LIBRARY
OF THE
MASSACHUSETTS INSTITUTE
OF TECHNOLOGY
ANTICIPATIONS AND THE NON-NEUTRALITY OF MONEY II

by

Stanley Fischer

Number 207

July 1977
ANTICIPATIONS AND THE NON-NEUTRALITY OF MONEY II

by

Stanley Fischer

Number 207                July 1977

The views expressed in this paper are the author's sole responsibility and do not reflect those of the Department of Economics, the Massachusetts Institute of Technology or the National Science Foundation.
Anticipations and the Non-Neutrality of Money

Stanley Fischer*

This paper examines the non-neutrality of money in a context in which labor markets clear under full current information about current prices and quantities. In the model studied here, anticipated changes in the stock of money have (long-lived) real effects, while an unanticipated permanent increase in the money stock is neutral. The fundamental source of the non-neutrality of anticipated monetary changes is the Tobin effect—the effect of changes in the anticipated rate of inflation on the capital stock.

The interest in studying the non-neutrality of anticipated monetary changes arises from the recent emphasis on the role of unanticipated monetary changes, as for instance in the work of Sargent and Wallace (1975) and Barro (1977). In those studies, unanticipated monetary changes affect output through a Phillips-curve tradeoff, of the type formalized by Lucas (1972 and 1973), resulting from a confusion between real and monetary changes. While Phillips-curve type phenomena, whether arising from information confusions or wage and price inflexibilities, as in Fischer (1977) and Phelps and Taylor (1977), undoubtedly play a central role in the business cycle and in the non-neutrality of money, it is also worthwhile exploring the consequences of other non-neutralities of money in a rational expectations context. Further, as has been pointed out by Hall (1975), Lucas (1975) and Sargent (1977), the original Lucas mechanism does not explain the serial correlation of the level of output. The non-neutrality of money explored here does imply such serial correlation.

* Institute for Advanced Studies, Hebrew University of Jerusalem, on leave from M.I.T. I am indebted to Costas Azariadis and Alan Blinder for helpful discussions, and to members of the Faculty Seminar at the Hebrew University for comments. Research support from the National Science Foundation is gratefully acknowledged.
Through most of the paper, labor markets are assumed to clear under full current information. The Phillips curve source of the non-neutrality of (unanticipated) money is therefore removed. This enables us to concentrate on the underlying non-neutrality of money arising from the Tobin effect, and on the dynamic adjustment patterns it implies. However, since the Phillips curve is important, we discuss the implications of including it, in Section 12 of the paper.

The Tobin mechanism is present in the recent rational expectations paper by Lucas (1975), but the solution obtained there is for a particular money supply process and is not the same as that of this paper. The Tobin effect is, of course, the focus of money and growth models of James Tobin (1965), Miguel Sidrauski (1967) and others. This and other non-neutralities of money are discussed in more detail in Section 13.

1. The Model

The model studied can be thought of as a full employment version of the IS-LM model, and/or a stochastic version of a money and growth model. However, population is assumed constant. The government does not purchase goods, intervening only to fix the money stock each period, increasing it through transfer payments and decreasing it through taxes. Individuals can hold money and capital.

Although the model is not formally derived from micro-foundations, in that the labor supply, asset demand, and consumption functions are postulated rather than derived from explicit maximiation, it is useful to think of it as arising from the following overlapping generations setting. Individuals live two periods and work only in the first. Their initial wealth is zero; their first period work finances first period consumption and saving. They can save by carrying money and capital (equities) from the first period to the second. We refer to those in the first period
of their lives as young, or workers, and to those in the second period as retired or old. In each period, the labor of the young combines with the capital of the old to produce output. The old consume the value of their capital and its fruits as well as the real value of the money balances carried from the previous period. Money may be thought of as being carried from one period to the next for portfolio diversification reasons (in the absence of nominal bonds) and because transaction costs are reduced by having some initial second period wealth in the form of real balances.

We now specify in turn the market clearing conditions in the labor, goods, and assets markets.

A. The Labor Market

Each period the labor market clears with full information about current wages and prices. Current labor supply is a (log-linear) function of the current real wage.

\[ n_s^t = \phi_0 + \phi_1(w_t - p_t) + \varepsilon_{1t}, \quad \phi_1 > 0 \]

\( N \) is the quantity of labor, \( W \) the nominal wage, \( P \) the aggregate price level, and \( \varepsilon \) a disturbance term. Lower case letters are the logarithms of the relevant variables, whose levels are denoted by the equivalent upper case letters. Real rates of return on assets, whose role in labor supply and the business cycle has been emphasized by Sargent (1977), are omitted for simplicity.¹

The aggregate production function is Cobb-Douglas with constant returns to scale:

\[ Y_t = K_t^{a/(a-1)} \epsilon_{2t} \]

\( Y \) is output and \( K \) the capital stock. The resultant labor demand function is:

\[ n^d_t = \frac{1}{a} \ln(1-a) + k_t - \frac{1}{a}(w_t - p_t) + \frac{1}{a} \varepsilon_{2t}, \text{ where } \varepsilon_{2t} = \ln\varepsilon_{2t}. \]

¹. The supply of labor should in general be a function of the expected rates of return on each of the two assets in the model.
Now, equating the supply and demand for labor, and substituting the resultant quantity of labor into the production function, the (logarithm of the) level of output produced is

\[ y_t = \alpha_0 + \alpha_1 k_t + \varepsilon_{1t} + \varepsilon_{2t} \]

where \( \alpha_1 = \frac{(1+\phi_1)\alpha}{1+\alpha\phi_1} < 1, \varepsilon_{1t} = \frac{1-a}{1+\alpha\phi_1} \hat{\varepsilon}_{1t}, \varepsilon_{2t} = \frac{1+\phi_1}{1+a\phi_1} \hat{\varepsilon}_{2t} \)

and \( \alpha_0 \) is a constant of no further significance.

Because there is full current information, unanticipated inflation plays no role in determining the level of output.

Total resources available for consumption or investment (AS) is the sum of produced output and the capital stock:

\[ AS_t = Y_t + K_t \]

The (logarithm of the) real return to capital, \( r_t \), in period \( t \), can be derived from the relationship:

\[ r_t = \ln a + y_t - k_t \]

Thus

\[ r_t = \alpha - (1-\alpha_1)k_t + \varepsilon_{1t} + \varepsilon_{2t}, \text{(where } \alpha = \alpha_0 + \ln a) \]

is the real return to capital in period \( t \).

B. The Goods Market

The aggregate supply of goods is given in equation (5). The aggregate demand consists of the demand for consumption by the old \((C^R)\), plus the workers' demands for consumption \((C^W)\) and capital \((K_{t+1})\). Thus aggregate demand is

\[ AD_t = C_t + I_t = C^R_t + C^W_t + K_{t+1} \]
The consumption of the retired consists of the value of their capital and the rentals earned in this period plus the value of their real balances \( \left( \frac{M_{t-1}}{P_t} \right) \):

\[
C_t^R = (1 + r_t)K_t + \frac{M_{t-1}}{P_t}
\]

Hence:

\[
AD_t = (1 + r_t)K_t + \frac{M_{t-1}}{P_t} + C_t^W + K_{t+1}
\]

Equating aggregate supply and demand:

\[
Y_t = r_tK_t + \frac{M_{t-1}}{P_t} + C_t^W + K_{t+1}
\]

is the goods market clearing condition.

We shall not work explicitly with the goods market clearing condition, choosing to use rather the two asset market clearing conditions, the satisfaction of which implies goods market clearing.

C. The Assets Markets

The demand for capital is assumed to be:

\[
k_{t+1} = \beta_0 + \beta_1 r_{t+1} + \beta_2 (P_{t+1} - P_t) + Y_t + \epsilon_t, \quad \beta_1, \beta_2 > 0
\]

Notations of the type \( t_{t+1} \) indicate the expectation at time \( t \) of the random variable \( r_{t+1} \) conditional on information available at time \( t \). The demand for capital by the young is assumed proportional to current labor income, which is the workers' total income. By virtue of the Cobb-Douglas technology, labor income is proportional to total income—hence the unit coefficient on \( Y_t \) in (9). Since the inflation variable is one plus the expected rate of inflation and the interest rate variable is the real rate of return, \( \beta_2 \) would generally be greater than \( \beta_1 \) if the elasticities of capital demand with respect to the real interest rate and expected inflation rate respectively were similar. The stochastic term \( \epsilon_t \) represents animal spirits.
The demand for capital is assumed to be an increasing function of the real return on capital and of the expected rate of inflation. The expected rate of inflation (actually one plus the expected rate of inflation) enters to represent portfolio shifts towards capital as the expected real rate of return on the holding of money falls.

