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USES OF TECHNOLOGY TO ENHANCE EDUCATION

by

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This paper is the substance of a proposal to the N.S.F. for support of research on children's thinking and elementary education.

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The basis of organization into the four sections is:

Section 1:	Schematic outline of project and what we want. Hardly any intellectual content.
Section 2:	Statement of our goals in general terms. This statement is intended to have serious intellectual content but lacks meaty examples. Readers who find it too abstract for comfort might like to read at least part of §3 first.
Section 3:	A series of extended examples intended to give more concrete substance to the generalities in §2.
Section 4:	This is the real "proposal". It sets out specifically a list of concrete "goals" on which we want to work in the immediate future.
Appendix:	Papers by Jeanne Bamberger, Marvin Minsky, Seymour Papert and Cynthia Solomon.

Section 1

WHAT WE WANT

1.1 We Want Time

Over the past five years a certain style of research on Elementary Education has developed within the Artificial Intelligence Laboratory at M.I.T. The funding of the research has been on a relatively short erm and project-oriented basis. We feel that the work has matured to a point that requires somewhat longer term planning and that the research style has proven itself sufficiently to justify this confidence. The need for longer term stability comes from three sources:

1.1.1 People Need Time To Develop

A key feature of the research style is a much deeper than usual melding of competence and creativity from different areas, such as mathematical sciences/cognitive science/computer science/educational practice. In order for this to take place satisfactorily, individuals need to be immersed in the project for a sufficient period of time. We feel that less than a three year period is inadequate.

1.1.2 Ideas Need Time to Mature

Our most successful concepts, such as LOGO itself and Turtle Geometry, have typically taken about three years to progress from inception to maturity.

1.1.3 Children Need Time to Develop

We need badly to study the effects of exposing children to our learning environments over larger periods of time. We have often noticed in working with children over periods of a half or whole year that their development into the kind of thinking we try to foster goes at an increasing rate over the period.

1.2 We Want More People

We have demonstrated the richness for Education of a thorough integration of imaginative competence in several fields. Our team is still below critical mass for this. We believe that three new research associates (post-doctoral) and more research assistants (graduate students) would make a vastly more than proportional pay-off in productivity and in writing about what we produce. The possibility of attracting high calibre people in these categories has been enhanced by the recent formation at M.I.T. of an "Education Division" which will make it much easier for graduate students at M.I.T. to choose Research in Education as their primary academic focus. This research project is very closely associated with the new Division through overlap of people and of intellectual focus.

1.3 We Want a Workshop

We have built some effective educational devices using general facilities of the Artificial Intelligence Laboratory. Several factors make this arrangement increasingly unsatisfactory and we feel that this aspect of the project has proven itself sufficiently to deserve its own facility.

1.4 We Want Our Own Experimental School

We have been able to develop ideas and materials through working with children "lent to us" for a few hours a week by their normal public school. We are ready to move to the next stage of testing how the intellectual development of children will respond to a totally redesigned learning environment.

Section 2

We Want to Make Progress on the GENERAL Goals Described Below

The sub-sections numbered 2.x will convey the form of our research style and the structure of general subgoals on which work is either actively in progress or in an advanced stage of planning. Each section describes in general terms a kind of research goal, illustrated, where possible, by examples drawn from our previous work.

> More specific information will be found in subsequent sections (3 & 4). Although we claim the right and the need to work in an open-ended spirit, we do, of course, have a clear and very specific map of what we shall be doing in the immediate future. But to communicate one's reasons for thinking there is more gold in those hills is a much more complicated and uncertain business than to show the nuggets one has already found. However we must try. Section 4 will sketch specific research projects in progress or advanced states of planning. Sections 2 and 3 will develop enough of our philosophy of education to provide a perspective for these projects.
>
> The research plans mentioned in section 4 contain some projects we would not be able to

carry out entirely with funds requested in this proposal. We include them because they contribute to the conceptual coherence of our plans and because they do overlap projects for which we are here seeking support from the N.S.F.

The general goals will be described in this section under the following sub-headings:

2.1 Compelling Examples of the Uses of Technology to Enhance Education.
2.2 New Conceptualizations of Knowledge
2.3 A Cognitive Theory About and FOR Children
2.4 Studying Heuristics
2.5 Relationship of Our Work to Schools
    2.5.1 Material Suitable for Use Within the Traditional Schoo
    2.5.2 New Concepts of "School"
    2.5.3 Intellectual Centers Parallel to Schools
    2.5.4 Opening Avenues for the Severely Physically Handicappe
    2.5.5 Remedial Mathematics for Adults
    2.5.6 Paradigm for Experiments on Developmental Psychology
2.6 A New Kind of Professional for Research on Education
2.7 Mundane Aspects of Computers

2.1 Compelling Examples of the Uses of Technology to Enhance Education

One of our explicit goals is to provide compelling examples to show how technology can be used in education more profoundly and more imaginatively than has previously been done. It is frequently asserted that informational technology ought in principle to be a great boon to education but that current ideas on how to use it are insufficiently developed if not frankly superficial. (See for example the Carnegie Commission Report: "The Fourth Revolution"; James Koerner's recent article in Saturday Review and many others.) We offer our work as an example to show that while this stricture might apply to current practice in C.A.I., the bottleneck to progress is not a lack of ideas.

The main thrust of our examples has been to show that the experience of program controlled devices can be used to give children, to a quite unprecedented degree, a sense of the power of ideas in general, of science in particular, and especially of mathematical science. In a suggestive aphorism we might say: we have been able to give children a mathematical experience more like an engineer's than like a bookkeeper's.

To do this it is, of course, not sufficient merely to have a computer. It is necessary to develop contexts in which the computer can be used by a child to serve real, personal purposes. Such a context needs be both material and conceptual. The material facet of our work consists of making computer-controlled devices a child can use for projects with a high potential for personal involvement, intellectual adventure and cognitive enhancement. Devices of this nature already constructed are: a music generator which enables a child to embark on experiments in composition, musical games, etc.; a graphics system with the capability necessary for simple animated cartoons; cybernetic "animals" -- turtles, spiders and worms; motors, relays, etc., etc.

The conceptual facet of our work consists of making intellectual tools designed to give children the power to use the devices. The obvious item in our tool kit is a programming language; but this is far from enough; to make a computer generate music, pictures or mechanical processes one must also have the mental tools to think about temporal, tonal, geometrical and physical matters and much more beside these.

A big component of what one needs besides specific technical knowledge (of music, geometry, etc.) is general heuristic knowledge related to the skill of carrying out a complex project. Concepts related to this include: planning, de-bugging, modular structure, hierarchical structure, model. . . All this is what is common to the scientists' task of making a theory that works, the engineers' task of making a machine that works and the administrators' task of making an organization that will work. And all this is what is perhaps most disastrously missing from the traditional school experience -- especially of the sciences, most especially of mathematics.

Let us focus on one particular component of this knowledge: the art and techniques of experience of "de-bugging". The school experience of mathematics is dominated by the normative attitude implied by "right answer vs. wrong answer". The mathematician's experience of mathematics is dominated by the purposeful-constructive attitude implied by the struggle to "make it work." He abandons an idea not because it happened to go wrong, but because he has understood that it is unfixable. Dwelling on what went wrong becomes a source of power rather than a piece of masochism (as it would appear to most fifth graders in traditional math classes). We contend that ours is the only clearly defined proposal for producing this shift of attitude in the elementary school. To do this we change the contex of mathematical work from "work-book exercises" to using mathematical ideas to dominate a powerful technology.

> The spirit of what is being said can be enriched by some quick references to points of contact with people we consider as intellectual allies. Using a phrase from Illich we would say we are fashioning the computer into a convivial tool, and science in general into a convivial mental tool.

Simon's concept "The Sciences of the Artificial" has also greatly influenced us. These sciences are important both in their application and in their inherent simplicity and intelligibility. They should be at least as explicitly represented in the lives of children as "Natural Science". Simple but completely functioning models and "mini-theories" help immensely a child know "where he is at" in exploring the complex network of ideas about real, natural systems.

There is an obvious flavor of Dewey in our thinking. Learning is best when embedded in living experience. Dewey's followers fall into hollow romanticism for lack of the technical means to embed the learning of complex modern knowledge in meaningful experiences. Edith Biggs, Dienes, Gattegno and the blocks, rods and sticks we see in the infant schools are steps in the direction we like; we are trying to take giant leaps in the direction they have defined. Piaget is too close for brief comment except to say that we see much of the "Piaget and Education" community as standing him on his head by emphasizing the negative aspect of his work, namely his demonstration that children of certain ages have surprising "deficiencies" (which some say should be "remedied" by the schools). Much more important in our view is the demonstration that normal kids "remedy" these "deficiencies" all by themselves, without formal teaching. We'd rather see more knowledge, acquired in the way children (successfully) acquire "conservations", than see "conservations" foisted on children in the way schools (usually unsuccessfully) try to teach mathematics.

2.2 New Conceptualizations of Knowledge

Under this heading comes the real challenge for the future. Disappointingly few of those who quote, or even adopt, our work see this, or view the steps taken so far as pointing to it. We are quoted (and praised or criticized) as making "turtles", giving children computer graphics and designing programming languages. Well we are pleased to have done that. But the real tasks were more fundamental and more *mathematical* than technological. A typical question was: how could children make computer controlled displays do anything? For example, would they have to use Cart coordinates? Our turning point came when we decided to stop scratching around in the mathematician's cupboards looking for already elaborated geometries that might do the job. Instead, we decided to make our own. And after a number of false starts and many ideas from many people in various corners of M.I.T., there gradually took form a new piece of mathematics, now called Turtle Geometry. Once it is made, we see many points in common with established geometry. Of course; nothing is really new. But this does not undermine our thesis that we have stumbled on a new paradigm for research in education. Most such research accepts a given body of knowledge and worries about how to deliver it into the heads of the children. We say: no, what you want is to create new, more suitable knowledge. This leads into research that looks more like -- indeed actually *is* -- mathematical research.

More generally: we said we want to give the child a mathematical experience like an engineer's and we cited the new technologies as providing a material a child _can_ engineer. But to carry this idea into practice we were faced with the problem of giving a child access to necessary knowledge of mathematics, physics, control theory, programming, etc., etc. The problem is: _what_ knowledge is really necessary, and can it be formulated in learnable sequences for our purposes? Here we find ourselves in virgin territory. What knowledge, what intellectual structures do you really need to be an engineer, to dominate and manipulate the physical world? There is at least one aspect of the answer to this question about which we feel firm: the traditional knowledge-set followed by all the high schools and engineering schools _may_ be sufficient (for _some_ students) but it is neither necessary nor optimal. If it were necessary our enterprise would be hopeless!

Two of the more compelling examples of what we have been able to achieve so far in this direction are:

> Turtle Geometry which provides a conceptual frame for manipulating geometric objects without the algebra needed for doing the same job in the Cartesian frame.
>
> Our qualitative physics projects (which interlocks with Turtle Geometry) which has shown how to develop enough mechanics for a usable subset of control theory and

> planetary theory without anything resembling the familiar theories of differential equations or integration. We do not mean anything as trivial as replacing conceptual understanding of integration by "number crunching"! Physicists who glance at §3.3 will see why we claim that our mechanics is as theoretical and abstract as the classical ones; it is merely different, conceptually clearer and much more accessible.

How far one can go remains to be seen. We see the fragments of progress we have made as compelling evidence for our thesis that much of science can be reconceptualized to become vastly more accessible. But scarcely anyone ever tries when the existing conceptualizations are perfectly satisfactory for working scientists.

> Some readers might protest that this is unfair to the physicists and mathematicians who worked so hard on the curriculum reform movements (PSSC, SMSG, etc.). We are not trying to devalue the intellectual quality of their work. But we are trying to draw a distinction between making local changes in the exposition of, say, traditional physics, and globally changing the structure and conceptual foundations of the subject. To a first approximation, the PSCC physics book defines the same concepts,

> states the same "laws" and uses the same mathematical formalisms and theorems as the traditional books. Whether it does this very much better than they did is not the point at issue here. We are trying to define the conceptually different enterprise of defining other principles, other concepts, other theorems.. . . to arrive by a different route at the same ultimate conceptual and instrumental mastery of the physical world.

The project of re-conceptualizing areas of knowledge applies also to some usually regarded as extra-scientific. Music is one which several members of our research team (particularly Jeanne Bamberger) are pursuing. In principle vast new horizons can be opened for any child who likes music by having access to a computer with a music generator. In particular he is no longer prevented by lack of dexterity from exploration of composition and other musical experiments. The computer becomes an obedient orchestra and will play any piece the child can describe. But in what formalism will he describe it?

As Turtle Geometry gives a child a grasp on movement in space (which he may use in geometric or physical applications) so we need to develop ways to think about relations in time and in "tonal space" to give him a similar control over music. Work in this area started in our laboratory later than work on geometry, and has not reached the same level of development. But we see it as exceedingly important for a number of reasons of which we mention only two. The first is the importance of music for many children. The second is the importance of being able to think clearly about time for many purposes other than music. It is curious that the study of the temporal is so superficially represented in thinking about Education despite its importance and the difficulties students manifestly have when dealing with dynamic problems.

> There is a minor paradox which we must draw to the attention of any reviewer who might have missed it. If you want to make mathematics for children you work terribly hard to make the mathematics as simple and transparent as possible. You would like to make it almost unnoticable, so that children would learn it like we all learned our mother tongue, without even knowing there was anything difficult. Fine. But then people (like reviewers) pick it up without noticing it; and they say: what on earth have you been doing? In our case (as we have already noted) no one quite says that; but people often praise us for machines and don't notice what we really are

proud to have done. Even when they talk about the machines, they fail to notice what is mathematically interesting about, say, the turtle, namely the independence of forward and rotational motions.

2.3 A Cognitive Theory About and FOR Children

Much of our work strikes educators (of very different theoretical persuasions) as valuable "in its own right" without reference to an explicitly spelled-out cognitive theory. We have shown that children, even mathematically recalcitrant ones, learn geometry very well through working on computer graphics. Since everyone believes that it is desirable for children to learn geometry, there is from one point of view, nothing more to be said in justification except to ask whether the results are generalizable. But from another point of view there is a great deal to be said about the relation of these experiments to theoretical questions. The purpose of this section is to explain the latter point of view and indicate what we mean to do about it. We do this under a number of sub-headings.

The questions of generalizability, cost, etc. will be discussed in §2.7.

