ELECTRO-MAGNETIC EFFECTS IN DETONATION WAVES

BY

EDELBERT PLETT

B.A.SC., University of British Columbia (1962)

Submitted in partial fulfillment of the requirements for the degree of Master of Science at the Massachusetts Institute of Technology January, 1964

Signature of Author

Department of Mechanical Engineering January 20, 1964

Certified by

Thesis Supervisor

Accepted by

Chairman, Departmental Committee on Graduate Students
ELECTRO MAGNETIC EFFECTS IN DETONATION WAVES

By
Edelbert Plett
B.A.Sc., University of British Columbia, 1962

Submitted to the Department of Mechanical Engineering on Jan. 20, 1964, in partial fulfillment of the requirements for the Degree of Master of Science.

ABSTRACT.
A theoretical investigation of the effect of an electro-magnetic field on the propagation speed of a detonation wave is presented. The purpose of the investigation was to determine how the speed of propagation of a detonation wave might be changed by an electro-magnetic field, if the wave had attained a steady velocity before entering the field. The first approach was to examine what effect might be expected on the basis of the changes taking place between the wave front and the Chapman-Jouguet point. The results indicate that slight changes in the wave speed could be realized, and the magnitude of these changes depends on the size of fields applied as well as on the electrical conductivity of the gases within the wave. The second part of the analysis examined the effect of the electro-magnetic fields on the product gases. The object was to determine whether the classic Chapman-Jouguet point would remain unperturbed, or whether the fields could affect the product gases enough to allow disturbances to overtake the wave front. A numerical analysis was begun for several values of the fields, and for the cases examined, it appeared certain that the disturbances would eventually overtake the wave front.

The electrical conductivity of the gases in the detonation wave were briefly examined theoretically, based on simplifying assumptions. The exploratory aspects of an experimental program to measure the electrical conductivity within the detonation waves was reported.

Thesis Supervisor: Tau-Yi Toong
Professor of Mechanical Engineering
ACKNOWLEDGEMENTS.

The author wishes to express his appreciation to his thesis supervisor, Professor T.Y. Toong for the encouragement as well as the patient and critical appraisals with which he guided this work.

Words of appreciation go to the other members of the Gas Dynamics, and Magneto-Gas Dynamic Research Groups for the many valuable discussions which led to the shaping of this work. Particular words of thanks go to; Jon Kelly for sharing of his experience in shock tube work, to Roger Lucheta for his ever ready advice, to John McVey and Tom Blake for the many discussions of theory pertaining to this work and to Paul Wassmout and his staff in the shop for their co-operation in making and repairing parts for experimental apparatus.

Chord Ohnbush deserves a word of thanks for his enthusiastic help in assembling the electrical circuitry and bringing it to operational stages.

Many thanks to Joe Palermo of Lincoln Lab for applying his skills to producing a metallic film in the glass shock tube.

And a sincere thank-you to Miss Irene Stewart for so kindly giving of her time to the typing of this manuscript.

This work was supported by the U.S. Army Research Office (Durham) AROD Project No. 3149-E, Grant DA-ARO (D) 31-124-G190, and National Science Foundation Grant G-20880.
<table>
<thead>
<tr>
<th>CONTENTS</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chap. 2 The Effect of an Electro-Magnetic Field on the Product Gases of a Detonation Wave</td>
<td>23.</td>
</tr>
<tr>
<td>Chap. 3 Electrical Conductivity Theoretical Considerations</td>
<td>40.</td>
</tr>
<tr>
<td>Chap. 4 The Induced Magnetic Field</td>
<td>50</td>
</tr>
<tr>
<td>Chap. 5 Electrical Conductivity - Experimental Investigation</td>
<td>57</td>
</tr>
<tr>
<td>Appendix A. Graphical Relation Between the Induced Magnetic Field Parameter $g$, and the Change in the Wave Speed.</td>
<td></td>
</tr>
<tr>
<td>Appendix B. X-t Diagrams For Detonation In Electro-Magnetic Field.</td>
<td></td>
</tr>
<tr>
<td>&quot; C-1</td>
<td></td>
</tr>
<tr>
<td>&quot; C-2</td>
<td></td>
</tr>
<tr>
<td>&quot; C-3</td>
<td></td>
</tr>
<tr>
<td>&quot; C-4</td>
<td></td>
</tr>
<tr>
<td>&quot; C-5 Conductivity Profiles Corresponding to Exact Solution of Detonation Wave Structure.</td>
<td></td>
</tr>
<tr>
<td>&quot; C-6 Conductivity Profiles Corresponding to the Zeroth Order Solution of Detonation Wave Structure.</td>
<td></td>
</tr>
<tr>
<td>&quot; D-1 Conductivity Probe Circuitry</td>
<td></td>
</tr>
<tr>
<td>&quot; D-2 &quot; Assembly</td>
<td></td>
</tr>
<tr>
<td>&quot; D-3 Probe Calibration Curve</td>
<td></td>
</tr>
<tr>
<td>&quot; D-4 Log-Log Plot of Voltage VS. Conductivity</td>
<td></td>
</tr>
<tr>
<td>&quot; D-5 Weighting Function</td>
<td></td>
</tr>
<tr>
<td>&quot; D-6 Output Expected on Basis of Current Filament Hypothesis</td>
<td></td>
</tr>
<tr>
<td>&quot; D-7 Comparison of Output obtained with Output Predicted for Unshielded Tube.</td>
<td></td>
</tr>
<tr>
<td>&quot; D-8 Comparison of Output obtained with Output Predicted for Shielded Tube</td>
<td></td>
</tr>
<tr>
<td>&quot; D-9 Some Qualitative Results obtained in the Shock Tube.</td>
<td></td>
</tr>
</tbody>
</table>
Chapter I.

THE EFFECT OF AN ELECTRO-MAGNETIC FIELD ON THE PROPAGATION SPEED OF A DETONATION WAVE.

A QUASI-STeady ANALYSIS

Combustion, in general, involves simultaneous occurrence of chemical reaction and transfers of heat, mass and momentum. In some cases, it is possible to successfully analyze a combustion wave by considering only the transfers of heat, mass, and momentum, allowing the chemical reaction to serve as a heat source, but giving no consideration to the chemical kinetics. This approach has been extremely successful in predicting wave speeds of detonation waves, under ordinary conditions, with no electric or magnetic fields present. With this in mind, it is assumed that a simple extension of the ordinary governing equations of mass, heat and momentum transfer to include the influence of the electro-magnetic fields will provide the necessary information regarding the effect of these fields on the combustion process; in particular here, a detonation wave.

The fundamental electro-magnetic relations, Maxwell's equations "in vacuo", for a non-relativistic system will be introduced:

\[ \nabla \cdot \vec{E} = \rho^e / \varepsilon_0 \]  \hspace{1cm} (1.1)

\[ \nabla \cdot \vec{B} = 0 \]  \hspace{1cm} (1.2)

\[ \nabla \times \vec{B} = \mu_0 \vec{J} \]  \hspace{1cm} (1.3)

\[ \nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \]  \hspace{1cm} (1.4)
The first two are Gauss' law for electricity and magnetism. In the usual mhd approximation $\rho = 0$, due to assumed charge neutrality. Eqn. (1.3) describes the magnetic effect of a current, and (1.4) gives the electrical effect of a changing magnetic field.

Finally, the generalized Ohm's law may be written as

$$\mathbf{j} = \nabla \mathbf{E}^+$$  \hspace{1cm} (1.5)

where $\mathbf{E}^+ = \mathbf{E} + \nabla \times \mathbf{B}$  \hspace{1cm} (1.6)

and $\mathbf{E}^+$ is the total or effective electric field experienced by the fluid if $\mathbf{E}$ is the electric field experienced by an observer who sees the fluid move with a velocity $\mathbf{V}$ in the magnetic field $\mathbf{B}$.

The gas dynamic equations governing the fluid motion will take the following form. The continuity equation remains unchanged since the electro magnetic field does not introduce matter into the system.

$$\frac{D\rho}{Dt} + \mathbf{e} \nabla \cdot \mathbf{V} = 0$$  \hspace{1cm} (1.7)

The conservation of momentum, neglecting viscosity, is expressed by

$$\rho \frac{D\mathbf{V}}{Dt} = -\nabla \mathbf{p} + \mathbf{j} \times \mathbf{B}$$  \hspace{1cm} (1.8)

where $\mathbf{j} \times \mathbf{B}$ is the body force term resulting from the interaction of the currents in the gas with the magnetic field.

Conservation of energy is conveniently written as an enthalpy relation, neglecting again the viscous dissipation term.

$$\rho \frac{DH^*}{Dt} = \mathbf{E} \cdot \mathbf{j} - \nabla \cdot Q + \frac{\partial P}{\partial T} + mg$$  \hspace{1cm} (1.9)

where $H^* = c_p T + \frac{V^2}{2}$ is the stagnation enthalpy

$\mathbf{E} \cdot \mathbf{j}$ is the electrical energy addition

$Q = \kappa \nabla T$ is the heat conduction term.

The equation of state for a perfect gas will be used throughout.

$$p = \rho RT$$  \hspace{1cm} (1.10)
It will further be assumed that the specific heats at constant pressure, and constant volume are constant and their ratio $k$ is, constant, and the same for combustible, and products mixture alike.

Theoretical Model

The model adopted for the study of the effect on a steady state detonation wave when suddenly acted upon by an electro-magnetic field could conceivably be varied in nature. Two strong possibilities suggest themselves however.

If the change of velocity in time is relatively small with respect to its original velocity, then a quasi-steady analysis, neglecting time derivatives could be performed, and evaluated at various instants after the wave has entered the electro-magnetic field. If this type of analysis showed however that changes were occurring too fast to justify neglecting the transient effects, then a nonsteady analysis would be a better approach.

A quasi-steady analysis was used in ref. 2, to predict the effect of a magnetic field only, on the wave speed. This analysis will now be extended to include electric fields, and then examined from a nonsteady viewpoint to determine at least qualitatively, whether the quasi-steady model is adequate for predicting the effect on wave speed.
The analysis will be performed for the one dimensional problem of a fully developed detonation wave in a constant area channel. The region of interest begins where the wave enters a crossed transverse electric and magnetic field.

The problem will be analyzed for the case of a detonation travelling to the left in the laboratory frame of reference, so that an observer on the wave sees the conditions as in Fig.1. There is an applied magnetic field of intensity \( B_0 \) in the positive \( Z \) direction. All velocities are in the \( X \) direction and are positive when the vector direction is to the right. The electrodes are parallel to the \( X-Z \) plane, and since they will be considered to be perfect conductors, no electric field can exist in any direction on their surface, so the only applied electric field possible is in the \( y \) direction, and since \( \nabla \cdot \mathbf{E} = 0 \), it will be independent of \( y \), and considered positive when directed in the positive \( y \) direction. Before solving for the effect the electric and magnetic fields have on the detonation wave, or even specifying the boundary conditions to the problem, it will be of interest to determine in what manner the fields are altered by the flow of ionized gases.
How the Flow Affects the Field

To determine whether the boundary conditions being applied to a problem are qualitatively correct, regarding the behavior of the magnetic field, an analysis of how the flow affects the field can be very useful. Chu (ref. 1.) has formulated a method of determining the behavior of fields in the presence of moving matter. In his formulation he uses a cause and effect type of reasoning which allows him to use a method of successive approximations to determine the final field values. The final field value will simply be the sum of the successive orders of approximation used. (For a detailed development, see ref. 1, Chapter 6 and 9.) Without presenting his development in detail, the technique will be applied to the problem of a pressure disturbance, with accompanying velocity disturbance which is propagating into a region of zero velocity, in the laboratory reference frame, in the presence of a transverse magnetic field. The same configuration will be used as is used in Fig. 1, for the detonation wave, but fields will be analyzed in the laboratory co-ordinate system.

Following the derivation in ref 1, Chapter 9, the fields will be analyzed in the laboratory co-ordinate system, so that derivatives with respect to time do exist.

Zeroth Order

The zero-order fields are those which do not depend on time derivatives and are generally uncoupled from the fluid dynamic relations. For the case considered here, a zero order electric field is one applied to the problem, and by Maxwell's relation,
\[ \text{curl } \mathbf{E}_o = 0 \]  \hspace{1cm} (0.11)

From Ohm's law then,
\[ \mathbf{j}_o = \nabla \mathbf{E}_o \]  \hspace{1cm} (1.12)

which gives zero order currents due to the zero order electric field. Then the zero-order magnetic field is given by
\[ \text{curl } \mathbf{B}_o = \mu_0 \mathbf{j}_o \]  \hspace{1cm} (1.13)

Thus the zero-order magnetic field can consist of an irrotational component \( \mathbf{A} \), plus a contribution due to the zero order currents. In the case when \( \mathbf{E}_o \) is zero, it becomes apparent that \( \mathbf{B}_o \) is everywhere uniform.

**First Order**

![Diagram of wave in the electro-magnetic field as seen by a stationary observer in the laboratory.](image)

The first order equations are as follows:
\[ \text{curl } \mathbf{B}_i = \mu_0 \mathbf{j}_i \]  \hspace{1cm} (1.14)
\[ \mathbf{j}_i = \nabla (\mathbf{E}_i + \mathbf{u} \mathbf{B}_o) \]  \hspace{1cm} (1.15)

where \( \mathbf{u} \) is the local gas velocity, and
\[ \text{curl } \mathbf{E}_i = -\frac{\partial \mathbf{B}_o}{\partial t} = \nabla \cdot \frac{\partial \mathbf{B}_o}{\partial x} \]  \hspace{1cm} (1.16)

In order to fully determine \( \mathbf{E}_i \), the external circuit must be specified. The \( \nabla \) appears in eqn. 1.16, since it is assumed that \( \nabla = 0 \) upstream of the wave, so even though there is a zero order electric field, currents will flow only after passage of the wave.
Since the B field begins to change at the passage of the wave front, the curl in the electric field will be given by eqn. 1.16, in spite of the fact that the actual particle velocity is different from $V_a$.

The fact that the particle velocity is different from $V_a$ is brought into eqn. 1.15, so that this is manifest in the first order magnetic field, which in turn affects the second order electric field.

The second and subsequent orders of approximation are arrived at just as the first order with 1. being replaced by n, and o by n-l for the $n^{th}$ order. The final field values are comprised of the sum of the individual orders.

Since further general discussion of eqns. 1.11 to 1.16 could go on to great length, it seems advantageous to consider specific examples instead. First to be considered will be an infinitesimal pressure pulse propagating through a magnetic field. Then Chapter 4 will consider the application to a detonation wave, using the electrical conductivity results of Chapter 3.

**Small Pressure Pulse**

Since the analysis of a sound wave will be of interest in examining boundary conditions at a later point in this chapter, it will be useful to apply the foregoing analysis to a small pressure pulse, propagating at sonic velocity into a stationary zone. There will be no applied electric field in the laboratory system, relative to which the gas is initially stationary. The gas will be assumed to have a uniform electrical conductivity $\sigma_0$, of some magnitude which can be achieved in a laboratory. (i.e. not infinite.)
Before writing the equations in the steady state system, i.e. (b) of fig. 1-3, some observations can be made in the non-steady system.

