Essays on Insurance and Taxation

by

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Submitted to the Department of Economics
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Abstract

Chapter 1 analyzes Pareto optimal non-linear taxation of profits and labor income in a private information economy with endogenous firm formation. Individuals differ in both their skill and their cost of setting up a firm, and choose between becoming workers and entrepreneurs. I show that a tax system in which entrepreneurial profits and labor income must be subject to the same non-linear tax schedule makes use of general equilibrium effects through wages to indirectly achieve redistribution between entrepreneurs and workers. As a result, constrained Pareto optimal policies can involve negative marginal tax rates at the top and, if available, input taxes that distort the firms' input choices. However, these properties disappear when a differential tax treatment of profits and labor income is possible, as for instance implemented by a corporate income tax. In this case, redistribution is achieved directly through the tax system rather than "trickle down" effects, and production efficiency is always optimal. When I extend the model to incorporate entrepreneurial borrowing in credit markets, I find that endogenous cross-subsidization in the credit market equilibrium results in excessive (insufficient) entry of low-skilled (high-skilled) agents into entrepreneurship. Even without redistributive objectives, this gives rise to an additional, corrective role for differential taxation of entrepreneurial profits and labor income. In particular, a regressive profit tax may restore the efficient occupational choice.

In chapter 2, which is joint work with Nick Netzer, we show that, in the presence of a time-inconsistency problem with optimal agency contracts, competitive markets can implement allocations that Pareto dominate those achieved by a benevolent planner, and they induce more effort. In particular, we analyze a model with moral hazard and two-sided lack of commitment. After agents have chosen a hidden effort and the need to provide incentives has vanished, firms can modify their contracts and agents can switch firms, resulting in an adverse selection problem at the ex-post stage. As long as the ex-post market outcome satisfies a weak notion of competitiveness and sufficiently separates individuals who choose different effort levels, the market allocation is Pareto superior to a social planner’s allocation with a complete breakdown of incentives. In addition, even when a planner without commitment is able to sustain effort incentives, competitive markets without commitment implement more effort in equilibrium under general conditions. We illustrate our findings with standard market equilibrium concepts.

Chapter 3 studies Pareto-optimal risk-sharing arrangements in a private information economy with aggregate uncertainty and ex ante heterogeneous agents. I show that any such arrangement has to be such that ratios of expected inverse marginal utilities across different agents are independent of aggregate shocks. I use this condition to show how to implement Pareto-optima as equilibria when agents can trade claims to consumption contingent on aggregate shocks in financial markets. If aggregate shocks affect individual outputs only, the implementation of optimal allocations does not require interventions in financial markets. If they also affect probability distributions over idiosyncratic risk, however, transaction taxes need to be introduced that are
higher for claims to consumption in states with a more volatile distribution of likelihood ratios in the sense of second-order stochastic dominance. Two implementation results are provided. If transaction taxes are constrained to be linear, they need to condition on individual outputs in addition to aggregate shocks. To prevent double-deviations, they induce additional risk for agents who buy financial claims and provide additional insurance to those who sell them. Finally, an implementation with non-linear transaction taxes that do not depend on idiosyncratic shocks is constructed.

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Chapter 2 of this thesis is co-authored with Nick Netzer. Interacting with him throughout these years has been a true privilege. I have also been fortunate to be able to work with Emmanuel Farhi, Casey Rothschild, Robert Townsend and Iván Werning on joint research projects. I would like to thank my class- and officemates at MIT who made these years a pleasant experience, especially Suman Basu, Dan Cao, David Cesarini, Laura Feiveson, Luigi Iovino, Pablo Kurlat, Jen-Jen La’O, Monica Martinez-Bravo, Mike Powell and Alp Simsek. I am grateful to my advisors at the University of Konstanz, notably Friedrich Breyer and Mathias Kifmann, for spurring my interest in economics and encouraging me to pursue a PhD at MIT. It was made possible by financial assistance from the MIT Economics Department, the Studienstiftung des deutschen Volkes and the German Academic Exchange Service (DAAD).

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Chapter 1

Entrepreneurial Taxation, Occupational Choice, and Credit Market Frictions

1.1 Introduction

The question at what rate business profits should be taxed – notably relative to the tax rates on other forms of income such as labor earnings – is a recurring and controversial theme in the public policy debate. On the one hand, it is often argued that individuals who receive business profits, such as entrepreneurs, tend to be better off than those who do not. Therefore, arguments based on direct redistribution, or “tagging,” seem to justify the taxation of profits at a higher rate than other forms of income, as for instance implemented by a corporate income tax and the resulting double taxation of profits both at the firm and individual level. On the other hand, proponents of “supply side” or “trickle down economics” typically emphasize the general equilibrium effects of the tax treatment of businesses. In particular, they point out that a reduction in the entrepreneurs’ tax burden encourages entrepreneurial activity and labor demand. It thereby increases wages and hence “trickles down” to medium or lower income workers, achieving redistribution indirectly. Moreover, entrepreneurs invest and therefore have to borrow funds in credit markets that are typically subject to imperfections. This raises concerns about a too low number of individuals setting up firms due to borrowing constraints from credit market frictions. From both of the latter perspectives, a reduced taxation of firm profits, or even a subsidization of entrepreneurial activities, appears optimal.

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Underlying these opposing arguments is the question to what degree an optimal tax system should rely on indirect general equilibrium, or "trickle down" effects to achieve redistribution and affect occupational choice. To study this issue formally, I construct a simple model in which the production side is managed by entrepreneurs and both wages and the decision to become a worker or an entrepreneur are endogenous. In particular, I consider a population of individuals characterized by two-dimensional heterogeneity: Agents differ in their cost of setting up a firm, and in their skill, both of which are private information. They can either choose to become a worker, in which case they supply labor at the endogenous wage rate, or select to be an entrepreneur. In this case, they hire workers and provide entrepreneurial effort, which are combined to produce the consumption good.

I characterize Pareto optimal allocations in this economy and demonstrate that the resulting multidimensional screening problem is tractable and allows for a transparent analysis of the issues raised above. The key result is that it crucially depends on the set of available tax instruments whether a Pareto optimal tax system uses general equilibrium effects to achieve redistribution indirectly through "trickle down." I start with characterizing constrained Pareto optimal allocations when the government imposes the same, non-linear tax schedule on both entrepreneurial profits and labor income. In fact, this appears particularly appealing in view of the general presumption that introducing wedges between different forms of income is distorting and should therefore be avoided.

However, even though such a tax policy does not explicitly distort the occupational choice margin, it puts severe limitations on the amount of redistribution that can be achieved between entrepreneurs and workers. Due to two-dimensional heterogeneity, the income distributions of workers and entrepreneurs have overlapping supports: There are high-skilled agents who remain workers since they have a high cost of setting up a firm, low-skilled agents who enter entrepreneurship because of their low cost of doing so, and vice versa. It is therefore impossible for a tax system to distinguish workers and entrepreneurs just based on their income. Formally, a policy that does not condition tax schedules on occupational choice puts a no-discrimination constraint on the Pareto problem, since it rules out discriminating between entrepreneurs and workers of different ability levels that are related by the endogenous wage rate.

In the presence of this restriction, a Pareto optimal tax schedule indeed reflects some "trickle down" logic. I show that, if wages are not fixed by technology, the tax system explicitly manipulates incentives in order to induce general equilibrium effects through wages and thus achieve redistribution between entrepreneurs and workers indirectly, given that direct redistribution based on income is not possible. For instance, I provide
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conditions under which, if the government aims at redistributing from entrepreneurs to workers, top earning entrepreneurs are subsidized at the margin, as this encourages their effort and raises the workers' wage. This relaxes the no-discrimination constraints and therefore allows for additional redistribution in this case. As a result, optimal marginal tax rates not only depend on the skill distribution and wage elasticities of effort, as in standard models, but also on the degree of substitutability of labor and entrepreneurial effort in production. Moreover, I show that if the government has access to additional tax instruments, such as (non-linear) input taxes, it is generally optimal to distort marginal rates of substitution across firms in order to affect wages.

It turns out, however, that these non-standard properties of optimal tax systems, such as negative marginal tax rates at the top and production inefficiency, crucially rely on the restriction that there is only a single tax schedule for both entrepreneurs and workers. In fact, I show that they disappear as soon as the government can make firm profits and labor income subject to different non-linear tax schedules. A Pareto optimal tax policy can now achieve redistribution directly through differential taxation rather than indirectly through general equilibrium effects. For this reason, optimal marginal tax rate formulas no longer depend on substitution elasticities between different inputs in the firms' production function. Furthermore, even if the government could impose distorting input taxes in addition to the non-linear tax schedules on profits and labor income, this is not needed to implement constrained Pareto optima: With differential taxation, production efficiency is always optimal. I also show that, with differential taxation, the "trickle down" logic does not apply. In fact, when redistributing from entrepreneurs to workers, for instance, a Pareto optimal tax system does so in a way that depresses the workers' wage, who are of course more than compensated by tax transfers.

I compare the optimal tax schedules for profits and labor earnings in an economy that is calibrated to match income distributions and occupational choice between entrepreneurship and employment in the 2007 Survey Consumer Finances. Under various assumptions on the government's redistributive objectives, there robustly emerges an "excess profit tax," i.e. a higher taxation of entrepreneurial profits compared to labor income for individuals of the same skill level, as for instance implemented by a corporate income tax or a separate tax schedule for self-employed persons. I also simulate the effects of optimal tax policy on wages and entrepreneurship for various parameter combinations, with the finding that wages decrease and entrepreneurship is discouraged for most skill levels even when the government only aims at redistributing across different ability types, not between entrepreneurs and workers directly.

Finally, I introduce entrepreneurial investment and borrowing into the analysis. Indi-
individuals are assumed to be wealth constrained and therefore have to borrow funds from banks in a competitive credit market in order to set up a firm. Since there is privately known heterogeneity, credit markets are affected by adverse selection. This raises the concern that, due to the credit market imperfections, an insufficient number of agents choose to set up a firm, calling for a lower taxation, or even a subsidization, of entrepreneurship compared to the preceding analysis. I characterize the credit market equilibrium and show that it takes the form of a pooling equilibrium that involves a single debt contract being offered to all entrepreneurs. The resulting cross-subsidization from high to low quality borrowers provides excessive incentives for low skilled individuals to enter entrepreneurship, but insufficient incentives for high ability agents.

Even without redistributive objectives, credit market frictions therefore give rise to an additional, corrective role for a differential tax treatment of entrepreneurial profits. In particular, the occupational misallocation can be removed by a regressive profit tax, which involves negative average tax payments for high-skilled entrepreneurs and positive ones for low ability entrepreneurs, thus counteracting the cross-subsidization in the credit market equilibrium. This demonstrates that credit market imperfections do not necessarily justify a general subsidization of entrepreneurship, as raised at the beginning. Rather, they induce the wrong mix of individuals in the two occupations, with too many and too few entrepreneurs at the same time. More sophisticated tax interventions, such as the regressive profit tax suggested here, are therefore required to mitigate inefficiencies in occupational choice.

Related Literature. This paper contributes to a large literature that has studied the effects of tax policy on economies explicitly incorporating entrepreneurship. In particular, there has been considerable interest recently in using calibrated dynamic general equilibrium models with an entrepreneurial sector, such as those developed by Quadrini (2000), Meh and Quadrini (2004), and Cagetti and De Nardi (2006), to quantitatively explore how various stylized tax reforms affect the equilibrium wealth distribution, welfare, and investment. For instance, Meh (2005) and Zubricky (2007) have studied the effects of moving from a progressive to a flat income tax system in such economies, Cagetti and De Nardi (2009) have analyzed how an elimination of estate taxation would affect wealth accumulation and welfare, and Panousi (2008) and Kitao (2008) have computed the effects of capital taxation on entrepreneurial investment and capital accumulation. Yet none of these studies have aimed at characterizing and computing optimal tax systems in entrepreneurial economies, which is the focus of the present paper.\(^2\)

\(^2\)There is also related research that has focused on how taxes affect more specific aspects of entrepreneurial activity. For example, Kanbur (1981), Kihlstrom and Laffont (1979), Kihlstrom and Laffont
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In characterizing optimal allocations, my work therefore shares a common goal with Albanesi (2006) and, more relatedly, Albanesi (2008), who has extended the framework of optimal dynamic taxation to account for entrepreneurial investment. More precisely, she considers a moral hazard model where entrepreneurs exert some hidden action that affects a stochastic return to capital. Her focus is on characterizing the optimal savings distortions that entrepreneurs should face when the government provides insurance for entrepreneurial investment risk. Similarly, Chari, Golosov, and Tsyvinski (2002) examine optimal intertemporal wedges in a dynamic economy with start-up firms and incomplete markets. In contrast to this literature, I focus on characterizing the optimal taxation of profits and labor income in a static general equilibrium model that emphasizes how taxes affect the effort-leisure wedge of entrepreneurs versus workers and thus wages. Moreover, when incorporating entrepreneurial borrowing, I explicitly consider private credit markets and how tax policy interacts with the endogenous credit market equilibrium.

In this regard, the present paper is also related to the literature on the role of government intervention in credit markets with adverse selection, e.g. in Stiglitz and Weiss (1976), De Meza and Webb (1987), Innes (1991), Innes (1992), and Parker (2003). While this research has pointed out efficiency properties of credit market equilibria and scope for government intervention, there has been no systematic treatment of optimal entrepreneurial taxation in the presence of such credit market frictions and occupational choice. Most related is the contribution by De Meza and Webb (1987), who point out the possibility that adverse selection in credit markets leads to excessive entry into entrepreneurship, quite in contrast the credit rationing emphasized by Stiglitz and Weiss (1976). They suggest a tax on bank profits to deal with this inefficiency. Their result is a special case of the present setting when the second dimension of heterogeneity is removed. With two-dimensional heterogeneity, however, it turns out that there is excessive and insufficient entry into entrepreneurship simultaneously, so that a simple tax on bank profits is not sufficient (nor necessary) to correct the occupational misallocation. The analysis here therefore points at the role of entrepreneurial tax policy in undoing cross-subsidization in credit markets and restoring efficiency of occupational choice.

The paper also builds on earlier research on optimal income taxation in models with endogenous wages and occupational choice, such as Feldstein (1973), Zeckhauser (1977), Allen (1982), Boadway, Marceau, and Pestieau (1991), and Parker (1999). This literature (1983), Christiansen (1990) and Cullen and Gordon (2007) have examined the effects of taxation on entrepreneurial risk-taking. Moreover, the consequences of a differential tax treatment of corporate versus non-corporate businesses (or of its removal) for investment have been the focus of Gordon (1985), Gravelle and Kotlikoff (1989) and Meh (2008). See Gentry and Hubbard (2000) for an overview of these issues. I abstract from a distinction of firms in corporate and non-corporate in this paper.
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has restricted attention to linear taxation and typically ruled out a differential tax treat-
ment of the occupational groups. An exception is the work by Moresi (1997), who consid-
ers non-linear taxation of profits. However, in his model, the occupational choice margin
is considerably simplified and heterogeneity is confined to affect one occupation only, not
the other. Stiglitz (1982) and Naito (1999) study optimal non-linear taxation in economies
with two ability types and endogenous wages. While some of their results translate to
properties of Pareto optimal tax systems with uniform taxation of profits and income,
their models do not include different occupational groups. Therefore, neither of these pa-
pers allow for the comparison of uniform and differential taxation of profits and income,
and of the optimal (non-linear) tax schedules of workers and entrepreneurs in the case of
differential taxation, which is performed here.

In addition, restricting heterogeneity to affect one occupation only, or tax schedules
to be linear, sidesteps the complexities of multidimensional screening, which emerges
naturally in the present model. In fact, few studies in the optimal taxation literature have
attempted to deal with multidimensional screening problems until recently. Most related
to the formal modelling approach used here is the recent contribution by Kleven, Kreiner,
and Saez (2009) with an application to the optimal income taxation of couples. More
generally, this paper builds on the large literature on optimal income taxation following
the seminal contributions by Diamond and Mirrlees (1971) and Mirrlees (1971). However,
rather than focusing on allocations that maximize some utilitarian social welfare criterion,
I aim at characterizing the set of Pareto optimal tax policies, sharing the spirit of Werning
(2007).

The structure of the paper is as follows. Section 1.2 introduces the baseline model and
the equilibrium without taxation. In Section 1.3, I start with characterizing Pareto optimal
tax policies when the same (non-linear) tax schedule is applied to both entrepreneurial
profits and labor income. Properties of Pareto optimal tax schedules and the optimality of
production distortions are discussed. As I show in Section 1.4, these properties disappear
when profits and income can be made subject to different tax schedules. Section 1.4 also
computes the two tax schedules for a calibrated economy. Section 1.5 then introduces
entrepreneurial borrowing in credit markets, and shows that this gives rise to another,
corrective role for entrepreneurial taxation. Finally, Section 1.6 concludes. Most of the
proofs are relegated to the appendix.
1.2 The Baseline Model

1.2.1 Preference Heterogeneity and Occupational Choice

I consider a unit mass of heterogeneous individuals who are characterized by a two-dimensional type vector \((\theta, \phi) \in [\overline{\theta}, \overline{\theta}] \times [0, \overline{\phi})\), where \(\theta\) will be interpreted as an individual’s skill, and \(\phi\) as an individual’s cost of becoming an entrepreneur, as explained in more detail below.\(^3\) \(F(\theta)\) is the cumulative distribution function of \(\theta\) and \(G_{\theta}(\phi)\) the cumulative distribution function of \(\phi\) conditional on \(\theta\), both assumed to allow for density functions \(f(\theta)\) and \(g_{\theta}(\phi)\). Note that this allows for an arbitrary correlation between \(\theta\) and \(\phi\). Both \(\theta\) and \(\phi\) are an individual’s private information.

Agents can choose between two occupations: They can become a worker, in which case they supply effective labor \(l\) at the (endogenous) wage \(w\). Abstracting from income effects, I assume preferences over consumption \(c\) and labor to be quasi-linear with

\[
U(c, l, \theta) \equiv c - \psi(l/\theta).
\]

An individual’s disutility of effort \(\psi(.)\) is assumed to be twice continuously differentiable, increasing and convex. A particular specification, used later, is given by \(\psi(l/\theta) = (l/\theta)^{1+1/\epsilon}/(1 + 1/\epsilon)\), which implies that the individual’s elasticity of labor supply with respect to the wage is constant and equal to \(\epsilon\). \(\theta\) captures an individual’s skill type in the sense that a higher value of \(\theta\) implies that the individual has a lower disutility of providing a given amount of effective labor \(l\).

Alternatively, an agent may select to become an entrepreneur. In this case, she hires effective labor \(L\) and provides effective entrepreneurial effort \(E\) to produce output of the consumption good \(Y\), where \(Y(L, E)\) is a concave neoclassical firm-level production function with constant returns to scale. An entrepreneur’s profits are then

\[
\pi = Y(L, E) - wL,
\]

and her utility is given by

\[
U(\pi, E, \theta) - \phi \equiv \pi - \psi(E/\theta) - \phi.
\]

\(\phi\) is a heterogeneous utility cost of becoming an entrepreneur, which is distributed in the population as specified above, possibly depending on the skill type \(\theta\). Thus, \(\theta\) de-
1.2. The Baseline Model

terminates an individual’s skill in both occupations, but in addition, people differ in their idiosyncratic preferences for one of the two occupations, as captured by $\phi$. The cost $\phi$ can therefore be interpreted as a shortcut for heterogeneity in the population that is not otherwise captured in the present model explicitly, such as differences in setup costs, attitudes towards entrepreneurial risks, or access to entrepreneurial capital (see Section 1.5 for more on the latter). As a result of the two-dimensional heterogeneity, there will not be a perfect ranking between occupational choice and skill type (and thus income): For a given $\theta$, there are individuals who enter entrepreneurship and others who become workers due to their different $\phi$-type. This is an empirically attractive implication of the present specification, since it is true that, in reality, the income distributions of workers and entrepreneurs have overlapping supports.

1.2.2 The Equilibrium without Taxes

In order to introduce the mechanics of this basic model, let me start with briefly discussing the equilibrium without taxes. Taking the wage $w$ as given, conditional on becoming a worker, an individual of skill-type $\theta$ solves $\max l \psi l - \psi(l/\theta) \psi$ with solution $l^*(\theta, w)$ and indirect utility $v_W(\theta, w) \equiv wl^*(\theta, w) - \psi(l^*(\theta, w)/\theta)$. Similarly, conditional on becoming an entrepreneur, type $\theta$ solves $\max Y(L, E) - wL - \psi(E/\theta) \psi$ with solution $L^*(\theta, w), E^*(\theta, w)$ and indirect utility $v_E(\theta, w)$. Then the occupational choice decision for individuals of type $\theta$ is determined by the critical cost value

$$\bar{\phi}(\theta, w) \equiv \begin{cases} 0 & \text{if } v_E(\theta, w) - v_W(\theta, w) < 0 \\ \frac{v_E(\theta, w) - v_W(\theta, w)}{\bar{\phi}_\theta} & \text{if } v_E(\theta, w) - v_W(\theta, w) > \bar{\phi}_\theta \\ v_E(\theta, w) - v_W(\theta, w) & \text{otherwise,} \end{cases}$$ (1.1)

so that all $(\theta, \phi)$ with $\phi \leq \bar{\phi}(\theta, w)$ become entrepreneurs, and the others workers. With this notation, an equilibrium without taxes can be defined as follows:

---

4While I assume $\phi \geq 0$, i.e. that entrepreneurship is associated with some cost for all individuals, the following analysis does not rely on this assumption. Rather, I could allow for the support of $\phi$ to include negative numbers, accounting for the fact that some individuals value non-pecuniary benefits from being an entrepreneur, such as flexibility of schedules and being one’s own boss. The only advantage of assuming $\phi$ to be non-negative is that, in equilibrium, entrepreneurs receive a higher return on their effort than workers. See Section 1.4.2 for a detailed discussion of evidence on this.

5This is in contrast to models where occupational choice is only based on skill heterogeneity, such as Boadway, Marceau, and Pestieau (1991) and Moresi (1997), and where it is assumed that one occupation rewards ability more than the other. Then there exists a critical skill level such that all higher skilled agents select into the high-reward occupation, and lower-ability agents into the other. This results in income distributions for the two occupations that occupy non-overlapping intervals (see e.g. Parker (1999)).
1.2. The Baseline Model

Definition 1. An equilibrium without taxes is a wage $w^*$ and an allocation $\{l^*(\theta, w^*), L^*(\theta, w^*), E^*(\theta, w^*)\}$ for all $\theta \in \Theta \equiv [\theta, \bar{\theta}]$ such that the labor market clears, i.e.

$$\int_{\Theta} G_{\theta}(\tilde{\phi}(\theta, w^*))L^*(\theta, w^*)dF(\theta) = \int_{\Theta} (1 - G_{\theta}(\tilde{\phi}(\theta, w^*)))l^*(\theta, w^*)dF(\theta).$$ (1.2)

In fact, the entrepreneurs' utility maximization problem can be decomposed as follows. Since their labor demand $L$ only affects profits and not the other components of their utility, for given $E$ and $w$, entrepreneurs of all types $\theta$ solve the same problem $\max_L Y(L, E) - wL$ with the conditional labor demand function $L^c(E, w)$ as solution such that $Y_L(L^c(E, w), E) = w$. Under the assumption of constant returns to scale, Euler's theorem implies

$$Y(L^c(E, w), E) = Y_L(L^c(E, w), E)L^c(E, w) + Y_E(L^c(E, w), E)E,$$

and thus an entrepreneur's profits are given by

$$\pi = Y(L^c(E, w), E) - wL^c(E, w) = Y_E(L^c(E, w), E)E.$$

Hence, entrepreneurs can be thought of just receiving a different wage $\bar{w} \equiv Y_E$ on their effort. Moreover, there exists a decreasing one-to-one relationship between the workers' and the entrepreneurs' wage $\bar{w}(w)$: The entrepreneurs' wage $\bar{w}$ is high if the entrepreneurial effort to labor ratio used in production is low, which means that the marginal product of labor and thus the workers' wage is low.

With these insights, the following properties of the equilibrium without taxes can be established:

Proposition 1. Consider the no tax equilibrium as defined in Definition 1. Then

(i) the entrepreneurs' wage exceeds the workers' wage, i.e. $\bar{w}^* \equiv \bar{w}(w^*) > w^*$, and for all $\theta \in \Theta$, $E^*(\theta, \bar{w}^*) > l^*(\theta, w^*)$,

(ii) the critical cost value for occupational choice $\tilde{\phi}(\theta, w^*)$ is increasing in $\theta$, and

(iii) the share of entrepreneurs $G_{\theta}(\tilde{\phi}(\theta, w^*))$ is increasing in $\theta$ if $G_{\theta}(\phi) \geq_{\text{FOSD}} G_{\theta}(\phi)$ for $\theta' \leq \theta$.

Proof. (i) Recall that $v^*_W(\theta, w^*) = \max_{l} w^*l - \psi(l/\theta)$ and $v^*_E(\theta, \bar{w}^*) = \max_E \bar{w}^*E - \psi(E/\theta)$. Suppose, by way of contradiction, $\bar{w}^* \leq w^*$. Then $v^*_E(\theta, \bar{w}^*) \leq v^*_W(\theta, w^*)$, and hence by (1.1), $\tilde{\phi}(\theta, w^*) = 0$ for all $\theta \in \Theta$. Therefore (1.2) cannot be satisfied. To see that $E^*(\theta, \bar{w}^*) > l^*(\theta, w^*)$, note first that, since the function $\bar{w}(w)$ is decreasing function because $\bar{w} = Y_E(x) = Y_E(Y^{-1}_L(w))$ and $Y_E(x)$ is decreasing and $Y_L(x)$ increasing in $x$ by concavity of $Y$ (and therefore the inverse $Y^{-1}_L(w)$ from $Y_L(x) = w$ is a decreasing function).

6This is because, by linear homogeneity of $Y$, both $Y_L$ and $Y_E$ are homogeneous of degree zero and hence functions of $x \equiv E/L$ only. Then $\bar{w}(w)$ is a decreasing function because $\bar{w} = Y_E(x) = Y_E(Y^{-1}_L(w))$ and $Y_E(x)$ is decreasing and $Y_L(x)$ increasing in $x$ by concavity of $Y$ (and therefore the inverse $Y^{-1}_L(w)$ from $Y_L(x) = w$ is a decreasing function).
1.2. The Baseline Model

$wl - \psi(l/\theta)$ is supermodular in $(w, l)$, $l^*(\theta, w)$ is increasing in $w$ by Topkis' theorem (see Topkis (1998)). By the same argument, since $\bar{w}^* > w^*$ from (i), $E^*(\theta, \bar{w}^*) > l^*(\theta, w^*)$ for all $\theta \in \Theta$.

(ii) Using the results from (i),

$$\frac{\partial \bar{\phi}(\theta, w^*)}{\partial \theta} = \psi'\left(\frac{E^*(\theta, \bar{w}^*)}{\theta}\right) E^*(\theta, \bar{w}^*) - \psi'\left(\frac{l^*(\theta, w^*)}{\theta}\right) l^*(\theta, w^*) > 0 \quad \forall \theta \in \Theta$$

by the envelope theorem and convexity of $\psi$.

(iii) If $G_{\phi}(\phi) \geq_{FOSD} G_{\phi}(\phi)$ for $\theta' \leq \theta$, then

$$G_{\phi}(\phi(\theta', w^*)) \leq G_{\phi}(\phi(\theta, w^*)) \leq G_{\phi}(\phi(\theta, w^*))$$

for $\theta' \leq \theta$,

where the first inequality follows from (ii) and the second from first-order stochastic dominance.

Proposition 1 summarizes intuitive properties of wages and occupational choice in equilibrium: First, the entrepreneurs' wage $\bar{w}^*$ must be higher than that of the workers $w^*$ in equilibrium. The reason is that, when deciding whether to become a worker or an entrepreneur, an individual of a given skill type considers two variables: The different wage that she can earn when becoming an entrepreneur rather than a worker, and the cost $p$ she has to incur when doing so. Clearly, if the entrepreneurs' wage were lower than that of workers, there would be no trade-off and nobody would choose to enter entrepreneurship, which cannot be an equilibrium. The entrepreneurs' higher wage then immediately implies that they exert more effort and earn higher profits than workers of the same ability level. While this is a direct consequence of the assumption that $\phi \geq 0$, it is in line with empirical evidence on returns to entrepreneurship. For instance, De Nardi, Doctor, and Krane (2007) find that entrepreneurs have higher incomes than workers, and Berglann, Moen, Roed, and Skogstrom (2009) confirm this pattern for wages, controlling for hours. Moreover, based on data from the 2007 Survey of Consumer Finances (SCF), I find the same relationship between returns to entrepreneurship and employment, as will be discussed in Section 1.4.2.7 In addition, since this is a static model, Proposition 1 can be interpreted in terms of lifetime incomes, or wealth. There is strong evidence that entrepreneurs have more wealth than workers, for instance in Quadrini (2000) and Cagetti and De Nardi (2006).

The second result in the proposition is that, the higher the skill type $\theta$, the more the wage difference matters compared to the cost, which is why the critical cost value $\bar{\phi}(\theta, w^*)$ increases with $\theta$. Finally, the same holds for the share of entrepreneurs in equilibrium as a

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7 Hamilton (2000) and Blanchflower (2004) find lower returns to entrepreneurship than to employment. However, their concept of entrepreneurship is different, setting it equal to self-employment. As I will discuss in Section 1.4.2, I consider individuals as entrepreneurs if they are not only self-employed, but also own and actively manage a business and hire at least two employees.
function of skill whenever skill and disutility from entrepreneurship are independent or such that higher skills tend to have a lower disutility from being an entrepreneur in the first-order stochastic dominance sense. More generally, while such a correlation between \( \theta \) and \( \phi \) may strike as plausible, the model is flexible enough to generate more complicated relationships between income and the share of entrepreneurs through the dependence of the cost distribution on \( \theta \), as captured by \( G_\theta(\phi) \).\(^8\) Proposition 1 thus demonstrates that, while the basic model is admittedly stylized and quite different from other models of entrepreneurship, it is able to produce reasonable predictions about empirical relationships, and to point out how they depend on the underlying heterogeneity in the population.

### 1.3 Uniform Tax Treatment of Profits and Income

#### 1.3.1 A Constrained Pareto Problem

While the no tax equilibrium represents a particular point on the Pareto-frontier, other Pareto optimal allocations can be implemented by suitable tax policies. Let me start with characterizing the resulting Pareto-frontier under the assumption that the government imposes a single non-linear tax schedule \( T(\cdot) \) that applies to both the workers’ labor income \( y = wl \) and the entrepreneurs’ profits \( \pi \) in the same way. Such a tax system may seem particularly appealing on the grounds of neutrality, since it does not explicitly distort the occupational choice margin. Then the question is to what degree a Pareto-optimal tax policy makes use of general equilibrium (“trickle down”) effects through the workers’ wage to achieve redistribution indirectly.

With a tax on profits \( T(\pi) \), entrepreneurs solve \( \max_{L,E} Y(L,E) - wL - T(Y(L,E) - wL) - \psi(E/\theta) \) and thus their labor demand is always undistorted such that \( Y_L = w \) for all skill types \( \theta \). This implies that, by the same arguments as in the preceding section, entrepreneurs can be viewed as just receiving a different wage \( \bar{w} = Y_E \) than workers on their effort \( E \). Hence, entrepreneurs of type \( \theta \) choose their effort so as to solve \( \max_E \bar{w}E - T(\bar{w}E) - \psi(E/\theta) \), and workers of type \( \theta \) solve \( \max_l wL - T(wl) - \psi(l/\theta) \). Since they face the same tax schedule \( T(\cdot) \), it immediately follows that the profits generated by an entrepreneur of type \( \theta \) and the income earned by a worker of type \( \theta' \) such that \( \bar{w}\theta = w\theta' \) are the same:

\[
\bar{w}E(\theta) = wL \left( \frac{\bar{w}}{w} \right)
\]  

\(^8\)In Section 1.4.2, \( G_\theta(\phi) \) will be calibrated to match the relationship between income and entrepreneurship found in the data.
1.3. Uniform Tax Treatment of Profits and Income

for all $\theta \in [a,b]$ with $a = \max \{\bar{\theta}, (w/\bar{w})\bar{\theta}\}$ and $b = \min \{\bar{\theta}, (w/\bar{w})\bar{\theta}\}$. This is a no-discrimination constraint on the Pareto-problem that results from the restriction that both profits and income must be subject to the same tax schedule $T(.)$: With this instrument, it is impossible for the government to discriminate between entrepreneurs of skill $\theta$ and workers of the rescaled skill $(\bar{w}/w)\theta$, whereby the rescaling factor $\bar{w}/w$ is endogenous and corresponds to the ratio between the marginal products of entrepreneurial effort and labor.

By the revelation principle, any allocation that can be implemented with the single non-linear tax schedule $T(.)$ must therefore satisfy the no-discrimination constraints (1.3) and the incentive compatibility constraints as specified in the following. Suppose the social planner assigns labor supply $l(\theta)$ and consumption $c_W(\theta)$ to each individual of skill type $\theta$ who chooses to become a worker, and a labor demand and entrepreneurial effort bundle $L(\theta), E(\theta)$ and consumption $c_E(\theta)$ to each $\theta$-type who selects into entrepreneurship. Then the incentive constraints can be written as

$$c_W(\theta) - \psi \left( \frac{l(\theta)}{\theta} \right) \geq c_W(\bar{\theta}) - \psi \left( \frac{l(\bar{\theta})}{\bar{\theta}} \right) \quad \forall \theta, \bar{\theta} \in \Theta, \quad (1.4)$$

$$c_E(\theta) - \psi \left( \frac{E(\theta)}{\theta} \right) \geq c_E(\bar{\theta}) - \psi \left( \frac{E(\bar{\theta})}{\bar{\theta}} \right) \quad \forall \theta, \bar{\theta} \in \Theta \quad (1.5)$$

and

$$Y_L(L(\theta), E(\theta)) = w \quad \forall \theta \in \Theta. \quad (1.6)$$

Constraint (1.6) is a result of the fact that the profit tax $T(.)$ does not distort the entrepreneurs’ labor demand, and so all firms set it so as equalize the marginal product of labor to the workers’ wage. Hence, the marginal products of entrepreneurial effort are also equalized across firms with

$$Y_E(L(\theta), E(\theta)) = \bar{w} \quad \forall \theta \in \Theta. \quad (1.7)$$

Defining the indirect utility functions as

$$v_W(\theta) \equiv \max_{\bar{\theta} \in \Theta} c_W(\bar{\theta}) - \psi \left( \frac{l(\bar{\theta})}{\bar{\theta}} \right) \quad \text{and} \quad v_E(\theta) \equiv \max_{\bar{\theta} \in \Theta} c_E(\bar{\theta}) - \psi \left( \frac{E(\bar{\theta})}{\bar{\theta}} \right) \quad \forall \theta \in \Theta,$$

9Since the cost $\phi$ enters utility additively, it is straightforward to see that, conditional on occupational choice, individuals cannot be further separated based on $\phi$. Hence, indexing the allocation $\{l(\theta), c_W(\theta), L(\theta), E(\theta), c_E(\theta)\}$ by $\theta$ only is without loss of generality.
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and observing that preferences satisfy single-crossing, it is a standard result that the incentive constraints (1.4) and (1.5) are satisfied if and only if the envelope conditions

\[
v'_W(\theta) = \psi' \left( \frac{l(\theta)}{\theta} \right) \frac{l(\theta)}{\theta^2} \quad \text{and} \quad v'_E(\theta) = \psi' \left( \frac{E(\theta)}{\theta} \right) \frac{E(\theta)}{\theta^2} \quad \forall \theta \in \Theta \tag{1.8}
\]

hold and

\[
l(\theta) \quad \text{and} \quad E(\theta) \quad \text{are non-decreasing.}^{10}
\tag{1.9}
\]

Finally, incentive compatibility requires that the critical cost values for occupational choice are given by

\[
\tilde{\phi}(\theta) = v_E(\theta) - v_W(\theta) \quad \forall \theta \in \Theta. \tag{1.10}
\]

Summarizing these insights, the Pareto problem can be written as follows. Let the social planner attach Pareto-weights to individuals depending on their two-dimensional type vector, as captured by cumulative distribution functions \( \tilde{F}(\theta) \) and \( \tilde{G}_\theta(\phi) \). Then the program is

\[
\max \left\{ \tilde{G}_\theta(\tilde{\phi}(\theta)) v_E(\theta) - \int_{\phi} \tilde{\phi}(\phi) d\tilde{G}_\theta(\phi) + (1 - \tilde{G}_\theta(\tilde{\phi}(\theta))) v_W(\theta) \right\} d\tilde{F}(\theta)
\]

subject to

\[
\int_{\Theta} G_\theta(\tilde{\phi}(\theta)) L(\theta) d\tilde{F}(\theta) \leq \int_{\Theta} (1 - G_\theta(\tilde{\phi}(\theta))) l(\theta) d\tilde{F}(\theta), \tag{1.11}
\]

\[
\int_{\Theta} G_\theta(\tilde{\phi}(\theta)) \left[ Y(L(\theta), E(\theta)) - v_E(\theta) - \psi(E(\theta/\theta)) \right] d\tilde{F}(\theta) - \int_{\Theta} (1 - G_\theta(\tilde{\phi}(\theta))) \left[ v_W(\theta) + \psi(l(\theta/\theta)) \right] d\tilde{F}(\theta) \geq 0, \tag{1.12}
\]

and constraints (1.3), (1.6), (1.7), (2.6), (1.9) and (1.10). Inequality (1.11) requires the total amount of labor demand assigned to entrepreneurs not to exceed the total amount of labor supply assigned to workers. Similarly, (1.12) is the resource constraint that makes sure that the total amount of resources produced by the entrepreneurs in the economy covers the consumption allocated to entrepreneurs and workers.\(^{12}\)

\(^{10}\)See, for instance, Fudenberg and Tirole (1991), Theorems 7.2 and 7.3, and Kleven, Kreiner, and Saez (2009), online appendix.

\(^{11}\)Again, additive separability of \( \phi \) implies that any incentive compatible allocation must take a threshold form such that, for all \( \theta \), there is some critical value \( \phi(\theta) \) such that all \( \phi \leq \phi(\theta) \) become entrepreneurs and the others workers.

\(^{12}\)As is standard in the screening literature, I solve the Pareto problem ignoring the monotonicity constraint (1.9), assuming that it is not binding. Otherwise, the Pareto optimum would involve bunching of
1.3. Uniform Tax Treatment of Profits and Income

1.3.2 Properties of Constrained Pareto Optimal Tax Systems

Inspection of the constrained Pareto problem reveals that the wages \( w \) and \( \bar{w} \) enter the program through the no-discrimination constraints (1.3), a property that is referred to as a pecuniary externality. Intuitively, wages have first-order effects on welfare as their ratio determines to what extent the income distributions of the two occupations overlap, and hence which workers and entrepreneurs must be treated the same as a result of the non-discriminating tax treatment of profits and labor income. This has consequences for the amount of redistribution that can be achieved with a single tax schedule. For this reason, whenever wages are not fixed by technology, the optimal tax policy exhibits some non-standard properties. The following two propositions summarize characteristics of constrained Pareto optimal tax systems.

**Proposition 2.** (i) At any Pareto-optimum, \( \bar{w} > w \), and \( \bar{w} E(\theta) > w l(\theta) \) for all \( \theta \in \Theta \).

(ii) \( T'(wl(\theta)) = T'(\bar{w} E(\bar{\theta})) = 0 \) if \( Y(L,E) \) is linear.

(iii) Otherwise, \( T'(wl(\theta)) \) and \( T'(\bar{w} E(\bar{\theta})) \) have opposite signs whenever \( (1.3) \) binds for some \( \theta \in \Theta \).

**Proof.** See Appendix 1.7.1.

The first part of Proposition 2 holds for the same reason as in the equilibrium without taxes: Since profits and labor income are subject to the same tax treatment, the entrepreneurs' marginal product must be higher than the workers', because otherwise nobody would choose to set up a firm. This implies that the top earner at any Pareto optimum is an entrepreneur, and the bottom earner a worker. \(^{13}\)

Part (ii) establishes that the standard results are obtained for the bottom and top marginal tax rates if technology is linear so that wages are fixed: Both the bottom and the top earners should face a zero marginal tax rate, as in Mirrlees (1971). However, this is no longer necessarily true when technology is not linear, as shown in part (iii) of Proposition 2. In this case, since the tax system is restricted not to treat labor income and profits differently, and the ratio of wages determines which types of workers and entrepreneurs have to be treated the same as a result, the optimal policy manipulates effort incentives and thus wages to relax these no-discrimination constraints. This then allows for additional redistribution depending on the set of Pareto-weights.

\(^{13}\) It also implies that the no-discrimination constraints (1.3) do not bind at the top of the skill distribution: There does not exist a worker who achieves the same labor income as the highest skill entrepreneurs' profits, since \( \bar{w} \theta > w \theta \) for all \( \theta \in \Theta \). Hence \( a = \bar{\theta} \) and \( b = (w/\bar{w})\bar{\theta} \).

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some types. In the numerical analysis in Section 1.4.2, I check whether the monotonicity constraint is satisfied at the optimum, and find that bunching does not arise.
The tax system can increase the workers' relative to the entrepreneurs' wage (i.e. decrease $\bar{w}/w$) by encouraging entrepreneurial effort and discouraging labor supply. Therefore, and since part (i) has shown that the set of top earners is exclusively given by entrepreneurs and the lowest income is only earned by workers, the optimal tax schedule involves a negative marginal tax rate at the top and a positive marginal tax rate at the bottom in this case. In addition to redistributing across income/profit-levels directly through the tax schedule $T(.)$, the tax system thus makes use of the indirect general equilibrium effects through wages to achieve redistribution indirectly. This shows that optimal marginal tax rates depend on the degree of substitutability between the inputs of the two occupations in the firms' production function. While most of the public finance literature has typically focused on wage elasticities of effort and the skill distribution to derive optimal tax rates (e.g. Saez (2001)), Proposition 2 demonstrates that production elasticities are similarly important when tax policy is restricted to a single schedule.

This intuition is similar, although more intricate, to earlier models of taxation with endogenous wages, notably Stiglitz (1982). He considers a two-class economy where high and low ability workers' labor supply enter a non-linear aggregate production function differently. Then the top marginal tax rate is negative if the government aims at redistributing from high to low skill agents, because subsidizing the high ability individuals' labor supply reduces their wage and thus relaxes the binding incentive constraint preventing high skill agents from imitating low skill agents. In the present occupational choice model with two-dimensional heterogeneity, however, the income distributions of entrepreneurs and workers overlap, so that the no-discrimination constraints can bind in either direction. In particular, higher ability workers may have to be prevented from mimicking lower skilled entrepreneurs, but since $\bar{w} > w$, it is also possible that lower skilled entrepreneurs want to imitate higher ability workers given that the tax system does not condition on occupational choice, even if the Pareto-weights imply redistribution from high to low skill individuals.

The next proposition contains two results on the effects and desirability of additional tax instruments.

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14 If, by contrast, the Pareto-weights are such that the no-discrimination constraints are relaxed by increasing $\bar{w}/w$, the opposite pattern holds.

15 Allen (1982) analyzes optimal linear taxation with endogenous wages. In this case, the incentive effects of taxes on wages through the labor supply of different income groups are less clear, since all agents face the same marginal tax rate.

16 In section 1.4, I provide conditions that pin down the direction in which the no-discrimination constraints bind, and the optimal top marginal tax rate is indeed negative. They essentially require Pareto-weights such that redistribution from low-$\phi$ agents to high-$\phi$ agents is desirable, and thus from entrepreneurs to workers.
Proposition 3. (i) If, in addition to the non-linear tax $T(.)$ on profits and income, the government can impose a proportional tax on the firms' labor input, then a Pareto-optimal tax system satisfies $T'(wl(\theta)) = T'(\bar{w}E(\theta)) = 0$.

(ii) Moreover, if the government can distort $Y_L(L(\theta), E(\theta))$ across firms, e.g. through a non-linear tax on labor input, then it is optimal to do so whenever $Y(L, E)$ is not linear and the no-discrimination constraints (1.3) bind for some $\theta \in \Theta$.

Proof. See Appendix 1.7.1.

The first part of Proposition 3 demonstrates that the properties derived in the last part of Proposition 2 disappear when the government disposes of an additional instrument. With a proportional tax on the firms' labor input, entrepreneurs face a wage cost of $\tau w$ on their labor rather than the wage $w$ that workers receive. This decouples the scaling factor $\bar{w}/w$ in the no-discrimination constraints (1.3) from the marginal products of entrepreneurial effort and labor in constraints (1.6) and (1.7), so that there remains no need to affect them through the nonlinear tax schedule $T(.)$. As a result, the top and bottom marginal tax rates are again zero at any Pareto optimum, even if technology is not linear.

Whereas a pure profit tax, even when complemented by a proportional tax on labor inputs, always implies that marginal products of labor (and thus of entrepreneurial effort) are equalized across all firms, part (ii) shows that such production efficiency is not necessarily optimal in this framework. Intuitively, by distorting marginal products of labor and effort across firms, e.g. through a non-linear tax on the firms' labor input, the government can make the entrepreneurs' wage $\bar{w}$ vary with skill type. As a result, the rescaling factor $\bar{w}/w$ in the no-discrimination constraints can also vary with $\theta$, depending on how much (and in which direction) the no-discrimination constraint binds at that skill level. Then the government faces a trade-off between production efficiency and relaxing the no-discrimination constraints, which generally involves some degree of production inefficiency at the optimum.\(^\text{17}\)

\section*{1.4 Differential Tax Treatment of Profits and Income}

In this section, I relax the assumption that the government can only impose a single non-linear tax schedule that applies to both labor income and entrepreneurial profits. In contrast, suppose the government is able to condition taxes on occupational choice and thus

\(^{17}\)This is in contrast to the well-known Diamond-Mirrlees Theorem (Diamond and Mirrlees (1971)) in settings without pecuniary externalities. See also Naito (1999) for a related result in the two-class economy introduced by Stiglitz (1982), where production inefficiency is shown to be optimal in an economy with a private and public sector.
1.4. Differential Tax Treatment of Profits and Income

set different tax schedules \( T_y(.) \) for labor income \( y \equiv w_l \) and \( T_\pi(.) \) for profits \( \pi \). Moreover, suppose the government can use any additional tax instrument that is contingent on observables, such as the firms' outputs or labor inputs. Then the main results compared to the previous section will be that (i) the non-linear tax schedules \( T_y \) and \( T_\pi \) are enough to implement the resulting constrained Pareto optima, so that production distortions are no longer desirable, and (ii) redistribution is no longer achieved indirectly through general equilibrium effects, but directly through the tax system. As a result, optimal marginal tax rate formulas for workers and entrepreneurs no longer depend on elasticities of substitution in production. This will be shown in the following.

