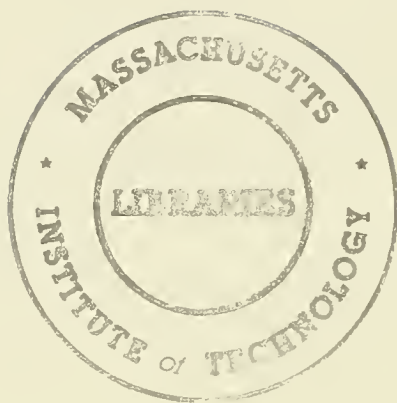



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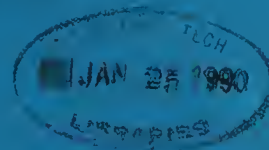


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Allocating and Abrogating Rights:  
How Should Conflicts Be Resolved  
Under Incomplete Information?

Joseph Farrell

Number 381

May 1985  
Revised

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Allocating and Abrogating Rights:

How should Conflicts be Resolved Under Incomplete Information?

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Revised, May 1985. Comments are solicited. I am grateful for helpful comments and encouragement from Oliver Hart, Suzanne Scotchmer, and Jean Tirole.



## 1. Introduction.

In a highly influential paper, Ronald Coase (1960) formulated what has become known as "the Coase Theorem". This is the proposition that, when negotiation is perfect, conflicts can be efficiently resolved by giving one of the parties a property right in the decision; and that, for efficiency, it does not matter who has the right.

Coase's analysis assumed that agents know each other's costs and benefits. In many conflicts, however, neither side precisely knows the other's payoffs. Indeed, if information were complete in this sense, then the court that assigns and enforces a right could equally well dictate and enforce an outcome. Therefore, we need to ask whether, with incomplete information, assigning rights is a good (not necessarily perfect) way to resolve externalities; and whether it matters for efficiency which party gets the rights.

We know that assigning rights will not be perfectly efficient, since negotiation under incomplete information is not a perfectly efficient process<sup>1</sup>. But we can ask, for instance, whether rights do better than enforcing a rigid norm set optimally ex-ante. In Section 3 below, we study this question in a simple model. We show that assigning rights is sometimes better than the rigid norm, and sometimes worse. In general, it matters which party gets the rights: in this model, efficiency is enhanced if the party that cares less about the decision, or whose tastes are more predictable, has the right.

In Section 4, we compare property rules with liability rules, in which compensating payments are not negotiated by the parties but are fixed by a

court which estimates damages. We show that, in the model considered in Section 3, liability rules outperform property rules, despite the court's ignorance of true damages. In other cases, this result may be reversed.

In Section 5, we consider another approach to rights. It has been proposed<sup>2</sup> as a desirable feature of a social choice rule that if a decision is especially important to a person, then he should have a right to choose the outcome. In the social choice literature, side payments are not considered, and so any assignment of a right in a decision that affects other people (even a little) will not yield Pareto efficient outcomes. This result has been called the "impossibility of a Paretian liberal".

We examine the use of rights as a decentralization device<sup>3</sup> when side payments are ruled out. We show that, if private information is important and there is little conflict between the parties, then rights are useful; if there is a great deal of conflict and little private information, then it is better to enforce the rigid norm. Rights should be assigned to the party who cares more about the decision, or whose preferences are less predictable.

In Section 6, we consider assigning limited rights: an agent is allowed to choose the outcome subject to limits. In general, this scheme will do strictly better than unlimited rights. We characterise optimal limits on rights, and compare limited rights with the rigid norm.

In Section 7, we calculate individual payoffs from the various rules, and show that the more efficient rules are not necessarily preferred by both agents.

In Section 8, we briefly summarize some related work. Section 9 concludes the paper.

## 2. The Basic Model.

There are two agents, A and B. One decision, represented by a real number  $x$ , must be taken. A and B have different preferences over the choice of  $x$ . Specifically, if  $x$  is chosen, the agents get payoffs

$$u_A = -\alpha (x - a)^2, \quad u_B = -\beta (x - b)^2 \quad (2.1)$$

where the numbers  $a$  and  $b$  are private information, known only to A and to B respectively. We think of  $\alpha$  as measuring the importance of the choice of  $x$  to agent A. For convenience, we assume that  $\alpha + \beta = 1$ .

