Nutation in the Spinning SPHERES Spacecraft and Fluid Slosh

by

Caley Ann Burke

Submitted to the Department of Aeronautics and Astronautics in partial fulfillment of the requirements for the degree of Masters of Science in Aeronautics and Astronautics at the MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2010

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Abstract

Spacecraft today are often spin-stabilized during a portion their launch or mission. Though the basics of spin stabilization are well understood, there remains uncertainty in predicting the likelihood of rapid nutation growth due to onboard liquids. Solely analytical methods of prediction are mainly unsuccessful and physical tests to gather slosh data have only been done for a few specific spacecraft. Data from past spacecraft is subject to a number complex physical factors and anomalies during the launch or mission.

This study verifies a ground based method to test fluid tanks horizontally and obtain the first fundamental frequency of the tank. Horizontal tanks have the gravitational acceleration vector applied in the same direction as the acceleration experienced by an offset tank on a spinning spacecraft.

The study also performs tests on the Synchronized Position-Hold, Engage, Reorient Experimental Satellites (SPHERES) satellites to characterize their nutation. In the tests, the satellite is spun about a single axis and then allowed to drift. Each principal axis is tested by at least one test. Two configurations of the satellite are tested: the satellite by itself and the satellite with an additional rigid mass attached to alter the inertia matrix of the system.

These two efforts can be combined in the future to perform spinning slosh testing on the SPHERES satellites, with knowledge of the frequency of the fluid tanks. The potential for the SPHERES Testbed to add to the generic fluid slosh data is due to it having a relatively simple spacecraft system capable of both software and hardware modifications and being located in the visually observable microgravity environment of the International Space Station (ISS).

Thesis Supervisor: David W. Miller

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Acknowledgements

NASA - Kennedy Space Center (KSC) for providing the funding for this research via the Kennedy Graduate Fellowship Program.

NASA Launch Services Program at KSC for encouraging my pursuit of a graduate degree and academic research. In particular, James Sudermann, for our many discussions relating to this research.

Dr. David Miller for guidance throughout this study.

The SPHERES team for working with me to perform the research on the Testbed and my fellow SSLers for being there with me through this process.

My family and friends for all their support of me in this endeavor.
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# Nomenclature

## Acronyms

<table>
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<th>Description</th>
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<tbody>
<tr>
<td>3DOF</td>
<td>Three Degrees of Freedom</td>
</tr>
<tr>
<td>6DOF</td>
<td>Six Degrees of Freedom</td>
</tr>
<tr>
<td>CAD</td>
<td>Computer-Aided Design</td>
</tr>
<tr>
<td>CM</td>
<td>Center of Mass</td>
</tr>
<tr>
<td>CO₂</td>
<td>Carbon Dioxide</td>
</tr>
<tr>
<td>GC</td>
<td>Geometric Center</td>
</tr>
<tr>
<td>IMU</td>
<td>Inertial Measurement Unit</td>
</tr>
<tr>
<td>ISS</td>
<td>International Space Station</td>
</tr>
<tr>
<td>KSC</td>
<td>Kennedy Space Center</td>
</tr>
<tr>
<td>LOS</td>
<td>Loss of Signal</td>
</tr>
<tr>
<td>LV</td>
<td>Launch Vehicle</td>
</tr>
<tr>
<td>MAMS</td>
<td>Microgravity Acceleration Measurement System</td>
</tr>
<tr>
<td>MESSENGER</td>
<td><strong>Mercury Surface, Space ENvironment, GEochemistry, and Ranging</strong></td>
</tr>
<tr>
<td>MIT</td>
<td>Massachusetts Institute of Technology</td>
</tr>
<tr>
<td>MOI</td>
<td>Moment of Inertia</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>NTC</td>
<td>Nutation Time Constant</td>
</tr>
<tr>
<td>PMDs</td>
<td>Propellant Management Devices</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
</tr>
<tr>
<td>S/C</td>
<td>Spacecraft</td>
</tr>
<tr>
<td>SN</td>
<td>SPHERES Number</td>
</tr>
<tr>
<td>SPHERES</td>
<td>Synchronized Position-Hold, Engage, Reorient Experimental Satellites</td>
</tr>
<tr>
<td>SSL</td>
<td>Space Systems Laboratory</td>
</tr>
<tr>
<td>SwRI</td>
<td>Southwest Research Institute</td>
</tr>
<tr>
<td>TS</td>
<td>Test Session</td>
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</table>
Mathematical Variables

1-g  sea level gravity
a  acceleration
A  surface area
Bo  Bond number
Cd  coefficient of drag
cm  centimeter
d  depth
E  rotational kinetic energy
F  force
f  frequency
F_d  drag force
g  gravitational acceleration
h  height
h  height of liquid within the container
Hz  Hertz (1/s)
I  inertia
i  imaginary number (square root of 1)
km  kilometer
L  angular momentum vector
L  characteristic length scale
l  bottle length
M  force moment
m  meters
mL  milliliter
N  length of signal in time
n  nth parameter of that variable
P  principal axis
psi  pressure per square inch
R  radius of the cylinder
R  distance of from the center of mass of a system to the center of
mass of the fluid container
rad  radians
s  seconds
t  time
v  velocity
w  width
x  signal of length N
z  nutation motion
\gamma_n  nth fluid frequency parameter
<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>(\theta)</td>
<td>nutation angle</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>nutation frequency</td>
</tr>
<tr>
<td>(\rho)</td>
<td>density</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>surface tension of the interface</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>sigma (distribution)</td>
</tr>
<tr>
<td>(\tau)</td>
<td>nutation time constant (NTC)</td>
</tr>
<tr>
<td>(\tau)</td>
<td>torque</td>
</tr>
<tr>
<td>(\omega)</td>
<td>angular acceleration</td>
</tr>
<tr>
<td>(\dot{\omega})</td>
<td>angular velocity</td>
</tr>
<tr>
<td>(\omega)</td>
<td>instantaneous rotation axis</td>
</tr>
<tr>
<td>(\Omega)</td>
<td>rotation rate</td>
</tr>
<tr>
<td>(\omega)</td>
<td>angular frequency</td>
</tr>
<tr>
<td>(\omega_1)</td>
<td>first longitudinal natural frequency</td>
</tr>
<tr>
<td>(\omega_N)</td>
<td>Nth root of 1</td>
</tr>
</tbody>
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Chapter 1

Introduction

This chapter contains the motivation of the work achieved in this thesis. It reviews the literature for previous work performed. It summarizes the SPHERES Testbed, on which some of the experiments in this thesis were executed. It concludes with the approach of the study of nutation of spinning spacecraft and fluid slosh.

1.1 Motivation

Propellant fluid slosh in spacecraft can couple into the dynamics of the vehicle leading to performance degradation in the propulsion and attitude stabilization sub-systems. Spacecraft are frequently rotated for several reasons: to provide gyroscopic stability; and to allow sensors or other hardware on the spacecraft to rotate. The rotation for stability often required on spacecraft with solid propellant upper stages. The rotation for sensors/hardware may be either for thermal reasons, for field of view purpose, or other reasons specified by the scientific mission. Spacecraft tanks are often offset from
this spin axis so that the rotation induces a local acceleration that forces the fluid into one side of the tank. This acceleration also provides an effective “stiffness” to the fluid’s motion which, when coupled with the fluid’s inertia, results in slosh. This oscillatory fluid motion then couples with the motion of the vehicle.

Understanding the dynamics of this fluid slosh is essential to the design of mitigation techniques such as attitude control, fluid baffles, etc. Of particular interest are conditions leading to resonance between the nutation motion and liquid modes, which can result in rapid nutation growth. Rapid growth nutation can lead to a loss of control of the spacecraft; a state where either the control system is either unable regain control of spacecraft attitude or the control system has used most to all of its system’s fluids overcoming the nutation, leaving minimal or no fluids available to perform the mission. Tanks with cylindrical sections have been shown particularly susceptible to this.

1.2 Literature Review

Purely analytical methods of predicting the effect of fluid slosh in microgravity have had very limited success at this point. Capturing this understanding through the use of dynamic models requires test data in order to accurately calibrate the models and ensure that all of the important physics involved are properly captured. Since rotation rates can differ, and these spacecraft are operating in the microgravity of space, it is important to obtain test data at different acceleration levels. However, some of these
acceleration levels are less than 1-g and therefore difficult to create the reduced gravity environment for long durations in a terrestrial laboratory.

There has been significant fluid slosh work performed today. Much of the work takes place on the ground. [1] There is also extensive knowledge of how to analytically predict the slosh of spheres and vertical cylinders. However, cylinders with the acceleration applied horizontally are significantly more difficult to model and require testing, once the tank changes beyond being a simple cylinder filled halfway. [2] [3] [4] [5]

In spacecraft where rapid nutation growth is of high concern, physical modeling of the tanks or spacecraft is often conducted. However, due to constraints, these tests are often conducted either in the 1-g or simulated microgravity via a drop tower on the miniature scale (capable of only ~2-3 seconds data per run). Additionally, the tests are conducted for highly complex spacecraft that using a variety of tanks, system configurations, and going to a variety of locations throughout the solar system. The technical community is lacking global, non-mission specific data regarding fluid slosh in microgravity. [6]

1.3 SPHERES Testbed

The Synchronized Position-Hold, Engage, Reorient Experimental Satellites (SPHERES) Testbed was developed and is maintained by the Space Systems Laboratory (SSL) at Massachusetts Institute of Technology (MIT). The testbed consists of six degrees
of freedom (6DOF) microsatellites capable of communicating with each other and the computer. [7] [8]

The satellites are each approximately 4 kg and have a diameter of 20 cm (about the size of a basketball).

![Figure 1.1 SPHERES Satellite (with battery pack attached to left hand side) on the ISS][9]

At the time of the testing described within this paper, there were two sets of three SPHERES satellites, located in two places: the SSL and the U.S. Laboratory of the International Space Station (ISS). The SPHERES Testbed in the SSL is operated with three degrees of freedom (3DOF) in a 1-g environment. The SPHERES Testbed on the ISS is operated at a 6DOF in a microgravity environment.
Testing on the ISS is performed as a voluntary science, which occurs as allowed by astronauts’ schedules. Multiple tests are grouped together and performed on the same day to form a Test Session (TS).

1.4 Approach

This thesis focuses on the nutation of spinning spacecraft in microgravity with fluid slosh. The test data analyzed divides into two sections: horizontal fluid slosh tests performed in 1-g and nutation testing of the SPHERES satellites in microgravity. The fluid slosh microgravity data gathered previously has been on functional spacecraft. This study intends to use the SPHERES Testbed, to gather data in microgravity for a relatively simple spacecraft system and be able to perform maneuvers specifically designed with slosh in mind. The system is visibly observable via video cameras and in person (an astronaut) and is able to be reset, both software wise and physically.

The approach to the experiments discussed within this thesis is as follows:

1. Horizontal fluid slosh experiments performed on the ground

   - Develop method of determining the fundamental frequency of a container via a test in the lab in 1-g
   - Perform the method on containers with known fundamental frequencies
     - Multiple containers
     - Multiple fill fractions of fluid (ratio of liquid in container to volume of the container)
• Verify that the method achieved the expected fundamental frequencies
• Perform the method on the fluid tank(s) to be used in microgravity testing [future work]

2. Nutation testing on the ISS

• Develop a test to be run on a SPHERES satellite that can measure the nutation of a spinning spacecraft
• Perform the test on the ground
• Perform the test on the ISS for multiple configurations
  • SPHERES satellite
  • SPHERES satellite with a rigid mass attached
• Verify the test performed as expected
• Determine the mass properties of the SPHERES satellite
• Perform the test on the ISS [future work]
  • SPHERES satellite with fluid tank

The thesis covers the nutation of spacecraft (Chapter 2), the setup of the tests (Chapter 3), the performance and data of the tests (Chapter 4), and the analysis of the test data (Chapter 5).
Chapter 2

Building a Model of the Problem (generic)

This chapter derives the equation for nutation frequency from the Euler Equations of Motion, which can be applied generically to a rigid body rotating about a single principal axis in microgravity. It then discusses how that nutation can be calculated, why knowledge of nutation is important in a spacecraft system and how it can interact with fluid slosh.