The Goose and the Golden Eggs

Turtle Geometry can be (has been!) judged and found acceptable by educators of many theoretical persuasions. But they didn't discover it. We did and we see its discovery as having been guided not merely by technological and mathematical thinking, but also by a general cognitive theory, which has gradually emerged in the Artificial Intelligence Laboratory. We are very unwilling to let the goose starve while we enjoy the golden eggs.

Artificial Intelligence and the Philosophy of Education

The cognitive theory underlying our work draws on ideas from the Piagetian tradition of thinking about children and from these aspects of Artificial Intelligence concerned with thinking about thinking in general. The next paragraphs will give some general insight into the nature of this theory. For detailed discussion we have to refer the reader to other sources. Some sources are appended. Readers are asked to bear in mind that the theory is in rapid evolution and documents appended are very preliminary statements. A considerably more extensive literature is on the threshold of availability. See:

Appended --

S. Papert, "Teaching Children Thinking"

M. Minsky & S. Papert, "Artificial Intelligence" (A Progress Report available from the A.I. Lab)

Available but not appended --

T. Winograd, Understanding Natural Language, Academic Press, 1972. (A Ph.D. Thesis from the A. I. Lab)

T. Winograd & S. Papert, Lecture notes in Process Models for Psychology, Rotterdam University Press, 1973. (Lectures at a NUFFIC International Summer Course, 1972)

P. Winston, "Learning Structural Descriptions from Examples" (A Ph.D. Thesis in the A. I. Lab)

(A Cognitive Theory . . .)

Available by late summer 1973 --

M. Minsky & S. Papert, "Thinking About Thinking"
(An extensively revised and expanded version
of item 2.)

G. Sussman, "A Theory of Skill Acquisition"
(A Ph.D. Thesis in the A. I. Lab)

I. Goldstein, "An Intelligent Model for LOGO"
(A Ph.D. Thesis in the A. I. Lab)

Does Knowing Impede Doing?

The children's rhymes on the next page illustrate a popular form of the trend in cognitive theory and educational practice with which we are in most direct conflict. The most immediately relevant manifestation of this trend is the prevalent idea that "verbalized" knowledge is undesirable for elementary mathematics; instead, it is said, children should "discover concepts" through "non-verbal intuitive" processes. More fundamental manifestations of this trend are found in the writings of J. S. Bruner (for example his doctrine of the "impotence" of "words and diagrams" in the acquisition of "enactive knowledge"), of M. Polyani, of H. Furth and many other currently influential authors.

The existence and importance of "non-symbolic" knowledge has received considerable attention in the Artificial Intelligence Laboratory quite apart from our interest in education. If real it would obviously deeply affect the general enterprise of representing knowledge in computers: if true it

The Chair

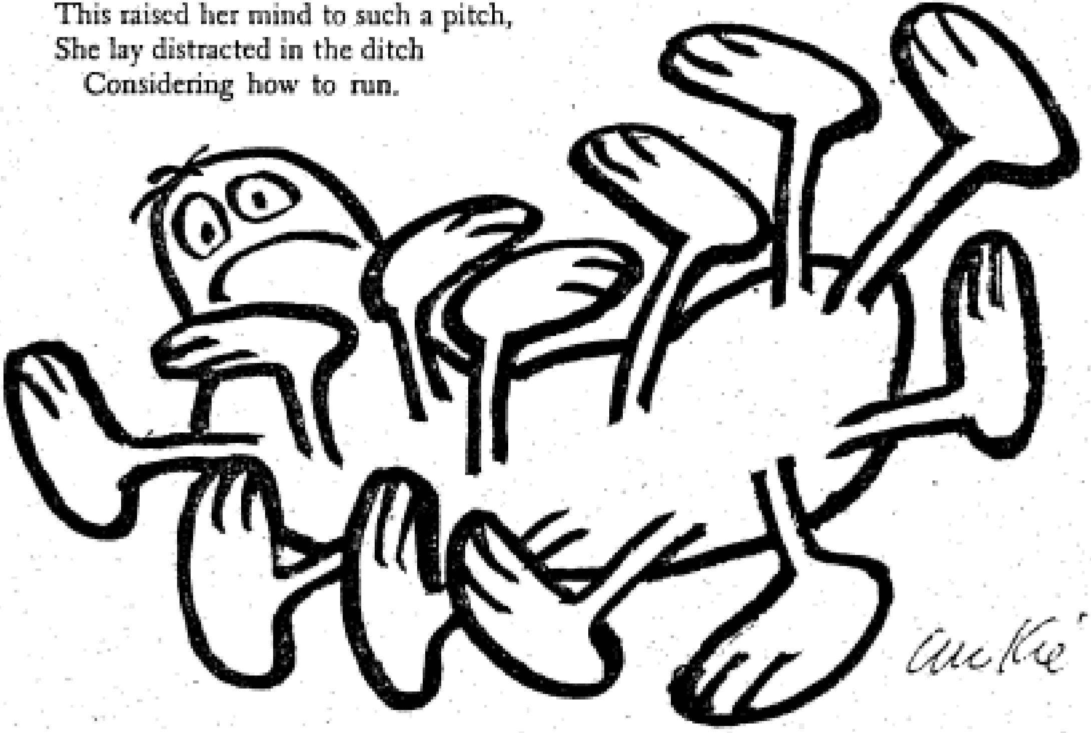
THEODORE ROETHKE

A funny thing about a Chair:
You hardly ever think it's there.
To know a Chair is really it,
You sometimes have to go and sit.

A Centipede Was Happy Quite

ANONYMOUS

A centipede was happy quite,
   Until a frog in fun
Said, "Pray, which leg comes after which?"
This raised her mind to such a pitch,
She lay distracted in the ditch
   Considering how to run.



from The Illustrated Treasury of Poetry for Children, Edited by David Ross,
Grosset & Dunlap, New York, 1970.

implies the need either to seek "non-symbolic" representations in machines or to recognize that certain intellectual functions could only be achieved in machines (if at all!) through processes fundamentally different from those operative in human intelligence. But everything we know about machine intelligence runs counter to both those suggestions.

From these studies has emerged a more sophisticated concept of what it is to be "symbolic" or "verbal" and a specific theory of a methodological trap which caught Bruner, Furth and others. Stated simplistically this theory is: _Bruner's language_ quite possibly _is_ impotent for the purpose, say, of telling someone how to ride a bicycle. But it does not follow that _language as such_ is impotent for this purpose. Indeed a major contribution of the Information Sciences (including control theory, computer science, A. I., etc.) is to have made more powerful languages for describing complex processes. Thus we ask the following question of anyone (and there are many) who concludes from formal experiment or from ordinary life experience that "you can't _tell_ someone how to do so-and-so":

> Do you mean _you_ failed to tell him in the available versions of ordinary natural language or do you mean that he could not _learn_ a technical language in which he could be told? If anyone is brave enough to take the second option we ask further: "can you explain to us how you managed to consider _all possible_ languages?"

Translated into Educational terms these ideas lead to the enterprise of developing a language in which to talk to children about cognitive matters about which one is normally silent. Examples will emerge in the following sections.

Time Constraints of Experiments on Learning

The idea of constructing special languages to express ideas and procedures usually classified as "intuitive" leads to an important methodological point about the form of experiments in cognitive psychology and education.

Consider a typical experiment on a question like: Does Verbal Instruction Help or Hinder Learning a Physical Skill such as Juggling? The standard methods for such an experiment take about as much time as is needed to acquire measurable proficiency in juggling. At most a few hours.

Now consider our more complex question about whether (and how much) verbal instruction can help subjects who have acquired a technical language and a whole set of concepts and intellectual skills. If the experiment includes learning the technical language it might need very much more time. In fact experiments in our laboratory use children who have had many months of experience in programming and talking about, computers and computational processes.

The difference is relevant also to experiments on the difficulty of accelerating (or changing the order of) "Piagetian stages" through teaching. The typical traditional experiment involves at most several hours of exposure to an experimental condition.

> Our informal experiments cast very serious doubts on conclusions (especially negative ones) drawn from all reported experiments on verbalization and on the invariance of Piagetian stages. A different time scale for the experiment, and a very different experimental setting seems to produce qualitatively different results. Over the next few years we will give major attention to translating these impressionistic observations into rigorous experiment. It might be argued that we should wait until then before saying anything. However a large part of our critique of classical experiments really uses the new experimental methodology in the spirit of the gedanken experiment. A sufficient part is of this form to make mounting further experiments a matter of great urgency.

Cognitive Theory as a SUBJECT in Elementary School

We have a double interest in working towards simpler and more explicit statements of our cognitive theories. One is the ordinary scientific interest in doing so to confront it more clearly with experimental reality. The other comes from a specially reflexive sub-thesis of our theory, namely the contention that a child's cognitive development greatly benefits from the child knowing about cognition processes.

> This is closely related to the previous point. It is stated in somewhat rhetorical form in "Teaching Children Thinking", the first paper in the Appended collection "New Educational Technology" to which the reader who is not familiar with our thinking on this issue is referred. See especially the sections of that paper headed:
>
> The Don't-Think-About-Thinking Paradox
> The Pop-Ed Culture
> Computer Science as a School Subject.

What does it mean for a fifth grader to study cognitive theory? Certainly we do not mean: send him off to read Piaget or Bruner or Newell and Simon! And this not only because these authors have a style of writing which it is hard enough for a well informed adult to penetrate. Even if they wrote more plainly, what they say is too purely theoretical. We are interested in a more applied, more practically oriented version of cognitive theory.

The simplest image of this is nothing more than a more articulate and more sophisticated ability to discuss such activities as learning skills, memorization, solving problems. Most children are so poor in their ability to do so that a very little positive knowledge can make a big difference. For example, if a child really believes (as some do!) that the best way to memorize material is "to make your mind a blank and say it over and over" then any slight knowledge of active mnemonic skills will put him in a better position to memorize . . . (if he has to; perhaps he doesn't; but this is just an example that happens to be easy to state). Similarly for the even greater number of children who know nothing explicitly about heuristic knowledge, very elementary heuristic skills make a noticable difference.

The educational problem is: how to involve the child in such knowledge. Lectures are surely hopeless! We have explored (rather informally so far) a two-pronged approach, using two kinds of "experience".

The Concept of a Learning Lab

Schematically the idea is to give children many experiences in learning "very learnable and discussable" skills; in teaching these skills to other children; in experimenting with different ways of teaching and learning. The selection of such skills took us some time and trouble. But we now have a large set,of which we concentrate here on a favorite subset: the CIRCUS ARTS (Juggling, Bongo Boards, Tight Rope, Unicycle, Circus Ball, etc.). These have these advantages:

-- almost everyone likes to learn them

-- once one has understood a suitable "technology" of learning, they are quickly learned

-- once one has a proper descriptive language it is easy to analyze, discuss, compare notes, transfer, etc. And it soon becomes evident that doing so ACTUALLY HELPS ONE TO LEARN.

(A Cognitive Theory . . .)

- -- they have enough in common so that a student who had to be taught one of them didactically, can transfer this knowledge and so increase his chances of mastering the next through self-generated learning strategies.
- -- they also have enough in common to allow a child to make simple mini-theories to cover more than one.

The Computer as a Model Pupil

It is obviously possible to conduct "learning labs" without computers. Nevertheless we believe (on admittedly very impressionistic and intuitive evidence which we hope to examine more carefully) that the interaction of a "learning lab" experience and a suitable computer experience has very particular value. The point is that "teaching the computer" objectivizes the process; it develops the concept of formal description; and it plants ideas such as "bug", "sub-procedure", "state", "control variable" ("input" in our jargon), etc. which enormously help the analytic aspects of the work in the learning lab.

2.4 Studying Heuristics

We assume the reader will accept that Polya is right in principle about the value of studying heuristic knowledge explicitly. But much work is needed to make this idea practically useful at the Elementary level or more useful at Advanced Levels. In particular:

(1) Polya is at his best in discussing relatively advanced and even somewhat esoteric topics (for example the harder problems in Euclidean Geometry). So the least we need is to "do a Polya" on more elementary topics -- and more commonly intuitive ones.

(2) Some problem domains exemplify heuristic principles much better than others. So the best strategy to learn heuristics for problem domain A might be to first learn problem domain B, study heuristics there, and then transfer them to A. If B is worth while in itself the gain is all the greater. And for quite fundamental reasons we believe that topics in computational mathematics are superb training grounds for heuristic thinking.

(3) Polya's particular set of heuristic principles is, of course, far from complete. The tradition of work on "Heuristic Programming" or "Artificial Intelligence" has pursued more deeply than he in certain directions, which happen to be especially suited to writing programs. Since our laboratory

is a leader in this area of work, it is not surprising that we should choose as an area of special interest to develop more systematic and fundamental formulations of heuristic knowledge.

2.5 Relationship of Our Work to Schools

(Applications, Demonstrations; Evaluations)

We classify the ways our work relates to schools into two broad categories and an intermediate one:

Reformist:	Producing new materials and methods which can be incorporated into traditional schools without fundamentally changing their nature.
Revolutionary:	Producing a total alternative to the "school" as it is known today.
Intermediates:	Developing new forms of "supplementary" learning centers in the spirit of Science Camps, Oppenheimer's Exploratorium and so on.

Although the "revolutionary" goals figure largely in the thinking of most members of our group, our practical work with children has necessarily been confined to the other two categories. Most belongs to the first category. The pattern we have used most is exposing children in a public elementary school to our materials for about two hours a week over a half or whole school year, that is to say, about as much time as they give to normal subjects like "math" or "social studies". The most interesting deviation from this pattern was a spectacularly successful project in which children were given free access to computers, devices and counsellors over a three week period last summer. From the "revolutionary" point of view we see all this as a necessary preparatory step towards more radical experiments in the global re-design of learning environments for children. The conceptual and technological products of our work are designed to serve as modules of the total alternate school from we hope to have brought to an operational status within two years from now. The following paragraphs will survey our current plans under each of these headings.

2.5.1 Material Suitable for Use Within the Traditional School

Our most elaborated module is a "course" designed to be used (in slightly different versions) somewhere between the fifth and the ninth grades (although it can also be adapted for remedial teaching of adults!). Its educational objectives include:

(i) The fundamental ideas and skills of programming; fluency in LOGO; experience in planning, developing and debugging a programming project.

(ii) Elements of Turtle Geometry (which includes a large part of the geometric knowledge normally taught at these grade levels . . . and much more).