The demand for real balances is a demand of the young, who are assumed to acquire money in two ways. First, they acquire money by selling goods or labor to the old. Second, by assumption, they acquire money from the government in the form of transfer payments. The transfer payments are assumed to be proportional for each individual to the quantity of money acquired from sales to the old, with the factor of proportionality being announced before trading takes place.

The manner in which new money balances are injected into the economy has significant effects on the neutrality of money. In particular, if the money were introduced only by being given to the old, in proportion to their holdings of money carried over from the previous period, money would be neutral. In general, however, money injections are not neutral, and the method we have chosen to introduce new money has the virtue of great tractability. In Section 14 below we discuss the relationship between the way the money supply is increased and its neutrality.

Letting $M_t$ be the amount of nominal money with which the young leave period $t$, the demand for real balances is given by:

$$ m_t - P_t = y_0 - y_1 t + y_2 (P_{t+1} - P_t) + \gamma_t + z_t $$

The demand for real balances is proportional to income and negatively related to the expected real rate of return on capital and to the expected inflation rate. It is assumed that the expected real return on capital has a greater influence on the demand for capital than on the demand for real balances, relative to the influence of the expected rate of inflation on the respective asset demands. Specifically, we assum
\[
\frac{\beta_1}{\beta_2} \geq \frac{\gamma_1}{\gamma_2}
\]

Equations (4), (6), (9) and (10) constitute the model with which we shall be working. However, before proceeding, it is worthwhile checking briefly that no budget constraints have been violated, and examining the consumption function for the young implied by the budget constraint plus the two demand functions, (9) and (10).

The budget constraint for the young is

\[
(11) \quad C_t^W + \frac{M_t}{P_t} + K_{t+1} = \frac{W_t}{P_t} N_t + \frac{T_t}{P_t}
\]

where \( T_t \) is the transfer payment of new money to the workers.

Since \( T_t = M_t - M_{t-1} \), we can rewrite (11) as (11)':

\[
(11)' \quad C_t^W + \frac{M_{t-1}}{P_t} + K_{t+1} = \frac{W_t}{P_t} N_t
\]

Comparing now (11)' with (8), it is seen that clearing of the capital and money markets implies the clearing of the goods market (since \( Y_t - r_t K_t = \frac{W_t}{P_t} N_t \)).

Using the budget constraint (11), and the two asset demand functions, we obtain the implied consumption function:

\[
(12) \quad C_t^W = [1 - \frac{\beta_0}{1-a} (r_{t+1})] (r_{t+1})^{\beta_2} e_{ut} - \frac{\gamma_1}{1-a} \frac{M_{t-1}}{M_t} (r_{t+1})^{\gamma_2} e_{zt} \frac{W_t N_t}{P_t}
\]

Consumption is proportional to labor income, with the proportion depending on the expected rates of return on the assets, along with the within (first) period return on money that is bought from the old and augmented (or reduced) by the monetary authority. Since labor income is a constant proportion of total income, consumption could also be written as proportional to total income. It is assumed that \( \beta_0 \) and \( \gamma_0 \) are sufficiently small that the average propensity to consumer is always positive.
The effects of both anticipated rates of return on the propensity to consume are of ambiguous sign, as can readily be checked. Given the anticipated rates of return, an increase in the current period rate of growth of money unambiguously increases the propensity to consume of the workers. The reason is that the increase in the current period money supply is essentially a redistributive mechanism between the old and the young. The higher the current growth rate of money, the smaller the proportion of their nominal balances the young have to acquire from the old, and therefore the smaller will be the consumption of the old. This statement is, of course, conditional on the anticipated rate of inflation, and thus will not necessarily hold in full equilibrium when the determination of the current and expected price levels is endogenised.

D. The Macro-Growth Model

The model with which we work consists of the following equations.

\[ y_t = a_0 + a_1 k_t + \varepsilon_t, \quad \text{where} \quad \varepsilon_t = \varepsilon_{1t} + \varepsilon_{2t} \]

\[ r_t = \alpha - (1-\alpha)k_t + \varepsilon_t \]

\[ k_{t+1} = \beta_0 + \beta_1 r_{t+1} + \beta_2 (tP_{t+1} - p_t) + \gamma_1 y_t + u_t \]

\[ m_t - P_t = \gamma_0 - \gamma_1 (t^2P_{t+1}) - \gamma_2 (tP_{t+1} - p_t) + \gamma_3 y_t + z_t \]

The stochastic disturbances, the \( \varepsilon_t, u_t, \) and \( z_t \), are each assumed to be serially independent and identically distributed over time, with expectation zero. Serial independence is imposed to ensure that any serial correlation in the endogenous variables is a result of the interactions within the model rather than exogenous.
2. A Separable Growth Model

If the demand for capital were not a function of the expected rate of inflation, i.e. if \( \beta_2 \) were zero, then equations (4), (6) and (9) would constitute a simple model of capital accumulation, separable from monetary considerations. Setting \( \beta_2 = 0 \), and substituting (4) into (9), we obtain

\[
k_{t+1} = a_0 + \beta_0 + \beta_{1t}r_{t+1} + a_1k_t + \epsilon_t + u_t
\]

(13)

It remains only to specify \( r_{t+1} \). Using (6), we obtain the rational expectation

\[
t^r_{t+1} = \alpha - (1-\alpha_1)k_{t+1} = \alpha - (1-\alpha_1)k_t + \epsilon_t + u_t
\]

(14)

The second equality in (14) follows from the fact that the stochastic terms that affect the level of \( k_{t+1} \) are all known at time \( t \), as can be seen from (13). Thus the value of \( k_{t+1} \) is known at time \( t \). Substituting (14) into (13) produces the difference equation in the capital stock:

\[
k_{t+1} = [1 + \beta_1(1-\alpha_1)]^{-1}[(a_0 + \beta_0 + a\beta_1) + a_1k_t + \epsilon_t + u_t]
\]

The solution of the difference equation is

\[
k_t = k^* + \left(\frac{a_1}{1+\beta_1(1-\alpha_1)}\right)^t(k_0 - k^*) + \frac{1}{1+\beta_1(1-\alpha_1)} \sum_{i=1}^{t} \left(\frac{a_1}{1+\beta_1(1-\alpha_1)}\right)^{i-1} (\epsilon_{t-i} + u_{t-i})
\]

(15)

where \( k^* = (a_0 + \beta_0 + a\beta_1)(1-\alpha_1)(1+\beta_1)^{-1} \).

If the world has an infinite past, which is equivalent to assuming the model is in its stochastic steady state,

\[
k_t = k^* + \frac{1}{1+\beta_1(1-\alpha_1)} \sum_{i=1}^{\infty} \left(\frac{a_1}{1+\beta_1(1-\alpha_1)}\right)^{i-1} (\epsilon_{t-i} + u_{t-i})
\]

(15)'

1. This is a very strong assumption; its relaxation would lead to the possibility of serial correlation arising from lags in obtaining information on the size of the capital stock.
Since \( 0 < \alpha_1 < 1 \), the capital submodel is stable.

There is first-order serial correlation of the level of the capital stock, the level of output, and the level of employment, even though the underlying disturbances have been assumed to be serially uncorrelated. The source of the serial correlation is the presence of the income term in the demand for capital. Today's animal spirits affect tomorrow's capital stock and thus tomorrow's level of income, in turn affecting the demand for capital in the following period. Similarly, a disturbance increasing today's output (whether through productivity, \( \varepsilon_{2t} \), or labor supply, \( \varepsilon_{1t} \)) increases the demand for capital tomorrow, thus increases tomorrow's output, and hence increases the output of future periods. These mechanisms will play a role in the serial correlation properties of output in the full model.