2.1 Euler Equations of Motion

The Euler Equations of Motion are three differential equations relating force moments \( (M) \), angular velocities \( (\omega) \), and angular accelerations \( (\dot{\omega}) \) of a rotating rigid body:

\[
\begin{align*}
I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 &= M_1 \\
I_3\dot{\omega}_3 + (I_1 - I_3)\omega_3\omega_1 &= M_2 \\
I_2\dot{\omega}_2 + (I_2 - I_1)\omega_1\omega_2 &= M_3
\end{align*}
\]

The reference body fixed axes are assumed to be the principal axes of inertia. [10]
The Euler equations will be applied to the generic system with several assumptions:

1. The force moments exerted on the system are zero. \( M_{1,2,3} = 0 \)

2. The angular velocity for each axis consists of a set rotation rate \( \Omega_i \) and incremental variance in that rotation rate \( \omega_i \). The angular acceleration is only applied to the variance in the rotation rate.

\[
\omega_i \Rightarrow \Omega_i + \omega_i
\]

3. The set rotations for the other two principal axes are zero. \( \Omega_{2,3} = 0 \)

4. The set rotation rate for the first principal axis is to be significantly larger than the variance in any three of the rotations. \( \Omega_i \gg \omega_{1,2,3} \)

5. The value of any variances in rotation values times another variance is zero (i.e. remove nonlinearities from the system).

\[
(\omega_1)(\omega_2) = (\omega_1)(\omega_3) = (\omega_2)(\omega_3) = 0
\]

As the first two of these assumptions are applied to the Euler Equations, Equation 2.1 becomes:

\[
\begin{align*}
I_1 \dot{\omega}_1 + (I_1 - I_2) (\Omega_3 + \omega_3) (\Omega_3 + \omega_3) &= 0 \\
I_2 \dot{\omega}_2 + (I_1 - I_3) (\Omega_2 + \omega_2) (\Omega_2 + \omega_2) &= 0 \\
I_3 \dot{\omega}_3 + (I_2 - I_1) (\Omega_1 + \omega_1) (\Omega_1 + \omega_1) &= 0
\end{align*}
\]

With the third assumption, Equation 2.4 becomes:

\[
\begin{align*}
I_1 \dot{\omega}_1 + (I_3 - I_2) \omega_2 \omega_3 &= 0 \\
I_2 \dot{\omega}_2 + (I_1 - I_3) \omega_1 (\Omega_1 + \omega_1) &= 0 \\
I_3 \dot{\omega}_3 + (I_2 - I_1) (\Omega_2 + \omega_2) \omega_2 &= 0
\end{align*}
\]

With the final two assumptions, Equation 2.5 becomes:
\[ I_1 \dot{\omega}_1 = 0 \]
\[ I_2 \dot{\omega}_2 + (I_1 - I_3) \omega_3 \Omega_1 = 0 \]
\[ I_3 \dot{\omega}_3 + (I_2 - I_1) \Omega_1 \omega_2 = 0 \]

By taking the Fourier transform of Equation 2.6, the equations are converted to state space. The response is assumed to be oscillatory and to correspond to the frequency of the variance in the rotation rates.

\[ s = i \lambda \]
\[ s^2 = -\lambda^2 \]
\[ sI_1 \omega_1 = 0 \]
\[ sI_2 \omega_2 + (I_1 - I_3) \omega_3 \Omega_1 = 0 \]
\[ sI_3 \omega_3 + (I_2 - I_1) \Omega_1 \omega_2 = 0 \]

Equation 2.7 can be manipulated to

\[ \omega_2 = \frac{(I_3 - I_1) \omega_3 \Omega_1}{sI_2} \]
\[ \omega_3 = \frac{(I_1 - I_2) \Omega_1 \omega_2}{sI_3} = \frac{(I_1 - I_2) \Omega_1}{sI_3} \frac{(I_3 - I_1) \omega_3 \Omega_1}{sI_2} \]
\[ \Rightarrow s^2 I_2 I_3 + (I_1 - I_2)(I_3 - I_1) \Omega_1^2 = 0 \]

By moving out of state space and solving for the variance frequency, Equation 2.8 becomes

\[ \lambda = \pm \Omega_1 \sqrt{\frac{(I_1 - I_2)(I_3 - I_1)}{I_2 I_3}} \]

This variance frequency is known as the nutation frequency (\( \lambda \)).

### 2.2 Nutation Prediction

Nutation references in this study will be defined in terms of common spacecraft usage. This differs from the classic mechanics description, which involves a prominent gravitational field causing vertical wobble in a spin axis [11].
2.2.1 Nutation Definition

Nutation is rotational motion for which the instantaneous rotation axis is not aligned with a principal axis. There are three vectors involved in nutation:

$L$: the angular momentum vector. It is fixed in inertial space.

$\hat{p}$: the principal axis which is rotating about $L$. It is fixed to the spacecraft, due to inertia properties.

$\omega$: instantaneous rotation axis. It rotates about both inertial space and the spacecraft.

![Figure 2.1(a) Nutation Motion (Z is a geometric axis) (b) Nutation Angle [11] [6]]

The angle between the principal axis and angular momentum vector is called the nutation angle ($\theta$); it measures the magnitude of nutation. Given the following conditions, the nutation angle will ideally remain constant: the other two of the principal moments of inertia are equal, the nutation angle is small, there are no external forces/torques on the system, and there is no energy dissipation in the system. The movement of $\omega$ follows the intersection of a space cone and body cone, as shown in Figure 2.2.
The body cone rolls on the space cone for (a) $I_1 = I_2 < I_3$ (b) $I_1 = I_2 < I_3$ [11]

Once the secondary principal moments of inertia are no longer equal, cone cross-sections become elliptical instead of circular and the nutation angle loses the possibility of remaining constant.

For any spacecraft with its principal moments of inertia such that $I_1 < I_2 < I_3$, the spacecraft may spin stably about either $I_3$ (the major axis) or $I_1$ (the minor axis). In either case, small perturbations produce bounded deviations from the nominal state. In cases
where $I_1 \approx I_2$, major axis spin is commonly referred to as Frisbee spin. In cases where $I_3 \approx I_2$, minor axis spin is commonly referred to as pencil spin.

However, spin about $I_2$ (the intermediate axis) is unstable and small perturbations produce unbounded deviations from the nominal state. The nutation frequency is always an imaginary value. The axis of rotation should not be the intermediate axis.

### 2.2.2 Excitation of Nutation

Torques can affect the nutation of the system. The type of torque and how it is applied determines the effect.

External torques on the system come in two main forms: disturbance torques from the environment (i.e. drag) and control torques from attitude control (i.e. gas jets). If the torque is applied parallel/anti-parallel to the angular momentum vector, the magnitude of the angular momentum will increase/decrease. A torque applied perpendicular to the angular momentum vector will change the direction of the vector along with its magnitude, which is called precession.

Internal torques cannot change the value of the angular momentum vector in inertial space. However, they can affect angular momentum vector in relation to the spacecraft frame. Possible causes of energy dissipation due to internal forces in spacecraft are the system (damping by the structure), spacecraft components (including fluids), and nutation damping hardware.

If internal torques cause energy dissipation, the rotational kinetic energy of the spacecraft will decrease. Equation 2.10 shows how the rotational kinetic energy ($E$)
relates to the spacecraft principal moments of inertia \( (I) \) and those axes' corresponding angular velocity \( (\omega) \) and momentum vector components \( (L) \) in a rigid spacecraft [11].

\[
E_k = \frac{1}{2} \left( I_1 \omega_1^2 + I_2 \omega_2^2 + I_3 \omega_3^2 \right) = \frac{1}{2} \left( \frac{L_1^2}{I_1} + \frac{L_2^2}{I_2} + \frac{L_3^2}{I_3} \right)
\]

For a constant value of \( L \), rotational kinetic energy decreases by transferring the angular momentum to its component corresponding to the largest principal moment of inertial (major principal axis). Once rotational kinetic energy is at its minimum, the energy dissipation ceases and the angular momentum vector is aligned with the major principal axis.

In cases where the intended axis of rotation is the major principal axis, this is known as nutation damping and excess kinetic energy is removed from the system. In cases where the intended axis of rotation is not the major principal axis, this could be disastrous for the spacecraft.

Rapid nutation growth can lead to loss of control. This may cause the spacecraft to use excess propellant to control the spacecraft, which shortens the life of the spacecraft. It could also result in loss of the spacecraft or mission. The intent of spin stabilization is usually to give gyroscopic stability. If an object spinning about a minor axis has any flexibility, it will need to have nutation damping capabilities, either active or passive, on board.

### 2.2.3 Slosh

The energy dissipation that this study will focus on is fluid slosh, specifically propellant slosh. Propellant slosh “refers to free surface oscillations of a fluid in a
partially filled tank resulting from translational or angular acceleration of [the] spacecraft.” [11] The acceleration could be caused by the control system, flexibility in the spacecraft structure, or an environmental disturbance.

The oscillations are movement of the liquid propellant within the tank, caused by the balance of inertia and surface tension forces. The Bond Number ($Bo$) characterizes the ratio between acceleration ($a$) to surface tension forces, as seen in Equation 2.11. [12] The surface tension forces are defined by the density of the fluid ($\rho$), surface tension of the interface ($\sigma$) and the characteristic length scale ($L$). On a spinning spacecraft in microgravity, the acceleration input is the centrifugal acceleration and not gravity, since generally $\Omega^2 R >> g$ ($R$ is the distance from the spin axis to the free surface).

$$Bo = \frac{\rho a L^2}{\sigma} \tag{2.11}$$

For cases when $Bo << 1$, surface tensions dominate and the fluid free surface climbs the tank walls. As the Bond number decrease, so do the components of natural frequency.

The following trends have high risk for significant slosh energy: large propellant tanks (the fluid is a significant percentage of the total spacecraft [S/C] mass) and intermediate fill fraction (i.e. ~60%) [13]. Each trend can independently create significant slosh energy, but having both trends present is what really creates the high risk situation. A tank filled 100% with fluid does not allow motion of the location of the fluid within the tank and an empty tank has no fluid to impact force, which is why these fill fractions are not high risk conditions for slosh.
Since standard tank walls generally provide minimal damping effects, propellant management devices (PMD) are often placed within the tank to increase slosh damping. Though this study will not be addressing PMDs, some examples are: baffles, diaphragms, vanes.

2.2.4 Coupled Slosh and Nutation

By determining the Nutation Time Constant (NTC) early in the design, it can be used to determine if the current control system will be able to keep nutation under control or if additional PMDs or other changes are necessary. The accuracy of the NTC is important, as it is required as an input to the stability analysis for spinning launch vehicle upper stage or spacecraft flight.

The upper stages spin to provide gyroscopic stabilization during orbital transitions. Due to geometric constraints imposed by the launch vehicle, this spin is about the minimum moment of inertia. The flexibility in the spacecraft leads to energy dissipation, which then lead to instability. Fluid slosh is a source of flexibility and therefore is also a source of energy dissipations.

Coupled resonance occurs when the nutation frequency is at or near the liquid modal frequency. There is rapid energy dissipation and nutation change. The liquid modal frequency changes as the propellant is depleted, so all fill levels of the tank must be considered for coupled resonance.

Since the frequencies of both nutation and the slosh are proportional to spin rate (as discussed in relation to Equation 2.1 and later demonstrated in 5.1.1.2), coupled
resonance cannot be avoided in a system simply by altering the spin rate. However, it may be less severe at lower spin rates.