(iii) Various other formal and heuristic mathematical ideas including: use of variables, functions, recursive definitions, etc. on the formal side; and some explicit Polya-like heuristic principles.

We have sufficient mimeographed material on this to enable an imaginative teacher (with access to appropriate computational facilities) to operate such a course. We hope by next year to have a book on Turtle Geometry ready for publication.

Modules on other topics are less developed but advancing rapidly. Within one year (optimistically) or two years (pessimistically) we expect to see the development in our Laboratory and elsewhere of modules on Music, Linguistics, Physics, Biology, Heuristic Programming (or elementary Artificial Intelligence), Perceptual Psychology, Circus Arts and other topics.

> Note: The reference to "elsewhere" is based on very specific plans involving the Urbana "Uni-High" Project and some vaguer ideas about the possibility of some loose coordination of work at several other centers.

2.5.2 New Concepts of "School"

New Methodologies of Education Research

We believe that the major impediment to the emergence of a truly modern theory and practice of Elementary Education is the methodology of experiment which makes small changes to a large and complex on-going system. If the experiment is lucky, a small effect is produced, just big enough to be distinguished from the noise by dint of ingenious statistics. If it is unlucky, the natural equilibria of the system assert themselves so well that no effect is seen beyond the initial "Hawthorne" transient. The consequence is the self-reinforcing syndrome of gloom and pessimism one might call "Jencksenism".

We propose a different paradigm for research on Education (and Developmental Psychology in general):

1. Take a theory of Education (such as ours, but people with other theories should also follow this paradigm!).
2. Develop the consequences of this theory far enough to design what it projects as a really good set of conditions for the intellectual growth of children.
3. Implement these conditions on a minimal viable scale (in our case this is a community of children on the scale of a small school; other theories might require working with large communities)
4. Equip your experimental "school" (or "school substitute"!) with all the resources of people, technology and ideas required by the design. IGNORE THE PROBLEMS OF COST PER STUDENT AND OF PERSUADING PRINCIPALS, TEACHERS, SCHOOL COMMITTEES AND EVEN COLLEAGUES.

5. Run your experiment for the time required by your theory (in our case: 2-3 years). Then one of two things happen:

   SUCCESS: The results are so qualitatively different from what would normally be expected that no sane observer says: "how do you measure that?". In this case the next problem is to study why the experiment worked, whether it can be generalized, what can be learned from it.

   FAILURE: If under these "ideal conditions the results are so poor that the statisticians want to test them for significance you declare the experiment a failure, try to understand why it did not work, perhaps try another.

2.5.3 Intellectual Centers Parallel to Schools

"Science Museums" and "children's museums" do notssucceed in actively engaging young minds to anything like the extent we think is possible. Frank Oppenheimer's Exploratorium takes a big step in a good direction. An Exploratorium based on our computer controlled devices could take a much bigger step.

(Relationship of Our Work to Schools)

We are particularly interested in a type of "Resource Center" at which children can spend larger slices of time then is usual in "museums". We imagine patterns such as: all day for three weeks; a full day once a week for half a year; etc. We have carried out one experiment of this sort with results we have no hesitation in describing as "mind-blowing". The experiment was conducted in Exeter, England as part of the U.S. National Presentation at the International Congress on Mathematical Education. Fifteen children (aged 10-12) participated and achieved a degree of involvement and sophistication far exceeding what we have seen under the more cramped conditions of working during forty minute school "periods" subdivided between other school activity. Part of this might have been "cultural" (it was a different country!) and "individual" (the group of children was more -- but not altogether -- self-selecting). However, we have seen an effect in the same direction when we were able to give children in the Bridge School (where we normally work) exceptional freedom of access. More work of this sort is planned.

2.5.4 Opening Avenues for the Severely Physically Handicapped

A computer controlled music generator opens new horizons for children to experiment with music and especially to compose it. Most adults and almost all children simply cannot make any attempt at musical composition for lack of the performance ability needed to get feedback by playing one piece. Physically normal people can overcome the lack of dexterity. Severely handicapped people cannot. For them availability and mastery of a computer with musical and graphic capabilities could produce an even greater improvement in the quality of life.

We are rather ashamed of our slowness in pursuing this application of our work and have firm resolutions to make up for this by energetic exploration in the coming year. The first phase of this will be to deepen contacts we have already made with individuals and institutions engaged in teaching handicapped children, to train some of these teachers in the use of our materials and to gain some experience with suitable terminal devices (such as those developed by Kaffafian's Cybernetics Organization in Washington, D.C.).

2.5.5 Remedial Mathematics for Adults

Most people emerge from high school without ever having had a joyful or personally meaningful mathematical experience. No wonder they hate it and refused to learn it! We think it is important and easy to remedy this for college students in academic trouble, for future teachers (especially!), for students at universities following policies of "Open Admissions" . . . etc. With small modifications our fifth grade courses seem to be as enjoyable and as instructive to adults as to children.

2.5.6 Paradigm for Experiments on Developmental Psychology

The concept of a totally re-designed learning environment of §2.5.2 and §2.5.3 provides new possibilities of significant experiments on Developmental Psychology. Some psychologists argue that *their* work has to be chronologically and logically prior to work on education. We maintain, on the contrary, that developmental psychology has reached a cul-de-sac out of which it can emerge *only* by adopting an experimental technique such as we propose.

> Aphoristically one might say: in the past "curriculum reform" has looked to "psychology" as a source of ideas; we propose to use "curriculum design" as an experimental technique for psychology. Or, better, we might say: previous psychologists have sought to observe children; the point however, is to change them. As long as psychology will not take *effective* steps to change them it will never become a truly experimental science.

2.6 A New Kind of Professional for Research on Education

[Missing section on need for a program of graduate studies. See Appended "Progress Report" of the M.I.T. Education Division.]

2.7 Mundane Aspects of Computers

There are people who say that our work is admirable but too expensive to be used for the vast majority of children. They are diametrically wrong. It is expensive _only_ because it is _not_ used for the vast majority of children. We are convinced (and are preparing a closely reasoned position paper on this) that mass production of a standardized computer really could bring the cost down to a very economical level.

> If every child had a computer, computers
> would be cheap enough for every child
> to have a computer.

Section 3

Concrete Examples to Illustrate the General Concepts and Goals Mentioned in §2.

The previous sections discussed general ideas somewhat abstractly. In following sections the reader will meet the same ideas but this time in a very concrete form, as they appear in particular instantiations. The choices of examples are somewhat arbitrary and far from exhaustive. Objections of the form: "But that's not all there is to biology (or physics or math)" are irrelevant. We have chosen little corners of these large subjects in order to illustrate a method, a way of looking.

The first example has an unusually large degree of futurism. We include it as an "ideal case" in which the concept is seen more clearly. The second example describes the technological environment in which we have actually worked with children. The third and fourth discuss topics in geometry, physics and biology. The sixth and seventh sketch designs for learning environments for children.

3.1 Alan Kay's Concept of the Dynabook
3.2 The "LOGO Turtle Lab"
3.3 The Power of the Idea of Powerful Ideas
3.4 Ramifications of Turtle Geometry into Physics and Biology

3.1. Example 1: Alan Kay's Concept of the Dynabook

The first example involves a piece of technology which does not quite exist yet. It almost certainly will; however, I use it here not as a basis for futuristic predictions, but rather as a conceptual example (a kind of *gedanken experiment*) to make some conceptual distinctions more rapidly and clearly. The validity of the distinctions is not contingent on making the Dynabook.

The Concept of a Mathematical Social Context

The first of these has to do with how different learning mathematics would be in a different cultural-technological context. To bring this idea into focus I want the reader to imagine a world in which sophisticated but accessible mathematical technology is as thoroughly diffused into the lives of elementary aged children as it now is in the lives of engineers and scientists. The Dynabook enters this discussion as a simple way to define what such a world might be like.

Description of the Dynabook

The Dynabook is (in the intention of its designer) a computer about the size and shape of an average book and inexpensive enough for mass use. Most of one side is a display panel with enough resolution for print-quality text and half-tone pictures. (That is to say much better quality than most readers of the essay will have seen in computer graphics).

The little computer is self-contained and easily portable by a child. In computational capacity and memory it exceeds anything one would normally

(Dynabook)

class as a "mini-computer". Think of it as rather more computer than a PDP-11. One of its simplest uses is, as the name suggests, as a book. It has more than enough memory capacity to hold the contents of a book. And if the book has been written with this use in mind it would be a very special book with interactive abilities. For example, if it is a "book" about space navigation it might include a program to simulate a space-craft. Or the owner of the Dynabook might make his own program to settle a question about planetary physics. Or he might use the Dynabook to modify a piece of music and play back the result. And carrying this idea further, he might use the Dynabook to compose and play a complex piece of music. Or an animated cartoon.

> I shall discuss later on reasons to believe that computer aids can make it possible for "ordinary" people to compose music or cartoons.
>
> Dr. Kay is head of the Learning Research Group at the Xerox Palo Alto Research Center. The Dynabook project is described in detail in his paper "A Personal Computer for Children of All Ages", an internal Xerox paper; LRG-1/XPARC, August, 1972. Abstracted in the Proceedings of the ACM Conference, Boston, August, 1972.

Learning Mathematics in a Mathematically Rich Context

It is foolish to speculate in detail about what would happen in a world in which everyone has a Dynabook. But it is not foolish to formulate questions about what *might* happen.

Such a question is: do we have any *firm reason* to believe that in such a world the development of mathematical thinking in children would follow the pattern whose manifestations include Piaget's stages and the grade levels of mathematical achievement? I think the answer must be negative. Certainly, it would be extremely dangerous to extrapolate any *negative* results of experiments of the kind one sees reported in journals of child development and educational research. For these experiments bear on the consequences of changes in the child's learning situation which are undeniably much smaller than the massive introduction of Dynabooks and very plausibly of a qualitatively different nature. Typically, a "learning experiment" on Piagetian structures consists of exposing the child to at most a few hours of some special treatment. So these experiments are extremely *local* in time. Experiments involving "curriculum reform" do expose the child to experimental conditions over a larger time period. But the experimental conditions are usually constrained to be very similar to the standard ones for reasons related to the general philosophy of the curriculum reform movement (as practiced over the past two decades). These changes are *conceptually local*.

As a gedanken experiment in math education, the massive diffusion of Dynabooks differs along several dimensions from the kind of experiments people actually do. One of these, and a possibly critically important one, is sheer scale: every adult who has occasion to learn new subjects, especially new styles of thinking, knows that a few hours of contact time is very little. But quite apart from time scale, the *quality* of relationship to the new material is vitally important, and the Dynabook brings into clear focus an issue about the quality of relationship between a child and mathematics.

Please accept for a moment the hypothesis that at least some, say eight year old, children might become involved in using the Dynabook to compose by writing music-generating programs. Please also accept that children who are doing such work can usually exploit any advances in their own mathematical understanding to obtain more interesting musical effects. Within these assumptions it is easy to see a sense in which the child's relation to mathematics could be described as active, personal, involved, experimental . . . in contrast to the relationship observed between children in even the best contemporary schools and the work-book exercises characteristic as much of the "new math" as of the "old". The exploration of this difference is a central part of the research perspective I am trying to develop through this discussion. There are many facets to this exploration. Not least of these is the need to define more carefully the nature of the difference, or even to be more certain that there is one. Other, more instrumentally defined, facets involve exploring the possibility of actually realizing the assumptions I asked you please to accept for the sake of argument. How much of this can be done without waiting for the Dynabook will be discussed in detail later and indicated in a general sense through the next example.

How to Make Dewey Less Romantic

I'd like to relate the approach to education being suggested here to the great thinkers of the philosophy of education. Dewey is a good example. He is often accused of being "romantic" when he calls for education to be part of living experience and sadly contrasts the "bookish", "formal" learning in the schoolroom with the way children in earlier societies learned hunting by going to the hunt or by playful imitation of the hunt. He is romantic in that he was unable to say how the complex knowledge of modern industrial society could be acquired in this pattern. How can a child learn serious mathematics by participating or by playfully imitating?

In my view he was perfectly right in his insights and intentions, but lacked the technical means (and perhaps the appropriate intellectual disposition) to realize these intentions. But these technical and intellectual deficiencies can now be remedied. The image of the Dynabook shows one way (and there are others) in which children could indeed learn some very sophisticated mathematics by direct and real use of it . . . including quite serious playful imitation of the daily activity of a real grown-up mathematician.

> Thus, paradoxically, modern technology may have carried us through a full circle by allowing us to enjoy on our technological level advantages of primitive societies that were quite impossible for so many intervening centuries.

3.2. Example 2: The "LOGO Turtle Lab"

The Dynabook is a very pure example. But it does not (yet) exist. The next example is contaminated by reality-dictated compromises of many sorts, but it does exist. In its most visible form it is a time-shared computer facility (a PDP-11/45) equipped with an unusual variety of "peripheral" devices: graphic devices (CRT displays, plotter), sound making devices ("music box", phoneme generator) and mechanical devices ("turtles", motors).

Children working in this lab have done most of the things mentioned before as possible uses of the dynabook, though under far from ideal conditions. In particular this computer is more like the chained hand-written books in a pre-Guttenberg monastic library than like the free use of printed paperbacks in our society. The child must come to the computer for his rationed time and cannot carry it away in his pocket for use when the interest arises. Nevertheless, we are sometimes able to see the kind of phenomena one would expect in the Dynabook world. Examples will come. To appreciate them (and us!) some understanding is needed of the intellectual means needed by a child to use this computer.

Consider a child in the fifth grade, low over-all scholastic achievement, lower mathematical achievement, classified by the school psychologist as having learning difficulties. The child is introduced to the computer through forty minute sessions twice a week. After a month he is extremely involved in writing a program to draw a truck on the CRT and make it move across the screen.

(LOGO Turtle Lab)

How do you think you can tell a computer how to draw a truck? The problem I want to bring into focus is how one can talk about geometric objects and relationships in a formal enough way for a computer to follow. The obvious answer is to follow Descartes, who showed how to translate geometry into algebra through the use of coordinates. But the child in our story doesn't know algebra. We could try to entice him to learn it by promising that he will eventually use it to operate the computer. But this is exactly what we do not want to do. We do not want to use "applications" as an external motivation for book-ish learning of "pure math". Rather, we want to embed the mathematics so thoroughly in the application that the two are indistinguishable. This is true "applied mathematics" from which pure mathematics can be abstracted later (if one wishes).

