Exotic $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ Oscillations in Double Chooz

by

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Submitted to the Department of Physics in partial fulfillment of the requirements for the Degree of

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Abstract

In this thesis, we estimate the sensitivity of Double Chooz, a reactor $\bar{\nu}_e \rightarrow \bar{\nu}_e$ experiment, to detect “early” neutrino oscillations based on a three-active plus one-sterile, or $(3 + 1)$, neutrino mixing model by implementing a least-squares fit to the simulated electron antineutrino reactor spectrum\textsuperscript{1}. By comparing the expected spectra from the null hypothesis and the $(3 + 1)$ oscillation hypothesis at the Double Chooz near detector $L = 200$ m, we expect a modest sterile neutrino discovery potential, limited by the $\sim 2\%$ reactor flux uncertainty. This potential may be expanded by employing a Double Chooz-like detector at a very short baseline $L = 6$ m from the reactor. At both baselines, the $p$-value for the null hypothesis was extremely small $p \sim 0.004$, which is compelling evidence for rejecting this hypothesis. Very short baseline antineutrino oscillation reactor experiments may help to resolve the current global incompatibilities between neutrino and antineutrino data sets.

\textsuperscript{1}The analysis was carried out in ROOT and RooFit with MINUIT

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Chapter 1

Introduction

Recent results [1, 2] have shown that despite tension between different data sets, there exists a globally allowed region in parameter space for (3+1) neutrino mixing, a theory with one extra heavy neutrino, in short baseline (SBL) antineutrino experiments. In other words, all antineutrino experiments to date have not ruled out the possibility that there exists a heavy sterile neutrino. This possibility has deep implications for the Double Chooz reactor antineutrino disappearance experiment in France, which aims to measure the neutrino mixing angle $\theta_{13}$ or limit it to $\sin^2(2\theta_{13}) < 0.025$ [3].

Double Chooz will utilize two identical detectors at 200 m and 1.05 km. Since the form of the neutrino survival probability is a sinusoid squared depending on $\Delta m^2 L/E_\nu$, the far detector is placed at a local minimum in the survival probability$^2$. The purpose of the near detector is to precisely measure the full neutrino flux in order to constrain the flux prediction at the far detector (as well as reduce the systematic error on the $\theta_{13}$ measurement)[3]. However, the existence of large mass splittings of order $\Delta m^2 \sim 1$ eV result in “early oscillations” transforming the neutrinos before they have propagated to the near detector. Not only would the diminished flux at the near detector have ramifications for the Double Chooz analysis methodology, but it would give experimentalists a keen opportunity to observe new physics using an

---

$^1$Double Chooz seeks electron antineutrino disappearance, which we denote as $\bar{\nu}_e \to \bar{\nu}_e$

$^2$Assuming realistic values for the parameters, $\sin^2(2\theta_{13}) = 0.1, \Delta m_{13}^2 = 2.5 \times 10^{-3}$ eV$^2$, and the average energy $E_\nu = 2$ MeV
existing experiment.

In this thesis, we present a study of a \((3+1)\) neutrino mixing model near \(\Delta m^2_{14} \simeq 1\) eV\(^2\) and \(\sin^2(2\theta_{ee}) = |U_{e4}|^2(1 - |U_{e4}|^2) \simeq 0.04\) at the Double Chooz near detector. The goal is to determine the sensitivity of the experiment to the parameters associated with the heavier *sterile neutrino* (which means it does not interact through the weak force). The method we use is to simulate the detected reactor \(\bar{\nu}_e\) spectrum under the assumption of \((3+1)\) neutrino mixing multiply it by the survival probability, include detector effects, and finally fit to the probability density function (p.d.f.) by \(\chi^2\) minimization.

The rest of this chapter summarizes the history of neutrino experiments and provides the motivation for studying \((3+1)\) sterile neutrino models given the current conflicted state of neutrino oscillation data.

Chapter 2 provides an overview of the theory of neutrino oscillations. Then, chapter 3 details the Double Chooz experimental design, with emphasis on the detection system. Chapter 4 outlines the basic physics of nuclear reactors.

Chapter 5 gives some statistics background and describes the least-squares method as it applies to this analysis, specifically, the construction of the p.d.f. for the number of detected electron antineutrinos and the \(\chi^2\) function.

Chapters 6 explains the details of the analysis for the sensitivity to a \((3+1)\) neutrino model at the Double Chooz near detector. Chapter 7 proposes a Double Chooz-like detector at a very short baseline of \(L = 6\) m, and repeats the sensitivity analysis. Chapter 8 summarizes this work and discusses the significance of this analysis for Double Chooz and future neutrino experiments.

### 1.1 History of Neutrino Experiments

Wolfgang Pauli first proposed the existence of the neutrino to reconcile the principle of energy conservation with the process of beta decay in 1930 [4].

The data on beta decay had long since perplexed physicists. In 1914, Chadwick
measured the energy spectrum for beta particles coming from the apparent beta decay.

\[
\ ^{214}_{82}\text{Pb} \rightarrow ^{214}_{83}\text{Bi} + \beta^-
\]  

(1.1)

According to energy conservation, if the nuclei are at rest (before and after the decay) and no other particle is produced, then every beta particle should leave the decay with the same energy, \( E_0 \), equal to the difference in rest mass energies between the parent and daughter nuclei. Chadwick’s result, however, was a continuous energy spectrum, showing that the beta particles were almost always ejected with energy \( E_\beta \) less than \( E_0 \). The experiment demonstrated that there was some energy missing [5].

Pauli’s insight was that perhaps another particle (later dubbed the neutrino) was escaping the decay undetected, and taking with it, the missing energy \( E_0 - E_\beta \). Based on this assumption, Fermi developed a very successful theory of beta decay, which predicted that the probability of neutrinos interacting was very small [6, 7]. After two decades without experimental confirmation, Reines and Cowan were able to use the Hanford nuclear reactor to measure the elusive particle [8]. The reactors of the 1950s were expected to produce \( 10^{19} \) antineutrinos per second [5]. Reines and Cowan hoped to use this large flux to overcome the small cross section barrier in order to detect the neutrino via the process of inverse beta decay (see Ch. 3). In 1953, they succeeded in providing the first tentative evidence for the existence of the neutrino, but it wasn’t until an improved experiment ran at the Savannah River reactor in 1958 that they obtained conclusive evidence of the existence of the neutrino[9].

In the following years, a new problem emerged: detecting neutrinos from the Sun. Solar neutrinos are produced in the core of the Sun, through the nuclear reactions that power the Sun [10]. In 1968, Ray Davis made the first measurement of solar neutrinos deep underground in the Homestake mine (to reduce cosmic-ray backgrounds) using an enormous tank of chlorine [11]. The detection reaction involved a neutrino being absorbed by the chlorine to produce argon.

\[
\nu_e + ^{37}\text{Cl} \rightarrow ^{37}\text{Ar} + e
\]  

(1.2)
Davis accumulated argon atoms over a period of several months, but only measured 1/3 of the neutrinos that the standard solar model predicted [12]. Explaining this large deficit became known as the solar neutrino problem [5].

Before the formulation of the solar neutrino problem, it was observed by E. P. Hincks and B. Pontecorvo in 1950 that the muon decays into an electron and two neutrinos [13].

\[ \mu \rightarrow e + \nu + \nu \]  

(1.3)

In 1957, B. Pontecorvo first suggested the possibility of neutrino-antineutrino \( \nu \leftrightarrow \bar{\nu} \) oscillations [14]. This suggestion had the necessary consequence that the neutrino had to be massive [15, 16]. Then in 1962, Z. Maki, M.Nakagawa and S. Sakata (MNS) suggested a two-neutrino mixing model, that is a model with two different types, or flavors of neutrinos [17]. Shortly thereafter, an experiment at Brookhaven produced conclusive evidence of a neutrino associated with the muon, called a muon neutrino \( \nu_\mu \), which was different from the \( \nu_e \) [18]. It wasn’t until 2000 that the DONUT collaboration confirmed the existence of a third neutrino associated with the tau lepton, \( \nu_\tau \) [19].

The proposed solution put forth by MNS in [17] and Pontecorvo in [20] to the solar neutrino problem, was that electron neutrinos produced at the Sun were transforming into a different flavor, such as a muon neutrino, due to quantum mechanical mixing. The muon neutrino doesn’t interact via inverse beta decay and thus was not being detected.

Besides nuclear reactions, cosmic ray interactions in the Earth’s upper atmosphere also produce neutrinos. Pion decay followed by muon decay

\[ \pi \rightarrow \mu + \nu_\mu \]

\[ \downarrow \]

\[ e + \nu_e + \nu_\mu \]  

(1.4)

result in an expected ratio of two muon neutrinos for every electron neutrino coming from the atmosphere, known as atmospheric neutrinos. In addition, due to the
weakness of neutrino interactions, it was thought that equal numbers of atmospheric neutrinos should be detected from above as from below (from the other side of the Earth).

