that may be a brittle fault. I also compare the effective stress exponent with other criteria and theories of localization by Rudnicki and Rice [1975] and Bercovici [1993]. Chapter 2 is based on Montési and Zuber [2001] with the addition of §2.8 and the discussion of rate- and state- dependent friction with two state variables in §2.6.2.

The concept of effective stress exponent allows simplified models of localization in the lithosphere to be produced. Instead of considering the full complexity of the rheological system undergoing localization, these models can use a simpler rheology, which would have a similar $n_e$. An effective stress exponent was already used by Smith [1977] in buckling and necking models of the lithosphere. However, $n_e$ was always positive in his models, to correspond to the ductile behavior of the lithosphere. In Chapter 3, I modify the buckling/necking theory to allow $n_e$ to be negative, thereby accounting for localizing behavior. The theory is also modified to handle depth-dependent strength profiles. The formulation of the instability analysis is presented in the Appendix. I
show that a layer of thickness \( H \) and \( n_e < 0 \), lying over an infinite substrate with \( n_e > 0 \), undergoes two superposed instabilities. One is the buckling/necking instability, with wavelength \( \lambda_{B/N} \). The other, that I call the localization instability, produces regularly-spaced faults. The instability wavelength is \( \lambda_L \). Both \( \lambda_{B/N} \) and \( \lambda_L \) are related to resonances between the different deformation modes. From these resonances, I introduce the approximate relations:

\[
\frac{\lambda_{B/N}}{H} = \frac{2}{j + 1/2 - a_{B/N}} \times (1 - \frac{1}{n_e})^{1/2}, \quad (1.2)
\]

\[
\frac{\lambda_L}{H} = \frac{2}{j + a_L} \times (-\frac{1}{n_e})^{1/2}, \quad (1.3)
\]

with \( j \) an integer, and \( a_{B/N} \) and \( a_L \) two parameters called the spectral offsets that indicate the effect of the density structure of the lithosphere and the depth-dependent profile. If the strength of the brittle layer increases linearly with depth and the strength of the ductile substratum decreases quasi-exponentially with depth, as is thought to be the case in the lithosphere, I find

\[
0 < a_{B/N} < 1/4 \quad \text{in compression},
\]

\[
-1/4 < a_{B/N} < 0 \quad \text{in extension}, \quad (1.4)
\]

\[
1/4 < a_L < 1/2 .
\]

In the last two chapters, the localization instability is applied to two examples of compressive tectonics with regularly-spaced faults. The first is the Central Indian Basin, on Earth [Chapter 4], where reverse faults have an average spacing of 7 to 11 km [Weissel et al., 1980; Bull, 1990; Van Orman et al., 1995]. Laboratory experiments on rock deformation as well as earthquake focal depths indicate that the brittle-ductile transition of the lithosphere, which marks the limit of brittle failure, is about 40 km deep in that region. If the faults penetrate to 40 km, the effective stress exponent of the brittle lithosphere must be about -300. The inferred \( n_e \) is that of an equivalent layer in with \( n_e \) does not vary with depth. However, the situation in the Central Indian Basin is probably more complex, as the faults active at
depth are probably not inherited structures; the localization mechanism, and therefore \( n_e \) is probably different between a near-surface region where faults are reactivated structures [Bull and Scrutton, 1990] and the deeper lithosphere, where faults probably formed during the current compressive tectonics episode. The inferred value of \( n_e \sim -300 \) is compatible with localization during frictional sliding on pre-existing faults. The instability analysis with negative \( n_e \) predicts simultaneous buckling and development of regularly-spaced fault patterns, as is indeed observed in the region.

Finally, I focus in Chapter 5 on the case of wrinkle ridges on Mars. On Mars, we do not have independent information on the lithosphere structure that would limit the possible range of thickness of localizing layer. However, the spacing of wrinkle ridges seems to vary systematically from highland ridged plains, where is it \( \sim 40 \) km, to the Northern lowlands, where it is \( \sim 80 \) km. The lowlands ridges have been only recently recognized from MOLA altimetric data. The difference of ridge spacing between these two regions corresponds to a difference of crustal thickness: the lowland crust is 20 to 30 km thick, whereas the crust in the ridged plains is 50 to 60 km thick. There is a range of geotherms for which the highland crust becomes ductile at depth, but the lowland crust is entirely brittle. Postulating that the difference of ridge spacing corresponds to a control by the brittle-ductile transition of mantle rocks in the lowlands and by crustal rocks in the highlands, I constrain the effective stress exponent to be \( \sim -16 \), compatible with brittle failure, and the geotherm between more than 9 K.km\(^{-1}\) in the highlands and less than 15 K.km\(^{-1}\) in the lowlands. This geotherm would be sufficient to carry the heat produced radiogenically approximately at the time of ridge formation. I also present finite element models that show how slight gradients of crustal thickness or of geotherm can preferentially select a fault vergence, as is observed in several regions of Mars.

### 1.5 Future directions

The material presented in this thesis is but the beginning of a new series of tectonic models that include localization. Hopefully, many more applications will be devised in
the years to come. The localization instability may be responsible for the arrangement of tectonic features with a characteristic spacing that may be underlain by faults in provinces other than the Central Indian Basin and the martian ridged plains discussed in Chapters 4 and 5, such as:

- the Basin-and-Range province of the Western United States [Ricard and Froideveau, 1986; Zuber and Parmentier, 1996]
- Mountain belts in Asia, in particular the Tien Shan [Burov et al., 1993]
- Thrust sheet and accretionary prism [Davis et al., 1983; Hoffman et al., 1988]
- Abyssal hills near Mid-Ocean ridges [Goff et al., 1997; Buck and Poliakov, 1998]
- Wrinkle ridges and ridge belts on Venus [Zuber and Parmentier, 1990; Solomon et al., 1992; Banerdt et al., 1997; Bilotti and Suppe, 1999]
- Grooved terrain on Ganymede [Collins et al., 1998; Pappalardo et al., 1998; Patel et al., 1999].
- Ridges, bands and folds on Europa [Head et al., 1999b; Pappalardo et al., 1999; Prockter and Pappalardo, 2000].

In addition, the instability analysis ought to be modified to relieve some of the simplifications that have been made up to now. Addressing the following questions would significantly improve our understanding of the formation of patterns of localized structures in planetary lithospheres:

- What is the effect of finite strain and progressive yielding of the lithosphere on the patterns of faulting?
- What patterns result from strain weakening instead of strain-rate weakening?
- How does weakening following plastic volume changes differ from weakening of shear deformation? Can this instability analysis be modified to produce patterns of rock joints or dikes?
- How does elastic deformation, which includes volume changes, influence localized patterns?

- Is a fault direction preferentially selected if the weakening is anisotropic or has a memory of past deformation?

- How does a pre-existing fault influence the pattern of faulting?

Furthermore, the approach to localization that I develop in this thesis, including the recognition of the apparent behavior of a rheological system and its characterization with the effective stress exponent, can be applied to other tectonic regimes. For instance a similar analysis may be conducted with strike-slip loading, which would lead to pattern of strike-slip faults [Bourne et al., 1998; Molnar et al., 1999; Roy and Royden, 2000a, b]. Melt forms channels with feedbacks that include the dynamic pressure, and mineral composition [Stevenson, 1989; Aharonov et al., 1995; Kelemen et al., 1995; Aharonov et al., 1997; Hall and Parmentier, 2000]; this system should be characterized and simple models of melt channel formation ought to be devised. The importance of large-scale feedback processes should be investigated, with the possible implications for the formation of plate boundaries and their influence on global convection and tectonic patterns [Bercovici, 1993; Bercovici et al., 2001b; Tackley, 2000a].
Chapter 2

A unified description of localization for application to large-scale tectonics

Abstract

Localized regions of deformation such as faults and shear zones are ubiquitous in the lithosphere of the Earth. However, we lack a simple unified framework of localization that is independent of the mechanism or scale of localization. We address this issue by introducing the effective stress exponent, $n_e$, a parameter that describes how a material responds to a local perturbation of an internal variable being tested for localization. The value of $n_e$ is based on micromechanics. A localizing regime has a negative $n_e$, indicating a weakening behavior, and localization is stronger for more negative $1/n_e$. We present expressions for the effective stress exponent associated with several mechanisms that trigger localization at large scale: brittle failure with loss of cohesion, elasto-plasticity, rate- and state- dependent friction, shear heating, and grain-size feedback in ductile rocks. In most cases, localization does not arise solely from the relation between stress and deformation but instead requires a positive feedback between the rheology and internal variables. Brittle mechanisms (failure and friction) are generally described by $n_e$ of the order of -100. Shear heating requires an already localized forcing, which could be provided by a brittle fault at shallower levels of the lithosphere. Grain size reduction, combined with a transition from dislocation to diffusion creep, leads to localization only if the grain size departs significantly from its equilibrium value, either because large-scale flow moves rocks through different thermodynamic environments, or new grains are nucleated. When shear heating or grain-size feedback produce localization, $1/n_e$ can be extremely negative and control lithospheric-scale localization.
2.1 Introduction

Extensive geological and geophysical observations indicate that tectonic deformation in the Earth’s lithosphere is not uniform but localized. Localized shear zones range in scale from microscopic cracks and foliation to brittle faults, ductile shear zones, even to entire plate boundaries. Whereas the microscopic processes leading to shear localization have been studied in the field and in the laboratory, the manner by which these processes influence large-scale tectonics has only rarely been assessed [Hobbs et al., 1990; Burg, 1999], partly because of the great variety of these localization processes and partly because of their complexity. To remedy this situation, we derive a general framework for the study of localization based on the effective stress exponent of a rheological system, $n_e$. This quantity characterizes the non-linear behavior of a rheological system and provides a measure of localization efficiency. As this measure is defined independently of the actual variables involved during localization, it allows the importance of different localizing processes for large-scale tectonics to be evaluated.

We will show how to compute the effective stress exponent for processes associated with localized shear zones or faults in the lithosphere.

Defining a localized shear zone is subjective. In contrast to fluid-like behavior, it implies that some measure of deformation (e.g., strain, strain rate) is significantly heterogeneous in a given region, often being greatly enhanced within a narrow area. The scale of observation is important to define localization. For instance, the 1000 km wide deformation area in the Central Indian Basin is a diffuse, or non-localized, plate boundary [Gordon, 2000], but deformation within it is concentrated on localized faults [Weissel et al., 1980]. Faults themselves reveal a fluid-like gouge [Sibson, 1977]. At yet smaller scales, localized slip surface and microcracks are apparent [Simpson, 1984; Scholz, 1990]. In this study, we address localization from the point of view of large-scale tectonics. When deriving the effective stress exponent, we consider a micromechanical localization process, but we are interested only in its apparent behavior at larger scales. In this paper, the effective stress exponent characterizes a material with a fault or a shear zone in it, not the fault or the shear zone itself.
Faulting has been included in numerous geodynamical models, but rarely in a self-consistent way. Frequently, faults are considered as pre-defined boundaries in otherwise elastic, viscous or plastic models [Dahlen and Suppe, 1988; Melosh and Williams, 1989; Beaumont and Quinlan, 1994; Beekman et al., 1996; Zhong and Gurnis, 1996]. Although pre-existing structures have a great importance in tectonic deformation, this approach lacks generality in that it concerns only a specific geometry and does not address the origin of these faults. In other studies, faulting is included only \textit{a posteriori}, by applying a failure criterion to a continuum model [Anderson, 1905; Hafner, 1951; Comer et al., 1985; Zuber, 1995]. The results of these models are questionable beyond initial faulting as they neglect how the development of these faults influences the stress field [Buck, 1990; Schultz and Zuber, 1994; Gerbault et al., 1998]. Finally, a few studies applied slip-line theory to lithospheric deformation [Odé, 1960; Tapponnier and Molnar, 1976; Lin and Parmentier, 1990; Regenauer-Lieb and Petit, 1997]. In this approach, the trajectory of discontinuities of the flow field is predicted. However, slip line theory implies a continuum of slip lines rather than individual faults. Slip-line solutions suffer from non-uniqueness and are restricted to rigid-plastic materials [Hill, 1950]. None of these approaches consider the dynamic evolution of the stress field during the development of these faults.

Recently-developed damage theories [Bercovici, 1998; Tackley, 2000b] and elasto-visco-plastic models [Poliakov et al., 1994; Buck and Poliakov, 1998; Gerbault et al., 1998; Lavier et al., 1999; Branlund et al., 2000] do follow the dynamic evolution of localized deformation, but they require numerical methods to solve realistic problems. Hence, the need is great for quasi-analytical analyses that provide physical foundations for more complex models. For instance, 1- and 2-dimensional shear zone models [Tackley, 1998] as well as a boundary layer theory of mantle convection including dynamic localization [Bercovici, 1993] were constructed using the self-lubricating rheologies introduced by Bercovici [1993]. The effective stress exponent defined in this study provides a general framework of localization that allows the comparison of analytical models, numerical models, and geological observations, although they may not use the same localization mechanism. A first quasi-analytical model that utilizes
the concept of effective stress exponent is presented elsewhere [Montési and Zuber, 1998, Chapter 3].

Before introducing the effective stress exponent, we define three categories of localizing behavior: inherited, imposed, and dynamic localization. The effective stress exponent is important in the context of dynamic localization. We present how the effective stress exponent relates to other approaches to localization, such as the bifurcation analysis [Rice, 1976; Needleman and Tvegaard, 1992] and self-lubricating rheologies [Bercovici, 1993]. Then, after reviewing briefly the different microscopic mechanisms of localization, we present the mathematical expressions of the effective stress exponent for several of these mechanisms. We choose to group these mechanisms into: 1) localization during failure, including the laws of elasto-plasticity often studied in relation with the bifurcation approach to localization; 2) localization during frictional deformation, including rate- and state-dependent friction laws and processes linked to the granular aspects of fault gouge; and 3) localization in the ductile regime, including shear-heating or grain–size feedbacks, which might contribute to the formation of ductile shear zones at deeper levels in the lithosphere.

2.2 Effective Stress Exponent and Other Concepts of Localization

2.2.1 Types of Localization

We define three ways by which localized deformation may occur. In the first, termed inherited localization, the deforming medium is initially heterogeneous. Localization is controlled by the pre-existing structure of the lithosphere, especially by pre-existing zones of weakness such as alteration and damage zones, plutonic intrusions, sutures, or greenstone belts. The second, imposed localization, occurs when the medium is homogeneous, but its boundaries are not, resulting in local stress enhancements. Again, localization depends on the pre-existing configuration. For instance, major faults in Tibet originate at the corners of the Indian tectonic indenter [Tapponnier and Molnar,
Finally, in *dynamic localization*, deformation localizes as the properties of the medium evolve the deformation. Dynamic localization requires that deformation is “easier” at locations where deformation has been enhanced. We quantify what “easier” means with the effective stress exponent. Dynamic localization does not require that the geometry of the localizing material or its surrounding be specified *ad hoc*. Certainly, the lithosphere is not a perfectly uniform medium and it is not stressed uniformly, but we assume that at large scale, these heterogeneities are sufficiently regular to appear as a uniform fabric. In that case, dynamic localization results in one of the embedded heterogeneities dominating the deformation.

### 2.2.2 Localization and effective stress exponent

Dynamic localization occurs via a feedback between a material’s rheology and its deformation field. In general, the rheology relates a set of variables \{\chi_i\}, the *rheological system*, that describe the state of a parcel of the material, to its strength \(\sigma\)

\[
\sigma = \sigma (\{\chi_i\}). \tag{2.1}
\]

The most commonly used variables \(\chi_i\) include strain, strain rate, chemical composition, temperature, and pressure. They may also include variables describing the integrated history, or equivalently the current physical or chemical state of the material element. Here, each \(\chi_i\) and \(\sigma\) are scalars which may be invariants of more general tensorial quantities.

One of these variables, \(\chi_0\), is identified as the localizing quantity. Dynamic localization of \(\chi_0\) occurs if, when \(\chi_0\) is perturbed from an equilibrium value by \(d\chi_0\), the system of internal variables adjusts in such a way that an additional increment of \(\chi_0\) is generated, which pushes \(\chi_0\) even further from its original equilibrium value. In particular, if both \(\sigma\) and \(\chi_0\) are positive, the response of the material that brings localization is a weakening one; the sign of the strength changes, \(d\sigma\), is opposite to the sign of \(d\chi_0\).

For the case where \(\chi_0\) is the plastic strain \(\varepsilon^p\), *Drucker* [1952] proposed that a
material is stable if the work done by an increment of strain is positive, \( d\sigma_{ij} d\varepsilon_{ij}^p > 0 \). Localization occurs otherwise. This criterion was subsequently extended by Hill [1958] and Drucker [1959]. On the other hand, Mandel [1966] favored the use of the tangent modulus \( A \), a fourth-order tensor that relates increments of strain and stress, in a criterion for localization. Stability is ensured if \( A \) is positive definite. A stable material in the sense of Mandel is also stable in the sense defined by Drucker [1952, 1959]. However, Mandel’s definition implies that a material is unstable if any eigenvalue of \( A \) is negative, even if Drucker’s criterion for stability is verified. Identifying localization with material instability, Mandel’s criterion is a necessary condition for localization, whereas Drucker’s criterion is sufficient for localization. Rudnicki and Rice [1975] and Rice [1976] extended the definition of \( A \) to include the effects of a planar discontinuity imbedded in the deforming medium. When \( |A| = 0 \), the deformation field undergoes a bifurcation expressed by different deformation states across the discontinuity. Localization arises through the coupling between different components of the strain and stress increment tensors by the discontinuity, although the constitutive behavior of the material is stable.

These analyses indicate that not all the components of the strain and stress tensors become unstable under the same conditions and that the coupling between the variables involved in the rheology is important. Building on Mandel [1966] and Rudnicki and Rice [1975], we consider that two scalar measures of the strain and stress tensors localize when the tangent modulus relating them — considering coupling of other internal variables or geometrical relations such as a potential discontinuity — indicates weakening. For instance, we refer to the localization of the shear strain \( \varepsilon_{zz} \) with the normal stress \( \sigma_{zz} \), both being positive values, if \( d\sigma_{zz}/d\varepsilon_{zz} < 0 \). Coupling is implicit in the use of the total derivative if more than one variable appears in Eq. 2.1. Moreover, the use of strain in the localization criterion is generalized to any quantity \( \chi_0 \) of the rheological system. Although we present only examples where \( \chi_0 \) is the strain or the strain rate in this paper, it is conceptually any variable, such as the temperature or the chemistry of a pore fluid.

The generalized tangent modulus, \( A = d\sigma/d\chi_0 \), is computed from the rheol-
ogy (Eq. 2.1), where $\sigma$ is either a component or an invariant of the stress tensor. Rephrasing Mandel’s criterion, which assumes $\sigma > 0$, quantities $\chi_0 > 0$ for which $A < 0$ localize.

Beyond a general criterion for localization, we seek a measure of localization efficiency to compare different localization processes. Hence, we scale the generalized tangent modulus by the current state of deformation, $\sigma$ and $\chi_0$ and define the effective stress exponent, $n_e$ by

$$\frac{1}{n_e} = \frac{\chi_0}{\sigma} \frac{d\sigma}{d\chi_0}.$$  \hfill (2.2)

The effective stress exponent gives the relative importance of the dynamic evolution of strength with respect to its current value. Therefore dynamic localization occurs for negative $n_e$ and is strongest for more negative $1/n_e$.

When localization is weak ($1/n_e \to 0^-$), it has little effect on the stress field; localized shear zones may be traced a posteriori on a stress field derived from continuum mechanics. If on the other hand, localization is strong ($1/n_e \to -\infty$), localization dominates the deformation. That regime is not accessible by continuum mechanics. The perfectly plastic limit, where strength is invariant of $\chi_0$, corresponds to $1/n_e = 0$. It bridges the localizing and non-localizing regimes. The condition $1/n_e = 0$ indicates also the bifurcation points of the deformation history, where two deformation states coexist, one being continuous, the other may have an active discontinuity [Rudnicki and Rice, 1975; Rice, 1976; Needleman and Tvegaard, 1992]. Hobbs et al. [1990] pointed that in that case, localization requires dynamic weakening of the deformation field that includes an active discontinuity, i.e., $1/n_e < 0$ for the relationship between the loading and strain when the discontinuity is active.

When $0 < 1/n_e < 1$, the ratio $\sigma/\chi_0$ decreases with $\chi_0$. This ratio correspond to the apparent modulus or apparent viscosity if $\chi_0$ is a strain or a strain-rate, respectively. Therefore, the material softens where $\chi_0$ is increased, so that further increases of stress result in enhanced deformation in that region. Changing $\chi_0$ by $d\chi_0$ creates a heterogeneity that can bring inherited localization (§2.2.1) even though the material strengthens and dynamic localization is not possible. We call the softening-related
localization that is possible at $0 < 1/n_c < 1$ progressive localization. It may result in a localized shear zone, but unlike dynamic localization, the stress in that shear zone is higher than in its surroundings. The stress in some natural ductile shear zones is indeed higher than in its surrounding [Jin et al., 1998] and localization in the ductile field may be progressive rather than dynamic, as our micromechanics-based approach also indicates (§2.7).

Although he did not consider the possibility that $n_e$ be negative, [Smith, 1977] showed that a general non-linear viscous rheology is characterized by an effective stress exponent. If $\eta \equiv \sigma / \dot{\varepsilon}$ is the (secant) viscosity at a given strain rate, small perturbations of strain rate behave as if they obey an effective viscosity $\eta / n_e$ [Smith, 1977]. If $n_e < 0$, the negative effective viscosity of these perturbations results in a weaker perturbed material.

The use of the total derivative $d\sigma / d\chi_0$ in Eq. 2.2 indicates that although we address the localization of a particular variable $\chi_0$, the response of the whole system of internal variables $\{\chi_i\}$ is considered (Fig. 2-1). We define the apparent rheology as the relation between $\chi_0$ and $\sigma$ that takes into account the response of the full system $\{\chi_i\}$ to changes in $\chi_0$. In practice, it is often required to truncate the system to a limited number of suitably chosen variables. Poirier [1980] and Hobbs et al. [1990] emphasized previously the difference between the direct response of the strength to a variation of $\chi_0$ and the total response of the system $\{\chi_i\}$. In most of the cases presented below, deformation does not localize solely from the direct relation between $\chi_0$ and $\sigma$, which is usually strengthening, but requires the feedback of other internal variables (Fig. 2-1). Some of the coupling may result from the geometry of the material considered, especially when a planar discontinuity is imbedded in the material [Rudnicki and Rice, 1975; Rice, 1976].

In fact, only a subset of the system of internal variables depends on $\chi_0$. For those, it is possible to write $\chi_i$ as a function of $\chi_0$ and to define $d\chi_i / d\chi_0$. The other variables cannot participate in the localization of $\chi_0$ and may be considered constant as the perturbation is applied. However, these variables may be important in determining the strength $\sigma$, and therefore the intensity of localization. As an example, let us
Figure 2-1: Yield surface $\sigma(\chi_0, \chi_1)$, where $\chi_0$ and $\chi_1$ are two internal variables that represent for instance strain rate and temperature. The strength increases if only $\chi_0$ is perturbed (dashed arrow, $\partial \sigma / \partial \chi_0 > 0$), but the system response may include also a change of $\chi_1$ induced by $\chi_0$, so that the total response is weakening (thick arrow, $d\chi_0 / d\chi_1 < 0$). The darker region of the yield surface indicate negative stress exponent, or localization.

consider a system where the strength depends on the strain rate, temperature, and pressure and examine its stability with respect to the strain rate. If the strain rate increases, the temperature increases as well because a fraction of the mechanical work is converted into heat, but the pressure may be buffered. Therefore, the derivative in Eq. 2.2 includes the variation of strength with strain (a strengthening term) and temperature (a weakening term), but not on pressure, which is to a large extent uncoupled from the strain rate. Localization is possible if the temperature change dominates over the strain rate change, but the intensity of localization depends on the initial strength of the material, which depends on the pressure as well as the initial temperature and strain rate.
2.2.3 Arresting localization

Using an effective stress exponent, $n_e$, to characterize the behavior of a rheological system suggests that its apparent rheology can be approximated by a power-law relation $\chi_0 \propto \sigma^{n_e}$, ignoring the details of the coupling between the internal variables. Such a rheology follows the same spirit as for instance the dislocation creep law, a power law between stress and strain rate that does not explicit the details of dislocation motions. While this approach can be useful in infinitesimal perturbation analyses [Smith, 1977; Montesi and Zuber, 1998], one must exert caution when the system is locally far from its original configuration. Indeed, a power law with a negative stress exponent predicts that localization continues until one of the two singularities $\sigma = \infty$ or $\sigma = 0$ is reached [Melosh, 1976; Tackley, 1998], resulting in one infinitely thin shear zone.

However, the concept of tangent modulus and effective stress exponent imply that the perturbation of the variable undergoing localization is infinitesimal. Therefore, the evolution of $n_e$ as localization progresses must be considered if the end-result of localization is sought. For instance, the thickness of a shear band or a plate boundary may be dictated by the point where $n_e$ becomes positive within the localized deformation zone. Once non-linearities in the system are taken into account, the effective stress exponent of a localizing system certainly becomes positive before the unrealistic singularities mentioned above are reached. In addition, other parameters that may be considered fixed at the onset of localization might become important when localization is ongoing. For instance, grain growth [Kameyama et al., 1997] and volatile incorporation [Regenaurer-Lieb, 1999] might stabilize shear zones formed by shear heating. Fault rotation may stabilize frictional sliding [Sibson, 1994]. The stress exponent must be negative only over a specific domain of the $\chi_0-\sigma$ space.

2.2.4 Self-lubricating Rheology

At the largest scale relevant for Earth sciences, the global deformation field is localized at plate boundaries, which appear weaker than plate interiors. The associated
viscosity heterogeneities allow for toroidal motion as well as the poloidal motion driven by convection in the mantle [Bercovici et al., 2000]. To produce dynamically the weakening associated with a plate boundary, Bercovici [1993, 1995] introduced a self-lubricating (modified Carreau) rheology:

\[
\eta = (\gamma + \dot{\varepsilon}^2)^{\frac{1}{2}}(\frac{\alpha}{\beta} - 1),
\]

where \( \dot{\varepsilon} \) is the second invariant of the strain rate tensor, \( \eta \) the apparent viscosity, and \( \gamma \) and \( \beta \) are constitutive parameters. The strength of such a material is \( \sigma = 2\eta\dot{\varepsilon} \) from which we compute:

\[
\frac{d\sigma}{d\dot{\varepsilon}} = \frac{\sigma}{\dot{\varepsilon}} + \frac{(1/\beta - 1)\sigma\dot{\varepsilon}}{\gamma + \dot{\varepsilon}^2}.
\]

Then, the inverse effective stress exponent for self-lubricating rheologies is:

\[
\frac{1}{n_e} = \frac{\gamma/\dot{\varepsilon}^2 + 1/\beta}{\gamma/\dot{\varepsilon}^2 + 1}.
\]

Dynamic localization occurs when \( 1/n_e < 0 \), or \( -\gamma/\dot{\varepsilon}^2 < 1/\beta < 0 \) (Fig. 2-2).

The self-lubricating rheology is actually an apparent rheology that approximates the weakening due to shear heating [Whitehead and Gans, 1974; Bercovici, 1993] or damage accumulation [Bercovici, 1998; Tackley, 1998, 2000b]. An alternative effective rheology could be a power law relation between strain rate and stress, with stress exponent \( n_e \). However, self-lubricating rheologies have an advantage over such power laws in that they strengthen at low strain rate [Bercovici, 1995; Tackley, 1998]. Neither the power law nor the self-lubricating rheology are valid at high strain rates, as they are cannot be differentiated in the limit \( \dot{\varepsilon} \to \infty \), with \( n_e \sim r < 0 \). Non-linearities in the feedback mechanism or additional physical mechanisms should produce \( n_e > 0 \) in that limit.
Figure 2-2: Inverse effective stress exponent for self-lubricating rheology with $r = 100$ (solid line), $r = 1$ (dashed line), $r = -1$, (dotted line), $r = -100$ (dash-dot). Circles show the limit of localizing domain when $r$ is negative ($\dot{\varepsilon}/\gamma = -r$).

### 2.2.5 Coupling Between Localizing and Non-Localizing Systems.

It is not realistic to consider all the possible couplings relevant for a particular rheological system. For instance, elastic strains are always present in a deforming continuum, but are not always included in the analyses to follow. As elasticity is strengthening, it might prevent localization in an otherwise localizing system. In this section, we show how two subsets of the rheological system $\{X_i\}$, one localizing and the other non-localizing with respect to a deformation variable $X_0$, can be coupled. Then, we address the particular case where the non-localizing system is elastic.

Rheological sub-systems parameterized by the variable $X_0$ may be coupled in two ways: internally, in which case $X_0$ is additive, or externally, in which case $\sigma$ is additive. If coupling is internal, the total $X_0$ is the sum of the contribution from the localizing
system $\chi^l_0$, and the contribution from the non-localizing system $\chi^{nl}_0$:

$$\chi_0 = \chi^l_0 + \chi^{nl}_0.$$  \hfill (2.6)

The effective stress exponents for the localizing and non-localizing parts of a rheological system are respectively $n^l_\varepsilon < 0$ and $n^{nl}_\varepsilon > 0$, defined by Eq. 2.2 using successively $\chi^l_0$ and $\chi^{nl}_0$ as $\chi_0$. The effective stress exponent for the total system is:

$$n_\varepsilon = \frac{\chi^l_0}{\chi_0} n^l_\varepsilon + \frac{\chi^{nl}_0}{\chi_0} n^{nl}_\varepsilon.$$  \hfill (2.7)

If on the other hand, coupling is external $\chi^l_0 = \chi^{nl}_0 = \chi_0$ and the total strength of the material is the sum of a contribution from the localizing system, $\sigma^l$ and a contribution from the non-localizing system $\sigma^{nl}$:

$$\sigma = \sigma^{nl} + \sigma^l.$$  \hfill (2.8)

Then, the effective stress exponent becomes

$$\frac{1}{n_\varepsilon} = \frac{\sigma^{nl}}{\sigma} \frac{1}{n^{nl}_\varepsilon} + \frac{\sigma^l}{\sigma} \frac{1}{n^l_\varepsilon}.$$  \hfill (2.9)

In the particular case where $\chi_0$ is the strain, the non-localizing part of the system may be elastic. The elastic strain $\varepsilon^e$ is related to the strength by:

$$\sigma = G\varepsilon^e.$$  \hfill (2.10)

where $G$ is an apparent rigidity modulus appropriate for the loading conditions. This gives $n_\varepsilon^{nl} = 1$. When coupling is internal, the total strain, $\varepsilon$, is the sum of $\varepsilon^e$ and a strain from the localizing part of the rheological system, $\varepsilon^l$. Eq. 2.7 becomes:

$$n_\varepsilon = 1 + \frac{\varepsilon^l}{\varepsilon} (n^l_\varepsilon - 1) = n^l_\varepsilon + \frac{\sigma}{G\varepsilon} (1 - n^l_\varepsilon).$$  \hfill (2.11)

The inverse effective stress exponent $1/n_\varepsilon$ is more negative for finite $G$ than it is in
the rigid limit \((G \to +\infty)\): when coupled internally, elasticity enhances localization. Physically, the weakening associated with an increase of \(\varepsilon^l\) decreases the supported stress, and with it the elastic strain. As the total strain is the localizing variable, this is compensated by higher \(\varepsilon^l\), and further weakening: the system is more unstable than in the absence of elastic effects.

When elasticity is coupled externally to a localizing system, \(\varepsilon = \varepsilon^e = \sigma^e/G\), with \(\sigma^e\) the stress supported by the elastic system. The localizing system supports a stress \(\sigma^l\). The total strength of the coupled system is \(\sigma = \sigma^e + \sigma^l\) and its effective stress exponent is

\[
\frac{1}{n_e} = 1 + \frac{\sigma^l}{\sigma} \left( \frac{1}{n^l_e} - 1 \right) = \frac{1}{n^l_e} + \frac{G\varepsilon}{\sigma} \left( 1 - \frac{1}{n^l_e} \right).
\] 

(2.12)

Although Eq. 2.12 is quite similar to Eq. 2.11 for internal coupling, the coupled system does not localize if \(G\) is large, \(i.e.,\), when the elastic sub-system is too stiff to accommodate the enhanced deformation. Indeed stiff machines have been used in experimental rock mechanics to determine the full strain-stress curves around yield in spite of the instability related to strain weakening [Cook, 1981].
## 2.3 Microscopic Localization mechanisms

Table 2.1: Parameters and variables used

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Symbol</th>
<th>Quantity</th>
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<tbody>
<tr>
<td>Plasticity and brittle failure</td>
<td>$\sigma$</td>
<td>Stress</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon^e$</td>
<td>Elastic strain</td>
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### Parameters

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Many micromechanical processes occur in association with localized shear zones and faults in natural examples and laboratory studies. We will present in the next four sections the mathematical expressions for the effective stress exponents of these mechanisms. In many of these examples, localization does not arise solely from the rheology, but from the coupling of several internal variables. Therefore, it is useful to first review how these processes lead to localization. We group the localization

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<td>Stress exponent of the equilibrium grain size</td>
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<td>$R$ for grain size $d_0$</td>
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<td>$\varepsilon_T$</td>
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Many micromechanical processes occur in association with localized shear zones and faults in natural examples and laboratory studies. We will present in the next four sections the mathematical expressions for the effective stress exponents of these mechanisms. In many of these examples, localization does not arise solely from the rheology, but from the coupling of several internal variables. Therefore, it is useful to first review how these processes lead to localization. We group the localization
mechanisms according to whether they occur with brittle failure, elasto-plastic deformation, frictional sliding, or ductile creep. Additional feedback processes can be imagined, some enhancing localization, such as the development of anisotropy with strain [Poirier, 1980] or the growth of ductile voids [Bercovici, 1998; Bercovici et al., 2001b], others reducing it, such as strain-hardening [Burg, 1999] or fault rotation [Sibson, 1994]. However, we limit this study to the processes discussed below. The parameters and variables involved in the localization processes considered herein are compiled in Table 2.1.

2.3.1 Failure-related Processes

Failure of intact rocks is associated with localized features such as cracks, faults, and plastic shear bands. Upon failure, a rock may lose some of its strength, either by growth of microcracks in low-porosity rocks or breakdown of a diagenetic matrix for less-consolidated sedimentary rocks [Paterson, 1978; Lockner, 1995]. In particular, the cohesion of the rock is reduced. Weakening in that type of brittle failure is explicit, and it may seem redundant to compute effective stress exponents in this case. We will do so only for comparison with other mechanisms and because the mathematical simplicity of this example provides a good introduction to the procedure involved in computing effective stress exponents.

2.3.2 Localization in elasto-plastic materials

Upon failure, a material may deform by a combination of elastic and plastic strains. Even if plastic flow does not reduce the yield strength, the combination of elastic and plastic deformation can produce a bifurcation in the deformation field and result in localized deformation [Rudnicki and Rice, 1975; Needleman and Tvegaard, 1992]. We will show that an elasto-plastic material may have negative effective stress exponent even though its yield strength is not reduced.

Localization in elasto-plastic materials occurs because the volume changes associated with loading are different from that provided by increments of plastic strain. The
plastic volume change and shear strain are related by the dilation angle $\psi$, whereas
the loading shear stress and pressure are related through the yield criterion by the
angle of internal friction, $\varphi$. Unlike the majority of metals considered in mechanical
engineering, $\varphi \neq \psi$ for rocks [Mandl, 1988] (however, Mandl [1988] noted that con-
jugate faulting brings an additional degree of freedom, so that an apparent dilation
angle at large scale may be such that $\psi = \varphi$). As $\psi$ is smaller than $\varphi$, shearing on
a failure plane does not produce enough dilation, resulting in elastic unloading of
the normal stress [Vermeer and de Borst, 1984]. Therefore, plastic shearing leads to
more failure, and deformation localizes. The occurrence and efficiency of localization
depends on the elastic properties of the material as well as on the plastic flow laws.

Because the plastic strain is the integral of strain increments that occur when
the yield criterion is verified [Hill, 1950; Lubliner, 1990], the quantities $\sigma$ and $\chi_0$
tested for localization depend on the loading history of the material and on the angle
$\theta$ between the principal directions of stress and the plane along which deformation
localizes. Fractures in rocks are observed at $\theta = 0$ (axial spitting) at low confining
pressure [Griggs and Handin, 1960; Paterson, 1978] and at $\theta_\varphi = \pi/4 - \varphi/2$, which
is the angle where the failure criterion is maximum [Anderson, 1951; Sibson, 1994].
However, in laboratory experiments using sand, shear bands are also observed at the
angles $\theta_\psi = \pi/4 - \psi/2$ and $\theta_\alpha = \pi/4 - (\varphi + \psi)/4$, which correspond respectively
to the directions where the strain increment in the shear band is collinear to the
loading direction, and where the post-bifurcation macroscopic weakening is maximum
[Vermeer, 1990]. Recently, compaction bands ($\theta = \pi/2$) have been observed in the
field and in the laboratory [Antonellini et al., 1994; Olsson, 1999]. We will show that
the effective stress exponent either changes sign, or passes through extrema at each
of these angles.

Bifurcation analysis predicts that discontinuities in an elasto-plastic material arise
spontaneously at given points in the loading [Rudnicki and Rice, 1975; Rice, 1976].
However, to be visible at large scale, the discontinuous state must be weaker that the
continuous material [Hobbs et al., 1990], i.e., have negative stress exponent. Hence,
we will determine $n_e$ for cases where a discontinuity divides a given material into an
elastic part and an elasto-plastic part. Bifurcation analysis predicts localization at \( \theta_\varphi \) and \( \theta_\psi \) [Vermeer, 1990].

### 2.3.3 Friction and Gouge Processes

Once a fault is formed, it slides at a stress level dictated by the laws of frictional sliding. With its long geological history, the brittle outer layer of the Earth, or schizosphere [Scholz, 1990], is riddled with faults to the point that it may be viewed as a continuum obeying the laws of friction at the tectonic scale [Brace and Kohlstedt, 1980]. However, not all faults are equally active and localization for a frictional material is expressed as an increase of the deformation taken by a given fault. Thus, localization is insured by the relative weakness of the most active faults. Indeed, laboratory studies indicate that active faults are weaker than inactive ones [Rabinowicz, 1951]. At the geological scale, some studies indicate that major faults such as the San Andreas Fault may be weaker than their surroundings [Zoback et al., 1987], although a consensus is still lacking [Scholz, 2000].

A probable microscopic explanation of why active faults are weak involves the evolution of fault gouge [Scholz, 1990]. As fault gouge becomes indurated and stronger if stationary, activity on a fault — quantified by the fault offset — reduces its coefficient of friction [Rabinowicz, 1951]. Alternatively, the combination of time-dependent healing and strain-dependent weakening produces apparent strain-rate weakening [Dieterich, 1978], and fault “activity” may be better described by a sliding velocity rather than a shear strain since activation [Scholz, 1990].

The most successful constitutive laws of friction to date are the rate- and state-dependent frictions laws (RSDF) [Scholz, 1990]. Beyond steady-state velocity weakening or strengthening effects, RSDF laws produce transient effects that are important for earthquake mechanics [Dieterich, 1992; Marone, 1998] and make the effective stress exponent time-dependent. This is because they involve two rheological variables: the instantaneous sliding velocity and a state variable \( \theta \) that evolves either with slip [Ruina, 1983] or time [Dieterich, 1979].

After giving the expressions for the effective stress exponent of a material that
obeys the RSDF laws, we will address two physical processes in the granular fault
gouge that may be at the origin of the state variable evolution. The first is purely mechanical. The fault gouge dilates as it shears, requiring more work than needed to overcome only the frictional resistance. Therefore, the apparent coefficient of friction is higher than the actual one [Frank, 1965; Orowan, 1966]. Localization can occur if the dilation rate decreases with shear, so that the apparent coefficient of friction decreases, although the actual coefficient of friction does not. However, this process needs to be modulated by the effect of fluids within the gouge that, if undrained, resist dilation. The second localization mechanism is thermal. The gouge, and especially its fluid portion, dilates when heated. Hence, the heat produced by shearing may weaken the fault [Shaw, 1995] and lead to localization.

2.3.4 Ductile Mechanisms

At sufficiently high temperatures, rocks flow like viscous fluids through dislocation and diffusion creep. Although experimentally-derived rheologies are usually strain-rate-strengthening [Evans and Kohlstedt, 1995], localized shear zones are a common occurrence in ductile rocks [Ramsay, 1980]. Localization is made possible by the response to a perturbation of strain rate of internal variables other than the strain rate. Two possible localization mechanisms are shear heating and recrystallization, where localization occurs through a feedback in temperature or grain size.

Because rocks are weaker at high temperature and shearing produces heat, temperature and deformation rate can feed back on one another to produce localization in the ductile regime. Shear heating has been proposed to explain ductile shear zones [Brun and Cobbold, 1980; Fleitout and Froideveau, 1980; Hobbs et al., 1986] and has been studied in relation to plate formation and orogeny [Froideveau and Schubert, 1975; Melosh, 1976; Bercovici, 1993; Schott et al., 1999]. Heat must accumulate to overcome the direct strain rate strengthening of the creep laws [Poirier, 1980; Hobbs et al., 1986], so that localization though shear-heating is favored by near-adiabatic conditions [Bai and Dodd, 1992]. However, adiabatic conditions are an instantaneous approximation of heating, whereas a finite time is required to accumulate sufficient
heat in a potential shear zone. Therefore a high heating rate is needed, which is favored by low temperature and high strain rate. We will show in §2.7.1 that a pre-existing heterogeneity such as a brittle fault is needed for shear heating to localize deformation. However, we ignore the additional coupling of grain size evolution \cite{Kameyama et al., 1997} or stored elastic energy \cite{Regenauer-Lieb and Yuen, 1998} to shear heating, which could make localization easier. \cite{Regenauer-Lieb and Yuen, 1998} show in particular that when elasticity is considered, localization can be so fast that an adiabatic approximation is reasonable.