This educational goal immediately leads into a mathematical enterprise. Since a survey of developed conceptualization of geometry did not yield a suitable one, we were led to develop some new mathematics known now as "turtle geometry".

> Note: This is the place to draw attention to the fact that there is no new mathematics in "The New Math" as practiced in contemporary elementary schools. Sets, number bases and the like were new only in relation to the practice of elementary schools; mathematicians knew about them long ago. But mathematicians did not know (certainly not explicitly) about turtle geometry. So it really is new math.

(LOGO Turtle Lab)

> This comment is part of an important theme which will be emphasized later: in the past "research in education" has been concerned with the transmission of already existing knowledge; I see a new paradigm slowly emerging of research directed at creating new knowledge for children. Why should the best math for kids be some piece of math made by mathematicians for their own esoteric research purposes? Most people balk at the suggestion that there should be children's math . . . "surely mathematician's math is the only good math." But they don't say this about literature!

The flavor of turtle geometry can be tasted by comparing typical descriptions of a geometric figure, say a circle, in Turtle Geometry and in Cartesian Geometry:

Cartesian: $y - b + \sqrt{R^2-(x-a)^2}$

Turtle:

```
TO CIRCLE
FORWARD 1
RIGHT 1
CIRCLE
END
```

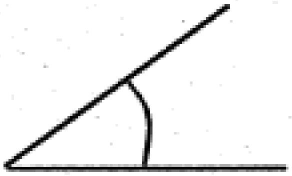
This is a program written in LOGO to generate a particular sized circle.

This is not the place for a detailed exposition of Turtle Geometry. (See next example.) But it should be apparent from this simple case that it is different in spirit from other familiar elementary geometrical systems. The aspects we want to emphasize here are:

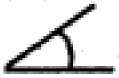
> Although completely formal and rigorous (even more so than elementary forms of Euclid) Turtle Geomtry is very close to the intuitive geometry one can assume a child will have acquired informally.
>
> Although one needs very little to get started, it is possible to obtain very powerful results.

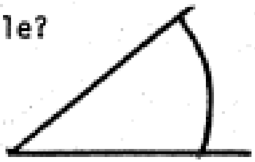
To illustrate the matching between Turtle Geometry and intuitive spatial notions contrast two images of "angle":

In Euclidean Geometry

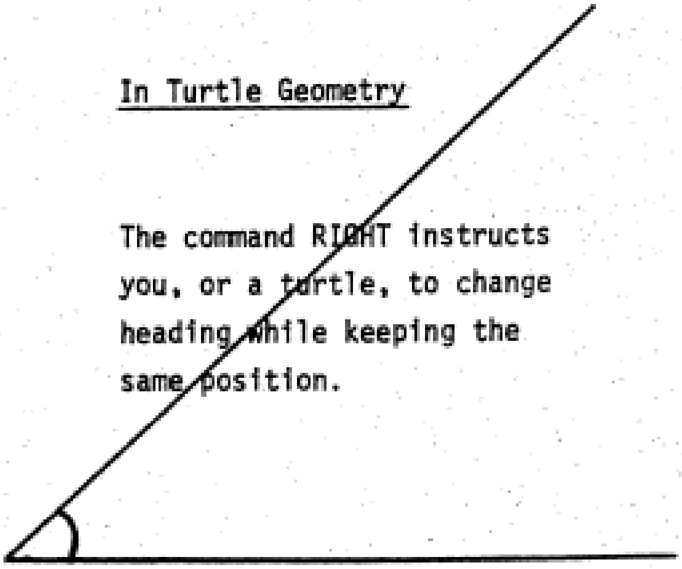


This diagram represents an angle. Which aspects of the diagram are relevant? Do the four diagrams represent the same angle?





In Turtle Geometry



The command RIGHT instructs you, or a turtle, to change heading while keeping the same position.

In Turtle Geometry the primitive concepts are commands and actions, perfectly intelligible things. In Euclid they are much queerer entities. Of course Euclidean Geometry is very beautiful (at least some parts are!) and surely every child who comes to love mathematics will want to know it. But these are not reasons for forcing a child to deal with the special conceptual complications of Euclidean Geometry in his very first encounter with any formal geometry.

Another important way in which Turtle Geometry meets Intuitive Geometry comes out in the use we can make of the child's own movements in space as models for solving problems in Geometry. For example suppose you want to make a turtle draw a circle (a possible solution would be the program CIRCLE mentioned above). How do you start solving the problem? Turtle Geometry lends itself to such heuristics as: walk in a circle yourself and watch very carefully what you are doing! This advice has led many (not all) children to observe that walking in a circle can be broken down into "rounds" consisting of "forward a little, turn a little" and so to rediscover the program CIRCLE.

The power in the idea comes, for many children, from having real turtles which obey these commands and will draw circles and much more. There are several kinds of turtle: floor turtles are mechanical devices equpped with motors, wheels, pens and sense organs; light turtles live on television screens, obey the same commands and draw "light pictures" with the accuracy needed to make complex designs and the speed necessary to make animated movies.

Each new idea in Turtle Geometry (or in programming) opens up new possibilities of action and elementary aged children in fact are very easily drawn into working on projects or making pictures on the CRT.

* * * * * * * * * * *

The Music Box is another device in the LOGO Turtle Lab. It can generate 60 pitches (in 5 chromatic scales of 12 pitches) and 2 drum sounds in each of four voices. In the same way as simple figures can be put together to form complex ones on the CRT, so simple tunes can be put together, successively or simultaneously to form complex pieces of music.

3.3. The Power of the Idea of Powerful Ideas

I assume that it is not necessary to explain what is meant by an idea being powerful -- with it you can think things you couldn't think before or do things you couldn't do before.

The examples in this section show how children can be brought into contact with particular powerful ideas . . . and hence (if it is done right) with the idea of powerful ideas (which might well be the most powerful of all). The two examples are the ideas of State and of Local. Both are fundamentally important in mathematical physics.

Aspects of the Idea of State in Turtle Geometry

Recall that the specific primitives of turtle geometry are:

(1) The Turtle

(2) The (geometric) state of the turtle which has two components: position and heading.

(3) The state change operators:
FORWARD which changes position but not heading
RIGHT which changes heading but not position.

The focus in this section is on the use of the concept of state.

Let us first observe that although this concept is one of the crucially powerful ideas in scientific thinking (and elsewhere in the culture as well) it has no representation at the elementary school level and only fragmentary representation in the high school curricula. This omission is not due to perversity or ignorance and cannot be corrected by merely adding curriculum units explaining the word "state". The deeper reason for its omission is that it has no natural place in the total structure of ideas and activities of elementary school children. I do not mean that it is not relevant to the work in the elementary school -- it certainly is at very many places; even (to take an extreme case) those dull algorithms for multi-digit manipulations are elucidated by the notion of finite state process. The issues of what can be introduced naturally are more complex than mere relevance. The point is that the ways in which the concept is relevant are too sophisticated or indirect to serve as natural entry points.

In Turtle Geometry the concept is used in very concrete practical ways as an immediately relevant means of intellectual grasp. Some examples will illustrate this.

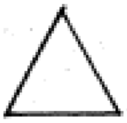
State Manipulation in the Total Turtle Trip Theorem (T^5)

One can draw an equilateral triangle using the following program:

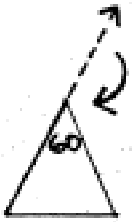
```
TO TRI

1 FORWARD 100
2 RIGHT 120
3 FORWARD 100
4 RIGHT 120
5 FORWARD 100
  END
```

How does one know that the turtle should turn 120°? No doubt adult readers will (if their minds are actively engaged at all!) have gone through some such process as:



The sum of the angles of a triangle is 180.
So 180/3 = 60. But that's the internal angle!

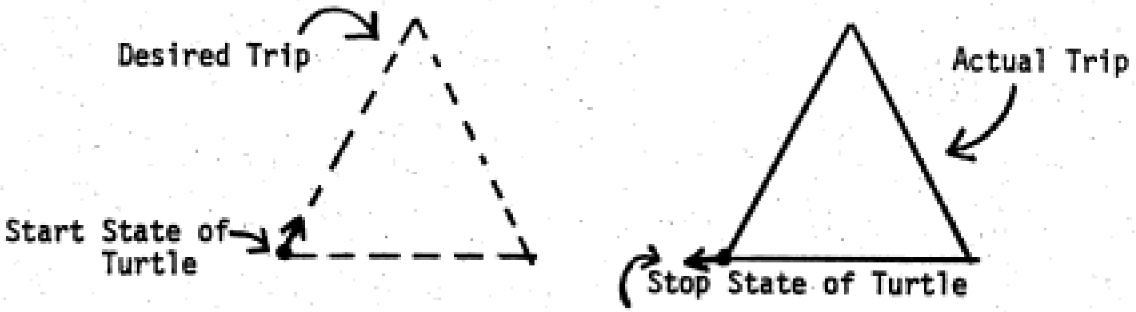


The external angle is 180-60 = 120.

(State in Turtle Geometry)

This process uses two well known theorems from Euclid. But our fifth grade kids haven't read Euclid and we have no intention of making them do so. Instead, we offer them a much more powerful and elegant method of thinking about the problem. It goes like this.

Think about the Turtle Trip around the Triangle:

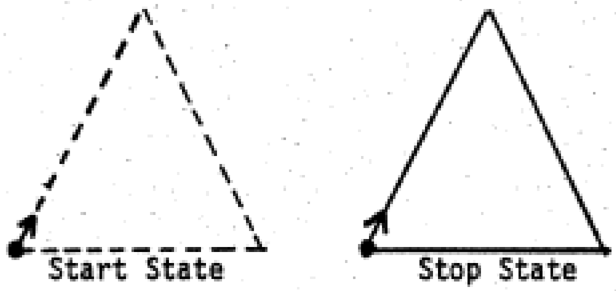

Desired Trip
Actual Trip
Start State of Turtle
Stop State of Turtle

It is easier to think about Turtle Trips if they are Total Turtle Trips (that means: the stop state is the same as the start state.) So we add an extra line onto our procedure TRI. Like this:

```
EDIT TRI
6 RIGHT  120
```

(State in Turtle Geometry)

Now it draws the *same* triangle but leaves the turtle where it found it. . .(much tidier!)



How much did the turtle turn all the way around the trip?

Obviously one complete rotation! That's 360°.

So the turtle did a RIGHT 360 "in three goes", so each was 360/3. That's 120 or we could even write the procedure like this:

```
TO TRI
FORWARD 100
RIGHT 360/3
FORWARD 100
RIGHT 360/3
FORWARD 100
RIGHT 360/3
```

Many kids can write programs like this, who can't reliably do the division of 360 by 3.

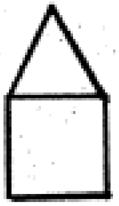
We'll come back later to discuss the exact statement and the power of the Total Turtle Trip Theorem. Meantime just observe that while the Euclid theorem is about triangles, this analysis applies in an obvious way to all polygons and in a slightly less obvious way to all closed figures. (It really is a topological theorem!) Thus the turtle turns 360/4 at each vertex to draw a square, 360/6 for a hexagon (and if it turns 360/360 it will draw something that for most imaginable purposes is a circle!)

```
TO POLYGON :SIZE  :NUMBER

1 FORWARD   :SIZE
2 RIGHT  360/:NUMBER
3 POLYGON   :SIZE  :NUMBER          (Notice the "recursion")
  END
```

State Manipulation in Debugging

Let's look at an even simpler problem of the sort we have used to introduce Turtle Geometry at the second and third grade level. Suppose that a child already has procedures TRI and BOX written so as to make a total trip. Consider the project of writing a procedure to draw a "house" like this:



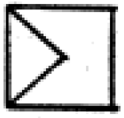
This is to be done by making a superprocedure which uses TRI and BOX as subprocedures. For example:

(State in Turtle Geometry)

```
TO HOUSE

1 BOX                 (Draw a square)
2 TRI                 (Draw a triangle)
END
```

Unfortunately this produces:

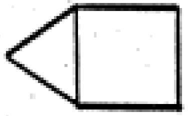


Clearly both sub-procedures are fine, yet they don't "fit together".

How are we to understand this so we can debug our procedure? The heuristic clue is: look carefully at the start and stop states of the turtle. If we do so we see we need a <u>state fix</u> in between the two subprocedures. So:

```
TO HOUSE

BOX
LEFT 60
TRI
END
```



So we make another state fix at the beginning:

```
TO HOUSE

RIGHT 90
BOX                     (Get the turtle in the proper state for BOX.
LEFT 60                 Do BOX.  Get the turtle in the proper state for
TRI                     TRI.   Do TRI.)
END
```

(State in Turtle Geometry)

The example is perhaps too simple for adults without a keen ability to see the world through third grade eyes. But a considerable body of experience shows:

(1) That the problem of putting together a square and a triangle is a significant one at that level

(2) "Think about the state" is helpful advice

(3) Not only does it help the child for the specific problem, but provides a vivid example of what it is to have a very general heuristic rule for solving a large class of problems.

(4) And, as we shall show later, the experience plants in the child's mind a concept -- namely state -- with wide applications in mathematics, physics, etc.

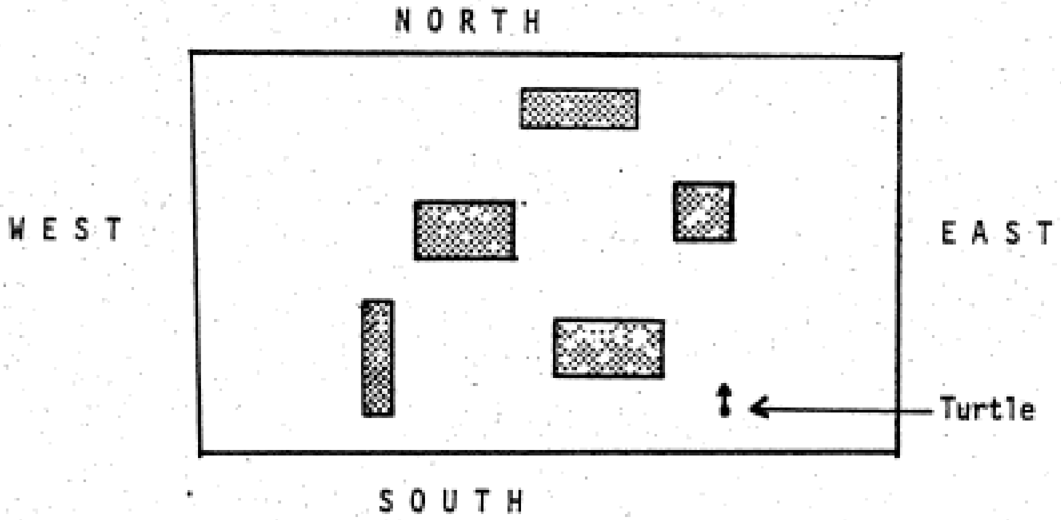
State of a Process; A More Sophisticated Example

This example is about how a 12 year old boy, Jonathan Pledge, in one of our Turtle Lab projects discovered a quite new and interesting algorithm for exploring mazes. Thus it also illustrates the idea that Turtle Geometry is a "discovery rich" area of mathematical work for young people. The example presupposes a touch-sensor turtle.