In 1989, the Kamiokande\(^3\) II experiment provided the first confirmation of the results of the Homestake experiment (the deficit of solar neutrinos) and measured the ratio of electron neutrinos to muon neutrinos from the atmosphere [21]. The Kamiokande experiment was intended to search for proton decay in the Kamioka mine in Japan. This experiment used a water detector to measure the particles' Čerenkov radiation. This form of radiation occurs when a particle traverses a medium at faster than the speed of light, causing lagging rings of light as the particle travels (like a Mach cone in a sonic boom). Solar and atmospheric neutrinos were detected by their elastic scattering off the water's electrons

\[
\nu_e + e^- \rightarrow \nu_e + e^-
\]

by observing the Čerenkov rings of the recoil electron. This reaction is sensitive to all three flavors \(\nu_e, \nu_\mu,\) and \(\nu_\tau,\) but with reduced sensitivity to \(\nu_\mu\) and \(\nu_\tau.\) Simultaneously, another proton decay experiment, the IMB\(^4\) experiment, was running that was also sensitive to atmospheric neutrinos. Both Kamiokande and IMB measured a deviation from 2 in the ratio of muon-to-electron atmospheric neutrinos observed, indicating either an excess of electron-like events or a deficit of muon-like events from the atmosphere [22].

The Super Kamiokande experiment succeeded the Kamiokande experiment with a water detector 25 times larger in the same mine. In 1998, Super Kamiokande found that the atmospheric deficit was in muon neutrinos coming from the other side of the Earth [23]. Further, Super Kamiokande measured the deficit of neutrinos from the Sun with the highest statistics of any solar experiment to date [24].

Finally, in 2002, the SNO\(^5\) collaboration presented compelling evidence that the

\(^3\)Kamioka Nucleon Decay Experiment
\(^4\)Irivne, Michigan, Brookhaven Experiment
\(^5\)Sudbury Neutrino Observatory
solar neutrino problem was due to neutrino flavor change. Using a 1 kiloton volume of heavy water, D$_2$O, SNO measured solar neutrinos through the rates of the three interactions$^6$:

\[
\begin{align*}
\nu_e + d &\rightarrow p + p + e^- \quad \text{(CC)} \\
\nu_x + d &\rightarrow p + n + \nu_x \quad \text{(NC)} \\
\nu_x + e^- &\rightarrow \nu_x + e^- \quad \text{(ES)}
\end{align*}
\]

In the CC interaction, which is sensitive to only the electron flavor neutrino, SNO measured a deficit, whereas in the NC interaction, which is sensitive to all three flavors equally, SNO's measurement is consistent with the Standard Solar Model [25, 12].

The final source of neutrinos to be tapped was that of accelerators [26]. In principle, accelerator neutrino experiments can control both the neutrino beam energy and the distance to the detector, leading to more refined experimental conditions. Neutrino beams are created through the decay of pions and kaons and thus are mostly $\nu_\mu$ or $\bar{\nu}_\mu$ with a small $\nu_e$ background. The accessible experimental channels are $\nu_\mu$ disappearance, $\nu_e$ appearance, and $\nu_\tau$ appearance$^7$. In 1996, the short baseline experiment LSND$^8$ measured $\bar{\nu}_e$ appearance in a $\bar{\nu}_\mu$ beam with $20 \text{ MeV} \leq E_\nu \leq 60 \text{ MeV}$ and $L = 30 \text{ m}$ for an $L/E_\nu \sim 1$. They observed an excess consistent with $0.2 \text{ eV}^2 \leq \Delta m^2 \leq 10 \text{ eV}^2$ over a range of $0.003 \leq \sin^2 2\theta \leq 0.03$. The best fit corresponded to $\Delta m^2 = 1.2 \text{ eV}^2$ and $\sin^2 2\theta = 0.003$ [27].

Other accelerator experiments have probed a different region of parameter space. Two long baseline experiments, MINOS and K2K, designed to be sensitive to the allowed atmospheric oscillation region, observed oscillation signals with $\Delta m^2_{23} \sim 3 \times 10^3 \text{eV}^2$ and $\sin^2 2\theta_{23} \sim 1.0$ despite having different values of $L$ and $E_\nu$ (they had the same $L/E_\nu$) [28, 29].

In 2001, the MiniBooNE$^9$ experiment, designed to confirm or refute the LSND re-

---

$^6$Charged Current (CC), Neutral Current (NC), and Elastic Scattering (ES)

$^7$Oscillation searches can be categorized as appearance or disappearance. In an appearance analysis, one searches for an excess of a particular flavor of neutrino in a beam of a different flavor (such as $\nu_e$ in $\nu_\mu$), while in a disappearance analysis one searches for a reduced rate of interactions of a particular flavor with respect to the expectation (usually in a beam of the same flavor).

$^8$Los Alamos Scintillator Neutrino Detector

$^9$Mini Booster Neutrino Experiment
1.1. HISTORY OF NEUTRINO EXPERIMENTS

result, began running at Fermilab. The MiniBooNE detector (500 tons of CH$_2$ Čerenkov spherical detector) was placed at $L = 550$ m from the neutrino beam ($E_\nu \sim 700$ MeV) for an $L/E_\nu \sim 1$. MiniBooNE’s initial search for $\nu_e$ appearance gave a null result, consistent with no oscillations [30]. Subsequent searches for $\nu_\mu$ and $\bar{\nu}_\mu$ disappearance and $\bar{\nu}_e$ appearance$^{10}$ also gave null results [31, 32, 33]

Coming full circle (from Reines and Cowan’s experiment), reactors are still a very attractive neutrino source due to their extremely high flux. The CHOOZ experiment set the best current limit on $\theta_{13}$ (see Ch.2) in 1998 with a single detector 1 km from the Chooz nuclear reactor. CHOOZ provides the upper bound $\sin^2 2\theta_{13} > 0.18$ assuming $\Delta m^2_{\text{atm}} = 2.0 \times 10^{-3}$ eV$^2$ (this limit is strongly correlated to the assumed value of $\Delta m^2_{\text{atm}}$) [34, 35].

Multiple reactor experiments also ran at baselines ranging from 10 m to 1 km, but none saw a significant deficit [10]. Finally, KamLAND, the first long baseline reactor experiment, situated in the same Kamioka mine as Kamiokande, became the first reactor antineutrino experiment to detect a deficit of antineutrinos from a reactor [36]. Japan has 55 nuclear reactors along the coast while the Kamioka mine is in the center of the island. Due to this geometry, the 55 reactors can be thought of as one very large antineutrino source at $L \sim 200$ km from KamLAND. This large distance requires that KamLAND have a very massive target: 1 kiloton of liquid scintillator (used to detect inverse beta decay). Interpreted as neutrino oscillations, the best fit to the KamLAND data gives a maximal mixing angle $\sin^2 2\theta = 1.0$ and $\Delta m^2 = 6.9 \times 10^{-5}$ eV$^2$ [37].

The results of all neutrino oscillation experiments until 2008 are shown in figure 1-1. Almost all of the experimental evidence to date, ranging many orders of magnitude in parameter space, favors neutrino oscillations to be the dominant mechanism causing neutrino flavor mutations during neutrino propagation. The three-neutrino PMNS$^{11}$ matrix, defined in Ch.2, embodies the mixing in the leptonic sector and can be parametrized in terms of 3 mixing angles ($\theta_{12}, \theta_{13}, \theta_{23}$) and a complex CP violat-

---

$^{10}$Searches for $\bar{\nu}_\mu$ ($\nu_e$) disappearance (appearance) were conducted in a $\bar{\nu}_\mu$ beam.

$^{11}$Pontecorvo-Maki-Nakagawa-Sakata
ing phase ($\delta$). Our knowledge of this matrix is far from complete—and we are most ignorant of the values of $\theta_{13}$ and $\delta$. Double Chooz is a reactor antineutrino disappearance experiment, which is starting data collection in 2010 at the CHOOZ site, using two identical detectors, aims to minimize this ignorance by measuring $\theta_{13}$ or at the least limit its value down to $\sin^2 2\theta_{13} \leq 0.025$. In order to do so, Double Chooz will need unprecedented precision, including a thorough understanding of the reactor antineutrino flux at the near and far detectors. For this reason, an exploration of the possibility of a heavy sterile neutrino, which would distort the spectrum at the near detector, is necessary.

1.2 Motivation for a $(3+1)$ Model

A global analysis of all short baseline (SBL) neutrino oscillation data sets reveals conflicting results. The most significant tension lies in reconciling the antineutrino and neutrino data. However, fits to antineutrino-only data sets, including appearance and disappearance experiments, are found to be significantly more compatible within a $(3+1)$ sterile neutrino model. In figure 1-2, considering only antineutrino short baseline data sets (except LSND), the allowed 90%, 99%, and 3\(\sigma\) confidence level regions of $\sin^2(2\theta)$ and $\Delta m_{44}^2$ parameter space are plotted [1].

