Name:.....

Massachusetts Institute of Technology

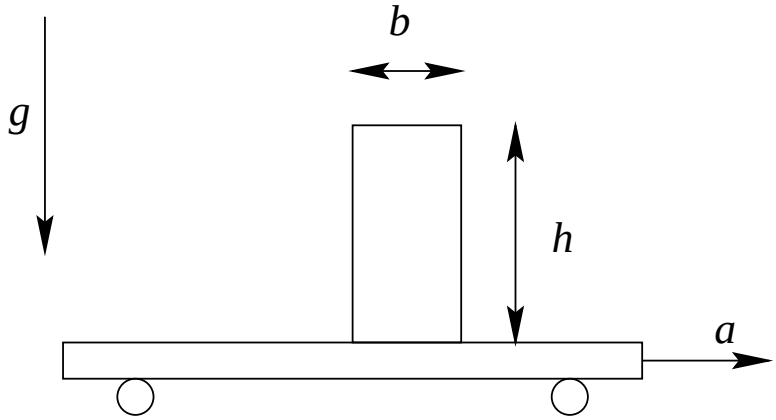
16.07 Dynamics

Final Exam

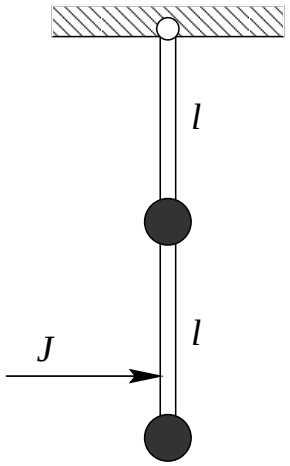
December 15, 2004

	Points
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7	
Problem 8	
Problem 9	
Problem 10	
Problem 11	
Total	

1. (9 points) What is the maximum acceleration which can be given to the cart without tipping over the container with dimensions h and b? Assume that friction is large enough to prevent any slipping.



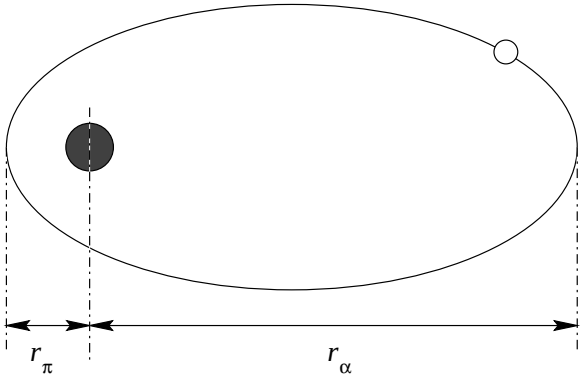
2. (9 points) A pendulum consists of a massless rigid rod of length $2l$ and two equal masses. Where should the impulse J be applied so that there is no horizontal reaction at the point from which the pendulum is hanging?



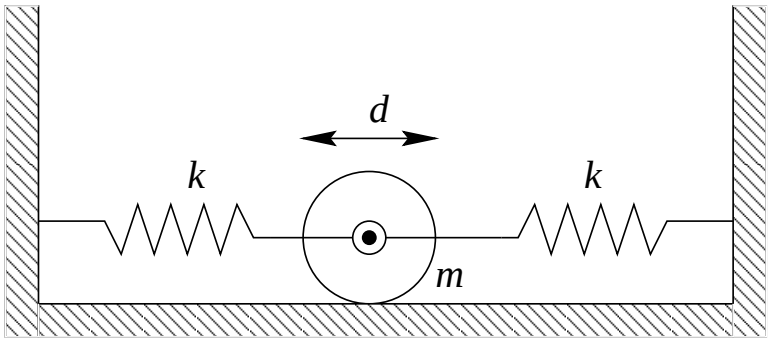
3. A small satellite orbiting the Earth has an elliptical orbit with distances from the Earth center to the perigee and apogee being r_π and r_α, respectively. The radius of the Earth is R and and acceleration of gravity on the surface is g.

   a) (5 points) What is the minimum Δv needed for the satellite to escape the gravitational influence of the Earth?