First, \(E_0=0\), so \(\mathbf{j}_e=0\) and \(\text{curl} \, B_0=0\), so \(B_0\) is constant everywhere.

Second, since \(E_0=0\), and the gas is stationary before the wave arrives, there will be no electric field induced, so there can be no currents upstream of the wave. Since electric current is independent of co-ordinate systems, there will be no currents upstream of the wave in the steady state system either, so the magnetic field remains unperturbed until the wave arrives.

Since the electrodes are assumed to be perfect conductors, all electric fields are in the \(y\) direction. All velocities are in the \(x\) direction, and so it follows, as in ref 1, that the induced magnetic field will be in the \(z\) direction.

Now assuming a position on the wave front, so that everything appears to be at a steady state. The gas now appears to be approaching the wave at a velocity \(c\), and leaving at \(c+\delta v\). The generalized Ohm's law then becomes

\[
\mathbf{j} = \nabla (\mathbf{E} + \nabla \times \mathbf{B})
\]
and since there is no electric field in the system where \( v=0 \) upstream of the wave, \( \mathbf{j} \) is zero here. To the observer on the wave this gas is moving at a velocity \( \mathbf{a} \), so that
\[
\mathbf{j}_{\text{upstream}} = \nabla \cdot (\mathbf{E}_1 - c \mathbf{B}_0) = 0
\]
or \( \mathbf{E}_1 = c \mathbf{B}_0 \).

Now since this electric field is measured in the steady state co-ordinates, derivatives with respect to time are zero, so \( \nabla \times \mathbf{E}_1 = 0 \) and \( \mathbf{E}_1 = c \mathbf{B}_0 \) everywhere. Downstream the current will be given by
\[
\mathbf{j}_1 = \nabla \cdot (c \mathbf{B}_0 - (c - \delta v) \mathbf{B}_0) = \nabla \cdot \mathbf{B}_0 \delta v
\]
Now evaluating the first order magnetic field,
\[
\frac{dB}{dx} = -\mu \mathbf{j}_1 - \mu \nabla \times \mathbf{B}_0 \delta v
\]
Since each term on the right side is positive as used, it follows that the magnetic field will have a negative gradient. Since the induced field only begins at the wave front, the total field will be a constant value \( \mathbf{B}_0 \) until the wave arrives, and then slowly decrease through the wave. This is shown schematically in Fig. 1-4.

![Fig. 1-4 Schematic showing changing magnetic field due to the currents induced in the wave.](image)

The first order magnetic field is determined from eqn. (1.18). This will in turn give rise to a second order electric field, and so on, but for moderately low conductivities, even the first order effect is small, so the second order effect is usually negligible.
An appreciation for orders of magnitude will be obtained from a specific example considered in Chapter 4 of this report.

**Boundary Conditions**

The solution of any problem in fluid flow is usually strongly dependent on the boundary conditions applied. The problem under consideration here is no exception. Much of the development thus far will be used to justify the boundary conditions to be applied.

The upstream boundary conditions can be specified almost at will, with some qualifications. The usual conditions of no physical changes upstream of the wave will be applied here.

This implies

\[ \frac{\partial T}{\partial x} |_{x} = 0, \quad \frac{\partial Y}{\partial x} |_{x} = 0, \quad \frac{\partial \phi}{\partial x} |_{x} = 0, \quad \frac{\partial p}{\partial x} |_{x} = 0 \]

Since in all practical cases, the upstream gas is quite cool, we can safely say that the electrical conductivity will be zero in this region, hence disallowing any current flow in the gas upstream of the wave. Since \( J = 0 \) upstream, from Ampere's law \[ \frac{\partial B}{\partial x} = 0 \] upstream.

The hot boundary conditions are less obvious. The usual classical approach requires that all derivatives again go to zero on the downstream side. While this condition may be realized to a good approximation in the ordinary gas dynamic-chemically reacting system, it is less likely for the case of interest here where electric and magnetic fields are included. The reason is the following; Since \[ \frac{\partial B}{\partial x} = \mu_0 J \] we require \( J \) to go to zero to have no gradient in the magnetic field.

The current as given by Ohm's law is, (6)

\[ J = -\nabla (E + \nabla \times B) \]
We cannot justifiably let $\nabla = 0$ at this boundary, since this is the hottest part of the wave, and if $\nabla = 0$ here, it could hardly have any significant value elsewhere. The other alternative is for $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$. This is the restriction most commonly placed on the problem. This, however, does not give the usual physical condition in a wave. From the examination of a pressure pulse in a magnetic field, it was found that $\mathbf{E} = \text{constant} = c \mathbf{B}$. More generally, for a similar wave propagating at velocity $\mathbf{v}$, it follows that $\mathbf{E} = c \mathbf{B}$. Then for the configuration used in Fig. 1-3b, it follows that

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = c \mathbf{B} - u \mathbf{B} \quad (1.19)$$

It is well known that for this system, the downstream velocity, $\mathbf{V}_d$, is less than $\mathbf{V}_a$ for any compression wave. It was also shown, see Fig. 1-4, that $\mathbf{B}$ decreases through the wave in this type of wave. Therefore since both the velocity and magnetic field strength decreases through the wave, their product cannot be the same as it was upstream. Thus for the case of no applied electric field in the laboratory system i.e. that system where upstream velocities are zero, it is impossible to realize the condition that $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$ at the downstream boundary. The condition of $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$ can be attained by applying a small electric field, but then $\mathbf{E} \neq c \mathbf{B}$, so the usual "frozen flux" relationship is still not applicable, although there are now no gradients of fields at the hot boundary.

The question then arises, is it necessary to have all derivatives of the physical quantities go to zero in order to have a steady state wave speed? The answer, supplied by Zeldovich (ref 7, p.133) is that this condition is not necessary.
In reference to the Chapman-Jonquet condition, he points out that if no losses are taken into account, the point of tangency at which the velocity of the substance is equal to the velocity of sound at a given point, was attained at the instant that the reaction went to completion, i.e. at the instant when no more heat was being liberated. With thermal losses taken into account this point is attained not at the instant when the reaction has been completed, but when it has been attenuated to the extent that the heat liberated is equal to the heat dissipation rate. Similarly here, it seems justifiable to assume that the C-J point will determine the hot boundary, and again, this point will be marked by an energy balance between energy added by the chemical reaction plus electrical energy, and the dissipative losses. It remains, however, to demonstrate which sound speed will most likely be the governing one.

**Speed of Sound at the C - J. Point**

An order of magnitude analysis will now be used to determine whether the presence of the transverse magnetic field will affect the sound speed in the C - J plane.

\[
\frac{B_c}{c} \left( \frac{c+dv}{\rho+dp}, \frac{c+dv}{\rho+dp} \right)
\]

**Continuity:**
\[
\rho c = (\rho + dp)(c + dv)
\]
\[
= \rho c + \rho dv + cd \rho + dp dv
\]
or.
\[
\rho dv + cd \rho = 0 \quad (1.20)
\]

**Momentum:** (assuming a steady state wave speed and taking only the first order effect of the magnetic field)
\[
\rho c \frac{dv}{dx} = -\frac{\partial P}{\partial x} + J \times B \quad (1.21)
\]

Integrating across the wave
\[ \rho c \, dv + dp = (\mathbf{j} \times \mathbf{B}) \Delta x \quad (1.22) \]

Using the results of the preceding analysis eqn 1.17 we obtain

\[ \rho c \, dv + dp = - \nabla \cdot B \cdot \nabla \cdot d \nu \Delta x \]

Combining with (1.20)

\[ -c^2 \, dp + dp = - \nabla \cdot B \cdot c \, dp \Delta x \]

Or \[ c^2 = \frac{dp}{\rho} + \nabla \cdot B \cdot \frac{c}{\epsilon} \Delta x \quad (1.23) \]

It will be recalled that this derivation was for small conductivity only, so only a first order approximation was made. For infinite conductivity, the series would not converge, and an alternate derivation would be required.

To determine the effect of the magnetic field on the wave speed, an order of magnitude argument seems appropriate. The following values will be used in eqn. 1.23.

1.) Since it is possible to show that a shock wave occurs in the distance of the order of a mean free path, (ref. 6 Chapter 5) it seems that a sound wave would be no wider than a few mean free paths. The width of the wave will be taken to be \( \Delta x \sim 10^{-6} \text{ cm}. \)

2.) The electrical conductivity in the hot gases of a detonation wave could assume a value of

\[ \nabla = 1 \text{ mho/cm} = \frac{10^{-2}}{\mu_0} \text{ sec/cm}^2. \]

3.) The magnetic pressure, \( \frac{B^2}{2\mu_0} \) would characteristically be of the order of magnitude of the gas pressure \( P \).

4.) For a perfect gas model, as will be used here, \( k \frac{\rho}{P} = \frac{(dp)}{dp} = a^2 \), the square of the ordinary sound speed.

Substituting these values into eqn. 1.23 yields, for \( c \) and \( a \) in cm/sec.

\[ c^2 = a^2 \left( 1 + 2 \times 10^{-7} \frac{c}{k} \right) \quad (1.24) \]
The solution is of quadratic form, and by inspection, it can be seen that the second term is small. So putting $c \sim a$ inside the bracket, and using a typical value of $a$ for detonation products,

$$a \sim 3000 \text{ ft/sec} \sim 9 \times 10^4 \text{ cm./sec}$$

Then

$$c^2 = a^2 \left(1 + \frac{1.3 \times 10^{-2}}{k}\right)$$

expanding this as:

$$\sqrt{1 + x} \approx 1 + \frac{x}{2} \text{ for } x \ll 1$$

$$c = a \left(1 + \frac{9.1 \times 10^{-3}}{k}\right)$$

For the conditions as cited then, we could expect about a half a percent change in sound speed due to the magnetic field. For most detonation waves the conductivity would be more of the order of $10^{-2} \text{ mho/cm.}$ unless it was seeded, so the additional term in this case would become negligible.

In the development which follows, only the ordinary sound speed at the C - J point, will be used keeping in mind the possibility that it could possibly be modified by a few percent if the magnetic field was sufficiently strong.

A quazi-steady analysis of the detonation wave, will now be developed to determine how the propagation rate will be altered when it enters the magnetic field.
Quazi - Steady Analysis

The action of the electro-magnetic field on the conducting gases associated with a detonation wave will cause electric currents to flow in the gas. These currents, given by the generalized Ohm's law, will have a magnetic field associated with them. This field, commonly known as the induced field will modify the total magnetic field in the gas. When the currents in the gas are small compared to the currents in the coils which produce the applied field, the induced field will be small compared to that applied, and can be treated as a perturbation quantity.

The development which follows is for such a case where the induced field is small compared to that applied, and Chap. 4 of this report will justify the use of such an assumption by computing the relative sizes of fields for some realizable cases.

The total magnetic field is given by

\[ \mathbf{B} = \mathbf{B}_0 + \mathbf{B} \]  

where \( \mathbf{B}_0 \) is the applied field, and \( \mathbf{B} \) is the induced field.

Then

\[ \nabla \times \mathbf{B} = \nabla \times \mathbf{B}_0 \]  

since \( \mathbf{B}_0 \) is a constant. Up to this point we have made no assumption about the relative sizes of \( \mathbf{B} \) and \( \mathbf{B}_0 \), but if we now assume \( \mathbf{B} \ll \mathbf{B}_0 \)

\[ \mathbf{j} \times \mathbf{B} = -\nabla (\mathbf{B}_0 \cdot \mathbf{B}) \]  

This analysis is for a one dimensional problem with magnetic fields only in the \( x \) direction, velocities only in the \( x \) direction and electric fields only in the \( y \) direction. Hall currents will be neglected, so all currents will also be in the \( y \) direction. Consequently subscripts denoting direction will be omitted.
The continuity relation can be written
\[ \rho_a v_a = \rho_w v_w = \rho v = \text{const.} = m \]  
(0.29)
where subscripts denote upstream values, and \( \omega \) the downstream observation point.

The momentum equation in differential form becomes
\[ \rho v \frac{d\rho v}{dx} + \frac{d\rho}{dx} + \frac{d}{dx} \left( \frac{\overrightarrow{B}_a \cdot \overrightarrow{B}_a}{\mu} \right) = 0 \]  
(1.30)

Now using a discontinuity type analysis, eqn. 1.30 can be integrated from the point \( \alpha \) upstream to point \( \omega \) downstream, yielding
\[ m v_\alpha + p_\alpha + \frac{\overrightarrow{B}_a \cdot \overrightarrow{B}_a}{\mu_\alpha} = m v_\omega + p_\omega + \frac{\overrightarrow{B}_\omega \cdot \overrightarrow{B}_\omega}{\mu_\omega} \]  
(1.31)
Since we are dealing with a wave which gives rise to a conducting gas, we have zero conductivity upstream of the wave, so that no currents will flow in this region and hence \( \overrightarrow{B}_\alpha = 0 \). Further we note that \( \overrightarrow{B}_a \cdot \overrightarrow{B}_\omega \) is the dot product of two vectors, so could take on both positive and negative values. For further development it is convenient to define the quantity
\[ \beta = \frac{\overrightarrow{B}_a \cdot \overrightarrow{B}_\omega}{\mu_\omega \rho_\omega} \]  
(1.32)
The momentum equation (1.31) can now be written
\[ m v_\alpha + p_\alpha (1 - \beta) = m v_\omega + p_\omega \]  
(1.33)
Now introducing the further notation
\[ M = \frac{\sqrt{\beta}}{\sqrt{k RT}} \]  
(1.34)
\[ k = \frac{C_P}{C_V} \]  
(1.35)
where \( M \) is the velocity of the gas divided by the ordinary speed of sound, and \( k \) is the ratio of specific heats in the gas, assumed constant throughout, then (1.33) can be further simplified to give
\[ p_\alpha (1 - \beta + k M^2) = p_\omega (1 + k M^2) \]  
(1.36)
The conservation of energy is now written in the form of an enthalpy relation, neglecting viscosity and heat loss.

\[ \rho v \frac{\partial H_0}{\partial x} = \vec{E} \cdot \vec{j} + q \rho v \]  

(1.37)

where \( H_0 = C_p T + \frac{v_z^2}{2} \)

\( q \) is the chemical source term

\( \vec{E} \cdot \vec{j} \) is the contribution to the change in stagnation enthalpy due to the electric field. This form of equation places in evidence the fact that the stagnation enthalpy is a function of the co-ordinate system in which it is evaluated, as is the electric field.

