The L- to H-Mode Transition and Momentum Confinement in Alcator C-Mod Plasmas

by

John Reel Walk, Jr.

Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of

 Bachelor's of Science

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2010

© John Reel Walk, Jr., MMX. All rights reserved.

The author hereby grants to MIT permission to reproduce and
distribute publicly paper and electronic copies of this thesis document
in whole or in part.

Author

Certified by

Earl S. Marmar

Senior Research Scientist, MIT Plasma Science and Fusion Center

Thesis Supervisor

Certified by

John E. Rice

Principal Research Scientist, MIT Plasma Science and Fusion Center

Thesis Supervisor

Accepted by

David E. Pritchard

Senior Thesis Coordinator, Department of Physics
The L- to H-Mode Transition and Momentum Confinement in Alcator C-Mod Plasmas

by

John Reel Walk, Jr.

Submitted to the Department of Physics on May 7, 2010, in partial fulfillment of the requirements for the degree of Bachelor's of Science

Abstract

Using a spatially-resolving x-ray spectrometer system, toroidal impurity rotation in Alcator C-Mod plasmas was measured. The propagation of the rotational velocity from the edge to the core of the plasma column was measured during the L- to H-mode transition. Momentum transport was measured in both Ohmic and ICRF-heated discharges, which produced EDA and ELM-free H-modes. The momentum transport was modeled by a simplified diffusion model, in which momentum diffusivity was substantially higher than neoclassically predicted values.

Thesis Supervisor: Earl S. Marmar
Title: Senior Research Scientist, Department of Physics

Thesis Supervisor: John E. Rice
Title: Principal Research Scientist, MIT Plasma Science and Fusion Center
Acknowledgements

I would like to thank several people for their support of my thesis work and undergraduate career. First, I would like to thank John Rice for introducing me to fusion work as a UROP student, and for supervising this thesis. I look forward to working with him again in my graduate studies. Likewise, I wish to thank Earl Marmor for his support for the UROP program on C-Mod, and for serving as the Course 8 advisor for this thesis. I owe a great deal to several of C-Mod’s graduate students as well. The assistance of Alex Ince-Cushman and Matt Reinke was invaluable in getting me started in my work on C-Mod. It has been a great pleasure to work with Yuri Podpaly as well; his assistance was invaluable in completing this thesis. I would also like to thank the entire Alcator C-Mod staff for their support of x-ray spectroscopy research on C-Mod. For their support and friendship over my undergraduate career, I owe a great deal to my brothers in the Zeta Psi fraternity. Last but certainly not least, I would like to thank my family for their support in getting me to where I am today, and Aria for the good years to come.

Work on the Alcator C-Mod project is funded by Department of Energy contract number DE-FC02-99ER54512.
Contents

1 Introduction
   1.1 Controlled Fusion Power ........................................ 15
   1.2 The Alcator C-Mod Tokamak .................................... 17
   1.3 Plasma Confinement Modes ...................................... 19

2 Theoretical Background ........................................... 21
   2.1 X-ray Spectroscopy and Velocity Measurement ............... 21
   2.2 Plasma Dynamics and Transport ................................ 24
      2.2.1 Transport .................................................. 25
      2.2.2 Neoclassical Theory ...................................... 26
   2.3 The L-H Transition and Toroidal/Poloidal Flow ............. 28
      2.3.1 H-mode Classification ..................................... 29
      2.3.2 Theories of Toroidal/Poloidal Rotation .................. 30
      2.3.3 Central Rotation and Momentum Input ..................... 32
      2.3.4 Rotation Profiles and Anomalous Momentum Transport ... 33

3 Experimental Observations and Conclusions .................... 39
   3.1 Experimental Setup ............................................. 39
      3.1.1 Spectrometer ............................................... 39
      3.1.2 Computer Model ............................................ 40
   3.2 Observations .................................................... 41
      3.2.1 EDA H-Modes .............................................. 41
      3.2.2 ELM-Free H-Modes ...................................... 47
3.3 Conclusion ................................................. 52
  3.3.1 Results ............................................... 53
  3.3.2 Future Work ........................................ 53

A Alcator C-Mod parameters ................................ 55

B Argon Emission Spectra .................................... 57
  B.1 He-like Spectra ......................................... 58
  B.2 H-like Spectra .......................................... 59

C Energy Confinement Time .................................. 61
List of Figures

1-1 The Alcator C-Mod Tokamak, shown in cutaway through the cryovessel. The shaping magnet coils are also visible. 16

1-2 Cross-section view of C-Mod. The last closed magnetic flux surface is shown (the ribbon line) within the vacuum chamber. 18

2-1 Bragg reflection off a crystal lattice. 22

2-2 Sight lines in a Rowland Circle. Paths from two slit sources are shown corresponding to separate focused points on the detector. 23

2-3 The spherical Johann crystal-detector configuration allowing spatial (sagittal direction) and spectral (meridional direction) resolution. 24

2-4 Plasma parameters during an EDA H-mode shot. Shown are the plasma current $I_p$ in MA, ICRF heating power (MW), particle density in $10^{20}$ m$^{-3}$, and plasma stored energy in kJ. Parameters are from shot 1070830010. 29

3-1 Plasma stored energy (kJ), toroidal rotation (km/s), and ICRF power (MW) for EDA H-mode in shot 1070830010. 42

3-2 Plasma stored energy (kJ), toroidal rotation (km/s), and ICRF power (MW) for EDA H-mode in shot 1070830029. 42

3-3 Fitted curve on the toroidal velocity for shot 1070830029. The dotted line is the low-$D$ fit, the dashed line the high-$D$ fit, and the dot-dash line the best-fit model. 43
3-4 Rotation profile for EDA H-mode on shot 1080213004. (a) The fitted curve for the diffusive model. The dotted line is the low-$D$ fit, the dashed line the high-$D$ fit, and the dot-dash line the best-fit model. (b) Plasma stored energy, toroidal rotation, and ICRF power.

3-5 Rotation profile for EDA H-mode on shot 1080213015. (a) The fitted curve for the diffusive model. The dotted line is the low-$D$ fit, the dashed line the high-$D$ fit, and the dot-dash line the best-fit model. (b) Plasma stored energy, toroidal rotation, and ICRF power.

3-6 Average and final $\tau_E$ vs. measured $\tau_\phi$ for EDA H-modes. The stars indicate low-fit values of $\tau_\phi$, the diamonds high-fit values, and the plus-signs best-fit values.

3-7 Plasma stored energy (kJ), toroidal velocity (km/s), and ICRF heating power (MW) for ELM-free H-mode in shot 1070830012.

3-8 Plasma stored energy (kJ), toroidal velocity (km/s), and plasma current (MA) for ELM-free H-mode in shot 1080124003.

3-9 Rotation profile for ELM-free H-mode on shot 1080124004. (a) The fitted curve for the diffusive model. The dotted line is the low-$D$ fit, the dashed line the high-$D$ fit, and the dot-dash line the best-fit model. (b) Plasma stored energy, toroidal rotation, and plasma current.

3-10 Rotation profile for ELM-free H-mode on shot 1080124030. (a) The fitted curve for the diffusive model. The dotted line is the low-$D$ fit, the dashed line the high-$D$ fit, and the dot-dash line the best-fit model. (b) Plasma stored energy, toroidal rotation, and plasma current.

3-11 Average and final $\tau_E$ vs. measured $\tau_\phi$ for ELM-free H-modes. The stars indicate low-fit values of $\tau_\phi$, the diamonds high-fit values, and the pluses best-fit values.

3-12 Average and final $\tau_E$ vs. measured best-fit $\tau_\phi$. The average $\tau_E$ are represented by squares, while final $\tau_E$ are represented by X's.

B-1 He-like argon emission spectrum.
B-2  H-like argon emission spectrum.

C-1  Energy confinement time $\tau_E$ for EDA H-mode discharge 1070830010.
    The H-mode duration is from approximately 0.7 s to 1.5 s.

C-2  Energy confinement time $\tau_E$ for ELM-free H-mode discharge 1080124006.
    The H-mode duration is from approximately 0.6 s to 1.5 s.
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Relevant He-like and H-like argon emission spectra</td>
<td>40</td>
</tr>
<tr>
<td>3.2</td>
<td>$D_\phi$ [m²/s] values for EDA H-mode discharges</td>
<td>45</td>
</tr>
<tr>
<td>3.3</td>
<td>$\tau_\phi$ [s] values for EDA H-mode discharges</td>
<td>45</td>
</tr>
<tr>
<td>3.4</td>
<td>Average and final $\tau_E$ [s] values for EDA H-mode discharges</td>
<td>46</td>
</tr>
<tr>
<td>3.5</td>
<td>$D_\phi$ [m²/s] values for ELM-free H-mode discharges</td>
<td>50</td>
</tr>
<tr>
<td>3.6</td>
<td>$\tau_\phi$ [s] values for ELM-free H-mode discharges</td>
<td>50</td>
</tr>
<tr>
<td>3.7</td>
<td>Average and final $\tau_E$ [s] values for ELM-free H-mode discharges</td>
<td>50</td>
</tr>
<tr>
<td>A.1</td>
<td>C-Mod reactor parameters</td>
<td>55</td>
</tr>
<tr>
<td>B.1</td>
<td>He-like argon lines in the wavelength range 3.94 - 4.00 Å</td>
<td>58</td>
</tr>
<tr>
<td>B.2</td>
<td>H-like argon lines in the wavelength range 3.72 - 3.80 Å</td>
<td>59</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

The world's energy demands are growing rapidly due to expanding population pressure and increased demand from consumers in large developing nations like China and India, even as worldwide supplies for traditional fossil-fuel energy sources dwindle. With increasing cost, technical difficulty, and political fallout involved in tapping remaining petroleum and natural-gas resources, the pressure to develop new, sustainable energy sources is greater than ever. Hydroelectric, wind, and solar power have a certain "green" appeal, but each is subject to strict limitations increasing the difficulty of implementing them on a national scale. Wind and hydroelectric power suffer constraints on viable locations, while solar power requires tolerance of large variations in output. Though this makes the low emissions and high efficiency of traditional (fission) nuclear power plants, these likewise suffer from difficulties in acquiring fissible fuel and storage of radioactive reactor waste. In this light, power production through nuclear fusion is an attractive concept for satisfying the world's growing energy demands in an efficient, environmentally sound manner.