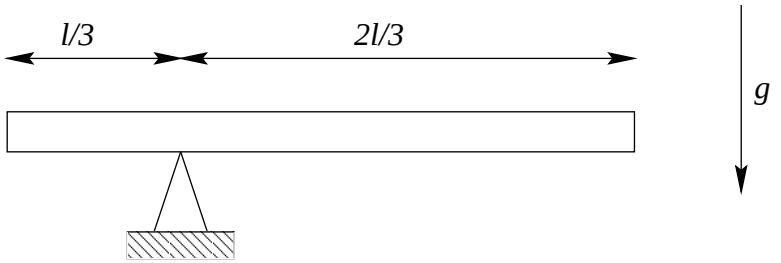
   b) (4 points) What is the radius of curvature of the elliptical orbit at apogee?



4. (9 points) The axis of a uniform disk of diameter d and mass m is connected to two springs with stiffness k as shown in the figure. What is the period of small oscillation of the system? Assume that the disk does not slip.



5. (9 points) A bar of total length l and mass m is released from rest at the position shown. What is the reaction force (magnitude and direction) at the support immediately after the release?



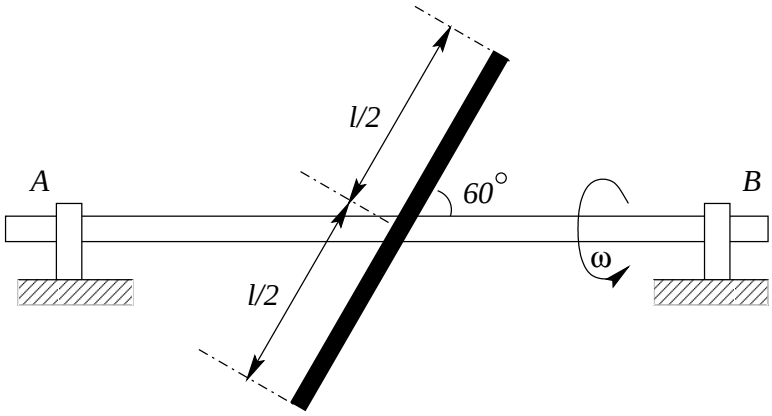
6. A solid bar of length l is welded to a shaft rotating with constant angular velocity ω. The mass of the bar is m and the bar is in the plane of the figure at the instant shown .

   a) (3 points) Find the inertia tensor I of the bar.

   b) (3 points) Find the angular momentum $\vec{H}$ of the bar.

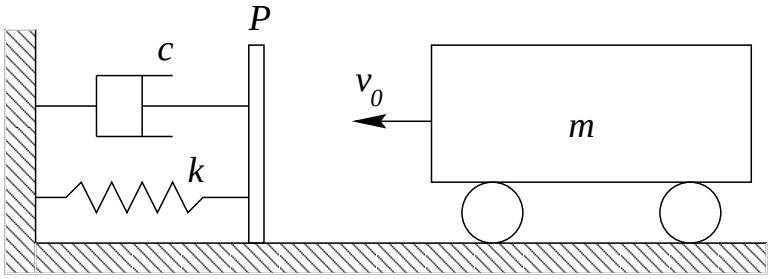
   c) (3 points) Find the kinetic energy T of the bar.

   Specify the coordinate system you are using.