Integrating equation (1.37) between points \( \alpha \) and \( \omega \)

\[ \int_\alpha^{\omega} dH_0 = \int_\alpha^{\omega} \vec{E} \cdot \vec{j} \, dx + \int_\alpha^{\omega} q \, dx \]

or

\[ C_p T_\omega + \frac{v_z^2}{2} = C_p T_\alpha + \frac{v_z^2}{2} + \frac{1}{c_v} \int_\alpha^{\omega} \vec{E} \cdot \vec{j} \, dx + Q' \]  

(0.38)

where \( Q' \) is the change in stagnation enthalpy due to the chemical reaction.

Now to deal with the electrical term, the most general case will be applied. It is convenient to write the electric field experienced by the observer on the wave in terms of what a stationary observer would measure. Then if \( \vec{E} \) is the field measured by the moving observer, we can write

\[ \vec{E} = \vec{E}_0 + \vec{v}_e \times \vec{B} \]

where \( \vec{E}_0 \) is the field measured by a stationary observer in the lab.

This can be expressed as

\[ \vec{E}_0 = \vec{v}_e \times \vec{B} \]

so that the direction of field can be controlled by the sign of the fictitious velocity \( V_e \).
In general then
\[ \vec{E} \cdot \vec{j} = (\vec{v}_e \times \vec{B}) \cdot \vec{j} + (\vec{v}_a \times \vec{B}) \cdot \vec{j} = -(\vec{v}_a + \vec{v}_e) \cdot (\vec{j} \times \vec{B}) \]
by simple vector manipulation. Then from eqns. (1.27) and (1.28)
\[ \vec{E} \cdot \vec{j} = (\vec{v}_a + \vec{v}_e) \cdot \nabla (\vec{B}_e \cdot \vec{B}) \quad (1.39) \]
Then performing the remaining integration step in equation (1.38)
taking into account vector directions yields, since \( \vec{v}_a = -\omega \vec{v}_e; \vec{v}_e = -\omega \vec{v}_a \),
\[ C_p T_w + \frac{v_e^2}{2} = C_p T_a + \frac{v_a^2}{2} - (v_a + v_e) (\vec{B}_e \cdot \vec{B}_w) + Q' \quad (1.40) \]
Using the equation of state for a perfect gas (1.10) and eqn. (1.32)
this if further simplified to
\[ C_p T_w + \frac{v_e^2}{2} = C_p T_a + \frac{v_a^2}{2} - \left(1 + \frac{v_e}{v_a}\right) R T_a \beta + Q' \]
Dividing through by \( C_p T_a \)
\[ \frac{T_w}{T_a} \left(1 + \frac{v_e}{v_a}\right) M_w^2 = 1 + \frac{v_e}{v_a} M_a^2 + Q' \left(1 + \frac{v_e}{v_a}\right) \beta \quad (1.41) \]
Now combining eqns. (1.36) and (1.29) and (1.10) an expression of \( \frac{T_w}{T_a} \)
is obtained
\[ \frac{T_w}{T_a} = \frac{M_w^2 \left(1 + \frac{v_e}{v_a} M_a^2 - \beta\right)^2}{(1 + \frac{v_e}{v_a} M_a^2)^2} \quad (1.42) \]
Putting this value into eqn. (1.41) results in
\[ \frac{M_w^2 \left(1 + k \frac{M_e^2}{2} - \beta\right)^2 \left(1 + \frac{v_e}{v_a} M_a^2\right)}{(1 + \frac{v_e}{v_a} M_a^2)^2} = 1 + \frac{k - 1}{2} M_w^2 + Q' \left(1 + \frac{v_e}{v_a}\right) \beta \quad (1.43) \]
This equation then relates the upstream and downstream conditions,
in terms of the chemical enthalpy added, the electrical contribution,
and the induced magnetic field parameter. For the ordinary gas
dynamic case of \( M_w = 1 \), and \( \beta = 0 \), this reduces to the simple relation
\[ \frac{(1 - M_e^2)^2}{2(k+1) M_a^2} = \left(\frac{Q'}{C_p T_a}\right)_{M_w=1} \]
which is the familiar relation of a simple \( T_0 \) change given in
Shapiro I, Chapter 7.
PARTICULAR CASES

(a) \( V_0 = 0 \)

The particular case of \( V_0 = 0 \) corresponds to connecting the two electrodes with an external short circuit. (Fig. 1) This will provide an external current path, but no energy is dissipated outside the gas, and no energy is added from this source. For this case it was shown that for an infinitesimal pressure pulse, the induced magnetic was of opposite sign to the applied field, so the parameter \( \theta \) will take on only negative values. This is also so for a detonation wave. When the usual Chapman - Jouquet condition that \( M_{\infty} = 1 \) is applied, it is found that for \( V_0 = 0 \), the velocity of the detonation wave will be slightly decreased. As will be shown in Chapter 4 of this paper, the value of \( \theta \) obtainable for this case is very small, so that it is likely that no effect on the wave speed will be observed for this case.

(b) A second case of interest is that for a resistor connected across the electrodes externally. This will be equivalent to \( \frac{V}{\sqrt{\rho}} < 0 \) but, the lower limit is not well defined. If the resistor is so chosen that the potential drop across it is about the same as the potential drop in the gas, then the equivalent of near optimum operating condition of an electric generator is depicted. For this case the currents in the gas have not been reversed although they are reduced, and energy is taken out of the flow, which is dissipated in the resistor.
Thus for the same value of $\beta$, the detonation velocity will be reduced more than with the external short circuit. Since the currents which are responsible for the induced magnetic field have been reduced, the value of $\beta$ obtainable, will have been reduced so that again it is doubtful whether the total effect will be measurable.

(c) External Voltage Source.

The most versatile case is one in which an external power source is used. This can correspond to $\frac{V}{V_a} \geq 0$. When $\frac{V}{V_a} > 0$, the external field is helping the current to flow in the direction it would naturally flow, so that $\beta$ will still be negative. There is an opposing effect however due to the energy addition so that when $\frac{V}{V_a}$ exceeds a certain value the energy addition will tend to increase the velocity of the wave more than the dissipative action of the $\mathbf{F}$ force will tend to decrease it, so that the result can be an acceleration.

When $\frac{V}{V_a}$ is negative, and its absolute value is some minimum value determined by the gas velocities, then the currents in the gas will be reversed. This will yield positive values for $\beta$, and together with the energy added by the electric source, combine to give the best possible acceleration of the wave.

The accompanying graph, appendix A, illustrates these
various regimes. It deals with the case of $\frac{\omega}{\omega_n}=6$, $M_w=1$, $k=1.25$. It is essentially a state plane, in that it relates change in detonation Mach number to the values of $\beta$, with $\nu_k$ as parameters, without specifying explicitly how large a value of $\beta$ is obtainable.

Line OB represents the case for an external short circuit, illustrating the slight decrease in velocity for the negative $\beta$'s attainable.

Line OC represents a certain value of external resistor giving a value $\frac{\nu_x}{\nu_k}=-1/4$. It illustrate the point made in (c) that greater deceleration results for the same $\beta$ compared with the short circuit.

The other lines to the left of center marked 1, 2, 4, 8, are for values of $\frac{\nu_x}{\nu_k}$ of 1, 2, 4, 8 respectively. It is evident how the energy predominates after $\frac{\nu_x}{\nu_k} \geq 2$.

Line OA to the right of center is a fictitious line corresponding to $\nu_x=0$ and $\beta$ positive. As mentioned earlier, this is not possible, but marks a lower limit of region to the right of the axis. The lines above it showing values of $\frac{\nu_x}{\nu_k}$ of -1, -2, -4, -8, should be realizable and they demonstrate the increased effect when the $jx\overline{B}$ force is pushing to complement the energy addition in increasing the wave speed.
Conclusions and summary of the quazi-steady analysis.

An analysis has been performed of the effect of an electro-magnetic field on the propagation speed of a detonation wave. By considering the wave as a discontinuity, and using the usual hot boundary condition of $M_t = 1$, it has been shown that the wave speed is slightly affected by the action of the electro-magnetic fields. Since the value of $\Theta$ obtainable in any real system is dependent on the process occurring in the wave, it will be necessary to examine the wave further for this reason.
CHAPTER II

A NON-STEADY ANALYSIS OF THE EFFECT OF ELECTRO-MAGNETIC FIELD ON PRODUCT GASES OF A DETONATION WAVE.

The usual model of a steady state detonation wave is one which, for the observer on the wave, the gas enters the wave supersonically, relative to the upstream conditions, and leaves at a velocity equal to the sonic velocity of the product gases. Thus the steady state wave speed is determined by the sonic velocity of the product gases. The reasons for this are discussed in various text books on combustion and gas dynamics, ref. 5, and will not be considered in detail here.

The point of interest, however, is that normally a small disturbance occurring behind the wave will not overtake the wave, since it can travel only at the sonic velocity relative to the particle velocity, and at this rate will always remain the same distance behind the wave front. If the disturbance was large enough, for example a weak shock it could overtake the wave front, and if significantly strong, could do so in a short enough distance to be noticed in a laboratory apparatus.

It is the purpose of this development to examine the possibilities of creating a sufficiently strong disturbance behind the wave front, by means of an electromagnetic interaction, to affect the wave speed.
Since a problem of this kind is essentially nonsteady, it will have to be examined from such a viewpoint. The region will be treated as one having a finite, constant, electrical conductivity with no chemical reactions occurring, i.e. frozen equilibrium, and being acted upon by a combined electromagnetic field. No heat transfer to the walls of the tube or friction will be considered. To further simplify the analysis, it will be assumed that the rarefaction wave normally associated with the detonation wave is far enough behind the wave so as not to be considered in the problem. All changes will occur in x dirn. or time t only.

**Governing Equations**

The general equations, subject to the above mentioned assumptions, will be of the following form:

\[
\frac{\partial p}{\partial t} + \frac{\partial(pu)}{\partial x} = 0
\]  
\[\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} = \vec{j} \times \vec{B} \]  
\[\rho \frac{\partial}{\partial t} (c_p T + \frac{u^2}{2}) + \rho u \frac{\partial}{\partial x} (c_p T + \frac{u^2}{2}) - \frac{\partial p}{\partial t} = \vec{j} \cdot \vec{E} \]  
\[p = \rho RT \]

For the configuration:

![Diagram](image-url)  
**Fig. 2-1**
\[ \mathbf{j} = \nabla (E - uB) \]  
(2.5)

Since these equations are all written for the laboratory co-ordinate system, the $E$ and $B$ appearing in the above equations are those applied fields which can be assigned independent values, and the effect of the flow on the fields is being neglected. It is convenient, however, to relate their magnitudes by the relation $E = V_0 B$  
(2.6)

Now if (2.6) is combined with (2.5) and together with (2.4) is introduced into (2.3) the equations becomes

\[ \left( \frac{\partial}{\partial t} + \rho u \frac{\partial}{\partial x} \right) \left( \frac{k}{k-1} \left( \frac{\partial}{\partial t} \left( \frac{\partial}{\partial x} + u \frac{\partial}{\partial x} \right) \right) - \frac{2P}{\partial x} = \nabla V_0 B^2 (V_0 - u) \]

expanding the left side and grouping the terms somewhat and using the relation $C^2 = kP$, results in

\[ \frac{1}{k-1} \frac{\partial P}{\partial t} + u \frac{k}{k-1} \frac{\partial P}{\partial x} - C^2 \left( \frac{\partial}{\partial t} \left( \frac{\partial}{\partial x} + u \frac{\partial}{\partial x} \right) \right) + u \left( \rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} \right) = \nabla V_0 B^2 (V_0 - u) \]  
(2.7)

Noting that the last bracket is contained in the momentum equation (2.2) or

\[ \rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} = \nabla B^2 (V_0 - u) - \frac{\partial P}{\partial x} \]  
(2.8)

Inserting (2.8) in place of the last bracket of (2.7) results in, after multiplying through by $k-1$

\[ \frac{\partial P}{\partial t} + u \frac{\partial P}{\partial x} - C^2 \left( \frac{\partial}{\partial t} \left( \frac{\partial}{\partial x} + u \frac{\partial}{\partial x} \right) \right) = \nabla B^2 (V_0 - u)^2 (k-1) \]  
(2.9)

All the properties (e.g., $u, p, \rho$) must vary continuously with $x$ and $t$ giving three additional relations the six basic relations being written thus;

\[ + dx \frac{\partial u}{\partial x} + dt \frac{\partial u}{\partial t} = d\mu \]

\[ + dx \frac{\partial P}{\partial x} + dt \frac{\partial P}{\partial t} = dP \]
These equations are now in the form used in Shapiro (ref. 6) Chap. 24 for nonsteady flow. The method of characteristic curves can now be applied to study the behaviour of the solutions. Solving for one of the derivatives, say $\partial u / \partial x$ in determinant notation

$$
\begin{vmatrix}
du & dt & 0 & 0 & 0 & 0 \\
dp & 0 & dx & dt & 0 & 0 \\
d\rho & 0 & 0 & 0 & dx & dt \\
0 & 0 & 0 & 0 & u & 1 \\
\frac{\sqrt{\gamma} B^2(V_o - u)}{\rho} & 1 & \frac{1}{\rho} & 0 & 0 & 0 \\
(k-1)\frac{\sqrt{\gamma} B^2(V_o - u)^2}{\rho} & 0 & u & 1 & -uc^2 & c^2 \\
\end{vmatrix}
$$

$$
\frac{\partial u}{\partial x} =
\begin{vmatrix}
dx & dt & 0 & 0 & 0 & 0 \\
0 & 0 & dx & dt & 0 & 0 \\
0 & 0 & 0 & 0 & dx & dt \\
\rho & 0 & 0 & 0 & u & 1 \\
u & 1 & \frac{1}{\rho} & 0 & 0 & 0 \\
0 & 0 & u & 1 & -uc^2 & c^2 \\
\end{vmatrix}
$$

(2.10)
Physical Characteristics.

The condition that \( \partial u / \partial x \) be indeterminate is that the denominator of equation (2.10) be equal to zero. Since \( \partial u / \partial x \) is in general finite, it is necessary that the numerator also be equal to zero. By setting the two determinants of eqn. 2.10 equal to zero, the differential equations of the characteristic curves is obtained.

Setting the denominator equal to zero, and simplifying yields the roots

\[
\left( \frac{dx}{dt} \right)_{I, II} = u \pm c \quad \text{(Mach lines)} \quad (2.11)
\]
\[
\left( \frac{dx}{dt} \right) = u \quad \text{(Path lines)} \quad (2.12)
\]

Equation 2.11 signifies that discontinuities in the derivatives may occur on lines which travel either rightward or leftward with the local speed of sound relative to the fluid. This is the same as saying that the disturbances are propagated on such lines.

Equation 2.12 indicates that the trajectories of the fluid particles themselves may be the loci of discontinuities. Physically, this means that the fluid particles can behave individually in a nonsteady motion so that the path lines are characteristic curves on which the entropy or temperature gradients may have discontinuities.