1.1 Controlled Fusion Power

In any plasma, simple statistical-mechanics arguments dictate that the vast majority of ion collisions will be restricted to Coulomb interactions, as the particles do not have sufficient energy to overcome the potential-energy barrier for fusion to occur.
In order for a reactor to be efficient, it must ensure (1) that collisions have a high enough average kinetic energy that a relatively high fraction of collisions result in fusion, and (2) that the collisions are frequent enough that that fraction gives a substantial output of fusion reactions. At a macroscopic scale, this corresponds to maintaining the plasma at high temperature and density, while maintaining thermal separation between the plasma and the innermost physical wall of the device.

There are two general concepts for confined fusion devices - inertial confinement and magnetic confinement. Inertial confinement utilizes lasers to compress and implode pellets of fuel, producing pulses of high-density plasma before dissipation. This type of device is currently being operated at the National Ignition Facility at Lawrence Livermore National Laboratory. Magnetic confinement, on the other hand, uses a variety of electromagnet configurations to create closed magnetic surfaces capable of trapping plasmas.

Figure 1-1: The Alcator C-Mod Tokamak, shown in cutaway through the cryo vessel. The shaping magnet coils are also visible.

A common configuration for magnetic confinement devices is called the tokamak, from the Russian abbreviation for “toroidal chamber with magnetic coils.” The Alcator C-Mod tokamak is shown in cutaway in figure 1-1. In such machines, nested
closed magnetic flux surfaces are created within a toroidal chamber, trapping plasma particles. A central solenoid induces current in the plasma to generate Ohmic heating and induce poloidal magnetic fields for confinement. The plasma resistance, which determines the power absorbed from Ohmic heating, is given by

\[ \eta = \frac{\pi e^2 m^{\frac{1}{2}}}{(4\pi\varepsilon_0)^2 (KT_e)^{\frac{3}{2}}} \ln \Lambda \]  

(1.1)

with \( m \) the ion mass, \( K \) the Boltzmann constant, \( T_e \) is the electron temperature, and \( \ln \Lambda \approx 15 \) is the Coulomb Logarithm, a plasma-dynamic constant. Note that the dependence of the resistance (and thus the Ohmic heating efficiency) is inversely dependent on the temperature; thus Ohmic heating alone becomes inefficient for plasma temperatures above a few keV. However, for a reactor plasma to be self-sustaining, the plasma must obey the Lawson Criterion

\[ p\tau_E = \frac{24}{E_\alpha} \frac{T^2}{\langle \sigma v \rangle} \]  

(1.2)

for plasma pressure \( p \), energy confinement time \( \tau_E \), alpha-particle heating energy \( E_\alpha \), and a collisionality parameter \( \langle \sigma v \rangle \). For ideal conditions, this requires temperatures in the range of 15-20 keV [1], thus additional heating methods are required.

Neutral-beam heating fires a very high energy (\( \sim 100 \) keV) stream of neutral atoms (often the same atom as the dominant ion species in the plasma) into the plasma. The beam dissipates energy into the plasma by collisions and electron exchanges between the higher-energy beam atoms and the lower-energy ions. Wave resonance uses guided beams of radiation at one or more resonant frequencies, such as the resonant frequency of electron or ion gyromotion about a magnetic-field line, to heat the plasma through Landau damping.

### 1.2 The Alcator C-Mod Tokamak

This experiment was conducted entirely on the Alcator C-Mod tokamak, the third iteration of the tokamak experiment at MIT's Plasma Science and Fusion Center.
Alcator C-Mod is a compact high-field device with major radius 0.67 meters and minor radius 0.21 meters. Toroidal magnetic fields can reach as high as 8 Tesla, and discharges reach a total plasma current $I_p$ of 2 MA [2]. Ion-cyclotron (ICRF) wave-resonance heating is available, and was examined in this experiment in addition to Ohmic heating. An additional wave-heating system tuned to the Lower Hybrid resonant frequency has been developed, but was not used in the discharges examined for this experiment. Typical discharges last ~ 2 seconds, with plasma current of 1 MA and toroidal magnetic field of 5-6 T. A more complete compilation of C-Mod’s operating parameters is given in Appendix A.

Figure 1-2: Cross-section view of C-Mod. The last closed magnetic flux surface is shown (the ribbon line) within the vacuum chamber.

A number of observational tools examining radiation effects in the plasma has been installed on the machine. These include Thomson scattering diagnostics to probe plasma temperature and density, reflectometer arrays measuring radial-resolution plasma density, and (most interestingly for this experiment) an array of x-ray spectrometers capable of collecting temperature, impurity density, and rotational-velocity data. In particular, this experiment used the High Resolution x-ray spectrometer with Spatial Resolution (HiReX SR, read “Senior”) to measure toroidal flow velocity in the
plasma. HiReX SR uses a spherically-bent quartz crystal to reflect the x-ray signal from the plasma onto an array of detectors. The system is arranged in the Johann configuration, spatially resolving the signal around the emission lines of hydrogen-like and helium-like argon impurities at 3.7 - 4.0 Å. By measuring the Doppler shift from predicted frequencies of prominent emission lines, SR allows the calculation of impurity flow velocity at any point in its field of view [3, 4]. The design and use of the HiReX SR spectrometer system is discussed more fully in the Background (2.1) and Setup (3.1.1) sections.

1.3 Plasma Confinement Modes

During discharges on the Alcator C-Mod tokamak, the plasmas are held in two general confinement modes. In low-energy confinement (L-mode), the particle and energy confinement times $\tau_p$ and $\tau_E$ deteriorate over the course of the discharge, resulting in low values of $\beta$. This is contrasted with high-energy confinement (H-mode), first documented on the ASDEX machine in 1981 [5], a sudden increase in plasma density and pressure occurs; the resulting plasma exhibits greatly improved particle and energy confinement and a higher value of $\beta$. The mechanism by which the H-mode forms is complex and not fully understood, as the observed values for energy confinement are substantially lower than those predicted by neoclassical theory.

During H-modes, a sharp increase in toroidal and poloidal rotation in the plasma has been observed [6, 7, 8]. This rotational momentum increase propagates from the edge of the plasma column inwards to the core. It has been theorized that edge velocity shear plays a role in the improved confinement in H-mode plasmas [9] by suppressing edge turbulence. This paper will examine the diffusion of toroidal rotation momentum inward from the initial edge increase at the transition between L- and H- modes.
Chapter 2

Theoretical Background

2.1 X-ray Spectroscopy and Velocity Measurement

The principle behind x-ray spectroscopy for fluid velocity measurement is elegantly simple: highly-ionized impurities in the plasma emit peaked radiation profiles at predictable wavelengths in the x-ray spectrum. The observed spectrum measured by spectrometers trained on the plasma can then be used for three key measurements. By means of a Doppler shift calculation on the observed mean wavelength of a known peak, coordinated flow velocity of the impurities may be measured. The peak will have some measurable width due to Doppler broadening, allowing impurity (and by extension plasma) temperature calculations. Finally, the total intensity of spectra associated uniquely with a certain impurity ion species allows measurement of impurity density, useful for intrinsic impurities in the plasma from the main ion species and ash byproducts. For this study, only the Doppler-shift velocity data are necessary; the shifted frequency is given by

$$\lambda = \sqrt{\frac{1 + \beta}{1 - \beta}} \lambda_0$$

(2.1)

where $\beta = v/c$, $v$ is the source velocity, and $\lambda_0$ is the rest wavelength.

The wavelengths for emission lines from hydrogen-like ions are given approximately by the Rydberg formula,
\[ \frac{1}{\lambda} = R_\infty Z^2 \left( \frac{1}{n^2} - \frac{1}{n'^2} \right) \]  

(2.2)

where \( Z \) is the nuclear ion charge, \( R_\infty \) is the Rydberg constant, and \( n, n' \) are the energy levels of the transition. The wavelength of the emission from a given transition may be found by the simple relation

\[ E = h\nu = hc/\lambda \]  

(2.3)

for speed of light \( c \) and Planck's constant \( h \). Emission wavelength calculations for non-hydrogen-like ions are well-established as well.

