THERMAL CONTACT RESISTANCE WITH NON-UNIFORM INTERFACE PRESSURES

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Cambridge, Massachusetts 02139

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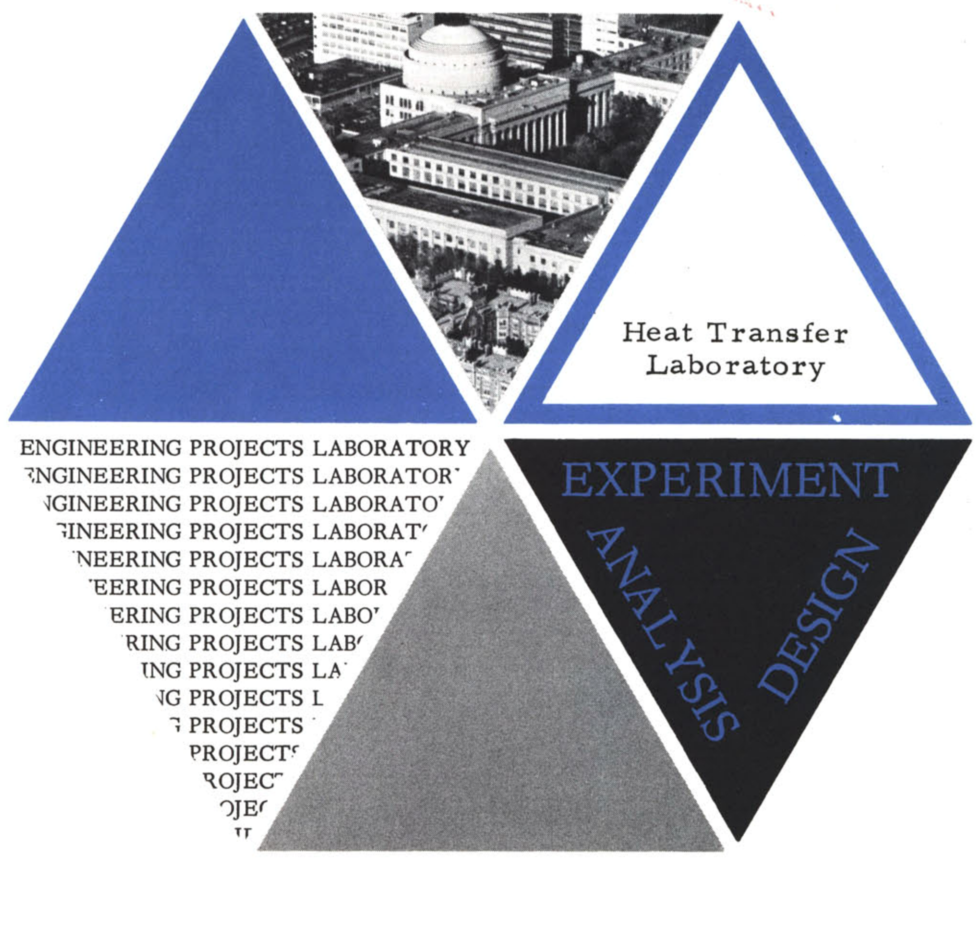

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ABSTRACT

This work considers the effect of roughness and waviness on interfacial pressure distributions and interfacial contact resistance. It is shown that for moderate roughness the contour area could be substantially different from the contour area calculated using the Hertzian theory. The model for pressure calculation assumes plastic deformation of surface irregularities and elastic deformation of a spherically wavy base. The calculations of pressure distributions cover the range of parameters of practical interest. Experimental contact resistance values have been determined and are compared with theoretical predictions. It was calculated that contact conductance for wavy surfaces can be increased for certain ranges of parameters by making surfaces rough.

ACKNOWLEDGMENTS

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NOMENCLATURE

a_H	Hertzian radius
$\overline{a}_H$	Dimensionless Hertzian radius, $a_H/(R_i\sigma)^{1/2}$
a_w	Contour radius
A	Area
A_{app}	Apparent contact area
A_c	Contour area
$\overline{A}$	$A/R_i\sigma$
b	Radius of heat channel
C.L.A.	Centerline average
C_n	Constant inside a summation
E_1, E_2	Moduli of elasticity of specimens in contact
$\overline{E}$	$\equiv (\frac{1 - \nu_1^2}{\pi E_1} + \frac{1 - \nu_2^2}{\pi E_2})^{-1}$
F	Force
$\overline{F}$	$F/H\sigma R_i$
$f(\lambda)$	h/h_{av}
H	Microhardness of the softer of two materials in contact
h	Wave amplitude; Heat transfer coefficient
h_{av}	Average heat transfer coefficient
J_n	Bessel function of order n
k	Thermal conductivity--if two materials are in contact, $k = 2k_1k_2/(k_1 + k_2)$
P	Pressure
P_{av}	Average pressure

p_a	Average pressure over contour area
p_H	Hertzian pressure
P_L	Local pressure over Hertzian area
$\overline{P}$	P/H
Pl	Plasticity index $\equiv (\overline{E}/H)\ (\sigma/R_i)^{1/2}$
Q	Heat
q/A, Q/A	Heat flux over surface A
R	Thermal resistance; Radius of curvature
R_c	Constriction resistance
R_i	$R_1R_2/(R_1 + R_2)$
R_1, R_2	Radii of curvature of undeformed, spherically wavy surfaces
R_{MAC}	Macroscopic resistance term
r	Radial coordinate parallel to contact interface plane
$\overline{r}$	$r/(R_i\sigma)^{1/2}$
r_1	A quarter-wavelength
r_o	Radius where pressure becomes zero
T_c	Local surface temperature
T_o	Ambient temperature
T_s	Extrapolated surface temperature
w(r)	Deformation of spherical object at point r
Y	Distance between the mean lines of specimen surfaces, measured in a direction perpendicular to the interface plane
$\overline{Y}$	Y/σ
y_i'	Slope of a surface profile at position i

z	Vertical distance coordinate

Greek Letter Symbols

λ	r/b
ν_n	Roots of $J_1(\nu_n) = 0$
ν_1, ν_2	Poisson's ratio of specimens in contact
ρ	Distance between any point r and any elemental area dA on the interfacial area
$\bar{\rho}$	$(\rho)/(R_i\sigma)^{1/2}$
σ	Root mean square deviation of roughness heights-- $\sigma = (\sigma_1^2 + \sigma_2^2)^{0.5}$ when two surfaces are in contact
$\mathrm{Tan}\theta$	Mean absolute value of profile slopes--$\mathrm{Tan}\theta = (\mathrm{Tan}_1^2\theta + \mathrm{Tan}_2^2\theta)^{0.5}$ if two surfaces are in contact. More specifically, $\mathrm{Tan}\theta_i \equiv \lim_{L \to \infty} \frac{1}{L} \int_o^L \lvert y_1' \rvert dx$; $\bar{i} = 1, 2, \ldots$
ϕ_w	Contact resistance factor

1. INTRODUCTION

Thermal resistances discussed in this work are assumed to occur in a vacuum environment, or in an environment where an interstitial fluid has a very low thermal conductivity. Thermal contact resistance is defined by

$$R = \frac{\Delta T}{q/A} \tag{1}$$

where (q/A) is the heat flux based on the apparent area, and ΔT is defined in Figure 1. Consider two pieces of smooth metal that are pressed together. As seen in Figure 2A, there is a finite number of contact points, A_c. The total actual contact area is often much smaller than the apparent area, A_{app}. Resistance to heat flow across such a joint is called contact resistance due to macroscopic constriction. The term "constriction" is used because heat-flow lines must squeeze together to pass through the contact area (see Figure 2B). Of course, the constriction would be present only in a vacuum, or where the interstitial fluid has a lower conductivity than the base material. If the surfaces are rough (Figure 2C), the true contact area will be even smaller than A_c. For this reason, A_c will from now on be referred to as a "contour area" rather than a "contact area." The additional area reduction due to roughness causes "microscopic constriction" of the heat-flow lines.

If $A_c = A_{app}$, only microscopic constriction is present, and the pressure distribution over the contour area is uniform. When both macroscopic and microscopic constriction are present, the pressure distribution is nonuniform, because of the clustering of contour areas at discreet locations.

Holm [6], Kragelski [7], Clausing [1], and Greenwood [3] developed equations for contact resistance under these conditions. They superimposed a microscopic constriction relation over another equation that applies to the macroscopic case. These relations, however, require a detailed knowledge of the contour area. Consider heat flow between two cylindrical solids whose channel radii are greater than the radius of their contour area (as in Figure 2B). In addition, let the surfaces originally be spherically wavy. There are three ways that one can model the heat transfer across the interface:

1. Assume constant temperature across the contour area;
2. Assume constant flux across the contour area; or
3. Assume that the heat flux through any point on the contour area is proportional to the microscopic conductance, which is in turn a function of the local pressure between the two surfaces.

In the first case, the maximum heat flux occurs at the outer rim of the contour area. For this reason, heat-flow lines that are outside the contour radius as $Z \rightarrow \infty$ must change their direction a minimal amount. The third case, however, places the maximum heat flux at the center of the contour area, making it necessary for the outer heat-flow lines to change direction substantially more. In the second case above, heat-flow is distributed in some intermediate fashion.

Resistance to heat flow is highest when the heat-flow lines must redistribute themselves to the greatest extent. For this reason, a contact resistance formula employing assumption (3) will be an upper bound for the actual contact resistance. Case (1) will consequently be a lower bound, and case (2) will fall somewhere in the middle.

A simple formula, e.g., Reference [2], predicts the macroscopic constriction resistance from the radius of the contour area:

$$R_{MAC} = \frac{\phi_w \pi b^2}{2k\ a_w} \tag{2}$$

where a_w is the contour radius, b is the channel radius, and k is the thermal conductivity. ϕ_w, the contact resistance factor, is given by

$$\phi = (1 - \frac{a_w}{b})^{1.5}$$

where constant temperature is assumed and by

$$\phi = \frac{32}{3\pi^2} (1 - \frac{a_w}{b})^{1.5}$$

for the case of constant flux. A similar resistance equation includes the effect of microscopic constriction resistance [9]:

$$R = \frac{b^2}{a_w^2} \frac{1}{h_c} + \frac{\phi_w \pi b^2}{2ka_w} \tag{3}$$

where

$$h_c = 1.45 \left(\frac{P_a}{H}\right)^{0.985} \frac{k\ \mathrm{Tan}\theta}{\sigma} \quad . \tag{4}$$

Tanθ is the mean absolute value of the profile slopes, and σ is the root-mean square deviation of roughness heights. P_a is the average pressure over the contour area, πa_w^2.

We now have equations for models (1) and (2), assuming constant temperature or constant heat-flux across the contour area. Mikic [8] has developed an equation that falls into the third category, where contact resistance is a function of the contour area pressure distribution, P(r):

$$R = 0.345 \frac{\sigma}{k \operatorname{Tan}\theta} \left[\int_o^1 \lambda \left(\frac{P}{H}\right)^{0.985} d\lambda\right]^{-1} + \frac{8b}{k} \sum_{n=1}^{\infty} \frac{\left[\int_o^1 \lambda (P/P_{av})^{0.985} J_o(\nu_n \lambda) d\lambda\right]^2}{\nu_n J_o^2(\nu_n)} \qquad (5)$$

b, σ, and Tanθ have been defined above. Hardness and thermal conductivity are denoted by H and k, respectively, and λ is the dimensionless radial coordinate $\lambda \equiv r/b$. J_n is the Bessel function of order n, and ν_n are the roots of

$$J_1(\nu_n) = 0 .$$

P_{av} is the average pressure over the apparent area. A summary of the derivation of this equation appears in Appendix A. Knowledge of the interfacial pressure distribution is required for this equation. The Hertzian pressure distribution for a smooth sphere pressed against a rigid flat plane cannot, in general, be used as an approximation. Greenwood [4] has shown that in many instances the pressure distribution under a rough sphere is substantially different from the Hertzian approximation.

This work evaluates numerically the required pressure distribution curves for rough, wavy surfaces in contact. The curves are compiled in terms of convenient dimensionless parameters. In addition, experimental values of contact resistance are presented and compared with theoretical results.

The model used in this work assumes that microscopic surface irregularities are random and normally distributed, and their deformation is plastic. Deformation of the spherically wavy base surface is elastic.

2. DESCRIPTION OF SURFACES

The surfaces considered in this investigation are both wavy and rough. A surface is wavy if its profile has a finite radius of curvature along some finite length (see Figure 3). Roughness appears as a zig-zag pattern of surface heights superimposed over the waviness. The surfaces considered are assumed to have a Gaussian distribution of heights. It has been shown in Reference [2] that for the purpose of determining contact resistance, the parameters σ, Tanθ, and R provide a sufficient description of the surfaces involved. σ is the root-mean-square deviation of roughness heights, and Tanθ is the absolute slope of these irregularities. In a wavy surface, R is the radius of curvature of a half-wave. A convenient way of finding R is by charting the surface profile and using the following equation (refer to Figure 3):

$$R = \frac{(\frac{1}{4}\text{ wavelength})^2}{2\text{ (wave amplitude)}} = \frac{r_1^2}{2h} \quad , \quad (\text{for } \frac{h}{R} << 1) \ . \tag{6}$$

A description of surface-profile measuring devices is given later on in this paper.

3. INTERFACE PRESSURE DISTRIBUTION

3.1 Governing Equations

The following equations determine the pressure distribution between two rough and wavy surfaces in contact under a load F:

$$P = \frac{H}{2} \text{ ERFC } \left(\frac{Y}{\sqrt{2}\,\sigma}\right) \tag{7}$$

$$Y(r) = Y(0) + \frac{r^2(R_1 + R_2)}{2\,R_1 R_2} - \frac{2\pi}{\overline{E}} \int P\,dr + \frac{1}{\overline{E}} \iint_A \frac{P}{\rho}\,dA \tag{8}$$

$$F = \iint_A P\,dA\ . \tag{9}$$

Non-dimensional versions of these equations are:

$$\overline{P} = \frac{1}{2} \text{ ERFC}\left(\frac{\overline{Y}}{\sqrt{2}}\right) \tag{7a}$$

$$\overline{Y}(\overline{r}) = \overline{Y}(0) + \frac{1}{2}\,\overline{r}^2 - \frac{2\pi}{P\ell} \int \overline{P}\,d\overline{r} + \frac{1}{P\ell} \iint_{\overline{A}} \frac{\overline{P}}{\overline{\rho}}\,d\overline{A} \tag{8a}$$

$$\overline{F} = \iint_{\overline{A}} \overline{P}\,d\overline{A}\ . \tag{9a}$$

The variables in these equations are defined in the Nomenclature and Figures 4A, 4B, and 5.

Equations (7) and (7A) come from surface-height distribution theory and the assumption of plastic deformation of surface asperities [2]. Equations (8) and (8A) result from geometry and an assumed elastic deformation [11] as seen in Figure 4A. The final equation is a simple force balance which must be satisfied. The solution of these equations is described in Section 3.2.

The non-dimensional version of these equations results in (dimensionless) pressure vs. (dimensionless) radius curves that are functions of only two parameters, PI and $\overline{F}$. For practical purposes, the most useful plasticity index values range from 0.0 to 0.6. These values correspond to metals such as copper, aluminum, and stainless steel, with root-mean-square roughnesses from 20 micro-inches to 200 micro-inches, and whose wavy surfaces have radii of curvature between 25 inches to infinity.

3.2 Method of Solution

This section describes the technique used to find a pressure distribution employed by the contact resistance equation (Equation (5)). A pressure distribution is determined by an iterative procedure that refines a rough approximation until all three of the governing equations (7), (8), and (9) are simultaneously satisfied.

The Hertz solution for a smooth sphere pressed against a rigid flat wall [11] is taken as a first approximation for Y(r). This Y(r) is substituted into Equation (7) whose pressure is in turn placed in Equation (8) along with an arbitrary constant value of Y(0). The new Y(r) found from Equation (8) is then placed in Equation (7) and checked by (9) to determine if its load matches the actual load. At this point, Y(0) is repeatedly modified until the pressures in (9) yield a force that is within 10 per cent of the correct load.

In some cases the calculated load will be lower than the actual load even when Y(0) = 0. Thus it would be impossible to reach the correct load by merely modifying Y(0). When this condition exists, Y(0) is immediately called zero, and the resultant pressures are sent directly to Equation (8).

Here the shape of the pressure distribution is changed until the correct load can be obtained. When the iteration process passes through Equations (7) and (8) twice in a row yielding approximately the same pressures (within 1 per cent), the computation is complete, and the desired pressure distribution is printed.

Computations were made using an I.B.M. 360. For the cases presented in this work, twenty-four radial increments have been used in the finite difference approximations. To be sure that twenty-four increments were sufficient, a test case was run at forty-eight increments also. Comparison of the two resulting pressure curves shows a maximum discrepancy of 5 per cent and an average discrepancy of less than 1 per cent. Based on the above, it was concluded that twenty-four increments yield sufficient accuracy.

In the calculation procedure, symmetry was imposed on the bulk elastic deflection by setting $w(0) = w(\Delta r)$ where $w(0)$ is the deflection at the center and $w(\Delta r)$ is the deflection at the first radial point. In this way, the slope of the deflection curve at the center line will be zero.

3.3 Pressure Distribution Curves

The Hertzian pressure distribution between two spherically wavy surfaces is given by

$$\overline{P} = \frac{1.5\,\overline{F}}{\pi\,\overline{a}_H^2}\left(1 - (\overline{r}/\overline{a}_H)^2\right)^{0.5} \qquad (10)$$

where $\overline{F}$ is the dimensionless applied load, and $\overline{a}_H$ is the dimensionless Hertzian radius, $\overline{a}_H \equiv 1.333\ (\overline{F}/P1)^{1/3}$.

When a surface is rough, the pressure distribution may or may not differ significantly from the Hertzian solution. The region of P1 vs. $\overline{F}$

plane where roughness is significant has been determined in this work and is presented in Figure 6. Roughness causes pressure distributions to differ significantly from Hertzian predictions in the following regions:

$$\overline{F} < 12.8\ (P1)^{1.25},\ 0 < P1 < .10 \tag{11}$$

$$\overline{F} < 11.7\ (P1) - 0.25,\ \ .10 < P1 < .60\ . \tag{12}$$

Even outside of these regions, edge effects will cause the contour radius to be greater than the Hertzian radius.

Dimensionless pressure curves for the region defined above appear in Figures 7-16. Coordinates for these and other pressure curves are listed in Appendix C. Pressure curves close to the transition line in Figure 6 closely resemble Hertzian shapes, whereas far below this line, curves are much flatter. Figures 17-19 illustrate this by depicting pressure curves at extreme values of $\overline{F}$ along with the associated Hertzian pressures. In each of these pictures, the curve with the higher maximum is the Hertzian pressure (p_H).

Let r_o be defined as the radial position where the pressure drops to zero. r_o is therefore the contour radius and is available from the enclosed pressure distribution solutions. The ratio of r_o over the Hertzian radius (a_H) becomes less as the applied load is increased. r_o/a_H is plotted as a function of P1 and $\overline{F}$ in Figure 20. r_o/a_H approaches a constant value greater than one as $\overline{F}$ values leave the region described by Equations (11) and (12). Outside this region, r_o should probably be used instead of a_H as the contour radius in Equations (2) and (3), whereas the Hertzian pressure distribution is in this case acceptable for Equation (5).

The r_o/a_H curves in Figure 20 are obtained from values of r_o which are read directly from Figures 8-16. There is some subjective interpretation as to where the pressure actually reaches zero. The behavior of the P1 = .1 curve should be accepted with caution because (as seen in Figure 8) only three r_o values were used to construct that curve.

4. EXPERIMENTAL PROGRAM

4.1 Preparation of Specimens

Experiments are performed with solid circular cylinders 1-1/2 inches long and 1 inch in diameter. The specimens are cut from stainless steel 303 bar-stock. Four holes (size 55 drill) are drilled to the centerline of each specimen so that thermocouples can be inserted for measurement of the axial temperature drop. The first hole is 1/4 inch from the interface, and the rest of the holes are 3/8 inch apart.

The contact resistance interface surfaces are lapped nominally flat. Waviness is created by the following method: A specimen is rotated in a lathe while a hard rubber block is used to press an abrasive (emery paper or diamond paste) against the test surface. The velocity distribution of the abrasive relative to the interface surface causes wear to be an increasing function of radius. In this way, the longer the abrasive is held in contact, the more convex the specimen becomes.

After the degree of waviness has been measured by a surface profilometer, the surface is blasted with glass beads to provide a desired roughness.

Waviness measurements are taken using a device specially built to accommodate the 1-inch diameter, 1-1/2-inch long specimens used in contact resistance experiments (see Figure 21). This device consists of a specimen holder that slides slowly beneath a diamond stylus that is connected to the core of a linear variable differential transformer. As the specimen passes underneath the stylus, the stylus moves up and down to follow the specimen surface. The profile shape is traced out on a Sanborn 150 recorder. This profilometer is capable of magnifying the vertical deflection by a factor of five hundred. A detailed description of this device may be found in Reference [5].

Roughness is measured on a Taylor-Hobson Talysurf IV. The "Talysurf" provides a profile chart and a direct reading of the centerline average (C.L.A.). The root-mean-square roughness (σ) can be determined by the following relation:

$$\sigma = [\text{C.L.A.}]\ (\pi/2)^{0.5} \cong 1.25\ [\text{C.L.A.}] \tag{13}$$

The absolute value of the slope, Tanθ can be computed graphically from the profile chart.

4.2 Description of the Apparatus

Contact resistance data have been obtained from the apparatus in Figure 22. In a few words, the apparatus passes heat through two specimens in a vacuum environment. The three main sections of the apparatus are:

1. a vacuum chamber;
2. a refrigeration unit; and
3. a lever arrangement for applying an adjustable force to the test interface.