These payoff functions are concave in  $x$ , so the optimal choice of  $x$  can be represented by the first-order condition alone. We assume that the agents' risk attitudes are embodied in these payoff functions, so that they will maximise expected payoff; and also that utility is transferable, so that if  $x$  is chosen and B pays A an amount  $P$ , the net payoffs are:

$$-\alpha (x - a)^2 + P, \quad -\beta (x - b)^2 - P \quad (2.2)$$

We will write "welfare" for the sum of A's and B's payoffs: since utility is transferable, this is legitimate. A more convenient expression for a monotone transform of "welfare" can be defined as follows.

$x^*(a,b)$ , the efficient decision in state  $(a,b)$ , maximizes:

$$W = -\alpha (x - a)^2 - \beta (x - b)^2 \quad (2.3)$$

and so we can write

$$x^*(a,b) = \alpha a + \beta b \quad (2.4)$$

Note that the individual payoffs in the first-best are given by

$$Eu^A = -\alpha \beta^2 (C^2 + \text{var}(a) + \text{var}(b)) \quad (2.5)$$

$$Eu^B = -\alpha^2 \beta (C^2 + \text{var}(a) + \text{var}(b)) \quad (2.6)$$

where we write  $C$  for the expected degree of conflict,  $C = (E_b - E_a)$ .

Now consider a decision rule  $R$  which leads to the choice  $x = x(a,b)$  in state  $(a,b)$ . Under  $R$ , ex-ante expected welfare is

$$- E(\alpha(x - a)^2 + \beta(x - b)^2)$$

$$= - E((x - x^*)^2 + \alpha a^2 + \beta b^2 - x^{*2}) \quad (2.7)$$

$$= - L(R) + \text{terms independent of } R \quad (2.8)$$

where

$$L(R) = E((x - x^*)^2) \quad (2.9)$$

We sometimes describe  $L(R)$  as the efficiency loss (relative to the first-best) from rule  $R$ .

We assume that  $a < b$  with probability 1. This means that  $A$  and  $B$  are always in conflict, and communication as studied by Crawford and Sobel (1982) will not occur. We assume that the random variables  $a$  and  $b$  have finite means and variances. Sometimes, for simplicity, we assume that they are independent. In Sections 3 and 4, we assume that they are uniformly distributed.



### 3. Efficiency of Property Rules with Side Payments under Incomplete Information.

In this section we evaluate the efficiency of the following rule: A has the right to choose  $x$ , but B may offer him bribes to affect his choice. Thus, B (after observing  $b$ ) can offer A a bribe  $P(x) \geq 0$  which may depend on A's choice  $x$ . A then chooses  $x(a, P(\cdot))$ , balancing his intrinsic desire to set  $x = a$  against his desire for money from B. We assume that B is a Stackelberg leader in these negotiations.

If B knew the value of  $a$ , he would choose a bribe function that would lead A to select  $x^*(a, b)$ : this result is the Coase theorem, and will emerge as a limiting case of our analysis. Since B does not know  $a$ , however, he does not know just how much he needs to bribe A. In maximizing his own expected net payoff he will "shade" his bribe schedule in such a way (we will find) that efficiency results only if  $a$  in fact has its maximum possible value. For smaller values of  $a$ ,  $x$  is chosen too close to  $a$ , and this is an efficiency loss. We compare this loss against the losses incurred by enforcing a rigid norm, or by reversing the roles and giving B the right to choose  $x$  and letting A bribe him.

Our calculation will assume for simplicity that  $a$  and  $b$  are independently uniformly distributed on  $(a_{\min}, a_{\max})$  and  $(b_{\min}, b_{\max})$  respectively. We also assume that there is just one offer  $P(\cdot)$ . By the revelation principle<sup>4</sup>, any sequence of offers and proposals can be modeled this way, provided that B can commit himself to a scheme. Models of bargaining in which parties cannot commit themselves (e.g. Peter Cramton, 1984) also show ex-post inefficient outcomes: indeed, Cramton shows that outcomes without commitment are (in an average sense) even less efficient, so that we are not biasing the analysis against the property

rule: rather the reverse.

