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Massachusetts Institute of Technology

### 16.07 Dynamics

## Problem Set 3

Out date: Sep 22, 2004
Due date: Sep 29, 2004

|  | Time Spent [minutes] |
| :---: | :---: |
| Problem 1 |  |
| Problem 2 |  |
| Problem 3 |  |
| Problem 4 |  |
| Study Time |  |

Turn in each problem on separate sheets so that grading can be done in parallel

## Problem 1

## Part A

An amusement ride called the "corkscrew" takes the passengers through the descending cylindrical helix. The equation of the helix is given, in cartesian coordinates, by $\boldsymbol{r}(\theta)=$ $R \cos \theta \boldsymbol{i}+R \sin \theta \boldsymbol{j}-h \theta \boldsymbol{k}$. If we know that the speed of the passenger car is constant and equal to $v_{0}$, give an expression for the velocity, the acceleration and the binormal vectors.

## Part B

The large wheel of the quick-return mechanism rotates in the counterclockwise direction at a constant speed of 120 rpm (revolutions per minute). Determine the velocity and acceleration of point $P$ as well as $s, \dot{s}$ and $\ddot{s}$, when $\theta=30^{\circ}$. Use $R=1 \mathrm{~m}, h=1.5 \mathrm{~m}$ and $L=3 \mathrm{~m}$.


## Problem 2

## Part A

The chain is released from rest with the length b of overhanging links just sufficient to initiate motion. The coefficients of static and kinetic friction between the links and the horizontal surface have essentially the same value $\mu$. Show that the velocity of the chain when the last link leaves the edge is

$$
v=\sqrt{\frac{g L}{1+\mu}}
$$

Neglect any friction at the corner.


## Part B

The figure shows a rigid regular cube, of size $4 \mathrm{~m} \times 4 \mathrm{~m} \times 4 \mathrm{~m}$ at time $t=0$. The cube is rotating about the $z$ axis with a constant angular velocity of $\boldsymbol{\omega}=5 \boldsymbol{k} \mathrm{rad} / \mathrm{s}$. Calculate the time derivative of the vectors $\overrightarrow{A B}, \overrightarrow{A A^{\prime}}$ and $\overrightarrow{D C^{\prime}}$, which are attached to the cube (you can think of them as being painted on the cube). You should do this problem in two ways:

- Calculate the positions of the points $A, A^{\prime}, B, C^{\prime}$ and $D$ as a function time. Obtain an expression for the vectors $\overrightarrow{A B}, \overrightarrow{A A^{\prime}}$ and $\overrightarrow{D C}^{\prime}$ and calculate their derivative directly.
- Review your notes on vectors and utilize the appropriate vector equation involving the angular velocity vector.



## Problem 3

Part A (Meriam/Kraige)
The motor unit $A$ is used to elevate the 300 kg cylinder at a constant rate of $2 \mathrm{~m} / \mathrm{s}$. If the power meter $B$ registers an electrical input of 2.30 kW , calculate the combined electrical and mechanical efficiency $e$ of the motor.


Part B (Meriam/Kraige)
The $3-\mathrm{kg}$ sphere is carried by the parallelogram linkage where the spring is unstretched when $\phi=90^{\circ}$. If the mechanism is released from rest at $\phi=90^{\circ}$, calculate the velocity vector, $\boldsymbol{v}$, of the sphere when the position $\phi=135^{\circ}$ is passed. The links are in the vertical plane and their mass is small and may be neglected.


## Problem 4

## Part A

Determine the minimum height $h$ at which the 200 gram toy car may be released to successfully traverse the elliptical hoop. Friction is negligible. Determine the force that the track exerts on the car the instant before the car reaches point $A$ and the instant after the car passes through $A$. Point A is the point on the track at the end of the hoop.


Hint : Recall that the equation of an ellipse of of semi-axes a and b is given by $\boldsymbol{r}(\theta)=$ $(a \cos \theta, b \sin \theta)$.

## Part B

A 100 kg payload is dropped from a height of 20km above the earth's surface. The drag force is given by $D=k_{2} v^{2}$ where $k_{2}=(1 / 36) \mathrm{kg} / \mathrm{m}$, and $v$ is the velocity. Determine the magnitude of the terminal velocity, $v_{f}$ (The terminal velocity is the value of the velocity for large values of time). Calculate the total energy dissipated by the drag force assuming:

- a constant gravity acceleration of $g=9.8 \mathrm{~m} / \mathrm{s}^{2}$.
- a gravity acceleration which is a function of the height above the earth's surface (consider $g=9.8 \mathrm{~m} / \mathrm{s}^{2}$ on the earth's surface and an earth radius of 6400 km ).

Hint : you do not need to integrate the equation of motion to solve this problem, even though, for the constant gravity case, it is not hard to do.