A different process leading to localization in the ductile regime is the interplay of grain size and grain-size-sensitive diffusion creep. The transition from grain-size-insensitive dislocation creep to diffusion creep has been observed in natural shear zones, concurrent to grain size reduction \cite{Handy, 1989; Jaroslow et al., 1995; Jin et al., 1998}. However, it is important to note that although the combination of diffusion and dislocation creep mechanisms results in a material that is weaker than if only dislocation creep operated, each mechanism is strain-rate-strengthening, so that localization is not ensured. In fact, as long as the grain size is constrained to follow its recrystallized equilibrium value, localization is not predicted from our analysis (§2.7.2). As in natural shear zones, stress increases with localization \cite{Jin et al., 1998}, localization may indeed be progressive rather than dynamic (§2.2.2). However, dynamic localization is possible if the grain size is initially out of equilibrium and evolves towards the recrystallized equilibrium value at a deformation-controlled rate (§2.7.2). The departure from the equilibrium grain size can result from change of tectonic environment or transitions in microstructure \cite{Tullis et al., 1990; Rutter, 1999}, due for instance to metamorphism or neocrystallization \cite{White and Knipe, 1978; Beach, 1980; Rubie, 1983; Brodie and Rutter, 1985; Fitz Gerald and Stünitz, 1993; Brown and Solar, 1998; Newman et al., 1999].
2.4 Effective Stress Exponents for Brittle Failure

A rock may be loaded elastically up to a yield stress \( \sigma_y \) beyond which the total strain, \( \varepsilon \), is the sum of the elastic strain \( \varepsilon^e = \sigma/G \) and a plastic strain \( \varepsilon^p \):

\[
\varepsilon = \varepsilon^e + \varepsilon^p
\]  

(2.13)

The elastic strain \( \varepsilon^e = \sigma/G \), with \( G \) a general elastic modulus as in Eq. 2.10.

Deformation commonly localizes as plastic strain accumulates, especially when plastic flow is accompanied by a decrease of the yield strength from \( \sigma_y \) to a final strength \( \sigma_f = \sigma_y - \Delta \sigma \) over a critical plastic strain \( \varepsilon_c \). The weakening may betray a loss of cohesion in the rock and could be linear

\[
\sigma = \sigma_y - \Delta \sigma \varepsilon^p/\varepsilon_c
\]  

(2.14a)

or exponential [Buck and Poliakov, 1998]

\[
\sigma = \sigma_y - \Delta \sigma \left[ 1 - \exp\left(-\varepsilon^p/\varepsilon_c\right) \right].
\]  

(2.14b)

To compute the effective stress exponents, we introduce Eq. 2.14a or 2.14b in Eq. 2.2, using the total strain \( \varepsilon \) as the localizing variable \( \chi_0 \).

\[
n_e = 1 - \varepsilon_c \sigma_y/\varepsilon \Delta \sigma,
\]  

(2.15a)

\[
n_e = 1 - \frac{\varepsilon_c}{\varepsilon} \left[ \frac{\sigma}{\sigma - \sigma_f} - \ln \left( \frac{\sigma}{\sigma - \sigma_f} \right) \right].
\]  

(2.15b)

Fig. 2-3 illustrates the variation of the stress and the inverse effective stress exponent \( n_e \) with strain. Both laws give the same effective stress exponent at yield:

\[
n_e = 1 - \varepsilon_c G/\Delta \sigma,
\]  

(2.16)

which may be expected as Eq. 2.15a is an approximate version of Eq. 2.15b, for \( \sigma \sim \sigma_y \).
Figure 2-3: Evolution of (a) stress and (b) the effective stress exponent during loss of cohesion upon failure, following the linear loss law Eq. 2.14a (solid line) or the exponential law Eq. 2.14b (dashed line). The parameters \((\Delta \sigma = 1/3 \text{ and } \varepsilon_c = 7)\) give \(n_e = -20\) at the yield point. Stress and strain are scaled to 1 at the first yield.
The effective stress exponent is negative only if $\varepsilon_c$ is greater than $\Delta \sigma / G$, the elastic strain corresponding to the unloading by $\Delta \sigma$. Therefore, elastic unloading must occur faster than plastic deformation for deformation to localize. As the coupling between elastic and plastic strains is internal, elasticity enhances localization (§2.2.5).

For $\Delta \sigma = 20$ MPa and $G = 30$ GPa, the minimum $\varepsilon_c$ is of order $0.6 \times 10^{-3}$. Buck and Poliakov [1998] used $\varepsilon_c = 0.03$ to 0.3, or $n_e \sim -450$ to -45 to study localization of deformation at mid-ocean ridge spreading centers and produced realistic-looking abyssal hills.

2.5 Effective Stress Exponent for Elasto-Plastic Materials

2.5.1 Flow Theory of Plasticity

Before addressing localization for elasto-plastic materials, we find it useful to review the basics of the flow theory of plasticity in two dimensions [Hill, 1950; Lubliner, 1990]. First, we define the stress and strain vectors

\[
\sigma = \begin{pmatrix} \sigma_{xx} \\ \sigma_{xy} \\ \sigma_{yy} \\ \sigma_{yx} \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{xy} \\ \varepsilon_{yy} \\ \varepsilon_{yx} \end{pmatrix}.
\] (2.17)

The strain in Eq. 2.17 is the total strain, the sum of an elastic strain, which is related to the instantaneous stress by the stiffness matrix, $D$, and a plastic strain. The plastic strain accumulates when a yield criterion is verified. Here, we assume that yielding follows a Mohr-Coulomb failure law

\[
f = \sigma_{11} - \sigma_1 \sin \varphi - c \cos \varphi = 0,
\] (2.18)
where $\sigma_1$ and $\sigma_1$ are the first and second invariants of the stress tensor, $\varphi$ is the internal friction angle, and $c$ is the cohesion [Vermeer and de Borst, 1984, Fig. 2-4]. The more familiar criterion $f = \sigma_s - \sigma_n \tan \varphi - c$, with $\sigma_s$ and $\sigma_n$ the shear and tangential stresses on a failure plane, is equivalent to Eq. 2.18, assuming that there exists a fabric of planes in the material with their normal orientated at $\theta_\psi = \pi/4 - \varphi/2$ from the direction of the least compressive stress, the most favorable orientation for failure. If the fabric is pervasive, the material is considered a continuum. We do not include explicit hardening or weakening in the failure criterion.

The direction of plastic strain increments is determined by the flow potential $g$; plastic strain increments are proportional to $\nabla g$ where $\nabla$ denotes the gradient taken in an orthonormal base describing the space of stress vectors. It is customary to take:

$$ g = \sigma_1 - \sigma_1 \sin \psi, \quad (2.19) $$

where $\psi$ is the dilation angle [Vermeer and de Borst, 1984; Mandl, 1988]. The strain field resolved on a plane includes generally shear and dilatant motion. Hence, the plastic strain increment is coaxial to the stress increment only if the plane is at an angle $\theta_\psi = \pi/4 - \psi/2$ from the least compressive stress (Fig. 2-4a). If plasticity is non-associated ($\varphi \neq \psi$), that plane is different from the failure plane. Therefore, the strain and stress increments are incompatible, which leads to localization [Rudnicki and Rice, 1975; Vermeer and de Borst, 1984].

The amplitude of the elasto-plastic strain increment is such that the material stays at yield. Hence, increments of stress and total strain are related by $d\sigma = M \cdot d\varepsilon$ [Vermeer and de Borst, 1984, §2.10.1;], with:

$$ M = D \cdot \left[ 1 - \nabla g f^T \cdot D \nabla f \cdot D \cdot \nabla g \right], \quad (2.20) $$

2.5.2 Localization in a Continuous Elasto-Plastic Medium

We first address localization in a continuous medium loaded elastically to yield. Potential discontinuities will be considered in the next section. Loading is achieved by
Figure 2-4: a) Geometry used in the treatment of elasto-plastic materials. \( \theta \) is the angle between the principal stress directions (large arrows) and the normal to a reference plane (thick line). Shearing on that plane is accompanied by dilation, so that the direction of motion on that plane makes an angle \( \psi \), the dilation angle, with the plane. The principal strain directions (smaller arrows) differ from the principal stress direction unless \( \theta_\phi = \theta_\psi \). b) Loading trajectories. The pre-stress \( \sigma_0 \) and \( \rho \) is the ratio between the first and second invariants of stress increments up to the yield envelope (thick line). When the loading is defined as a strain (§2.5.2), it is parameterized by \( \sigma_0 \) and \( \nu \), the ratio of first to second strain, which is related to \( \rho \) before yielding by \( \rho = (1 + \lambda/G)\nu \), with \( \lambda \) and \( G \) the Lamé parameters.
cumulative strain increments \( d\varepsilon = E \, de \), where \( E \) is a shape vector and \( de \) is the magnitude of each increment. The second strain invariant is used as \( de \), and \( E \) is parameterized by \( \nu \), the ratio of volume change to \( de \), and by the angle \( \theta \) between the reference axes and the principal directions of strain:

\[
E = \frac{1}{2} \begin{pmatrix}
\nu + \cos 2\theta \\
\sin 2\theta \\
\nu - \cos 2\theta \\
\sin 2\theta
\end{pmatrix}.
\]

(2.21)

Loading in pure shear corresponds to \( \nu = 0 \) and uniaxial loading to \( \nu = 1 \) (one principal strain increment is null). Using the elastic stiffness matrix \( D \), and a possible pre-stress \( \sigma_0 \), the stress at yield is \( \sigma_0 + n \, D \cdot E \, de \), with \( n \) the number of increments needed to reach the yield point. Any value of \( \nu \) is admissible if the cohesion is non-zero (Fig. 2-4b). The Lamé parameters \( \lambda \) and \( G \) and their ratio \( r = \lambda/G \) characterize \( D \) (§2.10.1). No localization is possible during the elastic loading stage \( (n_e = 1) \).

At yield, a new strain increment brings a stress increment \( d\sigma = M \cdot E \, de \), with \( M \) defined in Eq. 2.20. As \( \sigma \) is a tensor, we must define a measure \( s \) against which we test for localization. Hence, the effective stress exponent \( n_s \) measures the possible weakening of \( s \cdot \sigma \) that arises from a change of \( de \). Using Eq. 2.2 and 2.20, we obtain

\[
\frac{1}{n_s} = \frac{s \cdot M \cdot E}{s \cdot \sigma_0 + n \, D \cdot E}.
\]

(2.22)

For the sake of simplicity, we assume in what follows that \( \sigma_0 \) is 0, or at least transparent to the measure \( s \), as is the case for hydrostatic pressure if \( s \) measures a shear stress. We investigated several measures of stress and present our results for the localization of shear stress \( \sigma_{xy} \) \((s^T = [0,1,0,0])\) and normal stress \( \sigma_{zz} \) \((s^T = [1,0,0,0])\). The effective stress exponents for these measures are respectively:

\[
n_{xy} = \frac{1 + (1 + r) \sin \psi \sin \varphi}{(1 + r) (\sin \psi - \nu)},
\]

(2.23a)


\[ n_{xx} = n_{xy} \frac{\cos 2\theta - \nu (1 + r)}{\cos 2\theta + \sin \varphi / (1 + r)} \]  

(2.23b)

Each effective stress exponent addresses the localization of the external strain imposed on an elasto-plastic material, in relation to a given internal measure of stress. Hence, it does not concern the behavior of a potential shear zone inside the material, but rather how the material that includes that shear zone is seen from a larger scale.

Localization of strain and \( \sigma_{xy} \) occurs when \( \sin \psi < \nu \), regardless of whether the material is associated or not; localization demands that the volume changes associated with shearing on the plane are different for the loading system (expressed by \( \nu \)) and the elasto-plastic material (expressed by \( \sin \psi \)). In agreement with our result, Vermeer and de Borst [1984] determined that \( \psi \) must be negative (compaction) for localization to occur in a pure shear strain field (\( \nu = 0 \)). For a Poisson solid \( (r = 1) \) with \( \varphi = 30^\circ \) and \( \psi = 0^\circ \), as is typical of rocks, and \( \nu \) between 0 and 1, \( 1/n_{xy} \) is between 0 and \(-1/2\).

Although localization using \( \sigma_{xy} \) does not depend on the orientation of shearing planes, localization using \( \sigma_{xx} \) does. Hence, the apparent weakening of the material is anisotropic. Fig. 2-5 shows how \( n_{xx} \) varies with \( \theta \) for different values of \( \nu \). Axial splitting \( (\theta = 0) \) and compaction bands \( (\theta = \pi/2) \) are favored over shear failure, as \( n_{xx} \) passes through extrema at \( \theta = j\pi/2 \) with \( j \) an integer. Compaction bands are further unlikely as they require \( \nu < -1/(1 + r) \). On the other hand, axial splitting is a real possibility as it needs \( 1/(1 + r) > \nu > \sin \psi \). However, the effect of the pre-stress remains to be determined: in experiments, axial splitting is prevented by a small confining pressure.

It may be argued that localization requires \( n_{xx} = n_{xy} < 0 \) as, in that case, the strength lost during localization is a tensor coaxial with the stress increments during loading. However, \( n_{xx} = n_{xy} < 0 \) is verified only if \( -1/\sin \psi < (1 + r) \sin \psi \), which requires that plastic flow be compactive \( (\psi < 0) \).
Figure 2-5: Effective stress exponent of an elasto-plastic material without discontinuity for localization of strain and either shear stress (left panel) or normal stress (right panel). Stress is resolved on a plane with its normal at an angle $\theta$ to the least compressive stress. $\nu$ is the ratio of volumetric strain over the second invariant of strain. Solid line: $\nu = 1$; dashed line: $\nu = 0.2$; dotted line: $\nu = 0$; dash-dots: $\nu = -1$. Figure prepared for $\phi = 30^\circ$, $\psi = 6^\circ$, $r = 1$, and no pre-stress.

2.5.3 Localization in an Elasto-plastic Medium with a Planar Discontinuity

We now consider the case of an elasto-plastic material with a potential planar discontinuity, as in the bifurcation analysis of Rudnicki and Rice [1975]. The material is loaded elastically and uniformly to yield, at which point the discontinuity may separate a portion of the material that behaves elasto-plastically from another that remains elastic [Vermeer, 1990]. Elasto-plastic strain increments are given by Eq. 2.20. No explicit weakening or hardening is included in the flow law, but the activa-
tion of the discontinuity may reduce the overall strength of the material in which it is embedded; as in the previous section, we address only the apparent behavior of the material as seen from a larger scale.

The loading is now defined by stress increments \( d\sigma = Sd\sigma_{II} \), with \( d\sigma_{II} \) the increment of the second invariant of the stress tensor, and \( S \) a shape vector parameterized by \( \rho \), the ratio between the variation of first and second stress invariants, and \( \theta \), the angle between the normal of the plane and the direction of least compressive stress (Fig. 2-4b).

\[
S = \frac{1}{2} \begin{pmatrix}
\rho - \cos 2\theta \\
\sin 2\theta \\
\rho + \cos 2\theta \\
\sin 2\theta
\end{pmatrix},
\]

(2.24)

We define a reduced plastic increment matrix \( M' \) that relates the free components of strain increments within the band, \( d\varepsilon_{xx} \) and \( d\varepsilon_{xy} \), to \( d\sigma_{II} \). The other component of the strain increment tensor, \( d\varepsilon_{yy} \), and the components \( d\sigma_{xx} \) and \( d\sigma_{xy} \) of the stress increment tensor are transmitted across the potential discontinuity. [Rudnicki and Rice, 1975; Vermeer, 1990, §2.10.2]. We then test for the localization of \( d\varepsilon_{xy} \) and \( d\varepsilon_{xx} \) for which we define the effective stress exponents \( n_{xy} \) and \( n_{xx} \), respectively.

Assuming no pre-stress \( (\varepsilon = D^{-1} \cdot Sd\sigma_{II}) \) and using \( d\varepsilon = M' \cdot Sd\sigma_{II} \) in Eq. 2.2, we obtain (§2.10.2, Fig. 2-6)

\[
n_{xy} = \left\{ \frac{\cos^2 2\theta - (\sin \varphi + \sin \psi) \cos 2\theta}{\sin \varphi \sin \psi + \frac{\mathbf{r}_1 + \mathbf{r}_2}{1 + \mathbf{r}_1} (1 + \rho \sin \varphi)} \right\}
\]

(2.25a)
Figure 2-6: Contours of inverse effective stress exponent for localization of a) shear strain and b) normal strain for an elasto-plastic material with a discontinuity at angle θ from the principal directions of loading. The loading parameter ρ is the ratio of first to second invariant of the stress perturbation. The thick lines mark changes of sign of $n_e$, with contours at every 0.01 to a maximum of 2. The region of negative stress exponent is shaded. The darker shading shows where both $n_{xy}$ and $n_{xx}$ are negative. The figure assumes $r = 1$, $φ = 30°$, and $ψ = 6°$ and no pre-stress.
\[
\begin{align*}
n_{xx} &= \frac{(1 + r) \cos^2 2\theta}{\left\{ \left( \cos 2\theta - \sin \varphi \right) \left( \cos 2\theta - \sin \psi \right) \right.} \\
&\hphantom{=} \left. \times (\rho - (1 + r) \cos 2\theta) \right) \\
&\hphantom{=} + \left[ \rho - (1 + r) (\sin \varphi + \sin \psi) \right] \cos^2 2\theta \\
&\hphantom{=} + \left[ 2 + (1 + r) \sin \varphi \sin \psi + \rho (\sin \varphi - \sin \psi) \right] \times \cos 2\theta \\
&\hphantom{=} + 2 (1 + r) \sin \psi + \rho (3 + 2r) \sin \varphi \sin \psi.
\end{align*}
\] (2.25b)

Both \(1/n_{xy}\) and \(1/n_{xx}\) are null when the determinant of \(M'\) is zero, that is, when the discontinuity is oriented at \(\theta_\varphi = \pi/4 - \varphi/2\) and \(\theta_\psi = \pi/4 - \psi/2\) from the loading directions. This condition marks the bifurcation criterion of Rudnicki and Rice [1975]. Although the material does not weaken at this point, it allows the formation of a discontinuity [Vermeer, 1990]. In addition, \(1/n_{xx}\) is null as the elastic strain increment change signs at \(\cos 2\theta = \rho/(1 + r)\). The numerator of \(n_{xx}\) can also be zero for realistic conditions (Fig. 2-6b), but the numerator of \(n_{xy}\) does not change sign unless \(\varphi\) approaches 90° and \(\psi\) approaches -90°, which are not unrealistic values (§2.10.2).

Localization of \(\varepsilon_{xy}\) is most efficient at \(\theta_\alpha = \pi/4 - (\varphi + \psi)/4\), where \(1/n_{xy}\) is minimum [Vermeer, 1990]. The localization of \(\varepsilon_{xx}\) is strongest localization where \(1/n_{xx} \to -\infty\). However, this drastic localization may not occur spontaneously because \(n_{xy}\) is positive for these values (not all components of the deformation field tend to localize) and, as it is not related to to the condition for spontaneous discontinuity formation, \(|M'| = 0 [Rudnicki and Rice, 1975]\), the required discontinuity may be unavailable in the material. If there is a pre-existing discontinuity at the angle for which \(1/n_{xx} \to \infty\), the drastic localization of the normal strains may happen. This might explain the compaction bands documented by Antonellini et al. [1994] and Olsson [1999], as \(1/n_{xx} \to \infty\) at \(\theta = 90°\) for \(\rho = 1\) and \(r = 0\). However, this explanation is unlikely because compaction bands are observed in materials where \(r \neq 0\) and because the divergence of \(n_{xx}\) is suppressed with confining pressure, whereas compaction bands are observed only with significant confining pressure [Olsson, 1999].
Compaction bands are more likely to originate at the vertex between two yield envelopes [Issen and Rudnicki, 2001; Wong et al., 2001], a feature of the yield criterion that we do not consider herein.

We present in Fig. 2-7 a section through the maps of effective stress exponent of Fig. 2-6 at \( \theta = \theta_u \), where both \( n_{xy} \) and \( n_{xx} \) are negative, and \( 1/n_{xy} \) is minimum. For realistic rock parameters \( (r = 1, \phi = 30^\circ, \psi = 6^\circ) \), \( 1/n_{xy} \) increases with \( \rho \) from roughly -0.0525 to -0.0175, while \( 1/n_{xx} \) decreases to -0.02. A good value where both effective exponents are similar is \( n_e \sim -50 \), and \( \rho \sim 0.75 \).
2.6 Effective Stress Exponents for Localization in Frictional Materials

2.6.1 Static and Dynamic Friction

A frictional material is one that deforms by sliding on many faults, so that it may be viewed as a continuum with a strength obeying the friction laws. Its deformation field localizes as a particular fault becomes more active than the others. This requires that active faults be weaker than less active or inactive ones, possibly because the dynamic coefficient of friction, $\mu_d$, is lower than the static coefficient of friction, $\mu_s$. Rabinowicz [1951] proposed that the transition from $\mu_s$ to $\mu_d$ occurs over a critical sliding distance $D_C$. If the coefficient of friction decreases linearly with sliding distance $d$, we obtain:

$$n_e = 1 - \frac{\mu_s}{\mu_s - \mu_d} \frac{D_C}{d}, \quad d < D_C,$$

(2.26)

with $d$ the imposed displacement, different from $d_s$ because of elastic deformation. This formula is similar to Eq. 2.14a for localization upon failure (Fig. 2-3) with the loss of strength interpreted as a decrease of coefficient of friction rather than as a loss of cohesion. In either case, external coupling with a stiff loading system can prevent localization (§2.2.5), as is done experimentally to study the details of the weakening occurring upon failure or fault activation [Cook, 1981]. From Eq. 2.12, stabilization occurs when $k$, the rigidity of the loading apparatus, exceeds $(\mu_s - \mu_d) / D_C$, which is the rate at which $\mu$ decreases with displacement [Scholz, 1990, Eq. 2-24].

In order to model the healing of faults as their activity ceases, it may be better to associate the weakening of active faults with their sliding velocity $V$. Dieterich [1972, 1978] proposed that the coefficient of friction $\mu$ during steady sliding depends on the shearing velocity $V$ as

$$\mu = \mu_0 - c \cdot \ln \left( \frac{V}{V_0} \right)$$

(2.27)

with $\mu_0$ and $V_0$ the reference coefficient of friction and velocity, and $c$ a material...
constant. Using Eq. 2.2 with $V$ as $x_0$ and $\mu$ as $\sigma$, the effective stress exponent is:

$$n_e = -\mu/c.$$  \hfill (2.28)

For values typical of rocks ($\mu \sim 0.75$ and $c \sim 0.003$), $n_e$ is $\sim -250$.

Localized velocity can be expressed by localized regions that deform more or less uniformly in time (faults, intense shear zones) or non-localized regions where deformation is temporally localized (stick-slip motion, earthquakes). In reality, both occur, and the interpretation of localized velocity may differ depending on whether one is interested in earthquake mechanics or tectonic modeling.

### 2.6.2 Rate- and State-Dependent Friction

Transient effects of frictional behavior can be incorporated through the formalism of rate- and state-dependent friction laws (RSDF) [Scholz, 1990; Marone, 1998], where the coefficient of friction $\mu$ depends on the displacement velocity $V$ and on a state variable $\theta$, which may indicate the age of granular contacts [Dieterich, 1979] or the integrated slip history of the fault [Ruina, 1983]. The friction law

$$\mu = \mu_0 + a \ln \left( \frac{V}{V_0} \right) + b \ln \left( \frac{V_0 \theta}{D} \right)$$ \hfill (2.29)

is coupled to an evolution law for the state variable, of the form proposed by Dieterich [1979]:

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{D}$$ \hfill (2.30a)

or alternatively by Ruina [1983]:

$$\frac{d\theta}{dt} = -\frac{V\theta}{D} \ln \left( \frac{V\theta}{D} \right),$$ \hfill (2.30b)

where $D$ is a critical distance related to the macroscopic structure of the fault gouge and possibly to $D_C$ encountered in §2.6.1. The parameters $V_0$ and $\mu_0$ are reference values of sliding velocity and friction coefficient, and $a$ and $b$ are material parameters.
Figure 2-8: Evolution of the inverse effective stress exponent after a jump of sliding velocity using the rate- and state-dependent friction laws. Solid line: \( b = 3a/2 \), steady-state weakening. Dashed line: \( b = a/2 \), steady-state velocity strengthening. The thick horizontal segments indicate the steady-state limit (Eq. 2.27 and 2.28). The vertical line is the approximate te limit from Eq. 2.32. Figure assumes \( a = 1 \), \( \mu_0 = 1 \), and \( \delta v/v = 0.01 \).

In steady state, \( \theta = D/V \) in either case, and Eq. 2.29 becomes Eq. 2.27, with \( c = b - a \).

The rheological system involved in the RSDF laws is composed of the two variables, \( \theta \) and \( V \). The velocity is the quantity that localizes in our treatment of frictional materials obeying the RSDF laws. As the state variable depends on the integrated history of velocity, we must specify a study case, whereby a perturbation in velocity is sustained for all time \( t > 0 \). Initially, the state variable is at equilibrium. We obtain (§2.11)

\[
\frac{1}{n_e} = \frac{1}{\mu} \left\{ a - b \frac{D}{\theta V} \left[ 1 - \exp \left( -\frac{V t}{D} \right) \right] \right\} .
\] (2.31)

An illustrative example evolution of \( n_e \) upon perturbation of sliding velocity is given in Fig. 2-8. Initially, the effective stress exponent is positive because of the stabilizing direct effect \( (a > 0) \). However, if \( b > a \), \( n_e \) changes sign after a critical
time \( t_c \) because of the evolution of the state variable, with

\[
t_c \sim -\frac{D}{V} \ln \left(1 - \frac{a}{b}\right).
\]  \hspace{1cm} (2.32)

For velocity to localize in a material obeying the RSDF laws, the velocity perturbation must be held for at least \( t_c \).

In the infinite time limit, the effective stress exponent tends towards the value for steady-state sliding (Eq. 2.28, \( n_e \sim -250 \)). This value is probably the most relevant for large-scale geodynamics, where long time scales are involved. However, earthquakes mechanics are dominated by the transient behavior [Dieterich, 1992; Marone, 1998].

Laboratory experiments determined that, as temperature is increased above 400°C, frictional sliding of granite becomes stable [Brace and Byerlee, 1970]. The stability of sliding corresponds to velocity-strengthening behavior [Stesky et al., 1974; Stesky, 1977], which, in the context of RSDF laws, indicate that the parameter \( b \) increases with temperature, and that ultimately, \( b > a \) [Tse and Rice, 1986; Lockner et al., 1986; Blanpied et al., 1991]. However, a more accurate description of the transition from velocity-weakening to velocity strengthening behavior is that there are two state variables in the rheological system, \( \theta_1 \) and \( \theta_2 \), with associated critical distance \( D_1 \) and \( D_2 \) and state-dependence coefficients \( b_1 \) and \( b_2 \). Experiments on granite [Blanpied et al., 1995, 1998] as well as preliminary tests on gabbro [Montési et al., 1999] indicate that \( a \), \( b_1 \), and \( D_1 \) are constant at all temperatures, with \( b_1 < a \), but that \( b_2 = 0 \) below a threshold temperature and subsequently decreases with temperature: \( b_2 < 0 \). Above a critical temperature \( T_c \), \( a - b_1 - b_2 > 0 \), and the system is velocity strengthening in steady-state. However, \( D_2 \) is at least an order of magnitude greater than \( D_1 \) [Blanpied et al., 1998; Montési et al., 1999]. Hence, \( \theta_2 \) may be considered constant in a short interval after a perturbation of velocity, over which \( \theta_1 \) evolves. As \( a - b_1 < 0 \), it may be possible that the system becomes unstable in spite of the steady-state velocity-strengthening.
Figure 2-9: Evolution of the inverse effective stress exponent after a jump of sliding velocity using the rate- and state-dependent friction laws with two state variables. Solid line: two variables, $b_1 = 2a$, $b_2 = -2.5a$, $D_1 = 0.1$, $D_2 = 1$; Dashed line: first state variable only; dotted line: second state variable only. Figure assumes $a = 1$, $\mu_0 = 1$, $\delta v/v = 0.01$.

The effective stress exponent for RSDF laws with two state variables is

$$\frac{1}{n_e} = \frac{1}{\mu} \left\{ a - b_1 \frac{D_1}{\theta_1 V} \left[ 1 - \exp \left( -\frac{Vt}{D_1} \right) \right] - b_2 \frac{D_2}{\theta_2 V} \left[ 1 - \exp \left( -\frac{Vt}{D_2} \right) \right] \right\} .$$

(2.33)

In Fig. 2-9, we show how the effective stress exponent varies for a two-variables case. Initially, $1/n_e = 1/\mu$. Then, the first state variable evolves, which may lead to weakening, as in our example. In the long range, the second variable dominates, and stabilizes the system. However, the deformation localizes as soon as $1/n_e < 0$, which leads to more perturbations of the sliding velocity. Therefore, the long-term stabilization will not be observed in nature.

This analysis implies that seismic events may be generated even under conditions where frictional sliding is velocity-strengthening. For temperatures immediately above $T_c$, where $a - b_1 - b_2$ is positive but small, the minimum value of $1/n_e$...
is negative, which allows localization. In the laboratory, the relatively high stiffness of the testing apparatus prevents the instability to develop, but in the much softer loading assemblage that is the Earth [Walsh, 1971], an earthquake may occur. Wiens and Stein [1983] and Chen and Molnar [1983] showed that earthquakes in intraplate settings are limited by an isotherm around 600 to 800°C. For comparison, Granite displays velocity weakening up to 400°C [Blanpied et al., 1998], and gabbro is velocity-strengthening at 400°C [Montési et al., 1999]. Stesky et al. [1974] studied the velocity-dependence of friction for two types of peridotites and found velocity strengthening above 200°C. However, these experiments might have been influenced by the presence of serpentine, which, even in small amounts influences greatly the brittle behavior of ultramafic rocks [Escartín et al., 2001]. It remains that olivine-rich rocks are probably velocity-strengthening in the 400 to 600°C temperature range, were earthquakes are generated. The seismic and laboratory data can be reconciled if friction is characterized by two state variables in that temperature range.

2.6.3 Fault Gouge and Pore Fluid Effects

Apparent Coefficient of Friction and Dilatancy

One physical aspect of fault gouge mechanics that may explain the weakening of active faults is the dilation of gouge upon shearing [Frank, 1965; Orowan, 1966; Marone et al., 1990]. Gouge dilation works against the effective normal stress, \( \sigma_e = \sigma_n - p_f \), where \( \sigma_n \) is the normal stress imposed on the fault and \( p_f \) the pressure of a pore fluid within the gouge. The energy needed for dilation is provided by the loading, in particular by the shear stress \( \tau \). Therefore, the apparent friction coefficient \( \mu_a = \tau / \sigma_e \) is higher than the actual friction coefficient \( \mu \). Based on such thermodynamic considerations, [Frank, 1965] proposed:

\[
\mu_a = \mu + \frac{dv}{d\gamma}, \tag{2.34a}
\]

with \( v \) the volume of the gouge and \( \gamma \) the shear strain [Edmond and Paterson, 1972; Marone et al., 1990, see also]. In contrast, Orowan [1966], taking into account the
change of direction of the microscopic friction force between grains as they climb onto 
one another, derived:

\[ \mu_a = \frac{\mu + \frac{dv}{d\gamma}}{1 - \frac{dv}{d\gamma}}. \quad (2.34b) \]

Localization is possible if the dilation rate \( \frac{dv}{d\gamma} \) decreases with strain [Frank, 1965; Edmond and Paterson, 1972; Fischer and Paterson, 1989; Marone et al., 1990].

On the other hand, dilation decreases the pore fluid pressure, and therefore increases the effective normal stress [Reynolds, 1885; Frank, 1965], thereby stabilizing slip [Rice, 1975; Rudnicki, 1979]. The capacity of the fluid is defined as \( K_f = -\frac{dp_f}{dv} \). Physically, the fluid capacity represents not only the compressibility of the fluid, that is very small if the pore fluid is liquid, but also the ability of the fluid to circulate in the gouge and to exchanged with the surrounding wall rocks [Rudnicki, 1979; Rudnicki and Chen, 1988]. If the gouge is highly permeable, fluid pressure gradients cannot be sustained and \( K_f \) tends towards zero, even if the fluid is incompressible.

This rheological system is composed of the gouge volume, the shear strain, and the pore fluid pressure. We test for the localization of the shear strain, \( \gamma \), with respect to the shear stress. In addition, we consider only the initiation of sliding, so that \( \gamma = \tau / G \), with \( G \) the shear modulus of intact gouge. Hence, the effective stress exponent is

\[ \frac{1}{\eta_e} = \frac{1}{G} \left( \frac{\frac{d\eta_a}{d\gamma} \sigma_e - \eta_a \frac{dp_f}{d\gamma}}{\sigma_e - \eta_a \frac{dp_f}{d\gamma}} \right) \quad (2.35) \]

Replacing \( \mu_a \) by Eq. 2.34a and 2.34b respectively, and using \( K_f = -\frac{dp_f}{dv} \), we obtain

\[ \frac{1}{\eta_e} = \frac{\sigma_e}{G} \frac{d^2v}{d\gamma^2} + \frac{K_f}{G} \frac{dv}{d\gamma} \left( \mu + \frac{dv}{d\gamma} \right) \quad (2.36a) \]

for Frank’s theory, and

\[ \frac{1}{\eta_e} = \frac{\sigma_e}{G} \frac{1 + \mu^2}{(1 - \mu \frac{dv}{d\gamma})^2} \frac{d^2v}{d\gamma^2} + \frac{K_f}{G} \frac{dv}{d\gamma} \left( \mu + \frac{dv}{d\gamma} \right) \quad (2.36b) \]
Figure 2-10: Inverse effective stress exponent for a fault gouge as a function of the compaction/dilation according to Frank [1965] (thick lines) or Orowan [1966] (thin lines) theories with $\mu = 0.8$ and several values of the dilation rate $d^2v/d\gamma^2$, pore fluid pressure $p_f$, and fluid capacity $K_f$.

for Orowan’s theory. Fig. 2-10 shows how the effective stress exponent varies in function of the dilation rate for several values of the parameter $d^2v/d\gamma^2.\sigma_e/K_f$. The dilation rate is limited to $dv/d\gamma > -\mu$ for Frank’s theory and $1/\mu > dv/d\gamma > -\mu$ for Orowan’s theory by the condition that $\mu \alpha > 0$.

If the gouge is fully drained ($K_f = 0$), localization requires that $d^2v/d\gamma^2 < 0$, i.e., that the dilation rate decrease with strain [Frank, 1965; Marone et al., 1990]. However, for Frank’s theory, the stabilizing effect of pressure changes brings an upper limit to the dilation rate that can localize. Beyond that, the pressure change dominates and localization is impossible. However, changes of pore fluid pressure can help the system localize if the gouge compacts. Indeed, the pore fluid pressure increases as the
gouge compacts, supporting more of the normal stress. Hence, localization is possible in a compacting gouge \((dv/d\gamma < 0)\) if \(d^2v/d\gamma^2 \cdot \sigma_n/K_f < \mu^2/4\) (Fig. 2-10). Indeed, dynamic weakening of an undrained fault gouge undergoing compaction was observed by Blanpied et al. [1992]. Changes of pore fluid pressure has less effect in Orowans’ theory. In particular, they cannot change the domain of dilation rate that localize or not, although localization is generally less strong when the pore fluid effects are taken into account.

More rigorous treatments of fluid flow in a deforming matrix exist [Rudnicki and Chen, 1988; Segall and Rice, 1995]. For instance, Sleep and Blanpied [1994] showed that compaction, resulting from ductile creep in the fault gouge driven by the overpressurization of the pore fluid, is sufficient to destabilize fault slip. At high temperatures, the matrix is assumed to deform viscously [McKenzie, 1984; Spiegelman, 1993; Bercovici et al., 2001a]. In that case, porosity (or damage) and surface tension interact to concentrate a fluid phase to a localized band in Bercovici et al. [2001b]. How the band would affect the strength of the material in which it forms has not yet been discussed.

**Pore Fluids Heating**

Pore fluids promote localization if their pressure increases with either strain or strain rate. In the previous paragraph, pressure increased if the gouge compacted. Alternatively, pore fluid pressure increases as the fluid dilate if it is heated by the deformation. Shaw [1995], following Sibson [1973] and Lachenbruch [1980], proposed that the normal stress \(\sigma_n\) on a fault gouge decreases linearly with the heat \(Q\) as

\[
\sigma_n = \sigma_n^0 - \alpha Q,
\]

with \(\alpha\) a coefficient and \(\sigma_n^0\) the value without deformation. Heat may diffuse away over the time scale \(t_d\) but is replenished at a rate proportional to \(\mu v\), where \(v\) is the shearing velocity on the fault. The approximations that \(t_d\) is small (steady state conductive) or that \(t_d\) is large (adiabatic heating) result in weakening with velocity \(v\)
or slip $s$, respectively [Shaw, 1995]. The corresponding expressions for $\sigma$ are

$$\sigma = \frac{\mu \sigma_n^0}{1 + t_d \mu \alpha V}, \quad (2.38a)$$

$$\sigma = \mu \sigma_n^0 \exp(-\alpha \mu s), \quad (2.38b)$$

with effective stress exponents

$$\frac{1}{n_e} = \frac{-1}{1 + 1/t_d \mu \alpha V}, \quad (2.39a)$$

$$1/n_e = -\alpha \mu s. \quad (2.39b)$$

The effective stress exponent is negative for all finite $\alpha$. As $\alpha$ is not determined, it is difficult to address whether elastic effects would stabilize this mechanism of localization or not.

### 2.7 Effective Stress Exponent for Localization During Ductile Creep

#### 2.7.1 Shear Heating

**General analysis**

Shearing a rock at the rate $\dot{\varepsilon}$ and stress $\sigma$ (rock strength) releases energy by viscous dissipation, a fraction $\beta$ of which is converted into heat $H$:

$$H = \beta \sigma \dot{\varepsilon} \quad (2.40)$$

As the strength of ductile rocks is temperature-activated, the heat anomaly due to a local increase of strain rate can weaken the rock and localize the strain rate. The rheology of ductile rocks is such that:

$$\sigma = B \dot{\varepsilon}^{1/n} \exp(1/n \theta), \quad (2.41)$$
where $n$ is the stress exponent (not to be confused with the effective stress exponent $n_e$) and $B$ a material constant. The rheological temperature $\theta$ is defined as $\theta = TR_G/Q$, with $T$ the absolute temperature, $R_G$ the gas constant, and $Q$ the activation energy. We ignore for now additional dependences of $B$ on grain size (for diffusion creep), chemical activity, etc. Then, Eq. 2.41 introduced in Eq. 2.2 with $\dot{\varepsilon}$ as $\chi_0$ gives

$$
\frac{1}{n_e} = \frac{\dot{\varepsilon}}{\sigma} \left( \frac{\partial \sigma}{\partial \dot{\varepsilon}} + \frac{\partial \sigma}{\partial \theta} \frac{\partial \theta}{\partial \dot{\varepsilon}} \right) = \frac{1}{n} \left( 1 - \frac{\dot{\varepsilon}}{\theta^2} \frac{\partial^2 \theta}{\partial \dot{\varepsilon}^2} \right).
$$

(2.42)

The temperature is influenced by the strain rate through the heat produced by shearing. Perturbing the heat production by $dH$ changes the temperature by

$$
dT = T dH,
$$

(2.43)

where the coefficient $T$ is the heat retention factor that may depend on the size of the perturbation and the time $t$ since its onset, as well as on the thermal properties of the rock and the geometry of the perturbation. For instance, if heat retention is adiabatic, we have:

$$
T = t/\rho C,
$$

(2.44)

with $\rho$ the density and $C$ the thermal capacity of the rock. If heat conduction is included, the temperature anomaly depends on distance from the perturbation [Carslaw and Jaeger, 1959]. Only the temperature change at the location of the perturbation, $dT$, is important for shear-heating feedback. We define $dT$ as the average temperature anomaly within a thickness $d$, small but finite, of the perturbation origin, which is identified with the perturbation “size”. More specifically, we use the formula for a planar heat source continuously active from time 0 (onset of the perturbation) to $t$ [Carslaw and Jaeger, 1959, Eq. 10.4.9 of] to compute

$$
T = \frac{t}{\rho C} \left[ \text{erf} \frac{d}{2\sqrt{\kappa t}} + \frac{d}{2\sqrt{\kappa t}} \exp \left( -\frac{d^2}{4\kappa t} \right) \right. - \left. \frac{d^2}{2\kappa t} \text{erfc} \frac{d}{2\sqrt{\kappa t}} \right],
$$

(2.45)

with $\kappa$ the thermal conductivity of the material.
The linear relation between the anomalies of heat production and temperature (Eq. 2.43) is valid when the $dH$ is small. It gives
\[
\frac{\partial T}{\partial \dot{\varepsilon}} = T \frac{\partial H}{\partial \dot{\varepsilon}} = \beta T \sigma \left(1 + \frac{1}{n}\right). \tag{2.46}
\]

Then, the effective stress exponent (Eq. 2.42) becomes
\[
\frac{1}{n_e} = \frac{1}{n} \left[1 - \left(1 + \frac{1}{n}\right) \frac{\mathcal{H}}{\theta^2}\right], \tag{2.47}
\]

where $\mathcal{H}$ is a non-dimensional heat production defined as
\[
\mathcal{H} = \frac{T H R_G}{Q}. \tag{2.48}
\]

In the $\theta$-$\mathcal{H}$ space, contours of effective stress exponent follow parabolas (Fig. 2-11). High temperatures are stable because the low strength of the rock generates only little heat; shear heating is not efficient enough to overcome the direct strengthening effect of increasing the strain rates. As $\mathcal{H}$ depends on stress and thus on temperature by Eq. 2.41, $\mathcal{H}$ and $\theta$ are not independently controlled.