When B offers a bribe function  $P(x)$ , each possible "type"  $a$  of A will choose some  $x(a)$  and receive a side-payment  $p(a) = P(x(a))$ . By the revelation principle, we can regard B as choosing the pair  $(x(a), p(a))$  for each type  $a$ , subject to the "incentive compatibility constraints"<sup>5</sup> for all  $a$  and all  $a'$ :

$$-\alpha (x(a) - a)^2 + p(a) \geq -\alpha (x(a') - a)^2 + p(a') \quad (3.1)$$

From (3.1) we derive a first-order condition:

$$2\alpha (x(a) - a) x'(a) - p'(a) = 0 \quad (3.2)$$

Since the necessary condition (3.1) implies (3.2), we will solve B's optimization problem subject to (3.2) and then check that the solution satisfies (3.1). B's objective is to minimize the expected value of

$$\beta (x(a) - b)^2 + p(a)$$

or, in other words, to minimize

$$\int_{a_{\min}}^{a_{\max}} (\beta (x(a) - b)^2 + p(a)) da \quad (3.3)$$

subject to (3.2). If  $y(a)$  is the shadow price of (3.2) at  $a$ , B minimizes

$$\int_{a_{\min}}^{a_{\max}} (\beta (x(a) - b)^2 + p(a) + y(a) (-p'(a) + 2\alpha(x(a)-a)x'(a))) da \quad (3.4)$$

Applying the calculus of variations to (3.4), we get



$$2\beta(x - b) + 2\alpha y x'(a) = d/da (2\alpha(x - a)y) \quad (3.5)$$

$$- 1 = dy/da \quad (3.6)$$

Simplifying,

$$\beta(x - b) = -\alpha y + \alpha(x - a) dy/da \quad (3.7)$$

$$dy/da = -1 \quad (3.8)$$

whence

$$x = \alpha a + \beta b - \alpha y \quad (3.9)$$

Now we know that  $y(a_{\max}) = 0$ , since B incurs no cost<sup>6</sup> by increasing  $dp/da$  at  $a = a_{\max}$ . Hence, since  $dy/da = -1$ , we know that  $y(a) = (a_{\max} - a)$ , and (3.9) becomes

$$x(a,b) = (\alpha a + \beta b) - \alpha (a_{\max} - a) \quad (3.10)$$

In (3.10), the first term represents  $x^*(a,b)$ , the socially efficient value of  $x$ . We observe that, when  $a = a_{\max}$ ,  $x$  is chosen efficiently<sup>7</sup>. When  $a < a_{\max}$ ,  $x$  is chosen too low: B's bribe scheme is not persuasive enough. Intuitively, giving enough marginal incentive to a low type entails giving away unnecessary inframarginal money to all higher types. This purely private effect vanishes at  $a = a_{\max}$ , which is why efficiency holds there. Incidentally, we also see that if B knows  $a$ , then efficiency will result. This is the Coase theorem.

Writing  $R^A$  for the rule analysed above, we can now calculate  $L(R^A)$ .

$$\begin{aligned}
 L &= E \left( (x(a,b) - x^*(a,b))^2 \right) = \alpha^2 E \left( (a_{\max} - a)^2 \right) \\
 &= \alpha^2 r_a^2 / 3 \quad (3.11)
 \end{aligned}$$

where  $r_a$  means  $(a_{\max} - a_{\min})$ . By symmetry, the loss from the opposite system  $R^B$  (B has the rights and A can bribe him) is:

$$L(R^B) = \beta^2 r_b^2 / 3 \quad (3.12)$$

Finally, under the rigid norm  $x^n = E(x^*)$ ,  $L(R^n) = \text{var}(x^*)$ , or

$$\begin{aligned}
 \alpha^2 \text{var}(a) + \beta^2 \text{var}(b) &= \\
 (\alpha^2 r_a^2 + \beta^2 r_b^2) / 12 &\quad (3.13)
 \end{aligned}$$