The vacuum system is basically a hollow aluminum cylinder which has been fabricated from four main sections. An upper cylinder is welded to a top plate. When the rig is in use, a removable lower cylinder with a flange seals up against an "O" ring and a bottom plate. The bottom plate is bolted to the supporting structure. The upper and lower cylinders are bolted together at their interface where another "O" ring allows for a vacuum seal. Connections with the vacuum pumps and the refrigeration unit are made through the bottom plate. Thermocouple wires, power lines for the specimen heater, and a bellows for the loading mechanism enter through the top plate.

The vacuum is created by a forepump (Cenco HYVAC 14 rotary mechanical pump) and a 4-inch diameter diffusion pump (NRC model H4SP). A three-way valve allows the forepump to bring the system pressure down to 50 microns of mercury where activation of the diffusion pump will continue to lower the pressure to 15 microns of mercury.

The refrigeration unit, of course, supplies the low temperature sink for the heat fluxes that are passed through the test specimens. The unit is a 1-1/2 horsepower, Model 155 WFC, built by the Copeland Corporation. With its evaporator at 25 ^{o}F, it can receive up to 16,840 BTU/HR. Freon 12 serves as the refrigerant fluid. The magnitude of the heat-flux produced is crucial to contact resistance studies because with large loads too low a heat-flux will produce a negligible ΔT across the test interface.

A series of levers, supported by a welded steel frame, permits the application of force to the specimens at a ratio of 100 to 1. This deadweight loading is transmitted into the vacuum system via a 15-convolution, 3-3/8-inch I.D. stainless steel bellows, manufactured by the Flexonics Division of the Universal Oil Products Company. At atmospheric pressure, the applied load can be adjusted between 0 and 20,000 pounds. When the system is evacuated, the minimum load is 163 pounds, due to the atmosphere pressing down on the 3-3/8-inch diameter bellows device.

In addition to the apparatus listed above, there is a water-flow cooler between the bellows and the heater. This cooler prevents the heater from raising the temperature of the rest of the chamber to a level that would destroy the vacuum seals. The heater is powered by a 220 volt d.c. power source. A more detailed description of this apparatus may be found in Reference [12].

4.3 Experimental Procedure

Chromel-alumel thermocouples are covered with "Silver Goop" and inserted into the specimens. ("Silver Goop," manufactured by the Crawford Fitting Company, is a substance that provides a good thermal contact.) The thermocouple wires are then sealed in position with "White Epoxy," a product of the Hysol Division of the Dexter Corporation.

With the specimens inside, the vacuum chamber is sealed. The mechanical pump is switched on, and the system pressure is brought down to 50 microns of mercury. At this point the diffusion pump is activated to further lower the pressure to 15 microns of mercury. The applied load is now the minimum force of 163 pounds. After the water-flow cooler and the refrigeration unit are turned on, the heater is powered up to pass a heat-flux through the specimens.

Temperature readings are recorded from a thermocuple potentiometer every thirty minutes. When two successive readings are the same, it is assumed that steady state has been reached. The applied load is now increased, and the temperature-recording procedure is repeated. The applied load is always increased rather than decreased because the specimens may undergo a plastic (irreversible) deformation.

5. EXPERIMENTAL RESULTS AND DISCUSSION

The results of contact resistance experiments appear in Tables I and II and are plotted in Figures 23A and 23B. Also appearing are the predictions using Equation (2) for contact resistance with wavy surfaces, Equation (3) which includes the effects of both roughness and waviness, and Equation (5), an integral formula which assumes that the local interfacial flux is proportional to the local microscopic conductance and hence is a function of the local interfacial pressure. Equation (5) was solved on an I.B.M. 360 computer, using forty-four and eighty-nine radial increments (for the two different cases involved in the experiments) in a finite difference approximation of the integrals. The summation appearing in the macroscopic resistance term was evaluated using the first six terms of the series. The computer program for this equation appears in Appendix A. Equations (2) and (3) are given for the case of constant flux over the contour area, using two separate choices for the contour radius, a_w.

a. $a_w = a_H$, the Hertzian (smooth surface) approximation; and

b. $a_w = r_o$, the rough-surface contour radius determined from the pressure curves in Figures 7-16.

It has been stated earlier that Equation (5) is an upper bound for resistance (a lower bound for conductance). This indeed appears to be the case, as Equation (5) is the lowest curve in both Figures 23A and 23B. Using $a_w = a_H$, Equation (3) is very close to Equation (5). For the particular parameters involved in the experiments (for which the macroscopic conductance was the dominant factor), Equation (3) with $a_w = r_o$

gives the best prediction for the actual contact conductance (to within 25 per cent accuracy).

It is important to notice that in certain ranges of parameters, a wavy surface will yield a higher h if it is roughened. This is caused, as it is shown in this work, by an increase in the contour area. (The engagement of the two surfaces covers a larger area when roughness is present as in Figure 4B.) Experimentally, this was also observed by Clausing [1]. He, however, did not explain the phenomena.

Experiments performed in this work dealt with surfaces that were both rough and wavy. There was no need to experiment with rough, flat surfaces, because this topic has already been covered sufficiently, both experimentally and theoretically, in Reference [2], for example. Similarly, the case of smooth, wavy surfaces has been covered amply by authors such as Clausing [1].

6. CONCLUSIONS

From pressure distribution curves given in Figures 7-16, one can determine the actual contour area between two rough and wavy surfaces. In a certain range of parameters, this contour area is substantially larger than the value calculated from the Hertzian theory. Experiments were performed in this range, and three basic approaches for the calculation of contact conductance were applied to the results, including formulas based on

a. the Hertzian contour area;

b. the contour area predicted by this work; and

c. an integral relation which uses the complete interfacial pressure distribution.

It is suggested that Equation (3) (which assumes constant flux over the the contour area) using contour radii predicted in this work is the best prediction for the cases involved in the experiments (in which the predominant resistance comes from macroscopic constriction). For those cases where microscopic resistance is primarily controlling the value of contact resistance, it is believed that the integral relation (Equation (5)) would yield the correct prediction. (For the case of a uniform pressure distribution between two flat, rough surfaces, Equation (5) reduces to one term, the microscopic constriction resistance.)

The main conclusion is that contact conductance can be increased for certain ranges of parameters by making surfaces rough. This thesis also identifies the range of parameters where roughness will substantially affect the interfacial pressure distribution between rough and wavy surfaces.

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APPENDIX A

DERIVATION OF CONTACT RESISTANCE EQUATION

The equation developed by Mikic [8] for contact resistance (Equation 5) was derived as follows:

Figure 24A depicts a solid circular cylinder with a non-uniform heat transfer coefficient, h, on the z = 0 face. The sides are insulated, and heat-flow is assumed to be one-dimensional as $z \to \infty$. The flow of heat at the top surface is

$$Q = \int_A h\,(T_o - T_c)\,dA \tag{A1}$$

where T_o and T_c are defined in Figure 24A. The heat-flux over that surface is therefore

$$Q/A = T_o h_{av} - \frac{1}{A}\int_A T_c\, h dA \tag{A2}$$

where

$$h_{av} \equiv \frac{1}{A}\int_A h dA \quad .$$

The total resistance from the surface to the ambient is

$$R \equiv \frac{T_o - T_s}{Q/A} \tag{A3}$$

where T_s is defined in Figure 24B.

To transform (A2) into the form of (A3), the term

$$\frac{1}{A}\int_A h\; T_s dA$$

can be subtracted from and added to the second and third terms of (A2), respectively to yield:

$$Q/A = (T_o - T_s)\ h_{av} - \frac{1}{A} \int_A (T_c - T_s)\ h dA\ . \tag{A4}$$

Thus we now have:

$$R = \frac{1}{h_{av}} + \frac{1}{Q} \int_A \frac{h}{h_{av}} (T_c - T_s)\ dA\ . \tag{A5}$$

The second term on the right-hand side of (A5) is the constriction resistance:

$$R_c \equiv \frac{1}{Q} \int \frac{h}{h_{av}} (T_c - T_s)\ dA\ . \tag{A5.A}$$

This represents the difference in total resistance between the two cases pictured in Figure 25A.

Figure 25B illustrates the basic model for which Equation (5) was developed. At $z = 0$, h is a function of radius, and the sides are insulated. Also, $\frac{\partial T}{\partial z}$ = constant as $z \to \infty$.

The governing differential equation for the situation is

$$\nabla^2\ T = 0 \tag{A6}$$

where ∇^2 is the Laplacian operator. For the given boundary conditions, the steady-state solution is

$$T_s = T_c - \frac{Q}{k\pi b^2} z + \sum_{n=1}^{\infty} C_n\ e^{-\nu_n\ z/b}\ J_o(\nu_n\ r/b) \tag{A7}$$

where ν_n are the roots of

$$J_1(\nu_n) = 0\ . \tag{A8}$$

Using relation (A8) and the approximation

$$\frac{[-k(\frac{\partial T}{\partial z})_{z=0}]_{at\ r}}{Q/A} \cong \frac{h(r)}{h_{av}}\ , \tag{A9}$$

Equation (A7) becomes

$$T_c - T_s = \frac{2Q}{\pi bk} \sum_{n=1}^{\infty} \frac{\int_o^1 \lambda\ J_o(\nu_n\lambda)\ f(\lambda)\ d\lambda}{\nu_n\ J_o^2(\nu_n)} J_o(\nu_n\lambda) \tag{A10}$$

where $\lambda \equiv r/b$ and $f(\lambda) \equiv h/h_{av}$. Combining (A5.A) and (A10) will yield

$$R_c = 4\ \frac{b}{k} \sum_{n=1}^{\infty} \frac{[\int_o^1 \lambda f(\lambda)\ J_o(\nu_n\lambda)\ d\lambda]^2}{\nu_n\ J_o^2(\nu_n)} \quad . \tag{A11}$$

Contact conductance for purely rough surfaces with Gaussian surface height distributions has been shown in Reference [2] to be

$$h_c = 1.45\ \frac{k\ \mathrm{Tan}\theta}{\sigma} \left(\frac{P_a}{H}\right)^{0.985} \tag{A12}$$

where the variables have been defined in the nomenclature. By combining (A11) and (A12), Equation (A5) becomes Equation 5:

$$R = 0.345\ \frac{\sigma}{k\ \mathrm{Tan}\theta} \left[\int_o^1 \lambda\ \left(\frac{P}{H}\right)^{0.985} d\lambda\right]^{-1}$$

$$+ \frac{8b}{k} \sum_{n=1}^{\infty} \frac{[\int_o^1 \lambda (P/P_{av})^{0.985}\ J_o(\nu_n\lambda)\ d\lambda]^2}{\nu_n\ J_o^2(\nu_n)} \tag{5}$$

APPENDIX B

COMPUTER PROGRAMS

List of Fortran Variables Used in Programs

Notation of This Thesis	Fortran Symbol
a_H	AH
$\bar{a}_H$	AHB
b	B
$\bar{F}$	PLOAD,FBAR
H	H
$J_o(x)$	BESEL(X)
k	AK
p, or P	PRES,PRSS,PREZ, HRTZP (Hertzian) PRESH (Due to Hertzian Y)
$\bar{p}$, or $\bar{P}$	PBAR,PBAZ
Pl	PL
R	RES
R_i	RADI,RAD
R_1,R_2	R1,R2
r	R,RR,RHOM (radial position of dA)
$\bar{r},d\bar{r}$	RB,DRB
$\bar{Y}(\bar{r})$	YB(I)
$\bar{Y}(0)$	YNB
$\lambda,d\lambda$	ALAM,DL
ν_n where $J_1(\nu_n) = 0$	ANU(J)

Notation of This Thesis	Fortran Symbol
$\rho\ \bar{\bar{\rho}}$	S
Tanθ	TAN
Tan$_1\theta$,Tan$_2\theta$	TAN1,TAN2
σ	SIGMA
σ_1,σ_2	SIGM1,SIGM2

PROGRAM TO DETERMINE DIMENSIONLESS PRESSURE

DISTRIBUTIONS BETWEEN ROUGH AND WAVY SURFACES

INPUT DATA	LINE NO.
a. MAX = Number of radial increments	31
b. $P\ell$, $\overline{F}$	33
c. YNB = first approximation for $\overline{Y}$ (0)	38

OUTPUT DATA	
a. $P\ell$, $\overline{F}$	41
b. Dimensionless Hertzian pressures	57
c. Dimensionless Hertzian radius	68
d. First approximation for $\overline{Y}(\overline{r})$	69
e. First approximation for $\overline{P}(\overline{r})$	78
f. Second approximation for $\overline{P}(\overline{r})$	96
g. If any values of $\overline{Y}(\overline{r})$ are negative, they are written here.	151
h. Final values, $\overline{P}(\overline{r})$	229
i. Final values, $\overline{Y}(\overline{r})$	230
j. $\overline{F}$ calculated from $\overline{P}(\overline{r})$	232
k. Dimensionless radial coordinates	235
l. $P\ell$, $\overline{F}$	239

```
      DIMENSION PRSS(50),HRTZP(50),YB(50),PRESH(50),PBAR(50),Z(50),PBAZ(
     150),W(50),PREZ(50),RTER(50),PR1(50)
C
C     ****************************************************************
C     PROGRAM TO DETERMINE DIMENSIONLESS PRESSURE DISTRIBUTIONS BETWEEN
C     ROUGH AND WAVY SURFACES
C     ****************************************************************
C
  102 FORMAT(25X,F15.4)
  103 FORMAT(1H ,6E15.4)
  105 FORMAT(25X,15HHERTZIAN RADIUS)
  107 FORMAT(1H ,40X,15HCALCULATED LOAD/)
  108 FORMAT(1H ,40X,E15.4)
  109 FORMAT(1H2,30X,27HFINAL PRESSURE DISTRIBUTION/)
  112 FORMAT(E15.4)
  114 FORMAT(1H ,30X,34HINTERMEDIATE PRESSURE DISTRIBUTION/)
  117 FORMAT(1H ,14HPLASTICITY(PL),6X,4HFBAR/)
  118 FORMAT(1H ,E15.4,5X,E15.4)
  122 FORMAT(2F10.3)
  480 FORMAT(1H ,22HPURE HERTZIAN PRESSURE/)
 1001 FORMAT(I10)
 1005 FORMAT(1H ,8HYBAR(0)=,E15.4)
 1008 FORMAT(1H ,8HYBAR(I)=/)
 1011 FORMAT(1H ,25HPBAR DUE TO HERTZIAN YBAR/)
 4755 FORMAT(1H ,8HPBAZ(I)=/)
 4756 FORMAT(1H ,30HPBAR(I)=(PBAR(I)+HRTZP(I))/2.0/)
      A=0.0
      COUNT=0.0
      CHANGE=0.0
      G=0.0
      READ(5,1001) MAX
      PI=3.14159
      READ(5,122) PL,FBAR
      AEAP=0.0
      DO 8010 I=1,29
      HRTZP(I)=0.0
```

```
8010 CONTINUE
     READ(5,112) YNB
8000 CONTINUE
     WRITE(6,117)
     WRITE(6,118) PL,FBAR
     DO 75 I=1,MAX
75   PRSS(I)=0.0
     PLOAD=FBAR
     RMAX=MAX-1
     MAXM1=MAX-1
     AHB=1.333*((FBAR/PL)**.3333)
     AH=AHB
     AHSQ=AHB*AHB
     LL=(RMAX/4.0)+1.5
     DRB=(4.0*AHB)/RMAX
     DO 491 I=1,LL
     RB=FLOAT(I-1)*DRB
     HRTZP(I)=.477*(FBAR/AHSQ)*SQRT(1.0-(RB*RB/AHSQ))
491  CONTINUE
     WRITE(6,480)
     WRITE(6,103) (HRTZP(I),I=1,LL)
     DO 1 I=1,MAX
     RB=FLOAT(I-1)*DRB
     IF(RB-AHB)4,4,5
4    YB(I)=0.0
     GO TO 1
5    SIDE=SQRT(RB*RB-AHSQ)
     YB(I)=.5*(RB*RB-2.0*AHSQ*(1.0-.318*(((2.0-((RB/AH)**2.0))*ATAN(AH/
    1SIDE)+(((RB*RB)/(AHSQ))-1.0)**0.5))))
1    CONTINUE
     WRITE(6,105)
     WRITE(6,102) AHB
     WRITE(6,103) (YB(I),I=1,MAX)
     DO 200 I=1,MAX
     PRESH(I)=0.5*(1.0-ERF(YB(I)/(1.414)))
     PTEST=.001*PRESH(1)
```

```
      IF(PRESH(I).LT.PTEST) GO TO 1984
200   CONTINUE
1984  MAX=I
      MAXM1=MAX-1
      WRITE(6,1011)
      WRITE(6,103) (PRESH(I),I=1,MAX)
      TEST1=.01*PRESH(1)
      AEAR=0.0
      DO 486 I=1,MAX
      RB=FLOAT(I-1)*DRB
486   AEAR=AEAR+PRESH(I)*RB*DRB
      FLOAD=2.0*PI*AEAR
      GO TO 195
1898  CONTINUE
      G=G+1.0
      IF(G.EQ.1.0) GO TO 8001
      GO TO 8003
8001  CONTINUE
      DO 8002 I=1,MAX
      PBAR(I)=(HRTZP(I)+PBAR(I))/2.0
      PR1(I)=PBAR(I)
8002  CONTINUE
      WRITE(6,4756)
      WRITE(6,103) (PBAR(I),I=1,MAX)
8003  CONTINUE
      DO 8004 I=1,MAX
      PBAR(I)=(PR1(I)+PBAR(I))/2.0
      PR1(I)=PBAR(I)
8004  CONTINUE
11    TEST1=.01*PBAR(1)
      COUNT=COUNT+1.0
      IF(COUNT-15.0)1972,1972,1979
1972  CONTINUE
      DO 85 I=1,MAX
85    PRSS(I)=PBAR(I)
      TEST=0.1*PBAR(1)
```

```
      N=MAX                                                             PRES0109
      AREA=0.0                                                          PRES0110
      DO 25 I=2,N                                                       PRES0111
      RB=FLOAT(I-1)*DRB                                                 PRES0112
      SUM=0.0                                                           PRES0113
      ADD=2.0*SQRT(PI)*DRB*PBAR(I)                                      PRES0114
      DO 26 K=1,N                                                       PRES0115
      RHOM=FLOAT(K-1)*DRB                                               PRES0116
      SUM1=(2.0*RHOM*DRB*PBAR(K))/(RHOM*RHOM+RB*RB)**.5                 PRES0117
      EP=2.0*RB*RHOM/(RB*RB+RHOM*RHOM)                                  PRES0118
      IF(EP-1.0)600,601,601                                             PRES0119
600   S3=0.0                                                            PRES0120
      DO 300 J=1,5                                                      PRES0121
      THETA=FLOAT(J-1)*PI/90.0+PI/180.0                                 PRES0122
      S=SQRT(1.0-EP*COS(THETA))                                         PRES0123
      S3=S3+(PI/90.0)/S                                                 PRES0124
300   CONTINUE                                                          PRES0125
      S4=0.0                                                            PRES0126
      DO 301 M=2,18                                                     PRES0127
      THETA=FLOAT(M-1)*PI/18.0+PI/36.0                                  PRES0128
      S=SQRT(1.0-EP*COS(THETA))                                         PRES0129
      S4=S4+(PI/18.0)/S                                                 PRES0130
301   CONTINUE                                                          PRES0131
      S1=S3+S4                                                          PRES0132
      GO TO 602                                                         PRES0133
601   S1=(2.0**0.5)*ALOG(8.0*RB/DRB)                                    PRES0134
602   SUM=SUM+(SUM1*S1)                                                 PRES0135
26    CONTINUE                                                          PRES0136
25    W(I)=(SUM+ADD)/PL                                                 PRES0137
      W(1)=W(2)+(DRB*DRB)/2.0                                           PRES0138
      AEAP=0.0                                                          PRES0139
      DO 50 I=1,MAX                                                     PRES0140
      RB=FLOAT(I-1)*DRB                                                 PRES0141
      RTER(I)=RB*RB/2.0                                                 PRES0142
      YB(I)=RTER(I)-W(1)+W(I)+YNB                                       PRES0143
      PBAR(I)=0.5*(1.0-ERF(YB(I)/(2.0**.5)))                            PRES0144
```

```
50    CONTINUE                                                          PRES0145
      DO 950 I=1,MAX                                                    PRES0146
      IF(YB(I).LT.0.0) GO TO 8880                                       PRES0147
950   CONTINUE                                                          PRES0148
      GO TO 1099                                                        PRES0149
8880  CONTINUE                                                          PRES0150
      WRITE(6,103) (YB(I),I=1,MAX)                                      PRES0151
482   CONTINUE                                                          PRES0152
483   CONTINUE                                                          PRES0153
      DO 484 I=1,MAX                                                    PRES0154
      PBAR(I)=0.5*(1.0-ERF(YB(I)/(2.0**.5)))                            PRES0155
484   CONTINUE                                                          PRES0156
      GO TO 1099                                                        PRES0157
195   CONTINUE                                                          PRES0158
495   CONTINUE                                                          PRES0159
      DO 295 I=1,MAX                                                    PRES0160
      AEAP=0.0                                                          PRES0161
295   YB(I)=YB(I)+YNB                                                   PRES0162
      DO 99 I=1,MAX                                                     PRES0163
      PBAR(I)=0.5*(1.0-ERF(YB(I)/(2.0**.5)))                            PRES0164
99    CONTINUE                                                          PRES0165
1099  CONTINUE                                                          PRES0166
      DO 97 I=1,MAX                                                     PRES0167
      IF(ABS(PRSS(I)-PBAR(I)).GT.TEST1) GO TO 76                        PRES0168
97    CONTINUE                                                          PRES0169
      AF=(FLOAD-PLOAD)/PLOAD                                            PRES0170
      IF(ABS(AF).GT.0.1) GO TO 76                                       PRES0171
      GO TO 16                                                          PRES0172
76    CONTINUE                                                          PRES0173
      DO 44 I=1,MAXM1                                                   PRES0174
      RB=FLOAT(I-1)*DRB                                                 PRES0175
44    AEAP=AEAP+PBAR(I)*RB*DRB                                          PRES0176
      FLOAD=2.0*PI*AEAP+(PBAR(1)*PI*DRB*DRB)/4.0                        PRES0177
      AF=(FLOAD-PLOAD)/PLOAD                                            PRES0178
      AEAZ=0.0                                                          PRES0179
      DO 196 I=1,MAX                                                    PRES0180
```

```
      Z(I)=YB(I)-YB(1)                                                  PRES0181
      PBAZ(I)=0.5*(1.0-ERF(Z(I)/(2.0**.5)))                             PRES0182
  196 CONTINUE                                                          PRES0183
      DO 190 I=1,MAXM1                                                  PRES0184
      RB=FLOAT(I-1)*DRB                                                 PRES0185
  190 AEAZ=AEAZ+PBAZ(I)*RB*DRB                                          PRES0186
      FLOAZ=2.0*PI*AEAZ+(PBAZ(1)*PI*DRB*DRB)/4.0                        PRES0187
      CHECK=FLOAZ-PLOAD                                                 PRES0188
      DO 5000 I=1,MAX                                                   PRES0189
 5000 YB(I)=Z(I)                                                        PRES0190
      IF(CHECK)9011,9011,192                                            PRES0191
 9011 CONTINUE                                                          PRES0192
      DO 9012 I=1,MAX                                                   PRES0193
      PBAR(I)=PBAZ(I)                                                   PRES0194
 9012 CONTINUE                                                          PRES0195
 9013 CONTINUE                                                          PRES0196
      GO TO 11                                                          PRES0197
  192 IF(PLOAD.GT.FLOAD) GO TO 193                                      PRES0198
      GO TO 191                                                         PRES0199
  193 AF=(FLOAD-PLOAD)/PLOAD                                            PRES0200
      IF(ABS(AF).LT.0.1) GO TO 1898                                     PRES0201
      IF(ABS(AF).LT.0.4) GO TO 476                                      PRES0202
      IF(ABS(AF).LT.0.6) GO TO 470                                      PRES0203
      IF(ABS(AF).LT.1.0) GO TO 477                                      PRES0204
      YNB=YNB*0.8                                                       PRES0205
      GO TO 471                                                         PRES0206
  476 YNB=YNB*0.99                                                      PRES0207
      GO TO 471                                                         PRES0208
  470 YNB=YNB*0.96                                                      PRES0209
      GO TO 471                                                         PRES0210
  477 YNB=YNB*0.90                                                      PRES0211
  471 CONTINUE                                                          PRES0212
      GO TO 195                                                         PRES0213
  191 AF=(FLOAD-PLOAD)/PLOAD                                            PRES0214
      IF(ABS(AF).LT.0.1) GO TO 1898                                     PRES0215
      IF(ABS(AF).LT.0.4) GO TO 478                                      PRES0216
```

```
      IF(ABS(AF).LT.0.6) GO TO 473                                      PRES0217
      IF(ABS(AF).LT.1.0) GO TO 479                                      PRES0218
      YNB=YNB*1.2                                                       PRES0219
      GO TO 474                                                         PRES0220
  478 YNB=YNB*1.01                                                      PRES0221
      GO TO 474                                                         PRES0222
  473 YNB=YNB*1.04                                                      PRES0223
      GO TO 474                                                         PRES0224
  479 YNB=YNB*1.1                                                       PRES0225
  474 CONTINUE                                                          PRES0226
      GO TO 195                                                         PRES0227
   16 WRITE(6,109)                                                      PRES0228
      WRITE(6,103) (PBAR(I),I=1,MAX)                                    PRES0229
      WRITE(6,103) (YB(I),I=1,MAX)                                      PRES0230
      WRITE(6,107)                                                      PRES0231
      WRITE(6,108) FLOAD                                                PRES0232
      DO 1970 I=1,MAX                                                   PRES0233
      RADIM=FLOAT(I-1)*DRB                                              PRES0234
      WRITE(6,103) RADIM                                                PRES0235
 1970 CONTINUE                                                          PRES0236
 1979 CONTINUE                                                          PRES0237
      WRITE(6,117)                                                      PRES0238
      WRITE(6,118) PL,FBAR                                              PRES0239
 1973 CONTINUE                                                          PRES0240
 1978 CONTINUE                                                          PRES0241
 4901 CONTINUE                                                          PRES0242
 1971 CALL EXIT                                                         PRES0243
      END                                                               PRES0244
```