Application to Earth Materials

The importance of localization by shear heating in the lithosphere is evaluated using a set of experimentally-derived rock rheologies. We use the flow laws of Gleason and Tullis [1995] for quartzite, Caristan [1982] for diabase, Mackwell et al. [1998] for ultra-dry diabase, and Karato et al. [1986] for Olivine in the dislocation creep regime. In addition, we use the results of [Dimanov et al., 1998] for Plagioclase deforming in the diffusion creep regime, assuming a fixed grain size of 0.1 mm.

The effective stress exponent for shear heating is shown in Fig. 2-12 as a function of temperature, assuming a strain rate of $10^{-15}$ s$^{-1}$ and $T \sim 10^4$ K.W$^{-1}$. If the heat retention is total (adiabatic conditions), $T = t/\rho C \sim 10^4$ K.W$^{-1}$ corresponds roughly to a perturbation active during $\sim 1000$ years, in a rock of density $\rho \sim 3000$ kg.m$^{-3}$ and heat capacity $C \sim 1000$ J.kg$^{-1}$.K$^{-1}$. Although very strong localization is pre-
Figure 2-11: Contours of inverse effective stress exponent $1/n_e$ for shear heating with $n = 3$, as a function of the generalized coordinates $\theta$ and $H$ (see §2.7.1). Contours are plotted every 0.1 down to -2. Contours are dashed where $n_e > 0$ (no localization). The thick line marks the transition to localization ($1/n_e = 0$).

dicted ($1/n_e < -1$), it requires temperatures below 300 °C except for the example of diffusion creep. However, ductile flow is replaced by brittle failure or a high stress deformation mechanism such as Peierl’s creep at these temperatures. If $\mathcal{T}$ was higher, the maximum temperature for localization would increase, but heat losses by conduction could be important, as the implied time to hold the perturbation would be longer. Higher strain rates also help localization, but they do not occur spontaneously in the Earth’s lithosphere.

A more interesting question is what heat retention factor $\mathcal{T}$ is needed for localization, given a flow law, the deformation rate and the ambient temperature. Once $\mathcal{T}$ is known, we invert Eq. 2.44 or 2.45 to obtain the minimum time—or equivalently
Figure 2-12: Application of shear heating feedback to several rock types, assuming \( \dot{\varepsilon} = 10^{-15} \text{ s}^{-1} \) and \( T = 10^4 \text{ K.W}^{-1} \). Flow laws are GT: Quartzite without melt [Gleason and Tullis, 1995]; Ca: Diabase [Caristan, 1982]; Di: Plagioclase in diffusion creep regime, with 0.1mm grain size [Dimanov et al., 1998]; MC: Dry Columbia diabase [Mackwell et al., 1998]; Kw: Synthetic olivine aggregates [Karato et al., 1986].

the critical strain \( \varepsilon_c \)— during which the perturbation has to be active to allow localization (Fig. 2-13). Because localization is favored at low temperature, we use the temperature at the brittle-ductile transition for a given flow law, calculated following Bruce and Kohlstedt [1980] for a geotherm of 30 K.km\(^{-1}\) and assuming horizontal shortening. Therefore, the temperature increases with strain rate and varies with rock type.

Under the adiabatic assumption, localization may occur after 3 to 6 % strain, with little dependence on the strain rate (Fig. 2-13). Heat loss can only increase the time during which a perturbation must be held before it triggers localization. To include heat losses in the analysis, we assume a planar perturbation (Eq. 2.45),
Figure 2-13: Critical strain for localization by shear heating as a function of strain rate with various flow laws (style matching Fig. 2-12) at the brittle-ductile transition assuming a 30 K.km$^{-1}$ geotherm and horizontal compression. The lowest set corresponds to adiabatic heating. The others include conduction, either assuming fixed perturbation size (critical strain increasing at low strain rate), or fixed boundary velocity (critical strain increasing at high strain rate). The thin diagonal lines show the time needed to reach the instability at the unperturbed strain rate.

\[ \kappa = 1 \text{ mm}^2\text{s}^{-1}, \] and follow one of two assumptions regarding the perturbation size \( d \):

1. The perturbation width does not depend on strain rate: \( d = 1 \text{ km} \). The adiabatic assumption is valid for large strain rates (Fig. 2-13), but for \( \dot{\varepsilon} < 10^{-14} \text{ s}^{-1} \), the time for which the perturbation must be sustained before localization occurs is prohibitively long.

2. The perturbation width is inversely proportional to the strain rate so that the velocity across the perturbed region is a fraction \( f = 1\% \) of a total velocity
$v = 30 \text{ mm.yr}^{-1}$ typical of plate tectonics:

$$d = f v/\dot{\varepsilon}. \quad (2.49)$$

The adiabatic limit occurs at small strain rate, where the perturbation width is large. The time for localization is always larger than 1 Ma.

Localization is improbable at low strain rates. The adiabatic limit requires perturbation widths in excess of 100 km at $10^{-16} \text{ s}^{-1}$. Even if the thin plane approximation made to derive Eq. 2.45 remained valid for shear zones so wide, holding such a perturbation for at least 1 Ma, the minimum time required for localization, would probably indicate a pre-existing material heterogeneity. Hence, the localization should be regarded as inherited rather than dynamic. On the other hand, high strain rates permit localization of thin perturbations after a time short compared to tectonic time scales. A kilometer-thick shear zone can be considered adiabatic at strain rates in excess of $\sim 10^{-12} \text{ s}^{-1}$. Such strain rates occur in the continuity of brittle faults. Then, localization requires perturbations held for only 1000 years, which is of the same order than the earthquake cycle.

We conclude that shear heating does not arise spontaneously in the lithosphere, but requires pre-existing heterogeneities. in particular, it is possible beneath seismogenic faults. The relation of ductile shear zones and brittle faults has been pointed previously from field evidence [Ramsay, 1980; Sibson, 1980; Wittlinger et al., 1998]. Certainly, some additional feedback processes may increase the likelihood of localization. In particular, Regenauer-Lieb and Yuen [1998] included elasticity and showed that a thermal crack can develop less than 1 Ma, fed by the release of stored elastic strain energy. However, other feedback processes that we also neglected here can stabilize localization.

### 2.7.2 Onset of Diffusion Creep

Several deformation mechanisms can coexist in ductile rocks. For instance, dislocation creep and diffusion creep involve the motion of two different types of microscopic
defects [Ranalli, 1995]. They probably add to one another, although one often dom-
inates the total strain rate. As diffusion creep depends on grain size, and the grain size may evolve due to dislocation motions, it is possible to devise a feedback mechanism by which ductile rocks soften. But do they weaken as well? Is the effective stress exponent negative? In what follows, we first assume that the grain size adjust instantaneously to its stress-dependent recrystallized equilibrium value, and find that dynamic localization is impossible. When the equilibrium assumption is relaxed, localization through grain size feedback is allowed.

**Grain Size at Equilibrium**

The total strain rate of a rock undergoing simultaneously dislocation and diffusion creep, $\dot{\varepsilon}$, is the sum of a dislocation creep strain rate

$$\dot{\varepsilon}_D = A_D \sigma^n,$$

(2.50)

and a diffusion creep strain rate

$$\dot{\varepsilon}_G = A_G d^{-m} \sigma^p,$$

(2.51)

where $A_D$, $A_G$, $n$, $m$, and $p$ are material parameters, and $d$ is the grain size of the material. Additional dependencies on temperature, chemical composition, water and oxygen fugacity, etc. are ignored.

Stress-dependent dislocation motion induces recrystallization of grains that competes with kinetic grain growth, so that the grain size evolves towards an equilibrium value $D$:

$$D = D_0 \sigma^{-r}.$$  

(2.52)

If grain size evolution is rapid, $d = D$. The effective stress exponent is then derived from Eq. 2.2 with $\dot{\varepsilon} = \dot{\varepsilon}_D + \dot{\varepsilon}_G$ as the localizing variable:

$$n_e = \frac{\sigma}{\dot{\varepsilon}} \left( \frac{\partial \dot{\varepsilon}}{\partial \sigma} + \frac{dD}{d\sigma} \frac{\partial \dot{\varepsilon}}{\partial D} \right).$$
with $R$ the ratio of strain rate accommodated by diffusion creep over the strain rate accommodated by dislocation creep.

$$
R = \frac{\dot{\varepsilon}_D}{\dot{\varepsilon}_D} = \frac{A_G D_0^{-m}}{A_D} \sigma^{p+mr-n}.
$$

The effective stress exponent is always positive, regardless of the stress or the temperature—the latter enters in the coefficients $A_D$ and $A_G$. In fact, $n_e$ is bounded by $n$ at low stress (dislocation-dominated limit) and $p + mr$ at high stress (diffusion-dominated limit) (Fig. 2-14). Dislocation creep dominates at low stress if $p + mr > n$ because the large equilibrium grain size makes diffusion creep inefficient. The alternative assumption that the grain size is fixed at an initial value gives Eq. 2.53 as well, with $r = 0$. However, diffusion creep dominates at small strains if $d$ is fixed [Karato et al., 1986; Handy, 1989; Jin et al., 1998]. Weakening ($n_e < 0$) never occurs because neither the dislocation creep law nor the diffusion creep law weaken individually.

Although dynamic localization is not possible, the effective stress exponent is always more than 1, and increases with stress. Hence, progressive localization is possible. We recall from §2.2.2 that progressive localization requires that a perturbed material is softer than in its initial configuration. Hence, a further overall increase of stress activates preferentially the perturbed location. However, the initial perturbation is not able to start a runaway process in that material. A localized shear zone may develop, but only if externally applied stress increases. Indeed, Jin et al. [1998] documented that the stress in a natural shear zone was higher than in its surroundings; localization by grain-size feedback may be progressive. In that case, localized shear zones are little more than flow markers, and do not influence significantly large-scale tectonics unless another localization process is simultaneously active.
Non-equilibrium Grain Size

In the previous section, the grain size was fixed at its equilibrium value \( D \) (Eq. 2.52). However, this assumption may not be valid if the stress or the temperature vary rapidly due to tectonic motions [Kameyama et al., 1997; Braun et al., 1999] or if grain nucleation is important. We now suppose instead that the grain size of the material is initially at \( d_0 \neq D \) and evolves towards equilibrium. A perturbation in strain rate changes both the equilibrium grain size and the rate of grain size evolution, so that a perturbed location may appear weaker after a given time.

Following Kameyama et al. [1997] and Braun et al. [1999], we assume that the
grain size evolves towards the equilibrium value $D$ according to:

$$\frac{dd}{dt} = \frac{\dot{\varepsilon}_D}{\varepsilon_T} (d - D), \quad (2.55)$$

where $\varepsilon_T$ is a critical strain. Unlike previous authors, we include only the strain rate accommodated by dislocation creep in Eq. 2.55: grain size evolves only due to dislocation motion. Then, the grain size $d$ at time $t$ is:

$$d = d_0 - \int_{0}^{t} \frac{\dot{\varepsilon}_D}{\varepsilon_T} (d - D) \, dt \approx d_0 - \frac{\dot{\varepsilon}_D t}{\varepsilon_T} (d_0 - D). \quad (2.56)$$

The approximation is valid for $A_D \sigma^n t / \varepsilon_T < 1$. By making this assumption, we underestimate the time required to reach a certain $d$ starting from $d_0$. If larger values of $t$ are of interest, the system of equations must be solved numerically [Braun et al., 1999].

Inserting Eq. 2.50, 2.51, and 2.56 into Eq. 2.2 the effective stress exponent becomes

$$n_e = \frac{n + R_0 p - A_D \sigma^n m L t / \varepsilon_T}{1 + R_0}, \quad (2.57)$$

with $R_0$ the value or $R$ for grain size $d_0$:

$$R = A_G d_0^{-m} \sigma^p / A_D \sigma^n. \quad (2.58)$$

and $L$ a localization parameter given by

$$L = \frac{D_0}{d_0} \sigma^{-r} \left( p + n - r - \frac{R_0 (n + R_0 p)}{1 + R_0} \right) \left( -p - n + \frac{R_0 (n + R_0 p)}{1 + R_0} \right). \quad (2.59)$$

In deriving Eq. 2.57 to 2.59, we used several times the approximation that $A_D \sigma^n t / \varepsilon_T \ll 1$.

At the onset of the stress perturbation, the effective stress exponent is positive, as demanded by the dislocation and diffusion creep laws, but the evolution of grain
size towards equilibrium, being enhanced in regions of high $\dot{\varepsilon}$, results in a weaker rock after a critical time $t_C$ (Fig. 2-15) given by

$$t_C/\varepsilon_T = (n + Rp)/A_D \sigma^n mL.$$  \hspace{1cm} (2.60)

Localization through grain size feedback is possible only when $L$ is positive. This excludes a region near the equilibrium grain size; not only the perturbation must be held for a time longer than $t_C$, but the initial grain size must be sufficiently far
from equilibrium. Immediately outside of the range of initial grain sizes for which localization does not occur, the approximation of small time made in Eq. 2.56 is not valid, which makes the time needed for localization even longer than shown in Fig. 2-15. Hence, the domain where localization is not possible is larger than for strictly $L > 0$. The small time approximation corresponds to $Lm/n + Rp \gg 1$ but $Lm/n + Rp \leq 3$ if the initial grain size is larger than its equilibrium value. Localization may be possible only if the initial grain size is at least one order of magnitude smaller than its equilibrium value.

Departure from the equilibrium grain size may arise from rapid variations of the thermo-chemical environment of the rock. For instance, localization in the grain size evolution models of Braun et al. [1999] requires a thermal history corresponding to uplift to drive the system out of equilibrium. Alternatively, small grain sizes arise as new grains are nucleated. For instance, Rutter [1999] documents a transient weakening in laboratory experiments on marble at the onset of recrystallization, which could lead to localization at tectonic scale. New grains are also nucleated during metamorphism and noecrystallization, all of which often been documented in conjunction with natural shear zone [White and Knipe, 1978; Beach, 1980; Rubie, 1983; Brodie and Rutter, 1985; Fitz Gerald and Stünnitz, 1993; Newman et al., 1999].

2.8 Localization at regional scale

2.8.1 Regional Coupling

In addition to the microscopic feedback processes explored in the last four sections, localization can arise from regional coupling, which affects the overall flow of a studied region. Indeed, pressure, temperature, and even the chemistry of a location depend on the geodynamic flow. These parameters change the rock rheology, thereby influencing the flow.

Although beyond the scope of this paper, one may derive feedback scenarios through which large-scale dynamics influence the environment and are influenced
by rock strength. We already mentioned how large-scale motions such as uplift are crucial for localization through grain-size feedback in the ductile regime [Braun et al., 1999], but this is only one of the many aspects whereby regional feedbacks can produce localized flow.

As different mineral assemblages have different domains of stability in function of temperature and pressure, strength and density jump at phase transitions can produce runaway effects. In mantle convection, the ca. 670 km endothermic phase transition from spinel to perovskite acts as a barrier for thermal downwellings and upwellings [Davies, 1999]. Cold and heavy material accumulates above the transition, until its buoyancy is sufficient to traverse the phase transition, forming avalanches into the lower mantle that are localized both in time and in space [Tackley et al., 1993; Solheim and Peltier, 1994].

Melt is another important quantity that may produce large-scale feedbacks. Many studies explored how melt localizes in a porous viscous rock [Ribe, 1987; Spiegelman and McKenzie, 1987; Stevenson, 1989], but few explored the consequence of melting on lithosphere dynamics [Braun et al., 2000]. In tensile environments, decompression produces melting, which weakens the surrounding rocks [Kohlstedt and Zimmerman, 1996] and facilitates extension. Seafloor spreading might result from such a localization instability. However, hardening of the rocks by extraction of water from the matrix into the melt might more than compensate the melt-weakening effect [Hirth and Kohlstedt, 1996; Braun et al., 2000; Hall and Parmentier, 2000].

### 2.8.2 Localization and plate tectonics

At the largest scale, localization is the essence of plate tectonics. On Earth, mantle convection is organized in almost rigid regions (plates) separated by narrow boundaries [Davies and Richards, 1992; Gordon, 2000]; deformation is localized at plate boundaries. Plate tectonics cannot arise from conventional models of mantle convection [Bercovici et al., 2000; Tackley, 2000b]. Somehow, plate boundaries must be weaker than plate interior and it is debated whether that weakness arises dynamically due to a complex strain-rate weakening rheology of the lithosphere [Bercovici,
1993, 1998; Tackley, 2000a] or whether it is inherited from previous deformation [Zhong and Gurnis, 1996; Gurnis et al., 2000]. Lateral variations of lithosphere viscosity also permits toroidal motion, which is an important aspect of the current plate velocity field [Hager and O’Connell, 1978; Bercovici et al., 2000].

Models where plate boundaries arise dynamically utilize a self-lubricating rheology, often with $r = -1$ [Bercovici, 1993, 1995, 1998; Tackley, 1998, 2000b, a]. The resulting $n_e = -1$ (§2.2.4) indicates much stronger localization than most mechanisms explored above. Such strong localization sweeps the transient effects of the implied localization mechanisms by damage accumulation [Tackley, 2000a].

Although an Earth-like toroidal flow is obtained with self-lubricating rheology and imposed poloidal circulation [Bercovici, 1993, 1995], self-consistent convection models indicate that self-lubrication may not be the real key to plate tectonics. Models that include melt weakening at upwellings and yielding (without weakening) at downwellings gives good numbers for plateness diagnostics, with only minor improvement once self-lubrication is included [Tackley, 2000a]. Hence, plate-like behavior may depend more on large-scale feedbacks produced by melt-weakening and yielding rather than on the local weakening implied by damage accumulation models. In line with that argument, the rheology used by [Trompert and Hansen, 1998] did not produce negative stress exponents, but produce episodic plate-like circulation. Plate tectonic was associated with collapse of the rigid lid, when regional-scale feedback mechanisms are important. We note that whereas melt was assumed to weaken the mantle rocks in [Tackley, 2000a], water extraction upon melting might cancel that weakening or even turn it into melt-strengthening at mid-ocean ridges [Hirth and Kohlstedt, 1996].

No self-consistent model to date produced pure strike-slip plate boundaries. These may only require a memory effect on lithospheric strength [Gurnis et al., 2000; Tackley, 2000a]. Alternative, we speculate than strike-slip boundaries may result from anisotropic weakening of the lithosphere. Indeed, the viscosity is the fourth rank tensor, and it is possible that only certain components of the tensor weaken when the strain rate is perturbed.
### 2.9 Conclusions

Table 2.2: Effective stress exponents

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Expression</th>
<th>Eq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-lubricating rheology</td>
<td>$n_e = \frac{\gamma/\bar{\varepsilon}^2 + 1}{\gamma/\bar{\varepsilon}^2 + 1/r}$</td>
<td>2.3</td>
</tr>
<tr>
<td>Loss of cohesion</td>
<td>$n_e = 1 - \varepsilon \sigma_y / \varepsilon \Delta \sigma$</td>
<td>2.15a</td>
</tr>
<tr>
<td></td>
<td>$n_e = 1 - \frac{\varepsilon_c}{\varepsilon} \left[ \frac{\sigma}{\sigma_f} - \ln \left( \frac{\sigma - \sigma_f}{\Delta \sigma} \right) \right]$</td>
<td>2.15b</td>
</tr>
<tr>
<td>Elasto-plasticity without discontinuity</td>
<td>$n_{xy} = \frac{1 + (1 + r) \sin \psi \sin \varphi}{(1 + r) \sin (\psi - \nu)}$</td>
<td>2.23a</td>
</tr>
<tr>
<td></td>
<td>$n_{xx} = n_{xy} \frac{\cos 2\theta - \nu (1 + r)}{\cos 2\theta + 1/ \sin \varphi}$</td>
<td>2.23b</td>
</tr>
</tbody>
</table>
|                                   | $n_{xx} = \left\{ \begin{array}{l} 
(1 + r) \cos^3 2\theta \\
+ [\rho - (1 + r) (\sin \varphi + \sin \psi)] \cos^2 2\theta \\
+ \left[ 2 + (1 + r) \sin \varphi \sin \psi + \frac{\rho (\sin \varphi - \sin \psi)}{1 + r} \cos 2\theta \right] \\
+ 2 (1 + r) \sin \psi + \rho (3 + 2r) \sin \varphi \sin \psi \end{array} \right\} \frac{(\cos 2\theta - \sin \varphi) (\cos 2\theta - \sin \psi)}{(\cos 2\theta - \sin \psi) (\cos 2\theta - \sin \varphi) \times (\rho - (1 + r) \cos 2\theta)}$ | 2.25a |
| Static to dynamic friction       | $n_e = 1 - \frac{\mu_s}{\mu_s - \mu_d} \frac{D_C}{d}$ | 2.26   |

continued on next page
The importance of localization of deformation can be addressed within a unified framework with the help of the effective stress exponent, $n_e$. That quantity measures how the apparent rheology of a system of internal variable, $\{x_i\}$, responds to a perturbation of one of these variables, $x_0$. To compute the effective stress exponent (Eq. 2.2), the coupling between $x_0$ and the other internal variables must be assessed, as well as the direct response of the strength of the material to a perturbation of $x_0$. In our examples of §2.4 to 2.7, $x_0$ has been the strain or the strain rate undergone by a rock.

The sign of $n_e$ is a criterion for localization: localization requires $n_e < 0$, or dynamic weakening of the material. The value of $n_e$ indicates the efficiency of localization: localization is more efficient for $1/n_e$ more negative. The transition from

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Expression</th>
<th>Eq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate- and state-dependent friction</td>
<td>$n_e = -\mu/c$</td>
<td>2.28</td>
</tr>
<tr>
<td>Gouge dilatancy</td>
<td>$\frac{1}{n_e} = \frac{1}{\mu} \left{ a - b \frac{D}{\theta V} \left[ 1 - \exp \left( -\frac{-Vt}{D} \right) \right] \right}$</td>
<td>2.31</td>
</tr>
<tr>
<td>Shear heating</td>
<td>$\frac{1}{n_e} = \frac{\sigma_e}{G} \frac{d^2v}{d\gamma^2} + \frac{K_f}{G} \frac{dv}{d\gamma} \left( \mu + \frac{dv}{d\gamma} \right)$</td>
<td>2.36a</td>
</tr>
<tr>
<td>Grain size feedback</td>
<td>$n_e = \frac{n + R(p + mr)}{1 + R}$ if $d = D$</td>
<td>2.53</td>
</tr>
<tr>
<td></td>
<td>$n_e = \frac{n + R_0 p - A_D \sigma^n mL r / \varepsilon_T}{1 + R_0}$ if $d \neq D$</td>
<td>2.57</td>
</tr>
</tbody>
</table>
non-localizing behavior to localizing behavior occurs at \( \frac{1}{n_e} = 0 \), called the perfectly plastic limit.

Most instances of localized shear deformation in the Earth’s lithosphere rely on the feedback between the rock strength and an internal variable that is not always explicit when the rheology is described. It could be the elastic strain, coupled with plastic deformation; the geometry of a potential discontinuity; a variable describing the state of fault gouge in frictional sliding regime; temperature or grain size variations in ductile rocks. All these processes give negative \( n_e \), although in some cases (rate- and state-dependent friction laws, shear heating, and grain size feedback), it is necessary for the perturbation to be held for longer than a critical time for the material to be weaker than in the unperturbed state. In Table 2.2, we compile the expressions for the effective stress exponents derived in §2.4 to 2.7. The main aspects of localization by these mechanisms are summarized below.

1. Brittle failure. Upon failure, the strength of a rock decreases, either from loss of cohesion (§2.4) or through the interaction between plastic and elastic strains (§5) with \( n_e \sim -50 \) to -2.

2. Friction constitutive behavior. The rate-and state dependent friction laws gives \( n_e \sim -250 \) (§2.6.1). As velocity localizes, both spatial and temporal localization are predicted. Perturbations must be maintained for more than a critical time \( t_c \) (§6.2) to overcome the stabilizing direct velocity effect.

3. Ductile regime. Localization arises through feedbacks between the flow laws and temperature (§2.7.1) or grain-size (§2.7.2). However, localization through shear heating requires a pre-existing heterogeneity, such as a shallower brittle fault. Localization by grain size feedback requires large deviations from the equilibrium recrystallized grain size. If it can be initiated, localization in the ductile regime is potentially strong.
2.10 Appendix 1: Elasto-plastic Medium with Discontinuity

2.10.1 Elasto-plastic increment matrix

The elastic compliance matrix relating the strain and stress vectors defined in Eq. 2.17 is parameterized by the Lamé parameters $\lambda$ and $G$:

\[
D = G \begin{pmatrix}
2 + r & 0 & r & 0 \\
0 & 2 & 0 & 0 \\
r & 0 & 2 + r & 0 \\
0 & 0 & 0 & 2
\end{pmatrix},
\]  
(2.61a)

\[
D^{-1} = \frac{1}{G} \begin{pmatrix}
\frac{2+r}{4(1+r)} & 0 & \frac{2+r}{4(1+r)} & 0 \\
0 & 1/2 & 0 & 0 \\
\frac{2+r}{4(1+r)} & 0 & \frac{2+r}{4(1+r)} & 0 \\
0 & 0 & 0 & 1/2
\end{pmatrix},
\]  
(2.61b)

where $r = \lambda/G$.

The elasto-plastic strain increment matrix, $M$, has two parts (Eq. 2.20). The first is the elasticity matrix $D$. The second stems from the plastic strain increment. The direction of the stress increment due to plastic flow is

\[
D \cdot \nabla g = \begin{pmatrix}
(1 + r) \sin \psi + \cos 2\theta \\
\sin 2\theta \\
(1 + r) \sin \psi - \cos 2\theta \\
\sin 2\theta
\end{pmatrix},
\]  
(2.62)
and its magnitude is \( \nabla_f^T \mathbf{D} \varepsilon/d \), so that the material stays at yield. Using

\[
\mathbf{D} \cdot \nabla f = \begin{pmatrix}
(1 + r) \sin \varphi + \cos 2\theta \\
\sin 2\theta \\
(1 + r) \sin \varphi - \cos 2\theta \\
\sin 2\theta
\end{pmatrix},
\]

we obtain

\[
d = \nabla f^T \cdot \mathbf{D} \cdot \nabla g = 1 + (1 + r) \sin \varphi \sin \psi.
\]

2.10.2 Effect of a discontinuity

Let us consider an external stress increment as defined in Eq. 2.24, and a planar discontinuity with its normal oriented in the \( x \)-direction separating a region deforming elastically from a region deforming elasto-plastically. The components of the stress increment \( d\sigma_{xy} \) and \( d\sigma_{yy} \) are transmitted across the discontinuity. Therefore, in the elasto-plastic domain, \( d\sigma_{xy} \) and \( d\sigma_{yy} \) are given by Eq. 2.24 but \( d\sigma_{xx} \) is unknown.

In the elasto-plastic domain, the stress increment verifies \( d\sigma = \mathbf{M} \cdot \varepsilon \), with \( \mathbf{M} \) defined in Eq. 2.20 and \( \varepsilon \) a strain increment yet be determined. If there is no slip, \( d\varepsilon_{yy} \) is transmitted across the discontinuity:

\[
d\varepsilon_{yy} = \frac{1}{2G(1 + r)} (\rho - (1 + r) \cos 2\theta).
\]

The two remaining components of the strain increment, \( d\varepsilon_{xx} \) and \( d\varepsilon_{xy} \) are determined from the equations for \( d\sigma_{xy} \) and \( d\sigma_{yy} \). We obtain a set of two equations with two unknowns:

\[
\begin{cases}
S_1 d\sigma_{ii} = AGd\varepsilon_{xx} + BGd\varepsilon_{xy} \\
S_2 d\sigma_{ii} = CGd\varepsilon_{xx} + DGd\varepsilon_{xy}
\end{cases}
\]

where

\[
S_1 = \rho + \cos 2\theta - \frac{\rho - (1 + r) \cos 2\theta}{2(1 + r)} \times
\]
The solutions of this system of equations are given by:

\[
S_1 = B - S_2 D
\]

\[
A - B
\]

\[
C - D
\]

We compute:

\[
\begin{vmatrix}
A & B \\
C & D
\end{vmatrix} = \frac{(1 + r) \cos 2\theta - \sin \varphi \cos 2\theta - \sin \psi}{d},
\]

\[
\begin{vmatrix}
A & S_1 \\
C & S_2
\end{vmatrix} = \frac{(1 + r) \sin 2\theta}{d} \left[ \frac{2 + r}{1 + r} \right] \left[ 1 + \rho \sin \varphi \right]
\]

+ \sin \varphi \sin \varphi - (\sin \varphi + \sin \psi) \cos 2\theta + \cos^2 \theta,

\[
\begin{vmatrix}
S_1 & B \\
S_2 & D
\end{vmatrix} = \frac{1}{d} \left[ 2 (1 + r) \sin \psi + \rho (3 + 2r) \sin \varphi \sin \psi \right.
\]

+ \left[ 2 + (1 + r) \sin \varphi \sin \psi + \rho (\sin \varphi - \sin \psi) \right] \cos 2\theta
\]

+ \left[ \rho - (1 + r) (\sin \varphi + \sin \psi) \right] \cos^2 2\theta.

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The elasto-plastic strain increments are undetermined if \( \frac{1}{n_{xx}} = 0 \), or \( 1/n_{xy} = 0 \). This occurs when the discontinuity is oriented at \( \theta_\varphi = \pi/4 - \varphi/2 \) and \( \theta_\psi = \pi/4 - \psi/2 \). These conditions correspond to the bifurcation analysis of [Rudnicki and Rice, 1975].

The condition \( \begin{vmatrix} A & S_1 \\ C & S_2 \end{vmatrix} = 0 \) indicates drastic localization of the shear strain, \( n_{xy} = 0 \). It occurs when

\[
\cos 2\theta = \cos 20 \sin 20 \sin (\theta + 2\theta) = \\
= -\sqrt{\frac{(\sin \varphi - \sin \psi)^2}{2}} - \frac{2 + r}{1 + r} (1 + \rho \sin \varphi) + \frac{\sin \varphi + \sin \psi}{2},
\]

where the angle \( \theta \) is real only for

\[
\rho < \frac{1 + r}{4(2 + r)} \frac{(\sin \varphi - \sin \psi)^2 - 1}{\sin \varphi}.
\]

The other determinant can also be zero, indicating drastic localization of the normal strain, \( n_{xx} = 0 \), but we could not derive a simple expression for the critical angle.

### 2.11 Appendix 2: Rate- and State-Dependent Friction

The RSDF laws include two rheological variables, \( V \) and \( \theta \) (Eq. 2.29). Using \( V \) as the localization parameter, the effective stress exponent is in general:

\[
\frac{1}{n_c} = \frac{V}{\mu} \left( \frac{\partial \mu}{\partial V} + \frac{\partial \mu}{\partial \theta} \frac{d\theta}{dV} \right) = \frac{1}{\mu} \left( a + b \frac{V}{\theta} \frac{d\theta}{dV} \right).
\]
As \( \theta \) is an integral function of the velocity history (Eq. 2.30), the effective stress exponent depends on time and on the specific \( \theta-V \) history. We consider two specific cases. In the first (§2.6.1), \( \theta \) remains at equilibrium \( \theta_0 = D/V \), which corresponds to the limit \( t \to \infty \) or \( D \to 0 \). In that case:

\[
d\theta/dV = -\theta/V, \tag{2.72}
\]

\[
1/n_e = (a - b)/\mu = c/\mu \tag{2.73}
\]

In the other case (§2.6.2), the system is initially at equilibrium at velocity \( V \) and \( \theta = D/V \), but for \( t > 0 \), the sliding velocity is \( V + dV \). As the velocity does not depend on time for \( t > 0 \), the state evolution laws (Eq. 2.30) can be integrated and give

Dieterich law: \( \theta = \frac{D}{V + dV} + \frac{D dV}{V (V + dV)} \), \tag{2.74a}

Ruina law: \( \theta = \frac{D}{V + dV} \left(1 + \frac{dV}{V}\right)^x \), \tag{2.74b}

with \( x = \exp \left[-t (V + dV)/D\right] \). For \( dV \ll V \) both evolution laws give

\[
\frac{d\theta}{dV} \approx \frac{D}{V} \left[-1 + \exp \left(-\frac{V t}{D}\right)\right], \tag{2.75}
\]

which, substituted in Eq. 2.71, gives Eq. 2.31.
Chapter 3

Instability analysis of simple lithosphere models with dynamic localization

Abstract

The structure of several orogens displays regularly-spaced faults and localized shear zones. Numerical models reproduce this phenomenon. To understand how fault sets organize with a characteristic spacing, we present a semi-analytical instability analysis of an idealized lithosphere composed of a brittle layer over a ductile half-space undergoing horizontal shortening or extension. The rheology of the layer is characterized by an effective stress exponent, $n_e$. The layer is pseudo-plastic if $1/n_e = 0$ and forms localized structures if $1/n_e < 0$. The tendency for localization is stronger for more negative $1/n_e$. Two instabilities grow simultaneously in this model: the buckling/necking instability that produces broad undulations of the brittle layer as a whole, and the localization instability that produces a spatially periodic pattern of faulting. The latter appears only if the material in the brittle layer weakens in response to a local perturbation of strain rate, as indicated by $1/n_e < 0$. Fault spacing scales with the thickness of the brittle layer and depends on the efficiency of localization. The more efficient localization is, the more widely spaced are the faults. The fault spacing is related to the wavelength at which different deformation modes within the layer enter a resonance that exists only if $1/n_e < 0$. Depth-dependent viscosity and the model density offset the instability wavelengths by an amount that we determine empirically. The wavenumber of the localization instability, is $k^L_j = \pi (j + a_L) (-1/n_e)^{-1/2} / H$, with $H$ the thickness of the brittle layer, $j$ an integer, and $1/4 < a_L < 1/2$ if the strength of the layer increases with depth and the strength of the substrate decreases with depth.
3.1 Introduction

Much of our theoretical understanding of tectonics stems from continuum mechanics [Turcotte and Schubert, 1982]. For instance, certain large-scale patterns of deformation resemble a mode of folding of the strong layers of the lithosphere called buckling in compression, and necking in extension [Biot, 1961; Fletcher and Hallet, 1983; Richard and Froideveau, 1986; Zuber et al., 1986; Zuber, 1987a]. The buckling or necking theory predicts a preferred wavelength of deformation that is controlled by the structure of the lithosphere. Faults and shear zones constitute a primary indicator of the structure of orogens or rifts that can also be related to continuum mechanics models [Beaumont and Quinlan, 1994]. Faults often present a preferred spacing or characteristic scale [Weissel et al., 1980; Zuber et al., 1986; Davies, 1990; Watters, 1991; Bourne et al., 1998]. The buckling/necking theory might be used to model fault spacing [Watters, 1991; Brown and Grimm, 1997], but faults represent a localized style of deformation that is not accessible using the continuum theories implied in the buckling/necking analysis; deformation occurs mostly—if not entirely—within a narrow band.

Buckling and necking produce broad undulations of the lithosphere in which deformation is distributed. The pseudo-plastic rheology used to model the brittle levels of the lithosphere [Fletcher and Hallet, 1983] assumes distributed failure or faulting. However, faulting has a tendency to localize, to abandon a distributed style of faulting to concentrate deformation on a few isolated major faults [Sornette and Vanneste, 1996; Gerbault et al., 1999, Chapter 2]. As stress heterogeneities induced by buckling favor faulting in the hinge of large-scale folds [Lambeck, 1983; Martinod and Davy, 1994; Gerbault et al., 1999], localized fault patterns can be controlled by buckling if they develop after the folds have reached sufficient amplitude. However, faulting may occur from the onset of deformation in the buckling/necking theory. Hence, the faults have the possibility to organize themselves without the influence of finite-amplitude buckling. Indeed, some tectonic provinces display faults with a characteristic spacing unrelated to the buckling wavelength. To cite only examples in compressive environ-
ments, faults in the Central Indian Basin [Bull, 1990; Van Orman et al., 1995], in Central Asia [Nikishin et al., 1993], or in Venusian fold belts [Zuber and Aist, 1990] are more closely spaced than the wavelength of folds in the same region. Faulting and buckling appear as superposed deformation styles, each with a characteristic length scale.

Buckling and necking were first studied in Earth sciences as a mechanism to form folds or boudins in outcrop-scale layered sequences [Johnson and Fletcher, 1994]. While originally derived using a thin plate approximations of viscous and/or elastic materials [Ramberg, 1961; Biot, 1961], the buckling/necking theory was later developed as a thick plate formulation [Fletcher, 1974; Smith, 1975] and was applied to non-Newtonian materials [Fletcher, 1974; Smith, 1977]. For a non-Newtonian fluid, the stress supported by the fluid, $\sigma$, is related to the second invariant of strain rate, $\dot{\varepsilon}_{II}$, by $\dot{\varepsilon}_{II} \propto \sigma^{n_e}$, with $n_e$ the effective stress exponent, which is a measure of the non-linearity of the rock rheology [Smith, 1977, Chapter 2]. Non-Newtonian creep with $1 < n_e < 5$ is the rheology that describes rocks at sufficiently high temperature to behave in a ductile manner.

At low temperature, rocks behave in a brittle manner. The stress that they can support is instead limited by a yield strength, at which failure, faulting, and plastic flow occur. Yielding can be included in thin-plate analyses of folding by limiting the bending stresses to the yield strength and reducing the apparent flexural rigidity of a elastic or viscous plate accordingly [Chapple, 1969; McAdoo and Sandwell, 1985; Wallace and Melosh, 1994]. The thick-plate formulation of the buckling theory is particularly well adapted to an alternative treatment of failure, by which the yielding material is approximated as a highly non-Newtonian fluid in the limit $n_e \to +\infty$ [Smith, 1979; Chapple, 1969]. Most lithospheric-scale applications of buckling follow that approximation [Fletcher and Hallet, 1983; Zuber et al., 1986; Zuber, 1987a]. More accurate treatments of yielding have been included in buckling/necking theory. Leroy and Triantafyllidis [1996] and Triantafyllidis and Leroy [1997] studied the necking behavior of an elastic–plastic medium at yield using the strain rate–stress rate relations of the deformation theory of plasticity. However, deformation remained
distributed in their model; no localized faulting was predicted [Leroy and Triantafyllidis, 1996; Triantafyllidis and Leroy, 1997]. Davies [1990] modeled the buckling of a rigid–plastic layer with associated flow law surrounded by a rigid basement and a viscous half–space. Faults were forced by a local cusp in the model interface but an initially distributed perturbation remains distributed. Neumann and Zuber [1995] found that a localized perturbation triggers macroscale shear bands in a power–law medium with $n_\varepsilon \to +\infty$ as well. Fletcher [1998], who also included the effects of pore fluids and pressure–solution on a non–Newtonian porous fluid with $n_\varepsilon \to +\infty$, showed that the shear bands produced by a localized forcing are ephemeral. He speculated that the bands would be stabilized and therefore may correspond to faults if strain–weakening were included [Fletcher, 1998].

All previous treatments of yielding in buckling theory fail to produce localized deformation; faulting remains distributed throughout the material. Hence, any fault pattern develops late in the folding history, with a spacing that is controlled by the buckling or necking wavelength. However, this is contrary to many geological observations [Weissel et al., 1980; Nikishin et al., 1993]. In order to understand how fault spacing may differ from the buckling/necking wavelength, we study the buckling behavior of simplified lithosphere models with a rheology that weakens with deformation upon yielding. The weakening behavior is characterized by a negative effective stress exponent. Such an effective rheology brings a tendency to localize deformation and faulting [Chapter 2].

The effective stress exponent, $n_\varepsilon$, indicates how a material responds to a perturbation in the deformation field [Smith, 1977]. In this analysis, deformation is quantified by the second strain rate invariant, $\dot{\varepsilon}_{ii}$. When $n_\varepsilon < 0$, increasing $\dot{\varepsilon}_{ii}$ decreases the material strength, which ensures localization [Chapter 2]. The (algebraic) value of $n_\varepsilon$ is determined from the physical process that produces localization. It incorporates not only the direct effect of perturbing the strain rate, but also the possible feedback of internal variables that control the rheology. In particular, a frictional material such as the pervasively faulted brittle lithosphere, requires a higher stress to deform more rapidly. However, a higher sliding velocity on faults changes also the physical state.
of a granular gouge inside the fault and results in apparent weakening [Dieterich, 1979; Scholz, 1990]. In Chapter 2, we derive the conditions for which this feedback mechanism and others produce localization, and we give values for the corresponding effective stress exponents. For frictional sliding, $-300 < n_e < -50$.