Equations (3.11) - (3.13) yield a number of interesting results<sup>8</sup>: in Figure 1, we display the efficient rule as a function of  $r_b$  and of  $\alpha$ , when we normalize  $r_a$  to 1. Some of the information in Figure 1 can be summarized as:

Proposition 1: When  $a$  and  $b$  are independently uniformly distributed,

(i) If  $\alpha = \beta$ , and  $a$  and  $b$  are equally unpredictable, then the rigid norm is strictly more efficient than assigning rights to either party.

(ii) If  $\alpha \gg \beta$ , and  $a$  and  $b$  are about equally unpredictable, then B (sic) should be given the right to choose  $x$ .

(iii) If  $a$  is much more unpredictable than  $b$ , while  $\alpha \approx \beta$ , then B (sic) should be given the right to choose  $x$ .

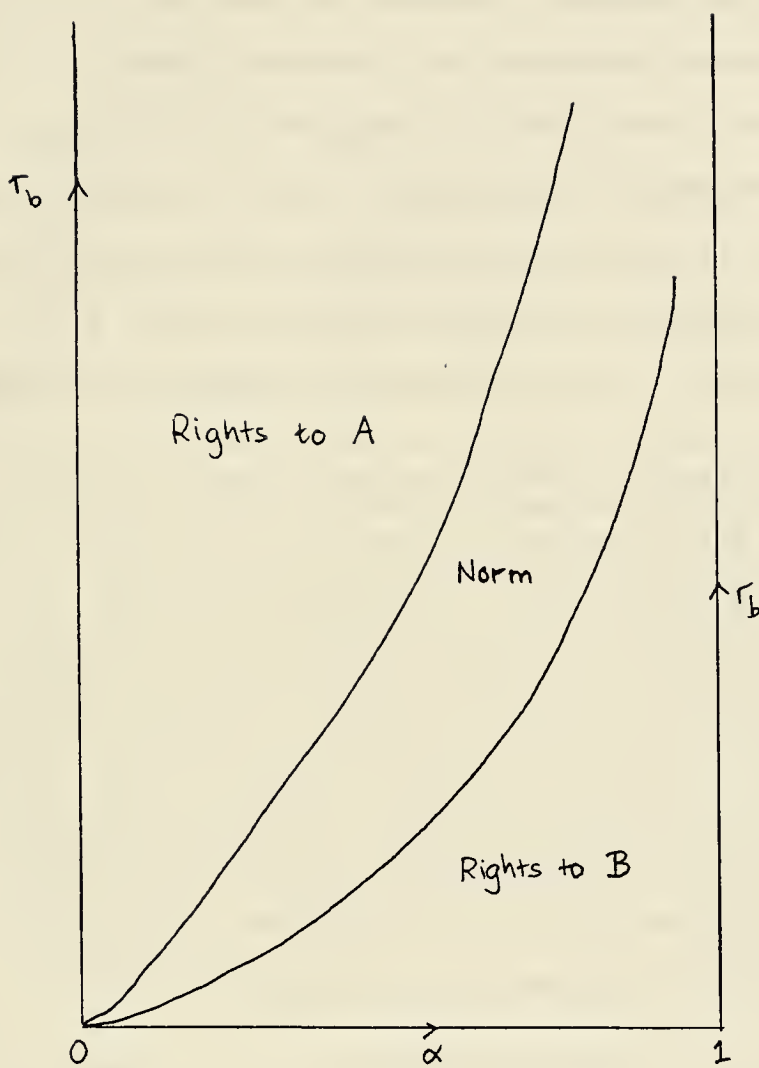


Figure 1.

Each of the results of Proposition 1 may be surprising to some readers. Part (i) states that, in a symmetric conflict, the rigid norm's unresponsiveness to private information is less harmful than the bargaining inefficiencies that result from giving one party the right to choose  $x$  (and letting the other bribe him). Part (ii) states that a decision which is especially important to one party should be assigned to the other; and part (iii) states that a decision which should depend a great deal on information only available to one party should be assigned to the other.