USING EQUATION (5)

INPUT DATA	LINE NO.
a. H, σ_1, σ_2, $Tan_1\theta$, $Tan_2\theta$	34
b. k (thermal conductivity)	35
c. R_1, R_2, MAX = Number of pressures to be read in, N = number of roots of $J_1(\nu_n) = 0$ to be read in	36
d. Dimensionless pressures	37
e. Roots of $J_1(\nu_n) = 0$	38
f. Dimensionless radial increment	39
g. Average pressure $\equiv F/A_{app}$	40

OUTPUT DATA	
a. INPUT DATA	93-100
b. Microscopic resistance term	102
c. Macroscopic resistance term	104
d. Total contact resistance	106

NOTE: The number appearing in the inequality in card 107 should be one less than the number of sets of input data.

```
      FUNCTION BESEL(X)
C
C     ******************************************************************
C     BESSEL FUNCTION SUB-ROUTINE USED IN THE PROGRAM FOR CONTACT RESIS-
C     TANCE USING EQUATION (5)
C     ******************************************************************
C
      IF(X-3.0)10,10,11
 10   BESEL=1.0-2.24999*((X/3.0)**2.0)+1.26562*((X/3.0)**4.0)-.31638*((X
     1/3.0)**6.0)+.04444*((X/3.0)**8.0)-.00394*((X/3.0)**10.0)+.00021*((
     1X/3.0)**12.0)
      RETURN
 11   FZ=.79788-.00552*((3.0/X)**2.0)-.00009*((3.0/X)**3.0)+.00137*((3.0
     1/X)**4.0)-.00072*((3.0/X)**5.0)+.00014*((3.0/X)**6.0)
      THETA=X-.78539-.04166*(3.0/X)-.00004*((3.0/X)**2.0)+.00263*((3.0/X
     1)**3.0)-.00054*((3.0/X)**4.0)-.00029*((3.0/X)**5.0)+.00014*((3.0/X
     1)**6.0)
      BESEL=(FZ*COS(THETA))/(X**0.5)
      RETURN
      END
```

```
      DIMENSION PBAR(200),PRES(200),ANU(200)                             RESI0001
C                                                                        RESI0002
C     ****************************************************************** RESI0003
C     PROGRAM TO DETERMINE THERMAL CONTACT RESISTANCE USING EQUATION (5) RESI0004
C     ****************************************************************** RESI0005
C                                                                        RESI0006
  100 FORMAT(5E15.4)                                                     RESI0007
  101 FORMAT(2F15.4,2I10)                                                RESI0008
  103 FORMAT(6F10.4)                                                     RESI0009
  104 FORMAT(1H ,6X,1HH,8X,5HSIGM1,10X,5HSIGM2/)                         RESI0010
  106 FORMAT(3F15.4)                                                     RESI0011
  107 FORMAT(1H ,4HTAN1,10X,4HTAN2,11X,1HK/)                             RESI0012
  108 FORMAT(5X,3HMAX,5X,1HN,5X,2HR1,13X,2HR2/)                          RESI0013
  109 FORMAT(1H ,I7,I6,2E15.4)                                           RESI0014
  110 FORMAT(1H ,23HDIMENSIONLESS PRESSURES/)                            RESI0015
  111 FORMAT(1H ,18HCONTACT RESISTANCE/)                                 RESI0016
  112 FORMAT(1H ,E15.4,1X,16H(HR-F-FT-FT)/BTU/)                          RESI0017
  115 FORMAT(E15.4)                                                      RESI0018
  116 FORMAT(1H ,4HDRB=,E15.4)                                           RESI0019
  117 FORMAT(1H ,E15.4)                                                  RESI0020
  118 FORMAT(23HMAXP IS GREATER THAN LB/)                                RESI0021
  119 FORMAT(1X,10HSIGMA TERM/)                                          RESI0022
  120 FORMAT(1X,6HB TERM/)                                               RESI0023
  130 FORMAT(1X,17HAVERAGE PRESSURE=,E15.4/)                             RESI0024
  131 FORMAT(E15.4)                                                      RESI0025
  132 FORMAT(1X,6HP/PAV=)                                                RESI0026
  133 FORMAT(1H,5X,2HY=,9X,10HJ-ZERO(Y)=)                                RESI0027
  134 FORMAT(2E15.4)                                                     RESI0028
  135 FORMAT(F15.2)                                                      RESI0029
      B=0.5                                                              RESI0030
      BNUM=0.0                                                           RESI0031
 3333 CONTINUE                                                           RESI0032
      BNUM=BNUM+1.0                                                      RESI0033
      READ(5,100) H,SIGM1,SIGM2,TAN1,TAN2                                RESI0034
      READ(5,117) AK                                                     RESI0035
      READ(5,101) R1,R2,MAX,N                                            RESI0036
```

```
      READ(5,100) (PBAR(I),I=1,MAX)
      READ(5,103) (ANU(J),J=1,N)
      READ(5,115) DRB
      READ(5,135) PAV
      PTOT=0.0
      TAN=(TAN1*TAN1+TAN2*TAN2)**0.5
      SIGMA=(SIGM1*SIGM1+SIGM2*SIGM2)**0.5
      RADI=(R1*R2)/(R1+R2)
      DR=DRB*((RADI*SIGMA)**0.5)
      MAXP=MAX+1
      LB=B/DR
      IF(LB-MAXP)51,51,49
49    CONTINUE
      DO 50 I=MAXP,LB
      PBAR(I)=0.0
50    CONTINUE
      GO TO 52
51    CONTINUE
      WRITE(6,118)
   52 CONTINUE
      MAX=LB
      PAVI=0.0
      DO 2 K=1,MAX
      RR=FLOAT(K-1)*DR
      PRES(K)=H*PBAR(K)
      PAVI=PAVI+PRES(K)*RR*DR
2     CONTINUE
      WRITE(6,130) PAV
      CK2=(0.345*SIGMA)/(AK*TAN)
      CK3=(8.0*B)/AK
      SUM=0.0
      DO 3 I=1,MAX
      R=FLOAT(I-1)*DR
      ALAM=R/B
      DL=DR/B
      SUM=SUM+ALAM*(PBAR(I)**0.985)*DL
```

```
 3    CONTINUE
      TERM1=CK2/SUM
      SUMMA=0.0
      DO 5 J=1,N
      X=ANU(J)
      SUM1=0.0
      DO 4 I=1,MAX
      R=FLOAT(I-1)*DR
      ALAM=R/B
      Y=ANU(J)*ALAM
      SUM1=SUM1+ALAM*((PRES(I)/PAV)**0.985)*(BESEL(Y))*DL
 4    CONTINUE
      SUMMA=SUMMA+(SUM1*SUM1)/(ANU(J)*(ABS(BESEL(X))**2.0))
      WRITE(6,131) SUM1
 5    CONTINUE
      TERM2=CK3*SUMMA
      RES=(TERM1+TERM2)/12.0
      ANS1=TERM1/12.0
      ANS2=TERM2/12.0
      WRITE(6,104)
      WRITE(6,106) H,SIGM1,SIGM2
      WRITE(6,116) DRB
      WRITE(6,107)
      WRITE(6,106) TAN1,TAN2,AK
      WRITE(6,108)
      WRITE(6,109) MAX,N,R1,R2
      WRITE(6,110)
      WRITE(6,100) (PBAR(I),I=1,MAX)
      WRITE(6,119)
      WRITE(6,112) ANS1
      WRITE(6,120)
      WRITE(6,112) ANS2
      WRITE(6,111)
      WRITE(6,112) RES
      IF(BNUM.GT.6.0) GO TO 3334
      GO TO 3333
```

```
 3334 CONTINUE
C      K IS IN (BTU/HR-F-FT))
C      SIGMA AND B ARE IN INCHES
C      RESISTANCE IS IN (HR-F-FT-FT)/BTU
       CALL EXIT
       END
```

APPENDIX C

TABLES OF COORDINATES FOR PRESSURE DISTRIBUTIONS

Pl = .002

$\overline{F}$ = .005

$\overline{r}$	$\overline{P}$
0.000	.00066
0.302	.00066
0.603	.00061
0.905	.00054
1.206	.00046
1.508	.00034
1.809	.00019
2.111	.00007
2.412	.00000

P1 = .004

$\overline{F}$ = .005		$\overline{F}$ = .010	
$\overline{r}$	$\overline{P}$	$\overline{r}$	$\overline{P}$
0.000	.00097	0.000	.00132
0.239	.00097	0.302	.00132
0.478	.00089	0.603	.00121
0.718	.00081	0.905	.00109
0.957	.00068	1.206	.00092
1.197	.00052	1.508	.00069
1.436	.00034	1.809	.00040
1.675	.00016	2.111	.00014
1.915	.00005	2.412	.00002
2.154	.00001	2.714	.00000
2.393	.00000		

P1 = .006

$\bar{F}$ = .010		$\bar{F}$ = .015	
$\bar{r}$	$\bar{P}$	$\bar{r}$	$\bar{P}$
0.000	.00164	0.000	.00199
0.263	.00164	0.302	.00199
0.527	.00152	0.603	.00183
0.790	.00136	0.905	.00165
1.054	.00115	1.206	.00139
1.317	.00087	1.508	.00103
1.580	.00054	1.809	.00062
1.844	.00024	2.111	.00023
2.107	.00006	2.412	.00004
2.371	.00000	2.714	.00000

$P1 = .01$

$\overline{F} = .02$

$\overline{r}$	$\overline{P}$
0.000	.00293
0.279	.00293
0.559	.00270
0.839	.00243
1.120	.00204
1.400	.00153
1.679	.00093
1.959	.00039
2.239	.00009
2.519	.00000

P1 = .02

$\overline{F}$ = .040

$\overline{r}$	$\overline{P}$
0.000	.00590
0.279	.00590
0.559	.00545
0.839	.00492
1.120	.00416
1.400	.00316
1.679	.00199
1.959	.00089
2.239	.00023
2.519	.00003
2.799	.00000

$\overline{F}$ = .060

$\overline{r}$	$\overline{P}$
0.000	.00685
0.320	.00685
0.641	.00631
0.961	.00568
1.282	.00476
1.602	.00350
1.922	.00198
2.243	.00068
2.563	.00010
2.884	.00000

$\overline{F}$ = .080

$\overline{r}$	$\overline{P}$
0.000	.00765
0.353	.00765
0.705	.00701
1.058	.00629
1.411	.00528
1.763	.00384
2.116	.00203
2.469	.00054
2.821	.00005

P1 = .04

$\overline{F}$ = .13		$\overline{F}$ = .16	
$\overline{r}$	$\overline{P}$	$\overline{r}$	$\overline{P}$
0.000	.01412	0.000	.01556
0.320	.01412	0.353	.01556
0.641	.01305	0.705	.01433
0.961	.01182	1.058	.01294
1.282	.01004	1.411	.01090
1.602	.00763	1.763	.00807
1.922	.00469	2.116	.00458
2.243	.00189	2.469	.00145
2.563	.00036	2.821	.00016
2.884	.00002	3.174	.00000
3.204	.00000		

$P1 = .05$

$\overline{F} = .2$

$\overline{r}$	$\overline{P}$
0.000	.01861
0.353	.01861
0.705	.01708
1.058	.01530
1.411	.01273
1.763	.00916
2.116	.00487
2.469	.00140
2.821	.00014
3.174	.00000
3.527	.00000

P1 = .1

$\overline{F} = .25$

$\overline{r}$	$\overline{P}$
0.000	.03056
0.302	.03056
0.603	.02825
0.905	.02540
1.206	.02138
1.508	.01609
1.809	.00992
2.111	.00429
2.412	.00104
2.714	.00011
3.015	.00000

$\overline{F} = .5$

$\overline{r}$	$\overline{P}$
0.000	.04072
0.367	.04072
0.734	.03758
1.100	.03397
1.467	.02878
1.834	.02169
2.201	.01282
2.567	.00439
2.934	.00053
3.301	.00002
3.668	.00000

$\overline{F} = .6$

$\overline{r}$	$\overline{P}$
0.000	.04503
0.415	.04503
0.829	.04122
1.244	.03693
1.658	.03079
2.073	.02210
2.488	.01103
2.902	.00232
3.317	.00009
3.731	.00000

P1 = .20

$\overline{F} = .2$

$\overline{r}$	$\overline{P}$
0.000	.03741
0.222	.03741
0.444	.03543
0.667	.03010
0.889	.02685
1.111	.02071
1.333	.01446
1.555	.00859
1.777	.00409
1.999	.00145
2.222	.00036
2.444	.00006
2.666	.00000

$\overline{F} = .4$

$\overline{r}$	$\overline{P}$
0.000	.05564
0.280	.05564
0.560	.05174
0.840	.04685
1.120	.04008
1.400	.03131
1.679	.02105
1.959	.01096
2.239	.00378
2.519	.00073
2.799	.00007
3.079	.00000

$\overline{F} = .6$

$\overline{r}$	$\overline{P}$
0.000	.06558
0.320	.06558
0.641	.06071
0.961	.05472
1.282	.04625
1.602	.03506
1.922	.02189
2.243	.00954
2.563	.00223
2.884	.00021
3.204	.00000

$\overline{F} = .8$

$\overline{r}$	$\overline{P}$
0.000	.07312
0.353	.07312
0.705	.06748
1.058	.06068
1.411	.05091
1.763	.03767
2.116	.02191
2.469	.00788
2.821	.00119
3.174	.00005
3.527	.00000

P1 = .20

$\overline{F} = 1$

$\overline{r}$	$\overline{P}$
0.000	.08200
0.379	.08200
0.759	.07567
1.114	.06825
1.520	.05759
1.899	.04296
2.279	.02486
2.659	.00818
3.039	.00090
3.419	.00002
3.799	.00000

$\overline{F} = 1.3$

$\overline{r}$	$\overline{P}$
0.000	.09035
0.404	.09035
0.807	.08344
1.211	.07551
1.615	.06403
2.018	.04827
2.422	.02826
2.826	.00897
3.229	.00079
3.633	.00001
4.037	.00000

$\overline{F} = 1.4$

$\overline{r}$	$\overline{P}$
0.000	.09551
0.425	.09551
0.850	.08810
1.275	.08002
1.700	.06825
2.125	.05113
2.550	.02863
2.975	.00791
3.400	.00049
3.825	.00000

$\overline{F} = 1.6$

$\overline{r}$	$\overline{P}$
0.000	.09919
0.444	.09919
0.889	.09136
1.333	.08232
1.777	.06892
2.222	.05033
2.666	.02679
3.110	.00611
3.554	.00023
3.999	.00000

$P1 = .20$

$\overline{F} = 1.7$

$\overline{r}$	$\overline{P}$
0.000	.10280
0.462	.10280
0.924	.09467
1.386	.08538
1.848	.07131
2.310	.05126
2.773	.02578
3.235	.00489
3.697	.00011
4.159	.00000

P1 = .30

$\overline{F} = .2$

$\overline{r}$	$\overline{P}$
0.000	.04341
0.194	.04341
0.388	.04109
0.582	.03840
0.776	.03203
0.970	.02575
1.164	.01922
1.359	.01283
1.553	.00742
1.747	.00357
1.941	.00137

$\overline{F} = .4$

$\overline{r}$	$\overline{P}$
0.000	.06279
0.245	.06279
0.489	.05844
0.734	.05259
0.978	.04461
1.223	.03477
1.467	.02403
1.712	.01364
1.956	.00588
2.201	.00174
2.445	.00032
2.690	.00003
2.934	.00000

$\overline{F} = .6$

$\overline{r}$	$\overline{P}$
0.000	.07933
0.280	.07933
0.559	.07374
0.837	.06653
1.120	.05658
1.400	.04384
1.679	.02921
1.959	.01511
2.239	.00525
2.519	.00103
2.799	.00099
3.079	.00000

$\overline{F} = .8$

$\overline{r}$	$\overline{P}$
0.000	.10440
0.308	.10440
0.616	.10020
0.924	.08434
1.232	.06819
1.540	.05281
1.848	.03449
2.157	.01671
2.465	.00484
2.773	.00066
3.081	.00003
3.389	.00000

P1 = .30

$\overline{F} = 1.2$

$\overline{r}$	$\overline{P}$
0.000	.11040
0.353	.11040
0.705	.10230
1.058	.09235
1.411	.07819
1.763	.05913
2.116	.03619
2.469	.01456
2.821	.00268
3.174	.00015
3.527	.00000

$\overline{F} = 1.5$

$\overline{r}$	$\overline{P}$
0.000	.11940
0.379	.11940
0.759	.11030
1.140	.09936
1.520	.08357
1.899	.06189
2.279	.03556
2.659	.01198
3.039	.00144
3.419	.00004
3.799	.00000

$\overline{F} = 2$

$\overline{r}$	$\overline{P}$
0.000	.13300
0.418	.13300
0.836	.12230
1.254	.11000
1.672	.09227
2.091	.06731
2.509	.03580
2.927	.00912
3.345	.00054
3.736	.00000

$\overline{F} = 2.6$

$\overline{r}$	$\overline{P}$
0.000	.15240
0.450	.15240
0.901	.14070
1.351	.12800
1.802	.10950
2.252	.08245
2.702	.04636
3.153	.01238
3.603	.00060
4.054	.00000