3. The Rational Expectations Solution for the Price Level

It is apparent from the structure of the model that the current price level must be a function of the nominal money stock, the expected price level, the capital stock, and the disturbances. But the capital stock itself is a function of past and future expected rates of inflation and disturbances. To solve for the price level as a function of current and lagged values of money stocks, expected price levels, and disturbances, it is necessary to remove the terms involving the current and expected capital stocks (through the \( y_t \) and \( r_{t+1} \) terms respectively) from equation (10). For that purpose, equation (9) is solved for the current capital stock as a function of current and lagged expected rates of inflation and the disturbances. The result is the equation for the price level:

---

1. The coefficient on \( \varepsilon_{t-1} \) in (15) is the same as that on \( u_{t-1} \) because the demand for capital is assumed unit elastic with respect both to income (and thus indirectly \( \varepsilon_t \)) and to animal spirits (\( u_t \)).
\begin{equation}
    p_t = b_0 + b_1 p_{t-1} + b_2 t p_{t+1} + b_3 t p_{t-1} + b_4 m_t + b_5 m_{t-1} + b_6 \xi_t +
    b_7 \pi_t + b_8 u_t + b_9 \pi_{t-1} + b_{10} u_t.
\end{equation}

where

\begin{align*}
    b_0 &= -\zeta \left( \gamma_0 - \gamma_1 + \alpha_1 \right) \left( 1 + \beta_1 \right) (1-\alpha_1) + (\alpha_1 + \gamma_1 (1-\alpha_1))(\beta_0 + \beta_1 + \alpha_0) \\
    b_1 &= \zeta \alpha_1 (1 + \gamma_2 + \beta_2) \\
    b_2 &= \zeta \left[ \gamma_2 (1 + \beta_1 (1-\alpha_1)) - (1 - \alpha_1) \gamma_1 \beta_2 \right] < 1 \\
    b_3 &= -\zeta \alpha_1 (\gamma_2 + \beta_2) \\
    b_4 &= \zeta (1 + \beta_1 (1-\alpha_1)) \\
    b_5 &= -\zeta \alpha_1 \\
    b_6 &= -\zeta (1 + (\beta_1 + \gamma_1) (1 - \alpha_1)) \\
    b_7 &= -\zeta (1 + \beta_1 (1-\alpha_1)) \\
    b_8 &= -\zeta \gamma_1 (1-\alpha_1) \\
    b_9 &= \zeta \alpha_1 \\
    b_{10} &= -\zeta \alpha_1 \\
    \zeta &= \left[ (1 + \gamma_2) (1 + \beta_1 (1-\alpha_1)) - \gamma_1 \beta_2 (1-\alpha_1) \right]^{-1} > 0
\end{align*}

In (16), the current price level is higher, the higher the lagged price level and the higher the expected price level. While the negativity of the coefficients \(b_3\) and \(b_5\) seems at first glance to show that the price level is inversely related to the lagged levels of the expected price level and money stock, it should be realized that those variables affect also \(p_{t-1}\). Taking expectations as fixed, the full effect of a change in \(m_{t-1}\) on the current price level is given by \((b_1 b_4 + b_5)\), which is positive. Similarly, the net effect of a higher lagged expectation of today's price level on the current price level is positive. An increase in today's money stock increases the current price level, but less than proportionately, since \(b_4\) is less than one. Each of the three current disturbances -- in output, animal spirits, and
money demand—tends to reduce the current price level. Of these, only the negative effect of an increase in animal spirits on the current price level appears counter-intuitive. In this connection, it should be recalled that by assumption any increase in animal spirits producing an increase in investment demand reduces consumption demand by the same amount. Thus an increase in animal spirits has no impact on aggregate demand in the current period. However, increased animal spirits increase the expected capital stock, thus reduce the anticipated real rate of return on capital, and make money more attractive to hold. Hence an increase in animal spirits—a rise in \( u_t \)—reduces the current price level.

From (16), the current price level depends on the expected price level, and on the current money stock. Since an equation of the form of (16) will describe the determination of the price level next period, it is clear that current expectations of future money stocks are relevant to the determination of the current price level. We shall assume that there is a common set of expectations at time \( t \) of all future money stocks, \( \{ t_m t+i \} \), \( i = 0, 1, 2, 3, \ldots \) where \( t_m t \) is today's (known) money stock. The source of the expectations is not considered in this paper, though they could be inferred from the operation of some money feedback rule, or from announcements by the monetary authority. In any event, it will be seen that the rational expectations determination of the current price level requires the public to have such expectations of all future money stocks.

The parameter values in (16) imply certain properties of the price level. First,

\[
\sum_{i=1}^{5} b_i = 1
\]

(16)

This implies that if all nominal variables on the right hand size rise by a given proportion, then the price level too will rise in that proportion. Such a neutrality
property is to be expected of any macro model without money illusion. Second, in this particular case
\[(16)\]
\[b_2^2 + b_4 = 1\]
Equation (16) implies that, given the predetermined nominal variables, \(p_{t-1}\), \(t_{-1}p_t\) and \(m_{t-1}\), an equal increase in today's money stock \(m_t\) and the expected price level \(t_{P_{t+1}}\) is neutral. This will be seen to imply that an unanticipated change in the current money stock, accompanied by equal proportional changes in all expected money stocks, is neutral. This is not a general property of money-illusion-free models, as will be shown in Section 12.

To obtain a rational expectation solution for the price level from (16), conjecture at a solution of the form:
\[(17)\]
\[p_t = \delta + \sum_{i=0}^{\infty} \pi_i t^m_{t+i} + \sum_{i=0}^{\infty} \theta_i t^{-1} m_{t-1+i} + \lambda p_{t-1} + \sum_{i=0}^{\infty} \eta_i \epsilon_{t-i} + \sum_{i=0}^{\infty} \psi_i z_{t-i} + \sum_{i=0}^{\infty} \xi_i u_{t-i}\]
A rational expectations solution to (16) in the form (17) is not the final expression for the price level in terms of actual and expected money supplies and the disturbances, since it includes the autoregressive term \(\lambda p_{t-1}\), but it is a simple matter to proceed from (17) to such an expression for \(p_t\) dependent only on anticipated and actual money stocks and the disturbances. In choosing a solution of the form (17) we work with the stochastic steady state of the model in which initial conditions have no impact on the current price level.

To solve for the parameter values in (17), take expectations in (17) to obtain \(t_{P_{t-1}}\) and \(t_{-1}p_t\), and then equate coefficients between (16) and (17). The parameter on which we focus attention initially is the autoregressive coefficient, \(\lambda\). After equating coefficients, we obtain
\[(18)\]
\[b_2 \lambda^2 + (b_3 - 1) \lambda + b_1 = 0\]
This equation for \( \lambda \) has two solutions. Both roots are positive, with one of them greater than unity and the other less than unity. We shall assume that the economy is stable and therefore that the smaller value of \( \lambda \) describes the behavior of the economy. In Section 13 below we return to the question of the multiplicity of rational expectations paths for the price level.

Working now with the stable root, equations (19) provide bounds on \( \lambda \), and also give the value of \( \lambda \) for the special case of neutral money, when \( \beta_2 = 0 \).

\[
0 \leq \lambda \leq 1
\]
\[
b_1 + b_3 \leq \lambda \leq b_1
\]

(19) \[
\lambda = \frac{a_1}{1 + b_1 (1 - a_1)} = b_1 \quad \text{when} \quad \beta_2 = 0
\]

The second inequality in (19) is equivalently

(19)' \[
\zeta a_1 \leq \lambda \leq \zeta a_1 (1 + \gamma_2 + \beta_2)
\]

It is clear from (19)' that the fundamental determinant of the value of \( \lambda \) is \( a_1 \), the elasticity of output with respect to the capital stock. If that elasticity were zero, then \( \lambda \) too would be equal to zero.