1.4.1 A Theoretical Characterization

Pareto Optimal Tax Formulas

When the planner is not restricted to a single tax schedule on profits and income, the no-discrimination constraints (1.3) disappear, as do the constraints (1.6) and (1.7) that required the equalization of marginal products across all firms. I am therefore left with the following relaxed Pareto problem:

\[
\max_{E(\theta), L(\theta), \lambda(\theta), v_E(\theta), v_W(\theta), \phi(\theta)} \int_\Theta \left[ \tilde{G}_\theta(\tilde{\phi}(\theta)) v_E(\theta) + (1 - \tilde{G}_\theta(\tilde{\phi}(\theta))) v_W(\theta) \right] d\tilde{F}(\theta) - \int_\Theta \int_{\tilde{\phi}} \phi d\tilde{G}_\theta(\phi) d\tilde{F}(\theta)
\]

s.t. \( \tilde{\phi}(\theta) = v_E(\theta) - v_W(\theta) \quad \forall \theta \in \Theta \)

\[
v'_E(\theta) = E(\theta) \psi'(E(\theta)/\theta)/\theta^2, \quad v'_W(\theta) = l(\theta) \psi'(l(\theta)/\theta)/\theta^2 \quad \forall \theta \in \Theta
\]

\[
\int_\Theta G_\theta(\tilde{\phi}(\theta)) L(\theta) dF(\theta) \leq \int_\Theta (1 - G_\theta(\tilde{\phi}(\theta))) l(\theta) dF(\theta)
\]

\[
\int_\Theta G_\theta(\tilde{\phi}(\theta)) [Y(L(\theta), E(\theta)) - v_E(\theta) - \psi(E(\theta)/\theta)] dF(\theta)
\]

\[
- \int_\Theta (1 - G_\theta(\tilde{\phi}(\theta))) [v_W(\theta) + \psi(l(\theta)/\theta)] dF(\theta) \geq 0
\]

Clearly, the remaining incentive, labor market clearing and resource constraints are the same as before. It can be seen from this formulation that the wages \( \bar{w} \) and \( \bar{w} \) have now dropped out of the planning problem. In other words, the pecuniary externality that resulted from ruling out differential tax treatment in the previous section has disappeared. This leads to the following proposition characterizing the Pareto-optimal tax policy.
1.4. Differential Tax Treatment of Profits and Income

**Proposition 4.**

(i) At any Pareto optimum, $Y_L(L(\theta), E(\theta))$ is equalized across all $\theta \in \Theta$.

(ii) If there is no bunching, $T'_\pi(\pi(\theta))$ and $T'_y(y(\theta))$ satisfy

\[
\frac{T'_\pi(\pi(\theta))}{1 - T'_\pi(\pi(\theta))} = \frac{1 + 1/\epsilon(\theta)}{\theta f(\theta) G_\theta(\phi(\theta))} \int_0^\theta \left[ \tilde{G}_\theta(\phi(\theta)) f(\theta) - G_\theta(\phi(\theta)) f(\theta) + g_\theta(\phi(\theta)) \Delta \Gamma(\theta) f(\theta) \right] d\theta
\]

\[
\frac{T'_y(y(\theta))}{1 - T'_y(y(\theta))} = \frac{1 + 1/\epsilon_y(\theta)}{\theta f(\theta) (1 - G_\theta(\phi(\theta)))] \int_0^\theta \left[ (1 - \tilde{G}_\theta(\phi(\theta))) f(\theta) - (1 - G_\theta(\phi(\theta))) f(\theta) - g_\theta(\phi(\theta)) \Delta \Gamma(\theta) f(\theta) \right] d\theta
\]

with $\Delta T(\theta) \equiv T'_\pi(\pi(\theta)) - T'_y(y(\theta))$.

(iii) $T'_\pi(\pi(\bar{\theta})) = T'_\pi(\pi(\bar{\theta})) = T'_y(y(\bar{\theta})) = T'_y(y(\bar{\theta})) = 0$.

**Proof.** See Appendix 1.7.1. \hfill \square

Proposition 4 shows first that, when allowing for different tax schedules $T_\pi$ and $T_y$, production efficiency is always optimal, since the marginal products of labor and entrepreneurial effort are equalized across all firms. Thus, the non-linear profit and income taxes are actually sufficient to implement any Pareto optimum: No additional tax instruments distorting the firms’ input choices are required.\(^{18}\)

Part (ii) of the proposition derives formulas for the optimal marginal profit and income tax rates. As usual, the optimal marginal tax rate faced by skill type $\theta$ is negatively related to the elasticity of profits (income) with respect to the after-tax wage

\[
\epsilon(\theta) = \frac{\partial \pi(\theta)}{\partial w(1 - T'_\pi(\pi(\theta)))} \frac{w(1 - T'_\pi(\pi(\theta)))}{\pi(\theta)}
\]

(and analogously for income) and the mass of entrepreneurs $f(\theta) G_\theta(\phi(\theta))$ at $\theta$ (this mass is $f(\theta)(1 - G(\phi(\theta)))$ for workers). This accounts for the local effort (labor supply) distortion generated by the marginal tax. The first two terms in the integral, in turn, capture the redistributive effects of the tax schedule, comparing the mass of Pareto-weights $\tilde{G}_\theta(\phi(\theta)) f(\theta)$ for all skill types $\tilde{\theta}$ below $\theta$ to that of the population densities $G_\theta(\phi(\theta)) f(\theta)$ (and again equivalently for workers). The last term in the integrals, finally, captures the effect of differential profit and labor income taxation on occupational choice. Specifically, the mass of agents of skill $\theta$ driven out of entrepreneurship by an infinitesimal increase in profit taxation $T_\pi$ is given by $g_\theta(\phi(\theta)) f(\theta)$, i.e. those individuals who were just indifferent between entrepreneurship and employment before the change. The resulting effect on

\(^{18}\)In a response to the results by Naito (1999), Saez (2004) has argued that the optimality of production inefficiency disappears when the individuals’ decision is not along an intensive (effort) margin, but along an extensive (occupational choice) margin. The present model includes both margins, and points out that it is the availability of tax instruments that is crucial for whether there exists a pecuniary externality, which in turn is the underlying reason for the desirability of production distortions.
the government budget is captured by the excess entrepreneurial tax \( \Delta T(\theta) \), which is the additional tax payment by an entrepreneur of type \( \theta \) compared to a worker of the same skill. Of course, this budget effect appears with opposite signs in the optimality formulas for the entrepreneurial profit and labor income tax schedule.\(^{19}\)

As can be seen from the formulas in Proposition 4, key properties of the restricted tax schedule characterized in the preceding section disappear as soon as differential taxation is allowed. Notably, the tax formulas no longer depend on whether technology is linear or not. Hence, no knowledge about empirical substitution elasticities in production is required to derive optimal marginal tax rates. Differential taxation thus justifies the focus of much of the public finance literature on estimating labor supply elasticities and identifying skill distributions, quite in contrast to the case of uniform taxation considered in the preceding section.

In fact, the wages \( \bar{w} \) and \( w \) earned by entrepreneurs and workers do not even appear in the formulas. Moreover, the bottom and top marginal tax rates are always zero, both for workers and entrepreneurs. In the present setting with a bounded support of the skill distribution, these results show that differential taxation generally allows for a Pareto improvement compared to uniform taxation: Since any Pareto optimum with differential taxation must be such that the bottom and top marginal tax rates for both workers and entrepreneurs are zero, any allocation that does not satisfy these properties must be Pareto inefficient. But Proposition 2 has shown that, whenever uniform taxation leads to binding no-discrimination constraints, the bottom and top marginal tax rates are not zero. Hence, starting from such an allocation, there must exist a Pareto improvement using differential taxation.

The following result is an immediate corollary of Proposition 4.

**Corollary 1.** With a constant elasticity \( \varepsilon \),\(^{20}\) the average marginal tax across occupations satisfies

\[
G_\theta(\bar{\phi}(\theta)) \frac{T'_{\pi}(\pi(\theta))}{1 - T'_{\pi}(\pi(\theta))} + (1 - G_\theta(\bar{\phi})) \frac{T'_{y}(y(\theta))}{1 - T'_{y}(y(\theta))} = \frac{1 + 1/\varepsilon}{\theta f(\theta)} (F(\theta) - F(\theta)). \tag{1.13}
\]

Note that the formula for the average marginal tax rate across entrepreneurs and

\[^{19}\text{See Kleven, Kreiner, and Saez (2009) for similar results and interpretations in a model with a secondary earner participation margin. Rather than tracing out the Pareto-frontier, however, they work with a concave social welfare function, which gives rise to different optimal tax formulas.}\]

\[^{20}\text{Even without a constant elasticity, a modified version of (1.13) holds, with is that}\]

\[
G_\theta(\bar{\phi}(\theta)) \frac{T'_{\pi}(\pi(\theta))}{1 + 1/\varepsilon_{\pi}(\theta) - T'_{\pi}(\pi(\theta))} + (1 - G_\theta(\bar{\phi})) \frac{T'_{y}(y(\theta))}{1 + 1/\varepsilon_{y}(\theta) - T'_{y}(y(\theta))} = \frac{\bar{F}(\theta) - F(\theta)}{\theta f(\theta)}.
\]

Thus, except for the nicer expression, Corollary 1 does not depend on a constant elasticity.
workers of a given skill type is given in closed form on the right-hand side of equation (1.13): It only depends on the elasticity parameter ε, the distribution of skill types as captured by \( f(\theta) \) and \( F(\theta) \), and the redistributive motives of the government in the skill dimension, determined by the cumulative Pareto-weights \( \bar{F}(\theta) \). In particular, the distribution of cost types \( \phi \), or redistributive motives in the cost dimension as captured by the Pareto-weights \( \bar{G}(\phi) \), play no role. This implies a separation result for the implementation of Pareto optima: Average marginal taxes across occupational groups are set so as to achieve the desired redistribution in the skill dimension. Then any redistribution across cost types and hence between entrepreneurs and workers of the same skill is achieved by varying the marginal profit and income taxes, leaving the average tax unaffected. In fact, the formula for a Pareto optimal average marginal tax rate in (1.13) is the same as the one that would be obtained in a standard quasi-linear Mirrlees-model without occupational choice and with only one-dimensional heterogeneity in \( \theta \).\(^{21}\)

**Testing the Pareto Efficiency of Tax Schedules**

Rather than determining the optimal shape of tax schedules for a given specification of Pareto-weights, the results in Proposition 4 can also be used as a test for whether some given tax schedules \( T_\pi \) and \( T_y \) are Pareto optimal. This approach has been pursued by Werning (2007) in the standard Mirrlees model, and provides an interesting reinterpretation of the formulas in Proposition 4 in the present framework. In fact, since the Pareto-weights \( \bar{G}(\bar{\phi}(\theta)))f(\theta) \) and \( (1 - \bar{G}(\bar{\phi}(\theta)))f(\theta) \) must be non-negative, the following corollary can be obtained immediately from Proposition 4:

**Corollary 2.** Given the utility function \( u(c,e) = c - e^{1+1/\epsilon}/(1 + 1/\epsilon) \), a skill distribution \( F(\theta) \) and cost distribution \( G_\theta(\phi) \), the tax schedules \( T_\pi, T_y \) inducing an allocation \( (\pi(\theta), y(\theta)) \) and occupational choice \( \bar{\phi}(\theta) \) are Pareto optimal if and only if

\[
\frac{\theta f_E(\theta)}{1 + 1/\epsilon} \frac{T'_\pi(\pi(\theta))}{1-T'_\pi(\pi(\theta))} + F_E(\theta) - \int_\theta^\infty \frac{g_\theta(\bar{\phi}(\theta))f(\theta)}{G} \Delta T(\theta)d\theta \quad \text{and} \quad (1.14)
\]

\[
\frac{\theta f_W(\theta)}{1 + 1/\epsilon} \frac{T'_y(y(\theta))}{1-T'_y(y(\theta))} + F_W(\theta) + \int_\theta^\infty \frac{g_\theta(\bar{\phi}(\theta))f(\theta)}{1-G} \Delta T(\theta)d\theta
\]

are non-decreasing in \( \theta \), where \( \bar{G} = \int_\Theta G_\theta(\bar{\phi}(\theta))dF(\theta) \) is the overall share of entrepreneurs in the population, \( f_E(\theta) = G_\theta(\bar{\phi}(\theta))f(\theta)/\bar{G} \) and \( f_W(\theta) = (1 - G_\theta(\bar{\phi}(\theta)))f(\theta)/(1 - \bar{G}) \) are the

\(^{21}\)See Diamond (1998) for such an analysis. However, since in his model redistribution is determined by a concave social welfare function rather than by Pareto-weights that trace out the entire Pareto-frontier, a closed form solution for the optimal marginal tax rates as in (1.13) cannot be obtained.
skill densities for entrepreneurs and workers, and $F_E(\theta)$ and $F_W(\theta)$ the corresponding cumulative distribution functions.

For a given elasticity $\varepsilon$, conditions (1.14) and (1.15) can be tested after identifying the skill and cost distributions from the observed income distributions and shares of entrepreneurs and workers for a given tax system. This identification step has been pioneered by Saez (2001) in a one-dimensional taxation model, and will be extended in Section 1.4.2 to the setting with two-dimensional heterogeneity and occupational choice considered here.

Two remarks on Corollary 2 are in order. First, adding conditions (1.14) and (1.15) yields another test for Pareto optimality, which is weaker but requires less information to be implemented. In particular, a necessary condition for $T_\pi, T_y$ to be Pareto optimal is that

$$\frac{\theta f(\theta)}{1 + 1/\varepsilon} \left[ G_\theta(\bar{\phi}(\theta)) \frac{T'_\pi(\pi(\theta))}{1 - T'_\pi(\pi(\theta))} + (1 - G_\theta(\bar{\phi}(\theta))) \frac{T'_y(y(\theta))}{1 - T'_y(y(\theta))} \right] + F(\theta)$$

is non-decreasing in $\theta$. This condition, relying on the average marginal tax rate of entrepreneurs and workers at a given skill level, only requires the identification of the skill distribution $F(\theta)$, not of the cost density $g_\theta(\phi)$ (note that $G_\theta(\bar{\phi}(\theta))$ can be easily inferred from the share of entrepreneurs at a given profit and hence skill level). However, this test is obviously weaker since some tax systems that pass it may fail the test in Corollary 2 and thus be Pareto inefficient.

Another special case of conditions (1.14) and (1.15) occurs when there is no occupational choice, so that whether an individual is an entrepreneur or a worker is a fixed characteristic. This can be thought of as a special case of the general formulation considered so far, where the cost $\phi$ has a degenerate distribution with only two mass points, at 0 and $\bar{\phi}$, and $\bar{\phi}$ is sufficiently high. Then $T_\pi$ must be such that

$$\frac{\theta f_E(\theta)}{1 + 1/\varepsilon} \frac{T'_\pi(\pi(\theta))}{1 - T'_\pi(\pi(\theta))} + F_E(\theta)$$

is non-decreasing, and analogously for $T_y$ replacing the (fixed) skill distribution for entrepreneurs by that for workers, $F_W(\theta)$. This coincides with the integral version of the condition derived in Werning (2007) for a standard Mirrlees model. Hence, the key difference arising from the present framework are the terms $-\int_\theta^\phi g_\theta(\bar{\phi}(\hat{\theta})) f(\hat{\theta}) \Delta T(\hat{\theta}) d\hat{\theta}/G$ and $\int_\theta^\phi g_\theta(\bar{\phi}(\hat{\theta})) f(\hat{\theta}) \Delta T(\hat{\theta}) d\hat{\theta}/(1 - G)$, reflecting the effects of taxation on occupational choice and thus on the resource constraint. Note that, since these terms enter conditions (1.14) and (1.15) with opposite signs, whenever one term is increasing in $\theta$, the other is decreasing, so that ceteris paribus it becomes harder for differential taxation with $\Delta T(\theta) \neq 0$. 

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to pass the test for Pareto efficiency the more elastic the occupational choice margin (and thus the higher the cost density at the critical level \( \phi(\theta) \)).

**Comparing Optimal Profit and Income Tax Schedules**

How do the optimal tax schedules for entrepreneurial profits and labor income compare under given redistributive objectives and thus Pareto-weights? To shed light on this question, I make the following two assumptions.

**Assumption 1.** \( \theta \) and \( \phi \) are independent and \( g(\phi) \) is non-increasing.

These assumptions are strong, and will be relaxed in the numerical explorations that follow in Section 1.4.2. Nonetheless, they allow me to obtain a theoretical characterization of the pattern of differential taxation of profits and income. I start with the case where the government aims at redistributing from entrepreneurs to workers.

**Proposition 5.** Suppose that \( \tilde{F}(\theta) = F(\theta) \), \( \tilde{g}(\phi) < g(\phi) \) for all \( \phi \leq \phi(\bar{\theta}) \) and Assumption 1 holds. Then

(i) \( T'_y(y(\theta)) < 0 \), \( T'_\pi(\pi(\theta)) > 0 \) for all \( \theta \in (\theta, \bar{\theta}) \),

(ii) \( \Delta T(\theta) > 0 \) and \( \Delta T'(\theta) > 0 \) for all \( \theta \in \Theta \), and

(iii) compared to the no tax equilibrium, \( w \) decreases, \( \bar{w} \) increases and \( L(\theta)/E(\theta) \) rises for all \( \theta \in \Theta \).

**Proof.** See Appendix 1.7.1.

The assumptions in Proposition 5 focus on the benchmark case where the government does not aim at redistributing across skill types (since \( \tilde{F}(\theta) = F(\theta) \) for all \( \theta \)), but puts a lower social welfare weight on low \( \phi \)-types (who end up as entrepreneurs) than their density in the population. This generates a redistributive motive from low to high cost types, and thus from entrepreneurs to workers. Corollary 1 immediately implies that, in this case, the average marginal tax rate must be zero for all skill types. The first part of Proposition 5 shows that, in fact, workers face a negative marginal tax rate and entrepreneurs a positive one at the optimum.\(^{22}\) Moreover, as a result of the redistributive motive from entrepreneurs to workers, there is a strictly positive excess profits tax \( \Delta T(\theta) \), which increases with the skill level.

\(^{22}\)This implies that \( E(\theta) < (w/\bar{w})((\bar{w}/w)\theta) \) for all \( \theta \in \Theta \). Hence, under the assumptions in Proposition 5 and Assumption 1, the no-discrimination constraints (1.3) in the previous Section 1.3 all bind in the same direction and such that the optimal restricted tax schedule involves a positive bottom and a negative top marginal tax rate.
1.4. Differential Tax Treatment of Profits and Income

It also turns out that the optimal policy involves a decrease in the workers’ wage, and makes the input mix of all firms more labor intensive compared to the no tax equilibrium. This is quite in contrast to the intuition based on a “trickle down” argument, which would have suggested a policy that increases the workers’ wage in order to benefit them indirectly. Here, however, this is not necessary since workers can be overcompensated for the decrease in their wage through the differential tax treatment directly, as captured by the positive excess tax on entrepreneurs. The reason for the depressed wage $w$ is that the excess profit tax discourages entry into entrepreneurship, and therefore the workers’ wage must fall so that each firm hires more labor and the labor market remains cleared.

If the Pareto-weights are such that $F(O) \neq F(\theta)$ for some $\theta$, so that redistribution across skill types is also desirable, then a comparison of the tax schedules for entrepreneurs and workers becomes more involved. A theoretical result is available for the following benchmark case. Suppose that $G(\phi) = G(\phi)$ for all $\theta \in \Theta$, but $F(\theta) \neq F(\theta)$. Also, suppose there is no occupational choice margin, but each individual’s occupation is in fact fixed and independent of the skill type, so that $G_\theta = G$ for all $\theta \in \Theta$. Then Proposition 4 implies

$$\frac{T'_\pi(\pi(\theta))}{1 - T'_\pi(\pi(\theta))} = \frac{T'_y(y(\theta))}{1 - T'_y(y(\theta))} = 1 + \frac{1}{e} \frac{1}{\theta f(\theta)} (F(\theta) - F(\theta))$$

for any $w, \bar{w}$. Hence, when the occupational choice margin is removed, the optimal marginal tax rates are the same for entrepreneurs and workers (and equal to the average marginal tax rate from Corollary 1), independently of the different wages in the two occupations. This makes clear that any difference in the optimal tax schedules for profits and income must be the result of an active occupational choice margin or a non-zero correlation between ability and occupational choice, which will be further explored in the subsequent numerical simulations.

1.4.2 A Quantitative Exploration

To further explore the importance of a differential tax treatment of profits and income, I provide a quantitative illustration of the analysis so far by computing optimal tax systems under various redistributive objectives. Notably, the formulas in Proposition 4 can be used to compute the tax schedules $T_\pi$ and $T_y$ once distributions for $\theta$ and $\phi$, Pareto-weights, a production function and preferences are specified.\textsuperscript{23} To calibrate the model, I use data on income, profits, and entrepreneurship from the 2007 Survey of Consumer

\textsuperscript{23}I use an iterative numerical procedure that is adapted from Kleven, Kreiner, and Saez (2009) and specified in Appendix 1.7.2.
1.4. Differential Tax Treatment of Profits and Income

Finances (SCF). I restrict the sample to household heads aged between 18 and 65 who are not unemployed/retired, and define the empirical counterpart of entrepreneurs in my model as those individuals who (i) are self-employed, (ii) own a business, (iii) actively manage it, and (iv) employ at least two employees. This is a widely used empirical definition of the notion of entrepreneurship. All other individuals in the sample are considered as workers.

Table 1 presents descriptive statistics of the resulting sample. A share of 7.4% of the sample ends up being classified as entrepreneurs according to the above criteria. Consistent with the theoretical findings so far, entrepreneurs have higher incomes than workers, even though the higher means come at the price of a higher income variability than for workers, as captured by the standard deviation. This suggests that entrepreneurship is more risky than employment, an aspect that will be accounted for explicitly in the next section. Entrepreneurs also work more than workers, as measured by yearly hours. Still, their wage, computed as the ratio of yearly income and hours, is higher than that of workers. This is consistent with the evidence on entrepreneurial incomes and wages in De Nardi, Doctor, and Krane (2007) and Berglann, Moen, Roed, and Skogstrom (2009).

<table>
<thead>
<tr>
<th></th>
<th>Entrepreneurs</th>
<th>Workers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>48.4</td>
<td>42.1</td>
</tr>
<tr>
<td>Yearly Income (in 1000$)</td>
<td>88.5</td>
<td>69.5</td>
</tr>
<tr>
<td>Hours per Week</td>
<td>48.3</td>
<td>43.4</td>
</tr>
<tr>
<td>Weeks per Year</td>
<td>50.2</td>
<td>50.4</td>
</tr>
<tr>
<td>Wage per Hour (in $)</td>
<td>55.5</td>
<td>34.6</td>
</tr>
</tbody>
</table>

For the baseline calibration, I work with the following parametric specifications: The disutility of effort takes the iso-elastic form \( \psi(e) = e^{1+1/\epsilon}/(1 + 1/\epsilon) \), and, based on the empirical labor supply literature, the wage elasticity of effort is set to be \( \epsilon = .25 \). The constant returns to scale technology used by entrepreneurs is captured by a Cobb-Douglas

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25In contrast, Hamilton (2000) and Blanchflower (2004) find lower returns to entrepreneurship than employment, but their definition of entrepreneurship is only based on self-employment and thus less restrictive than the concept used here.
1.4. Differential Tax Treatment of Profits and Income

production function \( Y(L, E) = L^a E^{1-a} \) with the parameter \( a \) set to equal the workers' share of income in the SCF data, so that \( a = .63 \).

To identify the skill distribution \( F(\theta) \), I use the empirical income distributions of entrepreneurs and workers in the SCF. However, since the SCF does not include information on marginal tax rates faced by individuals, which is required to perform the identification step, I impute marginal tax rates as follows. I adopt the flexible functional form for average taxes \( \tau(y) \) as a function of profits/income \( y \) suggested by Gouveia and Strauss (1994):

\[
\tau(y) = b - b \left[ sy^p + 1 \right]^{-1/p}.
\] (1.16)

The parameters \( b, s \) and \( p \) are estimated by Cagetti and De Nardi (2009) using PSID data for entrepreneurs and workers separately, with point estimates \( b = .26, s = .42 \) and \( p = 1.4 \) for entrepreneurs and \( b = .32, s = .22 \) and \( p = .76 \) for workers. Then I obtain marginal tax rates from the average tax rates in (1.16). With this information, I am able to identify \( w_\theta \) for workers and \( w_\theta \) for entrepreneurs from the first order conditions of the individuals' utility maximization problem

\[
1 - T^\pi_\theta(\pi) = \frac{\pi^{1/\epsilon}}{(w_\theta)^{1+1/\epsilon}} \quad \text{and} \quad 1 - T^\theta_\pi(y) = \frac{y^{1/\epsilon}}{(w_\theta)^{1+1/\epsilon}}
\]

for entrepreneurs and workers, respectively. Finally, \( \bar{w} \) and \( w \) are found such that \( \bar{w}/w \) equals the ratio of the mean wages of entrepreneurs and workers in the SCF data, using the fact that \( \bar{w} = (1 - \alpha) (\alpha/w)^{\frac{\alpha}{1-\alpha}} \) with Cobb-Douglas technology.

![Figure 1.1: Skill density and share of entrepreneurs](image)

The left panel of Figure 1.1 depicts a kernel estimate of the resulting inferred skill density, truncated at the 99 percentile. The smoothed approximation of it, also depicted, is used as \( f(\theta) \) in the simulations to obtain smoother optimal tax schedules. The right panel
in turn shows the share of entrepreneurs as a function of the skill level $\theta$, which is the result of a locally weighted regression of the indicator variable for whether an individual is an entrepreneur on $\theta$. As can be seen from the graph, the share of entrepreneurs is increasing in $\theta$, except for the lowest skill levels.\footnote{This U-shaped pattern is in line with the evidence in Parker (1997), who finds that entrepreneurs are over-represented at both the highest and lowest ends of the overall income distribution in the UK.} I use this pattern to calibrate the cost distribution $G_\theta(\phi)$. In particular, I assume an iso-elastic specification with $G_\theta(\phi) = (\phi/\bar{\phi}_\theta)^\eta$ and $\eta = .5$, implying an elasticity of occupational choice of .5. Then the upper bound of the support $\bar{\phi}_\theta$ is adjusted to generate the pattern of the share of entrepreneurs in the right panel of Figure 1.1.

![Graph showing marginal tax rates, tax schedules, share of entrepreneurs, and excess profit tax as functions of skill, both for the no tax equilibrium as well as for the case with taxation.](image)

Figure 1.2: Pareto weights $\tilde{G}_\theta(\phi) = G_\theta(\phi)^{\rho_\phi}$, $\rho_\phi = 2$

Figure 1.2 starts with the case of redistribution across cost types only, and hence from entrepreneurs to workers, with Pareto weights $\tilde{F}(\theta) = F(\theta)$ and $\tilde{G}_\theta(\phi) = G_\theta(\phi)^{\rho_\phi}$, $\rho_\phi = 2$. It depicts the marginal tax schedules $T'_\pi$ and $T'_y$, the tax schedules $T_\pi$ and $T_y$, the excess profit tax $\Delta T$, and the share of entrepreneurs $G(\phi(\theta))$ as a function of skill, both for the no tax equilibrium as well as for the case with taxation. The figure illustrates the results from Proposition 5: The marginal tax rates for entrepreneurs are positive, for workers negative, and there is a positive and increasing excess profit tax. Entry into entrepreneurship is discouraged for individuals of all skill levels compared to the no tax equilibrium. Moreover, the workers' wage falls by 11% from the no tax equilibrium to the equilibrium with taxation.
1.4. Differential Tax Treatment of Profits and Income

Figure 1.3: Pareto weights $\bar{F}(\theta) = F(\theta)^{1/\rho_\theta}, \rho_\theta = 2$

Figure 1.3 illustrates the other benchmark case, when the Pareto weights are such that redistribution across skill types only is implied. In particular, it assumes $\bar{F}(\theta) = F(\theta)^{1/\rho_\theta}$ with $\rho_\theta = 2$. In this case, both marginal tax schedules are positive (as is the average marginal tax) and such that entrepreneurs of low skill levels face a higher marginal tax rate than workers of the same skill, and the opposite relation at high skill levels. The excess profit tax remains positive and increasing, but is considerably smaller than in the case where redistribution from entrepreneurs to workers directly is desired. Again, the wage falls (by 3% compared to the no tax equilibrium) and entry into entrepreneurship is discouraged slightly, notably for higher skill levels for whom the excess profit tax is higher.

Figure 1.4 depicts the solution when the planner aims at redistributing in both dimensions of heterogeneity, so that both $\bar{C}_\theta(\phi) \leq G_\theta(\phi)$ and $\bar{F}(\theta) \geq F(\theta)$. Such redistributive objectives turn out to justify entrepreneurs facing both higher levels of taxation as well as higher marginal tax rates than workers for all skill levels, while all agents face positive marginal tax rates. Finally, Figure 1.5 shows a robustness check from increasing the elasticity of effort from $\varepsilon = .25$ to $.5$. This makes lower marginal tax rates in both occupations optimal, holding Pareto-weights fixed. Again, the workers’ wage falls (by 10%) as a result of the tax policy compared to the no tax equilibrium, entry into entrepreneurship is discouraged, and there emerges a positive and increasing excess profit tax $\Delta T$. All these effects appear as robust properties of optimal tax schedules from these quantita-
1.5. Entrepreneurial Taxation with Credit Market Frictions

The previous sections have shown that entrepreneurial taxation, in the form of a differential tax treatment of entrepreneurial profits and labor income, plays a role for redistribution and for dealing with pecuniary externalities that arise without such discrimination. However, the analysis so far has ignored the entrepreneurs’ investment and borrowing decisions, which are important aspects of entrepreneurial activity. The concern is that, when entrepreneurs have to borrow the funds required for investment in credit markets that are subject to imperfections, then the aggregate level of entrepreneurship may be inefficiently low.\(^{27}\) This may then generate an economic force that pushes towards a lower

\(^{27}\)See e.g. Evans and Leighton (1989), Hurst and Lusardi (2004) and De Nardi, Doctor, and Krane (2007) for evidence on the importance of borrowing frictions for entrepreneurship.
taxation of entrepreneurial profits compared to what the previous analysis has implied. However, it turns out that credit market frictions do not necessarily discourage entry into entrepreneurship across the board. While I do find that the no tax equilibrium, which involves adverse selection, is no longer efficient, in contrast to the no tax equilibrium characterized in Section 1.2.2, it provides incentives to enter entrepreneurship to the wrong mix of individuals, with too many and too few entrepreneurs at the same time. Even without redistributive objectives, this generates scope for a corrective role of entrepreneurial taxation, but it does not take the form of a general subsidization of entrepreneurship, as I show in the following.

1.5.1 Entrepreneurial Investment and Borrowing

Suppose that, to set up a firm, each entrepreneur has to make a fixed investment $I$. Yet, agents are born without wealth, and hence have to borrow these funds from banks in a competitive credit market. Since I am mainly interested in the efficiency of occupational choice in the following, I will simplify the model slightly in another dimension and assume that there is no intensive margin, i.e. entrepreneurial effort and labor supply are fixed. More precisely, let all workers supply some fixed amount of labor $l$ and receive
utility \( v_W = wL \). As before, entrepreneurs hire labor, but now produce stochastic profits

\[
\pi = Y(L) - wL + \epsilon,
\]

where \( \epsilon \) has some cdf \( H_\epsilon(\epsilon|\theta) \) that depends on the entrepreneurs’ skill \( \theta \). In particular, I assume that \( H_\epsilon(\epsilon|\theta) \geq \text{MLRP} \) \( H_\epsilon(\epsilon|\theta') \) for \( \theta > \theta' \), i.e. a higher skilled entrepreneur has a distribution of \( \epsilon \) that is better in the sense of the monotone likelihood ratio property.\(^{28}\)

There is a large number of risk-neutral banks, offering credit contracts that supply funding \( I \) in return for a repayment schedule \( R_\theta(\pi) \).\(^{29}\) The expected utility of an entrepreneur of ability type \( \theta \) and cost type \( \phi \) from such a contract is then

\[
\int_{\mathcal{E}} [Y(L) - wL + \epsilon - R_\theta(Y(L) - wL + \epsilon)] dH_\epsilon(\epsilon|\theta) - \phi,
\]

where \( \mathcal{E} \) is the support of \( H_\epsilon \). Both \( \theta \) and \( \phi \) are private information as before.

Let me first observe that, given this specification, for any given set of contracts \( \{ R_\theta(\pi) \} \), all entrepreneurs hire the same amount of labor such that \( Y'(L) = w \), and hence I can work directly with the resulting distribution of profits \( \pi \sim H(\pi|\theta) \), with the support denoted by \( \Pi \). I drop the additional dependence of \( H \) and \( \Pi \) on the wage \( w \) for notational simplicity. This leads to the following definition of a credit market equilibrium, taking a wage \( w \) as given.

### 1.5.2 Credit Market Equilibrium

**Definition 2.** A credit market equilibrium is a set of contracts \( \{ R_\theta(\pi) \} \) such that

(i)

\[
\int_{\Pi} (\pi - R_\theta(\pi)) dH(\pi|\theta) \geq \int_{\Pi} (\pi - R_{\theta'}(\pi)) dH(\pi|\theta) \quad \forall \theta, \theta' \in \Theta,
\]

(ii)

\[
\int_{\Theta} G(\tilde{\phi}(\theta)) \left[ \int_{\Pi} R_\theta(\pi) dH(\pi|\theta) - I \right] dF(\theta) \geq 0
\]

with

\[
\tilde{\phi}(\theta) = \int_{\Pi} (\pi - R_\theta(\pi)) dH(\pi|\theta) - v_W \quad \forall \theta \in \Theta, \quad \text{and}
\]

\(^{28}\)A stochastic component of profits is introduced to obtain non-trivial credit market equilibria. Otherwise, competitive banks will have entrepreneurs just repay \( I \) after profits have been earned.

\(^{29}\)I introduce the index \( \theta \) since, in a separating equilibrium, banks may offer different credit contracts to entrepreneurs of different quality levels \( \theta \). Of course, any such assignment has to be incentive compatible, as specified below.
(iii) there exists no other set of contracts \( \{ \tilde{R}_\theta(\pi) \} \) that earns strictly positive profits when offered in addition to \( \{ R_\theta(\pi) \} \) and all individuals select their preferred occupation and preferred contract from \( \{ \tilde{R}_\theta(\pi) \} \cup \{ R_\theta(\pi) \} \).

Equation (1.17) is the set of incentive constraints, which require that each entrepreneur is willing to select the credit contract \( R_\theta(\pi) \) intended for her. Constraint (1.18) makes sure that the set of equilibrium credit contracts make non-negative profits in aggregate when taken up by the agents who select into entrepreneurship, as given by the critical cost values \( \hat{\phi}(\theta) \) for all \( \theta \in \Theta \). Finally, the last part of the definition rules out profitable sets of deviating contract offers. Note that, although the structure of this definition is similar to Rothschild and Stiglitz (1976), it is considerably more general by letting banks offer sets of contracts, thus allowing for cross-subsidization between different contracts in equilibrium.

Following Innes (1993), I restrict attention to contracts \( R_\theta(\pi) \) that satisfy the following two properties: First, \( 0 \leq R_\theta(\pi) \leq \pi \) for all \( \theta \in \Theta \) and \( \pi \in \Pi \), which is a standard limited liability constraint. Second, \( R_\theta(\pi) \) is non-decreasing in \( \pi \) so that, when the entrepreneur earns higher profits, the repayment received by the bank \( R_\theta(\pi) \) is also higher. This monotonicity constraint can be motivated by noting that banks may have means to reduce firm profits, if they want to. For instance, they may compel the entrepreneur to prove her output level to an impartial third party and to bear the related audit costs, thus reducing the amount of profits available for distribution.\(^{30}\)

Under these assumptions, the following characterization of a credit market equilibrium as defined in Definition 2 can be obtained.

**Proposition 6.** Under Assumption 1, the credit market equilibrium is such that only the single contract \( R_{z^*}(\pi) = \min\{\pi, z^*\} \) is offered and \( z^* \) solves

\[
\int_{\Theta} G(\tilde{\phi}_{z^*}(\theta)) \left[ \int_{\Pi} \min\{\pi, z^*\} dH(\pi|\theta) - 1 \right] dF(\theta) = 0 \tag{1.19}
\]

with

\[
\tilde{\phi}_{z^*}(\theta) = \int_{\Pi} (\pi - \min\{\pi, z^*\}) dH(\pi|\theta) - v_w \quad \forall \theta \in \Theta. \tag{1.20}
\]

*Proof.* See Appendix 1.7.3. \( \square \)

Proposition 6 shows first that an equilibrium always exists. Second, it is such that entrepreneurs of all ability types are pooled in the same contract. Both of these results are very different from the canonical competitive screening model by Rothschild and Stiglitz.\(^{30}\)

\(^{30}\)In addition, it is straightforward to see that whenever the stochastic profit component \( \epsilon \) has only two possible realizations, any repayment scheme that guarantees the bank non-negative profits in equilibrium must be weakly increasing in \( \pi \).
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(1976). In particular, in their model, even when restricting each bank to offer a single contract only, an equilibrium may fail to exist, and it can never take the pooling form. Moreover, an equilibrium as specified in Definition 2, allowing for cross-subsidization, would fail to exist under an even larger set of parameters in the Rothschild-Stiglitz model, as shown by Wilson (1977) and Miyazaki (1977). However, the present model is quite different due to risk-neutrality of all parties and endogenous entry into the credit market.

The second key result in the proposition is that the equilibrium credit contract takes the very simple form of a debt contract: It specifies a fixed repayment level $z^*$, which the entrepreneur has to return to the bank whenever she can, i.e. whenever $\pi \geq z^*$. Otherwise, the firm goes bankrupt, and the entire amount of profits goes to the bank, with the entrepreneur hitting her liability limit and thus being left with zero consumption. This results in the contract $R_{z^*}(\pi) = \min\{\pi, z^*\}$, where $z^*$ is such that the banks' expected profit is zero given the set of agents who enter the credit market when anticipating the equilibrium contract $R_{z^*}(\pi)$. The intuition for this debt contracting result is as follows: By the monotone likelihood ratio property, low-skill entrepreneurs have a larger probability weight in low-profit states. Among all contracts satisfying the monotonicity constraint, debt contracts in turn are the ones that put the maximal repayment weight in these low-profit states. As a result, debt contracts are least attractive to low-skill borrowers, and hence any set of deviation contracts that do not take the debt contract form would attract a lower quality borrower pool and generate lower profits for banks.

1.5.3 Inefficiency of Occupational Choice and Entrepreneurial Tax Policy

The preceding subsection has characterized credit market equilibria for any given wage $w$. Of course, an equilibrium in the entire economy is then given by a wage $w^*$ such that the labor market clears, i.e.

$$\int_{\Theta} G(\tilde{\phi}^*_{z^*(w^*)}(\theta))L(w^*)dF(\theta) = \int_{\Theta} (1 - G(\tilde{\phi}^*_{z^*(w^*)}(\theta)))dF(\theta),$$

(1.21)

where I have now written $z^*(w)$ to clarify the dependency of the credit market equilibrium on the wage, and where $L(w)$ comes from the entrepreneurs solving $Y'(L) = w$.

I now ask whether the no tax equilibrium in this economy involves the efficient occupational choice. In fact, efficiency would require that a type $(\theta, \phi)$ becomes an en-

\[31\] See also Innes (1990) and Innes (1993) for related results in models with moral hazard and one-dimensional private heterogeneity.
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trepreneur if and only if
\[ \int_\Pi \pi dH(\pi|\theta) - I - \phi \geq v_W, \]
i.e. her expected profits minus the investment outlays minus the utility cost \( \phi \) exceed the utility from being a worker \( v_W \).\(^{32}\) This can be solved for the efficient critical cost value
\[ \bar{\phi}_e(\theta) \equiv \int_\Pi \pi dH(\pi|\theta) - I - v_W \] (1.22)
for any \( \theta \in \Theta \). Then the following result is a corollary of Proposition 6:

**Corollary 3.** There exists a skill-type \( \bar{\theta} \) s.t.
\[ \int_\Pi \min\{\pi, z^*\} dH(\pi|\bar{\theta}) = I \]
\[ \bar{\phi}_z(\theta) > \bar{\phi}_e(\theta) \quad \forall \theta < \bar{\theta} \quad \text{and} \quad \bar{\phi}_z(\theta) < \bar{\phi}_e(\theta) \quad \forall \theta > \bar{\theta}. \]

**Proof.** First, \( \int_\Pi \min\{\pi, z^*\} dH(\pi|\theta) \) is increasing in \( \theta \) by the monotone likelihood ratio property. Second, \( \bar{\theta} \) exists by the aggregate zero profit constraint (1.19). Third, by (1.20) and (1.22), \( \bar{\phi}_e(\theta) \geq \bar{\phi}_z(\theta) \) if and only if
\[ \int_\Pi \min\{\pi, z^*\} dH(\pi|\theta) \geq I. \]

Since the credit market equilibrium is a pooling equilibrium, it involves cross-subsidization across entrepreneurs of different quality \( \theta \). In particular, by the monotone likelihood ratio property, banks make higher profits with higher ability entrepreneurs, and thus by the zero profit condition (1.19), there exists some critical skill level \( \bar{\theta} \) such that banks make profits with all higher quality entrepreneurs and negative profits with all the others. But this cross-subsidization implies that, compared to the efficient occupational choice defined in (1.22), low skilled agents have too strong incentives to set up a firm, and too many high skill agents stay in the workforce. In other words, the credit market equilibrium generates occupational misallocation such that there is excessive entry of low ability types into entrepreneurship, but insufficient entry of high-skilled types. This can be seen most easily by substituting the equilibrium zero profit condition (1.19) into equation (1.22), solving the former for \( I \):

\[ \bar{\phi}_e(\theta) = \int_\Pi \pi dH(\pi|\theta) - \frac{\int_\Theta G(\bar{\phi}_z(\theta)) \int_\Pi \min\{\pi, z^*\} dH(\pi|\theta)dF(\theta)}{\int_\Theta G(\bar{\phi}_z(\theta))dF(\theta)} - v_W \]

and comparing it with the equilibrium critical values for occupational choice in (1.20):

\[ \bar{\phi}_z(\theta) = \int_\Pi \pi dH(\pi|\theta) - \int_\Pi \min\{\pi, z^*\} dH(\pi|\theta) - v_W. \]

\(^{32}\)This definition holds for any given wage, as does all of the following analysis.
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Notably, this comparison clearly identifies the importance of cross-subsidization as the source of the inefficiency: If $\Theta$ is singleton, for instance, then $\hat{\phi}_e^*(\theta) = \hat{\phi}_e(\theta)$.

A special case of this misallocation obtains when $\phi$ is the same for all agents, so that there only remains one-dimensional heterogeneity in ability. Then the equilibrium involves an excessive entry into entrepreneurship, with too many low-skill types receiving funding in the credit market. This observation has been made first by De Meza and Webb (1987) in a model where agents choose between a safe investment and a risky project (entrepreneurship) with binary output. This extreme case is quite in contrast to the seminal analysis by Stiglitz and Weiss (1976), who emphasized credit rationing and thus insufficient entry into entrepreneurship in a model where entrepreneurs differ in the riskiness of their projects rather than expected returns. The present model demonstrates that, with two-dimensional heterogeneity, the occupational inefficiency can take both forms simultaneously, as there are too many and too few entrepreneurs of different skill types. It makes clear that, most generally, if different occupations are affected by different degrees of cross-subsidization, this makes the equilibrium occupational choice decisions inefficient.

In the following, I show that there is a simple entrepreneurial tax policy that may eliminate this occupational misallocation. Suppose the government introduces a (possibly non-linear) entrepreneurial profit tax $T(\pi)$, so that an entrepreneur’s after-tax profits are given by $\hat{\pi} = \pi - T(\pi)$. Banks and entrepreneurs, taking the tax schedule $T(\pi)$ as given, then write contracts contingent on these after-tax profits $\hat{\pi}$, and I can define the resulting credit market equilibrium for any given tax policy just as in Definition 2, replacing $\pi$ by $\hat{\pi}$. Moreover, I keep assuming that contracts $R_\theta(\hat{\pi})$ must satisfy the limited liability constraint $0 \leq R_\theta(\hat{\pi}) \leq \hat{\pi}$ and the monotonicity constraint that $R_\theta(\hat{\pi})$ is non-decreasing in $\hat{\pi}$.\footnote{The idea behind this is that the government has no superior ability to extract tax payments from a firm in case of bankruptcy compared to banks, so that is must always hold that $\pi - T(\pi) - R \geq 0$, where $R$ is the repayment to the bank. In addition, when $T(\pi)$ is negative, it is assumed that banks can capture this transfer from the government in case of bankruptcy, so that $R \leq \pi - T(\pi)$. In other words, tax payments are fully pledgeable.} Under these conditions, it is known from Proposition 6 that the credit market equilibrium is a pooling equilibrium with only a debt contract being offered if the after-tax profits $\hat{\pi}$ satisfy the monotone likelihood ratio property with respect to $\theta$, so that $\hat{H}(\hat{\pi}|\theta) \succeq_{MLRP} \hat{H}(\hat{\pi}|\theta')$ for $\theta > \theta'$, where $\hat{H}(\hat{\pi}|\theta)$ is the cdf of after-tax profits for type $\theta$. The following lemma provides a condition on the tax schedule $T(\pi)$ for this to hold.