It may be of interest to note that the equation of
of the characteristic curves are identical to those found in ordinary gas dynamics, with no new terms introduced by the electro-magnetic field. The reason for this follows from the method of setting up the governing equations. Implicit in equations 2.6 through 2.9 is the assumption that \( \mathbf{B} \) is a fixed, constant quantity. This allows the magnetic field to affect the fluid dynamic problem as here developed, but does not allow for any interaction of the fluid with the magnetic field. It can be shown that the assumption is valid as long as the Magnetic Reynolds number, \( R_m = \frac{\mu \nabla \mathbf{u}}{\mathbf{B}} \), is less than 1. For the examples considered later, the value of \( R_m \) is \( \approx 0.0015 \) for a length of 1 meter, so this assumption seems quite valid. Further justification for using the ordinary sound speed may be found in chap. 1, where it has been shown that the contribution of the magnetic field is small. For a case such as studied here, a small change, such as 1/2\%, is not necessary characteristically different from no change since the sound speed has been assigned some reasonable value to start the numerical solution. The purpose of the solution carried out here is to see if, using the ordinary sound speed, it is able to predict some significant change in conditions behind the wave.
Consequently, no other modes of electromagnetic or hydrodynamic waves have been considered here either. This is also justifiable on the grounds that these wave interactions with the fluid occur only when the velocity field and magnetic field are strongly coupled, which they are not for small R_m, so no account has been taken here, of any of these modes.

**State Characteristics**

Setting the numerator of equation 2.10 equal to zero expanding the determinant and eliminating dx by means of equation 2.11, the following differential equation of the Mach lines in terms of the state co-ordinates p and u is obtained

\[
\frac{dw}{dx} = \frac{d}{dt} \frac{dp}{dx} - \frac{M dt}{c} + N dt \quad (2.18)
\]

where

\[
M = \nabla B^t (V_0 - u)^t (k-1)
\]

\[
N = \frac{c}{c} B^t (V_0 - u)
\]

Following the development of Shapiro Chap.24, it can be shown that

\[
\frac{dp}{dt} = \frac{2k}{k-1} \frac{p c}{c} \frac{dc}{dx} \quad (2.14)
\]

having made use of the relations

\[
\frac{dp}{dt}_{path} = \frac{k}{k-1} \frac{p c}{c} \frac{dc}{dx}_{path} - \frac{M}{k-1} \frac{dt}{path} \quad (2.15)
\]

\[
\frac{dp}{dt}_{I,II} = \frac{dp}{dt}_{path} \pm \frac{dp}{dx}
\]

\[
\frac{dc}{dt}_{I,II} = \frac{dc}{dt}_{path} \pm \frac{dc}{dx}
\]
Insertion of equation 2.14 into 2.13, after simplification yields

\[(du)_{I,II} = \tau \frac{2}{k-1} (dc)_{I,II} \left[ \frac{c^2}{k} \frac{\partial}{\partial x} \left( \frac{k}{\xi} \left( \frac{2k}{P-1} \right) \right) \right] + k \frac{V_N}{c} + N (1 + k \frac{\partial u}{\partial x}) dt_{I,II} \quad (2.16)\]

For computation sake, it is convenient to expand the first term in the bracket, so that

\[\frac{c^2}{k} \frac{\partial}{\partial x} \left( \frac{k}{\xi} \left( \frac{2k}{P-1} \right) \right) = \frac{2c}{k-1} \frac{\partial c}{\partial x} - \frac{c^2}{kP} \frac{\partial P}{\partial x} \quad (2.17)\]

**Solution of the Equations**

Since the equation to be solved (2.16) contains terms which depend upon t and x as well as c and u it is not possible to integrate the state characteristics in general form. Instead, the net of state characteristics is different for each physical problem, and it must be constructed simultaneously with the physical Mach net by means of a stepwise interactive procedure. At the same time that the state and physical characteristics nets are under construction, it is necessary to integrate equation 2.15 stepwise along the path lines. This not only provides useful information concerning the pressure at each value of x and t but also leads to the determination of equation 2.17 which is necessary for the solution of 2.16.

**Graphical Method of Solution.**

An iterative procedure for the solution of equation 2.16 is outlined in Shapiro II, Chap. 24. An outline of how
this is applied to this particular case will be given here.

For the purpose of solution, Eq. 2.16 is written in the finite difference form

\[(\Delta u)_{\text{r},\text{II}} = \frac{1}{k-1} (\Delta c)_{\text{r},\text{r}} + (\Delta u_s)_{\text{r},\text{II}} + (\Delta u_{\text{e},\text{c}})_{\text{r},\text{II}} + (\Delta u_{\text{t},\text{t}})_{\text{r},\text{II}} \]  

(2.18)

where \(\Delta u_s, \Delta u_{\text{e},\text{c}}, \) and \(\Delta u_t\) represent the terms in equation 2.16 connected with the effects of \(E \cdot J\) heating, \(J \times B\) forces, and longitudinal entropy gradients brought about by the other effects. For example:

\[(\Delta u_{\text{e},\text{c}})_{\text{r},\text{r}} = \pm \frac{kV_s N}{c} (\Delta t)_{\text{r},\text{r}}\]

where the coefficient of \((\Delta t)_{\text{r},\text{r}}\) is evaluated at its mean value over the interval. That this is indeed the term due to \(E \cdot J\) can be verified by substituting for \(N\) and \(V_s\) and simplifying.

The basic step in the construction of the characteristics net is this: Given data \((u, c, x, t)\) at two points 1 and 2, it is necessary to find similar data at the intersection point 3 of the two characteristics passing through points 1 and 2. Repeated application of this step, given suitable initial data, leads to the construction of the complete nets in the two planes.
At any step in the iteration, it is necessary to know the value of $N$ and $V_c$ in the neighborhood of the particular locations $(x, t)$ and states $(u, p, c)$ under consideration. It is also assumed that equation 2.15 has been integrated along the path lines as far as the current stage of the calculations permits, and that the $x$ derivative in the expression for $\Delta u_s$ is therefore known in the neighborhood of points 1 and 2. (That is, the points from which the next iteration will begin).

The method then is as follows:

i) A crude first approximation to point 3 in the state plane is found by ignoring the terms $\Delta u_{j1}$, $\Delta u_{jB}$ and $\Delta u_s$. It is then only necessary to find the intersections of the lines passing through points 1 and 2 with the slopes $\gamma(k-1)/2$, respectively. For the first point in this problem 1 and 2 have identical conditions, so the first approximation to 3 is the same as 1 and 2. This is because it is assumed
that no change in the conditions occur behind the wave, except that caused by the electromagnetic field, and since point 1 is the beginning of the field, and 2 is immediately behind the wave front where the chemical reaction is complete but no other appreciable changes have occurred, the two points have identical properties.

ii) Using the mean values of $\bar{u}_{1-2}$ and $\bar{c}_{1-2}$ between $t_0$ and $t$, the physical I characteristic is drawn through point 2 with the slope $\bar{u}_{1-2} + \bar{c}_{1-2}$. Similarly, the physical II characteristic is drawn through point 2 with the slope $\bar{u}_{2-3} - \bar{c}_{2-3}$. The intersection of these two lines establishes a first approximation to point 3 in the physical plane, and yields tentative values of $(\Delta t)_{1-3}$.

iii) The values of $(\Delta u_{1-3})_1$, $(\Delta u_{1-3})_2$, and $(\Delta u_{1-3})_3$ are calculated from equation 2.13 and the given data, the coefficient of $(\Delta t)_{1-3}$ in each case being taken at the mean conditions between points 1 and 3. The sum of these values, $\Sigma (\Delta u_{1-3})_1$, is then laid off horizontally (and algebraically) from point 1, thus yielding point 1a. Similarly, $\Sigma (\Delta u_{1-3})_2$ is calculated and laid off horizontally from point 2 thus giving point 2a. Straight lines with slopes $(k - 1)/2$ are then drawn through points 1a and 2a respectively. (See fig. 2.2). Their intersection at point 3' gives a second, and better, approximation to the desired point. This construction satisfies Eq. 2.13.

iv) Using the mean values of $u$ and $c$ between 1 and 3'
and between 2 and 3, point 3 is established in the physical plane as an improvement over point 3.

v) Steps (iii) and (iv) are repeated with the values of 1 and 2 and 3 and lead to the establishment of point 3" (a still better approximation) in the state and physical planes. This interactive procedure is continued until satisfactory and convergence is obtained.

Results

The governing equation in this problem is not solvable in general terms, so a qualitative understanding of the results can only be obtained by doing particular solutions and generalizing from them. Furthermore, a complete solution would entail plotting the entire characteristic net behind the wave. This has not been done, but only a beginning has been made to determine the trend of the solution.

The reasoning used was as follows: since a steady state detonation wave travels at constant speed, it will be a straight line on the x-t diagram. The I characteristics in the absence of electro-magnetic fields and other dissipative effects will likewise be straight lines of slope \( u_2 + c_2 \) which is a line parallel to the detonation wave front locus. As long as the disturbance behind the wave cannot overtake the wave, it will propagate at a steady velocity, and the values of the properties will be known along the line marking the hot boundary of the wave. One other point is needed to start the solution. This point is selected
at the point where the wave enters the magnetic field. Since it is assumed that no changes occur in the product gases, except those brought about by the electro-magnetic field, there is a line along \( x = 0 \) for which the properties are known. So there are now two lines along which starting conditions are known, so the solution may begin anywhere along these two lines.

**Solutions.**

For the solution a wave having the following properties was selected for examination. The temperature of the products directly behind the wave was taken to be 4700\(^\circ\)K, so that

\[ C_2 \approx 1300 \text{ m/sec and } U_2 \approx 1210 \text{ m/sec. } \]

This gives the slope of I characteristic as \( u_+ + c_2 = 2510 \text{ m/sec.} \) which also corresponds to the wave front speed. The pressure used was \( p_2 = 60 \text{ dmHg or 0.79 atm.} \) The magnetic field used was \( B = 1 \text{ web/m}^2. \) The electrical conductivity was assigned a value of 1 mhos/m, and a value of \( k = 1.25 \) was used.

Three cases of interest were examined briefly:

1) \( V_0 = 10,000 \text{ m/sec} \)
2) \( V_0 = -10,000 \text{ m/sec} \)
3) \( V_0 = 0 \)

The first case is with \( E \) in (eqn. 2.5) = 10,000 B m/sec.

or \( j = \nabla B (10,000 - u). \) The results are plotted on the x-t diagram No. 1. Only one I characteristic was examined
and it was assumed that the wave propagated at steady speed, so that values of properties are known along the line 0-A. The characteristic was found to bend noticeably toward the wave front, indicating that the interactions with the wave from behind is a strong possibility.

Since a complete solution has not been done, a quantitative prediction may be justified. However, the qualitative result seems significant, since it seems that if the characteristic nearest the wave bends towards it, that the others would behave similarly. A pressure increase is associated with this sense of bending of the characteristic, so physically one can imagine a pressure pulse building up behind the wave eventually overtaking it.

The second case was for the same electric field value, but in the opposite direction. To get a better insight into the solution, two L characteristics were followed. The line BC corresponds closely to the line BC in graph No.1, but was constructed after the solution of DE. Initially lines BC and DE bend almost an equal amount, but surprisingly, line DE begins to bend more toward the wave than the characteristic BC, so that in the region of x between 0.5 to 0.6 meters, the characteristics DE and BC are further apart than when the wave entered the field at x = 0. Correspondingly, there is a pressure rise between the wave and the characteristics DE, so that at point 11, the pressure is about 17.4 psi, as compared to 11.6 psi at point 10.
Continuing out it is found that point 105 has a pressure of 15.33 psi.

So it seems that there occurs a pressure rise directly behind the wave, with a decrease further back. The pressure calculated at the various points behind the wave are given on the graphs. The pressure for the two equivalent characteristics in the two graphs are very nearly the same. What the first graph fails to show however, and which is evident in the second, is the sudden build up of pressure immediately behind the wave, followed by a rarefaction. Judging from the better picture presented in the second case compared to the first, it seems a complete solution with a fine net of characteristics behind the wave would be of considerable interest.

The fact that the characteristics BC, for the two cases considered, act almost the same in spite of the reversed electric field is also interesting. In the following the solution through it becomes evident why this is so. The reason appears to be that the $\Delta u_{jB}$ is small compared to the $\Delta u_{ij}$ and $\Delta u_{s}$. Thus the heating effect of the currents gives rise to pressure changes and changes in the sound speed which cause the large changes in $\Delta u_{s}$.

From the graphs, it is further evident that with $\mathbf{j} \times \mathbf{B}$ directed against the flow the pressure builds up higher
for the equivalent position in the field. This is not caused by the $\mathbf{j} \times \mathbf{B}$ force, but by the fact that with $V_0$ negative, the current is given by $\mathbf{j} = \mathbf{v} \times \mathbf{B} (V_0 + u)$ whereas with $V_0$ positive it is $\mathbf{j} = \mathbf{v} \times \mathbf{B} (V_0 - u)$, so the current is increased enough so that the additional effect of the greater $E \cdot \mathbf{j}$ heating more than compensates for the reversed $\mathbf{j} \times \mathbf{B}$ force.

In the first case, it appeared that $\mathbf{j} \times \mathbf{B}$ force was pushing the gases, but here the $\mathbf{j} \times \mathbf{B}$ force is reversed and little change in the solution is apparent. Thus, the direct action of the $\mathbf{j} \times \mathbf{B}$ force is not responsible for the change, but it appears a combined interaction between $E$, $\mathbf{j}$ and $\mathbf{B}$ give the result.

The third case studied briefly was not plotted since the change was so small, but here again, the solution resulted in the characteristic bending toward the wave, although very slightly.

Conclusion.

This brief analysis indicates that the wave can be acted upon from behind when the electric fields of sufficient magnitude are used. The basic dissipation phenomena of viscosity and heat transfer to tube walls has been neglected in this work, so if it were considered, the results may be altered somewhat. One would expect both of these effects to cause the characteristic to bend away from the wave front, so the overall effect in a laboratory apparatus may be con-
siderably smaller than the predicted on the basis of this idealized model.
CHAPTER III

ELECTRICAL CONDUCTIVITY.

The results of the quasi-steady analysis of the effect of electro-magnetic fields on the propagation speed of a detonation wave were left in terms of a state relationship. That is, the effect of the field depended on the induced field. It would now be of interest to examine possible physical conditions.

The above mentioned analysis was based on changes occurring within the wave and so it will be necessary to examine the electrical conductivity within the wave. In previous reports (ref.10) the electrical conductivity within the wave was examined in connection with the structure of the wave. The results obtained will be presented here with further extension to determination of induced fields.

In order to predict the electrical conductivity from ionization equilibrium considerations it is necessary to know the temperature and pressure at points in question. In ref.(10) the structure of the wave as calculated by Hirschfelder et al (3) was used, and will again be used here to show some possible conductivity profiles for simplified cases.
To do a completely rigorous analysis of the ionization within the reaction zone, one would need very precise information regarding the chemical reaction. Then knowing the exact constituents at each point in the wave, one could calculate, from equilibrium considerations, what the electron concentration should be.