We next consider the circumstances under which \( \lambda = a_1 \). The root \( \lambda \) is equal to \( a_1 \) when

(19)" \[
\beta_2 - \beta_1 = (1 - a_1)(\beta_1 \gamma_2 - \gamma_1 \beta_2) > 0
\]

Because \( \beta_2 \) is the elasticity of capital demand with respect to (one plus the anticipate inflation rate) and \( \beta_1 \) is the elasticity with respect to the interest rate, there is a presumption that \( \beta_2 > \beta_1 \), even if capital demand responds more strongly to the own rate of return on capital than to the real rate of return on money. The implications of the special case, \( \lambda = a_1 \), may therefore be of some interest.
Once the value of $\lambda$ has been determined, it is relatively straightforward to solve for the remaining coefficients in (17). The values of $\pi_i$ and $\theta_i$ are given by:

$$\pi_0 = 1 - \frac{b_2^\lambda}{b_1} < 1$$

(20)

$$\pi_i = \frac{b_2^\lambda}{b_1} \pi_{i-1}, \quad i = 1, 2, 3,...$$

$$\theta_0 = \frac{b_2^\lambda}{b_1} < 0$$

$$\theta_i = \frac{b_2^\lambda}{b_2} \pi_i < 0, \quad i = 1, 2, 3,...$$

and where,

$$0 < \frac{b_2^\lambda}{b_1} < 1$$

Note also that

$$\sum_{i=0}^{\infty} \pi_i = 1$$

$$\sum_{i=0}^{\infty} \theta_i = -\lambda$$

The effects of the disturbances on the current price level are given by

$$\eta_0 = \frac{b_6}{1-b_2^\lambda} < 0; \quad \eta_1 = 0 = \eta_2 = ...$$

$$\psi_0 = -\pi_0; \quad \psi_1 = -\theta_0; \quad \psi_2 = 0 = \psi_3 = ...$$

(21)

$$\xi_0 = \left(\frac{b_2 b_1 10^\lambda}{b_1} + b_8\right) \frac{1}{1-b_2^\lambda} < 0;$$

$$\xi_1 = \frac{b_1 10^\lambda}{b_1} = \theta_0; \quad \xi_2 = 0 = \xi_3 ...$$

In solving for these coefficients it is once more necessary to invoke the assumption of stability, for the solution for the parameters, $\eta_i$, $\psi_i$, and $\xi_i$ is not unique;
however, only the values specified in (21) produce stability of the price level. Finally, for completeness, the constant term in (17) is given by

\[
\delta = \frac{\lambda b_0}{b_1 - b_2^\lambda}
\]

Before we proceed to study the behavior of the price level in detail, it is worth emphasizing that (17) shows how the rational expectations solution for the price level requires individuals to have expectations of all future money stocks. The rational expectations approach thus moves the focus of the formation of expectations from endogenous to exogenous variables.

4. Unanticipated Money and The Price Level

The price level is affected by the money stocks expected today to exist in all future periods, and also by all past expectations of future money stocks. For example, today's price level is affected by the expectation that was held at \((t-2)\) about the money stock that would exist at \((t-1)\). In each period, \(t\), the current money stock \(m_t\) is known and so is equal to \(m_t\). Thus the actual money stock - and hence the unanticipated component too - in each period in the past, as well as today, affects today's price level.

We start by considering the effect on unanticipated injection of money has on the current and subsequent price levels. We shall assume initially that the unexpected money does not affect current expectations of future money stocks—that is, that the monetary disturbance is regarded as temporary or transitory. The impact of an unexpected increase in today's money stock on today's price level is given by \(\pi_0\). Since \(\pi_0 < 1\), unanticipated money increases today's price level less than in proportion to the increase in the money stock.
Comparing now the value of $\pi_0$ in (17) with that of $b_4$ in (16), it can be shown that $\pi_0 \geq b_4$, that is, that the impact of an increase in the money stock on the current price level is greater in the rational expectations solution (17) than in the partial solution (16) that holds expectations fixed. This is not true, however, if money is neutral (that is when $\beta_2 = 0$); in the case of neutral money the rational expectations effect of a change in the money supply on the current price level is the same as in the partial solution (16). The reason for this is that, for $\beta_2 = 0$, when money is neutral, a change in today's money stock that does not affect expectations of future money stocks leaves future expected price levels unchanged. When money is not neutral, a change in today's money stock affects the capital stock expected for next period and thus affects the price level expected for next period.

Next we ask what effect today's unexpected transitory money has on tomorrow's price level. Examining (17) we see that

$$\frac{\partial p_{t+1}}{\partial m_t} = \theta_0 + \pi_0 \lambda = 1 - \frac{\lambda}{b_1} > 0$$

(23)

$$\frac{\partial p_{t+1}}{\partial m_t} < \frac{\partial p_t}{\partial m_t}$$

and, for all subsequent periods:

$$\frac{\partial p_{t+i}}{\partial m_t} = \lambda \frac{\partial p_{t+i-1}}{\partial m_t} \quad i = 2, 3, 4, \ldots$$

(24)

Thus, so long as money is not neutral, an unanticipated increase in today's money stock will, ceteris paribus, increase today's price level and the price level in all future periods. If money is neutral, an unanticipated transitory increase in today's money stock increases only today's price level (less than proportionately) but leaves all other price levels unchanged (since then $\lambda = b_1$). The details of the mechanism through which a change in today's money stock affects tomorrow's price level when money
is not neutral will become clearer when the response of the capital stock to unanticipated money changes is examined in Section 9. However, the basic mechanism can be simply stated: an unanticipated blip in today's money stock reduces the expected one-period rate of inflation, thereby reduces tomorrow's capital stock, and thus leads to an increase in tomorrow's price level. The future price level does not, though, rise enough to make the anticipated rate of inflation positive.

So far we have been discussing the impact of an unanticipated transitory change in the money stock on the current and future price levels, holding constant expected money supplies. A permanent change in the money stock can be represented by an equal change in $m_t$ and all $\Delta m_{t+i}$, where a given proportional change in all expected money stocks is implied (since we are working with the logarithms of money stocks). An unanticipated permanent change in the money stock is neutral, resulting in a proportional change in the current price level that leaves current real balances and all expected rates of inflation and capital stocks unaffected. This follows from the fact that $E\pi_i = 1$.

5. Anticipated Transitory Money and the Price Level

In this section we consider the impact on the current, past and future price levels of a fully anticipated transitory increase in the money stock in period $t$. From (17) it may be seen that the effect of an anticipated increase in the money stock on the price level depends both on the period in which the increase will occur, and on the period from which it is first anticipated that the increase will take place. Thus an announcement at $(t-5)$ that the money stock in period $t$ will increase by a given amount generally has a different effect on the price level in periods $(t-5)$ onwards than an announcement at time $(t-10)$ that the money stock in period $t$ will increase by the same amount. By fully anticipated, we mean an
increase in the money stock at time \( t \) that has been expected since time \( (t - \infty) \).

Let the notation \( \frac{\partial p_t}{\partial m_{t-1}} \) indicate the effects on the price level at time \( t \) of a fully anticipated transitory increase in the money stock at time \( (t-1) \). Then

\[
(\frac{\partial p_t}{\partial m_t})_a = \sum_{i=0}^{\infty} (\frac{\partial p_t}{\partial m_{t-i}})_a = \sum_{i=0}^{\infty} \frac{\pi i (\pi_i + \theta_{i+1})}{\pi_0 \frac{b_1 - b_2 \lambda^2}{b_1 - b_2 \lambda}} \leq \pi_0
\]

A fully anticipated increase in today's money stock accordingly has a smaller effect on today's price level than does an unanticipated increase, except in one case, that of neutral money. In that case, fully anticipated and entirely unanticipated transitory changes in the money stock have precisely the same effect on the current price level. The reason there is in general a smaller effect on the price level of perfectly anticipated money, compared with unanticipated money, is that in the former case the capital stock adjusts in response to the expected inflation induced by the anticipated monetary increase.

The later the first date at which the anticipation of an increase in the money stock occurs, the greater the effect of a given increase in that stock on the price level in the current period.

Next we ask what effects the anticipation of a blip in the money stock has on the price level in periods previous to those in which the change is expected to occur. That is, we are asking about

\[
(\frac{\partial p_{t-i}}{\partial m_t})_a = \left( \frac{b \lambda_i}{b_1} \right)^i (\frac{\partial p_t}{\partial m_t})_a^i, \quad i > 0
\]

We may thus describe the effects of future money stocks on the current price level as behaving like a Koyck lead: the effects of the anticipated increase in the money stock

1. One might also want to compare the effects of fully anticipated money on the current price level with the partial effects as given by the coefficient \( b \) in equation (16). I have not been able to show that there is any definite relationship between the magnitude of the two effects, although there appears to be a presumption that the fully anticipated effect is smaller.
on the price level are greatest in the period in which the change occurs, and are proportional to that change in earlier periods, with the weights declining geometrically the further in the future the change in the money stock.

It follows from (26) that an anticipated transitory increase in the money stock increases the inflation rate expected for every period up to the time the change takes place.