The parameters $\sin^2 2\theta_{\mu e}$ and $\sin^2 2\theta_{\mu \mu}$ in figure 1-2 represent “effective” mixing angles, that relate to the amplitudes of $\nu_\mu \rightarrow \nu_e$ and $\nu_\mu \rightarrow \nu_\mu$ transitions. Likewise a third parameter $\sin^2 2\theta_{ee}$ (not plotted) can be defined (see Eq. 2.12) which relates to the amplitude of $\nu_e \rightarrow \nu_e$ survival. In the same global analysis of all SBL antineutrino experiments except LSND, the best fit values for a $(3+1)$ model include $\sin^2 2\theta_{ee} = 0.043$ and $\Delta m_{44}^2 = 0.91 \text{ eV}^2$. In the rest of this thesis, we will assume the validity of this $(3+1)$ model, in which the parameters take on these values, in order to analyze Double Chooz’s sensitivity to this model.
1.2. MOTIVATION FOR A (3 + 1) MODEL

Figure 1-1: Summary of the regions of $\Delta m^2$ and $\tan^2 \theta$ parameter space favored or excluded by various experiments. From reference [38].
Figure 1-2: The allowed 90%, 99%, and 3σ CL regions from a combined analysis of all antineutrino SBL data sets. The left plot also shows the 90% CL allowed region obtained from a combined analysis of all antineutrino experiments except LSND (KARMEN, BNB-MB(ν)\textsuperscript{13}, Bugey, and CHOOZ). The right plot also shows the 90% CL exclusion limit from [31]. The MiniBooNE \( \bar{\nu}_\mu \) disappearance search excludes the parameter space to the right of the line at 90% CL.
Chapter 2

Theory of Neutrino Oscillations

Neutrino oscillations are the natural phenomena by which a neutrino of one flavor (such as an electron neutrino $\nu_e$) may transform into a neutrino of another flavor (such as a tau neutrino $\nu_\tau$). Theoretically, this occurs if (1) there is a non-zero mass difference between at least two mass eigenstates and (2) the mass eigenstates and the weak, or flavor, eigenstates do not perfectly coincide, that is there is a non-trivial mixing matrix relating the two bases. Currently, the standard model (SM)$^1$, represented in figure 2-1, does not incorporate massive neutrinos.

![Figure 2-1: The three flavors of elementary particles in the Standard Model. The neutrinos are in the raised blocks. This image is taken from Ref. [39].](image)
In the SM (with massless Dirac neutrinos), the neutrino mass eigenstates are degenerate and there is no difference between the mass and flavor eigenstates. Thus there is no mixing between mass and flavor eigenstates, electron number, muon number, and tau number are each strictly conserved in weak interactions, and there can be no neutrino oscillations [26, 40]. However, neutrino oscillations can be treated phenomenologically, by assuming nonzero mass splittings nontrivial mixing. For a full theoretical treatment see [41, 42, 40].

In general, if there are $N$ neutrino eigenstates of definite mass $\nu_1, \nu_2, \nu_3, \ldots$ with masses $m_1, m_2, m_3, \ldots$ and $N$ neutrino eigenstates of definite flavor $\nu_e, \nu_\mu, \nu_\tau, \ldots$, then an $N \times N$ unitary mixing matrix $U$, known as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, characterizes how these states overlap. Assuming the mass difference is too small to distinguish one mass eigenstate $\nu_j$ from another, each neutrino of definite flavor $\nu_\alpha$ (where $\alpha = e, \mu, \tau, \ldots$) produced by a source is the coherent superposition:

$$|\nu_\alpha\rangle = \sum_{j=1}^{N} U_{\alpha j}^* |\nu_j\rangle$$

In quantum mechanics, free particle states $|\Psi(x,t)\rangle$ obey the Schrödinger equation, which dictates how a state evolves in time [43].

$$i \frac{\partial |\Psi(x,t)\rangle}{\partial t} = H |\Psi(x,t)\rangle$$

where $H$ is the Hamiltonian. Suppose a neutrino $\nu_\alpha$ is born at position $x = 0$ and time $t = 0$ in the lab frame, and the $\nu_j$ component has momentum $p_j$ and energy $E_j$. Then, after a time $t$, the neutrino is in the state$^2$:

$$|\nu(x,t)\rangle = \sum_{j} U_{\alpha j}^* |\nu_j\rangle e^{ip_jx} e^{-iE_jt}$$

$^1$The standard model is a gauge theory of the strong ($SU(3)$) and electroweak ($SU(2) \times U(1)$) interactions with the gauge group $SU(3) \times SU(2) \times U(1)$. It does not take gravitation into account.

$^2$using natural units $\hbar = c = 1$
Since neutrinos are very light, they travel close to the speed of light. So, we may use the fact that $m_j \ll p_j$ to write $E_j = \sqrt{p_j^2 + m_j^2} \approx p_j + \frac{m_j^2}{2p_j}$. To very good approximation, we also have $p_j \approx E_\nu$ the beam energy. To zeroth order in the neutrino masses, we can approximate $t \approx x = L$, the fixed length from the source to the detector (in natural units) [44].

If we invert Eq. 2.1 to express $|\nu_j\rangle$ as a combination of $|\nu_\alpha\rangle$'s then we see

$$|\nu(L)\rangle = \sum_{\alpha'} \left[ \sum_j U_{\alpha j}^* e^{-im_j^2 L/2E_\nu} U_{\alpha' j} \right] |\nu_{\alpha'}\rangle$$  \hspace{1cm} (2.4)

We project the time evolved ket in Eq. 2.4 onto the detected neutrino bra to find the oscillation amplitude that a neutrino born as $\nu_\alpha$ will be detected as $\nu_\beta$.

$$\text{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \langle \nu_\beta | \nu(L) \rangle
= \sum_j U_{\alpha j}^* e^{-im_j^2 L/2E_\nu} U_{\beta j}$$ \hspace{1cm} (2.5)

Finally, we square Eq. 2.5 to find the oscillation probability. It is convenient to express the distance traveled $L$ in terms of meters, $E_\nu$ in MeV, and the squared mass differences $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$ in eV$^2$. Then the general probability that a neutrino born as $\nu_\alpha$ will oscillate to $\nu_\beta$ as a function of $L$ is

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha \beta} - 4 \sum_{i < j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2 \left( \frac{1.27 \Delta m^2_{ij} L}{E_\nu} \right) + 2 \sum_{i < j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin \left( \frac{2.54 \Delta m^2_{ij} L}{E_\nu} \right) \hspace{1cm} (2.6)$$

where $\Re$ and $\Im$ denote the real and imaginary components, respectively [38].
2.1 Sterile Neutrino Formalism

The standard theory of three neutrinos can be expanded by adding one or more sterile neutrinos, which do not couple to the weak vector bosons. We call a theory with three active and one sterile neutrino, a \((3+1)\) model. In this thesis, we will assume there is a fourth flavor state \(\nu_s\) and a fourth mass state \(\nu_4\), and that \(m_4 \gg m_3\), as portrayed in figure 2-2.

![Figure 2-2: The mass hierarchy in our \((3+1)\) neutrino mixing model. The approximate composite structure is represented by the colored fill patterns. White represents the sterile neutrino \(\nu_s\), which comprises only a small fraction of the standard three neutrino mass eigenstates.](image)

To determine the entries of the mixing matrix \(U_{\alpha j}\) in Eq. 2.6, we must address how to parametrize the unitary mixing matrix \(U\). To transform from the basis of flavor eigenstates to the basis of mass eigenstates requires a four-dimensional rotation matrix involving complex phases to account for the possibility of CP violation. A general rotation in four dimensions can be realized with six two-dimensional rotations, \(R_{ij}\) in the \(i - j\) plane. For SBL experiments assuming a \(3+1\) mixing model, a convenient parametrization is given in reference [45]. This form reduces to the standard three-dimensional matrix by setting \(\theta_{14} = 0\).

\[
U_{\text{SBL}}^{(3+1)} = R_{14}(\theta_{14})R_{24}(\theta_{24})R_{34}(\theta_{34})R_{23}(\theta_{23}, \delta_3)R_{13}(\theta_{13}, \delta_2)R_{12}(\theta_{12}, \delta_1)
\]  

(2.7)
We notice that each term in the probability formula oscillates as a sine squared with argument $1.27\Delta m^{2}_{12}L/E_{\nu}$. At the beam energy and short distance under consideration at Double Chooz, $L/E_{\nu} \sim 10$, we may ignore the standard mass differences $\Delta m^{2}_{12}, \Delta m^{2}_{13}, \Delta m^{2}_{23} \lesssim 10^{-3}$ compared to the extra mass splittings $\Delta m^{2}_{14} \sim 1$. Effectively, we are setting $m_{1} = m_{2} = m_{3}$. Equivalently, we could say that the undetectable angles and phases $(\theta_{12}, \delta_{1}), (\theta_{23}, \delta_{2}), (\theta_{13}, \delta_{3})$ decouple from the oscillation formula. Therefore, we ignore this physically degenerate three-dimensional subspace in the matrix $U^{(3+1)}_{SBL}$:

$$
U^{(3+1)}_{SBL} = \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} & U_{e4} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} \\
U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} \\
U_{s1} & U_{s2} & U_{s3} & U_{s4}
\end{pmatrix}
$$

where $s_{nm} = \sin \theta_{nm}$ and $c_{nm} = \cos \theta_{nm}$. Furthermore, at Double Chooz we are looking for $\bar{\nu}_{e}$ disappearance. CP-violating phases can only be detected in appearance channels, so we may neglect the complex phases altogether. To convert $\nu$ oscillation probabilities to $\bar{\nu}$ oscillation probabilities, we assume $CPT$ invariance holds [38].

$$
P(\bar{\nu}_{\alpha} \rightarrow \bar{\nu}_{\beta}) = P(\nu_{\beta} \rightarrow \nu_{\alpha}) \quad (2.9)
$$