Neurath and Smith [1982] showed that strain weakening reduces the value of $1/n_e$ in a non–Newtonian fluid, possibly resulting in $n_e < 0$. However, the materials that they studied never had negative $n_e$. While they recognized that $n_e < 0$ would lead to “catastrophic failure”, or localization, modulated by the interaction with the surroundings of the layer with negative exponent, Neurath and Smith [1982] did not present an analysis of buckling with $n_e < 0$.

In this paper, we derive the growth spectrum for simple models of the lithosphere. The growth spectrum indicates how fast a given wavelength growths in a particular model [Johnson and Fletcher, 1994]. Buckling and necking occur at wavelengths at which the growth rate is maximum. We identify a new instability when the material localizes and call it localization instability. Both the localization instability and the buckling/necking instability are associated with particular resonances within the localizing layer. We indicate how the wavelengths of the buckling and localization instabilities relate to the resonant wavelengths for several types of depth–dependent strength profiles that approximate the strength profile of a mono–mineralic lithosphere. Most of this study is conducted assuming uniform horizontal shortening. The generalization to horizontal extension is discussed in §3.6.2. In a companion paper, we show that the localization instability is a possible explanation for fault spacing in the Central Indian Basin, which is incompatible with a buckling instability.

### 3.2 Instability principle

#### 3.2.1 Solution strategy

The buckling/necking and localization instabilities develop in mechanically layered models of the lithosphere undergoing horizontal shortening or extension. In this
Figure 3-1: Schematics of lithosphere models. A layer of thickness $H$, viscosity $\eta_1(z)$, and effective stress exponent $n_1$ lies over a half-space of different mechanical properties $[\eta_2(z), n_2 = 3]$. The model is two-dimensional, incompressible, and subjected to pure shear shortening at the rate $\dot{\varepsilon}_{xx}$. Its density is $\rho$. For the instability analysis, the interfaces are perturbed by an infinitesimal sinusoidal topography of wavelength $\lambda$ and amplitude $\xi_1$ and $\xi_2$.

study, we concentrated on the behavior of a single brittle or plastic layer overlying a ductile half-space under horizontal shortening at the rate $\dot{\varepsilon}_{xx}$ (Fig. 3-1). Horizontal extension is discussed in §3.6.2. Each material is incompressible. As the deformation field is constrained to be two-dimensional, it is completely determined by the stream function $\varphi(x, z)$.

The deformation field in each layer is decomposed into a primary field and a secondary field of much smaller amplitude. The primary field is the flow solution when the interfaces in the model are perfectly flat and horizontal. It is invariant in $x$, with $\dot{\varepsilon}_{zz} = -\bar{\varepsilon}_{zz}$, and $\bar{\varepsilon}_{xx} = 0$. However, any relief on the interfaces perturbs the system and drives a secondary flow in the model. In turn, the secondary flow deforms
This analysis determines the rate at which interface perturbations grow in a given model. The flow equations for the secondary deformation field and the boundary conditions are linearized, assuming that the interface perturbations have a much smaller amplitude than the layer thickness [Johnson and Fletcher, 1994, Appendix A]. Thus, the Fourier components of an infinitesimal generic perturbation are decoupled from one another. Hence, we consider isolated interface perturbations

$$\xi_i = \xi_i^0 \exp(ikx),$$  \hspace{1cm} (3.1)

with $\xi_i^0$ the amplitude of the perturbation, $k$ its wavenumber, $i$ an index identifying an interface of the model, and $i = (-1)^{1/2}$. There are several modes of deformation in each layer, each characterized by a stream function $\varphi_j$

$$\varphi_j = \varphi_j^0 \phi_j(z) \exp(ikx),$$  \hspace{1cm} (3.2)

where $\phi_j$ is the depth kernel, $\varphi_j^0$ the amplitude of $\varphi_j$, and $j$ is an index identifying each deformation mode. The depth kernel is determined from the equations of Newtonian equilibrium and the mode amplitude is determined from the stress and velocity boundary conditions are each interface [Johnson and Fletcher, 1994, Appendix A]. Therefore, $\varphi_j^0$ depends on $\{\xi_i\}$. The depth kernel is discussed further in §3.3.

The velocity field from each deformation mode $j$ is given by

$$v_j^i(x, z) = -\varphi_j^0 \partial \phi_j / \partial z \exp(ikx),$$  \hspace{1cm} (3.3a)

$$v_j^i(x, z) = ik \varphi_j^0 \phi_j \exp(ikx).$$  \hspace{1cm} (3.3b)

It is a function of the amplitude perturbations.

The rate at which the interface perturbations grow has a kinematic contribution from the pure shear thickening of the model (primary flow field) and a dynamic contribution from the secondary flow field [Smith, 1975]. The latter comes from summing Eq. 3.3, evaluated at the depth of an interface, $z_i$, over the subset $\{j\}_i$ of
the deformation modes in the layers in contact with the interface $i$:

$$\frac{d\xi^0_i}{dt} = -\dot{\varepsilon}_{ix}\xi^0_i + ik \sum \varphi^0_j \phi_j.$$  \hspace{1cm} (3.4)

As the stream function amplitudes $\{\varphi^0_j\}$ are proportional to the amplitudes of all the interface perturbations, Eq. 3.4 can be written using the growth matrix $Q_{ij}$ [Appendix A]:

$$\frac{d\xi^0_i}{dt} = (-\dot{\varepsilon}_{ix} \delta_{ij} + Q_{ij})\xi^0_j,$$  \hspace{1cm} (3.5)

with $\delta_{ij}$ the Kronecker operator. The growth rate $Q$ is the eigenvalue of $Q$ that has the largest real part. The associated eigenvector describes the deformation mode of the model as a whole [Smith, 1975; Zuber et al., 1986]. For instance, it determines whether a given layer deforms by buckling (upper and lower interfaces in phase) or necking (upper and lower interfaces out of phase). We define the growth spectrum as the function $Q(k)$.

### 3.2.2 Effective rheology and effective stress exponent

The strength of each layer of the lithosphere model is an analytical function of depth. As we address the organization of strain rate within this model, we define the apparent viscosity $\eta$ by

$$\sigma_{ij} = -p\delta_{ij} + \eta\dot{\varepsilon}_{ij},$$  \hspace{1cm} (3.6)

where $\sigma$ is the stress supported by the material, $p$ the pressure, and $\dot{\varepsilon}$ the strain rate. In general, $\eta$ depends on the second invariant of the strain rate, $\dot{\varepsilon}_{ii}$ and depth $z$. In the brittle field, the material strength and therefore the viscosity may also depend on the strain undergone by the material. We chose in this analysis to ignore that additional complication, which would also require elasticity to be included in the model and make the analysis time-dependent [Neurath and Smith, 1982; Schmalholz and Podladchikov, 1999]

In the perturbation analysis, the flow field is decomposed into a primary field and
a secondary field. The primary field, which represents the state of uniform shortening is denoted by over-bars. It obeys

$$\sigma_{ij} = -p\delta_{ij} + \eta \dot{\varepsilon}_{ij},$$  \hspace{1cm} (3.7)

with $\eta$ the material viscosity at the externally imposed strain rate $\dot{\varepsilon}_i = |\dot{\varepsilon}_{xx}|$. It is an analytic function of depth in each layer. The secondary field (denoted with a tilde), which represents the perturbing flow obeys the apparent rheology:

$$\tilde{\sigma}_{xx} = -\tilde{p} + \frac{\eta}{n_e} \tilde{\varepsilon}_{xx},$$  \hspace{1cm} (3.8a)

$$\tilde{\sigma}_{zz} = -\tilde{p} + \frac{\eta}{n_e} \tilde{\varepsilon}_{zz},$$  \hspace{1cm} (3.8b)

$$\tilde{\sigma}_{xz} = \eta \tilde{\varepsilon}_{xz},$$  \hspace{1cm} (3.8c)

with $n_e$ the effective stress exponent,

$$\frac{1}{n_e} = 1 + \frac{\dot{\varepsilon}_i}{\eta} \frac{\partial \eta}{\partial \varepsilon_{ii}} = \frac{\dot{\varepsilon}_i}{\sigma} \frac{\partial \sigma}{\partial \dot{\varepsilon}_{ii}},$$  \hspace{1cm} (3.9)

calculated at the strain rate $\dot{\varepsilon}_i$. In deriving Eq. 3.8, we assumed that the amplitude of the secondary field is infinitesimal with respect to the primary field. This approximation is valid only for the onset of the instability.

The apparent viscosity of the secondary field is anisotropic, being reduced in the directions of the primary flow field by a factor $n_e$. As introduced by Smith [1977], the effective stress exponent is a local measure of the non-linearity of the rheology of a material. We extended this concept in Chapter 2 and used $n_e$ in a general framework of localization. When $n_e < 0$, the material is unstable with respect to local perturbations. A negative effective stress exponent indicates that the strength is reduced in locations where the strain rate is enhanced. This situation is unstable and results in a localized zone of high strain rate [Chapter 2]. The instability analysis of this study shows how these localized deformation areas organize at lithospheric scale.

Beyond the sign of the effective stress exponent, its algebraic value provides a
quantitative measure of the efficiency of localization [Chapter 2]. If $1/n_e = 1$ the material is effectively Newtonian, and does not localize at all. If $0 < 1/n_e < 1$, the material is well described by non–Newtonian creep. For instance, rocks deforming by ductile creep have $0.2 < 1/n_e < 1$. In this regime, a rock is softening in the sense that its apparent viscosity decreases with increasing strain rate. Local perturbations of strain rate are enhanced by the non–Newtonian behavior, but the material is stable: as its strength increases with strain rate, there is no dynamic weakening and no localization [Chapter 2]. In the limit $1/n_e \rightarrow 0^+$, the material is pseudo–plastic: its strength does not depend on strain rate. Localization requires $1/n_e < 0$. We showed in Chapter 2 that many mechanisms that are associated with localized shear zones in the laboratory or in nature have a negative effective stress exponent, often with $1/n_e \sim -10^{-2}$ to $-10^{-1}$ in the brittle field.

Often, the effective stress exponents is negative only when an internal feedback process is considered that may include a variable describing damage, or state of a fault gouge. This variable may require a finite time to respond to a local variation of strain rate. This results in an immediate strengthening response of the system, followed by weakening in the long-time limit [Chapter 2]. In this study, we ignore the transient response, arguing that perturbations may be held for long enough that steady–state is reached, and that the perturbation amplitude is so small that the strengthening “barrier” is easily overcome. However, this assumption should be relaxed in future work.

Most previous studies of lithospheric–scale instabilities treated the brittle upper crust and mantle as pseudo–plastic with $1/n_e \rightarrow 0^+$. These instabilities produce buckling in compression, and necking in extension [Fletcher and Hallet, 1983; Ricard and Froideveau, 1986; Zuber, 1987a]. In this study, we introduce the solutions for $1/n_e < 0$, which produce regularly–spaced shear zones, through a process that we call localization instability. The more negative $1/n_e$ is, the stronger localization is. Intuitively, a more efficient localized shear zone can accommodate the deformation from a wider non–localized region of the lithosphere. Hence, the spacing of localized shear zones should increase when $1/n_e$ is more negative.
3.3 Depth kernel

3.3.1 Fundamental equation

The first step in solving the instability problem defined above is to determine the expression of the depth kernel, \( \phi(z) \), which gives the depth–dependence of the stream function (Eq. 3.2) and therefore of the deformation field for each mode of deformation. Whereas the strength and the effective viscosity the lithosphere depend on depth, its effective stress exponent does not necessarily do so, as it measures the rate at which a rock weakens, scaled by its strength. In fact, \( n_e \) does not depend on depth for most processes of localization explored in Chapter 2. Therefore, we make the simplificative assumption that \( n_e \) is independent of depth, \( z \), within each layer.

We write the equations of Newtonian equilibrium for the secondary flow, using its apparent rheology (Eq. 3.8), and expressing the strain rate as a function of the stream function. These equations are then combined to eliminate the pressure term, and simplified as their dependence in \( x \) is \( \exp(ikx) \). We obtain [Appendix A]

\[
\frac{d^4 \phi}{dz^4} + 2r \frac{d^3 \phi}{dz^3} - (Ak^2 - s) \frac{d^2 \phi}{dz^2} - Ak^2 r \frac{d \phi}{dz} + (k^4 + k^2 s) \phi = 0, \tag{3.10}
\]

where we used the notation

\[
r(z) = \frac{d\eta/dz}{\eta}, \tag{3.11}
\]

\[
s(z) = \frac{d^2\eta/dz^2}{\eta}, \tag{3.12}
\]

\[
A = \frac{4}{n_e} - 2. \tag{3.13}
\]

As Eq. 3.10 is a fourth order partial differential equation, it admits four solutions for a given strength profile \( \eta(z) \), effective stress exponent \( n_e \), and wavenumber \( k \). Hence, there are four superposed deformation modes in each layer, for a given wavelength, each with its own amplitude that is determined from matching boundary
conditions at each interface [Appendix A].

### 3.3.2 Depth kernel for exponential and constant viscosity profiles

The depth kernel can be solved analytically if the viscosity varies exponentially with depth. Then, the function \( r \) does not depend on \( z \), \( s = r^2 \), and

\[
\eta = \eta_e \exp(rz),
\]

(3.14)

with \( \eta_e \) a constant. Constant viscosity layers are included as the special case \( r = 0 \).

For an exponential viscosity profile, Eq. 3.10 is solved by

\[
\phi = \phi_0 \exp \imath \alpha kz,
\]

(3.15)

with

\[
\alpha = \frac{1}{2} \left( \frac{1 - 2}{n_e} - \frac{R^2}{4} \pm \left( \frac{4}{n_e^2} - \frac{4}{n_e} - R^2 \right)^{1/2} \right)^{1/2},
\]

(3.16)

where \( R = r k \) [Fletcher and Hallet, 1983].

There are four values of the parameter \( \alpha \), each corresponding to a given deformation mode with spatial dependence: \( \exp \{ \imath k(x + \alpha z) \} \). Therefore, the amplitude of the stream function varies over a depth scale \( 1/\text{Im}(\alpha) \) and is correlated along lines with slope \(-\text{Re}(\alpha)\). Hence, \( \alpha \) is referred to as the mode slope.

For constant viscosity layers \( (r = 0) \), Eq. 3.16 becomes

\[
\alpha = \begin{cases} 
\pm \sqrt{-1 + 2 \left( 1 \pm \sqrt{1 - n_e} \right)/n_e}, & \text{if } 1 < 1/n_e, \\
\pm \sqrt{1 - 1/n_e} \pm i \sqrt{1/n_e}, & \text{if } 0 < 1/n_e < 1, \\
\pm \sqrt{1 - 2 \left( 1 \pm \sqrt{1 - n_e} \right)/n_e}, & \text{if } 1/n_e < 0.
\end{cases}
\]

(3.17)

These relations are depicted in Fig. 3-2. By convention, we define \( \alpha_1, \alpha_2, \alpha_3, \) and \( \alpha_4 \), by selecting in Eq. 3.16 or 3.17 the sign combinations \((+, +), (+, -), (-, +), \) and \((-,-)\).
Inverse Effective Stress Exponent 1/n_e

Figure 3-2: Mode slope parameters α in function of inverse effective stress exponent 1/n_e. Thick lines: Re(α); thin lines: Im(α).

Note that α is real when n_e is negative, i.e., when the material localizes. In that case, the stream function is correlated along four different slopes, but its amplitude is constant with depth: interface perturbations generate four wavelike deformation fields in each layer, none of which decays or grows with depth. This makes it impossible to solve for the behavior of a half-space with negative n_e, which requires the deformation field to vanish at infinity. Hence, the simplest solvable model that includes negative n_e is made of a layer of finite thickness over a half-space with positive n_e. Although our formulation can in principle handle any number of layers [Appendix A], only this type of model is considered in this paper.

If r \neq 0, the n_e−R space can also be divided into different domains as shown in Fig. 3-3 for cases where the different values of α − iR/2 are real, complex, or pure.
imaginary. The boundaries of these regions are

\[
\begin{align*}
\frac{1}{n_e} &= \frac{1 + \sqrt{1 + R^2}}{2} \quad \text{Boundary 1}, \\
\frac{1}{n_e} &= \frac{1 - \sqrt{1 + R^2}}{2} \quad \text{Boundary 2}, \\
\frac{1}{n_e} &= \frac{R^4 + 8R^2 + 16}{16R^2} \quad \text{Boundary 3}.
\end{align*}
\]

The domain where all \( \alpha \) are real is pushed to more negative \( 1/n_e \) when \( R \) increases, and vanishes altogether at \( R = 2\sqrt{2 + \sqrt{5}} \). This is a limit where the viscosity increases rapidly compared to the perturbation wavelength. However, even in the large wavelength limit \( \lambda > \pi/r\sqrt{2 + \sqrt{5}} \), there are more than two values \( \text{Re}(\alpha) \), one of which is zero. We will see that this condition is sufficient for the localization instability to develop. Fig. 3-4 presents the real and imaginary part of in function of of \( R \) and \( 1/n_e \).

### 3.3.3 General solution

For non-exponential viscosity profile, Eq. 3.10 must be solved numerically. We use a Runge–Kutta integration technique. By convention, the depth kernel has a value of 1 at the top of each layer. The four superposed modes of deformation are found by setting the initial values of the depth–derivatives of \( \phi \) in turn to each solution of \( \phi(z) \) for an exponential viscosity profile that approximates the actual viscosity profile at the top of the layer (Eq. 3.16). We verified that the solutions do not depend on the actual starting scheme chosen. The exceptional cases where the solutions are degenerate are ignored, and do not arise in practice except if \( 1/n_e = 1 \) or \( 1/n_e = 0 \).

In this study, we use the numerical solver only for the case where the viscosity depends linearly with depth. Only Bassi and Bonnin [1988b] have considered that case previously. They used the polynomial expansion of the depth kernel

\[
\phi(z) = \sum_{j=0}^{\infty} a_j z^j
\]

and determined a recurrence relation between the polynomial coefficients. Although
Figure 3-3: Solution domains for the mode slope parameters $\alpha$. The cross marks $R = 2\sqrt{2} + \sqrt{5}$ beyond which not all $\alpha - iR/2$ can be real.

technically an analytical solution, this scheme is subject to numerical errors and truncation of the expansion. We favored the numerical integration technique as it can handle any viscosity profile such as in the companion paper. The only limitation is that $\eta \neq 0$.

3.4 Constant viscosity analysis

3.4.1 Growth spectra

We first consider models where a layer of thickness $H$, effective stress exponent $n_1$, and viscosity $\eta_1$, independent of depth, lies over a half-space of lower viscosity, here $\eta_2 = \eta_1/10$, and effective stress exponent $n_2 > 0$. The exact value of $n_2$ matters little.
Figure 3-4: Two views of the real part (a and b) and the imaginary part (c and d) of $\alpha - iR/2$ as a function of $R$ and $1/n_e$, contoured every 0.5.

Here, we use $n_2 = 3$ (Fig. 3-1). This is the simplest approximation of the lithosphere strength structure that displays both localization and buckling instabilities. The model is shortened at the rate $\tilde{\varepsilon}_{xx} < 0$. For now, the material density is ignored. The coordinates $x$ and $z$ are normalized by $H$, the wavenumbers are normalized by $H^{-1}$, and the stresses are normalized by $\eta_1 \dot{\varepsilon}_{tt}$.

We present in Fig. 3-5 the growth spectra for various values of the effective stress exponent of the layer: $1/n_1 = 10^{-2}$, $10^{-6}$, and $-10^{-1}$. The flow fields for the last two
Figure 3-5: a) Growth spectrum for a layer of uniform viscosity $\eta_1$ overlying a half-space of uniform viscosity $\eta_2 = \eta_1/10$, with $\rho = 0$, $n_2 = 3$. Solid line: $n_1 = -10$; dashed line: $n_1 = 10^6$; dotted line: $n_1 = 100$. b) Viscosity profile. $K_{B/N} \approx \pi$ and $K_L \approx 10$ for the values of $n_1$ considered.

Cases are plotted in Fig. 3-6.

The cases with positive $n_1$ have been solved before [Ricard and Froideveau, 1986; Zuber, 1987a]. The growth spectrum passes through a first maximum at wavenumber $k \approx \pi/2$ that defines the preferred wavelength of the buckling instability of the layer as a whole (Fig. 3-6a). A growth spectrum characteristic of the buckling instability (Fig. 3-5) passes through successive maxima with a wavenumber scale $K_{B/N} - $standing for wavenumber of the buckling/necking instability. If $1/n_1$ is finite, the envelope of the growth spectrum decreases with wavenumber following the decay of the secondary field with depth indicated by $\text{Im}(\alpha) \neq 0$ in Eq. 3.17 [Ricard and Froideveau, 1986; Neumann and Zuber, 1995]. Accordingly, the magnitude of the growth rate does not change with $k$ in the pseudo-plastic limit $1/n_e \to 0$, represented here by $1/n_e = 10^{-6}$.

The growth spectrum for the case $n_e = -10$ is different from the other two (Fig. 3-5). It is best described as the superposition of a buckling-type spectrum and a sequence of doublets with infinite growth rate. These doublets denote the localization instability. The difference between the wavenumbers of consecutive doublets defines the wavenumber scale $K_L$, which is different from $K_{B/N}$. In Fig. 3-5, $K_{B/N} \sim \pi$. 

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Figure 3-6: Deformation fields corresponding to the growth spectra in Fig. 3-5, shaded as a function of strain rate. a) $n_1 = 10^6$; b) $n_1 = -10$. The velocity field is computed from a small-amplitude initial perturbation at the surface such that $\xi_1 \propto k^b$, with $b$ a real number. Then, the amplitude of the other interface is set to give the eigenvector of the growth matrix (Eq. 3.5) with the fastest growth rate. These modes are amplified by the growth rate (limited to an arbitrary value if $n_1 < 0$). The initial spectral shape, the amplification factor, and the limiting growth rate have been chosen to show clearly (a) the buckling deformation mode and (b) both the buckling and the localization modes.

and $K_L \sim 10$. The reconstructed deformation field has the same appearance of two superposed deformation modes, each having a specific length scale: buckling of the layer as a whole, and regularly-spaced localized shear zones. The infinite growth rate at the localization doublets (Fig. 3-5) is due to the unstable character of localization: a local perturbation of strain rate weakens the material locally, so the strain rate increases further. The strain rate perturbations trigger a positive feedback that results in an infinite growth rate. The localized shear zones have a large-scale organization given by the wavenumber at which a divergent doublet is present.
3.4.2 Resonance

The wavenumber scales $K_{B/N}$ and $K_L$ apparent in the growth spectra (Fig. 3-5) correspond to certain resonances between the superposed deformation modes in the layer of our model. If the wavelength of the interface perturbation is $\lambda$, potential shear zones are nucleated at the top surface ($z = 1$, as lengths are normalized by $H$) at $x = x_0 + j\lambda$, $j$ an integer (i.e., $j \in \mathbb{Z}$). At each location, four shear zones propagate into the medium, each corresponding to a particular deformation mode, or solution of Eq. 3.10. A shear zone nucleated at $x = x_0$, $z = 1$, reaches the bottom of the layer ($z = 0$) at $x_1 = x_0 - \text{Re}(\alpha_1)$, where $\alpha_1$ is an index between 1 and 4 that indicates which mode slope $\alpha$, or solution of Eq. 3.17, is considered. A second shear zone, identical to the first, is generated at $x = j\lambda$. It reaches the bottom of the layer at $x_2 = x_0 + j\lambda + \text{Re}(\alpha_2)$, where $\alpha_2$ is another index between 1 and 4, not necessarily the same as $\alpha_1$. A given wavenumber $k = 2\pi/\lambda$ is resonant if $x_1 = x_2$, or

$$k_{a_1,a_2}^j = 2\pi j/|\text{Re}(\alpha_1 - \alpha_2)|,$$  \hspace{1cm} (3.20)

where $j$ is the order of the resonance — the number of wavelength spanned between the surface intercept of the incipient shear zones. The resonant wavenumbers are plotted as a function of the effective stress exponent in Fig. 3-7.

At depth, each potential shear zone attempts to generate a discontinuous deformation field that is incompatible with the response of the substrate. Therefore, the localized shear zones can develop only if an additional degree of freedom is available, which happens when several shear zones interact at the bottom of the layer. Hence, the localized shear zones cannot develop in the model unless the perturbation wavelength is resonant. Indeed, the growth rate is finite at all wavenumbers that are not resonant, indicating that the model is stable and deformation remains distributed (Fig. 3-5). At resonant wavenumbers, there is a self-consistent network of faults and the growth rate is infinite.

The pattern of resonances compares well to the buckling and localization instabilities. We present in Fig. 3-8 a map of growth rate in the parameter space of $1/n_e-k$. 

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Figure 3-7: Resonant wavenumbers for a layer of thickness $H$ as a function of its effective stress exponent (Eq. 3.20). Thick line: Fundamental mode $j = 1$; other lines: higher order resonances. The labels on each branch indicate the mode slopes $\alpha$ of the involved deformation modes (see Eq. 3.17).

This representation is akin to a topographic map of the surface $Q(1/n_e, k)$. The growth spectra shown in Fig. 3-5 represent sections through that map taken with constant $1/n_e$ of -0.1, $10^{-6}$, and 0.01.

The buckling instability is best identified in the $1/n_e > 0$ domain by a broad maximum and can be followed into the negative $1/n_e$ domain (Fig. 3-8). The characteristic wavenumber of this instability, $K_{B/N}$, varies monotonically with $1/n_e$, following

$$K_{B/N} = k_{1,4} = \pi \left(1 - 1/n_1\right)^{-1/2}.$$  \hspace{1cm} (3.21)
The growth rate maxima are located at

$$k_j^B = (j + 1/2) K_{B/N}, \quad j \in \mathbb{Z}. \quad (3.22)$$

The localization instability is seen as a branch of very high growth rate that exists only when $1/n_e < 0$. Its characteristic wavenumber, $K_L$, follows a different resonance from the buckling instability

$$K_L = k_{1,2} = \pi (-1/n_1)^{-1/2}. \quad (3.23)$$

The wavenumber $K_L$ is real only if $1/n_1 < 0$, as expected if it arises due to localization.
The growth rate maxima for the localization instability are located near:

\[ k_j^L = (j) K_L, \quad j \in \mathbb{Z}. \]  

(3.24)

The relevant resonances are showed schematically in Fig. 3-9.

As was pointed out earlier, the effective stress exponent quantifies the efficiency of the localization process in the layer. Hence, the wavelength of the localization instability should depend on \( n_e \). The deformation imposed within one wavelength of a given fault is eventually localized on that fault. Therefore, if localization is efficient (\( 1/n_e \) more negative), a given fault can accommodate the deformation from a wider area than if localization is inefficient (\( 1/n_1 \) less negative). In the limit of perfect localization, all the deformation is localized on a single fault. Indeed the wavelength of the localization instability goes to infinity in the limit of drastic localization \( 1/n_e \rightarrow \)}
In the other case where the localization is marginal, \(1/n_e \to 0^-\), the instability wavelength goes to 0; many finely-spaced faults are predicted.

As the localization instability is characterized by a doublet of divergent growth rate, centered on the wavenumber given by Eq. 3.24, there are two preferred wavelengths of the localization instability for a given \(j\). The separation of the two branches of a given doublet is not predicted by the resonance analysis. We note that the doublets close when several resonances are superposed (compare Fig. 3-7 and 3-8), which might indicate the importance of the derivatives of the stream function. Whereas the resonance analysis uses only the stream function, the boundary conditions include velocity and stresses, which depend on the derivatives of the stream function. Two modes of deformation with different mode slopes have different values of stress and velocity at a given depth, even if their stream functions at that depth are identical. This might suffice to offset slightly the actual resonant wavelengths. The opening of the localization doublets is usually small enough to be ignored. In addition, the doublet structure may break down once the non-linearities in the system behavior and transient strengthening are taken into account. These additional aspects of a localizing system are needed to stabilize localization and probably limit the divergent growth rate of the localization instability to a finite value. Therefore, applications to the Earth need not consider the doublet opening and can replace it conceptually with a single peak at the preferred wavelength of the instability \(k^*_f\) (Eq. 3.24).

3.4.3 Effect of substratum viscosity

The buckling instability takes its origin in the viscosity contrast across the interfaces of the model [Johnson and Fletcher, 1994]. Therefore, the growth rate of the buckling instability increases with the viscosity contrast. Buckling requires that the layer be stronger than the substratum.

The localization instability, on the other hand, is linked to a resonance that is internal to the layer with negative stress exponent. Therefore, it can grow even if the viscosity of the half-space is higher than the viscosity of the layer. However, the spectrum is offset by one half of the characteristic wavelength (Fig. 3-10). When the
Figure 3-10: Map of growth rate as a function of substrate viscosity and perturbation wavenumber. Strength profile similar to Fig. 3-5b, except for varying substrate viscosity and $\eta_1 = -10$. The buckling instability vanishes when the substrate is more viscous than the layer, but the localization instability persists, although offset by one half of its characteristic wavenumber.

When the viscosity of the substrate is more viscous than the layer, the doublets are located at

$$k_j^L = (j + 1/2) K_L, \quad j \in \mathbb{Z}. \quad (3.25)$$

Physically, the offset is required because the viscous substrate cannot follow the localized deformation. At wavenumbers halfway between actual resonances, an incipient shear zone interacts with a negative image of itself. Therefore, these shear zones interact destructively at the interface, which is required by the stronger substratum. When the viscosity of the substrate is small, it provides no resistance to the localized deformation and the preferred wavenumber is exactly at the resonance.

The position of the doublet changes continuously from Eq. 3.24 to Eq. 3.25 as the
substrate viscosity increases (Fig. 3-10). In general, we write

$$k_j^L = (j + a_L) K_L, \quad j \in \mathbb{Z},$$  \hspace{1cm} (3.26)

with $a_L$ an empirical number called the spectrum offset, and $K_L$ the wavenumber scale defined in Eq. 3.23. The spectrum offset is 0 if $\eta_2/\eta_1 \ll 1$, $a_L = 1/4$ if $\eta_2/\eta_1 \approx 1$, and $a_L = 1/2$ if $\eta_2/\eta_1 \gg 1$.

For generality, we also define a spectrum offset $a_B$ for the buckling instability

$$k_j^B = (j + 1/2 - a_B) K_{B/N}, \quad j \in \mathbb{Z},$$  \hspace{1cm} (3.27)

with $K_{B/N}$ the wavenumber scale defined in Eq. 3.21. Eq. 3.26 and 3.27 are written so that $a_B = a_L = 0$ for models where both the layer and the substrate have constant viscosity and the layer is much stronger than the substrate (Fig. 3-5). This will change when depth-dependent viscosity is considered in the model layers.

### 3.4.4 Effect of model density

A density contrast at the surface of the model reduces the growth rate of the buckling instability and increases its preferred wavenumber because of the restoring force on the growing surface topography [Zuber, 1987a; Martinod and Molnar, 1995; Neumann and Zuber, 1995]. If $n_e > 0$, the growth rate is particularly reduced at small wavenumber (Fig. 3-11). If $1/n_e \to 0^+$, the density-induced reduction of the growth rate at small $k$ can result in the maximum growth rate being at a resonance with $j \geq 1$ [Neumann and Zuber, 1995]. In addition, there is no growth of the buckling mode over half of the wavenumbers, from $jK_{B/N}$ to $(j+1/2)K_{B/N}$ at small $j$ (Fig. 3-11). This produces a spectral offset in Eq. 3.27 up to $a_B = 1/4$ for $j = 1$ and large density of the model. The wavelength of buckling in the Central Indian Ocean may be larger to the north of the basin where the surface density contrast is reduced by the large sediment supply of the Bengal fan [Zuber, 1987a].

If the layer has localizing properties ($1/n_e < 0$), the model density influences
the buckling part of the growth spectrum in the same manner as described above, except that growth rate is enhanced near the divergent doublets of the localizing instability (Fig. 3-11). The effect is particularly pronounced at small wavenumber and increases with the density of the model (Fig. 3-12). However, scaling to Earth conditions indicates that the normalized density $\rho g / \eta \ddot{\varepsilon}_H$ should be of order 1 to 30 [Zuber et al., 1986], which is small. For these values, the density has little effect on the localization instability, although it does bring a spectral offset up to $a_B = 1/4$ for the longest preferred wavelength of the buckling instability ($j = 0$).

### 3.5 Models with depth–dependent viscosity

The models considered in the previous section are only crude approximations to the Earth’s structure. The layer represents the brittle crust and upper mantle, and the half–space represents the hotter ductile rocks leading to the asthenosphere. The interface between the layer and the half–space corresponds to the brittle–ductile transition.
Figure 3-12: Map of growth rate as a function of model density and perturbation wavenumber. Strength profile similar to Fig. 3-5b, with $n_1 = -10$. The value of the normalized density $\rho g/\eta \dot{\varepsilon}_{\parallel}$ is no more than 30 for terrestrial applications.

Unlike the models in the previous section, the strength profile of the Earth is continuous across the transition from brittle to ductile behavior. In idealized representation of the strength profile of the lithosphere the brittle–ductile transition occurs at a particular depth where the brittle and ductile strengths of rocks are identical [Brace and Kohlstedt, 1980], or be distributed over a depth range where the brittle and ductile rock strengths are comparable [Kirby, 1980; Kohlstedt et al., 1995]. In any case, the strength and therefore the apparent viscosity of the lithosphere varies continuously with depth within each layer, due to the pressure dependence of rock strength in the brittle regime [Scholz, 1990], and the temperature dependence of creep in the ductile regime [Kohlstedt et al., 1995].

In the following sections, we change progressively the simple strength profile of
the previous section to a more realistic strength profile, keeping track of the preferred wavelengths of buckling and localizing instabilities, as well as their growth rate. Our goal is to derive a simple prediction of the preferred wavelength of the localization instability relevant for a layer of rock undergoing a brittle–ductile transition at a specific depth with a strength profile similar to the Earth’s. The effect of having the brittle–ductile transition distributed over a finite depth range or density contrasts within the lithosphere will be considered in Chapter 4.

The wavenumber scaling of the instability is still valid when depth–dependent viscosity is considered. Therefore, we use Eq. 3.27 and 3.26 to describe the preferred wavenumbers of each instability. However, the value of the spectral offset parameter is empirically determined for each type of viscosity profile. The viscosity is scaled to 1 at the bottom of the brittle layer. We use $n_c = -10$ as an illustration for localizing behavior. In that case, $K_L \approx 10$. 

Figure 3-13: a) Growth spectrum for a layer of uniform viscosity $\eta_1$ overlying a half-space with exponentially–decaying viscosity $\eta_2 = \exp(r_2 z)$, with $r_2 = 10$, $\rho = 0$, $n_2 = 3$. Solid line: $n_1 = -10$; dashed line: $n_1 = 10^6$. b) Viscosity profile.
3.5.1 Exponential decay of viscosity in the substrate

Because the stream function for a layer of exponential viscosity profile is proportional to $\exp[ik(x + \alpha z)]$ (Eq. 3.15), a perturbation with wavelength $\lambda$ penetrates into a layer to a depth $z_d \sim \lambda/\text{Im}(\alpha)$. Hence, it senses a viscosity averaged over $z_d$. Therefore, a buckling instability can grow even if the strength profile is continuous at the boundary between the layer and the substrate if the viscosity of the substrate decreases exponentially with depth [Fletcher and Hallet, 1983; Zuber and Parmentier, 1986]. In fact, most applications of buckling or necking to the tectonics of terrestrial planets have used a strength profile made of a layer of uniform viscosity lying over a
substrate with viscosity profile

\[ \eta = \eta_e \exp(\tau z), \]  

(3.28)

A value of \( \tau \approx 10 \) is often appropriate [Fletcher and Hallet, 1983; Zuber and Parmentier, 1986].

There are two differences between the growth spectrum of a layer lying over a substrate with exponentially decaying viscosity and the previous case of a constant-viscosity half-space, even if the layer is plastic \((1/n_1 \rightarrow 0, \text{buckling instability only, Fig. 3-13})\). First, the envelope of the growth spectrum decreases at high wavenumber. This is because the short wavelength senses only the top of the substrate, which has higher viscosity than deeper levels, and therefore smaller density contrast [Ricard and Froideveau, 1986; Zuber and Parmentier, 1986; Neumann and Zuber, 1995]. Second, the instability grows only over one half of the range of wavenumbers, between \( j K_{B/N} \) and \((j + 1/2) K_{B/N}, j \in \mathbb{Z}\). Hence, the preferred wavenumbers of buckling become

\[ k_j^B = (j + 1/4) K_{B/N}, \quad j \in \mathbb{Z}, \]  

(3.29)

or, using Eq. 3.27, the spectrum offset \( a_B = 1/4 \) for this strength profile.

When the layer is undergoing localization \((n_1 < 0)\), the growth spectrum is described as the superposition of a buckling–like spectrum and a sequence of divergent doublets representing the localization instability (Fig. 3-13), as in the model with constant viscosity layers (Fig. 3-5). At the smallest wavenumbers, the substrate appears very weak, and the spectral offset \( a_L \sim 0 \) for \( j = 0 \). At larger wavenumbers, however, the substrate viscosity is similar to that of the layer, so that the spectral offset \( a_L = 1/4 \) (see below Eq. 3.26). In summary, the preferred wavelength of localization follows Eq. 3.26 with approximately

\[ a_L = 0, \quad \text{if } j = 0, \]  

(3.30a)

\[ a_L = 1/4, \quad \text{if } j > 0. \]  

(3.30b)
Fig. 3-14 shows how the growth spectrum varies as a function of the decay–depth of the viscosity profile. Note how the buckling instability vanishes when \( r < 0 \) (viscosity of the substrate increasing exponentially with depth) whereas the localization instability is still present. However, the substrate is now stronger than the layer, so that at \( a_L = 1/2 \) at small \( j \).

### 3.5.2 Depth–increasing strength of the layer

As the layer corresponds to rocks undergoing brittle deformation, its strength should increase with depth. We first consider models in which the viscosity of the layer increases exponentially with depth, which is mathematically more tractable, and then the more realistic case of a viscosity increasing linearly with depth in the layer. In both cases, the viscosity is \( \eta_1 = 1 \) at the base of the layer. The substrate has a constant viscosity of \( \eta_2 = 0.1 \) and a non–Newtonian behavior with \( n_e = 3 \).

#### Exponential viscosity profile

Having an exponential viscosity profile in the layer reduces its apparent viscosity. Accordingly, the growth rate of the buckling instability is reduced compared to the constant viscosity case but its preferred wavelength is unchanged (Fig. 3-15). When the material in the layer is pseudo–plastic (\( 1/\eta_1 \to 0^+ \)) The envelope of the growth spectrum does not depend on wavenumber because the penetration depth of the perturbation is infinite (\( \text{Im}(\alpha) \to 0 \)): the whole layer is sampled at all wavelengths. When the layer is localizing \( (\eta_1 < 0, \text{Fig. 3-15}) \), the preferred wavelength of the localization instability is offset by \( 1/4 \) of the characteristic scale \( K_L \). Interestingly, the amplitude of the buckling mode decreases at the smallest wavenumbers if \( 1/\eta_1 < 0 \) and the layer viscosity increases with depth (Fig. 3-15).

A complication arises because the mode slope \( \alpha \) depends on the decay parameter of the viscosity profile (Eq. 3.16). This changes the resonant wavenumbers (Fig. 3-16) and therefore the wavenumber scale of instabilities \( K_L \) and \( K_{B/\eta} \). Although the localization instability stills follows the resonance (Fig. 3-17), there is no longer
Figure 3-15: a) Growth spectrum for a layer with viscosity increasing exponentially with depth \( \eta_1 = \exp(r_1 z) \), overlying a half-space with constant viscosity \( \eta_2 = 0.1 \), with \( r_1 = -5 \), \( \rho = 0 \), \( n_2 = 3 \). Solid line: \( n_1 = -10 \); dashed line: \( n_1 = 10^6 \). b) Viscosity profile

an analytical expression for \( K_L \) or \( K_{H/N} \). However, Eq. 3.23 is approximately valid when \( r \approx 0 \), which is the case for realistic viscosity profiles. Therefore, the preferred wavelengths of localization are given approximately by 3.26 with

\[
a_L = \frac{1}{4}, \quad j \in \mathbb{Z}.
\]  

(3.31)

Note that the first localization doublet \( (j = 0) \) is wider than for other strength profiles.

The long wavelength limit \( r k > 2 \sqrt{2 + \sqrt{5}} \), beyond which not all the solutions of \( \alpha \) are real even if \( 1/n_e < 0 \), does not prevent the growth of a localization instability. This is because for \( 1/n_e \) sufficiently negative, one value of the mode slope, \( \alpha \), is pure imaginary: there are still two values of Re(\( \alpha \)), one being zero, and the resonance depicted in Fig. 3-9b is still defined.
Figure 3-16: Resonant wavenumbers for a layer of thickness $H$ and effective stress exponent $n_1 = -10$ as a function of the decays depth $r$ of viscosity in the layer. Thick line: fundamental mode $j = 1$; other lines: higher order resonances. The $j = 1$ branches are labeled with the mode slopes $\alpha$ of the appropriate deformation modes. Solution derived numerically from Eq. 3.16 and 3.20.

