Planar Feasibility Study for Primary Mirror Control of Large Imaging Space Systems Using Binary Actuators

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Abstract

The greatest discoveries in astronomy have come with advancements in ground-based observatories and space telescopes. Latest trends in ground-based observatories have been ever increasing size of the primary mirror, providing much higher apertures for more powerful image captures. The same trend can be envisioned for space telescopes. In fact, concepts for ultra-large space telescopes (ULST) on the order of hundreds of meters in size have been emerging since the late 1990’s and early 2000’s. Currently, James Webb Space Telescope (JWST) scheduled to be launched in 2014 only has primary mirror diameter of 6.5 m. An important issue in the ULST is correcting for optical errors caused by large thermal deformations expected due to exposure to radiation in orbit. As of now, there are no methods for solving technical complexities involved in correcting for such deformations. Furthermore, the costs associated with weight, deployability, and maintenance hinder advancements in large space telescopes. This thesis explores the idea of using binary actuators coupled with elastic elements to offer solutions to these problems. The feasibility of using binary actuators with elastic elements for correcting the focus of the deformed structure is investigated. The investigation begins with simple representations of the primary mirror structure in one-dimensional study, then in two-dimensional study for planar analysis. The analysis includes exploration of the workspace, demonstration of deterioration of superposition, and performance measured in precision of focus correction. In general, the number of actuators required for an acceptable level of correction is about three times the number of degrees-of-freedom in the system. Ultimately, it is concluded that in the planar domain it is feasible to use binary actuators in the control of primary mirror structure for large space telescopes.

Thesis Supervisor: Steven Dubowsky
Title: Professor of Mechanical Engineering
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Dedicated to my beloved parents, Kang Yeon Lee and Choo Young Lee

And to my loving God
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1. Introduction

1.1 Motivation

Space telescopes have played a prominent role in expanding human understanding of the universe. With the advent of the Hubble Space Telescope (HST) and the emerging James Webb Space Telescope (JWST), the technology for space imaging systems is continuing to advance. In astronomy today, efforts are made to upgrade angular resolution in space telescopes by implementing larger apertures [1]. Telescopes with larger apertures allow more remote objects to be captured and processed into an image. In fact, over the past century, apertures of ground-based telescopes have been increasing by a factor of two over each generation [2]. Today, the largest operational ground-based telescopes boast apertures greater than 10 m in diameter (Table 1.1). For future generations, design studies have been conducted for much larger telescopes with apertures ranging from 30 m such as the California Extremely Large Telescope (CELT), to 100 m like the OverWhelmingly Large (OWL) telescopes [3, 4]. In accordance with this progression, it is foreseeable to conceive such large systems for space telescopes (see Figure 1.1). Space telescopes have the advantage of being in orbit, away from any atmospheric disturbances and much closer to the skies. Table 1.2 shows basic specifications for some of the major space telescopes.
Table 1.1 Largest Ground-Based Telescopes [5]

<table>
<thead>
<tr>
<th>Telescope</th>
<th>Location</th>
<th>Diameter (m)</th>
<th>Primary f-ratio</th>
<th>Date Completed</th>
</tr>
</thead>
<tbody>
<tr>
<td>LBT¹</td>
<td>Arizona</td>
<td>11.8</td>
<td>1.14</td>
<td>2004</td>
</tr>
<tr>
<td>Keck I</td>
<td>Hawaii</td>
<td>10.5</td>
<td>1.75</td>
<td>1993</td>
</tr>
<tr>
<td>Keck II</td>
<td>Hawaii</td>
<td>10.5</td>
<td>1.75</td>
<td>1998</td>
</tr>
<tr>
<td>GTC</td>
<td>Canaries</td>
<td>10.4</td>
<td>1.65</td>
<td>2004</td>
</tr>
<tr>
<td>Hobby-Eberly</td>
<td>Texas</td>
<td>9.5</td>
<td>1.8</td>
<td>1999</td>
</tr>
</tbody>
</table>

1. Two 8.4 m primary mirrors resulting in effective aperture of 11.8 m.

Table 1.2 Major Space Telescopes [5]

<table>
<thead>
<tr>
<th>Telescope</th>
<th>Site</th>
<th>Diameter (m)</th>
<th>Mass (kg)</th>
<th>Launch Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRAS</td>
<td>900 km polar</td>
<td>0.57</td>
<td>1080</td>
<td>1983</td>
</tr>
<tr>
<td>HST</td>
<td>LEO</td>
<td>2.4</td>
<td>11000</td>
<td>1990</td>
</tr>
<tr>
<td>ISO</td>
<td>GEO elliptic</td>
<td>0.6</td>
<td>2200</td>
<td>1995</td>
</tr>
<tr>
<td>Spitzer</td>
<td>drift</td>
<td>0.85</td>
<td>950</td>
<td>2004</td>
</tr>
<tr>
<td>JWST</td>
<td>L2</td>
<td>6.5</td>
<td>6200</td>
<td>2014</td>
</tr>
</tbody>
</table>

Figure 1.1: Concept illustration of an ultra-large space telescope [6]

Launched in recent history. The diameter of the aperture is limited in space telescopes mainly due to the size of the launch vehicle. Therefore, space telescope with the largest aperture has been the Hubble Space Telescope, which was launched using the cargo hold of the Space Shuttle.
However, its 2.4 m diameter is significantly dwarfed by ground-based systems. Even JWST, scheduled to be launched in 2014 and considered to be the next generation of space telescopes, has a limited 6.5 m diameter. In order to develop space systems of much larger apertures, significant studies are required in delivery systems. As such, concept systems like the Magnum Launch Vehicle have been studied by Marshall Space Flight Center in the past [7]. However, much more significant problems arise in large space structures that first need to be solved for space telescopes with large apertures.

Implementing large apertures in telescopes means that the diameter of the primary mirror must be sufficiently large. Constructing monolithic primary mirrors become extremely costly beyond a couple of meters. As a result, ground observatories like Keck and Large Binocular Telescope (LBT) have used an arrangement of a number of hexagonal segmented mirrors to form a large primary mirror system. In order to function as a single primary mirror, the control algorithm for adjusting each one of the segmented mirrors requires precision on the order of micrometers [2, 8]. JWST (see Figure 1.2), at 6.5 m in diameter, utilizes all of these technologies and will become the largest and most powerful space telescope when launched.

Figure 1.2: JWST drawing (left) and three of the 18 segmented mirrors in its primary mirror construction (right) (www.jwst.nasa.gov)
significant challenge of a segmented mirror system is the underlying support structure. As the first space telescope to use segmented mirrors, JWST uses egg-crate structures (see Figure 1.3) for the individual mirrors themselves for its lightweight and stiffness properties critical in space systems [5, 9]. Ground-based Keck Observatory and the monolithic mirror of the HST use similar structures [10]. Supporting the egg-crate base typically calls for a web of truss-like construction underneath the mirrors spanning the entire system, similar to that shown in Figure 1.3. In the case of ultra-large space telescopes (ULST) being considered in this study, the underlying support structure would be on the order of hundreds of meters in size, no less than the diameter of the primary mirror. A space structure of this magnitude poses several problems. Any orbiting structure is subject to the harsh environments of space, most significant of which is exposure to thermal radiation. Thermal radiation effects cause static deformations in space structures that would distort the shape of the segmented primary mirror in space telescopes. There are three major sources of heat in space – direct solar radiation, direct Earth radiation, and Earth’s albedo [11, 12]. For space telescopes in the past, the considered deformations were small.

Figure 1.3: Egg-crate structure used for supporting mirrors (left) (www.jwst.nasa.gov) and underlying truss-like support structure on the backside of a segmented mirror system (right) (keckobservatory.org)
enough that it could be corrected using onboard actuators and sensors. In the case of JWST, seven actuators for each of the 18 segmented mirrors correct for any distortions in the primary mirror figure [9]. Furthermore, deployable heat shields and thermal insulators are used to minimize the thermal effects. However, for ultra-large space telescopes, the small deformations become amplified to the point where conventional methods will not work. A triangular truss space structure of approximately 200 m, constructed of Able Articulated Deployable Mast (ADAM™) experiences deformations resulting in approximately 1 m of bending [12] (see Figure 1.4). ADAM™ has been used in the construction of the International Space Station (ISS) for solar arrays and for the Shuttle Radar Topology Mission (SRTM). Deformations of this order will cause major distortions in the primary mirror and prevent functionality of the telescope.

![Figure 1.4: Thermal deformation of a triangular structure in space [12]](image)

Furthermore, for large structures of considered magnitude, using heat shields to reduce thermal effects would be physically impractical. A method for high precision correction of large deformations in the support structure of an ULST must be investigated for the concept to become a reality.
Application for this study is not limited to ultra-large space telescopes. The concept of large solar power satellites (SPS) (see Figure 1.5) that beam energy down to earth have been considered in the past by NASA and the Japan Aerospace Exploration Agency (JAXA) [13]. These power stations would also require structures on the order of thousands of meters and use reflecting mirrors to direct solar energy. Although the precision requirement would be significantly less, the control of these mirrors would benefit from this study. Furthermore, the problem of correcting for large deformations is not limited to space structures. Any large system requiring shape control, rather than deformation correction, would also be able to utilize the results from these studies.

Figure 1.5: Illustration of a solar power satellite in Earth orbit (www.space.com)

1.2 Approach

In the context of space telescopes, studies have been performed to evaluate the required range and precision of motions for control of segmented mirrors in large telescopes and found that actuators with large strokes and fine resolution would be required [6, 8, 14]. Several actuator concepts have been proposed to meet this need, including motorized differential micrometers and
ball screws, cable-driven elastic levers, impulse screws, voice coils, thermal/magnetostrictive/piezoelectric inchworms, and hybrids of the above. The drawbacks of these actuator solutions include: the need for active feedback control, short strokes, low output forces, high complexity, high costs, and low reliability [6]. Instead, the use of a large number of binary actuators (actuators having only two states — “0” or “1”) embedded in a highly compliant modular structure has been proposed [6]. This system is coined Binary Large Imaging Space Structures (BLISS) (see Figure 1.6). It is expected that high precision is achievable through binary actuators, much like binary numbers in digital computations. It can be argued that as the number of binary actuators in the system increases the precision of the device approaches that of a conventional continuous system [15]. Furthermore, because of its high redundancy, failure of individual binary actuators would result in graceful degradation of performance, thereby improving reliability.

![Figure 1.6: Mirror segment array for BLISS concept using binary actuator substructure element (insert) [6]](image)

Binary actuators can be considerably simple, cheap, and lightweight. Control algorithm is also simplified since active feedback control is not necessary; all computations for figure control can be conducted offline [16]. Cost savings from hundreds of sensors required between each mirror segment in conventional segmented mirror control would be significant as well. In addition,
using binary actuators removes the necessity for heavy, unreliable components such as motors, gears, and bearings [6]. Clearly, BLISS offers many advantages for application in ultra-large space telescopes; however, its feasibility remains to be analyzed.

1.3 Background Literature

Feasibility study of BLISS requires literature review from three areas: digital mechatronics, large space telescopes, and large space structures. Digital mechatronics studies provide insight towards other systems using binary actuation, while studies in large space telescopes provide relevant information on its current status and posing issues that this thesis is most concerned with. Literatures on large space structures are applicable in considering space environment factors and construction issues of large space telescopes.

1.3.1 Digital Mechatronics

There have been considerable amount of work performed on the use of binary actuators for various devices, with some dating back beyond the 1970’s [17, 18, 19, 20, 21, 22]. More recently, because of its simplicity, light-weight, and cost saving properties, applications have been explored for space robotics [17, 18, 23]. These studies involved concept designs for planetary exploration robots, primarily in regards to manipulator arm control and locomotion (see Figure 1.7). Furthermore, use of binary actuators in serpentine devices for probing of oil wells has been explored, and applications to medical devices have also been considered in surgery and examination environments [16, 19, 22]. In particular, work in [19] used binary actuation in the design of a device for precision control of a needle probe used for prostate cancer detection and treatment (see Figure 1.8). This work actually presents many parallels to the
BLISS concept in that it used elastic elements coupled with binary actuators to induce static deflections on the probe position using elastic averaging. Elastic averaging is the process in which the static equilibrium of the system is achieved from the combined effects of discrete linear displacements of actuators and compressions/elongations of the coupled elastic elements. The overall relationship resembles an averaged elastic effect on the system.

Figure 1.7: Planetary exploration robots using binary actuators for manipulator control (left) and locomotion (right) [18]

Figure 1.8: Design for prostate cancer detection device using binary actuators and elastic elements [19]
There has been progress in the study of binary actuators as well. In the past, conventional drive elements, such as pneumatic pistons and electromagnetic actuators (solenoids) have been used as mechanisms of bi-stable states [6, 16, 17, 19]. However, studies have shown the potential for artificial muscles using dielectric elastomers [18, 24, 25]. Dielectric elastomer (DE) actuators (see Figure 1.9) weigh less than half as much as comparable solenoid actuators and provide significantly more specific energy per unit mass. The force output can be customized to meet any requirements by combining multiple layers of DE actuators, and the stroke length is only limited by the geometric size of the actuators. With expected strain performance exceeding 100%, DE actuators can apply strokes twice its size. Furthermore, once automated, the construction of these actuators will result in lower costs compared to alternative actuators [26]. DE actuators have been developed to function as bi-stable elements for use in binary robotics. With further development, binary DE actuators seem quite suitable for application in ULST designs.

![Figure 1.9: Dielectric elastomer actuator [18]](image)

In the field of space robotics, binary robotic articulated intelligent device (BRAID) has been studied in detail (see Figure 1.10). BRAID is a binary actuated modular system that has been shown to be capable of executing practical tasks, such as manipulator control and
locomotion [18, 25]. A single stage consists of a three degrees-of-freedom parallel platform module and is capable of approaching the performance of a continuous system when increasing number of stages are used. BRAID is a much more kinematically complex device that has the potential to be applied to BLISS in influencing the direction of applied displacement loadings for a more robust control system.