**Lemma 1.** Suppose $H(\pi|\theta) \succeq_{MLRP} H(\pi|\theta')$ for $\theta > \theta'$, $\theta, \theta' \in \Theta$, and $T(\pi)$ is such that $\hat{\pi} = \pi - T(\pi)$ is increasing. Then $\hat{H}(\hat{\pi}|\theta) \succeq_{MLRP} \hat{H}(\hat{\pi}|\theta')$. 
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Proof. Since \( \hat{\pi} \equiv \Gamma(\pi) \equiv \pi - T(\pi) \) and \( \Gamma(\pi) \) is increasing, the following relation between \( \hat{H}(\hat{\pi}|\theta) \) and \( H(\pi|\theta) \) is true:
\[
\hat{H}(\Gamma(\pi)|\theta) = H(\pi|\theta) \quad \forall \pi \in \Pi, \theta \in \Theta
\]
and equivalently
\[
\hat{H}(\hat{\pi}|\theta) = H(\Gamma^{-1}(\hat{\pi})|\theta) \quad \forall \hat{\pi} \in \hat{\Pi}, \theta \in \Theta.
\]
Differentiating with respect to \( \hat{\pi} \), I therefore obtain
\[
\hat{h}(\hat{\pi}|\theta) = \frac{h(\Gamma^{-1}(\hat{\pi})|\theta)}{\Gamma'(\Gamma^{-1}(\hat{\pi}))} \quad \forall \hat{\pi} \in \hat{\Pi}, \theta \in \Theta. \tag{1.23}
\]
By assumption, \( H(\pi|\theta) \) satisfies MLRP, which means that \( h(\pi|\theta)/h(\pi|\theta') \) is increasing in \( \pi \) for \( \theta > \theta' \). Equation (1.23) yields
\[
\frac{\hat{h}(\hat{\pi}|\theta)}{\hat{h}(\hat{\pi}|\theta')} = \frac{h(\Gamma^{-1}(\hat{\pi})|\theta)/\Gamma'(\Gamma^{-1}(\hat{\pi}))}{h(\Gamma^{-1}(\hat{\pi})|\theta')/\Gamma'(\Gamma^{-1}(\hat{\pi}))} = \frac{h(\Gamma^{-1}(\hat{\pi})|\theta)}{h(\Gamma^{-1}(\hat{\pi})|\theta')},
\]
which is increasing in \( \Gamma^{-1}(\hat{\pi}) \) by the assumption that \( H(\pi|\theta) \) satisfies MLRP. Then the result follows from the fact that \( \Gamma^{-1}(\hat{\pi}) \) is an increasing function since \( \hat{\pi} = \Gamma(\pi) = \pi - T(\pi) \) is increasing. \( \square \)

Lemma 1 considers entrepreneurial profit tax schedules that involve marginal tax rates uniformly less than one, so that after-tax profits are increasing in before-tax profits. This is a weak restriction on tax policy that I assume to be satisfied in the following. The lemma shows that, under this condition, the fact that higher \( \theta \)-types have better before-tax profit distributions in the sense of the monotone likelihood ratio property translates into the same ordering of after-tax profit distributions. This is intuitive since such a profit tax preserves the ranking of before-tax profit levels and applies to all \( \theta \)-types equally. Combined with Proposition 6, Lemma 1 then implies that, whenever the government imposes a tax on entrepreneurial profits \( T(\pi) \) that involves marginal tax rates less than one, the resulting credit market equilibrium with this tax will be a single debt contract \( R_{z^*_T}(\hat{\pi}) = \min\{\hat{\pi}, z^*_T\} \), where \( z^*_T \) is such that banks make zero profits in aggregate:
\[
\int_{\Theta} G(\tau_{z^*_T,T}(\theta)) \left[ \int_{\Pi} \min\{\pi - T(\pi), z^*_T\} dH(\pi|\theta) - I \right] dF(\theta) = 0. \tag{1.24}
\]
Here,
\[
\tau_{z^*_T,T}(\theta) = \int_{\Pi} (\pi - T(\pi) - \min\{\pi - T(\pi), z^*_T\}) dH(\pi|\theta) - v_W \tag{1.25}
\]
denotes the critical cost value for entry into entrepreneurship at \( \theta \) when the tax policy \( T(\pi) \) is in place.

Now suppose the government sets the profit tax schedule \( T(\pi) \) such that, for all \( \theta \in \Theta \),
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\[
\int_{\Pi} T(\pi) dH(\pi|\theta) = - \left( \int_{\Pi} \min\{\pi - T(\pi), z_T^*\} dH(\pi|\theta) - I \right). \tag{1.26}
\]

Obviously, substituting equation (1.26) into (1.25) yields \(\tilde{\phi}_{z_T^*, T}(\theta) = \tilde{\phi}_e(\theta)\) for all \(\theta\), so that the policy is exactly counteracting the cross-subsidization in the credit market, providing the efficient incentives for entry into entrepreneurship to all agents.\(^{34}\) Note that equation (1.26) is a fixed point condition, since for any given profit tax schedule \(T(\pi)\), we can compute the equilibrium debt contract \(z_T^*\) from solving equations (1.24) and (1.25), and given \(z_T^*\), the tax policy must satisfy equation (1.26). Total revenue from the profit tax is then given by

\[
\int_{\Theta} G(\tilde{\phi}_{z_T^*, T}(\theta)) \int_{\Pi} T(\pi) dH(\pi|\theta) dF(\theta) = - \int_{\Theta} G(\tilde{\phi}_{z_T^*, T}(\theta)) \left( \int_{\Pi} \min\{\pi - T(\pi), z_T^*\} dH(\pi|\theta) - I \right) dF(\theta) = 0,
\]

where the first equality follows from equation (1.26) and the second from the zero profit condition (1.24). Hence, the government budget constraint is automatically satisfied with equality and the tax policy \(T(\pi)\) is feasible. The following proposition summarizes these insights:

**Proposition 7.** Suppose that an entrepreneurial tax policy \(T(\pi)\) is introduced that is such that \(\pi - T(\pi)\) is increasing and equation (1.26) is satisfied. Then

(i) the resulting credit market equilibrium is such that \(\tilde{\phi}_{z_T^*, T}(\theta) = \tilde{\phi}_e(\theta)\) for all \(\theta \in \Theta\), where \(\tilde{\phi}_{z_T^*, T}(\theta)\) and \(\tilde{\phi}_e(\theta)\) are given by (1.25) and (1.22), respectively, and

(ii) the government budget is balanced.

By (1.26), the efficient entrepreneurial tax policy is regressive in the sense that higher ability entrepreneurs face a lower expected tax payment. In fact, for all \(\theta > \tilde{\theta}_T\) with \(\tilde{\theta}_T\) such that \(\int_{\Pi} \min\{\pi - T(\pi), z_T^*\} dH(\pi|\tilde{\theta}_T) = I\), the expected tax payment is negative. This is because the entrepreneurial tax has to counteract the equilibrium cross-subsidization in the credit market, which is decreasing in \(\theta\) as argued above. By the monotone likelihood ratio property of \(H(\pi|\theta)\), this pushes towards a tax schedule \(T(\pi)\) that is itself decreasing in \(\pi\) and in that sense regressive as well. This makes the assumption in Lemma 1 that \(\pi - T(\pi)\) is increasing even less restrictive.

To see how the system of equation (1.24) to (1.26) can be solved for the efficient tax policy, it is useful to consider the following simple example.

\(^{34}\)From a bank’s perspective, of course, there is still cross-subsidization as net profits vary with \(\theta\).
Example 1. Suppose there are two ability types $\theta^k$, $k = g, b$, where the good type $g$ has the better profit distribution in the sense of MLRP than the bad type $b$. Also, suppose there are three possible profit levels $\pi_l < \pi_m < \pi_h$ with probabilities $h^k_l, h^k_m$ and $h^k_h$ for each type $k = g, b$. Let me fix some debt repayment level $z^*_T$ and assume that only the realization of the lowest profit level $\pi_l$ leads to bankruptcy. Then equation (1.26) becomes

\[
h^k_h T(\pi_h) + h^k_m T(\pi_m) = - \left[ (h^k_h + h^k_m) (z^*_T - I) + h^k_l (\pi_l - I) \right]
\]

for $k = g, b$. This is a system of two linear equations that can be solved for the two unknowns $T(\pi_h), T(\pi_m)$. Next, the critical cost values for occupational choice in equation (1.25)

\[
\tilde{\phi}_{z^*_T, T}(\theta^k) = h^k_h (\pi_h - T(\pi_h) - z^*_T) + h^k_m (\pi_m - T(\pi_m) - z^*_T) - \nu_W,
\]

$k = g, b$, are entirely pinned down by $z^*_T, T(\pi_h) and T(\pi_m)$. Hence, the zero profit condition (1.24) holds with equality when finding $T(\pi_l)$ such that

\[
\sum_{k=g,b} G(\tilde{\phi}_{z^*_T, T}(\theta^k)) [(h^k_h + h^k_m) z^*_T + h^k_l (\pi_l - T(\pi_l)) - I] f^k = 0.
\]

Finally, $z^*_T$ is adjusted to make $\pi_l - T(\pi_l) > \pi_m - T(\pi_m) > z^*_T > \pi_l - T(\pi_l)$ hold.

This example shows that it is easiest to find an entrepreneurial tax policy that solves equation (1.26) if the space of possible profit realizations is relatively rich compared to the type space $\Theta$, for example when $\Pi$ is an interval but there are only two groups of entrepreneurial abilities. Then the profit tax schedule $T(\pi)$ provides a high degree of flexibility, while only a small number of restrictions need to be satisfied. In the other extreme case, where there is a continuum of types but only a small number of possible profit realizations, it may not be possible to find an entrepreneurial tax policy that satisfies (1.26) and thus restores efficient occupational choice for all ability types $\theta \in \Theta$. However, in this case, one may think of grouping entrepreneurs in a finite set of ability levels, and providing the correct incentives for entry into entrepreneurship for those.

It is worth emphasizing that the entrepreneurial tax policy in Proposition 7 is quite different from the general subsidization of entrepreneurship that one may think of at first glance in view of credit market frictions. Even if there is only one-dimensional heterogeneity in entrepreneurial abilities, the resulting excessive entry into entrepreneurship in this case would require a lump sum tax on entrepreneurial profits, rather than a subsidy. Since the inefficiency with two-dimensional heterogeneity is more complicated, however, such a uniform tax turns out not to be optimal in general. The policy also differs from
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the tax on bank profits that De Meza and Webb (1987) propose in order to deal with the excessive entry into entrepreneurship that they find in their model with one-dimensional heterogeneity. As can be seen from the zero profit condition (1.24) and most clearly Example 1, a tax on bank profits and a lump-sum tax of entrepreneurial profits have equivalent effects. Thus, a tax on bank profits is not able (nor necessary) to restore occupational efficiency in the present setting. Instead, Proposition 7 points out the importance of entrepreneurial profit taxation as a more flexible corrective instrument.

1.6 Conclusion

This paper has analyzed the optimal non-linear taxation of profits and labor income in a private information economy with endogenous firm formation. I have demonstrated that it is optimal to apply different non-linear tax schedules on these two forms of income, removing the need for redistribution through indirect, general equilibrium effects and production distortions. Moreover, I have pointed out that a differential tax treatment of profits can also be justified based on corrective arguments, mitigating occupational misallocation that results from credit market frictions. In addition, the quantitative importance of differential taxation has been explored in a calibrated model economy.

Both the theoretical and numerical analysis, however, have abstracted from several potentially important aspects of entrepreneurship and its implications for tax policy. Notably, income effects and risk aversion, capital accumulation and additional choices available to entrepreneurs, such as the decision whether to incorporate or not, have been neglected in this paper. In addition, the role of entrepreneurs in fostering technological innovations and economic growth may generate yet other roles for entrepreneurial taxation, given that these activities are typically associated with externalities. Extensions of the present results to a more comprehensive exploration of these issues are left for the future.

1.7 Appendix

1.7.1 Proofs for Sections 1.3 and 1.4

Proof of Proposition 2

(i) Analogously to the proof of Proposition 1, if \( \bar{w} \leq w \), then \( \tilde{p}(\theta) = 0 \) for all \( \theta \in \Theta \) (with the only additional argument that, since both occupations face the same tax schedule on their profits
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(resp. income), there is also no tax advantage from entering entrepreneurship). This, together with the fact that (1.11) and (1.12) must hold as equalities at an optimum, implies \( l(\theta) = E(\theta) = v_E(\theta) = v_W(\theta) = 0 \) for all \( \theta \in \Theta \). Clearly, the no tax equilibrium characterized in Proposition 1 is Pareto-superior, demonstrating that \( \bar{\omega} \leq w \) cannot be part of a Pareto-optimum.

To see that \( \bar{\omega}E(\theta) > wL(\theta) \) \( \forall \theta \in \Theta \), define \( \pi(\theta) \equiv \bar{\omega}E(\theta) \) and \( y(\theta) \equiv wL(\theta) \). \( \pi(\theta) \) solves \( \max_{\pi} \pi - T(\pi) - \psi(\pi/(\bar{\omega}\theta)) \), and analogously for \( y(\theta) \), replacing \( \bar{\omega} \) by \( w \). Note that \( -\psi(x/(w\theta)) \) is supermodular in \((x, w)\). Then the result follows from Topkis’ theorem and \( \bar{\omega} > w \).

(ii) Using the result from (i) that \( \bar{\omega} > w \), let me recapitulate the Pareto problem as follows:

\[
\max_{\{E(\theta), l(\theta), l(\theta), v_E(\theta), v_W(\theta), \phi(\theta), w, \bar{\omega}\}} \int_{\Theta} \left[ \tilde{G}_\theta(\phi(\theta))v_E(\theta) - \int_{\Theta} \phi d\tilde{G}_\theta(\phi) + (1 - \tilde{G}_\theta(\phi(\theta)))v_w(\theta) \right] d\theta
\]

s.t. \( \tilde{\phi}(\theta) = v_E(\theta) - v_W(\theta) \) \( \forall \theta \in \Theta \)

\[
v'_E(\theta) = E(\theta)\psi'(E(\theta)/\theta) / \theta^2, \quad v'_W(\theta) = l(\theta)\psi'(l(\theta)/\theta) / \theta^2 \quad \forall \theta \in \Theta \quad \text{(IC)}
\]

\[
\int_{\Theta} G_\theta(\tilde{\phi}(\theta))L(\theta)dF(\theta) \leq \int_{\Theta} (1 - G_\theta(\tilde{\phi}(\theta)))l(\theta)dF(\theta) \quad \text{(LM)}
\]

\[
\int_{\Theta} G_\theta(\tilde{\phi}(\theta)) [Y(L(\theta), E(\theta)) - v_E(\theta) - \psi(E(\theta)/\theta)] dF(\theta)
\]

\[
-(1 - G(\phi(\theta)))) [v_W(\theta) + \psi(l(\theta)/\theta)] dF(\theta) \geq 0 \quad \text{(RC)}
\]

\[
v_E(\theta) = v_W ((\bar{\omega}/w)\theta), \quad E(\theta) = (w/\bar{\omega})l((\bar{\omega}/w)\theta) \quad \forall \theta \in \Theta (w/\bar{\omega})\Theta \quad \text{(ND)}
\]

\[
w = Y_L(\theta, E(\theta)), \quad \bar{\omega} = Y_E(\theta, E(\theta)) \quad \forall \theta \in \Theta. \quad \text{(MP)}
\]

Note that I have dropped the monotonicity constraint (1.9), assuming that it will not bind at the optimum (and thus ignoring problems of bunching). Attaching multipliers \( \mu_E(\theta) \) and \( \mu_W(\theta) \) to the incentive constraints (IC), \( \lambda_{LM} \) to the labor market clearing constraint (LM), \( \lambda_{RC} \) to the resource constraint (RC), \( \xi_v(\theta) \) and \( \xi_E(\theta) \) to the no-discrimination constraints (ND) and \( \kappa_L(\theta) \) and \( \kappa_E(\theta) \) to the marginal product constraints (MP), the corresponding Lagrangian, after integrating by parts,
can be written as

\[
\mathcal{L} = \int_\Theta \left[ \tilde{G}_\theta(\tilde{\phi}(\theta)) v_E(\theta) - \int_\hat{\theta} \tilde{\phi} d\tilde{G}_\theta(\phi) + (1-\tilde{G}_\theta(\tilde{\phi}(\theta))) v_W(\theta) \right] d\tilde{F}(\theta) \\
- \int_\Theta \left[ \mu'_E(\theta) v_E(\theta) + \mu_E(\theta) \psi' \left( \frac{E(\theta)}{\theta} \right) \frac{E(\theta)}{\theta^2} \right] d\theta - \int_\Theta \left[ \mu'_W(\theta) v_W(\theta) + \mu_W(\theta) \psi' \left( \frac{l(\theta)}{\theta} \right) \frac{l(\theta)}{\theta^2} \right] d\theta \\
+ \lambda_{LM} \left[ \int_\Theta (1 - G_\theta(\tilde{\phi}(\theta))) l(\theta) dF(\theta) - \int_\Theta G_\theta(\tilde{\phi}(\theta)) L(\theta) dF(\theta) \right] \\
+ \lambda_{RC} \left[ \int_\Theta G_\theta(\tilde{\phi}(\theta)) \left[ Y(L(\theta), E(\theta)) - v_E(\theta) - \psi' \left( \frac{E(\theta)}{\theta} \right) \right] d\theta \\
+ \left(\frac{w}{\theta^d}\right) \xi_v(\theta) \left[ v_E(\theta) - v_W(\frac{\tilde{w}}{\tilde{\theta}}) \right] d\theta + \int_\Theta \xi_E(\theta) \left[ E(\theta) - \frac{w}{\tilde{\theta}} \right] d\theta \\
+ \int_\Theta \kappa_L(\theta) \left[ w - Y_L(L(\theta), E(\theta)) \right] d\theta + \int_\Theta \kappa_E(\theta) \left[ \tilde{w} - Y_E(L(\theta), E(\theta)) \right] d\theta. \tag{1.27}
\]

The transversality conditions are \( \mu_E(\theta) = \mu_E(\bar{\theta}) = \mu_W(\theta) = \mu_W(\bar{\theta}) = 0 \). Note first that, due to quasi-linear preferences, \( \lambda_{RC} = 1 \). Then the necessary condition for \( L(\theta) \) is

\[
G_\theta(\tilde{\phi}(\theta)) f(\theta) [Y_L(L(\theta), E(\theta)) - \lambda_{LM}] - [\kappa_L(\theta) Y_{LL}(\theta) + \kappa_E(\theta) Y_{EL}(\theta)] = 0 \quad \forall \theta \in \Theta. \tag{1.28}
\]

Using the transversality conditions, the necessary conditions for \( E(\bar{\theta}) \) and \( l(\bar{\theta}) \) are

\[
G_\theta(\tilde{\phi}(\theta)) f(\bar{\theta}) \left[ \bar{w} - \frac{1}{\hat{\theta}} \psi' \left( \frac{E(\bar{\theta})}{\bar{\theta}} \right) \right] - [\kappa_L(\bar{\theta}) Y_{LE}(\bar{\theta}) + \kappa_E(\bar{\theta}) Y_{EE}(\bar{\theta})] = 0 \tag{1.29}
\]

and

\[
\lambda_{LM} - \frac{1}{\hat{\theta}} \psi' \left( \frac{l(\bar{\theta})}{\bar{\theta}} \right) = 0. \tag{1.30}
\]

If \( Y(L, E) \) is linear, then \( Y_{LL} = Y_{LE} = Y_{EE} = 0 \) and thus (1.28) and (MP) imply \( \lambda_{LM} = Y_L(\theta) = w \) for all \( \theta \). Therefore, by (1.29) and (1.30),

\[
\bar{w} = \frac{1}{\hat{\theta}} \psi' \left( \frac{E(\bar{\theta})}{\bar{\theta}} \right) \quad \text{and} \quad w = \frac{1}{\theta} \psi' \left( \frac{l(\theta)}{\theta} \right).
\]

Note that the first-order condition for the entrepreneurs' and workers' problem is

\[
\bar{w} (1 - T'(\bar{w}E)) = \frac{1}{\hat{\theta}} \psi' \left( \frac{E}{\theta} \right) \quad \text{and} \quad w (1 - T'(wl)) = \frac{1}{\hat{\theta}} \psi' \left( \frac{l}{\theta} \right),
\]

so I obtain \( T'(\bar{w}E(\bar{\theta})) = T'(wl(\bar{\theta})) = 0 \) at any Pareto-optimum if technology is linear.

(iii) There are 3 cases to be considered:

Case 1: \( \lambda_{LM} = w \).
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In this case, (1.28) together with (MP) implies

\[ \kappa_L(\theta)Y_{LL}(\theta) + \kappa_E(\theta)Y_{EL}(\theta) = 0 \quad \forall \theta \in \Theta. \]

Note that, with constant returns to scale,

\[ Y_{LL}(\theta) = -xY_{EL}(\theta) \quad \text{and} \quad Y_{EL}(\theta) = -xY_{EE}(\theta) \quad \forall \theta \in \Theta, \]

where \( x = E(\theta)/L(\theta) \) is independent of \( \theta \) by (MP). Thus

\[ \kappa_L(\theta)Y_{EL}(\theta) + \kappa_E(\theta)Y_{EE}(\theta) = 0 \quad \forall \theta \in \Theta, \]

and then (1.29) and (1.30) imply \( T'(\omega E(\theta)) = T'(wl(\theta)) = 0. \)

Case 2: \( \lambda_{LM} < w. \)

Now (1.28) and (MP) yield

\[ \kappa_L(\theta)Y_{LL}(\theta) + \kappa_E(\theta)Y_{EL}(\theta) > 0 \quad \forall \theta \in \Theta \]

and hence by (1.31)

\[ \kappa_L(\theta)Y_{EL}(\theta) + \kappa_E(\theta)Y_{EE}(\theta) < 0 \quad \forall \theta \in \Theta. \]

Then (1.29) and (1.30) yield \( T'(\omega E(\theta)) < 0 \) and \( T'(wl(\theta)) > 0. \)

Case 3: \( \lambda_{LM} > w. \)

This case is completely analogous to case 2 with all signs reversed.

Proof of Proposition 3

(i) With a proportional tax on the labor input of firms, entrepreneurs effectively face a wage \( \tau w \) rather than the wage \( w \) that workers receive, and hence the Pareto problem is the same as in Proposition 2 with the only difference that maximization is also performed over \( \tau \) and (MP) is replaced by

\[ \tau w = Y_L(L(\theta), E(\theta)), \quad \bar{w} = Y_E(L(\theta), E(\theta)) \quad \forall \theta \in \Theta. \] (MP')

The necessary condition for \( \tau \) yields \( \int_{\Theta} \kappa_L(\theta)d\theta = 0 \), and the necessary conditions for \( w \) and \( \bar{w} \) are

\[
\int_{\Theta} \kappa_L(\theta)d\theta = \frac{\bar{w}}{w^2} \int_{\Theta} \frac{v'_{\theta}}{v_{\theta}} \left[ \frac{1}{w} \frac{\bar{\bar{w}}}{\bar{w}} - \frac{1}{\bar{w}} \right] d\theta + \int_{\Theta} \frac{v'_{\theta}}{v_{\theta}} \left[ \frac{1}{w} \frac{\bar{\bar{w}}}{\bar{w}} - \frac{1}{\bar{w}} \right] d\theta
\]

\[
\int_{\Theta} \kappa_E(\theta)d\theta = -\frac{1}{w} \int_{\Theta} \frac{v'_{\theta}}{v_{\theta}} \left[ \frac{1}{w} \frac{\bar{\bar{w}}}{\bar{w}} - \frac{w}{\bar{w}} \right] d\theta - \int_{\Theta} \frac{v'_{\theta}}{v_{\theta}} \left[ \frac{1}{w} \frac{\bar{\bar{w}}}{\bar{w}} - \frac{w}{\bar{w}} \right] d\theta,
\]

which implies

\[ \int_{\Theta} \kappa_E(\theta)d\theta = -\frac{w}{\bar{w}} \int_{\Theta} \kappa_L(\theta)d\theta \] (1.32)
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and hence $\int_\Theta \kappa_E(\theta) d\theta = 0$. To obtain a contradiction, suppose $\lambda_{LM} < \tau w$. Then (1.28) and (MP') imply

$$\kappa_L(\theta) Y_{LL}(\theta) + \kappa_E(\theta) Y_{EL}(\theta) > 0 \quad \forall \theta \in \Theta$$

and rearranging yields (since $Y_{LL} < 0$)

$$\kappa_L(\theta) < -\frac{Y_{EL}(\theta)}{Y_{LL}(\theta)} \kappa_E(\theta) = x \kappa_E(\theta) \quad \forall \theta \in \Theta.$$  

Yet this contradicts the above result that $\int_\Theta \kappa_L(\theta) d\theta = \int_\Theta \kappa_E(\theta) d\theta = 0$. Similarly, if $\lambda_{LM} > \tau w$, then $\kappa_L(\theta) > x \kappa_E(\theta) \quad \forall \theta \in \Theta$, also yielding a contradiction. Hence, $\lambda_{LM} = \tau w$ must hold at a Pareto optimum. Then $Y_L - \psi'(wl(\theta))/\theta = Y_E - \psi'(\bar{w}E(\theta))/\theta = 0$ and thus $T'(\bar{w}E(\theta)) = T'(wl(\theta)) = 0$ follows from the proof of part (iii) of Proposition 2, case 1.

(ii) If the government can distort the marginal products of labor across firms, e.g. through a non-linear tax on labor inputs, then an entrepreneur of skill $\theta$ effectively faces a wage $\tau(\theta) w$, and (MP) is to be replaced by

$$\tau(\theta) w = Y_L(L(\theta), E(\theta)), \quad \bar{w}(\theta) = Y_E(L(\theta), E(\theta)) \quad \forall \theta \in \Theta,$$  

and (ND) becomes

$$v_E(\theta) = v_W \left( (\bar{w}(\theta) / w) \theta \right), \quad E(\theta) = (w / \bar{w}(\theta)) l \left( ((\bar{w}(\theta) / w) \theta) \right) \quad \forall \theta \in \left[ \theta, (w / \bar{w}(\theta)) \bar{\theta} \right].$$  

Now the necessary condition for $\tau(\theta)$ is $\kappa_L(\theta) = 0$ for all $\theta \in \Theta$, and for $\bar{w}(\theta)$

$$\kappa_E(\theta) = -\frac{1}{w \bar{v}(\theta)} v_W \left( \frac{\bar{w}}{w} \theta \right) - \zeta_E(\theta) \left[ \frac{1}{\bar{w}} \left( \frac{\bar{w}}{w} \theta \right) \right] \quad \forall \theta \in \Theta. \quad (1.33)$$

Note that it must hold that $\bar{w}(\bar{\theta}) > w$ (since otherwise $\bar{v}(\bar{\theta}) = 0$), and therefore (ND') does not bind at $\bar{\theta}$, which yields $\kappa_E(\bar{\theta}) = 0$. Then (1.28) implies (together with $\kappa_L(\hat{\theta}) = 0$) that $Y_L(\hat{\theta}) = \lambda_{LM}$. However, whenever there exists some $\theta < \hat{\theta}$ such that (ND') binds, then (1.33) implies $\kappa_E(\theta) \neq 0$ and thus, by (1.28), $Y_L(\theta) \neq \lambda_{LM}$. 

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Proof of Proposition 4

(i) After integrating by parts, the Lagrangian corresponding to the Pareto problem now becomes

\[
\mathcal{L} = C(\phi')v_E(\theta) - \int_\Theta \tilde{C}_\theta(\theta) v_E(\theta) d\tilde{\Phi}(\theta)
\]

\[
- \int_\Theta \left[ \mu'_E(\theta)v_E(\theta) + \mu_E(\theta)\psi' \left( \frac{E(\theta)}{\theta} \right) \frac{E(\theta)}{\theta^2} \right] d\theta - \int_\Theta \left[ \mu'_W(\theta)v_W(\theta) + \mu_W(\theta)\psi' \left( \frac{L(\theta)}{\theta} \right) \frac{L(\theta)}{\theta^2} \right] d\theta
\]

\[
+ \lambda_{LM} \left[ \int_\Theta (1 - G(\theta)) I(\theta) dF(\theta) - \int_\Theta G(\theta) L(\theta) dF(\theta) \right]
\]

\[
+ \lambda_{RC} \left[ \int_\Theta G(\theta) \left[ Y(\theta), E(\theta) - v_E(\theta) - \psi \left( \frac{E(\theta)}{\theta} \right) \right] - (1 - G(\theta)) \left[ v_W(\theta) + \psi \left( \frac{L(\theta)}{\theta} \right) \right] dF(\theta) \right].
\]

The necessary condition for \( L(\theta) \) immediately implies

\[
Y_L(L(\theta), E(\theta)) = \lambda_{LM} / \lambda_{RC} \quad \forall \theta \in \Theta
\]

and hence the result.

(ii) Note that (1.34) together with constant returns to scale implies that both \( Y_L(\theta) \) and \( Y_E(\theta) \) are equalized across all \( \theta \), and I can therefore again write \( \tilde{\omega} \equiv Y_E \) and \( \omega \equiv Y_L \). Hence \( \omega = \lambda_{LM} / \lambda_{RC} \) and the necessary condition for \( v_E(\theta) \) can be rearranged to

\[
\mu'_E(\theta) = \tilde{C}(\theta) \left[ \tilde{f}(\theta) - \lambda_{RC} G(\theta) \right] f(\theta) + \psi(\theta) f(\theta) \lambda_{RC} [ Y(\theta) - c_E(\theta) + c_W(\theta) - w (L(\theta) + I(\theta)) ]
\]

\[
(1.35)
\]

where \( c_E(\theta) \equiv v_E(\theta) + \psi(E(\theta)/\theta) \) and \( c_W(\theta) \equiv v_W(\theta) + \psi(I(\theta)/\theta) \). Note first that, by Euler's theorem, \( Y(\theta) - wL(\theta) = \tilde{\omega}E(\theta) \). Next, let me define the excess entrepreneurial tax (i.e. the additional tax payment by an entrepreneur of type \( \theta \) compared to a worker of type \( \theta \)) as

\[
\Delta T(\theta) \equiv T_{y_{\pi}}(\pi(\theta)) - T_{y_{\theta}}(y(\theta)) = \tilde{\omega}E(\theta) - c_E(\theta) - (wI(\theta) - c_W(\theta)).
\]

Then using the transversality conditions \( \mu_E(\theta) = \mu_E(\tilde{\theta}) = 0 \), I obtain

\[
0 = \int_\Theta \left[ \tilde{C}(\theta) \left[ \tilde{f}(\theta) - \lambda_{RC} G(\theta) \right] f(\theta) + \psi(\theta) f(\theta) \lambda_{RC} \Delta T(\theta) \right] d\theta.
\]

By the same steps, the necessary condition for \( v_W(\theta) \) can be transformed to

\[
0 = \int_\Theta \left[ (1 - \tilde{G}(\theta)) \left[ \tilde{f}(\theta) - \lambda_{RC} (1 - G(\theta)) \right] f(\theta) - \psi(\theta) f(\theta) \lambda_{RC} \Delta T(\theta) \right] d\theta.
\]

Adding the two equations yields \( \lambda_{RC} = 1 \). With this, I find that, for all \( \theta \in \Theta \),

\[
\mu_E(\theta) = \int_\Theta \left[ \tilde{C}(\theta) \left[ \tilde{f}(\theta) - G(\theta) f(\theta) + \psi(\theta) f(\theta) \Delta T(\theta) \right] d\tilde{\theta}
\]

(1.36)
and

\[ \mu_w(\theta) = \int_0^\theta \left[ (1 - G(\phi(\theta)))\bar{f}(\theta) - (1 - G(\phi(\theta)))f(\theta) - g(\phi(\theta))f(\theta) \Delta T(\theta) \right] d\theta. \tag{1.37} \]

Next, consider the necessary condition for \( E(\theta) \), which is given by

\[ G(\phi(\theta))f(\theta) \left[ \bar{w} - \frac{1}{\theta} \psi' \left( \frac{E(\theta)}{\theta} \right) \right] = \frac{\mu_E(\theta)}{\theta f(\theta) G(\phi(\theta))} \left[ \psi' \left( \frac{E(\theta)}{\theta} \right) \frac{1}{\theta} + \psi'' \left( \frac{E(\theta)}{\theta^2} \right) \frac{E(\theta)}{\theta} \right]. \]

Dividing through by \( \psi'(E(\theta)/\theta)/\theta \) and rearranging yields

\[ \frac{\bar{w} - \psi'(E(\theta)/\theta)/\theta}{\psi'(E(\theta)/\theta)/\theta} = \frac{\mu_E(\theta)}{\theta f(\theta) G(\phi(\theta))} \left( 1 + \frac{\psi''(E(\theta)/\theta)E(\theta)/\theta^2}{\psi'(E(\theta)/\theta)/\theta} \right). \tag{1.38} \]

Note that the entrepreneur’s first order condition from \( \max \bar{w}E - T(\bar{w}E) - \psi(E/\theta) \) is

\[ \bar{w}(1 - T'_{\pi}(\pi(\theta))) = \psi' \left( \frac{E(\theta)}{\theta} \right) \frac{1}{\theta}, \]

where \( \pi(\theta) \equiv \bar{w}E(\theta) \), and hence the elasticity of entrepreneurial effort \( E(\theta) \) with respect to the after-tax wage \( \bar{w}(1 - T'_{\pi}(\pi(\theta))) \) is

\[ \varepsilon_{\pi}(\theta) = \frac{\psi'(E(\theta)/\theta)/\theta}{\psi''(E(\theta)/\theta)E(\theta)/\theta^2}. \]

After substituting (1.36), this allows me to rewrite (1.38) as

\[ \frac{T'_{\pi}(\pi(\theta))}{1 - T'_{\pi}(\pi(\theta))} = \frac{1 + 1/\varepsilon_{\pi}(\theta)}{\theta f(\theta) G(\phi(\theta))} \int_0^\theta \left[ G(\phi(\theta))\bar{f}(\theta) - G(\phi(\theta))f(\theta) + g(\phi(\theta))\Delta T(\theta) \right] d\theta, \tag{1.39} \]

which is the result in Proposition 4. The derivation for \( T'_{y}(y(\theta)) \) proceeds completely analogously from the necessary condition for \( l(\theta) \) and using (1.37).

(iii) \( T'_{\pi}(\pi(\theta)) = T'_{\pi}(\pi(\theta)) = 0 \) immediately follows from (1.38) evaluated at \( \theta \) and \( \bar{\theta} \) and the transversality conditions \( \mu_E(\theta) = \mu_E(\bar{\theta}) = 0 \). Analogously, \( T'_{y}(y(\theta)) = T'_{y}(y(\bar{\theta})) = 0 \) is implied by the first order conditions for \( l(\theta) \) and \( l(\bar{\theta}) \) and the transversality conditions for \( \mu_w(\theta) \).

Proof of Proposition 5

(i) By way of contradiction, suppose there exists some \( \theta \in (\theta, \bar{\theta}) \) such that \( T'_{\pi}(\pi(\theta)) \leq 0 \) and \( T'_{y}(y(\theta)) \geq 0 \). By continuity of the marginal tax rates (from ignoring bunching issues), and the result that marginal tax rates are zero at the top and bottom, this implies that there must exist a subinterval \( [\theta_a, \theta_b] \) of \( \Theta \) such that \( T'_{\pi}(\pi(\theta)) \leq 0 \) and \( T'_{y}(y(\theta)) \geq 0 \) for all \( \theta \in (\theta_a, \theta_b) \) and \( T'_{\pi}(\pi(\theta)) = T'_{y}(y(\theta)) = 0 \) at \( \theta_a \) and \( \theta_b \). Using \( \bar{F}(\theta) = F(\theta) \), independence of \( \theta \) and \( \phi \) and the
optimality formulas in Proposition 5, this implies
\[
\int_{\theta}^{\phi} \left[ \bar{G}(\phi(\theta)) - G(\phi(\theta)) + g(\phi(\theta)) \Delta T(\theta) \right] dF(\theta) \leq 0
\]
on \((\theta_a, \theta_b)\), with equality at \(\theta_a\) and \(\theta_b\). Taking derivatives at \(\theta_a\) and \(\theta_b\), I must therefore have
\[
\bar{G}(\phi(\theta_a)) - G(\phi(\theta_a)) + g(\phi(\theta_a)) \Delta T(\theta_a) \leq 0 \quad \text{and} \quad \bar{G}(\phi(\theta_b)) - G(\phi(\theta_b)) + g(\phi(\theta_b)) \Delta T(\theta_b) \geq 0,
\]
which can be rearranged to
\[
\Delta T(\theta_a) \leq \frac{\bar{G}(\phi(\theta_a)) - G(\phi(\theta_a))}{g(\phi(\theta_a))} \quad \text{and} \quad \Delta T(\theta_b) \geq \frac{\bar{G}(\phi(\theta_b)) - G(\phi(\theta_b))}{g(\phi(\theta_b))}.
\] (1.40)
The assumption that \(T'_{\pi}(\pi(\theta)) \leq 0\) and \(T'_y(y(\theta)) \geq 0\) for all \(\theta \in (\theta_a, \theta_b)\) and \(\bar{w} \geq w\) imply by the agents’ first-order conditions
\[
\bar{w}(1 - T'_E(\bar{w}E(\theta))) = \frac{1}{\theta} \psi' \left( \frac{E(\theta)}{\theta} \right) \quad \text{and} \quad w(1 - T'_E(wE(\theta))) = \frac{1}{\theta} \psi' \left( \frac{l(\theta)}{\theta} \right)
\]
that \(E(\theta) > l(\theta)\) for all \(\theta \in [\theta_a, \theta_b]\) and hence that
\[
\bar{\phi}'(\theta) = v'_E(\theta) - v'_W(\theta) = \frac{E(\theta)}{\theta^2} \psi' \left( \frac{E(\theta)}{\theta} \right) - \frac{l(\theta)}{\theta^2} \psi' \left( \frac{l(\theta)}{\theta} \right) \geq 0 \quad \forall \theta \in (\theta_a, \theta_b),
\]
where I have used the local incentive constraints (2.6). Hence, I obtain \(\bar{\phi}(\theta_a) \leq \bar{\phi}(\theta_b)\). Next, note that by the assumption in the proposition that \(\bar{g}(\phi) \leq g(\phi)\) for all \(\phi \leq \bar{\phi}(\theta)\) and by the second part of Assumption 1, \((G(\phi) - \bar{G}(\phi))/g(\bar{\phi}(\theta))\) is non-decreasing in \(\phi\). With this, equation (1.40) yields \(\Delta T(\theta_a) \leq \Delta T(\theta_b)\). But recall that I assumed \(T'_{\pi}(\pi(\theta)) \leq 0\) and \(T'_y(y(\theta)) \geq 0\) for all \(\theta \in (\theta_a, \theta_b)\). Therefore,
\[
\Delta T'(\theta) = T'_E(\bar{w}E(\theta))\bar{w}E'(\theta) - T'_E(wE(\theta))wE'(\theta) < 0 \quad \forall \theta \in (\theta_a, \theta_b),
\]
where I have used (1.9) and thus \(E'(\theta), l'(\theta) \geq 0\). This implies \(\Delta T(\theta_a) > \Delta T(\theta_b)\) and hence the desired contradiction.

(ii) Note first that part (i) immediately implies
\[
\Delta T'(\theta) = T'_E(\bar{w}E(\theta))\bar{w}E'(\theta) - T'_E(wE(\theta))wE'(\theta) > 0 \quad \forall \theta \in \Theta.
\]
Next, at \(\theta\), I must have
\[
\bar{G}(\phi(\theta)) - G(\phi(\theta)) + g(\phi(\theta))\Delta T(\theta) \geq 0
\]
by the same arguments as in the proof for part (i). Since \(\bar{G}(\phi(\theta)) < G(\phi(\theta))\) by the assumption in the proposition, I obtain \(\Delta T(\theta) > 0\) and therefore \(\Delta T(\theta) > 0\) for all \(\theta \in \Theta\).

(iii) Suppose \(w = Y_L\) increases and thus \(\bar{w} = Y_E\) falls compared to the no-tax equilibrium. Then
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part (i) implies that $E(\theta)$ falls and $l(\theta)$ increases for all $\theta \in \Theta$ compared to the no-tax equilibrium. Moreover, by constant returns to scale, an increase in $Y_L$ implies an increase in $E(\theta)/L(\theta)$, and hence $L(\theta)$ must fall for all $\theta \in \Theta$. Finally, note that

$$
\tilde{\phi}(\theta) = \nu_E(\theta) - \nu_W(\theta)
$$

$$
= \left( \tilde{o}E(\theta) - T_\pi(\tilde{o}E(\theta)) - \psi \left( \frac{E(\theta)}{\theta} \right) \right) - \left( \tilde{w}l(\theta) - T_y(\tilde{w}l(\theta)) - \psi \left( \frac{l(\theta)}{\theta} \right) \right)
$$

Since $w$ increases and $\tilde{w}$ falls by assumption, and because of part (i), $\tilde{o}E(\theta) - \psi (E(\theta)/\theta)$ falls and $\tilde{w}l(\theta) - \psi (l(\theta)/\theta)$ increases compared to the no-tax equilibrium. Moreover, since $\Delta T(\theta) = 0$ in the no-tax equilibrium and $\Delta T(\theta) > 0$ by part (ii) in the Pareto optimum with redistribution, I conclude that $\tilde{\phi}(\theta)$ falls for all $\theta \in \Theta$. Putting this together with the above results for $E(\theta), L(\theta)$ and $l(\theta)$, this means that the labor market clearing constraint (1.11) is strictly slack in the Pareto optimum. This cannot be part of a Pareto optimum, however, since increasing $L(\theta)$ for some $\theta$ increases production and thus relaxes the resource constraint (1.12) without affecting any other constraint nor the objective of the Pareto problem. A slack resource constraint in turn cannot be Pareto optimal since consumption could be increased uniformly without affecting incentives nor occupational choice, increasing the objective for any set of Pareto weights. This completes the proof.

1.7.2 Computational Procedure for Section 1.4.2

To compute the optimal schedules $T_\pi$ and $T_y$ for any set of Pareto weights, I first fix some $x \equiv E(\theta)/L(\theta)$, equal for all $\theta$, which implies wages $\tilde{o} = Y_E(x)$ and $w = Y_L(x)$ for entrepreneurs and workers. Then I proceed as outlined in the following steps:

1. Start with an initial guess for the marginal tax schedules $T'_\pi(\pi(\theta))$ and $T'_y(y(\theta))$.

2. Given this, compute $E(\theta)$ and $l(\theta)$ from the individual first-order conditions

$$
\tilde{o}(1 - T'_\pi(\pi(\theta))) = \frac{1}{\theta} \psi' \left( \frac{E(\theta)}{\theta} \right) \quad \text{and} \quad w(1 - T'_y(y(\theta))) = \frac{1}{\theta} \psi' \left( \frac{l(\theta)}{\theta} \right).
$$

Also, $L(\theta)$ is obtained from $x = E(\theta)/L(\theta)$ and $E(\theta)$.

3. Note that the marginal tax schedules $T'_\pi(\pi(\theta))$ and $T'_y(y(\theta))$ pin down the actual tax schedules $T_\pi(\pi(\theta))$ and $T_y(y(\theta))$, except for the two intercepts, which in turn are given by $T_y(y(\theta))$ and $\Delta T(\theta)$. To find these two, proceed as follows:
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(a) First, find \( \Delta T(\theta) \) from solving the transversality condition

\[
\int_{\theta} [\tilde{G}_\theta(\bar{\phi}(\theta)) \bar{f}(\theta) - G_\theta(\bar{\phi}(\theta)) \bar{f}(\theta) + g_\theta(\bar{\phi}(\theta)) \Delta T(\theta) f(\theta)] d\theta = 0,
\]

using the fact that

\[
\bar{\phi}(\theta) = \left( \bar{w} E(\theta) - \psi \left( \frac{E(\theta)}{\theta} \right) \right) - \left( \bar{w} l(\theta) - \psi \left( \frac{l(\theta)}{\theta} \right) \right) - \Delta T(\theta)
\]

and

\[
\Delta T(\theta) = \Delta T(\bar{\theta}) + \int_{\theta} \left[ T'_\pi(\pi(\theta)) \tilde{w} E'(\theta) - T'_y(y(\theta)) \tilde{w} l'(\theta) \right] d\theta.
\]

(b) Then find \( T_y(y(\theta)) \) from solving the resource constraint (RC)

\[
\int_{\Theta} G_\theta(\bar{\phi}(\theta)) [Y(L(\theta), E(\theta)) - \nu_r(\theta) - \psi(E(\theta)/\theta)] dF(\theta) - (1 - G(\bar{\phi}(\theta))) [\nu_r(\theta) + \psi(l(\theta)/\theta)] dF(\theta) = 0,
\]

using \( \nu_r(\theta) = \bar{w} E(\theta) - T_\pi(\pi(\theta)) - \psi(E(\theta)/\theta) \) and \( \nu_l(\theta) = \bar{w} l(\theta) - T_y(y(\theta)) - \psi(l(\theta)/\theta) \).

4. Use the optimality formulas in Proposition 4 to compute updated marginal tax schedules \( T'_\pi(\pi(\theta)) \) and \( T'_y(y(\theta)) \). Repeat steps 2. to 4. until convergence.

For any given \( x \) and hence wages \( \bar{w} \) and \( w \), iterating on 1. to 4. yields tax schedules and an allocation that satisfy the optimality formulas as well as the transversality conditions and the resource constraint. Finally, I adjust \( x \) until the labor market clearing condition (LM) holds with equality.

1.7.3 Proof of Proposition 6

By construction, the proposed equilibrium contract \( R_{z^*}(\pi) \) satisfies conditions (i) and (ii) of Definition 2, and thus only requirement (iii) remains to be checked. I will do so by proving a series of lemmas, starting with the following result due to Innes (1993).

**Lemma 2.** Consider an arbitrary non-debt contract \( R(\pi) \) that satisfies the limited liability and monotonicity constraints, and let \( R_{z^*}(\pi) = \min\{\pi, z_\theta\} \) denote the debt contract such that

\[
\int_{\Pi} R(\pi) dH(\pi|\theta) = \int_{\Pi} R_{z^*}(\pi) dH(\pi|\theta)
\]

for some \( \theta \in \Theta \). Then

\[
\int_{\Pi} R(\pi) dH(\pi|\theta') \leq \int_{\Pi} R_{z^*}(\pi) dH(\pi|\theta') \quad \forall \theta' \leq \theta.
\]
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In words, whenever banks offer a debt contract $R_{z_{\theta}}(\pi)$ that involves the same expected repayment for entrepreneurs of ability $\theta$ as the non-debt contract $R(\pi)$, then the expected repayment from the debt contract $R_{z_{\theta}}(\pi)$ is at least as high as from the non-debt contract $R(\pi)$ for all entrepreneurs of a lower skill $\theta' \leq \theta$. This result immediately follows from the fact that the entrepreneurs' profit distributions are ranked by MLRP and that, among the contracts that satisfy the limited liability and monotonicity constraints, debt contracts put the maximal repayment in low profit states. Note that Lemma 8 also immediately implies

$$\int_{\Pi} [\pi - R(\pi)] dH(\pi|\theta') \geq \int_{\Pi} [\pi - R_{z_{\theta}}(\pi)] dH(\pi|\theta') \quad \forall \theta' \leq \theta,$$

i.e. all entrepreneurs of quality less than $\theta$ prefer the non-debt contract $R(\pi)$ to the debt contract $R_{z_{\theta}}(\pi)$. Clearly, this is independent of the cost type $\phi$.

Suppose that, in the presence of the equilibrium contract $R_{z_{\theta}}(\pi)$, a bank offers an arbitrary, incentive compatible set of deviation contracts $\{R_{d}(\pi)\}$. Let me denote the resulting critical cost values for occupational choice by $\bar{\theta}_d(\theta)$, i.e. for all $\theta \in \Theta$,

$$\bar{\theta}_d(\theta) \equiv \max \left\{ \int_{\Pi} [\pi - R_{d}(\pi)] dH(\pi|\theta), \int_{\Pi} [\pi - R_{\theta}^d(\pi)] dH(\pi|\theta) \right\} - v_W. \quad (1.41)$$

Next, the following auxiliary result is useful.

