TIME PATTERN ANALYSIS IN SCHOOL SCHEDULING

by

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S.B., Massachusetts Institute of Technology (1964)

S.M., Massachusetts Institute of Technology (1966)

SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June, 1975

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Submitted to the Alfred P. Sloan School of Management on May 2, 1975 in partial fulfillment of the requirements of the Degree of Doctor of Philosophy

ABSTRACT

Several times each year, an academic institution undertakes the process of choosing times for each of its classes and assigning instructors, rooms, and students to these classes. The school scheduling problem is to choose times and assign resources in such a way as to permit the school to function effectively and efficiently. The configurations of time used in establishing the schedule—time patterns—play a critical role in the scheduling decision process.

Time pattern analysis is an important part of school scheduling, focusing on the mechanics of the process with particular attention to the variables involving time. For any school, appropriate time pattern families can be identified. The thesis is that time pattern analysis—the identification of appropriate time pattern families—is a central and critical aspect of a sound systems approach to school scheduling, in that an error in time pattern design may detrimentally pervade (if not dominate) other aspects of school scheduling, and a proper design may beneficially pervade the system.

The school scheduling process is considered in relation to other scheduling problems, and in terms of general approach. The problem is formalized with emphasis on rigorous definition of the time variables and their interactions. Time pattern analysis is described in terms of impact of time variables upon the system. Following a discussion of intuitive arguments and some combinatorial analysis, a study is made of normative models. One of the models is identified as an ideal normative model, characterized by extremely well-behaved time pattern families.

Two case studies are introduced and documented. The Minuteman Regional Vocational Technical School, which opened in September 1974 in Lexington, Massachusetts, was the object of a two year study, culminating in an implemented time pattern system considered an improvement over the originally planned system. The Massachusetts Institute of Technology, which has used computer-assisted scheduling for more than a decade, was studied with respect to several alternative time pattern systems. The M.I.T. case study includes simplifications to as well as digressions from the current approach. The thesis concludes with an extrapolation from these two case studies to the general academic environment.

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Title: Assistant Professor of Management
ACKNOWLEDGEMENTS

I wish to express my appreciation to the administration of the Minuteman Regional Vocational Technical School in Lexington, Massachusetts, for their cooperation in the Minuteman case study. I am also grateful to the Office of the Registrar, the Schedules Office, and the Departmental scheduling officers at the Massachusetts Institute of Technology for their cooperation in the M.I.T. case study. The unexpected death, in 1973, of Irene Ezer from the M.I.T. Schedules Office was a loss to me of a friend and associate, as well as a loss to the Institute.

I appreciate the effort of Prof. Malcolm Jones who served as my thesis advisor and committee chairman during my first year of research in 1972-73. During that year, I was awarded an Education Research Center Fellowship, funded by a grant from the Carnegie Corporation of New York, and I am grateful for this support.

Throughout three years of research and writing, I have been the beneficiary of much patience and encouragement from friends, relatives, and associates. Space prohibits my thanking everyone individually here, but I do wish to single out my friends the Cannells, who gave me substantial incentive to organize the final version of the thesis. I also wish to thank Alice Drake for her assistance in proofreading the final draft.

Finally, I wish to thank my committee, whose patience and positive suggestions kept me going: Dr. Frederick Frick, Prof. J. Herbert Hollomon, Dr. Edwin Taylor, and my thesis advisor and committee chairman, Prof. Stuart Madnick.

May 1975

James Landon Linderman
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CHAPTER ONE: THE SCHOOL SCHEDULING PROBLEM

1.1.1 Introduction

Several times each year, an academic institution is concerned with the process of choosing times for each of its classes and then assigning instructors, rooms, and students to these classes. The school scheduling problem is to choose times and assign resources in such a way as to permit the school to function with some degree of satisfaction on the part of the community involved. For some schools, the emphasis is on the sheer feasibility of functioning at all. For others, refinement can be pursued among alternatives until a variety of objectives are satisfied.

This thesis is concerned with a central aspect of the school scheduling process—the configurations of time used in establishing a school's schedule. Like most decision processes, much can be said for taking a systems approach. A systems approach to school scheduling requires careful attention to the variables involving time, since the times chosen for classes directly affect many quantitative and qualitative measures: conflicts between classes, personal work schedules, resource utilization and loading, preparation time (for instructors), individual study time (for students), etc.

The process of scheduling a school must, of course, be subordinate to the overall educational objectives of the school. The unfortunate paradox of school scheduling is that the highest education aspirations of a school can be frustrated by a schedule supposedly designed to implement
them. Many schools are faced with the mechanics of scheduling at odds with the objectives of scheduling, creating a problem unduly out of proportion. Typically, a school has enough problems to cope with without getting bogged down in scheduling concerns.

Often seemingly remote from educational priorities, a systems approach to school scheduling—which focuses on the mechanics of the system—ultimately serves these high priorities. Like it or not, a school cannot function without a feasible schedule, and therefore, cannot take scheduling mechanics for granted. Ironically, many schools do take for granted those scheduling variables involving time, despite the powerful leverage they bring to bear on the system. Perhaps it is because of the many human factors also involved in scheduling, that the more abstract time factors tend to be underestimated. The focus of this thesis on scheduling mechanics, and time variables in particular, is intended to bring the role of these variables into clearer perspective.

1.2.1 Working Definition of Terms

The term \textit{school} is intended to cover most academic institutions, including primary and secondary schools, vocational schools, junior and senior colleges, and universities. A school has a curriculum of \textit{subject} offerings, which are taught during an academic year within a time frame we call a term (usually one or more grading periods). Subjects are occasionally divided into \textit{phases} according to style of presentation (e.g. lectures, laboratories, recitations, seminars). Subjects are ultimately divided into one or more \textit{classes}, a class being the simultaneous gathering of one or more \textit{instructors}, \textit{rooms}, and \textit{students}, involving the same people
and room(s) whenever it meets. The instructors (a term that does not imply rank or tenure), rooms (a term intended to cover equipment and facilities as well as space and seats), and students, are referred to as resources. A time pattern is a configuration of days and hours during which a class meets. The master schedule, or simply, the schedule, is the planned or de facto arrangement by which classes meet in a recurring cycle during a term.

The schedule cycle is the period of time over which the schedule does not repeat itself. It may be as short as a day or as long as the entire term. The cycle often coincides with the five-day calendar week. Since the cycle repeats itself throughout the term, it is the cycle that is scheduled. The cycle is composed of a number of days and a number of periods (modules) each day. It is often convenient to visualize the cycle as a two-dimensional time frame, with the days as the horizontal dimension, and the periods as the vertical dimension. A time pattern then becomes a two-dimensional configuration of days and periods with shape and orientation within the cycle.

A conflict is said to exist between time patterns when they overlap by one or more periods on any days. A conflict is said to exist between classes when the time patterns they are assigned conflict in any way. Conflicting classes pose a potential problem insofar as no resource (instructor, room, nor student) can be assigned to conflicting classes without coming up against the overlapping hours. One of the central objectives of school scheduling is to cope with conflicting classes.

Any school has one or more families of time patterns which it uses, deliberately or unconsciously. A time pattern family is usually characterized
by some sort of indifference among the member time patterns when it comes to assigning them to classes. For example, they usually involve the same number of modules during the cycle, and share similar shape, if not orientation within a cycle. Indifference is usually an a priori consideration, because once any time pattern is assigned to even one class, a precedent is set that must be taken as a context for other classes and their time patterns. Human factors tend to jeopardize theoretical indifference, and occasionally force decomposition of time pattern families into sub-families to cope with a variety of preferential concerns.

1.3.1 Purpose of Thesis

It is the claim of this thesis that for any school, there exist appropriate time pattern families, such that:

(1) elimination of time patterns from these families restrict feasibility;

(2) extensions to these families are, at least, fruitless, and in any case, introduce noise;

(3) such families are worth identifying.

The thesis is: that time pattern analysis—the identification of appropriate time pattern families—is a central and critical aspect of a sound systems approach to school scheduling, in that:

(a) an error in time pattern design may detrimentally pervade (if not dominate) other aspects of school scheduling;

(b) a proper design may beneficially pervade the system.
The purpose of the thesis is to draw the attention of school authorities to the importance of careful time pattern analysis, and to assist these authorities in performing a proper time pattern analysis.
CHAPTER TWO: RELATIONSHIP TO OTHER SCHEDULING PROBLEMS

2.1.1 Two Dissimilar/Irrelevant (Path Sequencing) Problems

To understand the school scheduling problem, and the role of time variables in the mechanics of solving it, it is useful to contrast this problem with other problems. Often a person unfamiliar with school scheduling arrives at the (correct) conclusion that it might have something to do with a number of other so-called scheduling problems. Two such problems familiar to many people are production (job shop) scheduling and transportation (e.g. airline flight) scheduling. While these two problems do indeed relate to school scheduling, the differences are even more important to note. The major distinction between school scheduling and these other problems lies in the predominant role that path sequencing plays in production and transportation scheduling, an all but absent consideration in school scheduling.

The order of events in a school is usually a question of prerequisites from one academic year or term to the next, or perhaps a phase sequencing issue requiring, for example, that a recitation follow a lecture. Seldom are such considerations anything but minor constraints on the immediate problem. In production or transportation scheduling, however, the whole point is to arrive at the correct sequence of events. In these problems, the order itself is the key issue which, once discovered, usually unlocks the solution.

Both of these two path sequencing problems are discussed below,
largely to illustrate why their solution approaches are somewhat irrelevant to this thesis.

2.1.2 Production (Job Shop) Scheduling

Production scheduling, or job shop scheduling as it is often called, is faced with the problem of deciding how work will flow through a series of events under important sequencing rules. An assembly line is a useful example, since it emphasizes the importance of doing things in the correct order. Like school scheduling, production scheduling involves people doing time-constrained things. An analogy can be drawn between skilled operators of specialized equipment, and trained teachers specialized in a field of expertise. An analogy can also be drawn between the physical equipment of both problems: laboratory rooms can be thought of as specialized equipment, and general classrooms as interchangeable tools. An analogy can even be drawn between raw material being processed, and students being educated.

The critical difference between the typical assembly line and the typical school lies in the role of sequencing. Although it is likely that upperclass subjects cannot be taken until after first-year prerequisites—admittedly, a sequencing issue—it is usually the case that no ordering whatsoever is demanded within a particular cycle; e.g. when first-year math is taken relative to first-year English, first-year physical education, or any other first-year subject. Within the specific time frame that school scheduling is most concerned with—the cycle—it is the exception rather than the rule to have any path constraints. Even the distance travelled between classes is hardly ever an issue except at the largest universities.

If the sequence of events plays such a small role during a school
cycle, then what plays the larger role? The combination of events, accomplished disjointly without conflict! Combinatorial, rather than sequential issues, underlie the school scheduling problem.

Under close scrutiny, a number of job shop analogies are seen to be strained. Seldom is a tool on an assembly line as flexible in purpose as a general classroom is in the school environment. Seldom are there as many process combinations and sequences admissible in a job shop as there are admissible student schedule combinations at a large university once electives are considered. Furthermore, the production scheduling problem is often vastly complicated by exogenous inputs, dynamically reconfiguring the original boundary conditions, whereas a school tends to be relatively stable, at least for the duration of a cycle. Even the extreme of continuous progress is seldom practiced on a moment-to-moment basis.

The notion of path, so critical to production scheduling, is usually irrelevant to school scheduling. Heuristics such as scheduling by shortest processing time (SPT), are of little help except where they suggest running an occasional class early in the cycle if some other subject or phase really is tied to it.

One lesson that does carry over from the job shop to the school environment is the value of interchangeable resources. The higher the percentage of interchangeable machines (rooms) one has, the more flexible they are when it comes to scheduling. It should come as no surprise to a school administrator that specialization carries with it certain allocation costs and restrictions. Perhaps the most important point to remember here is that a resource not only has to be logically suitable for assignment, but also has to be available from a timing standpoint. Idle resources are
costly resources, whether machines or rooms, skilled operators or faculty.

2.1.3 Transportation (e.g. Airline Flight) Scheduling

Any school administrator faced with school bus scheduling is already aware of at least one form of the transportation scheduling problem. While there are analogies to school scheduling, the approaches in general differ so much as to be of little help to each other. The notion of path plays a dominant role in transportation scheduling, and, as just discussed, very little role in school scheduling.

Both problems deal with people, physical facilities and timed events. The distinction is the different role played by geography. It is obviously very important in transportation scheduling to know geographically where people and facilities start from and go to, and movement between two points is the name of the game. In the academic environment, one generally could not care less where instructors or students have just come from or where they will go next; distance measures are seldom considered even at large universities. The key events (classes) are hardly characterized by physical movement between two points. Even if mental progress is suggested as a type of analogy, it soon breaks down when one remembers that the passengers on a trip are typically transient to the problem, unlike the constant and omnipresent student body. Even the flight crew, who stay involved not unlike instructors, are seriously constrained by their geographic location before and after flights, seldom a constraint worth mentioning in a school building.

The point of this section on transportation scheduling and the previous one on production (job shop) scheduling, is neither to belittle
the serious nature of these problem areas nor to imprudently dismiss useful analogies and lessons. Rather, the point is to warn that school scheduling is not readily attacked with the same weapons. It is very important to avoid lumping all scheduling problems under the same general heading.

2.2.1 Two Similar/Relevant (Assignment/Allocation) Problems

The foregoing sections were intended to show examples of so-called scheduling problems that fall short as useful analogies when dealing with school scheduling per se. The following sections show two examples of more powerful analogies, although ironically, neither enjoys the word "scheduling" in their usual nomenclature.

The first of these assignment/allocation problems is drawn from the mathematical sciences, and has already made contributions to final examination scheduling, a prototype subset of school scheduling. The second analogy is drawn from the computer sciences, and is included for the important single point it makes.

2.2.2 Graph Chromaticism

There is a concept in elementary graph theory known as chromatic number. Recall that a graph is characterized by a set of points (or nodes) connected in various combinations by lines (or arcs). In a so-called undirected graph, given any two distinct points, the two either are or are not connected by one or more lines. The chromatic number of such a graph is simply the number of colors needed to paint or label the points in such a way that no two adjacent points (directly connected by one or more lines)
are the same color.

If we think of the points as classes to schedule, and a line between two points as a means of indicating that a resource (instructor, room, or student) is assigned to both classes, the resulting graph connects pairs of classes involving at least one common resource. The chromatic number turns out to be the number of disjoint time patterns needed if one is to achieve a conflict-free schedule: so long as resource-connected classes involve disjoint time patterns, all is well. Mathematical techniques for finding this chromatic number have proven useful in determining the minimum number of final examination periods needed to support conflict-free final examinations. The complication in real life is usually that the chromatic number of a given school scheduling problem is so huge as to be infeasible. The real life problem normally places an upper bound on the number of colors (disjoint time patterns) and instead of negating conflicts, the question is one of minimizing them.

This analogy can be extended at the price of complication on the graph side. Not only do we have to multiply connect points (classes) for each resource in common (so that we can count the number of conflicts to avoid), but we also get into the highly political area of weighting certain connections: how many student conflicts are the equal of one instructor conflict, etc.

The key lesson afforded by the graph analogy arises once we realize that most school scheduling problems involve several families of time patterns and not merely one similar family. The corresponding complication on the graph side of the analogy makes a fundamental point. Now it is no longer appropriate to use a single color to label a point, but rather we
must use several colors per node to parallel the combinations of days and periods that make up the time patterns. To clarify this process, we can use several integers rather than colors to label points. If we also use integers to label each unique module in the school cycle, a time pattern is simply one or more of the integers and these same integers can be used to label a point (class) to represent a time pattern assignment.

Now we can make an important observation: it is no longer the case that the concept of conflict is binary. Conflict can also be partial, resulting from the sharing of even one cycle module (integer). The complications introduced by these partial conflicts on the graph side of the analogy are a sound warning of equivalent problems in the academic environment. A school administrator should realize that the majority of points of any school's graph are potentially connected; even if no instructor or student is involved in common, a general purpose classroom might well be. When a school moves from the traditional approach of seven or eight disjoint time patterns to the innovative realm of modular scheduling, the analogous graph moves from a palette of seven or eight colors to a rainbow of partial conflict possibilities where even a tinge in common (albeit a single module) can cause problems. Contrary to a popular myth, modular scheduling does not reduce scheduling problems, it increases them.

2.2.3 Computer Memory Allocation

Computer memory, or storage as it is sometimes called, is one of the most valuable resources requiring allocation in a computer system. This particular resource is highly interchangeable, is required in differing
configurations by literally every job in the system, and is the object of some contention. Beyond the questions of when to allocate memory, and how much, a major issue is where to allocate it. Location is important because, once allocated, that commitment affects all subsequent allocations. The potential problem is the danger of obstructive fragmentation, whereby subsequent requests for contiguous memory are frustrated by the particular location of an earlier commitment which has fragmented available memory into discontinuous blocks too small to support the new requests. This is particularly unfortunate when the total space is adequate, but the individual pieces are too highly fragmented. In modern computer systems, there is an eventual process of deallocation attending release of the memory resource, and in such a dynamic storage environment, the avoidance of obstructive fragmentation is even more important than in the permanent allocation environment.

The important observation for us regarding computer memory allocation, either permanent or dynamic, is to note what must be managed: the remaining space after each new allocation is committed. The objective is to maximize the utility of this remaining space. It is good to retain as much flexibility as possible for future use, and bad to so fragment the space as to limit its subsequent use. Incidentally, the profile of anticipated demands on remaining space is an important factor -- if certain kinds of demand are the most likely, they may direct the way in which we make our decisions.

The analogy to school scheduling is very important. Every time a portion of the cycle is allocated to a class and its resources, it eliminates and preempts that portion of the cycle from subsequent use by
those resources. The remaining portion of the cycle must be considered; subsequent allocations must occur in that remaining space. Particularly because of the variety of demands represented by the different resources involved, it is critical that the remaining space retain as much flexibility as possible for future use.

Allocation of cycle space is complicated by the non-contiguous and diverse time pattern shapes usually encountered in an academic environment. The more complex and disperse a time pattern is, the greater the danger of its contributing to obstructive fragmentation. As classes and their time patterns are assigned to a resource, the individual schedule of that resource is transformed from free space covering the entire cycle to the eventual schedule with most (if not all) of the time reserved. Throughout this cumulative process, each time a new class and its time pattern are assigned, the original free space is further and further diminished. Often, this remaining space is fragmented into holes, some of which may be permanently unusable for any further assignment. It is therefore very important to try to retain flexibility wherever we can, lest we implicitly preempt more cycle space than a class explicitly requires.

The parallel between cumulative computer memory allocation and the cumulative effect of time pattern assignment within the cycle, will be a motivating concern throughout this thesis. The shape and orientation of time patterns, the dimensions of the cycle itself, and the ways in which time patterns interact with each other are clearly critical to the cumulative cycle space allocation process, and therefore, to the overall school scheduling process. The next chapter elaborates further on the importance of prudent cycle space allocation.
3.1.1 Definition and Role in School Scheduling

A real master schedule, at any school, has at least two critical stages in its useful life: a development stage during which it is prepared, and an implementation stage during which it is in effect. The implementation stage is the "moment of truth" for the master schedule, and is so important that the earlier development stage must anticipate it properly. Eventual use is the prime motivation for a development stage, even when hypothetical schedules are simulated to study their value. The extent to which implementation is properly anticipated plays an important role in the success or failure of a master schedule.

Most schools encounter perturbations to their master schedule during implementation. Student requests change at the last minute—perhaps failing a subject requires its being repeated, or leads to a less ambitious program. An instructor may become ill or have to move out of the area. A room may show up as inadequate only after its first attempted use; it might lack audio/visual equipment (a requirement the instructor "forgot" to specify), more students might enroll than were expected, or a fire or accident might disqualify the room for a while. Each particular school has its own particular perturbations, some surprising, others predictable in the aggregate if not the details. Such perturbations handicap the accurate anticipation of demands upon a master schedule.
There is something that most schools can do to lessen the impact of last minute perturbations, a goal that turns out to be advantageous even in the presence of perfect information. A priori flexibility is an objective of school scheduling whereby as much as possible of the master schedule remain effective in the face of perturbations encountered during implementation. The "a priori" part of this objective suggests that this consideration be built into the schedule from the very beginning; the "flexibility" simply means keeping one's options open regarding future use of the cycle in accommodating new or different resource demands.

The importance of building flexibility into the schedule at the earliest opportunity should not be underestimated. It is not the last brush stroke that paints one into a corner. Rather it is a sequence of events, starting with free choice, using up degrees of freedom until the consequences occur. There is little difference between having to accommodate one unanticipated student request and having to accommodate any of the hundreds anticipated. Swapping two rooms around the day after classes start is only slightly more aggravating than having to swap them the day before. Finding out that an instructor really cannot teach after 4:00 P.M., once the schedule is published, is usually just an echo of earlier requests to avoid late afternoon hours. In one sense, perturbations encountered during implementation merely prolong the development stage, extending it throughout the life of the schedule.

A master schedule should be flexible at every point in its development, particularly during the "endgame". To be flexible in the end, it has to be flexible throughout, and this is one reason why a priori flexibility is such an appropriate goal—the pursuit of such an objective
tends to influence the entire decision process in a self-fulfilling way. The next section discusses why a priori flexibility is desirable in school scheduling.

3.2.1 Desirability

One of the strongest motivations for this thesis topic is the occurrence of obstructive fragmentation in individual resource schedules. Artificially constrained rooms can be readily found at the Massachusetts Institute of Technology, where they are often characterized by sitting empty about 40 percent of the time. And yet the rooms cannot easily be assigned to any more classes despite their being generally acceptable classrooms. The impediment to further assignment of such a room is the particular combination of days and hours left in its schedule—a combination which is not compatible with admissible time patterns. An instructor would have to be willing to teach in some such non-standard time pattern as 'M1,W2,F3', but more of a problem arises from having to fit such a "weed" time pattern into the individual schedules of all the students attending the class. So rooms sit empty—acceptable but unusable—a paradox brought on by lack of a priori flexibility.

Obstructive fragmentation of the cycle into unusable holes can occur when the time pattern system has been poorly designed—or has not been consciously designed at all. On the other hand, systematic allocation of coordinated time patterns can insure that wherever cycle space remains, it can—at least theoretically—be used. In this sense, non-obstruction is another synonym for flexibility.
A priori flexibility is a primary goal of time pattern systems, and should be appreciated for the role it can play in enhancing existing interchangeability of resources. Although this thesis is not directly concerned with decisions involving resource acquisition or analysis, it should be noted that flexible resources represent yet another kind of a priori flexibility important to a school; there are clearly many advantages to having multi-purpose resources. The thesis instead emphasizes that, in the presence of resources already acceptable for assignment to classes, an appropriate time pattern system can increase the likelihood that available time is usable time.
CHAPTER FOUR: APPROACHES TO THE PROBLEM

4.1.1 The Feasible-Desirable Spectrum

Before concentrating on the time variables involved in school scheduling, it is useful to understand a few of the approaches that can be taken to the problem as a whole. The word "approach" is a useful one, since a master schedule is often the net result of a series of successive approximations. Trial and error methods are normally employed over a number of iterations—perhaps over a number of years—with the dual purposes of seeking improvement and of gaining insight into the systems nature of the school being scheduled. The latter intent—learning about the problem at hand—often plays a major role in disclosing viable alternatives to the status quo.

There are two opposite directions from which the eventual schedule for a school can be approached. A school can try to accomplish all its goals at once. It can specify a schedule with all the "bells and whistles", reflecting all the resource preferences as well as the constraints. In other words, a school can take an optimistic ambitious approach. The problem usually is that such a highly desirable schedule often lacks feasibility. (A school cannot have all of its instructors teaching between 11:00 A.M. and 2:00 P.M., with Friday afternoons off, and still expect to come up with decent room utilization!) The opposite direction of approach is to start with modest expectations, requiring only the most serious constraints. In other words, a school can take a
conservative step by step approach. The problem here is that such a schedule, if feasible, is often less than desirable.

Despite difficulties with either direction of approach, the conservative approach is usually the best, because it deals from a fall-back position of feasibility. When an additional goal is superimposed on the system and it succeeds, it can be incorporated. But should it fail, the school still has the previous feasible schedule. Often when too much is asked of a schedule, and it shows to be infeasible, there is little information as to which of the myriad niceties should be compromised. There are psychological advantages to having a tangible schedule in hand -- albeit short of perfect -- throughout a schedule development stage.

Of course, any schedule should possess some degree of realism. It may be wishful thinking to temporarily ignore room assignments in the hope that "they can be added in later", or to blindly extrapolate an entire schedule from one that serves a single student year. Then again, there is a certain value to negative information. If a school cannot be feasibly scheduled even without rooms, the problem won't go away when the rooms are added. If first-year students cannot be handled by themselves, the upperclass students probably can't cure the problems. In such cases, it may be productive to investigate sub-problems in isolation.

The recommendation is that a school try to locate itself well over on the feasible end of the feasible-desirable spectrum, with the intention of progressing towards the desirable end. This is often the approach taken when manually scheduling a school, using such techniques as are discussed in the next sections.
4.2.1 Manual Techniques

School scheduling is inherently a manual process, involving as it does a multitude of human administrative decisions and compromises. There are three persuasive reasons why the task cannot be totally automated, given current technology:

1. The typical real life problem is combinatorially prohibitive of exhaustive optimization.
2. There are many qualitative (non-quantitative) considerations that make it difficult to even define "optimality".
3. It is difficult to formalize what is expected of a schedule without either over-specifying the parameters (thus resulting in infeasibility) or under-specifying them (thus resulting in a less than desirable "solution").

If one word had to be singled out to best describe the school scheduling process, it might well be "compromise". Often the problem is "solved" by changing the problem. There are also a few degrees of freedom in the solution space that can be capitalized upon by a knowledgeable scheduling officer—we call them discretionary leverages. Compromise and discretionary leverage are discussed in the next two sections.

4.2.2 Compromise--the Procrustean Approach

One way to "solve" a problem is to change the problem itself. This Procrustean approach is often valid in school scheduling. Every school administrator has faced such decisions. An extra room may be created by borrowing office space. A part-time instructor may be hired. A student
may be told certain courses are closed. Or an extra section can be added to accommodate unexpected student enrollment. In such cases, the original "constraints" may be relaxed—or tightened—more than a little.

The larger problem is not so much to unconditionally avoid compromise, but rather to identify where compromise can help and to assist in its justification. Most schools are willing to bend if the benefits of a compromise outway the costs.

One of the problems attending school scheduling is the complex interrelationship of the basic elements: classes, time patterns, and resources. An obstinate instructor can cause problems in room assignment, as when a limited number of rooms are available at choice hours. A poor time pattern can frustrate students who otherwise would attend a subject. An insufficient number of lecture halls can force classes to run at poor times. It is always helpful to have confidence in one or more sectors, that at least those sectors are not causing problems. Knowing that there really are not enough lecture halls, and that the problem is not fussy instructors or poor time patterns, represents valuable information permitting intelligent decision making.

4.2.3 Discretionary Leverage— the Systems Approach

The major difficulty with compromise is precisely the give and take involved—we would prefer to take without giving. This is where the discretionary leverage applies. The following example may help to illustrate the point. Suppose an instructor teaches two classes $c_1, c_2$ and, for reasons which we do not question, can only teach 'MTWRF9-11'.

Suppose he or she informs the scheduling officer that "$c_1$ is to run 'MWF9'
and \( c_2 \) is to run 'MWF10'. If these constraints are honored, it could happen that student attendance at these classes might be limited because of other student commitments 'MWF9-11'. It might be better to run \( c_1 'MWF10' \) and \( c_2 'MWF9' \) instead of vice versa. Notice that the "give" of compromise need not be involved here at all. If the true instructor requirement is "run \( c_1 \) and \( c_2 \) disjointly using time patterns contained in 'MTWRF9-11'", discretion and not compromise plays the key role.

A similar example arises when, for any of a number of reasons discussed in later chapters, a school finds it advisable to restrict three-day time patterns to MWF, and to use TR to run three-hour classes in two one and one-half hour sessions. For some, perhaps most, schools this might be a compromise, or even out of the question. But for others, it may be an equally acceptable alternative for all parties concerned. To continue the earlier example, maybe we should run \( c_1 'MWF10' \) and \( c_2 'TR9:30-11' \) in order to best accommodate the students involved. Again, so long as the instructor requirement really is "any time pattern contained in 'MTWRF9-11'", discretion provides the leverage.

In general, improvements can often be made to a master schedule simply by using discretion where allowed. This is a systems approach in the sense that otherwise equally acceptable alternatives are resolved in favor of the larger system. The above examples should make it clear that precise communication of constraints and preferences plays a key role in defining where discretion is allowed.

The beauty of many discretionary leverages is that they are both powerful and readily manipulated. A classic example of this is the synchronization of one and one-half hour classes throughout the day. At
the Massachusetts Institute of Technology, many such classes run each day. Very little pressure was brought on departments to coordinate these classes—some departments started them at 9, 10:30, 12, etc., other departments started them at 9:30, 11, 12:30, etc., still others started them at 10, 11:30, 1, etc., and occasionally even the same department was not consistent. When one considers the adverse impact of a '10-11:30' class on the schedule of a student wishing to take one and one-half hour classes using the other two conventions, it appears obvious that synchronization offers potential benefits. But the key point here is that for most departments such synchronization is an easy thing to do—an "equally acceptable alternative" ripe for exploitation as a discretionary leverage.

School schedules always "work by definition" as they go into effect. How satisfactory they are remains to be seen. Through systematic use of discretionary leverage and judicious use of compromise, master schedules often can be improved.

4.3.1 Computer-Assistance

Until now, no reference has been made to the role of a computer in school scheduling. Now that we introduce the computer, we qualify its role by referring to computer-assisted scheduling, as opposed to computerized or automated scheduling. There is good reason to do this, because much of the process requires human participation, not only because computer systems currently cannot tackle the entire job, but also because of the very nature of the problem. For this reason, the next sections start with limitations and only then cover capabilities.
4.3.2 Limitations

There are limitations on what computers can and cannot do to help solve the school scheduling problem—limitations typical of many application areas in which computers are involved. The most serious limitation is, ironically, often not appreciated: computers cannot do the impossible. Generally speaking, a computer cannot do what cannot be done (at least theoretically) by a human being. Faster, yes. More accurate, perhaps. But rabbits out of hats, no.

The computer is often a "red herring" in a school scheduling process, attributed powers it does not have. It is not unusual for a school to turn to some sort of computer-assistance at the same time a move is made to a new building, or when innovation is introduced for the first time. This is potentially the worst time to involve a computer, since it is very easy to expect too much of the machine at a time when not very much is humanly understood about the new scheduling problem. It is often better in the long run to manually work through a new schedule if for no other reason than to know what can and cannot be expected of it. The computer system might be used in parallel, or perhaps in simulation mode retroactively after the schedule is implemented. Under no circumstances should it be assumed the computer "will do the scheduling". Too many critical decisions require administrative discretion to warrant total abrogation of human responsibility.

If the foregoing applies to so-called sectioning or loading systems, which assign students (and sometimes other resources) to a manually determined time schedule, it applies even more so to time assignment systems. The combinatorial size of the problem explodes when times as
well as resources are to be chosen for classes.

Forewarned by this discussion of limitations, we can now address the capabilities of computer-assisted school scheduling.

4.3.3 Capabilities

Computer systems are fast, accurate, tireless, and good at keeping track of details, large and small. These are all advantageous traits when it comes to school scheduling. It is precisely these clerical powers that make the computer a valuable assistant in the scheduling process. In school scheduling, certain decision making tasks can be relegated to human clerks, and similar tasks can be relegated to a computer.

Given a complete list of all student requests for subjects, a computer system is in a very good position to appreciate potential conflicts. This kind of statistical analysis is a strong point of a computer system; few instructors or even a central scheduling officer have as ready access to exact tallies of "who wants to take what subjects in combination with which other subjects".

One of the most attractive capabilities of a computer-assisted approach is the ability to readily evaluate hypotheses. A schedule does not have to be implemented in order to be studied. The computer does not "know" whether the schedule it is working on will be implemented or not; it simply does the best it can with the schedule, and can provide useful evaluation. Accordingly, master schedules can be simulated by the system without the commitment associated with live implementation.
Clerical accuracy is an important backstop to a computer's decision making powers. Even if Jennifer Jones cannot be accommodated in Physics, at the very least the system can indicate the impasse. The problem may be due to a poor decision on the part of the computer—or on the part of the scheduling officer—but the impact of the poor decision need not be lost.

Once a satisfactory master schedule is obtained, the computer can play a very useful role in publishing the results. Computer systems are good at selecting, sorting, and formatting data in a variety of ways. What would be a tremendous effort if done by hand is almost an anti-climax for the computer system: printing individual student schedules showing classes, times, instructors, rooms; printing class lists showing students attending each class; generating loading statistics for instructors and rooms, individually or by "pools"; and so on. Needless to say, clerical accuracy is a blessing during this particular process.

An important role can be played by the computer system in choosing a time pattern for a class, given a set of acceptable time patterns and sets of appropriate instructors, rooms, and students who either must or may be assigned to the class. This capability will be discussed further in the next section.

4.3.4 GASP: an Implementation

The GASP (Generalized Academic Simulation Programs) system was developed in the mid 1960's at and for the Massachusetts Institute of Technology (M.I.T.). This system was designed to be general enough to handle primary and secondary school environments as well as colleges and universities. Development of the system was financed in part by the
International Business Machines Corporation and the Educational Facilities Laboratories.

One of the design features of GASP that distinguishes it from so-called sectioning or loading systems, is the ability to assist directly in the time assignment process of choosing time patterns for classes (as well as the resource assignment process of assigning resources to classes). GASP has enjoyed some degree of success in assisting the M.I.T. Schedules Office to prepare and implement master schedules for more than a decade, and has been used in a number of secondary school environments. No attempt will be made in this thesis to fully document the GASP system; however, this section will discuss the GASP approach to time variables and the time assignment process.

The GASP system requires a time pattern dictionary wherein are represented each and every time pattern that might be used anywhere in the system. These entries represent not only those time patterns which are preassigned to classes, but also any time patterns which potentially could be assigned to classes. Time patterns are categorized according to the families to which they belong, usually reflecting the number of days involved, the number of modules involved, and the way in which modules are distributed over the days. A school has a great deal of latitude in defining these families, and which time patterns do and do not belong to the same family.

During the time assignment phase of a GASP run, time patterns are assigned to each class, class by class. (During this phase, resource information is available and used in the decision process, but this is only incidental to the current discussion.) When each class is
encountered in turn, one of two situations occurs. Either the class has been preassigned a time pattern (in which case GASP has little to do) or else GASP is to choose a time pattern from a set of admissible time patterns. A set of admissible time patterns can be explicitly specified by listing individual time pattern dictionary entries, or it can be implicitly generated by generically specifying one or more appropriate families. Only those time patterns from the set can be assigned to the class, and one is chosen by GASP using techniques outside the range of this current discussion.

One of the earliest motivations for this thesis topic was the practice, frequently encountered at schools using GASP for the first time, of building a time pattern dictionary which contained almost every conceivable time pattern that appeared to belong in a family. The hope was that GASP would filter out the "bad" time patterns and home in on the "good" ones. Since this is possible only to a limited extent, many computer runs consumed too much time yielding too poor results. Often in the early stages of time assignment, a time pattern was chosen which, though admissible (because of set membership), was a very unfortunate choice leading to the worst kinds of obstructive fragmentation. At best such time patterns require effort to identify and avoid; at worst they are assigned.

It seemed desirable that the computer system be extended to analyze and select a good working subset of the time pattern dictionary, or perhaps generate the dictionary in the first place, but this is easier said than done. Given the limitations of GASP vis-a-vis "understanding" a school's objectives, it turns out that time pattern design and
specification is one of the most crucial vehicles for communicating the
nature of a school to GASP. Many aspects of a school's operating
requirements are built into the time pattern system it uses, and this
type of specification is a good example of where human intellect is
needed to guide a computer system. This does not mean that the process
of time pattern analysis and design will never by automated, only that we
need to know more about such processes first. It is hoped that this
thesis may contribute to the knowledge required to eventually allow at
least partial automation.
CHAPTER FIVE: NOTATION AND DEFINITIONS

5.5.1 The Basic School Scheduling Elements

The basic elements of school scheduling are time patterns, classes, instructors, rooms, and students. Time patterns (defined in greater detail below) are the configurations of time—days and hours—used in establishing the schedule. The instructors, rooms, and students of a school are called its resources. A class, sometimes called a class section, is the simultaneous gathering of one or more instructors, rooms, and students involving these same resources whenever it meets in such a dedicated manner as to prohibit their concurrent involvement with any other class. The class is the basic scheduling element of the curriculum, and is a subdivision of a subject. A subject is a course or area of study, which may be optionally divided into subject phases (lectures, laboratories, recitations, seminars, etc.) usually according to style of presentation. Subjects and their phases are always subdivided into one or more classes for purposes of scheduling.

One or more instructors satisfy a class's need for teaching staff; the term makes no distinction among professors, junior staff, or even student teaching assistants, but simply denotes individuals with teaching responsibilities. Usually a class requires one and only one instructor, but occasionally a team of several instructors needs be assigned. Such teams often split up the student population of large phases into smaller populations before, during, or after attendance at the larger phase.
Appropriate instructor load is an important scheduling consideration.

One or more rooms satisfy a class's need for physical facilities. The term is intended to cover equipment as well as space and seats. A room is categorized as special purpose when its use is dedicated to one or more specific subjects or subject phases due to equipment, location, etc. (e.g. laboratories, gymnasiums). A room is categorized as general purpose when there is no impediment (beyond size) to its use in support of a broad variety of different subjects. The set of all rooms can usually be partitioned into interchangeable pools, characterized by location, size, and equipment. Usually a class requires one and only one room.

A number of students satisfy a class's need for attendees to be taught. Students are typically characterized by student year (graduate, undergraduate, first-year, senior, 7th grade, etc.), and typically place demands on a broad cross-section of the curriculum. The appropriate number of students to be assigned to a given class is usually a function of the subject (phase) and is an important consideration.

5.2.1 The Master Schedule

The master schedule, the net result of the entire scheduling process, is made up of two collections of assignments determined as follows. A time pattern assignment is an ordered pair \((t, c)\) consisting of one time pattern \(t\) and one class \(c\); the time pattern is said to be assigned to the class, and the class is said to be assigned to the time pattern. The collection of all time pattern assignments is called the time pattern schedule; by convention, there is exactly one time pattern assignment per class, hence the cardinality of the time pattern schedule is equal to the
cardinality of the set of all classes.

A *resource assignment* is an ordered pair \((r, c)\) consisting of one resource \(r\) and one class \(c\); the resource is said to be assigned to the class, and the class is said to be assigned to the resource. The collection of all resource assignments is called the *resource schedule*. Because a typical resource is involved with several classes, the cardinality of the resource schedule is normally several times that of the classes (even if a few resources have no class assignments).

Determination of the master schedule is the result of two (interdependent) assignment processes: a time pattern assignment process establishing the time pattern schedule, and a resource assignment process establishing the resource schedule.

The simultaneous existence of a time pattern assignment \((t, c)\) and a resource assignment \((r, c)\) both involving the same class \(c\), can be thought of as generating a third ordered pair \((t, r)\) consisting of the time pattern \(t\) and the resource \(r\); we will call such an ordered pair a *time pattern resource assignment*, and say the time pattern is assigned to the resource and the resource is assigned to the time pattern. The collection of all time pattern resource assignments is not a particularly useful set in view of its redundancy given the time pattern schedule and the resource schedule.