The calculation of the nutation angle \( \theta \) over time requires knowledge of \( \tau \), the NTC of the spacecraft. Equation 2.12 assumes that the initial nutation angle \( (\theta_0) \) is small (no more than a few degrees).

\[
\theta = \theta_0 e^{\tau/\tau}
\]

Equation 2.12

The NTC can be either positive or negative, depending on if nutation is growing or decaying (minor or major axis spin). When there is fluid on board or flexibility in the S/C, the NTC is very difficult to calculate, especially in early S/C design. Simulations of propellant slosh often utilize mechanical analogs, such as pendulums and rotors, to replace full fluid modeling. Analytical methods of predicting liquid frequencies are possible, but usually underestimate the effect on nutation growth rates. Analytical methods of obtaining resonant NTC are often off by an order or two of magnitude. [6]

The most accurate determination comes from flight or test data. Prior to flight, the NTC can be calculated through forced motion (spin table in Figure 2.4), drop tower (free-fall), ballistic trajectory, and air-bearing tests.
Through testing, groups can be determined for off-axis fluid tanks with variances in the height and diameter of tank, as slosh is not very sensitive to these factors. These groups are classified by nondimensional constants, such as the length to diameter ratio of a cylindrical tank. However, NTC is very sensitive to factors such as fill fraction, inertia ratio, tank shape, internal hardware (i.e. PMDs), and the tank location within the S/C and so these dimension/values must be constant within a group.

If the tank is off-set from the spin axis of the spacecraft, the slosh modes will most likely be higher than the nutation frequency and not cause coupled resonance; any surface modes resulting from slosh would be small. However, if the tank is in-line, there is the possibility of coupled resonance between the two frequencies and therefore large surface waves: nutation synchronous motion. This complex motion is a rapidly
increasing rotating dynamic imbalance, visible in the jump of nutation angle, and the corresponding NTC in such cases is uncontrollable. [6]

2.3 Nutation Determination Problem

Nutation determination is an involved and expensive process, so it is generally not performed without a specific need. Testing in microgravity and other methods of nutation determination are usually only performed for spacecraft where there is a concern about rapid nutation growth. These spacecraft are usually very complicated, so the data obtained is difficult to apply to other systems.

This study observed nutation in a simple spacecraft system in microgravity. By characterizing nutation without and without additional fluid masses, the effect of the fluid masses can be determined. The data from the simplified system is then transferable. This thesis contains data only for the nutation of the spacecraft system without additional fluid masses.
Chapter 3

Physical Environments of the Experiments

This chapter describes the environments of the experiments performed and follows with the design of the experiments: horizontal fluid slosh ground tests and SPHERES Nutation in Microgravity. The location and assumptions of the ground tests performed are stated. Then the properties of the SPHERES satellite are detailed; this includes masses that will be added to the satellite system (solid mass and fluid tanks). The disturbance torques encountered in the ISS by the satellite are discussed. Finally the design of each experiment is outlined.

3.1 Environment of the Horizontal Fluid Slosh Ground Tests

The ground tests were performed in an MIT lab. Average sea level atmospheric conditions and gravity acceleration are assumed.
3.2 Geometry of a SPHERES satellite

Figure 3.1 is a Computer-Aided Design (CAD) rendering of a SPHERES satellite, with the body axes labeled. Through the center of the satellite is a CO\textsubscript{2} propellant tank, which aligns with the Z-axis. The propellant tank aboard the SPHERES satellite contains nominally 172 g of CO\textsubscript{2} when the tank is “full”, which is when the liquid fill is 68% of the capacity of the tank. The SPHERES satellite is powered by two battery packs, which are accessed by two magnetic doors located behind the +Y and -Y faceplates.
Having twelve thrusters allows the satellite to have six degrees of freedom capability. Figure 3.2 shows the locations of each thruster on a two dimension map of a SPHERES satellite. Table 3.1 shows the direction of thrust for each thruster.

**Table 3.1 Thruster Geometry (in body coordinate frame) [8]**

<table>
<thead>
<tr>
<th>Thruster</th>
<th>Thruster position [cm]</th>
<th>Nominal force direction</th>
<th>Nominal torque direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x</td>
<td>y</td>
<td>z</td>
</tr>
<tr>
<td>0</td>
<td>-5.16</td>
<td>0</td>
<td>9.65</td>
</tr>
<tr>
<td>1</td>
<td>-5.16</td>
<td>0</td>
<td>-9.65</td>
</tr>
<tr>
<td>2</td>
<td>9.65</td>
<td>-5.16</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>-9.65</td>
<td>-5.16</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>9.65</td>
<td>-5.16</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>-9.65</td>
<td>-5.16</td>
</tr>
<tr>
<td>6</td>
<td>5.16</td>
<td>0</td>
<td>9.65</td>
</tr>
<tr>
<td>7</td>
<td>5.16</td>
<td>0</td>
<td>-9.65</td>
</tr>
<tr>
<td>8</td>
<td>9.65</td>
<td>5.16</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>-9.65</td>
<td>5.16</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>9.65</td>
<td>5.16</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>-9.65</td>
<td>5.16</td>
</tr>
</tbody>
</table>
The SPHERES satellites have an inertial sensor suite: three rate gyroscopes and three accelerometers. They can record data at 1000 Hz and have respective ranges of ± 1.45 rad/s and ± 25.6 mg. The satellites also have a Position and Attitude Determination System global metrology system, which uses a global reference frame of either the laboratory or the ISS.

On the SPHERES satellites, the -X face has Velcro attach points, which can be used to attach the satellites to one another. Though this is most often used in docking experiments, the Velcro can also be used to attach two SPHERES satellites together or attach a used battery pack to a SPHERES satellite to perform tests with altered inertia. There is a used battery pack on the ISS that has Velcro applied in the corresponding pattern as on the SPHERES satellite.

3.3 Inertia Models

As when modeling any physical system, knowledge of the specifics of the system is an issue. The main source for the nutation of a SPHERES satellite is the inertia properties. With the exception of the CO₂ liquid inside the propellant tank, there is minimal flexibility within the SPHERES satellite system. The SPHERES satellite system was noted to experience fluid slosh within its propellant tank in [16]. However, the effect of noise produced by the slosh on the online mass property estimation was considered negligible in comparison to the gyro ringing and thruster variability. Since the majority of the tests performed on the SPHERES Testbed concern algorithms that employ state estimation, the negligible coupling of nutation with the fluid slosh on the
SPHERES satellite do not effect those tests and therefore has not been fully studied. Also, the spin rates have been very slow in all previous tests, making the nutation frequency negligible.

### 3.3.1 SPHERES Satellite

Prior to this study, there were three inertia models of the SPHERES satellites.

- **Analysis Model – CAD**
  - This model was developed by Payload Systems, Inc. [15]

- **Test Model – derived from Ground Based Measurements**
  - This model was developed by in MIT’s SSL at 1-g, using a test stand built in the lab. [16]
  - \( \sigma = 2 \times 10^{-3} \text{kg-m}^2 \)

- **Test Model – derived from KC-135 Microgravity Measurements**
  - A series of rotational and translational tests were performed in the KC-135 reduced gravity airplane microgravity environment. [7]
  - As the microgravity in this environment is only available in 10-15 second increments, these tests were severely limited in time.

The models are given in Table 3.2. The serial numbers of the SPHERES satellites are given as SN# (i.e. SN1). This notation indicates the specific SPHERES satellite that the inertia models given measured; the CAD model applies to all SPHERES satellites. Wet indicates that the model includes the CO₂ fluid in the propellant tank, at a fill fraction of 68% liquid by volume; dry indicates the model is without the fluid.
Table 3.2 Inertia Models of the SN1 SPHERES (wrt the geometric center [GC]) [16]

<table>
<thead>
<tr>
<th>(kg m²)*10²</th>
<th>CAD Model</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Dry)</td>
<td>(Wet)</td>
</tr>
<tr>
<td>I_{xx}</td>
<td>2.19</td>
<td>2.3</td>
</tr>
<tr>
<td>I_{yy}</td>
<td>2.31</td>
<td>2.42</td>
</tr>
<tr>
<td>I_{zz}</td>
<td>2.13</td>
<td>2.14</td>
</tr>
<tr>
<td>I_{xy}</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>I_{xz}</td>
<td>-0.03</td>
<td>-0.03</td>
</tr>
<tr>
<td>I_{yz}</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

A very notable difference in the inertia matrices is the designation of the major and intermediate axes. Although the CAD model and two test derived models give the Z-axis as the minor axis, the CAD model assigns the Y-axis as the major axis and the other two models assign the X-axis as the major axis.

Table 3.3 Center of Mass (CM) Models of the SN1 SPHERES (wrt the GC) [16]

<table>
<thead>
<tr>
<th>CM Offset from GC (mm)</th>
<th>CAD Model</th>
<th>Ground Test Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Dry)</td>
<td>(Wet)</td>
</tr>
<tr>
<td>X</td>
<td>0.49</td>
<td>0.48</td>
</tr>
<tr>
<td>Y</td>
<td>-1.24</td>
<td>-1.19</td>
</tr>
<tr>
<td>Z</td>
<td>3.98</td>
<td>1.08</td>
</tr>
</tbody>
</table>

The center of mass (CM) is close to but not exactly at the geometric center and varies with the models and any additional of mass to the system. However, for this system, the CM is never more than 4mm off and has a minimal effect on the system nutation. The measured ground model, found through hardware testing, lists individual values for each SPHERES satellite and is the most recently developed model.

Testing of the SPHERES satellites in microgravity on the ISS indicated the ground based testing model to be the closest to truth and it is used as the starting value for the
inertia matrices. However, the ground based testing model has only two fluid configurations of the SPHERES satellite: without fluid in the CO$_2$ tank and with a CO$_2$ tank at its maximum liquid fluid fill (68%). Therefore the variable CO$_2$ component in microgravity has to be added to the inertia matrix. The amount the moments of inertia (MOIs) increase is dependent on the amount of fluid in the tank and the configuration of the fluid within the tank. So the MOIs of the SPHEREs are constantly changing due to propellant usage, the accelerations exerted on the tank and free surface interactions of the fluid. When calculating the nutation, the particular configuration for that test must be considered (i.e. the fill level and location of the fluid within the CO$_2$ tank). At a maximum, the moments of inertia (MOI) for the X- and Y-axes will increase by 4.5% and the Z-axis by 0.2% due to the fluid mass.

### 3.3.2 SPHERES satellite with additional rigid mass

The addition of rigid mass(es) to the SPHERES satellite allows for observing the effect of adding mass without introducing flexibility to the system. The rigid mass alters the center of mass and the inertia matrix of the SPHERES satellite. As the inertia of a system is changes, so does the nutation. However, since all mass added is solid, the change in the nutation is due to only to the change inertia and not by adding energy dissipation to the system. This requires the attachment of the mass to have only minimal flexibility, so not to have energy dissipate through the attachment point.

The Velcro on the –X face provides a good connection between the objects, which the astronaut has the option of reinforcing with Kapton tape.
The mass and dimensions of the battery pack are well known. The mass is assumed to be evenly distributed, as there is minimal space between the batteries. Table 3.4 shows the resulting change in inertia of the SPHERES satellite system from adding a battery pack. It gives the inertia matrix for the cases where the battery pack is centered exactly over the -X face and where it is off by 1 cm each in the +Y and +Z directions, possible due to human error.