In particular, HiReX SR examines spectra from hydrogen-like and helium-like argon, \( \text{Ar}^{+17} \) and \( \text{Ar}^{+16} \), corresponding to spectra in the range 3.7-4.0 Å, using a spherically bent crystal to reflect incoming X-rays onto detectors arranged in the Johann configuration [3]. The basic principle for bent-crystal spectroscopy is Bragg reflection, in which EM waves are reflected off the lattice layers of a crystal.

Reflection off of different layers in a sheet-pattern lattice creates patterns of constructive and destructive interference at different angles for different wavelengths; as a wave reflected off a deeper surface travels an additional path length determined by the surface spacing \( d \) and the incident angle \( \theta \), path lengths that are an integer multiple of the wavelength \( \lambda \) will constructively interfere. Simply put, the relation for Bragg reflection is defined as
\[ n\lambda = 2d \sin \theta \] (2.4)

From the relation above, a given wavelength may only reflect off the crystal when incident at the correct angle. A flat crystal has the effect of splitting an unfocused radiant source (where all wavelengths are incident from a range of angles) into separated spectra. A curved crystal, on the other hand, can focus a range of wavelengths from an approximate point source into a single point when arranged in a configuration known as the Rowland Circle, shown in figure 2-2.

![Figure 2-2: Sight lines in a Rowland Circle. Paths from two slit sources are shown corresponding to separate focused points on the detector.](image)

The 3-D Johann configuration on HiReX SR expands this concept to a spherically bent crystal, enabling both spatial and spectral resolution [3]. A spherically bent crystal separates spectra by wavelength horizontally (as with a cylindrically curved crystal), while also maintaining vertical spatial resolution of the plasma by retaining distinct lines-of-sight into the plasma (though the spectrum is inverted through the crystal), creating a “slice” cross-section of the plasma profile. Earlier spectrometer models utilized only a cylindrically-curved crystal for spectral resolution; the HiReX SR system was the first to demonstrate accurate measurement of local plasma...
parameters using a spherical Johann configuration. The orientation of spatial- and spectral-resolution planes is shown in figure 2-3 [10].

![Diagram of spherical Johann crystal-detector configuration]

Figure 2-3: The spherical Johann crystal-detector configuration allowing spatial (sagittal direction) and spectral (meridional direction) resolution.

*Ab initio*, spectrometers in such a configuration do not have absolute calibration - that is, the channel on the spectrometer modules does not correspond inherently to a particular wavelength. The spectrometer is periodically calibrated using locked-mode discharges, in which nonrotating modes act as a "brake" of sorts on toroidal rotation. In the absence of any toroidal velocity, there is no Doppler effect shifting the spectral lines, and the correspondence between spectrometer channel and wavelength can be established. The zero toroidal rotational velocity is confirmed for locked-mode calibration shots with independently calibrated spectrometer systems.

### 2.2 Plasma Dynamics and Transport

In general, plasmas are modeled by extended fluid-dynamics (Navier-Stokes) equations governing a two-fluid electromagnetically-active system. These equations are generally intractible; many situations arising in reactor plasma behavior can be effectively modeled by a simplified system known as Magnetohydrodynamics, or MHD. However, the MHD model is insufficient to itself describe turbulent behavior in the plasma edge region or certain transport behaviors, both of which are necessary for this examination.
2.2.1 Transport

At its simplest, one may derive a model for the balance, movement, and eventual loss of physical particles, momentum, and energy in the plasma from the cylindrical (1D) MHD equations. A simple diffusive model gives equations of the form

\[
\frac{\partial Q}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r D \frac{\partial Q}{\partial r} \right) + S(Q, r, t)
\]  

(2.5)

for physical quantity \( Q \), corresponding diffusion coefficient \( D \), and aggregate source/sink term \( S \). By introducing the restrictions that: (1) the plasma passes through quasi-static MHD equilibria (2) Ohmic resistance can be separated into resistivity parallel and perpendicular to the magnetic field and (3) the adiabatic energy MHD equation is modified to include conduction, convection and compression effects. Thus modified, the MHD mass, momentum, and energy conservation equations reduce to

\[
\text{mass: } \frac{\partial n}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (rnv) = 0
\]

(2.6)

\[
\text{momentum: } \frac{\partial}{\partial r} \left( p + \frac{B_z^2}{2\mu_0} \right) + \frac{B_r}{\mu_0} \frac{\partial}{\partial r} (rB + \Theta) = 0
\]

(2.7)

\[
\text{energy: } 3n \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial r} \right) + \frac{2nT}{r} \frac{\partial}{\partial r} (rv) = -\nabla \cdot \mathbf{q} + S
\]

(2.8)

for particle density \( n \), flow velocity \( v \), plasma pressure \( p = 2nT \), and heat flux vector \( \mathbf{q} \), which will be discussed further below. Solving the mass and momentum conservation equations together allows a useful expression for the classical particle diffusion coefficient,

\[
D_{n}^{(CL)} = \frac{2nT\eta_\parallel}{B_0^2}
\]

(2.9)

Particle-collision analysis [1] gives the result

\[
D_{n}^{(CL)} = 2\frac{\nu_{ei}m_eT}{e^2B_0^2} \sim \frac{r_{Le}^2}{\tau_{ei}}
\]

(2.10)

which is in agreement with the fluid-model result to within an minor numerical factor.
The heat flux vector used in the energy balance equation may be expressed as
\[ q = -n\chi \frac{\partial T}{\partial r} e_r \] (2.11)
for energy diffusivity \( \chi \). A similar particle analysis (random-walk) model as used above yields results for the classical electron and ion thermal diffusivities,

\[ \chi_i^{(CL)} = \frac{1}{4} \frac{v_{T_i}^2}{\omega_{ci}^2} \sim \frac{r_{Li}^2}{\tau_{ii}} \] (2.12)
\[ \chi_e^{(CL)} = \frac{1}{4} \frac{v_{Te}^2}{\omega_{ce}^2} \sim \frac{r_{Le}^2}{\tau_{ee}} \] (2.13)

where \( v_{Tj} \) is the \( j^{th} \) species thermal velocity, \( \omega_{cj} \) is the species cyclotron frequency, \( \tau_{jj} \) is the mean collision time, and \( r_{Lj} \) is the species Larmor radius.

Using reasonable parameters for a tokamak plasma and a clever simplification of the heating profile, an approximate scaling relation [1] for the classical energy confinement time \( \tau_E \) may be found:

\[ \tau_E = \frac{3}{2} \frac{\overline{p}V}{P_{\Omega}} = 23 \frac{a^4 n_{20} T_{k}^{3/2}}{I_M^{1/2}} = 2.6 \frac{a^{3/2} B_0^{5/2} I_M^{1/2}}{n_{20}^{3/2}} \] (2.14)

where \( \overline{p} = 2n_{20} \overline{T} \) is the mean plasma pressure, \( V = \sqrt{\frac{T}{T_0}} \) is the square root of the normalized temperature, and \( P_{\Omega} \) is the Ohmic power. The latter expressions use "real" units for a reactor - \( n_{20} \) is density in \( 10^{20} \) particles/m\(^3\), \( T_k \) is temperature in keV, and \( I_M \) is current in MA.

### 2.2.2 Neoclassical Theory

The results in 2.2.1 were derived for a simple straight cylindrical plasma. However, the Coulomb collisions driving classical transport are influenced by particle drift velocities induced by the toroidal geometry in a tokamak. The modifications to the classical theory as described above are known collectively as neoclassical transport theory. In particular, classical transport assumes the Coulomb collision step size is on the order
of a gyro radius $r_L$ (as the particle is trapped within gyro motion about a magnetic flux surface); in a tokamak configuration, however, $\nabla B$ and curvature drifts may force the particle far off its initial flux surface, increasing the collisional step size and increasing transport.

Neoclassical particle drift may be examined by the simplification of a large-aspect-ratio tokamak, in which the ratio of path length $l$ to magnetic field $B$ is approximately equal over sufficiently short distances in both the parallel (along $B$ lines) and strictly poloidal directions:

$$\frac{dl}{B} \approx \frac{dl}{B_0} \approx \frac{dl_p}{B_0}$$  \hfill (2.15)

thus the path length is found in terms of the safety factor $q(r) = \frac{r_B}{R_B}$:

$$l(r) \approx \frac{B_0}{B_\theta} l_p = \frac{B_0}{B_0} \pi r = \pi R_0 q(r)$$  \hfill (2.16)

Motion over this path length at $v_\parallel$ yields the drift half-transit time $\tau_{1/2}$ and transit frequency $\omega_T = \frac{\pi}{\tau_{1/2}}$, which when used in the drift motion equation

$$\Delta r = r_i - r_f = 2\frac{|v_{DR}|}{\omega_T} \cos \theta_0$$  \hfill (2.17)

gives a mean-square step size

$$(\Delta l)^2 = 2\frac{|v_{DR}|^2}{\omega_T^2} \approx 4 \frac{q^2 v_T^2}{\omega^2} \sim q^2 r_L^2$$  \hfill (2.18)

a factor $q^2$ larger than the classically expected result. As the particle diffusion coefficient $D_n$ scales by $(\Delta l)^2$, then the neoclassical coefficient $D_n^{NC}$ likewise scales by $q^2$ over its classical counterpart, simply for passing-particle collisions. Extension of this analysis onto more complex flux surfaces adds an additional factor of $(\frac{R_0}{r})^{3/2}$ to account for trapped particles [1].

