NAME : ....

Massachusetts Institute of Technology

16.07 Dynamics

Problem Set 6

Out date: Oct 13, 2004

Due date: Oct 20, 2004

	Time Spent [minutes]
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Study Time	

Turn in **each problem** *on* **separate sheets** *so that grading can be done in parallel*

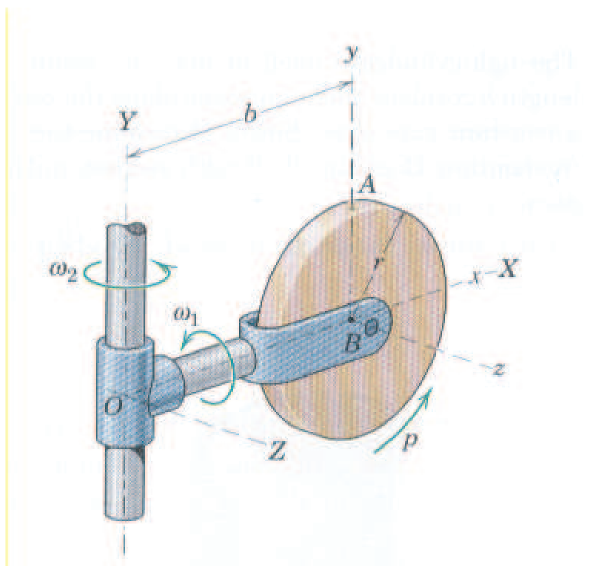
Problem 1

Part A

Suppose a particle follows a *planar* path such that the component of its velocity perpendicular to its position vector $\boldsymbol{r}$ is inversely proportional to the magnitude of $\boldsymbol{r}$. Show analytically that the particle's acceleration is always directed along $\boldsymbol{r}$.

Part B

The disk of radius r rotates about its z axis with a constant angular velocity p, and the yoke in which it is mounted rotates about the X axis trough OB with a constant angular velocity ω_1. Simultaneously, the the whole assembly rotates about the fixed Y axis through O with a an angular velocity ω_2 and and angular acceleration $\dot{\omega}_2$. Determine the velocity and acceleration of point A at the instant shown.



Problem 2

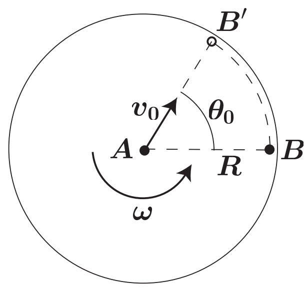
Part A

A projectile is thrown vertically upwards (as measured by an observer standing on the earth surface) with an initial velocity of 250 m/s at a latitude of $L = 45^o$. Determine the position where it will land relative to the position from which it was fired. Neglect air friction and the variations of gravity with the height.

Part B(MMS)

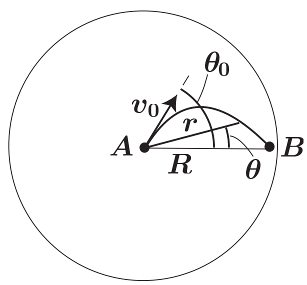
Two astronauts are in space inside a cylindrical spacecraft. The spacecraft is spinning at a constant $\boldsymbol{\omega}$ (out of the plane). Astronaut A who is at the center of the spacecraft wants to toss a screwdriver to astronaut B who is at the rim, at a distance R. Knowing about Newton's first law, A tosses ahead of B with velocity $\boldsymbol{v}_0$, such that B will be there (at B') to meet the screwdriver, i.e. astronaut A throws at an angle θ_0 such that

$$\frac{R}{v_0} = \frac{\theta_0}{\omega} \qquad (= \text{flight time})$$



VIEW FROM INERTIAL FRAME

In an inertial axis the motion is a straight line and everything is simple. However, we are interested in finding out what is the motion of the screwdriver as observed by the astronauts.



VIEW FROM ROTATING FRAME

Derive the equation of motion in cylindrical coordinates that will be observed by the astronauts.
Show, by substitution, that

$$r = \frac{v_0}{\omega}(\theta_0 - \theta) \quad ,$$

is the solution of the equations of motion you have derived. (This curve is known as a backwards Archimedes spiral.)

Problem 3

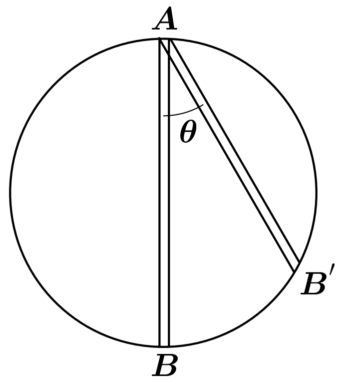
Part A
Suppose we drill a hole $A - B$ through the center of the earth and drop a mass at A.

1.- Write the equation of motion for the mass and solve it *(neglect friction effects and assume a spherical homogeneous earth).*

2.- Determine the time (in minutes) it takes for the mass to reach point B.

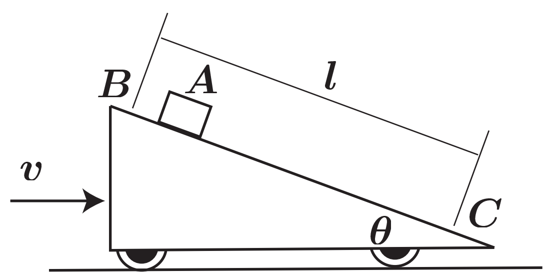
3.- If the mass is dropped with a downwards initial velocity of 100 m/s. How high will it go at the other end?

Repeat 1 and 2 above for a hole $A - B'$ which doesn't go through the center of the earth and which is at an angle θ with $A - B$.



Part B

The small slider A moves with negligible friction down the tapered block, which moves to the right with constant speed $v = v_0$. Use the principle of work and energy to determine the magnitude v_A of the absolute velocity of the slider as it passes point C if it is released at point B with no velocity relative to the block. Apply the equation, both as an observer fixed to the block and as an observer fixed to the ground, and reconcile the two relations.



Hint: $v_A = [v_0^2 + 2gl\sin\theta + 2v_0\cos\theta\sqrt{2gl\sin\theta}]^{1/2}$.

Problem 4

Part A

Find the angular acceleration of the disc D as a function of the angular velocities and accelerations given in the diagram. The angle of $\boldsymbol{\omega}_1$ with the horizontal is ϕ.

Part B

An pendulum of length l is installed inside an elevator. Determine the period of oscillation of the pendulum as a function of the upward acceleration, a, of the elevator. What happens when $a = -g$?