In this case, the oscillation formula in Eq. 2.6 for $\bar{\nu}_{e}$ disappearance becomes:

$$
P(\bar{\nu}_{e} \rightarrow \bar{\nu}_{e}) = 1 - 4 \sum_{j=1}^{3} |U_{ej}|^{2}|U_{e4}|^{2} \sin^{2} \left(\frac{1.27\Delta m^{2}_{14}L}{E_{\nu}}\right) \quad (2.10)
$$

Using the fact that the mixing matrix is unitary so that $|U_{e1}|^{2} + |U_{e2}|^{2} + |U_{e3}|^{2} + |U_{e4}|^{2} =$
then substituting in the actual terms we get:

\[
P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - 4|U_{e4}|^2 (1 - |U_{e4}|^2) \sin^2 \left( \frac{1.27 \Delta m^2_{14} L}{E_\nu} \right)
\]

\[
= 1 - 4 s_{14}^2 c_{24}^2 c_{34}^2 (1 - s_{14}^2 c_{24}^2 c_{34}^2) \sin^2 \left( \frac{1.27 \Delta m^2_{14} L}{E_\nu} \right)
\]

\[
\equiv 1 - \sin^2(2\theta_{ee}) \sin^2 \left( \frac{1.27 \Delta m^2_{14} L}{E_\nu} \right)
\]

(2.11)

where in the last equation we defined a new mixing angle \( \theta_{ee} \) by the equation

\[
\sin(\theta_{ee}) = \sin(\theta_{14}) \cos(\theta_{24}) \cos(\theta_{34})
\]

(2.12)

To demonstrate the qualitative differences between the standard three-neutrino theory and our (3 + 1) model, refer to figures 2-3 and 2-4. In figure 2-3, the \( \bar{\nu}_e \) survival probability is plotted as a function of distance from the reactor \( L \), assuming a monoenergetic \( E_\nu = 2 \text{ MeV} \) (characteristic energy of antineutrinos from nuclear reactors). The blue vertical lines indicate the positions of the Double Chooz near detector \( L = 200 \text{ m} \) and far detector \( L = 105 \text{ m} \). Thus, assuming these values for the parameters, it is clear why the Double Chooz near and far detector positions were chosen—the far site is at a maximum in the oscillation probability, whereas at the near site, the neutrinos haven’t had the chance to oscillate yet.

On the other hand, in figure 2-4, the result of additional mixing angles is a superimposed interference pattern. The “extra” oscillations have a small amplitude, but a much higher “frequency” (since it is inversely proportional to the very large \( \Delta m^2_{14} \sim 1 \text{ eV}^2 \)). In other words, within this (3 + 1), the neutrinos do have the chance to oscillate away before they reach the near detector. We now turn to describing the details of the Double Chooz detector.
2.1. STERILE NEUTRINO FORMALISM

The survival probability $P(P_{\nu_e}, \text{chosen})$ comes from [38, 3].

Figure 2-3: The survival probability $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ for the standard theory. The parameter values chosen come from [38, 3].

Figure 2-4: The survival probability $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ for a $(3+1)$ oscillation model. The values chosen for the standard five parameters (three mixing angles and two squares mass differences) are from [38, 3], while the remaining four parameters (three mixing angles and one squared mass difference) were chosen to agree with the best-fit results within a $(3+1)$ model from [1, 2].
Double Chooz

Chapter 3

Double Chooz is a reactor electron antineutrino disappearance experiment to run near the Chooz power plant in France. It aims to measure the neutrino mixing angle $\theta_{13}$, a basic parameter in the standard theory of neutrino mixing. Over three years of data collection, Double Chooz should be able to explore the range of $\sin^2 2\theta_{13}$ from 0.2 to 0.03. To better model the flux of reactor neutrinos and reduce the systematic errors, data collection will proceed in two stages: stage 1 will utilize a far detector at 1.05 km from the reactor (in the existing Chooz experiment hall) and stage 2 will employ an additional identical near detector at 200 m. In the standard theory of three neutrinos, the probability of $\nu_e$ oscillation $P(\bar{\nu}_e \rightarrow \bar{\nu}_x)$ is very small at 200 m, while it becomes nearly maximal at 1 km (see figure 2-3). Therefore comparing the two datasets allows for cancellation of systematic errors producing a more precise measurement of $\theta_{13}$ [3].

3.1 Detection Concept

The detector design is optimized to detect reactor $\bar{\nu}_e$ with very high efficiency. The detector consists mostly of Gadolinium-loaded liquid scintillator (LS) surrounded by photomultiplier tubes (PMTs), which covert the scintillation light into an electrical signal. Inverse beta decay (IBD), in which a $\bar{\nu}_e$ interacts with a proton to produce a
positron and a neutron, is the main detection reaction[3].

\[
\bar{\nu}_e + p \rightarrow e^+ + n
\]  

(3.1)

This process, represented by the Feynman diagram in figure 3-1, has an energy threshold of \( E_\nu \gtrsim 8.5 \text{ MeV} \). The positron scintillates until it annihilates with an electron to produce two 0.511 MeV \( \gamma \)'s. This is known as the prompt signal. The neutron thermalizes (i.e. loses kinetic energy through interactions with its environment) and captures on the Gadolinium (which has an exceptionally high thermal neutron cross section). This leads to an excited state of the Gadolinium, which then generates a cascade of \( \gamma \)-rays of total energy \( \sim 8 \text{ MeV} \). On average, the neutron capture occurs \( \sim 30 \mu s \) after the IBD event, so this is called the delayed signal.

\[
n + \text{Gd} \rightarrow \text{Gd}^* \\
\downarrow \\
\text{Gd} + \sum_i \gamma_i
\]

The coincidence of the prompt and delayed signals is the signature of a \( \bar{\nu}_e \) in Double Chooz [46].

Figure 3-1: Feynman diagram of inverse beta decay. The time axis points left-to-right.
3.2 Multi-volume Detectors

The Double Chooz experiment set the current trend in $\theta_{13}$ experimental design: using several, multi-volume (layered), cylindrical, small detectors [47]. The Double Chooz far detector is displayed in figure 3-2.

**Target and γ-catcher**

The far detector will consist of a target cylinder with a volume of 10.3 m$^3$. This volume is filled with liquid scintillator loaded with gadolinium (Gd-LS) at a concentration of approximately $1 \text{ g/ℓ}$. Surrounding this is a γ-catcher volume (21.5 m$^3$), which also contains LS (but no Gd). This scintillating buffer is necessary to effectively measure the γ-rays which constitute both the prompt and delayed signals from $\bar{\nu}_e$ events. Each of these volumes is contained in an acrylic shell, known as a vessel. The vessels will be transparent to photons with wavelengths above 400 nm, tightly contain the LS in both regions without leaks for 10 years, and be chemically compatible with the LS for at least 5 years. The last of these is the strongest constraint since no modification of the liquid properties of the LS nor any degradation of the acrylic material can be tolerated during the data collection.

**Non-scintillating buffer and PMT support**

A non-scintillating buffer encapsulates the double vessel (target and γ-catcher). This buffer, a 105 cm thick region of non-scintillating mineral oil (114.2 m$^3$), serves to decrease the level of accidental background (mainly from PMT radioactivity). A final vessel of 3 mm thick stainless steel sheets and stiffeners surrounds this non-scintillating buffer. A total of 390 (10 inch) PMTs are mounted in a uniform array on the interior surface of the buffer vessel.

**Inner and outer veto systems**

The inner veto is a 50 cm thick region filled with about 80 m$^3$ of LS for both the near and far detectors. Another stainless steel vessel (thickness 10 mm) houses the inner veto region and supports an additional 78 PMTs. At the far detector, the outer veto will be an active proportional tube tracking system that will identify and locate “near-miss” cosmogenic muons. At the near detector, the outer veto system will
consist of plastic scintillator strips with $x/y$ measurement capabilities. The reason for using different outer veto systems lies in the fact that the far and near detector sites have different overburdens: the far site has about 300 meters-water-equivalent (m.w.e.), whereas the near site has about 114 m.w.e. Shielding at the far site will consist of 15 cm steel (weighing approximately 300 tons).

To summarize, the Double Chooz experiment is equipped to detect the unique signal of $\bar{\nu}_e$ IBD events with very high efficiency ($\sim 61\%$ at the far detector and $\sim 44\%$ at the near detector) using multi-volume detectors [3]. Additionally, two identical detectors will be deployed at different baselines from the Chooz reactor (described in detail in the following chapter) to minimize the systematic error on the final measurement of $\theta_{13}$. 

Figure 3-2: The Double Chooz far detector.
Chapter 4

Nuclear Reactors

Nuclear reactors are constant, copious sources of electron antineutrinos up to energies of about 8 MeV. The high flux and low systematic uncertainty associated with the absolute flux ($\lesssim 2\%$) make reactors optimal for precision oscillation tests [3]. The production of $\bar{\nu}_e$'s is actually the result of a chain of nuclear reactions. In general, reactors are fueled by a highly radiocative isotope, which undergoes fission. Fission is the process by which a heavy nucleus of an element is split into two lighter nuclei, often releasing free neutrons or photons. These lighter nuclei, called fission products, undergo other nuclear reactions during which electron antineutrinos are eventually produced. It is the beta decay of the daughters of these fission products that lead to electron antineutrinos.

$$n \rightarrow p + e^- + \bar{\nu}_e$$  \hspace{1cm} (4.1)

The Chooz nuclear power station consists of two twin pressurized-water (PWR) reactors of the most recent generation (N4). The core of each reactor is made up of 205 fuel assemblies. The vessel is filled with pressurized water ($p = 155$ bars) at temperature ranging from $280^\circ C$ to $320^\circ C$. The water acts as a neutron moderator and cooling element [35].
4.1 Reactor Neutrino Production

Electron antineutrinos are produced by the two nuclear reactors located at Chooz-B nuclear power station operated by the French company Electricité de France (EdF) in partnership with the Belgian utilities Electrabel S.A./N.V. and Société Publique d’Electricité corés.