**Linear viscosity profile**

Although an exponential viscosity profile is only a poor approximation of the linear increase of strength with depth expected in the brittle layer from friction laws, there is little difference between the results of the previous section and the growth spectra obtained with the linear law. The amplitude of the buckling mode is reduced, and the preferred wavelength of the localization instability is offset by $1/4$ of the wavelength scale $K_L$ at small wavelengths. In addition, the resonance wavelength is close to the analytical value of Eq. 3.23 obtained for a constant viscosity layer. This is because the average strength of the layer is limited to one half of its maximum value when it
Figure 3-17: Map of growth rate as a function of decay length of layer viscosity and perturbation wavenumber. Strength profile similar to Fig. 3-15b, except for varying $r_1$ and $n_1 = -10$. Positive values of $r$ indicate that the layer viscosity decreases exponentially with depth.

increases linearly with depth. The decay parameter $r$ for exponential viscosity profile that produces the same characteristics is small. Indeed, the growth spectrum for a layer with linear viscosity profile is closest to the case $r = 2$ with an exponential viscosity profile. For these values the resonant wavenumbers cannot be differentiated from the limit $r = 0$. 
3.6 Discussion

3.6.1 Putting it all together: Growth spectrum for realistic strength profiles

A realistic strength profile for application to tectonics has a plastic or brittle layer with strength increasing linearly with depth, followed by a layer of half-space of ductile material with viscosity decreasing with depth. We learned from the previous sections that there are two superposed instabilities for a plastic or localizing layer overlying a half-space: the buckling instability that results in broad undulation of the layer as a whole when that layer is stronger than the substratum averaged over a wavelength–dependent penetration depth, and the localizing instability that produces regularly–spaced faults or shear zones. Reintroducing \( H \), the thickness of the brittle layer, as a length scale in Eq. 3.21, 3.23, 3.26, and 3.27, these instabilities grow preferentially at the wavenumbers

\[
k_j^B H = (j + 1/2 - a_B) K_B/N,
\]  

(3.32a)
$$k_j^L H = (j + a_L) K_L,$$  \hspace{1cm} (3.32b)

with $j$ an integer, $a_B$ and $a_L$ spectral offsets that depend on the strength profile, and $K_{B/N}$ and $K_L$ wavenumber scales that correspond to resonances in the brittle layer and are given by

$$K_{B/N} = \pi (1 - 1/n_1)^{-1/2},$$ \hspace{1cm} (3.33a)

$$K_L = \pi (-1/n_1)^{-1/2}$$ \hspace{1cm} (3.33b)

Depth-increasing viscosity in the layer, depth-decreasing viscosity in the half-space, and buoyancy forces all decrease the growth rate of the buckling instability (Fig. 3-11, 3-13, and 3-15). Hence, the buckling instability shows only modest growth rates for the most realistic strength profile used in this study (Fig. 3-18). Furthermore, the exponentially-decaying viscosity in the substrate suppresses the instability over half of the wavenumber range (Fig. 3-13), and a surface density contrast cancels the instability over the other half of the wavenumber range (Fig. 3-12). It results that buckling is not a likely expression of shortening in a layered lithosphere (Fig. 3-19) unless the surface density contrast is reduced. Indeed, natural examples of lithospheric-scale buckling are associated with deformation under a heavy fluid, which reduces the surface density contrast. In the Central Indian Ocean, this fluid stands for the sediments from the Bengal fan [Zuber, 1987a; Martinod and Molnar, 1995]. On Venus, the Ridge Belts grew in the same time that basaltic flood plains were emplaced [Zuber and Aist, 1990; Stewart and Head, 2000]. In Central Asia [Nikishin et al., 1993; Burov et al., 1993], the highly erosive conditions also reduced the surface density contrast. If buckling does grow, the relevant spectral offset is

$$0 < a_{B/N} < 1/4.$$  \hspace{1cm} (3.34)

Neither depth-dependent viscosity nor surface density contrasts reduce the growth rate of the localization instability. Depth-dependent viscosity in the layer and in the substrate each offsets the preferred wavelength by about $1/4 K_{B/N}$. The density of
Figure 3-19: Map of growth rate as a function of the model density $\frac{\rho g}{\eta L \varepsilon_{||}}$ and wavenumber. Viscosity profile similar to Fig. 3-18b. Lighter tone indicates high growth rate, with the contours indicated the color bar. As the model density increases, the buckling mode vanishes.

the model also increases the spectral offset (Fig. 3-19). All things considered, the spectral offset for a realistic viscosity profile is

$$1/4 < a_L < 1/2.$$  \hspace{1cm} (3.35)

A map of growth rate similar to Fig. 3-8 but for a realistic strength profile is presented in Fig. 3-20. It shows how the wavenumber of the localization instability varies with the effective stress exponent of the layer. The localization instability is seen to follow the resonant wavenumbers, $K_L$. The buckling mode of deformation all but vanishes when depth-dependent viscosity is taken into account. It is visible only
near the divergent doublets of the localization instability.

3.6.2 A note about extension

Although the previous sections assumed horizontal shortening of the model, the formalism is equally valid for horizontal extension, for which $\bar{\varepsilon}_{xx} > 0$. However, the kinematic contribution of the primary flow to the growth of interface perturbations (Eq. 3.5) has the tendency to erase the imposed perturbation under horizontal extension [Smith, 1975]. Hence, only wavelengths with $Q > 1$ can be observed.

We present in Fig. 3-21 the growth spectra for a pseudo-plastic or brittle layer of uniform viscosity over a weaker half-space under horizontal extension. The corre-
Figure 3-21: a) Growth spectrum for a layer of uniform viscosity $\eta_1$ overlying a half-space of uniform viscosity $\eta_2 = \eta_1/10$, with $\rho = 0$, $n_2 = 3$ undergoing horizontal extension. Solid line: $n_1 = -10$; dashed line: $n_1 = 10^6$; dotted line: $n_1 = 100$. b) Viscosity profile.

Figure 3-22: Deformation fields corresponding to the growth spectra in Fig. 3-21, shaded as a function of strain rate. a) $n_1 = 10^6$; b) $n_1 = -10$. Models undergoing extension. Construction otherwise similar to Fig. 3-6.
sponding deformation field are plotted in Fig. 3-22. The spectra are rather similar to the shortening case (Fig. 3-5). In particular, the wavenumbers of the growth rate maxima for a pseudo–plastic layer \( (n_e = 10^6) \) and of the divergent doublets for the localizing layer \( (n_e = -10) \) are similar to the shortening case. The major difference between horizontal extension and shortening is the shape of the most unstable deformation mode over the whole model: the pseudo–plastic is necking under extension rather than buckling (Fig. 3-22a). The localization instability gives rise to regularly–spaced localized shear zones (Fig. 3-22b).

In presence of depth–dependent viscosity and density, the approach of a spectral offset (Eq. 3.27 and 3.26) is still valid. However, the spectral offset for the necking instability is \(-1/4 < a_B < 0\) if the viscosity of the layer increases linearly with depth and the viscosity of the half–space decreases exponentially with depth. Hence, the wavelength of necking is generally smaller than the wavelength of buckling. As was observed in the shortening case, depth–dependent viscosity and model density conspire to reduce the range of wavelengths where growth of the necking instability is possible under horizontal extension, diminishing the likelihood that necking be observed in the tectonic record, unless the surface density contrast is small. Necking has been observed in nature, most prominently in the Basin–and–Range province [Fletcher and Hallet, 1983; Zuber et al., 1986; Ricard and Froideveau, 1986] and plays an important role in rifting [Zuber and Parmentier, 1986]. The spectral offset of the localization instability in extension is the same as in compression, \(1/4 < a_L < 1/2\).

### 3.6.3 Comparison with numerical studies

In early studies of fault patterns, faults were either \textit{a posteriori} markers of deformation or \textit{a priori} boundary conditions. In neither case is faulting a dynamic feature of the models or can the self–consistent pattern of faulting be determined. The instability of the localization process, which is expressed in our study by the fact that the effective stress exponent is negative, presents many analytical and numerical challenges. However, recent numerical methods have been able to present a continuum approach to localization, from microscopic scale [Hobbs and Ord, 1989; Poliakov et al.,
1994] to global scale [Bercovici, 1993; Tackley, 2000a]. With numerical models, it is possible to go beyond the instantaneous patterns of faulting explored in this paper to study how faulting evolves over time [McKinnon and Garrido de la Barra, 1998; Buck et al., 1999; Sornette and Vanneste, 1996; Cowie et al., 2000]. Our analysis provides a physical insight into the origin of the fault pattern observed.

Using the numerical method FLAC [Cundall, 1989], Poliakov and coworkers explored the patterns of faulting in elastic–visco–plastic models for different tectonic environments [Buck and Poliakov, 1998; Gerbault et al., 1998, 1999; Lavier et al., 2000]. Even in the absence of explicit weakening, an elastic–plastic rheology is characterized by a negative stress exponent, or dynamic strain–weakening, because the strain and stress increments upon failure are not collinear [Chapter 2]. Localization by strain–weakening may behave differently from the strain–rate–weakening used in our paper. However, the effective exponent provides a unifying measure of localization, and it is relevant to compare the numerical results in presence of strain–weakening to our model, provided that we use $-0.1 < 1/n_e < -0.01$, as appropriate for localization in elastic–plastic materials [Chapter 2]. The fault spacing predicted by our analysis ($0.4 < \lambda < 2.5$) is consistent with the spacing observed in numerical models. The localization instability (adapted for strain-weakening) is a likely origin of the fault pattern observed in numerical models.

Explicit strain–softening was shown to enhance faulting in elastic–visco–plastic models and to increase the fault spacing [Gerbault, 1999]. This is again consistent with the localization instability, which predicts larger fault spacings for more efficient localization. However, if the weakening is too strong, another transition occurs and a single fault develops in numerical models [Lavier et al., 2000]. Frederiksen and Braun [2001] also observed localization on a single fault and its conjugate in their models that include strain-softening of the viscous, rather than the plastic rheology. They also showed that the fault intensity depends on the rate of weakening, consistent with localization being controlled by the effective stress exponent rather than only the amount of weakening [Frederiksen and Braun, 2001]. Localization to a single fault is not predicted by our model. It may be due to second order or finite strain effects that
not yet modeled. Sornette and Vanneste [1996] and Cowie et al. [2000] also report on localization of strain over a single fault upon finite deformation. Rather than following an elastic-plastic rheology, their models are elastic, with fault slip accumulating when a yield criterion is verified. The initial pattern of faulting is dominated by the strong prescribed heterogeneity in these models, which prevents a characteristic length-scale to develop. As slip on a fault enhances the stress in the vicinity of the fault tip, the tendency to failure of faults in that region is enhanced. After finite slip, the fault pattern may localize because of the interaction between neighboring faults [Sornette and Vanneste, 1996; Cowie et al., 2000]. The interaction between several active faults may be described with a negative effective stress exponent. Future improvement of our model, in particular including a higher-order or time-dependent analysis may address this later instability of fault pattern.

The numerical studies closest to our study consider strain-rate softening in visco-plastic models [Neumann and Zuber, 1995; Montesi and Zuber, 1997, 1999]. They produce regularly-spaced faults superposed on either buckling or necking. Numerical results suggest that faults are mostly active in the anticlines of lithospheric-scale folds [Montesi and Zuber, 1997, 1999] or the necks of lithospheric scale boudins [Neumann and Zuber, 1995]. This interaction between the buckling/necking and faulting deformation fields is not predicted in our analysis, for which faults are present everywhere (Fig. 3-6 and 3-22), but may result from an higher-order interaction between the buckling/necking and localization instabilities. Numerical results indicate several faults in the growing anticlines or necks. Their spacing is consistent with the prediction of our model, showing again a control of the fault pattern by the localization instability. As deformation proceeds, some faults cease their activity, and others replace them [Montesi and Zuber, 1997, 1999]. Switches in fault patterns are discrete in time. They reuse recently active faults, with new faults formed in the front of the existing deformation zone, separated from it by the same spacing as within the deformation zone. Thus, the localization-instability-controlled fault spacing is prominent at finite strain. Propagation of a deformation front with characteristic fault spacing is also observed in elastic-plastic models [Hardacre et al., 2001], but to our knowledge, the
fault spacing in that case has not been studied.

In summary, fault sets produced by numerical models may show a preferred spacing that is consistent with the prediction of the localization instability. However, a high level of initial heterogeneity can prevent a regular spacing to form [Sornette and Vanneste, 1996]. With finite displacement, the fault pattern may collapse on a single fault [Sornette and Vanneste, 1996; Cowie et al., 2000; Lavier et al., 2000; Frederiksen and Braun, 2001] through a process that we cannot address here. All the models undergoing horizontal shortening show regularly-spaced fault sets even at finite strain [Gerbault et al., 1999; Montesi and Zuber, 1999]. Accordingly, compressive orogens often display a propagating deformation front with regularly-spaced faults [Hoffman et al., 1988; Meyer et al., 1998]. However, the importance of sub-horizontal decollements for this behavior remains to be evaluated. Eventual localization of compressive strain over a single shear zone may indicate the initiation of subduction.

Other numerical studies focused on the localization of shear within a fault gouge [Morgan and Boettcher, 1999; Place and Mora, 2000; Wang et al., 2000] or the spatio-temporal localization of slip over a seismogenic fault [Ben Zion and Rice, 1995; Miller and Olgaard, 1997; Lapusta et al., 2000; Madariaga and Olsen, 2000]. As our study does not resolve the temporal evolution of localization and assumes a different geometry than that relevant for seismogenic and fault gouge processes, we cannot compare our work and these studies. Similarly, our analysis cannot be applied directly to regularly-spaced fault sets in strike-slip environments [Bourne et al., 1998; Roy and Royden, 2000a]. However, the use of a negative stress exponent to build simple models of localization can be adapted to these problems. We hope that future developments of our model will address these different geometries as well as the interaction between the localization and buckling/necking instabilities.

3.7 Conclusions

We have presented new solutions of the perturbation analysis of mechanically layered models of the lithosphere undergoing shortening in which a brittle layer lies over a
ductile substrate. In addition to the classically-recognized buckling instability, the layer may undergo a localization instability that results in regularly-spaced faults or shear zones. Localization is possible when the effective stress exponent of the brittle layer, a general measure of the mechanical response of the material to local perturbations, is negative [Chapter 2]. However, localization of deformation produces incipient shear zones in which the deformation field tries to develop a discontinuity that is not compatible with coupling with a ductile substrate. Therefore, shear zones cannot develop unless there is a resonance between several incipient shear zones. This resonance is the basis for a scaling wavenumber, $K_L$, that controls the wavelengths of instability. The resonance that is at the origin of $K_L$ exists only if the effective stress exponent is negative (Eq. 3.23). The actual wavelength of the instability is linked to $K_L$ by Eq. 3.26 which also includes a “spectral offset” parameter, $a_L$, that is calibrated empirically in function of the strength profile in the model. For a realistic profile where the strength of the brittle layer increases linearly with depth and the strength of the substrate decreases with depth, $1/4 < a_L < 1/2$. Similar principles are used to describe the buckling instability except that the scaling wavenumber, $K_{B/N}$ is rooted in a different resonance that does not require a negative effective stress exponent (Eq. 3.21), and that the spectral offset $a_B$ (Eq. 3.27) is between 0 and $1/4$ in compression and between $-1/4$ and 0 in extension. Model density has only a minor effect on the localization instability. Buckling is much reduced when depth-dependent viscosity is included and is a likely expression of tectonic deformation only if the surface density contrast is reduced, for instance because of a high erosion or sedimentation rate.
Chapter 4

Fault spacing in the Central Indian Basin

Abstract

Tectonic deformation in the Central Indian Basin (CIB) is organized at two spatial scales: long-wavelength (\(\sim 200\) km) undulations of the basement and regularly-spaced faults. The fault spacing, of order 7 to 11 km, is too short to be explained by lithospheric buckling. We show that the localization instability derived in Chapter 3 provides a reasonable model for the fault spacing in the CIB. Localization describes how deformation focuses on narrow zones analogous to faults. The localization instability predicts that as they develop, localized shear zones form a regular pattern, with a characteristic spacing. The theoretical fault spacing is proportional to the depth to which localization occurs. It also depends on the strength profile and on the effective stress exponent, \(n_e\), which is a measure of localization efficiency in the brittle crust and upper mantle. The fault spacing in the CIB can be matched by \(n_e \sim -300\) if the faults reach the depth brittle-ductile transition, around 40 km, or \(n_e \sim -100\) if the faults do not penetrate below 10 km. These values of \(n_e\) are compatible with laboratory data on frictional velocity-weakening. Many faults in the CIB were formed during seafloor spreading. The pre-existing faults located near target locations dictated by the localization were preferentially reactivated during the current episode of compressive tectonics. The long-wavelength undulations may result from buckling of the brittle crust and upper mantle as a whole, but growth of lithosphere-scale buckling is slow for our preferred strength profile. Buckling in the CIB may be aided by interaction with the developing fault pattern or with the load of the Afanazy-Nikishin seamounts.
4.1 Introduction: Tectonics of the Central Indian Basin

The Central Indian Basin (CIB) is the region southeast of India delimited by the Ninetyeast ridge to the east, the Chagos-Laccadive ridge to the west, and the South-East and Central Indian Ridges to the south. To the north, the basin is covered by the Bengal fan that transports sediments from the High Himalayas (Fig. 4-1).

The best-studied oceanic diffuse plate boundary is located in the CIB [Wiens et al., 1985; Gordon et al., 1990]. The area was originally identified from a relatively high earthquake activity [Gutenberg and Richter, 1954; Sykes, 1970; Bergman and Solomon, 1985, also Fig. 4.2]. This intraplate deformation area is part of the Indo-Australian plate. Following the nomenclature of Gordon [2000], the composite Indo-Australian plate is made of three component plates: India, Capricorn, and Australia. The Capricorn plate rotates anti-clockwise with respect to the Indian plate about a pole of rotation located near the Chagos Bank [Royer and Gordon, 1997], compressing the CIB in a roughly N-S direction.

Previous studies showed that shortening in the CIB is expressed at two length-scales:

1. The basement is folded at ~ 200 km wavelength. The undulations, of amplitude up to 2 km, are visible both from long seismic reflection profiles [Weissel et al., 1980] and as E-W lineations of the gravity field [Stein et al., 1989, Fig. 4-3].

2. Reverse faults cut through the sediment cover and the crystalline basement [Eittreim and Ewing, 1972; Weissel et al., 1980; Chamot-Rooke et al., 1993], delimiting 5 to 20 km wide crustal blocks [Neprochnov et al., 1988; Bull, 1990; Van Orman et al., 1995; Krishna et al., 2001].

The long-wavelength deformation may correspond to buckling of a thick plastic plate [Zuber, 1987a]. Buckling requires that the density contrast between the lithosphere and the overlying fluid be small [Zuber, 1987a; Martinod and Molnar, 1995]. Hence, the overlying Bengal fan may be instrumental in allowing buckling as the mo-
Figure 4-1: Satellite-derived bathymetry of the Central Indian Basin region, from ETOPO5 [Smith and Sandwell, 1997]. ANS: Afanazy-Nikishin Seamounts.
bile fan sediments make for a heavier “fluid” than ocean water. Zuber [1987a] also documented an increase of wavelength of basement undulation towards the north, which is consistent with increased sediment supply. However, the Wharton basin, immediately to the east of the Ninetyeast ridge outside the CIB (Fig. 4-1), displays similar gravity lineations that may represent basement undulations beyond the reaches of the Nicobar fan [Cloetingh and Wortel, 1986; Tinnon et al., 1995]. The Wharton basin is seismically active [Robinson et al., 2001; Abercrombie et al., 2001], and fracture zones in it have been recently reactivated [Deplus et al., 1998]. In the absence of fan sediments, it is not clear how buckling developed in the southern parts of the Wharton basin. Folding of a thin elastic-plastic plate [McAdoo and Sandwell, 1985] is an alternative origin for the basement undulations. However, it requires that the flexural rigidity of the lithosphere be reduced, possibly by yielding or faulting above and below the strong elastic core of the strength envelope [McAdoo and Sandwell, 1985; Wallace and Melosh, 1994].

The origin of the pattern of reverse faults in the CIB has received much less attention than the lithospheric folds. Previous studies have focused on the geometry of these faults in order to constrain the magnitude of the N-S shortening [Eittreim and Ewing, 1972; Bull and Scrutton, 1992; Chamot-Rooke et al., 1993; Van Orman et al., 1995]. Faults may be divided into two separate populations, one north-verging and the other south-verging [Bull, 1990; Chamot-Rooke et al., 1993] and groups of faults may have been active at different times [Krishna et al., 1998, 2001]. Numerical models have shown that faults may be important in allowing buckling to develop, in

Figure 4-2 (facing page): Centroid Moment Tensors (CMT) of earthquakes over bathymetry in the Central Indian Basin region. Satellite-derived bathymetry from Smith and Sandwell [1997]. Compressive quadrants of CMT are shaded according to origin. From darker to lighter: relocated events from Bergman and Solomon [1985]; historic events from Petrov and Wiens [1989]; Harvard CMT catalogue with depth information within intraplate deformation area; other Harvard CMT events. When determined, the depth of an event is indicated immediately above CMT. The shaded region represents the study area of Bull [1990] and the two thick lines the seismic profiles from which Van Orman et al. [1995] reported the fault locations used in this study.
Figure 4-3: Gravity anomaly in the Central Indian Basin after Sandwell and Smith [1997].
particular by lowering the stress required for buckling [Wallace and Melosh, 1994; Beekman et al., 1996; Gerbault, 2000]. However, what controls the fault spacing, to our knowledge, has never been addressed.

While the basement undulations can be explained by buckling of the lithosphere, the fault pattern cannot: fault spacing is much less than the spacing of basement undulations, and faulting, being a localized rather than distributed deformation mode, is not accounted for in the usual buckling analysis. However, in Chapter 3, we modified the buckling analysis to account for the fact that brittle rocks have a tendency to localize deformation. In that case, two superposed instabilities grow simultaneously in lithosphere models undergoing horizontal shortening. One is the buckling instability [Fletcher, 1974; Zuber, 1987a], resulting in broad undulations of the lithosphere as a whole. The other, that we call the localization instability, results in a network of localized shear zones that we interpret as faults. The developing fault network has a characteristic fault spacing, which scales with the thickness of the brittle layer and is a function of the efficiency of localization.

Because fault spacing is seldom discussed, we first review the evidence for regularly-spaced faults in the Central Indian Basin, paying special attention to the occurrence of fault reactivation. Then, we apply the localization instability analysis derived in Chapter 3 to the CIB, comparing the theoretical wavelength of instability with the fault spacing. We explore different assumptions about the strength profile of the lithosphere at the brittle-ductile transition, focusing on the manner that they influence fault spacing, and the efficiency of localization that matches the observation. Finally, we discuss the localization mechanism implied for the CIB, the depth of localization, and how faulting and folding may be related. Our theory provides a unified model for creating basement undulations and regularly-spaced faults in the CIB.
4.2 Faults in the Central Indian Basin

4.2.1 Geometry and activity

Although some faults in the CIB are visible in the basement, their clearest expression is often a tight fold in the sedimentary sequence. The faults are sub-vertical in the sediments, but dip around 40° in the basement [Bull, 1990; Chamot-Rooke et al., 1993]. They are sometimes imaged to depths of 6 km, below which the available data loses resolution. It is thought that the faults penetrate into the upper mantle [Bull and Scrutton, 1992; Chamot-Rooke et al., 1993].

The faults originated in the basement and propagated into the sediment cover [Bull and Scrutton, 1992]. Geophysical evidence shows that folding in the CIB was episodic, with individual pulses of activity marked by a separate unconformity in the sediments [Krishna et al., 1998]. Faults propagated to the uppermost unconformity in a given area. The earliest unconformity was dated as Late Miocene (7.5 My, [Curray and Munasinghe, 1989]) and the latest as Late Pleistocene (0.8 My, [Krishna et al., 1998]). Krishna et al. [1998] suggested that deformation in the CIB is episodic and that the center of activity shifts over time [Krishna et al., 2001].

There has been no earthquake associated unequivocally with the observed faults. While the faults could be inactive, or the rate of seismicity too low for the existing record, the absence of a small aperture seismic network in the region prohibits relocation of events to the depth range where the faults are observed. The late Pleistocene unconformity spans the region that has been the most seismically active, hinting towards a relation between seismogenic faults and surface faulting [Krishna et al., 2001]. In addition, many earthquakes can be correlated with surface features, in particular ancient fracture zones [Stein and Okal, 1978; Bergman and Solomon, 1985]. Therefore, it is possible that the rupture planes of earthquakes down to 40 km are somehow related to the shallower faults, although no direct link between of the shallow and deep faults can be proven at this point.
### Table 4.1: Statistics of fault spacing

<table>
<thead>
<tr>
<th>Position (number of faults)</th>
<th>Measure</th>
<th>Mean</th>
<th>Median</th>
<th>Standard Deviation</th>
<th>Inter-Quartile Scale</th>
<th>Coefficient of skew</th>
</tr>
</thead>
<tbody>
<tr>
<td>78.8°E (127)</td>
<td>Δ₁</td>
<td>6.54</td>
<td>4.56</td>
<td>6.32</td>
<td>5.57</td>
<td>2.57</td>
</tr>
<tr>
<td>81°E (49)</td>
<td>Δ₂</td>
<td>8.40</td>
<td>8.24</td>
<td>3.87</td>
<td>4.84</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Fault positions from J. Van Orman, pers. comm., 2000

#### 4.2.2 Fault spacing

Van Orman et al. [1995] have compiled the location of faults along several seismic reflection profiles running roughly north-south across the intraplate deformation area. We use two different schemes to compute fault spacing, Δ, from their datasets:

\[
\begin{align*}
\Delta_1 &= x_{i+1} - x_i \\
\Delta_2 &= \frac{x_{i+1} - x_{i-1}}{2},
\end{align*}
\]  

where \(x_i\) is the distance from the fault \(i\) to a reference point, projected onto the north-south direction. The actual shortening direction is less than 10° from the projection direction [Cloetingh and Wortel, 1986; Tinnon et al., 1995], which introduces a systematic error in our spacing estimate of less than a few percent. Histograms of fault spacing are presented in Fig. 4-4 and spacing statistics are gathered in Table 4.1.

The first measure (Eq. 4.1a) is the usual fault spacing measure. It indicates the width of the block delimited by consecutive faults. The second measure (Eq. 4.1b) represents the distance between the midpoints of two consecutive blocks. If the deformation accommodated on a fault \(i\) were distributed over a region surrounding it, the width of that region would be roughly \(\Delta_2\). The spacing distribution is well described as log-normal, which might be expected, as spacing is constrained to be...
Figure 4-4: Histograms of fault spacing determined using the measures a) $\Delta_1$ and b) $\Delta_2$ (Eq. 1) from fault locations reported by Van Orman et al. [1995] on the two seismic lines marked in Fig. 4-1. Black: 78.8°E line; gray: 81°E line. Insets represent schematically the measure of spacing.

positive. As Bull and Scrutton [1992] already recognized, the cumulative frequency diagram is not fractal. The second measure, which corresponds to a moving average of the first, gives a tighter distribution of spacing (Fig. 4-4, Table 4.1).

A possible physical origin for the significant spread in the distribution of spacing is as follows. Starting from a set of regularly-spaced faults, the location of each fault is perturbed randomly. Spacing is measured on the perturbed fault location. We conducted Monte-Carlo simulations of this process and found that the skewness and median of the simulated distributions are compatible with the data from Van Orman et al. [1995] when the amplitude of the perturbation of fault location is at least half the original fault spacing. As the mean is conserved by this process, we favor using mean values in each dataset as the "characteristic fault spacing". Alternative explanations
for the distribution of fault spacing can be devised and should be explored in future studies.

Our mean spacing values (6.5 and 8.4 km) are consistent with the results of Van Orman et al. [1995], who found a \( \sim 7 \) km fault spacing using the same data set, and of Bull [1990], who determined a 6.6 km fault spacing in that region (Fig. 4.1). However, data from seismic lines more to the east of the CIB give longer average fault spacing, between 9.0 and 11.6 km [Krishna, pers. comm., 2001]. An eastward increase of fault spacing may be somehow related to higher strain in the eastern CIB [Van Orman et al., 1995; Royer and Gordon, 1997]. However, new estimates of shortening based on fault throw [Krishna, pers. comm., 2001] do not support the eastward increase of shortening documented by Van Orman et al. [1995].

In summary, we take the fault spacing in the CIB to be 7 to 11 km, depending on the location. Variations in fault spacing cannot yet be related to differences of tectonic history. At any given location, fault spacing is rather broadly distributed around the mean value.

### 4.2.3 Reactivation

Many of the observed reverse faults are probably reactivated normal faults that formed during seafloor spreading 65 to 90 My ago [Bull and Scrutton, 1990; Chamot-Rooke et al., 1993]. The strongest argument in favor of reactivation is the strike of the faults, parallel to magnetic lineations and perpendicular to fracture zones [Eittreim and Ewing, 1972; Bull, 1990]. Bull [1990] also argues that the north-facing and south-facing populations of faults are distinct and that only the north-facing faults are reactivated structures. Chamot-Rooke et al. [1993] also distinguish between two families of faults as a function of their dip in the basement. They argue that only the steepest faults are reactivated.

As many of the observed faults are reactivated, it is conceivable that the observed spacing is inherited from the original distribution of normal faults. However, as we show below, the observed fault spacing is not consistent with the expected configuration of pre-existing faults. The reactivated faults probably originated as abyssal
hill-bounding faults. Studies of modern spreading centers indicate that the width of abyssal hills decreases with increasing spreading rate [Goff et al., 1997]. As the half-spreading rate of the Mid-Indian Ridge was about 10 cm yr$^{-1}$ at the time of formation of the CIB, the width of the abyssal hills expected in that region is around 2 km, similar to the modern East Pacific Rise [Goff, 1991; Goff et al., 1997]. Faults themselves could be more closely spaced, as in the slower spreading Mid-Atlantic Ridge [Goff et al., 1997]. In addition, volcanism might mask some faults in morphological observations [Macdonald et al., 1996]. In any case, the pre-existing fault spacing derived from abyssal hills cannot be more than 2 km, a factor of 3 to 5 less than observed. Moreover, the spacing of reactivated structures is probably larger than the 7 km mean fault spacing because a significant fraction of faults are not reactivated features [Bull, 1990; Chamot-Rooke et al., 1993]. Therefore, pre-existing abyssal hill-bounding faults were only partially reactivated. Several basement faults have been identified in the CIB that do not affect the sedimentary cover. They are probably non-reactivated normal faults [Bull and Scrutton, 1992].

4.3 Origin of “regularly-spaced” fault sets

As the 7 to 11 km mean spacing of reverse faults in the CIB is probably not inherited from the pattern of pre-existing normal faults, we follow the hypothesis that it developed over the last 7.5 My, in the current compressive environment. As the geophysical evidence suggests that many reverse faults are reactivated pre-existing faults, we suppose that reactivation occurred preferentially in the vicinity of target locations. These target locations would have a characteristic spacing between 7 and 11 km, controlled by one of the two instabilities that grow in mechanically-layered lithospheres: the buckling instability and the localization instability [Chapter 3]. These deformation modes can be discriminated upon using the wavelength of deformation that they predict. First, we present how each instability develops and then show that only the localization instability is consistent with the observed fault spacing.
4.3.1 Buckling and localization instabilities

A lithosphere undergoing horizontal shortening can produce regularly-spaced faults in at least two different ways.

1. **Buckling instability:** a strong layer such as the upper lithosphere can buckle, producing broad undulations with wavelength $\lambda_B$ [Biot, 1959; Fletcher, 1974; Zuber, 1987a]. When the undulations reach sufficient amplitude, associated stress heterogeneities can force faults to achieve a spacing of $\lambda_B$ if faults develop at the crest of anticlines, or $\lambda_B/2$ if faults develop at the hinge of the folds [Lambeck, 1983; Gerbault et al., 1999].

2. **Localization instability:** when a material has certain properties (defined below and in Chapter 2), local perturbations are unstable and produce localized shear zones akin to faults. These localized shear zones organize themselves with a characteristic spacing, $\lambda_L$, that is controlled by the mechanical layering of the lithosphere and the efficiency of localization [Chapter 3].

The preferred wavelength of the localization instability and the buckling instability are derived as follows. The lithosphere is idealized as a sequence of horizontal layers of given mechanical properties. As long as the interfaces between these layers are flat and horizontal, the model deforms by pure shear shortening. However, any topography on the model interfaces drives a secondary flow, which in turn deforms the interfaces. Assuming that the secondary deformation field and the interface perturbations have small amplitude, each wavelength of deformation grows self-similarly at a rate, $q$, which depends on the perturbation wavelength $\lambda$. The wavelengths that have high growth rates are more likely to be expressed in the tectonic record. Maxima of the growth spectra $q(\lambda)$ are identified with the preferred wavelengths of lithospheric-scale instabilities.

The type of instability that develops in a given mechanical structure and its preferred wavelength depend on the effective stress exponent, $n_e$, of the brittle layer. The effective stress exponent is a general measure of the non-linearity of the apparent rheology of a material [Smith, 1977]. Although $n_e$ is defined for any rheology [Chapter
we assume in this study that the strength of a material, $\sigma$, depends only on the second invariant of strain rate, $\dot{\varepsilon}_{\text{II}}$. Then, the effective stress exponent has the form

$$n_e = \frac{\sigma}{\dot{\varepsilon}_{\text{II}}} = \frac{d\sigma}{d\dot{\varepsilon}_{\text{II}}}.$$  (4.2)

The strength of a material increases with the strain rate when $n_e > 0$. However, the apparent viscosity of a material with $n_e > 1$ decreases at a location where the strain rate is anomalously high. Although this leads to enhanced deformation at that location if the stress is further increased, deformation is otherwise stable: localized shear zones do not appear spontaneously but must be triggered by a localized forcing. An example of such materials is a rock deforming by dislocation creep, for which $n_e \sim 3$ to $5$.

For localized shear zones to appear spontaneously, the material must be characterized by a weakening behavior [Hobbs et al., 1990, Chapter 2]. If not only the apparent viscosity but also the strength of the material decreases when the strain rate is anomalously high, a local perturbation of strain rate grows unstably to form a localized deformation zone, even if the material was loaded uniformly. Such dynamic weakening is characterized by $n_e < 0$ [Chapter 2].

In addition to being a criterion for localization, the effective stress exponent provides a quantitative measure of localization efficiency. Localization is more effective for more negative $1/n_e$. We refer to the limit $1/n_e \to -\infty$ as drastic localization and $1/n_e \to 0^-$ as marginal localization. In the limit $1/n_e \to 0$, a material is regarded as perfectly plastic [Chapter 2].

The value of the effective stress exponent is loosely related to the localization mechanism [Chapter 2]. Localization by frictional velocity weakening leads to $-300 < n_e < -50$, whereas localization by cohesion loss upon failure or non-associated elastic-plastic flow give $-50 < n_e < -10$. In this study we suppose that $n_e$ is constant in the brittle layer; we do not account for a possible change of localization mechanism with depth, although the shallow faults where at least partially reactivated structures and the deeper faults formed during the current shortening episode. Hence, the effective
stress exponent inferred from our models is the value appropriate for an equivalent layer in which a single localization process is active. As we know that reactivation occurred in the shallow faults [Bull and Scrutton, 1990] and reactivation probably leads to localization by frictional velocity weakening, we use \(-300 < n_e < -50\) as an a priori range of admissible \(n_e\).

The buckling instability develops when a plastic layer lies over a weaker ductile substrate [Smith, 1979; Fletcher and Hallet, 1983]. The localization instability requires \(n_e < 0\) and arises regardless of the strength structure of the lithosphere [Chapter 3]. It has a preferred wavelength only if there is a substrate with \(n_e > 0\) beneath a layer with \(n_e < 0\). A reconstruction of the deformation field of layers undergoing the buckling and the localization instabilities is shown in Chapter 3.

### 4.3.2 Instability scaling

The growth spectra of the buckling and localization instabilities are characterized by successive growth rate maxima defining their preferred wavelengths [Chapter 3]. For each wavelength, there are four superposed deformation modes in each layer of the model. The instabilities appear at wavelengths where these deformation modes are resonant. Using the theoretical dependence of the resonant wavelengths to the effective stress exponent, \(n_e\) [Chapter 3], the preferred wavelengths of the buckling and localization instabilities, \(\lambda_B\) and \(\lambda_L\), are approximately

\[
\lambda_B/H = \frac{2}{j + 1/2 - a_B} \times (1 - 1/n_e)^{1/2},
\]

\[
\lambda_L/H = \frac{2}{j + a_L} \times (-1/n_e)^{1/2},
\]

with \(H\) the thickness of the plastic or localizing layer and \(j\) an integer that gives the order of the resonance involved in the instability. The parameters \(a_B\) and \(a_L\) are called the spectral offsets and depend on the viscosity profile of the lithosphere. Assuming that the strength increases with depth in the localizing layer, and that it
Table 4.2: Scaling depths

<table>
<thead>
<tr>
<th></th>
<th>Buckling, Eq. 4.6</th>
<th>Localization, Eq. 4.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_e )</td>
<td>1/( -\infty )</td>
<td>1/( -\infty )</td>
</tr>
<tr>
<td>( \lambda/H )</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>7 km(^\dagger)</td>
<td>1.8</td>
<td>0.9</td>
</tr>
<tr>
<td>11 km(^\dagger)</td>
<td>2.8</td>
<td>1.4</td>
</tr>
<tr>
<td>200 km(^\dagger)</td>
<td>50</td>
<td>25</td>
</tr>
</tbody>
</table>

\( \lambda/H \) decreases with depth in a substrate with \( n_e > 0 \), as on Earth, we find [Chapter 3]

\[
\begin{align*}
0 &< a_B < 1/4 \quad (4.5a) \\
1/4 &< a_L < 1/2. \quad (4.5b)
\end{align*}
\]

The instabilities are best observed in nature at their longest wavelength, given by Eq. 4.3 or Eq. 4.4 with \( j = 0 \). The smaller instability wavelengths (\( j \geq 1 \)) may also be expressed in the tectonic record, but are difficult to differentiate from noise in the longer wavelength deformation field. Therefore, they will be ignored in what follows. With the approximation \( 1/n_e \approx 0 \), Eq. 4.5a, and \( j = 0 \), Eq. 4.3 further reduces to:

\[
4 < \lambda_B/H < 8. \quad (4.6)
\]

The relations between predicted instability wavelength and effective stress exponent (Eq. 4.3 and 4.4) are plotted in Fig. 4-5.

Table 4.2 shows the thickness of brittle layer, \( H \), which is required for the buckling or the localization instability to match the 7 to 11 km spacing of faults in the CIB. We use \( n_e = -300 \) and \( n_e = -50 \) are reasonable bounds for the localization efficiency [Chapter 2]. The layer of thickness \( H \) implied in each instability is interpreted as a strong surface layer for the buckling instability and as a layer in which deformation localizes into faults for the localization instability. They are not necessarily the same

\(^{\dagger}\)Fault spacing

\(^{\dagger}\)Base ment undulations
physical layer.

From this simplified scaling analysis, we reject the buckling instability as controlling the fault spacing in the CIB. A layer that buckles with a 7 to 11 km wavelength is at most 3 km thick. Pre-deformation sediments are the only candidate layer of that thickness in the CIB, but as sediments are usually weaker than the basement, they would not develop buckling. Moreover, this would imply that faults are limited to the sediment cover whereas faults are imaged in the basement to 6 km depth. The maximum $\lambda/H$ from seismic data is 1, which is not compatible with the preferred wavelength of buckling (Fig. 4-5).

On the other hand, the localization instability can explain the spacing of faults in the CIB. For $-300 < n_e < -50$, which is most compatible with laboratory data on frictional velocity, and depending on the spectral offset $a_L$, the inferred depth of
faulting, $H$, varies from 6 to 50 km. If the efficiency of localization is maximal while compatible with frictional sliding ($n_e = -50$), $H$ corresponds to the minimum depth to which faults are observed. If on the other hand, the efficiency of localization is minimal, although within the bounds of frictional velocity weakening ($n_e = -300$), $H$ corresponds roughly to the depth of brittle-ductile transition predicted by the thermal age of the CIB, 65 to 90 My [Sclater and Fisher, 1974; Patriat and Segoufin, 1988; Krishna and Rao, 2000].

In summary, the scaling analysis based on Eq. 4.3 and Eq. 4.4 indicates that only the localization instability is compatible with the observed spacing and depth penetration of faults in the Central Indian Basin. In the next section, we present complete growth spectra for different lithosphere models that strengthen this statement by avoiding assumptions regarding the value of the spectral offset $a_L$.

## 4.4 Fault spacing as a function of the strength profile of the lithosphere

### 4.4.1 Model set-up

To better constrain the range of lithosphere structures and effective stress exponents that can match the fault spacing in the Central Indian Basin, we now present growth spectra for five models of the lithosphere that follow alternative assumptions on the strength profile and material stability at the brittle-ductile transition (Table 4.3). By presenting the full growth spectra, we avoid using the approximate value of the instability wavelength (Eq. 4.3 and 4.4) and assuming a value for the spectral offset (Eq. 4.5).

