

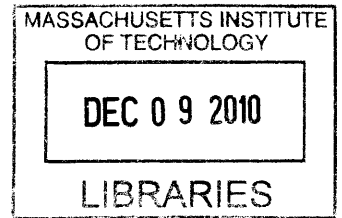
REACTIVITY-EQUIVALENT PHYSICAL TRANSFORMATION MODEL  
FOR PIN CELL ARRAYS

by

Steven T. Lynch

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Signature of Author: \_\_\_\_\_

  
Steven T. Lynch  
Department of Nuclear Science and Engineering  
May 17, 2010

Certified by: \_\_\_\_\_

  
Benoit Forget  
Assistant Professor, Nuclear Science and Engineering  
Thesis Supervisor

Accepted by: \_\_\_\_\_

  
Dennis Whyte  
Associate Professor, Nuclear Science and Engineering  
Chair, NSE Committee for Undergraduate Students

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**Abstract**

The behavior of TRISO fuel used in high temperature gas reactors in order to achieve high fuel performance is difficult to model using traditional lattice codes due to the double-heterogeneity effect created by the multi-coated fuel kernels in a graphite matrix. A simple volume-weighted homogenization does not accurately reduce the problem to one degree of heterogeneity as it does not properly account for the self shielding of the TRISO particles. The Reactivity-equivalent transformation (RPT) model, which condenses the TRISO fuel into a smaller fuel zone radius before homogenization, has been proposed as a possible solution to the problem of double-heterogeneity. The RPT method has been demonstrated to accurately model the reactivity of individual pin cells. While small, seven-cell RPT arrays are still highly accurate models of TRISO behavior, it is unclear if negligible error will extend to even larger arrays, especially in the presence of a B<sub>4</sub>C absorber. The validity of RPT array models was assessed by comparing the reactivity, thermal absorption, thermal utilization, resonance escape probability, fast fission factor, power, and neutron flux with reference values for an array containing all double-heterogeneous cells.

Thesis Supervisor: Benoit Forget

Title: Assistant Professor, Nuclear Science and Engineering

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# Chapter 1

## Introduction

High temperature gas reactors (HTGR) achieve high fuel performance utilizing multicoated TRISO fuel particles. And as HTGR technology continues to advance, it is essential to develop accurate, efficient models of TRISO fuel behavior under reactor operating conditions. TRISO fuel elements typically consist of a  $\text{UO}_2$ <sup>1</sup> fuel kernel surrounded by a porous carbon buffer region, an inner pyrolytic carbon coating, a layer silicon carbide, and another outer pyrolytic carbon coating. These coatings maintain both the integrity of the TRISO particles themselves as well as contribute to the overall safe operation of the HTGR. The buffer region, composed of low density pyrolytic carbon, serves two main roles: 1. protecting the inner pyrolytic carbon layer from radiation damage by capturing fission product recoils from the fuel kernel and 2. controlling the internal pressure of the fuel particle. The inner and outer pyrolytic carbon coatings are higher density than the buffer region and prevent both the escape of fission products as well as the failure of the pressure vessel by maintaining compression on the silicon carbide region located between them. The silicon carbide coating is responsible for providing structural support to the TRISO in addition to acting as the particle's primary pressure boundary and fission product barrier [1]. The fuel region in its entirety is created by dispersing the TRISO particles within a graphite matrix contained by either a cylindrical compact or spherical pebble. The volume packing fraction of the fuel elements within the graphite matrix is low and normally between 29-35% [2-4]. Unfortunately, these TRISO particle and fuel zone design specifications make modeling TRISO particle behavior in HTGRs difficult using conventional transport lattice codes.

---

<sup>1</sup> While  $\text{UO}_2$  is the most common composition of the TRISO fuel kernel,  $\text{PuO}_2$ ,  $\text{ThO}_2$ ,  $\text{UC}_2$ , and  $\text{UCO}$  are also possible [1].



## 1.1 Double-heterogeneity Effect

Double-heterogeneity arises from the distribution of TRISO particles within the graphite matrix and moderator. The first level of heterogeneity results from the multilayered nature of the TRISO particles themselves. An additional level of heterogeneity arises from the fuel element's placement within the graphite matrix and moderator. This so-called double-heterogeneity effect of TRISO fuel particles in high temperature gas reactors is difficult to model using conventional transport lattice codes as they cannot accurately represent the complicated geometries created by randomly dispersed TRISO particles within a graphite matrix. To address this issue, it is necessary to design a model that homogenizes the TRISO particles and graphite matrix within the fuel zone, thus eliminating the obstacle of geometric representation, while also accounting for the effects of double-heterogeneity and resonance self-shielding<sup>2</sup>. A volume-weighted homogenization of the fuel zone is not a solution to this problem as it significantly reduces the resonance self-shielding, resulting in an underestimation of reactivity at low burn-up and an overestimation at high burn-up; however a potential resolution to this issue may lie in the Reactivity-equivalent Physical Transformation (RPT) model.

## 1.2 Reactivity-equivalent Physical Transformation (RPT) Method

The RPT model creates a single-heterogeneous problem out of double-heterogeneous one. This is achieved by condensing the TRISO particles within their graphite matrix, resulting in a higher TRISO packing fraction and reduced surfaces area of the fuel zone. It has been proposed that the new fuel region, once smeared and homogenized, can accurately model the double-heterogeneous fuel zone using conventional transport lattice codes. The radius of the new fuel zone is selected such that the neutron multiplication factor matches the value in the double-heterogeneous case. Additionally, the reactivity of the fuel has a linear relationship with the effective fuel radius, making it simple to determine the RPT radius [2].

---

<sup>2</sup> Resonance self-shielding is important to the accurate modeling of TRISO particles due to the many small, scattered concentrations of the  $\text{UO}_2$  fuel; thus, its effect cannot be ignored in homogenized approximations of the fuel zone.

### 1.3 Project Summary

The objective of this thesis is to confirm the Reactivity-equivalent Physical Transformation method of modeling TRISO fuels in High Temperature Gas Reactors and investigate its applicability to depletion calculations in both individual pin cells and pin cell arrays. To date, both conventional and diluted fuel kernels have been modeled using the RPT method. These models have been evaluated using Monte Carlo depletion calculations, DRAGON, and SCALE [5]. Additional research has also demonstrated that the RPT radius is insensitive to material temperature fluctuations between 300 and 1500°K [5].

While double-heterogeneity has been previously modeled using both UO<sub>2</sub> [4] and PuO<sub>2</sub> [6] fuels at packing fractions between 29% and 35% [2-4], this paper will focus on undiluted UO<sub>2</sub> TRISO particles at a packing fraction of 29%.

