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Upper Bounds on Processing Loss for Wideband, Long-CPI Space-Time Adaptive Processing

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Abstract
The combination of synthetic aperture radar (SAR) and space-time adaptive processing (STAP) for moving target indication (MTI) radar applications allows the use of long, potentially sparse arrays, wide bandwidths, and long coherent processing intervals (CPIs), all of which enable detection of a greater variety of targets than is possible with traditional systems. In this paper, upper bounds on the signal-to-interference-plus-noise (SINR) loss are derived for post-SAR processing in the presence of three types of impairments: internal clutter motion, volumetric clutter, and antenna backlobes. These bounds are important for both the design and assessment of this type of MTI system, as impairments such as these can influence the entire system architecture.

1. Introduction
Space-time adaptive processing (STAP) has long been applied to multi-channel radar systems for the purpose of ground (or surface) moving target indication (MTI) [4,6,7]. In such systems, an antenna array is deployed along the direction of motion of a moving platform, such as an aircraft. Typically, the signal of a moving target is masked in any single channel by the presence of clutter at a variety of Doppler frequencies, due to the platform motion. By combining the returns of several channels with STAP, the moving target signal may be separated from that of the clutter.

The subject of this paper is a variant of STAP, introduced in [8], that is appropriate for systems employing wideband waveforms, long physical arrays that may be sparse, and long coherent processing intervals (CPIs). In this limit of radar system parameters, a judicious organization of the STAP processing chain is to form registered synthetic aperture radar (SAR) images across the multiple channels, and combine them adaptively to detect moving targets. This post-SAR form of STAP may be viewed as an adaptive extension of the displaced phase center array (DPCA) technique for canceling clutter, as it shares the concept of viewing the clutter from identical points with different elements.

Of the many possible performance metrics that can be applied to STAP, this paper will focus on signal-to-interference-plus-noise (SINR) loss, or processing loss, defined as the ratio of SINR to SNR in the absence of clutter, with both measured at the output of the detector. While the ultimate performance of a MTI system may be better captured by other metrics such as the receiver operating characteristic (ROC) curve, there are several reasons to consider SINR loss when designing and assessing a system.

SINR loss is a direct measure of how much of the target signal is lost due to clutter suppression. A value near unity demonstrates that the system performs near optimally, while a low value for targets of interest is an indicator of a poorly designed system, in that potentially available target signal power is being sacrificed. Furthermore, SINR loss is easily measured from experimental data and is independent of most details of the clutter, beyond its covariance. The detector ROC curve, by contrast, depends strongly on the form of the clutter statistics and other details, rendering it much less useful as a tool for system assessment.

Upper bounds on SINR loss are of evident utility in system design and assessment. While there are limits in which closed form expressions for $L_{SINR}$ can be derived, the expressions are less readily evaluated when various realistic impairments are introduced. One solution is numerical simulation, but the radar parameters considered in this paper incur a large computational burden. Instead, closed form expressions are derived for the frequency space clutter co-
variance, in the presence of a variety of impairments, that reduce the computation of SINR loss to a one- or two-dimensional quadrature. In addition to being easily evaluated numerically, these expressions provide great insight into the nature of the various impairments considered.

2. Post-SAR STAP

Post-SAR STAP, first proposed in [8], involves fixing a synthetic aperture in space, along the flight path, and processing only pulses from an element that are launched inside that synthetic aperture. Co-registered SAR images are formed for each channel, and adaptive processing is applied to the resulting multi-channel image.

A simple example will serve to clarify this variant of STAP and set the stage for the more involved calculations to follow. The detector assumed throughout will be the adaptive matched filter (AMF), defined as

$$\text{AMF}(x) = \left| v^\dagger_k R^{-1}_{c+n} z \right|^2,$$

(1)

where the data vector $z$ consists of one or more SAR pixels from each channel, $v_k$ is the target steering vector, and $R_{c+n}$ is the clutter plus noise covariance matrix. It is easily seen that the SINR loss is given by

$$L_{\text{SINR}} = \frac{v_k^\dagger R^{-1}_{c+n} v_k}{v_k^\dagger R^{-1} v_k}$$

(2)

where $R_n$ is the pure noise covariance. We assume that the clutter field is confined to the ground plane, temporally static, and spatially white, i.e.