Table 3.4 Inertia Effects on SN1 SPHERES satellite System due to the Addition of a Battery Pack

<table>
<thead>
<tr>
<th>(kg m$^2$) $10^{-2}$</th>
<th>No Additional Mass</th>
<th>Battery Pack Attached to -X face</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Centered</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1cm offset in +Y and +Z</td>
</tr>
<tr>
<td>$I_{xx}$</td>
<td>2.450</td>
<td>2.455</td>
</tr>
<tr>
<td>$I_{yy}$</td>
<td>2.150</td>
<td>2.424</td>
</tr>
<tr>
<td>$I_{zz}$</td>
<td>2.030</td>
<td>2.302</td>
</tr>
<tr>
<td>$I_{xy}$</td>
<td>-</td>
<td>-0.003</td>
</tr>
<tr>
<td>$I_{xz}$</td>
<td>-</td>
<td>-0.007</td>
</tr>
<tr>
<td>$I_{yz}$</td>
<td>-</td>
<td>0.000</td>
</tr>
</tbody>
</table>

An important note from the inertia change due to the battery pack is that the major and intermediate axes are now within $4\times10^{-4}$ kg-m$^2$ (~2%) of each other and the one sigma value for the model is $2\times10^{-3}$ kg-m$^2$. The built-in uncertainty means that it is
possible for the major and intermediate axes to actually be able to switch. The addition of a rigid mass 5.5% of the mass of the SPHERES satellite has dramatically changed the nutation properties of the system and which axes it can stably spin about. The effect on the inertia ratios must be considered when adding any mass to the SPHERES satellite system.

3.3.3 SPHERES Satellite with additional fluid mass

The addition of fluid mass to the SPHERES satellite system has all the same effects of adding a rigid mass plus it introduces a new source of flexibility and energy dissipation to the system.

Symmetry with respect to the spin axis is desirable so to keep the center of mass close to the geometric center of the SPHERES satellite system. The longitudinal axis of the cylindrical fluid containers is parallel to the spin axis. The setup in Figure 3.4 is for spin about the +Y axis; the +Y face of the SPHERES satellite in the figure has the SPHERES logo on it.
3.3.3.1 Availability of Tanks for Additional Fluid Mass

This study intends to test slosh in microgravity conditions by adding fluid tanks already available on the ISS. This additional liquid mass within the fluid tank could be added to the SPHERES satellite system to observe fluid slosh. This option is seriously considered for several reasons.

The modeling of off-axis tanks in spinning spacecraft is better understood by the community at this time than on-axis tanks. This would allow for better correlation of the test results to the models developed. Also, there would be a level of control over the amount of fluid and fill fraction of the additional liquid masses that does not exist for the onboard CO₂ tanks.

At the maximum propellant load, the fluid mass of the SPHERES satellite is never greater than 4% of the system mass. Since tanks are only changed on a needed basis and test order is predetermined for SPHERES test sessions, the mass of the CO₂ at the start of any test could vary between 26-172 g. The mass of the CO₂ is best known soon after
installation. After that, thruster use is tracked, but thruster variability leads to inaccuracy in the estimate of the remaining CO$_2$.

A list of desired attributes of the additional fluid mass has been drawn up. A few of the top priorities for the container were: rigid, cylindrical, clear, 8-12 oz fluid capacity, and low mass. By examining general objects on the ISS and looking at the catalog of crew provisions [18], the most suitable candidate container was the No Rinse shampoo bottle. Its drawbacks are that it is not clear and has a cap on one end that off sets the cylindrical shape (Figure 3.5). However, its original fluid has a viscosity near that of water and there was the possibility of filling used bottles with water to a set fill level.

![Figure 3.5 No Rinse Shampoo Bottle](image)

The organizations managing the SPHERES Testbed determined that objects not designated for use in the SPHERES experiment could not be attached to the SPHERES satellites. There are two options being considered for future tests: attaching unused CO$_2$ propellant tanks or launching to the ISS fluid tanks specifically designed to be used with the SPHERES project.

### 3.4 Disturbance Torques

#### 3.4.1 Atmospheric Drag

The atmosphere on board the ISS is maintained to have a composition and air pressure (14.7 psi) similar to that of the Earth at sea level. The density of the atmosphere
at 200 km is eight orders of magnitude smaller than the density of the air inside the ISS. [11] So while spacecraft in low Earth orbit do have to account from atmospheric drag, it is more often concerned with the lifetime of the mission and orbit maintenance. However, when tests consider the physical system, atmospheric drag (3.1, [19]) is a significant external force on the SPHERES satellite within the ISS and cannot be considered negligible. This is particularly true when increasing the velocity ($v$) of the SPHERES satellite, since that causes an exponential increase in drag. The other factors in drag force are the density of the fluid ($\rho$), surface area ($A$) and coefficient of drag ($C_d$).

$$F_d = -\frac{1}{2} \rho v^2 AC_d \vec{v} \quad 3.1$$

In this study, the SPHERES satellite spins in along each of its three principal axes. Additionally, it spins along its Y-axis with a configuration change that includes a battery pack attached at the +X face. Though the coefficient of drag has not been calculated for any of the cases, observation indicates that it would be lowest for the Z-axis spin and highest for the Y-axis spin with battery attachment, with the $C_L$ for X-axis and Y-axis spin just being a little smaller when without the battery pack attached. This is illustrated in Figure 3.6, which shows the obtrusions of the knob and tank in the +Y and +X spin and the additional obtrusion of the battery pack when it is attached. The system with the battery pack attached also has a higher surface area, as will any system with additional mass attachments, which increases the drag force.
As discussed in 2.2.2, atmospheric drag is an external force and will change the angular momentum vector of the spacecraft in inertial space.

### 3.4.2 Gravity

The ISS is in a free fall state and therefore objects within it results in microgravity levels of acceleration. The quasi-steady acceleration measured by the Microgravity Acceleration Measurement System (MAMS) indicates that the magnitude of the mean acceleration experienced by that instrument aboard the ISS was less than $0.3 \, \mu g$ in all directions, and never more than $4 \, \mu g$; this acceleration is due to the center of gravity offset from the ISS to the MAMS and vibrations within the ISS. The MAMS published data for two out of the three days in which SPHERES testing occurred for this study; the acceleration values given apply directly to those days and are assumed to be true for the third day. [21]

For a tank radius of 1 cm, as long as the SPHERES satellite was spinning at more than $0.0626 \, \text{rad/s}$, the centrifugal acceleration experienced by the fluid would be greater than the free fall acceleration. Since the rotation rates in this study are more than
an order of magnitude greater than that rate, the effect of gravity is not factored into the physical system.

3.5 Design of Experiment

The experiment plan has multiple phases:

- Observe fluid slosh in horizontal cylinders at 1-g
- Characterize nutation of the SPHERES satellite spinning in microgravity
- Characterize nutation of the SPHERES satellite spinning in microgravity with a rigid mass attached
- Characterize nutation of the SPHERES satellite spinning in microgravity with fluid tanks attached

This study executed the first three phases and discusses the results. Implementation of the final phase is discussed in the future work section.

3.5.1 Horizontal Fluid Slosh Ground Tests

The end phase of the experiment plan involves observing the fluid slosh in tanks in microgravity. To better understand the fluid slosh in the tanks, this study observed fluid slosh in horizontal cylinders in 1-g.

The configuration of horizontal cylinders was chosen for a couple of reasons. Fluid slosh in spheres is currently well understood and there are many models already available, so exploration of this shape was limited in possible contributions. Spacecraft propellant tanks are frequently cylinders with hemi-spherical ends. The acceleration experienced due to centrifugal force in off-axis tanks on spacecraft is in the same
direction as gravity for a horizontal tank on Earth. The MESSENGER spacecraft
launched in 2003 and was a spin stabilized spacecraft with multiple fluid tanks,
including cylindrical off-set fuel tanks as shown in Figure 3.7.

D.O. Lomen developed a digital method of using mechanical analogs to accurately
determine slosh frequencies for tanks of arbitrary shape, provided the acceleration force
is applied in parallel to the symmetric axis. Though the cylindrical tanks are symmetric,
the acceleration is perpendicular to the symmetric axis, and Lomen’s method does not
apply. In Figure 3.8, the containers pictured in (a) (b) can utilize Lomen’s method and
the container in (c) cannot. [23] [24]
Methods of determining slosh modes for partially filled horizontal cylinders are progressing in several studies. However, the curvature of the walls and misalignment of acceleration and axis of symmetry make the calculations difficult. The current methods have multiple restrictions on use, particularly in variability of fill fraction. See [1] [2]. For this study, the test approach was taken over analysis in observing the slosh modes in a container of varying fill levels.

The test plan was to impart a longitudinal shock to excite the slosh modes in the tank. This corresponds to an off-set tank in a spinning spacecraft that has an external force imparted on the system, upsetting the tank. For the entire duration of the test, the container remains horizontal, so that the acceleration vector due to gravity is constant with respect to the cylinder and that slosh observed is not caused by directional changes in the gravity vector. The slosh modes beginning several cycles after the shock event are observed.
Figure 3.9 1-g Slosh Testing Set-up

The figure above shows the test set-up. The container was filled with water to a specified fill level. It was attached to four strings from above to form a parallelogram. This formation allowed the container to be parallel to the ground at all times, so not to alter the direction of the gravity vector within its frame. The overhead string attach points formed a rectangle, to decrease sideways movement of the container as it swung. The strings attached to the container at two points equidistant from the container’s center of gravity. At the location where the parallelogram was a rectangle, one end of the container was touching a bracket mounted to a load cell. The bracket and container both had Velcro at their interaction points. The accelerations measured by the load cell
are amplified and recorded by passing through an amplifier to an oscilloscope to a computer with LabView.

The test was performed by drawing back the container a prescribed distance from the bracket/load cell configuration, ensuring it is level with the ground. Once the fluid had settled, the container was released and struck the bracket. The container and bracket fastened to one another. The fluid within the container was excited by the shock event. It then sloshed to its natural frequencies, which were observed via the accelerations recorded throughout the test. The test was performed between 2-8 times each for fill fractions ranging from 10% to 90% for Container A; the drawback distance was kept approximately the same, but was not measured. The test was performed three times each for three different draw back distances (5, 10, 15 cm) for fill fractions ranging from 10% to 60% for Container B.

Possible sources of error in the test included the flexibility the Velcro connection at the attach point, the difficulty in keeping the container level with ground and verifying this, and any angle between the end of the container and the bracket, caused by an indirect initial connection.

The results given for this study are for two cylindrical containers with smooth walls and some imperfections at the end caps. The fluid used was water.

<table>
<thead>
<tr>
<th>Container</th>
<th>Diameter</th>
<th>Length</th>
<th>Fluid Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>17</td>
<td>550</td>
</tr>
<tr>
<td>B</td>
<td>6.5</td>
<td>15</td>
<td>504</td>
</tr>
</tbody>
</table>
The tests performed are preliminary and demonstrate the method for obtaining 1-g slosh modes for a horizontal cylindrical container. The tests also show trends in slosh based on fill factor. Once go ahead is given to test attach fluid tanks to the SPHERES satellite on the ISS and the fluid container is definite, these 1-g tests should be performed again with that container and containing the fluid to be in it on the ISS.

3.5.2 SPHERES Nutation in Microgravity

In order to characterize the nutation of the SPHERES satellite, with and without a rigid mass attached, four separate test configurations were performed over three test sessions:

- SPHERES satellite spinning about the X-axis
- SPHERES satellite spinning about the Y-axis
- SPHERES satellite spinning about the Z-axis
- SPHERES satellite with a rigid mass attached spinning about the Y-axis

The intention of the tests was to observe a growing or decaying nutation angle (depending on spin axis), in order to better understand nutation of the SPHERES satellite system. Then, when additional fluid masses are later added, the nutation due to the fluid masses only can be extracted and therefore determine the impact of the fluid masses on the dynamics of the system as a whole. Also, the rigid body tests were intended verify the accuracy of the inertia ratios with the nutation frequency.

The design of each test followed the following general procedure:

1. Calibrate the estimator
2. Stop in the middle of the test volume
3. Spin up to a set rotation rate about the designated spin axis

4. Turn off all control for a short period of time (i.e. drift)

5. Torque the SPHERES satellite about the X-axis via a short thruster pair firing

6. Turn off all control for a long period of time (i.e. drift)

7. Bring the vehicle to a physical stop

The first two steps were the set-up portion of the test: getting the SPHERES satellite oriented and settling out all disturbances from astronaut release. The spin-up step brought the SPHERES satellite up to the condition that it is imitating in real-life spacecraft: gyroscopic stability. The drift steps removed all control from the system: the only forces on the system are the atmospheric drag and energy dissipation within it. The torque step acted as a disturbance force, to excite the nutation.