Part (i) shows that the Coase theorem genuinely depends on complete information, and that even a weak form ("rights are more efficient than a rigid norm") can easily fail otherwise. The counterintuitive parts (ii) and (iii) are not policy suggestions. Rather, they demonstrate the possibility that important information may be better encoded in the choice of bribe schedule than in the response to a given bribe schedule, and so it need not be true that the party more affected by or more informed about a decision should take the decision. The opposite, more intuitive, case is possible also, and always holds when there are no side payments (see Section 5).

#### 4. Property Rules versus Liability Rules.

Inefficient outcomes in a property rights system, as analysed above, result from "strategic behavior" : the agent (say, B) who is to bribe the other (A) deliberately sets a payment schedule that will not induce efficient behavior, in order to conserve on side payments. At the cost of not having B's information feed into the decision, we could stop this inefficiency by fixing exogenously the schedule of payments from B to A (contingent on  $x$ ), and then letting A choose  $x$ .

Except for a lump-sum transfer, this is equivalent to allowing A to choose  $x$  and making him pay court-determined damages to B. We expect a gain in efficiency to the extent that inefficient shading of the bribe schedule occurs above, but a loss in efficiency to the extent that B's private information  $b$  is important. In this section, we analyse this problem.

A court can only award estimated<sup>9</sup>, not actual, damages. We take that to mean that if A chooses  $x$ , he must pay enough damages to B to make whole an average type ( $b = E_b$ ) of  $B^{10}$ . This will induce<sup>11</sup> A to set

$$x(a,b) = x^*(a, E_b) \quad (4.1)$$

Therefore the expected loss (relative to the first best  $x^*(a,b)$ ) is<sup>12</sup>

$$L = \beta^2 \text{var}(b) \quad (4.2)$$

If we reverse the roles of A and B, we get a loss

$$L = \alpha^2 \text{var}(a) \quad (4.3)$$

Comparing (4.2) and (4.3), we immediately get:

Proposition 2: When using an estimated-damages rule, the court should give the decision to the party with the less-predictable tastes and/or the greater stake in the decision, and award estimated damages to the other party. However, even using the "wrong" liability rule is more efficient than enforcing a rigid norm.

Next, we ask how the liability system compares in efficiency with the property-rights system considered in Section 3. To do this, we assume that (as in Section 3)  $a$  and  $b$  are independently uniformly distributed. Then the loss from the liability rule in which A pays B is (from (4.2))

$$\beta^2 r_b^2 / 12 \quad (4.3)$$

This is better than the property-rights system  $R^B$  in which B has the rights and A bribes him, since in that system, by (3.12), the loss is

$$\beta^2 r_b^2 / 3 \quad (4.4)$$

Similarly the liability rule in which B chooses  $x$  and pays A estimated damages will do better than the property-rights rule  $R^A$  in which A chooses  $x$  and B must try to bribe him. Thus we get:

Proposition 3: When  $a$  and  $b$  are independently uniformly distributed, a suitable liability rule is always more efficient than allocating property rights in  $x$  and letting the parties negotiate, or enforcing a rigid norm.

Proposition 3 tells us that the strategic inefficiency that occurs when parties negotiate under incomplete information can be more damaging than the inefficiency resulting from the court's ignorance of the true level of damages. The opposite case can happen too. The clearest example is when  $a$  and  $b$  are perfectly correlated, so that the agents themselves can negotiate under complete information, but the court cannot observe  $a$  or  $b$ . In this case, the Coase theorem is both true and important: allocation of rights leads to efficient outcomes, while none of the other rules considered does so. In practice, this analysis suggests, allocating rights is most desirable when the parties themselves have better information about one another's preferences than the court has about either.