**Lemma 3.** For all $\theta \in \Theta$, let $\Delta \bar{\theta}(\theta) \equiv \bar{\theta}_d(\theta) - \bar{\theta}_{z_{\theta}}(\theta)$ denote the change in critical cost values for occupational choice due to the deviation. Then $\Delta \bar{\theta}(\theta)$ is decreasing in $\theta$.

**Proof.** Showing that $\Delta \bar{\theta}(\theta) \equiv \bar{\theta}_d(\theta) - \bar{\theta}_{z_{\theta}}(\theta)$ is decreasing in $\theta$ is, by (1.20) and (1.41), equivalent to showing that

$$\int_{\Pi} R^{d}_{\theta}(\pi)dH(\pi|\theta) - \int_{\Pi} R_{z_{\theta}}(\pi)dH(\pi|\theta)$$

is increasing in $\theta$. To see this, note that, by Lemma 8, if $\int_{\Pi} R^{d}_{\theta}(\pi)dH(\pi|\theta) = \int_{\Pi} R_{z_{\theta}}(\pi)dH(\pi|\theta)$ for some $\theta$, then $\int_{\Pi} R^{d}_{\theta}(\pi)dH(\pi|\theta') \leq \int_{\Pi} R_{z_{\theta}}(\pi)dH(\pi|\theta')$ for all $\theta' \leq \theta$, which implies that

$$\int_{\Pi} R^{d}_{\theta}(\pi)dH(\pi|\theta) - \int_{\Pi} R_{z_{\theta}}(\pi)dH(\pi|\theta) \geq \int_{\Pi} R^{d}_{\theta}(\pi)dH(\pi|\theta') - \int_{\Pi} R_{z_{\theta}}(\pi)dH(\pi|\theta') \quad (1.42)$$

whenever $\theta \geq \theta'$. Moreover, by incentive compatibility of $\{R^{d}_{\theta}(\pi)\}$,

$$\int_{\Pi} [\pi - R^{d}_{\theta}(\pi)] dH(\pi|\theta') \geq \int_{\Pi} [\pi - R^{d}_{\theta}(\pi)] dH(\pi|\theta')$$

and hence $\int_{\Pi} R^{d}_{\theta}(\pi)dH(\pi|\theta') \leq \int_{\Pi} R^{d}_{\theta}(\pi)dH(\pi|\theta')$, which, when combined with (1.42), completes the argument. □

This allows me to prove the following lemma:
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Lemma 4. Let $\Delta G(\theta) \equiv G(\bar{\phi}_d(\theta)) - G(\bar{\phi}_{z^*}(\theta))$ for all $\theta \in \Theta$. Under Assumption 1, $\Delta G(\theta)$ is decreasing in $\theta$.

Proof. First, observe that $\bar{\phi}_{z^*}(\theta)$ is increasing in $\theta$ by MLRP. Moreover, $G(\phi)$ is concave by Assumption 1. Therefore, the result from Lemma 3 that $\Delta \bar{\phi}(\theta) \equiv \bar{\phi}_d(\theta) - \bar{\phi}_{z^*}(\theta)$ is decreasing in $\theta$ implies that

$$\Delta G(\theta) \equiv G(\bar{\phi}_d(\theta)) - G(\bar{\phi}_{z^*}(\theta))$$

is also decreasing in $\theta$, proving the lemma. □

The deviating bank’s expected profits from offering $\{R^d_\theta(\pi)\}$ are given by

$$\Pi^d = \int_{\Theta} 1_{\{\bar{\phi}_d(\theta) > \bar{\phi}_{z^*}(\theta)\}}(\theta)G(\bar{\phi}_d(\theta))\int_{\Pi} (R^d_\theta(\pi) - I) dH(\pi|\theta)dF(\theta)$$

$$< \int_{\Theta} 1_{\{\bar{\phi}_d(\theta) > \bar{\phi}_{z^*}(\theta)\}}(\theta)G(\bar{\phi}_d(\theta))\int_{\Pi} (R_{z^*}(\pi) - I) dH(\pi|\theta)dF(\theta)$$

(1.43)

since $\int_{\Pi} R^d_\theta(\pi)dH(\pi|\theta) < \int_{\Pi} R_{z^*}(\pi)dH(\pi|\theta)$ whenever $\bar{\phi}_d(\theta) > \bar{\phi}_{z^*}(\theta)$ by (1.41). Aggregate profits in the proposed equilibrium are

$$\Pi^* = \int_{\Theta} G(\bar{\phi}_{z^*}(\theta))\int_{\Pi} (R_{z^*}(\pi) - I) dH(\pi|\theta)dF(\theta) = 0$$

(1.44)

by (1.19), and therefore, subtracting (1.44) from (1.43) yields

$$\Pi^d < \int_{\Theta} \left(1_{\{\bar{\phi}_d(\theta) > \bar{\phi}_{z^*}(\theta)\}}(\theta)G(\bar{\phi}_d(\theta)) - G_{z^*}(\theta)\right)\int_{\Pi} (R_{z^*}(\pi) - I) dH(\pi|\theta)dF(\theta)$$

$$= \int_{\Theta} \left(\Delta G(\theta) - 1_{\{\bar{\phi}_d(\theta) = \bar{\phi}_{z^*}(\theta)\}}(\theta)G(\bar{\phi}_{z^*}(\theta))\right)\int_{\Pi} (R_{z^*}(\pi) - I) dH(\pi|\theta)dF(\theta)$$

(1.45)

The following two lemmas establish that the RHS of (1.45) is non-positive.

Lemma 5. In equation (1.45),

$$\int_{\Theta} \Delta G(\theta)\int_{\Pi} (R_{z^*}(\pi) - I) dH(\pi|\theta)dF(\theta) \leq 0.$$  

(1.46)

Proof. Find $\bar{\theta}$ such that $\int_{\Pi}(R_{z^*}(\pi) - I) dH(\pi|\bar{\theta}) = 0$, which exists and is unique by (1.19) and MLRP. Also, find the constant $\delta$ such that $\delta G(\bar{\phi}_{z^*}(\bar{\theta})) = \Delta G(\bar{\theta})$. Then since $\Delta G(\theta)$ is decreasing and $\Delta G(\bar{\phi}_{z^*}(\theta))$ is increasing by Lemma 4, $\Delta G(\bar{\phi}_{z^*}(\theta)) \leq \Delta G(\theta)$ for all $\theta \leq \bar{\theta}$, and $\Delta G(\bar{\phi}_{z^*}(\theta)) \geq \Delta G(\theta)$ otherwise. Thus,

$$\int_{\theta}^{\bar{\theta}} \Delta G(\theta)\int_{\Pi} (R_{z^*}(\pi) - I) dH(\pi|\theta)dF(\theta) + \int_{\bar{\theta}}^{\bar{\theta}} \Delta G(\theta)\int_{\Pi} (R_{z^*}(\pi) - I) dH(\pi|\theta)dF(\theta)$$

$$\leq \int_{\theta}^{\bar{\theta}} \delta G(\bar{\phi}_{z^*}(\theta))\int_{\Pi} (R_{z^*}(\pi) - I) dH(\pi|\theta)dF(\theta) + \int_{\bar{\theta}}^{\bar{\theta}} \delta G(\bar{\phi}_{z^*}(\theta))\int_{\Pi} (R_{z^*}(\pi) - I) dH(\pi|\theta)dF(\theta)$$

$$= 0,$$  

(1.47)
where the inequality comes from $\int_{\Pi}(R_{z^*}(\pi) - l)dH(\pi|\theta) < 0$ for $\theta \leq \tilde{\theta}$ and $\int_{\Pi}(R_{z^*}(\pi) - l)dH(\pi|\theta) \geq 0$ otherwise, and the equality from (1.19).

**Lemma 6.** In equation (1.45),

$$\int_{\Theta} \mathbf{1}_{\{\hat{\phi}_d(\theta) = \hat{\phi}_{z^*}(\theta)\}}(\theta)\mathcal{G}(\hat{\phi}_{z^*}(\theta)) \int_{\Pi}(R_{z^*}(\pi) - l)dH(\pi|\theta)dF(\theta) \geq 0. \quad (1.48)$$

**Proof.** There are 3 cases to be considered. If $\hat{\phi}_d(\theta) = \hat{\phi}_{z^*}(\theta)$ for all $\theta \in \Theta$, then (1.48) holds with equality due to (1.19). If there does not exist a $\theta \in \Theta$ such that $\hat{\phi}_d(\theta) = \hat{\phi}_{z^*}(\theta)$, then (1.48) also holds as an equality trivially. Finally, if $\hat{\phi}_d(\theta) = \hat{\phi}_{z^*}(\theta)$ holds for some but not all $\theta \in \Theta$, there must exist some threshold value $\bar{\theta} \in (\underline{\theta}, \overline{\theta})$ such that $\hat{\phi}_d(\theta) > \hat{\phi}_{z^*}(\theta)$ for all $\theta < \bar{\theta}$ and $\hat{\phi}_d(\theta) = \hat{\phi}_{z^*}(\theta)$ otherwise. This follows from Lemma 3, which has shown that $\Delta \hat{\phi}(\theta)$ is decreasing in $\theta$ and, by the definition in (1.41), $\hat{\phi}_d(\theta) \geq \hat{\phi}_{z^*}(\theta)$. With this, (1.48) becomes

$$\int_{\bar{\theta}} \mathcal{G}(\hat{\phi}_{z^*}(\theta)) \int_{\Pi}(R_{z^*}(\pi) - l)dH(\pi|\theta)dF(\theta) > 0$$

since $\bar{\theta} > \theta$ and $\int_{\Pi}(R_{z^*}(\pi) - l)dH(\pi|\theta)$ is increasing in $\theta$ by MLRP. \qed

Lemmas 5 and 6 together with equation (1.45) show that $\Pi^d < 0$, and hence there does not exist a profitable deviation.
Chapter 2

Competitive Markets Without Commitment

2.1 Introduction

The question whether – and why – markets may perform better than centralized institutions, such as governments, has fascinated economists for a long time, at least since the work of Hayek (1945). However, despite the importance of this question for economics and beyond, it is still hard to find formal arguments for why markets may be able to outperform a benevolent government. Instead, the benchmark result is still provided by standard welfare theorems according to which a benevolent planner can always replicate the market outcome, or even improve upon it if the market is affected by failures such as adverse selection or externalities. In this paper, we compare markets and governments and show that a government, even though benevolent and facing the same constraints as competitive firms, may not be able to replicate the market equilibrium, but instead implements an allocation that is Pareto dominated by the market outcome.

In particular, the market dominates a central planner even though it is affected by an adverse selection problem, overturning the classic justification for efficiency enhancing government interventions in competitive markets. Market “failure” due to adverse selection is indeed a major argument in both the academic and political debate on markets versus governments. For instance, the need to provide mandatory social unemployment

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1This chapter is the product of joint work with Nick Netzer. We are grateful to Daron Acemoglu, Carlos Alós-Ferrer, Abhijit Banerjee, Helmut Bester, Peter Diamond, Dennis Gaertner, Mike Golosov, Jon Gruber, Bard Harstad, Martin Hellwig, Casey Rothschild, Armin Schmutzler, Robert Shimer, Jean Tirole, Robert Townsend, Iván Werning, seminar participants at FU and HU Berlin, Bonn, ETH, Kellogg, LSE, MIT, Princeton, UCL, Wharton, Wisconsin School of Business, Zurich, the 2009 North American Summer Meetings of the Econometric Society at BU and the 2010 ASSA Meetings in Atlanta for valuable suggestions.
insurance is usually derived from the argument that private unemployment insurance markets would suffer from adverse selection and fail (Chiu and Karni 1998). In contrast, we point out that markets with adverse selection can outperform a government as they provide greater incentives to exert effort, e.g. to find good jobs with low unemployment risk.

We consider a framework where an adverse selection problem arises endogenously from a moral hazard problem with lack of commitment. Ex-ante, a risk-averse agent is able to affect the probability distribution over output by choosing some hidden action, and optimal contracts reflect a tradeoff between providing incentives and insurance. However, such contracts are subject to a fundamental time-inconsistency problem: Whereas underinsurance is optimal ex-ante so that the agent has incentives to exert effort, it becomes suboptimal once effort has been chosen, but before output is realized. At this stage, the need to provide effort incentives has vanished and a risk-neutral principal might find it optimal to provide the agent with full insurance. The agent, anticipating this, then would have incentives to exert the least costly effort level.

We examine how governments and competitive markets perform in the presence of this time-inconsistency problem. A benevolent government offers incentive contracts to a population of agents who differ in their privately known disutility of effort, and is free to change them after agents have chosen their unobservable effort. We first show that, for a large class of benevolent planners, including utilitarian ones and those that aim at redistributing from agents who have taken high effort, and thus expect high output at the ex-post stage, to low effort, low expected output agents ex-post, the unique equilibrium is such that no agent provides effort and everybody is fully insured, because the planner will always implement an allocation that favors low effort agents ex-post, eliminating ex-ante incentives.

We then turn to the analysis of competitive markets, where contracts are offered by many risk-neutral firms and there is two-sided lack of commitment: Firms can offer new and modify their old contracts, and agents can switch to other firms once they have chosen effort. Hence, firms take the agents' effort decision and the composition of the population of agents as given at the ex-post stage, and the moral hazard problem becomes a standard problem of adverse selection. While there are many ways to model competitive markets with adverse selection, suppose, for example, that the equilibrium in the ex-post market results in the separating Rothschild and Stiglitz (1976) allocation. In anticipation of this outcome with separation and underinsurance, agents with low effort cost will find

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2 The underlying mechanism relates to what has been described as the Samaritan's dilemma (Buchanan 1975) or the problem of soft budget constraints (Kornai, Maskin, and Roland 2003).
it optimal to provide high effort ex-ante. Some incentives for effort provision can thus be sustained in contrast to the planner’s equilibrium allocation, since the ex-post adverse selection problem endogenously introduces commitment to refrain from full insurance and pooling.

Importantly, this result does not depend on the specifics of Rothschild-Stiglitz contracts, but turns out to be a robust implication of competition. We show that, whenever the outcome of an ex-post market satisfies a weak notion of competitiveness, called minimal contestability (Rothschild 2007), and it sufficiently separates agents who have taken different effort choices, it Pareto-dominates the government outcome. We provide a discussion of various circumstances – in addition to the standard Rothschild-Stiglitz setting – under which minimally contestable and separating allocations arise. We then focus on the Miyazaki-Wilson allocation, which includes the Rothschild-Stiglitz outcome as a special case and is well-founded game-theoretically. We show that it satisfies our axioms so that the Pareto result applies. Also, it can be interpreted as the outcome that a planner would choose who cares only about high effort agents ex-post and thus provides maximal incentives for effort ex-ante. Whereas the Pareto comparison depends on the social planner being concerned about a welfare criterion that makes redistribution towards high effort cost types desirable, we show based on this insight that, for any distribution of Pareto-weights that the planner may use to evaluate welfare, a Miyazaki-Wilson market still implements more effort than the social planner under general conditions.

Our formal model captures key characteristics of a variety of real-world allocation problems. For instance, the moral hazard problem can be interpreted as an individual’s education decision with subsequent labor markets, as in Boadway, Marceau, and Marchand (1996) and Konrad (2001). Significant parts of education are private information and are typically completed before binding contracts with employers are signed. Even if there are contracts, as in the case of executive education, agents cannot be prevented from moving to other employers some time after their education has been completed, and employers are able to modify employment contracts or eventually lay off employees. The model captures exactly this setting of two-sided lack of commitment.

Our analysis also applies to settings of insurance with ex-ante moral hazard. For instance, workers can exert effort to reduce their risk of becoming unemployed by finding a job with low unemployment risk, patients may affect their risk of illness by undertaking a precautionary effort, and bankers may affect the risk of default of their loan portfolio by monitoring borrowers. Depending on the institutional framework, unemployment, health or credit insurance is provided either through competitive markets or by the government, and both lack commitment. In particular, our model of markets captures a situ-
2.1. Introduction

ation where, after preventive effort has been chosen but before the risk is realized, agents can switch insurers, taking along their private effort type such as their layoff risk, health or quality of their loan portfolio, and insurance companies can modify contractual terms, resulting in an ex-post adverse selection problem.

For all these applications, we can draw both positive and normative conclusions from our analysis. On the positive side, it predicts that market economies generate more effort (e.g. in terms of education) and thus a larger per capita output than centrally planned economies in the presence of commitment problems, contributing to the long-standing questions raised at the very beginning. This reason is that governments cannot commit not to help the weak and poor at the ex-post stage, while markets endogenously generate a form of commitment to refrain from full insurance and pooling.

Beyond this general implication, there are also more specific insights to be drawn from the model. For instance, it makes predictions about the equilibrium level of education and the form of employment contracts, and in particular how they differ between competitive private firms and the public sector. There are several existing explanations for why private firms make use of explicit incentives more often than the public sector.\(^3\) It remains questionable, however, whether the lack of incentives in the public sector can indeed be considered as optimal (Burgess and Ratto 2003). Our model offers an explanation without making assumptions about exogenous differences between the public and the private sector, other than the difference in the implicit objective functions of governments and markets. As a consequence, the absence of high-powered incentives in the public sector is not interpreted as optimal.

On the normative side, our results have implications for market regulation. We emphasize that, for markets to be able to deal with the commitment problem successfully, firms must be allowed to offer separating contracts, some of which involve underinsurance and possibly strictly positive profits. These properties of the market equilibrium must not be regarded as a sign of market failure, and they do not provide support on their own for government interventions such as the provision of mandatory social insurance against unemployment or health risk, for instance. After having analyzed the model, we will return to these issues in section 4.4, where we will relate our insights to recent proposals for regulating the markets for health and credit insurance.

**Related Literature.** The paper most closely related to ours is the seminal contribution by Fudenberg and Tirole (1990). They observe the same time-inconsistency problem in a

\(^3\)For instance, public sector jobs might exhibit a multi-task nature (Holmström and Milgrom 1991), they might be affected by common agency problems (Dixit 1997), or they could be “mission-oriented” and occupied by intrinsically motivated agents (Besley and Ghatak 2005).
principal-agent economy with a monopolistic profit-maximizing principal. In particular, after the monopolist has offered an initial contract and the agent has exerted effort, renegotiation occurs subject to the constraint that the agent cannot be made worse off than with the initial contract. The principal can therefore use the initial contract offer to affect the agent’s reservation utility at the ex-post renegotiation stage, and thus to improve commitment. In contrast, we do not assume that initial contracts represent a constraint at the ex-post stage, neither for a government nor for firms. The social planner maximizes some weighted sum of ex-post utilities subject to a resource constraint, rather than profits subject to a set of reservation utilities as in Fudenberg and Tirole (1990). As for the market, we consider the case where several profit-maximizing principals are competing and there is two-sided lack of commitment: Firms are free to modify old or offer new contracts, possibly making some of their customers worse off than initially, but agents in turn are free to obtain a contract from a competing firm. The necessity of randomization between effort levels in Fudenberg and Tirole (1990) is replaced in our model by the assumption of ex-ante heterogeneity in effort costs.

Comparing the efficiency of markets and governments in a setting without commitment, our paper shares a common goal with the contributions by Acemoglu, Golosov, and Tsyvinski (2008a, 2008b). However, their modelling of both markets and governments is quite different from the approach taken here. The provision of insurance contracts by private firms in competitive markets is ruled out, and government policies are distorted by political economy constraints. Moreover, their equilibria crucially rely on reputational concerns in an infinitely repeated game. In contrast, we completely abstract from reputational effects, assume a benevolent government and consider markets where competitive firms can offer insurance contracts that are only restricted by informational and commitment constraints.

Bisin and Rampini (2006) consider the performance of a government without commitment, comparing the cases where agents can or cannot trade in anonymous markets. However, the time-inconsistency problem that they consider is different from the moral hazard problem analyzed here. It results from the flow of information that is revealed to the government through the agents’ initial contract choices and is therefore an application of the ratchet effect. Markets are helpful in such a setting because they may reduce the information flow to the government and therefore its commitment problem. In contrast, we explicitly rule out commitment problems from a ratchet effect by assuming that ex-

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4See Freixas, Guesnerie, and Tirole (1985) and Dewatripont (1989) for the standard treatments. Asheim and Nilssen (1996) study the consequences of such ratchet effect commitment problems for equilibrium in competitive insurance markets.
post contract offers cannot be conditioned on initial contract choices. Instead, we focus on the time-inconsistency problem related to moral hazard, and make predictions about how the commitment problem affects ex-ante effort incentives.

Our results also complement a vast literature on public versus private provision of goods and services in an incomplete contracts world (see Shleifer (1998) for an overview). Whereas this literature focuses on how privatization affects the asymmetry of information or the production technology in a firm, we derive a clear advantage of competitive markets over a benevolent government without assuming any differences in the technological, informational or commitment constraints faced by these different institutions. In contrast to Schmidt (1996) and Bisin and Rampini (2006), where privatization or the creation of anonymous markets, respectively, is assumed to conceal information from the government and to act as a constraint on the set of feasible policies, we show that the establishment of competitive markets can be interpreted as the choice of a specific, effort prone welfare function. Thus, markets result in superior allocations compared to a planner's allocation due to the different objective function that they implicitly maximize, not due to a difference in the informational or commitment constraints.

Finally, our work relates to the literature on tax competition (e.g. Kehoe (1989) or Conconi, Perroni, and Riezman (2008)). In the case of capital taxation, this research has studied the effect of competition between countries on the time-inconsistency problem that results from the fact that capital taxes are highly distortive ex-ante, but not ex-post, after capital has been accumulated. The focus of this literature is on the optimal degree of cooperation between countries, facing a trade-off between disciplining effects of non-cooperative behavior and an adverse race-to-the-bottom.

The paper is structured as follows. Section 2.2 introduces our model economy. We then proceed to compare competitive equilibria without commitment to those achieved by a social planner with the same commitment problem. In section 2.3, we provide a comparison based on a general, axiomatic treatment of competitive market outcomes. In section 2.4, we examine in greater detail some market outcomes that satisfy the axioms and are well-founded game-theoretically. Section 2.5 concludes.

2.2 The Model

We consider the following model economy. There is a continuum of risk-averse agents, indexed by the set [0,∞). Agents are expected utility maximizers with a Bernoulli utility function $U(c)$, where $c$ is consumption. $U(c)$ is twice continuously differentiable, with $U' > 0$ and $U'' < 0$. Following Fudenberg and Tirole (1990), we assume that
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both the domain and the range of $U$ are given by $\mathbb{R}$, so that $\lim_{c \to -\infty} U(c) = -\infty$ and $\lim_{c \to \infty} U(c) = \infty$. We also assume that the Inada condition $\lim_{c \to \infty} U'(c) = 0$ is satisfied. Let $\Phi(U)$ be the inverse function of $U$, which then satisfies $\Phi' > 0$, $\Phi'' > 0$, $\lim_{U \to -\infty} \Phi(U) = -\infty$, $\lim_{U \to \infty} \Phi(U) = \infty$ and $\lim_{U \to \infty} \Phi'(U) = \infty$.

Each agent faces idiosyncratic risk with respect to the amount of consumption good that she produces for a principal. Production output can either be high, $y_h$, or low, $y_l$, with $y_l < y_h$. The probability of either output depends on the level of effort $e \in \{e, \bar{e}\}$ that the agent has exerted. Since the ex-ante moral hazard problem becomes a problem of adverse selection ex-post, we say that an agent who chooses the high effort $\bar{e}$ becomes an ex-post good type $(g)$, whereas an agent who chooses the low effort $e$ becomes an ex-post bad type $(b)$. Good types produce the high output $y_h$ with probability $p_g$ and bad types with probability $p_b$, where $0 < p_b < p_g < 1$ holds. In an insurance application, $y_h$ represents each agent’s endowment, and $y_h - y_l$ is a possible damage that occurs with low probability $1 - p_g$ for low risks (who have exerted preventive effort) and with larger probability $1 - p_b$ for high risks.

The agents’ preferences are assumed to be separable between consumption and effort, so that overall utility is given by $U(c) - H(e)$, where $H(e)$ denotes effort cost. We normalize $H(e)$ to zero. Agents differ in their disutility of effort $H(\bar{e}) = d$, which is given by their index $d \in [0, \infty)$ and which we refer to as their ex-ante cost type. The composition of the population is described by a continuous distribution function $G$, defined on $\mathbb{R}$ with $G(d) = 0$ for all $d \leq 0$. We adopt the convention of extending $G$ to $G(\infty) = 1$ and we assume that $G$ has an associated density $g$ that satisfies $g(d) > 0$ for all $d \in [0, \infty)$. Note that, throughout the paper, neither effort cost nor effort choice will be observable.

For our analysis it is convenient to operate in the utility space. In this space, a contract that a planner or a firm offers to an agent is a tuple $(u_h, u_l) \in \mathbb{R}^2$ of consumption utilities that the agent obtains when producing the high and the low output, respectively. A contract allocation is a quadruple $(u_{b,h}, u_{b,l}, u_{g,h}, u_{g,l}) \in \mathbb{R}^4$ where $(u_{b,h}, u_{b,l})$ is the contract intended for bad types and $(u_{g,h}, u_{g,l})$ the one intended for good types.

In terms of the applications discussed above, the cost $d$ can be interpreted as an ex-ante skill type, affecting the disutility of effort such as obtaining education, looking for a high quality job, staying healthy or monitoring borrowers. At the ex-post stage, this then translates into an adverse selection problem with good and bad types, i.e. individuals who differ in their privately known human capital or the quality of their job, patients who differ in their health status or banks with different loan portfolios. Good types have the lower risk of being affected by the low outcome $y_l$, such as becoming unemployed, getting ill or suffering defaults of loans. Insurance contracts specify payments and thus
consumption utilities \( u_h \) and \( u_l \) contingent on these outcomes, which can be interpreted as a payment scheme for an employee, or an insurance contract with a premium to be paid in case of the high outcome \( y_h \) and an indemnity paid out when the bad outcome \( y_l \) is realized.

### 2.3 Markets Versus Governments

Throughout Section 2.3, we are concerned with the comparison between a benevolent planner and competitive markets without commitment. We consider the planner first, followed by an axiomatic analysis of markets in subsection 2.3.2.

#### 2.3.1 A Social Planner Without Commitment

Our first result, which predicts a complete breakdown of incentives for a large class of social planners, is a generalization of an analogous result by Boadway, Marceau, and Marchand (1996) in the setting of education and taxation under a utilitarian planner. We consider the following reduced timing:

*Stage 1*: Agents simultaneously choose their effort level.
*Stage 2*: The social planner announces a policy, i.e. a set of two contracts.
*Stage 3*: Agents simultaneously choose among the offered contracts.

One could think of stage 1 being preceded by an additional stage where the planner announces an initial policy. Then, the agents choose an effort level in anticipation of some final policy, possibly different from the announcement. If the planner is not committed to its initial announcement, she is free to change the policy ex-post, after effort choice has taken place (but remains unobservable). The initial announcement is then irrelevant, as captured by the reduced time structure. This argument assumes that initial announcements do not constitute binding reservation constraints to a benevolent planner (dictator), as opposed to the monopolistic firm in Fudenberg and Tirole (1990).

In an insurance application, the policy could be an optimally designed social insurance arrangement, such as a mandatory public health insurance where individuals can choose between different levels of coverage. In an education and job market application, the policy describes the payment structure of jobs in the public sector, such as in schools or prisons (Hart, Shleifer, and Vishny 1997) or in other firms owned by the state (La Porta, Lopez-De-Silanes, and Shleifer 2002).
2.3. Markets Versus Governments

We solve the game backwards. First, for any given unobservable effort choice in stage 1 and policy announcement in stage 2, each ex-post type $k \in \{g, b\}$ selects the best contract in stage 3, where we break ties in favor of the contract with a coverage closer to full insurance, i.e. smaller absolute difference $|u_h - u_l|$. Stage 3 can then be eliminated by subsuming this choice into the planner’s payoff function. We can next derive the planner’s optimal policy at stage 2 when effort choices have been made. Observe first, however, that optimal effort choices in stage 1 must be of a threshold type in any equilibrium, with a critical value $\hat{d} \in \mathbb{R}_0^+ \cup \{\infty\}$ such that the effort choice of ex-ante type $d$ is given by $\bar{e}$ if $d < \hat{d}$ and by $\check{e}$ if $d \geq \hat{d}$. Whenever an agent with effort cost $d$ finds it optimal to choose the high effort, in anticipation of some final policy, the same holds for any agent with $d' \leq d$. Thus, in any equilibrium, agents with small effort cost ($d < \hat{d}$) choose the high effort and those with high effort cost ($d \geq \hat{d}$) the low effort, and the share of good types in the society becomes $G(\hat{d})$.

Suppose then that the planner has formed a correct belief about $\hat{d}$ and the share of good types $G(\hat{d})$. Let $\Psi(\hat{d})$ denote the relative weight placed on good types in the planner’s ex-post welfare evaluation. For example, this weight equals the population share of good types for a utilitarian planner, so that $\Psi(\hat{d}) = G(\hat{d})$. In general, we can derive the whole ex-post Pareto frontier and, by an appropriate choice of weights, can even capture planners that are concerned about effort directly or take into account individual effort costs in a non-separable manner, as long as their implemented policy is ex-post efficient. Then, whenever $\hat{d} \in (0, \infty)$ so that both ex-post types exist, the planner solves the following problem, which we refer to as program SP($\hat{d}$):

\[
\max_{(u_{g,h}, u_{g,l}, u_{b,h}, u_{b,l}) \in \mathbb{R}^4} \Psi(\hat{d})[p_g u_{g,h} + (1 - p_g)u_{g,l}] + (1 - \Psi(\hat{d}))[p_b u_{b,h} + (1 - p_b)u_{b,l}] \tag{2.1}
\]

subject to the constraints

\[
p_g u_{g,h} + (1 - p_g)u_{g,l} \geq p_g u_{b,h} + (1 - p_g)u_{b,l}, \tag{2.2}
\]

\[
p_b u_{b,h} + (1 - p_b)u_{b,l} \geq p_b u_{g,h} + (1 - p_b)u_{g,l}, \tag{2.3}
\]

\[\text{Since effort choice remains unobservable, we do not need to derive the planner's optimal policy for effort choice profiles different from such threshold profiles, because deviations from an equilibrium candidate are not observed.}\]
2.3. Markets Versus Governments

\[ G(\hat{d})[p_g \Phi(u_{g,h}) + (1 - p_g)\Phi(u_{g,l})] + (1 - G(\hat{d}))[p_b \Phi(u_{b,h}) + (1 - p_b)\Phi(u_{b,l})] \leq R(\hat{d}). \]  

(2.4)

The planner maximizes a weighted average of the expected utilities of good and bad types subject to the two standard incentive constraints and the resource constraint. Here, \( R(\hat{d}) = [G(\hat{d})p_g + (1 - G(\hat{d}))p_b]y_h + [1 - G(\hat{d})p_g - (1 - G(\hat{d}))p_b]y_l \) are per capita resources. The following lemma characterizes the solution of this problem:

**Lemma 7.** Fix any \( \hat{d} \in (0, \infty). \)

(i) \( SP(\hat{d}) \) has a unique solution \( V_{SP}(\hat{d}) = (u_{b,h}^{SP}(\hat{d}), u_{b,l}^{SP}(\hat{d}), u_{g,h}^{SP}(\hat{d}), u_{g,l}^{SP}(\hat{d})). \)

(ii) If \( \Psi(\hat{d}) \geq G(\hat{d}) \), then \( u_{b,h}^{SP}(\hat{d}) = u_{b,l}^{SP}(\hat{d}) = u_{g,h}^{SP}(\hat{d}) = u_{g,l}^{SP}(\hat{d}) \). Furthermore, \( u_{b}^{SP}(\hat{d}) = p_b u_{b,h}^{SP}(\hat{d}) + (1 - p_b)u_{b,l}^{SP}(\hat{d}) \leq p_g u_{g,h}^{SP}(\hat{d}) + (1 - p_g)u_{g,l}^{SP}(\hat{d}) \) holds.

(iii) If \( \Psi(\hat{d}) \leq G(\hat{d}) \), then \( u_{b,h}^{SP}(\hat{d}) = u_{b,l}^{SP}(\hat{d}) \equiv u_{g,h}^{SP}(\hat{d}) = u_{g,l}^{SP}(\hat{d}) \). Furthermore, \( u_{g}^{SP}(\hat{d}) = p_g u_{b,h}^{SP}(\hat{d}) + (1 - p_g)u_{b,l}^{SP}(\hat{d}) \leq p_b u_{b,h}^{SP}(\hat{d}) + (1 - p_b)u_{b,l}^{SP}(\hat{d}) \) holds.

**Proof.** See Appendix 2.6.1. \( \square \)

Lemma 7 demonstrates how the planner’s optimal policy at the ex-post stage depends on the Pareto-weights assigned to ex-post good and bad types. In particular, the ex-post Pareto-frontier can be decomposed in two parts: The first regime, characterized in part (ii) of the lemma, involves redistribution from bad to good types (as captured by \( \Psi(\hat{d}) \geq G(\hat{d}) \), so that the planner attaches a larger weight to good types than their population share) and is therefore such that the bad types’ incentive constraint (2.3) binds, the bad types obtain full and the good types partial insurance. The other regime, which arises if \( \Psi(\hat{d}) \leq G(\hat{d}) \) (part (iii) of the lemma), involves the opposite direction of redistribution ex-post, from good to bad types. Therefore, the good types’ incentive constraint (2.2) binds and they obtain full and the bad types overinsurance. Clearly, good types obtain the higher ex-post expected utility than bad types in the first regime, and the lower one in the second regime. A special case arises when \( \Psi(\hat{d}) = G(\hat{d}) \) and thus both types are weighted at their population share. Then, at the optimum, both types get full insurance and there is complete pooling: \( u_{b,h}^{SP}(\hat{d}) = u_{b,l}^{SP}(\hat{d}) = u_{g,h}^{SP}(\hat{d}) = u_{g,l}^{SP}(\hat{d}) \). Note that, since the effort choice has already been taken, the planner without commitment does not care about providing effort incentives with her policy at the ex-post stage, but only about achieving optimal redistribution across the two ex-post types.

If \( \hat{d} \in \{0, \infty\} \), i.e. if all agents are either good types or bad types, the planner’s problem \( SP(\hat{d}) \) prescribes the utility maximization of the unique ex-post type, subject to a resource constraint. First, resources will clearly be exhausted in the solution. Second, convexity of
2.3. Markets Versus Governments

\( \Phi \) implies that the solution entails an output-independent payment. Hence for \( \hat{d} \in \{0, \infty\} \) we analogously define \( V^{SP}(\hat{d}) \) by \( u_{b,h}^{SP}(\hat{d}) = u_{b,l}^{SP}(\hat{d}) = u_{g,h}^{SP}(\hat{d}) = u_{g,l}^{SP}(\hat{d}) = U(R(\hat{d})). \)^6 Similar to Fudenberg and Tirole (1990, Definition 3.1), we now define an equilibrium with a social planner without commitment as follows.

**Definition 3.** An *equilibrium with a social planner (ESP)* is a pair \((d^*, V^*)\) where

(i) \( V^* = V^{SP}(d^*) \) and

(ii) \( d^* = p_g u_{g,h}^{SP}(d^*) + (1 - p_g) u_{g,l}^{SP}(d^*) - p_b u_{b,h}^{SP}(d^*) - (1 - p_b) u_{b,l}^{SP}(d^*) \).

If effort choice is described by some \( \hat{d} \) and the planner has formed a correct belief, she will implement \( V^{SP}(\hat{d}) \) ex-post. Condition (ii) captures that agents will in stage 1 anticipate this outcome, so that their actually optimal effort choice is described by the threshold

\[
D^{SP}(\hat{d}) = p_g u_{g,h}^{SP}(\hat{d}) + (1 - p_g) u_{g,l}^{SP}(\hat{d}) - p_b u_{b,h}^{SP}(\hat{d}) - (1 - p_b) u_{b,l}^{SP}(\hat{d}).
\]

This holds because each agent calculates her ex-post utility from being a good type (choosing the good type’s optimal contract) and the corresponding utility from being a bad type, and compares the difference to her effort cost \( d \). The function \( D^{SP} \) therefore yields the indifferent cost type given any threshold \( \hat{d} \), and no agent has an incentive to deviate if and only if the fixed point condition \( d^* = D^{SP}(d^*) \) and hence (ii) is satisfied.

Given the above results on the solution \( V^{SP}(\hat{d}) \) for varying levels of \( \hat{d} \in \mathbb{R}^+_0 \cup \{\infty\} \), the following result is immediate.

**Proposition 8.** For any distribution of Pareto weights \( \Psi \), \((0, V^{SP}(0))\) is an ESP. It is the unique ESP if \( \Psi \succeq_{FOSD} G \).

If the planner puts weakly overproportional weight on bad types ex-post for any given \( \hat{d} \), captured by \( \Psi \succeq_{FOSD} G \), continuation contracts are always such that the segment of the ex-post Pareto-frontier is attained which is described in part (iii) of Lemma 7. Since the planner aims at redistributing from good to bad types at the ex-post stage, bad types obtain the (weakly) higher continuation expected utility than good types, which of course eliminates any effort incentives from an ex-ante perspective. If \( \Psi \succeq_{FOSD} G \) is not satisfied, the complete breakdown of incentives remains an equilibrium: If all agents choose the low effort, any planner implements a full insurance allocation ex-post. Additional ESP might, however, emerge in that case. We examine such equilibria in Section 2.4.

---

^6We let \( V^{SP}(0) \) and \( V^{SP}(\infty) \) be elements of \( \mathbb{R}^4 \) for notational consistency, even though there is only one ex-post type if \( \hat{d} \in \{0, \infty\} \). One can still think of \((u_{g,h}^{SP}(\hat{d}), u_{g,l}^{SP}(\hat{d}))\) as the best contract for type \( k \in \{g, b\} \) among those offered, even though only one type actually exists and the planner offers a single contract only.
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2.3.2 Competitive Markets Without Commitment

We now turn to the case where contracts are provided by competitive firms rather than a social planner, and both firms and agents are unable to commit to contracts before the hidden effort choice. That is, we consider a time structure with two-sided lack of commitment, where, after an initial phase of contract offers and agents' choices of contracts and effort, firms are free to alter their existing contracts and offer additional ones, while agents are free to abrogate their contract and choose a new one, possibly switching between firms.

As argued in the introduction, we do not allow the firms' new contract offers or modifications to be conditioned on an agent's initial choice of contract. In the first place, this allows us to isolate our commitment problem from ratchet effects (Freixas, Guesnerie, and Tirole 1985). Second, it captures the realistic scenario that firms can modify concluded contracts only if they do not target specific individuals. For instance, insurance contracts can contain clauses that give the firm the right to modify terms of the contract without discriminating between customers. The decision whether to accept or to opt out of the contract is then left to the insurant, as in our model. On the other hand, the insured person can often cancel its policy with relatively short notice. In an education application, long-run contracts that arrange the terms of employment before educational choices have been made often do not exist at all, yielding an equivalent game theoretic structure:

\begin{itemize}
  \item \textit{Stage 1:} Agents choose an effort level.
  \item \textit{Stage 2:} Some market game takes place, resulting in a set of offered contracts.
  \item \textit{Stage 3:} Agents simultaneously choose a contract.
\end{itemize}

The structure is exactly the same as for the planner. In particular, we assume that ex-ante contracts (if they exist) do not constitute binding reservation constraints, and therefore we do not exogenously assume better commitment opportunities for private firms than for the planner. Also, all assumptions about information and observability are as before.

The key here is that the comparison between markets and governments that we derive in the following does neither require a detailed specification of the market game in stage 2, nor does it rest on the particular equilibrium notion for the ex-post market. We only assume that, after agents have chosen their effort according to a threshold \( \hat{d} \), firms form a correct belief about \( \hat{d} \) and some ex-post market game takes place, which results in an equilibrium set of contract offers. While the market game for given effort \( \hat{d} \) could be complicated, with respect to the timing of moves or its observability assumptions,
we restrict attention to its outcome, i.e. to the two contracts among the final offers that maximize the utility of the two different effort types and will thus be chosen in stage 3. We denote this outcome, for a fixed but yet unspecified ex-post market game, by $V^M(\hat{d}) = (u^M_{b,h}(\hat{d}), u^M_{b,l}(\hat{d}), u^M_{g,h}(\hat{d}), u^M_{g,l}(\hat{d}))$, and propose the following equilibrium definition, which is completely analogous to the ESP definition in Section 2.3.1.

**Definition 4.** An equilibrium with competitive markets (ECM) is a pair $(d^*, V^*)$ where (i) $V^* = V^M(d^*)$ is the ex-post market outcome given $d^*$ and (ii) $d^* = p_g u^M_{g,h}(d^*) + (1 - p_g) u^M_{g,l}(d^*) - p_b u^M_{b,h}(d^*) - (1 - p_b) u^M_{b,l}(d^*)$.

We now proceed to formulate plausible conditions on the outcomes $V^M(\hat{d})$, which essentially require informational and resource feasibility as well as a minimal degree of competitive pressure. This approach builds on the insights of Rothschild (2007), who has observed a similar robustness property in a setting of categorical discrimination in insurance markets. In particular, we impose the following axioms:

(C1) $V^M(\hat{d})$ is incentive compatible, i.e.

$$p_k u^M_{k,h}(\hat{d}) + (1 - p_k) u^M_{k,l}(\hat{d}) \geq p_k u^M_{k,h}(\hat{d}) + (1 - p_k) u^M_{k,l}(\hat{d}) \forall k, k' \in \{g, b\}.$$  

(C2) $V^M(\hat{d})$ is resource feasible, i.e.

$$R(\hat{d}) \geq G(\hat{d}) [p_g \Phi(u^M_{g,h}(\hat{d})) + (1 - p_g) \Phi(u^M_{g,l}(\hat{d}))] + (1 - G(\hat{d}))[p_b \Phi(u^M_{b,h}(\hat{d})) + (1 - p_b) \Phi(u^M_{b,l}(\hat{d}))].$$  

(C3) $V^M(\hat{d})$ is minimally contestable, i.e. there does not exist an incentive compatible outcome $\hat{V} = (\hat{u}_{b,h}, \hat{u}_{b,l}, \hat{u}_{g,h}, \hat{u}_{g,l})$ such that

1. $\pi_k(\hat{u}_{k,h}, \hat{u}_{k,l}) \geq 0 \forall k \in \{g, b\}$, and
2. $\pi_k(\hat{u}_{k,h}, \hat{u}_{k,l}) > 0$ and $p_k \hat{u}_{k,h} + (1 - p_k) \hat{u}_{k,l} > p_k u^M_{k,h}(\hat{d}) + (1 - p_k) u^M_{k,l}(\hat{d})$ for some $k \in \{g, b\},$

where $\pi_k(u_h, u_l) = p_k(y_h - \Phi(u_h)) + (1 - p_k)(y_l - \Phi(u_l))$ are the profits earned with one unit of type $k$ agents in contract $(u_h, u_l)$.

Clearly, any market outcome $V^M(\hat{d})$, whether competitive, monopolistic, or in between, has to satisfy (C1) and (C2). The third requirement (C3), introduced by Rothschild (2007), captures a minimal notion of competition. It implies that a market outcome fails
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to be minimally contestable only if a firm could offer a pair of incentive compatible deviation contracts that are such that they make non-negative profits no matter what types the firm attracts, and they earn strictly positive profits on some type that strictly prefers them to $V^M(\hat{d})$. This rules out market outcomes that do not survive even the slightest degree of competitive pressure. Let us provide an example for an outcome $V^M(\hat{d})$ that satisfies conditions (C1) to (C3).

Example. Consider the Rothschild-Stiglitz contracts $(u^{RS}_{g,h}, u^{RS}_{g,l})$ and $(u^{RS}_{b}, u^{RS}_{b})$, where $u^{RS}_{b} = U(p_b y_h + (1 - p_b)y_l)$ is the output-independent payoff for bad types, and the good type's contract satisfies $\pi_g(u^{RS}_{g,h}, u^{RS}_{g,l}) = 0$ and $p_b u^{RS}_{g,h} + (1 - p_b)u^{RS}_{g,l} = u^{RS}_{b}$. These contracts are independent of $\hat{d}$, and the outcome $V^{RS} = \left(u^{RS}_{b}, u^{RS}_{b}, u^{RS}_{g,h}, u^{RS}_{g,l}\right)$ satisfies conditions (C1) and (C2) for any $\hat{d} \in (0, \infty)$ by definition. It also satisfies (C3) because $V^{RS}$ simultaneously maximizes the utility of both types among the incentive compatible pairs of contracts that break even individually.

The example illustrates that axioms (C1) - (C3) are actually weak. Even though the Rothschild-Stiglitz contracts might not be considered a reasonable market outcome for all $\hat{d} \in (0, \infty)$ (e.g. due to equilibrium non-existence problems), they still satisfy the axioms. Also, we will later illustrate that the axioms do not rule out cross-subsidization between ex-post types, so that outcomes satisfying them might still be susceptible to cream-skimming behavior. But these considerations strengthen our following result, which is based on (C1) - (C3) only, and will thus hold a fortiori for market outcomes that satisfy even stricter requirements.\footnote{The axioms (C1) - (C3) coincide with those used by Rothschild (2007) except for two differences. First, in the definition of minimal contestability, Rothschild (2007) requires the deviation $\hat{V}$ to be resource feasible, but this is implied by property 1 in the definition of (C3) and can be omitted. Second, in addition to (C1) - (C3) Rothschild (2007) also requires a market outcome to be individually rational, which amounts to the assumption that $p_k u^M_{k,h}(\hat{d}) + (1 - p_k)u^M_{k,l}(\hat{d}) \geq p_k U(y_h) + (1 - p_k)U(y_l) \forall k \in \{g, b\}$. As we show in the proof of Theorem 1, this axiom is actually not independent, i.e. it is implied by (C1) - (C3) and can also be omitted.}

2.3.3 A Comparison

We will now compare markets based on properties (C1) - (C3) to a planner as analyzed above. Our concept of Pareto dominance applies to ex-ante utilities, i.e. we say that an equilibrium Pareto dominates another if it gives every agent (i.e. every ex-ante cost type) a weakly larger and at least one a strictly larger overall utility, including effort cost. The effort level might be different for some types between the two equilibria.
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**Theorem 1.** Any ECM \((d^*, V^*)\) in which \(d^* \in (0, \infty)\) and \(V^* = V^M(d^*)\) satisfies (C1)-(C3) Pareto dominates the ESP \((0, V^{SP}(0))\).