The next figure shows a room with compass orientation, a turtle and

(State in Turtle Geometry)

a cluster of objects. The turtle's goal is to touch the north wall.



At the time he began to work on this, Pledge had already solved the problem of making a turtle circumnavigate an object. His procedure for this was one that many people of ages from 7 to 70 have re-discovered in their initial contact with LOGO controlled touch-sensor turtles:

```
TO CRAWL

TEST LTOUCH                 (See whether you have contact with
                            the object on your left.)

IFFALSE LEFT 10             (If not, you are wandering away
                            from it, turn towards the object.)
```

(State in Turtle Geometry)

```
IFTRUE RIGHT 10            (A little more subtle:
                           to avoid running into the object)

FORWARD 10                 (Take a little step forward.)

CRAWL                      (Recursion line:  keep doing
                           the same cycle.)
```

Now Pledge's plan was to program the turtle as follows:

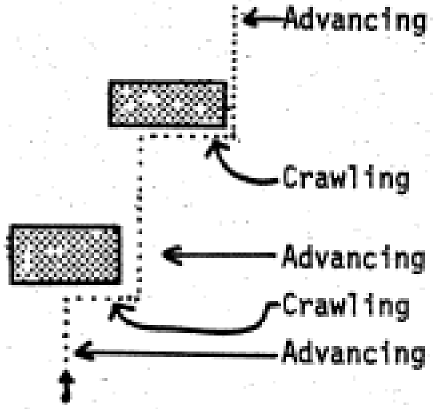
(1) When in the clear, move north.
Let's call this motion "Advancing".

(2) When contact is made with an object
start "crawling" around it.

(3) When you have crawled "enough" advance again.

The tricky part is to define "enough" and to modify the procedure CRAWL so that it would crawl "enough" (instead of going around forever as the original CRAWL procedure does).

Pledge's first attempt at defining "enough" used the simplest concept of the state of the turtle: whenever the turtle is heading North try to advance. Here is Pledge's picture of how the turtle might proceed:



(State in Turtle Geometry)

To achieve this, Pledge had to solve some small technical problems, whose details we'll gloss over except for one: how does the turtle "know" that it is facing north?

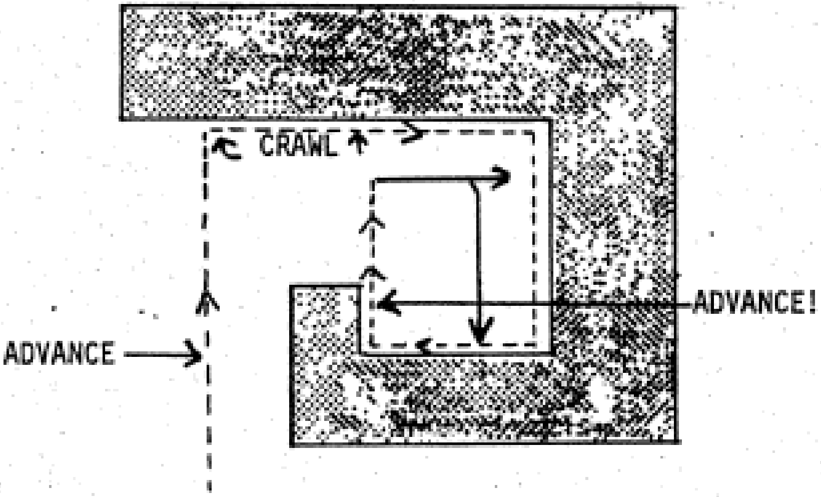
Pledge's solution is simple: assume it is facing north at the start; now since the turtle "knows" how much it has turned, it can keep track of its total rotation; let's call this "TOTROT". Whenever it turns right 10 (or 90) it adds 10 (or 90) to :TOTROT; when it turns left 10 it subtracts 10 from :TOTROT. And facing north can be expressed by:

:TOTROT = 0(modulo 360)

In general, the heading part of the turtle's state is given (in LOGO) by

(REMAINDER :TOTROT 360)= 0

The procedure conceived in this way provides very well for the kind of obstacle shown in the previous figures. But it does have serious bugs.



A Turtle Trap

(State in Turtle Geometry)

The "turtle trap" is an obstacle this procedure will not get around. It throws the turtle into a never-ending loop! What can be done?

Pledge's solution has a brutal mathematical elegance. If you really want to appreciate it, you should take time off at this point to find it yourself. It will be easier for you to do so than it was for young Pledge: you are grown-up and he is a kid; moreover, the turtle trap itself contains the germ of the idea (and the exposition above is full of little hints!).

The key observation is this: the state of the turtle contains no real memory of its past history. True, it is computed from :TOTROT; but when :TOTROT is smashed down modulo 360 all historical information is lost except what is embodied in the actual current position of the turtle. To make this clearer we use a very important concept: enriching the state. We'll do this using another important heuristic concept: inventing a "shadow turtle" (virtual turtle). The shadow turtle exists only "in the mind" -- which can include, fortunately, the "computer's mind". The state of the shadow turtle is position and :TOTROT (instead of position and heading). Let's make this more concrete by building a state table for a real and a shadow turtle. (Since position is not important here we shall ignore the position part of the turtle.)

(State in Turtle Geometry)

COMMAND	STATE OF REAL TURTLE (Heading)	STATE OF SHADOW TURTLE (:TOTROT)
	0	0
RIGHT 180		
	180	180
RIGHT 90		
	270	270
RIGHT 90		
	0	360
RIGHT 360	0	720
LEFT 720	0	0

Now we can state Pledge's idea very simply.

The old algorithm stopped crawling when the turtle's heading state is like it was at the start.

This means:
:TOTROT ≡ 0(mod360)

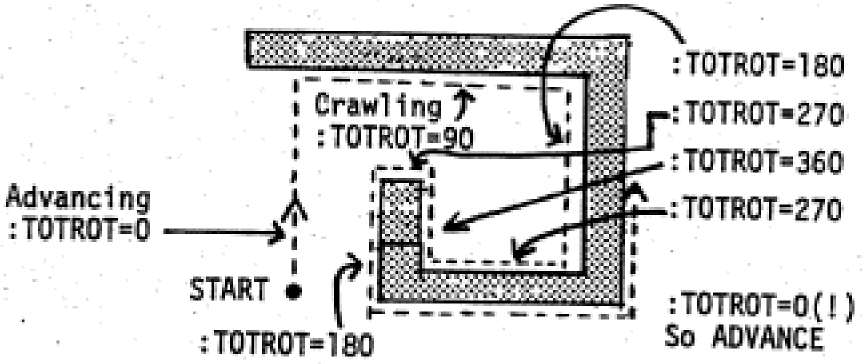
The new algorithm stops crawling when the heading part of the shadow turtle's state is like it was at the start.

This means when
:TOTROT = 0

Remark: The "more sophisticated" program looks simpler. In advanced topology one studies this idea more formally (under the "theory of covering spaces") but the beautiful basic idea here is available to children!

(State in Turtle Geometry)

The next figure shows :TOTROT as the turtle evades the turtle trap.


:TOTROT=180
Crawling
:TOTROT=90
:TOTROT=270
:TOTROT=360
Advancing
:TOTROT=0
:TOTROT=270
START
:TOTROT=180
:TOTROT=0(!)
So ADVANCE

Recall:
RIGHT 90 ADDS
LEFT 90 SUBTRACTS
to/from :TOTROT

The final program is:

```
TO ESCAPE

ADVANCE
LINEUP
CRAWL
ESCAPE
END
```

----- Turtle track
→ Pointers for Values of :TOTROT
▨ Obstacle

Aspects of the Idea of Local and Global in Turtle Geometry

The procedure:

```
TO CRAWL

TEST LTOUCH
IFTRUE  RIGHT 5
IFFALSE  LEFT 5
FORWARD  10
CRAWL
END
```

was taken for granted in the discussion leading up to Pledge's algorithm. But it is very instructive to look more closely at it and at how subjects of various ages work towards discovering it.

We have given many children the problem to which this is one possible answer: write a procedure to make the floor turtle go around an object.

The first step is to find a good, almost always rectangular object. The first attempt at a program is usually something like this:

```
TO CIRCUMNAVIGATE

TEST LTOUCH
IFTRUE  FORWARD 10
IFFALSE LEFT 90
CIRCUMNAVIGATE
END.
```

The intention of the procedure is obvious: the turtle "wants" to keep its left side in contact with the object. It uses its touch sensor to verify that this is the case. If not it "concludes" that the corner has been reached, so it turns 90°.

This kind of procedure will not work -- much to the surprise of many who have written them. We want to delve into three questions:

-- why doesn't it work?

(Local & Global in Turtle Geometry)

- -- how can it be made to work?
- -- is there some general idea that expresses the difference between the inadequate procedure CIRCUMNAVIGATE and the adequate one CRAWL?

The first question can be answered in several ways. One fundamental answer is that the procedure might work in an ideal dream world, but has no chance of working in the real world. The point here is that the procedure is based on the assumption that the turtle moves exactly straight alongside the edge of the object. It won't. Before it gets near the corner it will either drift off and get into the complicated situation shown in the left figure. Or it will run into the object and spin its wheels. If it did get to the corner it would have other troubles in the same spirit . . . which we leave the reader to sort out. A very different and equally fundamental answer refers to the way the writers of these programs often come to write them. This is by following our advice of pretending to be a turtle and walking around the object feeling it with the left hand (eyes closed of course, so as not to have an unfair advantage!). But in doing so they think of themselves as feeling only to determine whether they have reached the corner. They neglect another function of feeling the object: namely as a local control mechanism to keep themselves close to the table. More careful self-observation will eventually reveal this omission. Thus using oneself as a model generates the bug and then the solution. That is the way of progress toward truth!

Now the third question. Yes. One such general idea is that CIRCUMNAVIGATE "thinks globally" while CRAWL "thinks locally".

This is one of very many examples of how the turtle brings the student clearly into contact with the very fundamental and powerful idea of "local action". Without this idea (in some version) calculus is mere formalism and much of physics is quite unintelligible.

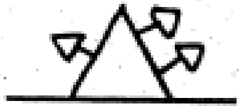
More examples will accumulate later.

3.4 Ramifications of Turtle Geometry into Physics and Biology

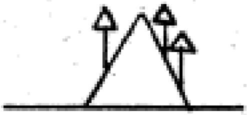
In the survey of Turtle Geometry we have dwelt on certain concepts -- "state","local control", "little-man models", etc. -- which we claimed, in passing, to be of great importance in scientific thinking generally, not merely in computation. This section will present some examples to develop this thought.

First let me make a quibbling qualification concerning the phrase: "other than computation". If "computation" refers specifically to machines like the ones made by IBM or DEC, this phrase can stand. But another, and deeper, way to describe the moral of the stories you are about to hear is to say that computation in a generalized ("metaphorical") sense is everywhere in nature: brains compute thoughts, seeds compute trees . . . sometimes it is heuristically valuable to say even that suns and planets compute orbits. By this I don't mean anything wild and controversial like denying that people are conscious, free agents or asserting that stones have minds. I merely want to focus on the very practical utility of seeing the world on occasion through "computational" (or "informational" or "cybernetic") conceptual spectacles.

For example, consider the way trees grow on mountain slopes. Piaget -- and others -- are struck by the fact that young children think they grow like this:

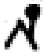
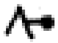


Older people know they grow more like this:



(Biology)

Why do you think they grow straight up? Or, perhaps I should ask: supposing you had never seen a tree on a mountain (or had forgotten) could you figure out which way of growing would be more likely?

Of course there are many kinds of "right" answers -- everything in nature is over-determined and so there are many reasons for it being the way it is. I want to consider a particular kind of reason brought out by the informationally anthropomorphic form of question "how does the tree know which way to grow?" Someone sensitized (perhaps through turtle geometry) to the set of issues we have been discussing will quickly recognize that it would be much easier to construct a local control process for vertical growth  than for growth at right angles to the ground  . Every part of the tree can -- in principle -- "know which way is up" . . . because "gravity is everywhere." But growth at right angles to the ground would require some specific mechanism to detect the lay of the ground and transmit it. And what would happen if the ground changed?

Some Examples of Areas for Systematic Work in Biology

On the Aggregation of Animals

If a population of wood lice (Porcellio scaber) is placed in a micro-ecology, with variable humidity, most will be aggregated in the moist regions. If a population of planaria (Dendrocoelum lacteum) is placed in a micro-ecology with variable light intensity, most will be aggregated in the dark regions. What kind of mechanism can do this?

From a turtle point of view there is an interesting duality. More careful observation will show that the rate of forward motion of the wood louse is affected by the humidity; in the case of the planarium light does not affect the velocity but the amount of turning. (These kinds of behavior are sometimes distinguished as "ortho-kinesis" and "klino-kinesis".) Just how does the stimulus affect the behavior? In some animals it is an absolute level; very often (necessarily for klino-kinesis to be effective!) it is the gradient. If an animal is to react to a gradient it must have an internal state (like the shadow turtle in the Pledge problem) with more information about the past than is embodied in the externally visible state.

The remarks are intended to transmit the flavor of an area of work in biology, consisting of formulating "turtle models" of actual animals with the goal of fitting a simulation to experimental observations. The appropriateness of the area is supported by the following observations:

(1) Very simple models do in fact account quite well for observed behavior.

(2) One finds in nature a variety of equally simple mechanisms, and the experimental task of differentiating between them is well within the technical reach of elementary aged children.

(3) The concepts needed to formulate the models are of great power and generality.