In order to assess the applicability of the RPT method, reference depletion calculations will be conducted using the DOUBLEHET cell capability in SCALE. SCALE accounts for double-heterogeneity by calculating appropriate flux disadvantage factors, resulting in properly self-shielded and homogenized cross-sections for fuel element materials.

The first models will be of an individual double-heterogeneous pin cell and a RPT pin cell. A simple volume-weighted pin cell will also be included in this comparison for reference and as a means of reaffirming its inaccuracy in estimating in cell reactivity over time.

After an accurate model of a RPT pin cell has been created, investigation will begin into the cell's behavior when placed in various array configurations. Three different array configurations of seven pin cells will be discussed in this paper: 1. a reference array consisting entirely of double-heterogeneous pin cells 2. an array made up of all RPT cells and 3. a hybrid array that contains a single RPT cell surrounded by double-heterogeneous cells. The purpose of these array configurations is to observe the behavior of RPT cells as they deplete in the presence of double-heterogeneous cells as well other RPT cells. Finally, using the basic array configurations described above, the response of RPT cells will be observed in the presence of B<sub>4</sub>C control rod.

In addition to allowing the modeling of double-heterogeneous problems using conventional lattice codes, a RPT pin cell that precisely accounts for self-shielding in the presence of other pin cells and neutron absorbers has benefits even for users of programs like

SCALE, which are now capable of modeling double-heterogeneity. Even simple double-heterogeneity calculations (e.g. a single pin cell model) using SCALE require a considerable amount of time to complete. RPT models have the potential to significantly decrease the run time of pin cell calculations, especially for more complicated geometries and lattices.

## Chapter 2

# Single Pin Cell Model

Currently, two models exist that provide approximations to the double-heterogeneous case. The first of these models is a simple volume-weighted approximation. The second method is a more complicated homogenization of the TRISO and graphite matrix, referred to as the Reactivity-equivalent Physical Transformation (RPT) model. The design specifications for each of these techniques are discussed below.

### 2.1 Double-Heterogeneous Pin Cell

The TRISO particles and graphite matrix region constitute the fuel zone of the pin cell. Surrounding the fuel zone is a helium gap. The helium gap separates the fuel zone from the graphite moderator. All of the pin cells used in this paper are hexagonal. The basic design specifications for this and all fuel-loaded cells are based on the specifications used by Yonghee Kim of the Korea Atomic Energy Research Institute [2]. The double-heterogeneous model will be used as a reference to which the other approximation methods will be compared.

#### 2.1.1 TRISO Particle Design

The TRISO dimensions used to model the double-heterogeneous pin cell are as follows: a 350 $\mu\text{m}$  diameter  $\text{UO}_2$  fuel kernel surrounded by layers of buffer, inner pyrolytic carbon, silicon carbide, and outer pyrolytic carbon (figure 2-1). The buffer region is a low-density pyrolytic carbon. The thicknesses of these layers are 100 $\mu\text{m}$ , 35 $\mu\text{m}$ , 35 $\mu\text{m}$ , and 35 $\mu\text{m}$ , respectively. 10.63  $\text{g}/\text{cm}^3$  was selected as the density of the  $\text{UO}_2$  fuel kernel (97% of the theoretical density). The material densities for the coatings of the fuel kernel are (in  $\text{g}/\text{cm}^3$ , from innermost to

outermost): 1, 1.87, 3.2, and 1.87. These material densities are valid for temperatures of approximately 1200 K [1,7,8]. Actual depletion calculations were conducted at 1171K [3].

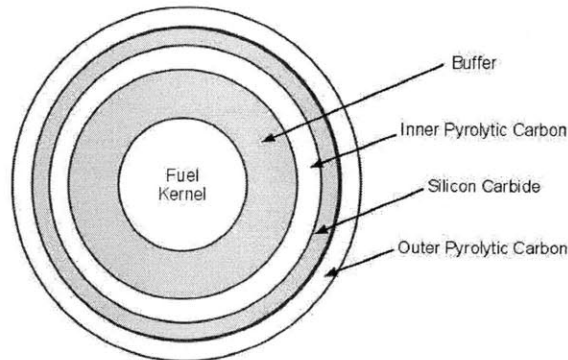


Figure 2-1: Coated TRISO particle

### 2.1.2 Pin Cell Layout

The hexagonal pin cells have a pitch of 2.2395 cm and consist of the cylindrical fuel zone and helium gap surrounded by graphite moderator. The graphite matrix, which contains the TRISO particles and completes the fuel zone has a radius of 0.6225 cm and a density of  $1.74 \text{ g/cm}^3$ . The packing fraction of the TRISO particles within the graphite matrix is 29%. Immediately outside of the fuel region is a  $125\mu\text{m}$ -thick helium gap. The remainder of the pin cell is filled with graphite moderator. The layout of the double-heterogeneous pin cell, neglecting He cooling channels, is depicted in figure 2-2.

## 2.2 Volume-weighted Pin Cell

The volume-weighted pin cell is a simple approximation of the double-heterogeneous case. In this pin cell model, all of the materials comprising the TRISO particles and matrix of the fuel zone are homogenized into a single mixture. In order to calculate the volumetric proportions of each material in the newly homogenized mixture, an arbitrary pin cell height of 10 cm was selected. The volume-weighted method has been demonstrated to be an inaccurate estimation of the behavior of the double-heterogeneous pin cell as it provides an underestimation of pin cell

reactivity at low burn-up and an overestimation at high burn-up due to its significant reduction of the resonance self-shielding within the fuel zone [3].

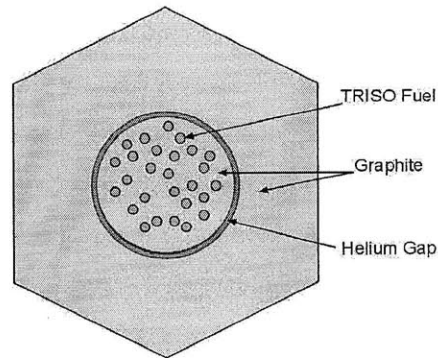


Figure 2-2: Layout of single hexagonal double-heterogeneous pin cell. The fuel zone consists of TRISO particles within a graphite matrix. A helium gap and graphite moderator surround the fuel zone.