Figure 1.10: BRAID kinematic structure [18]

Kinematics of binary actuated systems is a key area of study in binary mechatronics. Considerable amounts of literature exist investigating computational issues and trajectory planning of binary robots [25, 27, 28, 29]. It is noted that for binary actuated robots, a workspace (range of reachable locations of the end-effector of the manipulator) is a finite set of points in space rather than a continuous volume. As such, the inverse kinematics problem of finding an input configuration for the desired end-effector location involves searching through a discrete set of input configurations and finding the one that best matches the desired state. This is in contrast to solving geometric equations for continuous systems [27]. For serial manipulators, the inverse kinematics problem is often difficult to formulate closed form equations and requires search algorithms to solve. By contrast, the forward kinematics problem of finding the end-effector location based on an input configuration, is more convenient [30, 31]. For example, one can
easily formulate a transformation matrix that relates the inputs to the output end-effector position. Binary robots studied in [17], [18], and [23] primarily used BRAID, which is a serial chain of binary actuated parallel stages. Hence, the above observations hold true. However, BLISS operates with dominantly parallel manipulations, liken to the Stewart platform model [30]. This allows the system to have a stiff mechanical structure. For such systems using parallel manipulators, it is widely recognized that inverse kinematics is simple, while the forward kinematics is difficult to perform.

1.3.2 Large Orbiting Telescopes and Large Space Structures

Studies have been carried out considering the concept of ultra-large space telescopes. However, more notable are the previously mentioned ground-based OWL and CELT [2, 3, 6]. Throughout the late 1970’s and 1980’s, the term “large” space telescope referred to the likes of HST and those with aperture diameters of less than six meters [32]. It was not until the late 1990’s and early 2000’s that the concept of very large space telescope (VLST), proposing primary mirror sizes between 30 and 100 m in diameter, was published as the next generation system beyond JWST [33, 34].

Study of large space structures is relevant for ULST considerations. Because of their large size, ULST’s must be launched into space in modules and constructed in orbit. This poses new problems involving space construction methods, distortions in subassemblies, and structural vibrations when subassemblies are joined. Many studies have been conducted to address these issues, investigating the use of space robots for construction, implementing triangular truss modules for structure subassemblies, and exploring methods for estimating vibration of space structures [12, 35, 36, 37]. Although these are critical problems that must be faced in ultra-large space telescope architecture, they are outside the scope of this thesis.
1.4 Thesis Overview

This thesis studies the feasibility of using binary actuators to correct for thermal deformations on the underlying structure of ultra-large space telescopes. The feasibility is determined by investigating the effect of increasing the number of actuators and its ability to approach a desired level of precision.

1.4.1 Objectives

The primary objective is to determine the number of actuators required to achieve a desired level of precision in correcting for structural deformations caused by thermal radiations. Other objectives include providing general workspace characteristics of systems to aid in future studies. In the process, the feasibility of BLISS under designs considered in this thesis is established.

1.4.2 Approach

First, a one-dimensional study is performed using a single beam to represent the primary mirror surface. The behavior of the beam with perpendicular placement of actuators is studied to help understand more complex configurations. The effects of increasing the number of actuators are studied, including the validity of the assumption of superposition. Superposition is arriving at a solution using the summation of effects of each actuator individually, rather than using all actuations at the same time. Precision is determined by the system’s ability to approach a desired beam shape. The analysis uses both analytical results and computations from Finite Element Analysis (FEA). FEA results are obtained using commercial software called Automatic Dynamic Incremental Nonlinear Analysis (ADINA), developed by Dr. K.J. Bathe at the Massachusetts
Institute of Technology (MIT) [38]. Experimental results are used to validate the one-dimensional analysis.

A two-dimensional study, also using analytical approach and FEA, is presented. The beam is replaced by three rigid panels to simulate three segments of a segmented primary mirror assembly. Each panel is actuated at its endpoints. The actuators are placed at angles so that the resulting displacements of the structure is in both $x$ and $y$ directions. Complete workspaces of the system are obtained. The study begins with using six actuators. However, the BLISS concept dictates that the system is over-actuated, calling for more actuators than the actual number of degrees-of-freedom in the system. Therefore, the number of actuators is increased by including additional support structures to the panels. Ultimately, the process resembles a holistic design in which the actuators are embedded into the support structure itself. The effects of thermal deformations are investigated in the two-dimensional study. Here, performance and precision is determined by the system’s ability to improve its focus, deteriorated by the deformed support structure. Performance is measured by the minimization of lateral deviations from the original focal point by the reflected rays from the centers of each panel. Precision is defined as the magnitude of this minimization.

1.4.3 Assumptions

Several assumptions are used in this work. First, linear actuator stroke lengths are taken to be relatively small (less than 0.5% of the overall structure size). As a guideline, this is in direct correlation to the expected level of thermal deformations in a space structure, based on the studies conducted in [12].

In addition, wave-front aberrations are assumed to be corrected using adaptive secondary optics. Any optical distortions not accounted for by the primary mirror are delegated to the
secondary optics. This leads to the establishment of the desired precision level to be on the order of millimeters in considering the two-dimensional analysis of the study.

The linear actuators are assumed to be binary, having two states: extended and not extended. Furthermore, the actuators are assumed to be able to provide sufficient force to apply the entire stroke length. The focus of the study is in displacement loading rather than force loading. Combined with the promise of DE actuators, the actuators are assumed to be able to overcome any level of stiffness in the system.
2. One-Dimensional Study

One-dimensional study greatly simplifies the problem and offers insights and understanding for following studies of greater complexity. In this case, a simple beam represents the surface of the primary mirror. The segments of the primary mirror assembly are taken to be infinitely small. The actuators are placed perpendicular to the beam, with a uniform distribution along the length of the beam, and allow for only one degree-of-freedom (in the vertical direction) (see Figure 2.1). This configuration was considered to be the simplest form from which to study the behavior of a compliant system.

![Figure 2.1: Binary Beam Model (BBM)](image)

2.1 Model

The model using a beam with evenly distributed perpendicular linear actuators is named Binary Beam Model (BBM). Based on small displacement assumption, the beam is modeled by Euler beam equations with its stiffness defined by the Young’s Modulus, \( E \), and the area moment...
of inertia, \( I \). The beam is free at both ends, suspended in space by actuators that are in series with elastic elements (i.e. springs) (see Figure 2.1). The actuators are fixed to ground, preventing any rotations or translations at the base. In addition, the number of actuators is increased from two to \( N \). The elastic elements play a critical role in elastic averaging, as well as in preventing discontinuity in the shape of the beam.

### 2.1.1 Analytical Model

The analytical model is based on the Euler beam equations and methods for solving statically indeterminate systems [39] (see Figure 2.1). Primary equations of consideration are:

\[
\begin{align*}
\frac{v''(x)}{EI} &= \frac{M(x)}{EI} \quad (2.1) \\
\frac{v(x)}{EI} &= \frac{1}{EI} \int M(x) dx && M(x) = \text{Bending moment} \\
E &= \text{Young’s Modulus} \\
\sum_{i=1}^{N} F_i &= 0 \quad (2.2) \\
\sum M_0 &= 0 && I = \text{Area moment of inertia} \\
F_i &= \text{Force applied by actuator } i \\
M_o &= \text{Moment about } x=0
\end{align*}
\]

Equations in (2.1) state the relationship between deformations (or deflections in the \( y \) direction) and the bending moment along the beam. Equations in (2.2) are simply the summations of forces and moments on the beam. The free-body diagram of the system is illustrated in Figure 2.2. There are no boundary conditions at the ends of the beam, and only constraint requirements exist in deflection, \( v(x) \), and curvature, \( v'(x) \), along the beam. The actuators are fixed to the ground to prevent any translations or rotations at their bases. All of the forces, \( F_i \), applied on the beam are proportional to the compressions/elongations on the elastic elements in accordance with the
Hooke's Law – \( F_i = k_i(d_i - v(x_i)) \), where \( k_i \) is the stiffness of the elastic element \( i \), \( d_i \) is the state of the actuator \( i \) (0 or stroke length, \( \delta \)), and \( v(x_i) \) is the deflection of the beam at the \( x \) location of actuator \( i \). The bending moment equations along the beam for \( N \) actuators can be derived through

\[
\begin{align*}
F_i &= k_i(d_i - v_i) \\
F_1 &= \text{Force applied at location of actuator } i \\
d_i &= \text{Actuator extension (0 or } \delta) \\
v_i &= \text{y-deflection at location of actuator } i \\
M_i(x) &= \sum_{j=1}^{N} k_j(d_j - v_j) \left(x - \frac{(j-1)L}{N-1}\right) \\
M_i(x) &= \text{Bending moment equation between actuator (i-1) and } i
\end{align*}
\]

Let
\[
\begin{align*}
v_1 &= v(0) \\
v_2 &= v\left(\frac{L}{N-1}\right) \\
v_i &= v\left(\frac{(i-1)L}{N-1}\right) \\
\vdots \\
v_N &= v(L)
\end{align*}
\]

y-deflections at locations of actuator \( i=1 \) to \( N \), evenly distributed along the beam

\[
v = [v_1 \cdots v_N]^T
\]

Equations in (2.3), which results in \( N-1 \) equations. Following (2.1) gives equations shown in (2.4),

Chapter 2: One-Dimensional Study
which involves two integration constants per bending moment equation. These constants are found by equations in (2.5), solved from the continuity constraints on the slope and deflections along the beam. In total, there are \(2(N-1)\) integration constants.

\[
v'_i(x) = \frac{1}{EI} \int M(x)dx = \frac{1}{EI} \left( \sum_{j=1}^{i} \left[ -\frac{1}{2} k x^2 + k \left( \frac{(j-1)L}{N-1} \right) x \right] v_j + \sum_{j=1}^{i} \left[ \frac{1}{2} k x^2 - k \left( \frac{(j-1)L}{N-1} \right) x \right] d_j \right) + C_i \quad \text{for } x \leq \frac{j-1}{N-1} L \]
\[
v'_i(x) = \frac{1}{EI} \int M(x)dx = \frac{1}{EI} \left( \sum_{j=1}^{i} \left[ -\frac{1}{6} k x^3 + \frac{1}{2} k \left( \frac{(j-1)L}{N-1} \right) x^2 \right] v_j + \sum_{j=1}^{i} \left[ \frac{1}{6} k x^3 - \frac{1}{2} k \left( \frac{(j-1)L}{N-1} \right) x^2 \right] d_j \right) + C_i + D_i \quad \text{for } x > \frac{j-1}{N-1} L \]

Where \(C_i\) and \(D_i\) are integration constants.

From continuity of beam slope, \(v'_i(x)_{x=\frac{(i-1)L}{N-1}} = v'_{i+1}(x)_{x=\frac{(i-1)L}{N-1}}\)

\[
C_i = -\left[ -\frac{1}{2} k x^2 + k \left( \frac{(i-1)L}{N-1} \right) x \right] v_i + \left[ \frac{1}{2} k x^2 - k \left( \frac{(i-1)L}{N-1} \right) x \right] d_i \bigg|_{x=\frac{(i-1)L}{N-1}} + C_{i+1} \]

\[
= \sum_{j=1}^{i} \left[ -\frac{1}{2} k \left( \frac{(j-1)L}{N-1} \right)^2 v_j + \frac{1}{2} k \left( \frac{(j-1)L}{N-1} \right)^2 d_j \right] + C_i
\]

\[
C_1 = v'_1(x)_{x=0}
\]

Similarly,

From continuity of beam deflection, \(v_i(x)_{x=\frac{(i-1)L}{N-1}} = v_{i+1}(x)_{x=\frac{(i-1)L}{N-1}}\)

\[
D_i = -\left[ -\frac{1}{6} k x^3 + \frac{1}{2} k \left( \frac{(i-1)L}{N-1} \right) x^2 \right] v_i + \left[ \frac{1}{6} k x^3 - \frac{1}{2} k \left( \frac{(i-1)L}{N-1} \right) x^2 \right] d_i \bigg|_{x=\frac{(i-1)L}{N-1}} + D_{i+1}
\]

\[
= \sum_{j=1}^{i} \left[ \frac{1}{6} k \left( \frac{(j-1)L}{N-1} \right)^3 v_j - \frac{1}{6} k \left( \frac{(j-1)L}{N-1} \right)^3 d_j \right] + D_i
\]

\[
D_1 = EIv_1
\]
Finally,

From \( \sum_{F=0}^{N} \begin{array}{c}
\sum_{M=0}^{N} v_i = \sum_{i=2}^{N} (d_i - v_i) + d_i \\
\sum_{M=0}^{N} v_2 = \sum_{i=3}^{N} (d_i - v_i)(i-1) + d_2
\end{array} \)

Then, from continuity constraint between \( v_1 \) and \( v_2 \) we can define \( C_r \) to be:

\[
C_r = \frac{N-1}{L} \left[ \left( \frac{1}{6} k \left( \frac{L}{N-1} \right) \right)^3 - EI \right] v_i + EI v_2 - \frac{1}{6} k \left( \frac{L}{N-1} \right)^3 d_1
\]

(2.5 cont’d)

The procedures from (2.1) to (2.5) result in a closed-form solution for the shape of the beam based on inputs to the linear actuators for any \( N \). Unfortunately, the system of equations involved expand indefinitely with increasing number of actuators. This is due to the coupling effects between not only the actuators and the elastic elements, but also with the elasticity of the beam. The above solution can be modified to a matrix form of \( v = Av + Bd \) and manipulated as shown in (2.6) to arrive at \( v = [I - A]^{-1} Bd \), where \( [I - A]^{-1} B \) is the transformation matrix relating inputs to outputs.