$P1 = .30$

$\overline{F} = 3.1$		$\overline{F} = 3.4$	
$\overline{r}$	$\overline{P}$	$\overline{r}$	$\overline{P}$
0.000	.19610	0.000	.16930
0.479	.19610	0.499	.16930
0.957	.18000	0.998	.15550
1.436	.15500	1.497	.14050
1.914	.12160	1.996	.11900
2.393	.08924	2.495	.08791
2.872	.04903	2.994	.04557
3.350	.01158	3.493	.00824
3.829	.00035	3.992	.00012
4.307	.00000	4.491	.00000

P1 = .40

$\overline{F}$ = .1		$\overline{F}$ = .2	
$\overline{r}$	$\overline{P}$	$\overline{r}$	$\overline{P}$
0.000	.02907	0.000	.04654
0.140	.02907	0.176	.04654
0.279	.02770	0.353	.04411
0.419	.02567	0.529	.04010
0.559	.02301	0.705	.03503
0.699	.01985	0.882	.02884
0.839	.01638	1.058	.02233
0.979	.01281	1.234	.01584
1.120	.00941	1.411	.01008
1.260	.00644	1.587	.00561
1.400	.00405	1.763	.00265
1.540	.00231	1.940	.00103
1.680	.00118	2.116	.00032
1.820	.00054	2.292	.00008
1.959	.00021	2.469	.00001
2.099	.00007	2.645	.00000
2.239	.00002		
2.379	.00000		

P1 = .40

$\overline{F}$ = .3		$\overline{F}$ = .4	
$\overline{r}$	$\overline{P}$	$\overline{r}$	$\overline{P}$
0.000	.05887	0.000	.07284
0.202	.05887	0.222	.07284
0.404	.05519	0.444	.06905
0.606	.05004	0.667	.04606
0.807	.04320	0.889	.05324
1.009	.03497	1.111	.04192
1.211	.02603	1.333	.03019
1.413	.01720	1.555	.01883
1.615	.00972	1.777	.00963
1.817	.00450	1.999	.00378
2.019	.00162	2.222	.00106
2.220	.00044	2.444	.00020
2.422	.00008	2.666	.00002
2.624	.00001	2.888	.00000
2.826	.00000		

P1 = .40

$\overline{F}$ = .5

$\overline{r}$	$\overline{P}$
0.000	.07819
0.239	.07819
0.479	.07288
0.718	.06563
0.957	.05584
1.197	.04382
1.436	.03064
1.675	.01795
1.915	.00819
2.154	.00267
2.393	.00057
2.633	.00007
2.872	.00000

$\overline{F}$ = 1.0

$\overline{r}$	$\overline{P}$
0.000	.14330
0.302	.14330
0.603	.13600
0.905	.12600
1.206	.09847
1.508	.07404
1.809	.04977
2.111	.02613
2.412	.00886
2.714	.00154
3.015	.00011
3.317	.00000

$\overline{F}$ = 1.6

$\overline{r}$	$\overline{P}$
0.000	.16620
0.345	.16620
0.690	.15000
1.035	.13430
1.381	.10980
1.726	.08462
2.071	.05520
2.416	.02549
2.761	.00606
3.106	.00051
3.451	.00001
3.797	.00000

$\overline{F}$ = 2.0

$\overline{r}$	$\overline{P}$
0.000	.16250
0.380	.16250
0.760	.15070
1.140	.13660
1.520	.11630
1.899	.08867
2.279	.05445
2.659	.02113
3.039	.00327
3.419	.00012
3.799	.00000

P1 = .40

$\overline{F}$ = 2.3

$\overline{r}$	$\overline{P}$
0.000	.16910
0.409	.16910
0.818	.15580
1.228	.13980
1.637	.11670
2.046	.08486
2.455	.04595
2.864	.01283
3.274	.00097
3.683	.00001
4.092	.00000

$\overline{F}$ = 3

$\overline{r}$	$\overline{P}$
0.000	.18660
0.434	.18660
0.869	.17200
1.305	.15490
1.739	.13030
2.174	.09618
2.609	.05282
3.044	.01389
3.479	.00077
3.914	.00000

$\overline{F}$ = 3.5

$\overline{r}$	$\overline{P}$
0.000	.20270
0.462	.20270
0.924	.18720
1.386	.16970
1.848	.14390
2.310	.10610
2.773	.05679
3.235	.01355
3.697	.00051
4.159	.00000

$\overline{F}$ = 4.0

$\overline{r}$	$\overline{P}$
0.000	.21280
0.487	.21280
0.973	.19570
1.459	.17680
1.946	.14950
2.432	.10930
2.919	.05533
3.405	.01040
3.892	.00020
4.378	.00000

P1 = .50

$\overline{F}$ = 0.1		$\overline{F}$ = 0.2	
$\overline{r}$	$\overline{P}$	$\overline{r}$	$\overline{P}$
0.000	.03035	0.000	.04989
0.130	.03035	0.164	.04989
0.260	.02906	0.327	.04730
0.390	.02716	0.491	.04357
0.520	.02259	0.654	.03869
0.650	.02155	0.819	.02909
0.780	.01712	0.982	.02375
0.910	.01395	1.146	.01934
1.039	.01119	1.310	.01312
1.169	.00808	1.473	.00802
1.299	.00546	1.637	.00432
1.429	.00342	1.801	.00201
1.559	.00197	1.964	.00079
1.689	.00103	2.128	.00025
1.819	.00049	2.292	.00007
1.949	.00020	2.455	.00001
2.079	.00008	2.619	.00000
2.209	.00002		
2.339	.00000		

P1 = .50

$\overline{F} = 0.3$		$\overline{F} = 0.4$	
$\overline{r}$	$\overline{P}$	$\overline{r}$	$\overline{P}$
0.000	.06450	0.000	.07678
0.187	.06450	0.206	.07678
0.375	.06118	0.413	.07269
0.562	.05880	0.619	.06960
0.750	.04869	0.825	.05661
0.937	.03417	1.031	.04537
1.124	.03037	1.237	.03365
1.312	.02100	1.444	.02217
1.499	.01288	1.650	.01248
1.686	.00677	1.856	.00572
1.874	.00294	2.062	.00203
2.061	.00102	2.269	.00053
2.249	.00027	2.475	.00009
2.436	.00005	2.681	.00001
2.623	.00000	2.887	.00000

P1 = .50

$\bar{F} = 0.5$		$\bar{F} = 1$	
$\bar{r}$	$\bar{P}$	$\bar{r}$	$\bar{P}$
0.000	.08860	0.000	.12320
0.222	.08860	0.279	.12320
0.444	.08396	0.559	.11450
0.666	.07700	0.839	.10280
0.889	.06489	1.112	.08680
1.111	.05131	1.400	.06657
1.333	.03720	1.679	.04418
1.555	.02347	1.959	.02282
1.777	.01222	2.239	.00806
1.999	.00492	2.519	.00165
2.222	.00143	2.799	.00017
2.444	.00028	3.079	.00000
2.666	.00003		
2.888	.00000		

P1 = .50

$\bar{F} = 1.6$

$\bar{r}$	$\bar{P}$
0.000	.16010
0.320	.16010
0.641	.14910
0.961	.13520
1.282	.11580
1.602	.09038
1.922	.05998
2.243	.02954
2.563	.00861
2.884	.00112
3.204	.00005
3.524	.00000

$\bar{F} = 2$

$\bar{r}$	$\bar{P}$
0.000	.18160
0.353	.18160
0.705	.16870
1.058	.15280
1.411	.13030
1.763	.10030
2.116	.06408
2.469	.02833
2.821	.00624
3.174	.00045
3.527	.00000

$\bar{F} = 3$

$\bar{r}$	$\bar{P}$
0.000	.21710
0.404	.21710
0.807	.20130
0.211	.18240
1.615	.15520
2.018	.11810
2.422	.07195
2.826	.02652
3.229	.00334
3.633	.00007
4.037	.00000

$\bar{F} = 4$

$\bar{r}$	$\bar{P}$
0.000	.23810
0.443	.23810
0.889	.21990
1.333	.19850
1.777	.16740
2.222	.12400
2.666	.06894
3.110	.01859
3.554	.00101
3.999	.00000

P1 = .50

$\overline{F}$ = 5.3		$\overline{F}$ = 5.5	
$\overline{r}$	$\overline{P}$	$\overline{r}$	$\overline{P}$
0.000	.26970	0.000	.27400
0.497	.26970	0.485	.27400
0.994	.24820	0.969	.25380
1.491	.22350	1.455	.23050
1.988	.18800	1.940	.19600
2.485	.13720	2.425	.14760
2.982	.06998	2.909	.08466
3.479	.01309	3.394	.02249
3.976	.00023	3.879	.00083
4.473	.00000	4.364	.00000

P1 = .60

$\overline{F} = .5$

$\overline{r}$	$\overline{P}$
0.000	.09278
0.209	.09278
0.418	.08787
0.627	.08000
0.836	.06858
1.045	.05507
1.254	.04093
1.463	.02701
1.673	.01521
1.882	.00694
2.091	.00244
2.300	.00062
2.509	.00011
2.718	.00001
2.927	.00000

$\overline{F} = .9$

$\overline{r}$	$\overline{P}$
0.000	.13250
0.263	.13250
0.526	.12340
0.790	.11120
1.054	.09446
1.317	.07363
1.580	.05048
1.844	.02816
2.107	.01155
2.371	.00307
2.634	.00047
2.897	.00004
3.161	.00000

$\overline{F} = 1.4$

$\overline{r}$	$\overline{P}$
0.000	.16470
0.302	.16470
0.603	.15310
0.905	.13790
1.206	.11660
1.508	.08929
1.809	.05843
2.111	.02877
2.412	.00900
2.714	.00144
3.015	.00009
3.317	.00000

$\overline{F} = 2.2$

$\overline{r}$	$\overline{P}$
0.000	.24200
0.332	.24200
0.664	.23800
0.996	.20000
1.327	.16730
1.659	.12580
1.991	.08412
2.323	.04265
2.655	.01276
2.987	.00162
3.319	.00006
3.650	.00000

P1 = .60

$\overline{F}$ = 3

$\overline{r}$	$\overline{P}$
0.000	.23870
0.379	.23870
0.759	.22160
1.140	.20090
1.520	.17150
1.899	.13200
2.279	.08380
2.659	.03514
3.039	.00631
3.419	.00028
3.799	.00000

$\overline{F}$ = 4

$\overline{r}$	$\overline{P}$
0.000	.27230
0.418	.27230
0.836	.25280
1.254	.22970
1.672	.19650
2.091	.15090
2.509	.09325
2.927	.03470
3.345	.00414
3.763	.00007
4.181	.00000

$\overline{F}$ = 5

$\overline{r}$	$\overline{P}$
0.000	.28120
0.450	.28120
0.901	.25950
1.351	.23340
1.802	.19570
2.252	.14280
2.702	.07633
3.153	.01889
3.603	.00087
4.054	.00000

$\overline{F}$ = 6

$\overline{r}$	$\overline{P}$
0.000	.37270
0.479	.37270
0.957	.35000
1.436	.29740
1.914	.23730
2.393	.17740
2.872	.10300
3.350	.02892
3.829	.00129
4.307	.00000

TABLE I

EXPERIMENTAL DATA

Stainless Steel (303)

Specimen Pair No. 1
$k = 10.0$ BTU/HR.FT.^{o}F

Specimen 1.1	Specimen 1.2	Combined Values
$\sigma_1 = 81\ \mu''$	$\sigma_2 = 69\ \mu''$	$\sigma = 106\ \mu''$
$R_1 = 10.4''$	$R_2 = 8.4''$	$R_i = 4.65''$
$Tan_1\theta = .0512$	$Tan_2\theta = .0378$	$Tan\theta = .0635$

F(Applied Load in Lbs.)	h(BTU/HR.FT^{2o}F)
165	10.7
265	12.5
365	13.9
765	20.0
1165	26.0
2165	32.8
3165	41.0
5165	54.7
7165	62.5
9765	85.7

TABLE II

EXPERIMENTAL DATA

Stainless Steel (303)

Specimen Pair No. 2

$k = 10.0$ BTU/HR.FT^{o}F

Specimen 2.1	Specimen 2.2	Combined Values
$\sigma_1 = 69\ \mu''$	$\sigma_2 = 0\ \mu''$	$\sigma = 69\ \mu''$
$R_1 = 250''$	$R_2 = 156''$	$R_i = 96''$
$\tan_1\theta = .068$	$\tan_2\theta = 0$	$\tan\theta = .068$