Since the tests happened over three separate test sessions, the timing and execution of each of these events varied for every test. For example, the final stopping step was only implemented in the first test session. The other variations will be specified in 4.2.

The limits of testing, on both rotation rate of the SPHERES satellite and the length of time of the test, minimized the number of nutation cycles occurring within the test. It is preferable that 20 or more nutation cycles occur, since increasing the number of cycles allows for more nutation decay or growth to occur. One of the tests of major axis spin with the battery mass attached increased its rotation rate to 4 times the sensor limit (via open loop thrusting) and observed 11 nutation cycles, as opposed to the 1-4 nutation cycles observed by the other tests. The trade off was that the spin about the primary axis had to be calculated by observing the SPHERES satellite spin via video and counting
the number of frames per cycle. This increased the uncertainty in the rotation rate of the spin axis from $7.1 \times 10^{-4}$ to 0.21 rad/s, which also made determination of the angular momentum vector less certain.
Chapter 4

Experimental Data

This chapter details the experiments performed and data collected for both the horizontal slosh ground tests and the SPHERES nutation in microgravity tests. It discusses how the microgravity tests varied upon iteration.

4.1 Horizontal Slosh Ground Tests

There were a total of 60 runs performed on Container A and 72 runs performed on Container B at various fill fractions (ratio of liquid in container to volume of the container). The data from a single test is shown within this section, and correlation of results is covered in Chapter 5.

The raw data for a Container B test run is shown in Figure 4.1. Figure 4.1a is of the entire test, beginning a second before the container connects with the load cell and generates the shock event. Figure 4.1b shows the test data beginning ~2.5 seconds after
the shock event, after the shock has tampered out. Only the data from the second figure was processed to determine frequency modes and respective powers.

The force signal is then run through a Fast Fourier Transform (FFT). For this process, $x$ is a signal of length $N$ in the time domain and $\omega_N$ is the $N^{th}$ root of 1.

\[
\omega_N = e^{2\pi i/N} = \cos \frac{2\pi}{N} + i \sin \frac{2\pi}{N}
\]

\[
\omega_N^2 = \omega_m \text{ if } m = \frac{N}{2}
\]

\[
F_N = \begin{bmatrix}
\omega_N^{(N-N)} & \cdots & \omega_N^{(N-N)(N-1)} \\
\vdots & \ddots & \vdots \\
\omega_N^{(N-1)(N-N)} & \cdots & \omega_N^{(N-1)(N-1)}
\end{bmatrix}
\]

\[
y = F_N x
\]
By way of the Cooley-Tukey algorithm, $x$ is split into even ($x'$) and odd ($x''$) components. By using a half-matrix $F_m$ now instead of $F_N$, the number of multiplications when calculating the $y$ component can be reduced.

$$y_j = \sum_{k=0}^{N-1} \omega_N^{kj} x_k = \sum_{k=0}^{m-1} \omega_N^{2kj} x_{2k} + \omega_N^{(2k+1)j} x_{(2k+1)}$$

$$= \sum_{k=0}^{m-1} \omega_m^{kj} x' + \omega_N^{j} \sum_{k=0}^{m-1} \omega_m^{kj} x'' = y'_j + y''_j$$

$$j = 0, ..., N - 1$$

$$y_j = y'_j + \omega_N^j y''_j \quad y_{j+m} = y'_j - \omega_N^j y''_j$$

$$j = 0, ..., m - 1$$

The Power Spectral Density (PSD) represents the density of the power of the signal at the frequency $f$ in the spectrum. Essentially, the PSD is $y$ times its complex conjugate.

$$G(f) = |\tilde{x}(f)|^2 = \frac{y(f)\overline{y(f)}}{N}$$

The corresponding frequency is found by the following equation, where $t$ represents time.

$$f_j = \frac{j}{2N\Delta t}$$

The highest detectable frequency will be half of that data rate of the signal. [25]

The PSD from the test corresponding to the signal from Figure 4.1 is shown in Figure 4.2. The peaks indicate the modes in which larger percentages of the active mass are participating in that mode. The PSD value at 1.38 Hz ($10^{0.14}$) is nearly an order of magnitude larger than any other frequency.
Based on the individual PSD plots, the frequency mode of the first longitudinal mode associated with each run was found, with uncertainty in each run ranging from 0.25-0.4 Hz. Then, for each fill fraction, the runs were averaged to determine the first longitudinal frequency mode and are shown in Figure 4.3.

4.2 SPHERES Nutation in Microgravity ISS Tests - Execution and Results

This section of the study discusses the performance of the various tests in operational terms. The discussion of nutation and other model expectations are found
in the following chapter. Since this study is dependent on the interactions of the physical system, outlining events and disruptions occurring in the system is important.

A summary of the tests can be found in Table 4.1. SPHERES Number (SN) indicates which of the three SPHERES satellites on the ISS was used for the test. CO₂ Tank Fill Fraction indicates what percentage of the volume of the CO₂ tank is filled with liquid. The rotation information refers to which axis the satellite was spun around for that test and what the rate at the end of the controlled portion of the test. Test Session indicates which SPHERES Test Session the particular test occurred in; tests in later sessions were altered based on results from previous test sessions.

Table 4.1 ISS Nutation Test Matrix

<table>
<thead>
<tr>
<th>Test</th>
<th>Type of Spin</th>
<th>Configuration</th>
<th>SN</th>
<th>CO₂ Tank Fill Fraction</th>
<th>Axis of Rotation</th>
<th>Initial Rotation Rate rad/s</th>
<th>Test Session</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Minor Axis</td>
<td>SPHERE</td>
<td>2</td>
<td>49%</td>
<td>Z</td>
<td>0.74</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>Intermediate Axis</td>
<td>SPHERE</td>
<td>1</td>
<td>47%</td>
<td>Y</td>
<td>1.25</td>
<td>14b</td>
</tr>
<tr>
<td>3</td>
<td>Major Axis</td>
<td>SPHERE</td>
<td>3</td>
<td>68%</td>
<td>X</td>
<td>1.15</td>
<td>16</td>
</tr>
<tr>
<td>4</td>
<td>SPHERE + Battery Pack</td>
<td></td>
<td>2</td>
<td>5%</td>
<td>Y</td>
<td>0.74</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td>1</td>
<td>28%</td>
<td></td>
<td>1.25</td>
<td>14b</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td>3</td>
<td>66%</td>
<td></td>
<td>6.2</td>
<td>16</td>
</tr>
</tbody>
</table>

Four of the six tests were successfully executed and able to determine the nutation of the satellite. The exceptions are Test 2, the intermediate axis test, which was successfully executed twice but could not determine the nutation due to unstable spin, and Test 1, which was executed with partial success twice but did not have enough test data to determine the nutation. Common details of the tests are mentioned in 4.2.1; only variations on these details are mentioned in further sections.
The initial rotation rate was obtained by closed loop controlled rotation for all tests except Test 6. Test 6 incorporated open loop thrust in order to obtain a rotation rate of 6.2 rad/s. All closed loop rotation rates were obtained within 0.01 rad/s of the targeted value except Test 3, which is explained below. There were perturbations via +X torques by open loop thrust for 30ms increments, which were predicted to increase by the X-axis rotation rate by 25.4-28.7 mrad/s, depending on the SPHERES satellite. The tests experienced increases ranging from 21.2-28.4 mrad/s; thruster variability, the open and closing of the thrusters and inaccuracy of the inertia matrix are the causes of the small differences between predicted and actual rotation rate increase.

4.2.1 Minor Axis Spin

Test 1 was the only microgravity test that spun about the minor principal axis (+Z). The maneuver start times performed for this test around found in Table 4.2 and correspond to the telemetry figures within this section. For other tests, the main maneuver differences will be the axis of rotation and maneuver time lengths, though further changes are mentioned in 4.2.5.

In Table 4.2, the Inertial Measurement Unit (IMU) Data Recorded indicates which maneuvers obtained high frequency data from gyroscopes and accelerometers. Recording this data requires additional time for data download following the active portion of the test, which is why it is limited to only certain maneuvers and not collected over the entire test.
### Table 4.2 Minor Axis Spin Maneuvers

<table>
<thead>
<tr>
<th>Maneuver Performed</th>
<th>Length (s)</th>
<th>Start Time (s)</th>
<th>IMU Data Recorded</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimator Converge</td>
<td>17</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Attitude Stopping Maneuver</td>
<td>16</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>Controlled Spin about the Minor Axis (Z)</td>
<td>26</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>Controlled Spin about the Minor Axis (Z)</td>
<td>6</td>
<td>59</td>
<td>X</td>
</tr>
<tr>
<td>Drift without Control</td>
<td>26</td>
<td>65</td>
<td>X</td>
</tr>
<tr>
<td>Perturbation: +X-axis torque for 30ms</td>
<td>2</td>
<td>91</td>
<td>X</td>
</tr>
<tr>
<td>Drift without Control</td>
<td>41</td>
<td>93</td>
<td>X</td>
</tr>
<tr>
<td>Attitude Stopping Maneuver</td>
<td>16</td>
<td>134</td>
<td>X</td>
</tr>
<tr>
<td>End Test</td>
<td></td>
<td>150</td>
<td></td>
</tr>
</tbody>
</table>

As shown in Figure 4.4, the satellite spun up to 0.74 rad/s. From 65 to 134 seconds, it drifted without control except for two 30ms open loop thrust +X torques at 91 seconds. Following the drift, the satellite was brought to an attitude stop.

![Figure 4.4 Body Axis Rotation Rates for Test 1: Minor Axis Spin (+Z) Test](image)

Nutation is clearly visible in the rotation rates of the body axes. In Figure 4.4, it is most evident in the non-principal spin axes. However, the corresponding frequency is
visible in the principal spin axis (Z) also; there was a peak in the Z-axis rotation rate at 117 seconds, when the phasing of the non-principal axes has them at their combined lowest. This would be the point when the nutation angle was the smallest; the nutation angle was at its highest at just before the stopping maneuver (134 seconds).

For this particular test, a full uninterrupted nutation cycle was not observed. At the nutation frequency observed, just over one cycle could have been observed. However, the timing of the perturbation, which affects the magnitude and frequency of the nutation within the rotation rates, interrupted this cycle. Tests following this test session moved the perturbation to beginning of the drift period, so not to disturb the nutation cycles, and increased both the principal axis rotation rate and drift period to observe a greater number of nutation cycles.

The position of the SPHERES satellite moved throughout the entire test. In this particular case, the satellite moved generally closer to the center of the test volume. However, had the satellite’s velocity in the Z direction been reversed, it most likely would have encountered a wall before the completion of the active portion of the test.
The processed IMU data in Figure 4.6 matches the rotation rates and nutation values in the gyroscopic data with that given by the telemetry. The spikes in the accelerometer data indicate the opening and closing of thrusters. The data shows thruster activity for the end of the controlled rotation and the attitude hold maneuver at the end of the test. In between, there is no thruster activity during the long drift period except the two spikes for the perturbation of two 30ms +X torque separated by 1 second (at ~91 seconds of test time).
4.2.2 Intermediate Axis Spin

Test 2 was the only microgravity test that spun about the intermediate principal axis (+Y). The test was performed twice with very similar results. Therefore, only the first test execution will be shown here.

As shown in Figure 4.7, the satellite spun up to 1.25 rad/s. From 55 to 262 seconds, it drifted without control except for a single 30ms open loop thrust +X torque at 58 seconds. Following the drift, the satellite was brought to an attitude stop.