As mentioned, each N4 PWR has a thermal power of 4.27 GWth, a downtime of about one month per year, and a total of 205 fuel assemblies. Each fuel assembly is a square lattice structure of completed, pressurized fuel rods, which contain enriched uranium dioxide \( \text{UO}_2 \) encased in Ziracaloy tubes about 4 m long [48]. The fuel is mainly composed of the nonfissile, abundant isotope \( \text{U}^{238} \), which is enriched with \( \sim 5\% \) of the fissile isotope \( \text{U}^{235} \). During the burn-up process the \( \text{U} \) isotopes breed \( \text{Pu}^{239} \), and \( \text{Pu}^{241} \). Three of these, \( \text{U}^{235} \), \( \text{Pu}^{239} \), and \( \text{Pu}^{241} \), may undergo fission with just thermal neutrons, while \( \text{U}^{238} \) is only fissionable by fast neutrons. On average each fission releases 6 \( \bar{\nu}_e \)'s, which, upon reaching the detector and being detected, typically have energies between 2 and 8 MeV\(^1\). Together, both reactors emit about \( 10^{21} \bar{\nu}_e \text{s}^{-1} \) isotropically at full power [49].

Since each branch (full decay chain leading to \( \bar{\nu}_e \)) of each isotope has a different \( \bar{\nu}_e \) spectrum, the actual spectrum is a superposition of the different possible spectra. We use the “conversion” method to determine the \( \bar{\nu}_e \) spectrum, which is based on the fact that the antineutrino and the electron share the available energy \( E_0^i \) (also known as the endpoint energy for the \( i^{\text{th}} \) branch) [50]. When possible, the \( \beta \) spectrum is determined experimentally. This is the case for \( \text{U}^{235} \), \( \text{Pu}^{239} \), and \( \text{Pu}^{241} \), while Monte Carlo simulation is used for the \( \beta \) spectrum of \( \text{U}^{238} \).

\[
S_\beta(E_\beta) = \sum_i a_i S_\beta^i(E_\beta, E_0^i, Z) \quad (4.2)
\]

where \( a_i \) is the amplitude of the \( i^{\text{th}} \) branch, and \( S_\beta^i \) is the differential electron yield

\(^1\)This includes the fact that in order to be detected the energy of the neutrino must be above the inverse beta decay energy threshold
4.2. FISSION RATES AND DRAGON

Figure 4-1: Relative fission contribution for the four dominant isotopes that produce $\bar{\nu}_e$ as a function of reactor operating time [40].

per fission, which depends on the energy of the electron $E_\beta$, the endpoint energy $E_{0i}^i$, and the average charge $\bar{Z}$ of the $\beta$-decaying nuclei. From this, the $\bar{\nu}_e$ spectrum can be constructed.

$$S_\nu(E_\nu) = \sum_i a_i S_\beta^i (E_{0i}^i - E_\beta, E_{0i}^i, \bar{Z})$$

On the other hand, it has been found that we may parametrize $S_\nu$ on an isotope-by-isotope basis. Thus, the $\bar{\nu}_e$ spectrum (differential neutrino yield per fission) from each isotope $\ell$ is

$$S_\ell(E_\nu) = \exp \left[ \sum_{k=0}^{2} a_k^\ell E_\nu^k \right]$$

where $k$ is the power of $E_\nu$ and the coefficients $a_k^\ell$ can be found in reference [51].

4.2 Fission Rates and DRAGON

In order to proceed we must be able to calculate the fission rates $f_\ell(t)$, which are the number of fissions per second for the isotope $\ell$. In general, these fission rates vary as a function of time, since the quantity of each isotopes is changing in time. However, since these fission rates vary slowly, we may take $f_\ell(t) = f_\ell(0)$ to first order.
The calculation of the fission rates can be done using a simulation of the reactor implemented in a lattice code called DRAGON, written by École Polytechnique de Montréal. DRAGON simulates the neutron behavior of a unit cell or fuel assembly in a nuclear reactor [52]. In particular, DRAGON models the neutron transport equation, including neutron leakage, transport-transport, and transport-diffusion, in a user-defined geometry. We use the two-dimensional geometry, given in figure 4-2(a), in which each square cell represents a fuel assembly $16 \times 16$ structure of fuel rods. A schematic of a fuel assembly, as represented in DRAGON, is shown in figure 4-2(b).

(a) The nuclear reactor core map. Each square cell (b) $16 \times 16$ fuel assembly in represents a $16 \times 16$ fuel assembly.

Figure 4-2: Two-dimensional geometry of the nuclear reactor.

After simulating the reactor in DRAGON, the fission rates $f_\ell(t)$ over a long period of time can be calculated. Multiplying by the neutrinos per fission $S_\ell(E_\nu)$ and summing over the isotopes gives a double differential for the number of electron antineutrinos produced $N_\nu$.

$$\frac{d^2 N_\nu}{dE_\nu dt} = \sum_\ell f_\ell(t) S_\ell(E_\nu)$$  \hspace{1cm} (4.5)

Noting as before that we can take $f_\ell(t) \equiv f_\ell$ to be constant over a short period of time $\Delta t$, we can calculate the shape of the expected reactor antineutrino spectrum at the detector by taking into account the probability that an antineutrino will interact
4.2. FISSION RATES AND DRAGON

at the detector,

\[
\frac{dN_\nu}{dE_\nu} = \int_0^{\Delta t} dt \sum_\ell f_\ell(t) S_\ell(E_\nu) \sigma(E_\nu)
\]

\[
\propto \sum_\ell f_\ell S_\ell(E_\nu) \sigma(E_\nu)
\]

(4.6)

where \(\sigma(E_\nu)\) is the cross section for inverse beta decay as a function of the energy of the electron antineutrino. In the low energy limit, the cross section for inverse beta decay can be written as a function of the energy \(E_e\) and the momentum \(p_e\) of the outgoing positron [35].

\[
\sigma(E_e) = \frac{2\pi^2 h^3}{m_\nu c^7 f \tau_n} p_e E_e
\]

(4.7)

where the free neutron decay phase-space factor is \(f = 1.71465(15)\) [53] and the neutron lifetime is \(\tau_n = 885.7(8)\) s [42]. We recall that the energy of the neutrino and the positron are related by

\[
E_\nu = E_e + (M_n - M_p) + \mathcal{O}(E_\nu/M_n)
\]

(4.8)

where \(M_n\) and \(M_p\) are the masses of the neutron and proton respectively. Thus we may rewrite Eq. 4.6 in terms of the observable positron energy \(E_e\) as

\[
\frac{dN_\nu}{dE_e} \propto \sum_\ell f_\ell S_\ell(E_\nu) \sigma(E_e)
\]

(4.9)

where the \(E_\nu = E_\nu(E_e)\) is an implicit function of the positron energy \(E_e\). Though this equation does not give us an accurate prediction for the total number of detected antineutrinos \(N_\nu\), it does accurately predict the shape of the antineutrino spectrum at the detector. Figure 4-3 shows the \(\bar{\nu}_e\) reactor neutrino spectrum of Eq. 4.9, normalized to one for two different times, \(t = 0\) and \(t = 1015\) days (in blue and red, respectively)\(^2\).

The small difference in shape between these two spectra is justification for using \(f_\ell(t) = f_\ell\) constant fission rates to determine the neutrino spectrum shape for all time.

\(^2\)1000 days is characteristic of the longest continuous running time of a reactor
(a) The $\bar{\nu}_e$ reactor spectrum at $t = 0$ days (in blue) calculated using Eq. 4.9 and DRAGON as a function of $E_e$ and normalized to unity. The normalized reactor spectrum at $t = 1015$ days (in red) is plotted as well.

(b) The ratio of $\bar{\nu}_e$ reactor spectra at $t = 0$ days and $t = 1015$ days. Horizontal line at 1 is plotted (black dotted line).

Figure 4-3: The $\bar{\nu}_e$ reactor spectrum shape and ratio $(t = 0 \text{ days})/(t = 1015 \text{ days})$
Chapter 5

The Likelihood Model

In general, the likelihood model tests the agreement between theory and experiment. Given a set of very large set of measurements (say $N$ total) of a certain observable $x$ it is often convenient to bin the values into a histogram. In other words, we can group the observations into frequencies of identical observations. If there are $m$ distinguishable values of $x$, denoted $x_1, x_2, ..., x_m$, then one obtains a vector of "binned" counts $\vec{n} = (n_1, n_2, ..., n_m)$. Each $n_i$ (also known as the bin content) is the number of measurements that yielded $x \approx x_i$. So when one sums up all the bin contents, one gets $N$, the total number of measurements.

$$\sum_{i=1}^{m} n_i = N$$  \hfill (5.1)

$N$ is also known as the normalization of the histogram of $x$. The standard deviation, or the "error bar," $\sigma_i$ characterizes the uncertainty in the observed frequency $n_i$. If the distributions of frequency measurements can be approximated as Poisson, then we may use $\sigma_i = \sqrt{n_i}$ as the error [54].