Our analysis makes precise that discussed by Polinsky (1980)<sup>13</sup>. There may be an inefficiency in property rules if agents negotiate "strategically", and a court-imposed schedule of damages can avoid this problem. In Polinsky's analysis, the court has complete information about preferences. That will normally be true only if agents themselves have complete information; but then (in most models of bargaining) negotiation will be efficient. Accordingly, we have examined a model in which negotiation is inefficient for a clearly specified reason, and shown that the liability rule may still be superior, but is not necessarily so.



## 5. Rights Without Side Payments.

In many conflicts<sup>14</sup> the parties cannot effectively negotiate before the choice of  $x$ . Yet it may be possible to decide in advance who will control  $x$ . What does our model tell us about appropriate rules for such cases? In this section, we evaluate the ex-ante expected efficiency of some simple rules: "rights to A", in which A simply chooses  $x$  to suit himself; "rights to B", in which B chooses  $x$ ; and "rigid norm", in which the choice  $x = x^n$  is mandated.

Because of the simplicity of the calculations if side payments are ruled out, we can allow for general distributions of  $a$  and  $b$ , including the possibility that  $a$  and  $b$  are correlated. We show that, if  $\alpha = \beta$ , so that the problem is of equal importance to both parties, then the rigid norm is the most efficient unless there is "enough" correlation between  $a$  and  $b$ . However, if  $\alpha \gg \beta$ , then A should be given the right to choose  $x$ , as intuition suggests.

If A has the right to choose  $x$ , he will set  $x = a$ . So under this rule expected payoffs to the two agents are

$$Eu^A = 0 \quad (5.1)$$

$$Eu^B = -\beta E((a - b)^2)$$

Hence, ex ante, expected welfare is

$$EW = -\beta E((a - b)^2) \quad (5.2)$$

Similarly, if B has the right to choose  $x$ , he will set  $x = b$ , so that under that rule

$$EW = -\alpha E((a - b)^2) \quad (5.3)$$



From (5.2) and (5.3) we have:

Proposition 4: With no side payments, it is more efficient to assign rights to the agent with more at stake than to the other agent. No conditions on the joint distribution of (a,b) are needed for this result.

Next, we compare these rules with the rigid norm rule,  $x = x^n$ , which gives expected welfare:

$$EW = -\alpha E((x^n - a)^2) - \beta E((x^n - b)^2)$$

which we can expand to give

$$EW = -\alpha \beta C^2 - \alpha \text{var}(a) - \beta \text{var}(b) \quad (5.4)$$

In order to compare this with (5.2) and (5.3), we assume (without loss of generality) that  $\alpha \geq \beta$ , and expand the expression (5.2) for the expected welfare from the more efficient rights system,  $R^A$ :

$$EW(R^A) = -\beta (C^2 + \text{var}(a) + \text{var}(b) - 2 \text{cov}(a,b)) \quad (5.5)$$

Comparing (5.5) with (5.4), we see that the rigid norm is more efficient than either rights system if and only if

$$\beta^2 C^2 > (\alpha - \beta) \text{var}(a) + 2\beta \text{cov}(a,b) \quad (5.6)$$

From (5.6), we have, recalling that  $\alpha + \beta = 1$ :

Proposition 5: If  $\alpha = \beta$ , then the rigid norm is more efficient than the better allocation of rights if and only if:

$$C^2 > 4 \text{ cov}(a,b) \quad (5.7)$$

This tells us that a high degree of conflict (on average) argues for norms, while positive covariance between  $a$  and  $b$  argues for allocating rights. Independence or negative covariance guarantees that the rigid norm is more efficient than allocating rights, given that  $\alpha = \beta$ .

When  $\alpha \gg \beta$ , then (5.3) and (5.4) are worse than (5.2).

Thus,  $A$  should have the right to choose  $x$ , despite the externality. It is possible to be a Paretian liberal, provided the externality is sufficiently small. The impossibility result of Sen (1970) comes from his limited-information framework, in which we cannot express the idea that a decision is much more important to  $A$  than it is to  $B$ . But such judgments seem to underlie many of our ideas about why one person should take a decision alone. If we did not think that, in general, I am likely to care much more than you do about the color of my bedroom walls, then we should be less certain that I should have the right to choose. This argument does not capture all the reasons for rights, but plausibly there are many cases in which, as here, rights are a device for attaining utilitarian efficiency.