After the anticipated blip in the money stock has occurred, subsequent price levels are higher than they would have been had the change not taken place so long as money is not neutral. Specifically,

\[
\frac{\partial p_{t+1}}{\partial m_t} = \frac{(1-\lambda)(b_1-\lambda)}{b_1 - b_2\lambda^2} \geq 0
\]

\[
\frac{\partial p_{t+1}}{\partial m_t} < \lambda \frac{\partial p_t}{\partial m_t}
\]

\[
\frac{\partial p_{t+i}}{\partial m_t} = \lambda^{i-1} \frac{\partial p_{t+1}}{\partial m_t} \quad i = 1, 2, \ldots
\]

The effects of the change in the money stock die away at a geometric rate, though at a different rate than they build up as the anticipated change in the money supply comes closer. Whether the coefficient \( b_2/\lambda/b_1 \) in (26) is smaller or larger than \( \lambda \) in (27) obviously depends on the ratio \( b_2/b_1 \), which is not a priori determinate. That ratio will be greater than unity if \( b_2 - b_1 \) is positive. The sign is given by:

\[
b_2 - b_1 \sim (1-\alpha_1)[\gamma_2(1+\beta_1) - \gamma_1\beta_2] - \alpha_1(1+\beta_2)
\]

where \( \sim \) means "of the same sign as". Thus \( b_2 \) is more likely to be greater than \( b_1 \), the larger are \( \gamma_2 \) and \( \beta_1 \), and the smaller are \( \alpha_1 \), \( \gamma_1 \) and \( \beta_2 \). Treating \( \alpha_1 \) as the basic determinant of the degree of persistence in the model, we see that the Koyck "lead" coefficient, \( b_2/\lambda/b_1 \), is more likely to be larger than the Koyck lag coefficient, \( \lambda \), the smaller the degree of persistence in the model.
Since the effects of the anticipated monetary increase on the price level in subsequent periods decline geometrically, a past anticipated monetary blip reduces the anticipated inflation rate in all periods subsequent to the change, again provided money is not neutral.

Next, we compare the lead and lag distributions resulting from a given transitory change in the money stock on the price level, depending on whether the change is anticipated or unanticipated. Figure 1 shows the (smoothed) relationship between the effects of a change in the money stock in period $t$ on the price level in earlier and later periods. In drawing Figure 1, it is assumed that money is not neutral, that is, that $\beta_2 \neq 0$. For neutral money, neither anticipated nor unanticipated changes in the money stock have any effect on subsequent price levels. In the case of neutral money, the conclusions that would be reached by working with equation (16) and ignoring expectations coincide with the rational expectations analysis of the effects of unanticipated money. There is of course a difference between the effects of unanticipated and anticipated monetary changes on the price level in earlier periods than the change occurs, since an unanticipated change obviously cannot have any effects before it takes place.

Finally, we can briefly consider the effects on the price level of a transitory change in the money stock that becomes anticipated some finite number of periods before it takes place. In terms of Figure 1, this is intermediate between the two previous cases. Before the expected event, but after the change in expectations, the price level is higher than it would have been had the same change been expected earlier, unless money is neutral, in which case the effects on the price level are the same as if the change had been perfectly anticipated. In period $t$, the period the change in the money supply takes place, the price level is higher than it would have been had the change been perfectly anticipated and lower than it would have been had it been totally
FIGURE 1: Effects of a Transitory Increase in the Money Stock in Period \( t \) on the Price Level in Previous and Subsequent Periods (for Non-Neutral Money)
unanticipated—and similarly in subsequent periods. This case is shown as "Partially Anticipated" in Figure 1. Again, the behavior of the price level can be understood by realizing that the capital stock in each period before the change is higher, the earlier the change was anticipated.

6. The Impact of an Anticipated Permanent Change in the Money Supply on the Price Level

We have already seen that an unanticipated change in the current money stock, accompanied by a change in expectations such that all future money stocks are expected to increase in the same proportion as the current money stock, is purely neutral—the current price level rises proportionately to the change in the current and anticipated money stocks and future capital stocks are unaffected. What now if we consider two different paths for the money stock, on one of which the money stocks from period $t$ on are, and are expected to be, proportionately higher than on the other path? We refer to this as an anticipated permanent change in the money supply.

A fully anticipated permanent change in the money stock in general leads to an increase in the price level in the period in which the change occurs that is proportionately less than the increase in the money stock. Only asymptotically does the price level tend to rise in the same proportion as the money stock. Specifically:

$$\frac{\partial p_t}{\partial m_t} = \frac{\lambda(1 - b_2\lambda)}{b_1 - b_2^2} \leq 1$$

(28) $$\frac{\partial p_{t-i}}{\partial m_t} = \frac{b_2\lambda}{b_1} \frac{\partial p_t}{\partial m_t} \quad i = 1, 2, \ldots$$

$$\frac{\partial p_{t+i}}{\partial m_t} = 1 - \frac{\lambda^i(b_1 - \lambda)}{b_1 - b_2^2} > 0 \quad i = 1, 2, \ldots$$

where the subscript "sa" indicates an anticipated permanent change in the money stock.
It may be seen from (28) that in the case of neutral money, for which $b_1 = \lambda$, the price level increases in the period in which the change in the money supply takes place in the same proportion as the money stock change—and it remains higher by that same proportion in all subsequent periods. Thus the difference between the behavior of the price level in the face of anticipated and unanticipated changes in the money stock respectively, follows from the behavior of the capital stock. When money is not neutral, the expected inflation that the anticipated monetary increase induces causes the capital stock to rise, and thus exerts a deflationary effect on the price level.

In anticipation of our discussion of the behavior of the capital stock when there is an anticipated permanent change in the money supply, we note that it can be shown that the inflation rate increases up to the period in which the change takes place, and decreases thereafter.

Figure 2 provides a graphical comparison of the behavior of the price level in the face of anticipated and unanticipated permanent changes in the money supply, for non-neutral money. For neutral money, the price level increases proportionately to the increase in the money stock in the period of the change in both cases.

7. The Effects of Disturbances on the Price Level

Each of the three disturbances, the supply disturbances $\epsilon_t$, the money demand disturbance $z_t$, and the capital demand disturbance—or animal spirits—$u_t$, reduces the price level in the period in which it occurs, and in all subsequent periods.

It is hardly surprising that an output supply disturbance ($\epsilon_t$), which increases the level of output and thus the demand for money, reduces the price level. However, the effect of a supply disturbance on the price level is not confined to this mechanism. For, since a supply disturbance also increases the demand for capital, tomorrow’s expected capital stock is higher as a result of an output disturbance, and the real interest rate is therefore lower. Further, tomorrow’s anticipated price level is lower. These factors also tend to increase the demand for money and reduce the price level.
FIGURE 2: Effects of a Permanent Change in the Money Stock in Period $t$ on the Price Level in Previous and Subsequent Periods (for Non-Neutral Money).
A positive money demand disturbance acts in precisely the same way on the price level as an unanticipated transitory decrease in the money stock in the current and all subsequent periods.

Finally, we noted in Section 3 that the effects of an increase in \( u_t \) on the price level have to be understood as depending on the assumption that an increase in investment demand comes at the expense of consumption demand. An increase in investment demand therefore is not, in this model, an increase in aggregate demand. The increase in investment demand increases the capital stock expected for next period, thereby reduces the expected real interest rate and the expected price level, and so tends to increase the demand for money.

Each of the disturbances has long-lasting effects on the price level, except if money is neutral. If money is neutral, a money demand disturbance works itself out entirely within the period in which it occurs. The other, real, disturbances do have serially correlated effects on the price level since they have serially correlated effects on the capital stock, as can be seen from (15).

8. The Behavior of the Capital Stock

In studying the behavior of the capital stock, we derive an equation expressing the capital stock as a function of the disturbances and anticipated rates of inflation. To derive this equation, substitute (4) into (9) to express the current capital stock as a function of the anticipated return on capital and the lagged capital stock, and then use (6) to obtain the rational expectation of the real interest rate. Solving the resultant difference equation, we obtain

\[
(29) \quad k_t = \frac{\beta_0 + \beta_1 a + \alpha_0}{(1+\beta_1)(1-\alpha_1)} + \frac{\beta_2}{1+\beta_1(1-\alpha_1)} \sum_{i=1}^{\infty} \frac{\alpha_1}{l+\beta_1(1-\alpha_1)} (t-i P_{t-i+1} - P_{t-i}) \\
+ \frac{1}{1+\beta_1(1-\alpha_1)} \sum_{i=1}^{\infty} \frac{\alpha_1}{l+\beta_1(1-\alpha_1)} (\epsilon_{t-i} + u_{t-i})
\]
For $\beta_2 = 0$, the case of neutral money, this reduces to the expression (15) given earlier for the separable growth model. Since $a_1 < 1$, this model is stable, and the capital stock will be finite provided the rate of inflation has not been increasing too rapidly in the past.