**Strength profile**

Each model is composed of several horizontal layers. Each layer is given an analytical viscosity profile, an effective stress exponent, and a density (Fig. 4-6). The viscosity profile follows the lithospheric strength profile, which depends on the dominant
mechanism of deformation at each depth (see §4.7). We consider that the strength is controlled by the weakest of the resistance of frictional sliding or dislocation creep, with the strength possibly limited by a saturation value, \( S \). A new layer is defined each time that the dominant deformation mechanism, the density, or the effective stress exponent changes.

Rocks deform viscously at high temperature by dislocation creep [Evans and Kohlstedt, 1995]. The strength \( \sigma_d \) in this regime depends on the rate of deformation of the rock and the ambient temperature. In all our models, we assume a strain rate of \( 10^{-15.5} \) s\(^{-1}\) given by plate reconstructions [Gordon, 2000], and a linear geotherm of 15 K.km\(^{-1}\) appropriate for a 65- to 90-My old lithosphere as in the CIB [Parsons and Sclater, 1977]. The temperature field saturates at 1350°C. Because the earliest evidence of faulting and folding is only 7.5 My old whereas the age range of lithosphere in the CIB spans 30 My, a more precise estimate of the geotherm is not justified. With the assumed thermal profile, the crust is brittle throughout, and is therefore ignored in models 1 to 4. As the oceanic upper mantle is probably dry because of water extraction during formation of the crust at mid-oceanic ridges [Hirth and Kohlstedt, 1996], we use the flow law for dry olivine of Karato et al. [1986]. Neglecting thermal or grain size feedback processes, the effective stress exponent is 3.5 in this regime [Chapter 2].

Table 4.3: Model Parameters

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>crust</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>( S ) (MPa)</td>
<td>( \infty )</td>
<td>300(^\dagger)</td>
<td>300(^\dagger)</td>
<td>( \infty )</td>
<td>( \infty )</td>
</tr>
<tr>
<td>( T_w ) (°C)</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>150</td>
<td>( \infty )</td>
</tr>
<tr>
<td>( z_{lo} ) (km)</td>
<td>44</td>
<td>50</td>
<td>11</td>
<td>10</td>
<td>44</td>
</tr>
<tr>
<td>( n_1 ) for 7 km</td>
<td>-333</td>
<td>-700</td>
<td>-200</td>
<td>-100</td>
<td>-333</td>
</tr>
<tr>
<td>( n_1 ) for 11 km</td>
<td>-140</td>
<td>-350</td>
<td>-100</td>
<td>-50</td>
<td>-140</td>
</tr>
</tbody>
</table>

\( \dagger \)\( n_e < 0 \) in layer with saturated strength
\( \dagger \)\( n_e = 10^6 \) in layer with saturated strength
Overlying fluid
\[
\rho_s
\]
Substratum
\[
\eta_3(z_{\text{max}}), \rho_3
\]
Localizing layer
\[
\eta_1(z), \rho_1
\]
Plastic layer
\[
\eta_2(z), \rho_2
\]
Ductile layer
\[
\eta_3(z), \rho_3
\]

Figure 4-6: Schematics of lithosphere models. A sequence of layers undergoes shortening at the rate \( \dot{\varepsilon}_{xx} = 10^{-15.5} \text{s}^{-1} \) under an inviscid fluid of density \( \rho_s \). The strength profile follows the weakest of frictional resistance, ductile strength, or a saturation strength. Each layer \( i \) is characterized by a density \( \rho_i \), a thickness \( H_i \), an effective stress exponent \( n_i(z) \), and a viscosity profile \( \eta_i \) that correspond to the dominant deformation mechanism in that depth range (see §4.7). A substratum with rheology corresponding to the lowest level of the model is included for mathematical convenience. The shading differentiates between localizing layers (darkest shade, \( n_i < 0 \)), plastic layers (medium shade, \( n_i = 10^6 \)), and ductile layers (lightest shade, \( n_i = 3.5 \)). The same shading is used in the strength profile in the next figures.

At lower temperature and pressure, rocks are brittle. Because of the presence of pre-existing faults, the brittle strength of lithosphere is controlled by frictional sliding and is universal function of pressure, \( \sigma_f \) [Byerlee, 1978]. At low temperature, \( \sigma_f \) decreases with sliding velocity [Scholz, 1990] with \(-300 < n_e < -50 \) [Chapter 2]. It is possible that at depth, the brittle lithosphere behaves more plastically, with \( 1/n_e \rightarrow 0 \).

Because the brittle strength of the lithosphere increases with depth and its ductile strength decreases with depth, the brittle and ductile strengths are equal at a partic-
ular depth, the depth of the brittle-ductile transition (BDT) [Brace and Kohlstedt, 1980]. Model 1 follows such a simple strength profile, with only two layers, one brittle \( n_1 < 0 \), the other ductile \( n_2 = 3.5 \), separated by the BDT. The depth of the BDT is \( \sim 44 \) km in that model. However, the transition between the brittle and the ductile deformation may be spread over a large depth range because of the suppression of microcracking at high pressure [Kirby, 1980] and the progressive onset of ductility for the different minerals composing mantle rocks [Kohlstedt et al., 1995]. As the rock strength is roughly constant in the transition regime, models 2 and 3 include an intermediary layer with strength \( S = 300 \) MPa [Kohlstedt et al., 1995]. As it is uncertain whether localization occurs in the semi-brittle regime or not, we use \( n_e < 0 \) in Model 2 and \( n_e = 10^6 \) in Model 3. In addition, friction may become velocity-strengthening above a critical temperature \( T_w \) [Stesky et al., 1974; Tse and Rice, 1986; Blanpied et al., 1998]. Hence, in Model 4, the strength in the brittle regime increases with depth until it exceeds the ductile strength \((S \to \infty)\), but we use \( n_e = 10^6 \) if \( T \geq T_w = 150^\circ C \). We did not study models where saturation of the strength envelope and a transition to \( n_e = 10^6 \) occur at different depths. We report in Table 3 the maximum depth of localization, \( z_{loc} \), as well as the effective stress exponent needed in the localizing layers to produce a fault spacing of 7 or 11 km in each model.

The density contrast between a model and the overlying fluid is important for the development of buckling. Here, we assume that overlying fluid is sediment rich water with density \( \rho_s = 2300 \) kg.m\(^{-3} \), which favors buckling. However, only the high-density mantle is considered in models 1 to 4, which reduces buckling. The crust, being entirely brittle for the imposed geotherm, has only a minor effect on the strength envelope. However, it is included as a new layer in Model 5 because it reduces the surface density contrast and introduces a density contrast within the model. Model 5 is otherwise similar to Model 1, with the BDT occurring at a single depth.
Solution strategy

As described in Chapter 3, we derive the growth rate of perturbations of the models as a function of the perturbation wavelength. First, we solve for the shape of individual deformation modes in each layer using the depth-dependent viscosity profile and its depth-derivatives. The amplitude of each deformation mode is found by matching stress and velocity boundary conditions at each interface. This links the velocity field within the model to the amplitude of interface perturbations, thereby determining the growth rate of the perturbation.

4.4.2 Lithosphere strength models: Results

Model 1: Localization everywhere in the brittle regime

Model 1 assumes that there is no limiting strength to the failure law \( S \to \infty \) and that localization occurs whenever the Byerlee criterion for frictional sliding is reached \( T_w \to \infty \). Hence, the brittle-ductile transition is idealized as a point \([\text{Brace and Kohlstedt}, 1980]\), which is at a depth of 44 km for the assumed geotherm, strain rate, and mantle flow law, (Fig. 4-7).

In this model, the localization instability occurs at 7 km if \( n_1 = -333 \) and 11 km if \( n_1 = -166 \) (Fig. 4-7, Table 4.3), if we consider the longest wavelength of the instability \( j = 0 \). These values of \( n_e \) are within the bounds of experimental data \((-300 < n_e < -50)\) although on the low efficiency of localization side. The tradeoff between the efficiency of localization, expressed by \( n_1 \), and the preferred spacing of faults is shown in Fig. 4-8 by a map of the growth rate as a function of \( n_1 \) and the wavelength of perturbation. Higher growth rates are in white and correspond to the localization instability. Additional divergent peaks at smaller wavelengths mark harmonics of the localization instability \( (j \geq 1, \text{Fig. 4-7 and Fig. 4-8}) \). If present, they are harder to recognize in nature, because the expression of the the fundamental mode \( (j = 0) \) is noisier than our idealized model predicts. The high growth rate at long wavelengths \( (\lambda \approx 100 \text{ km, Fig. 4-7}) \) may be associated with the buckling instability. It will be discussed in §4.5.3.
Figure 4-7: a) Growth spectrum and b) strength profile for Model 1. Solid line: $n_1 = -333$; dashed lines: $n_1 = -60$; dotted line: $n_1 = 10^6$. The gray-shaded area in a) marks the range of fault spacing observed in the Central Indian Basin. The order of the localization instability, $j$, is indicated for the case $n_1 = -333$. Growth rate in the 100 to 300 km wavelength range may correspond to buckling (see §4.5.3).

Models 2 and 3: Limiting strength

The strength of the lithosphere may be limited at the approach of the brittle-ductile transition because of cataclastic flow and because the different minerals that constitute a rock become ductile at different depths [Kirby, 1980; Kohlstedt et al., 1995]. Therefore, we consider models where the strength is limited at 300 MPa, as advocated by Kohlstedt et al. [1995]. Then, the strength envelope is saturated at ~ 11 km depth.

In Model 2 (Fig. 4-9), we retain the effective stress exponent of frictional sliding in the region where strength is saturated (11 to 48 km deep). The localization instability requires $n_1 = -700$ for a 7 km fault spacing and $n_1 = -350$ for a 11 km fault spacing (Table 4.3). To match the fault spacing in the CIB, localization in Model 2 must be less efficient than any mechanism considered in Chapter 2. This may indicate that the effective stress exponent varies with depth, in the sense of the material being more plastic in the brittle-ductile transition zone.

Therefore, we present an alternative treatment of the broad brittle-ductile transition in which the region where strength is limited is plastic, with $n_e = 10^6$ (Model 3). In Fig. 4-10, we present the growth spectra for Model 3 with $n_1 = -200$ and
Figure 4-8: Map of growth rate as a function of effective stress exponent of brittle layer, $n_1$, and perturbation wavelength for the strength profile of Fig. 4-7. Higher growth rates correspond to lighter tones. The branches $j = 0$ and $j = 1$ of the localization traverse this portion of the parameter space. They indicate the predicted fault spacing as a function of $n_1$.

$n_1 = -100$, which produce fault spacings of 7 and 11 km, respectively (Table 4.3). Hence, limiting localization to the depth where the strength increases with depth brings $n_1$ back to the range of experimental values.

To produce the same fault spacing, $\lambda$, localization must be more efficient ($1/n_1$ more negative) in Model 3 than in Model 1. This is needed to compensate the smaller thickness of the localizing layer, $H_1$, in Model 3. Indeed, the deformation imposed over a given area can be accommodated by a smaller number of faults if there is is a greater tendency for localization, resulting in larger $\lambda/H$ (Eq. 4.4).

The tradeoff between the limiting rock strength, $S$, and the effective stress exponent is shown in Fig. 4-11, where we assume that the layer where strength is saturated is plastic, as in Model 3. If the limiting strength, $S$, is so small that the brittle layer
Figure 4-9: a) Growth spectrum and b) strength profile for Model 2. Solid line: \( n_1 = n_2 = -700 \); dashed lines: \( n_1 = n_2 = -350 \). There is no positive growth rate for \( n_1 = 10^6 \). The gray-shaded area in a) marks the range of fault spacing observed in the Central Indian Basin. The order of the localization instability, \( j \), is indicated for the case \( n_1 = -700 \). Growth rate in the 100 to 300 km wavelength range may correspond to buckling (see §4.5.3).

Figure 4-10: a) Growth spectrum and b) strength profile for Model 3. Solid line: \( n_1 = -200 \); dashed lines: \( n_1 = -100 \). There is no positive growth rate for \( n_1 = 10^6 \). The gray-shaded area in a) marks the range of fault spacing observed in the Central Indian Basin. The order of the localization instability, \( j \), is indicated for the case \( n_1 = -200 \). Growth rate in the 100 to 300 km wavelength range may correspond to buckling (see §4.5.3).
Figure 4-11: Map of growth rate for a 7 km wavelength as a function of saturation value of the strength profile, $S$, converted into a maximum depth of localization in the upper axis, and the effective stress exponent of brittle layer, $n_1$. The strength profile is similar to Model 3 (Fig. 4-10b) with variable $S$. High growth rate (lighter tones) indicates the region of the parameter space where the localization instability is at 7 km. The map shows what $n_1$ is needed to explain a 7 km fault spacing as a function of the limiting strength (or depth) of localization. Localization is more efficient towards the top of the figure.

is less than 20 km thick, the effective stress exponent needed to produce a 7 km fault spacing and $S$ are correlated: decreasing $H_1$ requires more efficient localization. However, if the brittle layer is between 20 and 40 km thick, $n_e \sim -350$, regardless of $S$. Increasing $H_1$ in that range still requires less efficient localization, but it also changes the shape of the strength profile significantly, decreasing the importance of the plastic layer where the strength does not depend on depth. In Eq. 4.4, changing the shape of the strength envelope influences the spectral offset $a_L$. This effect roughly compensates the change of $H_1$ in the 20 to 40 km depth range. Hence, the effective stress exponent needed to match a 7 km fault spacing is roughly constant for $H_1$ in that depth range.
Model 4: Temperature-limited localization

In Model 4, the brittle strength follows Byerlee’s law of friction down to the ideal brittle-ductile transition, as in Model 1. However, localization is limited to temperatures less than $T_w$ in Model 4, to model transition from velocity-weakening friction. For $T > T_w$, $n_e = 10^6$. Early experiments by Stesky et al. [1974] indicated that $T_w$ could be as low as 150°C for olivine-rich rocks. However, these experiments contradict the distribution of intraplate earthquake focal depths, which show a cutoff temperature of at least 600°C in the mantle [Chen and Molnar, 1983; Wiens and Stein, 1983]. Unfortunately, no other experimental study to our knowledge has explored how the velocity-dependence of friction changes with temperature for olivine-rich rocks. We explore with Model 4 how temperature-limited localization influences the localization instability.

If we consider that localization is limited to 150°C (10 km) but that the brittle strength always increases with depth, effective stress exponents of -100 to -50 are needed to explain fault spacings of 7 or 10 km, respectively (Fig. 4-12, Table 4.3). These values are consistent with laboratory data on frictional sliding, although more compatible with localization by cohesion loss.

To match the same fault spacing, localization must be more efficient in Model 4 than in the case where localization occurs everywhere in the brittle regime (Model 1), because of the smaller scaling depth $H_1$, and than in the case where the strength saturates in the plastic regime (Model 3), because of the different type of strength profile and corresponding spectral offset $a_L$ (Eq. 4.4).

We show in Fig. 4-13 how the effective stress exponent required for a 7 km fault spacing correlates with the depth limit of localization. In contrast with Fig. 4-11, the value of $n_e$ needed to match the 7 km fault spacing is correlated with the depth of localization, $H_1$, for all $H_1$. This is because the strength envelope is identical whether there is localization or not; there is no depth range where the change of $H_1$ changes the shape of the strength envelope and the spectral offset.
Figure 4-12: a) Growth spectrum and b) strength profile for Model 4. Solid line: \( n_1 = -100 \); dashed lines: \( n_1 = -50 \); dotted line: \( n_1 = 10^6 \). The gray-shaded area in a) marks the range of fault spacing observed in the Central Indian Basin. The order of the localization instability, \( j \), is indicated for the case \( n_1 = -100 \). Growth rate in the 100 to 300 km wavelength range may correspond to buckling (see §4.5.3).

Model 5: Effect of the crust

As the crust was ignored in Models 1 to 4, there was only one density contrast, at the surface of the model. In Model 5, the crust is present. It reduces the surface density contrast and introduces a second density contrast within the brittle layer. The model is otherwise identical to Model 1 (\( S \rightarrow +\infty \) and \( T \rightarrow +\infty \)). The density stratification produced by the crust has only a minor effect on the preferred wavelength of the localization instability, as is shown by comparing Fig. 4-14 and Fig. 4-7. However, the crust may be important for the development of the buckling instability, as will be discussed in §4.5.3.

4.5 Discussion

4.5.1 Localization mechanism and history of faulting

Using the scaling relations developed in Chapter 3, we have shown that the fault spacing in the Central Indian Basin is compatible with a localization instability, whereas
Figure 4-13: Map of growth rate for a 7 km wavelength as a function of limiting temperature of localization, $T_w$, converted into a maximum depth of localization in the upper axis, and effective stress exponent of localizing layer, $n_1$. The strength profile is similar to Model 4 (Fig. 4-12b) with variable $T_w$. High growth rate (lighter tones) indicates the region of the parameter space where the localization instability is at 7 km. The map shows what $n_1$ is needed to explain a 7 km fault spacing as a function of the limiting temperature (or depth) of localization. Localization is more efficient towards the top of the figure.

it cannot be explained by a buckling instability. We have used different lithospheric strength profiles to estimate more accurately the range of effective stress exponents, $n_e$, that produce a fault spacing of 7 or 11 km. Except for Model 2 (layer with saturated strength and negative $n_e$ at the brittle-ductile transition) the values of $n_e$ required to match a 7 to 11 km fault spacing are compatible with friction velocity-weakening. However, the constraints on the localization mechanism provided by the value of $n_e$ are sufficiently broad that no localization mechanism can be ascertained from our analysis. In addition, the mechanism may change with depth, which would result in the inferred $n_e$ being an average of the actual depth-dependent $n_e$.

Velocity-weakening localizes deformation in the following manner [Chapter 2].
Consider that the lithosphere is riddled with pre-existing faults. The faults are activated when the frictional resistance is overcome (i.e., when the stress exceeds a critical value). However, in the case of velocity-weakening, the coefficient of friction decreases with sliding velocity. If a fault slides slightly faster than its surrounding ones, it becomes weaker, so that it accommodates more shortening. Its sliding rate increases, which further weakens the fault, and so on.

An important aspect of this mechanism is that faults are present before the deformation begins. This is indeed pertinent to the tectonics of the Central Indian Basin, as many of the seismically-imaged thrust faults are reactivated structures [Bull and Scrutton, 1990]. Thus, the following history of faulting can be deduced. Faults were generated during seafloor spreading, 65 to 90 My ago, dependent on the location in the current CIB. These faults were probably abyssal-hill-bounding normal faults, with a spacing of order 2 km. About 7.5 My ago, the faults, now buried under the sediment cover, were reactivated in a reverse sense by north-south compression. Because of the intensity of localization associated with fault reactivation, a 7 to 11 km fault spacing attempted to develop in the region, constrained by the requirement of using
pre-existing faults. Thus, pre-existing faults in the vicinity of target locations separated by 7 to 11 km were preferentially reactivated. This explains why not all faults were reactivated, and also the high noise level in the fault spacing histograms. Occasionally, the reactivated fault pattern was complemented with a new fault. The faults propagated upward into the sediment cover and downward because of the deepening of the BDT between the time of fault creation and reactivation. Fault propagation requires the creation of new fault surfaces, which may result in different $n_e$ than reactivation. Again, our analysis can only indicate an average effective stress exponent for the CIB lithosphere.

4.5.2 Depth of localization

Even without considering that the actual temperature profile or background strain rate of the Central Indian Basin could be different from our model, there is a wide range of localization depths that are consistent with the observed fault spacing. We summarize in Fig. 4-15 how varying the thickness of the localizing layer is compensated by varying the efficiency of localization, expressed by the effective stress exponent $n_e$. The faults may penetrate no deeper than imaged (6 to 8 km) if $n_e \approx -50$ or reach the predicted depth of brittle-ductile transition around 40 km if $n_e \approx -300$. This makes it impossible to evaluate whether the assumed geotherm of 15 K.km$^{-1}$ is realistic. It has been argued that the lithosphere is anomalously hot under the CIB because of high heat flow measurements [Stein and Weissel, 1990]. For our analysis to constrain the geotherm better, we need tighter bounds on the effective stress exponent than currently available.

Another piece of information pertinent to the depth of faulting in the CIB comes from the focal depth of earthquakes (Fig. 4.1). Although the depth of shallow earthquakes cannot be confidently assessed from teleseismic data, information is available for deeper events. Histograms of centroid depth from two different studies (Fig. 4-16) show broad maxima between 10 and 30 km, and no earthquakes below 45 km. Although earthquake data were not used in choosing the flow law, geotherm, and strain rate of our models, the distribution of focal depths and our strength profiles
Figure 4-15: Tradeoff between the efficiency of localization (effective stress exponent) and depth of localization. Localization is more efficient towards the top of the figure. The solid lines mark the localization instability for a strength profile as in Model 3 (strength-limited localization, Fig. 4-10 and 4-11) and the dashed lines use a strength profile as in Model 4 (temperature-limited localization, Fig. 4-12 and 4-13). The localization instability passes between these two lines for intermediate strength profiles (shaded areas). Darker shade: 7 km fault spacing; lighter shade: 10 km fault spacing.

are remarkably consistent. As velocity-weakening is needed for earthquake generation [Rice, 1983; Dieterich, 1992] and is the most likely localization mechanism for the faults in the CIB, earthquake data imply that localization reaches 40 km depth. In the absence of a plastic layer around the brittle-ductile transition, models with saturated strength envelope imply $n_e$ out of the range compatible with velocity-weakening of friction (Table 4.3). Therefore, we argue that among the models presented herein, Model 1, in which the strength increases to an idealized brittle-ductile transition around 40 km and localization is not limited with depth, is most compatible with the data in the CIB. The continuous range of focal depths also argues against models that explain the long-wavelength deformation of the CIB by folding of the putative elastic core of the lithosphere [McAdoo and Sandwell, 1985].
However, the faults that were reactivated during the current shortening episode did not reach 40 km depth, as they formed during sea-floor spreading and rocks deeper than a few kilometers are ductile in mid-ocean ridge settings [Hirth et al., 1998]. Fault reactivation in the CIB was probably accompanied by formation of new faults in the 6 to 40 km depth range. Therefore, the most realistic model may have two different values of $n_e$, both being negative, above and below a $\sim 6$ km depth horizon. We have not explored this family of models due to the poor constraints on the values of $n_e$ to be used. The delay between the onset of shortening indicated from plate reconstruction [Royer and Gordon, 1997] and fault activation [Curray and Munasinghe, 1989; Krishna et al., 2001] might reflect the need to build enough stress to break the deeper levels of the lithosphere [Gerbault, 2000].

### 4.5.3 Origin of long-wavelength deformation

Although we focused in this study on the short wavelength of deformation in the CIB, expressed by regularly-spaced faults, the long wavelength of deformation, thought to
Figure 4-17: Map of growth rate as a function of density of overlying fluid $\rho_s$ and perturbation wavelength for the strength profile of Model 1 with $n_1 = 10^6$. Model density is 3300 kg.m$^{-3}$. Higher growth rates correspond to lighter tones. A high-density fluid is needed for growth of the buckling instability.

represent buckling of a strong plastic layer, gives important constraints on the lithospheric structure [McAdoo and Sandwell, 1985; Zuber, 1987a; Martinod and Molnar, 1995]. The $\sim 200$ km wavelength deformation is compatible with buckling of a strong plastic layer 25 to 50 km thick (Eq. 4.6 and Table 4.2), the same thickness that is compatible with the fault spacing.

Although the wavelength of basement undulations in the CIB is in agreement with the preferred wavelength of buckling, buckling alone may be insufficient to explain the undulations. Indeed, both depth-dependent viscosity in the lithosphere and the density contrast between the lithosphere and the overlying fluid reduce the growth rate of the buckling instability [Chapter 3]. Using the strength profile of Model 1 and $n_1 = 10^6$, the buckling mode all but vanishes for realistic strength profiles unless the surface density contrast is small (Fig. 4-17). In that model, the density
of the lithosphere is 3300 kg.m\(^{-3}\): significant growth of the buckling mode \((q \geq 10)\) requires that the density of the overlying medium be \(\sim 3000\) kg.m\(^{-3}\), which is excessive. If a lighter crust or pre-deformation sediments were included, buckling might have developed if the density of the overlying fluid was \(\sim 2000\) kg.m\(^{-3}\), which is compatible with syn-tectonic sediments. However, there should be an additional density contrast between the crust and the mantle or between the pre-deformation sediments and the crust. This internal density contrast increases the growth rate at short wavelength but decreases growth at long wavelength, as can be seen by comparing the growth spectrum of Model 1 with \(n_1 = 10^6\) (Fig. 4-14), with a 3000 kg.m\(^{-3}\) crust to that of Model 5 with \(n_1 = 10^6\) (Fig. 4-7), where the crust is absent. Hence, a higher-order buckling wavelength \((j \geq 1, \lambda \sim 80\) km) may be favored in models with density stratification. Its wavelength would not be consistent with the observed basement undulations in the CIB.

In view of these contradicting effects of density on buckling, we ask whether localization can enhance the growth rate of the buckling instability. For all the models presented herein, there is a window of wavelengths between 100 and 300 km where the growth rate is high when \(n_1 < 0\), in spite of the large density contrast that reduces the maximum growth rate to 1 or less if \(n_1 = 10^6\) (Fig. 4-7, 4-9, 4-10, 4-12, and 4-14). A map of growth rate as a function of effective stress exponent and perturbation wavenumber for Model 1 Fig. (4-18) shows that this long-wavelength domain of high growth rate is linked to the mode \(j = 0\) of the localization instability. However, this range of wavelengths is not predicted by the scaling relation of the localization instability (Eq. 4.4).

High growth rate in the 100 to 300 km wavelength range may be understood from the behavior at long wavelengths of the opening of the doublet that marks the localization instability. The average position of this doublet follows Eq. 4.4, which is based on the position of a resonance between deformation modes inside the layer undergoing localization [Chapter 3], but there is no similar relation for the separation between the branches, or opening, of each doublet. At short wavelength, the opening is small enough to be neglected, but at long wavelengths, the branches
are so distinct that one warps around and traverses the domain $\lambda \sim 200$ km and $n_e \sim -300$ (Fig. 4-18). As we do not have a physical explanation for the factors that control the opening of the doublets, we must be cautious in extrapolating it to natural conditions. However, it is possible that the warping the of $j = 0$ branch of the localization instability helps buckling to develop. In all the model that we studied, the warped localization branch reaches the highest $1/n_e$ at the wavelength of the buckling instability, suggesting an interaction of the buckling and localization instabilities. Consistent with the idea that localization may facilitate buckling, faults
were found to help buckling in finite element models [Wallace and Melosh, 1994; Beekman et al., 1996; Gerbault et al., 1999]. However, Krishna et al. [2001] point to areas where the basement buckled without faulting. Localization-aided buckling is possible for any surface density contrast. Hence, it can explain why buckling is observed in the Wharton Basin beyond the reaches of the Nicobar fan.

Alternatively, Karner and Weissel [1990] proposed that the basement undulations are due to some wavelengths of the flexural response of the lithosphere to the load provided by the Afanazy-Nikishin seamounts (Fig. 4-1) being selectively amplified by the north-south shortening. In support of this idea, the seamounts are centrally located in the intraplate deformation area [Karner and Weissel, 1990]. In addition, they are located at the margin of the region that was active during the three episodes of deformation defined by Krishna et al. [2001]. However, Karner and Weissel [1990] assume only elastic folding in the CIB, which is not consistent with the continuous distribution of earthquake focal depths. Karner and Weissel [1990] could explain the pattern of the geoid anomaly in the CIB if the surface density contrast is reduced by active sediment deposition. Hence, their analysis cannot apply to the Wharton basin, where geoid undulations are observed beyond the Nicobar fan. To our knowledge, the instability analysis for a plastic or localizing lithosphere in presence of pre-existing load or heterogeneity has not yet been conducted.

### 4.6 Conclusions

The pattern of faulting in the Central Indian Basin probably developed from a combination of fault reactivation and localization instability. Localization is defined as the process by which deformation in a homogenous lithosphere focuses on specific regions or localized shear zones that we identify with faults. This process is driven by local perturbations of the deformation rate, but develops a lithospheric-scale pattern through what we call the localization instability [Chapter 3]. The preferred wavelength of the localization instability can be matched to the 7 to 11 km spacing of fault in CIB. Alternative controls on faults spacing, such as plastic buckling or elastic
folding, cannot explain the observed fault spacing.

The preferred wavelength of the localization instability depends on the effective stress exponent, $n_e$, a parameter that measures the efficiency of localization. The values of effective stress exponent needed to explain a fault spacing of 7 to 11 km vary from $-300$ to $-100$ depending on the exact strength profile assumed in the lithosphere. They are consistent with experimental data on frictional sliding. Localization during frictional sliding is also consistent with reactivation of pre-existing faults, which is relevant for the CIB tectonics as many faults formed during seafloor spreading and were reactivated during the current north-south shortening episode. The pre-existing faults in the vicinity of target locations separated by the wavelength of the localization instability were preferentially reactivated. This can explain the significant spread in the fault spacing distributions.

Earthquake data indicate faulting to the brittle-ductile transition, around 40 km, leading us to favor a model in which localization occurs to the brittle-ductile transition with $n_e \sim -300$ and the strength of the brittle layer increases continuously with depth. However, the deepest faults were probably not reactivated structures, as the lithosphere was ductile during seafloor spreading. These faults must be formed during the current compressive tectonics, so that localization mechanism and $n_e$ may be different below a few kilometers depth. Our analysis gives only an average effective stress exponent for the whole lithosphere.

The long wavelength of deformation expressed by broad undulations of the CIB lithosphere is most likely associated with a buckling instability. The wavelength in that case is also compatible with the brittle-ductile transition being around 40 km depth. However, the density of the lithosphere and depth-dependent strength profile reduce the growth rate of buckling, so that buckling is unlikely to develop in the Central Indian Basin. Buckling may be able to grow because of an interaction with either the developing localized fault patterns, or the load of the pre-existing Afanazy-Nikishin seamounts. Both of these phenomena must be better understood before their possible relevance to the tectonics of the CIB can be ascertained.
4.7 Appendix: Construction of rheological models

4.7.1 Brittle law

Expressed in terms of stress invariants instead of the usual form of normal and shear stress resolved on a particular fault, Byerlee's law of friction states

\[ \sigma_{\|} = C + f \sigma_1, \quad (4.7) \]

with \( C \) the cohesion term, \( f = \sin(\phi) \) the coefficient of friction with \( \phi \) the friction angle, and \( \sigma_1 \) and \( \sigma_{\|} \) the first and second invariant of stress, respectively. The parameters \( C \) and \( f \) are related to the parameters \( S \) and \( \mu \) of Byerlee [1978] by

\[ C = \frac{S}{\sqrt{1 + \mu^2}}, \quad (4.8a) \]
\[ f = -\mu \sqrt{1 + \mu^2}. \quad (4.8b) \]

We use only the high pressure branch of Byerlee’s law, with \( S = 50 \) MPa and \( \mu = 0.6 \).

For a model undergoing horizontal shortening, we assume that \( \sigma_{zz} \) equals the weight of the overburden rocks, \( -p_{ov} \). Along the horizontal axis, we have

\[ \sigma_{xx} = -p + 2\eta \dot{e}_{xx}, \quad (4.9) \]

with \( p \) the pressure and \( \eta \) the apparent viscosity. The stress invariants becomes

\[ \sigma_1 = -p_{ov} + 2\eta \dot{e}_{xx}, \quad (4.10a) \]
\[ \sigma_{\|} = \eta |\dot{e}_{xx}|. \quad (4.10b) \]

Introducing these relations in Eq. 4.7, we obtain

\[ \eta = \frac{C - f p_{ov}}{|\dot{e}_{xx}| - 2f \dot{e}_{xx}}. \quad (4.11) \]
In each layer, the overburden pressure varies as

\[ p_{ov} = p_t - \rho g(z - z_t), \]  

(4.12)

with \( p_t \) and \( z_t \) the pressure and depth at the top of the layer and \( \rho \) the density of the material in that layer. Finally, we obtain

\[ \eta = \frac{C - f p_t - f \rho g z_t}{|\ddot{\varepsilon}_{xx}| - 2f \ddot{\varepsilon}_{xx}} + \frac{f \rho g}{|\ddot{\varepsilon}_{xx}| - 2f \ddot{\varepsilon}_{xx}} z. \]  

(4.13)

### 4.7.2 Ductile law

Dislocation creep flow laws have the form

\[ \dot{\varepsilon}_a = A \Delta \sigma^n \exp\left(-\frac{Q}{RT}\right), \]  

(4.14)

where \( \dot{\varepsilon}_a \) is the axial shortening rate of a sample, \( \Delta \sigma \) is the differential stress applied on a sample, \( n \) is the stress exponent, \( Q \) is the activation energy, \( R \) is the gas constant, and \( T \) is the absolute temperature. With the axially-symmetric configuration of the experimental studies, the second invariants of the strain rate and stress tensors are related to \( \dot{\varepsilon}_a \) and \( \Delta \sigma \) by

\[ \dot{\varepsilon}_\| = 3 \times 2^{-3/2} \dot{\varepsilon}_a, \]  

(4.15)

\[ \sigma_\| = 2^{1/2} \Delta \sigma. \]  

(4.16)

These relations are used to express Eq. 4.14 as a function of \( \dot{\varepsilon}_\| \) and \( \sigma_\|. \)

On the other hand, our models assume a pure-shear geometry. Hence, we express \( \dot{\varepsilon}_\| \) and \( \sigma_\| \) as a function of the horizontal normal strain rates and deviatoric stress, \( \dot{\varepsilon}_{xx} \) and \( \tau_{xx} \):

\[ \dot{\varepsilon}_\| = 3^{1/2} \times 2^{-1/2} \dot{\varepsilon}_{xx}, \]  

(4.17)

\[ \sigma_\| = 3^{1/2} \times 2^{-1/2} \tau_{xx}. \]  

(4.18)
They are related by the apparent viscosity, $\eta$, by

$$\dot{\varepsilon}_{xx} = 2\eta \tau_{xx},$$

(4.19)

from which we obtain

$$\eta = B\dot{\varepsilon}_{\text{ini}}^{1/n-1} \exp Q/nRT,$$

(4.20)

with

$$B = \frac{\sqrt{3}}{6} \left( \frac{A\sqrt{3}}{2} \right)^n.$$

(4.21)

Depth dependence of this viscosity profile comes from the geotherm, $T(z)$, herein taken as linear.
Chapter 5

Clues to the lithospheric structure of Mars from wrinkle ridge sets and localization instability

Abstract

Wrinkle ridges are a manifestation of horizontal shortening in planetary lithospheres. Deformation is localized on faults that underlies individual ridges. In ridged plains of Mars, such as Solis Planum or Lunae Planum, wrinkle ridges are spaced \( \sim 40 \) km apart, whereas in the martian northern lowlands, where ridges are identified only in MOLA altimetric data, the ridge spacing is \( \sim 80 \) km. We attribute ridge spacing to an instability of the lithosphere under compression. The localization instability, which results in periodically-spaced faults [Chapter 3], links the difference of ridge spacing in the northern lowlands and in the highland ridged plains to the difference of crustal thickness, via the depth of the brittle-ductile transition (BDT). In Solis and Lunae Plana, where the crust is 50 to 60 km thick, the crust may be ductile at depth, limiting faulting to the BDT of crustal rocks. In the lowlands, where the crust is only \( \sim 30 \) km thick, it may be brittle throughout. Thus, the depth of faulting may be controlled by the BDT of mantle rocks, which is roughly a factor of two deeper than that of crustal rocks. The geotherm can be identical in both regions, at \( 12 \pm 3 \) K.km\(^{-1}\), although differences of a few K.km\(^{-1}\) can be accommodated within this model. The heat flux implied by this geotherm is similar to heat produced by radiogenic decay 3 Gyr ago. Our analysis provides a rheological explanation for the difference in spacing between the highlands and the lowlands, in contrast to the suggestion of of Head et al. [2001], who proposed that half the ridges are buried by sediments. Using finite element models, we show that slight variations of either the geotherm or of the crustal thickness perpendicular to the ridge trend favors slip on
faults verging toward high-standing plains, as observed in Solis and Lunae Plana.

5.1 Introduction

The structure and evolution of planetary lithospheres helps to constrain the thermal history of a planet. As no direct subsurface information is available for objects other than the Earth and the Moon, the three-dimensional structure of the lithosphere of other planets must be inferred from modeling of remote sensing data such as multispectral and panchromatic images, surface topography, and the gravity and magnetic fields. In particular, the vertical strength profile of planetary lithospheres can be constrained from the horizontal length scale of tectonic features. The wavelength of the flexural response to surface or subsurface loads indicates the thickness of an equivalent elastic plate [Turcotte and Schubert, 1982; Comer et al., 1985; Banerdt et al., 1992; Wieczorek and Phillips, 1998]; fold wavelengths are proportional to the thickness of a strong layer sometimes associated with the brittle layers of the lithosphere [Fletcher and Hallet, 1983; Ricard and Froideveau, 1986; Zuber, 1987a; Zuber and Aist, 1990].

The spacing of wrinkle ridges has been used to infer the thickness of a mechanically strong layer at the top of the martian lithosphere [Saunders et al., 1981; Zuber and Aist, 1990; Watters, 1991], as will be done here. Depending on the model used, however, wrinkle ridge spacing can imply deep penetration of ridge-related deformation (several to tens of km), most consistent with the influence of global stress patterns on ridge orientation [Zuber and Aist, 1990; Banerdt et al., 1992; Zuber, 1995], or the presence of a shallow weak layer (1 to 2 km deep) related to a possible megaregolith or to the base of volcanic plains [Saunders et al., 1981; Watters, 1991].

As faulting is an important aspect of wrinkle ridge formation [Lucchita and Klockenbrink, 1981; Plescia and Golombek, 1986; Watters, 1988; Allemand and Thomas, 1992], faults have been defined a priori in previous theories of ridge formation [Zuber, 1995; Schultz, 2000]. However, fault formation, i.e. the localization of failure onto discrete shear zones, has not been included as a dynamic process. In order to relate ridge spacing and the mechanical structure of the lithosphere, the faulting layer has
sometimes been regarded as pseudo-plastic [Zuber and Aist, 1990]. This rheology includes a yield strength upon which faulting occurs, but also implies that deformation remains distributed instead of localizing on discrete shear zones [Chapter 2]. In this study, we show how the estimated thickness of the layer involved in faulting at martian wrinkle ridges is modified by the consideration of localization process, using the concept of a localization instability introduced in Chapter 3. That the localization instability is the most likely control on fault spacing is best seen when the ridge spacing of different regions is compared. In areas where wrinkle ridges were recognized in Viking images, ridge spacing is \( \sim 40 \) km [Saunders et al., 1981; Zuber and Aist, 1990; Watters, 1991], whereas in the northern lowlands, where ridges were recently recognized from Mars Observed Laser Altimeter (MOLA) data [Withers and Neumann, 2001; Head et al., 2001], their spacing is \( \sim 80 \) km [Head et al., 2001]. The localization instability can explain the difference of ridge spacing in relation with the relatively thin crust in the northern lowlands [Zuber et al., 2000].

Although the martian northern plains are technically ridged plains since ridges have been identified in the lowlands with MOLA data, we restrict herein the appellation “ridged plains” to those plains where ridges were recognized in Viking images. We present in the next section topographic models of plains with wrinkle ridges derived from MOLA data [Smith et al., 2001], showing in particular the newly-identified wrinkle ridges in the northern lowlands [Withers and Neumann, 2001; Head et al., 2001] and evidence for consistent fault vergence at regional scale [Golombek et al., 2001]. Then, the localization instability analysis developed in Chapter 3 is applied to wrinkle ridge sets. In our preferred model, the faults underlying martian ridges penetrate to the brittle-ductile transition (BDT), which is 30 or 60 km deep, depending on the crustal thickness and the geotherm. Finally, we present finite element models that address the origin of the vergence selectivity observed in Solis Planum and Lunae Planum [Golombek et al., 2001]. The implications of our study for the sedimentation history in the lowlands and for the thermal evolution of Mars are subsequently discussed.
5.2 Wrinkle ridge sets

5.2.1 Morphology of wrinkle ridge

Wrinkle ridges are elongated and sinuous topographic highs identified at the surface of all the terrestrial planets and satellites [Watters, 1988, 1992, Fig. 5-1]. They result from horizontal shortening [Lucchita and Klockenbrink, 1981; Plescia and Golombek, 1986; Golombek et al., 1991]. Individual wrinkle ridges on Mars are composed of a broad swell, or ridge (up to 10 km wide, 300 m high, and hundreds of km long) with a summit crenulation, or wrinkle (~2 km wide, 100 m high) [Watters, 1988; Schultz, 2000; Golombek et al., 2001, Fig. 5-1, 5-2]. The ridges have an asymmetric profile that indicates that faulting was important in their formation [Golombek et al., 1991; Plescia, 1991; Golombek et al., 2001], although the relative roles of folding and faulting in forming the ridge have been much debated [Plescia and Golombek, 1986; Golombek et al., 1991; Watters and Robinson, 1997]. Recent numerical models showed that the ridge morphology is consistent with folding of a near-surface layered sequence forced by a deeper blind thrust [Niño et al., 1998; Schultz, 2000, Fig. 5-1]. Layering in martian examples comes from the volcanic sequence of the ridged plains. In the models of Niño et al. [1998] and Schultz [2000], the wrinkle appear as no more than a secondary structure related either to fault propagation or to a back thrust generated by interbed slip. The origin of the wrinkle may vary along the strike of the ridge, which is consistent with observations of wrinkle switching from one side of a ridge to another or broken en échelon [Schultz, 2000, Fig. 5-1, 5-2].