Similarly, particle-random-walk arguments using toroidal geometry yield modified results for the thermal diffusivity - as above, the passing-particle component increases diffusivity by $q^2$, while trapped-particle interactions on closed flux surface
orbits require an additional \((\frac{R_o}{r})^{3/2}\) factor.

\[
\chi_e^{(NC)} = 0.89q^2 \left(\frac{R_o}{r}\right)^{3/2} \chi_e^{(CL)} \\
\chi_i^{(NC)} = 0.68q^2 \left(\frac{R_o}{r}\right)^{3/2} \chi_i^{(CL)}
\]  

(2.19)

(2.20)

However, neoclassical theory is based in the extension of modified MHD equilibria onto a toroidal geometry, and thus is unable to account for turbulent effects. Similar scaling arguments as in the classical case approximate the neoclassical energy confinement time as

\[
\tau_E \sim \frac{q^2}{\chi_i^{(NC)}}
\]

(2.21)

for reactor minor radius \(a\). However, this is found to be overly optimistic in light of experimental results.

### 2.3 The L-H Transition and Toroidal/Poloidal Flow

The high-confinement (H-mode) plasma, first observed on the ASDEX tokamak in Germany [5], is marked by rapid improvement of energy and particle confinement with increasing plasma stored energy. Triggering and sustaining H-mode confinement in a reactor enables high-temperature, high-\(\beta\) plasmas with minimal power input into magnetic field energy. Example parameters for an L-H transition are shown in figure 2-4.

The method by which a plasma discharge transitions from L-mode, in which particle and energy confinement degrade with increased plasma stored energy, to the H-mode’s increased energy confinement efficiency is not particularly well understood. Experimentally, a sharp increase in the edge density and temperature gradients and a strong pressure gradient are observed, suggesting that the suppression of turbulent behavior at the edge contributes to H-mode formation. A concurrent increase in
Figure 2-4: Plasma parameters during an EDA H-mode shot. Shown are the plasma current \( I_p \) in MA, ICRF heating power (MW), particle density in \( 10^{20} \text{ m}^{-3} \), and plasma stored energy in kJ. Parameters are from shot 1070830010.

toroidal and poloidal rotation during the LH mode transition is subject to a number of theories as contributing to suppressing edge turbulence. However, no adequate explanation for the mechanism triggering the rotation increase and subsequent momentum diffusion through the plasma has been reached for discharges with no external momentum input, as is the case for C-Mod plasmas.

### 2.3.1 H-mode Classification

For most plasma discharges reaching H-mode confinement in Alcator C-Mod, the dominant behavior is termed an Enhanced \( D_\alpha \) (EDA) H-mode. In EDA discharges, so named for the substantial increase in detected \( D_\alpha \) radiation, a quasi-coherent (QC) electromagnetic mode forms at the edge of the plasma column, which accelerates particle transport across the edge [11]. In normal H-modes, a steep pressure gradient forms at the edge, acting as a transport barrier; the EDA QC mode effectively acts as a “relief valve” for pressure and particle buildup at the edge. Though exhibiting higher particle transport, the EDA mode, by preventing the buildup of particles and
impurities at the plasma edge, enables the maintenance of a steady-state H-mode by limiting MHD instabilities triggered by the pressure gradient. EDA discharges exhibit flattened steady-state rotation in H-mode, with momentum propagating inward from the plasma edge [7].

In contrast with this, certain H-mode discharges are ELM-free, which lack any edge relaxation modes and maintain the steep pressure gradient, acting as an edge transport barrier. ELM-free modes exhibit substantially greater particle confinement. Impurity density in the plasma concurrently builds up rapidly; this triggers a rapid increase in radiated power (exhibiting faster growth than the plasma density), culminating in radiative collapse [11]. In contrast to H-modes, the rotation profiles of ELM-free H-modes exhibit a strong inward momentum pinch, rather than equilibrating into a steady state [8].

In certain H-modes (EDA or not), a region may form within the plasma column (though usually fairly near the edge) in which the ion thermal conductivity is greatly reduced. These internal transport barriers (ITBs) exhibit thermal conductivity approximately at the neoclassical value, triggering a sharply peaked core temperature profile and corresponding high value for $\tau_E$. The phenomenon by which ITBs form is not well understood, even raising difficulty in reliably triggering ITB modes on test discharges; however, the enhanced energy confinement is a very promising feature for sustained H-mode operation.

2.3.2 Theories of Toroidal/Poloidal Rotation

After the observation of H-mode confinement on ASDEX is 1982, a number of theories has been proposed regarding the mechanism of the L-H transition. Early theoretical models suggested that a sudden increase in radial electric field strength ($E_r$ becoming more strongly negative) suppresses turbulent fluctuations at the plasma edge.

Shaing and Crume [12] reject models for smoothly varying $E_r$, instead suggesting a poloidal momentum balance equation with bifurcated solutions, with a shift between bifurcations triggered by $E_r$ shifting to a more negative value and/or a more positive value for $\frac{dE_r}{dr}$. By solving the radial force balance equation for perpendicular flow,
Shaing reaches for the poloidal rotational velocity

\[ v_\theta = v_\phi \left( \frac{B_p}{B} \right) - \left( \frac{cE_r}{B} \right) + \left( \frac{c}{NeB} \right) \frac{dP}{dr} \] (2.22)

where \( B_p \) is the poloidal magnetic field, \( E_r \) is the radial electric field, and \( P \) is the ion pressure. From similar derivation, Groebner et al. [13] give for the radial electric field

\[ E_r = \frac{1}{n_iZ_i e} \nabla P_i - (V_i \times B)_r \] (2.23)

using the ion particle density \( n_i \), ion nuclear charge \( Z_i \), ion pressure \( P_i \), and ion flow velocity \( V_i \). Both derivations agree on the requirement that \( E_r \) and \( v_\theta \) shift preceding the L-H transition, either velocity jumps leading the transition by time scales on the order of 10 ms. Similarly, both Shaing and Crume and Groebner et al. observed similar values for \( E_r \) and \( v_\theta \). By solving the drift kinetic equation for the poloidal viscosity damping, Shaing and Crume establish from the poloidal momentum balance equation

\[ \frac{d}{dt} \left( NM \langle B_p v_\theta \rangle \right) = \frac{1}{c} \left\langle J \times B \cdot B_p \right\rangle - \left\langle B_p \cdot \nabla \cdot \pi \right\rangle - \frac{1}{c} e \Gamma_{orbit} \times B \cdot B_p \] (2.24)

(with poloidal viscous damping \( \langle B_p \cdot \nabla \cdot \pi \rangle \) and particle flux \( \Gamma \)) a bifurcated solution to \( U_p \).

The DIII-D discharges in Groebner et al. exhibit the bulk of the velocity shift perpendicular to the magnetic field, with a 200% change in \( v_\perp \), compared to 25% purely in \( v_\theta \) [13]. While neoclassical estimates for poloidal rotation were too small to explain the increase in \( v_\theta \), the ion-orbit-loss torque, calculated from \( e\Delta r (\partial n/\partial t) B_\phi \), is more than sufficient. On the other hand, Kim et al. [14] suggest that assumptions on the coupling between impurity and primary-ion rotation are incorrect, and attempt to rectify neoclassical models with observed poloidal and toroidal rotation.

Biglari et al. [9] suggests that coupling between poloidal flow and turbulent ra-
dial scattering, rather than poloidal flow alone as in Shaing, account for suppressing edge turbulent effects. More significantly, Biglari’s results were independent of directionality in the radial electric field or its shear. The shear measurements in Biglari et. al. agree in magnitude with those in Groebner et. al.

2.3.3 Central Rotation and Momentum Input

Theoretical models strongly correlate the poloidal momentum shear with the trigger mechanism for the L-H transition. However, there appears to be a strong link between the increased energy confinement in H mode with toroidal momentum confinement.

Initial observations of coordinated toroidal rotation in tokamak plasmas were in discharges with some external source of angular momentum, most commonly from directional neutral-beam-injection heating. Small (\(\sim 10\) km/s, an order of magnitude lower than NBI results) toroidal rotation has been observed in purely ohmic plasmas in L-mode steady states [15]. L-mode toroidal rotation is in good agreement with neoclassical theory; impurity toroidal rotation \(v_{\phi}^I\) runs with the electron toroidal drift counter to the plasma current direction, with magnitude given by

\[
v_{\phi}^I = 4.19 \times 10^2 f \frac{Z_i}{\sqrt{\mu_i}} \frac{V_i T_i^{3/2}}{R n_i} \text{ (km/s)} \tag{2.25}
\]

Where \(Z_i, n_i, \mu_i\) are the nuclear charge, particle density \((10^{20} m^{-3})\), and ion mass \((\text{amu})\) of the primary ion species, \(V_i\) is the loop voltage, \(T_i\) is the ion temperature \((\text{keV})\), and \(f\) is a scaling relation \(\sim 1\) between primary and impurity ion species.

During ohmic discharges, the toroidal countercurrent rotation is greatest (in magnitude) at the beginning of the shot \((\sim 60\) km/s), and settles to a lower steady state magnitude \((\sim 20\) cm/s) within 100-200 ms.