The probability density function (p.d.f.) for an observable $x$ is a nonnegative function which describes the probability of measuring a certain value (or range of values) of the observable. It usually depends on unknown parameters $\vec{\theta} = (\theta_1, \theta_2, ..., \theta_p)$. The
defining property of the p.d.f \( f(x|\bar{\theta}) \) is how it connects to the physical probability.

\[
P(a \leq x \leq b|\bar{\theta}) = \int_a^b f(x|\bar{\theta}) \, dx \tag{5.2}
\]

where \( P(a \leq x \leq b|\bar{\theta}) \) is the probability of measuring \( x \in [a,b] \) given the parameters \( \bar{\theta} \).

Note that Eq. 5.2 implies that the p.d.f must be normalized to unity since the observable \( x \) must have a definite real value (that is, with probability 1).

\[
\int_{-\infty}^{\infty} f(x|\bar{\theta}) \, dx = 1 \tag{5.3}
\]

An extended p.d.f \( F(x,\bar{\theta}) \) is one that has an intrinsic expected normalization \( \hat{N} \) (as part of the model parameters \( \bar{\theta} \)). We may write the expectation values for the number of events in each bin, based on the model parameters, as \( \hat{n}_i = F(x_i,\bar{\theta}) = \hat{N} f(x_i|\bar{\theta}) \).

### 5.1 The Method of Least Squares

We may define a function \( \chi^2(\bar{\theta}) \) of the parameters, which quantifies the level agreement between the extended p.d.f and the data.

\[
\chi^2(\bar{\theta}) = \sum_{i=1}^{m} \left( \frac{n_i - \hat{n}_i}{\sigma_i} \right)^2
\]

\[
= \sum_{i=1}^{m} \left( \frac{n_i - \hat{N} f(x_i|\bar{\theta})}{\sigma_i} \right)^2 \tag{5.5}
\]

The method of least squares proceeds by varying the set of parameters \( \bar{\theta} \) to achieve a minimum. The least squares (LS) estimators for the parameters, denoted by \( \hat{\theta} \), are determined by the minimum \( \chi^2 \).

\[
\frac{\partial \chi^2}{\partial \theta_k} = 0 , \quad k = 1,2,...,p \tag{5.6}
\]
For good agreement between data and theory, the average spread between the data and the model should be about equal to the expected spread (the average error on a data point). In fact, the expectation value for the $\chi^2$ is

$$\langle \chi^2 \rangle = m - p \equiv \nu$$

(5.7)

which is the number of degrees of freedom (d.o.f.). In other words for a good fit the normalized $\chi^2$/d.o.f. should be about equal to one.

The $\chi^2$, being a measured term itself, has a probability density distribution for $\nu$ degrees of freedom given by

$$f_\chi(x^2; \nu) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} (x^2)^{(\nu-2)/2} e^{-x^2/2}$$

(5.8)

The $p$-value, defined as the probability of observing a value of $\chi^2$ that is larger than the particular value for the random sample of $N$ observations with $\nu = m - p$ degrees of freedom is given by the following integral.

$$p_\chi(x^2; \nu) = \int_{x^2}^{\infty} f_\chi(z; \nu) dz$$

(5.9)

For a more thorough treatment of maximum likelihood and least squares methods see Refs. [55] [54].

### 5.2 Constructing the P.D.F. for $dN_\nu/dE_e$

In order to determine the p.d.f. for the observable positron energy $E_e$, we must first make an assumption about the parent distribution. We will proceed by positing two separate hypotheses and constructing the associated p.d.f.'s. The null hypothesis makes only the most general assumptions about the shape and normalization of the model. Specifically, the null hypothesis assumes the data (binned $dN_\nu/dE_e$ counts) are the result of neutrinos propagating to the detector from a reactor at a distance
5.2.1 Null Hypothesis

In order to determine the extended p.d.f. for $E_e$ we first determine the reactor neutrino spectrum shape from Eq. 4.9 as a function of positron energy $E_e$ and normalize it to 1.

\[
f^{\text{null}}(E_e) = \frac{\sum_{\ell} f_{\ell}S_{\ell}(E_e)\sigma(E_e)}{\int_0^\infty dE_e \sum_{\ell} f_{\ell}S_{\ell}(E_e)\sigma(E_e)}\quad (5.10)
\]

Then we simply multiply the p.d.f. in Eq. 5.10 by the proper expected number of events, $N_{\nu}^{\text{null}}$ to get the extended p.d.f.

\[
F^{\text{null}}(E_e|N_{\nu}^{\text{null}}) = N_{\nu}^{\text{null}} f^{\text{null}}(E_e)\quad (5.11)
\]
5.2. CONSTRUCTING THE P.D.F. FOR $D\bar{\nu}_v/DE_E$

5.2.2 (3 + 1) Neutrino Oscillation Hypothesis

The theoretical model outlined in Ch. 2 is the basis for constructing the p.d.f for the number of $\nu_e$ events observed at a detector from a reactor.

We can construct the oscillation hypothesis p.d.f. by multiplying the null hypothesis p.d.f. in Eq. 5.10 by the $\nu_e$ survival probability $P(\bar{\nu}_e \rightarrow \nu_e) \equiv P(E_\nu, \sin^2(2\theta_{ee}), \Delta m_{14}^2, L)$ in Eq. 2.11 and normalizing,

$$f^{(osc)}(E_e | \sin^2(2\theta_{ee}), \Delta m_{14}^2) = \frac{P(E_\nu, \sin^2(2\theta_{ee}), \Delta m_{14}^2, L) \sum_t f_t(t) S(t) E_\nu \sigma(E_\nu)}{\int_0^\infty dE_\nu P(E_\nu, \sin^2(2\theta_{ee}), \Delta m_{14}^2, L) \sum_t f_t(t) S(t) E_\nu \sigma(E_\nu)}$$

Next, we scale by the proper expected normalization $N^{(osc)}_\nu$, to get the extended p.d.f

$$F^{(osc)}(E_e | N^{(osc)}_\nu, \sin^2(2\theta_{ee}), \Delta m_{14}^2) = N^{(osc)}_\nu f^{(osc)}(E_e | \sin^2(2\theta_{ee}), \Delta m_{14}^2)$$

Further, given the expected number events in the absence of (3 + 1) oscillations, $N^{(null)}_\nu$, we can “oscillate away” events according to the survival probability $P(E_\nu, \sin^2(2\theta_{ee})$ to calculate $N^{(osc)}_\nu$. In other words, the fraction

$$\frac{N^{(osc)}_\nu}{N^{(null)}_\nu} = \int_0^\infty dE_\nu P(E_\nu, \sin^2(2\theta_{ee}), \Delta m_{14}^2, L) f^{(null)}(E_\nu) < 1$$

can be determined and it is always less than 1. This makes perfect physical sense since we expect to see a deficit of $\nu_e$ events in oscillation hypothesis.

Note that $L$, the distance from the reactor source to the detector, has the ability to change the shape of the oscillation spectrum (in Eq. 5.12) and the total normalization (in Eq. 5.14).
Chapter 6

Sensitivity at Double Chooz

The Double Chooz near detector, located at 200 m from the reactor neutrino source may be sensitive to the signal from the heavy fourth neutrino (embodied in the oscillation formula \( P(\bar{\nu}_e \rightarrow \nu_e) \)). This signal may be manifest in two distinct ways: (1) changing the total number of \( \bar{\nu}_e \) events and (2) changing the shape of the \( \bar{\nu}_e \) energy spectrum.

For effect (1) to be definite, the deficit of \( \bar{\nu}_e \) events must be greater than the uncertainty on the reactor flux, which is \( \sim 2\% \). In other words, the deficit \( 1 - N_{\nu}^{(\text{osc})}/N_{\nu}^{(\text{null})} \) must be greater than 0.02. Effect (2) is a more promising signal, because no matter how much the overall normalization floats it will never fit a drastically different shape.

As an added detector effect, each p.d.f is convoluted with a Gaussian of standard deviation \( \sigma_E = 0.1 \) MeV, to simulate the detector energy resolution\(^1\).

\[
\mathcal{F}^{(\alpha)}(E_e|\bar{\theta}_{(\alpha)}) = \int \mathcal{F}^{(\alpha)}(E_e - E|\bar{\theta}_{(\alpha)}) \cdot \left( \frac{\exp\left(\frac{-E^2}{2\sigma_E^2}\right)}{\sigma_E \sqrt{2\pi}} \right) dE \quad (6.1)
\]

where \( (\alpha) = (\text{null}) \) or (osc). The effect of this “Gaussian smearing” is to smooth out some of the subtle shape differences, which would be undetectable due to the finite

\(^1\)This is a worst case energy resolution, as Double Chooz expects to resolve finer energies than this [3]
energy resolution of a real detector.