In view of the complexity of the chemical reactions involved in a detonation wave, no attempt will be made here to evaluate what electron concentration may be associated with the reaction. Instead, the ionization of an inert gas in the mixture, as well as that of a salt of low ionization potential will be considered. Only equilibrium thermal ionization of these species will be considered.

Conductivity Calculations

The relation for the equilibrium degree of ionization of a monotomic gas is given by the Saha equation. The reaction for which it is applicable is of the type

$$B = B^+ + e$$

(3.1)

The equilibrium constant of this reaction is given by

$$K_e = \frac{P_{e}^+ P_e}{P_B}$$

where $P_e^+$, $P_e$, and $P_B$ represent a partial pressure in the atmospheres of the positive ions, electrons, and atoms respectively. The constant $K_e$ can be expressed as a function of temperature as

$$\log_{10} K_e = -\frac{U}{2.3RT} + \frac{5}{2} \log_{10} T + \log_{10} \frac{(2\pi m_e)^{3/2} k^{5/2}}{h^3} + \log_{10} \frac{g_{e^+} g_e}{g_B}$$

(3.2)
where \( g_i \), \( g_e \), and \( g_A \) are the statistical weights of the fundamental state of the ion, the electron and atom, \( k \) is the Boltzmann constant, \( h \) is Planck's constant, \( m \) is the electronic mass, \( T \) is the temperature in degrees Kelvin and \( U \) is the heat ionization of \( B \).

For the equilibrium ionization of potassium, the above equation reduces to the form

\[
\log_{10} K_e = -\frac{5040}{T} I_k + 2.5 \log_{10} T - 6.49
\]

(3.3)

where \( I \) is the first ionization potential in eV.

When more than one species are contributing electrons, the effect will be similar to a common ion effect in a chemical reaction, in that the electron pressure will be higher than the ion pressure of any one species. The equilibrium constant will still be determined by the temperature, but the ion pressure will now be suppressed by the greater electron pressure, thus yielding fewer electrons than if each species acted independently. If one of the species under consideration has a much lower ionization potential than any of the others present, then the electrons contributed by the others will be negligibly few, so that the electron pressure can be considered equal to the ion pressure of this species.

In the case under consideration here, when potassium is present in small but sufficient quantities, the assumption of dealing with a neutral gas yields the expressions \( p_e = p_{K^+} \),
so that \( K_e = P_e^2 / P_k \).

Then if

\[ X = \frac{\text{initial number of moles of } K}{\text{total number of moles of gas}} \]

then \( P_e + P_k = x(p) \), where \( p \) is the total pressure in the gas in the atmospheres. And,

\[ K_e = \frac{P_e^2}{P_k} = \frac{P_e^2}{(px - P_e)} \]

or

\[ P_e = -K_e + \sqrt{K_e^2 + 4Kexp} \]

(3.4)

The fraction ionized, \( \alpha \), is given by

\[ \alpha = \frac{P_e}{P} \]

(3.5)

The one case to be considered here is for the mole fraction of potassium \( X_k = .001\% \) or .000,01 and the pressure \( p \) corresponding to the Chapman-Jouguet point was 60 cm Hg or 0.79 atm.

The detonation to be considered is a stoichiometric oxy-acetylene mixture with argon added. The mixture by moles is then \( 2C_2H_2 + 5O_2 + 5A \), and the final temperature will be about 4000. For another case, for no potassium present, the ionization of the argon only will be considered predominant for the sake of simplicity. When the potassium is present, its contribution to the electron pressure will predominate, so that the ionization of argon will be negligible in that case.

The Saha equation for argon, due to a different partition
function becomes

\[ \log_{10} K_e = -5040 \frac{1}{T} + 2.5 \log_{10} T - 4.27 \]  \hspace{1cm} (3.6)

Equations (3.4) and (3.5) apply to the argon as well when the potassium is not present with the X referring to the mole fraction of argon.

Equation (3.4) can be simplified in certain cases. The first case of interest is for \( K_e \ll X_p \). Then

\[ p_e = \sqrt{K_e} x_p \]

and

\[ \alpha = \frac{p_e}{\rho} = \sqrt{\frac{K_e x_p}{\rho}} \]  \hspace{1cm} (3.7)

It can further be seen that Saha eqn (3.2) can be written in terms of exponential rather than a logarithmic equation.

\[ K_e = G^2 T^{5/2} \exp \left( \frac{-q_1}{kT} \right) \]  \hspace{1cm} (3.8)

where \( G \) is a constant dependent on the species and \( q_1 \) is the first ionization potential for the gas atom. Then substituting into (3.7) above

\[ \alpha = \sqrt{X} \frac{G T^{5/2}}{\rho^{5/2}} \exp \left( \frac{-q_1}{kT} \right) \]  \hspace{1cm} (3.9)

This relation is applicable to the ionization of argon everywhere in the wave, but useful for the salt only at low temperatures, due to the large difference in the ionization potential. (The Constant \( G = 1.73 \times 10^{-2} \) for argon if \( T \) is in \( \text{K} \), and \( \rho \) in \( \text{cmHg} \)).

For a slightly ionized gas mixture, the electrical conductivity is given by

\[ \sigma = \frac{0.532 \alpha \varepsilon^2}{(m_e k T)^{5/2}} \frac{1}{\sum x_j Q_j} \]  \hspace{1cm} (3.10)
where $\epsilon$ is the electronic charge, $m_e$ the electronic mass, $Q_j$ the effective electron-atom collision cross-section of the species $j$, and $x_j = \frac{n_j}{n}$ mole fraction of the neutral species $j$.

Combining equations (3-9) and (3-10) to give a non-dimensional equation yields

$$\frac{T}{T_m} = \frac{\Theta^{3/4}}{(F/F_m)^{1/2}} \exp \left[\frac{-Q_j}{2kT_m} \left(\frac{1}{\Theta} - 1\right)\right]$$

(3.11)

where $\Theta = T/T_m$, and the subscript $m$ will refer to the conditions at the hot boundary. Implicit in equation (3-11) is the assumption that $\sum x_j Q_j$ remains constant through the wave. This may not be a completely valid assumption in view of the fact that $\sum x_j$ is not constant, due to the chemical reaction, and $Q_j$ is a function of temperature but since data on the values of $Q_j$ at the low temperatures of interest here, is very scarce and uncertain, added to the fact that the detailed steps of the chemical reaction are also uncertain, it seems justifiable in the interest of simplicity to assume $\sum x_j Q_j$ a constant.

A possibly better approximation would be to evaluate $\sum x_j Q_j$ at the hot and cold boundaries and assume some simple variation through the reaction zone. This has not been done here however, and will be left for more sophisticated approach to the problem.

The value of $\sum x_j Q_j$ used here is $2.0 \times 10^{-14} \text{cm}^2$ evaluated at the hot and cold boundary. This compares not too
favorably, with the value of $2.5 \times 10^{-15}$ cm$^3$ used by Basu ref. (4), evaluated at the hot boundary. This seems to further support the need for a more careful treatment of this point, but again, it will not be attempted here.

For the salt additive, at intermediate temperatures where $K_e \sim 4 \times P$, the entire equation (3-4) is essential. At higher temperatures, however, the approximation $K_e \gg 4 \times P$ becomes useful. For this case, equation (3-4) can be rewritten

$$p_e = K_e \left\{ -1 + \sqrt{1 + \frac{4 \times P}{K_e}} \right\}$$

Now the term under the radical is of the form $\sqrt{1 + a}$, where $a$ is small in this case. Expanding this in binomial expansion yields $\sqrt{1 + a} = 1 + \frac{a}{2} + \text{higher order terms}$. Expanding (3-12) in this fashion yields

$$p_e = K_e \left\{ -1 + 1 + \frac{4 \times P}{2K_e} + \ldots \right\} \approx 4 \times P$$

and the degree of ionization

$$\alpha = \frac{p_e}{P} = x$$

This is the case where essentially all the salt added has been ionized. Results of calculations are tabulated and plotted with the wave structures in appendix C.

**Discussion of Results.**

The structure of a detonation wave has been the subject of much discussion in the past, and since there are, as yet, no existing solutions which have been experimentally verified,
it will be of interest to examine the conductivity results from different structural solutions. The two structures to be employed here are from solutions by Hirschfelder et al (3). The first is their so-called exact solution, which is accomplished by assuming Lewis number of unity, and Prandtl number of 3/4. This results in the chemical reaction occurring simultaneously with the initial pressure rise. This infers strong coupling between shock zone and reaction zone, with little resemblance to the Von Neumann-Döring-Zeldovitch model.

The second solution, their "zeroth approximation" is fashioned after the NDZ model, with no reaction occurring in the shock zone. The results of the two computations are tabulated and plotted in appendix C.

Two independent cases are plotted for each, with and without the potassium additive, both being plotted on the same graph in each case.

Since the hot boundary conditions imposed on both models are the same, the final value of conductivity is the same in each case, as one would expect. The points of interest are then how the conductivity distributions compare.

Both models show that the potassium will have significant ionization earlier in the wave, due to the lower ionization potential. Similarly, both eventually gives complete ionization of the potassium, thus having little further temperature dependence.
although these developments occur further from the wave front in the case of the zeroth approximation.

The ionization of the argon differs in the two models, due to its stronger dependence on the temperature variations. The exact solution has a slight temperature maximum near $\xi = 3$, so that this is amplified to give a significant ionization maximum in the region. The zeroth solution has the temperature approaching the hot boundary from below, so no maximums exist in the conductivity profile.

Several points of interest can be arrived at from this comparison.

First, if no species exists in the wave which predominantly controls electron density by becoming completely ionized, it is possible to get large variations in conductivity within the wave, with very slight temperature variation. Thus, what might be interpreted as non-equilibrium effects if detected by experiment may be due to just such a temperature maximum within the wave.

Secondly, if one had the intention of using the conductivity measurement to check the validity of the various models, and if the exact solution was modified only enough so that no local temperature maximums existed in the wave one could not distinguish the two models by measuring only the conductivity, but a separate measurement of pressure or density variation would need to be made simultaneously.
Conclusions

The cases studied in this chapter have been very specialized in the sense of the ions being considered. The two possible wave structures used represent two extremes, one with no coupling between the shock and reaction zone; the other with strong coupling. Since the conductivity profile was found to be quite different for the two cases when no easily ionizable material was present, it may be possible to determine which model is nearest reality by observing the conductivity profile within the wave. When the salt was present, there was no noticeable difference in profiles for the two structures, except for the position of the conducting zone relative to the initial pressure rise.
CHAPTER IV

THE INDUCED MAGNETIC FIELD.

In chapter 1, a method of determining the induced magnetic field was briefly outlined. The results of the quasi-steady analysis which followed were expressed in the terms of the parameter \( \beta \), which it may be recalled was

\[
\beta = \frac{\mathbf{B}_0 \cdot \mathbf{b}_w}{\mu_0} \quad \text{(1.32)}
\]

where \( \mathbf{b}_w \) is the magnetic field vector resulting from induction due to the currents in the gas between points \( \alpha \) and \( \omega \) of the discontinuity type analysis. At that point, little was said about magnitude \( b_w \) might be expected to assume, since this requires some knowledge of the electrical conductivity within the wave. Using the results of Chap.3, where some possible electrical conductivities were computed, it will now be possible to say more about the value of \( b_w \).

The general case of non-zero electric field will be applied here. From eqn (1.12) and (1.13) then

\[
\mathbf{j}_0 = \nabla \mathbf{E}_0 \quad \text{(1.12)}
\]

\[
\text{curl} \quad \mathbf{B}_0 = -\frac{\partial \mathbf{B}_0}{\partial x} = \mu_0 \mathbf{j}_0 \quad \text{(1.13)}
\]

Thus there is a zero order perturbation in the magnetic field due to zero-order currents, caused by the zero-order electric field.

The first order effect becomes

\[
\mathbf{j}_1 = \nabla (\mathbf{E}_1 + \nabla \times \mathbf{B}_0)
\]

\[
= \nabla (\nu_\alpha \mathbf{B}_0 - \mu_\omega \mathbf{B}_0) = \nabla \mathbf{B}_0 (\nu_\alpha - \mu_\omega) \quad \text{(4.1)}
\]
The induced magnetic field will consist of two components, a zero and a first order term. Hence \( b = b_0 + b_1 \)

\[
\mathbf{b} = -\int \mathbf{\mu}_0 \mathbf{j}_1 \, dx - \int \mathbf{\mu}_0 \mathbf{j}_2 \, dx
\]  
(4.2)

Using the results of chap. 3, an estimate of the induced field will now be made.

**Induced Magnetic Field.**

The results of chap. 3 give conductivity in the wave as a function of \( \xi \), the dimensionless distance through the wave. Since eqn (4.2) requires an actual dimensional distance, it is necessary to transform back to real dimensions.

The function \( \xi \) is obtained from

\[
\xi = m \frac{\dot{C}_p}{\lambda} \int \frac{dx}{\lambda}
\]  
(4.3)

where \( m = \rho u \) = mass flow rate = constant

\( \dot{C}_p \) = specific heat at constant pressure, also constant

\( \lambda \) = thermal conductivity which varies with temperatures.

Differentiating, this expression becomes

\[
dx = \frac{\dot{C}_p}{m \dot{C}_p} \, d\xi
\]  
(4.4)

Using this in eqn. (4.2) then

\[
b_\omega = \frac{\mu_0}{m \dot{C}_p} \int_\omega \lambda \nabla E_\omega \, d\xi
\]

\[
= \frac{\mu_0 \lambda_\omega E_\omega}{m \dot{C}_p} \int_\omega \lambda \left( \frac{\nabla}{\nabla} \right) d\xi
\]  
(4.5)
Results and Discussions:

The solution of eqns. (4.5) and (4.6) are easily accomplished by numerical integration. Typical values of $\lambda$ for high temperature gases were obtained in ref. 15. The mass flow rate was determined from the examples considered in chap. 3, for $T_\infty = 4000^\circ$K, and $P_\infty = 60$ cmHg, a flow rate of $V = 21.8$ lbm/sec ft$^2$ was calculated. The value of $C_p$ used was $0.2948$ Btu/lbm$^\circ$F Values of $\lambda$ ranged from $8 \times 10^{-6}$ to $31.2 \times 10^{-6}$ Btu/sec$^\circ$F for the temperature range under consideration. Then using the values of $r/r_\infty$ for the seeded detonation calculation of chap. 3, eqn. (4.5) was integrated numerically to the point $\xi = 7$.