![Figure 4.7 Test 2 Body Axis Rotation Rates](image)

**Intermediate Axis Spin (+Y) Test**

The test was able to maintain the +Y axis as the principal axis of rotation provided active control was on. Once the active control was turned off, the direction of the angular momentum vector began moving through the body. This was not the coning and nutation seen in major or minor axis spin; it was unstable spin. The direction of spin completely reversed directions four times.

At the time this test was performed, the CAD inertia model was being used as the reference inertia model, which indicates the Y-axis to be the major axis and the X-axis to
be the intermediate axis. That order of the principal axes clashes with the results of this test.

Following this test, the reference containing the ground measured inertia matrix model was discovered and it was put into use. These tests correlate with order of axes in that inertia model, which indicates the Y-axis to be the intermediate axis for the three SPHERES satellites on the ISS.

4.2.3 Major Axis Spin

Test 3 was the only microgravity test that spun about the major principal axis (+X).

As shown in Figure 4.8, the satellite achieves a spin rate of 1.115 rad/s at 67 seconds at the end of the controlled rotation period, which is within 0.14 rad/s of the targeted rotation rate of 1.25 rad/s. A previous attempt of the test attained 1.25 rad/s with a similar rotation profile just prior to running out of CO₂. The impact of the change of the CO₂ tank immediately before this test on the gas levels in the tank and the thruster value system is assumed to be the cause of the undershooting. The only disruption this caused was that since the rotation rate was lower, so was the nutation frequency and there were fewer nutation cycles to observe.
At 71 seconds, there was a single 30ms open loop thrust +X torque, resulting in a spin rate of 1.145 rad/s. This open loop torque was originally intended to be a perturbation, as it is in the other tests. However, a coding error resulted in only an increase in the rotation rate of the principal spin axis. The initial rotation rates of the other two axes were large enough to still observe nutation though. From 71 to 275 seconds, the satellite drifted without control. The satellite encountered a wall of the ISS at 232 seconds, disrupting all data following that point. The active portion of the test ended with the drift.

Four uninterrupted nutation cycles occurred within this test. Figure 4.8 also illustrates the decline of the rotation rate of the principal axis over time. The decline that is visible in this figure and others in this chapter was due to atmospheric drag on the satellite system as it rotated. If the system was in a vacuum with no external torques, the rotation rate of the major principal axis would be expected to increase as the rates of the other principal axis decreases, due to energy dissipation. This is analyzed further in Chapter 5.
4.2.4 Major Axis Spin of the SPHERE + Battery Pack Configuration

There were three tests in which the inertia matrix was altered by adding a solid mass (battery pack) to the SPHERES satellite (Test 4,5,6). The satellite spun about the major principal axis of the system (+Y). The primary difference between the tests was the rate of rotation and they will be identified by that value.

As shown in Figure 4.9, the satellite spun up to 0.74 rad/s for Test 4. From 65 to 134 seconds, it drifted without control except for two 30ms open loop thrust +X torques at 91 seconds. The satellite encountered a wall of the ISS at 112 seconds, disrupting all data following that point. Following the drift, the satellite was brought to an attitude stop. Due to the very low nutation frequency observed, a full nutation cycle could not have been observed even without the disturbances of the perturbation and the wall encounter.

![Figure 4.9 Test 4 Body Axis Rotation Rates](image)

**Figure 4.9 Test 4 Body Axis Rotation Rates**

**Major Axis Spin of the SPHERES + Battery Pack (+Y) Test: Rotation Rate 0.74 rad/s**

As shown in Figure 4.10, the satellite spun up to 1.25 rad/s (Test 5). From 55 to 262 seconds, it drifted without control except for a single 30ms open loop thrust +X torque
at 58 seconds. Following the drift, the satellite was brought to an attitude stop. Two full nutation cycles were observed.

As shown in Figure 4.11, the satellite spun up to 1.4 rad/s under closed loop control in Test 6. From 71 to 80 seconds, open loop thrust increased the rotation rate to 6.2 rad/s. From 80 to 282 seconds, the satellite drifted without control. The satellite encountered a wall of the ISS at 233 seconds, disrupting all data following that point. The active portion of the test ended with the drift. Eleven full nutation cycles were observed. The loss of signal (LOS) in broadcast of the video occurred at 175 seconds, which is why the rotation rate detected by the video ends there (discussed further in 4.2.4.1).
Since the sensor detection limit is 1.44 rad/s, obtaining a rotation rate above that value required open loop thrust and an alternate method of detecting the rotation rate. Two thrusters were commanded to open for 0.9 second periods nine times to create a +Y torque. By examining the IMU data, the thrusters were open for a total of 7.2 seconds; explanation of this shorter thrust time can be found in 4.2.4.2. For this thrust time, the expected increase in rotation of the satellite in a vacuum was be 5.21 rad/s. The actual increase, which was subjected to atmospheric drag, was 4.88 rad/s.

Unlike the other tests, the rotation rates of the non-principal axes were not controlled to 0.00 ±0.05 rad/s during the entire spin up process. The baseline X-axis rotation rate is four times larger than the maximum rate experienced by a non-principal axis in the other tests; the Z-axis rate is in family with the other tests. However, since the principal axis rotation rate is so high, the nutation angle is actually slightly lower than several other tests.
4.2.4.1 Detecting Rotation Rate by Video

In this test, video of the test session was examined to detect principal axis rotation over the sensor limit. The video was received was from a broadcast during the test session and LOS are common through test sessions. An LOS occurred from 175 seconds through the end of the test, which is why there is no available rotation rate data for the Y-axis during that period.

The satellite was oriented so that during the principal axis spin, the battery pack and CO₂ tank end moved in a circle whose plane was parallel to the camera view. The battery pack and tank end stuck out at approximately 90° from one another. By noting the frame numbers in which the two items line up either parallel or perpendicular to the side of the frame, it was possible to determine the frequency of that quarter wavelength of the rotation.

The uncertainty in rotation rate was the rotation rate divided by the frequency of frame rate divided by the number of wavelengths. The video received from NASA was 29.97 Hz. However, the effective rate was approximately 15 Hz, as every two frames were identical. During the drift period, the rotation rate ranged from 5.23-6.2 rad/s, which means the uncertainty in the rate was 0.35-0.42 radians-wavelength/s. During the portion of the test where the sensor data is still valid and the rotation rate is measured by the sensors to be 1.39-1.40 rad/s, error in the video data was within the predicted uncertainty of 0.12 rad/s when calculated at every three-fourths of a wavelength length. Calculating the rotation rate for wavelengths less than three-fourths resulted in errors significantly exceeding that of the predicted uncertainty (i.e. error was
twice the uncertainty at half a wavelength). This indicates that three seconds of data are necessary in this test for the error to be within the predicted uncertainty.

For Figure 4.11, the rotation rate during the drift was calculated every half a wavelength over a two and a half wavelength period. The uncertainty of the average rotation rate for the entire period was 0.17 rad/s. The disadvantage with averaging the rotation rate over a period so that uncertainty is known is that the ability to view fluctuations (i.e. the 23 second nutation cycles) within the rotation rate is reduced. However, these fluctuations were already difficult to detect due to the uncertainty and declining rate due to drag. The PSDs of the rotations rates from 85 to 175 seconds are shown in Figure 4.12. The figure illustrates how the nutation frequency dominates the non-principal axes frequency spectrums, but not for the principal axis rotation rate measured via video. Reducing the period of averaging the rotation rate from 2.5 to 1 wavelength did not cause the nutation frequency to peak on the PSD to appear.

4.2.4.2 Open Loop Thrust Anomaly

During Test 6, one or more of the thrusters did not fire as commanded. This occurred during the open loop thrusting portion of the test. During this part of the test,
two of the twelve thrusters were commanded to be open for 0.9 seconds out of every second for a 9 second period.

The thruster commands recorded match the expected firing of two X-axis direction thrusters, whose combined theoretical thrust results in Y-axis torque with no net force (Figure 4.13). The commanded thrust was to occur 0.9 seconds of every second. The accelerometers experienced a disturbance every time the thrusters opened or closed. The accelerometer data for this portion of the test is shown in Figure 4.14. During the first commanded thrust of 0.9 seconds, the thrusters were open for 0.9 seconds. During the following six commanded thrusts of 0.9 seconds, the thrusters were open for only 0.81 seconds. This shows that by commanding the thrusters to open and close once each per second, the maximum time that the thrusters can remain open is 0.81 seconds.

However, there was unexplained disruption in the thruster firings at the end of the open loop thrusting, as shown in Figure 4.14. From time 71 – 78 seconds, the disturbance forces occurred in the expected pattern. During the eighth thrust cycle, there were two additional disturbances, as if one or both thrusters closed and then reopened during the middle of a period when the thruster should have continually remained open. An astronaut not running the test took a picture during at approximately this time; this was the only noted variant at the time of the anomaly from the previous 7 seconds.
By examining the position and rotation rates of the body axes at this point, there was no obvious disruption. It is assumed that both thrusters simultaneously opened and closed. This would reduce the total time exerting the torque force to increase rotation rate, but have no other discernable effects. The actually increase in rotation rate was
93% that it would have been had this anomaly not occurred. With this being a physical test, it was more important to know the torque exerted and the resulting rotation rate than actually reaching a specific rotation rate. The results of the test were not affected by this anomaly.

### 4.2.5 Modifications of the Tests

Most of the modifications of the test format happened following the first test session. Of the three test sessions in which these tests took place, Tests 1 and 4 took place in the first one; Tests 2 and 5 took place in the second one; and Tests 3 and 6 took place in the third test session.

#### 4.2.5.1 Adding the Position Holds

In the first test session, the position of the SPHERES satellite is never controlled. The astronaut placed the satellite in the middle of the test volume with as little initial velocity as possible. For this experiment, the position is insignificant provided that is has minimal change and the SPHERES satellite is not near the bounds of the test volume. However, as found in Figure 4.15, the satellite is capable of reaching these bounds and encountering a wall (115 seconds) during this period.

The combination of the initial velocity acquired during initial placement in the volume, the velocity due to uneven thruster force during the controlled rotation from 33-60 seconds, and the velocity due to any air currents within the ISS imparted enough velocity for the satellite to impact the wall of the ISS for this particulate test run. Once this occurred, all test data following that point was not relevant, due to the significant external force. Since the two previous attempts of the test experienced satellite resets...
which interrupted the test or the data and it was necessary at that point to move on to another test within the test session, this test was limited in useable data collected. Since the nutation frequency was quite low, an entire nutation cycle was not captured in the data.

Figure 4.15 Test 4 Position and Velocity Data

Major Axis Spin of the SPHERES + Battery Pack (+Y) Test: Rotation Rate 0.70 rad/s

The other test sessions (Tests 2,3,5,6) employed a position hold during all controlled portions of the test. This way the velocity due to the three factors was minimized up until the point that control was stopped. The addition of the position did not stop the problem, as Figure 4.16 illustrates at 230 seconds. However, it did extend the length of time that the SPHERES satellite could drift while rotating from 50 to 165 seconds.
It is noted that over half of the tests from all sessions were able to run through completion without encountering the wall or other obstacles in the ISS. Two runs of the exact same test were performed one after another, with nearly identical initial positioning; however, the path of the satellite varied for both. Though the addition of position hold during controlled rotation increases the time before the satellite could encounter an obstacle, any future experiments should consider or watch for opportunities to reduce the position drift when the satellite is not being controlled.

4.2.5.2 Removing stopping maneuver

In the first two test sessions, there is a maneuver at the end of the test that brings the satellite to an attitude hold. The original intention of this maneuver is to cause whatever fluid is within the CO₂ tank to slosh. However, the control system was imparted and the noise from use of the estimator overwhelms what fluid slosh may be occurring. It was
removed from the diagnostic tests performed in this study to allow more time for the
drift and therefore nutation cycles.

For future experiments with additional fluid tanks, the stopping maneuver could be
imparted with open loop thrusting, removing the noise from the estimator. However,
other disturbances with this method will have to be considered, such as the uneven
thruster forces.