## 2.3 Reactivity-equivalent Physical Transformation Pin Cell

A proposed alternative to the volume-weighted pin cell approximation is the Reactivity-equivalent Physical Transformation method. The RPT technique creates a new fuel zone by first concentrating the TRISO particles within a smaller radius. The TRISO particles, now at a significantly higher packing fraction than in the original double-heterogeneous model, are homogenized with the graphite matrix contained within the new fuel zone radius as show in figure 2-3. In terms of composition, the ratio of fuel to matrix in this homogenized region is substantially increased compared to the mixture created for the simple volume-weighted model. Also, the general layout of the RPT pin cell is different than the double-heterogeneous cell. There is now a TRISO-free region of graphite matrix between the fuel zone and helium gap.

### 2.3.1 Calculation of RPT Radius

The RPT radius was determined such that there was no difference between the reactivity of the RPT pin cell and double-heterogeneous pin cell during depletion calculations. The RPT radius for these approximations was calculated to be 0.408842 cm. Figure 2-4 depicts the nearly linear

relationship between changes in the RPT radius and reactivity. This radius corresponds to a new TRISO packing fraction of 67%.

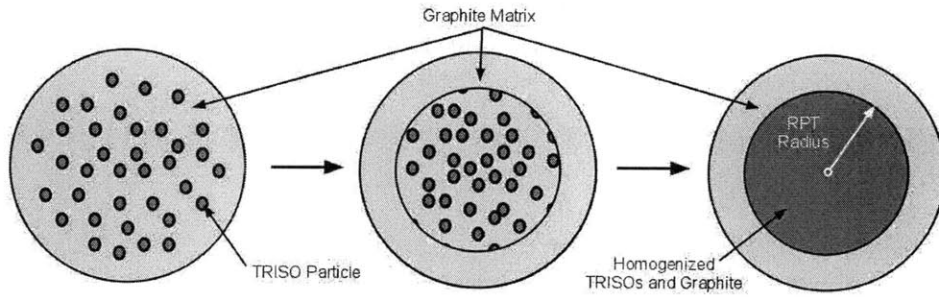


Figure 2-3: RPT method of fuel zone homogenization. TRISO particles are condensed at a higher packing fraction in the center of the fuel zone. The graphite and TRISOs in this new region are then homogenized into a single mixture.

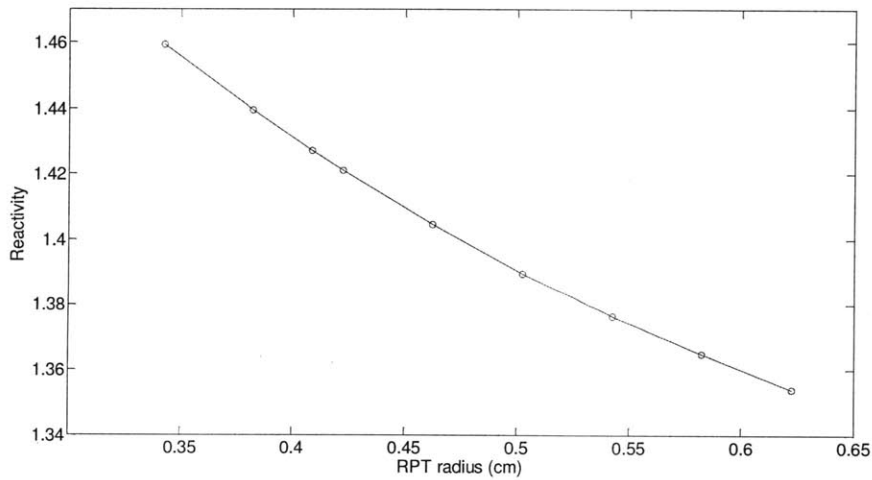


Figure 2-4: Reactivity as a function of RPT radius.

## Chapter 3

# Pin Cell Arrays

RPT pin cell behavior in arrays is also of interest in order to understand the effect these cell models have on each other and on non-RPT cells (double-heterogeneous pin cells). These effects were illustrated by three basic configurations: 1. an array of all double-heterogeneous pin cells, 2. an array of a single RPT pin cell surrounded by double-heterogeneous cells, and 3. an array consisting entirely of RPT cells.

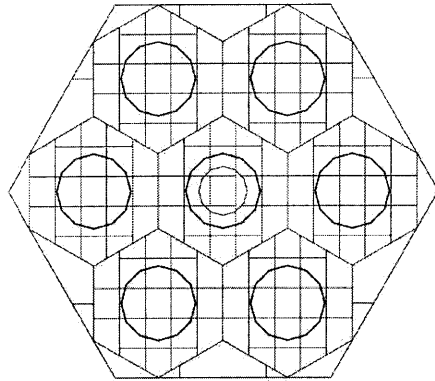
### 3.1 General Array Layout

As shown in figure 3-1, a simple array of seven pin cells was designed to investigate the interactions of RPT cells with both other RPT cells and double-heterogeneous cells. The regions of the array not occupied by pin cells have been filled with graphite moderator. White boundary conditions were selected for the individual cells and the array. Thus, the net angular neutron flux across all boundaries is zero,

### 3.2 Specific Array Configurations

An array consisting entirely of double-heterogeneous cells serves as the reference model to which the other simplified arrays are to be compared. The first of these simplified arrays is made up entirely of RPT pin cells. The intention of using this array configuration is to observe the behavior of RPT cells in the presence of other RPT cells during burn-up. Another configuration surrounds a single RPT cell with double-heterogeneous cells (figure 3-1) in order to test the effects RPT cells have on double-heterogeneous cells and vice versa.





*Figure 3-1: Array of seven pin cells. The center pin cell is a RPT model, while the surrounding six pin cells are double-heterogeneous cells. The regions not occupied by 1 of the seven pin cells are filled with graphite moderator.*

## Chapter 4

# Control Rod Arrays

Additional tests were performed on the pin cell arrays by replacing one fuel pin in each array with a  $B_4C$  absorber pin.

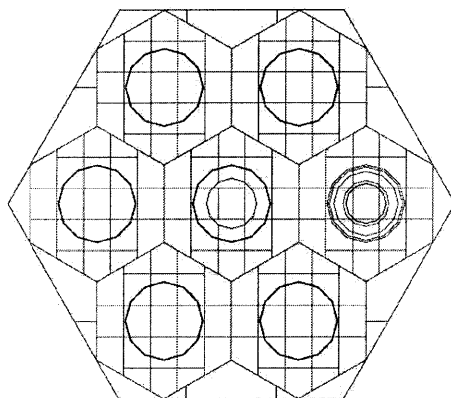
### 4.1 Control Rod Specifications

The basic design for the annular control rod used in these arrays is based on a design described by Y. Tachibana of the Japan Atomic Energy Research Institute [9]. However, it is important to note that for the geometric purposes of these tests the control rod design has been simplified and scaled down to approximately 1/10 of the original specifications. This was done so that the rod would occupy the same space as the fuel zone of its neighboring pin cells. In truth, a simple region of  $B_4C$  could have been used instead of an accurately designed control rod as the main interest of this paper is to ascertain the basic reaction of RPT cells in the presence of a neutron absorber. A suggestion of an HTGR control rod design is suggested in anticipation of future explorations of the RPT method involving a more complete modeling of an HTGR core.