In plasma discharges reaching H-mode confinement with isotropic ICRF heating (so there is no net source of angular momentum), substantially increased toroidal rotation has been observed [6]. Over a period of around 100 ms (on the order \(\tau_E\)) after the LH transition (as indicated by the increase in plasma stored energy), the impurity toroidal rotation increases to velocities on the order of \(\sim 100\) km/s in the co-current
direction, reversing its direction from the L-mode rotation. A strong correlation between the core toroidal rotation and the plasma stored energy is observed; the correspondence between the increase in stored energy $\Delta W$ and the impurity rotation parameter $\Delta n_I V_\phi$ leads to the conclusion that stronger toroidal rotation correlates with improved energy confinement.

### 2.3.4 Rotation Profiles and Anomalous Momentum Transport

As discussed above, the correlation between core rotation and energy confinement is fairly well-established. However, observations of toroidal rotation along multiple lines-of-sight to establish toroidal rotation in the plasma column between the core and the edge show momentum propagation profiles contrary to the predictions of neoclassical theory. The toroidal rotation increase in the LH transition is observed to begin as an edge effect (observed at $r/a = 0.6$ in C-Mod) and propagate inwards to the core at an anomalously high rate [7].

The increase in momentum diffusion is of particular interest for theoretical treatments of ion-temperature-gradient instabilities ($\eta_i$ modes), which produce theoretically equal levels of thermal and momentum diffusivity and could explain the overly rapid heat transport in tokamaks. Scott et. al. [16] found for unbalanced-NBI-heated discharges on the TFTR tokamak approximately that $\chi_{\phi} \approx 1.5 \chi_i$ (the momentum and thermal diffusivities, respectively) for the outward propagation of momentum from the torque-driven core. The diffusivities are found by analysis of the total ion- and electron-heat fluxes and the radial momentum flux, given by

\begin{align*}
Q_i &= -\chi_i n_i \nabla T_i + \frac{3}{2} \Gamma_i T_i = -\chi_i^{eff} n_i \nabla T_i \\
Q_e &= -\chi_e n_e \nabla T_e + \frac{3}{2} \Gamma_e T_e = -\chi_e^{eff} n_e \nabla T_e \\
\gamma &= -\chi_\phi \left( \sum_j n_j m_j \right) \nabla v_\phi + \Gamma (m_h) v_\phi = -\chi_\phi^{eff} \left( \sum_j n_j m_j \right) \nabla v_\phi
\end{align*}

(2.26)  \hspace{1cm} (2.27)  \hspace{1cm} (2.28)
where the $\chi^{eff}$ include diffusive and convective effects, $\Gamma$ is the particle flux. In examination of the velocity and temperature profiles on TFTR, it was determined that outside the core ($r/a \geq 0.2$) the convective effects were minimal, and that the momentum and thermal diffusivities had strikingly similar profiles in magnitude and variance with the minor radius [16]. Moreover, Scott et. al. established only a weak dependence for the diffusivity on the toroidal magnetic field and the plasma current. Similar results were reached on the ASDEX tokamak [17], operating on the assumption of a common mechanism for thermal and momentum transport (potentially the $\eta$ modes). Again using neutral-beam-injection heated discharges, Kallenbach et. al. found scaling laws for the momentum and energy confinement times and momentum diffusivity

$$\tau_\phi = 36P_{tot}^{-0.7}n_e^{-0.45}I_p^{0.85}A_{eff}^{1.0}A_{beam}^{-0.2} \left(\frac{B_T}{2.2}\right)^{-0.8}$$  \hspace{1cm} (2.29)

$$\tau_E = 37P_{tot}^{-0.58}n_e^{-0.35}I_p^{0.5}A_{eff}^{0.65}$$  \hspace{1cm} (2.30)

$$\chi_\phi = 1.69P_{tot}^{0.65}n_e^{-0.73}I_p^{-0.65}A_{eff}^{-0.8}$$  \hspace{1cm} (2.31)

In the above, $P_{tot}$ is the total heating power and $A$ is the mean plasma mass (amu) per electron, relating the electron density to ion mass.

More recently, momentum profiles in plasmas with no coordinated momentum input have been observed. The profiles for ICRF-heated and Ohmic plasmas are sufficiently similar to assume that the anomalous momentum diffusion is not due to ICRF-wave or fast-particle effects [8]. The lack of a momentum input enables the use of a simple source-free diffusion equation,

$$\frac{\partial}{\partial t}P + \nabla \cdot \left(-D_\phi \frac{\partial}{\partial r}P - v_c r P\right) = 0$$  \hspace{1cm} (2.32)

where $P = n_i n_e v_\phi$ is the ion momentum, $v_c$ is the convective velocity, and $D_\phi$ is the anomalous momentum diffusivity. Lee et. al. and Rice et. al. observed three distinct profile types for EDA, ELM-free, and ITB-forming H-modes. In EDA H-modes, the
momentum increase at the edge propagates inward to the core on a timescale on the order of $\tau_E$, equalizing to a flat momentum profile across the plasma column, with an overall velocity increase proportional to the plasma stored energy normalized by the total plasma current [7]. In addition to providing a trackable momentum diffusion profile, the flat steady-state profile implies that convection is a minimal effect in the momentum propagation. ITB modes reach a similar flat rotation profile, commonly referenced as a “hollow” profile; however, momentum seems to be ejected from the core, leading to loss of momentum confinement. This velocity profile corresponds to the centrally-peaked density profile, with velocities consistent with the neoclassical Ware pinch velocity [8]. ELM-free H-modes, on the other hand, exhibit flat electron-density and temperature profiles, while the velocity profile is strongly centrally peaked consistent with an inward momentum pinch driving transport up a velocity gradient.

As noted in Kallenbach et. al. and Lee et. al. [17, 8], the convective effects in momentum transport for the LH transition are minimal. Using this simplification, and isolating the radial component of 2.32, we may model the momentum diffusion by

$$\frac{\partial}{\partial t} P - \frac{1}{r} \frac{\partial}{\partial r} \left( rD_\phi \frac{\partial}{\partial r} P \right) = 0$$

(2.33)

again using $D_\phi$ for the anomalous momentum diffusivity. We may, for the purposes of simplification, assume a spatially and temporally constant $D_\phi$ and a boundary condition on $v_\phi$,

$$v_\phi(a,t) = \begin{cases} 
0 & t < t_{L \to H} \\
v_0 & t_{L \to H} \leq t \leq t_{H \to L} \\
0 & t > t_{H \to L}
\end{cases}$$

(2.34)

using the assumption of zero edge rotation outside the H-mode - not strictly true, but a useful simplification, and the L-mode rotation is substantially smaller than in H-mode. These assumptions allow us to reach a quite tractible model for the toroidal rotation,
\[ \frac{\partial}{\partial t} v_\phi - D_\phi \left( \frac{\partial^2}{\partial r^2} v_\phi + \frac{1}{r} \frac{\partial}{\partial r} v_\phi \right) = 0 \]  

which is solveable by an expansion in Bessel functions [7]. Specifically, we are interested in Bessel functions of the first kind, \( J_\alpha(x) \), given by

\[ J_\alpha(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m! \Gamma(m + \alpha + 1)} \left( \frac{x}{2} \right)^{2m+\alpha} \]  

where \( \alpha \) is the order of the function and \( \Gamma(z) \) is a generalized factorial function defined by

\[ \Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt \]  

For integer-order functions (that is, integer values of \( \alpha \)), we may alternately express the Bessel function \( J_\alpha(x) \) more simply as

\[ J_\alpha(x) = \frac{1}{\pi} \int_0^{\pi} \cos(\alpha \tau - x \sin \tau) \, d\tau \]  

The diffusive model (equation 2.35) can be solved by Taylor expansion into the series

\[ v_\phi(r, t) = v_0 + \sum_{i=1}^{\infty} c_i e^{-\lambda_i D t} J_0 \left( \sqrt{\lambda_i r} \right) \]  

where \( v_0 \) is the baseline velocity, \( J_0 \) is the zero-order Bessel function, and \( D \) is the anomalous diffusivity [18]. The Taylor expansion is done about the roots of the Bessel function, labeled \( z_j \). The roots are normalized by the reactor minor radius \( a \), in the form \( \lambda_i = (z_i/a)^2 \). The summation constant \( c_i \) is defined by

\[ c_i = \frac{\int_0^a v_0 J_0 \left( \sqrt{\lambda_i r} \right) r \, dr}{\int_0^a \left( J_0 \left( \sqrt{\lambda_i r} \right) \right)^2 r \, dr} \]  

normalizing the \( J_0 \) Bessel function with the baseline velocity \( v_0 \). Separating the solutions into the form \( v_\phi = v_{\phi R}(r)v_{\phi T}(t) \), the time dependence is of the form

\[ v_{\phi T}(t) \sim e^{-t/\tau_\phi}; \quad \tau_\phi = \frac{1}{\lambda_0 D} \]  

36
using $\lambda_0$, the dominant largest root of the Bessel function, thereby establishing the relation between the diffusivity $D$ in the model and the anomalous momentum confinement time.