F(Lbs.)	h(BTU/HR.FT^{2o}F)
165	49
265	71
365	88
565	101
765	124
1165	208

FIGURES

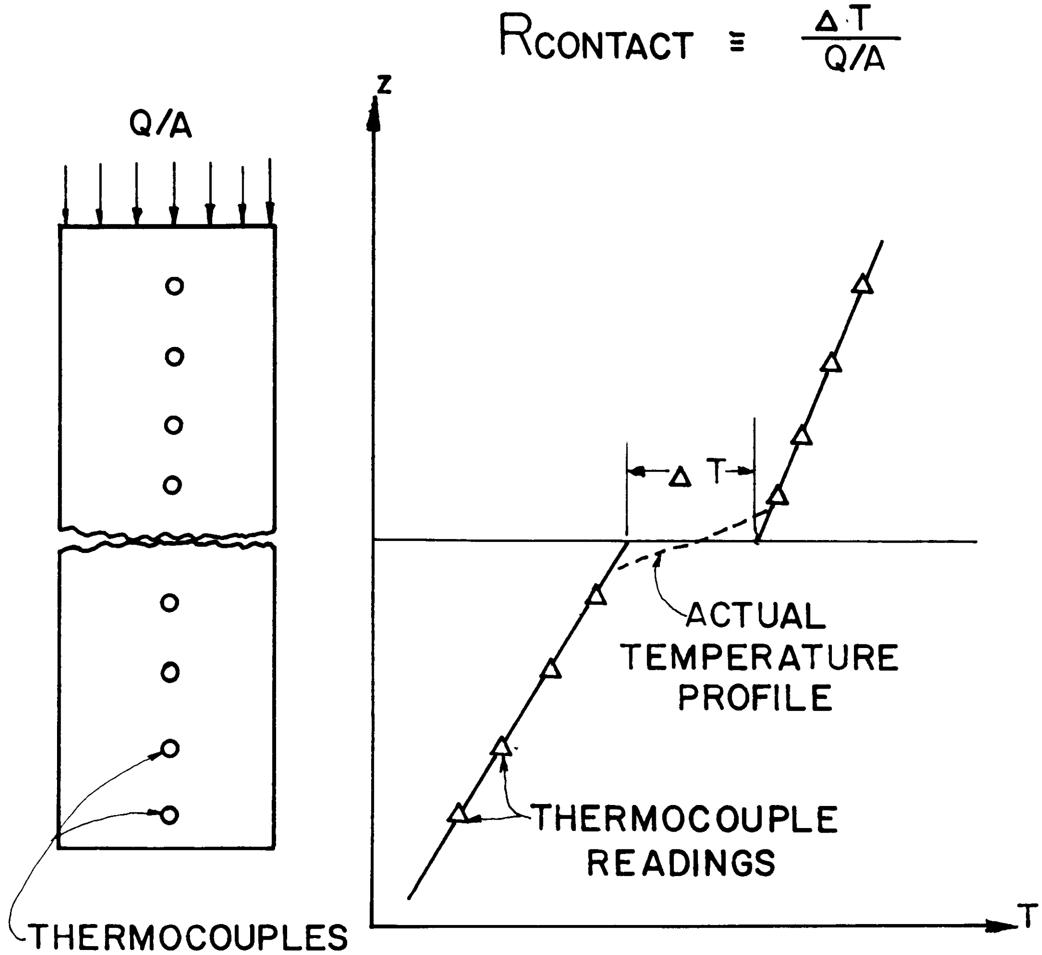


FIG. I DEFINITION OF CONTACT RESISTANCE

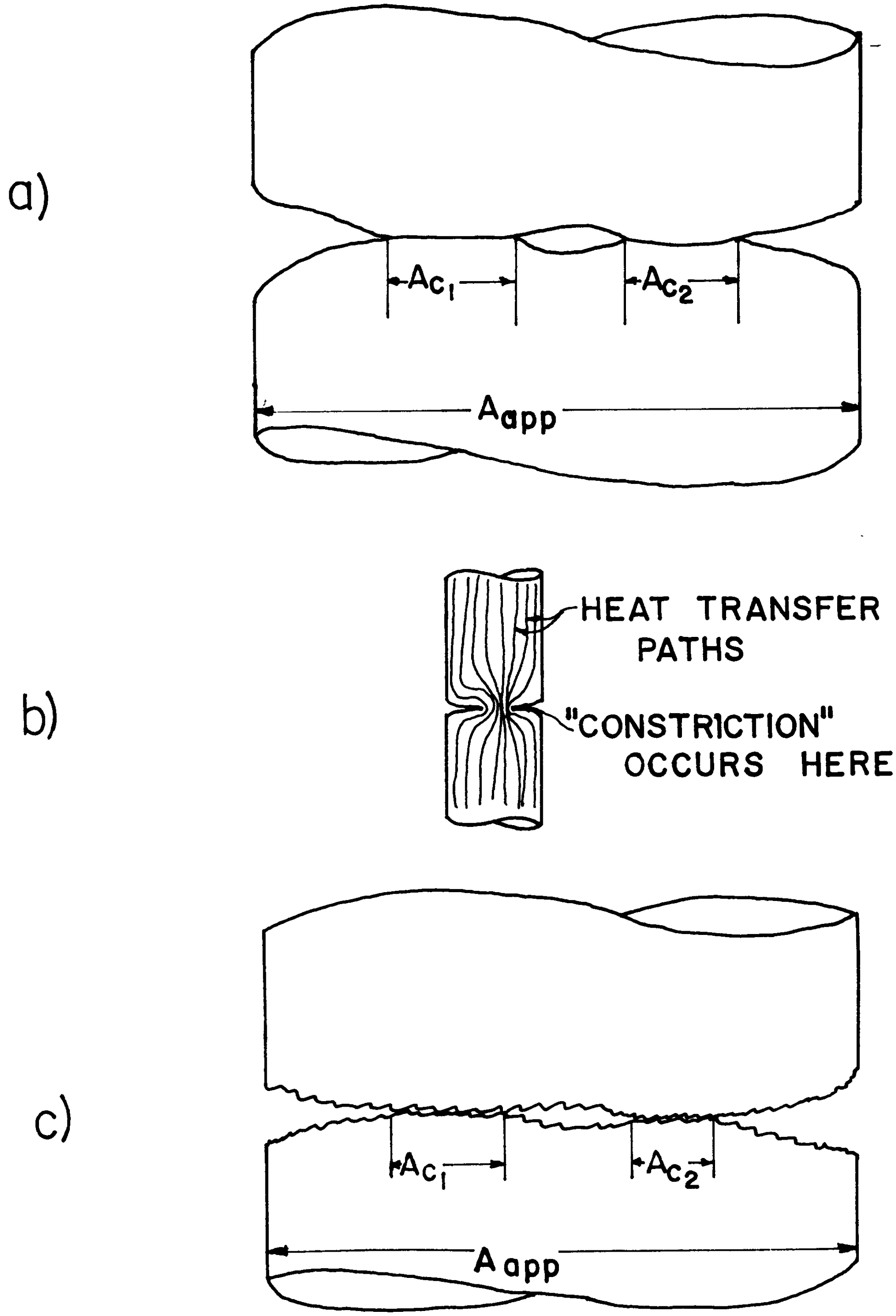


FIG. 2 SURFACE CONTACTS

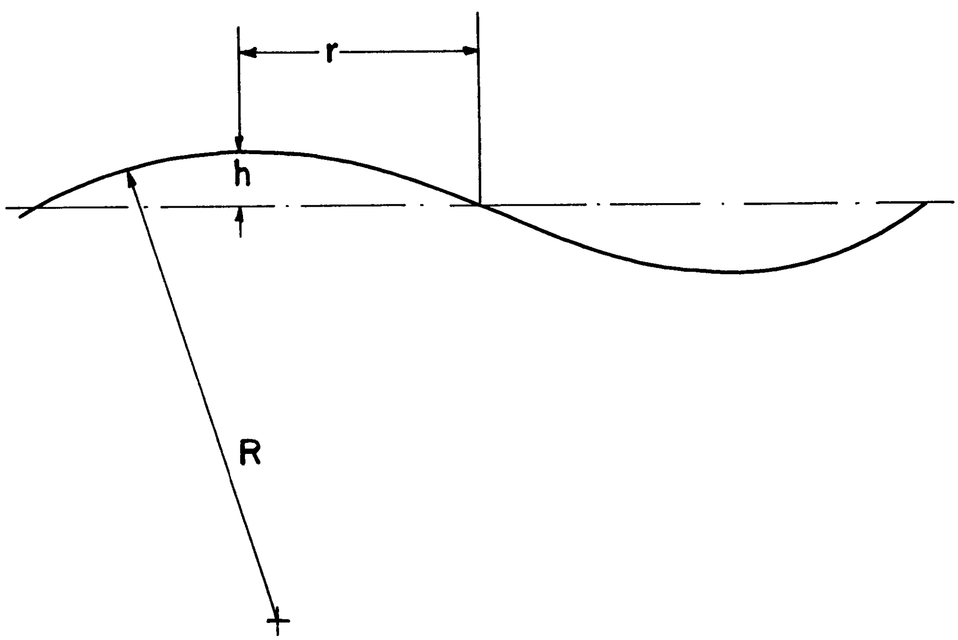


FIG. 3 SURFACE WAVINESS

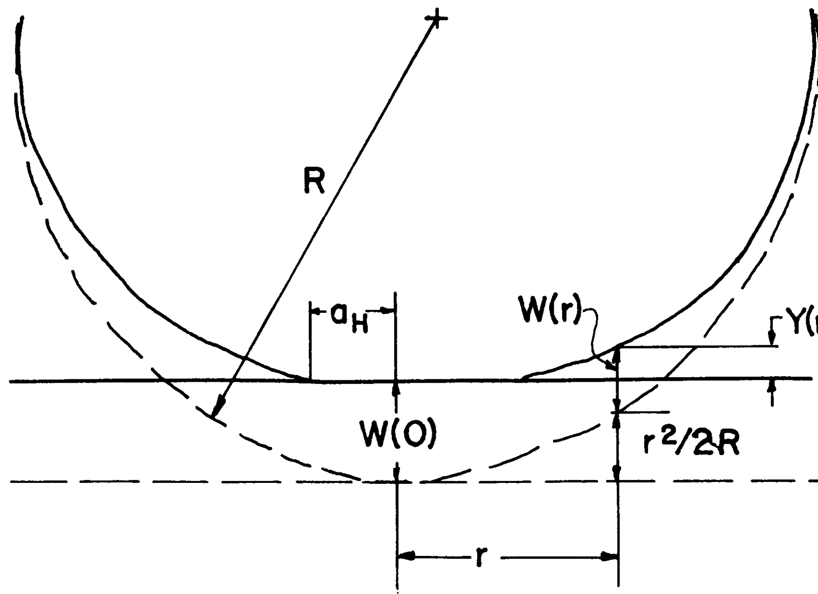


FIG. 4a SMOOTH SPHERE PRESSED AGAINST RIGID PLANE

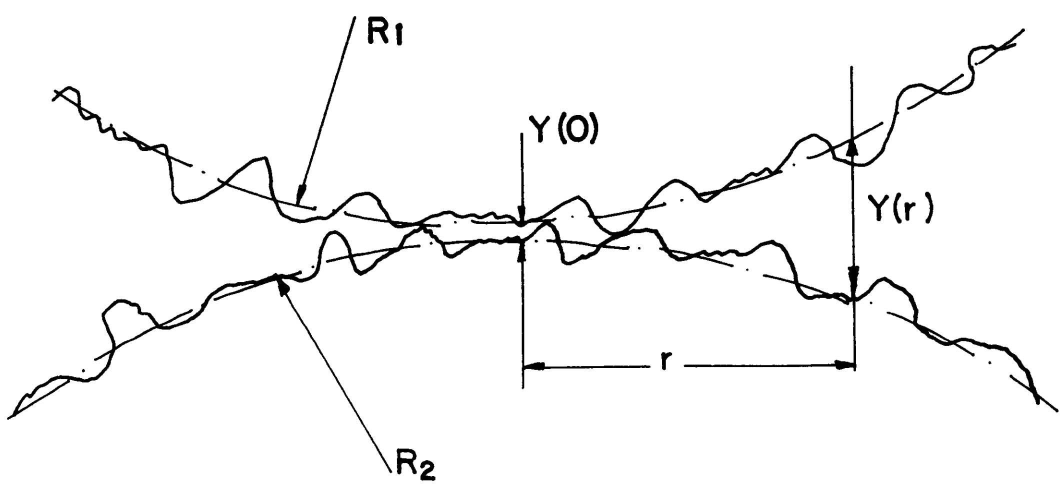


FIG. 4b TWO ROUGH SURFACES PRESSED TOGETHER

$$w(r) = \frac{1-\nu^2}{\pi E}\iint \frac{p}{\rho}\, dA$$

POINT A IS A POINT WHERE W(r) IS TO BE COMPUTED

POINT B IS THE ORIGIN OF A CIRCULAR COORDINATE SYSTEM

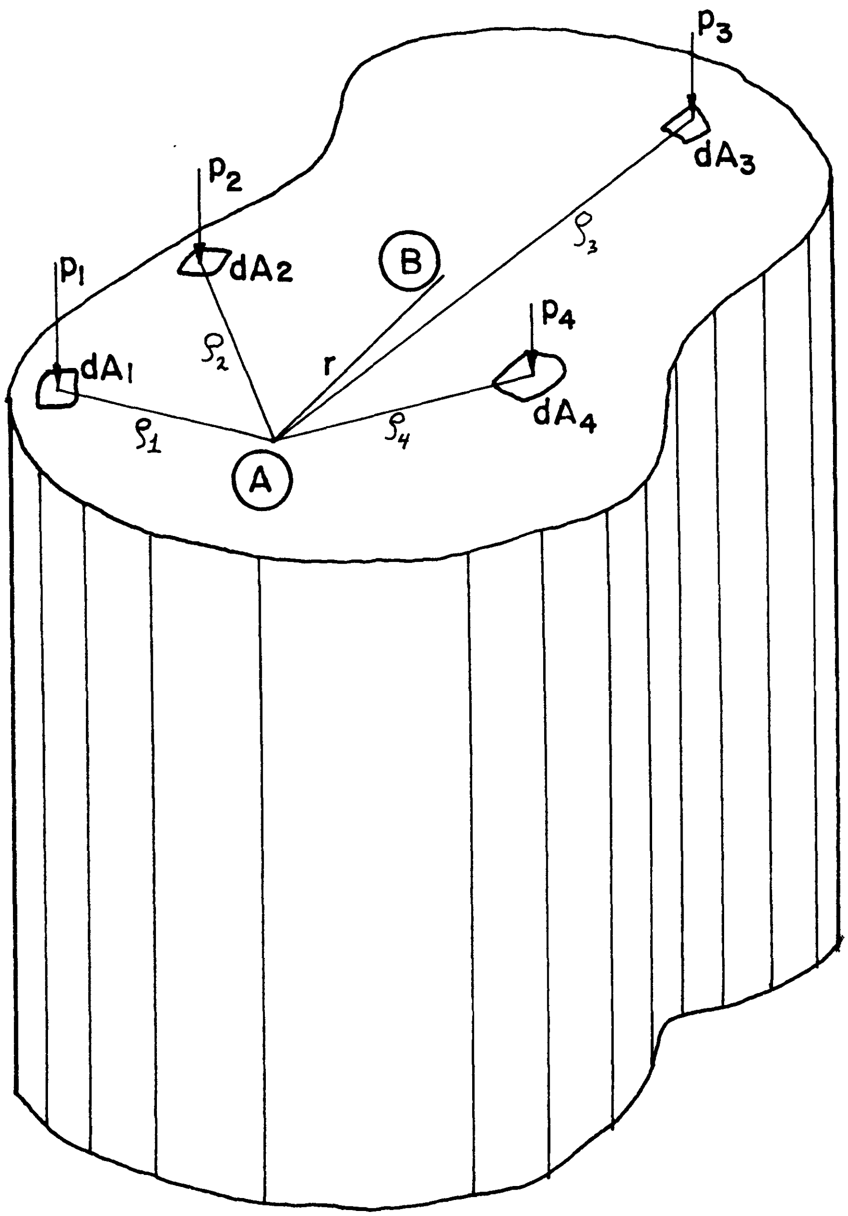


FIG. 5 TYPICAL CONTACT AREA

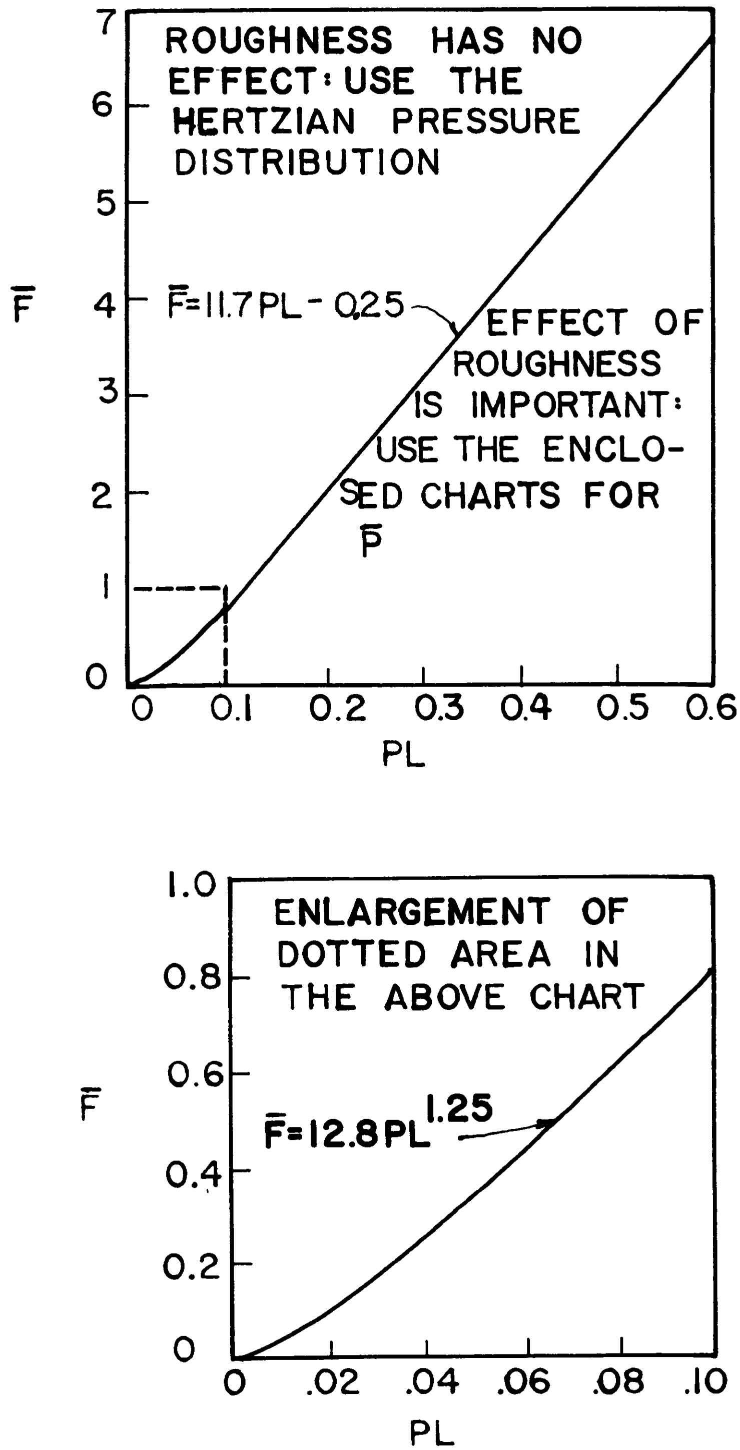
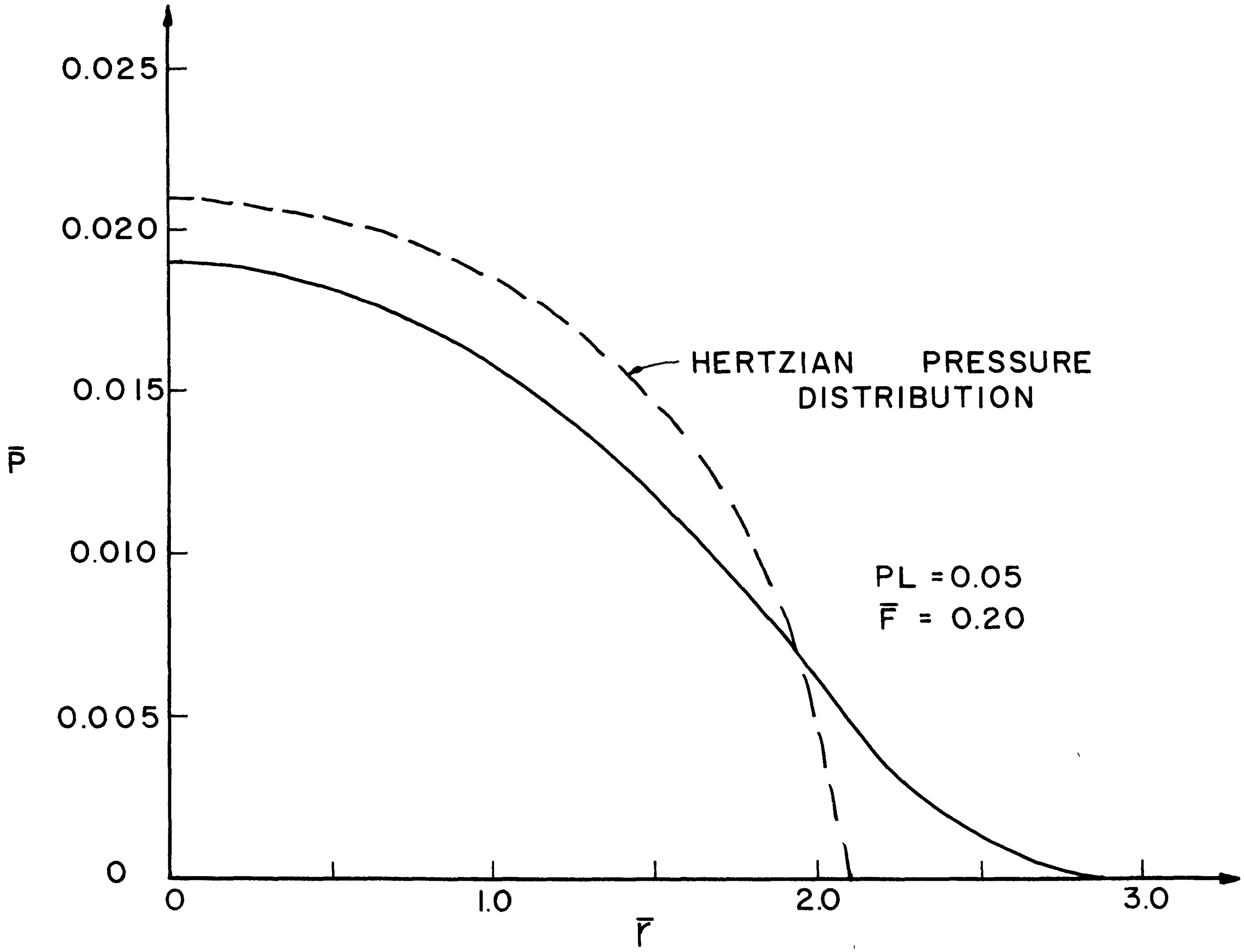


FIG. 6 REGION WHERE ROUGHNESS IS SIGNIFICANT



0.025
0.020
0.015
0.010
0.005
0
$\bar{P}$
HERTZIAN PRESSURE DISTRIBUTION
PL = 0.05
$\bar{F}$ = 0.20
0
1.0
2.0
3.0
$\bar{r}$