5.3.1 The Schedule Cycle

The schedule *cycle* is the period of time over which the master schedule does not repeat itself. This "academic week" is frequently a calendar week, but need not be restricted to five days. The cycle is made
up of \( d \) days, and \( m \) daily modules (sometimes called periods), for a total of \( dm \) cycle modules. Even if not all of the \( dm \) cycle modules are usable (as when Saturday classes are held, but only up to 1:00 p.m.), it is generally convenient to think of the cycle as a two-dimensional \( d \times m \) matrix of cycle modules, with the days as the horizontal dimension and the periods as the vertical dimension. Unusable modules can be marked as such (see figure 5-a).

| M   | T   | W  | R  | F  | S
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<tbody>
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<td>1</td>
<td>9</td>
<td>17</td>
<td>25</td>
<td>33</td>
</tr>
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<td>2</td>
<td>10</td>
<td>18</td>
<td>26</td>
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<td>11:00</td>
<td>3</td>
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<td>19</td>
<td>27</td>
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<td>12:00</td>
<td>4</td>
<td>12</td>
<td>20</td>
<td>28</td>
<td>36</td>
</tr>
<tr>
<td>m=8 modules</td>
<td>1:00</td>
<td>5</td>
<td>13</td>
<td>21</td>
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<td></td>
<td>4:00</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
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</tbody>
</table>

Fig. 5-a.--A \( d=6 \) (day) by \( m=8 \) (module) Cycle, Shown as a \( d \) by \( m \) Matrix of Hour Length Cycle Modules

5.4.1 Time Patterns and Time Pattern Families

A time pattern is any subset of the \( dm \) cycle modules. A time-pattern family is an aggregate collection of any number of related time patterns. The cardinality of a time pattern \( t \) is the number of cycle modules of which it is composed, and is designated \(|t|\). The cardinality of a time pattern family \( T \) is the number of member time patterns, and is designated \(|T|\).

The cycle is itself a time pattern \( t_{cycle} \) (of cardinality \(|t_{cycle}|=dm\)). The time pattern consisting of no modules (the vacuous subset of the \( dm \) cycle modules) is the null time pattern \( \lambda \) (of cardinality \(|\lambda|=0\)). Each individual cycle module is a unit time pattern (of cardinality 1). There are \( 2^dm \) unique time patterns (\( 2^n \) being the cardinality of the collection
of all subsets of a set of \( n \) elements). There are a variety of ways in which time patterns can be identified; four such ways are discussed in the next four paragraphs.

Once the \( dm \) cycle modules have been serially numbered \( i=1,2,\ldots,(dm) \) as in figure 5-a, a time pattern \( t \) (being a subset of the set of \( dm \) cycle modules) can be represented in ordered set notation

\[
t={i_1,i_2,\ldots,i_n},
\]

\( 1\leq i_1<i_2<\ldots<i_n \leq(dm) \). This is not a particularly useful notation, but does emphasize that a time pattern is a set.

By using \( d \) unique graphics as day names \( D_1,D_2,\ldots,D_d \) to represent the \( d \) days (e.g. M,T,W,R,F,S to represent Monday, Tuesday, Wednesday, Thursday, Friday, Saturday if the cycle corresponds to a calendar week; or perhaps simply A,B,C,...), and by using \( m \) unique character strings distinct from the day names as module names \( M_1,M_2,\ldots,M_m \) to represent the \( m \) daily modules (e.g. period numbers 1,2,\ldots,m, or starting clock times 8, 8:30, 9,9:30,\ldots,5:30,6,EVE), we can name time patterns by strings of day names qualified by module names. Examples of simple time pattern names are 'MTWRF10-11', 'MWF9:30-11', 'TR3-5', and 'F EVE'. The first example 'MTWRF10-11' can be abbreviated without the dash 'MTWRF10' once a school has established a so-called standard class length, in this case one hour. Finally we allow compound time pattern names, to handle different module configurations on different days, by use of commas: 'MWF9,T3-5' and 'M10, T11,W12,R1,F2'. For schools which use period numbers (as opposed to clock times) it is usually more convenient to interpret the dash as inclusive rather than exclusive; e.g. 'TR1-2' meaning "first and second period" rather than "1:00 to 2:00 P.M.". Either notation is acceptable, once the interpretation is fixed; in subsequent usage, the dash will be inclusive
when dealing with period numbers and exclusive when dealing with clock
times; which option is in effect should be clear from context.

By pictorial representation, we can visually identify time patterns
in their actual placement in the cycle matrix. When doing this, we will
adopt the convention of labelling each and every cycle module contained in
a time pattern with the same symbol. Alternatively, we may label the area
of the cycle space covered by a time pattern. This notation permits the
identification of several time patterns via a single cycle diagram.

Figures 5-b and 5-c illustrate pictorial representation.

<table>
<thead>
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<th>D3</th>
<th>...</th>
<th>Dd</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
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<th>Dd</th>
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</table>

Fig. 5-b.—Two Pictorial Representations of (the same) m Time Patterns over
a d(day) by m(module) Cycle
Although the following notation will not be subsequently used in this thesis, it is convenient and useful for storage and manipulation of time patterns by a computer. Under this notation, time patterns are represented by bit strings of binary zeros and ones. Each time pattern \( t \) is coded as \( d \) (the number of days) substrings of \( m \) (the number of daily modules) bit positions, the \( j \)th (module) bit position of the \( i \)th (day) substring being one if the time pattern \( t \) contains the \( j \)th module on the \( i \)th day, and zero otherwise. Whenever this notation is displayed, it is convenient to parse the day substrings via the string concatenation operator "|" to emphasize the day structure. As an example, in a 5(day) by 8(module) cycle, the time pattern 'MWF2,TR3-4' (second period MWF, third and fourth periods TR) is represented by the 40-bit-string:

\[
01000000 \ |
00110000 \ |\ | 01000000 \ |
01100000 \ |
01000000
\]
The motivation for such a coding is apparent when one considers some of the pattern recognition capabilities of computer systems. For example, observe that once two time patterns $t_1, t_2$ have been thus coded, it is a simple matter to determine if they conflict (have at least one cycle module in common); the two bit strings need only be logically AND-ed together and the resulting string $t = t_1 \land t_2$ will be all zeros ($t = \lambda$) if and only if there is no conflict. Ones in the resulting time pattern string $t$ pinpoint modules in conflict.

5.4.2 Conflicts and Other Set-Theoretic Concepts

Because a time pattern is a set of cycle modules, certain set-theoretic concepts can be defined. [Examples are given in brackets after each definition.]

**Definition** A time pattern $t_1$ is said to be contained in another time pattern $t_2$ if and only if every cycle module contained in $t_1$ is also contained in $t_2$. The notation $t_1 \subset t_2$ indicates that $t_1$ is contained in $t_2$. ['WF2' $\subset$ 'MWF2']

**Definition** The intersection of two or more time patterns $t_1, t_2, \ldots, t_n$ is also a time pattern $t$. We write $t = t_1 \land t_2 \land \ldots \land t_n = t$. ['W9:30-11' = 'MW9-12' $\land$ 'WF9:30-11']

**Definition** The union of two or more time patterns $t_1, t_2, \ldots, t_n$ is also a time pattern $t$. We write $t = t_1 \cup t_2 \cup \ldots \cup t_n = t$. ['TR9-12' = 'TR9-11' $\cup$ 'TR10-12']

**Definition** Two or more time patterns $t_1, t_2, \ldots, t_n$ are said to be collectively exhaustive with respect to a time pattern $t$ if and only if $t = \lor t_i$. We also say the $t_i$ cover $t$. When $t$ is the entire cycle
to cycle, we simply say the \( t_i \) are collectively exhaustive. ['MWF10' and 'TR10' are collectively exhaustive with respect to 'MTWRF10'; they cover 'MTWRF10']

**Definition** Two time patterns \( t_1, t_2 \) are said to be **disjoint** if and only if their intersection is null \( (t_1 \cap t_2 = \emptyset) \). ['MWF10' and 'TR10' are disjoint]

**Definition** Two time patterns \( t_1, t_2 \) which are not disjoint are said to **conflict** (be in conflict, be conflicting); the conflict (between two conflicting \( t_1, t_2 \)) is the non-null time pattern \( t \) which is their intersection \( t = t_1 \cap t_2 \). The conflict \( t \) is said to be a **partial conflict** if \( t \) is a proper subset of either \( t_1(t \neq t_1 \text{ or } t \neq t_2) \). If \( t_1 = t = t_2 \) the conflict \( t \) is said to be a **total conflict**. If \( t_1 = t \neq t_2 \) the conflict \( t \) is said to be total with respect to \( t_1 \) and partial with respect to \( t_2 \). ['TR10' is a total conflict with itself, and results in the partial conflict 'R10' with respect to 'MR10' (partial with respect to both 'TR10' and 'MR10')]

**Definition** Two or more time patterns \( t_1, t_2, \ldots, t_n \) are said to be **mutually disjoint** if and only if the \( t_i \) are pairwise disjoint (i.e. for any \( i, j \neq i \) \( t_i \cap t_j = \emptyset \)). ['MT9', 'TW9', 'WR9', 'R9' are not mutually disjoint; removing 'TW9' causes the other three time patterns to be mutually disjoint]

**Definition** The **sum** of \( n \) mutually disjoint time patterns \( t_1, t_2, \ldots, t_n \) is their union \( t = \bigcup t_i \); because they are mutually disjoint, we permit the notation \( t = t_1 + t_2 + \ldots + t_n = \Sigma t_i \). ['MTWRF3' = 'M3' + 'TR3' + 'WF3']

**Definition** Two or more mutually disjoint time patterns \( t_1, t_2, \ldots, t_n \) are said to be **supplementary** with respect to a time pattern \( t \) if and only
if \( t = \sum t_i \). When \( t \) is the entire cycle \( t_{cycle} \), we simply say the \( t_i \) are supplementary. Note that in this case the \( t_i \) are also collectively exhaustive; they cover the cycle. ['MWF3' and 'TR3' are supplementary with respect to 'MTWRF3']

Given the foregoing definitions, the concept of conflict may now be extended to classes and to resource assignments. [Again, examples are given in brackets after each definitions.]

**Definition** Two classes \( c_1, c_2 \) are said to conflict (be in conflict, be conflicting) if and only if the two time patterns \( t_1, t_2 \) to which they are assigned are in conflict. Given two time pattern assignments \( (t_1, c_1) \) and \( (t_2, c_2) \), \( c_1 \) conflicts totally/partially with \( c_2 \) if and only if \( t_1 \) conflicts totally/partially with \( t_2 \). Note that a conflict between classes is not necessarily a problem in and of itself, since common resources might not be involved; classes in conflict pose only a potential problem with respect to resource scheduling. [A math lecture assigned 'TR10' (totally) conflicts with a physics lecture assigned 'TR10' and (partially) conflicts with a physics laboratory 'T9-12']

**Definition** Two or more classes \( c_1, c_2, \ldots, c_n \) are said to be without conflict (be conflict-free, be mutually disjoint) if and only if the \( n \) time patterns \( t_1, t_2, \ldots, t_n \) to which they are assigned are mutually disjoint. [Three humanities recitations, assigned 'MWF10', 'W EVE', and 'TR9:30-11' are conflict-free]

**Definition** Two resource assignments \( (r, c_1) \) and \( (r, c_2) \) involving the same resource \( r \) are said to conflict (be in conflict, be conflicting) if
and only if the two classes $c_1, c_2$ are in conflict. In the presence of conflict, (either) one of the two resource assignments must be designated an invalid assignment; by convention, the higher-subscripted class is usually attributed the invalidity. [French III (assigned 'MTW10') is an invalid assignment for an instructor already assigned French II (assigned 'WRF10')]

Definition Two or more resource assignments $(r, c_1), (r, c_2), \ldots, (r, c_n)$ involving the same resource $r$ are said to be without conflict (be conflict-free, be mutually disjoint) if and only if the $n$ classes $c_1, c_2, \ldots, c_n$ are without conflict. If the $n$ classes are not without conflict, one or more of the $n$ resource assignments must be designated invalid; by convention, the smallest possible number $n'$ of the $n$ resource assignments are attributed the invalidity such that the remaining $(n-n')$ resource assignments are then without conflict; all other things being equal within this convention, the higher-subscripted class(es) is/are usually attributed the invalidity.

[Chemistry laboratory 'M9-12' is an invalid assignment to a student taking math lecture 'MWF9', physics recitation 'MWF10', and economics 'MWF11']

5.4.3 Time Pattern Structure

In subsequent discussions, we will categorize time patterns according to their structure. The structure of a time pattern is the quantitative aspect of the configuration of modules over days. Structure is indicated by an ordered expression of from 1 to $d$ (the number of days in the cycle) integral numeric elements, ordered largest to smallest, each representing
the number of consecutive modules on a corresponding day. We limit the concept of structure to time patterns with at most one consecutive string of modules per day; time patterns such as 'MW1,MW5' are thus disregarded. If a subject really requires such a split time pattern, this could be accommodated via artificial phases assigned normal time patterns. The expression between and including the brackets:

\[ [n_1, n_2, \ldots, n_k] \text{ (} m \geq n_1 \geq n_2 \geq \ldots \geq n_k \geq 1, k \leq d \) \]

denotes the structure of any time pattern involving \( k \) days, \( n_1 \) consecutive modules on one of the \( k \) days, \( n_2 < n_1 \) consecutive modules on another of the remaining \( (k-1) \) days, etc. For example, \([321]\) denotes the structure of any time pattern involving three days, 3 consecutive modules on one day, 2 on another, and 1 on the third.

Because this type of expression suggests "multiplication", we permit the use of "exponents" for convenience in handling cases where \( n_i^{i+1} \rightarrow n_i \). The expression between and including the brackets:

\[ [n_1, n_2, \ldots, n_j] \text{ (} m \geq n_1 > n_2 > \ldots > n_j > 1, i_1 + i_2 + \ldots + i_j = k \leq d \) \]

denotes the structure of any time pattern involving \( i_1 + i_2 + \ldots + i_j = k \) days, \( n_1 \) consecutive modules on each of \( i_1 \) days, \( n_2 < n_1 \) consecutive modules on each of \( i_2 \) other of the remaining \( (k-i_1) \) days, etc. As an example in a 5(day) by 8(module) cycle, the three time patterns 'MW2-4', 'MT2-4', and 'M2-4, W1-3' all have the same structure \([22]\) = \([2^2]\). In that cycle, the two time patterns 'M1-3, TWRL' and 'TWR2, F1-3' both have the same structure \([2111]\) = \([21^3]\). This notation will be heavily used in the remainder of the thesis. Remember that the exponents are a convenience for representing days. Hence \([x^y]\) means \( x \) modules on each of \( y \) days (and not vice-versa).
When designating families of time patterns, we will often choose a superscript for the family and its member time patterns which mnemonically suggests structure. For example, $A^{12}$ might be a family of $[1^2]$ time patterns $a_1^{12}, a_2^{12}, \ldots, a_n^{12}$. Likewise, $T^{321}$ might be a family of $[321]$ time patterns $t_1^{321}, t_2^{321}, \ldots, t_n^{321}$ or a family of $[3^21]$ time patterns. In such cases, the superscript is only mnemonic and may be somewhat ambiguous regarding the exact structure. $T^{xy}$ could represent either $[x^y]$ or $[xy]$ time patterns, although in this thesis the former is usually the case.

5.4.4 Time Pattern Shape (Congruency, Similarity)

The shape of a time pattern is a more intrinsic characteristic than its structure. The concept of shape is concerned with the two-dimensional orientation of the time pattern's component modules with respect to each other. Shape includes the concept of day separation—the way in which modules are distributed over the days, possibly being separated by one or more days. The simplest way to denote the shape of a time pattern is to use the pictorial representation of the time pattern.

The number of unused modules intervening between the modules involved in a time pattern is significant. It is up to the individual school to decide whether or not the cycle wraps around for purposes of comparing shapes; e.g. whether or not Monday follows Friday in the same contiguous fashion that Wednesday follows Tuesday. Unless stated to the contrary, we will generally assume that the cycle does wrap around its days. We now proceed to the definition of two time pattern relations involving shape: congruency, and similarity.
Definition Two or more time patterns \( t_1, t_2, \ldots, t_n \) are said to be congruent if and only if their pictorial representations can be superimposed (placed one upon the other) such that they coincide. Thinking of each shape as a rigid template, translation and/or rotation of the templates (along or about any axis) are allowed to achieve superimposition. (This definition of congruency more or less corresponds to the usage in plane geometry.) Note that a congruency transformation preserves structure and day separation.

Example In a five day cycle, 'M1-3,TWR1' is congruent to 'TWR2,F1-3' and, assuming wrap around of the cycle's days, it is also congruent to 'W1-3,RFM1'; it is never congruent to 'W1-3,TRF1'.

Example 'MW1,F2' is congruent to 'MW2,F3' and 'MW2,F1' but not 'MF1,W2'.

Definition Two or more time patterns \( t_1, t_2, \ldots, t_n \) are said to be similar if and only if their shapes can be transformed into congruent shapes by independent translations of modules within their days. The template is now thought of as \( d \) (the number of days in the cycle) vertically parallel day-templates, each of which can be translated vertically although now interchanged day-wise. (This definition of similarity unfortunately does not conform exactly to normal usage in plane geometry; we have no use for the plane geometry concept and the term does have mnemonic value. Like the plane geometry term, it is a weaker correlation between two shapes than is congruency.)

Example 'M1-3,TWR1' is similar to 'M1-3,T1,W2,R3' and 'TR2,W1,F2-4'.

Example In figure 5-d, each of the five time patterns \( t_5 \) have similar (but not congruent) shapes if we assume wrap around of the cycle's days. In addition, time patterns \( t_1, t_3, t_5 \) are congruent, as are \( t_2, t_4 \).
Note that a similarity transformation and a congruency transformation each preserve structure and day separation. However, the former relaxes the vertical rigidity of the templates by permitting independent daily translations.

5.4.5 Time Pattern Blocks

We now define several terms which will help us organize time patterns into interchangeable families.

Definition A straight time pattern is a time pattern where both of the following are true:

(a) the structure can be expressed as \([n^k]\); i.e. there are the same number \(n \leq m\) of daily modules on any day involved; and,

(b) the exact same consecutive daily modules \(\{M_j, M_{j+1}, \ldots, M_{j+n-1}\}\) are repeated on each day involved; i.e. the time pattern name can be expressed "\(D_{i_1} D_{i_2} \ldots D_{i_k} M_j - M_{j+n-1}\)".

Example 'Ml', 'MTl', 'MWFi' are straight time patterns. 'Ml,Tl-3' is not because of structure; 'Ml,T2' is not because of different daily modules.
Definition A straight time pattern involving all days (i.e. having a structure \([n^d]\)) is called a rectangular time pattern.

Example The entire cycle \(t_{\text{cycle}}\) is a rectangular time pattern, as are the time patterns \('D_1D_2\ldots D_d i'\).

Definition A time pattern block structure is any partition of the cycle into \(k\) time patterns \(b_i\) such that the \(b_i\) are supplementary (disjointly cover the cycle). If in addition the \(b_i\) are congruent, we call the partition a congruent block structure. The \(b_i\) are referred to as (congruent) time pattern blocks.

Example The \(k=m\) time patterns \('D_1D_2\ldots D_d i'\ \ i=1,2,\ldots,m\) are congruent time pattern blocks in the "finest" possible congruent rectangular block structure.

Example The \((m-1)\) pairs of time patterns \('D_1D_2\ldots D_d M_i-M_i', 'D_1D_2\ldots D_d M_{i+1}-M_m'\) corresponding to \(i=1,2,\ldots,(m-1)\) constitute the \((m-1)\) "coarsest" possible rectangular block structures, congruent in the one case that \(m\) is even and \(i=\left(\frac{m}{2}\right)-1\).

Definition A family of time patterns \(T=\{t_1,t_2,\ldots,t_n\}\) is said to be contained in blocks if and only if there exists a time pattern block structure of \(k\) blocks \(b_1, b_2, \ldots, b_k\) such that for any \(t_i \in T\) there is a unique \(b_j\) whereby \(t_i \in b_j\). In other words, \(T\) is contained in blocks if an adequate block structure can be found such that each \(t_i\) conflicts with exactly one block \(b_j\); if such a block structure cannot be found, or if time patterns cut across blocks, we do not have the required conditions.
CHAPTER SIX: TIME PATTERN ANALYSIS

6.1.1 Degrees of Freedom—Leverage in the Time Pattern Sector

Of the three major elements of school scheduling—time patterns, classes, and resources (instructors, rooms, students)—the time patterns play the dominant role in coordinating the master schedule. Powerful leverage, for better or for worse, lies in this sector; yet, positive manipulation of time pattern variables may be considerably easier here than elsewhere, in terms of both recognition and implementation. This is particularly true of the so-called discretionary leverages, whereby the overall scheduling problem can be rendered more tractable simply by adopting a systematic approach in choosing the correct time pattern families from a number of alternatives, any of which would otherwise be equally acceptable to the other sectors.

Many things are done in school scheduling because they must be; there is often no alternative given the context of previous decisions. Any school has a number of degrees of freedom, within which it may exercise options; once these options are determined, they in turn determine the remaining course of events by severely constraining further decisions. This chapter will study the degrees of freedom enjoyed in the time pattern sector, in terms of the bounds imposed on the solution space as options are determined. These dimensions are categorized as compromises and discretionary leverages according to the costs normally involved with establishing an option: compromises typically involve giving up something
in return (and as such are hardly arbitrary decisions), whereas discretionary leverages are often made among otherwise equally acceptable alternatives. In both cases the options, once determined, dramatically affect subsequent scheduling decisions, where the cycle and the time pattern families become common ground for the other variables.

6.2.1 The Compromises

The compromises in the time pattern sector represent decisions which, for most schools, discriminate between appreciably different alternatives. Whether subjects convene three versus four hours each week is likely to be a serious distinction; the length of a school day has ramifications for bus schedules, faculty morale, physical plant upkeep, etc.; and there are major operating differences between a five-day and a four-day week. For a given school, some of these decisions may not even be negotiable; in other cases, change from a status quo may be very difficult. In any case, these are sensitive decisions in establishing a master schedule, and as such cannot be treated lightly.

Because the costs and payoffs of compromises will vary from school to school, the intent of these sections is not so much to promote one particular option over another, as to admonish a school to question its own decisions and investigate the possibility and impact of alternatives. To emphasize the critical and dominant nature of these decisions, the discussions will cover the cause-effect nature of different options in terms of their impact on subsequent scheduling.
6.2.2 Contact Requirements and Time Pattern Structures

Very early in the scheduling process a school must decide, for each subject in the curriculum, how much contact time students shall have with that subject, and over what distribution of days and periods this time shall be configured. For some subjects, this process may focus upon subject phases, with contact time being configured for lectures, laboratories, recitations, etc. In some situations, specific durations—even particular clock hours—may be required by law or convention. In other cases, choice may be allowed within given bounds.

As a school establishes the contact requirement \( (CR_i) \) for its classes, it is implicitly establishing the resource loads of its instructors, rooms, and students. It is also implicitly—and sometimes unconsciously—starting to determine the free time of its resources: the time a resource will not be involved with formal curriculum. The most important impact that the \( CR_i \) have on the master schedule, and upon individual resource schedules, is in terms of the ratios which the \( CR_i \) bear with respect to each other and to the total cardinality of the cycle. Although cycle design (discussed in the following section 6.2.3) is a contributing factor, the \( CR_i \) of the curriculum play a major role in determining the combinations of subjects which can compositionally make up an individual resource schedule.

Beyond the raw statistic of resource load, determined by the \( CR_i \), the nature of an individual resource schedule is significantly affected by the time pattern structures \( (TS_i) \) represented, because this attribute of the classes determines their distribution over the two dimensions (days and periods) of the cycle. Choice of \( TS_i \), can significantly limit what is
feasible for a resource, even in the presence of theoretically feasible \( CR_i \). Days and modules are two very different dimensions, and both are quite sensitive with respect to the \( TS_j \). For example, if the \( TS_j \) are limited to a single structure \([x^y]\) over a cycle whose module-dimension \( m \) lies inclusively between \( kx+1 \) and \( kx+(x-1) \), mandatory free time must result even if the cycle cardinality is integrally divisible by the \( CR=xy \), as when only five \([3^1]\) time patterns can be accommodated on a 16-period day even though a \( d=3 \) day cycle (of 16 daily periods) has a cardinality \( (48) \) which is integrally divisible by the \( CR=3 \), suggesting a load of 16 such classes rather than only 15. Note that \([y^p]\) time patterns might be accommodated where \([x^y]\) time patterns are not; in the example just given, 16 \([1^3]\) time patterns fit perfectly, emphasizing the difference between the two dimensions involved.

In terms of a priori flexibility, the major role of the \( TS_j \), stems from the combinative relationships which can occur among them. Whereas it is true that a single class with a \( CR \) of 6 could theoretically be assigned in place of two classes whose \( CR_i \) are each 3, it becomes less likely (in some cases impossible) to do so if the single class requires a \([2^3]\) structure while the two classes each require a \( TS_j \), of \([3^1]\). In short: the interchangeability of a set of one or more classes for another set of one or more classes depends very much on the combinative compatibility of the \( TS_j \), represented, as well as the \( CR_i \) involved. Accordingly, in determining \( CR_i \) and \( TS_j \), for a curriculum, a school should be guided not only by the local constraints and preferences of each individual subject, but also by the global behavior of the \( CR_i \) and \( TS_j \).
As will be seen in the normative models, the best ratios of $CR_i$ are those involving multiples of one another. The best $TS_i$, are those which are, in turn, compositions of other $TS_j$, already required. These matters will be discussed in detail later; at this point we simply state as an objective that combinations of $TS_i$, be substitutable for one another in many ways. "Substitutability" is, in fact, a good synonym for the overall objective of interchangeability sought for classes and resources.

In summary, the first two degrees of freedom are (1) the $CR_i$ ratios of the curriculum, and (2) the $TS_i$, chosen to represent the $CR_i$. While it remains a question to what extent a school can vary them, it is definitely true that, once determined, these constraints will influence the remainder of the scheduling process. Their major impact will be upon admissible partitions and compositions over the cycle, a variable we consider next.

6.2.3 Cycle Shape (Days and Modules)

Cycle shape--determined by the numbers $d$ of days and $m$ of daily modules--is second only to the admissible $CR_i$ and $TS_i$, in terms of impact on the scheduling process. Because it limits the cardinality of time pattern families, cycle shape is of major importance to time pattern system design. Just as the $CR_i$ implicitly determine resource loads, the context of a fixed cycle cardinality implicitly determines resource free time, some of which may be mandatory given the $TS_i$.

The compatibility of the cycle shape with the $CR_i$ and $TS_i$, is very important, and this interaction is so critical that for some schools the $CR_i$ and $TS_i$, may themselves have to be retrofit based on a specified cycle. For such schools, the necessity of one particular cycle may be so dominant
as to constrain even the $CR_t$. Although time pattern design is theoretically more influenced by the $TS_t$, than by the cycle design, both are important and are so interconnected that some schools may be faced with a priority problem of deciding which comes first. Cycle shape is, in one sense, the "perimeter" of the problem, with the $TS_t$, controlling the "area", and it is difficult to separate the two components of the "geometry" in terms of functional primacy.

Of the two cycle dimensions, the day-dimension $d$ is usually the more sensitive, both in terms of impact on family design and, unfortunately, in terms of choice allowed most schools. Very rarely does a school have the freedom to choose, say, either a four-day week or a five-day cycle. On the other hand, the distribution of classes over the days is usually a more critical design consideration than distribution over the modules.

Cycle shape limits the cardinality of time pattern families in two ways. The first, rather obvious, way is that the cycle cardinality provides an upper bound on the number of modules which a family can cover (disjointly or otherwise). The other influence, less obvious but just as critical, is that the two-dimensional "perimeter" of the cycle encloses an "area" within which time pattern families must interact, a constraint which occasionally recommends reduction of cardinality within one family in order to interact better with another family. Sacrifice of cardinality within one family because of another is often the fault of the value of $d$ or $m$ for the particular cycle.

In summary, the third and fourth degrees of freedom are the two cycle dimensions: (3) $d$, the number of days in the cycle, and (4) $m$, the number of daily modules. These two variables jointly determine the
cardinality of the cycle $|t_{cycle}| = dm$. As with the first two degrees of freedom, it is not the case that a school will always be free to adjust these "variables"; it should be clear, however, that the number and nature of time patterns in the system will be very much influenced by these parameters. How these parameters influence class interchangeability and resource interchangeability is discussed next.

### 6.3.1 Relationship of Compromises to Overall Problem

The four compromise degrees of freedom in the time pattern sector are:

1. the $CR_i$ ratios of the curriculum,
2. the $TS_i$, chosen to represent the $CR_i$,
3. $d$, the number of days in the cycle, and
4. $m$, the number of daily modules.

These parameters are called compromises because only rarely could or would a scheduling officer unilaterally change one or more of them without the advice and consent of other sectors within the faculty and administration. They are still called degrees of freedom in the sense that change in one or more of these parameters theoretically alters the solution space of the scheduling problem in significant enough manner to warrant thinking of them as key variables. In most schools the resulting differences in the solution space are dramatic enough to at least recommend consideration of compromise. These variables have major impact upon class interchangeability and resource interchangeability. To understand this impact, we look at the resource assignment process (in contrast to the time pattern assignment process) from dual viewpoints: that of an individual resource and that of an individual class.
6.3.2 Impact of Compromises on Class Interchangeability

The resource assignment process can be thought of as assigning classes to resources, resource by resource. From this viewpoint, that of an individual resource, we would like to have broad accessibility to a variety of classes and class-combinations. To an instructor this might mean: "teaching my share of the English sections during my preferences of hours" or "being able to drop in on any of the recitation sections tied to my lecture" or "being able to audit the lectures being given by a certain visiting professor". For a room this might mean: "being able to cover any class for which the room is suited in terms of location, size, and equipment" or "being able to dedicate the room to all lab sections of a subject to simplify experiment setup". To a student this might mean: "flexibility in choosing one or more electives beyond my required program" or "ability to reserve hours after 4:00 for a job or athletics without sacrificing curriculum options".

In each of the above cases, and in similar situations, the desired objective is to achieve flexibility as to how time can be spent. Given any open hole of time in the cycle, small or large (in the extreme, the entire cycle itself), the intent is to have a broad spectrum of classes or class-combinations which fit that hole. The larger the number of classes or class-combinations the better; of course, quality counts along with quantity.

Any hole in the cycle is, by definition, a time pattern $t_h$, and as such has a cardinality and contact requirement $CR_h = |t_h|$, and a time pattern structure $TS_h$. Whether this hole is a unit time pattern of structure $[1^1]$, the entire cycle of structure $[m^d]$, or represents any
any structure in between, the utility of the time pattern \( t_h \) is a function of the compositions of admissible time patterns which are contained in \( t_h \).

It should be clear that such utility is highly dependent on the options chosen to represent the first two degrees of freedom: (1) the \( CR_i \) ratios, and (2) the corresponding \( TS_i \). For example, an odd \( CR_h \) must lead to one or more permanent holes if the chosen \( CR_i \) are all even; so must a \( TS_h = [x^y] \) if either \( x \) or \( y \) are less than the minima represented by the chosen \( TS_i \).

The role played by cycle shape (the other two compromise degrees of freedom) in determining the utility of a hole \( t_h \) is more indirect than that of the \( CR_i \) and \( TS_i \). Cycle shape influences which specific time patterns should be considered admissible, as opposed to whether a time pattern structure could theoretically fit a hole. By influencing specific time pattern design, cycle shape often rules out one or more time patterns which might otherwise fit a given \( t_h \). Furthermore, the cycle contributes immensely to the shape and orientation of the holes in the first place.

To the extent that classes can be assigned to resources only in compositions allowed by admissible time patterns, it follows that which classes and class-combinations are available to an individual resource is dependent on the utility of the cycle and its sub-spaces, in terms of coverage by admissible time patterns. If time patterns and combinations of time patterns are not substitutable, then neither are classes and class-combinations.

6.3.3 Impact of Compromises on Resource Interchangeability

The resource assignment process can be regarded not only from the viewpoint of the individual resources, but also from the dual viewpoint of
assigning resources to classes, class by class. Instead of thinking of resources requiring assignment to classes, we think of classes needing resources: each class requiring one or more instructor(s), at least one room, and a class-worth of students, in order to operate. This viewpoint recommends broad accessibility to resources and resource-combinations. It means: "having at least one qualified instructor available, perhaps an entire team", and "being able to find a room which satisfies the location, size, and equipment needs of the class", and finally "accommodating a reasonable number and composition of student participants".

To achieve all such resource objectives of a given class simultaneously—it is necessary that the time pattern $t_c$ assigned the class fit an open hole in all of the individual resource schedules involved. It is not enough that the instructor(s) be qualified, the room be functionally adequate, and the students compose a good group; all these resources must be free (timewise) to be assigned to the class. And so we are back to holes in resource schedules, and what their shapes are. The shapes of these holes are even more critical from this viewpoint because here we are dealing with many resource schedules, representing a variety of differing curriculum-combinations, all of which must share a common useful sub-space of the cycle. These differing curriculum-combinations result in some schedules (e.g. instructors and special purpose rooms) where the $TS_i$, may all be similar, and some schedules (e.g. students and general purpose rooms) where the $TS_i$, span a spectrum, yet all pertinent schedules must have holes containing $t_c$.

To the extent that resources can be assigned to a class only if appropriate holes exist in their evolving schedules, it follows that which
resources and resource-combinations are available to a given class is dependent on the shape and orientation of these holes. Holes, in turn, are a function of the $CR_t$, $TS_t$, and cycle shape, and therefore resource accessibility is a function of the four compromise degrees of freedom.

6.4.1 The Discretionary Leverages

In the previous sections, we saw how four compromise degrees of freedom influence the school scheduling process. Despite the importance of their impact—and hence the value of understanding their leverage—the usual problem with these four decision areas is precisely their compromise nature: changing the values of these parameters is likely to be difficult. Difficult, because such things as cycle shape are usually sensitive and controversial, and may, frankly, be non-negotiable. This does not mean that such compromise may not be called for, only that it may require justification far beyond its scheduling impact. The advantage of the so-called discretionary leverages is that, for many schools, the choice may be easy to implement, once understood. The alternatives may appear equally acceptable to the community at large, and hence a scheduling officer may be able to take effective action by fiat.

These discretionary leverages are often as powerful as the more controversial compromises; they occasionally result in major improvements, and can sometimes illuminate advantages of compromise. Two time pattern characteristics are discussed: (1) interchangeability, and (2) mesh. Each of these is discussed first within and then between time pattern families, and these sections conclude with a discussion of the impact of these discretionary leverages on the overall problem, again in terms of
class and resource interchangeability.

6.4.2 Time Pattern Interchangeability

_Time pattern interchangeability_ can be defined as the ability to substitute one or more time patterns from one family for one or more time patterns from either the same or different families, in such a manner that the resulting impact on the master schedule is minimal. Included in this concept is the notion of a _priori_ indifference among the time patterns of a family so far as individual resources are concerned. Two types of interchangeability are discussed: _intra_-family interchangeability within a family, and _inter_-family interchangeability between families.

6.4.3 Intra-Family Interchangeability

Four obstacles can stand in the way of time patterns achieving interchangeability within the same family (of similar structure):

1. disparity among the days represented,
2. disparity among the clock times represented,
3. substantial non-congruence,
4. different _priori_ conflict behavior within the family.

Disparity among the days represented occurs when one or more days of the cycle are biased for or against, and the time patterns that make up a family do not equitably distribute the bias. For example, if Monday holidays occur frequently, then \([x^1]\) classes will miss some weeks if their day is Monday instead of some other day; a similar disparity occurs for such structures as \([21]\) where one or more days are heavier than others and day bias exists--note that rectangular \([x^d]\) classes share such bias and hence are not disparate.
Disparity among the clock times represented is an obstacle similar to the first, and occurs when one or more daily modules are biased for or against, and the time patterns that make up a family do not equitably distribute the bias. The most common examples are: identification of some modules for lunch (thus lessening their usability for other purposes), and the case where early morning and/or late afternoon hours are considered undesirable.

Substantial non-congruence results from similar time patterns deviating so far from congruence as to render them effectively different structures, as when a \([x^2]\) time pattern family must be decomposed into those time patterns with two adjacent days, and those with the two days separated by one or more intervening days. Many schools using a four-day or five-day week would consider two-day time patterns involving consecutive days inferior to time patterns splitting the contact. Deviation from straightness might also be considered a flaw by some schools.

The fourth obstacle, different a priori conflict behavior within the family, can occur only in non-disjoint time pattern families, and is characterized by a subset of one or more time patterns in conflict with \(i\) family members, while another subset of one or more time patterns are in conflict with \(j \neq i\) members. The classic example of this is the "weed" time pattern 'M1,T2,W3,R4,F5' which cuts a diagonal swath across five straight 'MTWRFi' time patterns in a \([1^5]\) family, thus preempting five times its cardinality with respect to the straight members.

The most direct way to sidestep any of these four intra-family obstacles is simply to stop calling such time patterns the same family. In most cases, the biases or other differences partition the original
family into two or more sub-families within which the member time patterns really are interchangeable. If for no other reason than precise communication, it is important to discern these sub-families, interchangeable within themselves.

Although such factoring and redefinition is straightforward, and an enhancement of communication, it does not solve the larger problem of which sub-families to consider admissible: one, some, or all. This is where the discretion comes in.

For literally any structure, other than the trivial $[1^1]$ or $[m^d]$ extremes, the universal family $T^\infty$, containing all possible time patterns of that structure, will contain a large proportion of redundant time patterns. Restricted to but one structure, the ideal system is any family of time patterns which are supplementary (disjointly cover the cycle). Since several (usually many) such supplementary families exist, a judicious choice among these alternatives may resolve any interchangeability problems. Problems of substantial non-congruence will disappear if a congruent (or quite similar) sub-family can be found to represent the structure instead of the problem family. Different a priori conflict behavior must disappear if a disjoint subset of the problem family is adopted.

Several approaches may be taken in avoiding day disparity. It may be that less than total coverage is desirable in order to achieve, say, straight time patterns. In such cases (e.g. $[4^2]$ and $[3^2]$ time patterns in the five-day week of the M.I.T. case study) one entire day may turn out to be superfluous, and disparity among days might be sidestepped by simply singling out the appropriate day to avoid. Or perhaps the $d$-day cycle, even if $d=5$, might be loosened from lockstep with the five-calendar-day
week to compensate for holidays, snow closings, etc. and thus reduce day disparity (should a day or days be missed, the cycle simply picks up after the disruption where it left off).

It is a fact that the conflict behavior of any master schedule is unaffected by interchanging those (vertical) portions of the cycle represented by any two days; the resulting schedule may worsen or improve for other reasons, but not relative to conflicts. Accordingly, any number of pairwise day interchanges, and therefore any permutation of the days, is transparent with respect to conflicts (which can neither be created nor eliminated in the process). Such day permutation does influence the impact of calendar days on cycle days, and an alternative sequence of such permutation might eliminate day disparity through equitable distribution of time patterns over biased days. Care must be taken with respect to day separation lest substantial non-congruence result, but simple cycle reversal (the first day one cycle is the last day the next, etc.) avoids even this problem. Of course, if the cycle is already free of the calendar, cycle reversal could cause day separation problems. Note that community acceptance of any day permutation is more likely in schools where resources are fully loaded (as opposed to institutions where permuted light loads could look very irregular).