The matrix organization in (2.6) confirms the coupling between the effects of each actuator. A limitation of this formulation is that it only returns the deflections at points where the actuators are located. For the deflections of the beam between actuators, functions in (2.5) must be solved directly. An approximation would be to interpolate between solution points given by (2.6). Other limitation is that the solutions are restricted to the small deflections constraint of the Euler beam model.
\[ v = A v + B d \quad \text{where} \quad v = [v_1 \cdots v_N]^T \quad \nu \in \mathbb{R}^{N \times 1} \]
\[ d = [d_1 \cdots d_N]^T \quad \delta \in \{0, \delta\}^{N \times 1} \]
\[ \delta = \text{Stroke length} \quad A, B \in \mathbb{R}^{N \times N} \]

\[ A = \frac{1}{EI} \begin{bmatrix} 0 & -EI & -EI & \cdots & -EI \\ 0 & 0 & -2EI & -3EI & \cdots & -(N-1)EI \\ a_3 & b_3 & c_{33} & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & 0 \cdots 0 \\ a_j & b_j & c_{kj} & \cdots & c_{kj} & 0 \\ a_N & b_N & c_{Nj} & \cdots & \cdots & c_{NN} \end{bmatrix} \]

\[ B = \frac{1}{EI} \begin{bmatrix} EI & EI & EI & EI & \cdots & EI \\ 0 & EI & 2EI & 3EI & \cdots & (N-1)EI \\ -a_3 + EI & -b_3 + EI(3-1)(\zeta - 1) & -c_{33} & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & 0 \cdots 0 \\ -a_j + EI & -b_j + EI(i-1)(\zeta - 1) & -c_{ij} & \cdots & -c_{ij} & 0 \\ -a_N + EI & -b_N + EI(N-1)(\zeta - 1) & -c_{Nj} & \cdots & \cdots & -c_{NN} \end{bmatrix} \]

\[ a_i = \alpha_i + EI + (i-1)(\zeta - 1) \]
\[ b_i = \alpha_i - \eta_j \frac{(i-1)L}{N-1} + EI(i-1)(\zeta - 1) + \varepsilon_j \]
\[ c_{ij} = \alpha_j - \eta_j \frac{(i-1)L}{N-1} + \varepsilon_j \]

and
\[ \alpha_j = -\frac{1}{6} k \xi^3 + \frac{1}{2} k \left( \frac{(j-1)L}{N-1} \right)^2 \right|_{x=\frac{(j-1)L}{N-1}} \]
\[ \zeta = \frac{1}{6} k \left( \frac{L}{N-1} \right)^3 \]
\[ \eta_j = \frac{1}{2} k \left( \frac{(j-1)L}{N-1} \right)^2 \]
\[ \varepsilon_j = \frac{1}{6} k \left( \frac{(j-1)L}{N-1} \right)^3 \]

(2.6)
2.1.2 Finite Element Analysis

The analytical approach shows that the system of equations becomes more complicated as the number of actuators increase. Fortunately for the BBM, a linearized model was found. This was a complicated process even for such a simple system, yet relatively easy compared to what would be expected in two-dimensional study or much more complicated configurations. In preparation for further studies, a more effective method is required. This method is Finite Element Analysis. FEA is a numerical method for solving partial differential equations and integral solutions by dividing the entire system into many elements defined by nodes and constructing a stiffness matrix based on the structure. Matrix operations on this stiffness matrix and pre-described basis functions approximate the solutions [40]. For detailed theoretical explanations see references [41] and [42]. The availability of FEA software make construction of FEA models far more attractive than using analytical models. Furthermore, graphical interfaces featured on most FEA software allow for models to be created by researchers without an in-depth understanding of finite element analysis. For the studies conducted in this thesis, ADINA is used as the FEA software of choice [38]. This program is used in this thesis because it is readily available at MIT and it was also used in studies performed in [12].

In ADINA, components can be modeled based on their properties. Element groups such as trusses, beams, 2-D solids, 3-D solids, and shells are common model choices. Only beam elements are used to construct the model of the BBM. Elastic elements (i.e. springs) are often modeled using truss elements; however, this causes singularity issues due to non-positive definiteness of the stiffness matrix caused by pre-imposed boundary conditions on a truss element. Essentially, the boundary conditions on truss elements do not restrict the rotational motions of its nodes. Non-positive definiteness of the stiffness matrix implies that the system physically collapses on itself even without any loading [43]. For the models constructed here,
using beam elements eliminates this concern. A necessary property of the beam elements representing the elastic elements of BBM is the Young’s Modulus. This value is found by (2.7) based on the desired stiffness of the elastic elements. The actuators are modeled as rigid beam elements with its Young’s Modulus set artificially high. The actuations are created in ADINA by imposing artificial temperature loadings on the appropriate elements giving the desired extension. Expression in (2.8) shows the relationship between the user-defined thermal coefficient of the beam elements representing the actuators and the temperature load. The beam itself is modeled as a beam element. Figure 2.3 shows the model construction in ADINA for a four-actuator system. Figure 2.4 shows a typical solution in graphical display for an input with the second actuator extended (i.e. binary input [0 1 0 0]). In accordance with Figure 2.1, boundary conditions on the overall system include translation and rotation fixities to the ground on all actuators supporting the beam.

\[
E = \frac{kL}{A}
\]

\(E=\text{Young’s Modulus}\)

\(k=\text{desired stiffness of elastic element}\)

\(A=\text{cross sectional area of beam element representing the elastic element}\)

(2.7)

\[
\delta = \alpha \Delta T L
\]

\(\delta=\text{desired stroke length}\)

\(\alpha=\text{thermal coefficient}\)

\(\Delta T=\text{temperature loading (change in temperature)}\)

\(L=\text{length of the beam element representing the actuator}\)

(2.8)
Beam elements for elastic components
• Young's Modulus given by equations in (2.7)
• Cross-section selected to provide desired stiffness

Fixed translation and rotation on grounds

Each hash-mark represents an element division; i.e. nodes.

Figure 2.3: FEA model construction in ADINA for BBM using graphical interface

Figure 2.4: Typical solution in graphical display of ADINA for a four-actuator system.

For above case, L=3 m and δ=0.015 m

While ADINA makes computations for solutions simpler, there is a major issue. Minor changes in the system require that the entire model is reconstructed. In addition, for studying effects of increasing the number of actuators, loading definitions must be manually changed in ADINA for every input combination ($2^N$ times). Therefore, an automated computational tool is
required. The solution was to interface ADINA with MATLAB and run problems in batch mode. ADINA models can be created using text files written in programming syntax unique ADINA [38]. MATLAB has the capability to write these text files and to call the ADINA executable program. In this way, interface between MATLAB and ADINA was created, allowing for automated modifications of the FEA model. See Appendix A for details.

Figure 2.5 shows one comparison of results for a four-actuator BBM between analytical results based on section 2.2.1 and FEA solutions from ADINA. Actuation input is $[0 \ 1 \ 0 \ 0]$ (i.e. only $i=2$ engaged), length of beam is 3 m, and stroke length is 1.5 cm (0.5% of length of beam). Steel beam is subjectively selected with Young’s Modulus of 200 GPa and cross-section of 1 cm x 1 cm, resulting in the area moment of inertia of $8.333 \times 10^{-10}$ m$^4$ (using equation 2.9) [39]. The initial stiffness of the elastic elements is also arbitrarily chosen to be 2000 N/m. Clearly, the two

\[
I = \frac{wh^3}{12} \\
I = \text{Area moment of inertia} \\
w = \text{width of the beam cross section} \\
h = \text{height of the beam cross section (thickness)} \\
(2.9)
\]

Figure 2.5: Comparison of FEA and analytical solutions for $[0 \ 1 \ 0 \ 0]$
solutions are a close match. Comparing the results for the entire input domain, the maximum difference of deflections in $y$ at any point along the beam is found to be on the order of $1 \times 10^{-6}$ m. This difference is attributed to the fact that the elastic elements are modeled as beam elements in FEA, which accounts for its own bending and deformations. By contrast, the analytical solution assumes that the elastic elements only act in the $y$ direction. The error is acceptable considering that the maximum deflection of the beam for any actuator input is around 1.6 cm.

Figure 2.6 shows a comparison of results for a ten-actuator BBM (with other parameters held same as before) between analytical results and FEA solutions from ADINA. Actuator input is $[0 0 0 0 0 1 1 1 1 1]$, which is essentially a step input to the system. Comparisons show that the maximum difference between the two results is on the order of $1 \times 10^{-6}$ m. Again, this difference can be attributed to the fact that ADINA considers bending of the elastic elements in the system, while the analytical matrix form does not. Based on the two comparisons, it can be concluded that the results for the two models agree.

![Figure 2.6: Comparison of FEA and analytical solutions for step input to a ten-actuator BBM](image)
2.2 Experimental Verification and Results

A simple experimental setup was built to validate the models obtained from Euler beam equations and FEA. A beam of 89.9 cm in length and 2.19 cm in width is cut from a sheet of stainless steel of 1.02 mm in thickness. This thin beam is supported by ten actuators. Each actuator is a simple system of a bolt-and-screw concept in which one rotation of the bolt provides approximately 2.6 mm of movement on the screw. This movement represents the stroke of linear actuators.

2.2.1 Experiment Setup

For the experiment setup, material parameters are first established. The beam’s Young’s Modulus is measured by cantilevering the beam at various lengths and measuring the free end deflections caused by various weights applied to the free end. Twelve data points were taken and averaged to arrive at Young’s Modulus of 112 GPa. As the deflections were hand measured using a digital caliper, this value is prone to uncertainties. However, the measured value is consistent with the published range of 68.9 ~ 317 GPa for all grades of stainless steel, with structural grade being 200 GPa.

For elastic elements, vibration mounts were chosen. Vibration mounts are used in industry to isolate vibrations from machinery to the attached structure. Often called isolation mounts, these components can play the role of elastic elements as well as rotational joints, since it is able to bend and rotate. This is a favorable feature when considering future experimental systems, in which the actuator and elastic element assembly may not be installed strictly perpendicular to the main structure. The vibration mount should be chosen so that it is soft enough to allow for elastic averaging, yet stiff enough to prevent a structurally flimsy system.
To determine an estimate of the desired stiffness of the elastic elements, the following considerations were made. In a survey of existing electric linear actuators, a micro linear actuator manufactured by Firgelli Technologies in Canada offered advantages in size, stroke length, force, and cost. This device can overcome a maximum of 40 N in opposing force. Assuming that the actuator stroke length is 2.6 mm (stroke length of 2.6 mm would be appropriate for the experimental system of 89.9 cm long, in consistency with the established assumption of less than 0.5% ratio), and that the instance requiring the most force from the actuator is when all of the stroke provided by the actuator is absorbed by the elastic element, the desired stiffness of the elastic element can be found by application of the Hooke’s Law in (2.10). Therefore, the upper limit of the stiffness for the elastic elements would be 15,384 N/m in this case. A larger system would have a lower limit since greater stroke length would be imposed. By the same relationship, if actuators with greater force capabilities are chosen, stiffer elastic elements would be used. The bolt-and-screw actuators in the experimental BBM are capable of opposing greater than 40 N; therefore, the upper limit of the stiffness is adjusted accordingly. In order to ensure proper stiffness of the system, an isolation mount with spring constant near 20,000 N/m is selected for use.

\[
k = \frac{F}{\delta} = \frac{40}{0.0026} = k=\text{stiffness criteria (spring constant)} \quad F=\text{maximum force capable by linear actuator} \quad \delta=\text{assumed stroke length} \quad (2.10)
\]

Advanced Vibration Components, located in New York, has a wide selection of isolation mounts in its inventory. Of these, part V10Z-304A most closely meets the need. Based on the part data (see Figure 2.7), its spring constant is 21,014 N/m. If necessary, its stiffness can be
reduced by making incisions to the vibration mounts. The actual measured value for the part turned out to be 21,702 N/m. This is an average of ten trials on five different parts, measured by applying various axial loadings and recording its compression lengths with a digital caliper.

![Figure 2.7: Load plot (curve A) for vibration mount V10Z-304A, showing its compression stiffness (www.vibrationmounts.com)](image)

For the purposes of the BBM experimental setup, data for the mentioned Firgelli micro linear actuators were used strictly as the basis for selecting the stiffness of the elastic elements. Future, more sophisticated experimental systems would eventually use the micro linear actuators; however, for the BBM experimental setup much simpler bolt-and-screw type manual actuators are used. Because BBM is a simple system, its components are kept as simple as possible. Figure 2.8 shows a schematic for one of the manual actuators.
Ten holes are drilled on the beam, equidistant from each other. At each hole, the bolt-and-screw linear actuators are installed, as shown in Figure 2.8. One full clockwise rotation of the polymer block yields 2.6 mm of extension, resulting in the linear stroke. The final setup is shown in Figure 2.9. Approximating the beam to be 1 m in length, stroke lengths of up to 5 mm can be used and still be true to the assumption of less than 0.5% ratio. However, given the imprecise nature of the actuators, just one full rotation is used to actuate the beam, yielding in the 2.6 mm of stroke. A summary of the measured parameters of the system is shown in Table 2.1.
Table 2.1 BBM Experiment Setup Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Length ($L$)</td>
<td>89.9 cm</td>
</tr>
<tr>
<td>Beam Young’s Modulus ($E$)</td>
<td>112 GPa (steel beam)</td>
</tr>
<tr>
<td>Beam cross section</td>
<td>22.6 mm x 0.96 mm (width x height)</td>
</tr>
<tr>
<td>Beam Area Moment of Inertia ($I$)</td>
<td>$1.663 \times 10^{-12}$ m$^4$</td>
</tr>
<tr>
<td>Number of actuators ($N$)</td>
<td>10</td>
</tr>
<tr>
<td>Stroke length ($\delta$)</td>
<td>2.6 mm (one rotation)</td>
</tr>
<tr>
<td>Vibration mount stiffness ($k$)</td>
<td>21702 N/m</td>
</tr>
<tr>
<td>Vibration mount length</td>
<td>13.04 mm</td>
</tr>
<tr>
<td>Vibration mount cross section</td>
<td>Circular, 14.06 mm diameter</td>
</tr>
</tbody>
</table>

2.2.2 Verification and Results

Each experiment requires a manual input of the actuators, followed by a manual collection of data. Data collection involves measuring the deflections at points along the beam using a dial indicator. A baseline is taken with the dial indicator, which is subtracted from the measurements after the actuator inputs. For verification purposes, Figure 2.10 shows a step input to the system. Step input is engaging all of the actuators on one side of the system only. For the experimental system, this is engaging five actuators on the right side, while five actuators on the left are left idle. Binary input is $[0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1]$. 
The experimental results clearly support FEA results, which is supported by analytical solutions. At worst, the experimental data is off by approximately 10%. The discrepancies can be attributed to the uncertainties involved in manual actuation and in using the dial indicator. Furthermore, uncertainties in system parameter measurements would cause additional errors. Since the input was symmetric, the output also shows symmetry. There is a notable overshoot around the area of input discontinuity (i.e. transition from ‘0’ to ‘1’), followed by a subtle settling region. This is due to the elastic effects of the system. The deformations of the elastic elements can be seen through ADINA by noting its nodal displacements. As shown in Figure 2.11, some elastic elements are in compression and others are in tension to result in the final beam shape. Note that the region of greatest compression/tension (hence the region with greatest axial forces applied by the actuator), is at the location of the step. At 4.8 N, the axial forces on these actuators overshadow those on other actuators, which range from 0 to 1.3 N. Additionally, 4.8 N implies that the entire stroke of the actuator is not absorbed by the elastic elements, but rather transferred.
to the bending of the beam.

Figure 2.11: Axial forces on ten-actuator BBM for step input at 65x displacement magnification

The second input used for comparing experimental results to FEA results is the pulse input – [1 1 0 0 1 1 0 0 1 1]. Figure 2.12 shows that at worst, the experimental data is approximately 6% off from the FEA solution. In similar observation to the step input response, the peak deformation of the beam is slightly greater than the actuator stroke length of 2.6 mm. However, because the pulse input is periodic, the settling region is absent. Periodic symmetric input produced periodic symmetric beam deformation, resulting in sinusoidal behavior.
Because of the non-automated condition of the BBM experimental setup, the experiments are slow and cumbersome. Therefore, they are run purely as verification of the FEA model in ADINA. As such, the two experimental results above conclude that the ADINA model is correct. Further investigations on the characteristics of BBM will be conducted in software simulations.