The inner and outer sleeves of the rod are made of Alloy 800H, a corrosion and embrittlement resistant nickel-iron-chromium alloy designed for use in temperatures in excess of 1100 K. The inner and outer radii of these sleeves are 0.325 cm and 0.635 cm, respectively. The thickness of these sleeves is 0.035 cm. The annular  $B_4C$  absorber has inner and outer radii of 0.375 cm and 0.525 cm, respectively. All other regions of the control rod are filled with helium. The control rod is placed within a graphite moderator block of the same density used in the fuel-loaded cells.

## 4.2 Control Rod Placement

The control rod cell occupies the same geometric space as the fuel-loaded pin cells. It is positioned such that it replaces one of the fuel-loaded pins in the outer ring of pin cells in each array. Since the arrays are symmetric about each axis, it does not matter which of these outer fuel cells is replaced. Figure 4-1 depicts the configuration of the array containing double-heterogeneous cells, an RPT cell, and a control rod cell. The position of the control rod does not change within the array in either the all-double-heterogeneous or all-RPT array cases.



*Figure 4-1: Control rod placement in pin cell array. The control rod is placed within the outer ring of double-heterogeneous cells. The center pin cell is a RPT model.*

## Chapter 5

# Results and Discussion

Each of the individual pin cells and arrays discussed above were modeled using SCALE, which has the capability to handle double-heterogeneous cases. Depletion calculations were based on 18 month refueling cycles, with each pin cell remaining within the core for a total of 54 months. In order to observe cell behavior at high burnup (~200 MWd/MTU) the depletion calculations were made at an operating specific power of 120.4 MW/MTHM. In order to assess the validity of the RPT model, reactivity, thermal absorption, thermal utilization, resonance escape probability, fast fission factor, power, and neutron flux are considered.

### 5.1 Individual Pin Cells

A successful approximation of the double-heterogeneous pin cell was created using the RPT method. Only slight errors, not exceeding 0.13%, in the reactivity measurements occurred at high burnup in the RPT model. This is a vast improvement over the volume-weighted approximations, which underestimates the reactivity by nearly 6% at low burnup and overestimates the reactivity at high burnup by 7%. Figure 5-1 displays the trends of reactivity vs. burnup for each of the individual pin cell models.

In addition to comparing reactivity trends, examination of the four factors reveals again that the RPT model is valid for an individual pin cell. As shown in table 5.1, the differences between each of the four factors are at most a few tenths of a percent when comparing the double-heterogeneous and RPT cases. This is mostly due to the corrected self-shielding that is present in the RPT model as a result of decreasing the radius of the fuel zone. The absence of the self-shielding correction is obvious in the volume-weighted homogenization. While somewhat accurate estimates of the thermal absorption, thermal utilization, and fast fission factor

in the volume-weighted cell, the error of the resonance escape probability is considerable. At high burnup, this factor is off from the reference model by 10%. These results are consistent with the studies conducted by Kim [3] and confirms the ineffectiveness of simple volume-weighted homogenizations and the suitability of the RPT method for individual pin cells.

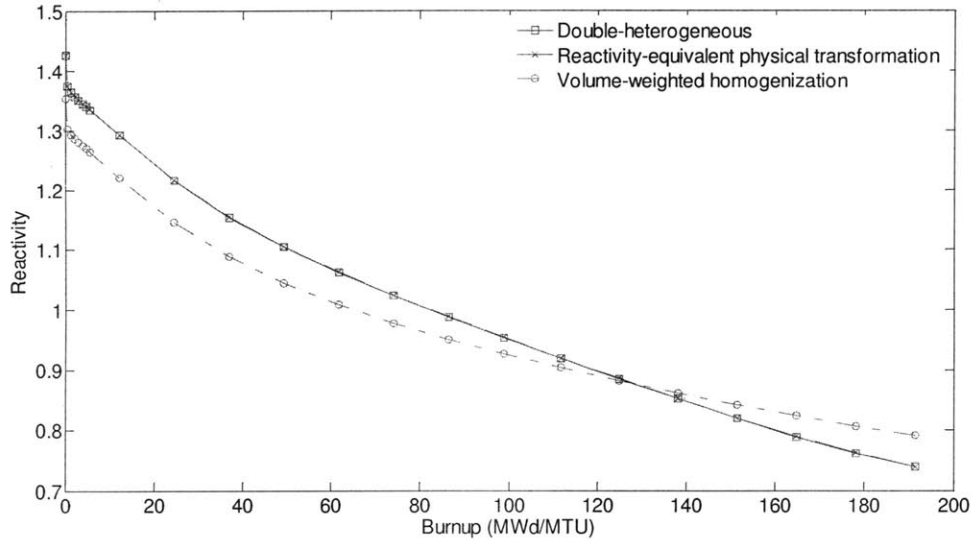


Figure 5-1: Comparison of double-heterogeneous, RPT, and volume-weighted cell reactivity during burnup.

Table 5.1: Effect of volume-weighted homogenization and RPT on the four factors.

0 days	$\eta$	f	p	$\epsilon$
DH	1.95192	0.98092	0.65775	1.13392
VWH	1.95223	0.98107	0.62110	1.13873
Error	0.00016	0.00015	-0.05573	0.00424
RPT	1.95962	0.97709	0.65760	1.13407
Error	0.00394	-0.00391	-0.00022	0.00013

101 days	$\eta$	f	P	$\epsilon$
DH	1.78454	0.98365	0.64846	1.13433
VWH	1.78090	0.98451	0.60983	1.14177
Error	-0.00204	0.00088	-0.05957	0.00656
RPT	1.79056	0.98037	0.64836	1.13455
Error	0.00337	-0.00334	-0.00016	0.00019

820 days	$\eta$	f	p	$\epsilon$
DH	1.53277	0.98190	0.58289	1.08729
VWH	1.58462	0.98674	0.53119	1.11578
Error	0.03383	0.00493	-0.08869	0.02620
RPT	1.53876	0.97828	0.58283	1.08757
Error	0.00615	-0.00558	-0.00007	0.00030

1588 days	$\eta$	f	P	$\epsilon$
DH	1.26147	0.97269	0.57385	1.04806
VWH	1.43830	0.98463	0.51409	1.08616
Error	0.14018	0.01228	-0.10414	0.03635
RPT	1.26924	0.96726	0.57381	1.04838
Error	0.00615	-0.00558	-0.00007	0.00030

## 5.2 Pin Cell Arrays

Since the volume-weighted homogenization was demonstrated to be a poor approximation for double-heterogeneity, it will not be treated in any of the subsequent array analysis.