$$\mathbb{E} \left( \alpha_c(x_g) \alpha_c^*(x'_g) \right) = \sigma^2_c \delta(x_g - x'_g),$$

(3)

where $\alpha_c$ is the complex scattering amplitude density of the clutter. If we further assume that the antenna array is perfectly aligned with the aircraft flight path, then the SAR image formed from each element will be identical. Neglecting SAR image sidelobes, the multi-channel clutter plus noise covariance is well approximated by

$$R_{c+n} = \sigma^2_{c,p} I \otimes 11^\dagger + \sigma^2_{n,p} I \otimes I,$$

(4)

where $\sigma^2_{c,p}$ and $\sigma^2_{n,p}$ are the mean single-channel clutter and noise power per pixel, the leftmost operator in the tensor product operates on pixel space, and the vector $1 = (1, \ldots, 1)^\dagger$ is a $N_{\text{ch}}$-dimensional channel space vector. With this expression, the SINR loss from (2) can be seen to be

$$L_{\text{SINR}} = 1 - (N_{\text{ch}} + \text{CNR}^{-1})^{-1} \left\| \sum_{n=1}^{N_{\text{ch}}} q^{(n)}_t \right\|^2 / \left\| \sum_{n=1}^{N_{\text{ch}}} \left\| q^{(n)}_t \right\|^2 \right\|^2,$$

(5)

where $\text{CNR} = \sigma^2_{c,p} / \sigma^2_{n,p}$, $q^{(n)}_t$ is the target response (SAR image) in channel $n$, and $\| \cdot \|$ is the pixel-space norm of a SAR image.

The purpose of this paper is to derive expressions for $L_{\text{SINR}}$ that generalize (5) to less ideal imaging conditions that cause additional loss. The impairments considered are models of internal clutter motion (ICM), which includes the effect of wind on clutter, volumetric clutter, and the presence of backlobes. While closed form expressions for $L_{\text{SINR}}$ will no longer be available, it will be shown that the calculation can be reduced to a one- or two-dimensional quadrature that is easily implemented.

3. Polar Format Algorithm

The multi-channel images that are fed into the adaptive processor are co-registered SAR images. The choice of the polar format algorithm (PFA) [3, 5] for analyzing SAR image formation leads to easily evaluated expressions, but imposes two constraints on the range of validity of the results. First, the focus area must simultaneously include the target images of all channels. Because moving targets may be blurred in SAR images, the targets considered must not have movement that results in blurred responses that exceed the area of focus. This is not a severe restriction for typical ground targets and airborne radar systems. Secondly, as discussed below, the image coordinate systems for the various channels may differ in the presence of array pitch or crab. The result mis-registration can bias the results. This is not expected to be an issue when the array is confined to a single platform, but could be for multi-platform systems.

Many details on the PFA may be found in [3, 5]. We will assume a flat ground plane on which the image is formed. The PFA image of a point target located at position $x$ close to the chosen aimpoint $x_0$ is the Fourier transform of a pure phase ramp:

$$\hat{I}(k_g; x) = \exp \left(-i k_g^0 (z_g(x) - z_g(x_0)) \right).$$

(6)

Several coordinate systems come into play in (6). The ground plane wave vector $k_g$ is in a Cartesian system that depends on the aimpoint; its relationship to the waveform frequency and element location, along with its region of support $\Omega_\zeta$, is explained below. The point target $x$ and aimpoint $x_0$, both in $\mathbb{R}^3$, are in a fixed Cartesian system, and $z_g(x)$ is a coordinate transformation to ground range-Doppler coordinates, also explained below.

The definition of the ground plane wave vector $k_g = (k_x, k_y)$ can be understood by referencing Figure 1. We express $k_g$ in the ground plane Cartesian coordinate system defined by the unit vectors $\hat{k}_x$ and $\hat{k}_y$. A vector $k_g$ is the orthogonal projection (in $\mathbb{R}^3$) of a slant plane wave vector $k_s = (k_u, k_v, k_p)$, which is expressed in coordinates defined by $\hat{k}_u$ and $\hat{k}_v$. This orthogonal projection induces a linear transformation $P_g$ between the two image planes given by
In each channel, there is a mapping \( \xi(\mathbf{k}_g) \) from a ground plane wave vector \( \mathbf{k}_g \) to the \( x \)-coordinate of the element at the time the corresponding pulse was launched, given by

\[
\xi(\mathbf{k}_g) = x_0 - \frac{k_g \cos \theta_0 + k_u \sin \theta_0}{k_g \sin \theta_0 - k_u \cos \theta_0} (y_0 - y_{ch})
\]

where the aimpoint is \( x_g = (x_0, y_0), \theta_0 = \arctan(y_0/x_0) \), and \( y_{ch} \) is the \( y \)-component of the synthetic aperture of the channel.