**Proof.** Suppose \(V^* = V^M(d^*)\) satisfies (C1)-(C3). We first show that \(p_k u^M_{k;j}(d^*) + (1 - p_k) u^M_{k,j}(d^*) \geq p_k U(y_h) + (1 - p_k) U(y_l) \forall j \in \{g, b\}\) must hold. Assume to the contrary that this is violated for a type \(j \in \{g, b\}\) and consider the outcome \(\tilde{V} = (U(y_h) - \epsilon, U(y_l) - \epsilon, U(y_h) - \epsilon, U(y_l) - \epsilon)\) for small \(\epsilon > 0\). \(\tilde{V}\) is incentive compatible and satisfies \(\pi_k (U(y_h) - \epsilon, U(y_l) - \epsilon) > 0 \forall k \in \{g, b\}\) by definition. Also, for \(\epsilon\) sufficiently small, we have that \(pj(U(y_h) - \epsilon) + (1 - pj)(U(y_l) - \epsilon) > pj u^M_{j,l}(d^*) + (1 - pj) u^M_{j,l}(d^*)\) still holds, so that \(V^* = V^M(d^*)\) violates (C3), a contradiction. Thus any outcome that satisfies (C1) - (C3) must be individually rational as defined by Rothschild (2007).

Consider the Rothschild-Stiglitz contracts as introduced before. They satisfy \(u^R_S > u^R_B\) and hence, since \(p_g > p_b, p_g u^RS_{g,h} + (1 - p_g) u^RS_{g,l} > u^RS_b\), Lemma 4 in Rothschild (2007), considering the special case of only two types, now implies that \(V^M(d^*)\) satisfies \(p_g u^M_{g,h}(d^*) + (1 - p_g) u^M_{g,l}(d^*) \geq p_g u^RS_{g,h} + (1 - p_g) u^RS_{g,l}\) and \(p_b u^M_{b,h}(d^*) + (1 - p_b) u^M_{b,l}(d^*) \geq u^RS_b\), i.e. both types are ex-post weakly better off in \(V^M(d^*)\) than in the Rothschild-Stiglitz contracts. Since \(u^RS = u^SP(0)\) and \(p_g u^RS_{g,h} + (1 - p_g) u^RS_{g,l} > u^RS_b = u^SP(0)\), this implies that \(V^M(d^*)\) ex-post Pareto dominates \(V^{SP}(0)\).

Condition (ii) in Definition 4 then implies that \(p_g u^M_{g,h}(d^*) + (1 - p_g) u^M_{g,l}(d^*) - d > p_b u^M_{b,h}(d^*) + (1 - p_b) u^M_{b,l}(d^*) \geq u^RS_b = u^SP(0)\) for all \(d < d^*\), where the first inequality follows directly from Definition 4(ii) and the other comparisons from the above argument for ex-post Pareto dominance. Hence all agents who prefer the high effort \((d < d^*)\) and subsequently contract \((u^M_{g,h}(d^*), u^M_{g,l}(d^*))\) in the ECM have a strictly larger utility, including effort cost, than they obtain as bad types in the ESP. Agents preferring low effort \((d \geq d^*)\) are weakly better off, from the above comparison.

We thus do not need to know exactly how markets work. The theorem shows that two properties of market equilibria are sufficient to make a Pareto comparison. First, the market has to be weakly competitive (minimally contestable). Second, the market must be able to sustain some incentives for effort provision, captured by the assumption of an interior share of good types \(d^* \in (0, \infty)\) in equilibrium. We have already illustrated the plausibility of minimal contestability in the previous section. To show that markets without commitment can actually be able to sustain incentives, we again consider the Rothschild-Stiglitz example.

**Example continued.** Assume that \(V^M(\hat{d}) = V^{RS}\) for all \(\hat{d} \in (0, \infty)\). Since \(V^{RS}\) is separating with \(p_g u^RS_{g,h} + (1 - p_g) u^RS_{g,l} > u^RS_b\), the optimal critical value for effort choice in anticipation of \(V^M(\hat{d}) = V^{RS}\), \(\Delta = p_g u^RS_{g,h} + (1 - p_g) u^RS_{g,l} - u^RS_b\), is strictly positive and independent of \(\hat{d}\). Thus \((\Delta, V^{RS})\) is an ECM that satisfies the prerequisites of Theorem 1.

The example illustrates that a sufficient degree of ex-post separation for all \(\hat{d} \in (0, \infty)\) is necessary to sustain incentives, which specifically requires that some separation is sus-
tained as the share of good types goes to zero. Moreover, the separation has to be such that good types are better off ex-post. For example, in regime (iii) from Lemma 7 the planner might also separate types, but bad types are better off, leading to a breakdown of incentives from an ex-ante perspective. Correct ex-post separation is indeed necessary to obtain an ECM in which good types do exist. But the existence of both types does not already guarantee that the allocation Pareto dominates the planner’s outcome. If the minimal notion of competition as captured by (C3) is not satisfied by a separating outcome, we cannot expect it to be Pareto superior to $V^{SP}(0)$. Consider, for example, a profit-maximizing monopolist that screens the population of agents ex-post and extracts as many resources as possible. Then, even if there is an equilibrium in which both types do exist and are separated, the outcome will generally not leave both types better off than in $V^{SP}(0)$. In fact, agents will be strictly worse off whenever their outside option is sufficiently unattractive.\footnote{We can ignore outside options in our analysis, because a planner maximizes welfare and implements contracts on which any reasonable outside option, such as the possibility to remain uninsured $(U(y_h), U(y_l))$, imposes no binding constraint. The same holds for weakly competitive markets from our previous results.} Hence, while our comparison between markets and governments does not depend on the details of the market game, it depends on the assumption that the market satisfies a minimal notion of competition.

2.4 Markets As Planners

2.4.1 Market Foundations

The results in the previous section were based on axioms on market outcomes. We have illustrated them using the standard Rothschild-Stiglitz separating allocation. Unfortunately, as is well-known, the Rothschild-Stiglitz equilibrium concept may run into non-existence problems. However, there are various ways to overcome this problem. For instance, Riley (1979) has restored existence by considering a reactive equilibrium concept that involves deviators anticipating their competitors' reaction, and Bisin and Gottardi (2006) show that the Rothschild-Stiglitz separating allocation is always a Walrasian equilibrium when agents are restricted to trade incentive-compatible consumption bundles contingent on the states $h, l$. Similarly, Guerrieri, Shimer, and Wright (2009) demonstrate that the non-existence problem vanishes in a setting with capacity constraints or search frictions.

Another problem with the Rothschild-Stiglitz equilibrium, even if it exists, is that it restricts firms to offer only one contract, ruling out cross-subsidization and therefore lead-
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ing to potential inefficiency.\(^9\) Since such a restriction is not imposed on our social planner, it may bias the comparison in favor of markets. Indeed, restricting the number of contracts that firms can offer amounts to a restriction of their ex-post deviation possibilities, reducing the scope for profitable deviations from initial announcements and therefore increasing commitment. However, we demonstrate in Netzer and Scheuer (2009) that the comparison of markets and governments does not rely on such a restriction. In particular, we provide a game-theoretic foundation for the Miyazaki-Wilson equilibrium (Miyazaki 1977), which includes Rothschild-Stiglitz as a special case and is always Pareto-efficient, by constructing an extensive form market game where firms can offer any finite number of contracts, and in which a unique robust subgame-perfect equilibrium always exists. We show that the resulting equilibrium allocation is the solution to a maximization problem, which we characterize and discuss in the following.\(^10\)

2.4.2 Miyazaki-Wilson ECM

Consider the contracts characterized by the following program, denoted by MW(\(\hat{d}\)):

\[
\max_{(u_{b,h}, u_{b,l}, u_{g,h}, u_{g,l}) \in \mathbb{R}^4} p_g u_{g,h} + (1 - p_g) u_{g,l}
\]

subject to the constraints

\[
p_g u_{g,h} + (1 - p_g) u_{g,l} \geq p_g u_{b,h} + (1 - p_g) u_{b,l},
\]

\[
p_b u_{b,h} + (1 - p_b) u_{b,l} \geq p_b u_{g,h} + (1 - p_b) u_{g,l},
\]

\[
G(\hat{d}) \left[ p_g \Phi(u_{g,h}) + (1 - p_g) \Phi(u_{g,l}) \right] + (1 - G(\hat{d})) \left[ p_b \Phi(u_{b,h}) + (1 - p_b) \Phi(u_{b,l}) \right] \leq R(\hat{d}),
\]

\[
\Phi(p_b u_{b,h} + (1 - p_b) u_{b,l}) \geq p_b y_{h} + (1 - p_b) y_{l}.
\]

In program MW(\(\hat{d}\)), the expected utility of good types is maximized under the ex-post

---

\(^9\)This assumption is shared by many approaches in the literature on competitive insurance markets, such as Wilson (1977) and Hellwig (1987).

\(^10\)Bisin and Gottardi (2006) show that the Miyazaki-Wilson allocation can also be obtained in their setting if agents have to buy consumption rights in a separate Walrasian market before they can go to insurance markets and trade contingent consumption bundles there.
incentive compatibility constraints (2.6) and (2.7), the resource constraint (2.8), and a constraint (2.9) that requires the certainty equivalent of the bad types’ contract to be at least as large as their expected endowment. This constraint makes sure that cross-subsidization can only go from good to bad types in any solution, because it implies that the resource cost of the bad types’ contract must always be weakly larger than their expected output, i.e. it earns zero or negative profits taken on its own. Note that, comparing with $SP(\hat{d})$, the only two differences are the additional constraint (2.9) and the objective function (2.5), which is a special case of (2.1) putting weight exclusively in good types.

Lemma 8. Fix any $\hat{d} \in (0, \infty)$.
(i) $MW(\hat{d})$ has a unique solution $V^{MW}(\hat{d}) = (u_{b,h}^{MW}(\hat{d}), u_{b,l}^{MW}(\hat{d}), u_{g,h}^{MW}(\hat{d}), u_{g,l}^{MW}(\hat{d}))$.
(ii) $V^{MW}(\hat{d})$ satisfies (C1) to (C3). Furthermore, $u_{b,h}^{MW}(\hat{d}) = u_{b,l}^{MW}(\hat{d}) = u_{g,h}^{MW}(\hat{d}) = u_{g,l}^{MW}(\hat{d}) > u_{g,l}^{MW}(\hat{d})$, and $u_{b}^{MW}(\hat{d}) = p_{b}u_{g,h}^{MW}(\hat{d}) + (1 - p_{b})u_{g,l}^{MW}(\hat{d}) < p_{g}u_{g,h}^{MW}(\hat{d}) + (1 - p_{g})u_{g,l}^{MW}(\hat{d})$.

Proof. See Appendix 2.6.2. \qed

The problem characterizes the so-called Miyazaki-Wilson contracts. There are two cases depending on whether constraint (2.9) does or does not bind. If it does, each contract individually makes zero profits, and we obtain the classical Rothschild-Stiglitz outcome. Otherwise, the flat contract for bad types makes negative and the incentive contract for good types makes positive profits, implying cross-subsidization from good to bad types. For completeness, we again briefly turn to the case where $\hat{d} \in \{0, \infty\}$ and simply define $V^{MW}(\hat{d})$ by $u_{b,h}^{MW}(\hat{d}) = u_{b,l}^{MW}(\hat{d}) = u_{g,h}^{MW}(\hat{d}) = u_{g,l}^{MW}(\hat{d}) = U(R(\hat{d}))$ for $\hat{d} \in \{0, \infty\}$.

Now suppose the ex-post market actually leads to Miyazaki-Wilson contracts, i.e. suppose that $V^{M}(\hat{d}) = V^{MW}(\hat{d})$ for all $\hat{d}$. Then, to characterize the set of equilibria of the complete game without commitment, called $MW$-ECM, we need to find the fixed points of the function

$$D^{MW}(\hat{d}) = p_{g}u_{g,h}^{MW}(\hat{d}) + (1 - p_{g})u_{g,l}^{MW}(\hat{d}) - p_{b}u_{b,h}^{MW}(\hat{d}) - (1 - p_{b})u_{b,l}^{MW}(\hat{d}),$$

which gives us the indifferent ex-ante cost type $D^{MW}(\hat{d})$ if agents anticipate the final outcome $V^{MW}(\hat{d})$. If $\hat{d} \in (0, \infty)$, we know from Lemma 8 that $u_{b,h}^{MW}(\hat{d}) = u_{b,l}^{MW}(\hat{d}) = p_{b}u_{g,h}^{MW}(\hat{d}) + (1 - p_{b})u_{g,l}^{MW}(\hat{d})$, so that we can simplify $D^{MW}(\hat{d})$ to

$$D^{MW}(\hat{d}) = (p_{g} - p_{b}) \left( u_{g,h}^{MW}(\hat{d}) - u_{g,l}^{MW}(\hat{d}) \right). \quad (2.10)$$

Also, we immediately obtain $D^{MW}(0) = D^{MW}(\infty) = 0$. Let us collect some useful properties of the function $D^{MW}$ in the following lemma. These properties are based on comparative static effects of varying levels of $\hat{d}$ on the solution $V^{MW}(\hat{d})$. 

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Lemma 9. (i) $D^{MW}$ is continuous on $(0, \infty)$.
(ii) $\lim_{d \to 0} D^{MW}(\tilde{d}) > 0$ and $\lim_{d \to \infty} D^{MW}(\tilde{d}) = 0$.
(iii) If
\[
\frac{d}{du} \frac{\Phi''(u)}{\Phi'(u)} \geq 0,
\]
there exists $\tilde{d} \in (0, \infty)$ such that $D^{MW}(\tilde{d})$ is flat on $(0, \tilde{d}]$ and strictly decreasing on $(\tilde{d}, \infty)$.

Proof. See Appendix 2.6.3. \hfill \Box

Properties (i) and (ii) together with $D^{MW}(0) = 0$ imply that, while $D^{MW}$ is continuous otherwise, it has a discontinuity at $\tilde{d} = 0$. This is because a contract with output-independent utilities and hence no incentive for effort provision is the unique ex-post outcome if $\tilde{d} = 0$, while for any positive $\tilde{d}$ the good type’s contract remains high-powered. Specifically, we show in the proof of the lemma that (2.9) is binding in $V^{MW}(\tilde{d})$ for sufficiently small but positive $\tilde{d}$, which is saying that the Rothschild-Stiglitz contracts obtain if the given share of good types is small. As $\tilde{d} \to \infty$, on the other hand, the good type’s contract converges to an output-independent, full insurance contract, which requires cross-subsidization to the bad types to preserve incentive-compatibility. Condition (2.11) was introduced by Fudenberg and Tirole (1990). We show that, if it is satisfied, there is exactly one critical value $\tilde{d}$ at which the transition from zero to positive cross-subsidization occurs, and an increase in $\tilde{d}$ above $\tilde{d}$ leads to an increased subsidy and lower-powered incentives $u^{MW}_{gh}(\tilde{d}) - u^{MW}_{gJ}(\tilde{d})$, such that $D^{MW}$ is strictly decreasing. Fudenberg and Tirole (1990) show that (2.11) is satisfied whenever risk-aversion is not decreasing too quickly in income (see their Lemma 3.2), for example for CRRA preferences with a coefficient of at least 1. Figure 2.1 depicts the function $D^{MW}(\tilde{d})$ for such preferences. We can now state the main result of this subsection, which is a direct implication of the previous results and standard fixed point theorems.

Proposition 9. (i) The pair $(0, V^{MW}(0))$ is an MW-ECM.
(ii) There exists a value $d^* > 0$ such that $(d^*, V^{MW}(d^*))$ is an MW-ECM.
(iii) Under condition (2.11), there are no additional equilibria.

Clearly, $\tilde{d} = 0$ is always a fixed point of $D^{MW}$, meaning that there is an MW-ECM where no agent exerts effort and everyone obtains a contract with an output-independent payment. However, there always exists at least one other fixed point $d^* > 0$ of $D^{MW}$ and hence an MW-ECM in which a non-zero mass of agents exert the high effort. Since $D^{MW}$ is weakly decreasing under condition (2.11), the positive fixed point is unique in this case. Otherwise, multiple non-zero fixed points and associated MW-ECM may exist.
2.4. Markets As Planners

Figure 2.1: Fixed Point Problem with Miyazaki-Wilson Markets

2.4.3 Comparing MW-ECM to ESP

We are now in a position to compare market equilibria based on ex-post Miyazaki-Wilson contracts to equilibria with a social planner. We will present two main results in this section. The first, Theorem 2 compares welfare between the two types of equilibria. The second, Theorem 3, compares the induced aggregate effort levels.

**Theorem 2.** Any MW-ECM \((d^*, V^{MW}(d^*))\) with \(d^* > 0\) ex-ante Pareto dominates the ESP \((0, V^{SP}(0))\), and strongly so if \(V^{MW}(d^*)\) satisfies (2.9) with slack.

**Proof.** We consider two cases, depending on whether constraint (2.9) is binding or not in \(V^{MW}(d^*)\).
Assume first that it does, implying \(u^M_w(d^*) = U(p_h y_h + (1 - p_b)y_l) = u^{SP}(0)\), i.e. bad types in \(V^{MW}(d^*)\) obtain the same utility as all agents in \(V^{SP}(0)\), where nobody exerts any effort. By definition of \(d^*\), we then have that \(p_s u^M_{gh}(d^*) + (1 - p_b) u^M_{gl}(d^*) - d > u^M_b(d^*) = u^{SP}(0)\) for all \(d < d^*\). Given that \(d^*\) is a fixed point of \(D^{MW}\), the critical value \(d^*\) determines optimal effort choice in \(V^{MW}(d^*)\), so that all good types in \(V^{MW}(d^*)\) are ex-ante (including effort cost) strictly better off than they are as bad types in \(V^{SP}(0)\). If (2.9) is slack in \(V^{MW}(d^*)\), then \(u^M_b(d^*) > u^{SP}(0)\), and both low and bad types are ex-ante strictly better off in \(V^{MW}(d^*)\), with the same argument.

The first part of Theorem 2 is a corollary of previous results (although we present a convenient direct proof above): \(V^{MW}(d^*)\) satisfies (C1) - (C3) according to Lemma 8, so
we can immediately apply our general Theorem 1 to obtain the Pareto comparison. The second part of Theorem 2 goes beyond the general insight of Theorem 1. If the market equilibrium involves cross-subsidization from agents who choose to become good types to those who prefer to become bad types, as captured by slackness of (2.9), even all agents are strictly better off in the market than under a social planner.

The intuition for Theorem 2 is that in the ESP $(0, V^{SP}(0))$ all agents end up being bad types in a contract with an output-independent payoff, equal in size to their expected output. In the market, only some agents remain bad types, obtaining a flat payment at the same or even a subsidized level, whereas the other agents prefer to become good types and choose a contract that makes them strictly better off. Hence it is the ex-post adverse selection problem in competitive markets, leading to underinsurance of some agents, which is crucial for the dominance of markets over governments. Konrad (2001), in an extension of Boadway, Marceau, and Marchand (1996), compares the optimal tax policy of an informed to an uninformed government and also finds that information rents due to ex-post asymmetric information can be welfare-enhancing. We emphasize, however, that we impose the same assumptions about information on both markets and the planner, and, as opposed to Schmidt (1996), Konrad (2001) and Bisin and Rampini (2006) do not compare different information structures. From our perspective, establishing a competitive market can be interpreted as the delegation of decisions to a planner who cares only for good types and faces the additional constraint (2.9). The similar idea that the creation of an independent agency and the subsequent delegation of decisions to this agency can be beneficial in the presence of a commitment problem is central to the research on central bank independence. In our model, the advantage of a market is that it acts as if it was a specific planner, while individual firms are still maximizing their real objective (profits). This avoids incentive problems that would be present with an independent agency such as a central bank (Walsh 1995).

According to Proposition 8, $(0, V^{SP}(0))$ is always an ESP, and the unique one whenever the planner is concerned about a utilitarian welfare criterion or aims at redistributing towards agents with high effort cost. If $\Psi \geq_{FOSD} G$ is not satisfied, then the welfare comparison in Theorem 2 can become inapplicable because ESP with a positive share of good types might emerge. It is indeed not possible to make a general Pareto comparison between markets and government in that case. For instance, a social planner who cares almost only for low effort cost agents might want to implement cross-subsidization from bad to good types, and thereby make bad types worse off than in an MW-ECM, where this direction of cross-subsidization is impossible. However, we are now going to show that it is possible to compare the aggregate level of effort implemented by a social plan-
ner with a general distribution $\Psi$ of Pareto-weights to that implemented by competitive markets. Despite the potential multiplicity of ESP, we still have the following result:

**Theorem 3.** Suppose condition (2.11) is satisfied and the interior MW-ECM $(d^*, V_{MW}^{MW}(d^*))$ satisfies $d^* > \bar{d}$. Then $d^* \geq d^{**}$ for any ESP $(d^{**}, V_{SP}^{SP}(d^{**}))$.

**Proof.** See Appendix 2.6.4.

According to the theorem, whenever condition (2.11) is satisfied and the MW-ECM involves cross-subsidization, the associated equilibrium share of good types $G(d^*)$ is higher than the share of good types $G(d^{**})$ in any ESP, irrespective of the distribution of Pareto-weights $\Psi$ that is used. Hence, in terms of incentives for effort, a social planner cannot do better than the market. The reason is again that Miyazaki-Wilson markets replicate an extreme planner who cares only about ex-post good types.

The restriction to the case that the MW-ECM involves cross-subsidization is necessary because, as argued above, our markets are constrained by the fact that there cannot be cross-subsidization from bad to good types, while the planner is not. A planner who is otherwise similar to the market, in that she puts a large weight on good ex-post types, may find it optimal to make these good types even better off, at the expense of bad types, which increases the incentive to exert effort. As Proposition 8 makes clear, this can only occur if the planner puts sufficiently overproportional welfare weight on low effort cost types.

### 2.4.4 Implications for Market Regulation

The previous results have important implications for market regulation. They highlight that competitive firms must be allowed to offer separating contracts for markets to be able to implement allocations that dominate the government outcome. More importantly, the market equilibrium involves underinsurance for some agents, and it may require cross-subsidization where firms use strictly positive profits that they earn with some contracts to finance the losses incurred with other contracts. All these three properties, separation, underinsurance, and cross-subsidization, are crucial for the above results that markets sustain higher effort incentives than a government, and that they implement an ex-post Pareto-efficient outcome that (strictly) dominates the social planner's allocation.

These insights cast doubt on regulatory policies that aim at achieving standardization of contracts, reduce underinsurance, or enforce actuarial fairness of individual contracts. For instance, our model applies to the moral hazard problem that banks face in monitoring their loan portfolio, and the time inconsistency problem related to credit insurance.
From an ex-ante perspective, less than full credit insurance is optimal in order to incentivize banks to monitor their borrowers, but ex-post, a private credit insurer or government may insure banks fully against the risk of default of their loan portfolio, e.g. in the form of a bailout. Competitive credit insurance markets, as for instance in the form of a credit default swap market, stay away from full insurance due to the adverse selection problem at the ex-post stage, where the quality of different banks' loan portfolios is private information. For this, however, it is important that more than one type of credit insurance contract can be traded in the market, notably that they can differ in the amount of coverage, and that they may not individually make zero profits in expectation. This is in contrast to recent discussions about reorganizing the credit default swap market (see for instance Stulz (2010)). There, a popular proposal is to move away from an over-the-counter market with individualized contracts to exchange trading with standardized contracts, where actuarial fairness is mechanically enforced.

As far as there is a similar ex-ante moral hazard problem in health insurance markets (see e.g. Dave and Kaestner (2006) for evidence), the results also make clear that underinsurance and strictly positive profits of some insurance policies in competitive health insurance markets must not be viewed as signs of market failure, calling for regulatory intervention. Instead, they are key to sustain preventive incentives and achieve Pareto-efficiency. Legal constraints on deductibles and coinsurance, as contained in the current US health care reform bill, may, if binding, be detrimental to achieving these objectives.\textsuperscript{11}

\section{Conclusion}

We have analyzed the performance of competitive markets in the framework of a time-inconsistency problem with incentive contracts. We have first pursued an axiomatic approach, based on weak properties that ex-post market outcomes should satisfy to be considered competitive. We have shown that such outcomes Pareto dominate the allocation that a large class of redistributive social planners can achieve, whenever markets are able to sustain some incentives for effort provision through separating agents ex-post (Theorem 1). We have then examined a specific but well-justified ex-post market outcome in greater detail. If the ex-post market produces Miyazaki-Wilson contracts, incentives for effort provision are preserved ex-ante and our general Pareto result applies. It turns out that, in this case, the market replicates a social planner who cares only about high effort agents and thus sustains maximal incentives for effort provision.

\textsuperscript{11}For a summary of these provisions in the Patient Protection and Affordable Care Act signed into law on March 23, 2010, see the report of the Kaiser Family Foundation (2010).
2.6 Appendix

Our further results identify the level of cross-subsidization between ex-post types in the market as an important property to assess outcomes. First, the comparison between markets and governments becomes a strong Pareto dominance result whenever the market equilibrium entails cross-subsidization (Theorem 2). Second, even if the Pareto comparison is not applicable, because the planner does not belong to the above-mentioned class, we can still compare the aggregate equilibrium effort between market and planner if the market cross-subsidizes. In that case, the market always performs better than a government in terms of incentives for effort (Theorem 3). Altogether, our results suggest that competitive markets are an institution that is able to deal with the commitment problem very successfully, even in a model that excludes any reputational mechanisms.

Our model provides a transparent framework in which a benevolent planner cannot replicate the outcome achieved by a market. This result is not due to exogenously assumed differences in technologies, commitment constraints, policy instruments or information, but is solely based on the different objectives that the two institutions pursue (implicitly, in case of the market). To transparently expose the effect of competition on the commitment problem, we have ruled out reputational effects in our analysis. Future research on how competition and reputation interact may produce further interesting insights.

2.6 Appendix

2.6.1 Proof of Lemma 7

We prove the lemma by proving a sequence of preliminary claims. We suppress dependency on $\hat{d}$ for notational convenience.

Claim 1. Constraint (2.4) must be binding in any solution $V = (u_{b,h}, u_{b,l}, u_{g,h}, u_{g,l})$ to SP.

Proof. Assume that $V$ does not satisfy (2.4) with slack. Consider $\bar{V} = (u_{b,h} + \epsilon, u_{b,l} + \epsilon, u_{g,h} + \epsilon, u_{g,l} + \epsilon)$ for $\epsilon > 0$. $\bar{V}$ satisfies (2.2) and (2.3) and leads to a strictly increased value of (2.1). Continuity of $\Phi$ implies that (2.4) is still satisfied by $\bar{V}$ for $\epsilon$ sufficiently small, so that $V$ was not a solution to SP.

Claim 2. Any solution $V$ must satisfy $u_{b,h} - u_{b,l} \leq u_{g,h} - u_{g,l}$.

Proof. Adding (2.2) and (2.3) yields, after rearranging, $(p_g - p_b)(u_{b,h} - u_{b,l}) \leq (p_g - p_b)(u_{g,h} - u_{g,l})$. Together with $p_g > p_b$, the claim follows.

Claim 3. Any solution $V$ must satisfy $u_{k,h} = u_{k,l}$ for at least one $k \in \{g, b\}$.

Proof. Suppose the claim is not true and consider all possible cases. First, $0 < u_{b,h} - u_{b,l} \leq u_{g,h} - u_{g,l}$ may hold. Define $\tilde{u}_b = p_b u_{b,h} + (1 - p_b) u_{b,l}$ and consider $\tilde{V} = (\tilde{u}_b, u_{b,l}, u_{g,h}, u_{g,l})$. By construction, $\tilde{V}$ satisfies (2.3), and the value of (2.1) is the same under $V$ and $\tilde{V}$. Since $p_g > p_b$ and
2.6. Appendix

It follows that \( p_g u_{b,h} + (1 - p_g) u_{b,l} > p_b u_{b,h} + (1 - p_b) u_{b,l} = \bar{u}_b = p_g \bar{u}_b + (1 - p_g) \bar{u}_b \), so that \( \bar{V} \) satisfies (2.2) as well, given that it is satisfied by \( V \). Strict convexity of \( \Phi \) implies that \( p_b \Phi(u_{b,h}) + (1 - p_b) \Phi(u_{b,l}) > \Phi(p_b u_{b,h} + (1 - p_b) u_{b,l}) = \Phi(\bar{u}_b) = p_b \Phi(\bar{u}_b) + (1 - p_b) \Phi(\bar{u}_b) \), so that \( \bar{V} \) satisfies (2.4) with slack given \( G \in (0,1) \). As in the proof of claim 1, the value of (2.1) can then be increased above its value for \( \bar{V} \) and \( V \), so that \( V \) was not a solution. An analogous argument reveals that a solution cannot satisfy \( u_{b,h} - u_{b,l} > u_{g,h} - u_{g,l} \),

Claim 4. Let \( V \) be a solution to \( SP \). If \( u_{k,h} = u_{k,l} \equiv u_k \) for \( k \in \{g,b\} \), then \( u_k = p_k u_{j,h} + (1 - p_k) u_{j,l} \) for \( j \neq k \), i.e. type \( k \)'s incentive constraint is satisfied as an equality.

Proof. Suppose first that \( u_{b,h} = u_{b,l} \equiv u_b \) but, to obtain a contradiction, \( u_b > p_b u_{g,h} + (1 - p_b) u_{g,l} \). Incentive compatibility then implies \( u_{g,l} < u_{g,h} \). Consider \( \bar{V} = (u_b, u_b, u_{g,h} - \varepsilon, u_{g,l} + \varepsilon) / (1 - p_g) \), for \( \varepsilon > 0 \). The value of (2.1) is the same under \( \bar{V} \) as under \( V \) and (2.2) is still satisfied. For \( \varepsilon \) sufficiently small, (2.3) is also still satisfied by \( \bar{V} \), given slackness in \( V \). Strict convexity of \( \Phi \) immediately implies that (2.4) is slack in \( \bar{V} \), so that \( \bar{V} \) was not a solution to \( SP \), as argued before. The case where \( u_{g,h} = u_{g,l} \) is proven analogously.

Claims 1 to 4 imply that any solution \( V \) to \( SP \) must exhaust resources and satisfy either \( V \in \{ u_{b,h}, u_{b,l}, u_{g,h}, u_{g,l} \} \in \mathbb{R}^4 | u_{b,h} = u_{b,l} = p_b u_{g,h} + (1 - p_b) u_{g,l} \leq u_{g,h} \) or \( V \in \{ u_{b,h}, u_{b,l}, u_{g,h}, u_{g,l} \} \in \mathbb{R}^4 | u_{g,h} = u_{g,l} = p_g u_{b,h} + (1 - p_g) u_{b,l} \geq u_{b,h} \} \}. Observe that any \( V \in \mathcal{I}_1 \) automatically satisfies constraint (2.2) because \( p_g > p_b \), and any \( V \in \mathcal{I}_2 \) satisfies (2.3). Hence we can formulate the program \( SP' \), which has the same solutions as \( SP \), as follows:

\[
\max_{(u_{b,h}, u_{b,l}, u_{g,h}, u_{g,l}) \in \mathcal{I}_1 \cup \mathcal{I}_2} \Psi[p_g u_{g,h} + (1 - p_g) u_{g,l}] + (1 - \Psi)[p_b u_{b,h} + (1 - p_b) u_{b,l}]
\]

subject to

\[
G[p_g \Phi(u_{g,h}) + (1 - p_g) \Phi(u_{g,l})] + (1 - G)[p_b \Phi(u_{b,h}) + (1 - p_b) \Phi(u_{b,l})] = R.
\]

Let \( u_{\max} = U(R) \) and \( V_{\max} = (u_{\max}, u_{\max}, u_{\max}, u_{\max}) \). It is immediate that \( V_{\max} \in \mathcal{I}_1 \), \( V_{\max} \in \mathcal{I}_2 \), and \( V_{\max} \) satisfies (2.13). Denote by \( SP'_1 \) the program given by (2.12) and (2.13) with the additional restriction that \( V \in \mathcal{I}_1 \) only, and by \( SP'_2 \) the analogous program where \( V \in \mathcal{I}_2 \).

Claim 5. \( SP'_1 \) has a unique solution \( V_1 \). It satisfies \( V_1 = V_{\max} \) if and only if \( \Psi \leq G \).

Proof. Any solution to \( SP'_1 \) must be of the form \( V = (u_b, u_b, u_{g,h}, u_{g,l}) \) with \( u_b = p_b u_{g,h} + (1 - p_b) u_{g,l} \), or equivalently \( u_{g,l} = (u_b - p_b u_{g,h}) / (1 - p_b) \). The condition \( u_{g,h} \geq u_{g,l} \) can then be reformulated as \( u_{g,h} \geq u_b \). We can therefore state the following modified problem \( SP''_1 \), which has the
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same solutions as SP':

$$\max_{(u_{g,h}, u_b)|u_{g,h} \geq u_b} \Psi \left[ \left( \frac{1 - p_g}{1 - p_b} \right) u_b + \left( \frac{p_g - p_b}{1 - p_b} \right) u_{g,h} \right] + (1 - \Psi) u_b$$

(2.14)

subject to

$$G \left[ p_g \Phi(u_{g,h}) + (1 - p_g) \Phi \left( \frac{u_b - p_b u_{g,h}}{1 - p_b} \right) \right] + (1 - G) \Phi(u_b) = R.$$  

(2.15)

Denote the LHS of (2.15) by $E(u_{g,h}, u_b)$. $E$ is continuously differentiable on $\mathbb{R}^2$, and straightforward calculations reveal that it is strictly increasing in $u_{g,h}$ whenever $u_{g,h} \geq u_b$, with $\lim_{u_{g,h} \to \infty} E(u_{g,h}, u_b) = \infty$ due to convexity. $E$ is strictly increasing in $u_b$ globally, with $\lim_{u_b \to -\infty} E = -\infty$.

We first claim that $u_{b}^{\text{max}}$ represents the largest possible choice of $u_b$. Consider the tuple $(u_{b}^{\text{max}}, u_{b}^{\text{max}})$, which satisfies (2.15) by definition. Any tuple $(u_{g,h}, u_b)$ with $u_{g,h} > u_{b}^{\text{max}}$ and thus $u_{g,h}$ can be reached from $(u_{b}^{\text{max}}, u_{b}^{\text{max}})$ by first increasing $u_{g,h}$ from $u_{b}^{\text{max}}$ to $u_{g,h}$ and then increasing $u_b$ from $u_{b}^{\text{max}}$ to $u_{b}^{\text{max}}$. Both moves strictly increase $E(u_{g,h}, u_b)$, so that $(u_{g,h}, u_b)$ violates (2.15), which proves the claim.

Now fix any $u_b \leq u_{b}^{\text{max}}$. It follows that $E(u_{b}^{\text{max}}, u_b) \leq E(u_{b}^{\text{max}}, u_{b}^{\text{max}}) = R$, with strict inequality whenever $u_b < u_{b}^{\text{max}}$. Since $E(u_{g,h}, u_b)$ is strictly increasing in $u_{g,h}$ in the relevant range, with $\lim_{u_{g,h} \to \infty} E(u_{g,h}, u_b) = \infty$, it follows that there exists a unique value $H(u_b)$ such that $E(H(u_b), u_b) = R$, where $H(u_b) \geq u_{b}^{\text{max}} \geq u_b$. The resulting function $H : (-\infty, u_{b}^{\text{max}}] \to [u_{b}^{\text{max}}, \infty)$ is continuously differentiable and thus continuous, by the implicit function theorem.

We can now reduce SP" to the one-dimensional problem

$$\max_{u_b|u_b \leq u_{b}^{\text{max}}} \left( 1 - \frac{p_g - p_b}{1 - p_b} \right) u_b + \frac{p_g - p_b}{1 - p_b} H(u_b).$$

(2.16)

We first claim that $H(u_b)$ is strictly concave. Let $(u''_{g,h}, u''_b)$ and $(u_{g,h}', u_b')$ satisfy $E(u''_{g,h}, u''_b) = E(u_{g,h}', u_b') = \max (u_{g,h}^\prime, u_b^\prime)$ and $(u_{g,h}', u_b') \neq (u_{g,h}^\prime, u_b^\prime)$. Define $u''_{g,h} = \lambda u_{g,h}' + (1 - \lambda) u_{g,h}''$ and $u''_b = \lambda u_b' + (1 - \lambda) u_b''$ for $\lambda \in (0, 1)$. Strict convexity of $\Phi$ then implies that $E(u''_{g,h}, u''_b) < \max (u_{g,h}^\prime, u_b^\prime)$, which in turn implies that $H(u''_b) = H(\lambda u_b' + (1 - \lambda) u_b'') > u''_b = \lambda u_{g,h}'' + (1 - \lambda) u_{g,h}'' = \lambda H(u_b') + (1 - \lambda) H(u_b'')$, which proves the claim. Second, implicit differentiation of (2.15) reveals that $H$ is strictly decreasing with slope

$$H'(u_b) = \frac{G(1 - p_g) \Phi'(u_{g,h}) + (1 - G)(1 - p_b) \Phi'(u_b)}{G(1 - p_g) p_b \Phi'(u_{g,h}) - G(1 - p_b) p_g \Phi'(u_{g,h})}'$$

(2.17)

where $u_{g,h} = H(u_b)$ and $u_{g,h}$ has been re-substituted for $(u_b - p_b u_{g,h})/(1 - p_b)$. Observe that $\lim_{u_b \to -\infty} H'(u_b) = 0$. As $u_b$ decreases, $u_{g,h} = H(u_b)$ increases and $u_{g,h}$ decreases. Therefore, both terms in the numerator and the first term in the denominator of (2.17) are decreasing as $u_b$ is decreasing (but they remain positive). Since $\lim_{u_b \to -\infty} E(u_{g,h}, u_b) = -\infty$, it follows that $\lim_{u_b \to -\infty} H(u_b) = \infty$ and thus the second term in the denominator of (2.17) grows without bound.

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as \( u_b \to -\infty \), due to the Inada condition \( \lim_{u_b \to -\infty} \Phi'(u) = \infty \). Hence \( \lim_{u_b \to -\infty} H'(u_b) = 0 \) holds.

Strict concavity of \( H(u_b) \) implies that the objective in (2.16) is strictly concave whenever \( \Psi > 0 \) and strictly increasing in \( u_b \) if \( \Psi = 0 \). Together with the fact that the objective must be strictly increasing in \( u_b \) for sufficiently small values of \( u_b \), due to \( \lim_{u_b \to -\infty} H'(u_b) = 0 \) and \( 1 - \Psi (p_g - p_b)/(1 - pb) > 0 \), this implies existence and uniqueness of a solution \( V_1 \).

We prove that \( V_1 = V^{\text{max}} \) if and only if \( \Psi < G \) by showing that the slope of the objective (2.16) evaluated at \( u_b = u^{\text{max}} \) is (weakly) positive if and only if \( \Psi < G \). The result then follows from strict concavity of \( H(u_b) \) and \( H'(u^{\text{max}}) = u^{\text{max}} \).

Lemma 7 now follows. Since \( V^{\text{max}} \in \mathcal{I}_1 \) and \( V^{\text{max}} \in \mathcal{I}_2 \), whenever \( V_1 \neq V^{\text{max}} = V_2 \), then \( V^{SP} = V_1 \in \mathcal{I}_1 \) is the unique solution to the unrestricted problem \( SP' \) and hence \( SP, \) satisfying all properties given in (ii). This is the case iff \( \Psi > G \). Analogously, \( V_1 = V^{\text{max}} \neq V_2 \) implies \( V^{SP} = V_2 \in \mathcal{I}_2 \) and \( V^{SP} \) satisfies the properties given in (iii), which is the case iff \( \Psi < G \). If \( \Psi = G \), Claims 5 and 6 immediately imply that \( V^{SP} = V^{\text{max}} \), which satisfies the properties given in (ii) and in (iii).

### 2.6.2 Proof of Lemma 8

MW is a special case of \( SP \) for \( \Psi = 1 \), with the additional constraint (2.9). Therefore, Claims 1 - 4 in the proof of Lemma 7 apply unaltered, because none of the arguments is affected by (2.9).

Ignore constraint (2.9) and consider Claim 6 above. It implies that \( V_2 = V^{\text{max}} \) is the unique solution to \( SP'_2 \) due to \( \Psi = 1 \geq G \). Since \( V^{\text{max}} \) obviously satisfies (2.9), it is also the unique solution when constraint (2.9) is imposed additionally. Then, since \( V^{\text{max}} \in \mathcal{I}_1 \) as well, solutions \( V^{MW} \) to MW are identical to solutions of \( SP'_1 \) for \( \Psi = 1 \) and under the additional constraint (2.9). Since any \( V \in \mathcal{I}_1 \) must be of the form \( V = (u_b, u_b, u_{gh}, u_{g1}) \), (2.9) can be reformulated as \( u_b \geq U(p_b y_h + (1 - p_b) y_l) \equiv u^{\text{min}} \), where \( u^{\text{min}} < u^{\text{max}} \) due to \( p_b < p_g \). Then, with the same arguments
as for Lemma 7, MW can be reformulated analogously to (2.16), as

$$u^{MW}_b = \arg \max_{u_b \in [u^{min}, u^{max}]} \left( \frac{1 - p_g}{1 - p_b} \right) u_b + \left( \frac{p_g - p_b}{1 - p_b} \right) H(u_b).$$  \( (2.19) \)

Existence and uniqueness of \( V^{MW} \) now follows as before, with the additional simplification of a lower bound \( u^{min} \) on the choice of \( u_b \). Also, the arguments for Claim 5 above imply that \( u^{MW}_b < u^{max} \) because \( \Psi = 1 > G \) due to \( \hat{d} \in (0, \infty) \). This in turn implies \( u^{MW}_b < u^{MW}_b < H(u^{MW}_b) = u^{g,h}_b \), the second inequality from the results for Claim 5 above, and the first from incentive compatibility. This establishes the strict inequalities in Lemma 8.

**Conditions (C1)-(C3).** To show that \( V^{MW} \) satisfies conditions (C1) to (C3), consider the Rothschild-Stiglitz contracts \( V^{RS} = (u^{RS}_{b,h}, u^{RS}_{b,l}, u^{RS}_{g,h}, u^{RS}_{g,l}) \), which solve

$$\max_{(u_{b,h}, u_{b,l}, u_{g,h}, u_{g,l}) \in \mathbb{R}^4} p_g u_{g,h} + (1 - p_g) u_{g,l}$$

subject to the constraints

$$p_k u_{k,h} + (1 - p_k) u_{k,l} \geq p_k u_{k',h} + (1 - p_k) u_{k',l} \forall k, k' \in \{g, b\},$$

$$G \left[ p_g \Phi(u_{g,h}) + (1 - p_g) \Phi(u_{g,l}) \right] + (1 - G) \left[ p_b \Phi(u_{b,h}) + (1 - p_b) \Phi(u_{b,l}) \right] \leq R,$$

$$\Phi(p_k u_{k,h} + (1 - p_k) u_{k,l}) = p_k y_h + (1 - p_k) y_l \forall k, k' \in \{g, b\}.$$ Comparing with MW, this program involves the same objective function, but a strictly smaller constraint set, implying \( p_g u^{MW}_{b,h} + (1 - p_g) u^{MW}_{b,l} \geq p_g u^{RS}_{b,h} + (1 - p_g) u^{RS}_{b,l} \). Moreover, constraint (2.9) in MW implies \( p_b u^{MW}_{b,h} + (1 - p_b) u^{MW}_{b,l} \geq \mathcal{U}(p_b y_h + (1 - p_b) y_l) = p_b u^{RS}_{b,h} + (1 - p_b) u^{RS}_{b,l} \). Therefore, \( V^{MW} \) weakly Pareto-dominates \( V^{RS} \). The result then follows from Lemma 4 in Rothschild (2007).

### 2.6.3 Proof of Lemma 9

**Property (i).** We will show that the solution \( V^{MW}(\hat{d}) \) to MW(\( \hat{d} \)) is continuous in \( \hat{d} \) on \((0, \infty) \). It then follows that \( D^{MW}(\hat{d}) \) is continuous as well. From the proof of Lemma 8 we know that the solution to MW(\( \hat{d} \)) for \( \hat{d} \in (0, \infty) \) can be found by solving the simplified problem (2.19):

$$u^{MW}_b(\hat{d}) = \arg \max_{u_b \in [u^{min}, u^{max}(\hat{d})]} (1 - p_g) u_b + (p_g - p_b) H(u_b, \hat{d}),$$  \( (2.20) \)

where \( u^{max}(\hat{d}) = U(R(\hat{d})) \), and for given \( \hat{d} \), the function \( H \) is continuously differentiable, strictly decreasing and strictly concave in \( u_b \) on \([u^{min}, u^{max}(\hat{d})] \). Let \( \mathcal{F} = (0, \infty), \mathcal{U} = [u^{min}, U(p_g y_h + (1 - p_g) y_l)], \) and \( \mathcal{C}(\hat{d}) = [u^{min}, u^{max}(\hat{d})] \subset \mathcal{U} \). Clearly, the correspondence \( C : \mathcal{F} \rightarrow \mathcal{U} \) is compact-
valued and continuous. Define $Z: \mathcal{U} \times \mathcal{F} \to \mathbb{R}$ by

$$Z(u_b, d) = \begin{cases} H(u_b, d) & \text{if } u_b \leq u_{\text{max}}(d), \\ u_{\text{max}}(d) & \text{if } u_b > u_{\text{max}}(d). \end{cases}$$

(2.21)

The function $Z$ is continuous on $\mathcal{U} \times \mathcal{F}$, because $H$ is continuous in $d$ and in $u_b \in [u_{\text{min}}, u_{\text{max}}(d)]$, $H(u_{\text{max}}(d), d) = u_{\text{max}}(d)$ holds, and $u_{\text{max}}(d)$ is continuous in $d$. We can now rewrite the maximization problem as

$$u_{b, \text{MW}}(d) = \arg\max_{u_b \in \mathcal{C}(d)} (1 - p_s)u_b + (p_s - p_b)Z(u_b, d),$$

(2.22)

and Berge’s maximum principle implies that $u_{b, \text{MW}}(d)$ is continuous. Then, $u_{g, h, \text{MW}}(d) = Z(u_{b, \text{MW}}(d), d)$ and $u_{g, l, \text{MW}}(d) = (u_{b, \text{MW}}(d) - p_b u_{g, l, \text{MW}}(d)) / (1 - p_b)$ are continuous as well.

Property (ii) Consider first the case where $d \to 0$. We will show that, as $d \to 0$, constraint (2.9) must eventually become binding in $V_{\text{MW}}(d)$, i.e. $u_{b, \text{MW}}(d) = u_{\text{min}}$ for $d$ small enough. Consider the slope of the objective in (2.20), evaluated at $u_b = u_{\text{min}}$. Using $H'(u_b)$ as given in (2.17), the condition that the objective is weakly decreasing already in $u_b = u_{\text{min}}$ (which is then the solution to (2.20) due to strict concavity), can be rearranged to

$$\left(1 - G(d)\right)(p_s - p_b)\Phi'(u_{\text{min}}) \geq G(d)(1 - p_s)p_s \left[\Phi'(H(u_{\text{min}}, d)) - \Phi'\left(\frac{u_{\text{min}} - p_b H(u_{\text{min}}, d)}{1 - p_b}\right)\right].$$

(2.23)

Fixing $u_b = u_{\text{min}}$, the budget constraint (2.8) can be simplified to

$$p_s \Phi(H(u_{\text{min}}, d)) + (1 - p_s)\Phi\left(\frac{u_{\text{min}} - p_b H(u_{\text{min}}, d)}{1 - p_b}\right) = p_s y_h + (1 - p_s)y_l,$$

which implies that $H(u_{\text{min}}, d)$ is independent of $d$ and satisfies $H(u_{\text{min}}, d) = u_{\text{min}}$. Hence the LHS of (2.23) converges to a strictly positive value as $d \to 0$, while the RHS converges to zero. Hence (2.9) must eventually become binding, so that $\lim_{d \to 0} u_{g, h, \text{MW}}(d) = H(u_{\text{min}}, d) > u_{\text{min}}$, and $\lim_{d \to 0} u_{g, l, \text{MW}}(d) < u_{\text{min}}$ (the latter by incentive compatibility). Hence we have $\lim_{d \to 0} (u_{g, h, \text{MW}}(d) - u_{g, l, \text{MW}}(d)) > 0$, which implies that $\lim_{d \to 0} D_{\text{MW}}(d) > 0$.

