Massachusetts Institute of Technology

16.07 Dynamics

Problem Set 1

Out date: Sep 08, 2004

Due date: Sep 15, 2004

	Time Spent [minutes]
Problem 1	
Problem 2	
Study Time	

Turn in **each problem** *on* **separate sheets** *so that grading can be done in parallel*

Problem 1 - Vectors Review

Part A

An observer on the ground sees an aircraft flying with velocity $(0m/s, 100m/s, -10m/s)$ relative to a fixed set of axis $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$. Another observer measures that the same aircraft is flying with velocity $(50m/s, 86.6m/s, 10m/s)$ using a different, but also fixed, set of axis $\boldsymbol{i}', \boldsymbol{j}', \boldsymbol{k}'$.

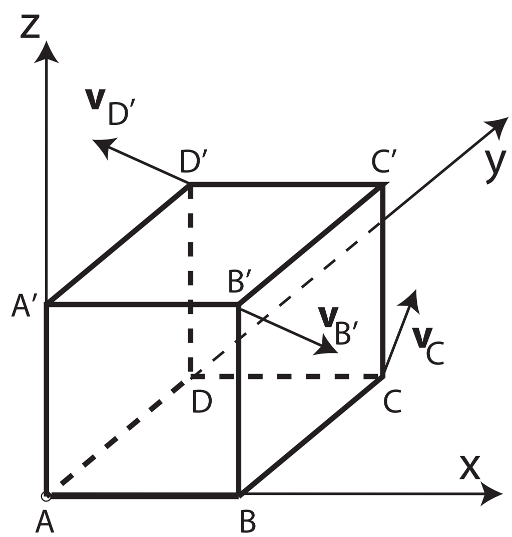
a) If we know that $\boldsymbol{k} = -\boldsymbol{k}'$, determine the components of the unit vectors $\boldsymbol{i}', \boldsymbol{j}', \boldsymbol{k}'$, referred to $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$.

b) Calculate the components of the matrix, $[T]$, that transforms $\boldsymbol{i}', \boldsymbol{j}', \boldsymbol{k}'$ coordinates, into $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ coordinates. Verify that $[T]^T = [T]^{-1}$.

c) If the *first* observer measures an acceleration of magnitude $3m/s^2$, along the direction of the vector $(-1, 1, 1)$, calculate what will be the acceleration (magnitude and direction) seen by the *second* observer.

Part B

The rigid cube $ABCDA'B'C'D'$ whose edges are $1m$ long is free to rotate about its fixed vertex A. At the instant shown the cube is aligned with the coordinate system xyz. What is the velocity $v_{D'}$ of vertex D' if the velocities of vertices C and B' are $v_C = (-4m/s, 4m/s, 0m/s)$ and $v_{B'} = (2m/s, 2m/s, -2m/s)$?



Part C
Consider the vector in cartesian coordinates $\boldsymbol{r}(t) = (e^t \cos t^2, e^t \sin t^2, 0)$.

1.- Calculate the derivative of this vector with respect to t and give its components in the directions parallel and normal to $\boldsymbol{r}$.

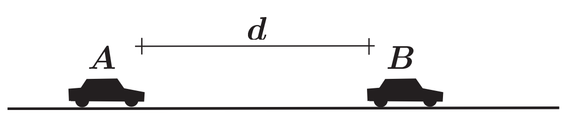
2.- Calculate the time derivative of the modulus of $\boldsymbol{r}$ with respect to t.

3.- Since the vector $\boldsymbol{r}(t)$ does not have a constant direction, calculate its angular velocity, $\boldsymbol{\Omega}(t)$.

4.- How do you relate the results of sections 2 and 3 to the components of the derivative calculated in section 1?

Problem 2 - Rectilinear Kinematics

Part A
Car B is travelling a distance d, ahead of car A. Both cars are travelling at the same speed of 70 ft/s, when the driver of B suddenly applies the brakes, causing his car to decelerate at 13 ft/s^2. It takes the driver of car A 0.8 s to react (this is a normal reaction time for drivers). When he applies his brakes, he decelerates at 16 ft/s^2. We want to determine the minimum distance d between the two cars so as to avoid a collision.



Part B
We consider a car of mass $m = 1000$ kg travelling with speed $v_0 = 50$ m/s on a horizontal straight road. The only force on the car is the aerodynamic drag force which is of the form $k_2 v^2$ (this is the usual form of the aerodynamic drag), where $k_2 = 6$ kg/m. Assume that at $t = 0$, $s = 0$. Determine,

- how long will it take to reach a distance $s = 500$ m?
- how long will it take to reach a distance $s = 1500$ m?

Now, assume that the drag force is of the form $k_1 v$ instead (this is the from of the drag force at *very* low speed) where $k_1 = 50$ kg s. Determine,

- how long will it take to reach a distance $s = 500$ m ?
- how long will it take to reach a distance $s = 1500$ m ?