## 6. Limiting Rights.

With no side payments, the reason for giving one agent (say, A) the right to choose  $x$  is that it makes  $x$  respond to shifts in  $a$ . We have compared the rule giving complete responsiveness (an absolute right) with the rule giving none (the rigid norm), and seen that either may do better. This suggests that there may be some intermediate optimal degree of responsiveness. Since we rule out negotiation between the parties, the only way to achieve that would be to give one side the right to choose  $x$ , but subject to restrictions. We now investigate the efficiency of such a rule. Because of the concavity of our problem, we restrict ourselves to rules of the form

$$\begin{aligned} & \text{"A chooses } x \text{ subject to } x \geq x^r \text{" or} \\ & \text{"B chooses } x \text{ subject to } x \leq x^s \text{"} \end{aligned} \quad (6.1)$$

Some limit on rights is always desirable if we care about both agents. For preventing (say) A from choosing very low values of  $x$  will be costless to him (to first order), while it will strictly benefit B.

In the rest of this section, we first analyse limited rights when  $a$  and  $b$  are independently uniformly distributed (as in Sections 3 and 4), and then find conditions for limited rights to be better or worse than the rigid norm in the general case.

We begin, then, with  $a$  and  $b$  independent and uniform. In Figure 2, we plot the locus of points  $(E(x - a)^2, E(x - b)^2)$  when  $y$  is between  $a_{\min}$  and  $b_{\max}$ , and  $x$  is given by

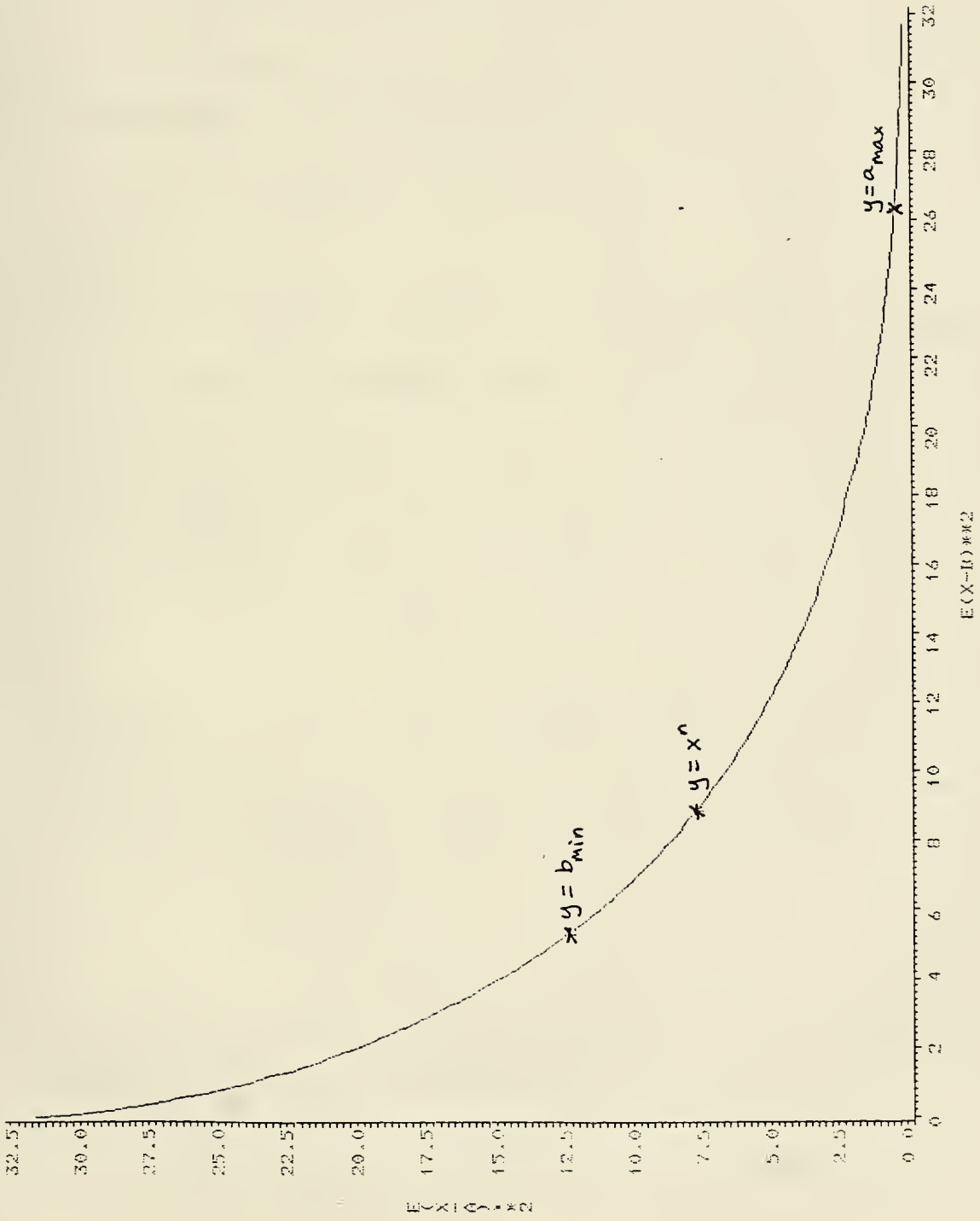
$$\begin{aligned} x &= \max(a, y) & \text{if } y \leq a_{\max} \\ x &= y & \text{if } a_{\max} \leq y \leq b_{\min} \\ x &= \min(b, y) & \text{if } y \geq b_{\min}. \end{aligned} \quad (6.2)$$