While the classical momentum diffusivity is estimated by

$$\chi_\phi \sim \frac{\rho_i^2}{\tau_{ii}} \sim 0.003 \, \text{m}^2/\text{s}$$

both Rice et al. and Lee et al. [7, 8] have observed momentum diffusivities $D_\phi \sim 0.1 \, \text{m}^2/\text{s}$, much higher than the classical or neoclassical predicted values. This examination will utilize the model in 2.35 to observe the anomalous momentum transport $D_\phi$ in C-Mod plasmas.
Chapter 3

Experimental Observations and Conclusions

3.1 Experimental Setup

3.1.1 Spectrometer

The spectral observations for this experiment were performed on the HiReX SR spectrometer array. Senior uses a pair of spherically bent crystals to separate the spectra from H-like and He-like argon emissions. Both crystals are constructed of \( \sim 0.15 \text{ mm} \) (102)-quartz, with \( 2d \) Bragg spacing of 4.56215 Å, layered onto a glass substrate. The H-like crystal is circular with radius 25 mm, while the He-like crystal is a 27 \( \times \) 64 mm rectangle. Both crystals are spherically bent with a radius of curvature of 1385 mm [10].

The crystals project the spectra onto a total of four Pilatus 100 spectrometer modules, one for the H-like spectra and three for the He-like spectra. Each module is a 487 \( \times \) 196 pixel array, each pixel of which reaching megahertz count rates for single-photon detection. Data readout times from all four modules allow the detector to image the plasma at a 200 Hz framerate throughout the discharge.

The detector modules and crystals are arranged within a sealed housing purged with helium at slightly above atmospheric pressure, to prevent attenuation of x-rays.
in normal air. The housing is separated from the reactor vacuum chamber by a 4" diameter, 0.001" thick beryllium window.

The HiReX SR spectrometer utilizes the He-like (3.94 < \( \lambda < 4.00 \) Å) and H-like (3.72 < \( \lambda < 3.80 \) Å) spectra for argon. These emissions produce relatively bright and simple spectra in the range of \(~3\) keV, making them quite useful for spectrographic measurements. Additionally, the argon impurity concentration in the plasma can be carefully controlled by gas puffing, in contrast to molybdenum and other impurities which can slough off the physical wall of the reactor. The relevant lines for velocity measurements are given in table 3.1. The He-like w, x, y, and z lines are used for Doppler-shift data; the H-like Lyman alpha line is used to check the tomographic inversion of the argon profile and provide on-axis measurements in the plasma.

<table>
<thead>
<tr>
<th>Spectral Line</th>
<th>Transition</th>
<th>Wavelength (Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>w</td>
<td>1s2p ( {}^1P_1 \rightarrow 1s^2 {}^1S_0 )</td>
<td>3.9492</td>
</tr>
<tr>
<td>x</td>
<td>1s2p ( {}^3P_2 \rightarrow 1s^2 {}^1S_0 )</td>
<td>3.9659</td>
</tr>
<tr>
<td>y</td>
<td>1s2p ( {}^3P_1 \rightarrow 1s^2 {}^1S_0 )</td>
<td>3.9692</td>
</tr>
<tr>
<td>z</td>
<td>1s2s ( {}^3S_0 \rightarrow 1s^2 {}^1P_0 )</td>
<td>3.9943</td>
</tr>
<tr>
<td>Ly-( \alpha_1 )</td>
<td>( 2p^2 {}^3P_{3/2} \rightarrow 1s {}^1S_{1/2} )</td>
<td>3.7311</td>
</tr>
</tbody>
</table>

Table 3.1: Relevant He-like and H-like argon emission spectra

More complete spectral data for H-like and He-like argon is available in appendix B.

3.1.2 Computer Model

The spectral data taken by HiReX SR is analyzed by an IDL program solving the diffusive model using a series of Bessel functions (equation 2.39).

For the computer model, the sum is truncated at 191 roots of the Bessel function. Using approximate inputs, the program produces a recursion fit for the velocity profile and anomalous diffusion constant \( D_\phi \). The model can produce some small artifacting at the beginning of the curve due to this truncation; however, this is a small effect isolated outside the L-H transition, so it does not affect the overall estimate for \( D_\phi \).

Tomographic inversion data are available for H-mode discharges, allowing the pro-
gram to produce velocity curves at multiple depths in the plasma column. An average value for the diffusivity $D_\phi$ through the plasma is found by fitting the tomographically-inverted central chord.

Using user-defined values for the initial and final velocities in the transitions, as well as deviations from those values (i.e., a spread on the initial and final velocities to allow for oscillations), the model produces three independent fits. High and low values for $D_\phi$ are found by fitting the low and high ends of the velocity spreads - high diffusivity from the fit $v_0 + \sigma(v_0) \rightarrow v_f + \sigma(v_f)$, low diffusivity from $v_0 - \sigma(v_0) \rightarrow v_f - \sigma(v_f)$, and a best-fit value for $D$.

### 3.2 Observations

Diffusivity measurements were made separately for EDA and ELM-free H-mode discharges. Using a value of

$$\lambda_0 = (z_i/a)^2 = (2.405/.22)^2 = 119.505 \text{ m}^{-2}$$

relation 2.41 gives values for low, high, and best-fit momentum confinement times $\tau_\phi$. Using the $\tau_{98}$ formula, the energy confinement time $\tau_E$ was calculated for the discharges. The average value of $\tau_E$ over the H-mode duration, as well as its final value (found from the midpoint of box-smoothed H-mode values) at the H-mode peak are compared to the $\tau_\phi$ values.

#### 3.2.1 EDA H-Modes

Enhanced $D_\alpha$ (EDA) H-modes exhibit strong co-current toroidal rotation and an increase in plasma stored energy. The rotation profile reaches steady state during the H-mode, as shown for example shots in figure 3-1 and 3-2.
Figure 3-1: Plasma stored energy (kJ), toroidal rotation (km/s), and ICRF power (MW) for EDA H-mode in shot 1070830010.

Figure 3-2: Plasma stored energy (kJ), toroidal rotation (km/s), and ICRF power (MW) for EDA H-mode in shot 1070830029.
In figures 3-1 and 3-2, the toroidal rotation lags slightly behind the H-mode trigger, as can be seen in the plasma stored energy and ICRF heating activation, in accordance with rotational-theoretic predictions. This corresponds to a shift in the boundary condition on the model, as given in equation 2.34, with the addition of nonzero boundary velocity at the plasma edge. By fixing the edge velocity, the model reduces to a three-parameter fit on the anomalous diffusivity \( D_\phi \) and initial and final velocities \( v_0 \) and \( v_f \). An example of the fitted curve is given in figure 3-3, using the same velocity profile as in figure 3-2.

![Figure 3-3](image)

**Figure 3-3:** Fitted curve on the toroidal velocity for shot 1070830029. The dotted line is the low-\( D \) fit, the dashed line the high-\( D \) fit, and the dot-dash line the best-fit model.

In the figure above, the low, high, and best-fit models are displayed on the relevant portion of the rotation profile. These correspond to fits in the initial and final velocities, as well as high and low deviations on the velocities in the profile, as described in 3.1.2. Further sample fits are given below (figures 3-4 and 3-5), with their rotation profiles.
Figure 3-4: Rotation profile for EDA H-mode on shot 1080213004. (a) The fitted curve for the diffusive model. The dotted line is the low-$D$ fit, the dashed line the high-$D$ fit, and the dot-dash line the best-fit model. (b) Plasma stored energy, toroidal rotation, and ICRF power.

Figure 3-5: Rotation profile for EDA H-mode on shot 1080213015. (a) The fitted curve for the diffusive model. The dotted line is the low-$D$ fit, the dashed line the high-$D$ fit, and the dot-dash line the best-fit model. (b) Plasma stored energy, toroidal rotation, and ICRF power.
The three-parameter fit \((v_0, v_f, \text{ and } D_\phi)\) produces diffusivity parameters separately for the three fits; fitting from the upper and lower deviations on the initial and final velocity produces lower and higher values for \(D_\phi\), respectively. The compiled low-fit, high-fit, and best-fit values for \(D_\phi\) from the model are given in table 3.2, with the corresponding \(\tau_\phi\) values in table 3.3.

<table>
<thead>
<tr>
<th>Shot</th>
<th>Low Fit (D_\phi)</th>
<th>High Fit (D_\phi)</th>
<th>Best Fit (D_\phi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1070830010</td>
<td>.0796621</td>
<td>.108703</td>
<td>.0931785</td>
</tr>
<tr>
<td>1070830024</td>
<td>.0616576</td>
<td>.137369</td>
<td>.0684106</td>
</tr>
<tr>
<td>1070830029</td>
<td>.0686925</td>
<td>.0941678</td>
<td>.0798608</td>
</tr>
<tr>
<td>1070830030</td>
<td>.0828506</td>
<td>.125432</td>
<td>.0897999</td>
</tr>
<tr>
<td>1070831021</td>
<td>.0816918</td>
<td>.141291</td>
<td>.103965</td>
</tr>
<tr>
<td>1070831028</td>
<td>.0497955</td>
<td>.127664</td>
<td>.0827891</td>
</tr>
<tr>
<td>1080110021</td>
<td>.0810190</td>
<td>.145465</td>
<td>.136198</td>
</tr>
<tr>
<td>1080213004</td>
<td>.0820199</td>
<td>.121257</td>
<td>.112882</td>
</tr>
<tr>
<td>1080213011</td>
<td>.108249</td>
<td>.164740</td>
<td>.119894</td>
</tr>
<tr>
<td>1080213015</td>
<td>.0917213</td>
<td>.151935</td>
<td>.110478</td>
</tr>
</tbody>
</table>

Table 3.2: \(D_\phi \text{ [m}^2\text{/s]}\) values for EDA H-mode discharges.