### 6.1 Generating Data

To generate realistic data based on the oscillation hypothesis we first need to set the “truth” values of the oscillation parameters. Reference [1] performs a global fit over short baseline antineutrino experiments in order to estimate various parameters of (3 + 1) and (3 + 2) oscillation models. Specifically, we take the best-fit values of \( \sin^2(2\theta_{ee}) = 0.043 \) and \( \Delta m^2_{14} = 0.91 \text{ eV}^2 \), extracted from a fit of antineutrino short baseline experiments assuming a unitary mixing matrix \( U_{\alpha j} \) for a (3 + 1) model.

Next, we can find the expected value of the parameter \( N^{(\text{null})} \) at \( L = 200 \text{ m} \). The Double Chooz proposal expects the integrated rate of \( \bar{\nu}_e \) events at the near detector to be 161,260 \( y^{-1} \), including detector efficiency, dead time, and reactor off periods averaged over a year[3].

Given the expected values of all the parameters, we may generate the correct number of events corresponding to a full Double Chooz run of 5 years: \( N^{(\text{null})} = 806300 \) and \( N^{(\text{osc})} = 788929 \). This data generation was done using a Poisson process in RooFit. Figure 6-1 displays the null hypothesis p.d.f. and the oscillation hypothesis p.d.f. as well as the data generated from the oscillation hypothesis.

### 6.2 Fitting with \( \chi^2 \) Minimization

The \( \chi^2 \) function in Eq. 5.5 was minimized with respect to the parameters over 25 bins of \( E_e \) using Poisson errors for each bin\(^2\). In the oscillation hypothesis fit there were three adjustable parameter: the overall normalization, the mixing amplitude, and the squared mass difference, \( \delta_{(\text{osc})} = (N^{(\text{osc})}, \sin^2(2\theta_{ee}), \Delta m^2_{14}) \). In the null (no oscillation) hypothesis fit, only one parameter was considered adjustable: the overall normalization \( \delta_{(\text{null})} = N^{(\text{null})} \).

\(^2\)We used ROOT 5.26/00 [56] and RooFit v2-00-05 [57] with MINUIT to construct the p.d.f.'s and \( \chi^2 \) as well as minimize the \( \chi^2 \).
Figure 6-1: The number of events per bin width for 25 bins of positron energy $E_e$ expected (before fitting) for the null hypothesis (blue) and the oscillation hypothesis (red). The data generated from the oscillation hypothesis is also shown (black data points).

Figure 6-2 and table 6.1 show the fit results for both the null hypothesis and the oscillation hypothesis. The null hypothesis p.d.f. has floated down to better fit the generated data. However, due to the slight shape differences that survive the Gaussian smearing, the $\chi^2$/d.o.f. for the null hypothesis is still 1.8 times greater than the $\chi^2$/d.o.f. for the oscillation hypothesis. A particularly significant measure is the $p$-value for each fit. The oscillation hypothesis has a minimum of $\chi^2_{\text{osc}}^{\text{min}} = 23.5$ and a $p$-value of $p^{\text{osc}} = 0.37$, while the null hypothesis has a minimum of $\chi^2_{\text{null}}^{\text{min}} = 46.1$ and a $p^{\text{null}} = 0.0043$. The small $p$-value of the null hypothesis is considered statistically significant and suggests that the null hypothesis is incompatible with the generated dataset [54].
Figure 6-2: The black points represent the generated data (based on the oscillation hypothesis). The red histogram represents the Gaussian-smeared prediction based on oscillation hypothesis. The blue histogram represents the Gaussian-smeared prediction based on null hypothesis. The fitted parameters have values of $\sin^2(2\theta_{ee}) = 0.037 \pm 0.007$ and $\Delta m_{14}^2 = 0.909 \pm 0.003$ eV$^2$, resulting in a $\chi^2_{\text{osc}} = 23.5$ and a $p$-value of $p^{(\text{osc})} = 0.37$. The null hypothesis fit results in $\chi^2_{\text{null}} = 46.1$ and a $p^{(\text{null})} = 0.0043$.

### 6.3 $\Delta \chi^2$ Confidence Levels

To find the confidence levels for our multiparameter fit, we minimize the $\chi^2$ function, keeping $\sin^2(2\theta_{ee})$ and $\Delta m_{14}^2$ fixed at particular values, but allowing $N_{\nu}^{(\text{osc})}$ to vary during the minimization. Thus we are plotting the variation in $\Delta \chi^2$ from the minimum $\chi^2_{\text{min}}$. As shown in [54], the variation in $\Delta \chi^2$ obeys the $\chi^2$ probability distribution for 1 degree of freedom. Recalling that the $\chi^2$ probability density distribution in Eq. 5.8 and setting the degrees of freedom $\nu = 1$, we may calculate the $p$-value using Eq. 5.8. Putting this all together, the probability that a certain region of $\Delta \chi^2$ covers the “true” parameter values is given by $1 - p$.

For example, since $\chi^2/(\nu = 1) \geq 1$ corresponds to 31.7% of the probability, our
### 6.3. $\Delta \chi^2$ CONFIDENCE LEVELS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Truth Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_\nu$</td>
<td>788927</td>
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<tr>
<td>$\sin^2(2\theta_{ee})$</td>
<td>0.043</td>
</tr>
<tr>
<td>$\Delta m^2_{14}$</td>
<td>0.91 eV$^2$</td>
</tr>
</tbody>
</table>

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<tr>
<th>Oscillation Fit Parameter</th>
<th>Fitted Value</th>
<th>Initial Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{N}_\nu^{(osc)}$</td>
<td>788066 ± 892</td>
<td>788929</td>
</tr>
<tr>
<td>$\sin^2(2\theta_{ee})$</td>
<td>0.037 ± 0.007</td>
<td>0.043</td>
</tr>
<tr>
<td>$\bar{\Delta}m_{14}^2$</td>
<td>0.909 ± 0.003 eV$^2$</td>
<td>0.91 eV$^2$</td>
</tr>
</tbody>
</table>

| $\chi^2$/d.o.f = 1.068 |
| $p = 0.37$              |

<table>
<thead>
<tr>
<th>No Oscillation Fit Parameter</th>
<th>Fitted Value</th>
<th>Initial Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{N}_\nu^{(null)}$</td>
<td>787674 ± 887</td>
<td>806300</td>
</tr>
</tbody>
</table>

| $\chi^2$/d.o.f = 1.922 |
| $p = 0.0043$              |

Table 6.1: This table summarizes the pre-fit and post-fit parameter values as well as the goodness of fit $\chi^2$ and $p$-values for $L = 200$ m.

$\Delta \chi^2 < 1$ corresponds to 68.3%. This $\Delta \chi^2 = 1$ defines the 1$\sigma$ confidence level contour, which represents a 68.3% probability of enclosing the true parameter values. Similarly, the confidence level contour for 2$\sigma$ corresponds to the $\Delta \chi^2 = 4$ contour (95.5% probability of enclosing the true parameter values) and 3$\sigma$ corresponds to $\Delta \chi^2 = 9$ contour (99.7% probability of enclosing the true parameter values). Figure 6-3 displays the 1$\sigma$, 2$\sigma$, and 3$\sigma$ confidence level contours in the parameter space of $(\sin^2(2\theta_{ee}), \Delta m^2_{14})$ for the oscillation hypothesis. As one can see, the 2$\sigma$ contour readily covers the true value of the parameters.
Figure 6-3: The allowed 1σ, 2σ, and 3σ confidence levels regions for the $\chi^2$ fit in figure 6-2. The black point represents the best fit $\chi^2$ minimum $\left(\sin^2(2\theta_{ee}), \Delta m^2_{14}\right) = (0.037, 0.909 \text{ eV}^2)$. The star represents the truth (initial) value $\left(\sin^2(2\theta_{ee}), \Delta m^2_{14}\right) = (0.043, 0.91 \text{ eV}^2)$.
Chapter 7

Sensitivity at a Very Near Detector

The results of the previous chapter indicate that the fitted number of events for the null hypothesis \( N^{(\text{null})} \) dropped by 2.3\% (with respect to the expected number of events for the null hypothesis) in order to fit the generated data based on the \((3 + 1)\) oscillation hypothesis. However, this parameter is still within the accepted range given the \( \sim 2\% \) uncertainty on the flux.

However, it can be seen from figure 2-4, that the first minimum of the \( \bar{\nu}_e \) survival probability for a monoenergetic neutrino source with \( E_\nu = 2 \text{ MeV} \) in a \((3 + 1)\) model occurs near \( L \sim 3 \text{ m} \). In fact with the reactor antineutrino energy spectrum shown in figure 4-3, the minimum occurs at approximately \( L = 6 \text{ m} \) from the reactor. Thus if we place a Double Chooz-like detector at the “optimal” distance \( L = 6 \text{ m} \), and repeat the analysis of Ch. 6, we will see an even greater discrepancy between the oscillation hypothesis and null hypothesis reactor neutrino spectra.