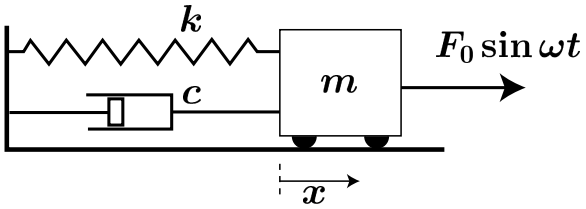


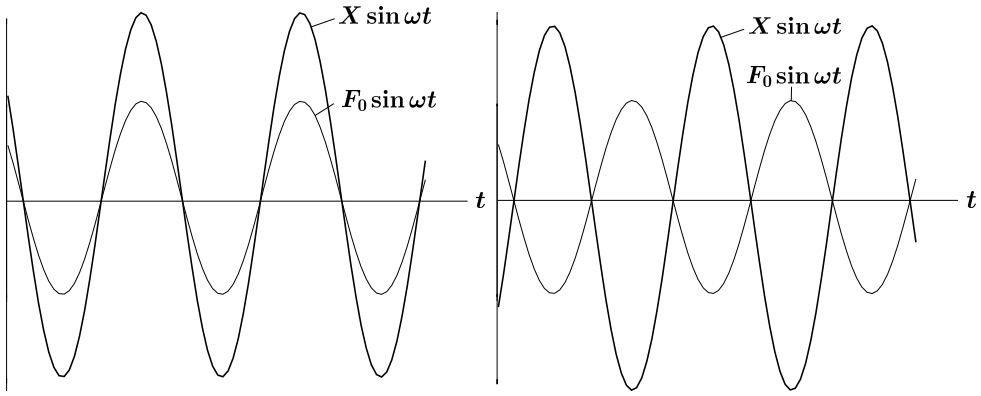
7. (9 points) A jet engine rotor has mass m and radius of gyration k_G. The center of gravity of the rotor is at distance l from the two supporting bearings. What are the **magnitudes** of the horizontal forces exerted by the two bearings on the rotor if the airplane is flying with velocity v along a horizontal circular trajectory with radius ρ. The dimensions of the airplane are small relative to the radius ρ.

8. (9 points) A cart of mass $m = 12$ kg is moving with velocity $v_0 = 8$ m/s when it hits the spring damper system shown in the figure. What is the resulting maximum deflection of the spring? The spring constant is $k = 48$ N/m and the damping coefficient is $c = 24$ Ns/m. Assume that the plate P has negligible mass so that there is no rebound.



9. (9 points) The lightly damped system shown below is forced by an external force $F = F_0 \sin \omega t$. For two different values of the mass the motion $x = X \sin \omega t$ looks as shown in A) and B). In which case is the mass m larger? Why?



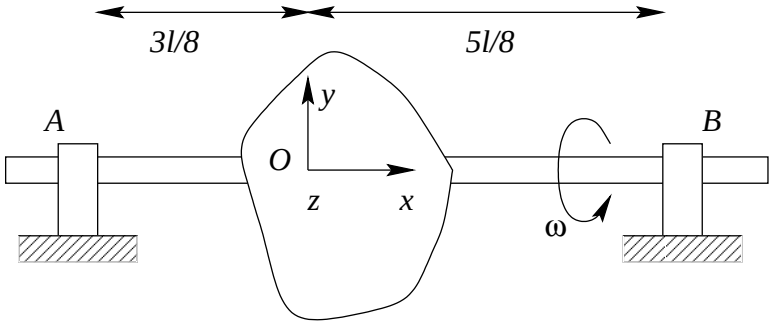


A) B)

10. (9 points) A body with mass m is welded to a shaft rotating at a constant angular velocity ω. The center of mass of the body O lies on the shaft and its inertia tensor in the body fixed $Oxyz$ frame shown in the figure has the form:

$$I = \begin{bmatrix} 3Q & -Q & 0 \\ -Q & 3Q & 0 \\ 0 & 0 & 2Q \end{bmatrix}$$

At the instant shown determine the **horizontal** reactions at the bearings A and B.



11. (10 points) A two degree of freedom system consists of a solid block with mass $m_1 = 4$ kg and a pendulum with length $l = 4.9$ m. The mass of the bob is $m_2 = 7$ kg and the spring stiffness is $k = 9$ N/m. Write down the equations of motion of the system. What are the natural frequencies of the system? As always, the acceleration due to gravity is $g = 9.8$ m/s^2.