Module disparity can be finessed in similar ways as day disparity, but additional care must typically be taken to avoid fragmentation within multiple-period time patterns. Of course, if one is able to simply single out and avoid biased modules in the normal course of time pattern design (as when less than complete cycle coverage is sought anyway), so much the better. But often use must be made of undesirable clock hours, and
in this case either a non-interchangeable family must result or some form of sharing the bias must occur. Module permutation is similar to day permutation, but care must be taken not to fragment any structures: all day permutations preserve structure, but most module permutations do not. (Interchanging the first and last periods of a day could result in a \([2^y]\) class splitting its sessions across the day, e.g. second and last periods; the comparable effect under day permutation merely affects day separation, usually a less sensitive transform.) Two particular forms of module permutation are singled out here: module rotation and module inversion.

\[
\begin{array}{cccccc}
M & T & W & R & F \\
1 & 1 & m & m-1 & m-2 & m-3 \\
2 & 2 & 1 & m & m-1 & m-2 \\
3 & 3 & 2 & 1 & m & m-1 \\
\text{modules} & \cdots & \cdots & \cdots & \cdots & \cdots \\
(m-1) & m-1 & m-2 & m-3 & m-4 & m-5 \\
m & m & m-1 & m-2 & m-3 & m-4
\end{array}
\]

Fig. 6-a.--Module Rotation in a 5-day \(m\)-module Cycle; Shift Factor of 1

A family of interchangeable \([1^5]\) time patterns such as shown in Fig. 6-a is adopted by some schools to sidestep the unpopular time syndrome. Note that there is a one-to-one correspondence between this arrangement and the straight 'MTWRFi' time patterns; we call this mapping module rotation, a transformation where the first period each day is shifted later and later in each succeeding day of the cycle, carrying all the next periods with it and wrapping around the chronologically latest and earliest times. Module rotation can be carried out even in the presence of time patterns involving more than one daily module, but such rotation must be carefully
designed, and the shift factor adjusted accordingly, i.e. the shift factor must be a multiple of the LCM (least common multiple) of the time pattern lengths.

A related transformation which works with a broader variety of structures (but which may not pay off as much as module rotation if the last period is held by some in equal contempt as the first period) is that of module inversion, whereby every other day (or some other subset of days) is reflected about a midday axis: the first period one day becoming the last, the second period becoming the penultimate, etc. Fig. 6-b illustrates an example of module inversion applied to every other day in a 5-day \( m \)-module cycle.

\[
\begin{array}{cccc}
  M & T & 5 \text{ days} & R \\
  1 & 1 & m & 1 \\
  2 & 2 & m-1 & 2 \\
  3 & 3 & m-2 & 3 \\
  m & \ldots & \ldots & \ldots \\
  (m-1) & m-1 & 2 & m-1 \\
  m & m & 1 & m \\
\end{array}
\]

Fig. 6-b.--Module Inversion Applied to Every Other Day in a 5-day \( m \)-module Cycle

Before leaving module permutation, several points should be made:

(a) Module rotation tends to complicate (and module inversion usually has little effect on) certain ways in which lunch may be accommodated by a school; since lunch tends to require straight time patterns, diagonal classes may preempt lunch on a few days if not careful.
Any module permutation may play havoc with free time requirements of resources. Extracurricular commitments in the outside world have a tendency to be symmetric with respect to days; e.g. a part-time instructor may have another commitment requiring the same clock hours daily, or students out for athletics may have to be free after 4:00 daily.

In those cases when module permutation will be used in conjunction with computer-assisted scheduling, the school may choose to perform the transformation(s) at the last possible occasion (say, just before actually printing output for distribution), and instead deal with straight time patterns throughout developmental or simulation computer runs. It is easier to conceptualize and cope with straight time patterns; for one thing, the name notation is more abbreviated ('MTWRF1' being shorter than 'M1,T2,W3,R4,F5') and more readily visualized.

6.4.4 Inter-Family Interchangeability

Five obstacles can stand in the way of time patterns from different families (of dissimilar structures) achieving interchangeability. Two of these obstacles amount to the first two just discussed as intra-family considerations: (1) disparity among the days represented, and (2) disparity among the clock times represented. A third obstacle is: (3) disparity among the $CR_i$ or $TS_i$, represented. The fourth is similar to the fourth intra-family obstacle, and is the problem of: (4) different a priori conflict behavior of time patterns from one family with respect to time patterns from the others. The fifth stems from: (5) differences in
compositional behavior.

One or both of the first two obstacles occurs whenever one or more days and/or clock times are biased either for or against, and the families involved do not equitably share the bias. Such would be the case if, for example, Monday was a poor day due to holidays, and one family (say, of structure $[2^3]$) was restricted to MWF, while another family (say, of structure $[3^2]$) was restricted to TR. Even should each family equitably distribute the bias within the family, the $[2^3]$ family shares all the bias against Monday, while the $[3^2]$ family has none of the bias. In this example, a class with a contact requirement of 6 would favor the $[3^2]$ family over the other (assuming no other distinction between the two structures). Similar disparity between two families would occur if one family involved poor clock times while another family did not (even if the bias was balanced within each family). To sidestep such an obstacle, one tries to design time pattern families in such a way as to distribute bias equitably among the families as well as within them. This may be more difficult than the intra-family cure, since different structures may make complete equality impossible. In the above example ($[2^3]$ and $[3^2]$ families with Monday poor), even if $[3^2]$ time patterns do use Monday, their basic structure forces use of "more Monday" than the $[2^3]$ time patterns—a true impasse. Nevertheless, a closer approximation to balance may be thus achieved, and disparity at least reduced if not eliminated. Unfortunately, other considerations may so dominate design as to militate against distribution of bias; after all, in contagiously spreading one family's problem to another to reduce disparity, the infection introduced may prove more serious than the original inequity. The objective is to:
all other things being equal, reduce disparity among time pattern families relative to days and clock times, using techniques similar to those used to reduce disparity within an individual time pattern family.

The third obstacle, disparity among the $CR_i$ or $TS_i$, represented, is less a problem of individual time patterns as it is a problem of the structures they represent. If two time pattern families have two different $CR_i$, it is very unlikely that they can be considered interchangeable; seldom could a class be able to use both. The more subtle problem occurs when the contact requirement is identical, but the $TS_i$, differ (such was the case is our example of $CR=6$, $TS_1=[2^3]$, $TS_2 = [3^2]$). Even here, many schools would discriminate between the two structures and few classes could use either interchangeably. There may be clear reasons for such a distinction, but there may also be less obvious bias. If a faculty member is to have a 2-day week, there is at least a chance of this with the $[3^2]$ time patterns, but no such possibility with even one $[2^3]$ time pattern. Carrying this example one step further, depending on the time pattern conventions a school uses, it may be better to achieve a 3-day week with $[2^3]$ time patterns given the risk of a 4-day week associated with $[3^2]$ time patterns. Note that this type of preference need have nothing to do with load—the issue here is one of specific days (or clock times) used by the school for the different families. It is not the intent of this thesis to either encourage or dissuade a school regarding policies on instructor preferences, but simply to point out that when one or more $TS_i$, serve the same $CR_i$, it is difficult to avoid disparity; whether or not the disparity is desirable is up to the school. At M.I.T. the same humanities subject
(with a contact requirement of 6 half-hour modules) might be taught in sections using \([2^3], [3^2],\) and \([6^1]\) (evening) time patterns, and this spectrum is apparently regarded desirable both for students and instructors; at some schools this flexibility might lead to a Pandora's box of bias. An important question to ask if several \(TS_j\), are interchangeable, is whether one structure could serve the needs of all, since it is easier to design ideal or near ideal systems with fewer different \(TS_j\).

The fourth obstacle to inter-family interchangeability is different a priori conflict behavior of time patterns from one family with respect to time patterns from the others. Suppose a school restricts its \([1^3]\) classes to MWF in a five day cycle. Further suppose that \([1^2]\) classes are allowed to run either TR or WF. Even if the \([1^2]\) time patterns in this example are congruent and mutually disjoint—and therefore potentially interchangeable within their family—they are not interchangeable with each other when one considers conflict behavior with the \([1^3]\) time patterns. One can conjure up numerous hypothetical cases where bias could result from such a situation, one of the simplest being a school using \([1^3]\) MWF time patterns for the majority of required subjects, in which case \([1^2]\) electives would tend to favor the TR time patterns—and bias against the WF time patterns—in order to minimize conflicts. Such an example illustrates a lack of inter-family interchangeability.

The easiest escape from such disparity may still be to stop calling \([1^2]\) time patterns a single family, and instead to reference the TR time patterns as one family and the WF time patterns as another. The ultimate objective is to achieve equitable conflict behavior, usually by pruning time pattern families down to a basic interchangeable nucleus, wherever
such simplification does not incur more serious difficulties. This might be done in our example by not using WF for \([1^2]\) time patterns to TR alone; unfortunately, both might be needed in some schools to minimize conflicts among \([1^2]\) classes.

The fifth and final obstacle to inter-family interchangeability stems from differences in compositional behavior. In the broadest sense, compositional behavior could cover any qualitative and quantitative aspect of schedules using time patterns from the pertinent families, including such considerations as the \(n\)-day work week discussed above as part of disparity among the \(CR_i\) or \(TS_i\), represented. In actuality, this obstacle involves a narrower interpretation of compositional behavior, namely the combinatorial restrictions placed by the families on admissible partitions and compositions of the cycle and its sub-spaces. For example, the straight \([1^5]\) time patterns 'MTWRFi' each can be decomposed into one \([1^3]\) time pattern 'MWFi' and one \([1^2]\) time pattern 'TRi'. In this case, there is a one-to-one correspondence between each \([1^5]\) time pattern and a related pair of time patterns (one \([1^3]\) and one \([1^2]\)). Hence there is a compositional trade-off in the spectrum of possible class combinations: any straight \([1^5]\) hole can be used for either one \([1^5]\) class or a pair of \([1^3]\) and \([1^2]\) classes. In this example, it is important that the cardinality of the \([1^5]\) structure (5) equals the sum (3+2) of the cardinalities of the \([1^3]\) and \([1^2]\) structures; i.e. that the contact requirement of \([1^5]\) classes is the same as the sum of the \(CR_i\) for the associated pair of classes. But it is also important that the decomposition works disjointly the way it does; if any one (or two) of the three pertinent families involves non-straight time patterns, the one-to-one
substitutability described above would fail.

A second example of compositional behavior is the case of forced consistency where once any time pattern is chosen from a family, it may be advisable to stay within that same family. Forced consistency is exemplified by two families of \(2^1\) time patterns, one involving time patterns starting with even numbered periods, the other with odd numbered periods. Note that both families have the same structure \(2^1\), but once either family is used, it may be well to consistently stay within it. Examples can also be contrived where consistent alternation between families is recommended rather than dedication to any one.

A final example of compositional behavior emphasizes the difference between the two cycle dimensions (days and periods). It was mentioned earlier that 16 \([1^3]\) time patterns can be designed to disjointly cover a \(d=3\) (day) by \(m=16\) (module) cycle, but only a maximum of 15 \([3^1]\) time patterns are disjointly attainable. If \([x^y]\) time patterns can differ from \([y^x]\) time patterns—despite the same contact requirement—how important it is to be careful with more complex structures.

6.4.5 Time Pattern Mesh

The American Heritage Dictionary of the English Language includes in the definition of the verb "mesh": "2. To be or become engaged or interlocked, as gear teeth. 3.a. To coordinate or fit harmoniously and effectively... b. To accord with another; harmonize." As a noun, the word means, among other things: "3. A net or network... 5. The engagement of gear teeth." Borrowing from these entries, we define
time pattern mesh as the extent to which time patterns engage and coordinate harmoniously with one another in individual resource schedules. At least three aspects of harmonious accord are involved if time patterns are to mesh well:

1. they must compose in mutually disjoint combinations (imagine gears with extraneous teeth);
2. they should engage or interlock without leaving holes (imagine gears with missing teeth); and
3. they should be systematic, possessing some sort of regularity, simplicity, and modularity (imagine gears designed by Rube Goldberg or at random).

The objective here is tantamount to designing each time pattern in the context of all the others. Being a recursive specification, this objective emphasizes the critical nature of where one starts. The normative models were chosen, in part, to demonstrate the role of time pattern mesh. Pursuit of appropriate mesh often provides useful evaluation for the compromises discussed earlier. Time pattern mesh is a major consideration in performing time pattern analysis.

Because mesh applies both to time patterns of the same structure and to those of dissimilar shapes, we can talk about both intra- and inter-family mesh, the subjects of the next two sections. What is at stake in all cases is the utility of the overall cycle, usually related directly to the utility of its sub-spaces. The utility of time patterns is little more than the quality and quantity of compositions they enjoy in combination, an interaction dominated by mesh.
6.4.6 Intra-Family Mesh

Intra-family mesh is the manner in which time patterns within the same family interact. Because a time pattern family is usually characterized by the same structure (almost always by the same contact requirement), and typically by similar shapes, it is easier to describe and attain good mesh within a family than with different shapes or structures. The fact that ideal time pattern systems can be readily identified for schools with only one time pattern family is offset by the scarcity of such environments in practice. The primary reason for understanding intra-family mesh is therefore to equip the time pattern designer with the motivation and techniques for preserving whatever good intra-family mesh can be retained, if and when local family ideals must be balanced against and traded for global objectives involving all families in the time pattern system. To be ideal, a time pattern family must be ideal in context, and this can mean less than ideal in terms of the family by itself.

As we will see in the normative models, good mesh is best locally attained by a time pattern family that is supplementary (disjoint covering the cycle). The gear analogy motivates the disjointness, but the coverage, though secondary, is still important. Once a supplementary family is designed, it should be intuitively clear, that were it the only family, it makes little sense to either extend or prune the member time patterns. By definition, extension must result in either duplication or other non-disjointness, and pruning sacrifices family cardinality and coverage without compensatory gain.

The characteristic of being supplementary satisfies (by definition)
the first two aspects of harmonious accord that (1) the time patterns compose in mutually disjoint combinations, and (2) they should (at least can) engage or interlock without leaving holes. The third aspect, that of being systematic, is usually satisfied within a family simply because it is a family, composed of related time patterns; if interchangeability considerations have been paid due respect, the family is likely to be regular.

When the time patterns within a family mesh well, it is a mark of such a family that we can usually identify adjacent time patterns which touch each other. Touching in this sense still requires disjointness, but emphasizes spatial proximity; the intuitive visual concept is probably adequate, but a reasonable informal definition is sharing one or more periods on adjacent days, or sharing adjacent periods on one or more days. Of course, time patterns can touch the perimeter boundaries of the two-dimensional cycle as well as each other, and this is good too, in that holes between the time patterns and the perimeter need not crop up.

The scheduling objective to keep in mind is that of avoiding obstructive fragmentation of the cycle and its sub-spaces; when time patterns touch each other and the cycle perimeter, it is at least possible to combine them in such a manner as to minimize the creation of fragmenting holes. The desired effect is analogous to close packing of similar objects in a two-dimensional physical environment. A strength of this physical analogy is the emphasis placed on the role of the objects' shape in determining the success or failure of total coverage (clearly useful in scheduling), and a simplicity and regularity of pattern used (a more subtle characteristic, the importance of which is only a matter of
aesthetics with a single family, but a growing concern as other families enter the scene). We have defined the utility of cycle sub-space in terms of the compositions of time patterns which can disjointly fit that space; in this case the close packing analogy corresponds strongly to the mesh desired within a single family. The problem, of course, is much more complicated as dissimilar shapes are introduced, and this is discussed in the next section.

6.4.7 Inter-Family Mesh

As dissimilar time pattern shapes are introduced, the "gears" must mesh well not only when they are the same shape but also when gears of diverse size and shape interact. The close packing analogy of the previous section must be expanded to involve a variety of shapes.

The scheduling objective remains that of avoiding obstructive fragmentation of the cycle and its sub-spaces. This goal is made considerably more complex and difficult by the presence of different time pattern shapes. The third aspect of harmonious accord, that time patterns be systematic, is now a major concern. It is not enough that each family stand on its own—each time pattern of every family must interact well with the time patterns of other families. This is particularly important to those resources, such as students and general purpose classrooms, involved with a broad cross-section of the curriculum. Such resources make it conceivable that any family may have to share cycle space with literally any other family, and this cooperation in doing so is essential.

In order to achieve harmonious accord among time pattern families, it may be necessary to compromise the locally ideal nature of one or more
families in order to attain systematic coordination of the entire system of families. Where possible, proper interaction should be preserved within individual families, but intra-family mesh is an equal concern. The best time pattern systems are those achieving inter-family mesh with a minimal disruption to mesh within individual families.

The need for simultaneous inter- and intra-family mesh is brought on by the spectrum of resource demands on the master schedule. Since some resources (instructors, special purpose rooms) tend to concentrate on one, perhaps two, families, while others cut across families, both types of mesh are usually important in the overall problem, as discussed in the next sections.

6.5.1 Relationship of Discretionary Leverages to Overall Problem

The two discretionary leverages in the time pattern sector are (1) time pattern interchangeability and (2) time pattern mesh. Each applies both within and among time pattern families. Both have major impact upon class interchangeability and resource interchangeability. We again consider the resource assignment process from dual viewpoints: that of an individual resource and that of an individual class.

6.5.2 Impact of Discretionary Leverages on Class Interchangeability

If we consider the resource assignment process from the viewpoint of an individual resource, we would like to have broad accessibility to a variety of classes and class combinations. Time pattern interchangeability and mesh are both critical if the resource is restricted to one time pattern family and even more so if a variety of time pattern families is involved.
A time pattern is usually chosen for a class because of its acceptability to all the resources involved. If an inconvenient time pattern must be assigned because the instructor or room schedule prohibits better choice, the students interested in the class may suffer. Or if only one time pattern is feasible because of student demands, it may be difficult to accommodate instructor preference or to find a room. Lack of interchangeability or mesh relative to one resource set can cause problems for other resources.

If instructors prefer morning hours, but students prefer afternoon hours, one of these resource sets is likely to limit broad accessibility to classes on the part of the other. If physical education classes must be assigned times solely to accommodate use of the gymnasium and/or coaches, student programs may be limited in scope. An exotic time pattern for a popular elective can cost the subject its popularity.

Whether or not a class can be assigned to a resource is usually a direct function of the compatibility of its time pattern with respect to the cumulative schedule of that resource. The discretionary leverages play a double role in such compatibility: (1) the class should fit into the schedule without conflict, and (2) the class should not lead to obstructive fragmentation unnecessarily limiting further choices.

6.5.3 Impact of Discretionary Leverages on Resource Interchangeability

When we consider the resource assignment process from the viewpoint of an individual class, we would like to have broad access to resources and resource combinations. This viewpoint is particularly important in order to achieve a priori flexibility, should perturbations during
implementation alter the profile of resource demands on the class.

Remember that a time pattern $t_o$ assigned the class must concurrently fit the individual resource schedules of all resources involved with the class. In discussing the impact of compromises on resource interchangeability, we emphasized the importance of the shape of holes in resource schedules, since these holes must all contain $t_o$. The shape and orientation of holes in resource schedules are functions of the way in which previous assignments to classes preempt cycle space, a process dominated by time pattern mesh. The fewer the holes, and the more flexible the cycle sub-space represented by holes, the more likely that holes will contain $t_o$. Since time pattern interchangeability and mesh apply to holes as well as the overall cycle space, they are pervasive concerns throughout the resource assignment process.
7.1.1. Trivial School Scheduling

There are a few types of trivial school scheduling problems, trivial in the sense that we can quickly "solve" them and turn our attention elsewhere. The first is the completely constrained problem where only one master schedule is admissible—the corresponding time pattern assignments and resource assignments are all absolute. In such a case, these permanent assignments determine the time pattern and resource schedules under an easily applied identity mapping, and we are done—done, because we had no choice whatsoever in what we were given to do. This "extreme" is not all that extreme in live applications; at the Massachusetts Institute of Technology, the instructor sector, itself essentially pinned down, almost completely determines the time pattern schedule and a substantial percentage of the room assignments, leaving only a percentage of room assignments and (of course) student allocation. These remaining aspects of resource assignment, although serious, are a small proportion of the larger problem. Note that the solutions determined by completely constrained problems, although "trivial because implicit", are not necessarily highly satisfactory!

At the other end of the scheduling spectrum, we have the (hypothetical) institution with infinite flexibility and unlimited resources. In contrast to the total absence of any choice which characterized the completely constrained problem, this time it is the total freedom of choice which
renders this problem trivial—we can do no wrong! This extreme is the farfetched one in real life.

An interesting case of trivial scheduling does exist, however, which not only has seen live application, but also which shares some of the characteristics of the two extreme cases just mentioned—the "little red schoolhouse" where there was one instructor, one room, and one conglomeration of students, brought together for one daily session. This school had only the one possible master schedule since it had such constrained resources, and hence the schedule was trivial by reason of the permanency of the assignments. But in a vastly more interesting sense, the schedule was also trivial because of the rich interchangeability of its resources. That one instructor, by default if not qualification, was assignable to any (all!) of the instructing duties; that room was assignable to any (all!) of the stations required; and the classes were conducted in parallel during the single daily time pattern. The interesting point here, important in extrapolation, is that schools with totally interchangeable resources (and time patterns) enjoy many of the advantages associated with infinite resources, largely because of the richness of the solution space. Of course, it is unrealistic to imagine instructors, any of whom can teach any subject; or general purpose rooms really suited for literally any subject. However, it is worth knowing the advantages of such a situation—before building a new physical plant, for example—and one of the key points of this thesis is that what resource interchangeability does exist can be greatly enhanced by time pattern interchangeability.
This last claim, that time pattern interchangeability can enhance resource interchangeability, can be intuitively justified by way of an example. In assigning, say, a room to a class, it is not sufficient that the room merely be suitable for the class; it also has to be available during a time pattern which is suitable for the class. In other words, just because the room is free (of other assignments) during five diverse modules does not mean the room can be assigned to just any five-module class for which it is suited. There might be no instructor available then, or perhaps no students could attend without conflict. The problem is, of course, one of time pattern conflict, which arises from the basic premise that resources can be assigned to at most one class at any one moment of time. (By judicious choice of our definition of resource, we can ensure this; e.g. if a physical room can hold two classes concurrently we logically define it to be two rooms.) When resources are available is as important in most cases as the fact that they are suitable for assignment.

7.2.1 The Multiple Jigsaw Puzzle Analogy

The multiple jigsaw puzzle analogy involves the configuration of jigsaw puzzle pieces for use in constructing combinative designs on not just one but several game boards, each requiring a different design. Once the shape of a piece is determined, that same piece must be reused on a number of boards in designs which sometimes call for similar pieces, sometimes for dissimilar ones.

The game is characterized by rules (constraints and preferences) on the number and shapes of pieces, the uses of pieces on game boards, the
designs permitted, etc. Two particular characteristics are:

(1) The uniform size and shape of all game boards, some of which are marked with forbidden zones which cannot be covered by pieces.

(2) The rule that the pieces, when used, must always be placed in the exact same coordinates on a board—-the pieces are really more like overlays with fixed shape and orientation.

While pieces must fit or mesh without overlap, we do allow (in some cases) less than total coverage of certain boards.

The pieces of our analogy are the classes, the shape and orientation of a piece representing the time pattern assigned to the class. There is exactly one game board for each and every resource (instructor, room, and student). The set of pieces used on a board are that individual resource's assignments to classes. The combinative design on a game board constitutes the individual resource schedule for that resource. The uniform size and shape of the boards reflect the schedule cycle, and forbidden zones are simply the modules in the cycle during which the resource cannot be assigned (the free time of the resource). (We might also allow avoidance zones during which the school prefers the resource not be assigned.) Rules governing the piece shapes parallel the time pattern needs of the classes; those governing use of pieces on boards parallel the acceptability of resource assignments to classes. The combinative designs are subject to evaluation in terms of daily loading, lunch, etc.

This jigsaw puzzle analogy emphasizes the importance of good piece design. In a normal jigsaw puzzle, the pieces mesh well in that (1) they are mutually disjoint, and (2) they engage and interlock without leaving
holes. In real puzzles, regularity of shape is usually unimportant. If anything, congruent shapes tend to make real puzzles more difficult, but this is because real puzzles deal with a single unique target design.

The multiple puzzle analogy differs from real puzzles on four important counts:

(1) Many different target designs are required.

(2) Several collections of (different shaped) pieces are involved. Even though each collection might cover boards (or part of them) by itself, the target designs will often call for shapes from more than one collection.

(3) It is unnecessary to use all the pieces; but once used, a piece must generally be reused (a class with an instructor but no room or students is unlikely).

(4) Boards often need not be completely covered, and in some cases should not be.

In view of the critical importance of shape in solving the multiple puzzle problem, the need for a systematic design should become evident. This is because the pieces must mesh well not only in one combination, but in many different target designs. Serving one sector well, as when instructors and special purpose rooms require coverage of the cycle by similar shapes, does not necessarily mean serving other sectors well. Students generally require a mix of shapes, and general purpose rooms can go either way.

This analogy is valuable towards intuitively understanding the role of time pattern analysis for many reasons. It is claimed to be a "natural" analogy, because there is an ease with which school scheduling
considerations can be translated to the puzzle game and vice versa without becoming too farfetched. For example, section balancing—the equitable distribution of students over multiple sections of the same subject—can be introduced into the game by counting the number of uses of a piece on boards, and allowing minima, maxima, and various other functions as extended rules of the game.

The analogy is particularly valuable because of the emphasis it places on the judicious shaping of the playing pieces. In the physical board environment it becomes patently obvious that success in the combinatorial configurations is directly related to intelligent choice of shapes for the pieces, and that poor pattern design will haunt the player throughout the game. An important parallel is suggested by the cogent arguments against playing this game "by committee", where different teams (departments) independently design piece shapes, while another team (the administration) independently establishes game board shapes.

The importance of resource interchangeability and its further enhancement by time pattern interchangeability are dramatized when we are confronted by such game rules as "each piece must be used exactly once on an instructor board, exactly once on a room board, and between \(x\) and \(y\) times on student boards". We are fortunate indeed to have flexibility in terms of board availability. Even if only one instructor board "works", it is nice to have a variety of room boards to choose from, any of which "work".
7.3.1 Heuristics

Several strategies and tactics recommend themselves for use during the time pattern assignment process of choosing times for classes. Those discussed in this chapter are heuristics (which tend to pay off), as opposed to algorithms (which are guaranteed to work). Heuristics often reflect common sense, and the three heuristics discussed in the following sections—time pattern reuse, block reuse, and block design and synchronization—have intuitive appeal. All three represent decision aids likely to be adopted if manually developing a master schedule, and the first two are incorporated into the logic of the GASP (Generalized Academic Simulation Programs) computer system used at the Massachusetts Institute of Technology for more than a decade.

7.3.2 Time Pattern and Block Reuse

Common sense suggests running two classes $c_1, c_2$ at the same time $t_1$ when there are no resources in common. For example, a first-year English class is unlikely to involve any of the same resources (instructor, room, or students) as a sophomore Physical Education class. Since there is no possibility of resource conflict, it does not matter that the classes conflict. Assuming one time pattern $t_1$ will serve both classes, assigning it in common is better than using two different time patterns $t_1, t_2$.

The reasoning behind reuse of $t_1$ is evident when one considers a third class $c_3$, for example an elective such as band or chorus. It is quite reasonable to expect that students attending either $c_1$ or $c_2$ may also attend $c_3$, and when we are faced with choosing a time for $c_3$, we have
more cycle space left to choose from when \( c_1, c_2 \) share \( t_1 \) than when two different time patterns \( t_1, t_2 \) preempt cycle space.

Not using a new time pattern until we have to is one way to reserve as large as possible a sub-space of the cycle for later use. The argument can be extended from reuse of an individual time pattern to the advisability of block reuse. Recall that a family \( B \) of blocks \( b_i \) represents a "gross" partition of the cycle, disjointly covering the cycle. If we are forced to choose between a time pattern \( t_i \) contained in a block \( b_i \) which has already been used, and a time pattern \( t_j \) contained in a block \( b_j \) which is not yet used, the better choice is to assign \( t_i \) so that \( b_j \) can retain maximum flexibility. Thus if a resource is already assigned classes during the first period, all other things being equal it is better to assign a second period time pattern than a third period one. If the third period time pattern is assigned, two holes are left in the resource schedule, one consisting of the second period, the other of the fourth through last periods. The second period hole can only be filled with a \([i^?]\) class, and the other hole is one less period in length than it need be. By contrast, a single, larger, and therefore more flexible, cycle sub-space is left by assigning the second period time pattern instead of the third.

As another example, in a five day cycle, where a resource is already assigned a class 'MWFl', given an otherwise equal choice of 'TR1' and 'TR3', the 'TR1' time pattern should be assigned to cumulatively preempt the rectangular time pattern 'MTWRFl' rather than risk obstructive fragmentation.
7.3.3 Block Design and Synchronization

The heuristics of time pattern and block reuse have particular validity if a single family $B$ of blocks $b_i$ can be designed such that each and every time pattern family is contained in the same blocks. To accomplish this, design of the time pattern system must consider both block design and design of the time pattern families, since each consideration may influence the other.

When all time pattern families are contained in a single family of congruent blocks, a valuable kind of time pattern mesh is thereby accomplished. A supplementary family $B$ of congruent blocks $b_i$ already enjoys ideal intra-family mesh, and this advantage is inherited by each time pattern family contained in the blocks. A time pattern system benefits considerably from the discipline of well-behaved time pattern blocks, because obstructive fragmentation can be attacked by such heuristics as time pattern reuse and block reuse.
8.1.1 Compositions and Partitions of a String $\overline{m}$

Time pattern analysis is concerned with the subdivision of cycle space, and its allocation to a variety of classes and class combinations. Because of the objective of avoiding conflicts—partial conflicts in particular—and because of the variety of target configurations sought through subdivision, two concepts from combinatorial analysis are pertinent: composition and partition. Both concepts deal with disjoint subdivision of an entity into a variety of different configurations.

This first section considers disjoint subdivisions of the $m$ modules from a single day. The $m$ consecutive modules can be thought of as a string of length $\overline{m}$, denoted $\overline{m}$ (by using a superscript bar). The string $\overline{m}$ can be subdivided into one or more disjoint substrings $\overline{t}_1\overline{t}_2\ldots\overline{t}_n$, where the length of each substring $\overline{t}_i$ is $l_i$, and $\sum l_i=m$.

In combinatorial analysis, a composition of the integer $m$ is defined to be an ordered collection of integers with given sum $m$. We define a composition of the string $\overline{m}$ (of $m$ consecutive modules on any one day) to be an ordered collection of $n$ ($1 \leq n \leq m$) disjoint substrings $\overline{t}_1, \overline{t}_2, \ldots, \overline{t}_n$ (of $l_i$ consecutive modules), the sum of whose lengths $\sum l_i=m$. For example, the six compositions of $\overline{4}$, restricted to substring lengths which are powers of 2, are: $\overline{4}$, $\overline{2^2}$, $\overline{21}$, $\overline{121}$, $\overline{12^2}$, and $\overline{4}$.

In combinatorial analysis, a partition of the integer $m$ is defined to be a collection of integers with given sum $m$, without regard to order.
We define a partition of the string $\overline{m}$ (of $m$ consecutive modules on any one day) to be an equivalence class of compositions of $\overline{m}$, characterized solely by the substring lengths and number of occurrences thereof in the member compositions. For example, the four partitions of $\overline{4}$, restricted to substring lengths which are powers of 2, are: $\overline{4}$, $\overline{2^2}$, $\overline{2^1}$ and $\overline{1^4}$. (Compositions $\overline{121}$ and $\overline{1^22}$ are both equivalent to $\overline{21^2}$.)

There are exactly $2^{(m-1)}$ unique compositions of $\overline{m}$, a cardinality which can be readily derived as follows. There are $m$ unit length substrings $\overline{I}$ within $\overline{m}$, and therefore $(m-1)$ places between adjacent pairs of unit length substrings where a subdivision of $\overline{m}$ could occur. There is a straightforward one-to-one correspondence between the $(m-1)$-bit binary representation of the integers $i=0,1,2,...,2^{(m-1)}-1$ and unique compositions of $\overline{m}$, whereby ones in the $(m-1)$-bit binary representation of the integer $i$ represent subdivisions between corresponding adjacent pairs of unit length substrings. Figure 8-a illustrates this one-to-one correspondence.

<table>
<thead>
<tr>
<th>integer $i$</th>
<th>$(m-1)$-bit binary representation of $i$</th>
<th>corresponding composition of $\overline{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 0 0 ... 0 0 0</td>
<td>$\overline{m}$</td>
</tr>
<tr>
<td>1</td>
<td>0 0 0 ... 0 0 1</td>
<td>$(m-1)$ $\overline{I}$</td>
</tr>
<tr>
<td>2</td>
<td>0 0 0 ... 0 1 0</td>
<td>$(m-2)$ $\overline{2}$</td>
</tr>
<tr>
<td>3</td>
<td>0 0 0 ... 0 1 0</td>
<td>$(m-2)$ $\overline{1^2}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$2^{(m-1)}-3$</td>
<td>1 1 1 ... 1 0 1</td>
<td>$\overline{1}^{(m-3)} \overline{2} \overline{I}$</td>
</tr>
<tr>
<td>$2^{(m-1)}-2$</td>
<td>1 1 1 ... 1 1 0</td>
<td>$\overline{1}^{(m-2)} \overline{2}$</td>
</tr>
<tr>
<td>$2^{(m-1)}-1$</td>
<td>1 1 1 ... 1 1 1</td>
<td>$\overline{1}^m$</td>
</tr>
</tbody>
</table>

Fig. 8-a. -- The $2^{(m-1)}$ Compositions of $\overline{m}$ in One-to-One Correspondence with Integers $i=0,1,2,...,2^{(m-1)}-1$. 
In order to attain all $2^{(m-1)}$ unique compositions of $\overline{m}$, it is necessary to involve all possible substrings of $\overline{m}$ at least once. However, individual substrings vary in compositional behavior. The string $\overline{4}$ has only one composition $\overline{121}$ which involves the substring $\overline{2}$ representing the second and third modules. In contrast, there are two compositions $\overline{22}$ and $\overline{212}$ which involve the substring $\overline{2}$ representing the first and second modules. There are two compositions $\overline{22}$ and $\overline{122}$ involving the third possible substring $\overline{2}$. Thus the particular substring $\overline{2}$ representing the second and third modules is seen to behave differently from the other two possible substrings $\overline{2}$, in that the former enters into only one composition, whereas the others each enter into two compositions. In other words, certain substrings $\overline{T}$ offer less compositional flexibility than other different substrings $\overline{T}$ of the same length $l$.

Note that the partition $\overline{212}$ is attainable with any substring $\overline{2}$, but that the particular substring $\overline{2}$ in the composition $\overline{121}$ is less flexible than either substring $\overline{2}$ in the composition $\overline{22}$. By restricting admissible substrings to those in the composition $\overline{22}$, we lose one composition $\overline{121}$, but do not lose the partition $\overline{212}$. The advantage of the retained substrings $\overline{T}$ from $\overline{T2}$ is that they are each well-behaved with respect to more than one composition, whereas the $\overline{2}$ from $\overline{121}$ was not.

This rather simple example illustrates the objective of time pattern analysis that a particular substring $\overline{T}$ of modules, if considered admissible for assignment to a class, should enter into a broad variety of compositional combinations. Those substrings $\overline{T}$ that do enjoy compositional flexibility are preferable to less flexible substrings $\overline{T}$. In order to avoid partial conflicts, we must often choose a disjoint set of substrings
$T$, with set cardinality equal to the integer part of $(n/l)$. Which substrings are chosen is largely determined by their flexibility in compositional reuse.

8.2.1 Compositions and Partitions of a Time Pattern $t$

The preceding section considered subdivision of the one-dimensional cycle sub-space represented by the string $m$ of $m$ modules on any one day. Because the cycle and the majority of its sub-spaces are two-dimensional, involving configurations over several days, the concepts of composition and partition are extended in this section to apply to two-dimensional time patterns.

We define a composition of a time pattern $t$ to be an ordered collection of $n$ ($1 \leq n \leq |t|$) time patterns $t_1, t_2, \ldots, t_n$ which are supplementary with respect to $t$. Since the $t_i$ are supplementary with respect to $t$, they are mutually disjoint, and collectively exhaustive with respect to $t$; hence $\Sigma t_i = t$ and the sum of their cardinalities $\Sigma |t_i| = |\Sigma t_i| = |t|$.

We define a partition of a time pattern $t$ to be an equivalence class of compositions of $t$, characterized solely by the structures and number of occurrences thereof in the member compositions.

The motivation for these definitions is the fact that once a time pattern $t$ is assigned to a class, that same time pattern $t$ must be reused in a variety of compositions of $t_{cycle}$ corresponding to the individual schedules of resources involved with the class. To the extent that $t$ can be reused in a broad variety of compositions, it is a good time pattern; to the extent that $t$ has limited compositional flexibility, it is a poor time pattern.
Often certain compositions are ruled out by restricting time pattern families to disjoint time patterns, in order to avoid partial conflicts. In such cases, good time pattern design tries to retain a broad spectrum of partitions of $t_{cycle}$. Furthermore, those time patterns that are considered admissible should enjoy flexibility for reuse in allowed compositions. The utility of an individual time pattern $t$ is directly related to the number of meaningful compositions of $t_{cycle}$ in which $t$ participates. The number of partitions of $t_{cycle}$ attainable with a given time pattern system is a measure of the utility of that system.

### 8.3.1 Probabilistic Effects of Block Structuring

If $t_{cycle}$ can be partitioned into $k$ congruent time pattern blocks, such that all time pattern families in the system are contained in the same blocks, the *a priori conflict behavior* of the system is more readily analyzed. This is particularly so if each block enjoys the same compositional behavior relative to other blocks. In this case, conflict behavior can be analyzed within an individual block, and then extrapolated to the entire cycle in terms of the probability $(1/k)$ that the block was involved.

The *a priori conflict* between two classes $c_1, c_2$ is the probability that they would conflict if each were assigned a time pattern at random from their appropriate time pattern families. When calculating this probability, congruent block structures enjoying equitable distribution of time patterns from all families allow us to concentrate our attention on the *a priori conflict behavior within a random block.*
Block structuring, useful in calculating a priori conflict behavior, is also useful in analyzing other aspects of time pattern behavior. When blocks are interchangeable, and time patterns are interchangeable within each block, the time pattern families must also be interchangeable. Furthermore, congruent time pattern blocks mesh well by definition, hence good overall mesh can be attained by insuring proper mesh within each congruent block. Occasionally several different congruent block structures can be found, each of which contain the time pattern system. Such organization is highly desirable.
CHAPTER NINE: NORMATIVE MODELS

9.1.1 Normative Model #1 (Unit Time Patterns)

In this chapter, we present several normative models of what we claim to be ideal time pattern systems, given qualified circumstances. Starting with systems involving only one time pattern structure (TS), and progressing to more complex arrangements, these systems are ideal in the sense that they serve broad combinatorial spectra of admissible contact requirements \((CR_i)\) of the classes (the ratios of contact time which classes require relative to each other) as regards the resources (instructors, rooms, and students) involved with the classes. Highest priority has been given to avoiding conflicts, including partial conflicts, between classes; specifically, to maximizing the number of desirable class combinations available for conflict-free assignment to the resources. Lowest priority has been given to contingencies having nothing to do with conflicts, such as a time pattern being required by a school because "the instructor refuses to teach at any other hours"; if every instructor poses such a constraint, they collectively define a required time pattern system where "ideal" is usually an accidental attribute.

Reasonably good but not necessarily optimal time pattern assignment and resource assignment processes are assumed. We assume that these processes do a decent job with the time pattern families as provided, but cannot fully compensate for improper design; i.e. any time pattern considered admissible not only can be assigned, but has a significant probability of
of being used somewhere. (M.I.T.'s computer-assisted system, GASP, exemplifies such "good, but not perfect" assignment processes.) Since each admissible time pattern in a family must be evaluated as a candidate for assignment, at best a poor time pattern costs effort to identify as such, and at worst it may be assigned. In our models, we hope to anticipate the time pattern and resource assignment processes, by simplifying evaluation wherever possible, and at least reducing the adverse impact of an injudicious choice.