4.2.5.3 Reducing time of IMU

The first test session recorded 1000Hz data from the gyroscope and accelerometer
sensors, beginning with the last 5 seconds of controlled rotation through the end of the
test. This is very costly in terms of test time, as the time to download the 1000 Hz data is
approximately four times the length of time the data was taken over (i.e. 15 seconds of
1000Hz data requires one minute of data download time). For the other test sessions,
1000Hz data was recorded beginning with the last 5 seconds of controlled rotation
through the first 10 seconds of the drift period. This allowed for the length of the drift
period to be increased without causing the length of the test to increase by five times
that time increase.
Chapter 5

Model Correlation

This chapter demonstrates that the horizontal slosh ground tests successfully correlated to previous experimental studies. The angular momentum and nutation frequency are used for the SPHERES Nutation in Microgravity tests to derive new inertia models for the SPHERES satellites. The Nutation Time Constant is also examined for the microgravity tests.

5.1 Horizontal Fluid Slosh Ground Tests

5.1.1 Frequency Modes

5.1.1.1 Previous Experimental Study

The results of this study were correlated to the experimental study performed by McCarty and Stephens [5]. In that study, horizontal cylindrical containers were agitated at gradually increasing frequencies until a modal frequency appeared and continued to resonant even without excitation. The containers in this study are cylinders with some
imperfections, within the same radius to length ratios as McCarty and Stephens, and half the actual length. The fluid in both studies was water.

The equations below refer to the experimentally determined fluid frequency parameter, $\gamma_n$, and can be used to calculate the natural modes of a horizontal cylinder among different acceleration values. Equation 5.1 is for longitudinal modes and Equation 5.2 is for transverse modes. The bottle length ($l$) and the height of the liquid within the container ($h$) are referenced. The frequency parameter for longitudinal modes found by McCarty and Stephens are shown with the results of this study in Figure 5.1.

$$\gamma_n = \omega_n \sqrt{\frac{l}{g \tanh \frac{n\pi h}{l}}} = 2\pi f_n \sqrt{\frac{l}{g \tanh \frac{n\pi h}{l}}}$$  \hspace{1cm} 5.1

$$\gamma_n = \omega_n \sqrt{\frac{R}{g}}$$  \hspace{1cm} 5.2

For Equation 5.2, Equation 5.3 and Figure 5.1, $R$ refers to the radius of the cylinder. For all other references to $R$ within this study, $R$ refers to the distance of from the center of mass of a system to the center of mass of the fluid container.

For case of $h = R$, Lamb applied the first transverse natural frequency in the equation below [26]. For Container B, the estimated first transversal natural frequency is 0.32 Hz, about a quarter of the first longitudinal natural frequency.

$$\omega_1^2 = \frac{8\pi}{48 - 3\pi^2} \frac{g}{R} = 1.36656 \frac{g}{R}$$  \hspace{1cm} 5.3

5.1.1.2 Correlation of Experiments

The longitudinal modes of the horizontal slosh tests are shown in Figure 5.1. The results are shown for the first longitudinal mode of each container, along with the faired curves of the parameter for the first four longitudinal modes from [5]. The uncertainty
in frequency and imperfections in the cylinders of this study account for the small variances from McCarty and Stephens. The method in this study of obtaining modes in containers with liquid is verified.

![Figure 5.1 Experimental 1st Longitudinal Modes](image)

Once the fluid frequency parameters are found here on Earth, they can be used to find the modal frequencies elsewhere by substituting the gravity term with the dominate acceleration source. On a spinning spacecraft in microgravity, this will be centrifugal force. Since the replacement of the acceleration force for gravity is \( \Omega^2 R \), a container for a given fill level will have its natural frequency by directly proportional to the spin rate, just like the nutation frequency.

\[
\omega_n = \omega_{n_{\text{Earth}}} \sqrt{\frac{\Omega^2 R}{g}}
\]

Assuming the offset tanks are 12.5 cm from the center of mass of the system and the system is accelerating at 1.25 rad/s, the acceleration exerted on the tanks will go from
9.807 m/s to 0.0195 m/s. The modal frequencies would be expected to decrease to approximately 4.5% of their values on Earth.

5.1.2 The Effect of Fill Fraction on Slosh Power

The force power is expected to increase both with increased fluid mass and increased draw back distance. However, the force power peaks at the 68% fill level Container A and the 50% fill level Container B, as shown in Figure 5.2. This is because the largest slosh mode/resonance is at this fill fraction; the largest active component of fluid occurs at this fill. Though the higher fill fractions have more overall fluid mass, a lower percentage of that mass is participating in the mode such that the amount of active mass is smaller.

Figure 5.2 Liquid Fill Fraction vs. Power Density of the 1st Longitudinal Mode

These power values are associated with the frequency that is the first longitudinal mode. This mode has the highest PSD for fill fractions between 20-90%; the outlying fill fractions had much higher frequencies than the highest PSD, but they were on the same order of magnitude as the PSD of the first mode. For Container B, the draw-back distance shown in Figure 5.2 is constant at 10 cm. The Container A distances were kept
approximately constant, but were not as rigorously controlled. This is why the trend in maximum power density is less ordered.

To determine the peak slosh power fill fraction, there should be greater detail around the fill fractions that have the highest power densities. The expected peak in active fluid is expected at ~60%.

5.1.3 Applicability

The horizontal slosh tests demonstrate a method of determining slosh frequency modes and the power associated with them. The tests match other experimental results of a horizontal cylindrical container with water. By altering the container with hemispherical ends and/or changing the fluid within the container, the ability to correlate with McCarthy and Stephens will wane. However, the new container(s) can be tested with the method established here to find its longitudinal modes, along with which fill fractions result in the highest slosh power for that container and fluid.

The method can also be altered so that transverse modes can be obtained. The main difference will be the side of the container contacting the load cell will switch, which will require set-up changes to accommodate.

However long the test is following shock event, 1/test length is the uncertainty that will result. For example, if the usable test data following the shock event is 5 seconds, then the PSD values are available at 0.2 Hz increments. Test data in this study results in frequency increments ranging from 0.25-0.4 Hz. This was caused by a coding issue in the test set-up for recording data that resulted in only 10 seconds of data per test. This
issue was not overcome at the time of the tests, but is expected to not be difficult to work through in the coding of the recording software.

The larger drawback tests had shorter usable test data, since it took the fluid longer to recover from the shock event, and so have larger uncertainty. Since the lower frequencies are of greater interest, it is vital to increase the time length of the usable data. By removing the restriction of the amount of time to end data recording, the amount of data following the shock event can increase and the fidelity in frequency modes will significantly increase.

The highest detectable frequency will be half of that data rate on the sensor. In this case, the sensor data rate was 1000 Hz, and 500 Hz is significantly higher than the frequencies of interest.

5.1.4 Angular Momentum Conservation and the Drag Effect

The drag force for the Z-axis is lower than that exerted on the other two axes. Since this is an external torque that acts as a disturbance to the test, it is preferable to minimize the drag when possible.

In the current configuration of the SPHERES satellite, the only practical locations to add mass to the system at the +X and −X faces of the satellite, since the Y faces are hinged battery doors and the Z faces have the tank and pressure knob protruding from the satellite. As the result, the moment of inertia (MOI) about the X-axis would only increase minimally compared to the MOIs about the Y-axis and Z-axis. When adding masses to the system, the impact on the MOIs must always be considered. For studies
involving gyroscopic spin, the axis of the spin can never be the intermediate axis, due to its inherent instability.

For the cylindrical offset fluid tanks this study is addressing, the symmetric axis of the tank is in-line with the axis of spin of the satellite. This orientation must be considered in determining the MOI of the new system. If the tanks are placed at the \(-X\) and \(+X\) faces in-line with \(Y\)-axis, the increase in the MOIs is greatest in \(Z\)-axis direction and least in the \(X\)-axis direction. If in-line with the \(Z\)-axis, then the \(Y\)-axis MOI will have the greatest increase and the \(X\)-axis will remain the least. This allows two options for spin axis:

- **Y-axis rotation** (major axis spin), provided the increase in the \(Y\)-axis MOI overcomes the \(X\)-axis MOI and the \(Z\)-axis MOI does not overcome the \(Y\)-axis MOI

- **Z-axis rotation** (minor axis spin), provided the increase in the \(Z\)-axis MOI does not overcome the \(X\)-axis MOI (unlikely for the \(Z\)-axis MOI to overcome the \(Y\)-axis MOI)

The inertia of the fluid tanks, including fluid within the tank, will dictate which, if either, of these options is possible. For tanks designed to be added to the system, this should be taken into consideration, to avoid unstable spin occurring during the test.

If both axes could be spun about without being the intermediate axis, the \(Z\)-axis is preferable so that the drag on the system is minimized. However, the drag due to the addition of the fluid tanks will have to be calculated for all configurations used.
5.2 Inertia Ratio Determination of Ground Based Test Model

Since the inertia values of each axis are so close in value due to the satellite’s inertia being near that of a sphere, very small changes in inertia dramatically change the inertia ratios of the satellites. The tests within this study are only able to determine the allowable inertia ratios. The ground measured inertia values show that the small variances between each SPHERES satellite due to manufacture and assembly result in very different inertia ratios (see Table 5.1), which vary up to 20%. Therefore, tests from different SPHERES satellites cannot be used together to determine the inertia ratio of a SPHERES satellite; this can only be done with the tests on the same SPHERES satellite.

<table>
<thead>
<tr>
<th>SN</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{xx}/I_{zz}$</td>
<td>1.207</td>
<td>1.247</td>
<td>1.297</td>
<td>1.075</td>
<td>1.142</td>
</tr>
<tr>
<td>$I_{yy}/I_{zz}$</td>
<td>1.060</td>
<td>1.103</td>
<td>1.108</td>
<td>1.131</td>
<td>1.016</td>
</tr>
<tr>
<td>$I_{xx}/I_{yy}$</td>
<td>1.139</td>
<td>1.131</td>
<td>1.170</td>
<td>0.950</td>
<td>1.124</td>
</tr>
</tbody>
</table>

The ground measured inertia ratios calculated by Berkowitz are used as a starting point; whenever possible, only the Z-axis dry inertia value is used. A point of uncertainty with using just the inertia ratios is that even for tests on the SPHERES satellite, the inertia of the system in the test has several opportunities for variance that are measured in tests of inertia value and not inertia ratio. The particular variances accounted for are the amount of fluid in the CO$_2$ tank and the addition of the battery pack.

In all calculations of the inertia matrix from the determined inertia ratios, it is assumed that Z-axis is the minimum axis. This is found in the dry ground measured
inertia values for all SPHERES satellites tested in microgravity for this study; the addition of CO$_2$ fluid increases the inertia in the other two axes more than the Z-axis and keeps it the minor axis. When adding mass on both the +X and -X face (i.e. a battery pack or fluid tank on each side), it is possible for the Z-axis to overcome one of the other axes in MOI and become the intermediate axis.

5.3 **Nutation Frequency of ISS Tests**

The nutation frequencies of the tests compared the predicted frequencies based off of the ground measured inertia model vary significantly from the test results, as shown in Table 5.2.