2.3 Analysis

The analysis of BBM produces three types of information: workspace, validity of superposition, and performance. They are produced through FEA using ADINA interfaced with MATLAB. For the purposes of analysis, beam length of 3 m is used with actuator stroke length of 1.5 cm. The Young’s Modulus of the beam is taken to be 200 GPa (stainless steel), with cross section of 20 mm x 2 mm (thin beam). Stiffness, $k$, of the elastic elements is 20,000 N/m.
2.3.1 Workspace

The workspace shows all of the possible solutions (or states) of a system. In the case of BBM, workspace represents all possible shapes of the beam resulting from deformations caused by every possible combination of binary inputs. For an $N$-actuator system, there are $2^N$ possible input combinations producing $2^N$ states for the beam. This relationship between the number of states and the number of actuators is expected in a system with binary actuators [19, 27]. Figure 2.13 shows the workspace for BBM from section 2.1.2, with $L=3$ m, $N=4$ and $\delta=1.5$.

For 16 states, the workspace becomes cluttered and difficult to discern detailed information. As $N$ increases, such workspace representation becomes nearly useless. However, some observations can still be made from the workspace in Figure 2.13. First, beam solutions are symmetric about $x = \frac{L}{2}$ and $y = \frac{\delta}{2}$. This is brought on by physical symmetry in BBM itself. Also, it is notable that some actuator inputs result in the deformation of the beam such that its peak reaches $y$ positions slightly greater than that of the stroke length. The maximum peak attainable is approximately 1.6 cm; with symmetry, the minimum attainable is -0.1 cm. As all actuators
displace either zero or $+\delta$, no further negative solutions are possible. In order to include negative solutions to the workspace, the bi-stable states of the binary actuators can be modified to 
\[
\left[-\frac{\delta}{2}, +\frac{\delta}{2}\right]
\] instead of $[0, +\delta]$. In this case, the entire workspace shifts down by $\frac{\delta}{2}$, with the horizontal line of symmetry at $y=0$. Although the exact maximum attainable peak may vary with stiffness parameters of the system, in general the range of the workspace is approximately $\delta$.

Figure 2.13 also shows that the curves on the workspace have areas of concentrations. Another attribute to the physical symmetry of the model, this is similar to the clustering effect seen in workspaces of other binary devices [19, 27]. Ideally, the workspace will contain solutions evenly distributed within the range of operation. In [19] it was found that the clustering can be alleviated by destroying the physical symmetry of the system with asymmetric variation of stiffness values in the elastic elements. Such effects are investigated in the two-dimensional study of this thesis. Observations above are the expected behaviors regardless of $N$.

Within the range established above, the workspace can be modified by changes in the beam behavior through several parameters. The behavior of the beam is most noticeably affected by the ratio between the uniform stiffness of the elastic elements (all elements having the same stiffness) and the beam stiffness (represented by $EI$) as shown in (2.11). The beam acts as a more flexible system when this ratio is high. In general, either increasing the stiffness of the elastic elements or decreasing the stiffness of the beam will cause greater deformations along the beam. As the ratio decreases, the beam behaves in a more rigid manner. Figure 2.14 shows these

\[
v \sim \frac{k}{EI}
\]

$v=$deformation of beam
$k=$stiffness of the elastic elements
$E=$Young's Modulus of beam
$I=$Area moment of inertia of beam

(2.11)
effects for a step input to a three-meter, ten-actuator system. BBM can be categorized as a *soft system* if the beam undergoes significant deformations and a *hard system* if the beam is dominated by rigid characteristics. In a hard system, the actuator strokes are largely absorbed by the elastic elements. By contrast, in a soft system, more of the displacements by the stroke are transferred to the beam.

![Graph showing beam deflection](image)

**Figure 2.14:** Step input [0 0 0 0 1 1 1 1 1 1] to a three-meter, ten-actuator system with varying flexibility

### 2.3.2 Superposition

Superposition solution is the summation of the deformations of the beam caused by each actuator separately, rather than calculating the effects of all of the actuators at the same time (2.12). For small enough actuator strokes, the two solutions yield the same answers. As it implies limited linearity in the system, the validity of such property offers many advantages in simplifying calculations. For instance, only $N$ finite element analysis followed by $2^N - N$
summation operations would be necessary to construct a complete workspace. This significantly reduces computation time compared to performing $2^N$ finite element computations. Furthermore, it allows for a very simple formulation of the transformation matrix for forward kinematics; each of its entries would correspond to the individual effects of the actuators.

\[
\begin{align*}
    v(x,d) &= \sum_{i=1}^{N} v(x,d_i) = v(x,d_1) + v(x,d_2) + \cdots + v(x,d_N) + e, \\
    e &= \text{error}
\end{align*}
\]

Figure 2.15 only shows results for up to $N=8$, but the trend is noticeable. The graph on the left plots the magnitude of maximum differences between normal and superposition FEA solutions, for deflections at any point along the beam, as the actuator stroke length is increased. It can be seen that the maximum difference grows as the stroke length increases, and also that its rate increases with increasing number of actuators. The graph on the right plots the same differences as a percentage of the deflections obtained from normal solutions. Below stroke length of 0.05 m, superposition yields solutions that are less than 1% off from the normal solutions. Furthermore, it can be said with confidence that a lower level of percentage differences will be maintained for much greater $N$ with stroke lengths less than 0.015 m (0.5% of the beam length). The ultimate conclusion of this study is that superposition holds for the binary beam model.
2.3.3 Performance

Performance of BBM is measured by how well the system can deform the beam to a pre-defined desired beam shape. Since the purpose of this study is to be able to modify the shape of the primary mirror of a space telescope, such an investigation is relevant. The desired shape is named the shape function. The binary input that results in the beam deformation closest to the shape function, as defined by (2.13), is the optimal input. Essentially, a deformed shape is compared to the shape function by the squared difference of the areas under each curve.

\[
M(s, v) = \left( \int s(x)dx - \int v(x, d)dx \right)^2
\]

\[
d^* = \arg \min_d \left( \int s(x)dx - \int v(x, d)dx \right)^2
\]

\[
v^* = v(x, d^*)
\]

The cost function to be minimized is:

\[
M(s, v) = \left( \int s(x)dx - \int v(x, d)dx \right)^2
\]

\[
v^* = \text{optimal solution based on optimal input } d^*
\]

(2.13)
For small number of actuators, an exhaustive search, in which every possible input combination is explored, is adequate. However, as the number of actuators increase, an optimization algorithm should be applied. Since the number of possible solutions exponentially grow with increasing number of actuators, exhaustive search becomes unreasonable. This will be investigated further in the next chapter.

First, BBM’s performance against an arbitrary shape function is investigated. Figure 2.16 shows the results for the same model studied previously. Plot on the left shows the best beam shapes attainable for 8, 10, and 12 actuators. Plot on the right shows the shaping error (cost function, $M$) versus number of actuators. In general, the error decreases with increasing $N$. Beyond $N=10$, there is very little improvement in the system’s ability to match the shaping function. To put the shaping error values into perspective, at $N=12$, the maximum deflection difference (or amplitude difference) between the shaping function and the best solution is approximately 0.25 cm. As improvements with increasing $N$ further become negligible, it can be concluded that the ability of BBM to match any arbitrary shaping function is not satisfactory. Figure 2.17 shows the same analysis for a half-wave shaping function, with its peak slightly greater than the stroke length of the actuators. Here, similar conclusions can be made as before. The shaping error quickly decreases with increasing $N$. However, even at $N=16$, the maximum amplitude difference along the beam is no less than 0.2 cm, with corresponding shaping error, $M$, on the order of $1 \times 10^{-6}$ m$^4$. In the system’s attempt to match the half-wave, note that the resulting optimal solutions are not able to reach peaks much larger than the stroke length of the actuators, regardless of higher values of $N$. The rapid decay of the shaping error is also noticeable by the decreasing increment between the peaks of the optimal solutions for increasing $N$. 
Figure 2.16: Optimal BBM solutions for matching arbitrary shape function (left) and progression of its error as the number of actuators increases (right)

Figure 2.17: Optimal BBM solutions for matching half-wave shape function $y=0.018\sin(\pi x/3)$ (left) and progression of its error as the number of actuators increases (right)

With its poor performance in matching neither arbitrary nor the half-wave shaping functions, it is questionable if it can even match the simplest shaping function – vertical translations of the beam in $y$. Figure 2.18 shows the best solution for an eight-actuator system with shaping function of $y=0.75$ cm. Because of the physical nature of the binary beam model, it
is expected that any best solution for vertical translations of the beam would result in rippled surfaces, except for the cases in which all actuators are either engaged or off (resulting in $y = \delta$ or $y = 0$ vertical translations, respectively). In some instances, the ripples are subtle, as in Figure 2.18.

![Figure 2.18: Optimal BBM solution for $N=8$ for vertical translation shaping function $y=0.75$cm](image)

In others, the best solution in terms of the cost function defined before, results in beam shapes that cannot be regarded as a simple translation of the beam. Table 2.2 summarizes the results for analyzing the system’s translation capability. For each value of $N$, a vertical translation shaping function is commanded, increasing from $y=0$ to 1.5 cm (equivalent to the actuator stroke length) in increments of 2 mm. For each increment the best solution is found. It turns out that the optimal binary input remains constant with incremental increase in $y$ until the translation shaping function reaches a value where a different input provides the best solution, starting a new range of constant optimal binary input. Each range of constant binary input can be considered as a discrete level. The maximum shaping error from the entire process (from all translation shaping
functions from \( y = 0 \) to \( 1.5 \) cm) is recorded for each \( N \), and this value signifies the degree of worst possible performance for matching \( \text{any} \) commanded vertical translation on the beam. According to Table 2.2, as \( N \) increases to 16, the maximum shaping error decreases to the order of \( 1 \times 10^{-8} \) m\(^4\). In perspective, this value corresponds to a maximum amplitude error on the order of millimeters. Considering that the vertical range of the beam is approximately 1.5 cm, amplitude errors on the order of millimeters is quite large; hence the performance is quite poor. Increasing the number of actuators further would not improve the situation much more.

<table>
<thead>
<tr>
<th>( N )</th>
<th>Number of discrete levels</th>
<th>Max shaping error ((\text{m}^4))</th>
<th>Max amplitude error ((\text{m}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4</td>
<td>(3.61 \times 10^{-6})</td>
<td>0.011</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>(1.25 \times 10^{-6})</td>
<td>0.011</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>(3.29 \times 10^{-7})</td>
<td>(6.81 \times 10^{-3})</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
<td>(7.36 \times 10^{-8})</td>
<td>(4.28 \times 10^{-3})</td>
</tr>
<tr>
<td>12</td>
<td>21</td>
<td>(4.28 \times 10^{-8})</td>
<td>(3.51 \times 10^{-3})</td>
</tr>
<tr>
<td>16</td>
<td>24</td>
<td>(1.95 \times 10^{-8})</td>
<td>(2.47 \times 10^{-3})</td>
</tr>
</tbody>
</table>

### 2.4 Summary

For all practical purposes, the BBM is a poor system. Its perpendicular, uniform actuator placement is inadequate for deforming the beam to an arbitrary shape to an acceptable level of precision. The fundamental flaw of the BBM is that it is under-actuated. Because the system is a continuous beam, it can be argued that the number of degrees-of-freedom in the system is infinite. A necessity of BLISS is that it has more actuators than the number of degrees-of-freedom in the system. In the case of BBM, an over-actuated system can never be fully achieved. Despite these
shortcomings, there are several main points to take away from the one dimensional study.

Most significantly, the validity of superposition was confirmed for a coupled system of binary actuators and compliant elements. For actuator stroke lengths less than 0.5% of the system size, superposition holds with errors less than 1%. Furthermore, it was demonstrated that precision (as determined by the shaping error) increases with increasing $N$, also showing that there is a limit to the improvements. In addition, it can be generalized that the range of motion for the system is limited to the range of the stroke length of the actuators. Lastly, the workspace within the range of the binary actuated system can be modified by adjusting the overall stiffness of the system. This stiffness can be varied by changing the ratio between the stiffness of the elastic elements coupled to the actuators and the stiffness of the beam itself. Beam stiffness is dependent on its material, cross-section, and length.

The insights gathered from the one-dimensional study will be revisited and applied to systems of additional complexity. In the next chapter, two dimensional studies with models more representative of a segmented primary mirror are investigated.
3. Two-Dimensional Study

Here, the complexity of the model is increased to allow for the linear actuators to act in both $x$ and $y$ components. First, the beam is replaced with three rigid panels to represent the mirrors on a segmented primary mirror of the space telescope. Each panel has actuators directly acting on its endpoints. Using three panels allows for the simplest representation of a segmented mirror system.

Initially, two simple problems are investigated: one in which the three panels are connected by joints and another in which they are connected by elastic links (see Figure 3.1). For each problem, complete workspaces are analyzed and the validity of superposition is tested. Following results, the second model is expanded to implement increasing number of actuators in a holistic design of the system, called a Gestalt System. Performance is tested on the Gestalt System by examining its capability to correct for a given thermal deformation on the support structure.

![Figure 3.1: Two simple problems for initial investigations](image)
In the two-dimensional study, the overall size of the model is approximately 10 m. Although this is hardly enough to represent an ultra-large space telescope system on the order of 100 m, it follows the logic that modular sections of 10 m diameter systems are more deliverable and can be joined in space to construct the ULST using methods studied in [35]. Each 10 m section is still expected to undergo the same magnitude of thermal deformation (less than 0.5% of the overall structure).

3.1 Simple Problem I

The first simple problem considered is shown in Figure 3.2. It consists of three rigid panels connected by free joints and six binary actuators in series with elastic elements. Each panel is chosen to be 3 m, making the overall structure just under 9 m. The outer panels are inclined at $\beta=5$ deg from the horizontal, creating curvature that results in the focal plane at $y=17.11$ m with the origin of the system at the midpoint of panel B. The focal plane is determined by tracing parallel rays from a source infinitely far away, reflecting off of the center of each panel. The intersection of the three reflected rays represents the focal point. This provides a primary $f$-ratio of 1.9, as given by equation (3.1), which is consistent with typical primary $f$-ratio values between 1 and 3 for space telescopes (primary $f$-ratio differs from system $f$-ratio in that the latter considers the focal length of the entire system, where as the primary $f$-ratio is only concerned with the primary mirror) [5, 44]. The primary $f$-ratio for JWST is approximately 1.5 [45]. Each actuator is positioned $\theta=45$ degrees with respect to the panel, attached at the endpoints of each panel. This causes each support structure with an actuator to be 2.12 m in length. Every joint in the system, including the grounded joints, are free to rotate. The displacements in the centers of the three panels and their rotations are the focus of this
investigation. As each panel is free to translate in $x$ and $y$ directions and rotate about its center, there are a total of nine degrees-of-freedom in this physically symmetric problem.