The following figure 9-1a defines the contact requirement (CR) and corresponding time pattern structure (TS) assumed for our first and most simple normative model, using a format which will be consistently used for subsequent models. In this trivial model, all classes are assumed to require exactly one cycle module (CR=1), which can be satisfied only by a unit time pattern structure (TS=[1^1], meaning one module on one day).

\[
\begin{array}{c|c}
\text{CR}_i & \text{TS}_i' \\
\hline
\text{1 (module)} & \text{1}^1 \\
\end{array}
\]

Fig. 9-1a.---Admissible CR and TS, for Normative Model #1

The next figure 9-1b exhibits the (unique in this case) ideal family \( U \) of unit time patterns \( u_1, u_2, \ldots, u_{dm} \) in a format which will be consistently used for subsequent models. In this model, a \( d \) (day) by \( m \) (module) cycle of cardinality \( |t_{cycle}|=dm \) is shown, with the understanding that all \( dm \) of the cycle modules are usable (an assumption which is made regarding all normative models).
There are $2^{\alpha m}$ possible families of $[1^I]$ time patterns over a $d$ by $m$ cycle, ranging in cardinality from zero to $\alpha m$, characterized by whether each $u_i$ is included or not. Although it is not obvious how few of the $u_i$ are absolutely necessary to support a particular schedule (e.g. having only a single class necessitates exactly one time pattern), it is safe to say that omission of one or more $u_i \in U$ cannot possibly expand the solution space of feasible schedules, but may very well constrict it. Given two classes randomly assigned time patterns $(u_i^+, u_j^+)$ from a disjoint family $U'$ of cardinality $|U'|$, the a priori probability that they are in conflict is the probability that $i=j$, namely $1/|U'|$. This probability is minimized by choice of largest $|U'|$, in this case $|U'|=\alpha m$. Since from a standpoint of conflict, we cannot hope to gain by leaving out time patterns, but only stand to lose, we choose the maximal subset $U$ as our ideal family.

An observation can be made which, although obvious for this trivial first model, is nevertheless worthwhile noting as a special case of a valuable target objective of time pattern analysis: each $u_i \in U$ preempts only its own fraction $(1/\alpha m)$ of the cycle. No matter how many (short of
of all \( \frac{dm}{dm} \) or which disjoint time patterns \( u_1, u_2, \ldots, u_{(n<dm)} \) are assigned in the individual schedule of a resource, the remaining fraction \( \frac{(dm-n)/dm} \) of the cycle is itself disjointly covered by \( (dm-n) \) admissible time patterns \( u_1, u_2, \ldots, u_{j(dm-n)} \), the maximum number we could hope for.

9.2.1 Normative Model #2

Figure 9-2a defines the contact requirement (CR) and corresponding time pattern structure (TS) assumed for our second normative model. Figures 9-2b and 9-2c exhibit two different families \( (T,T') \) of \( [1^2] \) time patterns, either ideal for this model. A specific cycle of \( d=5 \) days (named \( M,T,W,R,F \)) by \( m=8 \) modules (named \( 1,2,3,4,5,6,7,8 \)) is assumed.

<table>
<thead>
<tr>
<th>CR (_i)</th>
<th>TS (_i)'</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 (modules)</td>
<td>([1^2]) (one module on each of 2 days)</td>
</tr>
</tbody>
</table>

Fig. 9-2a. -- Admissible CR \(_i\) and TS \(_i\)' for Normative Model #2
Fig. 9-2b.--An Ideal Family $T$ of 20 [$1^2$] Time Patterns $t_1, t_2, \ldots, t_{20}$ over a $d=5$ (day) by $m=8$ (module) Cycle

<table>
<thead>
<tr>
<th></th>
<th>M</th>
<th>T</th>
<th>W</th>
<th>R</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$t_1$</td>
<td>$t_1$</td>
<td>$t_2$</td>
<td>$t_2$</td>
<td>$t_3$</td>
</tr>
<tr>
<td>2</td>
<td>$t_3$</td>
<td>$t_4$</td>
<td>$t_4$</td>
<td>$t_5$</td>
<td>$t_5$</td>
</tr>
<tr>
<td>3</td>
<td>$t_6$</td>
<td>$t_6$</td>
<td>$t_7$</td>
<td>$t_7$</td>
<td>$t_8$</td>
</tr>
<tr>
<td>4</td>
<td>$t_8$</td>
<td>$t_9$</td>
<td>$t_9$</td>
<td>$t_{10}$</td>
<td>$t_{10}$</td>
</tr>
<tr>
<td>5</td>
<td>$t_{11}$</td>
<td>$t_{11}$</td>
<td>$t_{12}$</td>
<td>$t_{12}$</td>
<td>$t_{13}$</td>
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<tr>
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<td>$t_{13}$</td>
<td>$t_{14}$</td>
<td>$t_{14}$</td>
<td>$t_{15}$</td>
<td>$t_{15}$</td>
</tr>
<tr>
<td>7</td>
<td>$t_{16}$</td>
<td>$t_{16}$</td>
<td>$t_{17}$</td>
<td>$t_{17}$</td>
<td>$t_{18}$</td>
</tr>
<tr>
<td>8</td>
<td>$t_{18}$</td>
<td>$t_{19}$</td>
<td>$t_{19}$</td>
<td>$t_{20}$</td>
<td>$t_{20}$</td>
</tr>
</tbody>
</table>

Fig. 9-2a.--An Alternate Ideal Family $T'$ of 20 [$1^2$] Time Patterns $t'_1, t'_2, \ldots, t'_{20}$ over a $d=5$ (day) by $m=8$ (module) Cycle

<table>
<thead>
<tr>
<th></th>
<th>M</th>
<th>T</th>
<th>W</th>
<th>R</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$t'_{20}$</td>
<td>$t'_1$</td>
<td>$t'_1$</td>
<td>$t'_2$</td>
<td>$t'_2$</td>
</tr>
<tr>
<td>2</td>
<td>$t'_3$</td>
<td>$t'_3$</td>
<td>$t'_4$</td>
<td>$t'_4$</td>
<td>$t'_5$</td>
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<tr>
<td>3</td>
<td>$t'_5$</td>
<td>$t'_6$</td>
<td>$t'_6$</td>
<td>$t'_7$</td>
<td>$t'_7$</td>
</tr>
<tr>
<td>4</td>
<td>$t'_8$</td>
<td>$t'_8$</td>
<td>$t'_9$</td>
<td>$t'_9$</td>
<td>$t'_{10}$</td>
</tr>
<tr>
<td>5</td>
<td>$t'_{10}$</td>
<td>$t'_{11}$</td>
<td>$t'_{11}$</td>
<td>$t'_{12}$</td>
<td>$t'_{12}$</td>
</tr>
<tr>
<td>6</td>
<td>$t'_{13}$</td>
<td>$t'_{13}$</td>
<td>$t'_{14}$</td>
<td>$t'_{14}$</td>
<td>$t'_{15}$</td>
</tr>
<tr>
<td>7</td>
<td>$t'_{15}$</td>
<td>$t'_{16}$</td>
<td>$t'_{16}$</td>
<td>$t'_{17}$</td>
<td>$t'_{17}$</td>
</tr>
<tr>
<td>8</td>
<td>$t'_{18}$</td>
<td>$t'_{18}$</td>
<td>$t'_{19}$</td>
<td>$t'_{19}$</td>
<td>$t'_{20}$</td>
</tr>
</tbody>
</table>
There are 640 possible \([I^2]\) time patterns over a \(d=5\) by \(m=8\) cycle, collectively representing the universal family \(T^m\). The cardinality of this universal family, \(|T^m|=640\), can be obtained two different ways. 640 = \(\binom{5}{2} \times 8 \times 8\) = (the number of ways of choosing 2 of the 5 days) \(\times\) (the number of ways of choosing one module on the earlier day) \(\times\) (the number of ways of choosing the other module on the later day). Also, 640 = \(\binom{40}{2} - 5 \times \binom{8}{2}\) = 780 - 140 = (the number of ways of choosing any 2 of the 40 modules) \(-\) (the number of such combinations where both modules fall on the same day); the latter expression is \((5\ \text{days}) \times \text{the number of ways of choosing 2 modules on the same day})\).

There are \(2^{640}\) possible subsets of \(T^m\) over the \(d=5\) by \(m=8\) cycle, and hence \(2^{640}\) possible families of \([I^2]\) time patterns, ranging in cardinality from zero to 640. This time we definitely do not propose the maximal set \(T^m\) as our ideal family, but rather claim that any disjoint subset covering the cycle is ideal. Either family \((T, T')\) shown above represents such an ideal subset.

Given two classes randomly assigned time patterns \(t_i, t_j\) from the ideal family \(T\) of cardinality \(|T|=20\), the a priori probability that they are in conflict is the probability that \(i=j\) (since \(T\) is a disjoint family), in this case \(1/20\). However, randomly assigning \(t_i^w, t_j^w\) from the maximal set \(T^m\) leads to a virtually doubled a priori probability of conflict of \(63/640\approx 1/10\). This happens despite the very low probability \((1/640)\) of total conflict (when \(i=j\)), due to the opportunities for partial conflict which occurs with probability \(62/640=(1/8\) of the time when there is only one day in common, an event of likelihood \(6/10\)) + \((14/64\) of the time when both days are common, an event of likelihood \(1/10\)).
Even one redundant time pattern extending $T$ (such as any $t_i \in T'$) increases the likelihood of a priori conflict from $1/20$ to $25/441 = 2/35$, since of the $(21)^2 = 441$ possible pair-wise time pattern combinations, 21 represent total conflict and 4 represent partial conflict. Combining $T'$ with $T$ to obtain a family $T'' = T \cup T'$ of cardinality $|T''| = 40$ leads to an a priori conflict probability of $3/40 = 120/1600$, since of the $(40)^2 = 1600$ pair-wise possibilities, 40 are total conflicts, and 80 are partial.

One can appreciate that there are a large number of disjointly covering families, by considering that any of the 640 time patterns in $T''$ can start off a family, leaving $7 \times 7 = 49$ possible second choices, disjoint from the first, on the same two days without even considering the other days. In the pursuit of disjointness, however, one should not lose sight of coverage, lest one arrive at a worst-case disjoint family such as $T''$, exhibited below in figure 9-2d, whereby a maximum of 8 uncovered cycle modules is achieved, with no more disjoint $[1^2]$ time patterns possible.

<table>
<thead>
<tr>
<th>$M$</th>
<th>$T$</th>
<th>$W$</th>
<th>$R$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$t_1^w$</td>
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<td>$t_1^w$</td>
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<tr>
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<td>$t_7^w$</td>
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</tr>
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<td>6</td>
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<td>$t_{12}^w$</td>
<td>$t_{11}^w$</td>
<td>$t_{12}^w$</td>
</tr>
<tr>
<td>7</td>
<td>$t_{13}^w$</td>
<td>$t_{14}^w$</td>
<td>$t_{13}^w$</td>
<td>$t_{14}^w$</td>
</tr>
<tr>
<td>8</td>
<td>$t_{15}^w$</td>
<td>$t_{16}^w$</td>
<td>$t_{15}^w$</td>
<td>$t_{16}^w$</td>
</tr>
</tbody>
</table>

Fig. 9-2d.--A Worst-Case Disjoint Family $T''$ of only 16 $[1^2]$ Time Patterns $t_1^w, t_2^w, \ldots, t_{16}^w$ over a $d=5$ (day) by $m=8$ (module) Cycle
When dealing with any single time pattern structure (TS=\{1^2\} in this case), it should be readily clear that both disjointness and maximal coverage are necessary conditions for a time pattern family to be ideal. Unless constrained by restricted resources, the introduction of a redundant time pattern (such as any \(t'_i \in T'\) as a 21st time pattern to extend \(T\)), cannot possibly eliminate any conflicts but can produce them. To understand this, consider any \(t'_i \in T'\). Each \(t'_i\) partially conflicts with two time patterns \((t_j, t_k) \in T\). Consider any class assigned to \(t'_i\). Each resource associated with this class is covered by exactly one of three cases:

**case 1:** the resource is free both \(t_j\) and \(t_k\) (the only case where \(t'_i\) doesn't result in a conflict); in this case, either of \((t_j, t_k)\) could have been assigned to the class instead of \(t'_i\).

**case 2:** the resource is free either \(t_j\) or \(t_k\), but not both; in this case \(t'_i\) results in a partial conflict whereas one of \((t_j, t_k)\) would have been a conflict-free assignment.

**case 3:** the resource is free neither \(t_j\) nor \(t_k\); in this case, all of the time patterns \((t'_i, t_j, t_k)\) represent total conflicts.

The same arguments in favor of the maximal set \(U\) in normative model \#1 apply to coverage in model \#2; so long as we stay disjoint, a case can be made for extending coverage to a maximum.

Again, as for model \#1, we observe that each \(t'_i \in T\) (each \(t'_i \in T'\), and in general each time pattern in any ideal family) preempts only its own fraction (2/40) of the cycle. No matter how many short of all 20) or which disjoint time patterns \(t'_1, t'_2, \ldots, t'_{(n<20)}\) are assigned in the individual schedule of a resource, thus preempting a fraction \((n/20)\) of
the cycle, the remaining fraction \(((20-n)/20)\) of the cycle is itself
disjointly covered by \((20-n)\) admissible time patterns \(t_1', t_2', \ldots, t_{(20-n)'}\)
the maximum number we could hope for.

The following figure 9-2e shows what could happen to an individual
resource schedule if time patterns were imprudently assigned from the
combined family \(T''=T\cup T'\) mentioned earlier. This worst-case fragmentation
of the cycle into holes that do not correspond to any time patterns \(t_i'\in T'\),
results in a fraction \((12/40)\) of the cycle becoming mandatory free time.

\[
\begin{array}{cccccc}
M & T & W & R & F \\
1 & t_1 & t_1 & - & t_2' & t_2' \\
2 & - & t_4 & t_4 & - & t_5' \\
3 & t_5' & - & t_7 & t_7 & - \\
4 & t_8' & t_8' & - & t_{10} & t_{10} \\
5 & t_{11} & t_{11} & - & t_{12} & t_{12} \\
6 & - & t_{14} & t_{14} & - & t_{15}' \\
7 & t_{15}' & - & t_{17} & t_{17} & - \\
8 & t_{18}' & t_{18} & - & t_{20} & t_{20} \\
\end{array}
\]

Fig. 9-2e.--A Worst-Case Fragmentation of an Individual Resource Schedule
using Time Patterns chosen from \(T''=T \cup T'\)

An occasional impulse in the face of such a situation might be to add six
more time patterns to \(T''\), such as: 'M2,W1', 'T3,R2', 'W4,F3', etc. Such
a posteriori time pattern family expansion, whereby one hopes to plug
holes in resource schedules, by tailor-making additional time patterns for
future use, is all too familiar a practice in real life, usually ending up as wishful thinking which makes matters even worse, particularly in cases involving a broad spectrum of time pattern structures.

Before leaving this second normative model, we emphasize that $T^b_0$ is a worst-case family, and that families such as $(T, T')$ are ideal, because there is only the one time pattern structure ($TS=[1^2]$) in the system. $T^b_0$ might well become superior (and $T, T'$ inferior) in the presence of other time pattern structures. The context is an important qualification.

9.3.1 Normative Model #3

Figure 9-3a defines the contact requirement (CR) and corresponding time pattern structures ($TS_i$) assumed for our third normative model. Figures 9-3b through 9-3e exhibit four families of time patterns $T^{32}, A^{23}, B^{23}, T^{321}$ appropriate to this model. A specific cycle of $d=4$ days (named $A,B,C,D$) by $m=9$ modules (named 1,2,3,4,5,6,7,8,9) is assumed. This is the first model involving more than one time pattern structure ($TS_i$), albeit restricted to a single contact requirement (CR=6).

<table>
<thead>
<tr>
<th>CR$_i$</th>
<th>TS$_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 (modules)</td>
<td>$[3^2]$ (3 modules on each of 2 days)</td>
</tr>
<tr>
<td></td>
<td>$[2^3]$ (2 modules on each of 3 days)</td>
</tr>
<tr>
<td></td>
<td>$[321]$ (3 modules on one day, 2 on a second day, 1 on a third day)</td>
</tr>
</tbody>
</table>

Fig. 9-3a.—Admissible $CR_i$ and $TS_i$, for Normative Model #3
Fig. 9-3b.—An Ideal Family $T^{32}$ of 6 $[3^2]$ Time Patterns $t_1^{32}, t_2^{32}, \ldots, t_6^{32}$ over a $d=4$ (day) by $m=9$ (module) Cycle

\[
\begin{array}{cccc}
A & B & C & D \\
1 & t_1^{32} & t_1^{32} & t_2^{32} & t_2^{32} \\
2 & t_1^{32} & t_1^{32} & t_2^{32} & t_2^{32} \\
3 & t_1^{32} & t_1^{32} & t_2^{32} & t_2^{32} \\
4 & t_3^{32} & t_3^{32} & t_4^{32} & t_4^{32} \\
5 & t_3^{32} & t_3^{32} & t_4^{32} & t_4^{32} \\
6 & t_3^{32} & t_3^{32} & t_4^{32} & t_4^{32} \\
7 & t_5^{32} & t_5^{32} & t_6^{32} & t_6^{32} \\
8 & t_5^{32} & t_5^{32} & t_6^{32} & t_6^{32} \\
9 & t_5^{32} & t_5^{32} & t_6^{32} & t_6^{32} \\
\end{array}
\]

Fig. 9-3c.—A Maximal Disjoint Family $A^{23}$ of 5 $[2^3]$ Time Patterns $a_1^{23}, a_2^{23}, \ldots, a_5^{23}$ over a $d=4$ (day) by $m=9$ (module) Cycle

\[
\begin{array}{cccc}
A & B & C & D \\
1 & a_1^{23} & a_1^{23} & a_1^{23} & a_1^{23} \\
2 & a_1^{23} & a_1^{23} & a_1^{23} & a_1^{23} \\
3 & a_2^{23} & a_2^{23} & a_2^{23} & a_2^{23} \\
4 & a_2^{23} & a_2^{23} & a_2^{23} & a_2^{23} \\
5 & a_3^{23} & a_4^{23} & a_4^{23} & a_4^{23} \\
6 & a_3^{23} & a_4^{23} & a_4^{23} & a_4^{23} \\
7 & a_5^{23} & a_5^{23} & - & - \\
8 & a_5^{23} & a_5^{23} & - & - \\
9 & - & - & - & - \\
\end{array}
\]
Fig. 9-3d.--An Alternate Disjoint Family \( B_{23} \) of only 4 \([2^3]\) Time Patterns \( b_1, b_2, b_3, b_4 \) over a \( d=4(\text{day}) \) by \( m=9(\text{module}) \) Cycle.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( b_{1}^{23} )</td>
<td>( b_{1}^{23} )</td>
<td>( b_{1}^{23} )</td>
<td>( b_{2}^{23} )</td>
</tr>
<tr>
<td>2</td>
<td>( b_{1}^{23} )</td>
<td>( b_{2}^{23} )</td>
<td>( b_{1}^{23} )</td>
<td>( b_{2}^{23} )</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>( b_{2}^{23} )</td>
<td>( b_{2}^{23} )</td>
<td>( b_{3}^{23} )</td>
<td>( b_{3}^{23} )</td>
</tr>
<tr>
<td>5</td>
<td>( b_{2}^{23} )</td>
<td>( b_{2}^{23} )</td>
<td>( b_{3}^{23} )</td>
<td>( b_{3}^{23} )</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>( b_{3}^{23} )</td>
<td>( b_{4}^{23} )</td>
<td>( b_{3}^{23} )</td>
<td>( b_{4}^{23} )</td>
</tr>
<tr>
<td>8</td>
<td>( b_{3}^{23} )</td>
<td>( b_{4}^{23} )</td>
<td>( b_{3}^{23} )</td>
<td>( b_{4}^{23} )</td>
</tr>
<tr>
<td>9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig. 9-3e.--An Ideal Family \( T_{321} \) of 6 \([321]\) Time Patterns \( t_{1}, t_{2}, ..., t_{6} \) over a \( d=4(\text{day}) \) by \( m=9(\text{module}) \) Cycle.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( t_{1}^{321} )</td>
<td>( t_{1}^{321} )</td>
<td>( t_{1}^{321} )</td>
<td>( t_{2}^{321} )</td>
</tr>
<tr>
<td>2</td>
<td>( t_{1}^{321} )</td>
<td>( t_{1}^{321} )</td>
<td>( t_{2}^{321} )</td>
<td>( t_{2}^{321} )</td>
</tr>
<tr>
<td>3</td>
<td>( t_{1}^{321} )</td>
<td>( t_{2}^{321} )</td>
<td>( t_{2}^{321} )</td>
<td>( t_{2}^{321} )</td>
</tr>
<tr>
<td>4</td>
<td>( t_{3}^{321} )</td>
<td>( t_{3}^{321} )</td>
<td>( t_{3}^{321} )</td>
<td>( t_{4}^{321} )</td>
</tr>
<tr>
<td>5</td>
<td>( t_{3}^{321} )</td>
<td>( t_{4}^{321} )</td>
<td>( t_{4}^{321} )</td>
<td>( t_{4}^{321} )</td>
</tr>
<tr>
<td>6</td>
<td>( t_{3}^{321} )</td>
<td>( t_{4}^{321} )</td>
<td>( t_{4}^{321} )</td>
<td>( t_{4}^{321} )</td>
</tr>
<tr>
<td>7</td>
<td>( t_{5}^{321} )</td>
<td>( t_{5}^{321} )</td>
<td>( t_{5}^{321} )</td>
<td>( t_{6}^{321} )</td>
</tr>
<tr>
<td>8</td>
<td>( t_{5}^{321} )</td>
<td>( t_{5}^{321} )</td>
<td>( t_{6}^{321} )</td>
<td>( t_{6}^{321} )</td>
</tr>
<tr>
<td>9</td>
<td>( t_{5}^{321} )</td>
<td>( t_{6}^{321} )</td>
<td>( t_{6}^{321} )</td>
<td>( t_{6}^{321} )</td>
</tr>
</tbody>
</table>
There are 294 possible \( \{3^2\} \) time patterns over a \( d=4 \) by \( m=9 \) cycle:

\[
294 = \binom{4}{2} \times 7 \times 7 = (\text{the number of ways of choosing 2 of the 4 days}) \times (\text{the number of ways of choosing three consecutive modules on the earlier day}) \times (\text{the number of ways of choosing three consecutive modules on the later day}).
\]

Of the \( 2^{294} \) possible families of \( \{3^2\} \) time patterns, ranging in cardinality from zero to 294, we have selected \( T^{32} \) as being ideal since its 6 time patterns are a disjoint coverage of the cycle. While other disjoint coverings are possible, \( T^{32} \) can be considered a canonical instance of such (equally ideal) families.

Given two classes randomly assigned time patterns \( t^{32}_i, t^{32}_j \) from the ideal family \( T^{32} \) of cardinality \( |T^{32}|=6 \), the a priori probability that they are in conflict is the probability that \( i=j \) (since \( T^{32} \) is a disjoint family), in this case \( 1/6 \). Note that this is the best one could hope for:

(a) Any other disjointly covering family exhibits the same (and no better) behavior.

(b) A disjoint family \( X \) which does not cover the cycle, and hence has cardinality \( |X|=(n<6) \), increases the probability of a priori conflict from \( 1/6 \) to \( (1/n) \).

(c) A non-disjoint family \( Y \), whether or not covering the cycle, increases the probability of conflict in a manner previously discussed.

To illustrate point (c) above, consider the case where two time patterns \( y^{\infty}_i, y^{\infty}_j \) are assigned at random from the maximal set \( Y^{\infty} \) of all possible \( \{3^2\} \) time patterns (of cardinality \( |Y^{\infty}|=294 \)). There is an a priori conflict probability of \( 7685/14406 = (29/49 \text{ of the time when there is only one day in common, an event of likelihood } 4/6) + (2001/2401 \text{ of the time when both} \)
days are in common, an event of likelihood 1/6). Partial conflicts cause almost all the problems here.

There are 2048 possible $[2^3]$ time patterns over a $d=4$ by $m=9$ cycle:

$$2048 = \binom{4}{3} \times 8 \times 8 \times 8 = (\text{the number of ways of choosing 3 of the 4 days}) \times (\text{the number of ways of choosing two consecutive modules on the earliest day}) \times (\text{the number of ways of choosing two consecutive modules on the latest day}) \times (\text{the number of ways of choosing two consecutive modules on the remaining day}).$$

Disjoint coverage cannot be achieved by any of the $2^{2048}$ possible families of $[2^3]$ time patterns, a fact which stems from the property that no odd number $m$ of modules (9 in this case) can be partitioned into strictly even substituting lengths (2 in this case). A maximal (although not covering) disjoint family of $[2^3]$ time patterns over a $d=4$ by $m=9$ cycle is represented by family $A^{23}$ of cardinality $|A^{23}|=5$. Note that any maximal disjoint family over the given cycle will exhibit 6 uncovered cycle modules: one module on each of three days and three (not necessarily consecutive) modules on the remaining day.

$A^{23}$ cannot be regarded as ideal since it does not cover the cycle; however, it is less than optimal for an even more critical reason: it meshes quite poorly with our initial family $T^{22}$. $B^{23}$, while only of cardinality $|B^{23}|=4$, may (depending on the time pattern structure profile of the classes) be a better family for reasons of mesh. To understand this, it is necessary to consider various ways in which time pattern families can interfere with one another.

Two major types of interference can occur between even the most ideal of time pattern families: dominance and transection. A family $W$ is said to dominate another family $X$ if and only if for any $x_i \in X$ there exists
exactly one $w_j \in W$ such that $x_i \subseteq w_j$ (although several $x_i$ may be contained in the same $w_j$). A family $Y$ is said to transsect another family $Z$ if and only if for any $y \in Y$ there exist at least two $z_i, z_j \in Z$ such that $y$ partially conflicts with both $z_i$ and $z_j$.

Occasionally dominance or transection is inevitable between two families because of the time pattern structures they represent -- the interference is in a sense between the two structures. In other cases, interference could and should be avoided. The ideal family $U$ of $[1^1]$ time patterns shown in figure 9-1b must be dominated by literally any family which is supplementary (disjointly covers the cycle). Another example of dominance: our definition of block containment implies that a collection of blocks dominates any time pattern family contained in the blocks. Subsequent normative models will give us better occasion to discuss dominance, but we can investigate transection within the current discussion.

The term transection was chosen because it suggests cutting across. Some instances of transection are unnecessary; the two ideal families $T, T'$ shown in figures 9-2b and 9-2c of the previous model transect one another, but -- precisely for that reason -- should not both occur in an application. Other instances are unavoidable, and the two structures $[2^2]$ and $[2^3]$ of our current model provide an example of this. Transection is always unavoidable between supplementary (disjointly covering) families with the same contact requirement ($CR_i$) but different time pattern structures ($TS_i$). (Observe that families whose $CR_i$ are relatively prime face similar problems.) Even if not supplementary, any reasonable sort of disjoint coverage leads to transection when the $CR_i$ are equal but the $TS_i$ differ; such is the case
between \( T_{32} \) and both \( A_{23} \) and \( B_{23} \).

The two time pattern structures \([2^3]\) and \([3^2]\) are simply not compatible in the given cycle, and neither \( A_{23} \) nor \( B_{23} \) escape transection with \( T_{32} \). However, \( A_{23} \) not only transects \( T_{32} \), but does so in such a way that the \( a_{i}^{23} \) are not even interchangeable in their partial conflicts with the \( t_{j}^{32} \). Three of the five \( a_{i}^{23} \) (\( i=1,4,5 \)) each partially conflict with exactly two of the six \( t_{j}^{32} \), but the other two \( a_{i}^{23} \) (\( i=2,3 \)) each partially conflict with three of the \( t_{j}^{32} \). From the standpoint of \( T_{32} \): two of the six \( t_{j}^{32} \) (\( j=5,6 \)) each partially conflict with only one of the five \( a_{i}^{23} \), two (\( j=1,4 \)) each partially conflict with two \( a_{i}^{23} \), but the remaining two (\( j=2,3 \)) each partially conflict with three of the five \( a_{i}^{23} \). Neither \( T_{32} \) nor \( A_{23} \) are interchangeable with respect to each other — the instructor of a \([3^2]\) class, who expects to accommodate students taking three \([2^3]\) classes, will undoubtedly prefer \( t_{5}^{32} \) or \( t_{6}^{32} \) over \( t_{4}^{32} \) and \( t_{3}^{32} \), and must avoid \( t_{2}^{32} \) and \( t_{3}^{32} \).

\( B_{23} \) does not eliminate transection with \( T_{32} \) — not even a family with only one \([2^3]\) time pattern can escape this fate — but the \( b_{i}^{23} \) are somewhat better behaved with respect to the six \( t_{j}^{32} \). Each of the four \( b_{i}^{23} \) partially conflict with exactly two of the six \( t_{j}^{32} \) (in contrast to two of the five \( a_{i}^{23} \) conflicting with three of the six \( t_{j}^{32} \)). From the standpoint of \( T_{32} \), \( T_{32} \) does not transect \( B_{23} \): four of the six \( t_{j}^{32} \) (\( j=1,3,4,6 \)) each partially conflict with only one of the four \( b_{i}^{23} \), although the other two (\( j=2,5 \)) each partially conflict with two \( b_{i}^{23} \). While \( B_{23} \) and \( T_{32} \) are not completely interchangeable with respect to each other, they mesh in a better fashion than do \( A_{23} \) and \( T_{32} \). The aforementioned instructor of a \([3^2]\) class, eager to have the class interact well with as many as three \([2^3]\) classes, must still avoid two of the six \( t_{j}^{32} \) (\( j=2,5 \)) but now is indifferent with
respect to the other four time patterns. Of course, we have increased
the a priori probability of conflict within the \([2^3]\) family from \(1/5\) to
\(1/4\), but this is a price which must be paid in the pursuit of mesh
between families; a school must weigh both concerns in the balance.

There are 12096 possible \([321]\) time patterns over a \(d=4\) by \(m=9\) cycle:
\[
12096 = \binom{4}{3} \times 3! \times 7 \times 8 \times 9 = (the \ number \ of \ ways \ of \ choosing \ 3 \ of \ the \ 4 \ days) \\
\times \ (the \ number \ of \ ways \ of \ permuting \ those \ days \ such \ that \ one \ supports \ 3 \\
consecutive \ modules, \ a \ second \ supports \ 2, \ and \ the \ third \ supports \ 1) \\
\times \ (the \ number \ of \ ways \ of \ choosing \ 3 \ consecutive \ modules \ on \ the \ one \ day) \\
\times \ (the \ number \ of \ ways \ of \ choosing \ 2 \ consecutive \ modules \ on \ the \ second \ day) \\
\times \ (the \ number \ of \ ways \ of \ choosing \ 1 \ module \ on \ the \ third \ day).
\]
Of the \(2^{12096}\) possible families of \([321]\) time patterns ranging in cardinality from zero
to 12096, we have selected \(T^{321}\) as being ideal since its 6 time patterns
are a disjoint coverage of the cycle. While other disjoint coverings are
possible, \(T^{321}\) can be considered a canonical instance of such (equally
ideal) families.

Given two classes randomly assigned time patterns \(t^i\), \(t^j\) from the
ideal family \(T^{321}\) of cardinality \(|T^{321}|=6\), the a priori probability that
they are in conflict is the probability that \(i=j\) (since \(T^{321}\) is a disjoint
family), in this case \(1/6\). Note that this is the best one can hope for.

We now investigate the mesh between \(T^{321}\) and \(T^{32}\), just as we previously
looked at that between \(T^{32}\) and \(A^{23}\) (and \(B^{23}\)). This time, despite transection,
the results are more satisfactory. Observe that there is a congruent
rectangular block structure obtained by partitioning the cycle into the
three time patterns: \('ABCD1-3', 'ABCD4-6', and 'ABCD7-9'. Both \(T^{321}\) and
\(T^{32}\) are contained in these same three blocks. Each \(t^i\) has a congruent
"buddy" $t_{ij}^{321}$ such that the two are supplementary with respect to their mutual block. Likewise, each $t_{ij}^{32}$ has its own congruent "buddy" $t_{ij}^{32}$ such that the two are supplementary with respect to their mutual block. Accordingly, assigning any $t_{ij}^{321}$ or $t_{ij}^{32}$ preempts exactly 50% of a block (1/6 of the cycle) leaving a hole which, while not usable by a time pattern from the other family, at least can be covered by its own buddy. Furthermore, the other two blocks are not affected in any way — the point of trying to achieve block structuring in the first place!

Given two classes $c_1, c_2$ such that $c_1$ is assigned a random $t_{ij}^{321} \in T^{321}$ and $c_2$ is assigned a random $t_{ij}^{32} \in T^{32}$, the a priori probability that $c_1$ and $c_2$ are in conflict is, in this case, the probability that $t_{ij}^{321}$ and $t_{ij}^{32}$ are contained in the same block, namely 1/3. Because the two families transect each other, interference within a block is to be expected, but the results here are as reasonable as possible. Note that a resource assigned a full load (of all 36 modules) and restricted to [321] and/or [32] classes must take an even number $k=0, 2, 4$ or 6 of [321] classes and an even number $k'=6-k$ of [32] classes because of the buddy restrictions. Given a random pair of classes subject to a distribution whereby there is a likelihood of $p$ that the pair require time patterns from the two different families, and $q=1-p$ that only one of the two families is involved, the a priori probability that the pair conflicts is: $p/3 + (1-p)/6 = (p+1)/6$.

It is important to realize that no matter what shape hole(s) remain after assigning any number of time patterns from the combined set $T=T^{321} \cup T^{32}$, all remaining space (if any) is coverable using elements of $T$. If holes contain intact blocks, elements from either $T^{321}$ or $T^{32}$ can be assigned; otherwise the buddy of an already assigned time pattern will still fit.
Because \( T^{321} \) and \( T^{32} \) mesh well and are each inherently ideal, whereas \( A^{23} \) meshes poorly with \( T^{32} \) (and \( T^{321} \) for that matter) and \( B^{23} \) is not supplementary, it might be a productive compromise for a school to abandon the \([2^3]\) structure altogether in favor of either or both \( T^{321} \) and/or \( T^{32} \).

The institution should question its motivation for the \([2^3]\) structure in the first place: were the 3 days critical; was a 3-period session too lengthy for 2 days but acceptable for one day; etc. Using only one time pattern family exclusively, or at most \( T = T^{321} \cup T^{32} \), may result in sufficient conflict reduction to warrant such a compromise.

9.4.1 Normative Model #4

Figure 9-4a defines the contact requirements (\( CR_i \)) and corresponding time pattern structures (\( TS_i \)) assumed for our fourth normative model. Figures 9-4b through 4e exhibit four families of time patterns (\( T^{15}_{i1}, A^{13}_{i1}, A^{12}_{i1}, B^{12}_{i1} \)) appropriate to this model. A specific cycle of \( d=5 \) days (named \( M,T,W,R,F \)) by \( m=8 \) modules (named \( 1,2,3,4,5,6,7,8 \)) is assumed. This is the first model involving more than one contact requirement (\( CR_i \)). (The somewhat staggered arrangement of the \( B^{12} \) time patterns will be explained in normative model #5).

\[
\begin{array}{ll}
\text{CR}_i & \text{TS}_i \\
5 \text{ (modules)} & [1^5] \text{ (1 module on each of 5 days)} \\
3 \text{ (modules)} & [1^3] \text{ (1 module on each of 3 days)} \\
2 \text{ (modules)} & [1^2] \text{ (1 module on each of 2 days)} \\
\end{array}
\]

Fig. 9-4a.--Admissible \( CR_i \) and \( TS_i \), for Normative Model #4
Fig. 9-4b.--An Ideal Family $T^{15}$ of 8 [15] Time Patterns $t_1^{15}, t_2^{15}, \ldots, t_8^{15}$
Partitioning a $d=5$ (day) by $m=8$ (module) Cycle into a Congruent Rectangular Block Structure

Fig. 9-4c.--A Disjoint Family $A^{13}$ of 8 [13] Time Patterns $a_1^{13}, a_2^{13}, \ldots, a_8^{13}$
over a $d=5$ (day) by $m=8$ (module) Cycle, Contained in the $T^{15}$ blocks
Fig. 9-4d.—A Disjoint Family $A^{12}$ of 8 [12] Time Patterns $a_1^{12}, a_2^{12}, \ldots, a_8^{12}$ over a $d=5$ (day) by $m=8$ (module) Cycle, Supplementing $A^{13}$ in the $T^{15}$ Blocks

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Fig. 9-4e.—A Disjoint Family $B^{12}$ of 8 [12] Time Patterns $b_1^{12}, b_2^{12}, \ldots, b_8^{12}$ over a $d=5$ (day) by $m=8$ (module) Cycle, Disjoint from $A^{12}$ and Contained in the $T^{15}$ Blocks

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There are $32768 = 2^5$ possible $[1^5]$ time patterns over a $d=5$ by $m=8$ cycle. Of the $2^{32768}$ possible families of $[1^5]$ time patterns, ranging in cardinality from zero to 32768, $T^{15}$ is the only congruent rectangular block structure. It is inherently ideal since its 8 time patterns are a disjoint coverage of the cycle.

There are 5120 possible $[1^3]$ time patterns over a $d=5$ by $m=8$ cycle: $5120 = \binom{5}{3} \times 8^3 = (\text{the number of ways of choosing 3 of the 5 days}) \times (\text{the number of ways of distributing 3 modules over 3 days})$. Disjoint coverage of the cycle is impossible, since the $CR$ of 3 does not evenly divide the $|T_{cycle}|=40$; however, it is possible to cover all but one cycle module disjointly, with a variety of time pattern families of cardinality $13=39/3$.

Figure 9-4f shows one such maximal disjoint family $T^{13}$.

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Fig. 9-4f.—A Maximal Disjoint Family $T^{13}$ of $13 [1^3]$ Time Patterns $t_1^{13}, t_2^{13}, \ldots, t_{13}^{13}$ over a $d=5$ (day) by $m=8$ (module) Cycle.
An important question is why we included $A^{13}$, (of cardinality $|A^{13}|=3$) in our normative model rather than $T^{13}$ (of greater cardinality $|T^{13}|=13$) despite the more complete coverage of the latter; by restricting $[1^3]$ classes to $A^{13}$, the a priori probability of conflict between two random $[1^3]$ classes is $1/8$ in contrast to the $1/13$ afforded by $T^{13}$. The answer is that we chose $A^{13}$ because of broader concerns, including mesh with $T^{15}$. Despite the local sub-optimality of conflict behavior within the $A^{13}$ family, it is important to consider intra-family conflicts. Six $t^{15}_i$ ($i=1,3,4,6,7,8$) preempt exactly two of the $t^{13}_j$, representing $2/13$ of the time patterns in $T^{13}$; $t^{15}_2$ and $t^{15}_5$ preempt three of the $t^{13}_j$, representing $3/13$ of the time patterns in $T^{13}$. In each instance conflict is only partial with respect to at least one $t^{13}_j$. Looking at the conflict situation from the standpoint of the $t^{13}_j$: eight of the $t^{13}_j$ ($j=1,3,5,6,8,10,11,13$) preempt exactly one $t^{15}_i$ ($1/8$ of the $T^{15}$ family), but five of the $t^{13}_j$ ($j=2,4,7,9,12$) preempt two $t^{15}_i$ ($1/4$ of the family). In view of the foregoing analysis, it can be stated that, with respect to $T^{15}$, the 13 time patterns of $T^{13}$ are not interchangeable. Eight time patterns in $T^{13}$ exhibit one kind of behavior, five exhibit another. Likewise, with respect to $T^{13}$, the eight time patterns of $T^{15}$ are not interchangeable; six exhibit one kind of behavior, two another. A class assigned $t^{13}_2='M2,RF1'$ preempts not merely $|t^{13}_2|=3$ cycle modules when it comes to $[1^5]$ classes, but rather 10 -- an impact of over three times its cardinality. Similarly a class assigned $t^{15}_2='MTWRF2'$ preempts not merely $|t^{15}_2|=5$ cycle modules when it comes to $[1^3]$ classes, but rather 9 -- almost twice its cardinality.