### 5.2.1 Reactivity Analysis

The reactivity measurements from the depletion calculations of each array are shown in figure 5-2. The reactivity of the double-heterogeneous/RPT hybrid array matched closely with the reference calculations performed on the entirely double-heterogeneous array. The greatest error between the models was 0.012%. The RPT pin cell model also aligned closely with the reference model. The maximum error between these models was 0.087%. Both of these errors are less than the reported error in the single pin cell comparison.

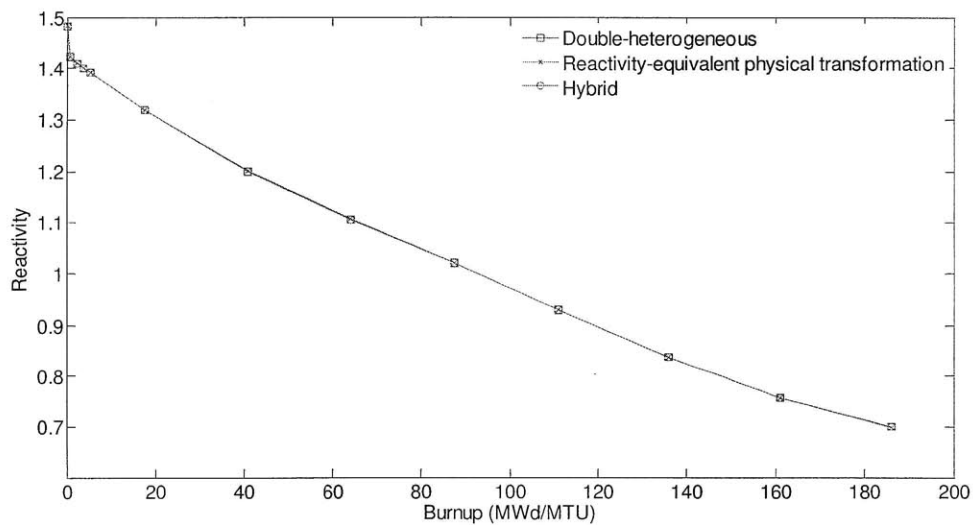


Figure 5-2: Reactivity during burnup for double-heterogeneous, RPT, and hybrid arrays. The hybrid array consists of a single RPT cell surrounded by 6 double-heterogeneous cells.

These results indicate that the RPT model accurately approximates the effects of double-heterogeneity during depletion when placed in arrays. This also demonstrates the RPT model's insensitivity to outside influences from neighboring cells and the moderator-filled border regions.

## 5.2.2 Four Factor Analysis

The values for the four factors in both RPT and hybrid arrays follow similar behavior as with the individual pin cell models. The error between the experimental arrays and the double-heterogeneous reference model are negligible in both cases (table 5.2). Maximum error occurs at high burnup, yet does not exceed 0.86%. The increase in four factor error between the hybrid and RPT arrays appears to be directly related to the number of RPT cells present in the array. The RPT array contains seven times the number of RPT pin cells as the hybrid array and the difference between each of the four factors of these models is approximately a factor of seven. This suggests that while the differences in the four factors are negligible for small RPT arrays, adjustments in the RPT radius may be necessary for larger arrays in which the error would become more significant.

Table 5.2: Four factor comparison of double-heterogeneous, RPT, and hybrid arrays.

0 days	$\eta$	f	p	$\epsilon$
Double-Het	1.95231	0.97584	0.70193	1.10949
RPT	1.96000	0.97200	0.70225	1.10957
Error	0.00394	-0.00393	0.00045	0.00007
Hybrid	1.95340	0.97529	0.70198	1.10950
Error	0.00056	-0.00056	0.00008	0.00001

340 days	$\eta$	F	p	$\epsilon$
Double-Het	1.68549	0.97781	0.66581	1.09188
All RPT	1.69162	0.97426	0.66633	1.09196
Error	0.00364	-0.00363	0.00078	0.00007
Hybrid	1.68636	0.97730	0.66589	1.09190
Error	0.00051	-0.00052	0.00011	0.00002

925 days	$\eta$	f	p	$\epsilon$
Double-Het	1.42289	0.96804	0.64177	1.05110
RPT	1.42964	0.96289	0.64240	1.05118
Error	0.00474	-0.00533	0.00098	0.00007
Hybrid	1.42384	0.96731	0.64186	1.05112
Error	0.00067	-0.00076	0.00014	0.00002

1541 days	$\eta$	F	p	$\epsilon$
Double-Het	1.11798	0.94983	0.63923	1.02802
RPT	1.12574	0.94169	0.63994	1.02821
Error	0.00694	-0.00857	0.00111	0.00018
Hybrid	1.11906	0.94867	0.63933	1.02805
Error	0.00097	-0.00122	0.00016	0.00003

## 5.2.3 Power Analysis

SCALE is capable of providing the power of individual cells during depletion, allowing for analysis of the interactions among pin cells within an array. Figure 5-3 compares the power of the center RPT cell of each experimental array during depletion with the center double-heterogeneous pin of the reference model.

The difference in power between the RPT and double-heterogeneous models is on the same order of magnitude as the errors in reactivity, with a maximum error of 0.059% at high burnup. The error of the center RPT cell, while no more than 0.26%, is noticeably greater than that of the RPT array cell. Also, the behavior of the center pin cell of the hybrid array over time is quite different than that of the RPT array. The RPT array cell closely follows the trends of the reference model, while the center cell for the hybrid array does not. The greatest error readings of the RPT cell within the hybrid array occur at high and low burnup, while the power almost plateaus at mid-burnup. This suggests that RPT pin cells most closely mimic the behavior of double-heterogeneous arrays when surrounded by other RPT pin cells.