![Diagram of panels and joints](image)

$\text{Figure 3.2: Simple Problem I for two dimensional study of BLISS}$

$$f\text{-ratio} = \frac{f}{D} \quad f=\text{primary mirror focal length}$$
$$D=\text{Diameter of primary mirror} \quad (3.1)$$

Initially, the stiffness of all elastic elements is made the same value. This value is determined in the same manner as in BBM, as described by equation (3.2). A general assumption is made that realistically, the actuators would produce 100 N of force to influence the space system. Maintaining approximately 0.5% of the overall structure size, stroke length is chosen to be 0.05 m. The resulting stiffness is 2000 N/m.

$$k = \frac{F}{\delta} = \frac{100}{0.05} \quad k=\text{stiffness criteria (spring constant)}$$
$$F=\text{maximum force capable by linear actuator}$$
$$\delta = \text{assumed stroke length} \quad (3.2)$$
3.1.1 Model and Verification

As with the one-dimensional study, an analytical model is compared to the finite element analysis from ADINA. The analytical approach involves a system of equations derived from truss analysis by applying force equalities and physical constraints on each of the joints. Figure 3.3 (a) shows the detailed diagram of the simple problem for joints A and B, while (b) and (c) shows its free-body diagrams. Equations in (3.3) show formulations for joint A. Equations for joints B through D are constructed in the same manner. There are two force balance equations (for x and y components) for each joint, two constraint equations for each panel, and six geometric equations for each actuator, for a total of twenty equations in the system. Furthermore, there are two displacement unknowns (for x and y components) for each joint, unknown rotations ($\Delta \beta$) for each panel, and unknown rotations ($\Delta \theta$) for each support member with actuators. Therefore, the system of equations involves twenty equations for twenty unknown variables. The centers of the panels are calculated from the displacements of joints A through D. Solutions are found numerically in MATLAB using the ‘fsolve’ function, which uses Gauss-Newton methods based on least-square algorithms.
Formulation for Joint A:

Truss analysis: Force balance on Joint A

\[ \sum F_{Ax} = -k_1(d_1 \cos(-\theta_1 - \Delta \theta_1) + \Delta A_x) - F_a \cos(-\beta a - \Delta \beta a) = 0 \]
\[ \sum F_{Ay} = k_1(d_1 \sin(-\theta_1 - \Delta \theta_1) + \Delta A_y) + F_a \sin(-\beta a - \Delta \beta a) = 0 \]

Constraints imposed by rigidity of the panel between joints A and B:

\( (A_x, A_y) = (x, y) \) coordinate of joint A:

\[ A_x + \Delta A_x + L_m \cos(-\beta a - \Delta \beta a) = B_x + \Delta B_x \]
\[ A_y + \Delta A_y + L_m \sin(-\beta a - \Delta \beta a) = B_y + \Delta B_y \]

Constraints imposed by geometry of the support structure with actuators:

\[ \theta_1 + \Delta \theta_1 = \arctan \left( \frac{A_y + \Delta A_y - H_y}{A_x + \Delta A_x - H_x} \right) \] (3.3)
With twenty simultaneous equations to solve for, the analytical model is very cumbersome even for such a simple problem with only six actuators. As more complexity is added, one can only expect the formulation process to become more arduous. Hence, Finite element model was created using ADINA to complement and verify the analytical model. Using same modeling techniques from the construction of the ADINA model for BBM, only beam elements are used and linear actuations are created using temperature loads. Free joints are created by implementing moment releases between beam elements (see Appendix B for more details). Again, the interface between MATLAB and ADINA was a critical tool in obtaining results (see Appendix A). Figure 3.4 shows the ADINA model displayed in ADINA User Interface (AUI) and Figure 3.5 shows one graphical output for binary input \([0 \ 1 \ 0 \ 0 \ 1 \ 0]\). This binary input results in the maximum axial forces in the system. As intended from the elastic element stiffness selection, the maximum axial force remains well below 100 N.

![Figure 3.4: FEA model construction in ADINA for Simple Problem I](image)

- Beam elements for elastic components
  - Young’s Modulus given by equation in (2.7)
  - Cross-section selected to provide desired stiffness
- Beam elements for actuator
  - Artificially high Young’s Modulus
  - Symmetric cross-section
  - Actuation by extensions due to temperature loading
- Boundary condition definition: Fixed translation and free rotation on grounds
- Each hash-mark represents an element division; i.e. nodes.

*Chapter 3: Two-Dimensional Study*
In comparing all $2^6=64$ solutions the maximum percentage error of the analytical solution to the finite element solution is less than 0.8% for displacements and rotations of all three panels. Therefore, conclusion is made that the two models agree and the solutions are accepted to be correct. The difference can be attributed in part to loss of accuracy as numerical errors build up in the search algorithm of ‘fsolve’ in MATLAB for the analytical solutions. However, more significant is the assumption of small displacements in using truss analysis method for the analytical solutions. FEA does not require this restriction, and is considered to be more accurate. Due to time constraint, an experiment system was not built as part of this thesis.

### 3.1.2 Workspace

In contrast to the BBM, and more inline with binary actuated systems, workspace construction for the simple problem involves sets of discrete points. The complete workspace is given by displacements in $x$ and $y$ directions for the centers of each panel, and the rotations of each panel versus $x$ and $y$ displacements (positive rotations are in counter-clockwise direction),
from all possible combination of binary inputs. Figure 3.6 gives the workspace for the system described above with stroke length, \( \delta \), equal to 0.05 m.

![Graph showing workspace for different panels]

**Figure 3.6: Workspace for Simple Problem I with stroke length = 0.05 m (solid line represents the initial panel orientation)**

Immediate observations are that the workspace is symmetric and that there is high degree of clustering of points. Both are attributes noticed in the study of the binary beam model and other binary actuated systems [19, 27]. Furthermore, the range of motion in \( x \) and \( y \) are both approximately 0.035 m, which is \( \delta \cos(\theta) \) and \( \delta \sin(\theta) \), respectively. The fact that stroke length of the actuators directly affects the range of motion was seen in the one-dimensional study. In
another similarity, displacements in $y$ only occur in the positive direction. Finally, the rotations of all three panels exhibit the same range of rotations ($-0.7 \rightarrow +0.7$ degrees), and in general the workspace distributions for the three panels are comparable. Figure 3.7 shows the workspace of the same system for stroke length of 0.025 m. At half stroke, the workspace is approximately half in all aspects. The range of displacements and rotations are half that of Figure 3.6, and the distribution is essentially preserved.

Figure 3.7: Workspace for Simple Problem I with stroke length = 0.025 m

In order to obtain a more uniformly distributed workspace, research in [19] noted that destruction of symmetry removed clustering of the discrete points. Using this information, the stiffness of the elastic elements in the problem is varied according to the uniform probability
distribution shown in Figure 3.8. Figure 3.9 shows the workspace for $\delta=0.05$ m, interval $p=1000$ N/m, and mean stiffness $k=2000$ N/m. With this random variation in stiffness of the elastic elements, clustering of points is relieved, albeit not completely. Despite stiffness variations, the range of motion is maintained. Determining the optimal stiffness to obtain the most uniformly distributed workspace is a topic of study worth investigating. However, for the purposes of this thesis, only a demonstration of the effect of symmetry destruction is performed.

$$f_d(k) = \text{uniform probability density function}$$

$$k = \text{random variable for elastic element stiffness}$$

Figure 3.8: Elastic element stiffness distribution
3.1.3 Superposition

Validity of superposition is tested in this section. As mentioned in the previous chapter, superposition is a valuable property for computational advantages. Furthermore, it simplifies the transformation matrix, $J$, describing the system input-output relationship. For the three-panel, six-actuator ($N=6$) system under consideration, the superposition solution is given by equation (3.4). $C_{Ax}$ is the panel $A$ center displacement in $x$, $C_{Ay}$ is the panel $A$ center displacement in $y$, and $R_A$ is the panel $A$ rotation (positive in counter-clockwise direction). Results for panels $B$ and $C$ are found in the same way, and the equations are applicable for future systems of greater $N$.

Figure 3.9: Workspace for Simple Problem I with stroke length=0.05 m and different stiffness for each elastic element
Equations in (3.4) can be simplified into a matrix form shown in (3.5) with the terms of (3.4) making up the entries of matrix A. Then, the formulation can be modified to create equation (3.6) for performing inverse kinematics to obtain inputs for desired displacement and rotations. For the purposes of this formulation, error \( e \) is disregarded. The physical meaning of matrix \( J \) was not investigated; therefore, its properties are not understood. Furthermore, equation (3.6) will only give non-discrete results for \( d \), making it unrealizable in binary systems. Mathematical investigations and optimization algorithms can be studied as part of future work related to this project.

\[
C_{A,x}(d) = \sum_{i=1}^{N} C_{A,x}(d_i) = C_{A,x}(d_1) + C_{A,x}(d_2) + \cdots + C_{A,x}(d_N) + e
\]

\[
C_{A,y}(d) = \sum_{i=1}^{N} C_{A,y}(d_i) = C_{A,y}(d_1) + C_{A,y}(d_2) + \cdots + C_{A,y}(d_N) + e
\]

\[
R_{A}(d) = \sum_{i=1}^{N} R_{A}(d_i) = R_{A}(d_1) + R_{A}(d_2) + \cdots + R_{A}(d_N) + e
\]

\[
x = \begin{bmatrix} C_{A,x} & C_{A,y} & R_{A} & C_{B,x} & C_{B,y} & R_{B} & C_{C,x} & C_{C,y} & R_{C} \end{bmatrix}^T
\]

\( d \in \{0,1\}^{1 \times N} \)

\( e = \text{error} \in \mathbb{R}^{9 \times 1} \)

\[
x = \begin{bmatrix} C_{A,x}(d_1) & C_{A,x}(d_2) & \cdots & C_{A,x}(d_N) \\
C_{A,y}(d_1) & C_{A,y}(d_2) & \cdots & C_{A,y}(d_N) \\
R_{A}(d_1) & R_{A}(d_2) & \cdots & R_{A}(d_N) \\
\vdots & \vdots & \ddots & \vdots \\
C_{C,x}(d_1) & C_{C,x}(d_2) & \cdots & C_{C,x}(d_N) \\
C_{C,y}(d_1) & C_{C,y}(d_2) & \cdots & C_{C,y}(d_N) \\
R_{C}(d_1) & R_{C}(d_2) & \cdots & R_{C}(d_N) \end{bmatrix}^T + e
\]
For Simple Problem I, Figure 3.10 and Figure 3.11 show the comparison between normal and superposition FEA solutions in the panels’ center displacements (in $x$ and $y$) and rotations, respectively. The left plots display absolute value differences between the two solutions, and the right plots display the percentage errors with the normal solutions as the basis. In both figures, the physical symmetry of the system can be observed from the results of panel $A$ and panel $C$ having identical curves. In general, superposition deteriorates quickly as the stroke length increases. Particularly in rotations, panel $B$ deteriorates with greater rate. As it was noted from BBM studies that superposition deteriorates with increasing $N$, this is likely due to the fact that more actuators are acting on panel $B$ than on $A$ or $C$ since it is located at the center. Furthermore, conclusions can be made that stroke length of 0.05 m, or approximately 0.5% of the overall structure size, is at the limits of acceptable discrepancies between normal and superposition solutions.
Figure 3.10: Panel center displacements comparison between normal and superposition FEA solutions given as absolute difference (left) and as a percentage of the normal FEA solution (right).

Figure 3.11: Panel rotations comparison between normal and FEA superposition solutions given as absolute difference (left) and as a percentage of the normal FEA solution (right).
3.2 Simple Problem II

The second problem considered is shown in Figure 3.12. It is the same as Simple Problem I in every aspect except that the free joints between the three panels are replaced with elastic links. There is no rotation allowed in the connection between the panel and the elastic link; however, all of the actuators are still connected to the endpoints of the panels by free joints. The elastic links are chosen to be 0.25 m long, making the overall structure 9.48 m. With the new focal plane at $y=18.53$ m, the primary $f$-ratio is 1.95. Including elastic links rather than free joints results in a more compliant system, having greater adherence to the BLISS definition. Furthermore, the system is more comparable to the BBM, by viewing Simple Problem II with $\beta=0$ deg, $\theta=90$ deg, and having infinitely small, infinitely many panels. Considering that the system will be launched into space, aluminum (Young’s Modulus = 75 GPa) is chosen as the elastic links material for its light-weight and compliant properties. Although there are better suited materials available today for space construction, this allows an experimental system to be built more readily, should such experiments ever take place. Furthermore, the elastic links are hollow in order to increase its compliant nature, having cross-section of 2 cm x 2 cm with wall thickness of 2 mm. As before, stiffness of the elastic elements in series with binary linear actuators is kept at 2000 N/m.
3.2.1 Model and Verification

The analytical model involves a combination of methods used in Simple Problem I and BBM. Figure 3.13 (a) shows a detailed diagram of the simple problem for this formulation, while (b) and (c) shows the free-body diagrams for joint A₁ and elastic link connecting joints A₂ and B₁, respectively. Equations in (3.7) are essentially identical to (3.3), involving force balance on joint A₁ (which is the same for C₂), constraints imposed by rigidity of the panels, and constraints imposed by the geometry of the support structure. For the elastic links connecting the panels, Euler beam equations for solving statically indeterminate systems are used to determine the deflections in the links [39]. Furthermore, axial compression/elongation effects are expressed by common application of the Hooke’s Law. Equations in (3.8) show force and moment balance on the link between A₂ and B₁ and application of these beam equations. The same processes are used for the link between B₂ and C₁. The resulting system of equations involves 28 equations and 28 variables. As before, the panels’ center displacements and rotations are calculated from the
displacements of joints $A_1$ to $C_2$. The solutions are found numerically in MATLAB using the 'fsolve' function.