It cannot be claimed that the lack of interchangeability in these two families $T^{15}, T^{13}$ is necessarily a bad thing; in an institution where
all or most of the resources associate almost exclusively with one family or the other but not both, the maximal coverage of $T^{13}$ may outway mesh considerations. (Such exclusiveness may be typical of instructors and special purpose rooms in many institutions, although students and general purpose rooms seldom are so restricted.) Even in schools where lack of interchangeability does not appear to be a major problem, it may be useful to factor $T^{13}$ into two disjoint subfamilies: $T^{13} = X^{13} + Y^{13}$ where $X^{13} = (t_1^{13}, t_3^{13}, t_6^{13}, t_9^{13}, t_{12}^{13})$ and $Y^{13} = (t_2^{13}, t_4^{13}, t_7^{13}, t_9^{13}, t_{12}^{13})$. Such a decomposition of $T^{13}$ into two subfamilies on the basis of interchangeability with respect to $T^{15}$ could be a very useful distinction for schools where there are some resources involved with both $[1^5]$ and $[1^3]$ classes. This decomposition provides a handle on partitioning $[1^3]$ classes into those interacting with $[1^5]$ classes and those not so interacting. The advisability of using time patterns from $Y^{13}$ must rely on the contact requirement profile of the individual school: $Y^{13}$ is a theoretically different family from $X^{13}$, and the impact of the theoretical distinctions depends on the school. It should be even more apparent why we chose $A^{13}$ to represent the $[1^3]$ structure once the remaining time pattern families in normative models #4 and #5 are considered.

Restricting our attention to $T^{15}$ and $A^{13}$, we can comment on a priori conflict given two random classes. Because of the block containment of $A^{13}$, the issue is reduced to one of block conflict no matter which of the two families are involved or how. In other words, each and every time pattern from either $T^{15}$ or $A^{13}$ preempts exactly one block, and the resulting a priori likelihood of conflict between two classes is $1/8$. Accordingly, at least from a conflict standpoint between $[1^5]$ and $[1^3]$ classes, we have
a certain interchangeability of $[1^5]$ and $[1^3]$ classes. A spectrum of contact requirement combinations is possible: from $i=0$ through $i=8$ $[1^5]$ classes, and $8-i[1^3]$ classes, can be assigned to any resource. Of course, $8[1^3]$ classes is not a full load, although a maximum using $A^{13}$. It is the one for one trade-off of $[1^5]$ for $[1^3]$ classes that makes this an interesting spectrum.

We previously investigated $[1^2]$ time patterns in normative model #2. Rather than choose a family of cardinality 20 (such as $T$ of Fig.9-2b) which by itself would be ideal, we must consider the context of $T^{15}$ and $A^{13}$ in this model. Accordingly, we focus our attention on $A^{12}$, of cardinality $|A^{12}|=8$ -- less than half the cardinality of $T$. $A^{12}$ has the notable property of supplementing $A^{13}$ in the $T^{15}$ blocks: $t^{15}_i = a^{13}_i + a^{12}_i$. Not only are the $a^{12}_i$ interchangeable with respect to the $t^{15}_i$ --and vice versa-- but moreover the $a^{12}_i$ are interchangeable with respect to the $a^{13}_i$ in a very special way: no $[1^2]$ class can possibly conflict with any $[1^3]$ class.

The presence of $A^{12}$ supplementing $A^{13}$ fills in our spectrum of contact requirement combinations rather nicely: given a one-to-one ratio of $[1^2]$ and $[1^3]$ classes, a full resource load of all 40 cycle modules can now be achieved with from $i=0$ to $i=8$ $[1^3]$ classes, by concurrently scheduling $i$ $[1^2]$ classes (in the same blocks as the $[1^3]$ classes) and $8-i$ $[1^5]$ classes (in the $8-i$ remaining blocks).

Looking at the composite set $A=A^{13}+A^{12}$ (and disregarding $T^{15}$ for now), we observe that each $a_i \in A$ preempts a constant $1/16$ of the total number of classes which can be scheduled ($1/8$ of the time patterns from its own family, representing half of the classes, and none of the time patterns from the other family, representing the other half; this constant $1/16$
averages the cardinalities of 2/40 for the $a_i^{12}$ and 3/40 for the $a_i^{13}$). No matter how many (short of all 16) or which disjoint time patterns $a_i^1, a_i^2, \ldots, a_i^n$ are assigned in the individual schedule of a resource, the remainder of the cycle is itself disjointly covered by (16-n) admissible time patterns $a_j^1, a_j^2, \ldots, a_j^{16-n}$ in a well-behaved manner.

For some institutions, the incomplete coverage of the cycle by $A^{12}$ (16/40 -- less than half) might be a drawback. For this reason, a secondary family of 8 [12] time patterns, $B^{12}$, has been included in this normative model. If we disregard $A^{13}$ for a moment, and consider the composite set $C^{12} = A^{12} + B^{12}$, we observe that $C^{12}$ is well-behaved both by itself and with respect to $T^{15}$. Although not collectively exhaustive of the cycle, $C^{12}$ does disjointly cover 32/40 of the cycle. Because $B^{12}$ is contained in the $T^{15}$ blocks, just as $A^{12}$ was, the $a_i^{12}$ are interchangeable with respect to the $t_j^{15}$ and vice versa. The trouble with $B^{12}$, and hence with $C^{12}$, is with respect to $A^{13}$.

Each $b_i^{12}$ is contained in a corresponding $a_i^{13}$, a conflict which is total with respect to $b_i^{12}$. Accordingly, each $b_i^{12}$ preempts the corresponding $a_i^{13}$ (1/8 of the time patterns in the [13] family). Note that each $b_i^{12}$ (despite a cardinality of 2 cycle modules) really does preempt 3 cycle modules and not merely its own 2. The reason this happens is that once a $b_i^{12}$ is assigned, within the four families we have chosen, no further use can be made of the $t_i^{15}$ block other than assigning the corresponding $a_i^{12}$: the three modules left in the $t_i^{15}$ block after $b_i^{12}$ is assigned do not correspond to an admissible [13] time pattern, and the [11] time pattern left after further assigning $a_i^{12}$ is a permanent hole. This means that a
full load of all 40 cycle modules cannot be achieved once one or more of the $b^{\frac{12}{2}}$ are used; however, we do obtain a more complete coverage of the cycle with $[1^2]$ time patterns (with or without also using $[1^5]$ time patterns), a benefit in some cases.

Note the flexibility we have within each of the eight $T^{15}$ blocks; each of these 5-module blocks can be used for any of the following partitions:

- a $[1^5]$ class;
- a $[1^3]$ class + a $[1^2]$ class;
- 2 $[1^2]$ classes (leaving a $[1^1]$ hole).

The above possibilities are intuitively optimal for five modules, but may seem inadequate if we consider partitions of ten modules (2 adjacent $T^{15}$):

- 2 $[1^5]$ classes;
- a $[1^5]$ class + a $[1^3]$ class + a $[1^2]$ class;
- a $[1^5]$ class + 2 $[1^2]$ classes (leaving a $[1^1]$ hole);
- $\dagger$ 3 $[1^3]$ classes (leaving a $[1^1]$ hole);
- 2 $[1^3]$ classes + 2 $[1^2]$ classes;
- a $[1^3]$ class + 3 $[1^2]$ classes (leaving a $[1^1]$ hole);
- $\dagger$ 5 $[1^2]$ classes.

Of course, five of these seven partitions of 10 modules are straightforward combinations of the three partitions of 5 modules already available, but the two combinations flagged with daggers are different and are unattainable with our four families $T^{15}, A^{13}, A^{12}, B^{12}$. The best the model can offer is 2, not 3, $[1^3]$ classes; and 4, not 5, $[1^2]$ classes (leaving two permanent $[1^1]$ holes!). A similar analysis of fifteen modules (3 adjacent $T^{15}$) would further point out the inability within the model to configure 5 $[1^3]$ classes, but rather only 3 of them. The question, then, is have we
lost something, and the answer is yes. What we have lost is the ability to achieve all meaningful partitions of the 40 cycle modules. This is a price which must often be paid in the pursuit of interchangeable time patterns and better conflict behavior.

Of the 5 $t^1_{13}$ time patterns required to pack 5 $[1^3]$ classes into the first 3 adjacent $t^1_{15}$ (see figure 9-4f), only 3 are contained in the $T^1_{15}$ blocks, and the other 2 are not as well-behaved with respect to the three $t^1_{15}$. Furthermore, even the $T^1_{13}$ family is not adequate to support all partitions involving $[1^3]$ classes, in the context of, say, a $[1^5]$ class assigned $t^1_{15}$. To achieve all possible meaningful partitions, many more inter-conflicting time patterns would have to be added.

It is not the intent of this model or the thesis to suggest that some meaningful partitions of the cycle should be unattainable by a school. The claim is that compromise, if possible, can lead to smaller time pattern families that are substantially better behaved with respect to conflicts. If a resource must be assigned to more than 8 $[1^3]$ classes, normative model #4 simply will not work. Even if no resource is assigned more than 8 $[1^3]$ classes, if it is more important to a school that $[1^3]$ classes be disjoint from one another than from $[1^5]$ or $[1^2]$ classes, again the model would be better off with $T^1_{13}$ than $A^1_{13}$. In straying from the model, a school should comprehend the impact of any non-interchangeable time patterns being introduced, and the strength of the original four families in this regard. Normative model #4 is above all a model of good conflict behavior on both an inter- and an intra-family level; it is also a good example of flexibility allowing the type of extension leading to

normative model #5.
9.5.1 Normative Model #5 (an Extension of Normative Model #4)

Normative model #5 is an extension of normative model #4; the same three contact requirements are involved, but one additional time pattern structure, \([21^2]\), is assumed. Figure 9-5a exhibits the \(CR_i\) and corresponding \(TS_i\). The same cycle \((d=5, m=8)\) used in the previous model is assumed here.

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<td>([21^3]) (2 modules on one day, 1 module on each of 3 other days)</td>
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<tr>
<td>3 (modules)</td>
<td>([1^5]) (1 module on each of 5 days)</td>
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<tr>
<td>2 (modules)</td>
<td>([1^3]) (1 module on each of 3 days)</td>
</tr>
<tr>
<td>1 (modules)</td>
<td>([1^2]) (1 module on each of 2 days)</td>
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Fig. 9-5a.--Admissible \(CR_i\) and \(TS_i\), for Normative Model #5

Figures 9-4b through 9-4e in the previous section exhibit four families of time patterns \(T^{15}, A^{13}, A^{12}, B^{12}\) appropriate to this model; figures 9-5b and 9-5c exhibit two alternative \([21^3]\) families of time patterns \(A^{213}, B^{213}\) pertinent to the model.
Fig. 9-5b.--An Ideal Family \(A_{213}\) of 8 \([21]^3\) Time Patterns \(a_{1, 213}, a_{2, 213}, \ldots, a_{8, 213}\) over a \(d=5(\text{day})\) by \(m=8(\text{module})\) Cycle, Favoring [1^3] Mesh

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Fig. 9-5c.--An Ideal Family \(B_{213}\) of 8 \([21]^3\) Time Patterns \(b_{1, 213}, b_{2, 213}, \ldots, b_{8, 213}\) over a \(d=5(\text{day})\) by \(m=8(\text{module})\) Cycle, Favoring [1^2] Mesh

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There are 71680 possible \( [21^3] \) time patterns of a \( d=5 \) by \( m=8 \) cycle:

\[
71680 = \binom{5}{1} \times 7 \times \binom{4}{2} \times 8^3 = (\text{the number of ways of choosing 1 of the 5 days}) \times \\
(\text{the number of ways of choosing 2 consecutive modules on that day}) \times \\
(\text{the number of ways of choosing 3 of the remaining 4 days}) \times \\
(\text{the number of ways of distributing 3 modules over those 3 days}).
\]

Either \( A^{213} \) or \( B^{213} \) is inherently ideal because the eight time patterns of each are a disjoint coverage of the cycle. They each exemplify families contained in four congruent rectangular blocks; the four blocks are

\[ b_i^{25} = t_i^{15} = t_{2i-1}^{15} + t_{2i}^{15} \quad \text{for } i=1,2,3,4 \]

(a coarser partition of the cycle than \( T^{15} \) by a factor of 2).

The \( a_i^{213} \) and \( b_i^{213} \) are interchangeable with respect to the \( t_{2j}^{15} \) and vice versa; each \( a_i^{213} \) or \( b_i^{213} \) preempts, and is preempted by, exactly 2 of the \( t_{2j}^{15} \), in each case 2/8 of the other family. This effect is understandable because of transection, although unfortunate since 10 modules are thus preempted in the other family by a single time pattern on cardinality 5. The major impact of this transection is that each 10-module block \( b_i^{25} \) can be used for either two \([1^5]\) classes or two \([21^3]\) classes, but not one class of each structure; in other words, if all classes had a contact requirement of 5, an even number of each of the two time pattern structures would have to occur in scheduling a full load.

The major differences between \( A^{213} \) and \( B^{213} \) emerge when considering them in the context of the \([1^3]\) and \([1^2]\) time patterns. Given \( A^{213} \) (and \( A^{13} \)) we can schedule a \([21^3]\) class and a \([1^3]\) class in a block (leaving a \([2^1]\) hole); but we cannot schedule a \([21^3]\) class and two \([1^2]\) classes given \( A^{213} \) (and \( C^{12} = A^{12} + B^{12} \)). If, however, we use \( B^{213} \) instead of \( A^{213} \), the situation reverses itself: now within each block the two \([1^2]\) classes can be scheduled (leaving a \([1^1]\) hole), but not the \([1^3]\) class. It should
be clear that a school would decide between $A_{213}^2$ and $B_{213}^2$ on the basis of which other structure, $[1^3]$ or $[1^2]$ respectively, it wishes to favor with respect to its $[21^3]$ classes. It should also be clear that one or the other, but not both, of these families should be used in support of $[21^3]$ classes.

Each $a_{i}^{213}$ preempts:

- one of two $a_{j}^{13}$ (total conflict with respect to $a_{i}^{13}$),
- both $a_{j}^{12}$ (partial conflicts),
- one of two $b_{j}^{12}$ (total conflict with respect to $b_{i}^{12}$).

In contrast, each $b_{i}^{213}$ preempts:

- both $a_{j}^{13}$ (partial conflicts),
- one of two $a_{j}^{12}$ (total conflict with respect to $a_{i}^{12}$),
- one of two $b_{j}^{12}$ (total conflict with respect to $b_{i}^{12}$).

The symmetric interchangeability of the two $b_{j}^{213}$ in each block $b_{i}^{213}$ is due to the staggered arrangement of the $B_{12}$ time patterns shown in figure 9-4e; if two days had been consistently used (say, W and F) instead of the three shown in figure 9-4e ($M, W, F$), the composite set with $A_{12}$ would have covered the same region of the cycle as did $T_{12}$ in figure 9-2d, and this would have impacted interchangeability between odd- and even-indexed $b_{j}^{213}$.

Figure 9-5d exhibits a compromise family $C_{213}^{2}$ of $[21^3]$ time patterns where half of the time patterns favor $[1^3]$ mesh and the other four favor $[1^2]$ mesh. Note that the $a_{i}^{213}$ are still interchangeable with respect to each other and the $t_{i}^{15}$, but decompose into two subfamilies (of 4 time patterns each) when it comes to interchangeability with $A_{13}$ and $C_{12} = A_{12} + B_{12}$, this being the primary objective of the compromise. A similar compromise family $D_{213}^{2}$ could have been obtained by combining the four $a_{i}^{213}$ from any
2 blocks with the four $b_{ij}^{213}$ from the other 2 blocks; in this case, however the blocks lose interchangeability.

\[
\begin{array}{cccccc}
M & T & W & R & F \\
1 & \sigma_1 & \sigma_1 & \sigma_1 & \sigma_1 & \sigma_1 \\
2 & \sigma_2 & \sigma_2 & \sigma_2 & \sigma_2 & \sigma_2 \\
3 & \sigma_3 & \sigma_3 & \sigma_3 & \sigma_3 & \sigma_3 \\
4 & \sigma_4 & \sigma_4 & \sigma_4 & \sigma_4 & \sigma_4 \\
5 & \sigma_5 & \sigma_5 & \sigma_5 & \sigma_5 & \sigma_5 \\
6 & \sigma_6 & \sigma_6 & \sigma_6 & \sigma_6 & \sigma_6 \\
7 & \sigma_7 & \sigma_7 & \sigma_7 & \sigma_7 & \sigma_7 \\
8 & \sigma_8 & \sigma_8 & \sigma_8 & \sigma_8 & \sigma_8 \\
\end{array}
\]

**Fig. 9-5d.** -- A Compromise Family $c^{213}$ of 8 $[21^3]$ Time Patterns $c_1^{213}, c_2^{213}, \ldots, c_8^{213}$ over a $d=5$ (day) by $m=8$ (module) Cycle, Balancing $[1^3]$ and $[1^2]$ Mesh

The matrix shown in figure 9-5e shows the a priori probability of conflict between two random classes, broken down by the families discussed in this model.
Fig. 9-5e.—Matrix of A Priori Conflict between two Random Classes, Broken Down by Time Pattern Families of Normative Model #5

9.6.1 Ideal Normative Model (Powers of 2)

Figure 9-6a defines the contact requirements \((CR_i)\) and corresponding time pattern structures \((TS_{ij})\) proposed for our ideal normative model, which we would promote as being ideal in the broadest sense of the word. Because it displays all of the advantages of sound time pattern analysis, this system can serve as a paradigm against which to measure other models and individual implementations. The claim is that if a school can adopt this arrangement, there is no reason to believe they can do better, particularly as regards a priori flexibility and interchangeability of classes and resources. We hope to substantiate this claim in the following discussion. Note that all of the contact requirements \((CR_i)\) are powers of 2 (namely, 1,2,4,8,16).
<table>
<thead>
<tr>
<th>$CR_i$</th>
<th>$TS_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 (modules)</td>
<td>$[4^4]$ (4 modules on each of 4 days)</td>
</tr>
<tr>
<td>8 (modules)</td>
<td>$[4^2]$ (4 modules on each of 2 days)</td>
</tr>
<tr>
<td></td>
<td>$[2^4]$ (2 modules on each of 4 days)</td>
</tr>
<tr>
<td>4</td>
<td>$[4^1]$ (4 modules on 1 day)</td>
</tr>
<tr>
<td></td>
<td>$[2^2]$ (2 modules on each of 2 days)</td>
</tr>
<tr>
<td></td>
<td>$[1^4]$ (1 module on each of 4 days)</td>
</tr>
<tr>
<td>2</td>
<td>$[2^1]$ (2 modules on 1 day)</td>
</tr>
<tr>
<td></td>
<td>$[1^2]$ (1 module on each of 2 days)</td>
</tr>
<tr>
<td>1</td>
<td>$[1^1]$ (1 module on 1 day)</td>
</tr>
</tbody>
</table>

Fig. 9-6a.—Admissible $CR_i$ and $TS_i$, for an Ideal Normative Model

Figures 9-6b through 9-6j exhibit the nine different time pattern families claimed to be ideal for this model, namely $T^{11}, T^{12}, T^{14}, T^{22}, T^{41}, T^{24}, T^{42}$, and $T^{44}$ (the notation $T^{xy}$ is intended to be mnemonic for the family of time patterns with structure $[x^y]$).
### Fig. 9-6b

The Ideal Family $T^{11}$ of 32 \([1^1]\) Time Patterns $t_1^{11}, t_2^{11}, \ldots, t_{32}^{11}$ over a $d=4$ by $m=8$ Cycle

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tr>
<td>1</td>
<td>$t_1^{11}$</td>
<td>$t_2^{11}$</td>
<td>$t_3^{11}$</td>
<td>$t_4^{11}$</td>
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<tr>
<td>2</td>
<td>$t_5^{11}$</td>
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<td>$t_7^{11}$</td>
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<tr>
<td>3</td>
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</tr>
<tr>
<td>4</td>
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<td>$t_{15}^{11}$</td>
<td>$t_{16}^{11}$</td>
</tr>
<tr>
<td>5</td>
<td>$t_{17}^{11}$</td>
<td>$t_{18}^{11}$</td>
<td>$t_{19}^{11}$</td>
<td>$t_{20}^{11}$</td>
</tr>
<tr>
<td>6</td>
<td>$t_{21}^{11}$</td>
<td>$t_{22}^{11}$</td>
<td>$t_{23}^{11}$</td>
<td>$t_{24}^{11}$</td>
</tr>
<tr>
<td>7</td>
<td>$t_{25}^{11}$</td>
<td>$t_{26}^{11}$</td>
<td>$t_{27}^{11}$</td>
<td>$t_{28}^{11}$</td>
</tr>
<tr>
<td>8</td>
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<td>$t_{30}^{11}$</td>
<td>$t_{31}^{11}$</td>
<td>$t_{32}^{11}$</td>
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</tbody>
</table>

### Fig. 9-6c

The Ideal Family $T^{12}$ of 16 \([1^2]\) Time Patterns $t_1^{12}, t_2^{12}, \ldots, t_{16}^{12}$ over a $d=4$ by $m=8$ Cycle

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
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<td>$t_{14}^{12}$</td>
<td>$t_{15}^{12}$</td>
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</tbody>
</table>

### Fig. 9-6d

The Ideal Family $T^{21}$ of 16 \([2^1]\) Time Patterns $t_1^{21}, t_2^{21}, \ldots, t_{16}^{21}$ over a $d=4$ by $m=8$ Cycle

<p>| | | | | |</p>
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<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
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</tr>
<tr>
<td>6</td>
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</tr>
</tbody>
</table>

Fig. 9-6e

Fig. 9-6f

Fig. 9-6g

Fig. 9-6e.--The Ideal Family $T^{14}$ of 8 $[1^4]$ Time Patterns $t_1^{14}, t_2^{14}, \ldots, t_8^{14}$ over a d=4 by m=8 Cycle

Fig. 9-6f.--The Ideal Family $T^{22}$ of 8 $[2^2]$ Time Patterns $t_1^{22}, t_2^{22}, \ldots, t_8^{22}$ over a d=4 by m=8 Cycle

Fig. 9-6g.--The Ideal Family $T^{41}$ of 8 $[4^1]$ Time Patterns $t_1^{41}, t_2^{41}, \ldots, t_8^{41}$ over a d=4 by m=8 Cycle
Fig. 9-6h — The Ideal Family $T^{24}$ of 4 $[2^4]$ Time Patterns $t_1^{24}, t_2^{24}, t_3^{24}, t_4^{24}$ over a $d=4$ by $m=8$ Cycle

Fig. 9-6i — The Ideal Family $T^{42}$ of 4 $[4^2]$ Time Patterns $t_1^{42}, t_2^{42}, t_3^{42}, t_4^{42}$ over a $d=4$ by $m=8$ Cycle

Fig. 9-6j — The Ideal Family $T^{44}$ of 2 $[4^4]$ Time Patterns $t_1^{44}, t_2^{44}$ over a $d=4$ by $m=8$ Cycle
The particular cycle of \( d=4 \) days (named \( A,B,C,D \)) and \( m=8 \) modules (named 1, 2, 3, 4, 5, 6, 7, 8) is quite important; note that \( d \) and \( m \) are both powers of 2. Observe that a halving decomposition has been repeated over the 8 daily modules in the cycle, thus partitioning the original 32 unit cycle modules first into two 16-module blocks, then into four 8-module blocks, and finally into eight 4-module blocks. In a similar halving decomposition, the 4 days split first into pairs, then into single days. This two-dimensional cleaving, schematized below in figure 9-6k, results in an extremely valuable symmetry whereby each cleaving can be thought of as creating twin buddies (a term deliberately chosen to suggest a parallel to the so-called buddy system of computer storage allocation/management). In figure 9-6k, the buddies are indicated by dotted lines; note that the 4-module area corresponding to the time pattern \( t_8^2 = 'CD7-8' \) can be considered a buddy of both \( t_6^2 = 'CD5-6' \) and \( t_7^2 = 'AB7-8' \).

![Figure 9-6k](image-url)

*Fig. 9-6k.*—Schematic Diagram of Halving Decomposition of \( d=4 \) by \( m=8 \) Cycle into Symmetric Buddies
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Before proceeding with the ideal attributes of these families, we present a few observations regarding the potential for real application of this model.

(1) The 8-module day corresponds nicely to the familiar 8-hour business day in widespread use (e.g. at M.I.T.), although some schools adopt a 6- or 7-hour school day for students if not for faculty. In any case, the actual length of these modules is left open as a variable, so long as one common length applies to all modules. A basic module length of between 40 and 60 minutes is common in real life.

(2) A basic module length of 40 to 60 minutes translates into familiar reasonable configurations according to the powers-of-2 contact requirement ratios: "one hour of Physical Education every other day", "two hour Science labs", "one hour daily for English, Math, Languages", "either all morning or all afternoon (4 modules daily) for vocational training", etc. A school interested in shorter sessions (e.g. "half-hour seminars or study halls") could adopt 16 half-length daily modules generated by one more cleaving; the original 8 modules of the model would then simply double as A.M. and P.M. blocks.

(3) These families are readily suitable for optional module rotation and module inversion. A shift factor of 4 (modules) could be used for module rotation. Inversion could be applied not only to the overall day, but also within blocks (e.g. the actual chronological sequence of modules could be 56782134 on one day, 12438765 the next, and so on, so long as buddies abut). These options are quite available as finishing touches to a schedule already satisfactory vis-a-vis conflicts, in order to finesse the popular/unpopular hour problem by sharing clock hours equitably.
(4) The days could be rearranged over the cycle; the need for every other day classes suggests that ACBD would likely be a more acceptable real life sequence than ABCD. Any day permutation exhibits equivalent conflict behavior (this is true of any time pattern system and cycle). The ABCD sequence was chosen here so that the figures would more clearly illustrate the buddy structure.

(5) The four day cycle may be a stumbling block to anyone who has never considered anything but a five calendar-day week. Implementation of a four day cycle may be hopelessly out of the question for schools tied to outside events such as shared resources involving other institutions, or when faculty loads are light and they desire the same days off each cycle. (Note that if every day is tied up with teaching duties, such day scrambling is likely to make less difference to faculty.) The four day cycle may be even more of a shock to a school than scrambling the modules. It is not, however, merely the figment of some combinatorial analyst's imagination; often implemented in Canada because of frequent snow closings, the independence of the schedule from lockstep with the calendar week permits the cycle to be picked up where it left off after a disruption is over, without imbalanced impact on classes that meet less often than daily. Given the trend towards Monday holidays in the U.S., cycle independence can contribute a great deal towards the interchangeability of non-rectangular time patterns. (Note that even a five day cycle need not be tied to a calendar week!) Finally, there is a trend in some areas towards a four day work week, in which case a four day cycle would retain correspondence to the calendar.

We now address ourselves to the intrinsic properties of these nine
time pattern families that render them ideal, and therefore recommend this model as a paradigm against which all other time pattern systems can be judged. Every one of these time pattern families is individually composed of the maximum number of disjoint time patterns collectively covering the cycle. In other words, each family would by itself be ideal were it the only time pattern structure in the model. Hence, if any resource is restricted to exactly one family, it is theoretically possible to cover the individual schedule of that resource with the maximum number of admissible time patterns, without conflict, by assigning each time pattern once and only once. In this case, there are no built-in holes left in the cycle after a family has been exhausted, yet no conflicts need result in the process so long as repetition is avoided. This fact is important to those resources in a school that rely heavily on similar time patterns, as when a lab must support several classes which are assigned similar time patterns, or when both the gym and the gym instructor(s) have to cover all (identically structured) Physical Education classes. Within its own family, each time pattern is totally interchangeable with respect to every other time pattern in the family; assigning any time pattern preempts only recurrence of that same time pattern in a resource schedule, with no possible conflict with the other family members.

These time pattern families are ideally well-behaved with respect to each other. Given any two different families, they interfere with each other as little as could possibly be hoped for, given the time pattern structures they represent and that each disjointly covers the cycle. Transection has been avoided wherever possible; when necessary, transection is at least minimal, in the sense that no time pattern transects
any more time patterns from a family than are inherently required for structural reasons. We have already pointed out that two supplementary (disjointly covering) families with the same contact requirement \((CR_i)\) but different time pattern structures \((TS_{i,})\) must inevitably transect each other. It is therefore understandable that \(T^{42}\) and \(T^{24}\) transect each other (as must \(T^{21}\) and \(T^{12}\)); each time pattern from the families must partially conflict with two time patterns from the other family.

The point is that no more than two time patterns need be transected, and this is the case with our families. In the case of the \([4^2]\) and \([2^4]\) structures, it is not hard to concoct examples of all four time patterns being preempted in the other family, as when module inversion is applied to any day in the \(T^{42}\) family. \(T^{41}, T^{22},\) and \(T^{14}\) must each transect the other two families, but no more than two of the eight time patterns from another family need be in conflict, and such is the case with our model.

To appreciate the advantages of minimal transection, it may help to study a worst-case example of preemption brought about by a poorly designed time pattern \(t^{14}_9\), added as an extension to \(T^{14}\). Figure 9-6l exhibits the \([1^4]\) time pattern \(t^{14}_9\), and figure 9-6m contrasts preemption by \(t^{14}_9\) to that of an original \(t^{14}_{i<9}\), in terms of fractions of the other families (and the cycle they cover).
Fig. 9-61.--A Poorly Designed $t_{14}$ Time Pattern $t_{14}$ Leading to Adverse Transection in Other Families

| Family | $|t_j|_{Family}$ | Preemption by $t_{14}$ | Preemption by $t_{i<9}$ |
|--------|-----------------|-----------------------|-----------------------|
| $T^{11}$ | 32 | 1 | 4/32 = 12.5% | 4/32 = 12.5% |
| $T^{12}$ | 16 | 2 | 4/16 = 25% | 2/16 = 12.5% |
| $T^{21}$ | 16 | 2 | 4/16 = 25% | 4/16 = 25% |
| $T^{14}$ | 8 | 4 | 4/ 8 = 50% | 1/ 8 = 12.5% |
| $T^{22}$ | 8 | 4 | 4/ 8 = 50% | 2/ 8 = 25% |
| $T^{41}$ | 8 | 4 | 4/ 8 = 50% | 4/ 8 = 50% |
| $T^{24}$ | 4 | 8 | 4/ 4 = 100% | 1/ 4 = 25% |
| $T^{42}$ | 4 | 8 | 4/ 4 = 100% | 2/ 4 = 50% |
| $T^{44}$ | 2 | 16 | 2/ 2 = 100% | 1/ 2 = 50% |

Fig. 9-6m.--Preemption by $t_{14}$ Contrasted to that of an Original $t_{i<9}$ in terms of Fractions of the Other Families
Not only are the nine families ideal from a standpoint of avoiding or minimizing transection, but also from a standpoint of dominance. Although dominance is a type of interference, it is not undesirable; on the contrary, when it is possible, dominance is a desirable objective of time pattern design. Previous examples have pointed out the disproportionately adverse effects of partial conflicts; thus it is an objective of good time pattern design that, given any two time patterns \( t_i, t_j \), either they be disjoint \( (t_i \cap t_j = \emptyset) \) or else conflict be total with respect to at least one of the two \( (t_i \subset t_j \text{ and/or } t_j \subset t_i) \). Dominance is desirable to the extent that it promotes the latter of these alternatives.

Each of our nine ideal families consists solely of straight time patterns; dominance can occur between two families of supplementary straight time patterns only when one family has a contact requirement \((CR_i)\) which is an integer multiple of the \( CR_j \) of the other family. As pointed out earlier, a family of \([1^1]\) time patterns, such as \( T_{11} \), must be dominated by any other supplementary family. In our model, \( T_{44} \) dominates all eight other families; \( T_{42} \) dominates \( T_{41} \) (which dominates \( T_{21} \)) and \( T_{22} \) (which dominates both \( T_{21} \) and \( T_{12} \)); and \( T_{24} \) dominates \( T_{22} \) and \( T_{14} \) (which dominates \( T_{12} \)). Dominance is a transitive relation, so \( T_{42} \) also dominates \( T_{21} \) and \( T_{12} \), etc. The nine families can be thought of as a lattice under the partial ordering of dominance, as diagrammed in figure 9-6n.
In figure 9-6n, a connecting path of downward lines indicates dominance, while the absence of such a path indicates "incomparable" families under the partial ordering of dominance, and in our model these "incomparable" families are precisely those which transect. To the extent that $T^{44}$ dominates all eight other families, and given that the $t^{44}_i$ are congruent rectangular time patterns, $t^{44}_1$ and $t^{44}_2$ constitute a congruent rectangular block structure for the cycle. However, the structural advantages of this cycle transcend those of mere block structuring: each and every time pattern $t$ of cardinality $|t| \geq 2$ is itself a kind of block, useful not only as the intact time pattern $t$ but also decomposing into other admissible time patterns in a most flexible manner.

We observed an imperfect example of time pattern decomposition in normative model #4, where a 5-module block $t^{15}_i$ could be decomposed either into $a^{13}_i + a^{12}_i$ or else into $a^{12}_i + b^{12}_i$ (leaving a permanent $[1^1]$ hole); this decomposition was flawed in that several meaningful partitions of the
cycle were unattainable, and further flawed by the permanent holes. The problem of the permanent \([1^1]\) holes might have been alleviated had we allowed \([1^1]\) or even \([1^2]\) or \([1^4]\) time patterns (as in the ideal model), but the partitioning flaw would still remain intractable—the given time pattern structures and the given cycle configuration inherently create the partitioning problem. In this ideal model, however, we not only avoid permanent holes (even without the \(T_{11}\) family, although these 32 \([1^1]\) time patterns enhance the model for other reasons), but we also achieve total flexibility with respect to partitions: all meaningful partitions of the 32 cycle modules are attainable, thanks to confining the \(CR_i\) to powers of 2. And furthermore, not only are all partitions achievable, they are each attainable in a variety of compositions which, while not exhaustive, afford rich flexibility.

A final ideal characteristic of these nine time pattern families is worth noting. Not only is each family inherently ideal by itself, and not only do the families mesh well, but also within every family, all time patterns are totally interchangeable with respect to every other family. No matter which families have the highest (or lowest) a priori usage, no time pattern in any family exhibits a priori bias either for or against usage. Each time pattern is equally sound in its family: each plays such a critical role that none can be omitted without detriment, nor can new time patterns contribute anything without disrupting the fundamental harmony of the overall system.
10.1.1 Case Study Objectives

In order to interpret the foregoing theory in terms of practical application, two case studies were embarked upon. The intention was to perform time pattern analysis in two inherently different real life situations. Two schools were chosen, not because either was typical of schools in general, but because of their positions near opposite ends of a school scheduling spectrum. These schools are the Minuteman Regional Vocational Technical School, henceforth referred to as Minuteman, a secondary school in Lexington, Massachusetts; and the Massachusetts Institute of Technology, henceforth referred to as M.I.T., a private university in Cambridge, Massachusetts.

The two schools contrast significantly in: (1) attitudes, predispositions, and expectations regarding scheduling; (2) type and availability of resources; (3) variety in student schedule requests and loads; and, (4) number of subjects offered and spectrum of $CR_i$ and $TS_i$, involved. A major difference is that the Minuteman case study transpired during the two years prior to the school's initial opening in September, 1974, whereas M.I.T. was and is an ongoing enterprise.

The primary objective of each case study was to demonstrate in practice the importance of the theory of time pattern analysis. The Minuteman study had the objective of identifying a sound time pattern system for actual implementation. The M.I.T. study had the objective of
identifying alternative time pattern systems, including simplifications to as well as digressions from the current approach. An underlying objective of both case studies was to document the processes of time pattern analysis that they could be extrapolated to the general academic environment.

10.2.1 Introduction to Minuteman

In order to provide a brief introduction to Minuteman, there follow excerpts from literature published by the school to introduce interested parties to their program. Choice of excerpts, and hence emphasis, is based upon scheduling considerations. The excerpts are all taken from a brochure "Minuteman Regional Vocational Technical School", distributed to potential students in the twelve appropriate school districts.

All programs at Minuteman are open to both girls and boys. ... Students are offered the opportunity to explore career alternatives while at the same time fulfilling the academic requirements for a high school diploma. ...

Students assume much of the responsibility for their own learning. ... Individualized programs and flexible scheduling are planned to make these experiences possible. ...

A compact, multi-level building, the school is characterized by openness. Wide areas flow into each other, providing a physical environment consistent with the academic one, reinforcing the idea that education is a continuing and evolving process. ...

Although the major aim of Minuteman is to graduate students with saleable vocational skills, the instructional programs also meet or exceed all the state requirements for a high school diploma. For those desiring it, Minuteman provides preparation and educational background necessary for further education.

The four-year program begins by introducing each student to a variety of vocational possibilities from which he or she will select for study those careers that are of the most interest.

The Freshman and part of the Sophomore years are devoted to a program designed to acquaint students with a variety of career possibilities and the basic elements of several career choices. This is also a period of evaluation and assessment of each student's capabilities and promise. During this introductory program, students are taught not only the differences among vocations, but also their similarities. ...
Twelve career programs are offered in the first year, from which each student selects eight for study.

The school year for ninth graders is divided into eight 5-week periods, corresponding to the areas of study selected by each student. Half the day is spent in an introductory vocational program, the other half in academic instruction. Academic programs are geared to the needs of each student and the careers being studied. ...

Following the introductory program, students concentrate on fewer career choices. In the second year, they select four activities for additional studies, of which three must have been studied in the first year. One of the four may be a career choice not yet explored. Either way, the depth of study is greater than in the introductory year.

In the third year, students select the specific career area they wish to follow. During this year, they undertake independent projects related to their chosen specialty.

In the fourth year at Minuteman, students are employed in actual work at the school or in cooperating industry, or continue with advanced skill development.

10.3.1 Introduction to M.I.T.

In order to provide a brief introduction to M.I.T., there follow excerpts from the 1974-75 "M.I.T. Bulletin, The General Catalogue Issue". Choice of excerpts and hence emphasis, is based on scheduling considerations.

The Massachusetts Institute of Technology is an independent, co-educational, endowed university committed to the extension of knowledge through teaching and research. It is organized into five academic Schools—Architecture and Planning, Engineering, Humanities and Social Science, Management, and Science—and a number of interdisciplinary groups and activities. There are about 8,000 students, more than half of them studying for undergraduate degrees; about 950 members of the faculty; and a teaching staff of 1,700. ...

The Institute has a single campus and a single faculty serving both undergraduate and graduate students. Most of the classrooms and laboratories are in an interconnected group of buildings which facilitates informal interchange between departments and disciplines. Members of the faculty group themselves for teaching and resource according to their interests.

Most faculty appointments are in one or more of the Institute's 24 academic departments, but there are also many interdisciplinary laboratories, centers, and divisions which provide support in numerous fields that extend beyond the traditional boundaries of a single department. Most undergraduate students major in specific departments, focusing their work according to their interests. There are ample opportunities for students to share in the
interdisciplinary activities of the faculty with whom they work and to major in fields which combine more than one discipline.

The academic programs of both undergraduate and graduate students are based upon a core of general Institute and departmental requirements. There is enough flexibility, however, to allow each student, in collaboration with a faculty advisor, to develop an individual program in response to his or her own interests and preparation.

Undergraduate subjects are offered by all of M.I.T.'s departments and Schools, and students' programs are generally made up of subjects from at least three of the Schools, some from all five. Graduate students frequently study in two or three of M.I.T.'s five academic Schools. Undergraduate upperclass students often register with graduate students for some of their classes; many undergraduates and almost all graduate students participate, often together, in advanced research. ...