Consider now the case where $d \to \infty$. From the same arguments as above it follows that (2.9) must become slack for sufficiently large value of $d$, because (2.23) will eventually be violated. Observe also that $u_b = u_{\text{max}}(d)$ can never be a solution to (2.20), for any $d \in (0, \infty)$, as this would imply $u_{g, h} = H(u_{\text{max}}(d), d) = u_{\text{max}}(d) = u_b$ and $u_{g, l} = u_b$, contradicting Lemma 8. Hence (2.20) must have an interior solution for large enough $d$. Again using $H'(u_b)$ from (2.17), the necessary and sufficient first order condition for (2.20) can then be rearranged to

$$\frac{G(d)}{1 - G(d)} = \frac{\Phi'(u_{b, \text{MW}}(d))}{\Phi'(u_{g, h, \text{MW}}(d)) - \Phi'(u_{g, l, \text{MW}}(d))} \frac{p_s - p_b}{p_s(1 - p_s)}.$$

(2.24)
2.6. Appendix

Clearly, \( u^\text{MW}(\hat{d}) \) is bounded below by \( u^\text{min} \) and above by \( u^\text{max}(\hat{d}) = U(R(\hat{d})) \). Since \( u^\text{max}(\hat{d}) \) itself is bounded above by \( U(p_b y_h + (1 - p_b) y_l) \), it must be that \( u^\text{MW}_b(\hat{d}) \in [U(p_b y_h + (1 - p_b) y_l), U(p_b y_h + (1 - p_b) y_l)] \) for all \( \hat{d} \in (0, \infty) \). Since \( \lim_{d \to \infty} (G(d)/(1 - G(d))) = \infty \) while \( \Phi'(u^\text{MW}_b(\hat{d})) \in [\Phi'(U(p_b y_h + (1 - p_b) y_l)), \Phi'(U(p_b y_h + (1 - p_b) y_l))] \) for all \( \hat{d} \in (0, \infty) \), we must have

\[
\lim_{d \to \infty} \left( \Phi'(u^\text{MW}_{g,h}(\hat{d})) - \Phi'(u^\text{MW}_{g,l}(\hat{d})) \right) = 0,
\]

since otherwise the first-order condition (2.24) would be violated for large \( \hat{d} \). Assume that \( \lim_{d \to \infty} u^\text{MW}_{g,h}(\hat{d}) = +\infty \). Then, incentive compatibility (2.7) requires \( \lim_{d \to \infty} u^\text{MW}_{g,l}(\hat{d}) = -\infty \), and the denominator on the RHS of (2.24) does not go to zero. Therefore, \( \lim_{d \to \infty} (u^\text{MW}_{g,h}(\hat{d}) - u^\text{MW}_{g,l}(\hat{d})) = 0 \) has to hold (because \( \Phi' \) is strictly increasing), i.e. the good types’ contract converges towards full coverage. This implies \( \lim_{d \to \infty} D^\text{MW}(\hat{d}) = 0 \).

**Property (iii).** Assume that condition (2.11) \( (d (\Phi''(u)/\Phi'(u)) / du \geq 0) \) is satisfied. Observe that this implies convexity of \( \Phi' \). We will now proceed in several steps.

First, we will show that under convexity of \( \Phi' \) (and thus under (2.11)), both \( p_g u^\text{MW}_{g,h}(\hat{d}) + (1 - p_g) u^\text{MW}_{g,l}(\hat{d}) \) and \( u^\text{MW}_b(\hat{d}) \) are weakly increasing in \( \hat{d} \), and strictly so if (2.9) is slack. As for \( p_g u^\text{MW}_{g,h}(\hat{d}) + (1 - p_g) u^\text{MW}_{g,l}(\hat{d}) \), this holds even without convexity of \( \Phi' \). Fix a value \( \hat{d}_0 \in (0, \infty) \) and let \( \hat{d} = \hat{d}_0 + \delta \) for any \( \delta > 0 \). In \( \text{MW}(\hat{d}) \), only the resource constraint (2.8) is affected. Straightforward calculations, using the fact that \( V^\text{MW}(\hat{d}_0) \) satisfies (2.8) for \( \hat{d}_0 \) with equality, reveal that \( V^\text{MW}(\hat{d}_0) \) is still feasible under \( \hat{d} \) iff

\[
(p_g - p_b)(y_h - y_l) - [p_g \Phi(u^\text{MW}_{g,h}(\hat{d}_0)) + (1 - p_g) \Phi(u^\text{MW}_{g,l}(\hat{d}_0)) - \Phi(u^\text{MW}_b(\hat{d}_0))] \geq 0,
\]

and satisfies the budget constraint with slack iff the inequality is strict. But the binding constraint (2.8) can be rearranged to

\[
G(\hat{d}_0) \left[ p_g \Phi(u^\text{MW}_{g,h}(\hat{d}_0)) + (1 - p_g) \Phi(u^\text{MW}_{g,l}(\hat{d}_0)) - \Phi(u^\text{MW}_b(\hat{d}_0)) - (p_g - p_b)(y_h - y_l) \right] \]

\[
+ \Phi(u^\text{MW}_b(\hat{d}_0)) = p_b y_h + (1 - p_b) y_l,
\]

which together with the fact that \( \Phi(u^\text{MW}_b(\hat{d}_0)) \geq p_b y_h + (1 - p_b) y_l \) from (2.9) implies that (2.25) is always satisfied, and as a strict inequality whenever \( \Phi(u^\text{MW}_b(\hat{d}_0)) > p_b y_h + (1 - p_b) y_l \). In this latter case, the optimal value of the objective under \( \hat{d} \) must be strictly larger than under \( \hat{d}_0 \), as argued in the proof of Lemma 7. Otherwise, given that the old contracts \( V^\text{MW}(\hat{d}_0) \) are still feasible under \( \hat{d} \), the optimal value of the objective cannot decrease. Now consider the bad type’s utility \( u^\text{MW}_b(\hat{d}) \). If (2.9) is binding, it is given by \( u^\text{MW}_b(\hat{d}) = U(p_b y_h + (1 - p_b) y_l) \) and is independent of \( \hat{d} \). Assume then that (2.9) is slack, such that \( u^\text{MW}_b(\hat{d}) \) satisfies the first-order condition (2.24). To arrive at a contradiction, suppose we increase \( \hat{d} \) and \( u^\text{MW}_b(\hat{d}) \) decreases weakly. The binding
self-selection constraint (2.7) can be rearranged to

\[ p_g u_{g,h}^{MW}(\hat{d}) + (1 - p_g) u_{g,l}^{MW}(\hat{d}) - u_b^{MW}(\hat{d}) = (p_g - p_b)(u_{g,h}^{MW}(\hat{d}) - u_{g,l}^{MW}(\hat{d})). \]

Given that \( p_g u_{g,h}^{MW}(\hat{d}) + (1 - p_g) u_{g,l}^{MW}(\hat{d}) \) strictly increases in \( \hat{d} \), as shown above, the term \( u_{g,h}^{MW}(\hat{d}) - u_{g,l}^{MW}(\hat{d}) \) must also be strictly increasing. If \( \Phi' \) is convex, this implies that

\[ \Phi'(u_{g,h}^{MW}(\hat{d})) - \Phi'(u_{g,l}^{MW}(\hat{d})) \]

is increasing in \( \hat{d} \), given that \( u_{g,h}^{MW}(\hat{d}) \) and \( u_{g,l}^{MW}(\hat{d}) \) cannot both decrease. Collecting results, we have that, by assumption, \( u_{g,h}^{MW}(\hat{d}) \) and hence \( \Phi'(u_{g,h}^{MW}(\hat{d})) \) weakly decreases, while \( \Phi'(u_{g,h}^{MW}(\hat{d})) - \Phi'(u_{g,l}^{MW}(\hat{d})) \) strictly increases. But this is a contradiction to (2.24), as it implies that the LHS of (2.24) strictly increases but the RHS strictly decreases. Hence \( u_b^{MW}(\hat{d}) \) is strictly increasing in \( \hat{d} \) whenever (2.9) is slack.

Second, if (2.9) is slack and \( u_b^{MW}(\hat{d}) \) is strictly increasing at some level of \( \hat{d} \), the same clearly holds for all \( \hat{d}' > \hat{d} \). Together with the previous result that (2.9) must be binding in \( V^{MW}(\hat{d}) \) for sufficiently small and slack for sufficiently large values of \( \hat{d} \), it follows that there exists a value \( \hat{d} \in (0, \infty) \) such that for all \( \hat{d} \leq \hat{d} \), constraint (2.9) will be binding in \( V^{MW}(\hat{d}) \) and neither \( V^{MW}(\hat{d}) \) nor \( D^{MW}(\hat{d}) \) change in \( \hat{d} \), while for all \( \hat{d} > \hat{d} \), (2.9) is slack and \( u_b^{MW}(\hat{d}) \) is strictly increasing in \( \hat{d} \).

Third, and finally, we are going to show that, for \( \hat{d} > \hat{d} \), \( u_{g,h}^{MW}(\hat{d}) - u_{g,l}^{MW}(\hat{d}) \) and thus \( D^{MW}(\hat{d}) \) are strictly decreasing in \( \hat{d} \). As \( \hat{d} > \hat{d} \) grows, the LHS of the first order condition (2.24) grows, and so must the RHS. The condition that the derivative of the RHS of (2.24) with respect to \( \hat{d} \) is strictly positive can be rearranged to

\[ \frac{\Phi''(u_b)}{\Phi'(u_b)} > \frac{\Phi''(u_{g,h}) u_{g,h}^{MW} - \Phi''(u_{g,l}) u_{g,l}^{MW}}{\Phi'(u_{g,h}) - \Phi'(u_{g,l})}, \]

where both the dependency on \( \hat{d} \) and the superscript \( MW \) have been suppressed for notational convenience, and primes denote partial derivatives of utilities with respect to \( \hat{d} \). To obtain a contradiction, suppose that \( u'_{g,h} \geq u'_{g,l} \). We want to find a condition under which the above inequality must be violated, that is, under which

\[ \frac{\Phi''(u_b)}{\Phi'(u_b)} \leq \frac{\Phi''(u_{g,h}) u_{g,h}^{MW} - \Phi''(u_{g,l}) u_{g,l}^{MW}}{\Phi'(u_{g,h}) - \Phi'(u_{g,l})}. \]

Since \( u'_b = p_b u'_{g,h} + (1 - p_b) u'_{g,l} \), we must have \( u'_{g,h} / u'_{g,l} \geq 1 \) and \( u'_{g,l} / u'_{g,h} \leq 1 \) given the assumption

\[ u'_{g,h} > 0. \]

\[ 12 \] It is straightforward to show that \( u_b^{MW}(\hat{d}), u_{g,h}^{MW}(\hat{d}) \) and \( u_{g,l}^{MW}(\hat{d}) \) are continuously differentiable if \( \hat{d} > \hat{d} \), given the properties of the function \( H(u_b, \hat{d}) \) used in (2.20), and the first order condition (2.24) for an interior solution. The derivation of the inequality makes use of the result that \( u'_b > 0 \).
2.6. Appendix

\[ u_{g,h}' \geq u_{g,l}'. \] Hence (2.26) is implied if

\[
\frac{\Phi''(u_b)}{\Phi'(u_b)} \leq \frac{\Phi''(u_{g,h}) - \Phi''(u_{g,l})}{\Phi'(u_{g,h}) - \Phi'(u_{g,l})}.
\] (2.27)

This can be rearranged to

\[
\Phi'(u_{g,h})\Phi'(u_b)\left[\frac{\Phi''(u_b)}{\Phi'(u_b)} - \frac{\Phi''(u_{g,h})}{\Phi'(u_{g,h})}\right] + \Phi'(u_{g,l})\Phi'(u_b)\left[\frac{\Phi''(u_{g,l})}{\Phi'(u_{g,l})} - \frac{\Phi''(u_b)}{\Phi'(u_b)}\right] \leq 0.
\]

Since \( u_{g,l} < u_b < u_{g,h} \), this is always satisfied under condition (2.11), which yields the desired contradiction and shows that, if (2.11) is satisfied, \( D^{MW}(\hat{d}) \) must be strictly decreasing if \( \hat{d} > \tilde{d} \).

2.6.4 Proof of Theorem 3

Fix a value \( \hat{d} \in (0, \infty) \) and consider two cases. First, suppose that \( \Psi(\hat{d}) \leq G(\hat{d}) \). Then, arguing as for Proposition 8, we obtain \( D^{SP}(\hat{d}) \leq 0 \). Second, consider the case \( \Psi(\hat{d}) > G(\hat{d}) \). Lemma 7 implies that the unique solution to \( SP(\hat{d}) \) is such that constraint (2.3) holds with equality, \( u_{b,h}^{SP}(\hat{d}) = u_{b,l}^{SP}(\hat{d}) \) and \( u_{g,h}^{SP}(\hat{d}) \geq u_{g,l}^{SP}(\hat{d}) \). From the fact that \( p_g > p_b \) it then follows that (2.2) is automatically satisfied.

Defining

\[
p(\hat{d}) \equiv p_g \Psi(\hat{d}) + p_b(1 - \Psi(\hat{d})),
\] (2.28)

the solution must therefore be such that \((u_{g,h}^{SP}(\hat{d}), u_{g,l}^{SP}(\hat{d}))\) solves the simplified problem

\[
\max_{(u_{g,h}, u_{g,l})|u_{g,l} \leq u_{g,h}} p(\hat{d})u_{g,h} + (1 - p(\hat{d}))u_{g,l}
\] (2.29)

subject to the resource constraint

\[
G(\hat{d}) \left[ p_g \Phi(u_{g,h}) + (1 - p_g)\Phi(u_{g,l}) \right] + (1 - G(\hat{d}))\Phi(p_b u_{g,h} + (1 - p_b)u_{g,l}) = R(\hat{d}),
\] (2.30)

which is binding as shown in the proof of Lemma 7. Suppose \( \hat{d} > \tilde{d} \). First, for \( \hat{d} = \infty \), \( D^{SP}(\hat{d}) = D^{MW}(\hat{d}) = 0 \) holds. Otherwise, if \( \hat{d} \in (\hat{d}, \infty) \), by Lemma 8 the outcome \( V^{MW}(\hat{d}) \) is such that \((u_{g,h}^{MW}(\hat{d}), u_{g,l}^{MW}(\hat{d}))\) solves

\[
\max_{(u_{g,h}, u_{g,l})|u_{g,l} \leq u_{g,h}} p_g u_{g,h} + (1 - p_g)u_{g,l}
\] (2.31)

subject to the same budget constraint (2.30), because (2.9) does not bind for \( \hat{d} > \tilde{d} \) as shown in the proof of Lemma 9. Since (2.30) is a convex constraint by the proof of Lemma 7, and \( p(\hat{d}) \leq p_g \) holds by (2.28), the solutions \((u_{g,h}^{SP}(\hat{d}), u_{g,l}^{SP}(\hat{d}))\) and \((u_{g,h}^{MW}(\hat{d}), u_{g,l}^{MW}(\hat{d}))\) must be such that

\[ u_{g,h}^{SP}(\hat{d}) \leq u_{g,h}^{MW}(\hat{d}) \quad \text{and} \quad u_{g,l}^{SP}(\hat{d}) \geq u_{g,l}^{MW}(\hat{d}). \]
This implies

\[ D^{SP}(\hat{d}) = (p_g - p_b)(u^{SP}_{gh}(\hat{d}) - u^{SP}_{gh}(\hat{d})) \leq (p_g - p_b)(u^{MW}_{gh}(\hat{d}) - u^{MW}_{gh}(\hat{d})) = D^{MW}(\hat{d}). \]  

(2.32)

Under property (2.11) and \( \hat{d} > d^* \), the function \( D^{MW} \) is strictly decreasing in \( \hat{d} \) above its unique interior fixed point \( d^* \), so that \( D^{MW}(\hat{d}) < \hat{d} \) for all \( \hat{d} > d^* \). Together with (2.32) and \( D^{SP}(\infty) = D^{MW}(\infty) = 0 \), this implies \( D^{SP}(\hat{d}) \leq D^{MW}(\hat{d}) < \hat{d} \) for all \( \hat{d} > d^* \), so that any fixed point \( d^{**} \) of \( D^{SP} \) must satisfy \( d^{**} \leq d^* \).
Chapter 3

Pareto-Optimal Taxation with Aggregate Uncertainty and Financial Markets

3.1 Introduction

Individual households face substantial economic risk over their lifetimes in the form of both idiosyncratic and aggregate uncertainty. For instance, individuals' employment, income, health status and mortality are all subject to idiosyncratic shocks and to economy-wide shifts in unemployment rates, wages, technology and life-expectancies. The two kinds of shocks have very different implications for risk-sharing, though. First, whereas individuals can influence their idiosyncratic uncertainty by taking unobservable actions, aggregate risk is not typically related to such private information problems. Moreover, idiosyncratic shocks can be insured by pooling risks in groups sufficiently large for laws of large numbers to apply, while aggregate uncertainty is harder to smooth. However, aggregate shocks usually have different effects on different agents in the economy. For instance, country-specific aggregate shocks may only affect agents in one country, not those abroad. A recession may increase unemployment rates in some sectors of the economy more than in others. Changes in wages and mortality rates have different impacts on elderly, retired agents than on young workers. Accounting for such heterogeneity among individuals, there therefore exist significant opportunities for smoothing even aggregate risk.\(^1\)

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\(^2\)Attanasio and Davis (1996) provide overwhelming empirical evidence for consumption insurance opportunities between birth cohorts and education groups in the US. Krueger (2006) and Storesletten, Telmer,
3.1. Introduction

In this paper, I ask how idiosyncratic and aggregate risk should be shared optimally among different groups in the economy. I consider a model where ex ante heterogeneous agents are subject to both aggregate and idiosyncratic shocks. Individuals can influence their probability distribution over idiosyncratic shocks by choosing some hidden effort, leading to a standard moral hazard problem. Aggregate shocks, by contrast, are assumed to be exogenous and to affect all agents’ outputs and probability distributions over idiosyncratic shocks, but in potentially different ways. I show that, if agents’ preferences between consumption and effort are separable, any Pareto-optimal risk-sharing arrangement in this private information economy has to satisfy an empirically testable condition, which requires the ratios of expected inverse marginal utilities between different agents to be independent of aggregate shocks. This efficiency condition does not depend on any assumptions about agents’ disutility of effort nor restrictions on the moral hazard problem that make the first-order approach valid.

I use this efficiency condition to study the role of financial markets in my economy, where agents exchange claims to consumption contingent on aggregate shocks. In practice, agents are able to insure considerable parts of the aggregate risk that they are exposed to by trading such financial assets. For instance, agents can hedge country-specific risk by buying foreign assets, and workers in a given sector can buy shares of companies in other sectors to reduce their overall exposure to the effects of aggregate shocks on their own sector. I explore under what conditions Pareto-optimal risk-sharing arrangements are consistent with agents having free access to such financial markets.

My first result is that any Pareto-optimum in my economy can be implemented without interventions in financial markets if aggregate shocks affect individual outputs in arbitrary ways, but not the distributions of idiosyncratic risk. Simple group-specific income transfers that condition on aggregate shocks and individual outputs are sufficient in this case. For instance, if aggregate shocks shift the distribution of wages across different sectors, but not the unemployment risk that individuals face who exert a given effort level, then free trading in financial markets does not conflict with efficiency. Interestingly, the result implies that it does not matter in this case whether trades in financial markets are observable or not: the same set of allocations can be implemented under both informational assumptions.

I then characterize optimal distortions in financial markets when aggregate shocks do affect distributions of idiosyncratic shocks. I show that, in this case, implicit transaction taxes in financial markets have to be introduced to implement Pareto-optima. These and Yaron (2007) also find that intergenerational sharing of aggregate risk is quantitatively important.

Thus, any correlations between aggregate shocks and idiosyncratic shocks are allowed for.
3.1. Introduction

Implicit taxes must be higher for those financial assets that provide consumption in aggregate states with a more volatile distribution of likelihood ratios in the sense of second-order stochastic dominance. Intuitively, since Pareto-optimal risk-sharing arrangements vary consumption according to likelihood ratios, consumption is more risky under aggregate shocks that induce a more risky distribution of likelihood ratios. With undistorted financial markets, agents would 'self-insure' against this risk by buying additional consumption for that aggregate state in financial markets. The optimal distortions are designed to prevent agents from doing so.

However, linear transaction taxes that condition on aggregate shocks only are in general not able to implement Pareto-optimals. The reason is that they can only make sure that agents have the correct trading incentives in financial markets if they choose the optimal hidden action. But agents may gain from a double-deviation, where they both choose another unobservable effort and trade in financial markets. To prevent such double-deviations, I show that linear transaction taxes in financial markets must be personalized, i.e. contingent upon both aggregate and idiosyncratic shocks. The way they introduce the optimal distortions in financial markets is by increasing or decreasing the idiosyncratic risk of the different market sides. For agents who buy claims for a given aggregate state and who therefore have a relatively high expected marginal utility in that state, they induce additional risk and thus reduce (after-tax) expected marginal utility. To agents who sell claims (which are those with a relatively low expected marginal utility), the optimal transaction taxes provide additional insurance. In this way, optimal linear transaction taxes lead to the equalization of after-tax expected marginal rates of substitution in financial markets, as is required for equilibrium, while still introducing the wedges between before-tax marginal rates of substitution implied by Pareto-optimality.

Finally, I show that, if transaction taxes are not constrained to be linear in trade volume, efficient allocations can also be implemented using output-independent transaction taxes that only rely on information about an agent’s trading strategy in financial markets. The way they prevent (double-) deviations from being profitable is by making agents indifferent between any trading strategy when they can choose their hidden action optimally given these trades. The resulting tax schedule does not rely on sharp penalties in the sense of discontinuous taxes, and marginal taxes are closely related to the implicit wedges between marginal rates of substitution at the optimum.

This paper builds upon the literature studying and testing optimal risk-sharing arrangements in economies with heterogeneous agents but without private information, as pioneered by Borch (1962), Diamond (1967), Wilson (1968), Townsend (1994) and Attanasio and Davis (1996). I demonstrate how the first-best risk-sharing rules derived
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there have to be modified when idiosyncratic risk is subject to moral hazard, so that risk-sharing has to be traded-off against the provision of incentives. In that respect, this paper shares a common goal with the contributions by Phelan (1994) and Demange (2008). Notably, Demange (2008) also considers a moral hazard model with aggregate uncertainty and discusses properties of risk-sharing rules under various assumptions on preferences and for a numerical example. However, neither the condition for Pareto-optimality nor its implications for optimal tax policy in financial markets presented here are derived. Moreover, all of the analysis in Demange (2008) is based on the first-order approach, which is not used throughout the present paper. This is particularly important since the double-deviations available to agents with access to financial markets can lead to failures of this methodology.\(^4\)

My analysis of a moral hazard model with aggregate uncertainty and of its implications for tax policy in financial markets is also related to a large literature that studies the optimal taxation of capital income in dynamic private information economies with idiosyncratic shocks.\(^5\) In these models, the Inverse Euler equation is derived as an intertemporal optimality condition and used to obtain implications for optimal savings distortions, similarly to the approach here. However, none of these papers consider ex ante heterogeneous agents nor aggregate uncertainty, so that they cannot address the question of how to optimally share aggregate risk across different groups in an economy. Also, due to the absence of aggregate shocks, no financial markets as discussed here emerge in these models.

An important exception is the contribution by Kocherlakota (2005), who generalizes the Inverse Euler equation to allow for aggregate shocks in a dynamic optimal taxation model. He also constructs a tax system with regressive capital taxes to implement efficient intertemporal allocations. However, since all agents are ex ante identical in his model, no restriction similar to the purely intratemporal Pareto-optimality condition for the sharing of aggregate risk across heterogeneous groups derived here can be obtained in his framework. Also, agents in his model can only trade capital, so that implications for optimal tax policy in financial markets do not arise.\(^6\) Golosov, Tsyvinski, and Werning (2006) also

\(^4\)Phelan (1994) considers a dynamic private information model with exponential utility and aggregate uncertainty and is mainly interested in how the dynamics of the cross-sectional distribution of consumption depend on aggregate shocks.


\(^6\)Kocherlakota and Pistaferri (2007) and Kocherlakota and Pistaferri (2008) consider models with aggregate shocks where heterogeneous agents can trade in financial markets. They are concerned with constructing stochastic discount factors based on CRRA-preferences that are able to explain empirically observed co-
consider optimal savings and labor wedges with aggregate uncertainty, but have to rely on numerical simulations to obtain results on how aggregate shocks affect wedges. In contrast, I analytically derive transparent conditions that allow me to characterize optimal distortions in financial markets with aggregate uncertainty.

The distortions in financial markets derived here are quite different from the savings distortions in dynamic taxation models. Whereas Golosov, Kocherlakota, and Tsyvinski (2003) show that the Inverse Euler equation always implies a downward distortion of savings to be optimal, the sign of optimal trading distortions in financial markets is zero if aggregate shocks affect outputs only, and depends on how aggregate shocks affect the distribution of idiosyncratic shocks otherwise. As I will show below, the optimality of a downward savings distortion in dynamic models can be regarded as a special case of my result that implicit transaction taxes must be higher for the aggregate states that induce a more risky distribution of likelihood ratios and hence consumption. Also, whereas the capital taxes in Kocherlakota's (2005) decentralization are regressive in the sense that they induce additional idiosyncratic risk, the linear transaction taxes constructed here are such that they provide additional insurance for one market side while inducing more risk for the other, as discussed above. Moreover, I show that transaction taxes can be made independent of individual outputs if they are allowed to be nonlinear in transaction volumes, based on insights from Werning (2009) for a dynamic economy without aggregate risk.

My result that Pareto-optima are not constrained by unobservable trades in financial markets if aggregate shocks affect outputs only complements a vast literature of models with hidden side trades in other settings. Golosov and Tsyvinski (2007), for instance, show for a dynamic model with unobservable trading in bond markets that the optimal capital tax is always positive if skill shocks are iid over time. In contrast, I provide a benchmark condition under which the planner finds it optimal not to distort unobservable trades in financial markets. In a moral hazard model with anonymous trading of unmonitored consumption goods, Acemoglu and Simsek (2007) show that the planner does not distort these trades if preferences are separable between consumption and effort. Although the condition obtained in my framework is not related to separability of preferences, it does make sure that marginal rates of substitution in financial markets are independent of effort, which is a separability property of its own kind. For a Bew-

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Footnote:
3.2. The Model

In order to address the issues discussed in the introduction formally, I consider an economic environment that builds upon those developed by Phelan (1994) and Demange (2008). There exists a continuum of agents of unit mass. Each individual belongs to one of $N$ groups. These groups may be thought of as workers in different sectors of the economy, individuals of different age (such as workers and retired agents) or even agents in different countries, so that the assignment of individuals to groups is public information. The mass of a given group $i \in I \equiv \{1, \ldots, N\}$ is given by $n_i$. Individuals within a group are ex ante identical in terms of both preferences and technology. In particular, let agents of group $i$ be endowed with separable preferences $U_i(c, a) = u_i(c) - v_i(a)$ over consumption $c \in C_i \subseteq \mathbb{R}$ and an action (effort) $a \in A_i$, where $A_i$ is a finite action set available to agents of group $i$. I assume $u_i(c)$ to be twice continuously differentiable with
3.2. The Model

\( u'(c) > 0, u''(c) \leq 0 \forall c \in C_i. \)

The agents' technology is affected by two kinds of shocks: aggregate shocks \( s \in S \) and idiosyncratic shocks \( \theta \in \Theta \), where both \( S \) and \( \Theta \) are finite sets. An agent of group \( i \) who experiences an idiosyncratic shock \( \theta \) in aggregate state \( s \) produces output \( y_i(\theta, s) \). In addition, if an agent of group \( i \) chooses some action \( a \) and the aggregate shock is \( s \), then the probability density function over idiosyncratic shocks is given by \( p_i(\theta|a, s) \). I assume both the realizations of idiosyncratic shocks \( \theta \) and of aggregate shocks \( s \) to be publicly observable, but an agent's action \( a \) to be private information. For instance, consider \( \theta \in \{\theta, \bar{\theta}\} \) as an (observable) unemployment shock, where \( \theta \) stands for unemployment and \( \bar{\theta} \) for employment. Suppose \( a \) is some (privately known) search effort to find or keep a job, and let \( i \) index workers in different sectors of the economy. Then (observable) aggregate shocks \( s \) may affect both the output \( y_i(\bar{\theta}, s) \) that a worker in sector \( i \) produces when employed (normalizing the output when unemployed \( y_i(\theta, s) = 0 \)) and the probability \( p_i(\theta|a, s) \) of a worker in sector \( i \) to stay or become unemployed when exerting effort \( a \). It is crucial to observe that both the dependency of outputs and of probability distributions on aggregate shocks are indexed by \( i \), so that aggregate shocks are allowed to have different effects on the different groups of the economy.

Let the probability density function over aggregate shocks \( s \in S \) be given by \( \pi(s) \). Idiosyncratic shocks \( \theta \) within a given group \( i \) are iid across agents conditional on the aggregate shock \( s \). I assume that a law of large numbers holds, which implies that the share of individuals in group \( i \) with output \( y_i(\theta, s) \) given that aggregate shock \( s \) is realized and agents in group \( i \) choose effort \( a \) equals \( p_i(\theta|a, s) \). This makes sure that all aggregate uncertainty in the economy comes from the aggregate shocks \( s \), not from idiosyncratic shocks \( \theta \).

The timing of events in the benchmark model is as follows. In a first stage, a social planner offers agents a consumption (or wage) schedule \( \{c_i(\theta, s)\} \) that specifies consumption levels for the agents in each group \( i \) contingent on both the realization of the aggregate shock \( s \) and the idiosyncratic shocks \( \theta \). Observing that, agents in each group privately choose an action \( a_i \in A_i \). Next, the aggregate shock \( s \in S \) is realized according to the distribution \( \pi(s) \) and, conditional on this realization, idiosyncratic shocks \( \theta \in \Theta \) are realized.

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\(^8\) No assumptions on the particular form of \( v_i(a) \) are required for the results in this paper.

\(^9\) While laws of large numbers for a continuum of random variables may fail due to technical complications (see Judd (1985)), they can be put back into force through a variety of approaches. These include the application of a weaker convergence criterion (Uhlig (1996)), the redefinition of the set indexing consumers (Green (1994)), or the relaxation of the strict independence assumption (Robson and Samuelson (2008)).

\(^{10}\) Note that since all agents within a group are ex ante identical, the assumption that the social planner offers the same contract \( c_i(\theta, s) \) to each agent in a given group is not restrictive for the following results. It is also clear that, faced with the same contract, all agents within a group choose the same action \( a_i \).
3.3. Constrained-Efficient Allocations

are drawn from the distribution $p_i(\theta|a_i, s)$ for all agents and all groups $i \in I$. This determines outputs $y_i(\theta, s)$, which are collected by the social planner and used to implement the consumption schedule $\{c_i(\theta, s)\}$ promised in stage one.

I define an allocation $\{c_i(\theta, s), a_i\}$ in this economy as a consumption schedule $\{c_i(\theta, s)\}$ and an action profile $\{a_i\}$ that specifies an action $a_i$ for each group $i \in I$. An allocation $\{c_i(\theta, s), a_i\}$ is feasible if it satisfies the resource constraint for each aggregate state $s$, i.e.

$$\sum_i n_i \sum_{\theta \in \Theta} c_i(\theta, s) p_i(\theta|a_i, s) \leq \sum_i n_i \sum_{\theta \in \Theta} y_i(\theta, s) p_i(\theta|a_i, s) \quad \forall s \in S. \quad (3.1)$$

It is incentive compatible if

$$\sum_{s \in S} \sum_{\theta \in \Theta} U_i(c_i(\theta, s), a_i) p_i(\theta|a_i, s) \pi(s) \geq \sum_{s \in S} \sum_{\theta \in \Theta} U_i(c_i(\theta, s), \bar{a}_i) p_i(\theta|\bar{a}_i, s) \pi(s) \quad (3.2)$$

$\forall i \in I, \bar{a}_i \in A_i$. I will say that a consumption schedule $\{c_i(\theta, s)\}$ implements an effort profile $\{a_i\}$ if it satisfies the incentive compatibility constraints (3.2) given $\{a_i\}$. The following definition introduces a notion of optimality in this economy:

**Definition 5.** An allocation $\{c_i^*(\theta, s), a_i^*\}$ is constrained-efficient if it solves

$$\max_{\{c_i(\theta, s), a_i\}} \sum_i \psi_i \sum_{s \in S} \sum_{\theta \in \Theta} U_i(c_i(\theta, s), a_i) p_i(\theta|a_i, s) \pi(s) \quad (3.3)$$

subject to the feasibility constraints (3.1) and the incentive compatibility constraints (3.2) for some set of Pareto-weights $\{\psi_i\}$, $\psi_i \geq 0 \forall i \in I$.

Hence, an allocation is constrained-efficient if it is Pareto-optimal within the class of feasible and incentive compatible allocations, treating identical agents within groups symmetrically. I also refer to a consumption schedule $\{c_i^*(\theta, s)\}$ as constrained-efficient given an effort profile $\{a_i\}$ if it solves (3.3) subject to (3.1) and (3.2) for some given $\{a_i\}$ (which may not be the optimal one), i.e. it is feasible and implements a given action profile optimally.

### 3.3 Constrained-Efficient Allocations

In this section, I ask how aggregate and idiosyncratic risk are shared optimally among the heterogeneous groups in the moral hazard economy described above. The main result is a restriction that any constrained-efficient allocation has to satisfy. Since effort is unobservable, the optimal consumption schedule $\{c_i^*(\theta, s)\}$ has to reflect a tradeoff between
3.3. Constrained-Efficient Allocations

risk-sharing and the provision of incentives. The incentive constraints (3.2), however, make the full moral hazard problem (3.3) subject to (3.1) and (3.2) hard to solve in general. Typical approaches to simplify the problem try to reduce the number of incentive constraints that have to be taken into account, e.g. by assuming that the agent’s effort is chosen from a continuum of possible effort levels and that it solves a first-order condition of the agent’s effort choice problem for a given contract. This first-order approach, however, is not generally valid without rather restrictive assumptions.\footnote{See Mirrlees (1999). As shown by Rogerson (1985a) and Jewitt (1988), for instance, the first-order approach is valid if the distributions of idiosyncratic shocks $p_i(\theta|a_i,s)$ satisfy the monotone likelihood ratio property and the convexity of the distribution function condition. These restrictions are not met by many plausible distributions, however.}

In contrast, I do not put additional restrictions on the maximization problem (3.3) subject to (3.1) and (3.2), but instead focus on deriving a necessary condition that any constrained-efficient allocation has to satisfy. The idea is to fix a given effort profile $\{a_i\}$ and characterize the optimal consumption schedule $\{c_i^*(\theta,s)\}$ that implements it.\footnote{See Grossman and Hart (1983) for a related approach to single agent moral hazard problems.} Based on a general variational argument, the following theorem shows that a condition on the optimal schedule $\{c_i^*(\theta,s)\}$ can be derived without further simplifying the set of incentive constraints (3.2).

**Theorem 4.** Any constrained efficient consumption schedule $\{c_i^*(\theta,s)\}$ that solves problem (3.3) subject to (3.1) and (3.2) for a given action profile $\{a_i\}$ must be such that

$$\frac{\mathbb{E}_i \left[ 1/u_i'(c_i^*(\theta,s)) | a_i, s \right]}{\mathbb{E}_i \left[ 1/u_i'(c_i^*(\theta,\bar{s})) | a_i, \bar{s} \right]} = \frac{\mathbb{E}_j \left[ 1/u_j'(c_j^*(\theta,s)) | a_j, s \right]}{\mathbb{E}_j \left[ 1/u_j'(c_j^*(\theta,\bar{s})) | a_j, \bar{s} \right]} \quad (3.4)$$

\[ \forall i, j \in I, s, \bar{s} \in S, \text{where} \]

$$\mathbb{E}_i \left[ 1/u_i'(c_i^*(\theta,s)) | a_i, s \right] = \sum_{\theta \in \Theta} \frac{p_i(\theta|a_i,s)}{u_i'(c_i^*(\theta,s))}. \]

**Proof.** Consider a constrained-efficient consumption schedule $\{c_i^*(\theta,s)\}$ that implements action profile $\{a_i\}$. Let me write $\bar{Y}(s) \equiv \sum_i n_i \sum_{\theta \in \Theta} p_i(\theta|a_i,s)y_i(\theta,s)$ for aggregate output in state $s$, and perform a change in variables from $\{c_i(\theta,s)\}$ to $\{u_i(\theta,s)\}$. Then $\{u_i^*(\theta,s)\}$ must solve

$$\max_{\{u_i(\theta,s)\}} \sum_i \psi_i \sum_{s \in S} u_i(\theta,s) p_i(\theta|a_i,s) \pi(s) \quad (3.5)$$
subject to

\[
\sum_i n_i \sum_{\theta \in \Theta} p_i(\theta|a_i,s)C_i(u_i(\theta,s)) \leq \bar{Y}(s) \quad \forall s \in S
\]  

(3.6)

and

\[
\sum_{s \in S} \sum_{\theta \in \Theta} u_i(\theta,s)p_i(\theta|a_i,s)\pi(s) - v_i(a_i) \geq \sum_{s \in S} \sum_{\theta \in \Theta} u_i(\theta,s)p_i(\theta|\bar{a}_i,s)\pi(s) - v_i(\bar{a}_i)
\]

(3.7)

\( \forall i \in I, \bar{a}_i \in A_i \), where I have defined the cost function \( C_i(u) \equiv u_i^{-1}(c) \).

Now consider the variation from \( \{u^*_i(\theta,s)\} \) to \( \{u^*_i(\theta,s) + \delta_i(s)\} \), i.e. I vary util in a parallel way across realizations of the idiosyncratic shock for all \( i \in I, s \in S \). Note that this leaves incentives for the given profile \( \{a_i\} \) unaffected due to the separability assumption:

\[
\sum_{s \in S} \sum_{\theta \in \Theta} [u^*_i(\theta,s) + \delta_i(s)]p_i(\theta|a_i,s)\pi(s) - v_i(a_i) \geq \sum_{s \in S} \sum_{\theta \in \Theta} [u^*_i(\theta,s) + \delta_i(s)]p_i(\theta|\bar{a}_i,s)\pi(s) - v_i(\bar{a}_i)
\]

is satisfied \( \forall i \in I, \bar{a}_i \in A_i \) if and only if

\[
\sum_{s \in S} \sum_{\theta \in \Theta} u^*_i(\theta,s)p_i(\theta|a_i,s)\pi(s) - v_i(a_i) \geq \sum_{s \in S} \sum_{\theta \in \Theta} u^*_i(\theta,s)p_i(\theta|\bar{a}_i,s)\pi(s) - v_i(\bar{a}_i)
\]

\( \forall i \in I, \bar{a}_i \in A_i \), which I assumed to start with. Thus, when considering this particular variation, the incentive constraints (3.7) can be ignored. If the original consumption schedule \( \{c_i(\theta,s)\} \) (and hence the associated util \( \{u_i(\theta,s)\} \)) was constrained-efficient given \( \{a_i\} \), it must not be possible to increase the objective function in problem (3.5) to (3.6) by any such variation \( \{\delta_i(s)\} \) for any set of Pareto-weights \( \{\psi_i\} \). Hence \( \delta_i(s) = 0 \) \( \forall i \in I, s \in S \), must solve

\[
\max_{\{\delta_i(s)\}} \sum_{i} \psi_i \sum_{s \in S} \sum_{\theta \in \Theta} (u^*_i(\theta,s) + \delta_i(s))p_i(\theta|a_i,s)\pi(s)
\]

subject to

\[
\sum_i n_i \sum_{\theta \in \Theta} p_i(\theta|a_i,s)C_i(u^*_i(\theta,s) + \delta_i(s)) \leq \bar{Y}(s) \quad \forall s \in S.
\]

(3.9)

Necessary first-order conditions (note also that the transformed problem is convex) evaluated at \( \delta_i(s) = 0 \) imply

\[
\frac{n_i}{\psi_i} E_i \left[ \frac{1}{u_i'(c_i^*(\theta,s))} | a_i,s \right] = \frac{n_j}{\psi_j} E_j \left[ \frac{1}{u_j'(c_j^*(\theta,s))} | a_j,s \right]
\]

(3.10)

\( \forall i, j \in I, s \in S \), where I used \( C_i'(.) = 1/u_i'(.) \). Condition (3.4) in the theorem follows from the fact that equation (3.10) holds for all aggregate states \( s \in S \). □

The risk-sharing condition (3.4) in the theorem requires any constrained-efficient consumption schedule \( \{c^*_i(\theta,s)\} \) to be such that the ratios of expected inverse marginal utilities between different aggregate states are equalized across all agents (equivalently, the condition could also be written as requiring that the ratios of expected inverse marginal
3.3. Constrained-Efficient Allocations

utilities between different agents be independent of aggregate shocks). Since this has to hold for arbitrary effort profiles \( \{a_i\} \), it must notably hold for any constrained-efficient allocation \( \{c_i^*(\theta, s), a_i^*\} \). The intuition behind the result becomes clear from the proof: 
\[ \mathbb{E}_i \left[ \frac{1}{u_i'(c_i^*(\theta, s))} \right] a_i, s \] is the expected marginal resource cost of providing additional utility to an agent in group \( i \) in aggregate state \( s \) without affecting incentives. In any given aggregate state \( s \), the constrained-efficient consumption schedule must equalize this cost across agents, weighted by population shares \( n_i \) and Pareto-weights \( \psi_i \). Taking ratios for different aggregate states, these weights cancel out and the condition in the theorem results.\(^{13}\)

It is interesting to compare this result to the benchmark case of observable effort. In this case, agents can be sufficiently punished when not choosing the optimal action so that the moral hazard problem disappears and the incentive constraints (3.2) can be ignored. An optimal consumption schedule is then designed purely to provide optimal risk-sharing. Using the necessary first-order conditions of problem (3.3) subject to (3.1), it is straightforward to show that any Pareto-optimal consumption schedule \( \{c_i^*(\theta, s)\} \) with observable effort must be such that the idiosyncratic risk is fully insured, i.e.

\[ c_i^*(\theta, s) = c_i^*(s) \quad \forall i \in I, \theta \in \Theta, \tag{3.11} \]

and aggregate risk is shared optimally across groups, namely

\[ \frac{u_i'(c_i^*(s))}{u_i'(c_i^*(\bar{s}))} = \frac{u_j'(c_j^*(s))}{u_j'(c_j^*(\bar{s}))} \quad \forall i, j \in I, s, \bar{s} \in S. \tag{3.12} \]

Condition (3.11) implies that, at the first-best optimum, individual consumption does not depend on individual output but only on aggregate output. Similar restrictions have been derived and tested by Townsend (1994) and Attanasio and Davis (1996). Condition (3.12) requires that the ratios of marginal utilities between different aggregate states are equalized across all agents.\(^{14}\) Notably, it implies that, if there exists a risk-neutral group, all the other groups are fully insured not only against their idiosyncratic, but also against

\(^{13}\)If I did not aim at characterizing the entire Pareto-frontier but focused only on the point that maximizes an unweighted utilitarian welfare function with \( \psi_i = n_i \quad \forall i \in I \), the condition in Theorem 4 could be strengthened to require that 
\[ \mathbb{E}_i [u_i'(c_i^*(\theta, s))] a_i, s = \mathbb{E}_j [u_j'(c_j^*(\theta, s))] a_j, s \quad \forall s \in S, i, j \in I \]
at the optimum given \( \{a_i\} \).

\(^{14}\)See, Borch (1962), Diamond (1967) and Wilson (1968) for an analysis of the resulting sharing rules for consumption.
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Theorem 4 shows that, in the presence of idiosyncratic uncertainty and moral hazard, it is not Pareto-optimal to equalize ratios of expected marginal utilities between aggregate states across agents, as one may naively expect in view of the first-best condition (3.12). This is because, in general, any variation that would lead to such a condition would violate incentive compatibility. The result that instead ratios of expected inverse marginal utilities have to be equalized in any Pareto-optimum is related to the Inverse Euler equation, which has received much attention in the framework of dynamic models with private information (see, for instance, Diamond and Mirrlees (1978), Rogerson (1985b), Ligon (1998), Golosov, Kocherlakota, and Tsyvinski (2003), Kocherlakota (2005) and Farhi and Werning (2006)). In dynamic models with idiosyncratic shocks and moral hazard, it can be shown that constrained-efficient allocations satisfy

\[ \frac{1}{u'(c_t)} = \frac{1}{R_t} \mathbb{E}_t \left[ \frac{1}{u'(c_{t+1})} \right], \quad (3.13) \]

where \( R_t \) is the marginal return to saving and \( \beta \) the discount factor.\(^{15}\) In these models, \( \mathbb{E}[1/u'(c)] \) is the expected marginal cost of providing additional utility to the agent in a given time period without affecting incentives, which is equalized across periods at the optimum by (3.13), accounting for discounting and the return to saving. Theorem 4 establishes a similar optimality condition with respect to sharing aggregate risk across heterogeneous agents in a completely static environment, which has not received attention so far.\(^{16}\)

Condition (3.4) provides an empirically testable restriction on any Pareto-optimal risk sharing arrangement. The validity of the test does not hinge upon information about the agents’ technology (i.e. outputs \( y_i(\theta, s) \) and probability distributions \( p_i(\theta|a, s) \)), preferences over effort \( v_i(a) \), nor the optimality of the agents’ effort choice, since (3.4) has to hold for arbitrary effort profiles. All that is needed to implement the test is individual-specific

\(^{15}\) Whereas Rogerson (1985b) obtains (3.13) in a dynamic moral hazard model, the economies considered in Golosov, Kocherlakota, and Tsyvinski (2003) and most of the ensuing literature on the Inverse Euler equation are dynamic optimal tax models with privately observed skill shocks. Both specifications of private information, however, lead to the same optimality condition (3.13).