There are many inherent difficulties in placing a detector very close to a nuclear reactor with such a high event rate. However, one experiment at the San Onofre Nuclear Generating Station (SONGS) shows that a detector at \( L = 25 \text{ m} \) is possible [58]. The antineutrino flux at the SONGS detector was \( 10^{17} \text{ m}^{-2} \text{s}^{-1} \). To reduce the extremely high muon-induced background rate, a muon veto time of \( t_\mu = 100 \mu\text{s} \) was used to reject \( \bar{\nu}_e \) IBD candidate events that occur within a 100 \( \mu\text{s} \) window of a muon hit. The detector trigger rate above a 1 \text{ MeV} \) threshold and the trigger rate for
the muon veto system were both $\sim 500$ Hz. Using a similar muon veto time at our proposed detector at $L = 6$ m may achieve similar results in reducing background events.

Since the expected rate of events scales as $1/L^2$, the flux is much larger at this short distance. Therefore, we may acquire data over a much shorter period to achieve the same high statistics expected at the Double Chooz near detector for a 5 year data set. Suppose we still expect $N_{\nu}^{(\text{null})} = 806300$ and $N_{\nu}^{(\text{osc})} = 776476$. Then, figure 7-1 shows the pre-fit expected distributions for the number of events per energy bin width for 25 bins of $E_e$ in the null hypothesis and the oscillation hypothesis at 6 m.

![Figure 7-1](image)

Figure 7-1: The number of events per bin width for 25 bins of of positron energy $E_e$ expected (before fitting) for the null hypothesis (blue) and the oscillation hypothesis (red). The data generated from the oscillation hypothesis is also shown (black data points).

Figure 7-2 displays the distributions after minimizing the $\chi^2$ and table 7.1 summarizes the results of the fit, including extracted parameter values. Finally figure 7-3(a) displays the allowed $1\sigma$, $2\sigma$, and $3\sigma$ confidence level regions in the $(\sin^2(2\theta_{ee}), \Delta m_{14}^2)$ parameter space.
Table 7.1: This table summarizes the pre-fit and post-fit parameter values as well as the goodness of fit $\chi^2$ and $p$-values for $L = 6 \text{ m}$.

<table>
<thead>
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<th>Parameter</th>
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<td>$N_\nu$</td>
<td>776474</td>
</tr>
<tr>
<td>$\sin^2(2\theta_{ee})$</td>
<td>0.043</td>
</tr>
<tr>
<td>$\Delta m_{14}^2$</td>
<td>0.91 eV$^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Oscillation Fit Parameter</th>
<th>Fitted Value</th>
<th>Initial Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{N}_\nu^{(osc)}$</td>
<td>775269 ± 880</td>
<td>776476</td>
</tr>
<tr>
<td>$\sin^2(2\theta_{ee})$</td>
<td>0.039 ± 0.008</td>
<td>0.043</td>
</tr>
<tr>
<td>$\Delta m_{14}^2$</td>
<td>0.839 ± 0.061 eV$^2$</td>
<td>0.91 eV$^2$</td>
</tr>
<tr>
<td>$\chi^2$/d.o.f</td>
<td>0.782</td>
<td></td>
</tr>
<tr>
<td>$p_X$</td>
<td>0.75</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>No Oscillation Fit Parameter</th>
<th>Fitted Value</th>
<th>Initial Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{N}_\nu^{(null)}$</td>
<td>775240 ± 880</td>
<td>806300</td>
</tr>
<tr>
<td>$\chi^2$/d.o.f</td>
<td>1.938</td>
<td></td>
</tr>
<tr>
<td>$p_X$</td>
<td>0.0038</td>
<td></td>
</tr>
</tbody>
</table>

After minimization, the fitted normalization for the null oscillation hypothesis drops by a full 3.9% with respect to the expected normalization. Furthermore, the null oscillation fit yields a $\chi^2$/d.o.f. of 2.5 times the $\chi^2$/d.o.f. from the oscillation hypothesis fit. Under these conditions, there is an even greater difference in the $p$-values for each fit. The oscillation hypothesis has a minimum of $\chi^2_{\text{osc}}^{\text{min}} = 23.5$ and a $p$-value of $p_{\text{osc}} = 0.75$, while the null hypothesis has a minimum of $\chi^2_{\text{null}}^{\text{min}} = 46.1$ and a $p_{\text{null}} = 0.0038$.

This wide discrepancy implies that under the right circumstances, our very near detector would be able to identify the $(3+1)$ model as more compatible with reality than the null hypothesis. As this example shows, the $(3+1)$ model is strongly favored when the difference between the number of expected events in the null hypothesis and the number of measured events differ by much more than 2% (the uncertainty in the flux).
Figure 7-2: The black points represent the generated data (based on the oscillation hypothesis). The red histogram represents the Gaussian-smearred prediction based on oscillation hypothesis. The blue histogram represents the Gaussian-smearred prediction based on null hypothesis. The fitted parameters have values of $\sin^2(2\theta_{ee}) = 0.039 \pm 0.008$ and $\Delta m^2_{14} = 0.839 \pm 0.061 \text{eV}^2$. 

\[
\sin^2(2\theta_{ee}) = 0.039 \pm 0.008 \\
\Delta m^2_{14} = 0.839 \pm 0.061 \text{eV}^2 \\
L = 6 \text{ m}
\]
(a) The allowed $1\sigma$, $2\sigma$, and $3\sigma$ confidence level regions for the $\chi^2$ fit in figure 7-2. The black point represents the best fit $\chi^2$ minimum $(\sin^2(2\theta_{ee}), \Delta m^2_{14}) = (0.039, 0.839) \text{ eV}^2$. The star represents the truth (initial) value $(\sin^2(2\theta_{ee}), \Delta m^2_{14}) = (0.043, 0.91 \text{ eV}^2)$.

(b) A wider view of the allowed parameter space in figure 7-3(a). Note that the $3\sigma$ contour extends to the end of the physical region.

Figure 7-3: Allowed $1\sigma$, $2\sigma$, $3\sigma$ confidence levels regions in $(\sin^2(2\theta_{ee}), \Delta m^2)$ parameter space at 6 m.
Chapter 8

Outlook and Conclusions

In this thesis, we examined the consequences of a plausible $(3 + 1)$ oscillation model at Double Chooz as well as a hypothetical very short baseline detector. We take into account much of the existing $\bar{\nu}$ data (but not the $\nu$ data) at short baselines in choosing the truth values of the $\bar{\nu}_e$ oscillation survival amplitude, $\sin^2(2\theta_{ee}) = 0.043$, and the relevant squared mass difference, $\Delta m^2_{14} = 0.91 \text{ eV}^2$. Using this oscillation hypothesis we generated 5 years of data at the Double Chooz near detector distance of 200 m. Then using the method of least-squares to minimize a $\chi^2$ function, we compared the fit results of the oscillation hypothesis and the null (or no oscillation) hypothesis. We concluded that the oscillation hypothesis fit both the rate and shape significantly better. Further, the $p$-value, or the probability of obtaining a $\chi^2$ at least as extreme as the one that was observed, for the null hypothesis is extremely small at 0.0043, which strongly suggests rejecting the null hypothesis. However, the normalization of the null hypothesis $N^{(\text{null})}_\nu$ floated down by only 2.3% in order to fit the data. This is just within the bounds of the known error on the reactor flux ($\sim 2\%$), so these conditions may not give us a conclusive signal of a heavy fourth neutrino at the Double Chooz near detector.

We repeated this analysis at the more optimal distance of 6 m. This time, $N^{(\text{null})}_\nu$ dropped 3.9% in order to fit the data. This is nearly twice the value of the flux uncertainty. This effect coupled with the even smaller null hypothesis $p$-value of
0.0038, may be enough to conclusively determine the existence of a heavy fourth neutrino. The added bonus of using a detector at such a short baseline is the extremely short period of data collecting necessary to reach this confidence level.

Though, to first approximation, these results are accurate and demonstrate that Double Chooz shows some promise in its sensitivity to "early oscillations," several effects may limit Double Chooz's ultimate sensitivity (and that of the very near detector). Firstly, at the very short baseline detector, the extremely high background rates would have to be tempered with corresponding selection cuts (such as the muon time veto discussed in Ch. 7) on the $\bar{\nu}_e$ interaction candidate sample. Luckily, though, the extremely high flux coming from the reactor at this distance, $\gtrsim 10^{18}\bar{\nu}_e$ m$^2$ s$^{-1}$, ensures that we would not be statistics-limited.

One approximation that may be rectified in future analysis is accounting for the burn-up affect in the reactor, embodied in the time-varying fission rates $f_\ell(t)$. Another condition currently neglected (at both $L = 200$ and $L = 6$ m) is that of the detector's dimensions. To correct for this, we would need to average the oscillation probability $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$, used in the p.d.f.'s (Eqs. 5.11, 5.13), over the finite extent of the detector (instead of evaluating at a single point).

Despite these ignored effects, our analysis suggests that Double Chooz would observe a noticeable distortion in the antineutrino spectrum at the near detector in the presence of a heavy fourth neutrino (within a neighborhood of our assumed parameter values). This signal we believe should be sufficient to prompt a more thorough investigation with a Double Chooz-like (i.e. highly sensitive to $\bar{\nu}_e$ IBD) detector at a much shorter baseline $L \simeq 10$ m from the reactor. Furthermore, if a definitive $(3 + 1)$ signal were not observed even under these ideal circumstances, then we could effectively rule out such a model and improve our understanding of neutrino flavor change physics. Such investigations, whether they turn up a positive or null result, may resolve the tension, and at times, outright contradictions, present in the current global neutrino and antineutrino oscillation data.
Bibliography