To correspond to the solution of chap. 1, the applied electric field was expressed in the terms of the magnetic field. Then for $\frac{V}{V_\infty} = 8$, and $B_0 = 1$ web/m$^2$, the zero order magnetic field induced the gas was found to be

$$b_{0w} = -5.8 \times 10^{-5} \ \text{web/m}^2$$

Since this is small compared to the applied magnetic field, it was neglected in the computation of the first order term. Using the same conditions as stated above, except that the electric field is now induced rather than
applied, the value obtained for the first order magnetic field was at \( \xi = 7 \), \( b/\omega = -3.83 \times 10^{-6} \) m/s.  \( (4.8) \)

From these figures it becomes obvious that the assumption made in chap.1, that the induced field was small compared to the applied field is good, within the wave even when a sizable electric field is applied. For a value of \( V_a \) of 2500 m/sec, this electric field corresponds to 200 volts/cm.

To determine the value of \( \beta \), using the induced fields given in eqn.(4.7) and (4.8), using \( p \) of 3 cmHg, after combining the two contributions, a value of \( \beta = -1.22 \times 10^{-2} \) was obtained. So looking again at the graph in appendix A, it becomes apparent that for this value of \( \beta \), the wave speed will be modified by less than 1/2%. So slight a variation would be difficult to detect.

The justification for not carrying the integration beyond \( \xi = 7 \) may be found in the discussion of boundary conditions, in chap.1. In reference to Chapman-Jouguet condition, it was assumed that the C-J point was realized at the point where energy addition just balanced energy loss due to dissipation. No attempt has been made to prove that this is at \( \xi = 7 \), but from the profile of reactants being depleted, it seems they are all gone well before \( \xi = 7 \).
Since intermediate reactions may still be going on however, a better indication of the completion of the reaction would be where the properties approach asymptotic values. Thus \( \xi = 7 \) seems a good estimate of proximity of the C-J point.

If the explanation by Zel'dovich, of how the C-J point is determined when dissipation phenomena exist, is correct, then it would seem that if the electro-magnetic fields added enough energy to overcome all dissipation, then the C-J point could not be realized while the wave was in the electro-magnetic field. When the wave first entered the field, however, the gases behind the wave were travelling at sonic speed relative to the wave front, so this condition may prevail at the entrance to the field, i.e. the gases enter the field at this same speed, so in effect, the C-J plane would remain at the entrance to the field. Thus the wave would become even wider and the wave speed could be predicted on the basis of the quasi-steady analysis, evaluating \( \delta \) from integration of the currents from the wave front to the entrance to the field, where the C-J point now remained. On this basis, one would be able to predict the wave speed at successive positions in the electro-magnetic field. The possibility of this above mentioned situation arising was the basis for terming the analysis of Chap.1, a quasi-steady analysis, for certainly in such
a case the change in time would be small compared to the changes in the flow direction. Similarly, when the structure is not significantly altered by the electro-magnetic field, the value of $\beta$ determined in a manner done here based on the structure of the ordinary wave, would predict the new detonation wave speed. Further since the structure for this case would possibly change slightly when the wave first entered the field, then not experience further change while the wave propagated through this field, the value of $\beta$ would reach a constant value, and the analysis of chap.1 would therefore predict another steady wave speed, slightly altered from the speed of the wave before it entered the field.

Another point of interest is that, if, as explained here, the C-J point remains at the entrance to the field, then the analysis of chap.2 becomes unapplicable and unnecessary. It would be unnecessary since the analysis of chap.1 would apply for the entire region in the field, and unapplicable since the nonsteady analysis was for the gases following the C-J plane in the electro-magnetic field, which for this case do not exist. When the wave structure is only slightly altered however, so that there is a region behind the C-J plane, within the electro-magnetic field, then the analysis as outlined in chap.2 becomes
of significance. For this case it might be expected to affect the wave speed by interactions from behind, if sufficiently large values for the fields were employed.
In the study of the electromagnetic phenomena in detonation waves, the electrical conductivity of the hot gases is of fundamental importance. Theory based on various assumptions, can predict electrical conductivities in regions of chemical and thermodynamic equilibrium. Thus the electrical conductivities in the detonation product gas can be predicted and checked by experiment as was done by Fay and Basu ref.8. The region within the detonation wave where the chemical reaction is occurring is so complex, and so little is known about the kinetics of the reaction, that any theoretical prediction is subject to considerable uncertainty, as was pointed out in chap.3.

Experimental measurements to date have been concerned generally with obtaining a maximum value of the electrical conductivity in the combustion gases, and due to the techniques, employed, have yielded some "average maximum" value corresponding to the conductivity of the combustion products. For a review of work done in this field see ref.9.

Interest in the electrical conductivity within the reaction zone was aroused by interesting results of the simplified analysis reported in chap.3, and was further stimulated by the results of the analysis in chap.1, which re-
quired a knowledge of the conductivity within the reaction zone. The interesting possibility of non-equilibrium ionization occurring within the wave was just another reason why it seemed important that some accurate, detailed measurements of the electrical conductivity within this region should be attempted.

Since such measurements had never been made in detonations it was necessary to critically examine the existing techniques available in an effort to determine the most suitable one for the problem at hand. The most successful, existing techniques have been critically examined by Lin et al, ref.12.

He chose to use a microwave transmission and reflection technique along with the magnetic induction technique which he had developed earlier, (ref.13). This combination proved successful, in spite of rather poor spatial resolution, since it was possible to obtain a wide shock zone by using very low initial pressures (0.02 ≤ \( p \leq 0.2 \text{mmHg} \)).

The magnetic induction technique was used by Basu, ref.4 to measure electrical conductivities in seeded detonation waves. Since results obtained by this method rely on a gas velocity interaction with the magnetic field, and since the local velocity is also unknown within the wave, a measure of the product of velocity and conductivity can be obtained at best.
The Langmuir probe was used by Kistiakowsky ref.11, to detect non-equilibrium ionization accompanying the acetylene-oxygen reaction in shock waves. Since the success of this work also depended largely on the utilization of low initial pressures, (about 1 Torr) and since it would be difficult to achieve steady state detonations at such low pressures it was decided to look for a more versatile method of attacking the problem.

A more sophisticated technique of magnetic interaction was introduced by Savic, ref.14. The high frequency coil produced a time varying magnetic field which could be used to measure electrical conductivities essentially independently of gas velocities. Since a static calibration could be used to determine spatial resolutions of this probe, it seemed that further investigations of its potential was warranted. It was decided to adopt this technique for further studies of the electrical conductivity within the detonation waves.

**Theory of Hydromagnetic Reactance.**

When an alternating current is made to flow through a coil, the inductance the latter presents to the current is governed by the electrical and magnetic properties of the coil core. If the core is made of conducting material, "image" currents will be induced in it which in turn react on the primary current in a direction tending to reduce the coil inductance.
The change in inductance is a non-linear function of core conductivity, permeability and diameter. Simultaneously, an apparent loss resistance appears which is a maximum for a certain value of core conductivity, decreasing thereafter until it disappears altogether at infinite core conductivity.

The change in inductance due to the presence of a conducting core is always small, except in the extreme case of the core having the same diameter as the coil, in which case the loss resistance attains prohibitively high values. However, with the aid of standard frequency discrimination equipment monitoring a self maintained tuned circuit, quite minute changes in reactance can be readily detected.

Since discriminators permit detection of steady frequency changes, the circuit may be calibrated statically by inserting samples of varying conductivity and noting deflection of meter or oscilloscope. Similarly the spatial resolution of the device can be determined by placing a sample of conducting material in various positions within the coil and noting the relative magnitudes of the output deflection. This is providing the carrier frequency is sufficiently high so that the effect of the moving gases in the shock or detonation is negligible in comparison to the effect of the time varying magnetic field.

**Inductance Change of a Coil.**

Since inductance is the consequence of the fact that
energy is reversibly transferred from the conductor to the magnetic field around it, any distortion of this field either by a change in the permittivity, conductivity or by hydro-magnetic means will react back on the circuit inductance.

Savic (ref.14) gave reference to a solution by Smythe (ref.16) of the change of inductance of a single-turn coil of radius \( a \) due to the presence of an infinitely long cylindrical core of radius \( b \), and permeability \( \mu \) equal to that of the surrounding medium (fig. 5-1), viz.

\[
\Delta L = -2 \mu a^2 \int_0^{\infty} I_1(kb) K_0(ka) dk = \mu(a+b)(\frac{a}{b})\left\{ (1-u^2)K - E \right\}
\]

where \( I_1 \) and \( K_0 \) are modified Bessel functions of order unity, \( K \) and \( E \) are the complete elliptic integrals with modulus \( u = 2(ab)^{1/2} (a+b) \). 

Fig. 5-1... Short Coil and Conducting Core.

The above formulas rest on two assumptions: (i) The conductivity of the core or applied frequency is so high that the image currents are confined to a thin skin on the surface of the core, the thickness of which is small compared with the core radius \( b \).

(ii) The radii \( a \) and \( b \) are of nearly equal magnitude so that all the field lines generated by the image currents may be assumed to link the coil.
Further, Eqn.(5-1) is the result of a Fourier cosine transform, giving recognition to the fact that the field is symmetrical around the plane of the coil.

Due to the above mentioned restrictions, and since no account has been taken of the hydromagnetic term introduced when there is motion of a conducting fluid in a magnetic field, Savic set about obtaining a more general solution, including the hydromagnetic term and for the case of finite conductivity.

For a more detailed development of the solution the reader is referred to ref.14. The general result obtained by Savic is obtained by determining the induced magnetic vector potential $A$, and integrating this over the space occupied by the primary current to obtain the change in inductance.

The solution takes the form

$$\Delta L = \frac{2\pi a}{I} \int_{-\infty}^{\infty} a_2 K_1|ka| dk$$

(5.2)

where $a$ is the coil radius

$I$ is the magnitude of the current in the coil. The current is given by $I \exp(jwt)$ but the exponential terms cancel out in the process of obtaining eqn.(5-2)

$$a_2 = \frac{I\mu_0 a}{2\pi} \left( k I_0(kb) \frac{I(b') - d'k I_0(d'b)}{d'k I_0(d'b) + k I_0(kb) I_0(d'b)} \right)$$

(5.3)

in which $K_1$ and $I_1$ are modified Bessel functions of order unity, $K_0$ and $I_0$ are the equivalent zeroth order functions; $b$ is the core radius; $k$ is the wave number resulting from the assumed form of solution of the vector potential, i.e.

$$A' = \exp(jwt + jkz)A$$
where $A'$ is the $A_0$ component of the vector potential; and $d'$ is defined as

$$d' = \left( k^2 + j\sigma \omega \nu k \right)^{1/2}.$$

The vector potential is nonsymmetrical about the plane $2 = 0$ (i.e. the plane of the coil) due to the hydromagnetic term, and consequently the solution must be obtained by integrating wave numbers over the whole core, as indicated by eqn.(5.2).

Implicit in eqn.(5.2) is the assumption that the core is infinite in extent, since the only boundary conditions imposed on the vector potential were at the outer radius of the core. Therefore this solution can be used to determine the conductivity of the core when the slug of conducting gas extends well beyond the plane of the coil, but a more sophisticated relation would be needed to calculate the conductivity at various stages through the slug, as the slug passed through the coil. It would seem that a better approach to determining the conductivity profile of a slug would be to actually calibrate the coil for this purpose by using known conductivity profiles. This was the approach attempted in this work, and a description of the results will follow in the sections discussing calibration of the probe.

**Circuit.**

The circuit of the frequency detecting equipment is a conventional i.f. amplifier limiter followed by a discriminator.
The oscillator is of the push-pull mutual-inductance coupled type. It derives its bias from a separate a.g.c. amplifier whose rectified and filtered output serves to stabilize the amplitude of the oscillator. As was shown by Savic, (ref.14) this variable bias control has a very marked effect on the frequency stability. This is important, as the oscillator has to work under conditions of large load variations. The automatic gain control thus makes the frequency of the oscillator largely independent of the purely resistive component of the load. Once the inductance change with conductivity of the core has been determined, the frequency change may be determined from LC product, provided the capacitance remains unchanged. This latter provision was apparently not realized in the work to be discussed here, and became a major cause of concern. The circuit diagram used for this probe and shown in appendix D-1, was obtained from Savic (ref.18).

The assembled circuit is shown in appendix D-2, along with the probe box mounted on the side of the plexiglass section of the shock tube. The coil consists of a two-turn, center tapped primary wound on the 2in. I.D., 2.5in. O.D. plexiglass shock tube, with a five turn secondary. The wire used was #26 formvar, and it was found that for this number of turns, it was necessary to push the coils very close together to achieve oscillation in the desired range of 10.7 mc. The overall coil length was about 1/8in.
Apparatus.

The shock tube used in this experimental work is the one described in ref.22, with a few modifications. Wu, ref.22 used the tube in ignition studies, and some of the gadgets he used were not required for detonation waves, and some instrumentation gadgets were added to simplify use of the tube.

The tube consists of a 42ft. length of 2in. I.D. steel pipe, divided into sections. For the work described here the first section was used as the driver section. At the closed end of this part, a spark plug was inserted to be used to ignite the combustible mixture. (See the sketch in appendix D-9). The short section containing the diaphragm puncturing mechanism, and the coupling to accommodate the diaphragm was attached at the end of the driver section. The remainder of the steel piping was next attached, followed by the test section at the very end.

Three spark plugs were fitted into the pipe wall just upstream of the test section. The first of these was appropriately connected to a battery and the oscilloscope to provide a trigger for the scope traces. The other two spaced 20.2cm apart were used to measure the wave speed, by means of a simple battery hook-up with the lower beam of the scope trace, (See appendix D-9).

A mixing tank was used to mix the combustible mixture before admitting it to the shock tube. The tank used here was 10in. I.D. extra strong steel pipe, 30in. long with flanges.
welded on both ends. A shaft was inserted through the center of the flanges, and the assembly mounted on bearings so that it could rotate freely. A length of stainless steel tubing was left inside the tank, so that when the shaft was rotated the tube would roll along inside the inside wall, thus helping the mixing process. Fittings to facilitate gas admission were inserted in the ends of the tank. The tank was designed for a maximum of 80 atm. pressure inside, so the combustible mixtures were kept to about 2 atm. or below, to allow a safety factor in case of accidental ignition.

The plexiglass test section was the one used by Wu. For all the runs made with detonation waves, the end of the section was rigidly closed. Shock waves were obtained to get at least a qualitative comparison with detonation waves regarding the direction of the output deflection of the probe. To obtain shocks of sufficient strength for such a comparison it was found most convenient to pump the tube down to a low pressure (p~0.1 in. Hg) and fill the driver section with detonable mixture at a pressure of about 1 atm. When this mixture was ignited, it easily burst the diaphragm separating the two sections, and produced shock waves having velocities comparable to the detonation velocities obtained. To prevent reflected shocks from the end of the test section building up the pressure to too high a value, the end was placed in the end, which removed and the diaphragm also burst easily upon impingement.
of the shock, producing a loud but harmless bang, and preventing reflected shocks from destroying the test section.

**Capacitative Coupling.**

Since the frequency of oscillation depends on the LC product, a change in the capacitance will also change the frequency. It has become apparent that with this probe, at conductivities at the order of electrolytes, the capacitance is increased, by the conducting core, more than the inductance is decreased. This results in a decrease of frequency which in turn reverses the direction of the output from what is obtained when a good conductor, such as metal is used as the core material.