4. SINR Loss with Impairments

4.1. Sources of Loss

Any departure from a perfect match of the clutter images between channels is potentially a source of loss. In this paper, we are concerned with loss due to the geometry of the array as it relates to properties of the clutter field. In particular, we will assume that the antennas and receivers are perfectly matched. In practice, there will always be some loss due to mismatch in these components, some of which can be mitigated by calibration, and some by adaptive processing. It is the additional loss that cannot be eliminated by these techniques that we address.

We will compute the loss due to the combination of three effects: internal clutter motion, antenna backlobes, and volumetric clutter. The first of these causes loss for any array geometry. The other two cause essentially no loss if the array is perfectly aligned with the flight path, and the latter is perfectly straight. Loss may result if either of these conditions is violated. We will retain the assumption of straight flight path throughout, but relax the assumption of alignment of the array, and allow arbitrary pitch and roll.

4.2. Internal Clutter Motion

The simplest of our three impairments to handle is that of internal clutter motion, which is modeled by adding an autocorrelation term to the clutter model (3):

\[
\mathbb{E} \left( \alpha_c(x_g, t)\alpha_c^*(x_{g}', t') \right) = \sigma_c^2 \delta(x_g - x_{g}') A(t - t').
\]

In this section, we continue to assume that the clutter field is confined to the ground plane. In (11), \( x_g \) and \( x_{g}' \) are, as before, points on the ground plane, and \( t \) and \( t' \) are the times at which the clutter is illuminated by the sensor. The autocorrelation term \( A(t - t') \) is arbitrary for our analysis; a common choice for windblown natural clutter is Billingsley’s model [2].

We will assume throughout that the platform moves at constant velocity \( \mathbf{v}_a \), with speed \( v_a \). In this section only, we will continue to assume that the array is perfectly aligned.
with the flight path, so it is characterized by the distances \(d_m\) between element \(m\) and the first element (so \(d_1 = 0\)). Using (6), the SAR image of clutter in channel \(m\) is given by

\[
\hat{I}_m(k_g) = \int dz_g \alpha_c(z_g, \tau_m(\xi_m(k_g))) e^{-i k_g^\dagger (z_g - z_0)},
\]

(12)

where the integral is over the ground plane, \(z_g\) is in ground range-Doppler coordinates, \(z_0\) is the aimpoint in these coordinates, and \(\tau_m(\xi)\) is the time at which the \(x\)-coordinate of element \(m\) reaches \(\xi\). With (12) we may immediately find the clutter covariance in ground plane wave vector space:

\[
R_c(m, k_g; n, l_g) = \sigma_c^2 A(\tau_m(\xi_m(k_g)) - \tau_n(\xi_n(l_g))) \times \int dz_g e^{-i(k_g - k_n)(z_g - z_0)} = 4\pi^2 \sigma_c^2 A_{m,n} \delta(k_g - l_g). \tag{13}
\]

Two comments about (13) are in order. First, the Dirac \(\delta\)-function in the final line is actually band-limited to the area of focus of the PFA around the aimpoint \(z_0\), as it is only in that region that the expression (6) is valid. The clutter outside this region may be safely neglected; although it is unfocused, it remains localized away from the region of focus in the SAR images. Second, we have used the fact that the autocorrelation matrix \(A_{m,n}\) is actually independent of \(k_g\) and \(l_g\) in this case. This can be seen by observing that \(\xi_m(k_g) = \xi_n(k_g)\) for all pairs of elements \((m, n)\), and that for any \(\xi\), \(\tau_m(\xi) - \tau_n(\xi) = (d_m - d_n)/v_a\), leading to the definition

\[
A_{m,n} = A \left( \frac{d_m - d_n}{v_a} \right). \tag{14}
\]