![Image](image_url)

$F_a$, $F_b$, $F_c$ = axial forces along panels $A$, $B$, $C$

$M_a$, $M_b$, $M_c$, $M_d$ = reaction moments at the ends of elastic links

$F_{s1}, \ldots, F_{s6}$ = forces applied by actuator $1 \ldots 6$

$\Delta \theta_1, \ldots, \Delta \theta_6$ = changes in angle of actuators (redefined to measure from the horizontal)

$\Delta \beta_a$, $\Delta \beta_b$, $\Delta \beta_c$ = panel rotation angle

$\Delta A_1$, $\Delta A_2$, ..., $\Delta C_1$, $\Delta C_2$ = joint displacements

**Figure 3.13: Free-Body Diagram for Simple Problem II**

Force balance equations for joint $A_1$; similar formulation for joint $C_2$:

$$
\sum F_{A_x} = -k_1(d_1 \cos(-\theta_1 - \Delta \theta_1) + \Delta A_1) + F_a \cos(-\beta a - \Delta \beta a) = 0
$$

$$
\sum F_{A_y} = \underbrace{k_1(d_1 \cos(-\theta_1 - \Delta \theta_1) + \Delta A_1)}_{F_{s1}} + \underbrace{F_a \cos(-\beta a - \Delta \beta a)}_{F_{s2}} = 0
$$

(3.7)

Constraints imposed by rigidity of the panel between joint $A_1$ and joint $A_2$:
\( (A_{1x}, A_{1y}) = (x, y) \) coordinate of joint \( A_1 \):

\[
\begin{align*}
A_{1x} + \Delta A_{1x} + L_m \cos(-\beta a - \Delta \beta a) &= A_{2x} + \Delta A_{2x} \\
A_{1y} + \Delta A_{1y} + L_m \sin(-\beta a - \Delta \beta a) &= A_{2y} + \Delta A_{2y}
\end{align*}
\]

Constraints imposed by geometry of the support structure with actuators:

\[
\theta_1 + \Delta \theta_1 = \arctan \left( \frac{A_{1y} + \Delta A_{1y} - H_y}{A_{1x} + \Delta A_{1x} - H_x} \right) 
\]  

(3.7 cont’d)

Analyzing elastic links

Force balance and moment balance about \( A_2 \) for elastic link between \( A_2 \) and \( B_1 \):

\[
\begin{align*}
\sum F_{x,\text{link}} &= F_a \cos(-\beta a) + k_5 (d_2 \cos(-\theta_2 - \Delta \theta_2) - \Delta A_{2x}) - F_a \cos(-\beta b) - k_3 (d_3 \cos(-\theta_3 - \Delta \theta_3) + \Delta B_{1y}) = 0 \\
\sum F_{y,\text{link}} &= -F_a \sin(-\beta a) + k_5 (d_2 \sin(-\theta_2 - \Delta \theta_2) - \Delta A_{2y}) + F_a \sin(-\beta b) + k_3 (d_3 \sin(-\theta_3 - \Delta \theta_3) - \Delta B_{1y}) = 0 \\
\sum M_{A_2} &= -M_A + M_B + F_{\text{link}} L_{\text{link}} \sin(-\beta b) + k_3 L_{\text{link}} (d_3 \sin(-\theta_3 - \Delta \theta_3) - \Delta B_{1y}) = 0
\end{align*}
\]

Now, apply Euler beam equations:

\[
\frac{\partial^2 v(u)}{\partial u^2} = \frac{M(u)}{E_{\text{link}} I_{\text{link}}}
\]

\[
v(u) = \frac{1}{E_{\text{link}} I_{\text{link}}} \int M(u) du
\]

which yields:

\[
M(u) = M_A + (-F_{a_y} + F_{s_{2y}}) u
\]

\[
\frac{\partial v(u)}{\partial u} = \frac{1}{E_{\text{link}} I_{\text{link}}} \left[ M_A u + \frac{1}{2} (-F_{a_y} + F_{s_{2y}}) u^2 \right] + K_1
\]

\[
v(u) = \frac{1}{E_{\text{link}} I_{\text{link}}} \left[ \frac{1}{2} M_A u^2 + \frac{1}{6} (-F_{a_y} + F_{s_{2y}}) u^3 \right] + K_1 u + K_2
\]

(3.8)

With integration constants \( K_1 \) and \( K_2 \).
Then, applying boundary conditions for fixed rotations at the endpoints of the elastic link gives:

\[
\frac{\partial v(u)}{\partial u} \bigg|_{u=0} = \tan(\Delta \beta_a) = K, \\
\frac{\partial v(u)}{\partial u} \bigg|_{u=L_{\text{link}}} = \tan(\Delta \beta b)
\]

where rotations of panels A and B directly affect the slopes at the ends of the elastic link.

Further applying boundary conditions for continuity between elastic link and panels A and B:

\[
v(u) \bigg|_{u=0} = K, \quad \Delta A, \\
v(u) \bigg|_{u=L_{\text{link}}} = \Delta B
\]

**Finally**, solving (1) and (2) for \( u=L_{\text{link}}: \)

\[
\tan(\Delta \beta b) - \tan(\Delta \beta a) - \frac{1}{E_{\text{link}} L_{\text{link}}^2} \left[M_A L_{\text{link}} + \frac{1}{2} \left( -F_a \sin(-\beta a) + k_2 \left( d_2 \sin(-\theta_2 - \Delta \theta_2) - \Delta A_2 \right) \right) L_{\text{link}}^2 \right] = 0 \]

\[
\Delta B \bigg|_{u=0} - \Delta A \bigg|_{u=L_{\text{link}}} - \frac{1}{E_{\text{link}} L_{\text{link}}^3} \left[ \frac{1}{2} M_A L_{\text{link}}^2 - \frac{1}{6} \left( -F_a \sin(-\beta a) + k_2 \left( d_2 \sin(-\theta_2 - \Delta \theta_2) - \Delta A_2 \right) \right) L_{\text{link}}^3 \right] = 0
\]

**Additionally**, compression/elongation effects by Hooke's Law gives:

\[
F = k \frac{E_{\text{link}} A_{\text{link}}}{L_{\text{link}}} \left( \Delta A_2 - \Delta B_1 \right), \quad A_{\text{link}} = \text{cross section area of the elastic link}
\]

\[
- F_b \cos(-\beta b) - k_3 \left( d_3 \cos(-\theta_3 - \Delta \theta_3) + \Delta B_3 \right) - \frac{E_{\text{link}} A_{\text{link}}}{L_{\text{link}}} \left( \Delta A_2 - \Delta B_1 \right)
\]

(3.8 cont’d)

Finite element model for Simple Problem II is also created using ADINA and the same modeling techniques as before. Figure 3.14 shows the ADINA model and Figure 3.15 shows one graphical output for binary input [0 1 0 0 1 0].
Each hash-mark represents an element division; i.e. nodes.

Beam elements for elastic links
- Young's Modulus equivalent to Aluminum (75 GPa)
- Cross-section 2 cm x 2 cm and 2 mm wall thickness

Elastic elements created same as for Figure 3.3

Beam elements for actuator
- Artificially high Young's Modulus
- Symmetric cross-section
- Actuation by extensions due to temperature loading

Boundary condition definition: Fixed translation and free rotation on grounds

Figure 3.14: FEA model construction in ADINA for Simple Problem II

Figure 3.15: One solution in graphical display of ADINA for Simple Problem II for input [0 1 0 0 1 0]

In comparing all 64 solutions, the results were not promising. While both models showed reasonable solutions (i.e. symmetric solutions giving symmetric results and ranges of motion being comparable to previous observations), they were not matching. In fact, for certain binary inputs, the solutions were off as much as 20% even for much smaller actuator stroke
lengths. The source of this error was not established for certain. It is possible that the elastic interactions are making this approach invalid. As the FEA results from BBM and Simple Problem I have proven to be accurate, and with the increasing confidence in ADINA model construction, the FEA solutions are accepted to be correct for continuing investigations.

3.2.2 Workspace

Workspace is constructed using FEA solutions. Figure 3.16 gives the workspace for Simple Problem II with stroke length $\delta=0.05$ m and uniform stiffness of 2000 N/m for the elastic elements in series with the actuators.

The greatest difference between the workspace of Simple Problem I and Simple Problem II is the considerable relief on clustering. Although evidence of clustering is still noticeable, replacing the free joints with elastic links spread out the points most significantly in rotations of panels $A$ and $C$. Furthermore, the range of motion in displacements is increased more than twofold to 0.08 m in both $x$ and $y$ directions. Since the panels are less constrained to the movements of other panels due to compliances in the links, the increased range seems reasonable. The range of rotations remains approximately the same as before. However, it appears that outer panels exhibit slightly more rotation outward (counter-clockwise for panel $A$ and clockwise for panel $C$) than inward. This is likely because these panels now experience more resistance to rotation from the elastic elements than free joints in the inner connections. Figure 3.17 shows the workspace for half stroke; $\delta=0.025$ m. Here, all of the observations from before hold true. The distribution remains the same, and the range for displacements and rotations are reduced by half.
Figure 3.16: Workspace for Simple Problem II with stroke length = 0.05 m (solid line represents the initial panel orientation)
Figure 3.17: Workspace for Simple Problem II with stroke length = 0.025 m

Figure 3.18 shows the workspace with stiffness of elastic elements varied according to the probability distribution from Figure 3.8. As before, $p=1000$ N/m, $k=2000$ N/m, and $\delta=0.05$ m. The elastic links are left unchanged. The symmetry of the system is destroyed and the points shift around within the range identified before. However, especially in the case of panel A, the clustering effect is not noticeably alleviated. As the stiffness values are determined randomly, the desired effect of uniform distribution is not fully attained. Nevertheless, random distribution of stiffness values still hold merit because destruction of symmetry increases the likelihood that there will not be overlapping solutions when studying performance issues in the next section.
Figure 3.18: Workspace for Simple Problem II with stroke length=0.05 m and different stiffness for each elastic element

3.2.3 Superposition

Superposition is tested in the same method as in section 3.1.3. Figure 3.19 and Figure 3.20 show the comparison between normal and superposition FEA solutions in the panels’ center displacements (in x and y) and rotations, respectively. The left plots display absolute value differences between the two solutions, and the right plots display the percentage errors with the normal solutions as the basis. Same characteristics observed from section 3.1.3 are seen here as
well. Curves for panels $A$ and $C$ are identical, and superposition deteriorates as stroke length increases. Again, this deterioration is more rapid for rotation of panel $B$. The most significant difference from Simple Problem I is that deterioration of superposition for rotations in panels $A$ and $C$ seems to occur at a slower rate. This may be attributed to the fact that the elastic links are removing the necessity for rotation of panel $B$ to directly influence the rotations on the other panels. It shows that the center panel, being affected the most by compliant elements, dominate the deterioration of superposition solutions. The final observation is that stroke length of 0.05 m (approximately 0.5% of structure size) continues to be at the limit of superposition validity.

![Figure 3.19: Panel center displacements comparison between normal and superposition FEA solutions given as absolute difference (left) and as a percentage of the normal FEA solution (right)](image)

Figure 3.19: Panel center displacements comparison between normal and superposition FEA solutions given as absolute difference (left) and as a percentage of the normal FEA solution (right)
3.2.4 Simple Problems Summary

In comparing the two simple problems, several insights can be obtained. First, range of the workspace is directly proportional to the stroke length of the actuators. This is a continuing theme from the one-dimensional study, and intuitively expected. Also, the inclusion of elastic links increases the displacement range of the workspace, as it reduces the physical constraints previously imposed by free joints between the panels (having effectively reduced the overall degrees-of-freedom in the system). In addition, elastic links appear to offer some alleviation in the clustering of discrete points in the workspace. Furthermore, the distribution of points within the workspace can be modified by varying the stiffness values of the elastic elements in each actuator. This is done by destroying the symmetry within the system. Although in general this action reduces clustering of points further, an optimized process for stiffness selection should be investigated in future studies. Finally, superposition holds for both Simple Problems I and II; however, actuator stroke length 0.5% of the overall structure size is at the limits of acceptable errors. In light of advantages held by Simple Problem II, this system is expanded for further
3.3 Gestalt Systems

The term Gestalt derives from the German word for ‘form’ or ‘shape’. In literature, it has roots from the idea of Gestalt psychology which operates on the principle that the brain is holistic, in that it processes information in whole forms [46]. In technical references, Gestalt is often associated with image analysis and computer vision for shape recognition as a whole form rather than individual lines and curves [47]. In expanding this thought to BLISS design, Gestalt is interpreted as a holistic design of the system, in which the actuators and elastic elements are embedded into the support structure. Problems investigated so far essentially incorporate this concept; however, this will be more apparent in the systems studied in this section.

3.3.1 Models

Since Simple Problem II provides greater range of motion and more uniform distribution of workspace, it is chosen to be expanded by systematically adding structures composed of triangular sections much like those studied in [12]. Each member forming the triangular sections is composed of an actuator and an elastic element, thereby embedding actuators into the support structure of the entire system. Furthermore, each member is connected to each other by free joints, allowing rotations. Figure 3.21 shows the three Gestalt models that will be investigated in this section. From (a) to (c), the structure is augmented by a layer of truss sections. The three panels are still 3 m and connected by 0.25 m long elastic links. The outer panels are inclined to $\beta=5$ deg. Model (a) is basically the same system as in Figure 3.12 with actuators included into supports $j$ and $k$. For construction simplicity, every diagonal member of
all three models is of the same length at 2.12 m. All other geometries are the result of this requirement.

For each Gestalt System, ADINA models are constructed using the same methods implemented before. At this point, analytical formulations become impractical; therefore, all subsequent results are computed using ADINA finite element analysis. For details on these models, see Appendix B. For each model, workspaces as established before are computed. In
addition, another depiction of the workspace, which is more applicable for determining imaging performance of the system, is defined and shown. Superposition is once again tested and deterioration characteristics are compared between the three Gestalt models. Finally, performance issues are discussed by studying the system's ability to correct for a 0.05 m thermal expansion (approximately 0.5% of the structure size) to one of its support structures.

3.3.2 Workspace

First, workspace for model (a) with δ=0.05 m and elastic element stiffness varying with uniform probability distribution from Figure 3.8 (p=1000 N/m and mean stiffness k=2000 N/m) is constructed and shown in Figure 3.22. With eight actuators, 256 discrete solutions are available. The range of displacements and rotations are comparable to that from Simple Problem II; however, distribution of points is considerably more dense and uniform, especially in the x component and in rotations. Noticeable clustering in the y direction can be seen in the workspace.