Ridges form sub-parallel sets [Maxwell, 1982; Watters and Maxwell, 1986, Fig. 5-1] with regionally-consistent inter-ridge spacing [Saunders et al., 1981; Watters, 1991]. At large scale, their pattern is concentric to global centers of tectonic and volcanic activity such as the Tharsis province on Mars [Banerdt et al., 1982, 1992; Maxwell, 1982; Chicarro et al., 1985; Watters and Maxwell, 1986] and Aphrodite Terra or Lada Terra on Venus [Banerdt et al., 1997; Bilotti and Suppe, 1999]. Local centers are also recognized on Venus [Tracadas and Zuber, 1998; Bilotti and Suppe, 1999]. Establishment of a Tharsis-centric pattern on Mars has been recently completed by the
Figure 5-1: Mars Observer Camera (MOC) a) narrow angle and b) wide angle images of a wrinkle ridge in Solis Planum (NASA/JPL/Malin Space Science Systems). c) Possible subsurface structure of a wrinkle ridge, after Schultz [2000] and Niño et al. [1998]. A blind thrust fault offsets vertically the basement underneath a layered plain unit of volcanic or sedimentary origin. A backthrust may be generated if interbed slip is possible in the layered unit. The crenulation, or wrinkle, on top of the ridge represents the shallower geometry of faults within the layered plains.
Figure 5-2: MOC a) narrow angle and b) wide angle images of a wrinkle ridge in Lunae Planum (NASA/JPL/Malin Space Science Systems). c) Digital Elevation Model of the area, from MOLA data gridded at 2' × 1' resolution [Smith et al., 2001].
recognition of numerous ridges in the northern lowlands using MOLA data [Withers and Neumann, 2001; Thomson and Head, 2001; Head et al., 2001].

5.2.2 Regional Topography

Two of the most intensely-studied ridged plain regions on Mars are Solis Planum and Lunae Planum (Fig. 5-3a, b). Both display sub-parallel ridge sets visible in Viking images [Watters and Maxwell, 1986]. They trend respectively N20°E and N-S. The ridge spacing is 40 to 50 km on average [Saunders et al., 1981; Watters, 1991; Head et al., 2001].

At regional scale, both Solis Planum and Lunae Planum are tilted down to the east. Using MOLA data, Golombek et al. [2001] showed that the plains have a staircase-like profile, being vertically offset across individual ridges and sub-horizontal to back-tilted between ridges (Fig. 5-4). This geometry indicates that the faults underlying individual ridges penetrate tens of km deep in the lithosphere and dip systematically to the same direction across the plains [Golombek et al., 2001]. A significant fraction of the current topography may reflect reverse motion on the stacked faults. Golombek et al. [2001] interpreted that topography as indicative of faults dipping toward the high-standing topography. On the other hand, Okubo and Schultz [2001], favored faults dipping in the opposite direction based on a model that assumes internal deformation of fault-bounded blocks.

In Lunae Planum, topographic data show a second set of ridges trending east-west, nearly perpendicular to the trend detected in Viking images (Fig. 5-5). These ridges are nearly invisible in the Viking and Mars Orbiter Camera (MOC) images because of the illumination direction and of the apparent lack of a summit crenulation (but see Fig. 5-5). As the tectonically-important feature of wrinkle ridges is the master fault, which is expressed by the ridge, and not the wrinkle [Schultz, 2000], these “wrinkle-less” ridges should be considered as a bona fide member of wrinkle ridges. The orientation of the second set of ridges in Lunae Planum is at odds with the Tharsis-dominated stress field that controls the nearly-perpendicular N-S ridges [Watters and Maxwell, 1986]. However, the E-W ridges are circumferential to a topographic high at the
Figure 5-3: Digital Elevation Models of a) Solis Planum, b) northern Lunae Planum, and c) Hesperia Planum. Terrain models from Mars Observer Laser Altimeter (MOLA) data gridded in 2' × 1' blocks [Smith et al., 2001].
Figure 5-4: Topographic profiles taken across wrinkle ridges in a) Solis Planum, b) Lunae Planum, c) Hesperia Planum, d) Alba Patera, e) Arcadia Planitia, f) Chryse Planitia. The location of the profiles is indicated in Fig. 5-3 and 5-6. Data from [Smith et al., 2001].
Figure 5-5: MOC a) wide angle images of a wrinkle ridges in Lunae Planum (NASA/JPL/Malin Space Science Systems). b) Digital Elevation Model of the area, from MOLA data gridded at 2' x 1' resolution [Smith et al., 2001]. The arrows indicate E-W oriented ridges.
southwest corner of Lunae Planum, near 5°N, 290°E; their orientation is controlled by a local stress field. The E-W ridges often end against a N-S ridge (Fig. 5-5)), indicating that they formed later. Hesperia Planum displays two perpendicular sets of ridges as well (Fig. 5-3c), but the ridges are so intertwined that they probably formed contemporaneously [Raitala, 1988; Mangold et al., 1999]. The spacing of each set of wrinkle ridges in Hesperia Planum is of order 40 km [Watters, 1991].

Although largely featureless in Viking data, the northern plains of Mars appear riddled with linear features in MOLA topography [Withers and Neumann, 2001]. Although the first of these new ridges were identified to the north of Alba Patera (Fig. 5-6a), where they were originally interpreted as ancient beach terraces [Head et al., 1999a], a more complete data set shows that these lineations are more consistent with degraded wrinkle ridges [Withers and Neumann, 2001; Thomson and Head, 2001; Head et al., 2001]. Fig. 5-6b shows another example of lowland ridges in Arcadia Planitia. Lowland ridges are sub-parallel and controlled both by global stresses such as the Tharsis province and local stress sources such as the Utopia basin [Head et al., 2001]. The lowland ridges parallel and complete previously-recognized ridge trends, implying a common origin and time of formation [Head et al., 2001].

Northern lowlands ridges have a smaller height than the ridges recognized in Viking images, which, combined with the identification of buried impact structures and the unique roughness properties of the northern lowlands [Aharonson et al., 1998; Kreslavsky and Head, 2000], led Head et al. [2001] to infer that ~100 m of sediments covers a Hesperian-age northern plain similar to Lunae Planum in age and morphology. Although controversial [Baker, 2001; Jakosky and Phillips, 2001], the northern ocean that might have been present in the Hesperian could have facilitated the sediment deposition [Head et al., 1999a]. MOC images also indicate that sediments cover the northern plains [Edgett and Malin, 2000]. These sediments are different from the older infilling of impact basins in the same region [Frey et al., 2001].

In addition to their smaller height, northern lowland ridges differ from the ridges in other ridged plains by a larger spacing, ~80 km, which may also indicate sedimentation of a pre-existing ridged plain [Head et al., 2001]. Interaction between
Figure 5-6: Digital Elevation Models of a) the northern flank of Alba Patera, b) south-east Arcadia Planitia, and c) Chryse Planitia. Terrain models from Mars Observer Laser Altimeter (MOLA) data gridded in $2' \times 1'$ blocks [Smith et al., 2001].
sedimentation and ridges is also evident in Chryse Planitia (Fig. 5-6c), where mass flows are guided by the ridges: the ridges are present in streamlined topographic highs and pre-existing plains kipukas, especially near the mouth of the channels. Debris flows were guided by the pre-existing ridges. However, we will argue that the sediment infilling has difficulties in explaining the difference in ridge spacing. Instead, we show that fault spacing may be controlled by a localization instability in a brittle layer of different thickness. The thickness of the brittle layer may reflect the different crustal thickness between the northern lowlands and the other ridged plains [Zuber et al., 2000, Fig. 5-7], is controlling fault spacing through a localization instability.
Figure 5-7: Map of crustal thickness derived from MGS data overlain on a shaded relief image of MOLA topography. The boxes mark the location of the Digital Elevation Models in Fig. 5-3 and 5-6. Contours every 10 km, thicker contours every 30 km. Hammer projection centered on −120°E. Redrawn from Zuber [2001].

5.3 Localization instability

5.3.1 The buckling and localization instabilities

A model lithosphere with vertically-stratified mechanical properties deforms by pure shear as long as the interfaces between different mechanical layers are perfectly flat and horizontal. However, any topography at these interfaces generates a secondary flow that in turn deforms the interfaces [Fletcher, 1974; Smith, 1977; Johnson and Fletcher, 1994]. The rate at which interface perturbations grow varies as a function of the perturbation wavelength. The growth rate may be maximum, or diverge at particular wavelengths, which may be expressed in the tectonic record [Biot, 1961; Johnson and Fletcher, 1994]. Lithospheric-scale instabilities are associated with these wavelengths.

Two simultaneous instabilities develop when the lithosphere is subjected to horizontal shortening. One of them, the buckling instability, results in broad undulations
of the lithosphere as a whole, with wavelength $\lambda_B$ [Fletcher, 1974; Ricard and Froideveau, 1986; Zuber, 1987a; Zuber and Aist, 1990]. The other, the localization instability, results in periodically-spaced localized zones of high deformation rate, that we interpret as faults [Chapter 3]. The wavelength of that instability, $\lambda_L$, gives the fault spacing. The buckling instability might also produce regularly-spaced faults if the undulations reach sufficient amplitude to produce significant stress heterogeneities before the fault pattern develops [Martinod and Davy, 1994, Chapter 4].

These two instabilities do not require exactly the same type of mechanical layering in the lithosphere. Buckling requires a strong layer overlying a weaker substrate [Fletcher, 1974; Smith, 1977]. It is favored if the strong layer is plastic. For tectonic applications, that layer may be identified with the brittle upper crust or upper mantle [Fletcher and Hallet, 1983; Zuber, 1987a]. In contrast, the localization instability requires a layer where faulting can occur, but it can be stronger or weaker than its substrate. In this study, the localizing layer is also identified with the brittle upper crust and mantle, but it is possible that it is only a part of it [Chapter 4].

Beyond its strength $\sigma$ or its apparent viscosity $\eta$, the rheology of a layer is characterized by the effective stress exponent $n_e$, a measure of the response of the overall strength of a material to a local perturbation of —for instance— the strain rate $\dot{\varepsilon}$ [Smith, 1977, Chapter 2]. The effective stress exponent is defined as:

$$\frac{1}{n_e} = \frac{\dot{\varepsilon} d\sigma}{\sigma d\dot{\varepsilon}}.$$  \hspace{1cm} (5.1)

If $n_e > 1$ (non-Newtonian behavior), increasing the strain rate decreases the apparent viscosity but not the strength of the material; the material is stable with respect to local perturbations. This is the case for rocks deforming in the dislocation creep regime. A material is termed plastic in the limit $n_e \to \infty$ [Chapple, 1978; Smith, 1979; Fletcher and Hallet, 1983; Zuber and Aist, 1990], or, more conveniently, $1/n_e \to 0$. When $n_e < 0$, the material weakens dynamically: a local increase of strain rates decreases not only the apparent viscosity but also the strength of the material. Hence, the material is unstable with respect to local perturbations of strain rate and generates
spontaneously localized shear zones that we identify with faults [Chapter 2].

While generated by local perturbation, these localized shear zones organize at the scale of the lithosphere into regularly-spaced fault sets as they interact with one another and with the non-localizing layers of the lithosphere [Chapter 3]. The lithospheric-scale fault pattern is controlled by the localization instability. The morphology of the ridges cannot be addressed with this analysis as the perturbations are infinitesimal, and near-surface interbed slip is not modeled.

5.3.2 Instability scaling

Both buckling and localization instabilities may result in regularly-spaced ridges [Chapter 4], but they differ in the relation between ridge spacing and brittle layer thickness that they predict [Chapter 3]. Physically, the wavelength of each instability is dictated by a resonance between certain deformation modes within the strong and/or localizing layer. The resonances involved in the buckling and localization instabilities are different in detail, and have different associated wavelengths, which can be determined analytically [Chapter 3]. Resonances occur at any integer multiple of a fundamental wavenumber. We define $j$ as that integer, and call it the order of the resonance and of the associated buckling or localization instability. The resonant wavelengths are the basis for a scaling relation between the instability wavelength, $\lambda_L$ and $\lambda_B$, the thickness of the strong and/or localizing layer $H$, and the effective stress exponent of that layer, $n_e$.

For the buckling instability, we obtain

$$\frac{\lambda_B}{H} = \frac{2}{1/2 - a_B} \times \left(1 - \frac{1}{n_e}\right)^{1/2}, \quad (5.2)$$

and for the localization instability, we have

$$\frac{\lambda_L}{H} = \frac{2}{a_L} \times \left(-\frac{1}{n_e}\right)^{1/2}, \quad (5.3)$$

where $a_B$ and $a_L$ are the spectral offsets, which depend on the type of strength profile.
assumed in the lithosphere. In Chapter 3, we have calibrated $a_B$ and $a_L$ for the case where the strength of in the layer increases with depth and the strength of the substrate decreases quasi-exponentially, as may be expected if there is only one brittle and one ductile layer in the lithosphere. We found

$$0 < a_B < 1/4,$$  \hspace{1cm} (5.4)

$$1/4 < a_L < 1/2.$$ \hspace{1cm} (5.5)

The density of the lithosphere has little effect on the localization instability, but reduces the maximum growth rate and the preferred wavelength of the buckling instability. The order $j = 0$ gives the longest preferred wavelength for each instability family, which we consider the most likely to be visible in nature. Higher order instabilities $j \geq 1$ may also be present, but are harder to differentiate from noise in the $j = 0$ instability. Hence, we consider only $j = 0$ in the application to large-scale tectonics. For equal brittle layer thickness, the wavelength of the localization instability is generally smaller than that of the buckling instability (Fig. 5-8).

The effective stress exponent that applies to the martian lithosphere cannot be specified \textit{a priori} because the exact localization mechanism is not known, and because extrapolating the values determined from laboratory studies requires reasonable caution. Based on rock mechanics experiments, the effective stress exponent may be between $-10$ to $-50$ for brittle failure mechanisms, and between $-300$ and $-50$ for localization over pre-existing faults [Chapter 2]. The same localization process is probably at work in the ridged plains and in the northern lowlands, as similar tectonic structures, the wrinkle ridges, are observed in these areas. Hence, we may assume that $n_e$, although unknown, is identical in both regions.

Table 5.1 shows what thickness of strong or localizing layer is needed to explain the ridge spacing in the ridged plains or in the northern lowlands according to each instability. The values $n_e = -50$ and $n_e = -15$ are chosen to illustrate the possible wavelengths of the localization instability. If the localization instability controls the ridge spacing, $n_e = -15$ and $a_B = 1/4$, the thickness of the layer undergoing localization
Figure 5-8: Wavelength of the buckling and localization instabilities as a function of the effective stress exponent. $H$ is the thickness of the material that develops the instability. Localization requires $n_e < 0$ and is more efficient for more negative $1/n_e$. The range of wavelengths for each instability correspond to the range of $a_B$ and $a_L$ in Eq. 5.2 and 5.3.
Table 5.1: Scaling depths

<table>
<thead>
<tr>
<th>Ridge spacing</th>
<th>Ridged plains</th>
<th>Northern lowlands</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 km</td>
<td>80 km</td>
<td></td>
</tr>
<tr>
<td>Bucking instability</td>
<td>5 to 10 km</td>
<td>10 to 20 km</td>
</tr>
<tr>
<td>Localization instability, $n_e=-50$</td>
<td>70 to 141 km</td>
<td>141 to 282 km</td>
</tr>
<tr>
<td>Localization instability, $n_e=-15$</td>
<td>38 to 77 km</td>
<td>77 to 144 km</td>
</tr>
</tbody>
</table>

5.3.3 Growth spectrum

Because the spectral offsets ($a_B$ and $a_L$) in Eq. 5.2 and 5.3 were determined empirically for simplified models of the lithosphere [Chapter 3], we need to verify the prediction of instability wavelengths for more realistic models of the martian lithosphere. This also results in a tighter range of $H$ compatible with a given effective stress exponent $n_e$. Hence, we compute the growth spectra using strength profiles derived from laboratory experiments on rock deformation. Following the method described in Chapter 4, the lithosphere is idealized as a sequence of horizontal layers (Fig. 5-9). The density and dominant rheology are specified for each layer. The dominant rheology is the weakest of the pressure-dependent brittle strength and temperature-dependent ductile creep law. The brittle strength follows Byerlee’s law of frictional sliding [Byerlee, 1978], which does not depend on rock type. In contrast, the ductile flow law does depend on the rock type. Meteorites as well as in situ analysis of martian rocks indicate that the crust is composed mostly of igneous rocks. Although andesite-like compositions have been recognized [McSween et al., 1999; Bandfield et al., 2000], most analyses indicate a basaltic composition [McSween, 1994; Zuber, 2001]. Hence, we use the dislocation creep laws of diabase [Caristan, 1982] for the crust. The upper mantle is probably olivine-rich [Longhi et al., 1992; Zuber, 2001] and may be dry if the crust is extracted from it by partial melting [Hirth and Kohlstedt, 1996]. Hence, we use the rheology of dry olivine [Karato et al., 1986] in the mantle. The ductile strength is a non-linear function of the strain rate, taken here as $10^{-16}$ s$^{-1}$. The thickness of the layers in the model used to compute the growth spectra Fig. 5-9 as well as the depth-dependent strength profile in the ductile regime are computed as for the
Figure 5-9: Schematic representation of lithosphere models. A sequence of layers undergoes shortening at the rate $\dot{\varepsilon}_{xx} = 10^{-16}\, s^{-1}$. The strength profile follows the weakest of frictional resistance or ductile flow law. Each layer $i$ is characterized by a density $\rho_i$, a thickness $H_i$, an effective stress exponent $n_i$, and a viscosity profile $\eta_i(z)$ that correspond to the dominant deformation mechanism in that depth range. A substratum with rheology corresponding to the lowest level of the model is included for convenience. The shading differentiates between localizing layers (lighter shade, $n_i < 0$) and ductile layers (darker shade, $n_i \geq 0$). The same shading is used in the strength profile in the next figures. The ductile crust and brittle layers are included only if the crustal thickness and geotherm allows it.
Figure 5-10: Depth of brittle-ductile transition for different materials as a function of the surface geotherm. Solid lines: diabase rheology [Caristan, 1982]; Dashed lines: dry olivine [Karato et al., 1986]. Thick lines: strain rate $10^{-15} \text{ s}^{-1}$; Thin lines: strain rate $10^{-18} \text{ s}^{-1}$. The brittle-ductile transition is defined as the point where the resistance to frictional sliding [Byerlee, 1978] equals the ductile strength. Horizontal shortening and a temperature profile in $erf(z)$ with surface temperature of $-20^\circ\text{C}$ and asymptotic temperature of $1350^\circ\text{C}$ are assumed.

prescribed the temperature profile [Chapter 4]. We assume that the temperature increases linearly with depth to a saturation temperature of $1350^\circ\text{C}$. The depth at which these flow laws are weaker than the brittle failure envelope is shown in Fig. 5-10. The effective stress exponent of the ductile layers is given by the flow law. In the brittle regime, we only constrain $n_e$ to be the same for the brittle crust and the brittle mantle. Localization requires $n_e < 0$. From the scaling analysis, $n_e \sim -15$ should explain the ridge spacing.

Stokes’ equation is solved in each layer and stress and velocity boundary conditions
matched to determine the growth rate of interface perturbations as a function of their wavelength, or growth spectrum [Chapter 3, Appendix]. The buckling instability appears as a finite maximum of growth rate, and results in smooth undulations of the brittle layers as a whole. The growth rate diverges at the localization instability, reflecting the unstable character of localization [Chapter 3].

The growth spectra of models with 30 and 60 km thick crust are presented in Fig. 5-11, along with the corresponding strength profiles. We used \( n_e = -16 \) and \( dT/dz = 12 \text{ K.km}^{-1} \). Under these conditions, the 30 km thick crust, corresponding to the northern lowlands, is brittle throughout. The localization instability is marked by divergent growth rate at a \( \sim 90 \) km wavelength, similar to the spacing of ridges in the lowlands. With a 60 km thick crust, as in Solis Planum, the localization instability has a wavelength of 40 km, similar to the ridge spacing in that region. The spectral offset is \( a_L = 1/4 \), consistent with the scaling derived in Chapter 3. In the thick crust case, the upper mantle is surrounded by weaker material. Hence the strong upper mantle can buckle, as indicated by the broad maximum of growth rate around 75 km wavelengths. However, this mode of deformation has very little expression at surface, and a moderate growth rate of order 10. This deformation mode has not been recognized in the tectonics of Solis Planum.

As localization becomes less efficient \( (1/n_e \to 0^-) \), more faults are needed to accommodate the deformation imposed over a given distance. Correspondingly, the wavelength of the localization instability decreases (Fig. 5-12). Constraining the ridge spacing to be 40 or 80 km, there is a tradeoff between \( n_e \) and the geotherm, which controls the depth to the BDT (Fig. 5-13). Indeed, a shallower BDT results in a longer \( \lambda/H \) to be modeled, which is achieved by increasing the efficiency of localization, or decreasing \( 1/n_e \). The 40 km ridge spacing in Solis Planum can be modeled by \( n_e = -16 \) if \( dT/dz = 15 \text{ K.km}^{-1} \), or \( n_e = -25 \) or \( dT/dz = 7 \text{ K.km}^{-1} \). For similar conditions, the ridge spacing in the northern lowlands is 80 to 90 km. In all these cases, localization is more efficient than expected for frictional velocity weakening, but is consistent with localization by other processes such as the loss of cohesion upon failure [Chapter 2].
Figure 5-11: a) Growth spectra of the lithosphere models for Solis Planum (thick line, 60 km thick crust) and the northern lowlands (thin line, 30 km thick crust). Strength profiles for the models of b) Solis Planum and c) the northern lowlands. The geotherm is 12 K.km$^{-1}$ and $n_e = -16$ in the brittle regime in both cases. The shaded regions in a) show the range of ridge spacing in Solis Planum (30 to 50 km), and the northern lowlands (80 to 100 km). The spikes in the growth spectra indicate the localization instability.
Figure 5-12: Map of growth rate as a function of effective stress exponent of localizing layer and perturbation wavelength for a lithosphere with a) a 60 km thick crust (as in Solis Planum, Fig. 5-11b) and b) a 30 km thick crust (as in the northern lowlands, Fig. 5-11c). High growth rate is in lighter tones. The localization instability is identified by the chains of very high growth rate. These graphs give the wavelength of the instability as function of the effective stress exponent.
Figure 5-13: Map of growth rate as a function of effective stress exponent of the localizing layers and geotherm for a) a perturbation wavelength of 40 km and a lithosphere with a 60 km thick crust (as in Solis Planum, Fig. 5-11b) and b) a perturbation wavelength of 80 km and a lithosphere with a 30 km thick crust (as in the northern lowlands, Fig. 5-11c). High growth rate is in lighter tones. The localization instability is identified by the chains of very high growth rate. These graphs give the geotherm needed to match the ridge spacing as a function of the effective stress exponent. The shaded parts of the graph indicate the range geotherms for which the ridge spacing in both the highlands and the lowlands cannot be matched with a similar $n_e$. 
5.3.4 Application to wrinkle ridge spacing

If buckling controlled ridge spacing, the faults underlying martian ridges would not necessarily penetrate deeply into the lithosphere. They would be limited by the thickness of the strong buckling layer, which would be at most 10 km thick in the highlands ridged plains and a factor of two thicker in the lowlands (Table 5.1). In both locations, this is less than the crustal thickness determined from MGS data [Zuber et al., 2000, Fig. 5-7]. First, we will assume that the thickness of the strong layer is limited by the brittle-ductile transition (BDT) of crustal rocks. The BDT is a function of the rock type and the geotherm (Fig. 5-10). Then, we will explore the possibility that the strong layer reflects intra-crustal stratification. Finally, as neither of these possibilities is likely to explain the systematic ridge spacing difference between ridged plains and northern lowlands, we explore the possibility that the ridge spacing is controlled by the localization instability. Then, the different crustal thickness between the highlands and the lowlands provides an explanation for the different ridge spacing.

Brittle-ductile transition controlled by different geotherms

If the ridge spacing is controlled by the buckling instability and the crust has similar composition in the highland and the lowlands, the ridge spacing implies that the geotherm in the lowlands was at least a factor of two lower than in the ridged plains at time of ridge formation. However, this situation is unlikely. The ridges formed roughly at the same times in both areas [Head et al., 2001], and the northern lowlands lithosphere is not older than that of the martian highlands, precluding significant differences in the mantle heat flow or in the heat loss by cooling of a immobile lithospheric plate between the two regions. The lowlands’ lithosphere might have lost heat more rapidly than the highlands during an early episode of plate tectonics in the lowlands [Sleep, 1994]. However, it is not known how strong the heat flux reduction would be, and for how long it would be preserved with a dynamic mantle, and the recognition of buried impacts basins [Smith et al., 1999; Frey et al., 2001;
McGill, 2001; Thomson and Head, 2001] makes it unlikely for plate tectonics to have occurred in the lowlands after the end of heavy bombardment. Degree-1 convection that caused excess heat loss and resurfacing in the lowlands [Zhong and Zuber, 2001] would cause lowland ridges to have smaller spacing than in the highlands, which is contrary to the observations.

It is possible that the lithospheres of Solis and Lunae Plana were reheated by mantle plumes and associated volcanism, but the temperature anomaly associated with thermal plumes on Earth is at most 200 K [Shen et al., 1998; Korenaga and Kelemen, 2000], or 20% of the temperature drop across the lithosphere. If a similar temperature change also happened on Mars, it would not be sufficient to explain the difference of depth of BDT. Radiogenic heating in the thicker highland crust, also cannot account for more than 2.1 K.km\(^{-1}\), assuming a concentration of heat-producing elements in the martian lower crust similar to that of tholeiitic basalts on Earth 3 Gyr ago [Turcotte and Schubert, 1982]. In summary, there is little justification to expect a significant difference between the lowland and highland heat fluxes during ridge formation.

**Brittle-ductile transition controlled by different crustal composition**

If instead of the geotherm, the composition of the crust was different in the lowlands than in the highlands, then the BDT, and by inference, the ridge spacing could be different between these two regions. Both data from Mars Pathfinder and Mars Global Surveyor Thermal Emission Spectrometer (MGS-TES) indicate the presence of silica-rich rocks such as icelandite or andesite [McSween et al., 1999; Bandfield et al., 2000]. Bandfield et al. [2000] showed the andesite-like component to be predominant in the northern lowlands. Although the spectral content is non-unique and sediment makes the link between surface and crustal composition ambiguous, especially in the lowlands, it is conceivable that the martian crust is more silica-rich in the lowlands than the highlands. However, silica-rich rocks such as granite are generally weaker than basaltic rocks such as gabbros [Tsenn and Carter, 1987; Evans and Kohlstedt, 1995]. Hence, if that compositional difference is real, the BDT of the lowlands would
be shallower than in the highlands, for identical geotherms, which would result in more closely-spaced ridges in the lowlands, contrary to the observations [Head et al., 2001]. Unless the crust of either the lowlands or the highlands is dominated by an unusual rock type, or the geotherm of the lowlands is much colder than that of the highland, for an unexplained reason, the BDT of the lowlands is unlikely to be a factor of two deeper than in the highlands. Hence, the ridge spacing, if controlled by the buckling instability, does not reflect the BDT of crustal materials.

Buckling controlled by intra-crustal stratification

The buckling layer may be limited by a change of rock type with depth within the crust rather than the BDT. Early models of ridge spacing assumed that the strong layer corresponds to basaltic flood plain units that are underlain by a weaker substrate usually identified with a lunar-type megaregolith [Saunders et al., 1981; Zuber and Aist, 1990; Watters, 1991]. Recent morphological studies of canyon walls showed that the plain thickness is at least 8 km [McEwen et al., 1999; Caruso and Schultz, 2001], which invalidates previous models using viscoelastic buckling requiring plain thicknesses of order 1 km [Saunders et al., 1981; Watters, 1991]. Models using a plastic layer [Zuber and Aist, 1990] are consistent with such a plain thickness. However, they also imply that the Hesperian plain unit would be between 20 and 40 km thick in the lowlands to explain the longer ridge spacing (Table 5.1). The plains would then constitute the major part of the 20 to 30 km thick crust of the lowlands [Zuber et al., 2000, Fig. 5-7], which is inconsistent both with the identification of basins underneath the volcanic plain unit in the lowlands [Frey et al., 2001] and with the presence of a significant megaregolith underneath the volcanic plains, as needed for buckling. A weak substrate would also decouple efficiently the surface from deeper stresses [Zuber, 1995], which is hard to reconcile with observations of consistent ridge orientation over thousands of km.

In summary, it is unlikely that the buckling instability controls the ridge spacing, regardless of whether the buckling layer is limited by the brittle-ductile transition of the crust, or an intra-crustal weak layer. If instead of the buckling instability, we
follow the hypothesis that ridge spacing is controlled by the localization instability, the thickness of the faulting layer is at least 40 km, depending on the value of effective stress exponent (Table 5.1). This value is consistent with the flexural rigidity derived from flexural modeling of Solis Planum [Zuber et al., 2000].

**Ridge spacing controlled by the localization instability**

If the ridge spacing is controlled by the localization instability, $H$ corresponds to the depth of faulting, which is indicated by the BDT of the lithosphere. As we discussed above, thermal effects alone are not sufficient to deepen the BDT by a factor of two in the lowlands, unless the crust has unusual compositions or these regions have undergone very different thermal evolutions. However, it is possible that the BDT is controlled by the mantle rheology in the lowlands and the crustal rheology in the highland ridged plains, because of the difference of crustal thickness [Zuber et al., 2000, Fig. 5-7].

As argued in §5.3.3, the rheology of dry olivine [Karato et al., 1986] provides a reasonable estimate of the strength of the mantle and the strength of the crust may be approximated by the strength of diabase [Caristan, 1982]. With a similar geotherm, the diabase and dry olivine rheologies predict a factor of two deeper BDT in the mantle than in the crust for a given geotherm (Fig. 5-10). Hence, a thin crust, as in the lowlands, may be completely brittle, whereas a thicker crust, as in the highlands, may be ductile at depth. The same factor of two increase in the depth of the BDT can be obtained if the crust in the highland is dominated by quartzite or granite, and the BDT of the lowlands is controlled by either dry diabase or wet olivine. However, these combinations of rock type are deemed less likely than the wet diabase/dry olivine pair assumed above because silica-rich rocks are found by remote sensing in the lowlands, not the highlands [Bandfield et al., 2000].

The geotherm is not necessarily the same in both regions: in the lowlands, the geotherm should be less than $\sim 15$ K.km$^{-1}$ for the 20 to 30 km thick lowland crust to be completely brittle, whereas in the highlands, the geotherm must exceed 7 K.km$^{-1}$ for the 60 km thick crust of the Solis Planum to be ductile at its deepest levels.
A higher geotherm in the highlands is needed if the 50 km thick crust in Lunae or Hesperia Plana is to be ductile as well. If the highlands’ geotherm is identical that of the lowlands and is $11 \pm 4 \text{ K.km}^{-1}$, the factor of two difference in ridge spacing between these regions is explained. Differences in geotherm between these areas would modulate this relation, but as argued above, they probably do not exceed 3 K.km$^{-1}$. The common geotherm implied by our analysis would change if rock types different from diabase and dry olivine controlled the BDT in the martian highlands and lowlands. For instance, if the rheology of the crust was controlled by quartzite [Gleason and Tullis, 1995], the implied geotherm would be less than 9 K.km$^{-1}$ in the lowlands and more than 4 K.km$^{-1}$ in the highlands. There is however no evidence for this rock type at the surface of Mars.

5.4 Finite element modeling of regionally-consistent fault vergence

Reliable topographic data from MOLA [Smith et al., 2001] demonstrated that plains are vertically offset across ridges, indicating that individual ridges are underlain by deeply rooted faults [Golombek et al., 2001, Fig. 5-4]. The sense of the topographic offset indicates the dip direction, or vergence, of the fault. In Solis and Lunae Plana, the faults dip consistently toward the high standing area. The instability analysis presented above can match the predicted fault spacing, but cannot explain regionally-consistent fault vergence. Indeed, if the region deforms by pure shear, conjugate faults are expected, with an approximately equal number of faults dipping toward of away from the high-standing areas. The symmetry of faulting can be broken if deformation has a component of simple shear in addition to the pure shear that represents horizontal shortening. We investigate using finite element models how such a component may arise from lateral variations in the mechanical properties of the lithosphere. Specifically, we model the effects of laterally-varying crustal thickness and geotherm.
5.4.1 Numerical Technique

The lithosphere is modeled as a visco-plastic material using the finite element method. [Chen and Morgan, 1990; Neumann and Zuber, 1995; Zuber and Parmentier, 1996; Behn et al., 2001]. The lithosphere is divided into contiguous elements that are initially rectangles. There are four nodes for each element. Velocity and forces are defined at each node of each element, and interpolated within the element. The physical relation between forces and velocities (or strain rate) is integrated over individual elements, so that each element is represented by an effective stiffness (or rather viscosity) matrix linking the velocity and the forces at each of its nodes. Then, displacement boundary conditions are imposed on selected nodes, so that, as the velocity in the model must be continuous and the forces at each node must be in equilibrium, it is possible to solve for the velocity and forces at every node [Bathe, 1996]. Our formulation is Lagrangian: computational nodes are advected with the velocity field. Concerns of finite distortion of the elements, time discretization, and regridding of the model are irrelevant for this work as we consider only the initial configuration of faulting.

In the results presented herein, the computational domain is 600 km × 120 km, discretized in 300 × 40 rectangular elements. Each element is 2 km wide in the horizontal direction. From top to bottom, the mesh is composed of 30 rows of 2 km high elements, 5 rows of 4 km high elements, and 5 rows of 8 km high elements. This stratification does not influence significantly the numerical results for the wavelengths of interest, but improves significantly the computation time. The top surface of the models is stress-free and the other boundaries have free-slip conditions. The vertical velocity is 0 at the bottom of the model and the horizontal velocity is 0 on the left wall and \( U = -60 \) km.Ma\(^{-1}\) on the right wall to provide the horizontal shortening rate of \( 10^{-7} \) yr\(^{-1}\) \( \sim 3.16 \times 10^{-15} \) s\(^{-1}\).

The temperature, \( T \), the pressure, \( P \), and the strain rate, \( \dot{\varepsilon} \) of each element are used to compute the stress needed for ductile creep, \( \sigma_d \), and its brittle strength, \( \sigma_b \). The weakest of \( \sigma_d \) and \( \sigma_b \) is retained to compute the apparent viscosity of the element,
\[ \eta = \min \{ \sigma_d, \sigma_d \}/2 \dot{\varepsilon}, \] which is used to compute the stress and velocity fields in the model. The strain rate is initially uniform throughout the model, but is then recomputed from the velocity solution. The procedure is repeated without deforming the model until self-consistent viscosity and strain rate fields are attained, producing the initial fault pattern that we discuss here. Convergence requires usually less than 200 iterations. In order to visualize the deformation field, we deform the grid producing the equivalent of 3\% horizontal shortening in one time increment.

The ductile flow law is
\[ \sigma_d = B \dot{\varepsilon}^n \exp T/T_R, \] (5.6)
where \( B, n, \) and \( T_R \) are constants determined from laboratory experiments and \( T \) the absolute temperature of an element. As before, we use the flow laws of dry olivine \([Karato et al., 1986]\) in the mantle and diabase \([Caristan, 1982]\) in the crust.

In the brittle regime, the rock strength follows Byerlee's law of frictional sliding \([Byerlee, 1978]\). Following \textit{Neumann and Zuber} [1995], the brittle rock strength decreases with the logarithm of strain rate to simulate the dynamic weakening required for localization:
\[ \sigma_b = (S + fP) \left( 1 - C \ln \dot{\varepsilon}/\dot{\varepsilon}_0 \right), \] (5.7)
where \( S \) and \( f \) are experimentally-determined constants, \( \dot{\varepsilon}_0 \) is a reference strain rate and \( C \) is a constant that controls the efficiency of localization. The dependence on strain rate mimics the velocity dependence of steady-state friction determined from laboratory experiments \([Dieterich, 1979; Ruina, 1983]\). The effective stress exponent in that regime is:
\[ \frac{1}{n_e} = -\frac{C}{1 - C \ln \dot{\varepsilon}/\dot{\varepsilon}_0} \sim -C \] (5.8)
For numerical reasons, localization requires \( C > 0.1 \), whereas we expect to match the ridge spacing for \( C < 0.06 \). Hence, we do not address the question of ridge spacing with the finite element model but only that of the selectivity of fault vergence.
Figure 5-14: Initial configuration of finite element models. Thick line shows the base of the crust, thin lines isotherm every 100°C. The shaded region shows element undergoing failure if the model deforms under uniform strain rate. However, the failure zone changes and forms discrete shear zones after convergence of the model (Fig. 5-16 to 5-18). a) no lateral gradients of crustal thickness and geotherm; b) lateral gradient of crustal thickness (1 km per 100 km); c) lateral gradient of geotherm (0.2 K.km⁻¹ per 100 km).

5.4.2 Results

In the first model, there is no lateral variation of crustal thickness or geotherm (Fig. 5-14a). The crustal thickness is 60 km, as in Solis Planum. The temperature is 253 K at the surface, and increases with depth as erf(z) with a surface gradient of 10 K.km⁻¹ and an asymptotic temperature of 1600 K. The crust is brittle to 40 km, and the mantle is brittle from 60 to 65 km (Fig. 5-14a and 5-15a).

Brittle failure, which is initially uniform (Fig. 5-14a), collapses onto discrete bands.
Figure 5-15: Stress supported in the left side (solid line) and right side (dotted line) of the finite element models before localization of the failure zones. The discretization of the profiles comes from the decomposition of the model into finite elements. The strength is given at the depth to the center of each element. a) no lateral gradients of crustal thickness and geotherm; b) lateral gradient of crustal thickness (1 km per 100 km); c) lateral gradient of geotherm (0.2 K.km$^{-1}$ per 100 km).

of enhanced strain rate and reduced stress as the model converges to a self-consistent solution (Fig. 5-16 a, b, c). The strain rate in the shear zones is about a factor of 10 higher than the strain rate of $10^{-7}$ yr$^{-1}$ imposed on the model as a whole (Fig. 5-16d). Between the bands, the strain rate is reduced to the point that the material is not undergoing failure (Fig. 5-16c). The BDT deepens by a few kilometers beneath the shear zones due to the enhanced strain rate. Even deeper, in the ductile regime, the shear zones become diffuse. The power spectrum of the surface strain rate shows a pronounced maximum a wavelength of $\sim$60 km (Fig. 5-16e), corresponding to the spacing of localized shear zones, and a secondary maximum near 200 km. That secondary maximum may correspond to the buckling instability but is not well resolved in this model. As the model is laterally-invariant, the pattern of localized shear zones is symmetric, with right-dipping and left-dipping faults of similar intensity (Fig. 5-16d).
Figure 5-16: a) strain rate field, b) stress field, c) failure zone (grey elements), d) surface strain rate, and e) power spectrum of the surface strain rate of the model without crustal thickness or geotherm gradients (Fig. 5-14a) after convergence and localization of the solution. In a), b), and c), a horizontal shortening of 3% was imposed in one time step after deformation field has converged to visualize the deformation field. The vertical arrows in d) indicate left-dipping faults. Strain rate is scaled by the overall strain rate of $10^7$ yr$^{-1}$. 
In the second model, the crustal thickness varies from 60 to 54 km over the 600 km of the model (Fig. 5-14b). As the weak lower crust is replaced by strong upper mantle, the strength of the lithosphere increases as the crustal thickness decreases (Fig. 5-15b). The resulting fault pattern structure is asymmetric (Fig. 5-17), with the strain rate being higher on faults dipping toward the thick crust region (Fig. 5-17d). The spectrum of the surface strain rate (Fig. 5-17e) shows a peak at the spacing of shear zones, around 70 km, and large amplitude at long wavelength that reflects a developing regional slope (Fig. 5-17). The regional slope results from the lithosphere being weaker, and therefore deforming faster, where the crust is thick. The preferred faults dip toward the growing high-standing area.

In the third model, the crustal thickness is 60 km throughout the model, but the surface geotherm decreases from 10 to 8.8 K.km\(^{-1}\) over the 600 km-wide model (Fig. 5-14c). The deeper BDT where the geotherm is high results in stronger lithosphere (Fig. 5-15c). As in the second model, the fault pattern is asymmetric, with the faults that dip toward the weak area being favored (Fig. 5-18). The spectrum of the surface strain rate (Fig. 5-18e) peaks at \(\sim 70\) km wavelength. Its high amplitude at long wavelengths reflects the developing slope that, as in the second model, results from the lithosphere being weaker on one side of the model. The preferred faults dip toward the growing high-standing area.

5.4.3 Applications

Gradients in crustal thickness and surface geotherm are both successful at producing an asymmetric pattern of faulting. The favored faults dip toward the weaker part of the lithosphere, being either of thicker crust or of higher geotherm. Shortening and thickening are concentrated in the weakest part of the model, resulting in progressive tilting of the surface, which may be part of the slope observed in Lunae and Solis Plana. This would be consistent with the observation that faults dip preferentially toward the high-standing areas [Golombek et al., 2001].