In addition to the clutter covariance, the other ingredient needed to compute SINR loss is the target steering vector, which is the multi-channel SAR image of a moving target as a function of ground plane wave vector. We will assume throughout that targets are confined to the ground plane, moving at constant velocity. The target position may be parameterized as a function of the \(x\)-component of element position \(\xi\) via \(x_{t,m}^{(m)}(\xi) = x_{t,0}^{(m)} + \xi \mathbf{u}_t\), where \(x_{t,0}^{(m)}\) is the target position when element \(m\) is at mid-aperture (\(\xi = 0\)), and \(\mathbf{u}_t\) is the target velocity normalized by the platform speed. The steering vector is then

\[
v_t(m, k_g) = -\frac{4\pi i f(k_g)}{c} \left( |x_{t,m}(\xi_m(k_g)) - p_m(\xi_m(k_g))| - |x_0 - p_m(\xi_m(k_g))| \right) \tag{15}
\]

Combining (2), (13), and (15), it follows that

\[
L_{\text{SINR}} = \frac{1}{N_{\text{ch}}|\Omega|} \text{Tr} \left( I + \frac{\sigma_c^2}{\sigma_n^2} A \right)^{-1} V \tag{16}
\]

where \(\Omega\) is the support of \(k_g\), explained below, and the elements of the matrix \(V\) may be written as

\[
V_{m,n} = \int d\Omega k_g \exp \left( -i k_g^\dagger \left( x_{t,0}^{(n)} - x_{t,0}^{(m)} \right) \right), \tag{17}
\]

which is just the SAR impulse response evaluated at \(x_{t,0}^{(m)} - x_{t,0}^{(n)}\). An expression similar to (16) was derived in [9] through a more involved stationary phase calculation in the context of back-projection SAR. The advantages of the PFA approach taken here are the simplicity of the derivation, which will be exploited in the various extensions of (16), and the fact that the PFA calculation naturally employs ground range-Doppler coordinates, which yield accurate results for apertures of any length.

If the integral in (17) is evaluated using polar coordinates for \(k_g\), the radial integral can be done analytically, leaving just the one-dimensional angular integral to be done numerically. The limits on the angular integral are determined by the synthetic aperture, while those on the radial integral are \((4\pi f_{\text{min}}, \text{max})/c \cos \theta_{\text{dep}}(k_g)\). Together, these limits determine the ground plane wave vector support region \(\Omega\).

4.3. Volumetric Clutter

When clutter returns are distributed over a volume instead of a plane, as is the case for forested regions [1], additional processing loss results above what would be observed due to planar clutter of the same CNR. This effect is due to layover, the artifact of SAR image image formation in which vertically displaced scatterers appear horizontally displaced in the image [5]. The effect can be summarized by the equation

\[
d = -h \frac{y_{ap}}{y_{ap} - y_{ap}} \hat{y}, \tag{18}
\]

valid for \(h \ll y_{ap}\), where the scatterer is located at a point \(x = x_{ap} + h\hat{z}\), and the aperture is defined by its intersection \((y_{ap}, z_{ap})\) with the \(y - z\) plane. An important point is that for perfectly linear flight paths focus is not affected by vertical displacement, considerably simplifying the volumetric clutter analysis.

If the planar clutter field defined by (11) is elevated to a height \(h\) above the ground plane, then due to layover the clutter covariance becomes

\[
R_c(m, k_g; n, l_g) = 4\pi^2 \sigma_c^2 A_{m,n}(k_g) \delta(k_g - l_g) \tag{19}
\]

where

\[
A_{m,n}(k_g; h) = A(\tau_m(\xi_m(k_g)) - \tau_n(\xi_n(k_g))) \times \exp \left( i h k_g^\dagger (t_m^{(l)} - t_n^{(l)}) \right) \tag{20}
\]
and \( t_m^{(l)} = \hat{y} z_m / y_0 \). In (19) we have replaced \( y_g - y_m \) from (18) with \( y_0 \), the \( y \)-component of the aimpoint, using the fact that the error incurred will be suppressed by a factor of \( 1/y_0 \), which is small for airborne systems.