While workspace of the form used until now is descriptive and provides great insight to the system characteristics, when considering imaging qualities of the system it is rather inadequate. This is because workspace as shown in Figure 3.22 does not take into account that there is coupling between the three panels. Each point in the workspace of one panel corresponds to a point in the workspace of each of the other panels, which in reality forms a set. In other words, the desired displacement/rotation state within the workspace of one panel may not be attainable with the desired displacement/rotation state of another panel at the same time.
A more comprehensive form of the workspace considers shifts in the focal point along the focal plane, $L_p$, and the associated defocus. Figure 3.23 shows the details of this approach. Parallel rays from a source infinitely far away are traced to reflect off of the centers of each panel to focus onto the focal plane at $y=18.53$ m. At zero input, the reflected rays converge to a single point. As the system is activated the panels move and rotate, causing the rays to hit different points along the focal plane. From (3.9) the new focal point, $f_c$, is defined as the centroid of these three points and defocus, $D$, is defined as the spread of the three points. The resulting workspace for Gestalt model (a) is shown in Figure 3.24.
Figure 3.23: Ray tracing for use in the development of a different type of workspace

\[ f_c(d) = \frac{D_A(d) + D_B(d) + D_C(d)}{3} \]

\[ D(d) = \max(D_A(d), D_B(d), D_C(d)) - \min(D_A(d), D_B(d), D_C(d)) \]

\[ f_c(d) = \text{shift in focal point along the focal plane due to input } d \]

\[ D(d) = \text{defocus in } f_c \text{ due to input } d \]  

(3.9)
Figure 3.24 shows that Gestalt model (a) is able to move the focal point along the focal plane to approximately $x=\pm 0.4$ m with a certain level in preservation of the focus. It is also able to move it to $x=\pm 0.2$ m with significant loss in focus. Despite destruction of symmetry with variance of the stiffness values in the elastic elements, the results here show that a large portion of the solutions are lost due to overlaps.

Before moving on to investigating Gestalt models (b) and (c), a discussion must be made on a rising issue. On a Dell Dimension 8100 personal desktop computer with 1.5 GHz Pentium 4 processor and 1.0 GB of memory, it takes approximately twelve hours to compute just $2^{12} = 4096$ solutions from ADINA. Even if superposition solutions are taken, and each computation takes fractions of a second, the problem will not be solved for a potential system with hundreds of actuators. To put things into perspective, if every electron in the universe (there are about $2^{266}$ of them) were a 1000 gigahertz computer that could evaluate each state for a trillion ($2^{40}$) states every second, and if those $2^{266}$ computers were run for a time equal to the age of the universe ($2^{58}$ seconds), they would still only visit $2^{364}$ states. It would take more than $2^{636} \approx 10^{190}$ universe ages
to elapse before $2^{1000}$ states had been visited [48]. A system with 364 actuators, or even 1000 actuators, is completely reasonable to envision for a large space telescope controlled by binary actuators. At this stage, developing a complete workspace becomes unrealistic. Monte Carlo methods are possible approaches to explore to develop a solution. For Gestalt models (b) and (c), there are 131,072 and 33,554,432 solutions, respectively. Considering the computing costs, in order to generate their workspaces, 4096 solutions are sampled by random selection of 4096 different binary inputs determined with uniform probability of the input domain. It is acknowledged that this is a grossly insufficient amount of data in order to represent the complete workspace; however, it gives an idea of its range, distribution, and characterization without loss of generality. Figure 3.25 shows the workspace for model (b) in displacements and rotations domain, and Figure 3.26 shows the workspace in the focus domain. Figures 3.27 and 3.28 show the same plots for Gestalt model (c).
Figure 3.25: Workspace for Gestalt model (b) with N=17, stiffness values varied, and δ=0.05 m

Figure 3.26: Workspace in focal plane of Gestalt model (b)
Figure 3.27: Workspace for Gestalt model (c) with $N=25$, stiffness values varied, and $\delta=0.05$ m

Figure 3.28: Workspace in focal plane of Gestalt model (c)
For both $N=17$ and $N=25$ systems, the range of motion in the three panels are comparable. The $x$ displacements are contained approximately between $\pm 0.1$ m, $y$ displacements between $-0.05$ m and $+0.3$ m, and rotations between $\pm 2$ deg. The outer panels appear to have slightly greater ranges than the center panel. This is more than double the range from the $N=8$ system. Adding the first layer of truss sections greatly augmented the displacement/rotation workspace for the Gestalt System, but the subsequent addition seems to primarily influence the density of points within the ranges. There are heavy concentrations of solutions at the centers of the workspaces with scattering of points around them. Although not shown, reducing the actuator stroke length by half also reduces the ranges of motions by approximately half. The focus domain workspaces for Gestalt models (b) and (c) in Figures 3.26 and 3.28, respectively, hold some interesting information. In contrast to model (a), (b) and (c) are able to shift the focus of the reflected rays along the focal plane for a more distributed range of locations. In general, there are much more points in which the defocus can be maintained below $0.1$ m for shifts in the focal point to within approximately $x=\pm 1$ m. Although defocus value of $0.1$ m is not very precise, the plots speak towards the systems’ increased capabilities to move the focal point along the focal plane, and also heightened potential to correct for defocus due to thermal deformations. The points in these workspaces seem to have concentration around $x=0$ m and defocus of $0.3$ m. However, as this is representative of only a fraction of all possible states, it is reasonable to assume that there would be more viable solutions for the purposes of this investigation. Furthermore, the deficiency of data may be attributing to the appearance of lack of symmetry in the focus domain workspaces for models (b) and (c).
3.3.3 Superposition

Superposition on the three Gestalt Systems is tested in the same way as before. Figure 3.29 shows a plot of the percentage differences between the superposition and normal solutions as actuator stroke length increases, with the normal solution as the basis. Only the worst deterioration curves are shown for each of the three systems; in all cases, the rotation of panel B exhibited the greatest differences. It should be noted that while all 256 solutions were compared for the $N=8$ model, only randomly selected 4096 solutions were compared for the $N=17$ and $N=25$ systems. As several trials were run to arrive at the final values, the results shown in Figure 3.29 have reasonable legitimacy.

Before, it was learned that superposition deteriorates as the number of actuators increases or as the stroke length of the actuators increases. Here, both situations can be observed. It turns out that Gestalt model (c), with twenty-five actuators, actually experiences 10%
discrepancies between the superposition and normal solutions even for stroke length as small as 0.001 m (0.01% of the overall structure size). For stroke length equal to 0.05 m, the discrepancies exceed 20%. In this case, superposition solutions cannot be accepted.

### 3.3.4 Genetic Algorithm

Prior to investigating the performance, a discussion on an optimization algorithm must first be made. Beginning with model (b), exhaustive search for finding the optimal solution becomes unreasonable. The most practical optimization method is the genetic algorithm. Genetic algorithm is a nonlinear, stochastic optimization process inspired by topics in genetics and the progression of evolution [49, 50]. It is particularly applicable for BLISS because the inputs to the system are in the form of binary strings of ones and zeros, which can be viewed as simplified sequences of genetic code corresponding to the “fitness” of the solution. The algorithm begins with an initial population of random binary inputs selected by a roulette wheel method (as the name suggests, it is the selection of the inputs based on the principles of probability in a roulette wheel). “Fitness” of each binary input is determined by the user-defined cost function. Inputs with the best fitness are “mated” by crossing over its binary codes, resulting in offspring to form up the new population. Each new population is called a generation and the crossover repeats for every generation, rejecting the “bad genes” from its population after each process. The stochastic element in the algorithm arises from the crossover percentage, which determines the crossover rate in the population. It also arises from the mutation factor, in which a bit of a binary input is flipped in some the population. Mutation probability is typically set low, but it accounts for the unfortunate situations where the best solutions may not surface from normal evolutionary process.

Although first introduced in the 1970’s, there are many speculations to the best
parameters to use in genetic algorithms. In fact, much of the mathematical proofs for why the algorithm works remain unexplained. Some text and publications suggest that the best population size to use is between 30 and 50, and the best number of generations is between 50 and 500 [49, 50]. Larger population size or greater number of generations does not necessarily result in better solutions. Crossover percentages are typically high (between 80% and 95%), while mutation percentages are generally low (between 0.5% and 1%).

It has been observed in past studies that genetic algorithms are unsuccessful in the optimization of systems with continuous variables [22]. However, in optimizations of distinctly discrete inputs like the systems studied in this thesis, it works quite well. Genetic algorithm using the parameters defined in Table 3.1 was developed and tested in the search for best solutions in the twelve-actuator BBM and the eight-actuator Gestalt model. The resulting best solutions were the same as the ones found through exhaustive search algorithm. For more details on genetic algorithms refer to references [49] and [50].

<table>
<thead>
<tr>
<th>Table 3.1 Parameters for Genetic Algorithm Used in Performance Analysis</th>
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<tbody>
<tr>
<td>Population size</td>
</tr>
<tr>
<td>Number of generations</td>
</tr>
<tr>
<td>Crossover percentage</td>
</tr>
<tr>
<td>Mutation percentage</td>
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3.3.5 Performance

Performance of Gestalt model is measured by the system's ability to correct its focus for disturbance due to thermal deformations on its support structure. As mentioned previously, large structures in space undergo deformations of approximately 0.5% of the overall structure size. In
reality, radiation causing the deformations would affect the entire structure. However, in this study the entire deformation will be applied to just one support member as shown in Figure 3.30. This allows for much simpler analysis and precludes the necessity for comprehensive orbital thermal analysis of this model, which could be a research endeavor in its own. Thermal loading applied to the support member causes the reflected rays from each panel to deviate from the center axis on the focal plane of the system (the original focal point), causing loss of focus. It is the intent of this section to see if the Gestalt System is able to correct for this loss of focus. By observation, the greatest deviations are produced with the thermal deformation entirely represented in the support member shown in Figure 3.30. It is expected that thermal deformations distributed elsewhere along the support structure would result in smaller deviations on the focal plane. Therefore, testing for thermal deformations as represented suggests that it will be able to handle other scenarios of structural deformations due to space radiations.

\[ \text{Initial average deviation of reflected rays} = 0.210 \text{ m} \]

\[ N = 25 \]

\[ N = 17 \]

\[ \text{Initial average deviation of reflected rays} = 0.232 \text{ m} \]

Figure 3.30: Depiction of thermal loading for performance analysis
Genetic algorithm discussed in section 3.3.4 is used to search for the best binary input that would minimize the cost function. The cost function (or fitness) is defined in (3.10) as the average of the three panels’ magnitude of deviations from \(x=0\) of the reflected rays onto the focal plane. It is similar to the defocus value introduced earlier; however, this method provides more insight to the focus of each individual panel. The goal is to reduce this value to the order of millimeters, beyond which it is assumed that the secondary optics can further refine the precision.

\[
M(d) = \frac{|D_A(d)| + |D_B(d)| + |D_C(d)|}{3}
\]

From the investigation of the focus domain workspace, it was apparent that Gestalt model (a) is not capable of much manipulation in the system’s focus. Therefore, only results for Gestalt models (b) and (c) are presented in Figure 3.31. With the thermal loading applied, the \(N=17\) system initially has average deviation value of 0.210 m. With actuator stroke length of 0.05 m, the system is able to reduce this value to 0.04 m for more than 80% improvement in the cost. Stroke lengths ranging from 0.005 m to 0.075 m were tested, but resulted in worse performance. By stroke length of 0.1 m, the system could not correct for the thermal deformation to any degree.

For the Gestalt System with twenty-five actuators, the initial average deviation is at 0.232 m. Actuator stroke length of 0.05 m resulted in reducing cost, \(M\), to 0.027 m, which is an 88% improvement. More importantly, when the stroke length of 0.025 m was used, the performance improved further. With average deviation value of 0.006 m, the system successfully achieved the goal of reducing the deviation to within millimeters. In general, Gestalt model (c)
performs better than (b) for all stroke lengths of the actuator.

![Graph showing performance results for Gestalt models (b) and (c).](image)

**Figure 3.31**: Performance results for Gestalt models (b) and (c), testing capability to correct for thermal deformation in the system

The performance of Gestalt model (c) is tested further with different values of extension on the support structure due to thermal loads. Results are shown in Figure 3.32. First, 0.1% thermal deformation (which is 0.01 m extension) is investigated. The initial average of deviations is 0.048 m. At best, with 0.005 m actuator stroke length, the average deviations are reducible to 0.0039 m, or nearly 92% improvement in the focus. At the other end of the spectrum, with 1% thermal deformation (which is 0.1 m extension), the initial average of deviations of 0.210 m is reducible to 0.033 m using 0.05 m actuator stroke length. Although this is not in the order of millimeters as desired, it is an 84% improvement in the system focus. Conclusion is that actuator stroke length of approximately 0.25% of the overall structure, or half of the expected deformation, provides the best performance results in correcting focus for thermal deformation.

For thermal deformations of 0.5% or less of the overall structure size, a BLISS system is capable of correcting its focus by reducing the average deviations to within millimeters.
Furthermore, it only takes actuator stroke length that is 0.25% of the overall structure size to provide this performance. For thermal deformations of 1% of the overall structure size, the system cannot reach the same level of precision; however, it is demonstrated that it has the capability to significantly reduce the average deviation of reflected rays from x=0 for each panel. Furthermore, it can be generalized that using actuator stroke length of greater than twice the expected thermal extension prevents the system from yielding any degree of performance; any actuation on the system will exacerbate the focus.

Figure 3.32: Performance results for Gestalt model (c), testing capability to correct for 0.1% deformation (left) and 1% deformation (right) in the supporting structure

3.3.6 Modularity

The analysis performed in this two-dimensional study concerns itself with structures of approximately 10 m. This approach was taken in order to consider the simplest arrangement of segmented primary mirror (using three panels) and to consider deployable sections. However, the original focus of this thesis was on ultra-large space telescopes. Holding true to this application, the above study can be extended and viewed in the following two ways.
First, Gestalt model (c) can be scaled to have 30 m panels, 2.5 m elastic links, and 94.8 m overall size to represent ULST. The results shown thus far can also be scaled proportionally to give the same understanding of results. Section 3.3.3 showed that superposition solution has 10% discrepancies from the normal solutions, even for small actuator stroke lengths. Although this was enough error to discard superposition results, the findings from that study also demonstrate presence of some linear relationships in the system. Furthermore, the couplings producing nonlinearities in the system are generally the result of interactions with the compliant elements in the system, not the dimensions of the system. As such, it is argued that scaling the model to larger dimensions will give proportional results. Nevertheless, segmented primary mirror system with 30 m panels is not realistic, both in terms of optics and deployability; hence another view is needed.