<table>
<thead>
<tr>
<th>Test</th>
<th>System</th>
<th>Axis of Rotation</th>
<th>Initial Rotation Rate $\Omega$: rad/s</th>
<th>Nutation Frequency $\lambda$: rad/s</th>
<th>$\lambda / \Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Predicted</td>
<td>Test</td>
</tr>
<tr>
<td>1</td>
<td>SPHERE</td>
<td>Z</td>
<td>0.74</td>
<td>0.126</td>
<td>0.103</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>Y</td>
<td>1.25</td>
<td>imag</td>
<td>imag</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>X</td>
<td>1.15</td>
<td>0.275</td>
<td>0.194</td>
</tr>
<tr>
<td>4</td>
<td>SPHERE + Battery Pack</td>
<td>Y</td>
<td>0.74</td>
<td>imag</td>
<td>0.052</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>Y</td>
<td>1.25</td>
<td>imag</td>
<td>0.087</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>Y</td>
<td>6.28</td>
<td>imag</td>
<td>0.459</td>
</tr>
</tbody>
</table>

A major difference between the predictions and the test results is that the ground measured model has the difference between the X-axis and Y-axis great enough that even with the addition of the battery pack, the Y-axis remains the intermediate axis. However, the tests demonstrate that the battery pack causes the Y-axis to be the major axis for all SPHERES satellites.
A trend in the results is that due to adding the battery pack, the possible nutation frequency decreases, since the intermediate and major axes are much closer in value. They also show that the inertia ratios for the SPHERES satellites are not dramatically different, although the frequencies for the 0.74 rad/s tests were very roughly estimated, due to the lack of nutation cycles.

Figure 5.3 The range of possible Inertia ratios for SN3

In Figure 5.3, the X-axis value is used as the reference and the inertia ratios based on the nutation frequency from two separate tests are used to calculate the possible Y- and Z-axes values. For the Test 6 (T3) value, the assumed battery inertia value is subtracted. So source of inertia. The varying directions of the possible inertia values are due to the X-axis being the major axis for Test 2 (T2) and the Y-axis bring the major axis for Test 6 (T3), due to the addition of the battery pack. Since the tests were one after another, there was minimal change in the amount of fluid within the tank. Based on these two tests, Z-
axis varies from 1.95-2.11*10^{-2} and Y-axis varies from 2.66-2.70*10^{-2} for the SN3 SPHERES satellite with a full tank.

Figure 5.4 Inertia Possible for SN1

Figure 5.4 assumes that the dry measured value of SN1 SPHERES satellite is correct for the Z-axis and that the effects of adding the battery and fluid at 30% fill the X-axis are known. Then the dry values for the X-axis is 2.288 and the Y-axis value is 2.180, as opposed to 2.471 as adding the battery and fluid slosh predict.

After compensating for the difference in fill fraction and the battery pack, this range of inertia values for the set the Z-axis value were used with the intermediate axis tests to find the inertia ratios that had the smoothest magnitude of the angular momentum. The angular momentum is expected to gradually decrease due to drag on the system. However, there should not be peaks or valleys at the times when the nutation angle
increases (i.e. when the rotation rate of the spin axis is at its lowest in the nutation cycle).

Figure 5.5 shows the angular momentum magnitude with the ground measured inertia values and with the inertia values from Figure 5.4 adapted from the major axis spin with the battery pack that results in the lowest magnitude of peaks/valleys.

![Angular Momentum Magnitude of the Test 2 for both the Ground and ISS Measured Inertia Values](image)

The method of optimizing the inertia ratios to best fit the magnitude of angular momentum does not work for other cases besides the intermediate axis spin. Since the intermediate axis spin is unstable, the satellite switches principal axes of rotation throughout the test and each axis crosses the zero value at least once in the test. The dramatic switching of axes allows for definite viewing of magnitude changes in the angular momentum vector that are due to inaccurate inertia ratios and not disturbances of the physical system.

In the minor/major axis spin tests, the peaks and valleys in the magnitude of angular momentum over time due to nutation are visible, but only slightly. Table 5.3 shows the calculated ISS best estimates of inertia of SPHERES satellite SN1 and SN3.
with the applicable fill fraction, based on all tests. Tests 1 and 4 were performed on SN2 and the data collected did not allow for calculation of the inertia matrix of that particular SPHERES satellite.

Table 5.3 ISS Measured Inertia Values

<table>
<thead>
<tr>
<th>SN</th>
<th>Fill Fraction of CO₂ Tank</th>
<th>(kg m²)*10²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>lxx</td>
</tr>
<tr>
<td>1</td>
<td>Dry</td>
<td>2.3245</td>
</tr>
<tr>
<td></td>
<td>48%</td>
<td>2.4357</td>
</tr>
<tr>
<td></td>
<td>30% + Battery</td>
<td>2.4939</td>
</tr>
<tr>
<td>3</td>
<td>Dry</td>
<td>2.7350</td>
</tr>
<tr>
<td></td>
<td>68%</td>
<td>2.8550</td>
</tr>
<tr>
<td></td>
<td>65% + Battery</td>
<td>2.8640</td>
</tr>
</tbody>
</table>

5.4 Nutation Time Constant Calculation

For major axis spin of the SPHERES satellite only (Test 3), the maximum nutation angle increases from 0.0715 to 0.073 radians from 92 seconds to 189 seconds (see Figure 5.6 (a)), resulting in a Nutation Time Constant (NTC) of 4672. Since the NTC is positive, this indicates that the nutation angle is growing, which is the opposite of what is expected for spin about a major axis. For that NTC, the nutation angle doubles every 3238 seconds.
Figure 5.6 Nutation Angle of (a) Test 3: Major Axis Spin (+X) Test (b) Test 5: Major Axis Spin of the SPHERES + Battery Pack (+Y) Test: Rotation Rate 1.25 rad/s (c) Test 6: Rotation Rate 6.28 rad/s

For major axis spin of the SPHERES satellite with battery pack at 1.25 rad/s (Test 5), the maximum nutation angle decreases from 0.0719 to 0.0708 radians from 80 to 224 seconds (see Figure 5.6 (b)), resulting in a NTC of -9340. Since the NTC is negative, this indicates that the nutation angle is decaying, which is as expected for spin about a major axis. For that NTC, the nutation angle halves every 6474 seconds.

When examining the nutation angle for every cycle, in both tests, the nutation angle goes both up and down. However, the NTCs listed above show the general trend for those tests.
This system has a prominent external force in the atmospheric drag and it does not affect the rotation rates about all axes the same. The drag is proportional to the square of the velocity, so the axes spinning faster will also not decrease at linearly the same rate as the slower spinning axes. Also, since the coefficient of drag is different for the axes, some will slow down faster than others. In the major axis spin of the SPHERES satellite only test, the coefficient of drag on the Z-axis is significantly lower than that of either the X- or Y-axis, and apparently that factor combines with the lower velocity to increase the nutation angle instead of decreasing it, as the inertia matrix of the SPHERES satellite would suggest. The major axis spin of the SPHERES satellite with altered inertia matrix at 1.25 rad/s test (Test 5) adds a battery pack to the SPHERES satellite. This would expectedly increase the coefficient of drag on all axes, but would have the greatest effect on the Z-axis. This evening out allows the inertia matrix to dominate over the atmospheric drag in effect on the nutation angle growth/decay.

5.5 Open Loop Thrust

Torque ($\tau$) is calculated from the cross product of the displacement vector ($r$) and the force vector ($F$). For calculating the open loop thrust of the SPHERES satellite, the displacement vector is the distance from the center of mass to the location of each firing thruster. The force vectors due to each thruster are assumed to be only in the Y plane (i.e. force vectors out of the Y plane are negligible). For constant torque, the angular acceleration $\dot{\omega}$ is also constant and can therefore be used to find the angular velocity.

$$\tau = r \times F$$
\[ \omega = \frac{\tau}{I} \]

5.6

Table 5.4 Various Calculations of Rotation Rate Increase

<table>
<thead>
<tr>
<th>Model for ( I_y ) Used</th>
<th>Angular Acceleration ( \text{rad/s}^2 )</th>
<th>Increase in Angular Velocity ( \text{rad/s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground Measured SN3 Full Tank + Battery</td>
<td>0.942</td>
<td>6.78</td>
</tr>
<tr>
<td>ISS Measured SN3 Full Tank + Battery</td>
<td>0.723</td>
<td>5.21</td>
</tr>
<tr>
<td>Test (Actual)</td>
<td>0.678</td>
<td>4.88</td>
</tr>
</tbody>
</table>

The results in Table 5.4 show how much closer the ISS Measured Inertia Values calculated increase in angular velocity is to the actual increase produced by open loop thrust than that calculated by the Ground Measured Inertia Values. The difference between the ISS Measured Inertia Values calculated increase in angular velocity and the actual increase in angular velocity are assumed to be due to thruster variance and atmospheric drag.

While the SPHERES satellite is in controlled rotation, the X- and Z-axis rotation rates are controlled to within 0.05 rad/s of 0.00 rad/s. Therefore, when the control is turned off and the SPHERES satellite is drifting, the centerline of the rotation rate for those axes is expected to be -0.05 to 0.05 rad/s. This occurs for all other tests, except for tests in which the rotation rate is set with an open loop thrust instead of closed loop estimation.
Chapter 6

Conclusions

This chapter includes the summary of this thesis and its contributions. It also makes recommendations for future work for this study.

6.1 Thesis Summary

This thesis discusses the issues involving fluid slosh and nutation on spinning spacecraft pertaining to prediction. It develops an approach to perform fluid slosh tests in microgravity on a simplified spacecraft system by using the SPHERES Testbed on the ISS. The work performed characterizes the nutation of the SPHERES satellite and the environment of testbed in preparation for performing spinning tests on the SPHERES with fluid tanks attached. The work also includes horizontal testing of fluid tanks on the ground, as a method of characterizing the fluid slosh frequencies of a tank prior to testing in microgravity (on the ISS).
6.2 Contributions

This work developed and verified a method of fundamental frequency determination of a fluid tank in a ground based lab. It has determined the inertia properties of the SPHERES satellites in microgravity. It also characterized the ISS environment and expected external disturbances. This will allow for future physical tests to better understand the mass properties of the spacecraft and the abilities of the SPHERES satellites to perform physical tests.

6.3 Future Work

This study can be continued by conducting microgravity tests with fluid tanks attached. There are two options for fluid tanks, based on the guidelines set for testing on the ISS: SPHERES CO₂ tanks and fluid tanks sent up to the ISS. Both of these tanks will need to be ground tested to determine their slosh frequencies prior to testing on the ISS.

By testing the fluid containers on Earth and knowing the expected location of the container and spin rate on the SPHERES satellite, the tests will be able to determine the first modes of longitudinal and transverse slosh expected to be observed in microgravity. These tests can then determine if slosh is expected to resonate with nutation and which mode it will be, or preferable use this information to develop tanks with fluid levels that will produce slosh to resonate with the nutation.

If possible, conduct the ISS tests with the SPHERES on-axis propellant CO₂ tank nearly empty. This will reduce its effect on the system, so that the attached off-set fluid tank(s) is the main contributor to the energy dissipation. Also, test on the ISS both with
the off-set tank empty or with a rigid mass inside in addition and with fluid in the tank. This allows observation of the effect of adding the mass of the tanks along with the increased atmospheric drag, with and without the motion due to the fluid.

1. Horizontal Slosh Ground Tests
   - Perform on containers intend to use as offset spinners on ISS (CO$_2$ or designed)
     - Longitudinal
     - Lateral
   - Make sure to:
     - Test heavily around fill fraction intended for use on ISS (i.e. test at a greater frequency in the range of fill fractions near the ISS fill fraction than for fill fractions outside of that range).
     - Collect data over a time length adequate to view low frequencies

2. Characteristics of the Instrumented Fluid Tanks (to be sent to ISS and then attached to SPHERES satellites for slosh testing)
   - Clear, to be able to see the slosh occurring in the tanks
   - Specially machined, to be able to choose a tank shape that will best correlate to the tank of interest (i.e. launch vehicle or spacecraft tanks)
   - Filled with specified fluid, to be able to choose a fluid for correlation to a chosen launch vehicle or spacecraft fluid and possibly color for viewing the slosh
   - Have attach points, to be able to have the best connection to the SPHERES satellite and reduce disturbances to the system and attenuation of the slosh forces to the sensors due to attachment
• Contain greater fluid than currently present in the SPHERES satellite, to be able to increase effect of slosh on the satellite

• Have sensors with capabilities beyond those present on the SPHERES satellites, to be able to better observe the fluid slosh
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    http://www.srh.noaa.gov/jetstream/atmos/pressure.htm