<table>
<thead>
<tr>
<th>Shot</th>
<th>Low fit (\tau_\phi)</th>
<th>High Fit (\tau_\phi)</th>
<th>Best Fit (\tau_\phi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1070830010</td>
<td>.105042</td>
<td>.076979</td>
<td>.089805</td>
</tr>
<tr>
<td>1070830024</td>
<td>.135715</td>
<td>.060915</td>
<td>.122318</td>
</tr>
<tr>
<td>1070830029</td>
<td>.121405</td>
<td>.088861</td>
<td>.104780</td>
</tr>
<tr>
<td>1070830030</td>
<td>.100999</td>
<td>.066712</td>
<td>.093183</td>
</tr>
<tr>
<td>1070831021</td>
<td>.102432</td>
<td>.059224</td>
<td>.080487</td>
</tr>
<tr>
<td>1070831028</td>
<td>.168044</td>
<td>.065545</td>
<td>.101074</td>
</tr>
<tr>
<td>1080110021</td>
<td>.103283</td>
<td>.057525</td>
<td>.061439</td>
</tr>
<tr>
<td>1080213004</td>
<td>.102022</td>
<td>.069009</td>
<td>.074129</td>
</tr>
<tr>
<td>1080213011</td>
<td>.077302</td>
<td>.050794</td>
<td>.069825</td>
</tr>
<tr>
<td>1080213015</td>
<td>.091231</td>
<td>.055075</td>
<td>.075742</td>
</tr>
</tbody>
</table>

Table 3.3: \(\tau_\phi \text{ [s]}\) values for EDA H-mode discharges.

These are compared with the average and final \(\tau_E\) values over the H-mode duration, calculated from the \(\tau_{98}\) procedure, as described in appendix C. The \(\tau_{98}\) procedure calculates time-varying values of \(\tau_E\). The momentum confinement time in the L-H
transition is compared to $\tau_E$ during the H-mode. The “average” $\tau_E$ is simply an average over the H-mode duration as determined by the plasma stored energy and (for ICRF discharge) heating power. Similarly, $\tau_E$ at the peak of the H-mode is determined from the midpoint of the box-smoothed $\tau_E$ values. These values are given in table 3.4.

<table>
<thead>
<tr>
<th>Shot</th>
<th>Avg. $\tau_E$</th>
<th>Final $\tau_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1070830010</td>
<td>0.0434186</td>
<td>0.0401247</td>
</tr>
<tr>
<td>1070830024</td>
<td>0.0459773</td>
<td>0.0463564</td>
</tr>
<tr>
<td>1070830029</td>
<td>0.0385722</td>
<td>0.0375959</td>
</tr>
<tr>
<td>1070830030</td>
<td>0.0373024</td>
<td>0.0388340</td>
</tr>
<tr>
<td>1070831021</td>
<td>0.0399177</td>
<td>0.0408890</td>
</tr>
<tr>
<td>1070831028</td>
<td>0.0342195</td>
<td>0.0355006</td>
</tr>
<tr>
<td>1080110021</td>
<td>0.0264287</td>
<td>0.0239203</td>
</tr>
<tr>
<td>1080213004</td>
<td>0.0464921</td>
<td>0.0491054</td>
</tr>
<tr>
<td>1080213011</td>
<td>0.0447176</td>
<td>0.0409528</td>
</tr>
<tr>
<td>1080213015</td>
<td>0.0437748</td>
<td>0.0401860</td>
</tr>
</tbody>
</table>

Table 3.4: Average and final $\tau_E$ [s] values for EDA H-mode discharges.

The observed values of $D_\phi \sim 0.1 \, \text{m}^2/\text{s}$ are in good agreement with the measured values obtained in previous experiments on Alcator C-Mod, as well as with measurements of anomalous momentum diffusion on other devices. Additionally, the values for the momentum confinement time given by

$$\tau_\phi = \frac{1}{\lambda_0 D_\phi}$$

are in qualitative agreement with the scaling with $\tau_E$ observed in other experiments, with $\tau_\phi \sim 2\tau_E$ - energy and momentum confinement times of the same order of magnitude, with the momentum confinement much shorter than neoclassically predicted values of $\tau_\phi$. A comparison of $\tau_E$ and $\tau_\phi$ for EDA discharges is given in figure 3-6, including the linear relation $\tau_\phi = 2\tau_E$ for comparison.
Figure 3-6: Average and final $\tau_E$ vs. measured $\tau_\phi$ for EDA H-modes. The stars indicate low-fit values of $\tau_\phi$, the diamonds high-fit values, and the plus-signs best-fit values.

### 3.2.2 ELM-Free H-Modes

H-mode plasma discharges that do not form edge-localized modes - termed ELM-free H-modes - exhibit similar stored-energy increases as in EDA H-modes. However, without the stabilizing effect of the QC mode in EDA discharges, the H-mode collapses. Many ELM-free H-mode discharges contain multiple short-duration H-mode formations. Two sample profiles are given below in figures 3-7 and 3-8. Note that in figure 3-7, the ELM-free mode occurred in an ICRF-heated plasma, while the plasma in 3-8 was Ohmic.
Figure 3-7: Plasma stored energy (kJ), toroidal velocity (km/s), and ICRF heating power (MW) for ELM-free H-mode in shot 1070830012.

Figure 3-8: Plasma stored energy (kJ), toroidal velocity (km/s), and plasma current (MA) for ELM-free H-mode in shot 1080124003.
Figure 3-9: Rotation profile for ELM-free H-mode on shot 1080124004. (a) The fitted curve for the diffusive model. The dotted line is the low-$D$ fit, the dashed line the high-$D$ fit, and the dot-dash line the best-fit model. (b) Plasma stored energy, toroidal rotation, and plasma current.

Figure 3-10: Rotation profile for ELM-free H-mode on shot 1080124030. (a) The fitted curve for the diffusive model. The dotted line is the low-$D$ fit, the dashed line the high-$D$ fit, and the dot-dash line the best-fit model. (b) Plasma stored energy, toroidal rotation, and plasma current.
Sample fits and the corresponding velocity profiles are given above in figures 3-9 and 3-10. As with the EDA H-modes, the compiled low-fit, high-fit, and best-fit values for $D_\phi$ from the model for ELM-free H-modes are given in table 3.5, with the corresponding $\tau_\phi$ values in table 3.6.

<table>
<thead>
<tr>
<th>Shot</th>
<th>Low Fit $D_\phi$</th>
<th>High Fit $D_\phi$</th>
<th>Best Fit $D_\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1070830012</td>
<td>.0670361</td>
<td>.122907</td>
<td>.104794</td>
</tr>
<tr>
<td>1080124003</td>
<td>.0722032</td>
<td>.135960</td>
<td>.0881051</td>
</tr>
<tr>
<td>1080124004</td>
<td>.0535646</td>
<td>.0886488</td>
<td>.0662685</td>
</tr>
<tr>
<td>1080124006</td>
<td>.0773065</td>
<td>.106233</td>
<td>.0834252</td>
</tr>
<tr>
<td>1080124030</td>
<td>.0335085</td>
<td>.0488121</td>
<td>.0405593</td>
</tr>
</tbody>
</table>

Table 3.5: $D_\phi$ [m$^2$/s] values for ELM-free H-mode discharges.

<table>
<thead>
<tr>
<th>Shot</th>
<th>Low Fit $\tau_\phi$</th>
<th>High Fit $\tau_\phi$</th>
<th>Best Fit $\tau_\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1070830012</td>
<td>.124826</td>
<td>.068083</td>
<td>.079850</td>
</tr>
<tr>
<td>1080124003</td>
<td>.115893</td>
<td>.061546</td>
<td>.094976</td>
</tr>
<tr>
<td>1080124004</td>
<td>.156220</td>
<td>.094398</td>
<td>.126272</td>
</tr>
<tr>
<td>1080124006</td>
<td>.108243</td>
<td>.078769</td>
<td>.100304</td>
</tr>
<tr>
<td>1080124030</td>
<td>.249723</td>
<td>.171430</td>
<td>.206312</td>
</tr>
</tbody>
</table>

Table 3.6: $\tau_\phi$ [s] values for ELM-free H-mode discharges.

As before, these are compared with the average and final $\tau_E$ values over the H-mode duration, calculated from the $\tau_{98}$ procedure. These values are given in table 3.7.

<table>
<thead>
<tr>
<th>Shot</th>
<th>Avg. $\tau_E$</th>
<th>Final $\tau_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1070830012</td>
<td>.0454404</td>
<td>.0480250</td>
</tr>
<tr>
<td>1080124003</td>
<td>.0606466</td>
<td>.0627681</td>
</tr>
<tr>
<td>1080124004</td>
<td>.0598785</td>
<td>.0511268</td>
</tr>
<tr>
<td>1080124006</td>
<td>.0592650</td>
<td>.0520816</td>
</tr>
<tr>
<td>1080124030</td>
<td>.0608863</td>
<td>.060203</td>
</tr>
</tbody>
</table>

Table 3.7: Average and final $\tau_E$ [s] values for ELM-free H-mode discharges.