FIG. 7

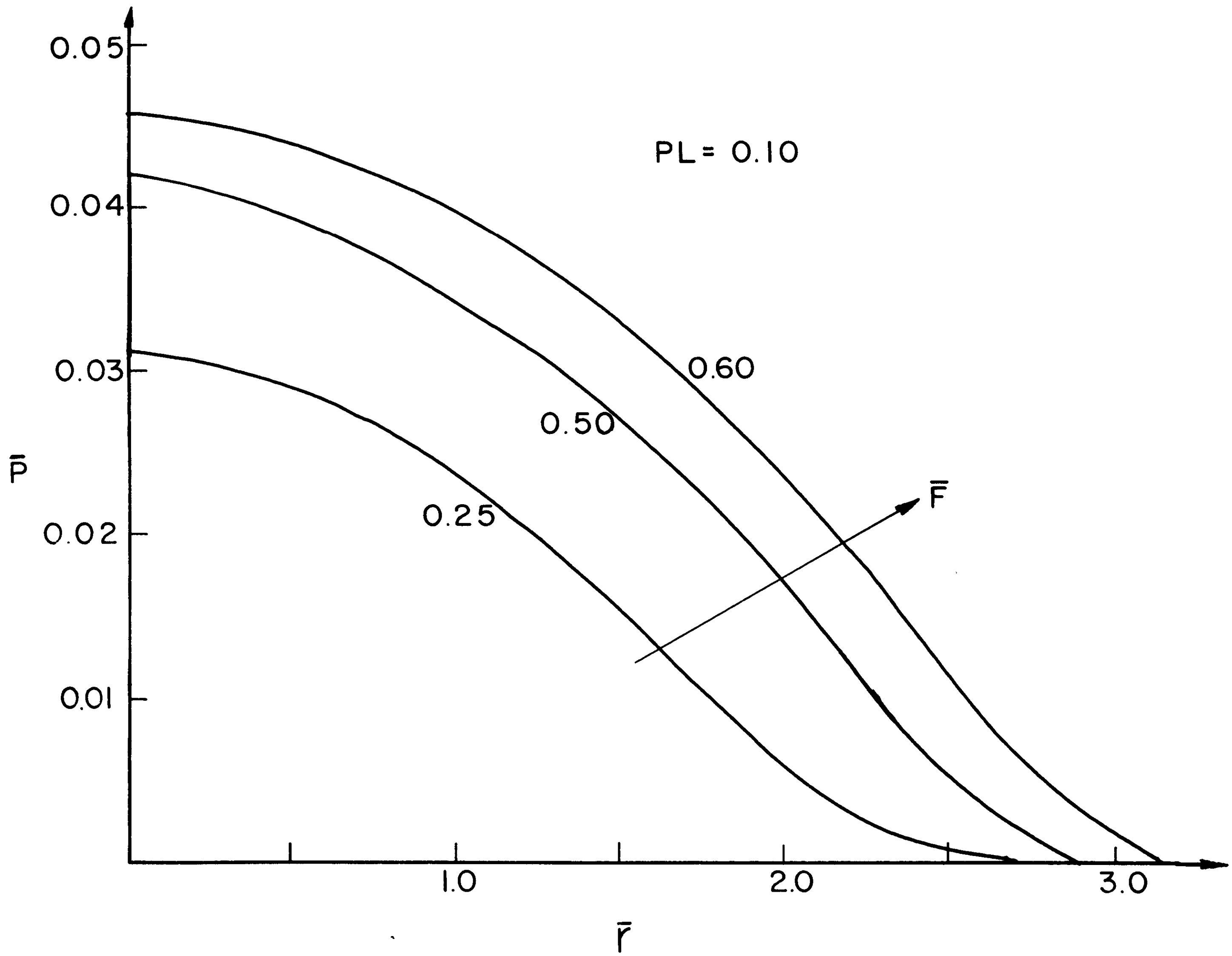


FIG. 8

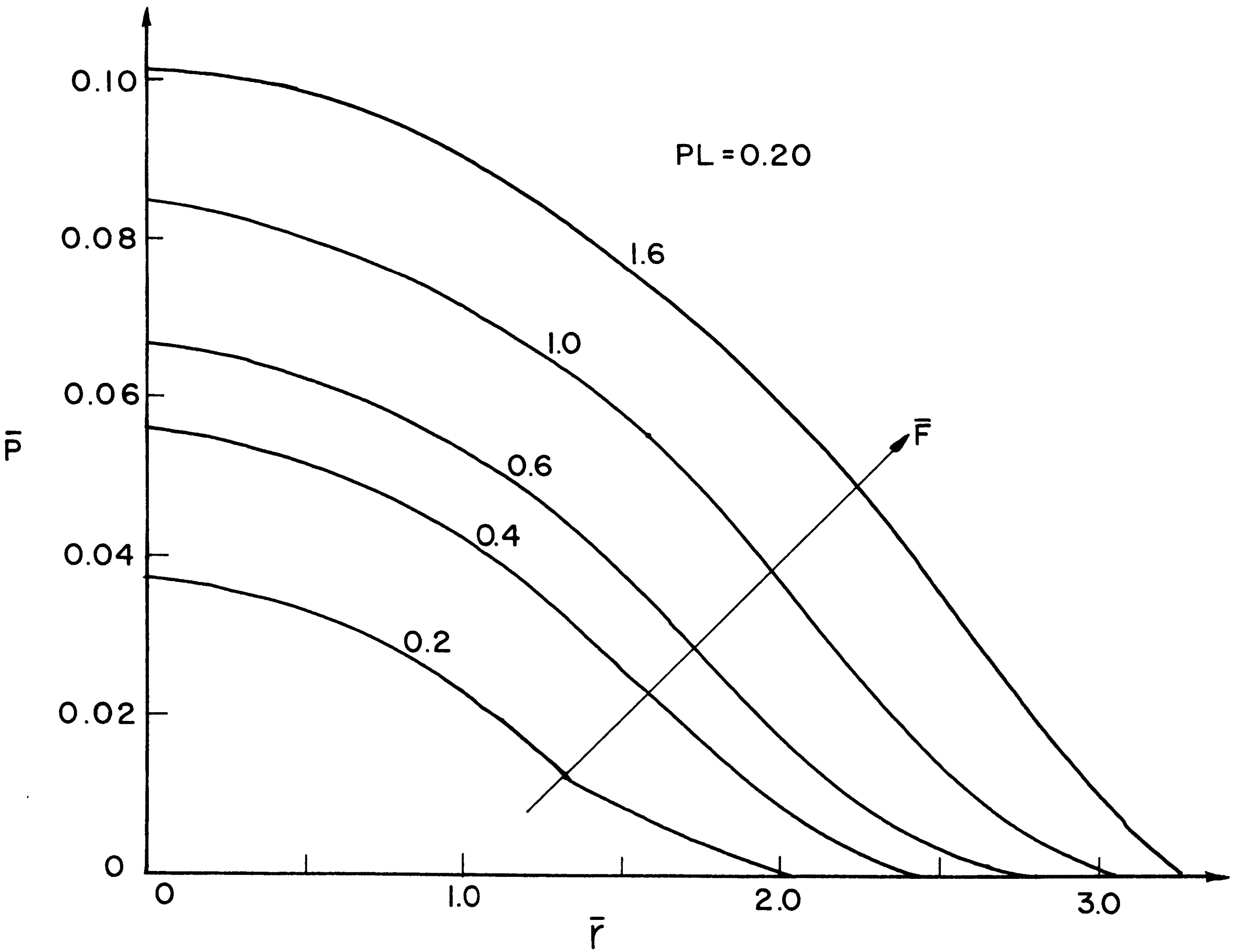


FIG. 9

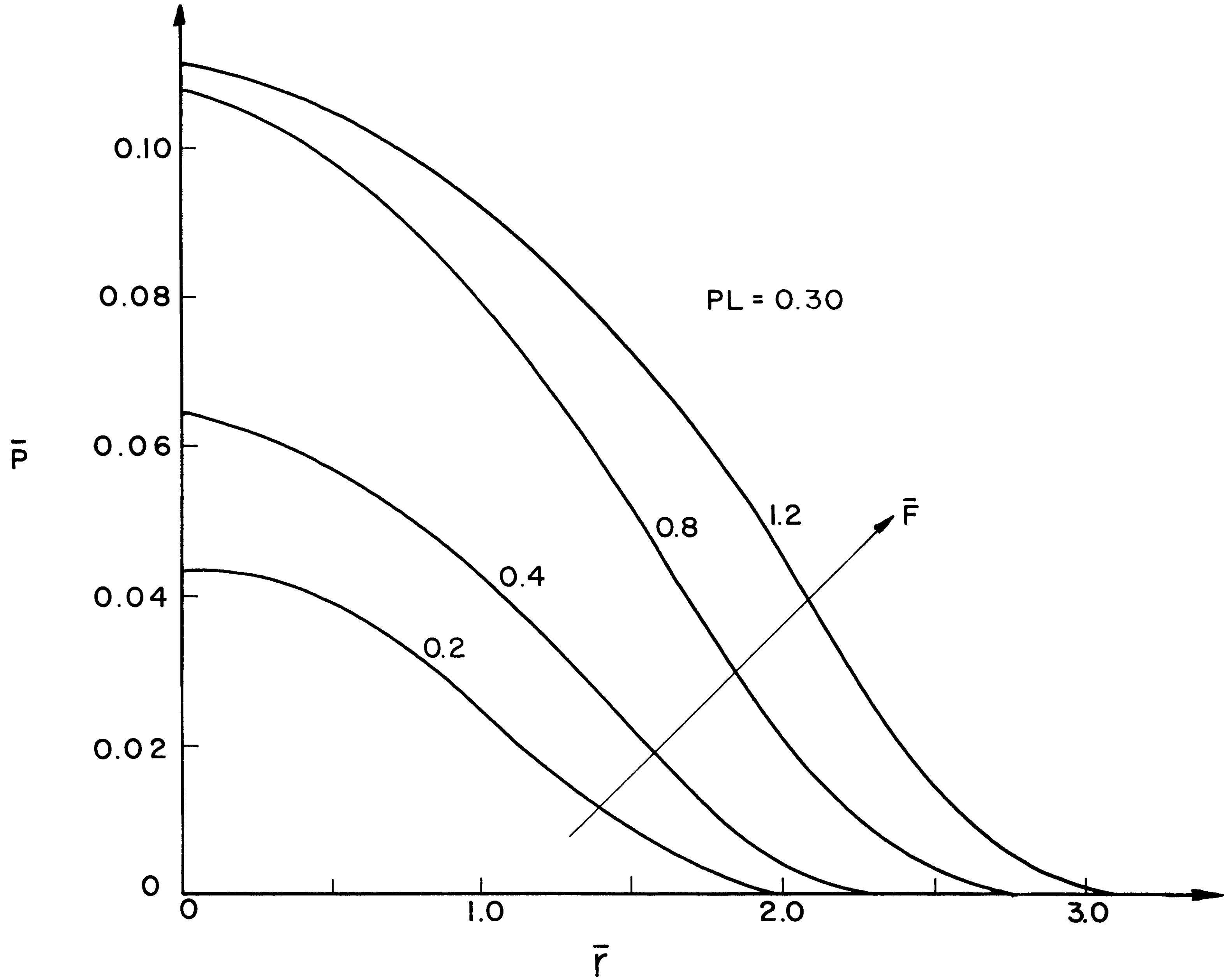


FIG. 10

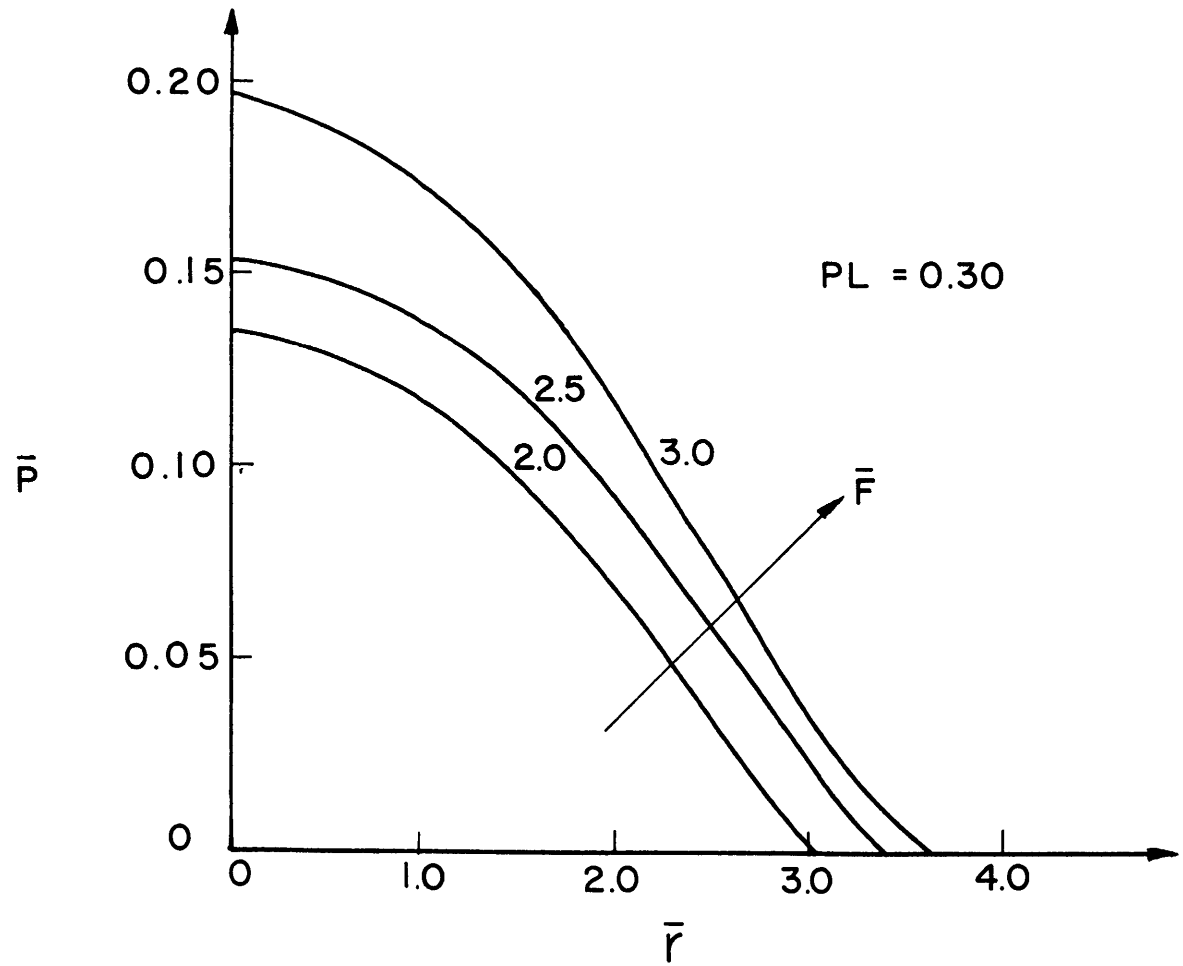


FIG. 11

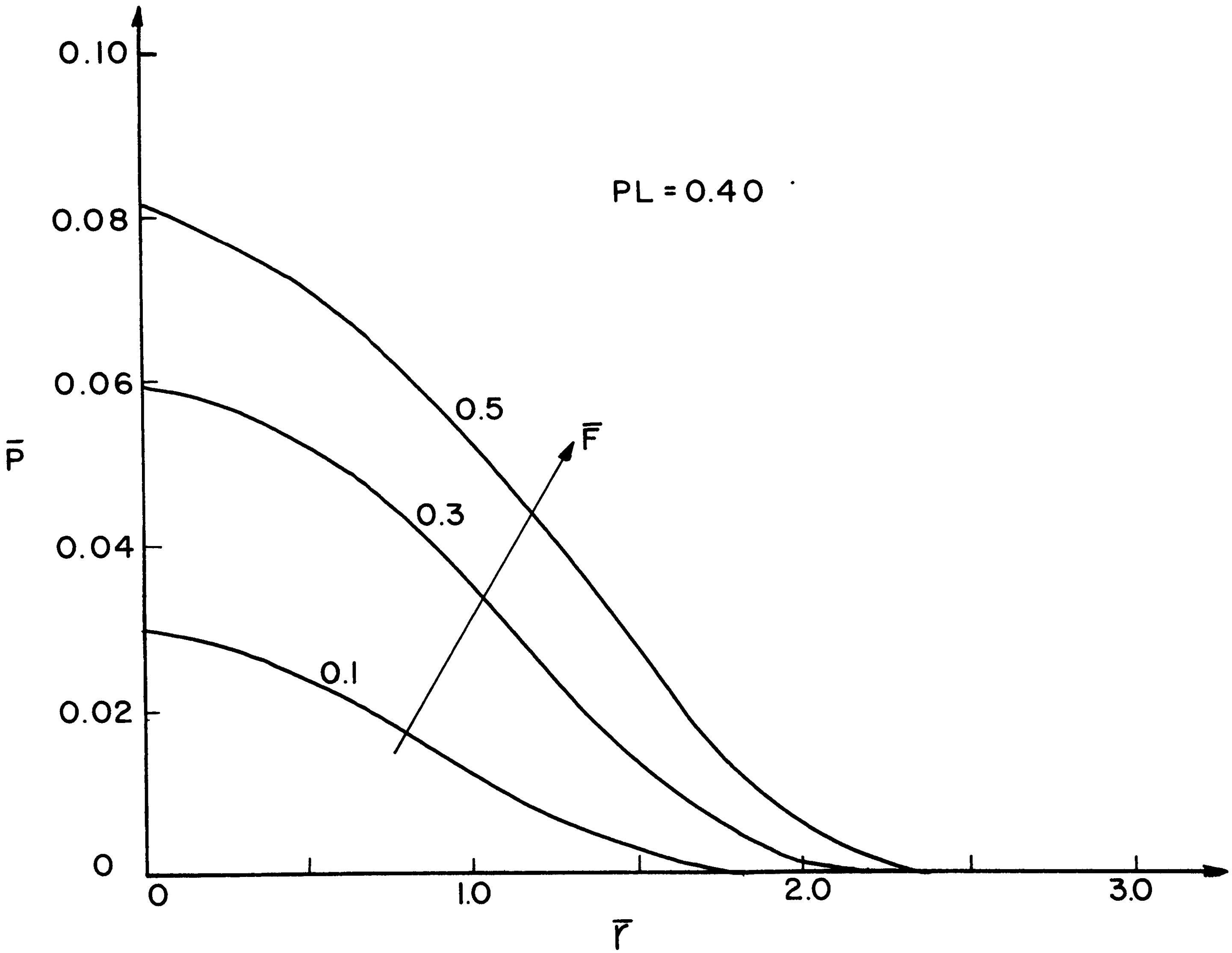


FIG. 12

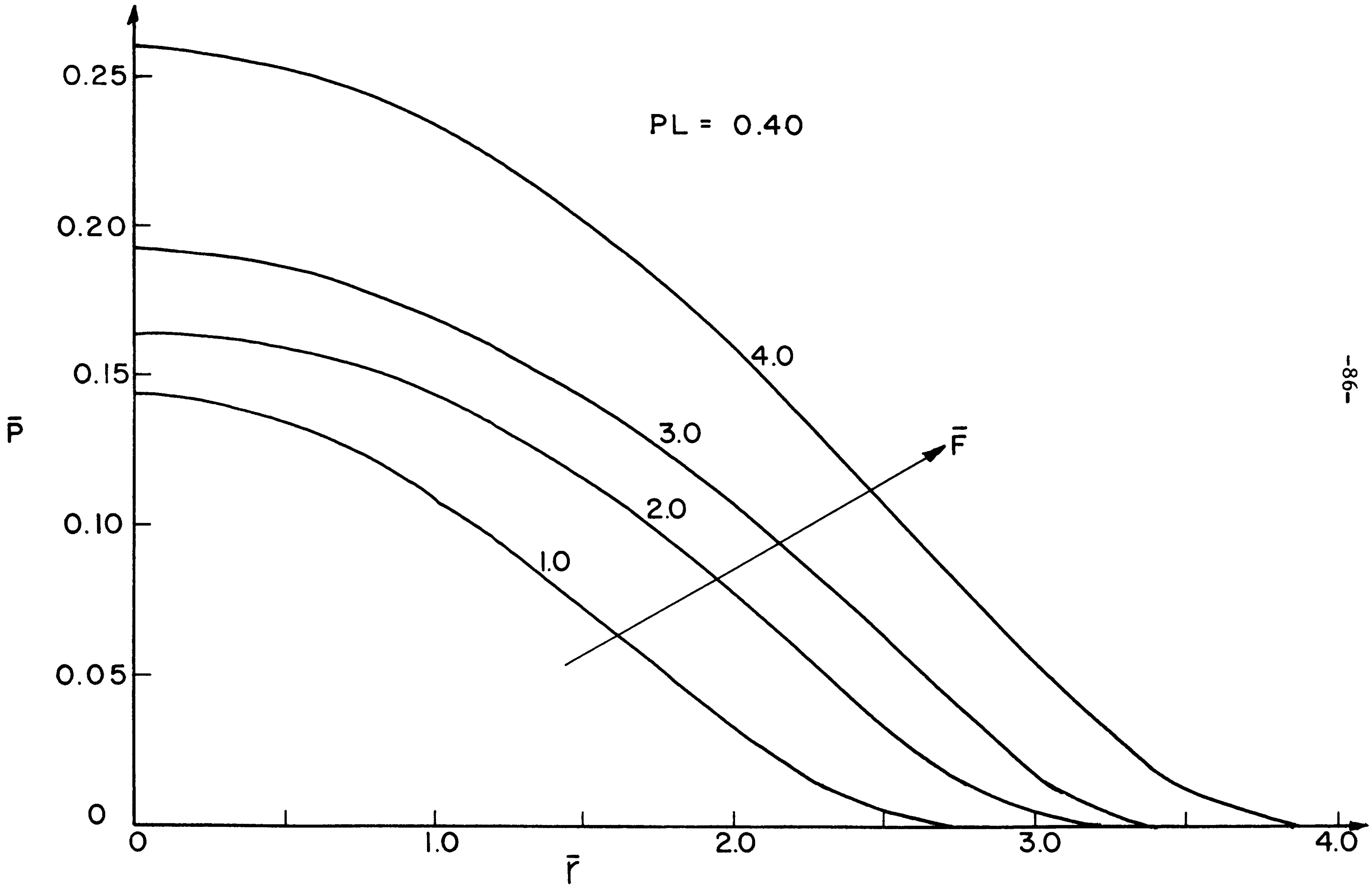

PL = 0.40
0.25
0.20
0.15
0.10
0.05
0
$\bar{P}$
4.0
3.0
2.0
1.0
$\bar{F}$
0
1.0
2.0
3.0
4.0
$\bar{r}$