Examining (29), it is seen that the capital stock is a function of the disturbances and all past anticipated rates of inflation. The latter in turn are functions of actual and anticipated money supplies and disturbances:

$$
tP_{t+1} - P_t = \sum_{i=0}^{\infty} \pi_i (t^{m_{t+1}+i} - t^{m_{t+i}}) + \sum_{i=0}^{\infty} \theta_i (t^{m_{t+i}} - t^{-1}m_{t-1+i})$$

$$+ \lambda (P_t - P_{t-1}) - \eta_0 \varepsilon_t + (\psi_1 - \psi_0) z_t - \psi_1 z_{t-1} + (\xi_1 \xi_0) u_t - \xi_1 u_{t-1}.$$  

Finally, the actual inflation rate is

$$p_t - p_{t-1} = \sum_{j=0}^{\infty} \lambda_j \sum_{i=0}^{\infty} \pi_i (t^{-j}m_{t+j+i} - t^{-j-1}m_{t-j-1+i})$$

$$+ \sum_{j=0}^{\infty} \lambda_j \sum_{i=0}^{\infty} \theta_i (t^{-j-1}m_{t-j-1+i} - t^{-j-2}m_{t-j-2+i})$$

$$+ \eta_0 \sum_{i=0}^{\infty} \lambda^i (\varepsilon_{t-i} - \varepsilon_{t-1-i}) + \psi_0 \sum_{i=0}^{\infty} \lambda^i (z_{t-i} - z_{t-1-i})$$

$$+ \psi_1 \sum_{i=0}^{\infty} \lambda^i (z_{t-1-i} - z_{t-2-i}) + \xi_0 \sum_{i=0}^{\infty} \lambda^i (u_{t-i} - u_{t-1-i})$$

$$+ \xi_1 \sum_{i=0}^{\infty} \lambda^i (u_{t-1-i} - u_{t-2-i}).$$

Although the expression for the capital stock and (30) and (31) appear formidable, the simplicity of the underlying autoregressive structure of the model makes it possible to study the properties of the behavior of the capital stock without undue difficulty.

The basic property of the equation for the capital stock is that the capital stock is higher, the higher have been anticipated inflation rates in the past. Examining only steady states, and assuming away the disturbances, the relationship between the capital stock and a given constant expected rate of inflation, $\mu$, is
The sensitivity of the capital stock to the expected rate of inflation depends chiefly on the properties of the capital demand function \( (9) \), being greater the larger the elasticity of capital demand with respect to the anticipated rate of inflation \( \beta_2 \) and the smaller the sensitivity of capital demand to the real rate of return \( \alpha_1 \). The larger the elasticity of output with respect to the capital stock \( \alpha_1 \), the greater the response of the steady-state capital stock to the anticipated rate of inflation—this latter feature resulting from the feedback effect of an increase in output resulting from an increase in the capital stock on the demand for capital.

One would also expect that, in the absence of disturbances, the actual and expected rates of inflation would be equal to the growth rate of money, if that had been, and was expected always to be, constant. This is indeed the case, as may readily be confirmed from \( (31) \) and \( (30) \).

9. The Effects of a Transitory Increase in the Money Supply on the Capital Stock

To derive the qualitative properties of the effects of an unanticipated transitory increase in the money stock on the capital stock in subsequent periods, it is necessary only to look at Figure 1 and equation \( (24) \). From Figure 1 we see that an unanticipated transitory increase in the money stock reduces the inflation rate expected for next period and all subsequent periods. Accordingly, from \( (29) \), an unanticipated blip in the money stock this period reduces the capital stock in all following periods. Further, the reduction in the capital stock can be shown to be largest in the period immediately following the increase in the money stock, with each subsequent decrease being smaller (in absolute value) than the preceding one.

More precisely, the effect of an unanticipated transitory increase in today's money stock on tomorrow's capital stock is given by:
(33) \[ \frac{\delta k_{t+1}}{\delta m_t} = \frac{\beta_2}{1+\beta_1(1-a_1)} \left[-\pi_0(1-\lambda) + \theta_0 \right] = \frac{\beta_2 \lambda}{a_1(1+\gamma_2 + \beta_2)} \]

Since \( \lambda \) is itself a function of the structural parameters, one cannot treat \( \lambda \) as fixed and then discuss the relationship between the above expression and the coefficients \( \beta_2, a_1 \) and \( \gamma_2 \). However, for the special case for which \( \lambda = a_1 \), the sensitivity of the capital stock to today's unanticipated money depends only on \( \gamma_2 \) and \( \beta_2 \). The change in the capital stock is greater the more sensitive the demand for capital is to the expected inflation rate (the larger is \( \beta_2 \)), and the less sensitive the demand for money is to the expected inflation rate (the smaller is \( \gamma_2 \)). The larger is \( \gamma_2 \), the smaller will be the decrease in the expected inflation rate occurring as a result of today's unanticipated money.

The effects of today's unanticipated transitory money on future capital stocks decay at the rate \( \lambda \):

(34) \[ \frac{\delta k_{t+i}}{\delta m_t} = \lambda^{i-1} \frac{\delta k_{t+1}}{\delta m_t} < 0, \quad i = 1, 2, \ldots \]

A fully anticipated blip in the money stock affects the inflation rate before it occurs; as can be seen from Figure 1, a fully anticipated transitory increase in the money stock causes the rate of inflation in each period before the increase occurs to be higher than it would otherwise have been. Accordingly, we can see from (29) that an anticipated increase in the money stock causes the capital stock to rise steadily in anticipation of the change. Calculating the effects of a fully anticipated transitory increase in the money stock on the capital stock in the period in which the monetary change occurs, we have:

(35) \[ \frac{\delta k_t}{\delta m_t} a = \frac{\beta_2 \lambda}{a_1(1+\gamma_2 + \beta_2)} \frac{b_1 - b_2 \lambda}{b_1 - b_2 \lambda^2} > 0 \]

In periods before the change in the money stock occurs, the capital stock is higher than it would otherwise have been, with the effects declining with a geometric
lag the longer away the anticipated event.

\[ \frac{3k_{t-i}}{\Delta m_t} a = \left( \frac{b_i}{b_1} \right) \frac{3k_t}{\Delta m_t} a, \quad i = 1, 2, ... \]

Next we consider the behavior of the capital stock subsequent to the anticipated transitory increase in the money stock. We saw in Section V that the inflation rate becomes negative immediately after the blip in the money stock. However, we also know that the capital stock is positively auto-correlated, and that the capital stock in period \( t \) is higher than it otherwise would be, if there is an anticipated transitory increase in the money stock in period \( t \). The decline in the inflation rate tends to reduce the capital stock in period \( (t+1) \), compared with what it would otherwise have been, and the autocorrelation of the capital stock tends to make it bigger than it would have been. The reduction in the anticipated inflation rate turns out to be unambiguously the more powerful effect:

\[ \frac{3k_{t+1}}{\Delta m_t} a = -\frac{b_2}{1+b_1(1-a_1)} \frac{b_4(1-\lambda)}{b_1 - b_2 \lambda^2} \]

Thereafter, the effects of the perfectly anticipated blip decay geometrically:

\[ \frac{3k_{t+i}}{\Delta m_t} a = \lambda^{i-1} \frac{3k_{t+1}}{\Delta m_t} a \]

There are two questions we want to ask about (38). First, is the decrease in the capital stock in period \( (t+1) \) resulting from an anticipated increase in the money stock at period \( t \) smaller in absolute value than the decrease in the capital stock at \( (t+1) \) resulting from an unanticipated increase in the money stock of the same magnitude at \( t \)? The answer to this is yes. Specifically,

\[ \frac{3k_{t+1}}{\Delta m_t} a < \frac{3k_{t+1}}{\Delta m_t} a \]

Second, is the decrease in the capital stock at \( (t+1) \) resulting from the anticipated monetary blip at \( t \) smaller in absolute value than the increase in the
capital stock at time \( t \) resulting from the same monetary change? The answer is "it depends". Specifically,

\[
\begin{align*}
\frac{\partial k_t}{\partial m_t} + \frac{\partial k_{t+1}}{\partial m_t} \sim b_1 - b_2
\end{align*}
\]

where \( \sim \) means "of the same sign as".