\(^{16}\) In a dynamic taxation model, Kocherlakota (2005) derives a generalization of the intertemporal condition (3.13) to allow for aggregate shocks. All agents are ex ante identical in his economy, however, and the purely intratemporal risk-sharing condition across groups (3.4) is not derived. For a utilitarian social welfare function, Mankiw and Weinzierl (2007) and Weinzierl (2007) obtain a Symmetric Inverse Euler Equation, which requires the equalization of average inverse marginal utilities across cohorts or height groups. None of these papers consider sharing rules for aggregate risk, however, and their results rely on the use of a utilitarian social welfare function as opposed to the general Pareto-criterion applied here.
consumption data and an assumption about preferences $u_i(c)$.\footnote{This would be conceptually similar to Townsend's (1994) test of the first-best conditions (3.11) and (3.12) in a model without moral hazard. Alternatively, (3.4) could be used to estimate coefficients of relative risk aversion, i.e. to identify the set of coefficients of risk aversion that are consistent with a given risk-sharing arrangement being constrained-efficient (see Ligon (1998) for such a study based on the Inverse Euler equation (3.13)).} With log-preferences, for instance, the following simple formula is obtained:

**Corollary 4.** Suppose preferences are given by $U_i(c,a) = \log(c) - v_i(a)$ for all $i \in I$. Then (3.4) reduces to

$$
\mathbb{E}_i [c_i^*(\theta, s) | a_i, s] = \mathbb{E}_j [c_j^*(\theta, s) | a_j, s] \quad \forall i, j \in I, s, \bar{s} \in S.
$$

Hence, constrained-efficiency in this case simply requires that the ratios of expected consumption between different aggregate states are equalized across groups. Another interesting special case arises when there exists a risk-neutral group in the economy (which may also be thought of as a financial sector outside of the economy that is able to perfectly diversify even aggregate risks or for which uncertainty is small).

**Corollary 5.** If there exists a risk-neutral group, (3.4) reduces to

$$
\mathbb{E}_i [1 / u_i'(c_i^*(\theta, s))] | a_i, s] = \mathbb{E}_i [1 / u_i'(c_i^*(\theta, \bar{s})) | a_i, \bar{s}] \quad \forall i \in I, s, \bar{s} \in S. \tag{3.14}
$$

In words, a risk-neutral principal faced with risk-averse agents and aggregate uncertainty equalizes the expected marginal resource cost of providing additional utility across aggregate states for each agent. This implies that the risk-averse agents do not necessarily obtain full insurance against aggregate risk, even though there is no moral hazard problem related to aggregate uncertainty.\footnote{See Demange (2008) for a related result in her framework.} For example, if there exists an aggregate state $s$ in which agents in some group $i \in I$ obtain full insurance (e.g. because the moral hazard problem disappears in that state), then (3.14) and Jensen's inequality imply that

$$
u_i'(c_i^*(s)) \leq \mathbb{E}_i [u_i'(c_i^*(\theta, \bar{s})) | a_i, \bar{s}] \quad \text{for all } \bar{s} \in S.
$$

Hence, the principal finds it optimal to provide the lowest expected marginal utility in the aggregate state with the least consumption uncertainty.\footnote{Whether this translates into expected consumption levels depends on whether $1 / u_i'(.)$ is convex or concave. Notably, by (3.14) and Jensen's inequality, I obtain

$$
c_i^*(s) \geq \mathbb{E}_i [c_i^*(\theta, s) | a_i, s] \quad \forall s \in S \quad \Leftrightarrow \quad p_i(c) / r_i(c) \leq 2,
$$

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I discuss these relationships between expected marginal utilities in constrained-efficient allocations in greater depth.

## 3.4 Optimal Trading Distortions in Financial Markets

### 3.4.1 Competitive Equilibria with Financial Markets

In this section, I ask under what conditions constrained-efficient allocations \( \{c^*_i(\theta, s), a^*_i\} \) are consistent with agents having undistorted access to financial markets, in the sense that they can buy and sell a complete set of claims to consumption contingent on aggregate shocks \( s \). I also characterize the optimal distortions if this is not the case. In particular, let me consider the following modified timing. In the first stage, a transfer system \( \{T_i(\theta, s)\} \) is announced that specifies group-specific transfers contingent on idiosyncratic and aggregate shocks. For the purpose of this paper, it is irrelevant whether this transfer system is provided by a social planner or the result of a competitive equilibrium in an insurance market where private firms offer contracts that specify the transfers \( \{T_i(\theta, s)\} \). In stage 2, agents then simultaneously choose an action \( a_i \in A_i \) and competitively trade \( s \)-contingent Arrow-Debreu securities among themselves, where a security for aggregate state \( s \) pays one unit of consumption if state \( s \) is realized and zero otherwise. Finally, risks are realized and consumption takes place as before, accounting for both transfers and traded financial assets.

Let \( \{q(s)\} \) be the set of prices of the \( s \)-contingent claims to consumption. Then, in stage 2, agents in group \( i \in I \) solve, taking transfers \( \{T_i(\theta, s)\} \) and prices \( \{q(s)\} \) as given,

\[
\max_{\{c_i(\theta, s), A_i(s)\}, a_i} \sum_{s \in S} \sum_{\theta \in \Theta} U_i(c_i(\theta, s), a_i) p_i(\theta|a_i, s) \pi(s) \tag{3.15}
\]

subject to

\[
\sum_{s \in S} q(s) \Delta_i(s) \leq 0, \tag{3.16}
\]

where

\[
c_i(\theta, s) = y_i(\theta, s) - T_i(\theta, s) + \Delta_i(s) \quad \forall \theta \in \Theta, s \in S,
\]

and \( \Delta_i(s) \) is the amount of securities for state \( s \) bought by agents in group \( i \). I call \( \{\Delta_i(s)\} \) a trading profile, which specifies a trading strategy \( \Delta_i(s) \) for each group \( i \in I \). Then an

where \( p_i(c) \equiv -u''_i(c)/u'_i(c) \) is the coefficient of absolute prudence and \( r_i(c) \equiv -u''_i(c)/u'_i(c) \) the coefficient of absolute risk aversion. In the log-case, \( p_i(c)/r_i(c) = 2 \), so that the knife-edge result \( c^*_i(s) = E_i[c^*_i(\theta, s)|a_i, s] \) \( \forall s \in S \) is obtained where the risk-averse agents must obtain the same expected consumption in all aggregate states.
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equilibrium without distortions in the financial markets is defined as follows.

**Definition 6.** An equilibrium in financial markets with the transfer system \( \{T_i(\theta, s)\} \) and without taxes is an allocation \( \{c^e_i(\theta, s), a^e_i\} \), a trading profile \( \{\Delta^e_i(s)\} \) and prices \( \{q^e(s)\} \) such that \( \{c^e_i(\theta, s), \Delta^e_i(s), a^e_i\} \) solves the agents' problem (3.15) to (3.16) taking prices \( \{q^e(s)\} \) and transfers \( \{T_i(\theta, s)\} \) as given, the financial markets clear, i.e.

\[
\sum_i n_i \Delta^e_i(s) = 0 \quad \forall s \in S, \tag{3.17}
\]

and the goods market clears, i.e.

\[
\sum_i n_i \sum_{\theta \in \Theta} p_i(\theta | a^e_i, s)c^e_i(\theta, s) = \sum_i n_i \sum_{\theta \in \Theta} p_i(\theta | a^e_i, s)y_i(\theta, s) \quad \forall s \in S. \tag{3.18}
\]

Note that the market clearing conditions (3.17) and (3.18) imply that the social planner's (or private insurers') budget constraints

\[
\sum_i n_i \sum_{\theta \in \Theta} p_i(\theta | a^e_i, s)T_i(\theta, s) = 0 \quad \forall s \in S
\]

are satisfied.\(^\text{20}\) From this definition and the necessary conditions of the agents' problem (3.15) subject to (3.16), it immediately follows that any equilibrium without taxes in the financial markets \( \{c^e_i(\theta, s), \Delta^e_i(s), a^e_i\} \) must be such that

\[
\mathbb{E}_i \left[ u_i'(c^e_i(\theta, s)) | a^e_i, s \right] = \frac{\mathbb{E}_j \left[ u_j(c^e_j(\theta, s)) | a^e_j, s \right]}{\mathbb{E}_j \left[ u_j'(c^e_j(\theta, \tilde{s})) | a^e_j, \tilde{s} \right]} \quad \forall i, j \in I, s, \tilde{s} \in S. \tag{3.19}
\]

Undistorted trading in financial markets thus leads to the equalization of ratios of expected marginal utilities between aggregate states across agents of different groups. This may conflict with the condition for constrained-efficiency (3.4), which requires the equalization of ratios of expected inverse marginal utilities. The following subsection explores this potential conflict in more detail.

\(^\text{20}\)The definition of equilibrium in financial markets used here parallels Golosov and Tsyvinski's (2007) concept of equilibrium in re trading markets, where agents can trade a risk-free bond. Notably, I could embed any equilibrium in financial markets in a competitive equilibrium where the insurance contracts \( \{T_i(\theta, s)\} \) are not offered by a social planner, but by competitive firms, as in their model. Given that trades are assumed to be observable, this would make no difference for any of the following results.
3.4.2 Optimal Wedges in Financial Markets

The comparison between the optimality condition (3.4) and the equilibrium condition (3.19) indicates that distortions may have to be introduced in financial markets to implement constrained-efficient allocations. In particular, implicit taxes (or wedges) may be required such that the agents’ trading in financial markets does not necessarily lead to an equalization of the marginal rates of substitution between aggregate states across groups, as in (3.19). Formally, fix a constrained-efficient allocation \( \{c_i^*(\theta, s), a_i^*\} \) and define the optimal wedge between the marginal rates of substitution of groups \( i \) and \( j \) for aggregate states \( s \) and \( \bar{s} \in S \) as

\[
\omega_{ij}(s, \bar{s}) = 1 - \frac{\mathbb{E}_i[u'(c_i^*(\theta, s))|a_i^*, s]}{\mathbb{E}_i[u'(c_j^*(\theta, \bar{s}))|a_j^*, \bar{s}]} / \frac{\mathbb{E}_j[u'(c_i^*(\theta, s))|a_i^*, s]}{\mathbb{E}_j[u'(c_j^*(\theta, \bar{s}))|a_j^*, \bar{s}]}.
\]

(3.20)

Hence, the wedge \( \omega_{ij}(s, \bar{s}) \) is zero if the marginal rates of substitution of groups \( i \) and \( j \) are the same, as would be the case with undistorted financial markets, and non-zero otherwise. It is therefore a measure of the distortion that the social planner has to introduce in the trading of groups \( i \) and \( j \) in the markets for \( s \)- and \( \bar{s} \)-contingent securities.

For example, one may imagine that the planner imposes group-specific implicit linear transaction taxes \( \{\tau_i(s)\} \) in the financial markets, so that the prices faced by agents of group \( i \in I \) become \( \{(1 + \tau_i(s))q(s)\} \) rather than \( \{q(s)\} \).\(^{21}\) The agents’ optimization problem would then be modified to

\[
\max \sum_{s \in S} \sum_{\theta \in \Theta} U_i(y_i(\theta, s) - T_i(\theta, s) + \Delta_i(s), a_i)p_i(\theta|a_i, s)\pi(s)
\]

subject to

\[
\sum_{s \in S} (1 + \tau_i(s))q(s)\Delta_i(s) \leq 0.
\]

and, by an argument analogous to (3.19) above, an equilibrium with such implicit taxes

---

\(^{21}\)I refer to these taxes as implicit since, as I will show in section 3.5, it turns out that it is not possible in general to implement a constrained-efficient allocation as an equilibrium based on them. What they capture is a relationship between marginal rates of substitution in financial markets that has to hold in any constrained-efficient allocation, as do wedges. A full implementation result will be derived in section 3.5.

\(^{22}\)An economically equivalent way to introduce implicit linear transaction taxes would be to let agents solve \( \max_{\{\Delta_i(s), a_i\}} \sum_{s \in S} \sum_{\theta \in \Theta} U_i(y_i(\theta, s) - T_i(\theta, s) + (1 - \tau_i(s))\Delta_i(s), a_i)p_i(\theta|a_i, s)\pi(s) \) such that \( \sum_{s \in S} q(s)\Delta_i(s) \leq 0 \), i.e. to impose taxes ex post on the return to the financial securities, rather than ex ante.
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would be such that

$$\frac{E_i \left[ u'_i(c^*_i(\theta, s)) | a^*_i, s \right]}{E_i \left[ u'_i(c^*_i(\theta, s)) | a^*_i, \bar{s} \right]} / (1 + \tau_i(s)) = \frac{E_j \left[ u'_j(c^*_j(\theta, s)) | a^*_j, s \right]}{E_j \left[ u'_j(c^*_j(\theta, \bar{s})) | a^*_j, \bar{s} \right]} / (1 + \tau_j(\bar{s})) \quad (3.21)$$

\(\forall i, j \in I, s, \bar{s} \in S\). A comparison of (3.20) and (3.21) reveals that, to be consistent with a given constrained-efficient allocation \(\{c^*_i(\theta, s), a^*_i\}\), the implicit linear transaction taxes \(\{\tau_i(s)\}\) have to satisfy

$$\frac{(1 + \tau_i(s))/(1 + \tau_j(\bar{s}))}{(1 + \tau_j(s))/(1 + \tau_j(\bar{s}))} = 1 - \omega_{ij}(s, \bar{s}) \quad \forall i, j \in I, s, \bar{s} \in S. \quad (3.22)$$

Clearly, for any given constrained-efficient allocation \(\{c^*_i(\theta, s), a^*_i\}\) and the resulting wedges \(\{\omega_{ij}(s, \bar{s})\}\), there exist many combinations of implicit linear transaction taxes \(\{\tau_i(s)\}\) that are consistent with it, which is why I focus on characterizing optimal wedges in the following.

I first identify a general condition under which wedges are all zero and no distortions in the financial markets are required to be consistent with constrained-efficient allocations:

**Proposition 10.** Consider a constrained-efficient allocation \(\{c^*_i(\theta, s), a^*_i\}\) and suppose that aggregate shocks affect outputs \(y_i(\theta, s)\) only, but not probability distributions, i.e. \(p_i(\theta | a, s) = p_i(\theta | a, \bar{s})\) for all \(i \in I, \theta \in \Theta, s, \bar{s} \in S\) and for \(a = a^*_i\) and all \(a = \bar{a}_i\) for which (3.2) is binding. Then \(\omega_{ij}(s, \bar{s}) = 0\) for all \(i, j \in I, s, \bar{s} \in S\).

**Proof.** The necessary first-order condition of the Pareto-problem (3.3) subject to (3.1) and (3.2) for \(c_i(\theta, s)\) can be rearranged to

$$\frac{1}{u'_i(c^*_i(\theta, s))} = \frac{\pi(s)}{n_i \xi(s)} \left[ \psi_i + \sum_{\bar{a}_i \in A_i} \mu_i(\bar{a}_i) \left( 1 - \frac{p_i(\theta | \bar{a}_i, s)}{p_i(\theta | a^*_i, s)} \right) \right],$$

where \(\xi(s)\) is the Lagrange-multiplier on the aggregate resource constraint in state \(s\) and \(\mu_i(\bar{a}_i)\) on the incentive constraint for group \(i\) and action \(\bar{a}_i \in A_i\). Taking expectations over \(\theta\) on both sides yields

$$E_i [u'_i(c^*_i(\theta, s)) | a^*_i, s] = \Psi(s) \Phi_i(a^*_i, s)$$

with

$$\Psi(s) = \frac{\xi(s)}{\pi(s)} \quad \text{and} \quad \Phi_i(a^*_i, s) = \sum_{\theta \in \Theta} \left[ \psi_i + \sum_{\bar{a}_i \in A_i} \mu_i(\bar{a}_i) \left( 1 - \frac{p_i(\theta | \bar{a}_i, s)}{p_i(\theta | a^*_i, s)} \right) \right].$$

If \(p_i(\theta | a, s) = p_i(\theta | a, \bar{s})\) for all \(i \in I, \theta \in \Theta, s, \bar{s} \in S\) and for \(a = a^*_i\) and all \(a = \bar{a}_i\) for which \(\mu_i(\bar{a}_i) > 0\).
3.4. Optimal Trading Distortions in Financial Markets

0, as the condition in the proposition makes sure, it is clear that $\Phi_i(a^*_i, s)$ is in fact independent of $s$. Let me therefore write $\Phi_i(a^*_i)$ in the following. Then (3.20) implies

$$\omega_{ij}(s, \bar{s}) = 1 - \frac{\Psi(s)\Phi_i(a^*_i) \Phi_j(\bar{s})}{\Psi(\bar{s})\Phi_i(a^*_i) \Psi(s)\Phi_j(\bar{s})} = 0 \forall i \in I, \bar{s} \in S.$$  

Proposition 10 implies that if aggregate shocks affect outputs but not probability distributions over idiosyncratic risk for the actions that agents choose or to which the may consider deviating, then all agents’ marginal rates of substitution are equalized in any constrained-efficient allocation. No distortions in the financial markets are therefore required, and the implicit transaction taxes $\{\tau_i(s)\}$ can all be set to zero. Note that, for this result to hold, no restrictions on how aggregate shocks may affect outputs nor conditions on individuals’ preferences are required.\textsuperscript{23} In addition, and less importantly, aggregate shocks are allowed to affect the probability distributions over idiosyncratic shocks induced by actions to which agents do not consider deviating.

The intuition of the result is the following. As in standard moral hazard models, it is optimal to allocate marginal utilities to agents according to the likelihood ratios $p_i(\theta|\bar{a}_i, s) / p_i(\theta|a^*_i, s)$.\textsuperscript{24} In this model with aggregate uncertainty, there is an additional effect of aggregate output (note that the only place where individual outputs enter the Pareto-problem (3.3) subject to (3.1) and (3.2) is in the feasibility constraints, so that individual outputs cannot affect the solution other than through aggregate output). However, variations in aggregate output only scale marginal utilities up and down uniformly across agents in a constrained-efficient allocation. Thus, if aggregate shocks leave the distribution of likelihood ratios unchanged but only affect outputs, the ratios of expected marginal utilities between different agents must be independent of aggregate states, which is the result in Proposition 10. In other words, if the distributions of likelihood ratios do not depend on $s$, aggregate states are symmetric in terms of the marginal resource costs of providing incentives. All that matters for incentives is therefore to provide a certain expected utility across aggregate states for each idiosyncratic shock, and it is optimal to do so without distortions in the marginal rates of substitution between aggregate states.

\textsuperscript{23}In a formal sense, Proposition 10 could be even strengthened slightly. In fact, aggregate shocks may affect probability distributions, but only in terms of ‘labelling’ idiosyncratic shocks. Formally, if for all $i \in I, s, \bar{s} \in S$ and $\theta' \in \Theta$, there exists a $\theta'' \in \Theta$ such that $p_i(\theta'|a, s) = p_i(\theta''|a, \bar{s})$ for $a = a^*_i$ and all $a = \bar{a}_i$ for which (3.2) is binding, then the result $\omega_{ij}(s, \bar{s}) = 0$ for all $i, j \in I, s, \bar{s} \in S$ goes through. Since the formulation in Proposition 10 already allows for an arbitrary dependency of outputs $y_i(\theta, s)$ on $\theta$ and $s$, however, this generalization is of limited economic value-added.

\textsuperscript{24}See, for instance, Holmström (1979) and Milgrom (1981).  

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Let me next discuss optimal wedges when aggregate shocks do affect probability distributions over idiosyncratic risk. The intuition one may derive from the previous result is that the sign of optimal wedges depends on how aggregate shocks change the distribution of likelihood ratios of the different groups considered. Indeed, it turns out that this idea can be formalized, as I show in the following. Fix again a constrained-efficient allocation \( \{c_i^*(\theta, s), a_i^*\} \) and define for all \( \theta \in \Theta \) and \( s \in S \) the likelihood ratio of group \( i \in I \) and for the deviation \( \tilde{a}_i \in A_i \)

\[
l_i(\theta|\tilde{a}_i, s) = \frac{p_i(\theta|\tilde{a}_i, s)}{p_i(\theta|a_i^*, s)}. \tag{3.23}
\]

The cumulative distribution function of \( l_i(\theta|\tilde{a}_i, s) \) given the constrained-efficient action \( a_i^* \) can be computed as follows:

\[
G_i(l|\tilde{a}_i, s) \equiv Pr_l(l_i(\theta|\tilde{a}_i, s) \leq l|a_i^*),
\]

\[
= Pr_l\left( \frac{p_i(\theta|\tilde{a}_i, s)}{p_i(\theta|a_i^*, s)} \leq l \right) \]

\[
= \sum_{\theta \in \Theta} p_i(\theta|a_i^*, s) 1 \left( \frac{p_i(\theta|\tilde{a}_i, s)}{p_i(\theta|a_i^*, s)} \leq l \right), \tag{3.24}
\]

where \( 1[.] \) is the usual indicator function. Let me denote the corresponding probability density function of \( l_i(\theta|\tilde{a}_i, s) \) by \( g_i(l|\tilde{a}_i, s) \). Note that the mean of \( l_i(\theta|\tilde{a}_i, s) \) given the constrained-efficient action is one for all groups \( i \in I \), states \( s \in S \) and deviations \( \tilde{a}_i \in A_i \) because

\[
\sum_l g_i(l|\tilde{a}_i, s) = E_i[l_i(\theta|\tilde{a}_i, s)|a_i^*, s] = \sum_{\theta \in \Theta} \frac{p_i(\theta|\tilde{a}_i, s)}{p_i(\theta|a_i^*, s)} p_i(\theta|a_i^*, s) = 1.
\]

Aggregate shocks therefore cannot shift the mean of the distribution of likelihood ratios, but only change higher moments. The following general result shows that it is the volatility of this distribution that is crucial for optimal wedges in financial markets.

**Theorem 5.** Consider a constrained-efficient allocation \( \{c_i^*(\theta, s), a_i^*\} \). Suppose that

\[
G_i(l|\tilde{a}_i, s) \preceq_{SOSD} G_i(l|\tilde{a}_i, \tilde{s})
\]

for all \( \tilde{a}_i \in A_i \) for which group \( i \)'s incentive constraint (3.2) is binding, and that

\[
G_j(l|\tilde{a}_j, \tilde{s}) \preceq_{SOSD} G_j(l|\tilde{a}_j, s)
\]

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for all $\tilde{a}_j \in A_j$ for which group $j$'s incentive constraint (3.2) is binding, with $i,j \in I, s, \tilde{s} \in S$. Then $\omega_{ij}(s, \tilde{s}) \geq 0$.\footnote{The result holds with strict inequality whenever at least one of the comparisons of distributions is strict, i.e. whenever in addition $G_i(l|\tilde{a}_i, s) \neq G_i(l|\tilde{a}_i, \tilde{s})$ for some $\tilde{a}_i$ for which (3.2) binds, or $G_j(l|a_j, s) \neq G_j(l|a_j, \tilde{s})$ for some $a_j$ for which (3.2) binds.}

Proof. See Appendix 3.7.1.

To understand the claim in Theorem 5, consider for simplicity the typical case where each group's incentive constraint is binding for only a single deviation $\tilde{a}_i \in A_i$, so that only one distribution of likelihood ratios needs to be considered. Then the claim is that group $i$ has the smaller marginal rate of substitution between states $s$ and $\tilde{s}$ than group $j$ in a constrained optimum if state $\tilde{s}$ leads to a riskier distribution of likelihood ratios than state $s$ for group $i$ in the sense of a mean-preserving spread, whereas the reverse is true for group $j$. In fact, under the given conditions, each group has the higher expected marginal utility in the aggregate state that induces the riskier distribution of likelihood ratios. To prevent agents from equalizing their marginal rates of substitution by trading in financial markets, higher implicit transaction taxes therefore have to be imposed on the securities for the riskier aggregate state for each group. Formally, by the result in the theorem and equation (3.22), if I normalize $\tau_i(s) = \tau_i(\tilde{s}) = 0$, then it must be that $\tau_j(s) \geq \tau_j(\tilde{s})$ (conversely, normalizing $\tau_i(s) = \tau_i(\tilde{s}) = 0$, it must be that $\tau_i(\tilde{s}) \geq \tau_i(s)$, so that it is always the riskier aggregate state that must be associated with the higher transaction tax).

Intuitively, since the social planner varies consumption according to likelihood ratios at the optimum, consumption will be more volatile in those aggregate states that involve a more volatile likelihood ratio.\footnote{By the results in Milgrom (1981), the likelihood ratio is a measure of the 'favorableness' of the information that output provides about the hidden effort choice. An output realization with a low likelihood ratio as defined in (3.23) is 'good news' about hidden effort choice and hence leads to higher optimal consumption. Note that this result and thus the theorems in this section do not depend on the likelihood ratio being monotone.} Whereas the planner spreads consumption in all aggregate states such that ratios of expected inverse marginal utilities are the same across states by Theorem 4, individuals' trading incentives in the financial market are determined by expected marginal utilities. By the convexity of the function $f(x) = 1/x$, individuals have a higher expected marginal utility and thus an incentive to buy additional consumption in financial markets for those aggregate states in which the likelihood ratio and hence consumption vary more (one may think of this as individuals buying additional consumption to 'self-insure' against their more volatile consumption in those states). To prevent this,
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the social planner needs to introduce a positive implicit transaction tax on the corresponding security.

The following proposition shows that the comparison of marginal rates of substitution in financial markets simplifies when there exists a risk-neutral group in the economy.

**Proposition 11.** Consider a constrained-efficient allocation \( \{c^*_i(\theta, s), a^*_i\} \) and assume there exists a risk-neutral group \( i = 1 \). Suppose that

\[
G_i(l|\tilde{a}_i, s) \succeq \text{SOSD} \ G_i(l|\bar{a}_i, \bar{s})
\]

for some \( i \neq 1 \) and all \( \tilde{a}_i \in A_i \) for which group \( i \)'s incentive constraint (3.2) is binding. Then \( \omega_{i1}(s, \bar{s}) \geq 0 \).

**Proof.** See Appendix 3.7.2. \( \square \)

Hence, if there exists a risk-neutral group, and normalizing the implicit transaction taxes for that group to zero \( (\tau_i(s) = 0 \forall s \in S) \), all the other, risk-averse agents in the economy must pay higher transaction taxes on the securities for the aggregate states that make their likelihood ratios more volatile, i.e. \( \tau_i(\bar{s}) \geq \tau_i(s) \forall i \neq 1 \). The intuition is very similar to that of Theorem 5, except that it now does not matter how the likelihood ratios of the risk-neutral agents are affected by aggregate shocks since their marginal rates of substitution in the financial markets are always one.

Let me briefly discuss a simple example of how the condition that the distribution of likelihood ratios in one state second-order stochastically dominates the distribution in another state in Theorem 5 and Proposition 11 translates into properties of the distributions over idiosyncratic shocks \( p_i(\theta|a, s) \):

**Corollary 6.** Consider a constrained-efficient allocation \( \{c^*_i(\theta, s), a^*_i\} \), assume there exists a risk-neutral group \( i = 1 \) and assume for simplicity that the incentive constraint (3.2) is binding for only one action \( \tilde{a}_i \in A_i \) for group \( i \neq 1 \). Suppose the idiosyncratic shock is binary with \( \Theta = \{\theta, \bar{\theta}\} \) and

\[
p_i(\bar{\theta}|\bar{a}_i, s) < p_i(\bar{\theta}|a^*_i, s).
\]

If aggregate state \( \bar{s} \) is such that

\[
p_i(\bar{\theta}|\tilde{a}_i, s) \leq p_i(\bar{\theta}|\bar{a}_i, s) \quad \text{and} \quad p_i(\bar{\theta}|a^*_i, \bar{s}) \geq p_i(\bar{\theta}|a^*_i, s),
\]

then \( \omega_{i1}(s, \bar{s}) \geq 0 \) (with strict inequality whenever one of the inequalities in (3.26) is strict).

This example is based on the following idea: Suppose that a realization of the high shock \( \bar{\theta} \) in aggregate state \( s \) is more likely when the agent has chosen the optimal action
3.4. Optimal Trading Distortions in Financial Markets

...than when he deviated to \(\bar{a}_i\). Now suppose that aggregate shock \(\bar{s}\) makes the high output even more likely compared to state \(s\) if the agent chooses the optimal action \(a^*_i\), but less likely if the deviation \(\bar{a}_i\) is chosen. Then Proposition 11 applies and there must be a higher implicit transaction tax on securities for state \(\bar{s}\) than for state \(s\).\(^{27}\) Intuitively, aggregate shock \(\bar{s}\) makes realizations of idiosyncratic shocks more informative about the hidden effort choice, so that the social planner finds it efficient to make consumption more variable depending on individual outputs. To self-insure against this risk, agents would want to buy additional consumption for aggregate state \(\bar{s}\) in financial markets without distortions, so that an implicit transaction tax is required to prevent them from doing so.

Corollary (6) nicely illustrates that it is the volatility of likelihood ratios and hence consumption, not that of idiosyncratic shocks and outputs, which matters for optimal wedges in financial markets. Notably, whereas the assumptions in the corollary imply that the variance of likelihood ratios is higher in aggregate state \(\bar{s}\) than in \(s\), i.e.

\[
\mathbb{V}_i \left[ \frac{p_i(\theta|\bar{a}_i, \bar{s})}{p_i(\theta|a^*_i, \bar{s})} \Big| a^*_i, \bar{s} \right] = \sum_{\theta \in \Theta} \frac{p_i(\theta|\bar{a}_i, \bar{s})^2}{p_i(\theta|a^*_i, \bar{s})} - 1 \geq \sum_{\theta \in \Theta} \frac{p_i(\theta|\bar{a}_i, s)^2}{p_i(\theta|a^*_i, s)} - 1 = \mathbb{V}_i \left[ \frac{p_i(\theta|\bar{a}_i, s)}{p_i(\theta|a^*_i, s)} \Big| a^*_i, s \right],
\]

nothing is implied about the relationship between the variances of \(\theta\) in the two aggregate states under any of the two actions \(a^*_i\) and \(\bar{a}_i\). Notably, it is possible that the variance of idiosyncratic risk

\[
\mathbb{V}_i[\theta|a_i, s] = p_i(\bar{\theta}|a_i, s)(1 - p_i(\bar{\theta}|a_i, s)) \bar{\theta} \theta
\]

is smaller in state \(\bar{s}\) than state \(s\) under the optimal action \(a^*_i\) and/or the deviation \(\bar{a}_i\), so that state \(\bar{s}\) looks less risky in terms of \(\theta\) than state \(s\). Nonetheless, consumption will be more variable in state \(\bar{s}\).

Let me finally move to another special case of Proposition 11 that is particularly interesting due to its relationship to the dynamic contracting models discussed in section 3.3. It arises when there exists an aggregate state \(s\) in which it is efficient to provide full insurance to some group of agents.

**Corollary 7.** Consider a constrained-efficient allocation \(\{c^*_i(\theta, s), a^*_i\}\) and assume there exists a risk-neutral group \(i = 1\). Suppose that agents in group \(i \neq 1\) obtain full insurance in aggregate

\(^{27}\)It is straightforward to see that the distribution of likelihood ratios in state \(\bar{s}\) is a mean-preserving spread of that in state \(s\): I can construct the distribution of \(l_i(\theta|\bar{a}_i, \bar{s})\) from the distribution of \(l_i(\theta|\bar{a}_i, s)\) as follows: Spread \(l_i(\theta|\bar{a}_i, s)\) to a lottery over \(l_i(\theta|\bar{a}_i, \bar{s})\) and \(l_i(\bar{\theta}|\bar{a}_i, \bar{s})\) with mean \(l_i(\theta|\bar{a}_i, s)\), and spread \(l_i(\bar{\theta}|\bar{a}_i, s)\) to a lottery over \(l_i(\theta|\bar{a}_i, \bar{s})\) and \(l_i(\bar{\theta}|\bar{a}_i, s)\) with mean \(l_i(\bar{\theta}|\bar{a}_i, s)\). Note that this is always possible since assumptions (3.25) and (3.26) imply that

\[
l_i(\theta|\bar{a}_i, s) \leq l_i(\bar{\theta}|\bar{a}_i, s) < l_i(\theta|\bar{a}_i, s) \leq l_i(\bar{\theta}|\bar{a}_i, s).
\]
3.4. Optimal Iraung Distortions in Financial Markets

state \( s \) in the constrained-efficient allocation, i.e. \( c^*_i(\theta, s) = c^*_i(s) \) for all \( \theta \in \Theta \). Then \( \omega_{11}(s, \bar{s}) \geq 0 \) for all \( \bar{s} \neq s \) (with strict inequality whenever group \( i \) does not obtain full insurance in state \( \bar{s} \)).

When there exists an aggregate state \( s \) where it is efficient to provide full insurance to agents in some group \( i \in I \), then that state must be associated with the lowest implicit transaction tax for group \( i \) in the financial markets, i.e. \( \tau_i(\bar{s}) \geq \tau_i(s) \) for all \( \bar{s} \in S \), normalizing the implicit transaction taxes for the risk-neutral group again to zero. The reason is that, if full insurance is provided to agents in group \( i \) in state \( s \), this means that their likelihood ratio is deterministic in that state.\(^{28}\) Then any other distribution of likelihood ratios is riskier than this degenerate distribution, so that the result follows from Proposition 11.

The special case with an aggregate state that leads to deterministic consumption is particularly interesting because it illustrates the relationship of the results about wedges in the present model to those that arise in the dynamic models discussed in the preceding section. There, it was noted that the Inverse Euler equation (3.13) implies the equalization of expected inverse marginal utilities across time periods. Similarly to financial markets here, agents equalize expected marginal utilities over time, however, when they can freely save, as implied by a standard Euler equation

\[
u'(c_t) = \beta R_t \mathbb{E}_t[u'(c_{t+1})].
\]

This conflict also generates a wedge between agents' intertemporal marginal rate of substitution and the marginal return to saving \( R_t \), which can be thought of as an implicit tax on the return to saving that has to be introduced to implement the optimum. It is straightforward to see (based on Jensen's inequality) that this implicit tax is always positive in these models.\(^ {29} \) The reason can be understood from Corollary 7: Current consumption \( c_t \) is deterministic from the point of view of period \( t \), whereas future consumption \( c_{t+1} \) is typically stochastic. Agents would buy too much consumption for the risky state (or the future) by buying securities in the financial market (or saving) if there were no distortions. The social planner therefore needs to tax the Arrow-Debreu securities for the risky state (the return to saving).

\(^{28}\) There are two extreme cases where this occurs. The first arises when the agents' action does not affect probability distributions in state \( s \), i.e. \( p_i(\theta|\bar{a}_i, s) = p_i(\theta|\bar{a}_i, s) \) \( \forall \bar{a}_i \in A_i, \theta \in \Theta \). Then the likelihood ratios are flat with \( l_i(\theta|\bar{a}_i, s) = 1 \) \( \forall \bar{a}_i \in A_i, \theta \in \Theta \), and full insurance is optimal since output contains no information about agents' effort. The second case results when output is deterministic for the optimal action in state \( s \), i.e. \( p_i(\theta|\bar{a}_i, s) = 1 \) for some \( \theta \in \Theta \), which immediately implies full insurance. The two cases mark opposite extremes in terms of how informative likelihood ratios are about the hidden action, but both are such that likelihood ratios are deterministic.

The previous results make clear why the result on intertemporal wedges is very special in the light of the present framework: It always involves the comparison of expected marginal utilities in a deterministic and a stochastic state, so that the sign of the wedge is unambiguous by Corollary 7.30. With aggregate uncertainty, however, different stochastic states have to be compared in general, and Theorem 5 shows that it is the volatility of likelihood ratios and hence consumption that more generally determines optimal wedges. Moreover, the static model with aggregate uncertainty considered here is different from the dynamic models as there exists no technology to transfer resources across aggregate states comparable to saving. What needs to be compared to determine wedges here are therefore the marginal rates of substitution of different agents, rather than a marginal rate of substitution and a marginal rate of transformation. The assumption of a risk neutral-group used in Proposition 11 and its corollaries therefore comes closest to the intertemporal framework with a linear savings technology.

3.5 Tax Implementation with Financial Markets

3.5.1 Taxes versus Wedges

In the previous section, I have characterized optimal wedges between marginal rates of substitution and associated them with implicit linear transaction taxes \( \{\tau_i(s)\} \) that the social planner should introduce in financial markets. I now ask whether it is possible to implement constrained-efficient allocations as equilibria with such transaction taxes and the transfers \( \{T_i(\theta, s)\} \) discussed in section 3.3.31.

The first insight is that this is in general problematic due to the possibility of double-deviations, where agents both deviate to an action \( \bar{a}_i \neq a_i^* \) and trade in financial markets. To see this, consider a constrained-efficient allocation \( \{c_i^*(\theta, s), a_i^*\} \) and linear transaction taxes \( \{\tau_i(s)\} \) in financial markets that satisfy condition (3.22). Suppose an agent in group \( i \in I \) considers deviating to an action \( \bar{a}_i \in A_i \) for which the incentive-constraint (3.2) is

\[ 30 \text{This is also the reason why a result comparable to the zero-wedge result in Proposition 10 does not arise in the dynamic contracting framework.} \\
31 \text{If insurance contracts are provided by competitive firms, the same distortions would need to be introduced in financial markets to implement Pareto-optima. Private insurers could do so by specifying the optimal trading distortions in their contracts.} \]
binding. Then I obtain

\[
\sum_{s \in S} \sum_{\theta \in \Theta} u_i(c_i^*(\theta, s)) p_i(\theta | a_i^*, s) \tau(s) - v_i(a_i^*) \\
= \sum_{s \in S} \sum_{\theta \in \Theta} u_i(c_i^*(\theta, s)) p_i(\theta | a_i, s) \tau(s) - v_i(a_i) \\
\leq \sum_{s \in S} \sum_{\theta \in \Theta} u_i(c_i^*(\theta, s) + \Delta_i(a_i, s)) p_i(\theta | a_i, s) \tau(s) - v_i(a_i),
\]

(3.27)

where \( \Delta_i(a_i, s) \) is the optimal trading strategy in financial markets given action \( a_i \) and inequality (3.27) is strict if \( \Delta_i(a_i, s) \neq 0 \) for some \( s \in S \). The agent is thus better-off after a double-deviation of this kind than in the constrained optimum. Note that the linear taxes guarantee that trading is not profitable if the agent chooses the optimal action \( a_i^* \) since

\[
\mathbb{E}_i \left[ \frac{u_i'(c_i^*(\theta, s)) | a_i^*, s}{u_i'(c_i^*(\theta, s)) | a_i^*, \bar{s}} \right] / (1 + \tau_i(s))
\]

(3.28)

is equalized across \( i \in I \) by equation (3.22), i.e. the taxes introduce the optimal wedges in financial markets correctly. But this does not necessarily hold when the agent deviates to action \( a_i \) unless the action does not affect the marginal rate of substitution in the financial markets and hence object (3.28). Thus, a constrained-efficient allocation \( \{c_i^*(\theta, s), a_i^*\} \) can be implemented as an equilibrium with linear transaction taxes in the financial markets \( \{\tau_i(s)\} \) that satisfy condition (3.22) if and only if the separability condition

\[
\mathbb{E}_i \left[ u_i'(c_i^*(\theta, s)) | a_i, s \right] / \mathbb{E}_i \left[ u_i'(c_i^*(\theta, \bar{s})) | a_i, \bar{s} \right] = \mathbb{E}_i \left[ u_i'(c_i^*(\theta, s)) | a_i^*, s \right] / \mathbb{E}_i \left[ u_i'(c_i^*(\theta, \bar{s})) | a_i^*, \bar{s} \right]
\]

(3.29)

holds for all \( a_i \in A_i \) for which (3.2) binds and all \( i \in I, s, \bar{s} \in S \).

Condition (3.29) requires the constrained-efficient allocation to satisfy a notion of separability in the sense that agents’ marginal rates of substitution in financial markets are independent of effort choice. Under what circumstances will an optimum exhibit this separability property, so that it can be implemented with the linear transaction taxes \( \{\tau_i(s)\} \)? The following proposition shows that conditions similar to those that led to zero wedges in the previous section also apply here:

**Proposition 12.** Consider a constrained-efficient allocation \( \{c_i^*(\theta, s), a_i^*\} \) and suppose that aggregate shocks affect outputs \( y_i(\theta, s) \) only, but not probability distributions, i.e. \( p_i(\theta | a, s) = p_i(\theta | a, \bar{s}) \) for all \( i \in I, \theta \in \Theta, s, \bar{s} \in S \) and \( a_i \in A_i \). Then it can be implemented as an equilibrium using the transfers \( \{T_i(\theta, s)\} \) only and without any interventions in financial markets.

**Proof.** Let the social planner set \( T_i(\theta, s) = y_i(\theta, s) - c_i^*(\theta, s) \) for all \( i \in I, \theta \in \Theta, s \in S \). Then I want
3.5. Tax Implementation with Financial Markets

to show that agents do not find it profitable to trade in financial markets. By Proposition 10, agents who choose the optimal action \( a_i^* \) do not find it profitable to trade since, at the constrained-efficient allocation,

\[
\frac{E_i \left[ u'(c_i^* (\theta, s)) \right] | a_i^*, s}{E_i \left[ u'(c_i^* (\theta, s)) \right] | a_i^*, \bar{s}} = \frac{\Psi(s) \Phi_i(a_i^*)}{\Psi(\bar{s}) \Phi_i(a_i^*)} = \frac{\Psi(s)}{\Psi(\bar{s})}
\]

is equalized across \( i \in I \) for any \( s, \bar{s} \in S \). But the same holds for any deviating agent choosing some action \( \bar{a}_i \in A_i \) since

\[
\Phi_i(\bar{a}_i, s) = \sum_{\theta \in \Theta} \frac{n_i p_i(\theta|\bar{a}_i, s)}{\Psi_i + \sum_{\bar{a}_i \in A_i} \mu_i(\bar{a}_i) \left( 1 - p_i(\theta|\bar{a}_i, s) / p_i(\theta|a_i^*, s) \right)}
\]

is in fact independent of \( s \) by the conditions in the proposition. Writing therefore \( \Phi_i(\bar{a}_i) \) yields

\[
\frac{E_i \left[ u'(c_i^* (\theta, s)) \right] | \bar{a}_i, s}{E_i \left[ u'(c_i^* (\theta, s)) \right] | \bar{a}_i, \bar{s}} = \frac{\Psi(s) \Phi_i(\bar{a}_i)}{\Psi(\bar{s}) \Phi_i(\bar{a}_i)} = \frac{\Psi(s)}{\Psi(\bar{s})}
\]

as well, which proves that the separability condition (3.29) is satisfied, so that there are no profitable double-deviations, and no agent trades in financial markets. Given that all agents therefore consume the constrained-efficient amounts \( \{c_i^* (\theta, s)\} \) by construction of the transfers, the fact that any constraint-efficient allocation satisfies the incentive constraints (3.2) implies that all agents choose the optimal action \( a_i^* \) in equilibrium. \( \square \)

Hence, not only are there no wedges when aggregate shocks affect outputs only, but also profitable double-deviations do not exist.\(^{32}\) No interventions whatsoever are therefore required in financial markets, and any constrained optimum is consistent with agents freely trading financial securities. This is particularly interesting in comparison to the dynamic contracting models, since a comparable result cannot be obtained there. Not only are wedges between the return to saving and marginal rates of intertemporal substitution always non-zero, as discussed in the preceding section, but also double-deviations (where agents deviate to a suboptimal action and at the same time save) are typically profitable, so that linear savings taxes equal to optimal wedges cannot implement the optimum (see, for instance, Kocherlakota (2005), Albanesi and Sleet (2006), and Golosov and Tsyvinski (2006)).

Interestingly, Proposition 12 implies that unobservability of individual trades in financial markets does not put a further restriction on Pareto-optimal risk-sharing arrangements: constrained-efficient allocations with observable and unobservable trades fall to-

\(^{32}\) Compared to Proposition 10, the conditions have been strengthened slightly in the sense that aggregate shocks must not affect probability distributions induced by any action \( \bar{a}_i \in A_i \), rather than just those for which (3.2) is binding. This is because, even though a simple deviation to \( \bar{a}_i \) makes the agent strictly worse-off, trading in financial markets may make her so much better-off that a double-deviation is still profitable.
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together if aggregate shocks affect outputs only. This contrasts with a large literature on unobservable side trades in other settings. For instance, Acemoglu and Simsek (2007) consider a moral hazard model with anonymous side trades and show that the planner does not distort these trades if preferences between effort and consumption are separable. Even with separable preferences, double-deviations are generally binding in my framework, however, unless Proposition 12 applies. Note also that the result in Proposition 12 is very different from those in dynamic models with unobservable savings such as in Golosov and Tsyvinski (2007). They show that, if the social planner cannot observe individual trades in a bond market, it is generally optimal to introduce a non-zero capital tax. An analogy to the result in Proposition 12 therefore does not exist in their framework. Moreover, the result does not rely on a specification of the risk preferences of individuals, such as constant relative risk aversion. In contrast, the result by Krueger and Lustig (2006), who find in a Bewley economy that bond prices are unaffected by aggregate uncertainty if aggregate risk is independent of idiosyncratic risk, crucially depends on the homogeneity property of CRRA preferences and assumptions on the stochastic process for aggregate consumption and borrowing constraints.

3.5.2 An Implementation with Linear Transaction Taxes

In this subsection, I develop a tax system that implements constrained-efficient allocations with financial markets even when the separability condition (3.29) is not satisfied and aggregate shocks affect individual probability distributions in arbitrary ways. Due to double-deviations, simple linear taxes on transactions in the financial markets that only depend on the group $i \in I$ and the aggregate state $s \in S$ then do not implement the optimum. It turns out, however, that any Pareto-optimum is implementable as an equilibrium with group-specific linear taxes in the financial markets that depend on both the aggregate and the idiosyncratic shock for each agent. Let me denote these taxes by $t_i(\theta, s)$. Then agents in group $i \in I$ take the linear taxes in the financial market $\{t_i(\theta, s)\}$ and the transfers $\{T_i(\theta, s)\}$ as well as the prices $\{q(s)\}$ of the Arrow-Debreu securities as given and solve

$$\max_{\{c_i(\theta, s), \Delta_i(s), a_i\}} \sum_s \sum_\theta u_i(c_i(\theta, s)) p_i(\theta|a_i, s) \pi(s) - v_i(a_i)$$

(3.30)

33 The main difference is that there is no aggregate uncertainty in Acemoglu and Simsek (2007) and they assume that all trades take place after all (idiosyncratic) uncertainty is realized.