(4) This is very genuinely mathematical biology -- the mathematical principles are simple, but necessary to understand the biological phenomena even in a qualitative way.

Posture, Reflexes, the Build-An-Animal Kit

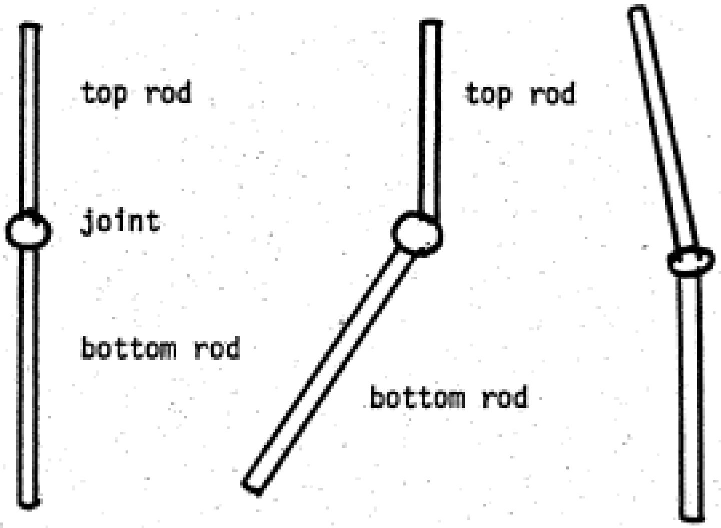
An Experience of Biology More Like a Creator's Than Like a Taxonomist's

Simulating orientation behavior in planeria or thinking about the control of growth in trees are small steps in a direction which leads to our favorite project: creating a "biology lab" in which students will be able to build cybernetic "animals" embodying the complex control tasks nature has solved through such devices as combining a central nervous system with a system of local reflexes, a particularly interesting (and physically fundamental!) collection of internal and external sense organs, the widespread use of quasi-independent sub-systems in its system architecture, ingenious coding methods and so on. We are not proposing to banish contact with real animals from the biological experience of children. But we do maintain that rediscovering the need for accelerometers provides at least as valuable an insight into the nature of the inner ear as does vaguely discerning the semi-circular cancels in the course of inexpert dissection. In any case, we are not proposing to make such choices, but to make it possible for others to make choices by providing them with more than one possibility.

The concept of the build-an-animal lab opens possibilities of work in many directions. We choose here one tiny example that has the advantages of being quickly described and linking well with other points we want to develop in this document. This latter, purely expository criterion, unfortunately biases the example towards looking more like "physics" than like "biology". But an imaginative reader will not have trouble seeing other possible directions.

Balance

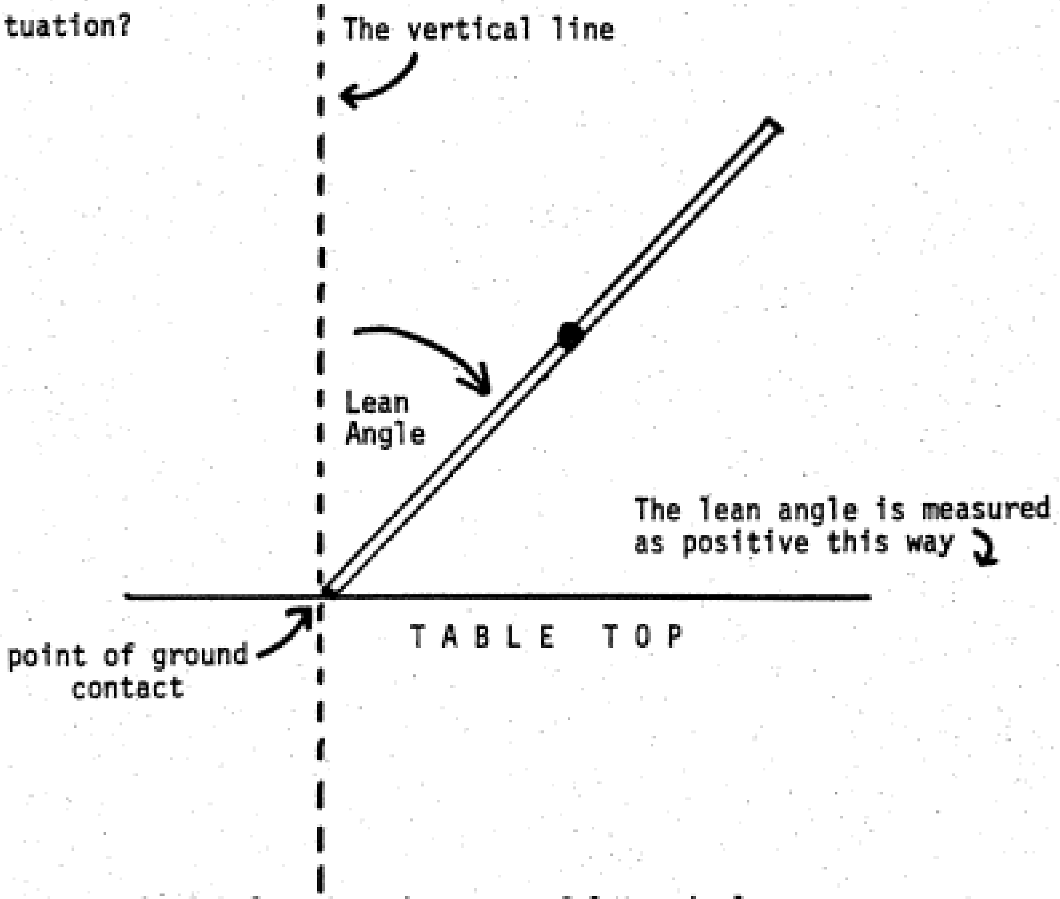
Consider a very simple challenge for thinking about how people or machines can balance. The figure shows a device consisting of two rods joined by a one-dimensional motorized hinge. The device is seen in several possible states.



(Balance)

The joint is controlled by a computer which also receives information from sensors attached to the rods. The problem to think about is how to make the system balance. To simplify the problem we treat it as planar: suppose that the system is constrained (by a hinge at the bottom) to move in the plane of the paper. Once you have understood the planar problem you can begin to ask whether the general system can be thought of as a super-position of planar systems.

The next figure shows a problem situation. The rods are leaning over and presumably falling. What action should be taken to recover from this situation?


The vertical line
Lean Angle
The lean angle is measured as positive this way
TABLE TOP
point of ground contact

An Imbalance, and some useful terminology

(Balance)

Clearly one has to turn the joint motor -- there is nothing else to do? But how much? And then what?

There are many possible solution strategies. The one chosen here is selected for reasons of exposition. To make it appear less arbitrary let's suppose that we do not know the center of mass of the upper rod (about which we assume only that it is heavy enough to be effective).

General Idea: Turn the motor counter-clockwise so as to bring the center of gravity across the vertical line. Gravity will then swing the whole system about the point of ground contact..

Sub-Problem: We do not even have direct knowledge of where the center of gravity of the system happens to be. So we need a control strategy based on getting this knowledge indirectly.

Strategy 1:
(Position Strategy)

Turn the motor counter-clockwise and watch the lean angle. When this becomes zero turn the motor clockwise until the rod angle is 180.

Bug: The over-shoot bug is evident from the series of seven framed pictures.

(Balance)

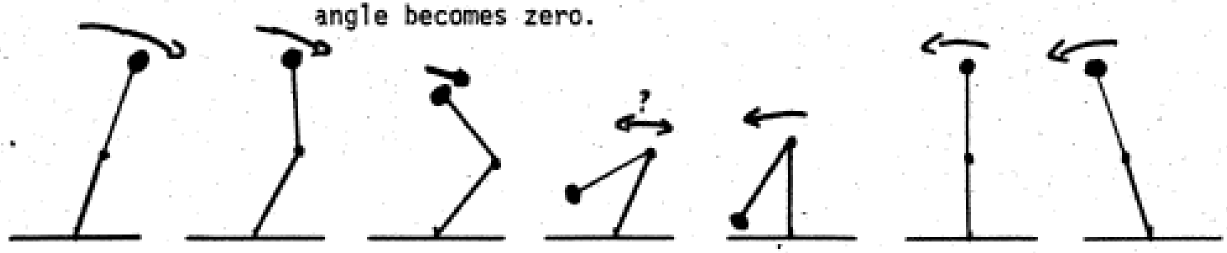
Strategy 2: (Velocity Strategy)

Instead of waiting until the lean angle actually becomes zero stop the motor when the lean angle stops increasing, in other words when it's velocity becomes zero.

Bug: Less disastrous but still there.

Strategy 3: (Acceleration Strategy)

Instead of waiting until the lean angle actually stops increasing, wait only until you see that the lean angle is increasing more slowly, then stop the motor. In other words stop the motor when the acceleration of the lean angle becomes zero.



State of Lean Angle	Lean angle still increasing.			Lean angle starts decreasing.	Lean angle zero.	Lean angle becoming negative because of momentum.	Toppling the other way.
State of Motor	Motor starts turning ↑	Motor turning.			Reverse Motor.		

(Balance)

Scenario of a Possible Project Sequence

Personae: Upper elementary or junior high student. A high school student as consultant.

Prior experience: Programming; turtle geometry; "non-dynamic control concepts" (e.g. as used for guidance of slow, sensor-turtles); a great deal of contact with the general art of choosing what information a procedure should use; slight contact with the idea of keeping "totals" and "gradients".

Phase 1: (First Shot) Student tries a procedure based on position strategy. No success. Appeals to consultant.

Phase 2: (Strategies for understanding) Consultant's advice is to see if the procedure will at least work in an easy case. Also to keep a trace of what happened. Student experiments with balancing sticks "by hand", soon finds that longer sticks are easier. Indeed, his procedure will work "slightly" with a very long rod. Why does it break down? The trace idea is translated into a very easy program to remember the history of an attempt at balance, and to play this back in slow motion on a CRT. He is intrigued enough to extend the power of the "play back" procedure to record a "balancing-by-hand" operation. The rest of the day is spent playing with records of his and other students' attempts to balance under varying conditions of difficulty.

Phase 3: (Insight) What is a difference between how people do it and how the procedure does it? An obvious one is "lead in" or "anticipation" i.e. the person does not keep pushing until the situation is corrected. How can this be done? *If you want to know when something will stop look at its speed*.

Phase 4: Getting it to work. (Perhaps with help from the consultant.)

Phase 5: (Playing with the Idea) Once it works it can be made to work better. Variations on the idea can be generated easily. For example: try to keep the rods in a steady oscillation; make the oscillation as big as possible; how *low* can you swing the rod and still recover? Can you manage *three* rods? Etc., etc.

Phase 6 to infinity: Applying these ideas to simulate, understand or reproduce other "balance" situations: walking, riding bicycles, tight ropes, studying animal forms with an eye on their adoption to effective balance, economic theory, metabolic equilibria. . . A typical mini-research project (combining elements one might normally classify as physics, psychology, math, simulation modelling and fun) is developing a theory of the limits of human performance in balancing skills. Anyone can balance a 2 meter rod on a finger; it is easy to learn to balance a 1/2 meter rod. I don't think that anyone can balance a 10 cm stick. Where is the cut-off? Why?

(Physics)

So What?

Different readers will surely have singled out different aspects of the possible educational advantages of learning science in context more like the Build-An-Animal Lab than like the traditional school laboratory. I want to emphasize some out of many facets which seem important to us. Each of these is a "working hypothesis" which must ultimately be justified by experiment, but, which, in the meantime, needs plausibility arguments to justify mounting the experiment.

(1) Observe the student's relationship to momentum. In the first strategy for balance momentum appeared as the villain of a mystery plot: why does the rod still fall? By the end of the project momentum has been mastered and made to serve the child's purposes. Contrast this with the presentation of the "concept" of momentum as an abstract principle or through "demonstration experiments". The line is admittedly not sharp; but there is reason to believe that our way might be more involving for many more students.

(2) In the school physics laboratory the student performs an experiment designed by someone else to prove someone else's point. When it's done it's ended. By contrast the balance projects are open ended, suggestive of personal ideas leading into "discovery-rich" areas of work.

(3) Observe a scene in which "momentum", "velocity", "acceleration", "force", etc. are used qualitatively. The term "qualitative" needs more careful definition and might, indeed, be dangerously misleading as used here. The point I want to make has to do with the fact that one can use the idea of, say, acceleration without the particular function-theoretic trappings in which classical calculus dresses it. In one sense that goes without saying: "step on the gas" is a way to "use acceleration". But I claim to see a deeper sense in which these physical ideas are used in ways that really capture their essential physical content. For the moment I ask the reader to contemplate the question. I'll return to explain more carefully what I have in mind. But, again, this point is being made too vaguely and needs clarification. The next section will help provide some.

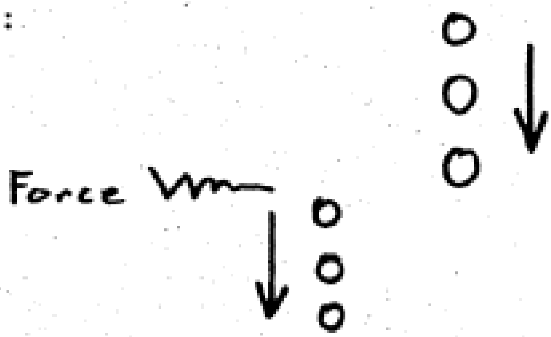
(4) Closely related to the previous point is the idea of developing more formal physics in closer interaction with intuitive physics. At least two aspects of this are apparent in the balancing projects. The algorithm eventually developed comes from examination and refinement of an existing intuitive idea.

(Physics)

The "qualitative" forms of physical "concepts" or "laws" are closer to intuitive ones than one (for example) the formation of calculus.

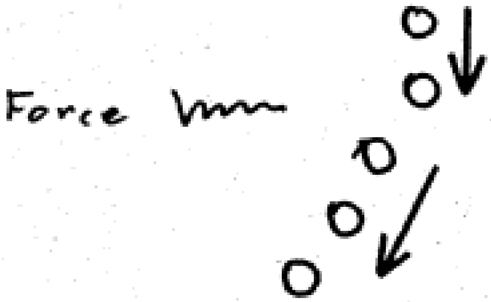
"Qualitative" Expression of (Part of) Newton's Laws.