Savic encountered similar complications, but he found that by shielding the coil from the core with a very thin metallic film he was able to solve his difficulties. The principle is to have an adequate film to effect electro-static shielding, but not so small that it would prevent the magnetic field of the coil from penetrating the core. All attempts to achieve this with the probe used here, failed to produce complete shielding. It was noted, however, that with partial shielding, that the output obtained was related to the conductivity of the core material, although the output deflection was in the opposite direction to that obtained with a metallic conductor. A brief description of the various attempted solutions to the shielding problem will now be given.

The first attempts to calibrate the probe were made with no shielding of any kind. Electrolytic solutions were used
as the known conductor, but it was noted immediately that
the output deflection was up. (The deflection was down for
metal sample, and to reduce repetition, the discussion will
be conducted assuming the reader can visualize the output
deflection on the oscilloscope face being down for a metal,
and up for the cases considered, using relatively poor
conductor, such as the electrolytic solution or the detonation
and shock waves obtained). It may be noted also, that when
a human hand was brought near the coil from the outside, that
the deflection was also up.

A first attempt at shielding was putting aluminium
foil inside the tube. The method used to attach it was inade-
quate, however, and the foil was easily destroyed by the
passing detonation wave.

A second method was up to paint the inside of the tube
with a metallic paint. The paint used was cold setting
silver #2, supplied by Engelhard, Hannovia Liquid Gold
Division. As the name implies, it is cold setting which
was the primary reason for using it. Considerable difficulty
was experienced in getting the paint to adhere to the plexiglass
tube walls. This problem was partially solved by roughening
the tube surface with sand paper. Some of the paint, how-
ever, remained loose enough that small particles came off
when the detonation passed, thus causing contamination of
the flow, as well as changing the film thickness, which is
further detected by the probe and is undesirable.

A third attempt was made, this time having a metallic film deposited in a glass pipe. A very thin stannic oxide film was deposited on both the inside and outside of the tube. (This work was done by Joe Palmero at Lincoln Lab.). Using this shielding the deflections were still up, but the calibration with the salt solutions gave a consistent relationship between output and conductivity of the electrolyte. Unfortunately, the pipe was broken in the process of mounting it on the shock tube, so no results with detonations were obtained.

Another attempt is now being made using a shielding of Teledeltos paper, wrapped around the outside of the tube. Little information has been obtained by this method to date.

Calibration.

The purpose of developing this probe was to be able to measure the electrical conductivity distribution within a detonation wave. The method of calibration will therefore be designed to answer the questions as to how the output is related to the conductivity of the material in the core, and how the output as a function of time is related to the conductivity of the slug, passing through the coil, as a function of position through the slug.

Although shielding failed to eliminate capacitive effects completely, it appeared that the effect measured
by the probe was dependent on the conductivity of the core. Since the theory developed by Savic was for inductance change only, it was decided that the only seemingly valid method of calibration would be to use samples of conducting material having conductivities in the range in which the probe would be used. Then even if the effect was not derived solely from the inductance changes, the required information regarding the relation between conductivity of the core and the probe output would be known. To accomplish this, samples of electrolytic solutions were prepared from data in ref. (19) to have conductivities ranging from $10^{-3}$ to $10^{-1}$ mho/cm.

The tube having no shielding was never fully calibrated by this technique since at the time the major concern was directed toward obtaining output deflections in the same direction as obtained from a metallic conductor. One electrolytic sample was used however having a conductivity circa $10^{-2}$ mho/cm. This solution gave an output deflection, up, of 2.15 volts, compared to 1.5 volts, up for distilled water. The sensitivity was reduced in the shielded tubes.

The glass tube having stannic oxide film was more completely calibrated. The results of this calibration are shown in appendix D-3, where the deflection on the scope is plotted against the conductivity of the core. These results were reproducible to good agreement using the same solutions.
again and again. The relationship is plotted on a log-log plot in D-4, where it is found that the relationship between output voltage \( V \) in millivolts and core conductivity \( \nu \) in mho/cm is, \( V \propto \nu^{0.44} \).

This relation was arrived at by plotting both quantities having specific dimensions. Since functional relations become comparable only when determined from dimensionless plots, there is little value in comparing this with the theoretical results for low conductivities obtained by Savic. It may be of interest, in future work with this probe to replot appendix D-4 using dimensionless quantities to see how the results compare with the theory.

It has not been demonstrated in this work that the change of frequency is a purely inductive effect, but as long as one is satisfied that the change of frequency is caused by the conductivity of the core, whether this affects the capacitance more than the inductance, or not, is of little consequence. The primary concern is to obtain a relationship between the output voltage and the core conductivity, and, as this has been accomplished for this case, the necessity of perfect shielding conditions seems to have been bypassed.

**Spatial Resolution of the probe.**

The detailed solution of the fields in the core as the core advances through the coil, and the subsequent de-
termination of the inductance change resulting, would be a very complicated undertaking. Instead, an intuitive approach was used to try to determine the spatial resolution of the probe.

If a current filament exists alone at a distance \( z \) from the coil, it will cause a certain inductance, or capacitance change in the coil. The current in this filament has been induced due to a changing magnetic field associated with the current in the coil. When more than one such filament exists at various places in the field, each will contribute to the change observed in the coil. When the current in the individual filaments is small compared to that in the coil, it will not cause a substantial modification in the fields around it, and the currents in all the other filaments will be essentially independent of each other, but depend on the driving current in the coil. In such a case, it would be expected that the total effect measured at the coil would be the sum of the effects of each current element, and this effect would be a weighted sum since current elements at various positions in the
field would have different amount of effect on the coil. This weighting function could be determined by devising an experiment to advance a current filament through the coil and noting the change of frequency as a function of position of the filament. Complete success of this approach relies on the assumption that the field is not perturbed by the filament, and it would be exact only in that limiting case. This would be true to good approximation, however, in case of interest here where the skin depth of the plasma to be measured is much greater than the dimensions of the coil, so the field will penetrate with ease.

The difficulty comes in, in finding a material to use as a conductor for the current filament which will produce a measurable output without distorting the field too much.

The material used here was a disc cut out of aluminium foil, .00075 in. thick, and attached at the end of a wooden dowel which just fit inside the tube. The change of frequency caused by this disc was so great that the filters in the circuitry cut it off before it reached the output. The change in frequency was noted however, by displaying the frequency, as taken from the "mixer out" terminal on the circuit, on the Tektronix 555 scope, and algebraically adding a constant 10.7mc. In this way, the beats appeared on the
scope and were counted to determine the change of frequency. The result of such a calibration is plotted in appendix D-5.

Now that a weighting function has been obtained, it is possible, on the basis of the assumptions outlined in this section to predict the type of output resulting from the slugs of various conductivity profile advancing through the coil. A simple model was chosen and the results plotted in appendix D-6. The simple case of $C = 1$ (i.e. a single step change in conducting profile) is realized in the calibration when a sample of electrolytic solution is advanced through the coil. It is therefore of interest to compare at least this case with some measured values obtained with salt solutions.

The first one to consider is shown in appendix D-7. From this one it can be seen that the output as predicted from the above model is not as steep as is actually measured for the tube having no shield. Appendix D-8 shows another comparison, this one taken in the tube having stannic oxide shield. This one shows the model predicting a steeper output gradient than was actually measured. The reason for this may be that the shield causes the field to be modified near the wall in such a way as to give a frequency change when conducting fluid is still considerably further from the coil than would be expected on the basis of the calibration. This may be further evidence of the ineffectiveness of this
shield since it appears that this is a purely capacitive effect.

The other curves in D-6 show that it would be difficult to detect very small, and very localized conductivity changes, even on the basis of this idealized model.

Results.

The results obtained from this probe to date have been rather inconclusive. The difficulty experienced with the shielding caused considerable delay and ultimate uncertainty in the data.

Three examples of data obtained are shown in appendix D-9. The first picture was taken when the tube had no shield. The calibration at that time was incomplete, but the one known value was for an output of 2.15 volts, the -2 core had a conductivity circa 10 mho/cm. This corresponds closely to the first plateau on the probe trace, so that a rough check of the conductivity of this detonation wave can be made.

The second and third pictures are meant merely as a qualitative comparison between a detonation wave and a shock. Note that these are both measured with the same shielding conditions, whereas the first picture was for a tube with no shield. For the second picture the end of the test section was closed so a reflected shock may partially account for the second rise, whereas the third picture was taken when a diaphragm was used in the end of the tube. This diaphragm
broke when struck by the wave so no shock would be reflected in this case. The two traces are initially quite similar here, although some of the other results seemed to show that the rise was considerably slower in the shock than in the detonation wave.

The results of the calibration in the shielded glass tube give rise to some optimism concerning the usefulness of the probe. The seemingly poorer spatial resolution for this tube is somewhat discouraging however, Perhaps, to get better spatial resolution it will be necessary to use a shield on the outside of the tube only, since this may cause less axial spreading of the field within the core.

Recommendations.

The most important problem, still not completely solved for this probe is that of shielding. Once that is satisfactorily solved, further use of the probe seems promising.

The method used to obtain spatial resolution of this probe was not completely satisfactory since the aluminium foil caused a noticeable distortion in the field. This was noted by placing two discs close together, and it was noted that the combined effect was not nearly as great as the sum of the two acting completely independently. So a sample of conducting material of considerably lower conductivity, or at least considerably thinner would be advisable.
The shielding noticeably reduced the sensitivity of the probe, so that the output deflections were rather small. Thus the drift in output signal which was negligible when no shield was used became of accountable size compared to the desired deflection since the desired signal was now much smaller. This implies that it may be necessary to use better quality resistors in the circuitry since it is suspected that the drift in the signal may be caused by the changing bias conditions due to the changing resistance of the low quality resistors.

Conclusions.

It seems the probe described in this report could be used to measure the conductivity profile in a detonation wave. Adequate shielding of the coil presents a problem, but not an insolveable one. The lack of dependence of the output on the velocity of the core material makes static calibration possible, which is a great advantage over previous techniques. The calibration further indicates that the conductivity profile is not obtained directly from the output signal, but can be determined by using the weighting function, provided the weighting function is determined correctly.
References.


17. McLachlan, Bessel Functions for Engineers,
Oxford Univ. Press. London, 1941

   Magnetohydrodynamic Effects on Exothermal Waves.
   J. Aero Space Sciences, Vol.27 No.4 April 1960
22. Wu, P. Cheng-Kang, A shock tube for the study of
   ignition in fuel-air mixtures.
Graphical Relation Between the Induced Magnetic Field Parameter $\beta$ and the Change in Wave Speed

\[ \frac{v}{v_0} = 6 \]

\[ k = 1.25 \]

\[ M = 1 \]

\[ \frac{v}{v_0} = 0 \]

\[ \frac{v}{v_0} = 1 \]

\[ \frac{v}{v_0} = -1/4 \]

\[ \frac{v}{v_0} = -2 \]

\[ \frac{v}{v_0} = -4 \]

\[ \frac{v}{v_0} = -8 \]
X-t DIAGRAM FOR DETONATION IN ELECTRO-MAGNETIC FIELD

\[ E/B = 10,000 \text{ m/sec.} \]
\[ B = 1 \text{ web./m}^2 \]
\[ \nabla = 1 \text{ mho/m} \]

APPENDIX B-1
**APPENDIX B-2**

*X-t DIAGRAM FOR DETONATION IN ELECTRO-MAGNETIC FIELD*

- \( E/B = -10,000 \text{m/sec} \)
- \( B = 1 \text{ weber/m}^2 \)
- \( \nabla = 1 \text{ mho/m} \)

Steady State Wave Front

Induced Current (I) Characteristic

Modified I Characteristic
**IONIZATION OF ARGON.**  
**APPENDIX (C-I)**

\[ x = \frac{5}{12} = .416 \quad \sqrt{x} = 0.646 \]

\[ q_1 = 15.69 \text{ eV} \]

\[ T_\infty = 4000 \text{ °K} \]

\[ \frac{q_1}{2kT_\infty} = 22.7 \]

<table>
<thead>
<tr>
<th>( \xi )</th>
<th>( \Theta )</th>
<th>( \frac{P}{P_\infty} )</th>
<th>( \frac{1}{\Theta} - 1 )</th>
<th>( \Theta^{3/4} )</th>
<th>( (\frac{P}{P_\infty})^{\frac{5}{4}} )</th>
<th>( \frac{q_1}{2kT_\infty} \left( \frac{1}{\Theta} - 1 \right) )</th>
<th>( \frac{\nabla}{\nabla_\infty} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>.18</td>
<td>.11</td>
<td>5.5</td>
<td>.267</td>
<td>.33</td>
<td>- 125</td>
<td>0</td>
</tr>
<tr>
<td>-3</td>
<td>.3</td>
<td>.2</td>
<td>3.3</td>
<td>.405</td>
<td>.45</td>
<td>- 75</td>
<td>0</td>
</tr>
<tr>
<td>-2</td>
<td>.5</td>
<td>.4</td>
<td>1.0</td>
<td>.595</td>
<td>.45</td>
<td>22.7</td>
<td>10</td>
</tr>
<tr>
<td>-1</td>
<td>.7</td>
<td>.95</td>
<td>.43</td>
<td>.765</td>
<td>.97</td>
<td>9.26</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>.9</td>
<td>1.7</td>
<td>.10</td>
<td>.94</td>
<td>1.3</td>
<td>2.27</td>
<td>.075</td>
</tr>
<tr>
<td>+1</td>
<td>1.0</td>
<td>1.5</td>
<td>0</td>
<td>1.0</td>
<td>1.22</td>
<td>0</td>
<td>0.82</td>
</tr>
<tr>
<td>+2</td>
<td>1.05</td>
<td>1.4</td>
<td>~0.055</td>
<td>1.037</td>
<td>1.18</td>
<td>1.2</td>
<td>2.91</td>
</tr>
<tr>
<td>+3</td>
<td>1.05</td>
<td>1.3</td>
<td>~0.55</td>
<td>1.037</td>
<td>1.14</td>
<td>1.2</td>
<td>3.02</td>
</tr>
<tr>
<td>+4</td>
<td>1.05</td>
<td>1.28</td>
<td>~0.55</td>
<td>1.037</td>
<td>1.13</td>
<td>1.2</td>
<td>3.04</td>
</tr>
<tr>
<td>+5</td>
<td>1.02</td>
<td>1.22</td>
<td>~0.02</td>
<td>1.015</td>
<td>1.1</td>
<td>0.45</td>
<td>1.57</td>
</tr>
<tr>
<td>+6</td>
<td>1.01</td>
<td>1.20</td>
<td>~0.01</td>
<td>1.01</td>
<td>1.1</td>
<td>0.22</td>
<td>1.14</td>
</tr>
<tr>
<td>+7</td>
<td>1.0</td>
<td>1.18</td>
<td>0</td>
<td>1</td>
<td>1.09</td>
<td>0</td>
<td>0.92</td>
</tr>
</tbody>
</table>

**Calculation of \( \nabla_\infty \) For Argon**

\[
\alpha = \frac{1/2}{x + X_G T} \left( \frac{5/4}{1/2} \right) \exp \left( \frac{q_1}{2kT_\infty} \right) = (0.646)(1.73 \times 10^{-2}) (4000)^{5/4} \exp^{-22.7}
\]

\[ = 6.7 \times 10^9 \]
\[ \nabla_\infty = \frac{3.84 \times 10^{-10}}{\sqrt{T} \xi_j Q_j} = 3.84 \times (6.7)^{10^{-19}} = 2.04 \times 10^{-3} \text{ mho/cm} \]
IONIZATION OF POTASSIUM

\( x = 0.00001 \)

\( q = 4.318 \text{ V} \)

\( P_{\infty} = 60 \text{ cm Hg} \hspace{1cm} T_{\infty} = 4000^\circ \text{ K} \)

FOR HIRSCHFELDERS EXACT SOLUTION.