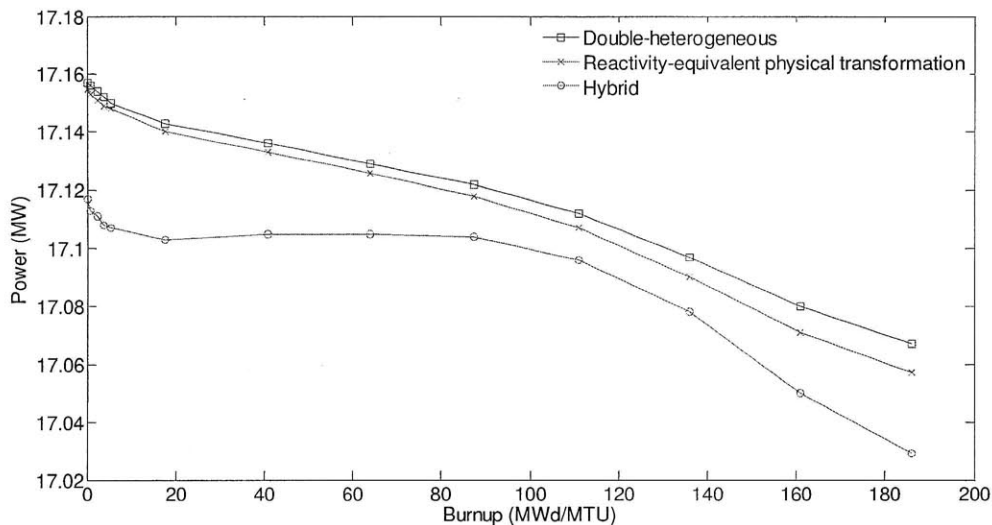


Figure 5-3: Comparison of center pin cell power among double-heterogeneous, RPT, and hybrid arrays.

### 5.2.4 Thermal and Total Flux Analysis

Examination of the thermal and total flux through the fuel zone reveals only slight inaccuracies between the experimental and reference arrays. Figures 5-4 and 5-5 compare the thermal and total fluxes of the center cell of each array. The RPT array never exceeds 0.75% error in thermal flux readings and stays beneath 0.42% for total flux. The hybrid has a somewhat higher thermal flux error (~1%) than the RPT array in the low to mid-burnup range, which corresponds to the



higher degree of error in the power calculations. This error increased, however, is not perceivable in the total flux, which only differs from the double-heterogeneous reference model by at most 0.99%.

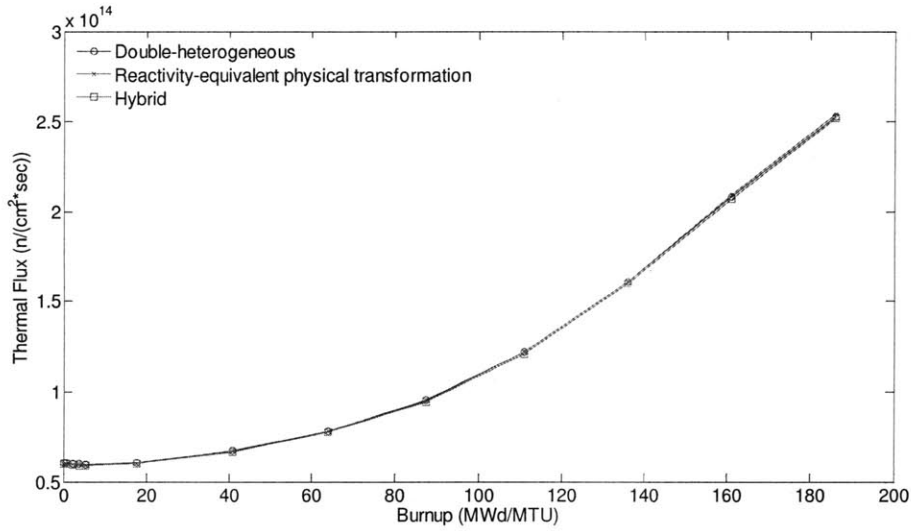


Figure 5-4: Thermal flux through the fuel region of center cells of double-heterogeneous, RPT, and hybrid arrays.

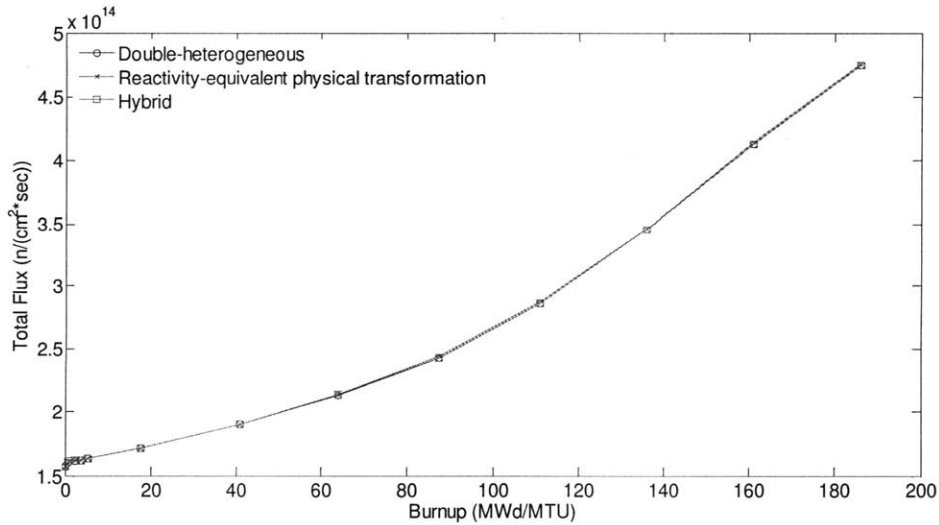


Figure 5-5: Total flux through the fuel region of center cells of double-heterogeneous, RPT, and hybrid arrays.

### 5.3 Pin Cell Arrays with Neutron Absorber

For the next set of calculations, one fuel-loaded cell from the outer ring of each array was replaced with an annular  $B_4C$  control rod in order to observe the behavior of RPT cell arrays in the presence of a neutron absorber. Aside from this change in array configuration, depletion calculations were conducted at 120.4 MW over a period of 54 months as before.

#### 5.3.1 Reactivity Analysis

The effects of a  $B_4C$  neutron absorber on the accuracy of either of the experimental homogenization arrays appears to be slight (figure 5-6). The most significant error occurs at low burnup and then decreases over time. The maximum reactivity differences between the RPT and reference arrays is now 0.25% (compared to 0.087% without the neutron absorber). Again, the maximum reactivity error of the hybrid array is approximately an order of magnitude less at 0.041%. This reaffirms that the error between experimental and reference models is partially dependent on the number of RPT cells present within the array. Thus, while the error measured for a small array of seven RPT cells is small, it may not be able to be neglected in larger arrays.