Suppose a perfect round steel ball is rolling on a perfect table at (presumably) uniform velocity. A force is applied -- say by turning on an electromagnet for a short while. How will the ball be effected? Like this:



Ball changes position, keeps heading fixed

or like this:



Ball changes heading with "no instanteous effect on position."

(Physics)

Everyone knows "intuitively" that the change in direction is somehow more correct, though people often forget to apply this knowledge in particular problem situations. We'll return to look at some problems in a moment. First let's ask for a general statement of the principle involved.

In Newton's mechanics a particle, like a turtle, has a state. The turtle's state is

state of turtle == (position, heading)

The state of a particle (with constant mass so we need not mention it) could be described as

state of particle = (position, velocity)

The turtle had state change operators to allow us to act symmetrically on each component of the state. What are the state change operators for the particle?

The symmetry of the turtle has vanished! In fact, the position operator (like forward) seems to be taken over by the particle itself . . . it changes its own position according to its velocity. We can get a handle on it only by changing the velocity. So there is really only one effective state change operator . . . it changes velocity, leaving position alone. This operator is usually called Force.

Let's compare this description of a fundamental phenomenon of mechanics with a more familiar one contained in Newton's Laws:

(Physics)

	Traditional Formulation	Our Formulation
Newton I	The particle stays in its state of uniform motion unless a force is applied.	There is only one state change operator, called Force.
Newton II	F = ma (With mass constant force is numerically proportional to acceleration.	Force changes the velocity component (so force is an "ACCELERATOR").

The point I want to emphasize is that "our" formulation on the right expresses in a qualitative form a substantial and useful part of what Newton's Laws express in the traditional formulation using essentially the quantitative equational notation F = ma. Is this an advantage?

Certainly the qualitative formulation has an advantage for students who have not achieved easy familiarity with algebraic notation. But we attach more importance to deeper hypotheses related to the idea that qualitative formulations (the one given above is just one) can capture aspects of the "intuitive", "physical thinking" of ordinary people and even of sophisticated physicists.

Towards an Elementary Planetary Theory -- A Challenge to Physicists

This final example illustrates a type of research challenge we see as conducive to fundamental enquiry in the area we are trying to define. The challenge to develop an "elementary" planetary theory. The sense of "elementary" is to be taken in a very relative spirit, illustrated by the following sketch of a sample topic.

The Earth's Orbit is Closed
"The Earth Takes a Total Trip Around the Sun"

All readers probably know the importance in the history of physics of Newton's demonstration that the inverse square law for gravity implies an elliptic orbit (with the sun at one focus) and conversely. School children are repeatedly told of this (and similar feats) in a spirit that can only encourage a sense of mystical awe. There is no hint at how *they* can see a connection between the inverse square law and the shape of the orbit. Physics texts at college level copy one another endlessly in repeating the standard proofs using vector differential equations (or worse). We wonder whether some other way of looking at it all would not yield a more accessible and more intelligible demonstration.

Following the kind of Polyan heuristic we advocate to children, we began by:

(Planetary Theory)

Look for a simpler form of the problem.

How can the problem be simplified? One way, which we abhor, is to replace it by the boring numerical-computational problem: calculate the orbit and you will see that it is elliptical. This reduction to numerical form adds little to understanding.

We would rather go in the opposite direction. So we apply the heuristic advice:

Look for a more QUALITATIVE form of the problem.

For people steeped in Turtle Geometry one step in this direction suggests itself immediately.

A simpler form of the problem is:

Instead of proving that the orbit is ELLIPTICAL
prove merely that it is CLOSED.

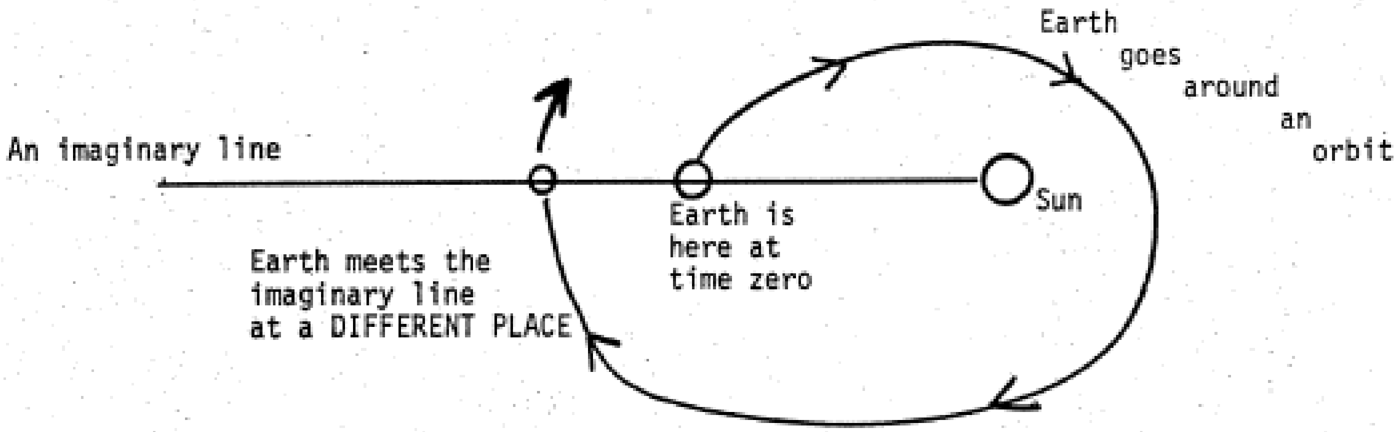
This is not the place to spell out all details; but any reader with a little imagination should be able to see that it is much, much more accessible than the standard proofs that the orbit is elliptical. We don't even need to pre-suppose a definition of ellipse!

> Anyone who wants to jeer and say: "sure it is easier to prove less", should go back to read Polya. Or just wait a little to see how our story grows!

(Planetary Theory)

Sketch of Proof

Let's try to prove the following to be impossible:


Earth goes around an orbit
An imaginary line
Sun
Earth is here at time zero
Earth meets the imaginary line at a DIFFERENT PLACE

What we shall actually prove is: if the earth crosses any half-line from the sun twice, it crosses it at the same place both times.

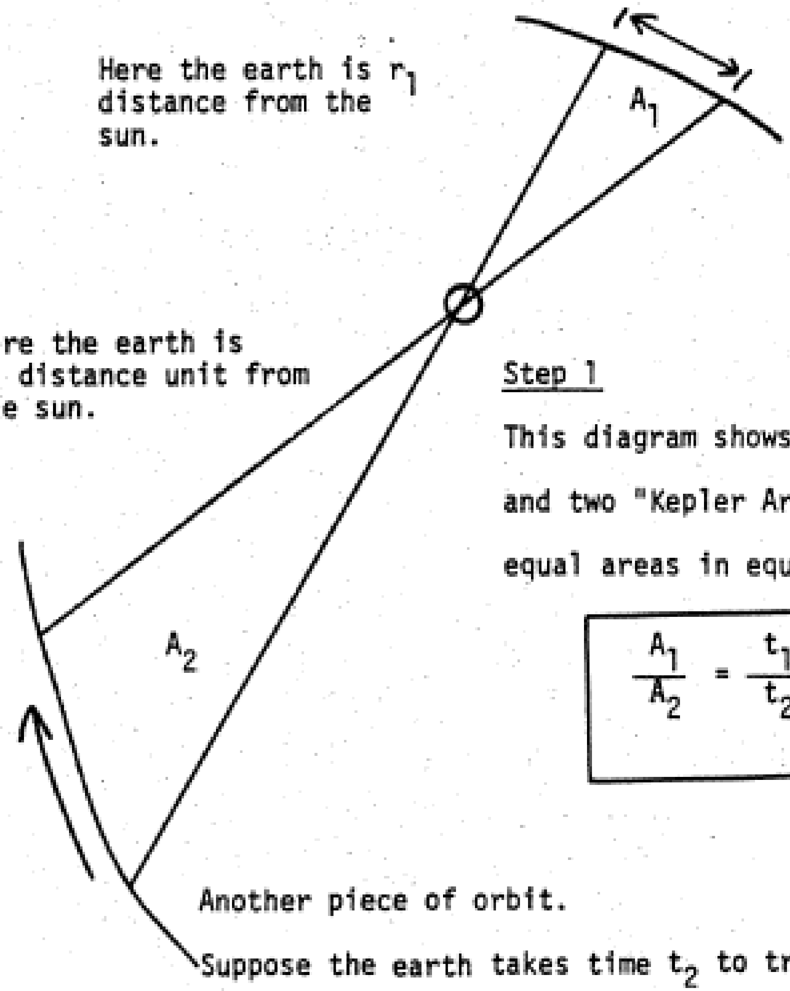
We'll use the following pieces of knowledge:

The attractive force is $\frac{1}{r^2}$

The angular momentum is conserved (we use this in the form of Kepler's equal areas in equal times law).

(Planetary Theory)

Energy is conserved. (We use this in the form of a principle of trade-off between potential and kinetic energy: if one changes, the other must as well. If the earth comes closer to the sun it must move faster.)



A piece of orbit

Here the earth is r_1 distance from the sun.

Suppose the earth takes time t_1 to traverse this piece.

Here the earth is r_2 distance unit from the sun.

Another piece of orbit.

Suppose the earth takes time t_2 to traverse this piece.

Step 1

This diagram shows two pieces of orbit and two "Kepler Areas", A_1 and A_2. The equal areas in equal time law says that

$$\boxed{\frac{A_1}{A_2} = \frac{t_1}{t_2}}$$

(Planetary Theory)

Now what else do we know about A_1 and A_2? Some simple geometry tells us

$$\boxed{\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}}$$

Now r_1^2 and r_2^2 are _very_ suggestive. What else do we know about them? The key fact is that the gravitational _forces_ are proportional to these.

So:

$$\frac{|F_1|}{|F_2|} = \frac{r_2^2}{r_1^2}$$

So: $$\frac{|F_1|}{|F_2|} = \frac{A_2}{A_1}$$

So: $$\frac{|F_1|}{|F_2|} = \frac{t_2}{t_1}$$

So: $$t_1|F_1| = t_2|F_2|$$

Now since F_1 and F_2 pull in opposite directions we see:

$t_1F_1 = -t_2F_2$.

But what is tF? It is the impulse of the "kick" or the charge of velocity! So we see that the charges of velocity on these two sectors are equal and opposite. So they cancel. Putting this together: the charges in velocity all the way around the orbit all cancel. So the velocity at the end must be the same as the velocity at the beginning.

Finally, our principle of conservation of energy says that if the earth ends up with its initial velocity it must also end up at its initial distance!

Exploring Further: A Turtle Theorem in Velocity Space

A cleaner but less elementary statement of this is:

<u>All Orbits (closed or not) are Circles</u>

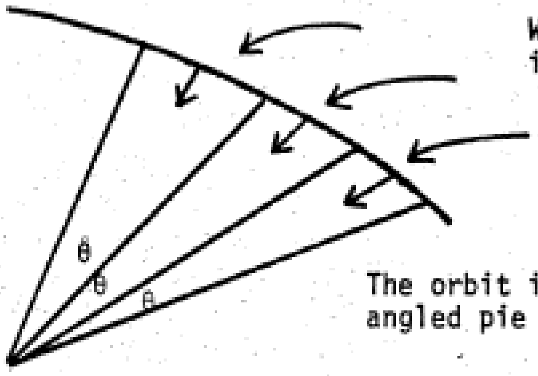
<u>in Velocity Space</u>.

Even elementary students in Turtle Geometry know that the following procedure generates a (close approximation to a) circle:

```
↻ FORWARD   :DISTANCE
  RIGHT     :ANGLE
```

↻ means: keep repeating

Now let's look at the successive "kick" vectors tF:



We are interested in these little vectors.

The orbit is divided into equal angled pie slices. Call the angle θ.

Think of placing the kick vectors end-to-end like the steps on a turtle trip. The rotation between each step and the next is constant: θ. How long are the steps?

(Turtle Theorem in Velocity Space)

Well the typical kick vector is tF.

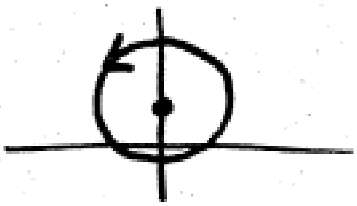
Now $\frac{t}{A}$ is constant.

So $\frac{t}{r^2}$ is constant. So $t|F|$ is constant.

So the kick vectors line up to make a circle!

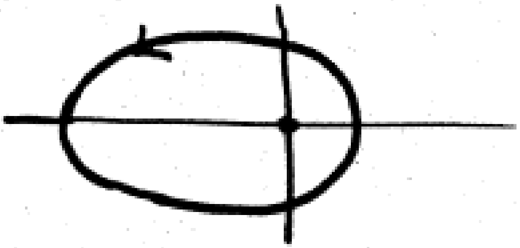
Qualitative Properties of the Orbit

The earth's orbit in velocity space is a circle. Like this:



Looking at the circle one can read off properties of the orbit in position space. Since conservation of energy means that a larger magnitude of velocity implies a smaller radius, and vice versa, we see that during one orbit the earth is once at a maximal distance (minimal velocity) and once at a minimal distance (maximal velocity). Also the axes of symmetry of the circle (with respect to the origin, zero velocity) carry over to symmetries in position space, and the following picture emerges of the orbit in position space corresponding to the velocity space orbit above.

(Turtle Theorem in Velocity Space)



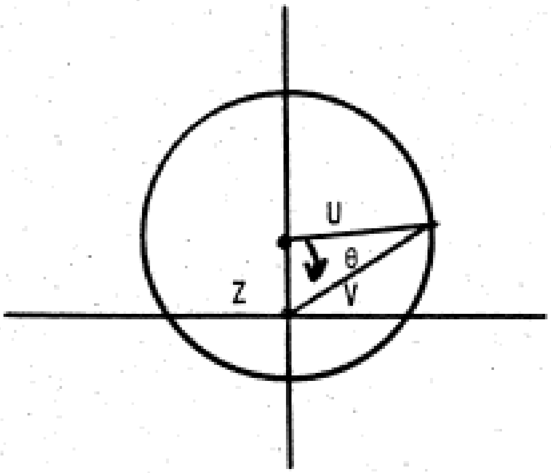
Towards a Theory of Astrogation.