In the second view, the structure of Gestalt model (c) can be seen as a modular section, which can be constructed to build larger structures as shown in Figure 3.33. Three sections form to create nearly 30 m in primary mirror size; ten sections will create nearly 100 m in primary mirror size, which is on the order of ultra-large space telescopes. Unfortunately, as the concept of superposition fails for Gestalt model (c), the coupling effects between sections would need to be thoroughly investigated. A ten-section Gestalt System would require 250 binary actuators. By the trends observed in superposition studies, it can be predicted that superposition solutions will have extreme errors in such system. Independently, however, the modular sections will be able to correct for their own thermal deformations, as demonstrated in the studies above. Therefore, if the modular sections can be decoupled, or its interactions minimized, it can be said with certainty that the complete structure would be able to adapt to the thermal loads applied to the space structure.
Both views extend the knowledge gained through this study from a 10 m system to that of ultra-large space telescopes. Modular approach gives the most practical application, and although it has its limitations, the analysis shows that it is possible to correct for thermal deformations using a system of binary actuators.

3.4 Summary

In summarizing the two-dimensional study, several key factors can be mentioned. The workspace in displacements/rotations domain is affected by the stroke length of actuators and the stiffness values of the elastic elements. The stroke proportionally modifies the range, and the stiffness values modify the distribution within the range. Destruction of symmetry in the system by varying stiffness values reduces the likelihood of overlapping solutions, and in general reduces clustering of the solutions. Using elastic links instead of free joints between panels results in increased range of motion for each panel, as well as providing more evenly distributed points in the workspace. From the focus domain workspace, the ability to shift the location of the focal point and the ability to maintain the degree of that focus can be observed. Eight-actuator system showed that it was not capable of much manipulation in the focus of the system, while
seventeen and twenty-five actuator systems showed more viable solutions.

A significant discovery is that superposition deteriorates with increasing actuator stroke length and with increasing number of actuators. Stroke length of 0.5% of the overall structure considerably reduces the validity of superposition. Also, as the number of actuators reaches twenty-five in a system of triangular truss configurations, superposition solutions become practically unusable.

Most importantly, it was found that a system with twenty-five actuators embedded in triangular truss organization is capable of correcting for thermal deformations applied to one of its support structure members. The deviation (due to the thermal load) of the reflected rays from the original focal point is reducible to the order of millimeters, beyond which secondary optics can further refine the focus. This precision is attainable using actuator stroke length that is half of the expected thermal deformation. In the cases studied, the expected thermal deformation was 0.5% of the overall structure size, or 0.05 m. This means that actuator stroke length of 0.025 m provides the best performance. Determining the best input that provides this performance required an optimization process using genetic algorithm.

Lastly, the concept of modularity was discussed in order to connect the findings of this thesis to a system on the order of 100 m. While the analysis conducted in this chapter is limited and the applied thermal conditions are the most basic, the studies show that it is feasible to use binary actuators to control the primary mirror structure in ultra-large space telescopes.
4. Conclusion

4.1 Summary

Research conducted in this thesis explored the mechanics of the BLISS concept. First, one-dimensional study investigated the binary beam model, using binary linear actuators in uniform distribution and perpendicular placements along an elastic beam. Analysis and experimental results served to provide basic understandings of a binary actuated compliant system, and also validated the construction of finite element models in ADINA. The research continued with two-dimensional studies, in which the actuators acted in both x and y components. Several three panel models of approximately 10 m primary mirror size were investigated. The first two investigated the use of free joints and elastic links as connectors between the panels. Following results, the model using elastic links was systematically expanded with layers of triangular truss sections. With the actuators embedded into the support members, the resulting structures were named Gestalt models. Three types of Gestalt models, having eight, seventeen, and twenty-five actuators, were studied. The ability to change the focus was explored for each model, and ultimately the ability to correct for thermal deformations to the system was tested for seventeen and twenty-five actuator systems. This ability was translated into a performance metric, from which it was determined that the primary objective of the thesis was met. In general, the simplest cases and designs were considered in this investigation for feasibility of BLISS. As such, there is significant room for improvements and in-depth analysis. Nevertheless, studies shown
here have value in that it offers numerical objectivity and foundations for future undertakings.

Some of the basic findings include the following. The range of the workspace in displacements and rotations is directly proportional to the actuator stroke length, while the distribution within the workspace is modified by the stiffness values of the elastic elements. Using elastic links instead of free joints between panels increases the range of motion for each panel, and aids in removing clustering of points.

One of the primary finds is the deterioration of superposition with increasing actuator stroke lengths and increasing number of actuators. Stroke length equal to approximately 0.5% of the structure size is at the limits of acceptable superposition solutions, and by twenty-five actuators superposition deteriorates to greater than 10% in error for even stroke length as small as 0.01% of the overall structure size. For the purposes of BLISS, where large number of actuators is key, superposition does not hold.

The main objective of this thesis was to determine feasibility for BLISS by studying the system’s ability to correct for thermal deformations induces by space conditions. For simplicity, the thermal deformation was generalized and entirely represented in one member of the support structure, rather than the support structure as a whole. The resulting loss of focus was quantified by averaging the magnitude of deviations along the focal plane from the original focal point of the reflected rays from the centers of each panel. It turns out that twenty-five actuators with actuators providing stroke length half of the expected thermal extensions reduces this average of deviations to a value on the order of millimeters. Beyond this level of precision, the secondary optics can be expected to further enhance the focus of the system. The results allude to the relationship that for such precision, the number of actuators required in a triangular truss configuration is approximately equal to three times the number of degrees-of-freedom in the system. For the final model considered in this study, the three panels having a total of nine
degrees-of-freedom (rotation and \( x \) and \( y \) translation freedoms for each panel) needed twenty-five actuators. In general, it was demonstrated that BLISS of the configuration studied is capable of reducing the effects of thermal deformations to the system. Finally, applicability and effectiveness of using genetic algorithm was discussed for finding the optimal input for correcting for the thermal deformations.

The final section of this study closed with an argument on modularity, relating the 10 m systems studied in this thesis with ultra-large space telescopes. The 10 m sections can be assembled to create a 100 m system. The resulting hundreds of actuators in the structure will cause significant deterioration of superposition, prompting the address of coupling effects between the modular sections. However, for this preliminary investigation, an assumption can be imposed that the space telescope is constructed with minimal influence of one section to the other. Because independent modular sections are able to give adequate performance, it can then be said that the 100 m system will also be able to provide adequate corrections to thermal deformations.

Ultimately, this thesis finds that it is feasible to use an arrangement of many binary actuators to control the primary mirror structure for large space telescopes in correcting for the effects of deformations caused by radiations experienced in orbit.

4.2 Future Work

As this is one of the initial studies in binary actuated large space imaging systems, there are considerable amounts of potential for future studies. The most critical topics are listed in the following. First, as this thesis does not include a three-dimensional analysis, verification of the findings for a three dimensional system should be in order. Experimental systems could also be
built for improved confidence in the conclusions and physical demonstrations. Furthermore, more comprehensive structural deformations due to orbital radiations should be applied for future studies to enforce the conclusions established for feasibility of BLISS. In addition, configurations other than triangular truss structures need to be investigated in order to explore possibilities for better design. Certainly, as systems investigated in this thesis did not involve an intelligent design process, there must be an optimal placement of binary actuators that can produce better performances. Essentially this calls for studies in design optimization. Lastly, the concept of modular construction should be investigated further so that system feasibility is maintained even in the presence of coupling between sections.

For more theoretical advancements, Monte Carlo methods can be explored in order to simplify workspace generation for systems with high number of actuators. In another area, stiffness values of the elastic elements proved to have impact in the distribution of points in the workspace. This was noticed in other studies involving binary actuation. In order to construct user defined workspace (mostly, the desired workspace would have uniform distribution within its range), these stiffness values can be obtained using an optimization algorithm. Such process could resemble that studied in [22] regarding design for serpentine robotic system.

In order to research advanced BLISS systems, the feasibility of incorporating BRAID sections into the support structure of the space telescope can be considered as part of establishing optimal design of BLISS systems [17, 18, 23]. Using BRAID sections has the potential for more robust control of the support structure, further enhancing the level of precision.

As mentioned in the introduction, the application of binary actuators for control of large structures extends beyond ultra-large space telescopes. Research in systems for solar energy and orbital power stations can be revisited with binary actuators, which offer benefits in cost, weight reduction, and technical simplicity. There is great potential for projects like BLISS and systems
using binary actuators. As demonstrated in this thesis, control of large structures using binary actuators coupled with elastic elements is quite feasible and offers advantages sought-after in many areas of technical developments. Therefore, continued research in this area and similar paths has significant merit and promise.
References


Appendix A:
ADINA Interface with MATLAB

ADINA stands for Automatic Dynamic Incremental Nonlinear Analysis. It is a finite element analysis software for structures, fluids, and heat transfer, initially developed in the 1980's by Dr. K. J. Bathe of MIT Department of Mechanical Engineering [38].

In utilizing ADINA for analysis of systems investigated in this thesis, an interface with MATLAB is necessary. This is because ADINA can only compute one solution at a time. For changing binary inputs, a new model must be manually constructed and processed. As the number of inputs increase exponentially with the number of actuators, an aid is required to automatically generate and run new models for each input. ADINA contains an executable file called ‘adinarun.exe’ that allows batch processing. This is a program that is accessible by a command prompt outside the ADINA software, and allows for the interface to be constructed.

Information flow in ADINA can be represented by Figure A.1. The program takes inputs from either a graphically generated model (via AUI: ADINA User Interface) with file extension *.idb, or a script with file extension *.in. The latter is a text file with the model depicted in a coding language unique to ADINA. Completed file of this type can also be interpreted by AUI and displayed graphically. Either input file is converted into a data file (extension *.dat) by ‘adinarun.exe,’ which is processed by the core software of ADINA. Processed information is output in two types of files with extensions *.por and *.out. Extension *.por files, called portholes, are viewable only by AUI in graphical representations. It allows for user-friendly examination of results, but the results are not extractable. Extension *.out files are text files which lists the user-selected results along with information relevant to ADINA calculations.
Figure A.1: ADINA information flow

The interface process can be divided into three components written in MATLAB scripts: writing, running, and retrieving.

- **Writing** involves generating the text file that will be read by ADINA. Each line of code for the *.in file can be written by MATLAB using the function ‘fprintf.’ The syntax is described by ADINA System Manuals [38]. This component also involves calculation of all geometric points and material parameters to be used for the construction of the model. By generating lines of coding with parameters reflecting the changes in the binary input to the system, the creation of the input file is automated.

- **Running** is the simplest of the three components. A single line of code using MATLAB function ‘system’ allows an outside program to be called by MATLAB. The simulation is executed using this feature.

- **Retrieving** involves extracting the relevant information from the *.out file. Although output information can be selected in the input file (for instance, a line of code can instruct ADINA to only record displacements for certain nodes as the results), there are a lot of useless text in the *.out file that pertains to the simulation parameters. These need to be discarded. Using MATLAB function ‘fscanf’ converts the text in the *.out file into a data of strings. Using function ‘strrep’ strips the string of the
unnecessary information, and 'sscanf' converts the remaining numbers into a matrix of doubles that can be used in MATLAB for further analysis.

The three components allow automatic generation of the finite element model and execution of the simulation. The functions readily available in MATLAB make it easy for the construction of this interface with ADINA. Refer to ADINA System Manuals under batch processing for more details.
Appendix B:
ADINA Finite Element Models for Gestalt Systems

This appendix describes the construction of finite element models for Gestalt models (b) and (c) in ADINA, but are applicable for construction of other systems used in this thesis.

ADINA includes many different finite element models that can be used to describe different components in the system. Known as element groups, the most commonly used of these are trusses, beams, 2-D solids, 3-D solids, and shells. In order to circumvent issues involving singularities in the stiffness matrix, the Gestalt models are constructed using only beam elements. A graphical depiction and porthole results in AUI are shown in Figure B.1 for Gestalt model (b). Figure B.2 shows the same for Gestalt model (c), without any annotations. There are five different components of beam elements used in the model.

- **Rigid panels:** The panels that represent the segmented mirrors of the primary mirror are modeled as rigid beam elements. Using methods used in ADINA User Interface Primer (a collection of example problems for ADINA), the Young’s Modulus of these beam elements are set artificially high in order to exhibit rigid effects [51].

- **Elastic links:** The panels are connected by elastic links. They are modeled as beam elements with Aluminum properties \((E=75 \text{ GPa})\), and hollow cross section of 2 cm x 2 cm and wall thickness of 2 mm. As beam elements adequately exhibit all elastic effects, this is appropriate.

- **Elastic elements:** The elastic elements are modeled as described in Chapter 2. Its material properties are based on the desired stiffness values of each elastic element.

- **Actuators:** The actuators are in series with the elastic elements. They produce linear
extensions by the use of temperature loads as described in Chapter 2. The beam elements representing the actuators have thermal coefficient values that reflect the desired extension. Its Young’s Modulus is also set artificially high in order to ensure that the entire length of the stroke is applied to the system.

- **Supports:** All other components are modeled as supports. Supports are beam elements with artificially high Young’s Modulus that simply connect the elastic elements and the actuators to the rest of the system. These supports connect to form joints in the system that are free to rotate. This includes the joints at the panels. By property of the beam elements in ADINA, when two beam elements are connected, they act as a single beam; the connection is not seen as a free-joint. In order to allow free rotation between the two pieces, end-releases must be applied. This process releases a specific connection between the two beam elements, preventing transference of moment. Without any reaction moments, the two beam elements act as if it was joined by a pin.
N=17

(i: Gestalt Model (b))

Beam elements for panels
- Artificially high Young's Modulus
- Symmetric cross-section

Beam elements for elastic components
- Young's Modulus given by equation in (2.7)
- Cross-section selected to provide desired stiffness

Beam elements for actuator
- Artificially high Young's Modulus
- Symmetric cross-section
- Actuation by extensions due to temperature loading

Boundary condition definition: Fixed translation and free rotation on grounds, applied on locations labeled by B

(ii: ADINA Model in AUI)

Beam elements for elastic links
- Young's Modulus equivalent to Aluminum (75 GPa)
- Cross-section 2 cm x 2 cm and 2 mm wall thickness

Beam elements for actuators
- Boundary condition definition: Fixed translation and free rotation on grounds, applied on locations labeled by B

(iii: ADINA Results in porthole shown in AUI)

All 17 actuators are engaged for 0.05 m extension

Figure B.1: Finite Element Analysis model and results shown in graphical representation using AUI for Gestalt model (b)
Figure B.2: Finite Element Analysis model and results shown in graphical representation using AUI for Gestalt model (c)