FIG. 13

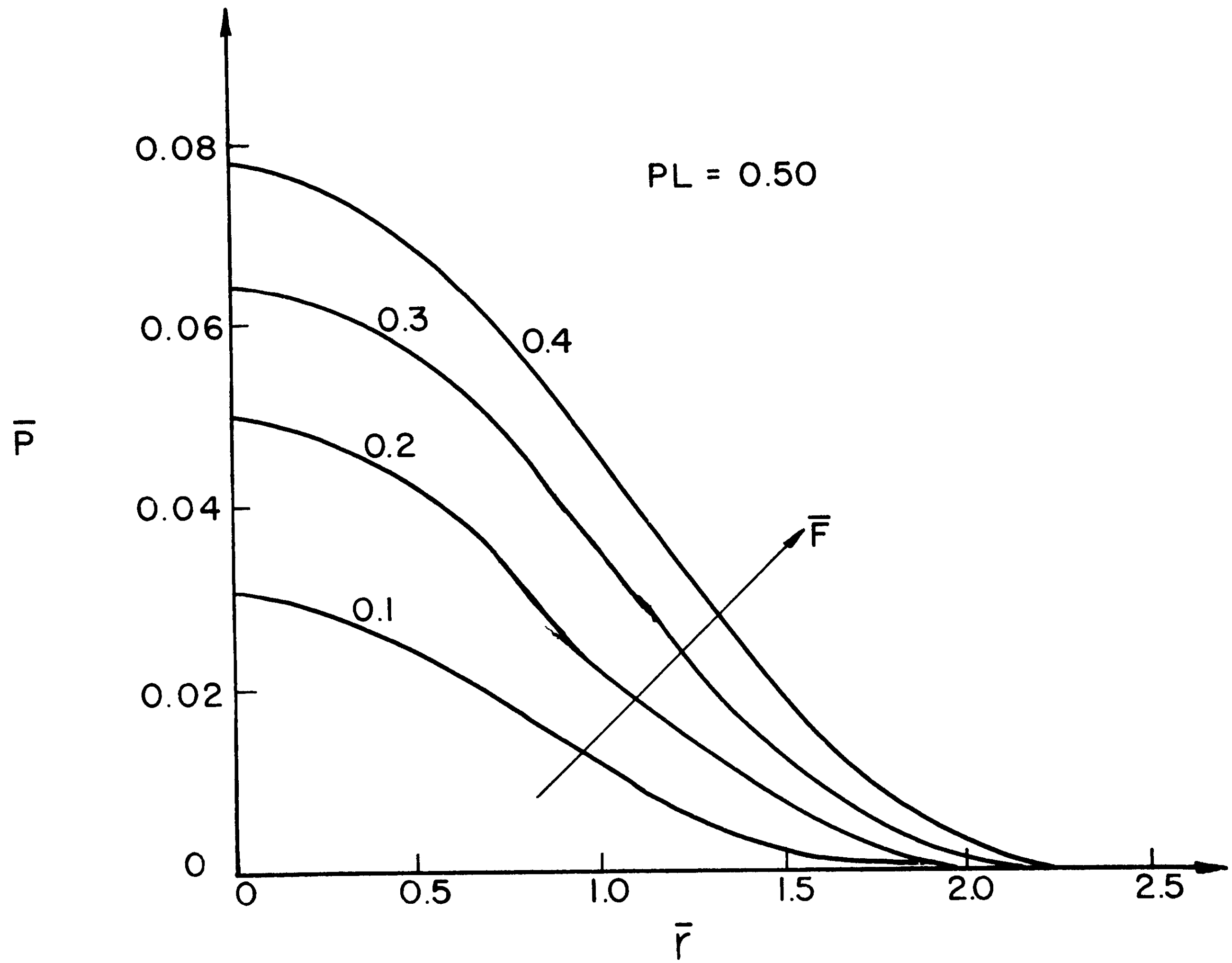


FIG. 14

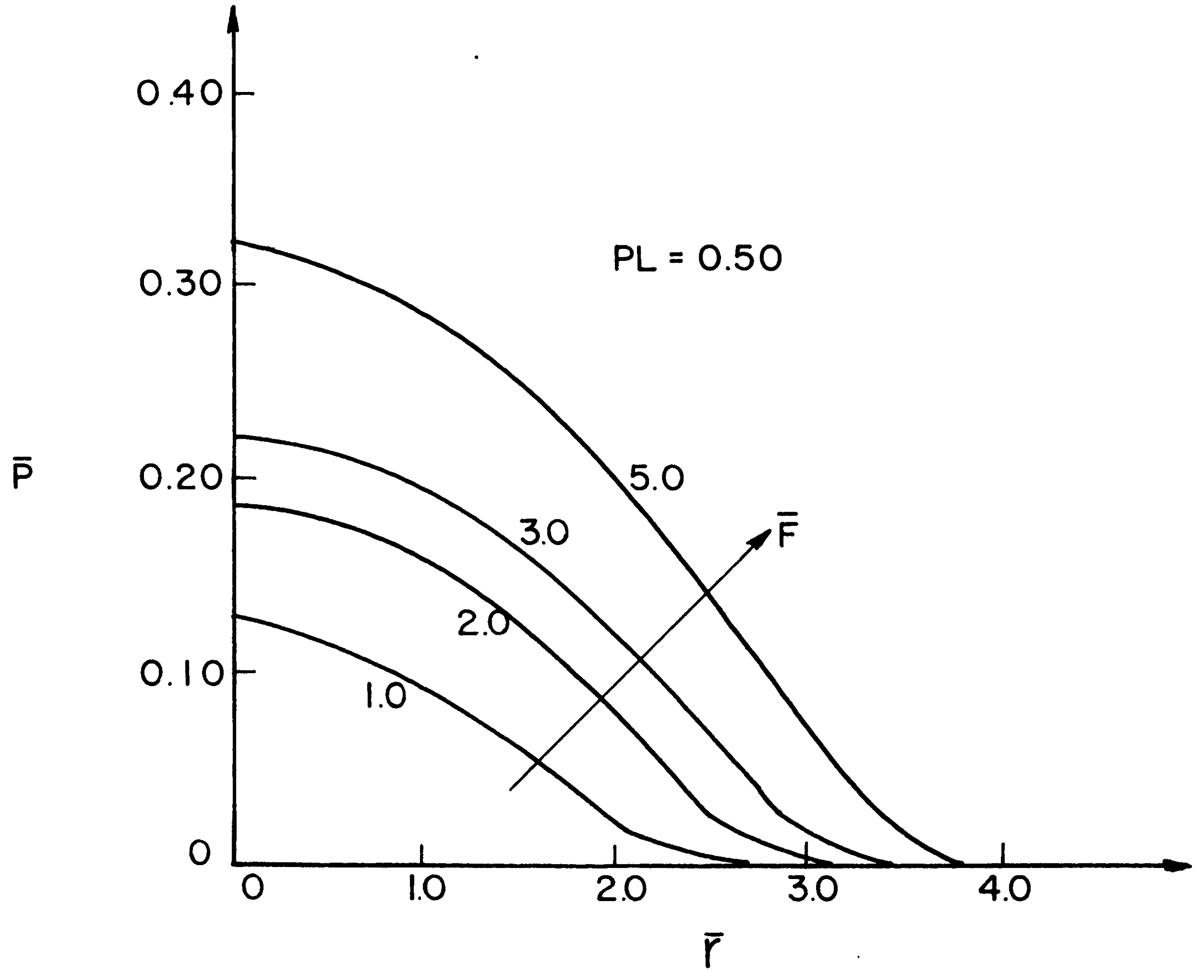


FIG. 15

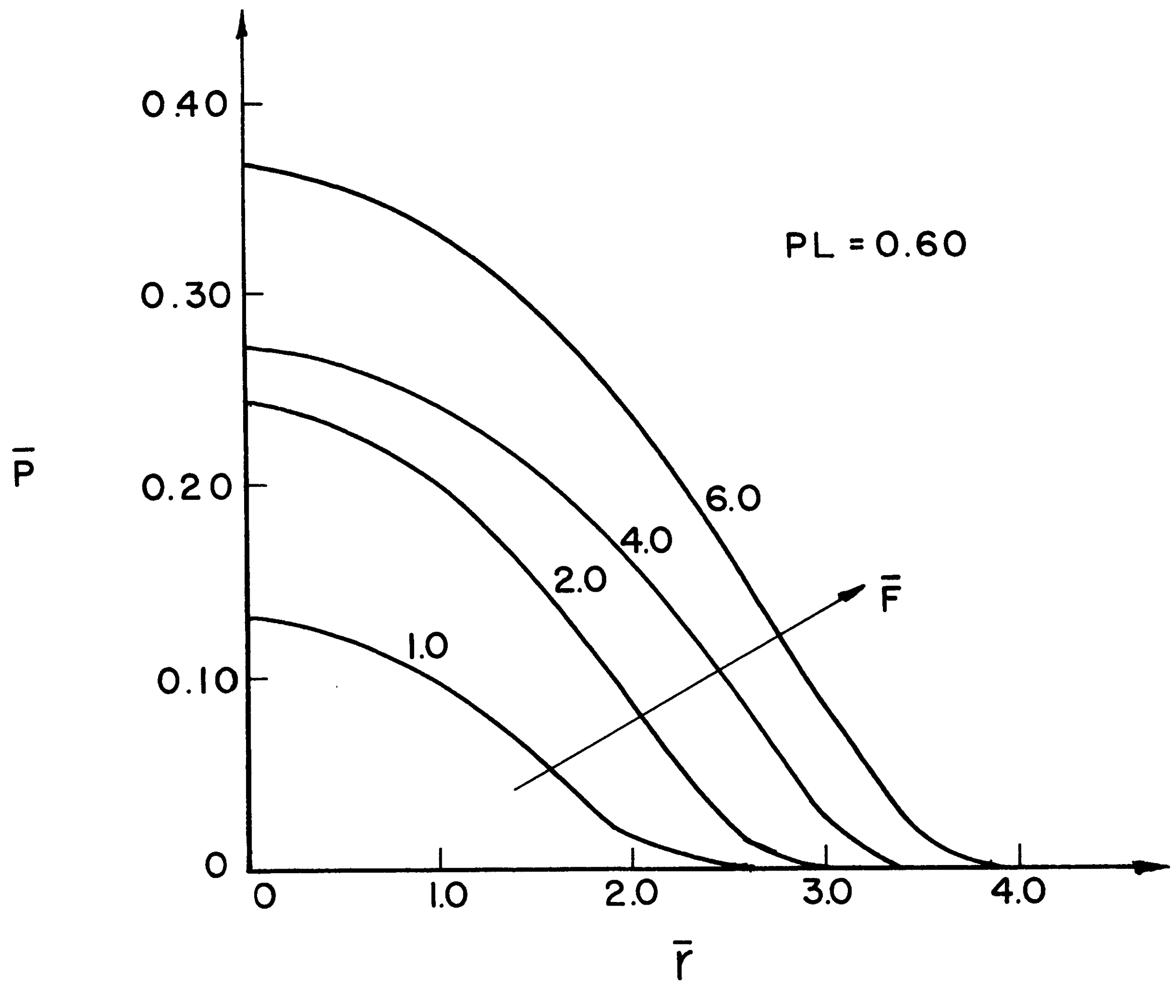


FIG. 16

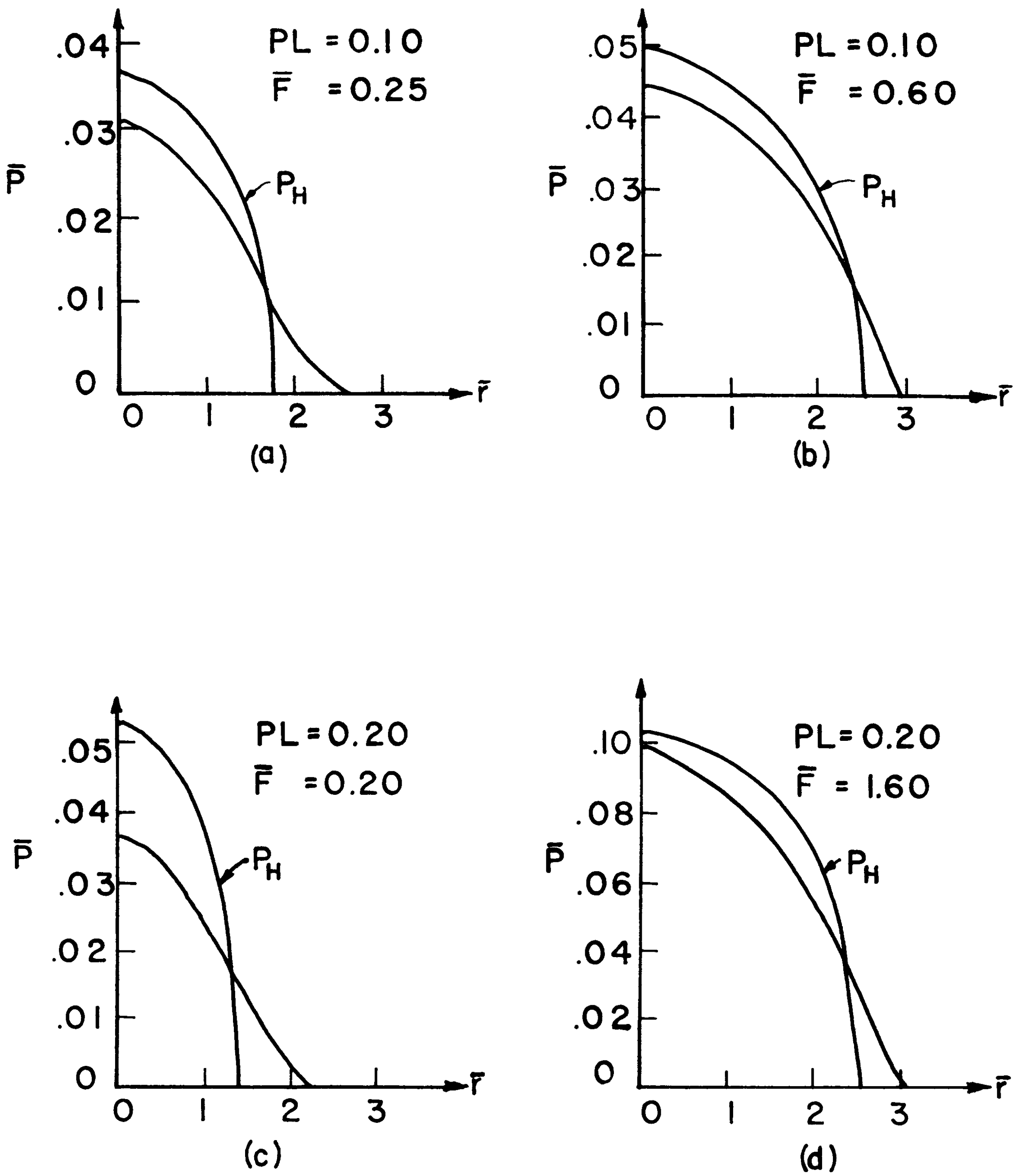


FIG. 17 COMPARISON OF HERTZIAN (P_H) AND ROUGH-SPHERE PRESSURE DISTRIBUTIONS

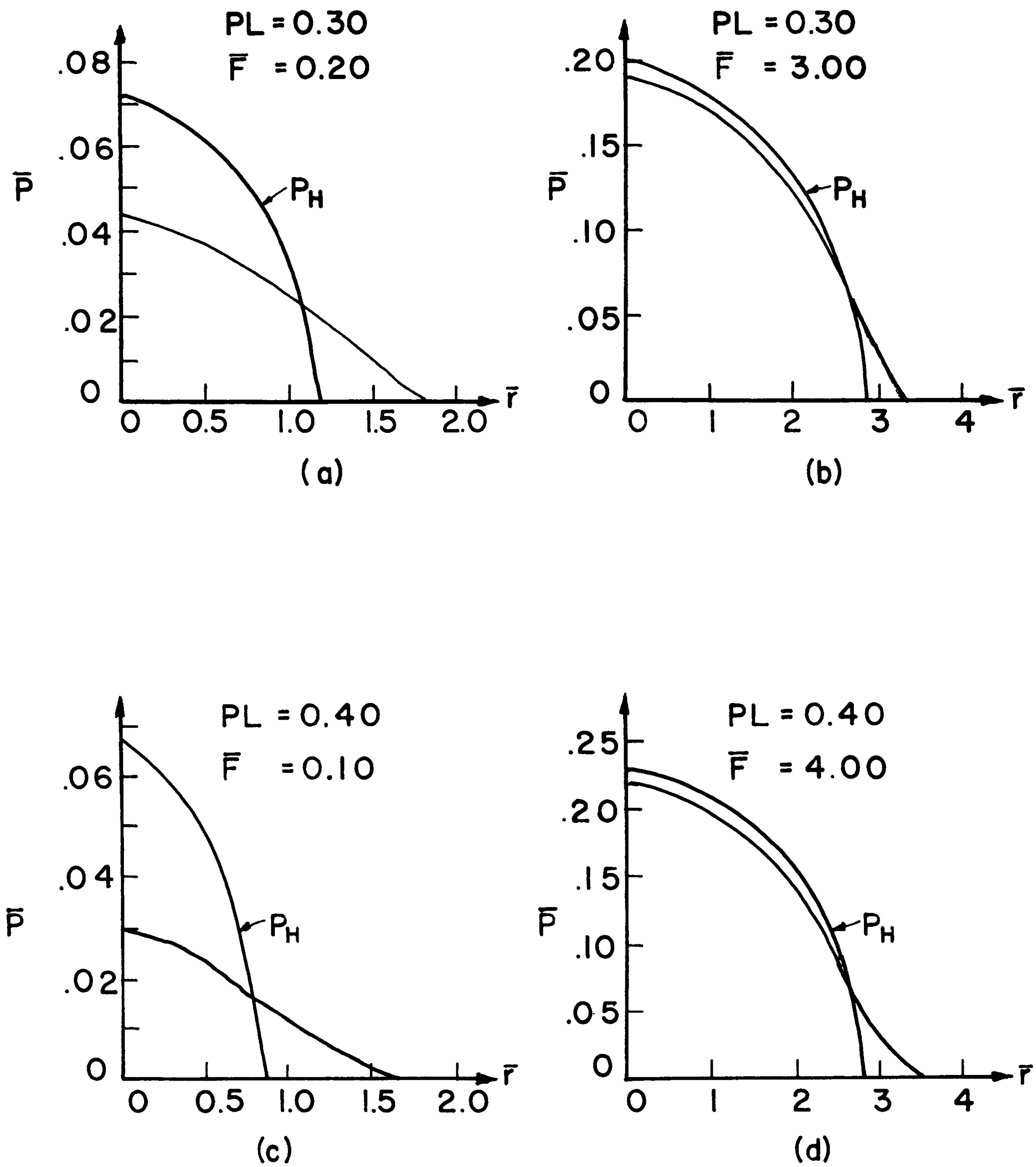


FIG. 18 COMPARISON OF HERTZIAN (P_H) AND ROUGH-SPHERE PRESSURE DISTRIBUTIONS

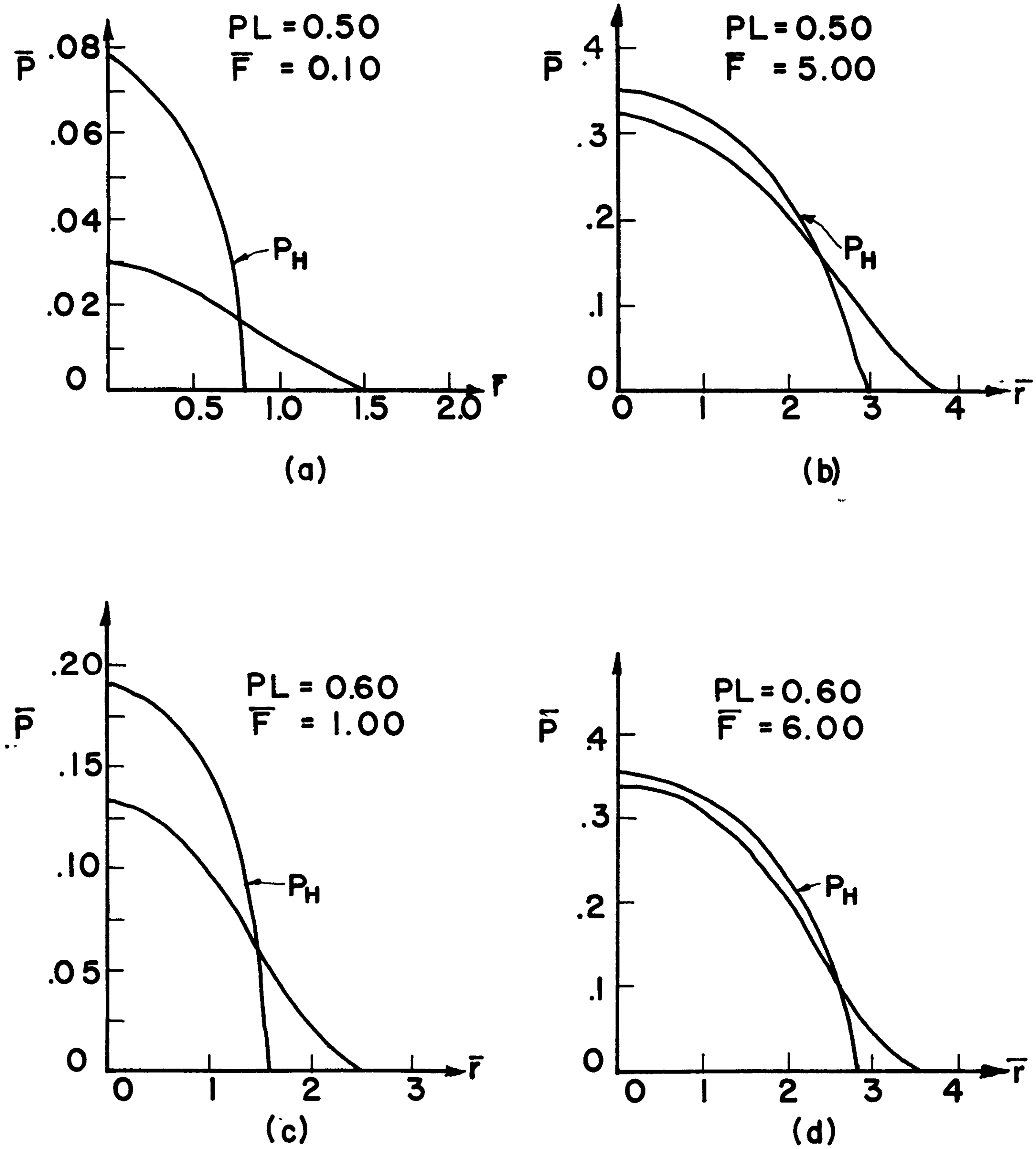


FIG. 19 COMPARISON OF HERTZIAN (P_H) AND ROUGH-SPHERE PRESSURE DISTRIBUTIONS

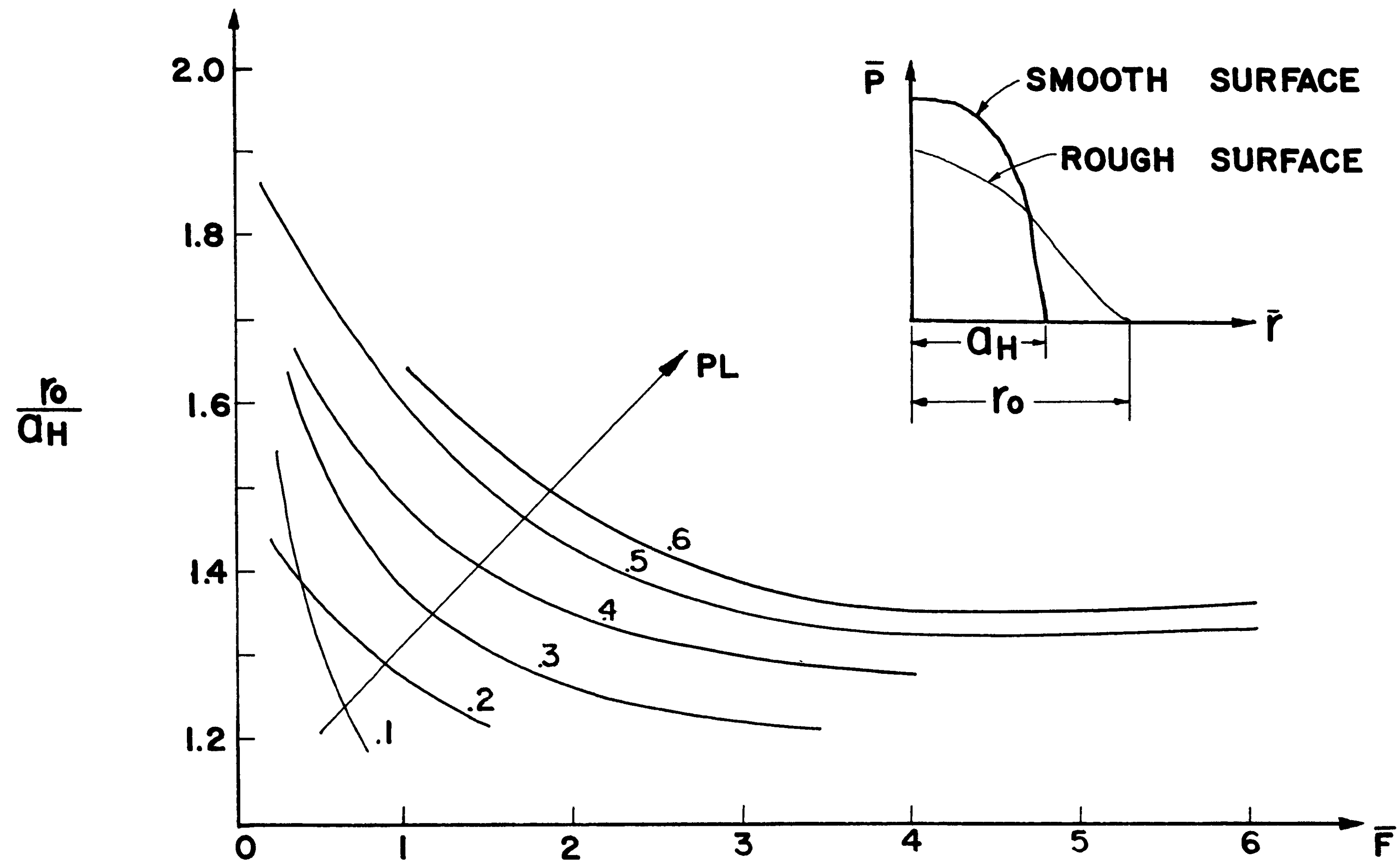