Let's shift perspective and think of our orbiting object as a spaceship going around the earth. Can we find out how a short rocket engine burn will effect the orbit from our qualitative knowledge? Yes! Though we won't take the time for details here, simple impulses either away from the earth or perpendicular to that direction can easily be shown to have well-determined effects on the position of the center of the circle and the radius of the circle in velocity space. Translated into position space these tell us what happens qualitatively to the orbit, with little effort.

In fact it is not much harder to see the effects of adding almost any small force to gravitation. A small constant force (solar wind) on the rocketship can be seen to make a circular orbit more and more eccentric -- but in a direction perpendicular to the wind! Small radial deviations from $1/r^2$ make the orbit precess. All this from shifting attention to velocity space!

Analytic Properties of the Orbit

Once we have established our Turtle Theorem in Velocity Space it is possible to show that the orbit really is an ellipse using only algebra and trigonometry, although of course this is necessarily less "elementary" than the qualitative theory -- we must introduce equations. Let me not make this a thesis on orbits with details, but the curious are invited to apply the law of cosines to the following triangle in velocity space, and use conservation of energy, $v^2 = A + B/r$.



Recap of Orbits:

The usual method of attack on the problem of orbits brings the student to abstract and intuitionless manipulations involving differential equations. We think the alternate route we have outlined develops and brings to bear concepts more closely related to the physics and therefore more useful on an intuitive level. Whether such concepts as velocity space and the modes of thinking about the shapes of paths used here are only generic to the physics in this problem or can fit comfortably into a more expansive and complete view of elementary physics remains to be seen. We can only hope to claim that we have shown that global changes in thinking about teaching elementary physics might be useful and deserve to be thought about.

SECTION 4

SPECIFIC RESEARCH PLANS

This section sets out specific goals under the following sub-headings:

4.1 Computer Controlled Devices

4.2 Knowledge for Children

4.3 Contexts for Work with Children

4.4 Dissemination of Results

The section lists describe identifiable goals. However we do want to emphasize the fact that the most important part of our work is too intengible to be expressed as specific goals. This is the building of an intellectual style and atmosphere.

4.1 Computer Controlled Devices

LOGO Animals

Visitors to our laboratory might get the impression that we mean to inundate schools with a whole zoo of cybernetic animals. In addition to our original "turtle" (now in its fourth year of life!) there are several other species of turtle with various touch and light sensors, rumors of turtles with ears and hands, a large pneumatic "worm" and pieces of a "spider". Where will it stop?

In our view it will stop when we have perfected a modular kit which will not only enable educators to design devices to suit their own needs but enable children to build "animals" as well as program them. The project of making this "Build-An-Animal Kit" is slowly progressing on a design level. The first modules will appear during the next academic year. Behind the proliferation of animals is an examination of a variety of possible technologies. The latest worm is diametrically different from the first turtle in many dimensions:

-- It's power is pneumatic (rather than directly electro-mechanical.

-- It's effectors are "muscles" whose length and rigidity changes in very "biological" ways (as opposed to the turtle'svery "un-biological" motors.

(Computer Controlled Devices)

- It's mode of operation is conceptually analog (as opposed to the highly digital, discrete turtle design).

The "spider" (not yet working) goes a step further in exploring, "soft", pneumatic technology. Like the worm it derives its motive power by creating changes of air-pressure in appropriate places. One projected version also derives its structural rigidity from air pressure. Without power it would become as limp as a deflated balloon -- which, indeed, it is.

Our research strategy in this area is dominated by two ideas:

- the idea of making computer controlled devices for children is so new that one must expect to grope a little before obtaining a firm grasp on the relative advantages of available technologies.
- exploration of this kind is easy and inexpensive, especially in the context of a technological university such as M.I.T. Making a new animal (or a new organ for an old one) has proved to be an excellent topic for a student research project.

So we propose during the next year:

GOAL (1) To set up a work-shop at M.I.T. in which students (and, of course, staff members) can build devices.

(Computer Controlled Devices)

The material resources of the workshop will be matched to computer-controlled devices; the atmosphere will be matched to the informal way in which students work most productively (as opposed to a professional shop full of machines that need specialist operators who expect professional technical drawings and so on). We are quite sure that a relatively small investment in this workshop will produce a large multiplier in our productivity.

GOAL (2) To construct a basic module for the build-an-animal kit consisting of two rods connected by a rotary joint with one degree of freedom. It is an open question how this joint should be powered (rotary motors vs. linear actuators; electric vs. hydraulic vs. pneumatic). During this summer we will review alternate designs and make a firm choice for the first module.

(Computer Controlled Devices)

Terminals

Input Side

As we progress towards computer systems better matched to the needs of children, the clumsiness of the standard "terminal" becomes more and more of a sore thumb. We really need something fundamentally better in human engineering. We do not intend to make general terminal development a focus of research in our laboratory (except that we will be receptive to proposals from students for low-profile, inexpensive exploratory projects and will keep in touch with other groups in universities and industry). However, we do have some specific projects on the development of terminals for special purposes. In particular:

GOAL (3) terminals designed for pre-literate children (or adults!)
GOAL (4) terminals designed for people with severe motor disabilities
GOAL (5) hand-held terminals
GOAL (6) "analog" terminals

Output Side

Displays

The project of developing low-resolution, low-cost displays has been taken as far as we need to go at the moment.

During the coming years we want to get some experience with the potentialities at the other extreme of the resolution spectrum. The best route we see into this area is through close collaboration with Alan Kay's group at the Xerox Palo Alto Research Center. We expect that this group will be able to lend us one of the computers they have developed as large scale simulators of the Dynabook. If this happens the first areas we shall explore are:

-- integrating Kay's style of freehand graphics with our style of programmed graphics

-- exploring new horizons opened by the greater potential of Kay's system for real time and multi-processing computations.

We already have a PLATO terminal on loan from the PLATO group at Urbana and are beginning to explore the potentialities of the plasma display in our context.

GOAL (7) As a research tool for immediate work we need a sufficiently portable display system to take to sites where we want to work with particular children on an ad hoc basis. We are studying commercially available systems for this purpose.

4.2 Knowledge for Children

This part of the work covers the essential educational content. It includes what has been described in previous sections as "re-conceptualizing" areas of knowledge such as physics, mathematics, music, physical skills, elementary cognitive science, elementary computer science (including programming). Specific sub-projects which we expect to emphasize next year are:

Developing Language and Images for Thinking About Programming

The idea of a flow-chart is a classical example of what falls in general under this heading. Flow charts are useful but not universally good pictures of computational processes. In particular they are quite inadequate as representations of recursion and other aspects of procedural interactions. One of our earliest contributions was the "little man model" for computation which has now been greatly refined and demonstrably helps beginners of all ages avoid quite needless conceptual difficulties which, for example, often make a bug-a-boo of the very simple idea of recursion.

The development of such conceptual devices is a continuingly important facet of the life of our research team. It has benefited in the past years from the fact that we are constantly engaged in teaching at almost all age levels (from age 4 to adulthood!) and so constantly brought face to face with deficiencies in our ability to present computational ideas with the utmost clarity.

We propose to maintain the following minimal teaching operations to support this (and other purposes simultaneously):

GOAL (8) Individual Work with Pre-Literate Children

This work is a major continuing interest of Radia Perlman who began it while an undergraduate and is now a beginning graduate student in the M.I.T. Mathematics Department and the A. I. Laboratory.

GOAL (9) Small Group Teaching at Mid-Elementary Ages and Varying Cultural Backgrounds

Before early 1973 most of our contact with children was in a suburban school setting (Lexington, Mass.). We have begun to work with children of more varied social background in Cambridge. Cynthia Solomon will continue this work, in the future, entirely with children from what is regarded as a "difficult" urban Cambridge school. There will be a special emphasis on integrating into the presentation of computational thinking the special interests, linguistic preferences and intellectual strengths of children from this area.

GOAL (10)* Remedial Teaching of Older Students

GOAL (11)* Teaching M.I.T. Undergraduates

Experience with teaching is necessary but far from sufficient for this work. Two other phases which we attach great importance to are:

GOAL (12) Experiments in the Very Clear Expression of Ideas Through Writing and Film

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*No cost to NSF.

GOAL (13) Interaction with Leading People Engaged in Creating New Programming Languages

Fortunately there is a high concentration of these in our immediate vicinity in the M.I.T. A. I. Laboratory, in particular: Carl Hewitt (originator of the language PLANNER), Gerald Sussman, Joel Moses and Drew McDermott (originators of the language CONNIVER).

New Conceptualizations, Images and Language for Mathematical Physics, Biology, Control Theory, etc.

Interest in this area has been steadily growing in and around our group over a period of a few years. This year its intellectual productivity has taken a qualitative jump. People involved include the following: A powerful group of three close collaborators, Hal Abelson (mathematician), Andy di Sessa (physicist), Lee Rudolph (mathematician and poet); Gerald Sussman whose recently completed Ph.D. thesis includes a new theoretical model of the intellectual content of acquiring mastery of intellectual "skills" such as ability to design circuits or solve problems in mechanics;

Ira Goldstein (who is on the verge of completing a Ph.D. thesis mentioned several times in this proposal). Abelson, Goldstein and Sussman will all have junior faculty appointments at M.I.T. in the next academic year and will thereby gain a large multiplier in effectiveness through drawing new students into this kind of work. A locus of activity will be:

GOAL (14)* Two New Seminars on "Intuitive Mathematical Sciences" and "Heuristics" to be offered by Abelson and Goldstein in M.I.T.'s new Education Division.

Music

The program of research on musical education has many facets including contributions to musicology, to the psychology of music and to teaching of music for its own sake. The facet we emphasize here draws on all of these and can be labelled "Integration of Music into the Child's Intellectual Life". We see it as an important part of our campaign against the separation, especially in Elementary School, of knowledge into compartments called "science" and "art" or "mathematics" and "physics". If a child is truly interested in music, then music should be a locus for the employment and exercise of his growing intellectual capacities, including those aspects

*No cost to NSF.

usually called "mathematical", "logical", "scientific", "experimental", etc., etc. Carrying this idea into practice has many components. The most obvious is to pursue the ideas we have already demonstrated in using computers and computational concepts to bring young children up to a level of mastery of music which will permit them to work freely in composition and other musical activities. Work along these lines is directed by Jeanne Bamberger, an experienced and creative musician and music educator who has joined our research team and has recently been appointed as Research Associate of M.I.T. jointly in the Education Division and the Music Section of the Humanities Department. It must be emphasized that even if this were all that we achieved a contribution would have been made to mathematics and science education as well as to music education. Children who learn in this way gain a sense of the power of mathematics and general heurstics to achieve personal goals. But this is not all. Bamberger and others are pursuing more ambitious plans of developing explicit formulation of conceptual structures -- of which those related to time are the most obvious -- which overlap music and scientific thinking. The music project also plays a key role in our general investigation into the nature of intuitive thinking.

GOAL (15) Work with children on integrated MUSIC-MATH-SCIENCE projects.

GOAL (15a) Conceptual studies of the relation of musical to more general conceptual structures.

Heuristics as a Subject for Elementary School

The incubation of this idea will continue during the coming year as a "background activity" in the laboratory. It is very probably near the threshold of emerging as a full-fledged activity with specific research goals and personnel, in which case it will figure very prominently in our next round of proposal writing.

4.3 Contexts for Work with Children

(a) The M.I.T. LOGO Lab

At M.I.T. we have a well-equipped LOGO Lab on the general lines described in §3.2. One mode of working with children is to bring them (or let them come!) in from neighboring areas of Cambridge.

(b) Outposts at Schools

At the Bridge School we have a small facility remotely operated from M.I.T. over telephone lines. It has one display and a variable number of other terminals and devices. This has been the site of by far the most profitable work with children (except perhaps, for our Exeter experience, see below.). The facility is nearing the end of its useful life partly because its hardware was hurriedly produced four years ago and is beginning to fail, partly because our methods of work have advanced. We do not plan to renovate it under this proposal. If the Lexington School system (or some other agency) will carry its renovation and local operational costs, we consider that our project would gain enough from its continued operation to justify providing remote computer service.

On the other hand, we do urgently want to create a more modern outpost in or near a Cambridge School. ("Near" might mean renting a store front or part of a house within a block or two of the school.)

GOAL (16) Set up a small computer facility with remotely controlled terminals and devices in conjunction with a Cambridge School.

(c) Planning A Demonstration School or Learning Center

Goal (16) will move us a step closer to a controlled experiment on the effectiveness of our ideas under realistic educational conditions. But the experiment will still be limited by the fact that the children involved spend the greater part of their time in a "normal school" whose intellectual style is different from, and even antagonistic, to ours. A really rigorous and convincing experiment will require setting up a coherent learning environment in which children can be more totally engaged. We are conceptually ready to design such environments and see the designing process as an immediate goal. We have not included the operation of such experiments in this proposal for three reasons:

- -- we have not settled on a final design, though we are close to doing so.
- -- the final design might be for an experimental school adminstered by a public school system (or consortium of school systems) rather than by our Laboratory.
- -- the necessary preparation time makes it quite impossible to begin operation of such an experiment during the present fiscal year.

We do consider designing such experiments as part of the work under the proposal and intend publishing a set of "Designs for Learning Environments". When we have formalized our intentions we may submit a new proposal on this project.

4.4 Dissemination and Publication

This phase of our work has included:

(a) Publications in scholarly Journals and through our internal memo system. See bibliography.

(b) Making movies of children at work in our classes. We maintain half a dozen copies of these which have been shown at numerous conferences, seminars and on one national and several local television programs. The movies are freely available for loan to interested people and institutions.

(c) Conferences and public lectures.

(d) System dissemination.

We expect scholarly publication to increase very substantially in the next year as a result of the maturation both of ideas and of members of the project. This proposal itself overlaps considerably with the draft of a book expressing the underlying ideas of our work. This aspect makes only small demands on the budget. However, we are sufficiently encouraged by the success of our movies to include a bigger budget item for movie-making.

Our contribution to system dissemination does not add significantly to costs, but we consider it to be very important and have included a small item for this. During the past few years our biggest contribution to public dissemination of LOGO took the form of making available to BBN* our additions to LOGO of Display, Turtle and Music primitives. We expect to deal more directly in the future with other institutions who may want our new LISP LOGO (which we now consider to be the best instrument for more universities who wish to experiment with LOGO) our PDP-11 system and several more experimental systems which will probably exist as add-on modules to the LISP LOGO.

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*Bolt, Beranek and Newman has been responsible for public dissemination of standard LOGO system for the PDP-10.