Using (19) we may express the SINR loss for a planar clutter field elevated above the ground plane by a height \( h \) as

\[
L_{\text{SINR}} = \frac{1}{N_{\text{ch}}[\Omega]} \int_{\Omega} d_{k_g} v_t^\dagger(k_g) M(k_g) v_t(k_g)
\]

where the aperture is defined by its intersection point \((p_{y,m}, p_{z,m})\) with the \( y-z \) plane. The first term in (22) is the mis-registration caused by the geometry of the backlobe clutter field; the second is the layover term.

The calculation of the backlobe clutter covariance proceeds exactly as in Section 4.3, with the substitution \( t^{(l)} \rightarrow t^{(b)} \). As with the mainlobe volumetric clutter, the backlobe clutter layers may be integrated over. Because the main and backlobe fields are independent, the covariances add, and the SINR loss is again given by (21), but with the substitution

\[
M(k_g) = \left( I + \frac{\sigma_c^2}{\sigma_n^2} A(k_g) + \frac{\sigma_{c,\text{back}}^2}{\sigma_n^2} A^{(\text{back})}(k_g) \right)^{-1}
\]

5. Examples

The various effects calculated in this paper will be demonstrated through numerical evaluation using a set of assumed sensor parameters. We considered a 10 m uniform linear array (ULA) of 10 receive elements, with a single transmit element, mounted on a platform that moves at 150 m/s at a height of 6 km. The waveform was assumed to have a bandwidth of 50 MHz centered at 450 MHz. ICM effects were included via Billingsley’s model for windblown forest clutter [2]. The target is assumed to be imaged at broadside, at a ground range of 10 km, with a CNR of 35 dB. Although post-SAR STAP is sensitive to both components of velocity, and the methods of this paper accommodate arbitrary target motion and the associated blurring of the SAR response, our examples are focused on the radial velocity component with steering vectors parameterized by ground radial velocity.

Three cases are shown in Figure 2. The ideal case of perfect array alignment, planar clutter, and no wind, is compared to perfect array alignment, planar clutter, and 10 mph wind, and array misalignment of \( \phi = 20^\circ \), volumetric clutter of height 25 m, and no wind. The effect of windblown clutter is, as expected, to cause a spread in the SINR loss curves. Volumetric clutter combined with array misalignment has a similar effect, with rather large crab angles \( (\phi = 20^\circ) \) required to reach the magnitude of a 10 mph wind. This effect is also sensitive to array pitch.

The effect of backlobes, without wind or volumetric clutter, is shown in Figure 3. The backlobe CNR was taken to be 5 dB, which is 30 dB lower than that of the mainlobe. It can be shown that, with a fixed ULA misalignment, the backlobe clutter causes a sharp notch in SINR loss corresponding to a particular target velocity along the \( y \)-axis. The case shown was a crab angle of \( \phi = 2^\circ \), which corresponds to a ground speed about 10.5 mps. Also shown is the combined effect of backlobes, with the same 2\(^\circ\) misalignment, with a wind of 10 mph and volumetric clutter height of 25 m. Most of the resulting loss is due to the wind, as volumetric clutter contributes little loss for small misalignment angles.
6. Conclusion

Post-SAR processing is a natural extension of standard post-Doppler STAP that enables long, sparse arrays, wide bandwidths, and long CPIs, all of which can enable detection of a greater variety of targets than is possible with traditional systems. The ability to compute SINR loss for such systems under realistic imaging scenarios is of fundamental importance at both the design and experimental assessment stages.

In this paper, three sources of additional loss were considered: that due to internal clutter motion, as modeled by a temporal autocorrelation function, volumetric clutter, and antenna backlobes. The results were obtained through application of the polar format algorithm, which leads to simple, closed form expressions of the clutter covariance in ground plane wave vector space. These expressions, in turn, lead to forms for the SINR loss that are easily evaluated by one- or two-dimensional numerical quadrature.

The simplicity of the derivations in this paper was due to the fact that we were able to evaluate all of the relevant clutter fields in the vicinity of the target image with the use of a single aimpoint for motion compensation. The use of a single aimpoint, in turn, was driven by the assumption of a perfectly straight flight path. It is known that out-of-plane clutter is defocused when this assumption is relaxed [5]. It is also the case that another important source of loss, that of ambiguous range returns, results from out-of-focus returns near the target image. A more involved calculation is necessary to compute SINR loss in these more general situations, which will be explained in a separate work.

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References