The inequality implies that the sign of the term in (41) depends on whether the fundamental forward autoregressive coefficient \( \frac{b_2}{b_1} \) is smaller than the ex post autoregressive coefficient \( \lambda \). We discussed the determinants of this relationship following equation (27).

Figure 3 shows the response of the capital stock to unanticipated and anticipated transitory changes in the money stock in period \( t \). The particular adjustment pattern can perhaps be understood better if it is realized that a transitory blip in the money stock—whether anticipated or unanticipated—is followed in each case by an anticipated permanent decline in the money stock. Following the current change in the money stock, it is anticipated that future money stocks will be lower than the current stock.

Further, in this model with full current market clearing, an unanticipated change in the money stock has no impacts on current output. In Section 12 we discuss how the adjustment patterns of Figure 3 are affected by the addition of a Phillips curve.

10. The Impact of A Permanent Change in the Money Supply on the Capital Stock

We have seen that an unanticipated change in the money stock that is expected to persist (with all expected money stocks rising in the same proportion as the current money stock) is neutral, and has no effects on the capital stock. However, an anticipated stock increase in the money supply does have effects on the behavior of the capital stock both before and after it takes place.
FIGURE 3: Effects of a Transitory Increase in the Money Stock in Period $t$ on the Capital Stock (for Non-Neutral Money)
Figure 2 shows that an anticipated increase in the money stock creates inflation both before and after it happens. We should therefore expect the capital stock to be higher in every period as a result of a fully anticipated permanent increase in the money stock in period \( t \). The impact of the change in the money stock in the period \( t \) occurs is given by:

\[
\frac{\partial k_t}{\partial m_t} = \frac{\lambda b_4}{b_1 - b_2 \lambda^2}
\]

As should be expected, this increase in the capital stock is larger than that occurring when there is a perfectly anticipated transitory increase in the money stock in period \( t \).

In earlier and later periods, the change in the capital stock is given by:

\[
\frac{\partial k_{t+i}}{\partial m_t} = \lambda^i \left( \frac{\partial k_t}{\partial m_t} \right) \quad i = 1, 2, ...
\]

\[
\frac{\partial k_{t-i}}{\partial m_t} = \left( \frac{b_2 \lambda}{b_1} \right)^i \left( \frac{\partial k_t}{\partial m_t} \right) \quad i = 1, 2, ...
\]

Thus the effects of a fully anticipated permanent change in the money stock in period \( t \) on the capital stock reach a maximum in the period in which the change first occurs, building up to that maximum in earlier periods, and decaying from that maximum in later periods. Whether the build-up is faster than the decay depends on whether \( b_2 \) exceeds \( b_1 \).

Figure 4 shows the effects of anticipated change in the money stock on the capital stock.

Summing up on the full information money model, it can be seen that anticipated changes in the money stock are not neutral, while an unanticipated permanent change in the money stock is neutral.


Each of the disturbances has long-lasting effects on the capital stock, and output. A positive disturbance—whether a supply disturbance, money demand, or animal spirits—
general makes the capital stock output, and employment, in each subsequent period higher than they would otherwise have been. The only exception is that of a money demand disturbance when money is neutral, in which case real variables are unaffected by the disturbance.

As can be seen from (29), each of the real disturbances affects the capital stock in subsequent periods through two mechanisms. There is first the direct real route for the disturbances, through their effects on current or future levels of real output and thus on saving, that was reflected in the separable growth model of Section 2, and is represented by the last set of terms in (29). Second, each of the disturbances affects expected inflation rates in the current and subsequent periods, and affects the demand for capital through that route. Each disturbance tends to reduce the current price level relative to future price levels, and thus to induce expectations of inflation (though future price levels are lower than they would otherwise have been). Thus the capital stock tends to increase as a result of this indirect route too.

A money demand disturbance has precisely the same effects on subsequent capital stocks as an unanticipated reduction in the current money stock that leaves expected money stocks unchanged.

More precisely, the effects of the real disturbances on future capital stocks are given by:

\[ \frac{\partial k_{t+i}}{\partial \varepsilon_t} = \frac{\alpha_1}{1 + \beta_1 (1 - \alpha_1)} \frac{\partial k_{t+i-1}}{\partial \varepsilon_t} - \frac{\beta_2 \eta_0 (1 - \lambda)^{i-1}}{1 + \beta_1 (1 - \alpha_1)} > 0 \quad i = 2, \ldots, \infty \]

(44)

\[ \frac{\partial k_{t+i}}{\partial u_t} = \frac{\alpha_1}{1 + \beta_1 (1 - \alpha_1)} \frac{\partial k_{t+i-1}}{\partial u_t} - \frac{\beta_2 \eta_0 (1 - \lambda)^{i-1}}{1 + \beta_1 (1 - \alpha_1)} [\xi_1 + \lambda \xi_0] > 0 \quad i = 2, \ldots, \infty \]

(45)
The effects of the output disturbance on future capital stocks are at a maximum in the period immediately following the shock, as seen in (44).\footnote{I have not been able to show that the same is true for an increase in animal spirits, though that appears to be the case.}

Examining (44) and (45), the direct effects of the disturbance on next period's capital stock are reflected in each case in the number one within the brackets in the first equation. The output disturbance has the same coefficient as the animal spirits disturbance because the demand for capital is assumed to be unit elastic with respect to income. The coefficient \((1 + \beta_1(1-\delta_1)^{-1}\) outside the brackets arises from the rational expectations feature that individuals take into account the effect of their present actions on the future real rate of return on capital in calculating the expected rate of return. In each case, the other term within the brackets reflects the indirect effects of the disturbances on the capital stock—through their influence on the expected rate of inflation.

12. **Adding a Phillips curve**

The model studied so far has included neither price inflexibilities nor any absence of information on current macro variables, in order to concentrate on the mechanism that produces serial correlation. However, Phillips curve phenomena play a central role in short term fluctuations. In this section we sketch the effects of the introduction of a Friedman-Phelps-Lucas type Phillips curve.

Specifically, suppose that the aggregate supply curve is replaced by\footnote{A simple method of generating (4)' in place of (4) is to assume—as argued by Friedman (1968)—that labor supply is a function of the expected price level rather than the actual price level, \(n_t = \phi_0 + \phi_1(w_t - t^{-1}p_t) + \epsilon_t\). Firms may be assumed to have an information advantage and thus the labor demand function continues to be represented by (3). An alternative rationalization is contained in Lucas (1973).}

\[
y_t = a_0 + a_1 k_t + a_2 (p_t - t^{-1}p_t) + \epsilon_t
\]

\[
y_t' = a_0 + a_1 k_t + a_2 (p_t - t^{-1}p_t) + \epsilon_t
\]
This necessitates replacing (6) by

\[(6)' \quad r_t = \alpha - (1-a_1)k_t + a_2(p_t - t_1p_t) + \varepsilon_t\]

The basic equation for the price level (16) may be written now in the same general form. All the coefficients except \(b_3\) and \(\zeta\) are unchanged. The new coefficients are

\[(16)' \quad b_3 = \zeta[a_2(1 + (\beta_1 + \gamma_1)(1 - a_1)) - a_1(\gamma_2 + \beta_2)]
\]

\[\zeta = [(1 + \gamma_2 + a_2)(1 + \beta_1(1 - a_1)) + (a_2\gamma_1 - \gamma_1\beta_2)(1 - a_1)]^{-1}\]

Since \(\zeta^{-1}\) is now larger than in (16), all coefficients but \(b_3\) are reduced in absolute value. The sum of the coefficients \(\frac{5}{2} b_3 = 1\), as before; however,

\[b_2 + b_4 < 1\] now.

The value of \(\lambda\) is unchanged, and there is accordingly a single stable root.

A number of the equations (19) through (45) no longer apply, since the equality

\[b_2 + b_4 = 1\] was used in their derivation. However, the qualitative changes in the behavior of the model can be simply explained.