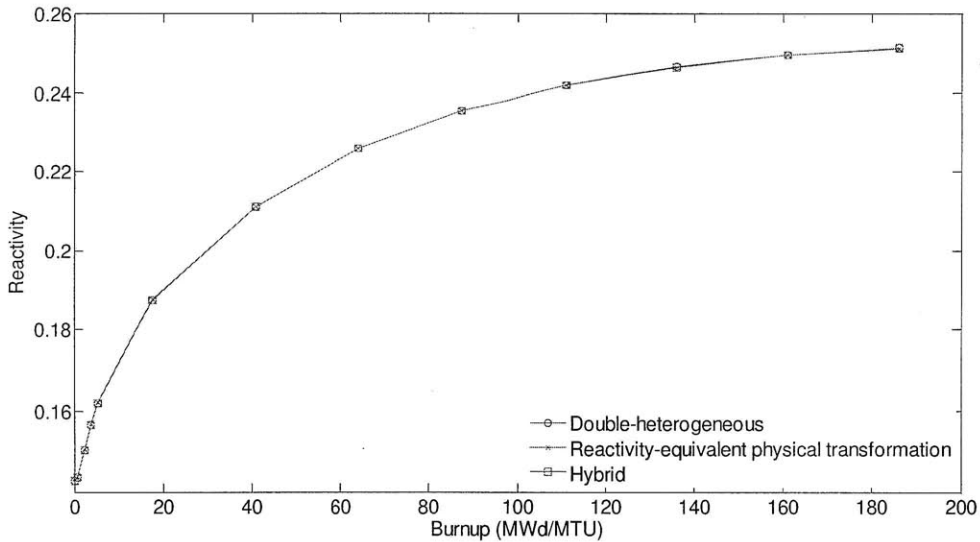


Figure 5-6: Reactivity during burnup for double-heterogeneous, RPT, and hybrid arrays in the presence of a  $B_4C$  control rod.

### 5.3.2 Four Factor Analysis

The four factor error when a  $B_4C$  neutron absorber is placed within each experimental array is minimal (table 5.3). While the error of each of the four factor increases with time there does not appear to be a clear factor by which the error increases based on the number of RPT within the array.

Table 5.3: Four factor comparison of double-heterogeneous, RPT, and hybrid arrays in the presence of a  $B_4C$  control rod.

0 days	$\eta$	f	P	$\epsilon$
Double-Het	1.94895	0.16087	0.19726	2.30942
RPT	1.95676	0.16013	0.19751	2.31347
Error	0.00401	-0.00456	0.00126	0.00175
Hybrid	1.95020	0.16074	0.19730	2.31008
Error	0.00064	-0.00079	0.00023	0.00028

340 days	$\eta$	f	p	$\epsilon$
Double-Het	1.69994	0.37392	0.17817	1.86266
RPT	1.70204	0.37233	0.17839	1.86668
Error	0.00124	-0.00424	0.00126	0.00216
Hybrid	1.70026	0.37366	0.17819	1.86342
Error	0.00019	-0.00070	0.00012	0.00041

925 days	$\eta$	f	P	$\epsilon$
Double-Het	1.66227	0.46620	0.16176	1.93092
RPT	1.66356	0.46409	0.16204	1.93461
Error	0.00077	-0.00452	0.00175	0.00191
Hybrid	1.66246	0.46585	0.16179	1.93165
Error	0.00012	-0.00075	0.00018	0.00037

1541 days	$\eta$	f	p	$\epsilon$
Double-Het	1.62895	0.50057	0.15340	2.00910
RPT	1.62994	0.49826	0.15368	2.01294
Error	0.00061	-0.00461	0.00177	0.00191
Hybrid	1.62909	0.50019	0.15343	2.00987
Error	0.00008	-0.00076	0.00017	0.00038

### 5.3.3 Power Analysis

The addition of the  $B_4C$  control rod does not substantially affect the accuracy of the center RPT fuel cell in either of the experimental arrays (figure 5-7). As was the case in the non-absorber models, the RPT array maintains a higher degree of accuracy than the hybrid array throughout burnup, with a maximum error of 0.054%. The hybrid array, while not exceeding a 0.33% difference from the reference model, becomes a poorer approximation of the center pin power over time, again indicating that RPT cells perform best when surrounded by other RPT cells.

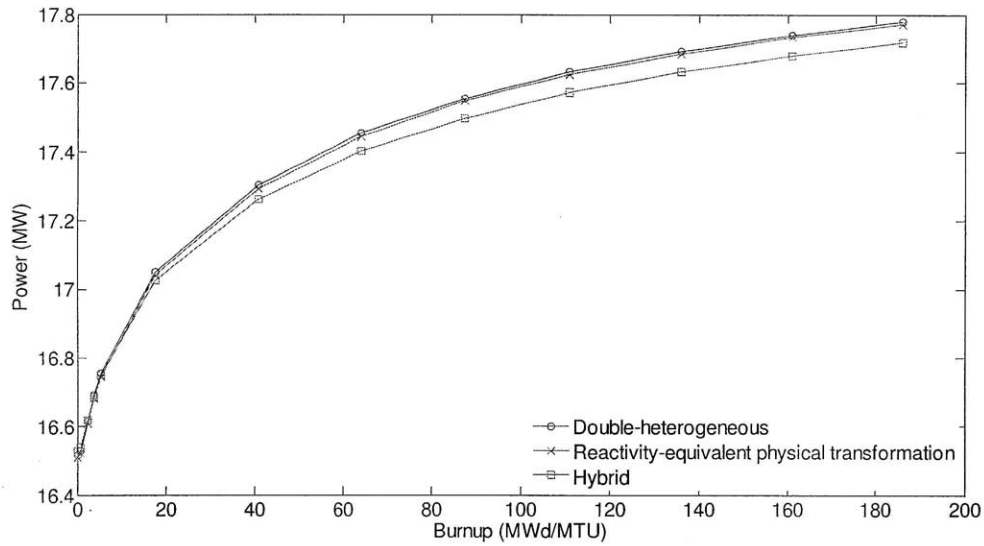


Figure 5-7: Comparison of center pin cell power among double-heterogeneous, RPT, and hybrid arrays in the presence of a  $B_4C$  control rod.

### 5.3.4 Thermal and Total Flux Analysis

Examination of the thermal and total flux in the  $B_4C$  array cases reveal the most significant discrepancies between the experimental and reference models. Error appears to increase as a function of time, but does not seem to have a significant dependence on the number of RPT cells within an array. Due to the similarity in fluxes of the RPT and hybrid arrays, it seems that the majority of the error in these models can be attributed to the presence of just one RPT cell in an array that contains a neutron absorber. The magnification of the thermal and total flux error due to additional RPT cells is minimal. The calculations of thermal and total flux reach maximum errors of approximately 4% and 1%, respectively at high burnup. The effects of the  $B_4C$  on the thermal and total flux of the center pin of each array is shown in figures 5-8 and 5-9.

