1. Complex Exponential Amplitude Modulation
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The Concept of Modulation

Why?
- More efficient to transmit E&M signals at higher frequencies
- Transmitting multiple signals through the same medium using different carriers
- Transmitting through “channels” with limited passbands
- Others...

How?
- *Many* methods
- Focus here for the most part on *Amplitude Modulation (AM)*

\[ y(t) = x(t)c(t) \]
Amplitude Modulation (AM) of a Complex Exponential Carrier

c(t) = e^{j\omega_c t}, \quad \omega_c \text{ – carrier frequency}

y(t) = x(t)e^{j\omega_c t}

Y(j\omega) = \frac{1}{2\pi} X(j\omega) * C(j\omega)

= \frac{1}{2\pi} X(j\omega) * 2\pi \delta(\omega - \omega_c)

= X(j(\omega - \omega_c))
Demodulation of Complex Exponential AM

$$e^{j\omega_c t} = \cos\omega_c t + j \sin\omega_c t$$

Corresponds to two separate modulation channels (quadratures) with carriers 90° out of phase
Sinusoidal AM

\[ Y(j\omega) = \frac{1}{2\pi} X(j\omega) * \pi \{ \delta(\omega - \omega_c) + \delta(\omega + \omega_c) \} \]

\[ = \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c)) \]

Drawn assuming \( \omega_c > \omega_M \)
Synchronous Demodulation of Sinusoidal AM

Suppose $\theta = 0$ for now, $\Rightarrow$ Local oscillator is in phase with the carrier.
Synchronous Demodulation in the Time Domain

\[ w(t) = y(t) \cos \omega_c t = x(t) \cos^2 \omega_c t = \frac{1}{2} x(t) + \frac{1}{2} x(t) \cos 2\omega_c t \]

Then \[ r(t) = x(t) \]

Now suppose there is a phase difference, \( i.e. \theta \neq 0 \), then

\[ w(t) = y(t) \cos (\omega_c t + \theta) = x(t) \cos \omega_c t \cos (\omega_c t + \theta) \]
\[ = \frac{1}{2} x(t) \cos \theta + \frac{1}{2} x(t) (\cos (2\omega_c t + \theta)) \]

Now \[ r(t) = x(t) \cos \theta \]

Two special cases:
1) \( \theta = \pi/2 \), the local oscillator is 90° out of phase with the carrier, \( \Rightarrow r(t) = 0 \), signal unrecoverable.
2) \( \theta = \theta(t) \) — slowly varying with time, \( \Rightarrow r(t) \equiv \cos[\theta(t)] \cdot x(t) \), \( \Rightarrow \) time-varying “gain”.
Synchronous Demodulation (with phase error) in the Frequency Domain

Demodulating signal – has phase difference $\theta$ w.r.t. the modulating signal

$$\cos(\omega_c t + \theta) = \frac{1}{2} e^{j\theta} e^{j\omega_c t} + \frac{1}{2} e^{-j\theta} e^{-j\omega_c t}$$

\[\downarrow \mathcal{F}\]

$$\pi e^{j\theta} \delta(\omega - \omega_c) + \pi e^{-j\theta} \delta(\omega + \omega_c)$$

Again, the low-frequency signal ($\omega < \omega_M$) = 0 when $\theta = \pi/2$. 
Alternative: Asynchronous Demodulation

- Assume $\omega_c >> \omega_M$, so signal envelope looks like $x(t)$
- Add same carrier with amplitude $A$ to signal

\[ y(t) = (A + x(t)) \cos \omega_c t \]

\[ A = 0 \Rightarrow \text{DSB/SC (Double Side Band, Suppressed Carrier)} \]
\[ A > 0 \Rightarrow \text{DSB/WC (Double Side Band, With Carrier)} \]
Asynchronous Demodulation (continued)

Envelope Detector

Disadvantages of asynchronous demodulation:
— Requires extra transmitting power $[A \cos \omega_c t]^2$ to make sure $A + x(t) > 0 \Rightarrow$ Maximum power efficiency = 1/3 (P8.27)

In order for it to function properly, the envelope function must be positive for all time, \textit{i.e.} $A + x(t) > 0$ for all $t$.

\textbf{Demo:} Envelope detection for asynchronous demodulation.

\textit{Advantages} of asynchronous demodulation:
— Simpler in design and implementation.

\textit{Disadvantages} of asynchronous demodulation:
— Requires extra transmitting power $[A \cos \omega_c t]^2$ to make sure $A + x(t) > 0 \Rightarrow$ Maximum power efficiency = 1/3 (P8.27)
Double-Sideband (DSB) and Single-Sideband (SSB) AM

Since $x(t)$ and $y(t)$ are real, from conjugate symmetry both LSB and USB signals carry exactly the same information.

DSB, occupies $2\omega_M$ bandwidth in $\omega > 0$.

Each sideband approach only occupies $\omega_M$ bandwidth in $\omega > 0$. 

USB

LSB
Single Sideband Modulation

Can also get SSB/SC
or SSB/WC
Frequency-Division Multiplexing (FDM)
(Examples: Radio-station signals and analog cell phones)

All the channels can share the same medium.
FDM in the Frequency-Domain

"Baseband" signals

Channel a

Channel b

Channel c

Multiplexed signals
Demultiplexing and Demodulation

Channels must not overlap $\Rightarrow$ Bandwidth Allocation

It is difficult (and expensive) to design a highly selective bandpass filter with a tunable center frequency

Solution – Superheterodyne Receivers
The Superheterodyne Receiver

**Operation principle:**
- Down convert from $\omega_c$ to $\omega_{IF}$, and use a coarse tunable BPF for the front end.
- Use a sharp-cutoff *fixed* BPF at $\omega_{IF}$ to get rid of other signals.

**Mathematical Expressions:**

\[
\frac{\omega_c}{2\pi} = 535 - 1605 \text{ kHz} \quad \text{— RF}
\]

\[
\frac{\omega_{IF}}{2\pi} = 455 \text{ kHz} \quad \text{— IF}
\]