The primary organization of the academic program is along the lines of the traditional disciplines. Each of the 24 academic departments offers one or more degree programs, or Courses of study. Most upperclass undergraduate and graduate students are registered in one of these Courses. ...

There is a growing number of students who concentrate their studies in areas that cross departmental lines.

10.4.1 Comparison of the Two Schools

Minuteman had no strong predispositions about its scheduling, in the sense that almost all degrees of freedom were considered fair game for compromise, and evaluation in terms of leverage. Particularly because the school had not yet started operation, Minuteman was quite open-minded to various alternatives. M.I.T., on the other hand, has had systematized scheduling for more than a decade, and there are strong feelings—in many sectors—about what is to be expected of the schedule. M.I.T. has many "ground rules" about faculty preferences, admissible day and module combinations, and acceptable room utilizations, which range from published standards to tacit "understandings". Whereas M.I.T. builds each new schedule on the foundation of previous experience, relying heavily on extrapolation from one year to the next, such an evolutionary approach was
not possible for Minuteman, facing what was for them a first time experience. For Minuteman, simulation and planning had to serve in place of experience, a mixed blessing (suggesting mixed metaphors): advantageous since no boat to rock, disadvantageous since no bird in the hand—precisely the reverse of the M.I.T. situation.

Minuteman (like the majority of secondary schools) has limited resources, whereas M.I.T., by comparison, has extensive resources. This is more than a matter of money, more even than size; it is a characteristic difference between higher and secondary institutions. Because the student sector at colleges and universities typically places less formal demand on the other scheduling resources (i.e. student load is usually smaller than at primary or secondary schools), instructors seldom are fully loaded with formal curriculum, and rooms can often serve a larger percentage and broader cross-section of classes.

There are only eight possible schedule configurations for first-year students at Minuteman (assuming, for this purpose, no scheduling distinction among the twelve different vocational options); the school could literally get by with eight rubber stamps to handle student schedules. In contrast, M.I.T. would require near as many rubber stamps as students in order to span the full range of student schedule possibilities. Students at Minuteman take full loads with only 10 percent of the cycle formally unassigned, and even this time is used for accountable activity; at M.I.T. even the heaviest loads (which occur for underclass students) seldom exceed 50 percent of the cycle.

Finally, subjects at Minuteman do not involve explicit phasing (lectures, labs, etc.), and most classes neatly fall into consistent daily
patterns of 8, 4, or 2 20-minute contacts involving the same periods each day. Accordingly, few time patterns are needed to serve the entire range of $CR_i$ and $TS_i$, (e.g. only two vocational time patterns: morning and afternoon; only four academic time patterns; etc.). At M.I.T., phasing is frequently encountered, with combinations of lecture, laboratory, recitation, seminar, tutorial, quiz and design phases. Each of these phasing components in turn permits a variety of $CR_i$ and $TS_i$. Typical M.I.T. student registration computer runs utilize about 500 dictionary time patterns (the standard repertoire), and about 500 non-dictionary time patterns each serving one or two unusual configurations, to support about 2000 classes.

It is not claimed that Minuteman and M.I.T. are typical schools. On the contrary, they may well be atypical! They were chosen because of their mutual differences, because they represented some extremes in scheduling mechanics, and in the expectation that they would offer fertile ground for concrete examples of applied theory.

Both case studies uncovered unanticipated information as they progressed, both called for change of tack during the study, and both turned out to provide useful contexts within which to emphasize important theoretical considerations. The author considers himself fortunate that both case studies worked out as well as they did in terms of concrete examples. Particularly in the case of Minuteman, it could not have been realistically predicted two years earlier that time pattern analysis would develop which not only would serve the theoretical purposes of this thesis, but also would have dramatic impact on actual school operation.
11.1.1 Contact Requirements at Minuteman

The first step at Minuteman (Minuteman Regional Vocational Technical School) involved determination of the intended student contact requirement (CR) profile. During this analysis, certain subject groupings were evident: the twelve vocational subjects had similar characteristics, math/science (M/S) and communications/human-relations (C/H) could be referred to as the two academic subject-pairs or academics (with common attributes), two types of instructor planning sessions were to be handled, and electives were conspicuously absent. As of June 1973, every first year student was to be assigned:

1 vocational subject..................160 minutes daily
2 academic subject-pairs (M/S, C/H).... 80 minutes each daily
1 physical education (PE) section......120 minutes per week
1 lunch each day....................... 20 minutes daily.

The remainder of each student's day was to be independently assigned time (IAT), which would be represented by holes in the schedule (even though the student would be busy during these holes).

Each of these expectations was basically a daily requirement, except for PE which was given as a weekly requirement. The school had already begun to think in terms of a cycle of five calendar days, with twenty 20-minute modules each day. Figure 11-a summarizes the CR by subject group relative to a weekly cycle, and also shows the time pattern structures
(TS$_i$) originally envisioned to satisfy each CR$_i$.

<table>
<thead>
<tr>
<th>CR$_i$ (modules)</th>
<th>TS$_i$</th>
<th>Subject Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>[8$^5$]</td>
<td>the 12 vocational subjects</td>
</tr>
<tr>
<td>20</td>
<td>[4$^5$]</td>
<td>the 2 academic subject-pairs (M/S, C/H)</td>
</tr>
<tr>
<td>10</td>
<td>[2$^5$]</td>
<td>the 2 planning sessions (instructors only)</td>
</tr>
<tr>
<td>6</td>
<td>[2$^3$]</td>
<td>8 interchangeable PE sections</td>
</tr>
<tr>
<td>5</td>
<td>[1$^5$]</td>
<td>lunch</td>
</tr>
</tbody>
</table>

Fig. 11-a.--The CR$_i$ and TS$_i$, Originally Planned for Minuteman

The CR$_i$ thus planned fall into the ratio 40>20>10>6>5. The second step was to consider the impact of these CR$_i$ on the resources. Figure 11-b is a "Sample Student's Schedule" published by Minuteman to illustrate these original plans.
Note that the two academic subject-pairs (M/S, C/H) are each really 80-minute blocks of time, to be divided up between the disciplines represented as the instructor teams see fit. This is a good example of block- or macro-scheduling, whereby a framework of time is allocated to a pseudo-class, within which two or more de facto classes (sub-sections) are actually held; such packaging can be a very useful conceptual simplification.

From the standpoint of an individual student, this arrangement appeared quite satisfactory. Each student was to partition the cycle according to figure 11-c, which also shows instructor and key room demands on the schedule.
<table>
<thead>
<tr>
<th>Resource Group</th>
<th>Required Partition of the 100 Cycle Modules</th>
</tr>
</thead>
<tbody>
<tr>
<td>all (first year) students:</td>
<td>1 [6⁵] vocational subject ........ 40 modules</td>
</tr>
<tr>
<td></td>
<td>+2 [4⁵] academic subject-pairs .. 40</td>
</tr>
<tr>
<td></td>
<td>+1 [2³] PE section ................ 6</td>
</tr>
<tr>
<td></td>
<td>+1 [1⁵] lunch ........................ 5</td>
</tr>
<tr>
<td></td>
<td>91 modules</td>
</tr>
<tr>
<td>vocational instructors:</td>
<td>2 [8⁵] vocational classes .... 80 modules</td>
</tr>
<tr>
<td></td>
<td>+2 [4⁵] planning sessions .......... 20</td>
</tr>
<tr>
<td></td>
<td>+1 [1⁵] lunch ........................ 5</td>
</tr>
<tr>
<td></td>
<td>105 modules</td>
</tr>
<tr>
<td>academic instructors:</td>
<td>4 [4⁵] academic classes ...... 80 modules</td>
</tr>
<tr>
<td></td>
<td>+2 [2⁵] planning sessions .......... 20</td>
</tr>
<tr>
<td></td>
<td>+1 [1⁵] lunch ........................ 5</td>
</tr>
<tr>
<td></td>
<td>105 modules</td>
</tr>
<tr>
<td>PE instructors:</td>
<td>8 [2³] PE sections ................ 48 modules</td>
</tr>
<tr>
<td></td>
<td>+2 [2⁵] planning sessions .......... 20</td>
</tr>
<tr>
<td></td>
<td>+1 [1⁵] lunch ........................ 5</td>
</tr>
<tr>
<td></td>
<td>73 modules</td>
</tr>
<tr>
<td>PE room(s):</td>
<td>8 [2³] PE sections ................ 48 modules</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>cafeteria:</td>
<td>3 [1⁵] lunch options ................ 15 modules</td>
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</tbody>
</table>

Fig. 11-c.--Key Resource Demands on Original Minuteman Cycle

There were to be two sections (a morning section and an afternoon section) of each vocational subject, four sections of each academic subject-pair (tied in student population to the vocational subjects), eight PE sections, and three lunch options (it was understood that PE instructors would eat lunch outside the normal three lunch options). Already there was a problem: the vocational and academic instructors each required
105 modules within a cycle which had only 100. This impass was permanently resolved by scheduling one of the two [25] instructor planning sessions outside the standard cycle, "after school" during 21st and 22nd periods. The cycle was therefore to be 5 days by 22 modules, with the last 2 modules standing in special status to be used solely by the instructor sector (plus whatever room was occupied by them) for cluster planning. This arrangement had the built-in benefit of allowing all the faculty to meet together daily without disrupting teaching duties.

11.2.1 Time Pattern Families Originally Proposed by Minuteman

The third step was to enumerate all of the time patterns which could produce schedules. The five originally proposed time pattern families \( P^{25}, P^{45}, P^{23}, P^{15} \) are shown as of June 1973 in figures 11-d through 11-h. Note the non-disjoint nature of the \( P^{45} \) and \( P^{23} \) families.
Fig. 11-d. The Disjoint Family $p^{85}$ of 2 $[8^5]$ Time Patterns $p_1^{85}, p_2^{85}$ Originally Proposed by Minuteman for Vocational Subjects

Fig. 11-e. The Non-Disjoint Family $p^{45}$ of 5 $[4^5]$ Time Patterns $p_1^{45}, p_2^{45}, \ldots, p_5^{45}$ Originally Proposed by Minuteman for Academic Subject-Pairs
<table>
<thead>
<tr>
<th></th>
<th>T</th>
<th>W</th>
<th>R</th>
<th>F</th>
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<tbody>
<tr>
<td>1</td>
<td></td>
<td>25</td>
<td>P₁</td>
<td></td>
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<tr>
<td>2</td>
<td></td>
<td>23</td>
<td>P₂</td>
<td>23</td>
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<tr>
<td>3</td>
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<td>P₁/P₂</td>
<td>P₂</td>
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<td>4</td>
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<td>P₂</td>
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<td>5</td>
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<td>25</td>
<td>P₃</td>
<td>23</td>
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<tr>
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<td>P₄</td>
<td>23</td>
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<tr>
<td>7</td>
<td></td>
<td></td>
<td>P₃/P₄</td>
<td>P₄</td>
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<tr>
<td>8</td>
<td></td>
<td></td>
<td>P₃</td>
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<td>9</td>
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<td>23</td>
<td>P₅</td>
<td>23</td>
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<td>P₆</td>
<td>23</td>
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<td>11</td>
<td></td>
<td></td>
<td>P₅/P₆</td>
<td>P₆</td>
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<td>12</td>
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<td>23</td>
<td>P₇</td>
<td>23</td>
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<td>13</td>
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<td>P₇/P₈</td>
<td>P₇</td>
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<td>14</td>
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<td>P₇</td>
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<td>P₇</td>
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<td>16</td>
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<td>P₈</td>
<td>23</td>
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<td>P₈</td>
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<td>21</td>
<td>P₃</td>
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<td>22</td>
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</table>

**Fig. 11-f**

The Disjoint Family P²₅ of 3 [2⁵] Time Patterns P₁, P₂, P₃, P₄, P₅, P₆, P₇, P₈ Originally Proposed by Minuteman for Major Planning Sessions.

**Fig. 11-g**

The Non-Disjoint Family P²₃ of 8 [2³] Time Patterns P₁, P₂, ..., P₈ Originally Proposed by Minuteman for PE Sections.
<table>
<thead>
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<th>M</th>
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<td>10</td>
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<td></td>
<td></td>
<td>$p_1^{15}$</td>
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<tr>
<td>11</td>
<td></td>
<td></td>
<td>$p_2^{15}$</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>$p_3^{15}$</td>
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<td></td>
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</tbody>
</table>

Fig. 11-h.--The Disjoint Family $p^{15}$ of 3 $[1^5]$ Time Patterns $p_1^{15}, p_2^{15}, p_3^{15}$

Originally Proposed by Minuteman for Lunch
The fourth step was to critique the proposed time pattern families as a system. The families $P_{45}^4$ and $P_{23}^2$ are flawed by their non-disjointness. In the case of $P_{45}^4$ (for academics), the understanding was that $P_{25}^1 = 'MTWRF1-4'$ would be unused for C/H and that these instructors would hold their [2\textsuperscript{5}] discipline planning during $P_{25}^2 = 'MTWRF1-2'$ to mesh with $P_{45}^1 = 'MTWRF3-6'$; conversely $P_{25}^2 = 'MTWRF3-6'$ would be unused for M/S, and the [2\textsuperscript{5}] M/S discipline planning would run $P_{25}^1 = 'MTWRF5-6'$ to mesh with $P_{45}^2 = 'MTWRF1-4'$. This arrangement allowed four disjoint sections of each academic subject-pair, and seemed to pose no major problem beyond that of restricting room interchangeability during the first six periods.

A more serious problem resulted from the non-disjoint nature of $P_{23}^2$, used for PE sections. Although the 8 $P_{23}^2$ mesh well enough with the academic $P_{45}^4$ -- and this was the major concern in their design -- they interfere with each other. Because 3 does not divide 5, 3-day straight time patterns partially conflict when imposed on the five-day cycle and this means that each of the eight PE sections double up once a week for a joint session. This in turn implies twice the student load on the instructors and facilities during these weekly joint sessions, a less than satisfactory arrangement. A further criticism of the $P_{23}^2$ family, relative to the three $P_{15}^3$ lunch times, was that 25\% of the student body would be taking PE immediately after lunch (actually a flaw in the $P_{15}^3$ family more so than in $P_{23}^2$).

The single most outstanding flaw in this original time pattern system is its lack of symmetry. This asymmetry is the root of other problems. Within the given time pattern families, students taking the vocational subjects in the morning sections must take their PE/IAT either 9-10 or
11-12 (because the morning sections of PE/IAT would conflict with the vocational class). To achieve equitable section balance, this would mean that 25% of the student body (half of the 50% taking morning vocational sections) must have lunch 11th or 12th period after PE and another 25% must have lunch 9th or 10th period before PE. The problem shows up when we consider the other half of the student body taking afternoon vocational sections: since their PE/IAT must be taken in the morning, with their academics running through the 10th period, all must have lunch 11th or 12th period, and the net result is 75% of the total student body taking lunch 11th or 12th period with only 25% assigned lunch 9th or 10th period.

To obtain a better lunch balance it would be necessary to imbalance PE/IAT and/or vocational sections. This problem can be traced to the asymmetry of the time pattern families.

It is a frequently encountered irony in academic institutions that the worst bottlenecks can occur in other than the highest priority sectors: (without intending to slight the value of physical education) it hardly seems appropriate that an entire school schedule be built around an accommodation of PE. For that matter, it seems disproportionate that any single subject area determine the overall schedule of a school, yet this so often happens. (One school that comes to mind had pre-arranged a popular elective at another institution, forcing a contortion of the remainder of the schedule to accommodate the one subject.) At Minuteman, it became clear that PE was going to be a headache; note that its \( CR = 6 \) single-handedly prevents further reduction of the \( CR \) ratio 40>20>10>6>5.
11.3.1 A Revised Time Pattern System for Minuteman

The fifth step was to see whether or not PE could be better handled. Two-day time pattern structures satisfying the \( CR=6 \) were investigated (straight \( 3^2 \) time patterns, and non-straight \( 4^2 \) structures), but mesh with the \( 4^2 \) academics suffered. Alternate configurations for the academics were evaluated and rejected. One suggestion was to distribute the PE requirement unevenly over a student's four years: more some years in order to have less in others. This is a sensitive type of decision, in that a school cannot easily reverse itself once started down such a path. Other vocational schools were known to have done this, but for Minuteman this would have only made the situation worse in the heavy years. It was brought to our attention that some vocational schools avoided similar problems only by alternating "all vocational" with "all academic/PE" days or cycles, or even by dedicating entire terms to one or the other alternative. Finally the question was asked in April 1974 whether or not Minuteman could operate on a two-day cycle, with PE every other day. In practice, this would be tantamount to a ten-day cycle, with PE meeting 5 times (200 minutes) each two weeks (an average of only 100 minutes per week) rather than 6 times (240 minutes, an average of 120 minutes per week). Given the advantages (discussed presently) of such a compromise, the question was taken seriously and the happy answer was that this would be a legitimate arrangement. The revised \( CR_i \) and corresponding \( TS_i \) for the new two-day cycle are summarized in figure 11-i.
The CR<sub>i</sub> as adopted fall into the ratio 8:4:2:1. Given the two-day cycle and the TS<sub>i</sub>, we are very close to our ideal normative model, but fail to completely attain it because the 20 standard periods do not represent a power of 2.

The sixth step was to introduce as much symmetry as possible into a revised time pattern system. Four, rather than three, lunch options were established (the school intended to do this anyway starting in the second year of operation; advantages of symmetry were demonstrated for starting all four immediately). Academic discipline planning was reconfigured such that one discipline (M/S) would meet mid-morning, and the other (C/H) mid-afternoon. PE classes no longer were restricted to the first twelve periods: they now ran throughout the day, although half still fell near lunch -- note, however, that no student need take PE immediately before or after lunch, as a result of adding the fourth lunch option. The resulting five time pattern families \( (M^8, M^4, M^2, M^1, M^0) \) were adopted in April 1974.
by Minuteman, and are shown in figures 11-J through 11-N. They are quite symmetric (relative to the two days, and the two morning/afternoon blocks of 10 modules each). The two alternating days are named 'A' and 'B', but have also been labeled 'M-W-F-t-r-' and '-T-R-m-w-f' in the figures to reflect the de facto 10-day operating cycle. The 21st and 22nd periods remain in special status, for the sole use of the [2^2] cluster planning session attended by all instructors. Note that this cycle would further collapse to only one day, were it not for the every-other-day [2^1] PE sections, and would further simplify to ten standard 40-minute modules, were it not for the 20-minute lunch. Figure 11-O illustrates four (of the eight) prototype student schedules under the adopted system.
<p>| A='M-W-F-t-r-' | B='T-R-m-w-f' |</p>
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**Fig. 11-j**

The Disjoint Family $M^2$ of $2 \{8^2\}$ Time Patterns $m_1, m_2, \ldots$ Ultimately Adopted by Minuteman for Vocational Subjects

**Fig. 11-k**

The Non-Disjoint Family $M^2$ of $6 \{4^2\}$ Time Patterns $m_1, m_2, \ldots, m_6$ Ultimately Adopted by Minuteman for Academic Subject-Pairs
<table>
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**Fig. 11-l**

Fig. 11-l.—The Disjoint Family \(M^{22}\) of 3 \([2^3]\) Time Patterns \(m_1^{22}, m_2^{22}, m_3^{22}\) Ultimately Adopted by Minuteman for Major Planning Sessions

**Fig. 11-m**

Fig. 11-m.—The Disjoint Family \(M^{21}\) of 8 \([2^4]\) Time Patterns \(m_1^{21}, m_2^{21}, \ldots, m_8^{21}\) Ultimately Adopted by Minuteman for PE Sections
\[ A = 'M-W-F-t-r-' \quad B = 'T-R-m-w-f' \]

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**Fig. 11-n.**—The Disjoint Family \( M^{12} \) of 4 \([1^2]\) Time Patterns \( m_1^{12}, m_2^{12}, m_3^{12}, m_4^{12} \) Ultimately Adopted by Minuteman for Lunch
Fig. 11-o.—Four (of Eight) Prototype Student Schedules Under the Ultimately Adopted Minuteman System
11.4.1 A Critique of the Time Pattern System Adopted by Minuteman

The seventh, and final, step at Minuteman was to critique the adopted time pattern families, such that both the strengths and the weaknesses of the system could be fully appreciated by all concerned. While not theoretically ideal, these five time pattern families are very well behaved for Minuteman's purposes.

The asymmetry that flawed the original system has been replaced with a rich symmetry in the adopted system. The school could literally get by with eight "rubber stamps" for all of their student schedules: the four prototype schedules shown in figure 11-o, plus four others obtained by interchanging PE and related IAT on A and B days. More than merely being aesthetically pleasing, this symmetry can be usefully exploited through a number of options. For example: the PE classes could now be segregated by sex with, say, four A-day and two B-day sections for boys and two B-day sections for girls; or the corresponding IAT holes (complementing the PE in a one-to-one manner) could be populated under some equitable distribution such as advanced students on A-day and others on B-day -- in either of these two hypothetical cases, morning versus afternoon vocational section balance need not be affected. While there is a rapidly reached limit on the number of such options which can be simultaneously exercised, the point is that there is flexibility for them in the system in the first place. Even if not immediately exploited, there are fundamental advantages for innovation in such flexibility, and it is a fortunate scheduling officer who has access to such options.

The Minuteman schedule was simplified by the fact that only first-year students (25% of the ultimate student body after four years) would
place demands on the initial schedule, but the school was wise to consider the impact of future years during time pattern design. Whereas faculty increase in step with student population, the physical plant changes very little (aside from increased equipment). Although the classes "rattle around" in the rooms the first few years -- one reason why rooms were pretty much taken from granted during first year simulation -- it was necessary from the very beginning to not underestimate the room sector, particularly the PE facilities. Note that the eight \([2^1]\) time patterns accommodating PE (disjointly) cover only 16 of the 40 standard cycle modules; the remaining \(24/40=6-\%\) of the cycle can still be (disjointly) covered by twelve additional \([2^1]\) time patterns congruent to the original \(m^{21}_L\) in a well-behaved and symmetric extension of \(H^{21}\) to eventually accommodate upperclass PE.

By staying within the original \([4^2]\) time patterns when scheduling upperclass academics, the school's two (rather different and almost conflicting) objectives of retaining discipline planning for instructors across all years and at the same time opening up new PE (and corresponding IAT) time patterns can both be achieved given that formal academic contact time can be traded for increased IAT, a tradeoff which, fortunately, the school actively seeks. In short: the first year time pattern families make sense relative to subsequent years of operation; this is an immensely valuable asset and a tribute to the farsightedness of the school.

On the more negative side, it must be mentioned that student schedules under this time pattern system are basically full schedules, with no more capacity for any other formal classes. Electives, in the usual sense of the word, such as: band, chorus, orchestra, debate, drama,
driver training, etc., are conspicuously absent from the formal curriculum of this school. It would be impossible to choose an ideal time for any such elective, during the standard school day, which would not bias the rest of the curriculum. This is particularly the case should more than one elective choice be offered to students, and the difficulty is even further aggravated by the characteristic of most electives that they span a cross-section of student years and categories (upperclass as well as first-year students, advanced and slower-paced, etc.). Running a popular elective in the morning could have a direct impact on the section balance of other morning classes. In this regard, the prevailing symmetry of the system could start to wield leverage against the best interests of the school.

It is essential to understand that the symmetric advantages of the Minuteman time pattern system are intimately wedded to the expected balance in student programs. An equal number of students was expected to end up in each of the 12 first year vocational options; substantial deviation from this projected equality among vocational subjects would have repercussions throughout the entire curriculum -- academics and hence PE/IAT -- due precisely to the underlying symmetry of the schedule and the school's desire to aggregate academic classes by like vocational groupings.

The potential problem with this schedule is, ironically, that the beautiful symmetry can turn against the school if the projected balance, upon which the time pattern system was postulated, fails to materialize. The point is that this schedule has little slack. In one sense this is a tribute to system design: form follows function; in another sense it is a potential long range flaw, in that if function changes so must form. On
the whole the plusses of this time pattern system -- for this school -- probably outway any minuses.

11.5.1 Summary of Steps in the Minuteman Case Study

The following is a summary of the seven steps followed to reach the ultimately adopted time pattern system for Minuteman:

(1) Determination of the intended contact requirement (CR) profile, including grouping of subjects where possible.

(2) Consideration of the impact of the CR on the resources: instructors, rooms, and students.

(3) Enumeration of all time patterns which could produce schedules.

(4) Development of a critique for the proposed time pattern system.

(5) Investigation of better ways to handle PE, a particular bottleneck for Minuteman.

(6) Introduction of as much symmetry as possible into a revised time pattern system.

(7) Appreciation of the strengths and weaknesses of the adopted time pattern system, via a critique of the system.

These seven steps are typical of the iterative process most schools would encounter in performing a good time pattern design; note the compromise which took place in the fifth step as a result of the critique at the fourth step. The critique in the seventh step could very well have exposed new aspects of the scheduling problem peculiar to Minuteman, with further compromises and redesigns as a result. In fact, a number of successive approximations were made at every step, and a number of false starts discarded throughout the process. Convergence, the ultimate and
elusive objective of any process involving successive approximation, attended this time pattern design process because of the increasing education obtained about the systems nature of this particular school. It is essential that a school be understood in terms of its "pressure points" where compromise will afford leverage towards obtaining a schedule; it is now a moot point whether or not compromise on the PE contact requirement was necessary for Minuteman -- the important lesson is that said compromise led demonstrably to a satisfactory schedule.
12.1.1 A Critique of the Historical M.I.T. Time Pattern System

The case study at M.I.T. began with a critique of the existing historical time pattern system. It is important to understand that M.I.T. has a complex scheduling environment; the following are significant:

(a) M.I.T. allows the student body, undergraduates as well as graduates, a great deal of elective freedom. Subjects in one department almost always draw cross-registration from other departments. Very few students take exactly the same subjects. It is practically the case that given any two subjects, where one is not a prerequisite or basis of the other, there is a significant possibility of some student taking both; this is increasingly true of undergraduates taking graduate level subjects, and even occasionally true in the case of prerequisites.

(b) A broad flexibility is offered faculty in choosing a time pattern structure to satisfy any given contact requirement (CR); for example, with M.I.T.'s half-hour modules, three contact hours can be structured [6], [4], [3], or [2]. Occasionally the same 3-hour subject will have multiple sections representing two or three different TS's, (as when a humanities subject's sections meet: 'MWF10', 'TR10-11:30', 'W EVE', etc.).

(c) A student is given multiple opportunities to change his or her program before, during and after the official registration day procedure; it is not mandatory for graduate students to pre-register subjects at all,
as many as 60 percent of the underclass students have been known to alter their schedules during registration day, and changes to a program are usually accepted through the 15th week of a term.

(d) A significant percentage (over 20 percent) of the 2000 class sections held each term are listed in the *class schedules booklet* (CSB) as "to be arranged", meaning that a workable time pattern must be adopted, after the normal scheduling and registration process, to the mutual satisfaction of the instructor(s) and students involved, usually along with an a posteriori room assignment.

(e) For any number of reasons, including unexpected shifts in registration and/or gross section imbalance, class sections are frequently added, merged, or simply cancelled throughout the ongoing term.

For such reasons as the five foregoing, time patterns chosen for M.I.T. classes must be judged not only on the soundness of the schedule originally published in the class schedule booklet (CSB), but also in terms of subsequent response to the many perturbations inevitably encountered during the term. The time pattern assigned to a given class must therefore make sense not only under conditions of projected enrollment but also with respect to dynamically changing reality. What has been referred to in this thesis as a priori flexibility -- the resilience of a schedule to perturbations -- is a major issue at M.I.T., and therefore a major objective of the M.I.T. scheduling process. Accordingly, the primary criticism of the historical time pattern system at M.I.T. is that it does not enhance a priori flexibility as well as it might.

A first-year student entering M.I.T. in 1960 fell 'nto one of 35
prototype sections: all required subjects were attended with the same
group of schoolmates; attending section $i$ of calculus meant also attending
section $i$ of chemistry, physics, etc., and the 35 section combinations were
each designed to be conflict free. Saturday classes meant that $[2^3]$
subjects could meet either 'MWF' or 'TRS', and the dozen or so "freshmen
electives" all fit by definition into a dedicated 'MW9' time pattern
during which no required subjects ran. With the advent of computer-assisted
scheduling, and some major changes in scheduling philosophy, Saturday
Four-hour $[4^2]$ subjects became popular with some departments (e.g.
economics). A de facto time pattern system evolved more or less on its
own: certain contiguous day-combinations were avoided because they didn't
spread out over the week, and time patterns were as a rule straight, but
within fairly broad guidelines, almost any meaningful time pattern was
allowed in the historical system. Figures 12-a through 12-c attempt to
formalize the de facto historical time pattern system followed by M.I.T.
in recent years. The official cycle is a calendar week of 5 days (named
M,T,W,R,F) by 16 half-hour modules (starting at 9,9:30,10,...,4,4:30); a
number of evening classes (module name EVE) are held, but otherwise classes
are supposed to fall between 9 and 5. At M.I.T., the dash in time pattern
names is exclusive since clock times (rather than period numbers) are used.
These figures do not attempt pictorial representation of historical time
pattern families, since they are highly non-disjoint, and overlapping time
patterns can be confusing when pictured. Instead, figures 12-a and 12-b
document admissible day- and module-combinations, and figure 12-c uses
cross-product notation to enumerate the three principal time pattern
Fig. 12-a.--Primary Sets $D_i$ (and Secondary Sets $D'_i$) of i-day Day-Combinations Historically Used in M.I.T. Time Pattern Names

$M_2 = \{9,10,11,12,1,2,3,4\}$

$M_3 = \{9-10:30,9:30-11,10-11:30,10:30-12,11-12:30,$
$\quad 11:30-1,12-1:30,12:30-2,1-2:30,1:30-3,$
$\quad 2-3:30,2:30-4,3-4:30,3:30-5\}$

$M_4 = \{9-11,10-12,11-1,12-2,1-3,2-4,3-5\}$

Fig. 12-b.--Primary Sets $M_i$ of i-module Module-Combinations Historically Used in M.I.T. Time Pattern Names

$H^{42} = D_2 \circ M_4 = \{'MW9-11', 'MW10-12', \ldots, 'WF3-5'\}$

$H^{32} = D_2 \circ M_3 = \{'MW9-10:30', 'MW9:30-11', \ldots, 'WF3:30-5'\}$

$H^{23} = D_3 \circ M_2 = \{'MTR9', 'MTR10', \ldots, 'TRF4'\}$

Fig. 12-c.--Principal Families $H^{42}, H^{32}, H^{23}$ (of $[4^2], [3^2], [2^3]$ Time Patterns respectively) Historically Used at M.I.T.
families in the historical system.

Evaluating the historical M.I.T. time pattern system involves understanding the a priori conflict behavior of the three principal time pattern families ($H_{42}^2, H_{32}^3, H_{23}^4$), both within each family and with respect to each other. Figure 12-d tabulates the number of days in common between all possible pairs of day-combinations chosen from \( D_2 \cup D_3 \). Figure 12-e tabulates the number of modules in common between all possible pairs of module-combinations chosen from \( M_2 \cup M_3 \cup M_4 \). Figure 12-f summarizes the probabilities of a priori conflict within and between the three families $H_{42}^2$, $H_{32}^3$ and $H_{23}^4$; the calculations used to derive these probabilities are shown in figure 12-g.

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Fig. 12-d.--Matrix of Days in Common, Pairwise between Day-Combinations of \( D_2 \cup D_3 \)
Fig. 12-e.--Matrix of Modules in Common, Pairwise between Module-Combinations of \{M_2\cup M_3\cup M_4\}
Fig. 12-f. -- Matrix of A Priori Conflict Probabilities Within and Between $H_{42}, H_{32}, H_{23}$

$$
\begin{align*}
H_{42} & \quad \frac{285}{1225} = 23.3\% \\
H_{32} & \quad \frac{570}{2450} = 23.3\% \\
H_{23} & \quad \frac{280}{1400} = 20\%
\end{align*}
$$

$$
\begin{align*}
H_{42} & \quad \frac{285}{1225} = 23.3\% \\
H_{32} & \quad \frac{960}{4900} = 19.6\% \\
H_{23} & \quad \frac{560}{2800} = 20\%
\end{align*}
$$

$$\begin{array}{|c|c|c|c|}
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& H_{42} & H_{32} & H_{23} \\
\hline
H_{42} & 10 & 10 & 5 \\
& \frac{25}{25} & \frac{25}{25} & \frac{25}{25} \\
\hline
H_{32} & 10 & 10 & 5 \\
& \frac{25}{25} & \frac{25}{25} & \frac{25}{25} \\
\hline
H_{23} & 5 & 5 & 10 \\
& \frac{25}{25} & \frac{25}{25} & \frac{25}{25} \\
\hline
\end{array}
$$

Fig. 12-g. -- Calculations Supporting Figure 12-f
To appreciate just how poorly $h_{23}^3$ behaves with respect to $h_{42}^2$ and $h_{32}^3$, consider the worst case resource schedule shown in figure 12-$h$, whereby the maximum of 8 $[2^3]$ classes has been achieved at the expense of so fragmenting the unassigned remainder of the cycle as to be unusable for any further assignments from any of the three principal families. This means that only 60 percent of the cycle is covered, while 40 percent becomes mandatory free time. In the room sector, this would result in particularly poor resource utilization.

![Figure 12-h](image-url)

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Fig. 12-h.--A Worst-Case Fragmentation of an Individual Resource Schedule using 8 Time Patterns chosen from $h_{23}^3$

12.2.1 Contact Requirements and Time Pattern Structures at M.I.T.

The second step at M.I.T. was an attempt to better understand the contact requirement (CR) and time pattern (TS) profiles determined by the rooms and students. The profiles of the instructor sector were ignored for two persuasive reasons: (1) the teaching load at M.I.T. is very light
relative to the cycle (only one or two classes), and therefore no matter what reasonable combinations of time pattern structures \( (TS_i) \) occur, instructor conflicts should be negligible; and (2) very little information on the instructor sector was available: none in machine-readable form, and the scant M.I.T. catalogue data is inadequate in dealing with multiple section subjects (since only the person nominally in charge is listed therein).

Two computer applications were developed. The first transforms an input deck of class schedules booklet (CSB) cards into an output deck of generic CSB cards, whereon the time pattern name is mapped back into an appropriate two character code 'xy' reflecting the time pattern structure \([xy]\) of the family to which the time pattern belongs. (During this process, room assignments are also mapped back into general pools, but this is only incidental to the current discussion.) The output of this first application -- a generic CSB deck relating each of M.I.T.'s 2000 classes to its assigned \( TS_i \), -- can then be read as input by the second application, together with an input file enumerating assigned classes by rooms and students. By storing the \( TS_i \) in a table by class, this application can then scan the class assignments file and produce an output file which enumerates \( TS \) combinations for each room and student. This output file can be grouped (through sorting) by \( TS \) combination, and subsequently used as input to a statistics program which tallies and reports the \( TS \) profile. These statistics can be broken down by student year (sophomores, graduates, etc.), department, etc.

These two applications were run against Spring Term 1973-74 data, and the second application was originally programmed to distinguish among
literally all possible $TS$ combinations. The resulting student statistics, broken down by student year, showed no meaningful pattern whatsoever. No significant percentages of the student body clustered around any particular combinations of $TS_i$, but instead the students spanned a wide variety of different combinatorial arrangements. In the hope that some sort of pattern might emerge, the second application was re-programmed to collapse multiple instances of any given $TS_i$, into a binary "present" versus "not-present" indication (rather than keep track of cardinality). However, the range of $TS$ combinations, even under this equivalence classing, still did not demonstrate significant groupings in the $TS$ profile. The conclusion drawn from this effectively random spectrum is simply a reinforcement of the need for a priori flexibility at M.I.T.: given any combination of $TS_i$, already present in a student's schedule, there appears to be no predictable pattern to those additional $TS_j$, which might be further assigned. In short: each time pattern family at M.I.T. should interact well both within itself and with respect to any other time pattern family.

Figure 12-i shows the range of $CR_i$ and $TS_i$, predominantly used at M.I.T. There are also a few irregular classes (e.g. $7^2$, $5^2$, $5^1$), but these are anomalies of little significance. A count is also shown of the number of classes using each $TS_i$, during the Spring Term 1973-74. "To be arranged" classes were not included in the data.
<table>
<thead>
<tr>
<th>CR&lt;sub&gt;i&lt;/sub&gt; (in terms of half-hour modules)</th>
<th>TS&lt;sub&gt;i&lt;/sub&gt;</th>
<th>Number of M.I.T. Classes using TS&lt;sub&gt;i&lt;/sub&gt; in Spring Term 1973-74</th>
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<td>EVE (2 nights)</td>
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<td>EVE (1 night)</td>
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Fig. 12-i.--Range of CR<sub>i</sub> and TS<sub>i</sub>, Predominantly Used at M.I.T.
12.3.1 A Simplified Time Pattern System for M.I.T.

The third step in the M.I.T. case study was to propose and critique a simplified time pattern system within the historical framework (i.e. the simplified families would be subsets of the historical families) which would enhance a priori flexibility. The approach was to substantially prune the principal families \( H_{42}, H_{32}, H_{23} \) in order to achieve disjointness, even at the expense of coverage (although this too remained as a secondary objective). Day-combinations and module-combinations were both severely restricted to achieve this; figure 12-j shows the basic sets \( D_i \) (of \( i \)-day day-combinations) and \( M_j \) (of \( j \)-module module-combinations) underlying the three simplified time pattern families. These three simplified families \( S_{42}^2, S_{32}, S_{23}^2 \) are shown in figures 12-k through 12-m both in cross-product notation and (since disjoint) via pictorial representation.

\[
D_2 = \{TR, WF\} \quad |D_2| = 2 \\
D_3 = \{MWF\} \quad |D_3| = 1 \\
M_2 = M_2 = \{9, 10, 11, 12, 1, 2, 3, 4\} \quad |M_2| = 8 \\
M_3 = \{9:30-11, 11:30-1, 1:30-3, 3:30-5\} \quad |M_3| = 4 \\
M_4 = \{9-11, 11-1, 1-3, 3-5\} \quad |M_4| = 4
\]

**Fig. 12-j.--The Basic Sets** \( D_i \) (of \( i \)-day Day-Combinations) and \( M_j \) (of \( j \)-module Module-Combinations) used in the Simplified M.I.T. Time Pattern Families \( S_{42}^2, S_{32}, S_{23}^2 \).
\[ S^{42} = D_2^b \otimes M_2^b = \{ 'TR9-11', 'TR11-1', \ldots, 'WF3-5' \} \]
\[ S^{32} = D_2^b \otimes M_3^b = \{ 'TR9:30-11', 'TR11:30-1', \ldots, 'WF3:30-5' \} \]
\[ |S^{42}| = |D_2^b| \times |M_2^b| = 2 \times 4 = 8 \]
\[ |S^{32}| = |D_2^b| \times |M_3^b| = 2 \times 4 = 8 \]

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Fig. 12-k

Fig. 12-k.--A Simplified Family \( S^{42} \text{CH}^{42} \) of 8[4\(^2\)] Time Patterns \( s_1^{42}, s_2^{42}, \ldots, s_8^{42} \) Considered for M.I.T.