34 With linear taxes, I thus mean that transaction taxes are linear in the transaction volume $\Delta_i(s)$. 

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subject to their budget constraint in the financial market

\[ \sum_s q(s) \Delta_i(s) \leq 0, \]  

(3.31)

where \( c_i(\theta, s) \) is given by

\[ c_i(\theta, s) = y_i(\theta, s) - T_i(\theta, s) + (1 - t_i(\theta, s)) \Delta_i(s) \]  

(3.32)

for all \( \theta \in \Theta, s \in S \). The idea is here that, since the linear taxes in financial markets must depend on the realization of idiosyncratic shocks, they are due \textit{ex post} and applied to the realized return of the \( s \)-contingent claims to consumption \( \Delta_i(s) \), as opposed to the \textit{ex-ante} distortions \( \{ \tau_i(s) \} \) considered so far. Based on these instruments, I define an equilibrium with taxes as follows.

**Definition 7.** An equilibrium with taxes \( \{ t_i(\theta, s) \} \) in the financial markets and transfers \( \{ T_i(\theta, s) \} \) is an allocation \( \{ c_i(\theta, s), a_i^* \} \), a trading profile \( \{ \Delta_i(\theta) \} \) and prices \( \{ q^*(s) \} \) such that \( \{ c_i(\theta, s), \Delta_i(\theta), a_i^* \} \) solve the agents’ problem (3.30) to (3.32) given prices \( \{ q^*(s) \} \) and taxes \( \{ t_i(\theta, s), T_i(\theta, s) \} \), financial markets clear for each aggregate state

\[ \sum_i n_i \Delta_i(\theta) = 0 \quad \forall \theta \in \Theta, \]  

(3.33)

and the goods market clears in each state

\[ \sum_i n_i \sum_\theta p_i(\theta|a_i^*, s)c_i(\theta, s) = \sum_i n_i \sum_\theta p_i(\theta|a_i^*, s)y_i(\theta, s) \quad \forall \theta \in \Theta. \]  

(3.34)

Note that the market clearing conditions (3.33) and (3.34) imply that the social planner’s budget constraints

\[ \sum_i n_i \sum_\theta p_i(\theta|a_i^*, s)[t_i(\theta, s)\Delta_i(\theta) - T_i(\theta, s)] = 0 \]  

are satisfied for all \( s \in S \). The following theorem shows how a tax-transfer system \( \{ t_i(\theta, s), T_i(\theta, s) \} \) needs to be designed in order to implement a constrained-efficient allocation \( \{ c_i^*(\theta, s), a_i^* \} \) as an equilibrium with financial markets.

**Theorem 6.** Consider any constrained-efficient allocation \( \{ c_i^*(\theta, s), a_i^* \} \) that solves (3.3) subject to (3.1) and (3.2). Fix some positive group- and state-specific constants \( p_i(s) \) such that \( p_i(s)/p_i(\bar{s}) \) is independent of \( i \) for all \( s, \bar{s} \in S \) and set linear taxes \( \{ t_i^*(\theta, s) \} \) in the financial markets as follows:
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\[ t_i^*(\theta, s) = 1 - \frac{\rho_i(s)}{u_i'(c_i^*(\theta, s))} \quad \forall i \in I, \theta \in \Theta, s \in S. \quad (3.35) \]

Set prices \( \{q^*(s)\} \) such that

\[ \frac{q^*(s)}{q^*(\bar{s})} = \frac{\pi(s)\rho_i(s)}{\pi(\bar{s})\rho_i(\bar{s})} \quad (3.36) \]

for some \( i \in I \). For each \( i \in I \), fix an arbitrary trading strategy in the financial market \( \gamma_i(s) \) that satisfies the budget constraint \( \sum_s q^*(s)\gamma_i(s) = 0 \quad \forall i \in I \) given the prices defined in (3.36) and financial markets clearing \( \sum_i n_i\gamma_i(s) = 0 \quad \forall s \in S \). Set the transfers \( \{T_i^*(\theta, s)\} \) as follows:

\[ T_i^*(\theta, s) = y_i(\theta, s) - c_i^*(\theta, s) + (1 - t_i^*(\theta, s))\gamma_i(s) \quad \forall i \in I, \theta \in \Theta, s \in S. \quad (3.37) \]

Then the allocation \( \{c_i^*(\theta, s), a_i^*\} \), the trading profile \( \{\gamma_i(s)\} \) and prices \( \{q^*(s)\} \) are an equilibrium given the tax-transfer system \( \{t_i^*(\theta, s), T_i^*(\theta, s)\} \) defined in (3.35) and (3.37).

Proof. See Appendix 3.7.3.

Note that, since \( \rho_i(s)/\rho_i(\bar{s}) \) is independent of \( i \) by assumption, equation (3.36) indeed defines valid equilibrium prices in the financial markets. Theorem 6 shows that transfers \( \{T_i(\theta, s)\} \) and linear transaction taxes \( \{t_i(\theta, s)\} \) whose levels depend on the realization of idiosyncratic shocks provide a sufficiently rich instrumentarium to implement any constrained-efficient allocation as an equilibrium with financial markets. The reason why double-deviations are no longer profitable when transaction taxes can be contingent on idiosyncratic shocks is that, by choosing them appropriately, the social planner can make sure that after-tax marginal rates of substitution between aggregate states

\[ \frac{u_i'(c_i^*(\theta, s))(1 - t_i^*(\theta, s))}{u_i'(c_i^*(\theta, \bar{s}))(1 - t_i^*(\theta, \bar{s}))} = \frac{\rho_i(s)}{\rho_i(\bar{s})} \]

are independent of \( \theta \) and thus non-stochastic constants. It is then clear that expected after-tax marginal rates of substitution

\[ \frac{\mathbb{E}_i[u_i'(c_i^*(\theta, s))(1 - t_i^*(\theta, s))|a_i, s]}{\mathbb{E}_i[u_i'(c_i^*(\theta, \bar{s}))(1 - t_i^*(\theta, \bar{s}))|a_i, \bar{s}]} = \frac{\rho_i(s)}{\rho_i(\bar{s})} \]

and hence the incentives for trading in financial markets are independent of the hidden effort choice \( a_i \) of agents, no matter how aggregate shocks impact probability distributions over idiosyncratic risks.

\[ ^{35}\text{For instance, } \rho_i(s) = 1 \quad \forall i \in I, s \in S, \rho_i(s) = g_i h(s) \text{ or } \rho_i(s) = 1/\mathbb{E}_i[1/u_i'(c_i^*(\theta, s))|a_i, s] \text{ are possible normalizations, whereby the last one has some particular properties that I will discuss below.} \]
The result in Theorem 6 that transaction taxes in financial markets must condition on idiosyncratic shocks in addition to aggregate shocks generalizes Kocherlakota’s (2005) implementation result in a dynamic optimal tax model with privately observed skill shocks to allow for trades in financial markets. There, the result is that, to implement optimal allocations, it is not enough to impose linear taxes on the return to saving from period $t$ to $t+1$ whose levels depend on labor income in the periods up to $t$ only. In addition, to prevent double-deviations, they must be contingent on labor income in period $t+1$ (and non-linear taxes on labor income are required as well, similarly to the transfers $\{T_i(\theta, s)\}$ here). Theorem 6 shows how to translate a similar intuition in the present framework with financial markets.

The optimal tax/transfer system $\{t_i^*(\theta, s), T_i^*(\theta, s)\}$ has a number of interesting characteristics that are different from those in Kocherlakota’s (2005) dynamic optimal tax model. First, let me emphasize that it is able to implement any trading profile $\{y_i(s)\}$ in the financial markets consistent with market clearing. Notably, the theorem includes a special case $y_i(s) = 0 \forall i \in I, s \in S$. For any constrained-efficient allocation, it is thus possible to find a tax-transfer system such that there is no trading at all in financial markets in the equilibrium that implements it. However, the theorem also makes clear that this property of an equilibrium is by no means required to implement a constrained-efficient allocation.

Second, let me ask who pays higher linear taxes in the financial markets. By inspection of (3.35), $t_i^*(\theta, s)$ is decreasing in $c_i^*(\theta, s)$ so that marginal taxes are decreasing in consumption. In terms of the levels of transaction taxes paid, however, the comparative statics depend on the agents’ direction of trading in financial markets. If a state $s \in S$ is realized such that $y_i(s) > 0$, agents pay a lower tax (or receive a higher subsidy) on their return from the Arrow-Debreu security if they experience an idiosyncratic shock that gives them higher consumption at the optimum (i.e. taxes payments are regressive). By contrast, if $s$ is such that $y_i(s) < 0$, so that agents need to pay a positive amount of consumption in the financial market, their after-tax payment is $y_i(s)(1 - t_i^*(\theta, s))$, which is increasing in consumption, and tax payments are thus progressive.

Whereas transaction taxes thus spread consumption of those agents who bought $s$-contingent claims to consumption ($y_i(s) > 0$), they provide additional insurance to those who sold such claims, i.e. for whom $y_i(s) < 0$ holds. To understand why this is optimal, note that those who find it optimal to buy consumption for state $s$ in the financial market are those with a high (before-tax) expected marginal utility in state $s$, whereas those who want to sell claims do so because they have a lower expected marginal utility in that state.

Note that the sign of the taxes is completely indeterminate, so that with the term taxes I in fact refer to both taxes and subsidies.
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at the constrained optimum. By making consumption more risky for the former and less risky for the latter market side, transaction taxes thus move (after-tax) expected marginal utilities closer together, which is exactly their purpose.\(^{37}\)

3.5.3 An Implementation with Nonlinear Transaction Taxes

While the linearity property of the transaction taxes constructed in the previous subsection is appealing due to the resulting simplicity, the fact that they need to condition on individual outputs in addition to the aggregate state for which the financial asset is sold or purchased may be regarded as making them unrealistically complex. In the following, using the insights of Werning (2009), I construct an alternative tax system to implement constrained efficient allocations that does not require transaction taxes in financial markets to be contingent on idiosyncratic shocks. Rather, this novel implementation has the nice property that transaction taxes depend only on an agent’s trading strategy \(\Delta_i(s)\), even though generally in a nonlinear way.

For the analysis in this subsection, it is useful to index the set \(S\) of aggregate states with \(h = 0, ..., H\), where \(H = ||S||\). Let me also use the Arrow-Debreu security for state \(s_0 \in S\) as the numeraire asset and normalize \(q(s_0) = 1\). Then the idea is that, when an agent chooses some trading strategy \(\Delta_i(s)\) in the financial markets, she needs to pay a transaction tax \(\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))\) in terms of the numeraire asset for state \(s_0\) (and the tax schedule does not condition on \(\Delta_i(s_0)\) without loss of generality). Agents in group \(i \in I\) take the transaction tax \(\kappa_i\) and the transfers \(\{T_i(\theta, s)\}\) as well as the prices \(\{q(s)\}\) of the Arrow-Debreu securities as given and solve

\[
\max_{\{c_i(\theta, s), a_i\}} \sum_s \sum_{\theta} u_i(c_i(\theta, s)) p_i(\theta|a_i, s) \pi(s) - v_i(a_i) \tag{3.38}
\]

subject to their budget constraint in the financial market

\[
\Delta_i(s_0) + \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) + \sum_{s \neq s_0} q(s) \Delta_i(s) \leq 0, \tag{3.39}
\]

where \(c_i(\theta, s)\) is given by

\[
c_i(\theta, s) = y_i(\theta, s) - T_i(\theta, s) + \Delta_i(s) \tag{3.40}
\]

\(^{37}\)Since Kocherlakota (2005) only allows for positive capital holdings by all agents in his dynamic economy, wealth taxes are always regressive in his framework and the fact that taxes provide additional insurance, as they always do here for one market side, can never arise.
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for all \( \theta \in \Theta, s \in S \). An equilibrium with financial markets can then be defined analogously to the previous subsection:

**Definition 8.** An equilibrium with the transaction tax \( \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) \) in the financial market and transfers \( \{T_i(\theta, s)\} \) is an allocation \( \{c^*_i(\theta, s), a^*_i\} \), a trading profile \( \{\Delta^*_i(s)\} \) and prices \( \{q^*(s)\} \) such that \( \{c^*_i(\theta, s), \Delta^*_i(s), a^*_i\} \) solve the agents’ problem (3.38) to (3.40) given prices \( \{q^*(s)\} \), the transaction tax \( \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) \) and transfers \( \{T_i(\theta, s)\} \), financial markets clear for each aggregate state (equation (3.33)), and the goods market clears in each state (equation (3.34)).

The following theorem constructs a tax system with a non-linear but output-independent transaction tax schedule \( \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) \) and transfers \( \{T_i(\theta, s)\} \) that implements a constrained-efficient allocation \( \{c^*_i(\theta, s), a^*_i\} \) as an equilibrium with financial markets.

**Theorem 7.** Consider any constrained-efficient allocation \( \{c^*_i(\theta, s), a^*_i\} \) that solves (3.3) subject to (3.1) and (3.2). Let

\[
W^*_i = \sum_{s \in S} \sum_{\theta \in \Theta} u_i(c^*_i(\theta, s)) p_i(\theta|a^*_i, s) \pi(s) - v_i(a^*_i) \quad \forall i \in I
\]  

(3.41)

and fix some arbitrary positive prices \( \{q^*(s)\} \). For each \( i \in I \) and each trading profile \( \{\Delta_i(s)\} \), let the transaction tax schedule \( \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) \) be implicitly defined by

\[
W^*_i = \max_{a_i \in A_i} \left\{ \sum_{\theta} u_i \left( c^*_i(\theta, s_0) - \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) - \sum_{s \neq s_0} q^*(s) \Delta_i(s) \right) p_i(\theta|a_i, s_0) \pi(s_0) + \sum_{s \neq s_0} \sum_{\theta} u_i(c^*_i(\theta, s) + \Delta_i(s)) p_i(\theta|a_i, s) \pi(s) - v_i(a_i) \right\}.
\]

(3.42)

If \( u_i(c) \) is strictly increasing and continuous, then there exists a unique and continuous transaction tax schedule \( \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) \) solving (3.42).

For each \( i \in I \), fix an arbitrary trading strategy in the financial markets \( \gamma_i(s) \) that satisfies financial markets clearing \( \sum_i v_i \gamma_i(s) = 0 \) \( \forall s \in S \) and the budget constraint (3.39) with equality given the prices \( \{q^*(s)\} \) and the transaction tax schedule \( \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) \) defined in (3.42). Set transfers \( \{T_i(\theta, s)\} \) such that

\[
T^*_i(\theta, s) = y_i(\theta, s) - c^*_i(\theta, s) + \gamma_i(s) \quad \forall i \in I, \theta \in \Theta, s \in S.
\]

(3.43)

Then the allocation \( \{c^*_i(\theta, s), a^*_i\} \), the trading profile \( \{\gamma_i(s)\} \) and prices \( \{q^*(s)\} \) are an equilibrium given the transaction tax \( \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) \) and transfers \( \{T^*_i(\theta, s)\} \).
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Proof. See Appendix 3.7.4. \(\square\)

The result in Theorem 7 is that, for any constrained efficient allocation, there exists a continuous transaction tax schedule that conditions only on an individual's trades, not on idiosyncratic shocks, and implements it as an equilibrium. Thus, when transaction taxes are not constrained to be linear, information about individual outputs is not necessary to impose transaction taxes. Moreover, the implementation does not rely on sharp penalties in the form of discontinuous taxes. This is in contrast to a direct mechanism that completely prevents agents from trading in financial markets, which can be thought of imposing an infinite tax whenever agents deviate from \(\Delta_i(s) = 0 \forall s \in S\).

The construction of the transaction tax in equation (3.42) makes clear how the implementation works: For any trading strategy \(\Delta_i(s)\), the transaction tax schedule \(\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))\) is such that, when choosing the optimal action given this trading strategy, the agent is just indifferent to the constrained efficient allocation \(\{c_i^*(\theta, s), a_i^*\}\). Therefore exists no (double-)deviation that could make the agent better off. Note that this construction is quite a reversal of the idea behind the implementation using linear but output dependent taxes in the preceding subsection. There, transaction taxes were designed such that, for any action that the agent may choose, she prefers the trading strategy that leads to consumption \(\{c_i^*(\theta, s)\}\). Together with incentive compatibility, this also ruled out any profitable deviations.

It is evident from the construction in equation (3.42) that there exist many more non-linear transaction tax schedules that would implement the constrained-efficient allocation. Namely, they would make agents worse off after any deviation, instead of just indifferent (so that (3.42) would hold as a weak inequality for any \(\Delta_i(s)\), and as an equality for the trading strategy \(\gamma_i(s)\) to be implemented). Thus, the implementation in Theorem 4 picks the lowest possible transaction taxes to implement a constrained-efficient allocation, leaving agents the greatest amount of choice possible.

It is even possible to derive further properties of the transaction tax \(\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))\) at this level of generality. Notably, it turns out that the marginal transaction taxes are quite closely related to the wedges derived in section 3.4. To see this, let me normalize \(q^*(s) = 1 \forall s \in S\) without loss of generality (Theorem 4 shows that any constrained efficient allocation can be implemented as an equilibrium with arbitrary positive prices). For any trading strategy \(\Delta_i(s)\), let the solution of the maximization in (3.42) be given by \(a_i^*(\Delta_i(s_1), ..., \Delta_i(s_H))\), so that \(a_i^*\) is the set of actions \(a_i \in A_i\) maximizing the RHS of (3.42) given \(\Delta_i(s)\). If \(a_i^*(\Delta_i(s_1), ..., \Delta_i(s_H))\) is single-valued, then a standard Envelope Theorem

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implies that for $h \in \{1, \ldots, H\}$,

$$\frac{\partial \kappa_i(\Delta_i(s), \ldots, \Delta_i(s_H))}{\partial \Delta_i(s_h)} = \frac{E_i[u'_i(c^*_i(\theta, s_h) + \Delta_i(s_h)) | a^*_i, s_h]}{E_i[u'_i(c^*_i(\theta, s_0) - \kappa_i - \sum_{s \neq s_0} \Delta_i(s)) | a^*_i, s_0]} - 1,$$

where $\kappa_i$ stands short for $\kappa_i(\Delta_i(s_1), \ldots, \Delta_i(s_H))$ and $a^*_i$ for $a^*_i(\Delta_i(s_1), \ldots, \Delta_i(s_H))$. The marginal tax is thus equal to the difference between the marginal rate of substitution and the price ratio along the budget constraint in the financial markets. Notably, at the implemented trading profile $\{\gamma_i(s)\}$ and with the transfers $\{T_i(\theta, s)\}$ defined in (3.43), this reduces to

$$\frac{\partial \kappa_i(\gamma_i(s_1), \ldots, \gamma_i(s_H))}{\partial \Delta_i(s_h)} = \frac{E_i[u'_i(c^*_i(\theta, s_h)) | a^*_i, s_h]}{E_i[u'_i(c^*_i(\theta, s_0)) | a^*_i, s_0]} - 1,$$

and therefore

$$\frac{1 + \partial \kappa_i/\partial \Delta_i(s_h)}{1 + \partial \kappa_i/\partial \Delta_j(s_h)} = 1 - \omega_{ij}(s_h, s_0).$$

The ratio of one plus the marginal transaction tax between different groups, evaluated at the equilibrium, equals one minus the wedge between these groups as characterized in section 3.4. All the results derived there about optimal wedges therefore immediately translate into properties of the marginal transaction taxes evaluated at the implemented allocation.

This analysis of marginal transaction taxes applies when $a^*_i(\Delta_i(s_1), \ldots, \Delta_i(s_H))$ is single-valued in equilibrium, as would be typically the case when $a_i$ is chosen from a continuum of possible actions and $a^*_i$ satisfies a first-order condition. However, when $A_i$ is a discrete set, this is not the case since the optimal allocation $\{c^*_i(\theta, s), a^*_i\}$ is such that an agent of group $i$ is just indifferent between $a^*_i$ and a deviation $\bar{a}_i \in A_i$. In this case, the transaction tax schedule actually has a kink and is not differentiable at the equilibrium trading strategy $\gamma_i(s)$. Nevertheless, left and right derivatives can still be computed using Envelope Theorems (see e.g. Milgrom and Segal (2002)), establishing the same relationship between (directional) marginal transaction taxes and wedges as derived above.

### 3.6 Conclusion

I have derived optimality conditions for allocations in a moral hazard economy with heterogeneous agents and aggregate shocks under very general circumstances: They do not rely on a particular social welfare functional, but only use the Pareto-criterion; they do not require the first-order approach to be valid; and they do not put restrictions on preferences other than separability between consumption and effort. It may be interesting
in subsequent work to test to what degree real-world risk-sharing arrangements satisfy these conditions, or what parameterizations of preferences make them consistent with them. For instance, a quantitative analysis based on the present second-best model may provide insights into why Attanasio and Davis (1996) find a clear failure of first-best consumption insurance between groups in the US.

In spite of the general framework in which they have been derived, the efficiency conditions have turned out to have strong implications for optimal tax policy in financial markets. I have shown that optimal distortions in financial markets depend in a transparent way on how aggregate shocks affect the informativeness of likelihood ratios. Financial claims whose return is high under shocks that make likelihood ratios more variable and hence more informative about hidden effort choice must be subject to a higher implicit tax than claims that pay off in uninformative states in which consumption reacts only little to variations in individual outputs. As a benchmark case, I have shown that financial markets are undistorted at the optimum if aggregate shocks do not affect the informativeness of likelihood ratios.

In order to decentralize Pareto-optimal allocations as competitive equilibria with financial markets, the government may impose linear transaction taxes in financial markets that are contingent upon individual outputs. Notably, tax payments must be positively correlated with consumption for agents who sell financial assets, but negatively for those who buy assets that achieve a high return under a given aggregate shock. The second possibility is to impose transaction taxes that do not condition on idiosyncratic shocks, but are nonlinear in transaction volumes. The resulting marginal taxes are closely related to the wedges between marginal rates of substitutions at the optimum.

Given that consumption is taken to be observable, the tax decentralization developed here is one of many possible implementations. For instance, Pareto-optima could also be implemented by private insurance companies that competitively provide insurance contracts, prohibiting their customers from trading in financial markets. Alternatively, the government could provide all the insurance and completely shut down financial markets. The implementation with agents trading in financial markets, but subject to tax distortions, is a case that may be considered as more realistic. For instance, in a multi-country setting, the transaction taxes discussed here share similarities with a Tobin tax on international financial flows. Moreover, the asymmetric treatment of gains and losses from financial investments implied by the optimal taxes here is shared by many real-world capital gains tax systems.
3.7 Appendix

3.7.1 Proof of Theorem 5

By the definition of optimal wedges in (3.20), \( \omega_{ij}(s, \bar{s}) \geq 0 \) if

\[
\frac{E_i[u'_i(c^*_i(\theta, s))|a^*_i, s]}{E_i[u'_i(c^*_i(\theta, \bar{s}))|a^*_i, \bar{s}]} \leq \frac{E_j[u'_j(c^*_j(\theta, s))|a^*_j, s]}{E_j[u'_j(c^*_j(\theta, \bar{s}))|a^*_j, \bar{s}]}
\]

i.e. the marginal rate of substitution in the financial markets for states \( s \) and \( \bar{s} \) of group \( i \) is smaller than that of group \( j \) in the constrained-efficient allocation. Let me start with the marginal rate of substitution of group \( i \). The necessary first-order condition of the Pareto-problem (3.3) subject to (3.1) and (3.2) for \( c_i(\theta, s) \) can be rearranged to

\[
u'_i(c^*_i(\theta, s)) = \frac{\zeta(s)}{\pi(s)} \frac{n_i}{\psi_i + \sum_{\hat{a}_i \in A_i} \mu_i(\hat{a}_i) \left(1 - \frac{p_i(\hat{a}_i, s)}{p_i(\hat{a}_i, \bar{s})}\right)},
\]

where \( \zeta(s) \) is the Lagrange multiplier on the resource constraint (3.1) and \( \mu_i(\hat{a}_i) \) the multiplier on the incentive constraint for action \( \hat{a}_i \). Hence,

\[
\frac{E_i[u'_i(c^*_i(\theta, s))|a^*_i, s]}{E_i[u'_i(c^*_i(\theta, \bar{s}))|a^*_i, \bar{s}]} = \frac{\zeta(s)}{\pi(s)} \frac{\sum_{\hat{a}_i \in A_i} \psi_i + \sum_{\hat{a}_i \in A_i} \mu_i(\hat{a}_i) \left(1 - \frac{p_i(\hat{a}_i, s)}{p_i(\hat{a}_i, \bar{s})}\right)}{\sum_{\hat{a}_j \in A_j} \psi_j + \sum_{\hat{a}_j \in A_j} \mu_j(\hat{a}_j) \left(1 - \frac{p_j(\hat{a}_j, s)}{p_j(\hat{a}_j, \bar{s})}\right)}
\]

and a completely analogous result can be obtained for group \( j \). A sufficient condition for (3.44) to be satisfied is therefore given by

\[
\sum_{\theta \in \Theta} \frac{p_i(\theta|a^*_i, s)}{\psi_i + \sum_{\hat{a}_i \in A_i} \mu_i(\hat{a}_i) \left(1 - \frac{p_i(\hat{a}_i, s)}{p_i(\hat{a}_i, \bar{s})}\right)} \leq \sum_{\theta \in \Theta} \frac{p_i(\theta|a^*_i, \bar{s})}{\psi_i + \sum_{\hat{a}_i \in A_i} \mu_i(\hat{a}_i) \left(1 - \frac{p_i(\hat{a}_i, \bar{s})}{p_i(\hat{a}_i, s)}\right)}
\]

and

\[
\sum_{\theta \in \Theta} \frac{p_j(\theta|a^*_j, s)}{\psi_j + \sum_{\hat{a}_j \in A_j} \mu_j(\hat{a}_j) \left(1 - \frac{p_j(\hat{a}_j, s)}{p_j(\hat{a}_j, \bar{s})}\right)} \geq \sum_{\theta \in \Theta} \frac{p_j(\theta|a^*_j, \bar{s})}{\psi_j + \sum_{\hat{a}_j \in A_j} \mu_j(\hat{a}_j) \left(1 - \frac{p_j(\hat{a}_j, \bar{s})}{p_j(\hat{a}_j, s)}\right)}
\]

Let me again focus on group \( i \) and inequality (3.47) (condition (3.48) can be dealt with in a completely analogous way). By the definition of \( l_i(\theta|\hat{a}_i, s) \) in (3.23), I can write

\[
\sum_{\theta \in \Theta} \frac{p_i(\theta|a^*_i, s)}{\psi_i + \sum_{\hat{a}_i \in A_i} \mu_i(\hat{a}_i) \left(1 - \frac{p_i(\hat{a}_i, s)}{p_i(\hat{a}_i, \bar{s})}\right)} = \sum_{\theta \in \Theta} \frac{p_i(\theta|a^*_i, s)}{\psi_i + \sum_{\hat{a}_i \in A_i} \mu_i(\hat{a}_i) \left(1 - l_i(\theta|\hat{a}_i, s)\right)},
\]
Let me define the new random variable

\[ L_i(\theta|s) = \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) l_i(\theta|\tilde{a}_i, s), \]  

which, for each \( \theta \in \Theta \), is a weighted sum of the likelihood ratios of the actions \( \tilde{a}_i \) for which the incentive constraint of group \( i \) binds.\(^{38}\) Its cumulative distribution function given the constrained-efficient action \( a_i^* \) is

\[
\Gamma_i(L|s) = \text{Pr}_i\left(L_i(\theta|s) \leq L|a_i^*, s\right) \\
= \text{Pr}_i\left(\sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) l_i(\theta|\tilde{a}_i, s) \leq L|a_i^*, s\right) \\
= \sum_{\theta \in \Theta} p_i(\theta|a_i^*, s) \left\{ \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) \frac{p_i(\theta|\tilde{a}_i, s)}{p_i(\theta|a_i^*, s)} \leq L \right\},
\]

and let me denote the corresponding probability density function by \( \gamma_i(L|s) \). The following result will be useful.

**Lemma 10.** Suppose \( G_i(l|\tilde{a}_i, s) \succeq_{\text{SOSD}} G_i(l|\tilde{a}_i, \tilde{s}) \) for all \( \tilde{a}_i \in A_i \) for which (3.2) is binding. Then \( \Gamma_i(L|s) \succeq_{\text{SOSD}} \Gamma_i(L|\tilde{s}) \).

**Proof.** Since \( G_i(l|\tilde{a}_i, s) \) is a mean-preserving spread of \( G_i(l|\tilde{a}_i, \tilde{s}) \), it can be constructed as a compound lottery, where in a first stage, \( l \) is drawn from \( G_i(l|\tilde{a}_i, s) \) and, subsequently, each possible outcome of \( l \) is further randomized so that the final likelihood ratio is \( l + z_{\tilde{a}_i} \), where \( z_{\tilde{a}_i} \) has a cumulative distribution function \( H_i^l(z_{\tilde{a}_i}|\tilde{a}_i) \) and a corresponding probability density function \( h_i^l(z_{\tilde{a}_i}|\tilde{a}_i) \) with mean zero for all \( l \) (i.e. \( \sum_{z_{\tilde{a}_i}} z_{\tilde{a}_i} h_i^l(z_{\tilde{a}_i}|\tilde{a}_i) = 0 \forall l \)). I want to show that \( \Gamma_i(L|s) \) is a mean-preserving spread of \( \Gamma_i(L|\tilde{s}) \), where \( \Gamma_i(L|s) \) is the cumulative distribution function of \( L_i(\theta|s) = \sum_{\tilde{a}_i} \mu_i(\tilde{a}_i) l_i(\theta|\tilde{a}_i, s) \) by (3.50). To see this, note that \( \Gamma_i(L|s) \) can be constructed as a compound lottery where, first, \( L \) is drawn from \( \Gamma_i(L|\tilde{s}) \), and, in a second stage, each possible realization of \( L \) is further randomized so that the final outcome is \( L + Z \) with \( Z = \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) z_{\tilde{a}_i} \). The mean of \( Z \) is \( \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) \sum_{z_{\tilde{a}_i}} z_{\tilde{a}_i} h_i^l(z_{\tilde{a}_i}|\tilde{a}_i) = 0 \forall L \), so that \( \Gamma_i(L|s) \) is a mean-preserving spread of \( \Gamma_i(L|\tilde{s}) \), as claimed in the lemma.

Substituting the definitions of \( L_i(\theta|s) \) and \( \gamma_i(L|s) \) from (3.50) and (3.51) in (3.49), I can write

\[
\sum_{\theta \in \Theta} \frac{p_i(\theta|a_i^*, s)}{\psi_i + \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) \left( 1 - \frac{p_i(\theta|\tilde{a}_i, s)}{p_i(\theta|a_i^*, s)} \right)} = \sum_{L \in L_i} \frac{\gamma_i(L|s)}{\psi_i + \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) - L} = \sum_{L \in L_i} \Lambda_i(L) \gamma_i(L|s)
\]

with

\[
\Lambda_i(L) = \frac{1}{\psi_i + \sum_{\tilde{a}_i \in A_i} \mu_i(\tilde{a}_i) - L}.
\]

\(^{38}\)In the generic case where the incentive constraint (3.2) only binds for one action \( \tilde{a}_i \in A_i \), \( L_i \) is just a rescaling of \( l_i \) with \( L_i(\theta|s) = \mu_i(\tilde{a}_i) l_i(\theta|\tilde{a}_i, s) \forall \theta \in \Theta \).
3.7. Appendix

Note first that $\psi_i + \sum_{i \in A_1} \mu_i(\bar{a}_i) - \mathcal{L} > 0$ because of (3.45) and $u'_i(c_1^*(\theta,s)) > 0$ for all $i \in I, \theta \in \Theta, s \in S$. Hence $\mathcal{L}$ must always lie in the interval $L_i \equiv [0, \psi_i + \sum_{i \in A_1} \mu_i(\bar{a}_i))$. It is then straightforward to verify that $\Lambda_i(\mathcal{L})$ is strictly convex in this domain.

Since $\Gamma_i(\mathcal{L}|\bar{s})$ is a mean preserving spread of $\Gamma_i(\mathcal{L}|s)$ by Lemma 10, it can be constructed from a compound lottery where, in the first stage, $\mathcal{L}$ is drawn from $\Gamma_i(\mathcal{L}|s)$ and, in the second stage, each possible outcome of $\mathcal{L}$ is further randomized so that the final likelihood ratio is $\mathcal{L} + \mathcal{Z}$, where $\mathcal{Z}$ has a cumulative distribution function $H(\mathcal{Z})$ and a corresponding probability density function $h(\mathcal{Z})$ with mean zero for all $\mathcal{L}$ (i.e. $\sum_{\mathcal{Z}} \mathcal{Z} h(\mathcal{Z}) = 0 \ \forall \mathcal{L}$). Then convexity of $\Lambda_i(\mathcal{L})$ and Jensen's inequality imply that

$$\sum_{\mathcal{L} \in L_i} \Lambda_i(\mathcal{L}) \gamma_i(\mathcal{L}|\bar{s}) = \sum_{\mathcal{L} \in L_i} \left( \sum_{\mathcal{Z}} \Lambda_i(\mathcal{L} + \mathcal{Z}) h(\mathcal{Z}) \right) \gamma_i(\mathcal{L}|s)$$

$$\geq \sum_{\mathcal{L} \in L_i} \Lambda_i \left( \sum_{\mathcal{Z}} (\mathcal{L} + \mathcal{Z}) h(\mathcal{Z}) \right) \gamma_i(\mathcal{L}|s)$$

$$= \sum_{\mathcal{L} \in L_i} \Lambda_i(\mathcal{L}) \gamma_i(\mathcal{L}|s)$$

since $\sum_{\mathcal{Z}} \mathcal{Z} h(\mathcal{Z}) = 0 \ \forall \mathcal{L} \in L_i$. Using this together with (3.52) yields the desired inequality (3.47) for group $i$. A completely analogous argument, replacing $i$ by $j$ and interchanging $s$ and $\bar{s}$, yields the desired inequality (3.48) for group $j$, thus establishing the result in Theorem 5.

3.7.2 Proof of Proposition 11

By risk-neutrality of group $i = 1$ and the definition of optimal wedges in (3.20), $\omega_{11}(s, \bar{s}) \geq 0$ if

$$\frac{\mathbb{E}_i[u'_1(c_1^*(\theta,s))]|a_1^*, s]}{\mathbb{E}_i[u'_1(c_1^*(\theta,s))]|a_1^*, \bar{s}] \leq 1.$$ 

From equation (3.46) in the proof of Theorem 5, this is true if

$$\frac{\zeta(s)}{\pi(s)} \sum_{\theta \in \Theta} \frac{p_i(\theta|a_1^*, s)}{\psi_i + \sum_{i \in A_1} \mu_i(\bar{a}_i)} \left( 1 - \frac{p_i(\theta|a_1^*, s)}{p_i(\theta|a_1^*, \bar{s})} \right) \leq \frac{\zeta(\bar{s})}{\pi(\bar{s})} \sum_{\theta \in \Theta} \frac{p_i(\theta|a_1^*, \bar{s})}{\psi_i + \sum_{i \in A_1} \mu_i(\bar{a}_i)} \left( 1 - \frac{p_i(\theta|a_1^*, \bar{s})}{p_i(\theta|a_1^*, s)} \right).$$

I will show in the following that, if there exists a risk-neutral group,

$$\frac{\zeta(s)}{\pi(s)} = \frac{\zeta(\bar{s})}{\pi(\bar{s})} \quad \forall s, \bar{s} \in S.$$

Then the result in Proposition 11 follows from the proof of Theorem 5. Note that, since $u'_1(.) =$ const. by risk-neutrality, equation (3.45) in the proof of Theorem 5 implies

$$\text{const.} = \frac{\zeta(s)}{\pi(s)} \psi_i + \sum_{i \in A_1} \mu_i(\bar{a}_i) \left( 1 - \frac{p_i(\theta|a_1^*, s)}{p_i(\theta|a_1^*, \bar{s})} \right).$$
Taking reciprocals on both sides and integrating over \( \theta \in \Theta \) yields

\[
\text{const.}^{-1} = \sum_{\theta \in \Theta} p_1(\theta|a_1^*, s) \frac{\pi(s)}{\xi(s)} \frac{1}{n_1} \left( \psi_1 + \sum_{\tilde{a}_1 \in A_1} \mu_1(\tilde{a}_1) \left( 1 - \frac{p_1(\theta|\tilde{a}_1, s)}{p_1(\theta|a_1^*, s)} \right) \right) = \frac{\pi(s)}{\xi(s)} \frac{\psi_1}{n_1},
\]

which implies

\[
\frac{\xi(s)}{\pi(s)} = \frac{\psi_1}{n_1} \times \text{const.} \forall s \in S
\]

and hence the desired result.

### 3.7.3 Proof of Theorem 6

Clearly, any constrained-efficient allocation \( \{c_i^*(\theta, s), a_i^*\} \) is feasible, and hence by (3.1)

\[
\sum n_i \sum_{\theta \in \Theta} p_i(\theta|a_i^*, s)c_i^*(\theta, s) = \sum n_i \sum_{\theta \in \Theta} p_i(\theta|a_i^*, s)y_i(\theta, s)
\]

for all states \( s \in S \), so that the market clearing condition (3.34) is satisfied. By construction of \( \{\gamma_i(s)\} \), the clearing condition for the financial market (3.33) is also satisfied. Hence, all that remains to be shown is that, given the taxes in (3.35) and (3.37) and the prices in (3.36), \( \{c_i^*(\theta, s), a_i^*\} \) and \( \Delta_i(s) = \gamma_i(s) \forall i \in I, s \in S \) solve the agents’ problem (3.30) to (3.32).

To prove this, I proceed in two steps: First, I show that, given any effort choice \( \tilde{a}_i \in A_i \), it is optimal for all agents to set \( \Delta_i(s) = \gamma_i(s) \forall i \in I, s \in S \). This implies by condition (3.32) in the agents’ problem and the design of the transfers \( \{T_i^*(\theta, s)\} \) in (3.37) that agents choose the constrained-efficient consumption schedule \( \{c_i^*(\theta, s)\} \forall i \in I, \theta \in \Theta, s \in S \) in equilibrium. The second step then involves demonstrating that agents also find it optimal to choose the constrained-efficient action \( a_i^* \forall i \in I \).

**Step 1.** Fix some \( \tilde{a}_i \in A_i \). Then the optimization problem for an agent of group \( i \in I \) reduces to

\[
\max_{\{\Delta_i(s)\}} \sum_s \sum_{\theta} u_i \left( y_i(\theta, s) - T_i^*(\theta, s) + (1 - T_i^*(\theta, s))\Delta_i(s) \right) p_i(\theta|\tilde{a}_i, s) \pi(s)
\]

subject to

\[
\sum_s q_i^*(s)\Delta_i(s) \leq 0.
\]

Assuming strictly risk-averse agents, this is a strictly convex optimization problem (a strictly concave objective function to be maximized over a linear constraint set) implying that first-order conditions are necessary and sufficient. I therefore only need to show that \( \Delta_i(s) = \gamma_i(s) \forall s \in S \) satisfies the budget constraint and first-order conditions. The budget constraint is satisfied by
construction of \( \{\gamma_i(s)\} \). The first-order conditions imply that, for all \( s, \bar{s} \in S \),

\[
\frac{\pi(s)E_i \left[ u'_i\left(y_i(\theta, s) - T_i^*(\theta, s) + (1 - t_i^*(\theta, s))\Delta_i(s)\right) \right]}{\pi(\bar{s})E_i \left[ u'_i\left(y_i(\theta, \bar{s}) - T_i^*(\theta, \bar{s}) + (1 - t_i^*(\theta, \bar{s}))\Delta_i(\bar{s})\right) \right]} = \frac{q^*(s)}{q^*(\bar{s})}.
\] (3.53)

Setting \( \Delta_i(s) = \gamma_i(s) \forall s \in S \) and substituting \( T_i^*(\theta, s) \) from (3.37) and \( t_i^*(\theta, s) \) from (3.35) in the left hand side of (3.53) yields

\[
\frac{\pi(s)E_i \left[ u'_i\left(c_i^*(\theta, s)\right)\rho_i(s)\right]}{\pi(\bar{s})E_i \left[ u'_i\left(c_i^*(\theta, \bar{s})\right)\rho_i(\bar{s})\right]} = \frac{\pi(s)\rho_i(s)}{\pi(\bar{s})\rho_i(\bar{s})} = \frac{q^*(s)}{q^*(\bar{s})}.
\]

But this is satisfied by my definition of prices (3.36) in the theorem.

If there exists a group of risk-neutral agents \( i \in I \) with \( u_i(c) = a; c \), their maximization problem is, after substituting the definition of taxes \( \{t_i^*(\theta, s)\} \) and prices \( \{q^*(s)\} \) from (3.35) and (3.36),

\[
\max_{\{\Delta_i(s)\}} \sum_s \sum_{\theta} \alpha_i \left(y_i(\theta, s) - T_i^*(\theta, s)\right) p_i(\theta|\bar{a}_i, s)\pi(s) + \sum_s \pi(s)\rho_i(s)\Delta_i(s)
\]

subject to

\[
\sum_s \pi(s)\rho_i(s)\Delta_i(s) \leq 0.
\]

For any effort choice \( \bar{a}_i \), the risk-neutral agents are thus indifferent between any trading strategy that satisfies \( \sum_s \pi(s)\rho_i(s)\Delta_i(s) = 0 \) and therefore willing to choose \( \{\gamma_i(s)\} \) as well, completing the proof of step 1.

Step 2. Step 1 implies that, for any action choice \( \bar{a}_i \in A_i \), agents find it optimal to set \( \Delta_i(s) = \gamma_i(s) \forall s \in S \) and hence, by construction of the transfers \( \{T_i^*(\theta, s)\} \), to choose the constrained-efficient consumption schedule \( \{c_i^*(\theta, s)\} \). But then the fact that the constrained-efficient allocation \( \{c_i^*(\theta, s), a_i^*\} \) is incentive compatible and satisfies (3.2) for all \( \bar{a}_i \in A_i \) implies immediately that the action that agents choose is the constrained-efficient action \( a_i^* \forall i \in I \). This completes the proof.

3.7.4 Proof of Theorem 7

I start with showing that the transaction tax schedule defined in (3.42) is unique and continuous if \( u_i(c) \) is strictly increasing and continuous. To do so, note that, for any given trading strategy \( \Delta_i(s), (3.42) \) is equivalent to requiring that

\[
\sum_{\theta} u_i \left(c_i^*(\theta, s_0) - \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) - \sum_{s \neq s_0} q^*(s)\Delta_i(s)\right) p_i(\theta|a_i, s_0)\pi(s_0)
\]

\[
\leq W_i^* - \sum_{s \neq s_0} \sum_{\theta} u_i(c_i^*(\theta, s) + \Delta_i(s)) p_i(\theta|a_i, s)\pi(s) + v_i(a_i)
\] (3.54)
for all $a_i \in A_i$, with equality for some $a_i \in A_i$. Given the consumption schedule $\{c_i^*(\theta, s_0)\}$ for state $s_0$, I define

$$\tilde{W}_i(\Delta_i(s_0), a_i) = \sum_{\theta} u_i(c_i^*(\theta, s_0) + \Delta_i(s_0)) p_i(\theta|a_i, s_0) \pi(s_0).$$

The function $\tilde{W}_i(\Delta_i(s_0), a_i)$ is continuous and strictly increasing in $\Delta_i(s_0)$ by the assumed properties of $u_i(c)$. It is therefore invertible w.r.t. its first argument and the inverse function $\tilde{W}_i^{-1}(W_i, a_i)$ is continuous and strictly increasing in $W_i$. Using this to rewrite (3.54) yields the following explicit expression for the transaction tax schedule:

$$\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) = \max_{a_i \in A_i} \left\{ -\tilde{W}_i^{-1} \left( W_i^* - \sum_{s \neq s_0} \sum_{\theta} u_i(c_i^*(\theta, s) + \Delta_i(s)) p_i(\theta|a_i, s) \pi(s) + v_i(a_i), a_i \right) ight.$$

$$- \sum_{s \neq s_0} q^*(s) \Delta_i(s) \right\}. \quad (3.55)$$

This proves that there is a unique solution $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ to (3.42) for each trading strategy $\Delta_i(s)$. Moreover, $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ is defined as a maximization and the RHS of (3.55) is continuous in both $\Delta_i(s_1), ..., \Delta_i(s_H)$ (by continuity of $u_i(c)$ and $\tilde{W}_i^{-1}$ in its first argument) and in $a_i$ (since $a_i \in A_i$ and $A_i$ is a finite and thus discrete set). Berge’s Maximum Theorem therefore implies that $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ is a continuous function.

I next prove the second part of the theorem. The market clearing conditions (3.33) and (3.34) are satisfied by feasibility of $\{c_i^*(\theta, s), a_i^*\}$ and construction of $\{\gamma_i(s)\}$. It thus remains to be shown that given the prices $\{q^*(s)\}$, the transaction tax schedule $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ defined in (3.42) and the transfers $\{T_i(\theta, s)\}$ in (3.43), the solution to the agents’ problem (3.38) to (3.40) is given by $\{c_i^*(\theta, s), a_i^*\}$ and $\Delta_i(s) = \gamma_i(s) \forall i \in I, s \in S$.

To see this, observe that, by construction of the transfers $\{T_i(\theta, s)\}$ in (3.43) and the fact that the budget constraint (3.39) is binding at the optimum, the agent’s problem given prices $\{q^*(s)\}$ can be written as

$$\max_{\Delta_i(s), a_i} \sum_s \sum_{\theta} u_i(c_i^*(\theta, s) - \gamma_i(s) + \Delta_i(s)) p_i(\theta|a_i, s) \pi(s) - v_i(a_i)$$

subject to

$$\Delta_i(s_0) + \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) + \sum_{s \neq s_0} q^*(s) \Delta_i(s) = 0. \quad (3.57)$$

Otherwise, continuity in $a_i$ could be guaranteed by imposing continuity of $p_i(\theta|a_i, s)$ and $v_i(a_i)$ in $a_i$, $\forall \theta \in \Theta, s \in S$. 

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Substituting $\Delta_i(s_0)$ from (3.57) yields the following problem that agents solve:

$$
\max_{\Delta_i(s), a_i} \left\{ \sum_{\theta} u_i \left( c_i^*(\theta, s_0) - \gamma_i(s_0) - \kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H)) - \sum_{s \neq s_0} q^*(s)\Delta_i(s) \right) p_i(\theta|a_i, s_0) \pi(s_0) 
+ \sum_{s \neq s_0} \sum_{\theta} u_i(c_i^*(\theta, s) - \gamma_i(s) + \Delta_i(s))p_i(\theta|a_i, s)\pi(s) - v_i(a_i) \right\}.
$$

(3.58)

The construction of the transaction tax schedule $\kappa_i(\Delta_i(s_1), ..., \Delta_i(s_H))$ in (3.42) implies that all agents are indifferent between any trading strategy $\Delta_i(s)$ when they are able to choose their optimal action given $\Delta_i(s)$. By incentive compatibility of the allocation $\{c_i^*(\theta, s), a_i^*\}$, the maximum in (3.58) is therefore attained for all $i \in I$ by setting $\Delta_i(s) = \gamma_i(s) \forall s \in S$ and $a_i = a_i^*$, which produces expected utility $W_i^*$ as defined in (3.41) and completes the proof.
Bibliography


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