<table>
<thead>
<tr>
<th>( \xi )</th>
<th>( \Theta )</th>
<th>( P/P_{\infty} )</th>
<th>( T^\circ \text{K} )</th>
<th>( p \text{ cm Hg} )</th>
<th>( K_e (\text{a}^3) )</th>
<th>( 4xP (10^{-6}) )</th>
<th>( \alpha (10^{-6}) )</th>
<th>( \nabla \text{mho/cm} )</th>
<th>( \nabla )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.4</td>
<td>0.18</td>
<td>0.11</td>
<td>720</td>
<td>6.6</td>
<td>2.75(-30)</td>
<td>0.35</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-1.3</td>
<td>0.3</td>
<td>0.20</td>
<td>1200</td>
<td>12</td>
<td>2.57(-18)</td>
<td>0.63</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-1.2</td>
<td>0.5</td>
<td>0.40</td>
<td>2000</td>
<td>24</td>
<td>4.57(-10)</td>
<td>1.26</td>
<td>1.72</td>
<td>5.1x10</td>
<td>1.7x10</td>
</tr>
<tr>
<td>-1.1</td>
<td>0.7</td>
<td>0.95</td>
<td>2800</td>
<td>57</td>
<td>1.58(-6)</td>
<td>3.0</td>
<td>3.65</td>
<td>1.32</td>
<td>0.433</td>
</tr>
<tr>
<td>0</td>
<td>0.9</td>
<td>1.7</td>
<td>3600</td>
<td>107</td>
<td>1.91(-4)</td>
<td>5.36</td>
<td>9.36</td>
<td>3.0</td>
<td>0.98</td>
</tr>
<tr>
<td>+1</td>
<td>1.0</td>
<td>1.5</td>
<td>4000</td>
<td>90</td>
<td>1.02(-3)</td>
<td>4.72</td>
<td>10</td>
<td>3.05</td>
<td>1</td>
</tr>
<tr>
<td>+2</td>
<td>1.05</td>
<td>1.4</td>
<td>4200</td>
<td>84</td>
<td>2.40(-3)</td>
<td>4.42</td>
<td>10</td>
<td>2.96</td>
<td>0.97</td>
</tr>
<tr>
<td>+3</td>
<td>1.05</td>
<td>1.3</td>
<td>4200</td>
<td>78</td>
<td>2.40(-3)</td>
<td>4.11</td>
<td>10</td>
<td>2.96</td>
<td>0.97</td>
</tr>
<tr>
<td>+4</td>
<td>1.05</td>
<td>1.28</td>
<td>4200</td>
<td>76.7</td>
<td>2.40(-3)</td>
<td>4.03</td>
<td>10</td>
<td>2.96</td>
<td>0.97</td>
</tr>
<tr>
<td>+5</td>
<td>1.02</td>
<td>1.22</td>
<td>4080</td>
<td>73.2</td>
<td>1.26(-3)</td>
<td>3.85</td>
<td>10</td>
<td>2.91</td>
<td>0.954</td>
</tr>
<tr>
<td>+6</td>
<td>1.01</td>
<td>1.20</td>
<td>4040</td>
<td>72</td>
<td>9.8 (-4)</td>
<td>3.79</td>
<td>10</td>
<td>2.90</td>
<td>0.95</td>
</tr>
<tr>
<td>+7</td>
<td>1.18</td>
<td>1.18</td>
<td>4000</td>
<td>71</td>
<td>9.2 (-4)</td>
<td>3.74</td>
<td>10</td>
<td>2.88</td>
<td>0.945</td>
</tr>
</tbody>
</table>

( * ) Term in brackets is the power of ten to which the number preceding it is raised \( (2.75(-30) = 2.75\times10^{-30}) \)
IONIZATION OF ARGON  
APPENDIX (C-3)
FOR HIRSCHFELDERS ZEROTH ORDER SOLN.

<table>
<thead>
<tr>
<th>$\xi$</th>
<th>$\Theta$</th>
<th>$P/P_\infty$</th>
<th>$\frac{1}{\Theta} - 1$</th>
<th>$\Theta^{3/4}$</th>
<th>$(P/P_\infty)^{1/2}$</th>
<th>$-\frac{q}{2\pi k T_\infty} (\frac{1}{\Theta} - 1)$</th>
<th>$\Sigma/\Sigma_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>0.1</td>
<td>0.05</td>
<td>9</td>
<td>.178</td>
<td>.222</td>
<td>-204</td>
<td>5 0</td>
</tr>
<tr>
<td>-4</td>
<td>0.15</td>
<td>0.08</td>
<td>5.75</td>
<td>.240</td>
<td>.282</td>
<td>-130</td>
<td>5 0</td>
</tr>
<tr>
<td>-3</td>
<td>0.23</td>
<td>0.15</td>
<td>3.35</td>
<td>.332</td>
<td>.386</td>
<td>-76</td>
<td>5 0</td>
</tr>
<tr>
<td>-2</td>
<td>0.30</td>
<td>0.30</td>
<td>2.32</td>
<td>.405</td>
<td>.55</td>
<td>-53.4</td>
<td>5 0</td>
</tr>
<tr>
<td>-1</td>
<td>0.45</td>
<td>1.2</td>
<td>1.21</td>
<td>.55</td>
<td>1.1</td>
<td>-27.5</td>
<td>5 0</td>
</tr>
<tr>
<td>0</td>
<td>0.48</td>
<td>1.95</td>
<td>1.08</td>
<td>.577</td>
<td>1.4</td>
<td>-24.5</td>
<td>5 0</td>
</tr>
<tr>
<td>1</td>
<td>0.50</td>
<td>1.95</td>
<td>1.0</td>
<td>.595</td>
<td>1.4</td>
<td>-22.7</td>
<td>5 0</td>
</tr>
<tr>
<td>2</td>
<td>0.55</td>
<td>1.85</td>
<td>0.81</td>
<td>.638</td>
<td>1.36</td>
<td>-18.4</td>
<td>5 0</td>
</tr>
<tr>
<td>3</td>
<td>0.68</td>
<td>1.80</td>
<td>0.47</td>
<td>.749</td>
<td>1.35</td>
<td>-10.7</td>
<td>1.2 x 10^{-5}</td>
</tr>
<tr>
<td>4</td>
<td>0.90</td>
<td>1.40</td>
<td>0.11</td>
<td>.924</td>
<td>1.18</td>
<td>2.5</td>
<td>.0642</td>
</tr>
<tr>
<td>5</td>
<td>1.00</td>
<td>1.20</td>
<td>0</td>
<td>1</td>
<td>1.10</td>
<td>0</td>
<td>.91</td>
</tr>
<tr>
<td>6</td>
<td>1.00</td>
<td>1.10</td>
<td>0</td>
<td>1</td>
<td>1.05</td>
<td>0</td>
<td>.95</td>
</tr>
</tbody>
</table>

Note: The calculation of $\Sigma_\infty$ will be identical to that in C-1, and the values listed above the table C-1 are also used for the computations in this table.
IONIZATION OF POTASSIUM
FOR HIRSCHFELDER'S ZERO TH ORDER SOLN.

APPENDIX (C-4)

<table>
<thead>
<tr>
<th>ξ</th>
<th>Θ</th>
<th>P/P₀</th>
<th>T*K</th>
<th>P cm.Hg.</th>
<th>Kₑ (a)</th>
<th>4xp atm. (10⁻⁶)</th>
<th>α (10⁻⁶)</th>
<th>T mho/cm</th>
<th>T/T₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>-.3</td>
<td>.23</td>
<td>.15</td>
<td>920</td>
<td>9</td>
<td>1.45(-23)</td>
<td>.474</td>
<td>364(-9)</td>
<td>2.28x10⁻⁶</td>
<td>.975</td>
</tr>
<tr>
<td>-.2</td>
<td>.30</td>
<td>.30</td>
<td>1200</td>
<td>18</td>
<td>2.57(-18)</td>
<td>.95</td>
<td>16.5(-6)</td>
<td>5.83x10⁻⁶</td>
<td>1.9x10⁻³</td>
</tr>
<tr>
<td>-.1</td>
<td>.45</td>
<td>1.2</td>
<td>1800</td>
<td>72</td>
<td>3.63(-11)</td>
<td>3.79</td>
<td>1.95(-2)</td>
<td>8.73x10⁻³</td>
<td>2.8x10⁻³</td>
</tr>
<tr>
<td>0</td>
<td>.48</td>
<td>1.95</td>
<td>1920</td>
<td>117</td>
<td>2.34(-10)</td>
<td>6.17</td>
<td>3.9(-2)</td>
<td>1.71x10⁻³</td>
<td>5.6x10⁻⁴</td>
</tr>
<tr>
<td>1</td>
<td>.50</td>
<td>1.95</td>
<td>2000</td>
<td>117</td>
<td>4.57(-10)</td>
<td>6.17</td>
<td>5.46(-2)</td>
<td>.235x10⁻²</td>
<td>.0077</td>
</tr>
<tr>
<td>2</td>
<td>.55</td>
<td>1.85</td>
<td>2200</td>
<td>111</td>
<td>9.5(-9)</td>
<td>5.83</td>
<td>0.254</td>
<td>0.104</td>
<td>.034</td>
</tr>
<tr>
<td>3</td>
<td>.68</td>
<td>1.80</td>
<td>2720</td>
<td>108</td>
<td>1.29(-6)</td>
<td>5.67</td>
<td>2.65</td>
<td>0.975</td>
<td>.32</td>
</tr>
<tr>
<td>4</td>
<td>.90</td>
<td>1.40</td>
<td>3600</td>
<td>84</td>
<td>1.91(-4)</td>
<td>4.42</td>
<td>9.5</td>
<td>3.05</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1.00</td>
<td>1.20</td>
<td>4000</td>
<td>72</td>
<td>1.02(-3)</td>
<td>3.79</td>
<td>10</td>
<td>3.05</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1.00</td>
<td>1.10</td>
<td>4000</td>
<td>66</td>
<td>1.02(-3)</td>
<td>3.48</td>
<td>10</td>
<td>3.05</td>
<td>1</td>
</tr>
</tbody>
</table>

(a) Term in brackets is the power of ten to which the number preceding it is raised (1.45(-23) = 1.45 x10⁻³)

The values tabulated at the top of table C-2 were also used in the calculation for this table.
CONDUCTIVITY PROFILES
Corresponding to Exact Solution of Detonation Wave Structure

\( \infty \) refers to values at the hot boundary

APPENDIX C-5
CONDUCTIVITY PROFILES
Corresponding to Zeroth Order Solution of Detonation Wave Structure

Shock Zone

Reaction Zone

\( \frac{\gamma}{\gamma_{\infty}} \)

\( \frac{P}{P_{\infty}} \)

\( \chi \)-mole fraction reactant

\( \frac{T}{T_{\infty}} \)

\( \frac{T_{f}}{T_{\infty}} \)

\( \frac{T_{f}}{T_{\infty}} \text{ near} \)

\( \frac{T_{f}}{T_{\infty}} \text{ hot boundary} \)

Refers to values at the hot boundary

Dimensionless Distance

APPENDIX C-6
CONDUCTIVITY PROBE CIRCUITRY
FRONT VIEW
PROBE CIRCUIT ASSEMBLY
BOTTOM VIEW

OSCILLATOR CIRCUIT
PROBE BOX MOUNTED ON PLEXIGLASS TEST SECTION and SHOWING COIL WOUND ON TUBE

APPENDIX D-2
This calibration is for the probe mounted on the glass pipe, which had the Stannic Oxide shield.
APPENDIX D-4

Log.-Log. Plot of Voltage output vs. Conductivity.
Using the same data as in appendix D-3.

\[ \ln V = k \ln \tau \]
\[ k = \frac{8}{15.2} = 0.44 \]

Therefore:
\[ V \propto \tau^{0.44} \]
WEIGHTING FUNCTION
Probe frequency change with varying position of a metallic wafer (current filament) inside the coil.

APPENDIX D-5
APPENDIX D-6

Output Expected as Slugs of various conductivity profile, (see model below) advance through the coil, based on the response plotted in appendix D-5.
Comparison of output response as obtained from the Current Filament Model and actual Calibration with Electrolytic Conductor in the plexiglass tube having no shield.

Actual measurements with a salt soln.

Theoretical response based on current filament model

Normalized voltage output

Height of soln. in in.
Comparison of output response as obtained from the Current Filament Model and actual Calibration with Electrolytic Conductor in the Glass Tube with Stannic Oxide Shield

Actual measurements with salt solns.

Theoretical response based on current filament model

APPENDIX D-8
Driver Section

Spark Initiation of Detonation

End Rigid for
No. 1 and No. 2,
Diaphragm in End
for No. 3.

Discriminator

SCOPE

Trig.

Upper Beam

Lower Beam

Oscillator Spark plugs

2 ft. 42 ft.

DETONATION WAVE

Stoichiometric Mix

\[ 2C_2H_2 + 5O_2 \]

3 in. Hg. initial press.

Probe: Upper Beam
Sens: 2 volt/cm.
Sweep Rate: 0.1 ms/cm.

Vel. Trace: L. Beam

\[ V_1 = 1820 \text{ m/sec.} \]

No Shield

(Traces swept twice giving
2 extra lines)

DETONATION WAVE

Stoichiometric Mix

\[ 2C_2H_2 + 5O_2 \]

3 in. Hg. initial press.

Probe: Upper Beam
Sens: 1 volt/cm.
Sweep Rate: 0.1 ms/cm.

Vel. Trace: L. Beam

\[ V_1 = 2410 \text{ m/sec.} \]

With Silver Paint Shield

SHOCK WAVE in AIR

Initial press. 0.1 in. Hg.

Probe: U. Beam, 1 volt/cm.
Sweep Rate: 0.1 ms/cm.

\[ V_2 = 2770 \text{ m/sec.} \]

With Silver Paint Shield

APPENDIX D-9