Fig. 12-l

Fig. 12-l.--A Simplified Family \( S^{32} \text{CH}^{32} \) of 8[3\(^2\)] Time Patterns \( s_1^{32}, s_2^{32}, \ldots, s_8^{32} \) Considered for M.I.T.
$S^{23} = D^b_3 \circ M^b_2 = \{ 'MWF9', 'MWF10', \ldots, 'MWF4' \}$

$|S^{23}| = |D^b_3| \times |M^b_2| = 1 \times 8 = 8$

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Fig. 12-m. -- A Simplified Family $S^{23} \subset R^{23}$ of 8 $[2^8]$ Time Patterns
$s_1, s_2, \ldots, s_8$ Considered for M.I.T.
The probabilities of a priori conflict -- a constant $1/8 = 12.5\%$ in every case -- within and between the three simplified families $S^{42}$, $S^{32}$ and $S^{23}$ are shown in figure 12-n.

\[
\begin{array}{ccc}
S^{42} & S^{32} & S^{23} \\
S^{42} & \frac{1}{8} = 12.5\% & \frac{1}{8} = 12.5\% & \frac{1}{8} = 12.5\% \\
S^{32} & \frac{1}{8} = 12.5\% & \frac{1}{8} = 12.5\% & \frac{1}{8} = 12.5\% \\
S^{23} & \frac{1}{8} = 12.5\% & \frac{1}{8} = 12.5\% & \frac{1}{8} = 12.5\%
\end{array}
\]

Fig. 12-n. -- Matrix of A Priori Conflict Probabilities Within and Between $S^{42}, S^{32}, S^{23}$

The a priori conflict situation using $S^{42}$, $S^{32}$, and $S^{23}$ is considerably better than the corresponding probabilities for $H^{42}$, $H^{32}$ and $H^{23}$ (compare figure 12-f); in only one case do the historical families perform as well as the simplified families ($H^{23}$ and $S^{23}$ each conflict within themselves with probability $1/8 = 12.5\%$), but there are at least 50% more opportunities for conflict in all other historical cases, and almost twice as many in several cases (e.g. within $H^{42}$).

An astounding realization is that the historical families suffer poorer a priori behavior without offering very much in return: in terms of disjoint coverage, $S^{42}$ and $S^{23}$ are the equals of $H^{42}$ and $H^{23}$, and $S^{32}$ is only slightly inferior to $H^{32}$. Despite the higher cardinality of the historical families, they foster so many partial conflicts that their disjoint coverage is little better than that of the (already disjoint)
simplified families. Of the $35 \pi^4_{22} \in \pi^4_{42}$, a maximum of 8 can be assigned to a resource without conflict; although this maximum can be reached in different ways, the subset $S^4_{42}$ (uniquely) achieves the same maximum with 27 fewer noise time patterns. Likewise, $S^2_{23}$ (uniquely) achieves the maximum of 8 conflict-free assignments with 32 fewer noise time patterns than $H^2_{23}$, which cannot do better (although $H^2_{23}$ can achieve this maximum of 8 in different ways). The only sacrifice encountered in the simplified families occurs in pruning $H^3_{32}$ to arrive at $S^3_{32}$ -- and here the compromise is deliberate and voluntary, for reasons of mesh.

The 70 $h^3_{32}$ partially conflict with each other in a particularly unproductive manner. Given an 8-hour (16-half-hour) day, only 5 disjoint $[3^2]$ time patterns can be accommodated by any one day, and accordingly $H^3_{32}$ permits a maximum of 10 $[3^2]$ classes to be assigned without conflict to any one resource (albeit in many ways). One can do just as well by using $D^5_{2b}$ in cross-product with any 5 disjoint 3-module combinations. While $S^3_{32}$ actually achieves only 8 $[3^2]$ disjoint classes as a maximum, in the process it obtains a much better mesh with the $[4^2]$ classes than would be possible with 10 disjoint $[3^2]$ time patterns: it is preferable that $S^4_{42}$ dominate $S^3_{32}$. In any case, it is easy to part with 60 of the 70 $h^3_{32}$ as being noisy without even looking outside the $[3^2]$ structure.

To illustrate the importance of synchronizing day- and module-combinations throughout the Institute, consider the possibility of one third of the departments starting $[3^2]$ classes at 9:00 and every 3 half-hours thereafter, a second third of the departments using 9:30 (and every 3 half-hours thereafter), while the final third uses 10:00, 11:30, etc. Even though each department might be internally content with its own set of
ten disjoint time patterns, a student would find that most of a department's $[3^2]$ classes preempt not merely one but two $[3^2]$ classes in the majority of other departments. This unpleasant situation would be further compounded if the departments fail to synchronize day-combinations.

The $8$ time patterns in $S^{42}$ and $S^{32}$ are each totally interchangeable with respect to the other family; the one-to-one containment $s^{32}_i \leq s^{42}_i$ means not only that $S^{42}$ dominates $S^{32}$, but also that the two families are effectively equivalent in their impact on the cycle (since M.I.T. does not run classes of only one half-hour duration). The use of one or more $S^{32}_i$ will always result in a corresponding number of permanent $[1^2]$ holes in a resource schedule, but it is important to realize that $[3^2]$ holes preempt at least one $[4^2]$ time pattern in the presence of $[4^2]$ classes, and that $[1^2]$ holes are therefore a price to pay to avoid more serious conflict behavior. Such half-hour gaps in M.I.T. schedules are not generally regarded as a flaw with respect to students, and actually serve a useful purpose in those room schedules where it is desirable that setup time be provided (e.g. before physics lectures).

The $8$ time patterns in $S^{23}$ are not interchangeable with respect to the other two families (and vice versa) but rather $S^{23}$ partitions each of the two other families into two sub-families such that the $8$ $S^{23}_i$ are totally interchangeable with the $4$ time patterns in each sub-family (and vice versa). Because $S^{23}$ is restricted to MWF, the $4$ TR members of $S^{42}$ or $S^{32}$ form a sub-family which in no way conflicts with any $s^{23}_i$, and the $4$ WF members form the other sub-family, of which each member conflicts with exactly two $S^{23}_i$. The behavior of these families and sub-families is similar to that discussed in normative model #4, wherein it was necessary
to configure \([1^2]\) and \([1^3]\) time patterns over a 5-day 8-period cycle; there are parallels between the design of \(S^{32}\) given \(S^{23}\) and the design of \(C^{12} = A^{12} + B^{12}\) given \(A^{13}\). Note that Monday was singled out in the M.I.T. case as being the day to avoid for two-day classes; since U.S. holidays tend to fall on Mondays, this means that two-day classes need not be impacted, while three-day classes each lose a third of that week's contact. Faculty (and rooms) associated exclusively with two-day classes have Monday off, and several M.I.T. departments actively seek such a four-day week for their faculty.

There is a congruent block structure within which the three simplified families fall: the week can be partitioned into 4 blocks of 2 hours (4 half-hours) yielding the four time pattern blocks 'MTWRF9-11', 'MTWRF11-1', 'MTWRF1-3', and 'MTWRF3-5'. Within each (interchangeable) block, the following partitions of the twenty modules are attainable given \(S^{42}\), \(S^{32}\) and \(S^{23}\):

- 2 \([4^2]\) classes (leaving a \([4^1]\) hole);
- a \([4^2]\) class + a \([3^2]\) class (leaving a \([4^1]\) and a \([1^2]\) hole);
- a \([4^2]\) class + 2 \([2^3]\) classes;
- 2 \([3^2]\) classes (leaving a \([4^1]\) and a \([1^4]\) hole);
- a \([3^2]\) class + 2 \([2^3]\) classes (leaving a \([1^2]\) hole).

Noting the shape of any holes left, one soon realizes that the historical families \((H^{42}, H^{32}, H^{23})\) add no further partitions (although they do increase the number of compositions, and they can partition a block of thirty modules somewhat differently, due to denser coverage by \([3^2]\) time patterns; in the case of thirty, however, the \([4^2]\) time patterns are less useful, and by going to partitions of sixty modules as a compensation, advantages of
block structure are abandoned in the process).

Finally, note that obstructive fragmentation, such as was illustrated in figure 12-1, is simply not possible given the three simplified families. If there is a \([2^3]\) hole in a resource schedule, there is also a \(s_i^{23}\) to fit it. If there is a \([4^2]\) hole, one \(s_j^{42}\) and one (corresponding) \(s_j^{32}\) will each fit it. It is true that \([1^2]\) holes are permanent, but this has already been justified; the \([1^4]\) holes are an unfortunate by-product of the \(TS_i\), themselves, and the particular families cannot really be blamed. Avoidance of obstructive fragmentation is particularly important with respect to the M.I.T. room sector, since most rooms fall into interchangeable pools (grouped mainly by location, size, and equipment) within which any member can serve a number of classes. Rooms -- and resources in general -- cannot support classes assigned time patterns that do not coincide with holes in their evolving resource schedule, and this fact alone can reduce the utility of an otherwise perfectly acceptable resource.

12.4.1 Alternative Time Pattern Systems Outside the Historical M.I.T. Framework

The fourth step in the M.I.T. case study was to consider time pattern systems that fall outside the historical framework -- to see if advantages beyond those of the simplified system might be gained through introduction of innovation in the time pattern approach. Two hypothetical systems were considered. The first of these systems involves expanding the cycle from 40 to 45 hours by extension of the school day (a half-hour at each end) to 9 rather than 8 hours, permitting 6 (rather than 5) possible \([3^2]\) time patterns on a day and supporting a congruent rectangular block
structure of three [6] blocks. The other hypothetical system retains a 40 hour cycle, but reconfigures it into a four day week: under this arrangement M.I.T. would run ten hours (20 half-hour periods) on each of four calendar days. Neither of these two hypothetical systems was regarded as frivolous: neither would significantly affect instructor or student load, but either could have dramatic impact on the room sector. Given the growing concern at M.I.T. over effective room utilization, particularly in view of a possible energy crisis, these two alternatives hold a great deal of interest in a number of M.I.T. sectors, since the first alternative might require fewer rooms, while the second would eliminate one entire day of room use each week. Furthermore, similar alternatives have been tried for one reason or another at other institutions. Accordingly, the case study included the design of time pattern systems, and critiques thereof, for these two alternatives.

Considering the 5-day 9-hour (18-half-hour) cycle first, it became evident that not a great deal of additional a priori flexibility would be introduced beyond that of the simplified families, thus theoretically lessening the chances for fewer rooms. Assuming the desirability of the [4] block structure, the extra hour adds very little -- two extra hours would yield a fifth interchangeable block, but just one extra hour extends only the \( S^{23} \) family (by one new time pattern \( s^{23}_9 \) at the end of the day). Note that neither \( S^{42} \) nor \( S^{32} \) can be extended, within their original design objectives, unless an entire new block of 4 half-hours is generated. The extra hour would therefore de facto apply only to three of the five days (M,W,F), and thus be limited in impact to improvement of the a priori conflict behavior within \( S^{23} \) from probabilities of 1/8 to 1/9 (not a very
impressive gain).

Instead of immediately rejecting the 5 by 18 cycle for the foregoing reason, the question was raised whether the $[4^5]$ block structure was so critical. 18 half-hours can be evenly partitioned into a congruent rectangular block structure of three $[6^5]$ time patterns: 'MTWRF8:30-11:30', 'MTWRF11:30-2:30' and 'MTWRF2:30-5:30'. A higher cardinality family $C^{32}$ of 12 $[3^5]$ time patterns formed by the cross-product of $D_2^b$ with six disjoint 3-module module-combinations is shown in figure 12-o.

$$C^{32} = D_2^b \cdot \{8:30-10,10-11:30,11:30-1,1-2:30,2:30-4,4-5:30\}$$

$$|C^{32}| = 2\times 6 = 12$$

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Fig. 12-o.--A Disjoint Family $C^{32}$ of 12 $[3^5]$ Time Patterns $a_1^{32}, a_2^{32}, \ldots, a_{12}^{32}$ over a $d=5$ (day) by $m=18$ (module) Cycle

It is a definite advantage of $C^{32}$ (of cardinality $|C^{32}|=12$) over $S^{32}$ (of cardinality $|S^{32}|=8$) that the a priori probability of conflict within the families is reduced from 1/8 to 1/12. Unfortunately, we are still stuck
with only four possible \([4^2]\) time patterns on any given day, and hence with something like \(S^{42}\) for a family. And \([4^2]\) time patterns mesh very poorly with \(C^{32}\) (recall the previous discussion as to why 10 disjoint \([3^2]\) time patterns were rejected in favor of only 8 in designing \(S^{32}\)).

Using \(C^{32}\) would also cost us interchangeability when it comes to \(S^{23}\) (as realigned on half-hour boundaries and extended by a ninth \(s^{23}_9\)): six \(s^{23}_i\) \((i=1,3,4,6,7,9)\) would conflict with exactly one \(c^{32}_j\), but three \(s^{23}_i\) \((i=2,5,8)\) would conflict with two \(c^{32}_j\). For the foregoing reasons, the theoretical conclusion was that M.I.T. would be little better off with an extra hour that would be hard to capitalize upon, than with the simplified families and an 8-hour day.

The second hypothetical system -- the four day week of 20 half-hour daily modules -- represents a major philosophical change from any of the other systems considered for M.I.T. It offers distinct scheduling advantages over these other systems, including the simplified historical one, but only in the presence of compromise. The critical compromise, required if the four day cycle is to enjoy its fullest flexibility, is the elimination of 3-day time patterns. The assumption is that, should M.I.T. adopt a four day week, it would be willing to constrain class sections to one, two or four days -- but not three. Two comments are called for immediately: (1) this does not mean that a subject cannot meet three times a week, only that its phases (lectures, laboratories, recitations, etc.) are so restricted; and, (2) the M.I.T. administrators with whom this prospect was raised allowed that such a restriction would be negligibly small in contrast to the general upheaval attending the changeover to a four-day operation, and that this was therefore not an unreasonable assumption in context.
For purposes of the following discussion, the four days will be named T, W, R, F and the twenty half-hour periods will be assumed to start at 8, 8:30, 9, ..., 5 and 5:30. Three families, $F^{44}$ (of 5 ideal $[4^4]$ time patterns establishing a congruent rectangular block structure), $F^{42}$ (of 10 ideal $[4^2]$ time patterns contained in the $F^{44}$ blocks), and $F^{32}$ (of 10 disjoint $[3^2]$ time patterns contained in the $F^{44}$ blocks), are shown in figures 12-p through 12-r. To emphasize the similarity of this time pattern system to the ideal normative model, the days are diagrammed in the sequence TRWF.

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Fig. 12-p.—An Ideal Family $F^{44}$ of 5 $[4^4]$ Time Patterns $f_{11}, f_{12}, ..., f_{55}$ Partitioning a $d=4$(day) by $m=20$(module) Cycle into a Congruent Rectangular Block Structure
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**Fig. 12-q.**—An Ideal Family $F^{42}$ of 10 $[d^2]$ Time Patterns $f_1^{42}, f_2^{42}, ..., f_{10}^{42}$ over a $d=4$(day) by $m=20$(module) Cycle, Contained in the $F^{44}$ Blocks.

**Fig. 12-r.**—A Disjoint Family $F^{32}$ of 10 $[s^2]$ Time Patterns $f_1^{32}, f_2^{32}, ..., f_{10}^{32}$ over a $d=4$(day) by $m=20$(module) Cycle, Contained in the $F^{44}$ Blocks.
It is not merely coincidental that the two principal families $F^{42}$ and $F^{32}$ look very much like extended versions of the simplified families $S^{42} H^{42}$ and $S^{32} H^{32}$ (compare figures 12-k and 12-l). The characteristics of $F^{42}$ and $F^{32}$ which render them so well-behaved were precisely the design objectives of $S^{42}$ and $S^{32}$. The unused fifth day in $S^{42}$ and $S^{32}$ was unused because of the basic soundness of the configuration over the remaining four days; the four day week eliminates the superfluous day and trades its unused -- and unusable! -- sixteen modules for a quite useful fifth interchangeable $[4^4]$ block. The two extra hours (on each of four days) buys us what one extra hour (on each of five days) could not: a 25 percent increase in time patterns consistent with sound design objectives. In this case, no net increase in cycle cardinality is required to achieve the increase in time pattern family cardinality: "length" and "width" of a cycle are two very different dimensions because $[x^y]$ structures are almost always considered different from $[y^x]$ structures. The four day environment does, however, require that M.I.T.'s $[2^3]$ classes adopt $[3^2]$ time patterns; this is the critical compromise already mentioned.

This time pattern system falls short of the ideal normative model on only two counts: one is unavoidable and the other--in this case--actually desirable. The unavoidable shortcoming lies in the $[3^2]$ structure, 3 not being a power of 2. It is this incompatibility that leads to the $[1^2]$ holes, since dominance of $F^{32}$ by $F^{42}$ is preferable to total (supplementary) coverage by $[3^2]$ time patterns. The second deviation from the ideal normative model is that 20 daily modules do not represent a power of 2. This is not a handicap given the M.I.T. TS, because the five individual $[4^4]$ blocks $f^{44}$ each represent a useful power of 2. For a time pattern system to be well-

behaved, it is not absolutely necessary that \( m \), the number of daily modules, be a power of 2, but rather than \( m \) be an integer multiple of the largest power of 2 represented by any \( TS_i \).

While it is true that M.I.T. does require a few structures \([x^y]\) where \( x>4 \), such structures are only lightly used, and very seldom will a case occur where \( x>8 \). Because the five \([4^4]\) blocks encompass a \( d=4 \) by \( m=16 \) sub-cycle, represented by blocks \( f_1^{44} + f_2^{44} + f_4^{44} + f_5^{44} \), this sub-cycle enjoys most of the advantages of the ideal normative model. The remaining \([4^4]\) block \( f_3^{44} \) not only yields extra time patterns, but also can lessen the impact of the very few \([x^y]\) time patterns where \( x>8 \) by preventing -- at its own expense -- use of more than one of the \([8^4]\) blocks surrounding \( f_3^{44} \). While \( f_3^{44} \) would then no longer be strictly interchangeable with the other \( f_i^{44} \), this is not such a bad arrangement in view of the lunch hours it contains.

One of the most interesting observations about the four day week is that, if M.I.T. were willing to restrict \([x^y]\) structures to \( x\leq8 \), it might be possible to still retain the eight hour operating day (and not go to ten hours). Even though the cycle cardinality would drop from 40 hours to only 32, the point is that most of the a priori conflict behavior would be no worse than even the simplified historical system. This still assumes, of course, a willingness to abandon \([2^3]\) structures in favor of \([3^2]\) configurations. The acequity of the 32 hour cycle is tied directly to the superfluous nature of the fifth day in the historical system, at least with respect to the two predominant structures \([4^2]\) and \([3^2]\). The most significant problem with losing the otherwise superfluous day -- a weakness in cutting any \( d \) day cycle down to \((d-1) \) days -- is with respect to \([x^1]\) time patterns, since such families must decrease in cardinality without direct compensation.
12.5.1 A Recommended Time Pattern System for M.I.T.

Willingness to restrict M.I.T. classes to one, two, or four days, by substituting one-, two-, or four-day structures for three-day structures, permits isomorphism between a four day system, approximating the ideal normative model, and a *recommended* five day system preserving the cycle in current use. Care must always be taken with such mappings to insure that all structures participate in the isomorphism. Such considerations as preserving day separation and preserving similar clock times are important. One thing that cannot be unconditionally preserved when altering the number of days in the cycles—understandably—is the day pattern of individual resource schedules. Two- or four-day schedules in a four day cycle may have to map into three- or five-day schedules in a five day cycle.

One straightforward isomorphism between a 4(day) by 10(module) cycle, and a 5(day) by 8(module) cycle, is illustrated in figures 12-8 and 12-t. For simplicity, these figures deal with hour length modules. Figure 12-8 shows the two cycles, each divided in half by congruent "staircase shaped" blocks, assembled as if cutting and sewing together a 4'x10' carpet to fit a 5'x8' room, or vice versa. Figure 12-t shows four \([4^2]\) blocks \(B=\{b_1, b_2, b_3, b_4\}\) which should dominate both cycles; the two "staircase shaped" blocks, though technically a congruent block structure, are not really useful as time pattern blocks since \(B\) is not contained in them. Note also that the four \(b_i\) do not completely cover the cycles—the four remaining modules can only be used for \([2^2], [2^1], [1^2]\) or \([1^1]\) time patterns which are not strictly interchangeable with their counterparts contained in the blocks—and thus \(B\) is technically not a block structure.
Fig. 12-s.--An Isomorphism between a 4(day) by 10(module) Cycle, and a 5(day) by 8(module) Cycle, using Congruent "Staircase Shaped" Blocks

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Fig. 12-t.--Four \([4^2]\) Blocks \(B=\{b_1, b_2, b_3, b_4\}\) Which Should Dominate Both Cycles of Figure 12-s
Although the "staircase shaped" blocks lead to a straightforward and easily visualized isomorphism between a 4 by 10 and a 5 by 8 cycle, there is a different isomorphism which maps the two cycles in a way better suited to M.I.T. This recommended isomorphism is illustrated in figures 12-u and 12-v, and details half hours rather than hours to emphasize M.I.T. needs. In figure 12-u, the ideal family $F_{42}^4$ of 10 $[4^2]$ time patterns is mapped from the 4(day) by 20(module) cycle shown in figure 12-q to a 5(day) by 16(module) cycle. Note that the first two blocks $f_{4}^{44}+f_{2}^{44}$ still reside on TWRF morning, while the last two blocks $f_{4}^{44}+f_{5}^{44}$ shift to MTWR afternoon; the two $[4^2]$ time patterns $f_{3}^{42}, f_{8}^{42}$ contained in $f_{3}^{44}$ share coverage of Monday morning and Friday afternoon. Figure 12-v defines the recommended isomorphism in terms of mapping individual half hour modules.

\[
\begin{array}{cccccc}
  & M & T & W & R & F \\
9:00 &  &  &  &  &  \\
9:30 & f_{3}^{42} & f_{4}^{42} & f_{6}^{42} & f_{1}^{42} & f_{6}^{42} \\
10:00 &  &  &  &  &  \\
10:30 &  &  &  &  &  \\
11:00 &  &  &  &  &  \\
11:30 & f_{8}^{42} & f_{2}^{42} & f_{7}^{42} & f_{2}^{42} & f_{7}^{42} \\
12:00 &  &  &  &  &  \\
12:30 &  &  &  &  &  \\
1:00 &  &  &  &  &  \\
1:30 & f_{3}^{42} & f_{9}^{42} & f_{4}^{42} & f_{9}^{42} & f_{9}^{42} \\
2:00 &  &  &  &  &  \\
2:30 &  &  &  &  &  \\
3:00 &  &  &  &  &  \\
3:30 & f_{10}^{42} & f_{5}^{42} & f_{10}^{42} & f_{5}^{42} & f_{8}^{42} \\
4:00 &  &  &  &  &  \\
4:30 &  &  &  &  &  \\
\end{array}
\]

Fig. 12-u.--Recommended Isomorphism of the Ideal Family $F_{42}^{42}$ to a 5(day) by 16(module) Cycle
The recommended time pattern system for M.I.T. is shown in figure 12-ω, using cross-product notation. Time pattern naming conventions apply to a 5(day) by 16(module) cycle where days are named M,T,W,R,F and modules are named by starting clock times 9,9:30,10,...,4,4:30. For sake of completeness, evening time pattern families are also defined. In three of the families \( R^{42}, R^{32}, R^{22} \), Monday morning and Friday afternoon are paired to yield non-straight (diagonal) time patterns outside the four \( R^{82} \) blocks. In three other families \( R^{41}, R^{31}, R^{21} \) the same diagonal cycle subspace is used to yield straight time patterns outside the four \( R^{82} \) blocks. Although this causes lack of total interchangeability for these
six families with respect to other families, their increases in cardinality contribute to the overall system. Each of these six families is at least interchangeable with respect to the other five.

| CR | TS | |family| |family definition |
|----|----|----|-------|----------------------------------|
| 16 | [8^2] | | | $R^{82} = \{TR9-1, WF9-1, TR1-5, MW1-5\}$ |
| 12 | [6^2] | | | $R^{62} = \{TR10-1, WF10-1, TR2-5, MW2-5\}$ |
| 8  | [8^1] | | | $R^{81} = \{(T, W, R, F) \in (9-1)\} + \{(M, T, W, R) \in (1-5)\}$ |
|   | [4^2] | | | $R^{42} = \{(TR, WF) \in (9-11, 11-1)\}$ + \{(TR, MW) \in (1-3, 3-5)\} + 'M9-11, F1-3' + 'M11-1, F3-5' |
| 6  | [6^1] | | | $R^{61} = \{(T, W, R, F) \in (10-1)\} + \{(M, T, W, R) \in (2-5)\}$ |
| 4  | [3^2] | | | $R^{32} = \{(TR, WF) \in (9:30-11, 11:30-1)\}$ + \{(TR, MW) \in (1:30-3, 3:30-5)\} + 'M9:30-11, F1:30-3' + 'M11:30-1, F3:30-5' |
|  | [4^1] | | | $R^{41} = \{(M, T, W, R, F) \in (9-11, 11-1, 1-3, 3-5)\}$ |
|  | [2^2] | | | $R^{22} = \{(TR, WF) \in (9, 10, 11, 12)\}$ + \{(TR, MW) \in (1, 2, 3, 4)\} + 'M9, F1' + 'M10, F2' + 'M11, F3' + 'M12, F4' |
| 3  | [3^1] | | | $R^{31} = \{(M, T, W, R, F) \in (9:30-11, 11:30-1, 1:30-3, 3:30-5)\}$ |
| 2  | [2^1] | | | $R^{21} = \{(M, T, W, R, F) \in (9, 10, 11, 12, 1, 2, 3, 4)\}$ |
| EVE nights | | | $R^{E2} = \{(MW, TR) \in (EVE)\}$ |
| EVE 1 night | | | $R^{E1} = \{(M, T, W, R) \in (EVE)\}$ |

Fig. 12-w.--The Recommended Time Pattern System for M.I.T.
It might be desirable to extend \( R^{82} \) via the additional time pattern 'M9-1,F1-5', and likewise extend \( R^{81} \) via 'M9-1' and 'F1-5', \( R^{62} \) via 'M10-1,F2-5', and \( R^{61} \) via 'M10-1' and 'F2-5'; however, such extension violates strict isomorphism with the original four day cycle. The \([8^2]\) time pattern 'M9-1,F1-5' is mapped from two \([4^2]\) blocks \( f_3^{42} + f_8^{42} \) which cannot be used for \([8^2]\) time patterns in the four day cycle, and the recommended system honors the strict isomorphism. Since the only real value of this strictness is that M.I.T. could theoretically switch back and forth between four and five day cycles at will, the advantages of extending \( R^{82}, R^{81}, R^{62} \) and \( R^{61} \) could outweigh any disadvantages.

Figure 12-x summarizes the a priori conflict behavior of the (unextended) recommended time pattern system, within and between families defined by the strictly isomorphic definition of figure 12-ω. Note that the behavior within and between \( R^{42} \) and \( R^{32} \) (a constant 10% a priori probability of conflict) is an improvement over even \( S^{42} \) and \( S^{32} \) (a constant 12.5%). The fifth and final step in the M.I.T. case study is to recommend adoption of this well-behaved time pattern system.
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**NOTE:** The notation \(x/y\) is used to indicate that each and every time pattern from the row family conflicts with \(x\) of the \(y\) time patterns from the column family;

the notation \(\frac{x}{y}\) is used to indicate that of the \(y\) possible pairwise combinations of time patterns from the two families involved, \(x\) represent conflict.

*Fig. 12-x.* Matrix of A Priori Conflict Within and Between Families in the Time Pattern System Recommended for Adoption at M.I.T.*
12.6.1 Summary of Steps in the M.I.T. Case Study

The following steps were followed in the M.I.T. case study:

1. Development of a critique of the historical time pattern system.

2. Analysis of the contact requirements (CR$_i$) and time pattern structures (TS$_i$) used at M.I.T.

3. Proposal and critique of a simplified time pattern system, consistent with but improving upon the historical system.

4. Consideration of time pattern systems outside the historical framework: a non-productive longer school day, and a four day cycle approximating the ideal normative model.

5. Recommendation that M.I.T. adopt a revised five day system shown to be isomorphic to the near ideal four day cycle.

These five steps are typical of the analytical process many schools could use to improve upon an existing time pattern system. Starting with a critique of the existing system as a first step serves the dual purposes of (1) uncovering potential flaws in the status quo, and (2) gaining insight into the systems nature of the problem in hand. For many schools, the second step of analyzing CR$_i$ and TS$_i$, profiles might yield positive information about time pattern family priorities; at M.I.T. it simply confirmed the need for proper interaction within and among all families.

The third step, whereby improvement was sought within the status quo through simplification, was rewarding at M.I.T. and often will be rewarding in other environments. This is very likely to be the case where de facto systems have evolved without conscientious planning, since such systems tend to be noisy with redundant time patterns.
The fourth step, whereby innovative time pattern systems were studied, is highly recommended. Even if such systems seem too compromising to actually implement, their study often leads to further insight into the systems nature of the particular problem. And, of course, there is always the outside chance that the results will be sufficiently attractive to warrant the associated compromise. In the M.I.T. case study, the four day cycle has many strong points, due largely to its approximation of the ideal normative model. Despite the compromise of using four days instead of five (a potential problem at M.I.T.), attractive leverage was obtained in this hypothetical system by reconfiguring three day classes to two or four days. Since isomorphism can be demonstrated between such a near ideal four day system, and an equally well-behaved revised five day system, the case study could conclude with the fifth step of recommending the revised time pattern system for adoption at M.I.T.
13.1.1 Objectives of Time Pattern Analysis in the General Case

There are three primary reasons for a school to perform time pattern analysis. All three apply when an initial analysis is performed, and further apply when a subsequent analysis is undertaken to update previous effort. These three objectives are:

1. Acquisition of new or better information about the systems nature of the school's scheduling problem.
2. Identification of areas where compromise leads to sufficient improvement in the scheduling approach to justify its consideration.
3. Identification of areas for discretionary improvement of the scheduling approach, through simplification of time pattern families and/or adjustments to time pattern mesh.

Any of the three objectives, if realized, offer a school opportunities to achieve a better master schedule. Although all three goals may not materialize during a particular time pattern analysis, it would be exceptional not to achieve at least the first objective, since the discipline of performing time pattern analysis provides ample opportunity to ask and answer questions about the mechanics of scheduling a school. Such an education is productive, particularly where time variables have been previously taken for granted. Even if only a little is learned about the time pattern system, a lot can be learned about the resources in their relation to time variables.
For many schools, time pattern analysis can lead to one or both of the objectives identifying areas for improvement—worthwhile compromise or discretionary leverage. In either case, typical analysis should not only identify potential improvement, but should also document and justify the improvement. The net result of a sound and convincing time pattern analysis should leave a scheduling officer much more comfortable with the school's time pattern system, even should it lead to discomforting information about resources or scheduling objectives. An honest appraisal of resources and scheduling objectives is considerably easier when there are few doubts about the time variables.

The case studies at Minuteman and M.I.T. involved different steps and different results, although it is claimed that all three primary objectives were realized in both studies. The following section extrapolates the two case studies to the general academic environment by proposing a number of steps which a school should consider in order to perform its own time pattern analysis.

13.2.1 The Steps in the General Case

A school about to perform a time pattern analysis—or trying to justify doing one—should consider the following steps. Not all of the steps need be pursued at a particular school, but each should be considered. The first three steps are recommended for any school.

(1) Familiarization with terminology and concepts discussed in this thesis, in particular the four compromise degrees of freedom and the two discretionary leverages. [Chapter Six]
(2) Familiarization with the normative models discussed in this thesis, with particular understanding as to why each time pattern family is or is not ideal in context. Appreciation that the ideal normative model is extremely well-behaved, and why. [Chapter Nine]

(3) Attention to the two case studies [Chapters Ten, Eleven, and Twelve] documented in this thesis, considering the following questions:

(3a) To what extent is your school similar to either case study institution? How is it different from each?

(3b) Do the steps taken at either case study institution make sense for your school? If not, why not?

(3c) What uses were made of compromise and discretionary leverage in the case studies that might apply to your school?

After taking the above steps, the following steps should be considered. The sequence is not essential, and steps might often be repeated in response to new insight into the overall problem. These next steps are basically a checklist of techniques discussed throughout the thesis.

(4) Determination of the contact requirement \( (CR_i) \) profile of your school [Section 6.2.2]. Analysis of resulting ratios:

(4a) Are the \( CR_i \) multiples of one another? Can the \( CR_i \) be adjusted to reduce the ratio? Recall how the Physical Education \( CR=6 \) single-handedly prevented reduction of the Minuteman ratio \( 40>20>10>6>5 \) [Section 11.1.1], and how changing the one \( CR \) to 5 reduced the ratio to \( 8>4>2>1 \) as in the ideal normative model [Section 11.3.1].
(4b) What combinations of CR\textsubscript{i} represent the highest priorities for your school? Do these critical CR\textsubscript{i} partition each other well? Recall that at M.I.T., almost any combination of CR\textsubscript{i} and TS\textsubscript{i} could occur [Section 12.2.1], and how this recommended treating \([s^y]\) time patterns as though they were tantamount to \([s^y]\) time patterns for reasons of mesh [Section 12.3.1].

(5) Determination of the time pattern structure (TS\textsubscript{i} \textsubscript{e}) profile of your school [Section 6.2.2]. Analysis of resulting compositional behavior:

(5a) Are the TS\textsubscript{i} \textsubscript{e} compositions of one another? Can the TS\textsubscript{i} \textsubscript{e} be adjusted to increase compositional utility? Recall the interchangeability of \([I^5]\) time patterns with a pair of \([I^3]\) and \([I^2]\) time patterns in normative model #4 [Section 9.4.1].

(5b) What combinations of TS\textsubscript{i} \textsubscript{e} represent the highest priorities for your school? Are these critical TS\textsubscript{i} \textsubscript{e} dominated by one or more congruent block structures? Recall the rich block structuring of the ideal normative model, resulting from dominance [Figure 9-6n].

(6) Investigation of cycle shape [Section 6.2.3].

(6a) Is \(d\), the number of days in your cycle, fixed? If not, could it be a power of 2? Even if \(d\) is fixed at 5, is there anything attractive about a two or four day cycle? Recall the isomorphism between four and five day cycles discussed in the M.I.T. case study [Section 12.5.1]. Also recall the improvement at Minuteman resulting from the
220

2 day cycle of 10 de facto days instead of 5 [Section 11.3.1].

(6b) Is *m*, the number of daily modules in your cycle, fixed?

Is there anything attractive about one or two more periods each day? Could periods be shortened or lengthened to let *m* become a power of 2? Could better *TS* result from doubling *m* and halving period length? Recall the low utility of adding a ninth hour to the eight hour M.I.T. day, but the value of extending it to ten hours [Section 12.4.1].

(6c) How compatible is the cycle cardinality *dm* with the *CR* ratio? With disjoint coverage by the *TS*? Recall the lack of compatibility between the \(2^3\) structure and the \(d=4\text{(day)} \times m=9\text{(module)}\) cycle of normative model #3 [Section 9.3.1].

(6d) Does the two-dimensional cycle space lend itself to coverage by one or more congruent block structures appropriate to the *TS*? Recall the importance of *d* and *m* being powers of 2 in the ideal normative model, and the rich block structuring of that cycle [Section 9.6.1].

(7) Analysis of time pattern interchangeability [Sections 9.4.2-4]:

(7a) Are certain days or clock hours biased for or against? If so, could day or module permutation reduce disparity? [Section 6.4.3]

(7b) Is it possible to restrict time pattern families to congruent or highly similar shapes? Recall the advantages of using either but not both \(3^2\) or \(32\) time patterns,
and not using $[2^3]$ time patterns, in normative model #3 [Section 9.3.1].

(7c) Can each time pattern family be restricted to disjoint
time patterns, even if non-coverage is the price? Recall
the advantages of the simplified M.I.T. time pattern system
over the "noisy" historical system, despite incomplete
coverage of the cycle by all three of the simplified
families [Section 12.3.1].

(7d) Are the $CR_i$ and $TS_i$, compatible when it comes to time
pattern family design? [Section 6.4.4]

(7e) Are all the time patterns from one family interchangeable
with respect to time patterns from other families?
[Section 6.4.4]

(7f) Can time patterns be composed in flexible manner without
favoring certain structures? Recall the multiple jigsaw
puzzle analogy [Section 7.2.1].

(8) Analysis of time pattern mesh [Sections 6.4.5-7]. Reexamine how
well-behaved the ideal normative model is on all these counts [Section 9.6.1];
how the recommended time pattern system for M.I.T. shares many of the same
advantages [Section 12.5.1]. Note also the lesson about symmetry taught
by the Minuteman case study [Sections 11.2.1, 11.3.1].

(8a) Can time patterns compose in mutually disjoint combinations,
or is a priori conflict high because of partial conflicts
and unwarranted transection?
(8b) Can time patterns engage or interlock without leaving holes, or does obstructive fragmentation result?

(8c) Are the time patterns and the families systematic? regular? simple? modular?

Perhaps the single most pressing question throughout the process is "can you make your school conform more closely to the ideal normative model?". In both case studies, the affirmative answer to this single question led to positive results. It is instructive to note that the ideal normative model is not appropriate "as is" for either school. Both schools, however, gained from a closer approximation to it. Minuteman gained a great deal from the two day cycle, and the model of symmetry offered. M.I.T. can benefit from a recommended five day system isomorphic to a four day system enjoying most of the advantages of the ideal normative model.

A school is strongly encouraged to address the issue of conforming to the ideal normative model, even if the compromises required to do so appear insurmountable. The advantages to be gained may justify compromise, or in any case may illuminate less controversial discretionary leverage.
14.1.1 Summary of Thesis

In the introductory chapter, it was stated that the purpose of the thesis is "to draw the attention of school authorities to the importance of careful time pattern analysis, and to assist these authorities in performing a proper time pattern analysis." In order to accomplish this purpose, the thesis concentrates on the definition and justification of time pattern analysis, involving both theoretical and practical considerations. The thesis has been organized into the following chapters:

Chapter One: The School Scheduling Problem.--The school scheduling problem and a working vocabulary of terminology are defined. The thesis is stated: that time pattern analysis--the identification of appropriate time pattern families--is a central and critical aspect of a sound systems approach to school scheduling.

Chapter Two: Relationship to Other Scheduling Problems.--School scheduling is contrasted with two other scheduling problems which are not particularly relevant to the thesis--production scheduling and transportation scheduling. School scheduling is compared to two allocation/assignment problems which offer useful analogies--graph chromaticism and computer memory allocation. These two analogies help motivate the avoidance of partial conflict and obstructive fragmentation.
Chapter Three: A Priori Flexibility.—A major objective of school scheduling is that as much as possible of the master schedule remain effective in the face of perturbations encountered during implementation. Such a priori flexibility requires avoidance of obstructive fragmentation.

Chapter Four: Approaches to the Problem.—A school can approach its eventual schedule from either of two opposite directions. It is recommended that the approach be from the feasible side, seeking increased desirability, rather than asking for too much too soon. There are advantages to a fall-back position of having a tangible schedule in hand. Compromise, the Procrustean attitude towards the problem, is often a valid approach so long as care is taken to justify the costs; discretionary leverage, where present, offers a scheduling officer a powerful and less controversial approach. The limitations and capabilities of computer-assisted scheduling are discussed.

Chapter Five: Notation and Definitions.—Scheduling variables involving time are formally defined, including such important concepts as: time patterns and their representation; conflict between time patterns, classes, and resource assignments; the structure and shape of time patterns, similarity and congruency of time pattern shapes; and time pattern blocks.

Chapter Six: Time Pattern Analysis.—Time pattern analysis is described in terms of the impact of time variables upon the system. Four compromise degrees of freedom are discussed, including contact requirements ($CR_i$), time pattern structures ($TS_i$), and cycle shape ($d$, the number of days...
involved, and \( m \), the number of daily modules). Two discretionary leverages—time pattern interchangeability and time pattern mesh—are discussed, their positive influence as well as impediments to their realization. These degrees of freedom and leverages are considered from the dual viewpoint of class interchangeability and resource interchangeability.

Chapter Seven: Intuitive Arguments.—The role and importance of time pattern analysis are intuitively justified. A multiple jigsaw puzzle analogy is introduced, valuable because of the emphasis it places on the careful design of individual and collective time pattern shapes.