First, there is no change whatsoever in the response of the system to anticipated changes in the money stock—and this applies to anticipations which go into effect as late as a single period before the expected change occurs.

Second, an unanticipated permanent change in the money supply is no longer neutral. The unanticipated rise in the price level in the current period causes an increase in output, in turn leading to a price level increase that falls proportionately short of the money stock increase. Today's output response causes future capital stocks to be higher, and the price level adjusts slowly over time to its new higher level, with the capital stock being higher in the interim period than it would otherwise have been.

Third, an unanticipated transitory monetary increase may not have the unusual consequences that it does as shown in Figure 3. In the analysis so far, a monetary shock affects no real variable in the current period, and results in lower capital stocks in subsequent periods. That effect is still present. However, the unanticipated
increase in the current price level in the present context results in higher output, leading—through the mechanism described in the previous paragraph—to the possibility that the capital stock in subsequent periods may be higher than it would have been otherwise. The relative strength of the two effects depends on the elasticity of current output with respect to unanticipated inflation. The larger this elasticity, the more likely is it that an unanticipated monetary blip today is expansionary in subsequent periods rather than contractionary. The capital stock does not necessarily approach its former level monotonically in this case.

This terse description makes it clear that the introduction of Phillips-curve type phenomena leads to the possibility of response patterns to unanticipated changes in the money stock that are more in accord with intuition and the results of standard analyses than those of the preceding sections. When the Phillips curve is included, neither anticipated nor unanticipated money changes are neutral.

13. Stability and Uniqueness

Four stability or uniqueness questions arise in the context of this model. The first is the familiar one of how or why the economy selects the unique stable rational expectations path. In the absence of an explicit maximization problem, of the type examined by Brock (1975), this question cannot be satisfactorily answered. The unstable paths are also rational expectations paths, in that the model implies that they may be followed—and if they are followed, expectations will turn out to have been "correct". If one uses the overlapping generations framework suggested in Section 1, optimization is not obviously a solution, as it is in the Brock case—for no one need look more than one period ahead. This is presented as an unsolved problem.

Second, there is a stability question arising from the behavior of the exogenous variable. It may be confirmed by examining (17) and the solution for the undetermined coefficients, that if the money supply is expected to grow "too fast"—more rapidly than
at the rate \( \frac{b_1}{\lambda b_2} \) --the current price level is not defined. This presumably represents the phenomenon of a flight from the currency if the cost of holding the currency is expected to be extremely high.

A third stability question concerns the existence of a stable root for the system. It is not inevitable, in an ad hoc model, that there be any stable root. For instance, if in the basic model of Section 1, the demand for capital is assumed to have a high income elasticity, then there may be no stable root for the price level equation (17).

The economics is simple: if the demand for capital has high income elasticity, then an increase in the current capital stock, increasing output, may lead to a larger increase in next period's capital stock, and so forth. Such difficulties are unlikely to arise in models derived from maximizing behavior, but it is probably worthwhile warning against any assumption that there is necessarily a single stable rational expectations path in every conventional macro-model.

The fourth question arises from a recent paper by Taylor (1977) who shows that in some rational expectations models, there may be non-uniqueness in the following sense: if individuals believe that some (extraneous) variable is relevant to the behavior of the economy, and form their expectations using information on that variable, it may turn out that the economy behaves in a manner consistent with their beliefs. This is a case of a self-fulfilling prophecy, as opposed to simple rational expectations. No such problem occurs in the present paper, and it appears from Taylor's example that the problem is in some sense pathological, existing only in a model which is inherently unstable.

14. The Source of the Non-Neutrality and Related Literature

The non-neutrality in the present paper arises from a combination of a distribution effect—between old and young—and the fixity of the nominal interest rate on money.

Inflation in the model of Section 1 is a tax on the holders of money, who accordingly
modify their asset holding behavior in the face of anticipated inflation. This is fundamentally the Tobin effect, as present in his 1965 paper. However, in Tobin (1965), money is introduced into the economy as a lump-sum transfer payment, and therefore affects disposable income. That particular mechanism is not included here, though it would probably produce similar results.

If money were introduced in this model by being given to existing money holders in proportion to their money holdings, it would be neutral. In that case, money holders would be compensated for inflation, and there would be no reason for them to modify their asset-holding behavior. Similarly, in the Tobin analysis, if changes in the growth rate of money meant only that existing holders of money received transfer payments at a higher rate, in proportion to their money balances, money would be neutral.

The model of Section 1 is essentially similar to the model of Section 2 of Lucas (1975). Lucas has the money enter the economy through government purchases of goods—this may also be regarded as a distribution effect. The solution Lucas obtains for his model is not the same as that obtained here. Lucas' solution appears to be valid only for certain special assumptions on the behavior of the money stock. His solution is correct if the money stock follows a random walk, or if, in the certainty case, alternative permanent fixed levels of the money stock are being considered. The non-neutralities of money considered in the remainder of Lucas's paper are an extension of the basic non-neutrality of unanticipated money analyzed in his seminal 1972 paper. Information lags play the key role, and the structure of the economy is such that money stock changes are only inferred gradually.

Sargent and Wallace (1975) include capital accumulation in their basic model but money is neutral in that model since the demand for capital depends only on the expected real rate of return on nominal bonds, and because aggregate demand is unaffected by the expected rate of inflation.
Sargent (1977) has analyzed a real trade cycle arising from the response of labor supply to changes in real rates of return on assets. These introduce inter-temporal labor substitution effects, that can be thought of as making the natural rate of unemployment move in serially correlated fashion from period to period. In the concluding section of his paper, Sargent briefly discusses the mechanism contained in the basic model of this paper.

Brock (1975) examines the response of an economy to pre-announced changes in the money stock, showing adjustment patterns in the case of perfect foresight that are similar to those obtaining here with full anticipations when money is neutral.

Aside from non-neutralities of money of the type analyzed in this paper, and particularly those associated with the Phillips curve—whether in the refined Lucas (1973) and Barro (1976) versions, or cruder versions—the prime non-neutrality of money usually discussed in the literature is that arising from changes in the ratio of bonds to money as resulting, say, from an open market operation. This non-neutrality is extensively discussed in Patinkin (1965). Its theoretical and validity hinges on the question of whether bonds are net wealth. The recent paper by Barro (1974) makes arguments for both sides on this issue, leaving the answer theoretically indeterminate.

Inventory accumulation is another potential source of the non-neutrality of money. It is central to the persistence effects analyzed by Phelps and Taylor (1977), and has recently been examined by Blinder (1977). Unless the stock of inventories affects the demand for labor—as Blinder argues it does—the Lucas supply curve continues to apply, and anticipated monetary changes remain neutral. Inventory accumulation and decumulation do undoubtedly play a role in the detailed mechanism through which unanticipated monetary disturbances affect future output levels.

Non-neutralities arising from the nature of non-indexed tax systems have recently been analyzed by a number of authors, including Feldstein, Green and Sheshinski (1976).
The interest in these non-neutralities stems from their potential for helping explain the impact of the behavior of money on the trade cycle. Those non-neutralities associated with the Phillips curve explain why unanticipated monetary changes are non-neutral, and the others play a role in explaining why anticipated changes in the money stock may have real effects.

15. Conclusions

The paper analyzes the impact of anticipated and unanticipated changes in the money stock in a model in which money is not neutral. For most of the paper, full current information is assumed. Under these circumstances, anticipated changes in the money stock have real effects, both before and after the change in the money stock takes place. An unanticipated change in the stock of money that is expected to be permanent is neutral. Monetary changes produce serially correlated changes in prices, output, employment and the capital stock. The serial correlation arises from the impact of anticipated inflation rates on the demand for capital, feeding back through the impact of capital on the level of output and income, to the level of investment and saving, and thus on to future capital stocks.

It is perhaps unnecessary to note explicitly that the form of monetary feedback rules can have real effects on the behavior of output (including its variability) when anticipated money is non-neutral.

Once the assumption of full current information is dropped, and a simple Friedman-Phelps-Lucas type Phillips curve added, unanticipated stock changes in the money supply are also non-neutral. Thus in general both anticipated and unanticipated monetary changes have real effects.
References


Blinder, Alan, "Inventory Behavior, Real Wages and the Short-Run Keynesian Paradigm", unpublished, Princeton University.


Sidrauski, Miguel "Inflation and Economic Growth", *Journal of Political Economy*, 75, 6 (December 1967), 796-810.