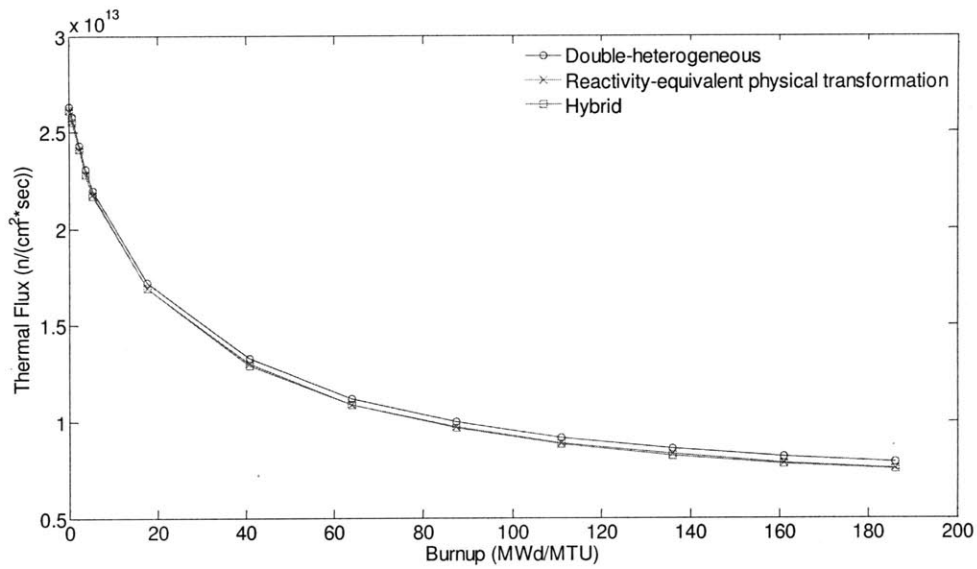


Figure 5-8: Thermal flux through the fuel region of center cells of double-heterogeneous, RPT, and hybrid arrays in the presence of a  $B_4C$  control rod.

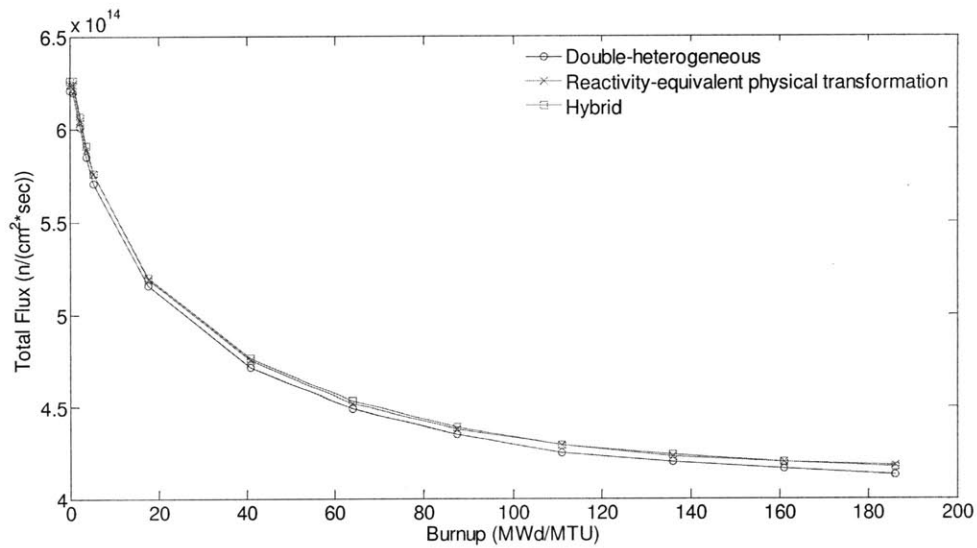


Figure 5-9: Total flux through the fuel region of center cells of double-heterogeneous, RPT, and hybrid arrays in the presence of a  $B_4C$  control rod.

## 5.4 Efficiency of the RPT Method

Creating an accurate homogenization model of double-heterogeneous cells not only allows for the use of conventional lattice codes, but also allows for shorter model runtimes of programs, like SCALE, that can handle double-heterogeneous models. The amount of time required to run models becomes particularly important for larger arrays when even a small reduction of in the total runtime could save hours. In the seven cell arrays discussed above, the all-RPT array models ran almost 88% faster than the double-heterogeneous models. Even the hybrid arrays, which replaced just one of the double-heterogeneous cells with an RPT cell ran just under 11% faster than the reference models. Table 5.4 lists the exact runtimes for each of the array models.

*Table 5.4: Comparison of runtimes for double-heterogeneous, hybrid, and RPT arrays.*

Array type	Runtime (min.)
Double-Het	591
Hybrid	528
RPT	73

## Chapter 6

# Conclusion

The Reactivity-equivalent Transformation method of homogenization has been demonstrated to accurately model the effects of double-heterogeneity caused by  $\text{UO}_2$  TRISO particles at 10% enrichment in a single pin cell. Furthermore, placement of the RPT pin cells within small arrays of other fuel cells and additional moderator does not appear to diminish its accuracy. However, the dependence of the reactivity and four factor error on the number of RPT cells present in an array suggests that the readjustments of the RPT radius may be necessary for large arrays when the error becomes significant. And the irregular behavior of the center cell power of the hybrid array indicates that RPT pin cells most closely mimic the behavior of double-heterogeneous arrays when surrounded by other RPT pin cells.

The presence of neutron absorbing  $\text{B}_4\text{C}$  seemed to significantly impact the thermal and total flux calculations. Yet, the inaccuracies in flux calculations appear to only be slightly amplified by the number of RPT cells present in the array, implying that the error in flux may not substantially increase in the case of larger arrays.

These results indicate that the RPT radius determined for individual fuel cells and small arrays may require readjustment once the fuel cells are placed within larger arrays when the cumulative error of the RPT cells is no longer negligible. And these fine adjustments become particularly important for correct thermal and total flux calculations, which appear to be particularly sensitive to slight inaccuracies in RPT radius when in the presence of a  $\text{B}_4\text{C}$  control rod.

Even as programs like SCALE are now able to properly handle the double-heterogeneity effects of TRISO particles explicitly, the amount of time required to run even simple arrays is

substantial. Further refinement of the RPT model will allow for more efficient modeling and analysis of core behavior in developing HTGR technology.



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