Chapter Eight: Combinatorial Analysis.—The concepts of composition and partition are borrowed from combinatorial analysis and extended from integers to strings of consecutive modules and two-dimensional time patterns. The probabilistic effects of block structuring are discussed, motivating the concept of a priori conflict.

Chapter Nine: Normative Models.—Six normative models, ranging from trivial to complex, are presented as paradigms of well-behaved time pattern systems. The sixth model is considered to be ideal, and is so well-behaved as to recommend its adoption or approximation whenever possible. These models include and exclude time patterns and time pattern families in accordance with degrees of freedom and leverages, and illustrate much of the theory of sound time pattern analysis.
Chapter Ten: Two Case Studies.--To support the foregoing theory, time pattern analysis was performed for two different academic institutions, the Minuteman Regional Vocational Technical School, and the Massachusetts Institute of Technology. The general objectives of the two cases studies are discussed, the two schools are introduced via excerpts from their publications, and the two schools are compared.

Chapter Eleven: The Process at Minuteman.--As the result of a two year study prior to the opening of Minuteman Regional Vocational Technical School in September 1974, time pattern analysis led to an implemented time pattern system considered an improvement over the originally planned system. A key aspect of this analysis was a compromise from a five day to a two day cycle, bringing the school much closer to the ideal normative model. The study reinforces the role of compromise degrees of freedom and the soundness of the ideal normative model. It also illustrates the role of symmetry in time pattern design.

Chapter Twelve: The Process at M.I.T.--The M.I.T. case study led to a deeper understanding of time pattern structures and priorities at the school. Improvements to the M.I.T. system are largely discretionary. Simplification of the historical system would reap benefits. The system actually recommended for M.I.T. is a revised five day system shown to be isomorphic to a near ideal four day system considered as one hypothetical alternative. The study emphasizes the productive value of considering a variety of hypothetical compromises, and once again demonstrates the soundness of the ideal normative model.
Chapter Thirteen: The Process in General.—Reasons are given for performing time pattern analysis at any school. Steps are recommended for consideration by a school about to perform or justify a time pattern analysis, with references to pertinent sections of the thesis. The role and value of the ideal normative model during the two case studies recommends its adoption or approximation through compromise or discretionary leverage.

Chapter Fourteen: Summary and Conclusion.—The thesis is summarized chapter by chapter. The conclusions of the thesis are reiterated. The thesis concludes with suggestions for future study.

Bibliography.—As part of the thesis research, a computer-assisted literature search was made to identify references to systematized school scheduling. The results of this search are included in addition to other supporting references.

Appendix A: Glossary of Notation and Terminology.—Because of the extensive notation and terminology used throughout this thesis, a glossary is included as a convenience for reading and research.

Appendix B: Reference List of Time Pattern Families.—To permit easy of reference to the various time pattern families documented in the thesis, each family is listed by symbolic name, together with its description, its cardinality, and the time pattern structure it represents.
14.1.2 Conclusions

It is hoped that the thesis will have a beneficial impact in the academic environment in terms of affording (1) insight into the systems nature of school scheduling, in particular into the role and importance of variables involving time, and (2) opportunities for, and justification of, improvements to master schedules, through time pattern analysis.

This hope is supported by the following conclusions:

(1) Theoretical time pattern analysis has been defined, and described in terms of impact of time variables on the system.

(2) Time pattern analysis can be appreciated intuitively.

(3) Normative models have been developed, including an extremely well-behaved ideal normative model, shown to have applicability in practice.

(4) The theory of time pattern analysis demonstrably applied during the two case studies, leading to practical application.

(5) A time pattern analysis which follows the theory, and parallels the practice, described in this thesis should have benefits in any academic environment, at least in terms of increased confidence and understanding, and often in terms of resulting improvement.

14.2.1 Suggested Areas for Further Study

When research began for this thesis, three years before its completion, the intent originally was to develop a "systems theory of school scheduling". Such a complete theory would focus on the class sector and individual resource sectors much as this thesis has focused on the time sector. The topic was narrowed and this thesis has covered only an aspect
of the overall problem. Although time pattern analysis is particularly important (in view of the pervasive nature of the time pattern system), it would be valuable to have similar work done in resource analysis and regarding class variables (e.g. the effects of phasing, ratios of class sizes, etc.). Curriculum development, and resource acquisition and allocation, are often considered for their educational merit, but the recommendation is that they be considered in depth in terms of scheduling mechanics.

To extend the theory of time pattern analysis, additional normative models could be developed and analyzed. Perhaps isomorphism could be demonstrated between the ideal normative model and a variety of different cycle shapes, following the logic used during the M.I.T. case study. It might be possible to rigorously prove that no other model can improve upon the ideal normative model. It would also be useful to investigate resource assignment practices that best capitalize on the inherently well-behaved nature of an ideal time pattern system— even the best time pattern systems fail to support poorly constrained resources.

Time pattern analysis could be made more attractive by a thorough cost/benefit analysis. Although the educational priorities vary from school to school so as to make this a difficult prospect in general, perhaps costs and benefits could be studied for a cross-section of cases, or in detail for a single institution.

Having spent some considerable time working with the two case study institutions, several suggestions can be made regarding these specific schools. Minuteman should seriously consider a detailed time pattern analysis for the projected full four year operation. As experience, and
better anticipatory statistics, illuminate their ongoing scheduling problem, their time pattern analysis should be updated accordingly. M.I.T. should further study the time pattern system recommended for them by this thesis.

Regarding the M.I.T. case study, it was originally hoped that a large number of computer simulation runs could be used to verify the expected results of implementing the various time pattern systems discussed. Time and money prevented carrying out these simulations. Should M.I.T. consider the recommended time pattern system--and there are strong reasons for doing so--the system should be simulated extensively, with particular attention to the role of, and impact upon, the various resource sectors. Even if the instructor sector could be "taken for granted" since the loads are light--and it is not at all clear that M.I.T. can do so because of the history of preferential considerations--the room sector should be reviewed as to the appropriateness of pool composition and request procedures. The student sector would probably serve as a stable sounding board in evaluating the proposals.

The final, and perhaps most important, suggestion for further study is that any academic institution that has never performed a time pattern analysis should seriously consider doing one on their own terms, for their own benefit.
The following four books served as reference material:


As part of the thesis research, a computer-assisted literature search was made to identify references to systematized school scheduling. This search was conducted through NASIC (the Northeast Academic Science Information Center), a regional resource developed by the New England Board of Higher Education (NEBHE). The data base searched was ERIC (Educational Resources Information Center), the educational data base developed and maintained by the U.S. Office of Education. The ERIC data base covers educational literature published since 1969 (a limited number of documents going back to 1956 is included), and contains all citations published in Research in Education (RIE) and Current Index to Journals in Education (CIJE), the two major monthly products of the ERIC system.

The search strategy was based on 16 data base descriptors; each item was to have at least one of the first five descriptors as a major category, and at least one of the other eleven descriptors. The first five descriptors deal with school scheduling: schedule modules, flexible schedules, school schedules, scheduling, flexible scheduling. The other eleven were intended to select items dealing with a systematized approach: systems approach, decision making, systems analysis, systems concepts, computers, automation, systems development, computer oriented programs, data processing, electronic data processing, management systems.

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APPENDIX A: GLOSSARY OF NOTATION AND TERMINOLOGY

italic letters.—Lower case italic letters are used to denote individual school scheduling elements: time patterns \( t_1, t_2, t_3, ..., \), classes \( c_1, c_2, c_3, ... \), and resources \( r_1, r_2, r_3, ... \). Occasionally the particular type of resource is indicated: instructors \( i_1, i_2, ... \), rooms \( r_1, r_2, ... \), or students \( s_1, s_2, ... \). Upper case italic letters are used to denote time pattern families, often with a superscript to mnemonically suggest structure. For example, \( A^{12} \) might be a family of \( n \) \([1^2]\) time patterns \( a_1^{12}, a_2^{12}, ..., a_n^{12} \), and \( T^{321} \) might be a family of \( 6 \) \([321]\) time patterns \( t_1^{321}, t_2^{321}, ..., t_6^{321} \).

Frequently \( i, j, k \) serve as general indices. See also: \( d \) and \( m \).

subscripts.—Used as a general distinguishing mark, as when discriminating between several time patterns \( t_1, t_2, ..., t_i, ..., t_n \). Occasionally used to indicate position in a sequence, as when the module names \( M_1, M_2, ..., M_m \) are to identify the corresponding sequence of \( m \) modules.

superscripts.—Used solely as a general distinguishing mark. Time pattern families and their member time patterns use superscripts to uniquely identify the family. Superscripts are usually chosen to mnemonically suggest the time pattern structure involved, as when \( T^{42} \) represents either a family of \([42]\) time patterns or a family of \([4^2]\) time patterns.

set-theoretic notation.—The following set-theoretic notation is used freely throughout the thesis:
the empty set

(...) braces, to indicate aggregation of members into sets

set containment: (is) contained in

intersection; \( n_{t_1} = t_1 \cap t_2 \cap \ldots \cap t_n \)

union; \( u_{t_1} = t_1 \cup t_2 \cup \ldots \cup t_n \)

set membership: (is) member of

summation of disjoint elements; \( \sum t_i = t_1 + t_2 + \ldots + t_n \)

\((u_{t_i} \text{ where for any } j, k \neq j \ t_j \cap t_k = \lambda)\)

cross product; \( A \times B \) is the set of all ordered pairs \((a, b)\)

where \( a \in A, b \in B \)

vertical bars, to indicate cardinality

Superscript bar---indicates a string (or substring) of consecutive modules on one day. The length of the string (or substring) is the value under the bar: the number of consecutive modules involved.

See also: composition and partition.

Brackets---to indicate time pattern structure.

\([x]^y\) indicates \( x \) consecutive modules on each of \( y \) days. The structure of the cycle \( t_{\text{cycle}} \) is \([m]^d\). The general form \([n_1, n_2, \ldots, n_j]\) denotes the structure of any time pattern involving \( i_1 + i_2 + \ldots + i_j = k \) days, \( n_1 \) consecutive modules on each of \( i_1 \) days, \( n_2 \leq n_1 \) consecutive modules on each of \( i_2 \) other of the remaining \((k-i_1)\) days, etc.

\( \lambda \) Lambda---used to indicate the null time pattern (of cardinality \(|\lambda| = 0\)) i.e. the time pattern consisting of no modules (the vacuous subset of the \( dm \) cycle modules).

\( \sigma, \sigma_i, \sigma_j \)---The lower case italic letter \( \sigma \) is used to denote a class:
subscripts are used to distinguish several classes.

\( CR, CR_i \) -- (See: contact requirements)

\( d \) -- The number of days in the schedule cycle. Choice of \( d \) is a compromise degree of freedom.

\( dm \) -- The product of \( d \) (the number of days in the cycle) and \( m \) (the number of modules on each day) determining the cardinality of the cycle:

\[
|t_{cycle}| = dm
\]

\( m \) -- The number of modules (periods) on each and every day of the schedule cycle. Choice of \( m \) is a compromise degree of freedom.

\( r, r_i, r_j \) -- The lower case italic letter \( r \) is used to denote an individual resource (instructor, room, or student); subscripts are used to distinguish several resources. Occasionally the lower case italic letters \( i, r, s \) may be used, with or without subscripts, to denote an instructor, room, or student, assuming the usage is clear in context.

\( t, t_i, t_j \) -- The lower case italic letter \( t \) is used to denote a time pattern; subscripts are used to distinguish several time patterns. Super-scripts may be used to denote the family to which the time pattern belongs. Because of the large number of time pattern families discussed, other lower case italic letters are also used to denote time patterns, usually with a superscript denoting the family. See also: time pattern and time pattern family.

\( TS, TS_i \) -- (See: time pattern structures)

a priori conflict. -- (See: conflict, a priori)

a priori flexibility. -- An objective of school scheduling that as much as possible of the master schedule remain effective in the face of perturbations encountered during implementation. It implies the
ability to add new classes, or alter the time pattern or resource assignments, with minimal disruption to existing individual resource schedules. The desirability of this objective stems from the often unstable nature of school scheduling constraints and preferences.

assignment, resource.—A resource assignment is an ordered pair \((r, c)\) consisting of one resource \(r\) and one class \(c\); the resource is said to be assigned to the class, and the class is said to be assigned to the resource. Typically, at least one instructor, exactly one room, and numerous students are assigned per class.

assignment, time pattern.—A time pattern assignment is an ordered pair \((t, c)\) consisting of one time pattern \(t\) and one class \(c\); the time pattern is said to be assigned to the class, and the class is said to be assigned to the time pattern. By convention, there is exactly one time pattern assignment per class.

assignments, valid versus invalid.—Two or more resource assignments \((r, c_1), (r, c_2), \ldots, (r, c_n)\) involving the same resource \(r\) are said to be without conflict if and only if the \(n\) classes \(c_1, c_2, \ldots, c_n\) are without conflict. If the \(n\) classes are not without conflict, one or more of the \(n\) resource assignments must be designated invalid; by convention, the smallest possible number \(n'\) of the \(n\) resource assignments are attributed the invalidity such that the remaining \((n-n')\) resource assignments are then without conflict; all other things being equal within this convention, the higher-subscripted class(es) is/are usually attributed the invalidity. The absence of conflict is necessary but not sufficient for validity; other constraints must not be violated: bounds on individual
resource load, required free time, limits on class section size, etc.

block, time pattern.—A time pattern which can be decomposed into two or more useful time patterns. Ideally all (or a significant majority of) useful time patterns either are totally contained in a block or else are disjoint from it. Thus a family of time pattern blocks can be thought of as a gross partition of the schedule cycle, with each useful time pattern family further partitioning the blocks into a fine partition of the cycle. If the cycle is disjointly covered by congruent time pattern blocks, and if useful time pattern families are both supplementary and contained in the blocks, then it is easier to discuss probabilistic conflict behavior by analyzing time pattern behavior within a typical block.

block- (or macro- ) scheduling.—A conceptual simplification whereby a framework of time is allocated to a pseudo-class, within which two or more de facto classes (sub-sections) are actually held. The assigned time pattern is then divided among the disciplines represented as the instructor teams see fit. Such a pseudo-class is typically assigned one room and at least one instructor for each de facto class represented.

buddy.—A pairwise relationship between two similar disjoint useful time patterns \( t_1, t_2 \) whose union (sum) \( t = t_1 + t_2 \) is a useful time pattern. The utility of \( t \) is enhanced by this ability to decompose into two useful time patterns \( t_1, t_2 \) because the fragmentation of \( t \) brought about by assigning either \( t_i \) results in a hole exactly coinciding with the useful buddy time pattern \( t_{j \neq i} \). The term is intended to bring to mind the so-called buddy system of computer storage
allocation where memory blocks of size $2^k$ are split in two (each of which is half as large as the original). Because the schedule cycle is two-dimensional (days by modules), time patterns often have two buddies, one with the same day(s), the other with the same module(s).

class (or class section).--The simultaneous gathering of one or more instructors, rooms, and students, involving these same resources whenever it meets in such a dedicated manner as to prohibit their concurrent involvement with any other class. The elementary scheduling entity of the curriculum. Multiple class sections may occur for the same subject. A subject may optionally have subject phases (lectures, recitations, laboratories, seminars, etc.) each running in one or more classes.

composition.--In combinatorial analysis, a composition of the integer $m$ is defined to be an ordered collection of integers with given sum $m$. We define a composition of the string $\overline{m}$ (of $m$ consecutive modules on any one day) to be an ordered collection of $n$ ($1 \leq n \leq m$) disjoint substrings $\overline{t}_1, \overline{t}_2, \ldots, \overline{t}_n$ (of $l_i$ consecutive modules), the sum of whose lengths $\Sigma \overline{t}_i = m$. For example, the six compositions of $\overline{4},$ restricted to substring lengths which are powers of 2, are: $\overline{4}, \overline{2^2}, \overline{2^1}, \overline{2^1}, \overline{2^2}$ and $\overline{4}$. We define a composition of a time pattern $t$ to be an ordered collection of $n$ ($1 \leq n \leq |t|$) time patterns $t_1, t_2, \ldots, t_n$ which are supplementary with respect to $t$. Since the $t_i$ are supplementary with respect to $t$, they are mutually disjoint, and collectively exhaustive with respect to $t$; hence $\Sigma t_i = t$ and the sum of their cardinalities $\Sigma |t_i| = |t|$. The number of compositions of $t_{cycle}$ attainable with a given time pattern system is a measure
of the utility of that system.

compromise degrees of freedom.—The four compromise degrees of freedom in the time pattern sector are: (1) the CR, ratios of the curriculum; (2) the TS, chosen to represent the CR; (3) d, the number of days in the cycle; and (4) m, the number of daily modules. These parameters are called compromises because only rarely could or would a scheduling officer unilaterally change one or more of them without the advice and consent of other sectors within the faculty and administration. Consideration of them, despite the difficulty usually encountered in changing them, is warranted by the leverage they exert on the system.

conflict (between two time patterns).—Two time patterns \( t_1, t_2 \) which are not disjoint are said to conflict (be in conflict, be conflicting); the conflict (between two conflicting \( t_1, t_2 \)) is the non-null time pattern \( t \) which is their intersection \( t = t_1 \cap t_2 \). The conflict \( t \) is said to be a partial conflict if \( t \) is a proper subset of either \( t_1 \) (\( t \subset t_1 \) or \( t \subset t_2 \)). If \( t_1 = t = t_2 \) the conflict \( t \) is said to be a total conflict. If \( t_1 = t \neq t_2 \) the conflict \( t \) is said to be total with respect to \( t_1 \) and partial with respect to \( t_2 \).

conflict (between two classes).—Two classes \( c_1, c_2 \) are said to conflict (be in conflict, be conflicting) if and only if the two time patterns \( t_1, t_2 \) to which they are assigned are in conflict. Given two time pattern assignments \( (t_1, c_1) \) and \( (t_2, c_2) \), \( c_1 \) conflicts totally/partially with \( c_2 \) if and only if \( t_1 \) conflicts totally/partially with \( t_2 \). Note that a conflict between classes is not necessarily a problem in and of itself, since common resources
might not be involved; classes in conflict pose only a potential problem with respect to resource scheduling.

conflict (between two resource assignments).—Two resource assignments \((r_1, c_1)\) and \((r_2, c_2)\) involving the same resource \(r\) are said to conflict (be in conflict, be conflicting) if and only if the two classes \(c_1, c_2\) are in conflict. Such conflict is total/partial to the extent that \(c_1\) totally/partially conflicts with \(c_2\). See also: assignment, valid versus invalid.

conflict, a priori.—The a priori conflict between two classes \(c_1, c_2\) is the probability that they would conflict if each were assigned a time pattern at random from their appropriate time pattern families. For example, if both classes require time patterns from the same disjoint family \(T\) of cardinality \(|T|=n\), their a priori conflict is \(1/n\) (the probability that the same time pattern is assigned to both).

congruency (among time patterns).—Two or more time patterns \(t_1, t_2, \ldots, t_n\) are said to be congruent if and only if their pictorial representations can be superimposed (placed one upon the other) such that they coincide. Thinking of each shape as a rigid template, translation and/or rotation of the templates (along or about any axis) are allowed to achieve superimposition. (This definition of congruency more or less corresponds to the usage in plane geometry.) Note that a congruency transformation preserves structure and day separation. Most schools treat congruent time patterns as interchangeable.

contact requirements \((CR)\).—The contact requirement \((CR)\) of a class is the number of cycle modules during which the class will meet; i.e. the cardinality of any time pattern which can be assigned to the class.
Most schools have a spectrum of different contact requirements, and
the ratios which these $CR_i$ bear with respect to each other, and to
the total cardinality of the cycle, are an important consideration
for interchangeability of classes and for time pattern design.
Choice of the $CR_i$ is a compromise degree of freedom.

cycle; schedule cycle.--The period of time over which the master schedule
does not repeat itself. This "academic week" is frequently a
calendar week, but need not be restricted to five days. The cycle
is made up of $d$ days, and $m$ daily modules (sometimes called periods),
for a total of $dm$ cycle modules. The cycle can be thought of as a
two-dimensional $d$ by $m$ matrix of cycle modules; it is itself a time
pattern $t_{cycle}$ of cardinality $|t_{cycle}| = dm$.
cycle reversal.--A specific type of day permutation whereby the first day
one cycle becomes the last day the next, the second day becomes the
penultimate, etc.
day; day names.--The $d$ days represent the horizontal dimension of the two-
dimensional schedule cycle (the $m$ modules represent the vertical
dimensions). Time patterns can be named by using $d$ unique graphics
$D_1, D_2, \ldots, D_d$ as day names (e.g. M, T, W, R, F, S to represent Monday,
Tuesday, Wednesday, Thursday, Friday, and Saturday), and by using $m$
unique character strings distinct from the day names as module names.
See also: time pattern names.
day permutation.--Any number of pairwise day interchanges (interchanging
those vertical portions of the cycle represented by any two days).
Each pairwise interchange, and hence any permutation of the days,
preserves structure and is transparent with respect to conflict
(which can neither be created nor eliminated in the process). One example is cycle reversal, whereby the $i$th day is mapped into the $(d+1-i)$th.

day separation.—A factor in determining the shape of a time pattern, emphasizing the way in which entire days intervene between days involved with the time pattern. Similar time patterns are often non-congruent for reasons of different day separation. For example, many schools using a four day or five day cycle would consider two day time patterns involving consecutive days inferior to time patterns splitting the contact.

discretionary leverages.—Often as significant as the compromise degrees of freedom, leverage is also afforded by: (1) time pattern interchangeability, and (2) time pattern mesh. These considerations, applicable both within and among time pattern families, represent alternatives which may appear equally acceptable to the community at large, and hence a scheduling officer may be able to take effective action by fiat.

disjointness; non-disjointness.—Two time patterns $t_1, t_2$ are said to be disjoint if and only if their intersection is null ($t_1 \cap t_2 = \emptyset$). Two or more time patterns $t_1, t_2, \ldots, t_n$ are said to be mutually disjoint if and only if the $t_i$ are pairwise disjoint (i.e. for any $i, j \neq i$, $t_i \cap t_j = \emptyset$). A family $T$ of time pattern $t_i$ is said to be disjoint if and only if the $t_i$ are mutually disjoint. Non-disjointness is simply the absence of disjointness.

dominance.—A major type of interference between even the most ideal of time pattern families. A family $W$ is said to dominate another family
X if and only if for any \( x \in X \) there exists exactly one \( \omega_j \in \mathcal{W} \) such that \( x \subseteq \omega_j \) (although several \( x \) may be contained in the same \( \omega_j \)). Although dominance is a type of interference, it is not undesirable; when it is possible, dominance is a desirable objective of time pattern design. See also: transection.

GASP.—Generalized Academic Simulation Programs, a system incorporating a computer-assisted approach to school scheduling, developed in the mid-1960's, and used at the Massachusetts Institute of Technology for more than a decade to assist in timetable construction and resource allocation.

hole; permanent hole.—A fragment of unassigned time in an individual resource schedule; cycle space during which no classes have (yet) been assigned to the resource. Holes are themselves time patterns, with the usual time pattern properties of cardinality, structure, shape, etc. A hole is said to be permanent when it neither coincides with, nor contains, any time patterns from the system of admissible time pattern families.

ideal (time pattern families).—A time pattern family is considered ideal when no improvement to the entire time pattern system can result from either adding or removing time patterns to or from the family. An ideal time pattern family is usually supplementary (disjointly covering the cycle), a necessary condition for its being ideal were it the only family in the system.

independently assigned time (IAT).—Those holes remaining in the individual schedule of a student after all formal classes are assigned. The usual understanding is that this time will be productively spent by
the student in study halls or resource centers performing personalized activity beyond the formal curriculum.

instructor.—A resource element, one or more of which satisfies a class's need for teaching staff. The term makes no distinction among professors, junior staff, or even student teaching assistants, but simply denotes individuals with teaching responsibilities. By convention, at most one class can be validly assigned to an instructor during any one cycle module. Usually a class requires one and only one instructor, but occasionally teams of several instructors need be assigned. Such teams often split up the student population of large phases into smaller populations before, during, or after attendance at the large phase. Appropriate instructor load is an important scheduling consideration.

interchangeability, class.—Broad accessibility to a variety of classes and class-combinations, an important requirement from the viewpoint of an individual resource during the assignment of classes to resources. The dual viewpoint is resource interchangeability (see: interchangeability, resource). Class interchangeability depends upon time pattern interchangeability and mesh, as well as upon the compromise degrees of freedom.

interchangeability, resource.—Broad accessibility to resources and resource-combinations, an important requirement from the viewpoint of an individual class section during the assignment of resources to classes. The dual viewpoint is class interchangeability (see: interchangeability, class). Resource interchangeability depends upon time pattern interchangeability and mesh, as well as upon the
compromise degrees of freedom.

interchangeability, time pattern.—The ability to substitute one or more time patterns from one family for one or more time patterns from either the same or different families, in such a manner that the resulting impact on the master schedule is minimal. Included in this concept is the notion of a priori indifference among the time patterns so far as individual resources are concerned. Intra-family interchangeability and inter-family interchangeability are both target objectives of time pattern analysis and are usually discretionary leverages.

interference.—See: dominance and transection.

load; resource load.—The load of an individual resource element \( r \) is the number of cycle modules during which \( r \) is assigned classes; i.e. the cardinality of that time pattern \( t \) which is the union of the \( n \) time patterns \( t_i \) assigned to the \( n \) classes \( \sigma_i \) in \( r \)'s individual resource schedule: \( (r, \sigma_1), (r, \sigma_2), \ldots, (r, \sigma_n) \). Load is also equal to \( |t_{cycle}| \) less the totalled cardinalities of all holes in \( r \)'s schedule.

mesh, time pattern.—The extent to which time patterns engage and coordinate harmoniously with one another in individual resource schedules. At least three aspects of harmonious accord are involved if time patterns are to mesh well: (1) they must compose in mutually disjoint combinations; (2) they should engage or interlock without leaving holes; and (3) they should be systematic (regular, simple, modular, etc.). Intra-family mesh and inter-family mesh are both target objectives of time pattern analysis and are usually discretionary leverages.
module; module names.—The $m$ daily modules represent the vertical dimension of the two-dimensional schedule cycle (the $d$ days represent the horizontal dimension). There are a total of $dm$ cycle modules. Time patterns can be named by using $m$ unique character strings $M_1, M_2, \ldots, M_m$ as module names (e.g. period number $1, 2, \ldots, m$ or starting clock times $8, 8:30, 9, 9:30, \ldots, 5:30, 6, \text{EVE}$) in combination with $d$ unique graphics distinct from the module names as day names. See also: time pattern names.

module inversion.—A specific type of module permutation whereby every other day (or some other subset of days) is reflected about a midday axis, the first period becoming the last, the second period becoming the penultimate, etc. Like module rotation, this technique can be used to avoid monopolization of unpopular clock times by a few time patterns.

module permutation.—Any number of pairwise daily module interchanges (interchanging those horizontal portions of the cycle represented by any two periods, on one day). Each pairwise interchange, and hence any permutation of the daily modules, is transparent with respect to conflicts (which can neither be created nor eliminated in the process). Care must be taken not to fragment any structures; whereas all day permutations preserve structure, most module permutations do not. See also: module inversion, module rotation.

module rotation.—A specific type of module permutation whereby the first period is shifted later and later each succeeding day of the cycle, carrying all the other periods with it, wrapping around the chronologically latest and earliest times. This technique can be
used to distribute unpopular clock times among several time patterns (e.g. 10:00 might be the third period the first day, the second period the second day, the first period the third day, etc.).

partition.--In combinatorial analysis, a partition of the integer \( m \) is defined to be a collection of integers with given sum \( m \), without regard to order. We define a partition of the string \( \overline{m} \) (of \( m \) consecutive modules on any one day) to be an equivalence class of compositions of \( \overline{m} \), characterized solely by the substring lengths and number of occurrences thereof in the member compositions. For example the four partitions of \( \overline{4} \), restricted to substring lengths which are powers of 2, are: \( \overline{4} \), \( 2^2 \), \( 21^2 \), and \( 1^4 \). (Compositions \( 121 \) and \( 12^2 \) are both equivalent to \( 21^2 \).) We define a partition of a time pattern \( t \) to be an equivalence class of compositions of \( t \), characterized solely by the structures and number of occurrences thereof in the member compositions. The number of partitions of \( t \) cycle attainable with a given time pattern system is a measure of the utility of that system.

period.--(See: module)

phase; subject phase.--An intermediate and optional division of a subject, normally according to teaching technique or style of presentation, as when the subject is run in lecture, recitation, laboratory, and/or seminar phases, or when run in large, medium, and/or small encounters. Each subject phase is run in one or more class sections. When students "request a subject", this means exactly one class is to be assigned from each subject phase. Beyond this interpretation, the concept of phase is further useful because multiple class sections
within a phase typically have similar time pattern, instructor, and room requirements, whereas different phases frequently allow or require different time pattern structures, instructors, or rooms.

pictorial representation (of one or more time patterns).—A visual means of identifying time patterns in their actual placement in the cycle matrix. In such figures we label with the same symbol each and every cycle module involved in a time pattern. Alternatively, we may label the area of the cycle space covered by a time pattern. This notation permits the identification of several time patterns via a single cycle diagram.

rectangular time pattern.—A rectangular time pattern is any straight time pattern involving all $d$ cycle days; i.e. any straight time pattern of structure $[n^d]$.

resource.—The resources of a school are its instructors, rooms, and students. An individual instructor, room, or student is referred to as a resource. The generic adjective simplifies referral to: (resource) schedules, (resource) conflicts, (resource) assignments, and any other terms shared by resources. By convention, at most one class can be validly assigned to a resource element during any one cycle module. See also: instructor, room, and student.

resource constraints versus preferences.—A sensitive but important distinction between requirements which must be honored (constraints) and weaker objectives which should be honored all other things being equal (preferences). Violation of a constraint is either disallowed or leads to invalidity. Violation of a preference is tolerated to honor constraints or other preferences. Precision in stating such
objectives is crucial.

resource required free time; resource preferred free time.—A specific portion of the cycle during which the resource cannot be assigned classes (or is preferred not to be assigned classes, in the case of preferred free time). Some resources may have both required and preferred free time, some may have neither. Free time is itself a time pattern. Violation of required free time leads to invalidity.

room.—A resource element, one or more of which satisfies a class's need for physical facilities. The term is intended to cover equipment as well as space and seats. By convention, at most one class can be validly assigned to a room during any one cycle module. A room is categorized as special purpose when its use is dedicated to one or more specific subjects or subject phases due to equipment or location (e.g. laboratories, gymnasiums). A room is categorized as general purpose when there is no impediment (beyond size) to its use in support of a broad variety of different subjects. The set of all rooms can usually be partitioned into interchangeable pools, characterized by location, size, and equipment. Usually a class requires one and only one room.

schedule, individual resource.—The set of all resource assignments \((r, c_1), (r, c_2), \ldots, (r, c_n)\) for the same resource element \(r\). An individual resource schedule is characterized by such quantitative measures as: load, validity, lunch hours missed, etc.; and such qualitative measures as: which days/modules are involved and which are free, etc.

schedule, master.—The master schedule (the net result of the entire school scheduling process) is made up of two collections of
assignments: the time pattern schedule of all time pattern assignments (of time patterns to classes), and the resource schedule of all resource assignments (of resources to classes).

school scheduling.--The problem and the process of acquiring a satisfactory master schedule: the assignment of time patterns to classes and classes to resources (instructors, room, students). This concept is more comprehensive than that of mere "sectioning", whereby resources (sometimes just students) are assigned or "loaded" to classes with predetermined time pattern assignments. The adjective "satisfactory" is used in this definition rather than "optimal" because of the non-quantitative nature of many problem constraints and preferences, making it difficult if not impossible to even define "optimality" let alone achieve it. A basic requirement of the master schedule is that it be feasible.

section.--(See: class; class section)

section balance.--An objective of school scheduling that the distribution of resources (particularly students) over multiple sections of the same subject (phase) be as equitable as possible. This is a default expectation wherever multiple sections exist, assumed desirable except in those few cases where disproportion is deliberately and explicitly requested.

shape (of a time pattern).--The two-dimensional orientation of a time pattern's component modules with respect to each other. A more intrinsic characteristic than time pattern structure. Best denoted via pictorial representation.

similarity (among time patterns).--Two or more time patterns $t_1, t_2, \ldots, t_n$
are said to be similar if and only if their shapes can be transformed into congruent shapes by independent translations of modules within their days. For example, 'M1-3, TW1' is similar to 'M1-3, TW2' and to 'T1-3, WR1' but not to 'MTWR1' nor to 'WL-3, MF1'. Note that the number of days involved, and day separation, are important.

straight time pattern.—A straight time pattern is a time pattern where both of the following are true: (a) the structure can be expressed as \([n^k]\); i.e. there are the same number \(n\) of daily modules on any day involved; and, (b) the exact same consecutive daily modules \(\{M_{j}, M_{j+1}, \ldots, M_{j+n-1}\}\) are repeated on each day involved; i.e. the time pattern name can be expressed \('D_{i_1} D_{i_2} \ldots D_{i_k} M_{j-M_{j+n-1}}'\).

structure (of a time pattern).—The quantitative aspect of a time pattern's configuration of modules over days. Brackets are used to indicate time pattern structure. \([x^y]\) indicates \(x\) consecutive modules on each of \(y\) days. The general form is \([n_{i_1} n_{i_2} \ldots n_{i_j}]\), denoting the structure of any time pattern involving \(i_{1}+i_{2}+\ldots+i_{j}=k\) days, \(n_{i_1}\) consecutive modules on each of \(i_{1}\) days, \(n_{i_2}<n_{i_1}\) consecutive modules on each of \(i_{2}\) other of the remaining \((k-i_{1})\) days, etc.

student.—A resource element, a number of which satisfy a class's need for attendees to be taught. This term contrasts with the concepts of instructor (who does the teaching), room (where the teaching is done), and time pattern (when the teaching is done). By convention, at most one class can be validly assigned to a student during any one cycle module. Students are typically characterized by student year (graduate, undergraduate, first-year, senior, 7th grade), and
typically place demands on a broad cross-section of the curriculum. The appropriate number of students to be assigned to a given class is usually a function of the subject (phase) and is an important consideration.

subject.—A course or area of study, representing a component of the formal curriculum. Subjects may optionally be divided into subject phases (lectures, laboratories, recitations, seminars, etc.); subjects are always subdivided into one or more classes for purposes of scheduling. Students typically request entire subjects (meaning exactly one class is to be assigned per subject phase), whereas instructors or rooms may often be assigned more than one class, or fewer than all phases, of a subject.

supplementary time patterns.—Two or more mutually disjoint time patterns \( t_1, t_2, \ldots, t_n \) are said to be supplementary with respect to a time pattern \( t \) if and only if \( t = \bigcup t_i \). When \( t \) is the entire cycle \( t_{cycle} \), we simply say the \( t_i \) are supplementary. Supplementary time patterns disjointly cover the cycle, and this is one requirement for a time pattern family to be ideal.

time pattern.—A configuration of time used in establishing the schedule. A subset collection of any number of unit cycle modules. Symbolized via lower case italic letters, e.g. \( t, t_1, t_2, a, b^{xy} \) etc. Subscripts provide unique identification. Superscripts, when used, denote time pattern family membership and are chosen to mnemonically suggest structure. Important properties include: cardinality (the number of unit cycle modules, denoted \(|t|\)), structure (the configuration of modules over days, denoted by using brackets, as
when \( [x^y] \) is used to denote \( x \) modules on each of \( y \) days), and shape (the specific orientation of the time pattern's component modules).

Extreme examples of time patterns include: the null time pattern \( \lambda \) \((|\lambda| = 0)\), and the entire cycle \( t_{cycle} \) of structure \([m^d]\) \((|t_{cycle}| = dm)\).

time pattern analysis.--The identification of time pattern families which are appropriate for a particular school. A central and critical aspect of a sound systems approach to school scheduling, in that an error in time pattern design may detrimentally pervade (if not dominate) other aspects of school scheduling, and a proper design may beneficially pervade the system.

time pattern family.--An aggregate collection of any number of related time patterns (usually) enjoying similar structure and (ideally) interchangeable with respect to a priori assignment to classes. Symbolized via upper case italic letters, e.g. \( T, T', A, T^{15}, T^{13} \) etc. Superscripts when used provide unique identification and are usually chosen to mnemonically suggest structure, as when \( T^{42} \) might be a family of \([4^2]\) time patterns or of \([42]\) time patterns. Important properties include: cardinality (the number of member time patterns, denoted \(|T|\)), disjointness or non-disjointness (whether or not the member time patterns are pairwise conflict-free), and degree of coverage (the extent to which the member time patterns collectively represent the entire cycle and its sub-spaces). By convention, \( T^\infty \) denotes any universal family of all possible time patterns with a stated relationship.
time pattern names.—One way of identifying a time pattern, by using $d$
unique graphics as day names and $m$ unique character strings (distinct
from the day names) as module names. Time patterns can thus be
named by strings of day names qualified by module names. See also:
day; day names and module; module names.
time pattern structures (TS$_i$).—Each and every admissible contact
requirement (CR$_i$) in the spectrum allowed by a school has associated
with it one or more admissible time pattern structures (TS$_{i'}$) which
the school allows for time patterns satisfying the CR$_i$. Choice of
the TS$_{i'}$, is a compromise degree of freedom. See also: structure
(of a time pattern).
transection.—A major type of interference between even the most ideal of
time pattern families. A family $Y$ is said to transect another
family $Z$ if and only if for any $y \in Y$ there exist at least two $z_i, z_j \in Z$
such that $y$ partially conflicts with both $z_i$ and $z_j$. Transection
cannot always be avoided, and usually results from structural
differences. When unnecessary, transection should be avoided.
See also: dominance.
validity versus invalidity.—(See: assignments, valid versus invalid)
wrap around.—The treatment of opposing boundaries of the schedule cycle
as contiguous, as in cycle wrap around where the first day of the
cycle is assumed to follow the last day in the same contiguous
fashion that the third day follows the second. Another example
involves module rotation, during which the first period of the day
is assumed to follow directly after the last period. Congruency of
time patterns is affected accordingly. See also: module rotation.
APPENDIX B: REFERENCE LIST OF TIME PATTERN FAMILIES

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</table>
BIOGRAPHICAL NOTE

James Landon Linderman was born in Eau Claire, Wisconsin, on February 15, 1942. He attended Regis High School in Eau Claire, Wisconsin, where he graduated in 1960. Mr. Linderman received a S.B. in Mathematics from the Massachusetts Institute of Technology in 1964. He received a S.M. in Industrial Management (Quantitative Option) from the Massachusetts Institute of Technology in 1966. He pursued his Doctoral Program in Computer Science, supervised by an Interdepartmental Committee, at the Massachusetts Institute of Technology, until 1968 in which year he interrupted his studies to take a full-time appointment as Assistant Director of the M.I.T. Office of Institutional Studies (now the Office of Administrative Information Systems).

In June of 1972, Mr. Linderman was granted a leave of absence from the Office of Administrative Information Systems, in order to complete his Thesis. During his first year of Thesis research, he was awarded an Education Research Center Fellowship, funded by a grant from the Carnegie Corporation of New York.

Mr. Linderman intends to return on a part-time Consulting basis to the M.I.T. Office of Administrative Information Systems, where he has been employed as a student throughout his undergraduate years, as a graduate research assistant during his graduate years, and as a full-time staff member since 1968. It is hoped that summer work will include a full-scale project investigating room utilization at M.I.T. Mr. Linderman is also a part-time Consultant in the area of school scheduling.

Mr. Linderman is an advocate of Wagnerian music drama, gourmet cooking, and applied Christianity.