FIG. 20 RADIUS RATIO: SMOOTH VS. ROUGH SURFACES

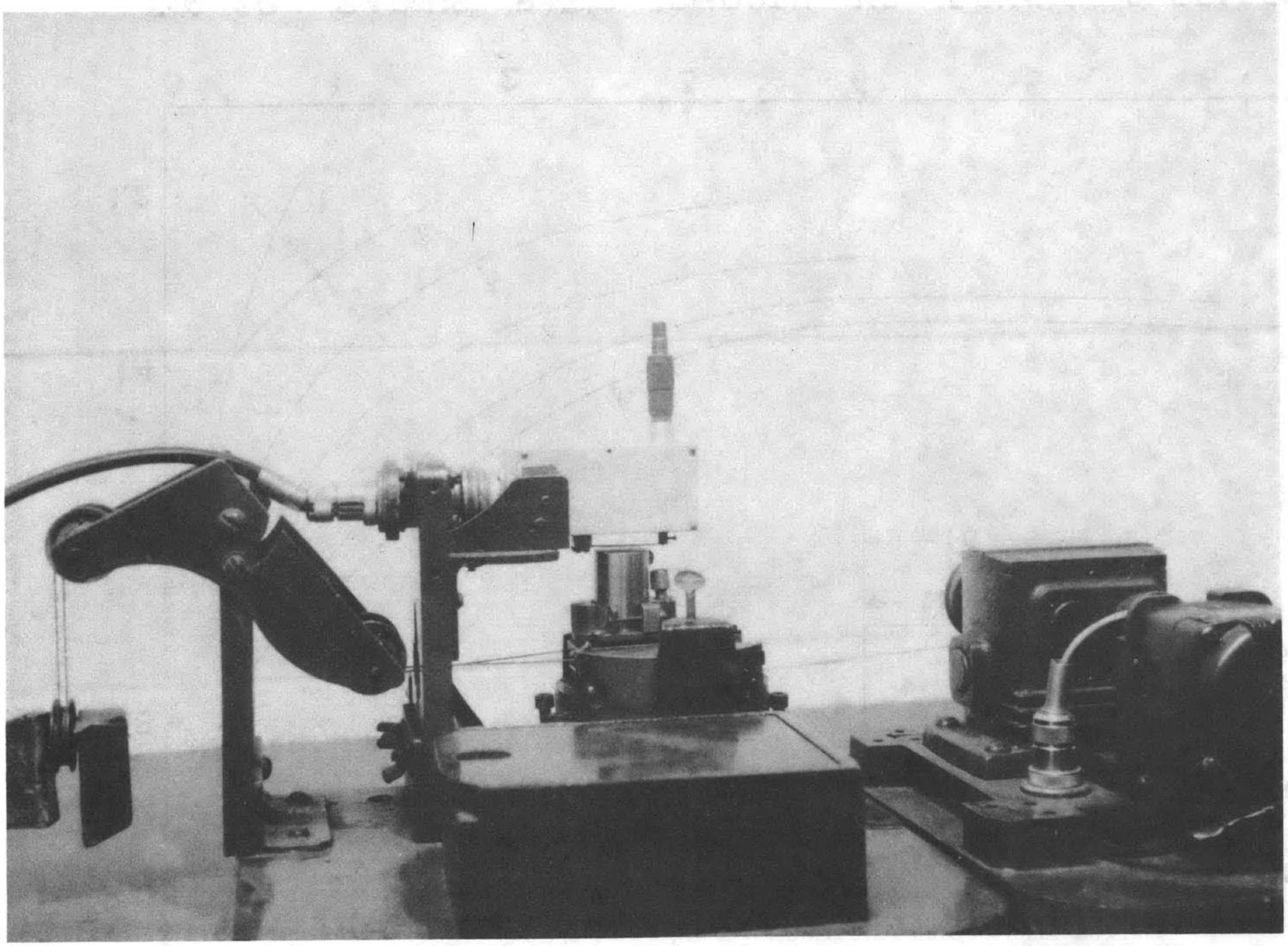


FIG. 21 SURFACE PROFILE MEASURING INSTRUMENT

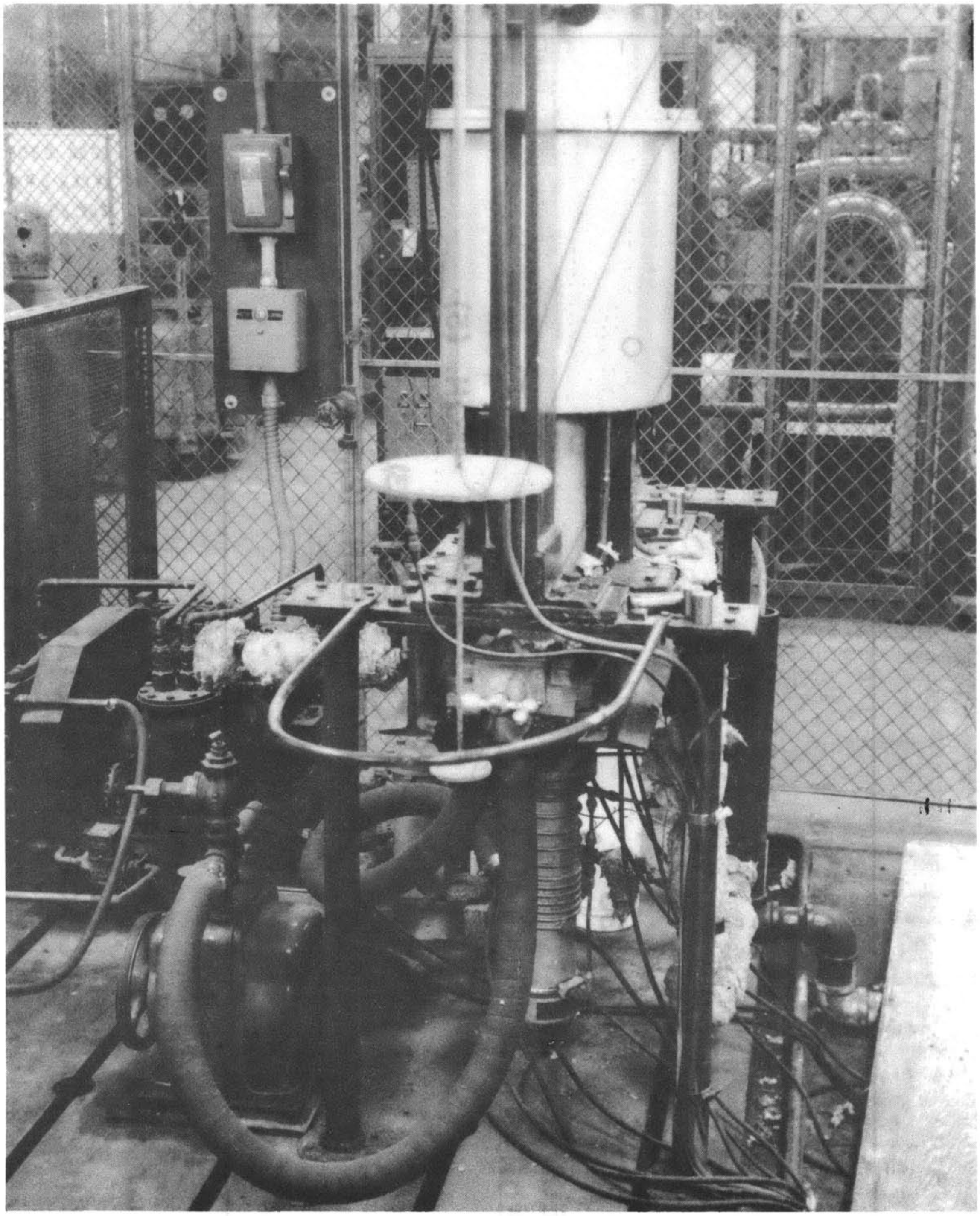


FIG. 22 CONTACT RESISTANCE APPARATUS

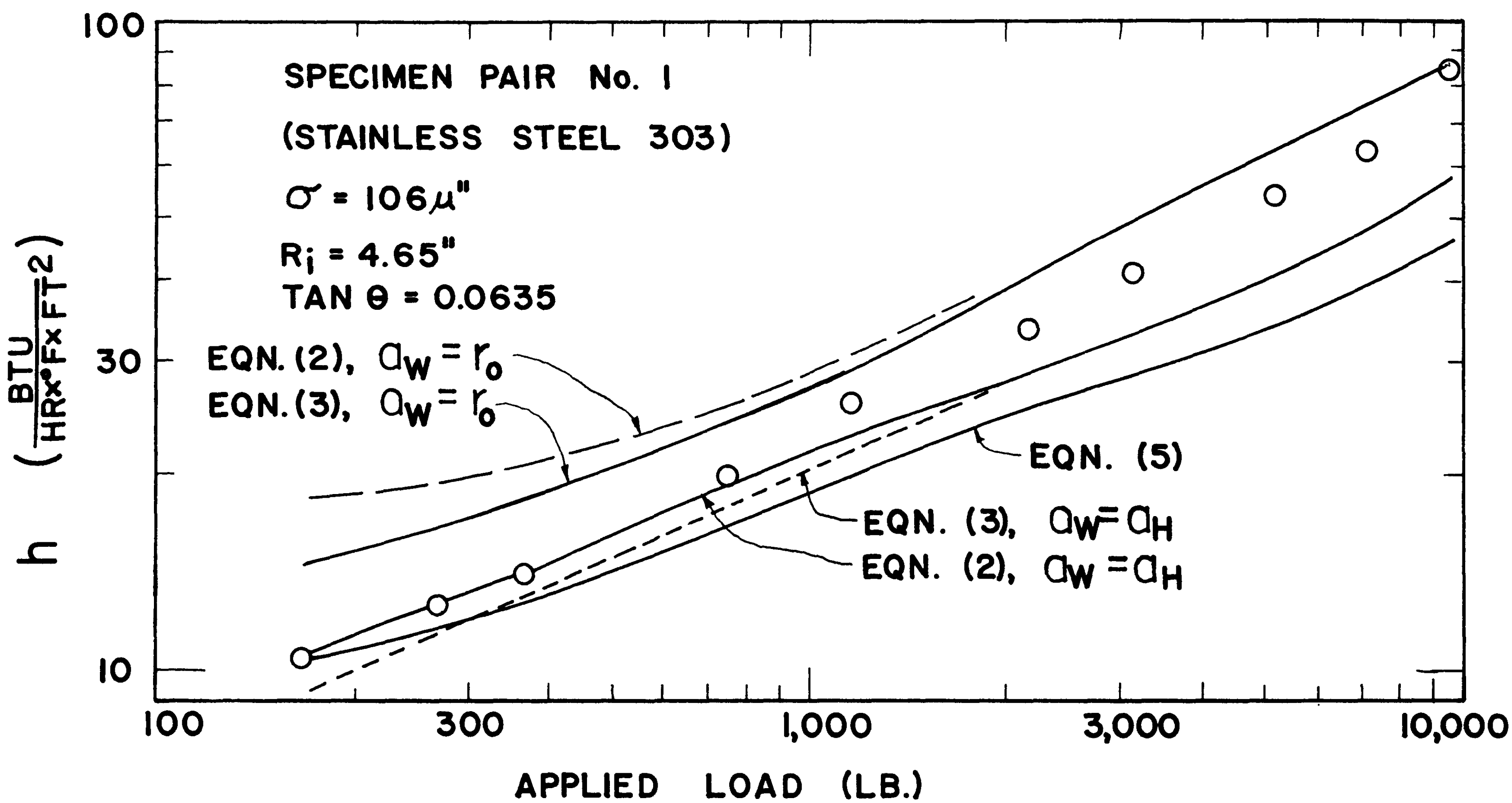


FIG. 23A. CONTACT CONDUCTANCE RESULTS

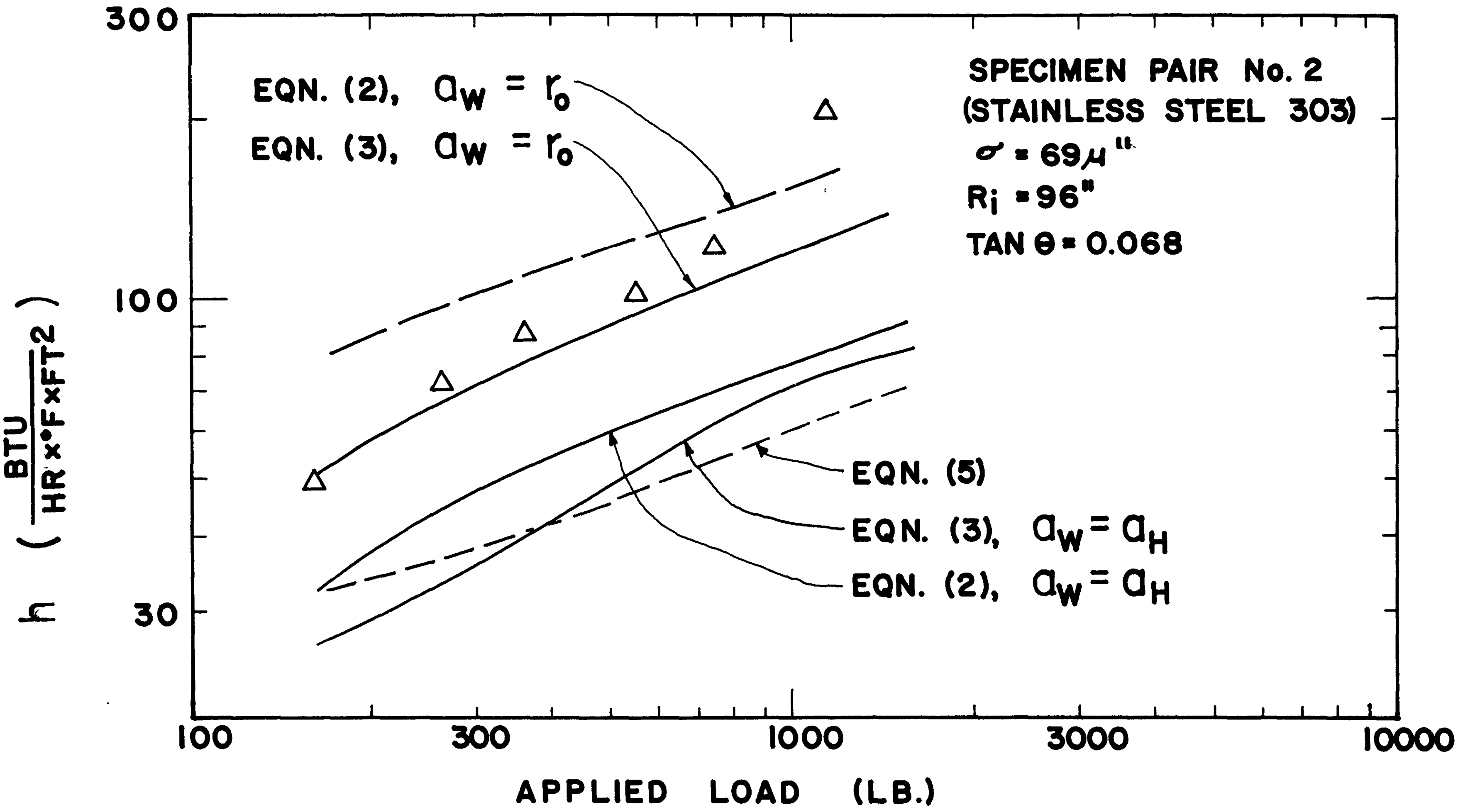


FIG. 23B. CONTACT CONDUCTANCE RESULTS

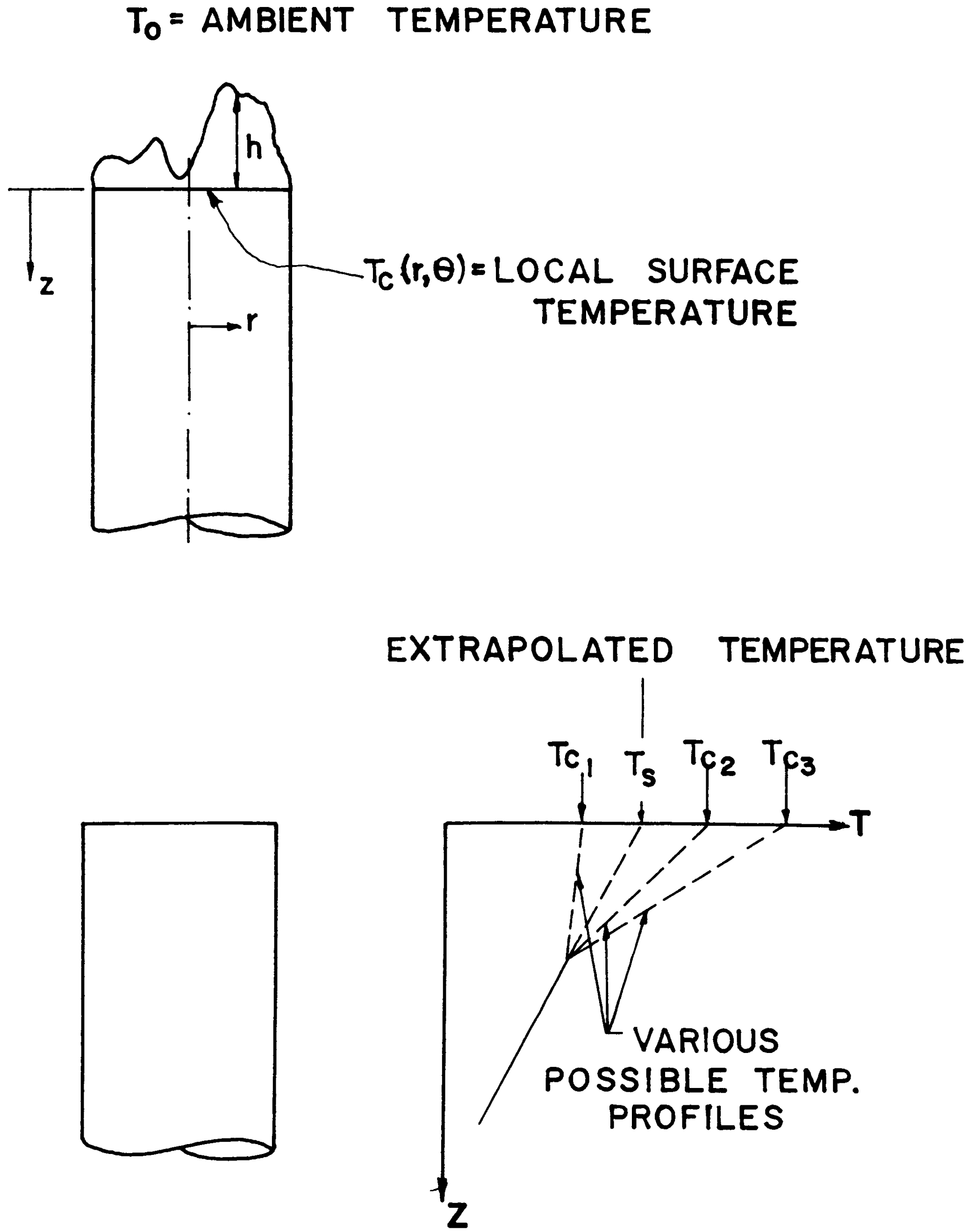


FIG. 24 NON-UNIFORM HEAT TRANSFER COEFFICIENT

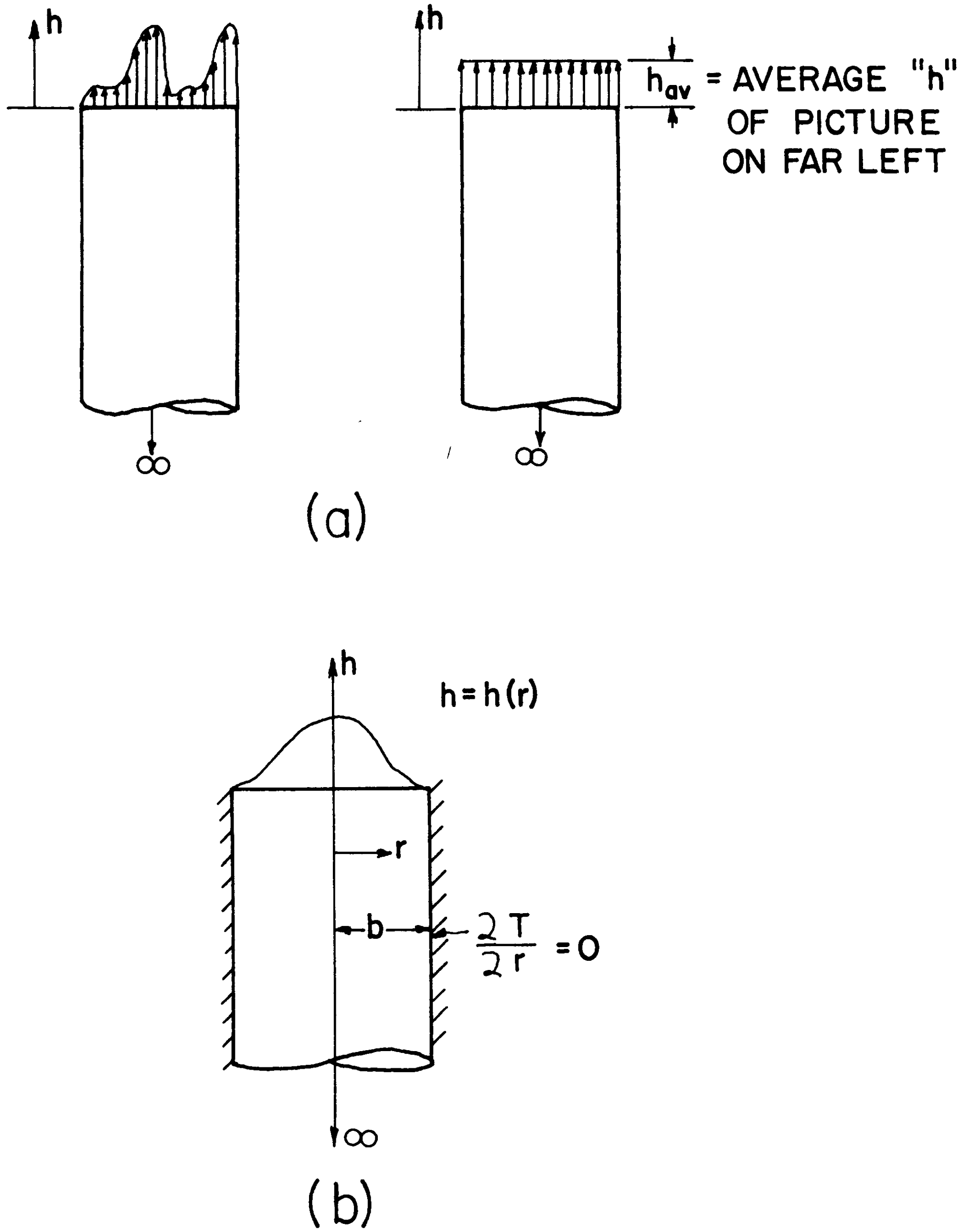


FIG. 25 NON - UNIFORM HEAT TRANSFER COEFFICIENT