The observed values of $D_\phi$ are again in good qualitative agreement with the measured anomalous momentum diffusivity from previous experiments on C-Mod. The
momentum diffusivity tends to be somewhat lower for ELM-free H-modes than in EDA H-modes, on the order of \( \sim 0.7 - 0.8 \) m²/s, as expected from previous observations. The momentum confinement time \( \tau_\phi \) scales approximately as \( \sim 2\tau_E \). A comparison of \( \tau_E \) and \( \tau_\phi \) for ELM-free discharges is given in figure 3-11, including the linear comparison \( \tau_\phi = 2\tau_E \) for comparison. The consistency of the correlation between \( \tau_E \) and \( \tau_\phi \) is rather poorer for the ELM-free discharges shown here than for the EDA discharges above. Particularly, there is substantial deviation by the ICRF-heated ELM-free mode (shot 1070830012) from the other Ohmic discharges. The poorer performance of the model is potentially due to larger convective effects in ELM-free H-modes, which have been found to be negligible in EDA H-modes and were ignored for the diffusive model.

![Graph of average \( \tau_E vs. \tau_p \)](image)

![Graph of final \( \tau_E vs. \tau_p \)](image)

Figure 3-11: Average and final \( \tau_E \) vs. measured \( \tau_\phi \) for ELM-free H-modes. The stars indicate low-fit values of \( \tau_\phi \), the diamonds high-fit values, and the pluses best-fit values.
For both the EDA and ELM-free H-modes, the anomalous momentum confinement time $\tau_\phi$ exhibits roughly linear scaling with the energy confinement time $\tau_E$. Though the momentum diffusivities $D_\phi$ differed for EDA ($\sim 0.1 \text{ m}^2/\text{s}$) and ELM-free ($\sim 0.7 \text{ m}^2/\text{s}$) H-modes, both scale approximately by a factor of 2 with the corresponding H-mode energy confinement times $\tau_E$. The best-fit values of $\tau_\phi$ for both discharge types are compared with $\tau_E$ in figure 3-12, demonstrating the linear relation:

![Graph showing $\tau_E$ vs. best-fit $\tau_\phi$](image)

Figure 3-12: Average and final $\tau_E$ vs. measured best-fit $\tau_\phi$. The average $\tau_E$ are represented by squares, while final $\tau_E$ are represented by X’s.

### 3.3 Conclusion

In EDA and ELM-free H-mode discharges on the Alcator C-Mod tokamak, tomographically-inverted core toroidal rotation data was measured using HiReX SR, a spatially-resolving X-ray spectrometer. Anomalous momentum transport during the LH tran-
position was measured using a simple diffusive model for the toroidal rotation profile, solved in an expansion of Bessel functions.

3.3.1 Results

For EDA H-modes, the anomalous diffusivity $D_\phi$ was found to be on the order $\sim 0.1 \text{ m}^2/\text{s}$, in agreement with previous measurements of momentum transport phenomena on C-Mod. For ELM-free H-modes, the anomalous diffusivity was found to be slightly lower ($\sim 0.07 \text{ m}^2/\text{s}$), in agreement with previous observations. For both, the momentum confinement time $\tau_\phi$ was observed to scale roughly linearly with the energy confinement time $\tau_E$, with a scale factor of $\sim 2$.

3.3.2 Future Work

The still not-perfectly-understood phenomena driving anomalous momentum transport leave this question open for further investigation. The purely diffusive model fit well for the EDA discharges, thus the body of data for such plasmas could be expanded further. However, the fit was somewhat less precise on ELM-free discharges. Additionally, there was substantial difference in the behavior of Ohmic vs. ICRF-heated ELM-free discharges. A potential improvement to the model could be the reintroduction of a convective velocity, as found in equation 2.32, for ELM-free discharges. The addition of convection substantially complicates the computational model, invalidating the model in equation 2.39. This may nevertheless be necessary for successful modeling of ELM-free H-modes.
Appendix A

Alcator C-Mod parameters

The parameters for the Alcator C-Mod tokamak are given below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value/Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>major radius</td>
<td>$R \sim 0.68m$</td>
</tr>
<tr>
<td>minor radius</td>
<td>$a \sim 0.22m$</td>
</tr>
<tr>
<td>plasma volume</td>
<td>$\sim 1m^3$</td>
</tr>
<tr>
<td>plasma surface area</td>
<td>$\sim 7m^2$</td>
</tr>
<tr>
<td>toroidal field</td>
<td>$B_T \leq 8T$</td>
</tr>
<tr>
<td>plasma current</td>
<td>$I_p \leq 2MA$</td>
</tr>
<tr>
<td>elongation</td>
<td>$\epsilon \leq 1.9$</td>
</tr>
<tr>
<td>ICRF power</td>
<td>8 MW, 50-80 MHz</td>
</tr>
<tr>
<td>LHRF power</td>
<td>3 MW, 4.6 GHz</td>
</tr>
<tr>
<td>normalized pressure</td>
<td>$\beta_N \leq 1.8$</td>
</tr>
<tr>
<td>absolute plasma pressure</td>
<td>$\leq 0.2MPa$ (avg)</td>
</tr>
</tbody>
</table>

Table A.1: C-Mod reactor parameters
Appendix B

Argon Emission Spectra

The HiReX SR spectrometer was designed to measure radiation from highly ionized argon impurities (Ar$^{+16}$ and Ar$^{+17}$). The spectrometer covers wavelengths in the range 3.7-4.0 Å. The He-like range, consisting of three spectrometer modules arranged on the Rowland circle, observes wavelengths in the spectral range 3.94 Å < λ < 4.00 Å. The H-like range, consisting of a single module, observes the spectral range 3.72 < λ < 3.80 Å.
B.1 He-like Spectra

<table>
<thead>
<tr>
<th>Line Name</th>
<th>Transition</th>
<th>Wavelength (Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>w</td>
<td>$1s2p^1P_1 \rightarrow 1s^2,^1S_0$</td>
<td>3.9492</td>
</tr>
<tr>
<td>x</td>
<td>$1s2p^3P_2 \rightarrow 1s^2,^1S_0$</td>
<td>3.9659</td>
</tr>
<tr>
<td>y</td>
<td>$1s2p^3P_1 \rightarrow 1s^2,^1S_0$</td>
<td>3.9692</td>
</tr>
<tr>
<td>q</td>
<td>$1s2s2p,^2P_{3/2} \rightarrow 1s^22s,^2S_{1/2}$</td>
<td>3.9815</td>
</tr>
<tr>
<td>r</td>
<td>$1s2s2p,^2P_{1/2} \rightarrow 1s^22s,^2S_{1/2}$</td>
<td>3.9839</td>
</tr>
<tr>
<td>a</td>
<td>$1s2p^2,^2P_{3/2} \rightarrow 1s2p,^2P_{3/2}$</td>
<td>3.9864</td>
</tr>
<tr>
<td>k</td>
<td>$1s2p^2,^2D_{3/2} \rightarrow 1s^22p,^2P_{1/2}$</td>
<td>3.9903</td>
</tr>
<tr>
<td>j</td>
<td>$1s2p^2,^2D_{5/2} \rightarrow 1s^22p,^2P_{3/2}$</td>
<td>3.9932</td>
</tr>
<tr>
<td>z</td>
<td>$1s2s,^3S_0 \rightarrow 1s^2,^1P_0$</td>
<td>3.9943</td>
</tr>
</tbody>
</table>

Table B.1: He-like argon lines in the wavelength range 3.94 - 4.00 Å

Figure B-1: He-like argon emission spectrum.
### B.2 H-like Spectra

<table>
<thead>
<tr>
<th>Line Name</th>
<th>Transition</th>
<th>Wavelength (Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ly-α₁</td>
<td>$2p^2 P_{3/2} \rightarrow 1s^1 S_{1/2}$</td>
<td>3.7311</td>
</tr>
<tr>
<td>Ly-α₂</td>
<td>$2p^2 P_{1/2} \rightarrow 1s^1 S_{1/2}$</td>
<td>3.7365</td>
</tr>
<tr>
<td>T</td>
<td>$2s2p^1 P_1 \rightarrow 1s2s^1 S_0$</td>
<td>3.7553</td>
</tr>
<tr>
<td>Q</td>
<td>$2s2p^3 P_2 \rightarrow 1s2s^3 S_1$</td>
<td>3.7611</td>
</tr>
<tr>
<td>J</td>
<td>$2s2p^2^1 D_2 \rightarrow 1s2p^1 P_1$</td>
<td>3.7718</td>
</tr>
</tbody>
</table>

Table B.2: H-like argon lines in the wavelength range 3.72 - 3.80 Å

Figure B-2: H-like argon emission spectrum.
Appendix C

Energy Confinement Time

The observed anomalous momentum confinement times $\tau_\phi$ are compared with energy confinement times $\tau_E$, calculated by the $\tau_{98}$ procedure. The procedure produces time-varying $\tau_E$; the momentum confinement time is compared to the energy confinement time over the duration of the H-mode. The average $\tau_E$ is specifically averaged over the H-mode duration, as determined by the plasma stored energy. The "final" $\tau_E$ at the H-mode peak is found from the median value of the box-smoothed confinement times. Example $\tau_E$ readings are given below for EDA and ELM-free H-mode discharges.
Figure C-1: Energy confinement time $\tau_E$ for EDA H-mode discharge 1070830010. The H-mode duration is from approximately 0.7 s to 1.5 s.

Figure C-2: Energy confinement time $\tau_E$ for ELM-free H-mode discharge 1080124006. The H-mode duration is from approximately 0.6 s to 1.5 s.
Bibliography