In Solis and Lunae Plana, lateral variations of geotherm similar to that considered in the third model may be linked to the Tharsis rise, if the volcano-tectonic province
Figure 5-17: a) strain rate field, b) stress field, c) failure zone (grey elements), d) surface strain rate, and e) power spectrum of the surface strain rate of the model with a crustal thickness gradient of 1 km per 100 km (Fig. 5-14b) after convergence and localization of the solution. In a), b), and c), a horizontal shortening of 3% was imposed in one time step after deformation field has converged to visualize the deformation field. The vertical arrows in d) indicate left-dipping faults. Strain rate is scaled by the overall strain rate of $10^7$ yr$^{-1}$. 

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Figure 5-18: a) strain rate field, b) stress field, c) failure zone (grey elements), d) surface strain rate, and e) power spectrum of the surface strain rate of the model with a geotherm gradient of 0.2 K.km$^{-1}$ per 100 km (Fig. 5-14a) after convergence and localization of the solution. In a), b), and c), a horizontal shortening of 3% was imposed in one time step after deformation field has converged to visualize the deformation field. The vertical arrows in d) indicate left-dipping faults. Strain rate is scaled by the overall strain rate of $10^7$ yr$^{-1}$. 
is due to mantle plumes. The crustal thickness varies approximately perpendicular
to the strike of ridges by the amount used in the second model [Zuber et al., 2000,
Fig. 5-7]. There are also gradients of crustal thickness and possibly geotherm along
the strikes of the ridges, but to evaluate their effect on the fault pattern requires
three-dimensional models that are not yet available.

Our models do not eliminate completely the faults conjugate to the dominant
vergence. This is probably an effect of the model formulation, which includes only
strain-rate weakening of the apparent viscosity of the lithosphere. Using either strain
weakening, or isotropic weakening may force the least developed shear zones to be
abandoned. Within our models, the asymmetry of faulting is enhanced if the crustal
thickness or geotherm gradients are higher than shown here. However, this comes
at the price of concentrating deformation to a region just a few tens of km wide
where the lithosphere is weakest. Awaiting improvements of the modeling technique,
we conclude that vergence selectivity is possible for realistic lateral gradients of the
crustal thickness or of the geotherm, although it remains to be determined whether
this consistent vergence selectivity can occur over an entire ridged plain.

5.5 Discussion

5.5.1 Fault geometry underneath wrinkle ridges

The depth to which the faults underlying wrinkle ridges penetrate has been a major
question in martian tectonics [Plescia and Golombek, 1986; Watters, 1991; Golombek
et al., 1991; Watters and Robinson, 1997]. Deep faulting models were recently pro-
moted by the recognition of elevation offsets in MOLA data [Golombek et al., 2001]
and models of ridge morphology as folding driven by a deeper blind fault [Schultz,
2000]. Our models of ridge spacing, which make no a priori assumption about the
depth of faulting, favor deep faulting as well. The comparison of ridge spacing in
two different provinces leads us to conclude that faulting penetrates to the shallowest
brittle-ductile transition (BDT) of the martian lithosphere. In Solis and Lunae Plana,
where the crust is 50 to 60 km thick, the BDT is determined from the crustal rheology
and is 30 to 50 km deep. In the northern lowlands, where the crust is no more than
30 km thick, the BDT is controlled by the mantle rheology and is 60 to 100 km deep.
The exact penetration depth depends on the geotherm, which should be less than
15 K.km\(^{-1}\) in the lowlands and more than 7 K.km\(^{-1}\) in the uplands to explain the
systematic difference of spacing of ridges in the northern lowlands compared to Solis
and Lunae Plana if the effective stress exponent is identical in both regions.

The effective stress exponents that can explain the ridge spacing are between -15
to -25. These values may indicate that localization is dictated by a processus active
during initial failure of rocks, such as cohesion loss upon failure, or non-associated
elastic-plastic flow [Chapter 2]. It is therefore unlikely that the fault underlying the
ridges existed before the horizontal shortening of the lithosphere that is expressed
by the ridges. This is in contrast with another area where a localization instability
appears to control fault spacing, the Central Indian Ocean [Chapter 4], or with the
terrestrial examples of basement uplift [Rodgers, 1987], which have been proposed
as terrestrial analogue of deeply-rooted wrinkle ridges [Plescia and Golombek, 1986;
Golombek et al., 2001]. Reactivation explains why the planform and orientation of
terrestrial basement uplifts is more irregular than that of martian wrinkle ridges.

Impact craters influence wrinkle ridges on Mars. Some ridges have a curvilinear
planform that probably reflects impact craters buried within or underneath the vol-
canic plain unit [Plescia, 1991; Raitala and Kauhanen, 1992] and wrinkle ridges are
statistically closer to impact craters than if random [Allemand and Thomas, 1995].
Although the influence of craters has been used to argue that ridges are only shallowly
rooted [Allemand and Thomas, 1992, 1995], we note that a deeply-rooted fault would
be likely to use the pre-existing heterogeneity constituted by an impact structure as
it propagates near the surface. The influence of craters on ridge trajectory does not
exclude the ridges being deeply rooted.
5.5.2 Sediment and ridge interaction in the northern plains

The morphology of the martian northern plains and outflow channels at their margin has been reported to support the idea that an ocean may have been present in the lowlands during the Hesperian, contemporaneous with ridge formation [Lucchita et al., 1986; Jöns, 1990; Baker et al., 1991; Scott et al., 1992; Parker et al., 1989, 1993; Moore et al., 1995; Head et al., 1999a; Parker and Currey, 2001]. However, these observations have alternative interpretations and the proposed ocean is still controversial [Malin and Edgett, 1999; Edgett and Malin, 2000; Baker, 2001; Jakosky and Phillips, 2001; McGill, 2001; Withers and Neumann, 2001]. The presence of a Hesperian ocean may be further tested by studying the effects of sedimentation on the northern plains topography. The characteristic roughness of the Vastitas Borealis formation [Kreslavsky and Head, 2000] is consistent with ~100 m of sediments covering a unit resembling ridged plains such Lunae Planum [Head et al., 2001]. However, these sediments are not necessarily marine; Edgett and Malin [2000] report on possible sediments in the northern plains using MOC images, but found no obvious submarine landform. The topographic roughness of Amazonis Planitia, also in the lowlands but at the limit of the proposed ocean, is compatible with extensive aeolian as well as submarine deposition [Aharonson et al., 1998].

The numerous ridges now recognized in the northern lowlands [Withers and Neumann, 2001; Thomson and Head, 2001; Head et al., 2001] lead to new ways to evaluate sedimentation in that region. Head et al. [2001] proposed that the spacing of ridges in the lowlands is a factor of two larger than in the ridged plains because of a sedimentary cover. If the sediments bury completely the smallest ridges, the average spacing of the exposed ridges increases [Head et al., 2001]. However, the ~100 m of sediments required to simulate the roughness characteristics of the Vastitas Borealis formation exceeds the average and median ridge height in non-sedimented plains such as Lunae Planum (Resp. 65 and 47 m [Head et al., 2001]). Most ridges should vanish under such a sediment cover. To increase the ridge spacing by a factor of two, half the ridges must be entirely buried. The level of flooding would then correspond to the median
of the distribution of ridge height in ridged plains, 47 m. Flooding would reduce
the maximum-likelihood value of the ridge height distribution by the thickness of the
sediments. However, the maximum-likelihood value for Lunae Planum ridge heights
is also \( \sim 50 \) km [Head et al., 2001]. Thus, the height distribution of the exposed
ridges in the lowlands should decay monotonically or have a small maximum likeli-
hood value, if a sediment flooding model was correct. In contrast, the histograms of
exposed ridge height in the lowlands are peaked around 45 to 50 m [Head et al., 2001],
similar to the value in the non-sedimented ridged plain. We conclude that burial of
ridges in the lowlands was not sufficient to explain the factor of two difference in ridge
spacing.

Our analysis shows that the deeper BDT in the lowland than the highlands, due
to the thinner crust, can explain the larger ridge spacing in this region. However, the
uncertainty in the rheology applicable to the martian lithosphere and the heat flux at
the time of ridge formation make it impossible to ascertain that the difference in BDT
is solely responsible for the large ridge spacing in the lowlands. The morphology of
these ridges indicates that they have been degraded and sedimented [Thomson and
Head, 2001; Head et al., 2001]. Sedimentation effects can certainly modulate the
observed ridge spacing, although we doubt that sedimentation alone can explain the
totality of the difference. Thomson and Head [2001] also showed that lowland ridges
are longer than highland ridges, which indicates, if the scaling relations between fault
length, displacement, and depth are verified at the scale of interest [Bonnet et al.,
2001], that the faults underlying lowland ridges penetrate more deeply than those
under highland ridges. This is consistent with our explanation of the ridge spacing
using a deeper BDT.

We noted that if the northern lowlands are covered by \( \sim 100 \) m, the majority
of the pre-existing ridges may be completely buried. Instead, a large fraction of the
population is visible, the sediments may be draped over the pre-existing ridge-scale
topography rather than flooding it. Draped sediments could preserve the ridge to-
pography while smoothing the smallest length scales. Summit crenulations may be
erased in that process, explaining the difficulty in identifying the ridges in visible
images. Crater remnants in the northern plain identified in topographic data [stealth craters, Head et al., 2001] are also more consistent with draping rather than flooding of the topography as they are filled to a level that is systematically below the surrounding plains. The sedimentation in the Earth’s ocean can be modeled as a layer of sediments over a pre-existing topography and subsequent downslope-diffusion [Webb and Jordan, 1993; Webb, 1997]. Inversion of seafloor bathymetry indicates that the diffusivity of the sediments is \(~0.1\text{m}^{-2}\cdot\text{yr}^{-1}\), which is fairly high; most sediments accumulate into local depressions. If the sediments of the martian lowlands plains are indeed draped on the topography, sediment transport must have been less efficient than on Earth. The effective diffusivity may be at least an order of magnitude smaller than on Earth. As scaling by the gravity, which is a factor of three smaller on Mars than on the Earth, may not be enough to account for this difference, this might reflect either that the putative ocean was short-lived, which reduces the time during which sediment transport occurs, or that the transporting medium was more viscous than water, as is mud. Both a mud ocean [Jöns, 1990; Tanaka et al., 2001] and an episodic short-lived ocean [Baker et al., 1991; Baker, 2001] have been proposed on the basis of surface morphology, but are subjective in their interpretation.

5.5.3 Surface geotherm and heat budget of Mars

The thermal evolution of Mars has important implications for the history of volcanism, volatile abundance, and climate [Weizman et al., 2001; Jakosky and Phillips, 2001]. The results of thermal models, however, are variable [Schubert et al., 1992]. Some predict intensive melting in present-day Mars, while others, appealing to an early episode of intense cooling [Nimmo and Stevenson, 2000; Zhong and Zuber, 2001] or differentiation of heat-producing elements [Kiefer, 2001; Weizman et al., 2001], allow the volcanic activity to be diminish over time. This is supported by geological observations [Greeley and Spudis, 1981].

Constraints on the thermal history of a planet come from estimates of the heat flux. Our modeling of ridge spacing implies a geotherm \(dT/dz = 12\pm3\text{K.km}^{-1}\) in the ridged plains and in the northern lowlands. We cannot resolve differences between the
Table 5.2: Heat flux

<table>
<thead>
<tr>
<th>Region</th>
<th>Epoch</th>
<th>Heat flux (mW.m⁻²)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cratered uplands</td>
<td>Noachian</td>
<td>38 to 57</td>
<td>[McGovern et al., 2001]</td>
</tr>
<tr>
<td>Highland Plana</td>
<td>Hesperian</td>
<td>18 to 26</td>
<td></td>
</tr>
<tr>
<td>Valles Marineris</td>
<td>Hesperian - Early Amazonian</td>
<td>13 to 26</td>
<td></td>
</tr>
<tr>
<td>Domal Rises</td>
<td>Hesperian - Early Amazonian</td>
<td>15 to 26</td>
<td></td>
</tr>
<tr>
<td>Tharsis Montes</td>
<td>Amazonian</td>
<td>13 to 22</td>
<td></td>
</tr>
<tr>
<td>Ridged plains,</td>
<td>Hesperian</td>
<td>27 to 47</td>
<td>This study</td>
</tr>
<tr>
<td>Northern lowlands</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

two regions. Hence, the corresponding heat flux, \( q_s \approx 37 \pm 10 \text{ mW.m}^{-2} \) may be taken as representative of Mars during the time of ridge formation, the Hesperian (3.7 to 3.0 Myr ago, [Hartmann and Neukum, 2001]). Heat flux can also be determined from modeling of elastic flexure, constrained by the topography and the gravity field of the planet [Turcotte and Schubert, 1982; Comer et al., 1985; Wieczorek and Phillips, 1998].

The results of flexural analyses of Mars Global Surveyor data [Zuber et al., 2000; McGovern et al., 2001] are reproduced in Table 5.2. The heat flux in the Hesperian-age Solis Planum is constrained to be between 18 and 26 mW.m⁻² [McGovern et al., 2001], slightly lower than our estimates for the same region. However, the flexural analysis indicates significant sub-surface loading [McGovern et al., 2001], the timing of which is not constrained. As our estimate of the Hesperian heat flux fits well into the time evolution of heat flux determined by elastic flexure analysis (Fig. 5-19), we consider that the flexural and ridge spacing analyses are broadly consistent.

The gradual decrease of heat flux with time, apparent in Fig. 5-19, has been attributed to cooling of the lithosphere [Zuber et al., 2000], as for the heat flux of the Earth’s ocean floor [Sclater et al., 1980]. If the lithosphere, initially at a temperature \( T_m \), is cooled such that its surface temperature is fixed at \( T_s \), its heat flux is \( q_s \) after a time

\[
t = \frac{k^2 (T_m - T_s)^2}{\pi \kappa q_s^2}
\]

with \( \kappa = 8 \times 10^{-7} \text{ m}^2 \text{s}^{-1} \) the thermal diffusivity, and \( k = 3.1 \text{ W.m}^{-1} \text{K}^{-1} \) the thermal
Figure 5-19: Estimates of martian surface heat flux. Shaded box: our analysis of ridge spacing; Open boxes: flexure analyses [McGovern et al., 2001]. Dashed boxes indicate analyses of Hesperian plains. Lines: heat flux needed to balance radiogenic heat production using the chemical models of Treiman et al. [1986] (solid line) and Laul et al. [1986] (dashes); prediction from lithosphere cooling (dash-dots). The bars above the graph indicate the time span of martian epochs, from Hartmann and Neukum [2001]
conductivity [Turcotte and Schubert, 1982]. If we take $T_m - T_s = 1300 \pm 200$ K, the heat flux $q_s = 37 \pm 10$ mW.m$^{-2}$, compatible with our observations of wrinkle ridges, is attained after only 90 to 300 Myr. Numerous impact basins, even in the northern lowlands [Frey et al., 2001], as well as Rb-Sr and Lu-Hf isotope systematics in martian meteorites [Borg et al., 1997; Blichert-Toft et al., 1999] attest that the lithosphere formed in the earliest martian history. The wrinkle ridges of Solis Planum and the northern lowlands, of middle to late Hesperian age, formed $\sim 1$ Gyr after formation of the lithosphere. The heat flux that we infer from the spacing is the ridges is higher than the simple cooling model predicts. The heat flux deduced from elastic flexure of Amazonian-age volcanoes [Arkani-Hamed, 2000; Zuber et al., 2000; McGovern et al., 2001] also exceeds by at least a factor of two the prediction from lithosphere cooling.

As the heat flux compatible with geological and geophysical observation exceeds that predicted by plate cooling relations, a contribution of radiogenic heating or secular cooling is required. We compare in Fig. 5-19 the heat flux deduced from flexure and ridge spacing models with the surface heat flux needed to evacuate the heat produced in the martian interior according to the composition models of Treiman et al. [1986] and Laul et al. [1986]. The modeled heat flux follows closely the decay trend of the radiogenic heat production. If the heat evacuated at the surface is roughly equal to the heat produced in the interior, the mantle temperature is roughly constant with time. If the mantle was cooled early on either by an episode of plate tectonics [Nimmo and Stevenson, 2000] or degree-1 convection [Zhong and Zuber, 2001], it could have remained at relatively low temperature, and volcanism would be progressively less important, due to the thickening of the lithosphere. A feedback between mantle temperature, viscosity and the heat flux may have permitted the sub-lithospheric convective system to adjust to the radiogenic heat production.

## 5.6 Conclusions

With the recent discovery of wrinkle ridges in the martian northern lowlands [Withers and Neumann, 2001; Head et al., 2001], the penetration depth of ridge-related faulting
as well as the structure of the lithosphere in the Hesperian can be constrained. The spacing of lowland ridges is a factor of two greater than in other ridged plains, which may reflect a two-fold increase in the depth of the brittle-ductile transition arising from variations of the crustal thickness. In the ridged plains, where the crust is relatively thick, the lower crust may be ductile, in which case faulting is limited by the brittle-ductile transition of crustal rocks. In the lowlands, the thinner crust may be brittle throughout, so that faulting is limited by the brittle-ductile transition of mantle rocks. If we use a combination of diabase [Caristan, 1982] and dry olivine [Karato et al., 1986] as controlling the ductile strength of the crust and the mantle, respectively, the factor of two increase is explained. The implied geotherm, tied to the crustal thickness derived from MGS data [Zuber et al., 2000] is 12±3 K.km⁻¹, without any resolvable difference between the two regions. Such a heat flux is comparable with the heat produced by radiogenic decay during the Hesperian.

Penetration of ridge-related strains to the brittle-ductile transition categorized the wrinkle ridges as thick-skinned tectonics, like basement uplifts on Earth [Rodgers, 1987]. The recent analogy of wrinkle ridges with folds forced by a blind thrust [Schultz, 2000] is also consistent with deep penetration of the faults underlying wrinkle ridges. The ratio of ridge spacing to depth of faulting is ~1, too small to be explained by buckling of the lithosphere. However, the localization instability developed in Chapter 3 can explain this ratio. The localization instability is alone among the possible origins of regularly-spaced tectonic features to consider the dynamics of localization, or formation of localized shear zones or faults. Hence, it is most adapted to the modeling of wrinkle ridge, which express a sub-surface fault. The efficiency of localization, determined from the ridge spacing, points towards failure of intact rock, as opposed to reactivation of a pre-existing structure. This inference is however subject to caution, because of the wide ranges of localization efficiency associated with localization mechanism [Chapter 2]. With a lithospheric structure similar to that implied by the ridge spacing analysis, faults dipping toward high-standing areas can be preferentially activated by even minor lateral variations of crustal thickness or geotherm. Golombek et al. [2001] observed such a pattern in Solis and Lunae Planum.
Appendix A

Formulation of the Localization Instability Analysis

Chapters 3, 4, and 5 are based on an instability analysis of lithosphere models undergoing horizontal shortening. The deformation field of these models is the superposition of a primary flow field, which is invariant in the horizontal direction, $x$, and a secondary field, of much smaller amplitude. We showed how this secondary field grows as a function of lithosphere structure and horizontal wavelength of perturbation. This appendix presents the mathematical formulation of the secondary flow field and of the instability analysis.

The primary flow was marked by over-bars and the secondary flow field by tildes in Chapter 3. In §A.1, we are concerned only about the secondary flow, and the tildes are dropped. In §A.2 and §A.3, the primary and secondary flows have respectively the superscripts 0 and 1, to better emphasize their order of approximation.
A.1 Fundamental equation for the depth kernel and secondary deformation field

A.1.1 General formulation with depth-dependent viscosity

The materials considered in Chapter 3 to 5 are incompressible and the model is two-dimensional. Hence, the secondary flow field is entirely by the stream function \( \varphi(x, z) \), with \( x \) the horizontal coordinate and \( z \) the vertical coordinate.

Per definition of the stream function, the velocity field is

\[
\begin{align*}
v_x &= -\frac{\partial \varphi}{\partial z}, \\
v_z &= \frac{\partial \varphi}{\partial x},
\end{align*}
\]  
(A.1)

and the strain rate field is

\[
\begin{align*}
\dot{\varepsilon}_{xz} &= \frac{\partial v_x}{\partial x} \\
&= -\frac{\partial^2 \varphi}{\partial x \partial z}, \\
\dot{\varepsilon}_{zz} &= \frac{\partial v_z}{\partial z} \\
&= \frac{\partial^2 \varphi}{\partial x \partial z}, \\
\dot{\varepsilon}_{xx} &= \frac{1}{2} \left( \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right) \\
&= \frac{1}{2} \frac{\partial^2 \varphi}{\partial x^2} - \frac{1}{2} \frac{\partial^2 \varphi}{\partial z^2}.
\end{align*}
\]  
(A.3)

The linearized rheology apparent for the secondary deformation field is given by Eq. 3.8 [Fletcher, 1974; Smith, 1977]. It gives

\[
\begin{align*}
\sigma_{xx} &= -p - \frac{1}{n_e} \eta \frac{\partial^2 \varphi}{\partial x \partial z}, \\
\sigma_{zz} &= -p + \frac{1}{n_e} \eta \frac{\partial^2 \varphi}{\partial x \partial z}, \\
\sigma_{xx} &= \frac{1}{2} \frac{\partial^2 \varphi}{\partial x^2} - \frac{1}{2} \frac{\partial^2 \varphi}{\partial z^2}.
\end{align*}
\]  
(A.6)

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The pressure, \( p \), is also a field and depends on \( x \) and \( z \).

We now write the condition of Newtonian equilibrium for a continuum [Malvern, 1969]

\[
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} = 0, \quad \text{(A.9)}
\]
\[
\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zx}}{\partial z} = 0. \quad \text{(A.10)}
\]

The body forces do not appear in the Eq. A.9 because they are balanced by the primary flow. Inserting the expressions for the stresses and rearranging, we obtain

\[
\frac{\partial p}{\partial x} = \frac{1}{2} \frac{d\eta}{dz} \frac{\partial^2 \varphi}{\partial x^2} - \left( \frac{1}{n_e} - \frac{1}{2} \right) \eta \frac{\partial^3 \varphi}{\partial x^2 \partial z} - \frac{1}{2} \frac{d\eta}{dz} \frac{\partial^2 \varphi}{\partial z^2} - \frac{1}{2} \frac{d\eta}{dz^2} \frac{\partial^3 \varphi}{\partial x \partial z^2}, \quad \text{(A.11)}
\]
\[
\frac{\partial p}{\partial z} = \frac{1}{2} \eta \frac{\partial^3 \varphi}{\partial x^3} + \frac{1}{n_e} \frac{d\eta}{dz} \frac{\partial^2 \varphi}{\partial x \partial z} + \left( \frac{1}{n_e} - \frac{1}{2} \right) \eta \frac{\partial^3 \varphi}{\partial x \partial z^2}. \quad \text{(A.12)}
\]

The derivatives of Eq. A.11 with respect to \( z \) and of Eq. A.12 with respect to \( x \) are combined to give

\[
0 = \frac{1}{2} \eta \frac{\partial^4 \varphi}{\partial x^4} - \frac{1}{2} \frac{d^2 \eta}{dz^2} \frac{\partial^2 \varphi}{\partial x^2} + \left( \frac{2}{n_e} - 1 \right) \frac{d\eta}{dz} \frac{\partial^3 \varphi}{\partial x^2 \partial z} + \left( \frac{2}{n_e} - 1 \right) \frac{d\eta}{dz} \frac{\partial^3 \varphi}{\partial x \partial z^2} + \frac{d\eta}{dz} \frac{\partial^3 \varphi}{\partial z^3} + \frac{1}{2} \frac{d\eta}{dz^2} \frac{\partial^3 \varphi}{\partial x \partial z}. \quad \text{(A.13)}
\]

### A.1.2 Separation of variables and spectral analysis

We seek solutions of Eq. A.13 of the form

\[
\varphi(x, z) = \phi^0 \phi(z) f(x), \quad \text{(A.14)}
\]

where \( \phi(z) \) is called the depth kernel. As Eq. A.13 is linear, is it useful to Fourier-transform \( f(x) \): each Fourier mode is uncoupled from the others. We can therefore consider each wavenumber separately. With the stream function of the form

\[
\varphi(x, z) = \phi^0_k \phi_k(z) e^{ikx}, \quad \text{(A.15)}
\]
the velocity, strain rate and stress field are

\begin{align*}
v_x &= -\phi'_ke^{ikx}, \\
v_z &= ik\phi e^{ikx}, \\
\dot{\varepsilon}_{xx} &= -ik\phi'ke^{ikx}, \\
\dot{\varepsilon}_{zz} &= ik\phi'_ke^{ikx}, \\
\dot{\varepsilon}_{xz} &= -\frac{1}{2} \left[ \phi'' + k^2 \phi \right] e^{ikx}, \\
p &= \frac{i}{2k} \left[ \eta \phi''' + \eta' \phi'' + \left( \frac{2}{n_e} - 1 \right) \eta k^2 \phi' + \eta' k^2 \phi \right] e^{ikx}, \\
\sigma_{xx} &= -\frac{i}{2k} \left[ \eta \phi''' + \eta' \phi'' + \eta k^2 \phi' + \eta' k^2 \phi \right] e^{ikx}, \\
\sigma_{zz} &= -\frac{i}{2k} \left[ \eta \phi''' + \eta' \phi'' + \left( \frac{4}{n_e} - 1 \right) \eta k^2 \phi' + \eta' k^2 \phi \right] e^{ikx}, \\
\sigma_{xz} &= -\frac{1}{2} \left( \eta \phi'' + \eta k^2 \phi \right) e^{ikx},
\end{align*}

with primes denoting derivatives with respect to \( z \). The pressure was found by integrating Eq. A.11, taking advantage that \( de^{ikx}/dx = ike^{ikx} \). For a particular wavenumber \( k \), Eq. A.13 becomes

\begin{equation}
0 = \left( k^4 \eta + k^2 \eta'' \right) \phi_k - Ak^2 \eta' \phi'_k + \left( \eta'' - Ak^2 \eta \right) \phi''_k + 2\eta' \phi'''_k + \eta \phi''''_k,
\end{equation}

with \( A = 4/n_e - 2 \). Eq. A.25 is the fundamental kernel equation (Eq. 3.10). Analytical solutions to Eq. A.25 exist for a limited number of functions \( \eta(z) \). In other cases, Eq. 3.10 is solved numerically, which also gives the first three derivatives of \( \phi_k \) with respect to \( z \). Hence, the velocity, strain rate, and stress fields are known in function of depth \( z \) and wavenumber \( k \). These functions can be backward transformed into physical space coordinates, \((x, z)\), is the amplitude of each deformation mode, \( \{\phi_k^0\} \), are known.
A.2 Boundary conditions at the model interfaces

A.2.1 Primary flow

Each interface links an upper and a lower medium, that we denote by the superscript \( u \) and \( l \). To prevent opening of the interface, the velocity field is continuous across the interface:

\[
\begin{align*}
    u_x^u &= u_x^l, \quad z = \zeta, \\
    u_z^u &= u_z^l, \quad z = \zeta,
\end{align*}
\]  

(A.26) 

where \( \zeta(x) \) is the position of the interface. The model interfaces are nearly flat and horizontal. Therefore, \( \zeta \) departs only slightly from an average value \( \zeta^0 \):

\[
\zeta = \zeta^0 + \zeta^1(x), \quad \zeta^1 \ll \zeta^0. \tag{A.28}
\]

To 0\(^{th}\) order, the boundary conditions become

\[
\begin{align*}
    u_x^{u_0} &= u_x^{l_0}, \quad z = \zeta^0, \\
    u_z^{u_0} &= u_z^{l_0}, \quad z = \zeta^0.
\end{align*}
\]  

(A.29) 

(A.30)

We assume perfect bounding at the interface. Hence, the stresses normal and tangential to the interface are identical in both bounding media:

\[
\begin{align*}
    \sigma_{nn}^u &= \sigma_{nn}^l, \quad z = \zeta, \\
    \sigma_{tt}^u &= \sigma_{tt}^l, \quad z = \zeta.
\end{align*}
\]  

(A.31) 

(A.32)

The stress conditions are expressed in a frame of coordinates that is rotated with the perturbed interface. If the interface slopes with an angle \( \theta \) from the horizontal, we have

\[
\sigma_{nn} = \sigma_{zz} \cos^2 \theta + \sigma_{xx} \sin^2 \theta - 2\sigma_{xz} \cos \theta \sin \theta, \tag{A.33}
\]

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\[ \sigma_{u} = (\sigma_{zz} - \sigma_{xx}) \cos \theta \sin \theta + \sigma_{xz} (\cos^2 \theta - \sin^2 \theta). \quad (A.34) \]

To 0\textsuperscript{th} order, the interfaces are flat and horizontal. Hence, \( \theta = 0 \). We obtain

\[ \sigma_{zz}^{0} = \sigma_{xz}^{0}, \quad z = \zeta^{0}, \quad (A.35) \]
\[ \sigma_{xx}^{0} = \sigma_{xz}^{0}, \quad z = \zeta^{0}. \quad (A.36) \]

A uniform horizontal strain rate \( \dot{\varepsilon}_{xx}^{0} \) is applied on the model. Because the material is incompressible, \( \dot{\varepsilon}_{zz}^{0} = -\dot{\varepsilon}_{xx}^{0} \), and is uniform as well, which verifies the continuity condition for the vertical velocity. As the viscosity may be different on either side of the interface, the pressure \( p^{0} \) and the shear strain rate \( \dot{\varepsilon}_{xx}^{0} \) must be discontinuous across the interface to satisfy the stress boundary conditions. In this study, we impose \( \dot{\varepsilon}_{xx}^{0} = 0 \) and obtain

\[ p^{u} - p^{f} = - (\eta^{u} - \eta^{f}) \dot{\varepsilon}_{xx}^{0}, \quad (A.37) \]
\[ \dot{\varepsilon}_{xx}^{0} = 0. \quad (A.38) \]

### A.2.2 Secondary flow

To determine the secondary flow, designated by the superscript 1, we linearize the boundary conditions. The velocity boundary conditions become

\[ v_{x}^{u1} + \zeta^{1} \dot{\varepsilon}_{xx}^{0} = v_{x}^{f1} + \zeta^{1} \dot{\varepsilon}_{xx}^{0}, \quad z = \zeta^{0}, \quad (A.39) \]
\[ v_{z}^{u1} + \zeta^{1} \dot{\varepsilon}_{xx}^{0} = v_{z}^{f1} + \zeta^{1} \dot{\varepsilon}_{xx}^{0}, \quad z = \zeta^{0}. \quad (A.40) \]

Using the strain rate of the primary field above, we obtain

\[ v_{x}^{u1} = v_{x}^{f1}, \quad (A.41) \]
\[ v_{z}^{u1} = v_{z}^{f1}. \quad (A.42) \]

To write the stress conditions, we first estimate the slope of the interface. As the
interface perturbation is small, we use

$$\cos \theta \sim 1.$$ \hspace{1cm} (A.43)

$$\sin \theta \sim d\zeta^1/dx.$$ \hspace{1cm} (A.44)

Note that this approximation breaks down if the high-frequency relief is present on the interface. This provides an upper bound to the wavenumber for which this analysis is valid. The rotated stresses become to first order

$$\sigma_{nn} = \sigma_{zz}^0 + \sigma_{zz}^1 - 2\sigma_{xz}^0 d\zeta^1/dx,$$ \hspace{1cm} (A.45)

$$\sigma_{tt} = \sigma_{zz}^0 + (\sigma_{zz}^0 - \sigma_{xx}^0) d\zeta^1/dx + \sigma_{zz}^1.$$ \hspace{1cm} (A.46)

A last correction term to the normal stress comes from the density of the material. The boundary condition is evaluated at the perturbed interface, but is more easily expressed at $\zeta_0$. Hence:

$$\sigma_{nn}(z = \zeta) = \sigma_{zz}^0(z = \zeta_0) + \sigma_{zz}^1(z = \zeta_0) - 2\sigma_{xz}^0(z = \zeta_0) d\zeta^1/dx + \rho g \zeta^1,$$ \hspace{1cm} (A.47)

Introducing these last expressions into the stress boundary condition, and using that $\sigma_{xz}^0 = 0$ and $\sigma_{zz}^0 - \sigma_{xx}^0 = -2\eta \ddot{\varepsilon}_{zz}$, we obtain

$$\sigma_{zz}^{u1} + \rho g \zeta^1 = \sigma_{zz}^{l1} + \rho g \zeta^1,$$ \hspace{1cm} (A.48)

$$-2\eta \ddot{\varepsilon}_{zz} d\zeta^1/dx + \sigma_{zz}^{u1} = -2\eta \dddot{\varepsilon}_{zz} d\zeta^1/dx + \sigma_{zz}^{l1}.$$ \hspace{1cm} (A.49)

**A.3 Interface growth rate**

The interface is advected with the deformation field:

$$\frac{d\zeta}{dt} = v_z + v_x \frac{\partial \zeta}{\partial x}, \hspace{1cm} z = \zeta(x).$$ \hspace{1cm} (A.50)
The velocity can be evaluated in either bounding medium because of the continuity of velocity at the interface. Again, we linearize this condition. Using \( \varepsilon_{zz}^0 = 0 \) and \( \partial \zeta^0 / \partial x = 0 \), we obtain

\[
\begin{align*}
\frac{d \zeta^0}{dt} &= v_z^0, \\
\frac{d \zeta^1}{dt} &= \varepsilon_{zz}^0 \zeta^1 + v_z^1, \\
\frac{dk}{dt} &= \varepsilon_{zz}^0 k,
\end{align*}
\] (A.51, A.52, A.53)

where \( \varepsilon_{zz}^0 \zeta^1 \) represents the kinematic contribution of the primary flow and \( v_z^1 \) the dynamic contribution of the secondary flow to the growth of interface perturbations \( \zeta^1 \) [Smith, 1975].

The growth rate of interface perturbations is controlled by the vertical velocity of the secondary strain field. The depth kernel and its derivatives tell us what the shape of the deformation field should be in each layer. The amplitude of each deformation mode is determined by the boundary conditions:

\[
\begin{align*}
\nu_x^{u1} - \nu_x^{l1} &= 0, \\
\nu_z^{u1} - \nu_z^{l1} &= 0, \\
\sigma_{zz}^{u1} - \sigma_{zz}^{l1} &= -\rho^u g \zeta^1 + \rho^l g \zeta^1, \\
\sigma_{zz}^{u1} - \sigma_{zz}^{l1} &= 2\eta^u \dot{\varepsilon}_{zz}^{u0} \zeta^1 / dx - 2\eta^l \dot{\varepsilon}_{zz}^{l0} \zeta^1 / dx.
\end{align*}
\] (A.54, A.55, A.56, A.57)

Both the amplitude of the interface perturbation \( \zeta^1 \), and its derivative \( \partial \zeta^1 / \partial x \) appear in the boundary condition. If the Fourier transform of \( \zeta^1 \) is defined as

\[
\xi(k) = \mathcal{F}(\zeta^1(x)),
\] (A.58)

we can use the fact that

\[
\mathcal{F}(\partial \zeta^1(x)/\partial x) = ik \xi(k).
\] (A.59)

Then, the boundary conditions involve only the Fourier coefficients of the interface
perturbations and the velocity and stress fields. Hence, there is a linear relation
between the amplitude of the deformation modes in the Fourier domain, $\phi_k^0$ and the
amplitude of the interface perturbation $\xi(k)$.

We now consider a given wavenumber $k$, and an interface $i$. We call $\xi_i$ the ampli-
tude of the perturbation of the interface $i$ with wavenumber $k$ and $\phi_j^0$ the amplitude of
the mode $j$ of the secondary flow, again with wavenumber $k$. The boundary conditions
are rewritten as

\begin{align}
0 &= ik \sum_j P_{ij} \phi_j^0, \quad (A.60) \\
0 &= - \sum_j P_{ij} \phi_j^0, \quad (A.61) \\
-2ikg \xi_i \sum_j P_{ij} \rho_j &= \sum_j P_{ij} \left[ \eta_j \phi_j'' + \eta' \phi_j' - (1 - 4/\eta_e) \eta k^2 \phi_j + \eta' k^2 \phi_j \right] \phi_j^0, \quad (A.62) \\
-4i\varepsilon_{xz} k \xi_i \sum_j P_{ij} \eta_j &= \sum_j P_{ij} \left[ \eta \phi_j'' + \eta k^2 \phi_j \right] \phi_j^0. \quad (A.63)
\end{align}

where $\eta_j$ and $\rho_j$ are the viscosity and density of the layer in which the mode $j$ is
defined, $P_{ij}$ is an operator that takes the value 1 if the interface $i$ is the lower limit
that layer, $-1$ if the interface $i$ is the upper limit of the layer in which the mode $j$ is
defined, and 0 if the layer and the interface are not in contact. All depth-dependent
quantities are estimated at the depth $z_i^0$. The interface evolution equation becomes

\begin{equation}
\frac{d\xi_i}{dt} = \dot{\varepsilon}_{zz}^0 \xi_i + ik \frac{1}{2} \sum_j |P_{ij}|^2 \phi_j^0 \phi_j^0. \quad (A.64)
\end{equation}

The boundary conditions can be written in a matrix form

\begin{equation}
\begin{bmatrix}
A^{11} & A^{12} \\
A^{21} & A^{22}
\end{bmatrix}
\begin{bmatrix}
\Phi \\
\Xi
\end{bmatrix} =
\begin{bmatrix}
0 & 0 \\
0 & I
\end{bmatrix}
\begin{bmatrix}
d\Phi/dt \\
d\Xi/dt
\end{bmatrix}, \quad (A.65)
\end{equation}

where $\Phi$ is a vector made of the coefficients $\phi_j^0$, $\Xi$ is a vector made of the coefficients
\( \xi_i \), and \( \mathbf{A}^{11}, \mathbf{A}^{12}, \mathbf{A}^{12}, \) and \( \mathbf{A}^{22} \) and \( \mathbf{I} \) are matrices with elements

\[
\begin{align*}
\mathbf{A}^{11}_{x,4(l-1)+1} &= ikP_{ij}\phi_j, \\
\mathbf{A}^{11}_{x,4(l-1)+2} &= -P_{ij}\phi'_j, \\
\mathbf{A}^{11}_{x,4(l-1)+3} &= P_{ij} [\eta_j\phi''_j + \eta'_j\phi'_j - (1 - 4/n_e) \eta k^2 \phi'_j + \eta' k^2 \phi_j], \\
\mathbf{A}^{11}_{x,4(l-1)+4} &= P_{ij} [\eta\phi''_j + \eta k^2 \phi_j], \\
\mathbf{A}^{12}_{x,4(l-1)+1} &= 0, \\
\mathbf{A}^{12}_{x,4(l-1)+2} &= 0, \\
\mathbf{A}^{12}_{x,4(l-1)+3} &= k\mathbf{S}_i, \\
\mathbf{A}^{12}_{x,4(l-1)+4} &= k\mathbf{R}_i, \\
\mathbf{A}^{21}_{x,l} &= -ik|P_{ij}|\phi_j/2, \\
\mathbf{A}^{22}_{x,l} &= -\varepsilon^0_{xx}\delta_{jl}, \\
\mathbf{I}_{x,l} &= \delta_{jl}.  
\end{align*}
\]  

with \( \delta_{jl} \) the Kronecker operator, and

\[
\begin{align*}
\mathbf{S}_i &= 2ikg \sum_m P_{im}\rho_m, \\
\mathbf{R}_i &= 4ie^0_{xx} \sum_m P_{im}\eta_m.
\end{align*}
\]

The models have \( n \) layers, the lowermost being a half-space. Therefore, the indices \( i \), which represents the interface perturbation amplitude, and \( l \), which represents the interface on which a set of boundary condition is imposed, run from 1 to \( n \), and the indices \( j \) and \( m \), which represent the deformation modes in the model, run from 1 to \( 4n \).

The uppermost surface \((i = 1 \text{ and } l = 1)\) requires a special treatment. Indeed, the medium above it (either air, water, or sediment-rich water) is considered inviscid. Velocity is not defined in it. Therefore, the first two rows of \( \mathbf{A}^{11} \) and \( \mathbf{A}^{12} \) must be ignored. In addition, we must prevent divergence of the flow field in the lowermost medium, which is a half-space. This can be achieved \textit{a priori} only if an analytical
solution for the depth kernels in the half-space is known. Hence, we restrict our
models as having an exponent viscosity profiles in the half-space. Then, the four
depth kernels in that layer are given by Eq. 3.15. If \( n_e < 0 \), no solution goes to 0 as
\( z \to -\infty \). Hence, we can solve only problems where the half-space does not localize.
For \( n_e > 0 \), only two depth kernels diverge as \( z \to -\infty \). The mode amplitudes
associated with them are set to 0. Hence, \( A^{11} \) is reduced to a \( 4n - 2 \times 4n - 2 \) matrix,
\( B^{11} \), \( A^{12} \) is reduced to a \( n \times 4n - 2 \) matrix, \( B^{12} \), and \( A^{21} \) is reduced to a \( 4n - 2 \times n \)
matrix, \( B^{21} \).

Manipulating Eq. A.65 by static condensation [Bathe, 1996; Zuber et al., 1986],
we obtain

\[
\frac{d\Xi}{dt} = M\Xi, \tag{A.79}
\]

with \( M = A_{22} - B_{21} (B^{11})^{-1} B^{12} \) the transfer matrix. The growth rate matrix, \( Q \), is
related to \( M \) by

\[
M = -\varepsilon_{xx}^0 (I + Q). \tag{A.80}
\]
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