With uniformly distributed  $a$  and  $b$ , the locus of points in Figure 3 is convex. (It is easy to check algebraically that the slope is a decreasing function of  $y$ .) This makes it easy to pick the point that minimizes  $\alpha E(x - a)^2 + \beta E(x - b)^2$ . The optimal choice of  $y$  is

$$y = x^r \equiv (2\beta Eb + \alpha a_{\min}) / (2\beta + \alpha) \quad \text{if } x^r < a_{\max}$$

$$y = x^s \equiv (2\alpha Ea + \beta b_{\max}) / (2\alpha + \beta) \quad \text{if } x^s > b_{\min} \quad (6.3)$$

$y = x_n$  otherwise



CASE 5

AMIN=0 AMAX=1 BMIN=4 BMAX=8

Figure 2



With more general distributions of  $a$  and  $b$  (we will continue to assume that  $a$  and  $b$  are independent), there is no reason to expect the locus of points in Figure 3 to be convex. Accordingly, efficiency is much less easy to calculate. We now turn to this more general case and see what we can say about optimal limits on rights.

If  $A$  chooses  $x$  subject to  $x \geq x^r$ , then the optimal "standard of reasonableness",  $x^r$ , will satisfy the first-order condition

$$E(x^*(a,b) \mid a \leq x^r) = x^r \quad (6.4)$$

If the left hand side of (6.4) is too small, then it pays to lower  $x^r$  a little; if too large, then  $x^r$  should be raised a little. Notice that the left hand side is too large when  $a = a_{\min}$ ; so if (6.4) never holds for  $x^r$  in the range of  $a$ , then it goes on paying to raise  $x^r$  until  $x^r = a_{\max}$ , when  $A$  has no discretion left in the choice of  $x$ . Then, clearly, it is better to insist on  $x = x^n$ . Thus we see that:

Proposition 6: If (6.4) has no solution  $x^r$  in the interior of the range of  $a$ , then  $A$  should not be given rights to choose  $x$ : it is more efficient<sup>15</sup> to insist on the ex-ante optimal choice  $x = x^n$ .

A similar proposition holds for giving rights to  $B$ . The corresponding equation for the maximum reasonable choice by  $B$ ,  $x^s$ , is

$$E(x^*(a,b) \mid b \geq x^s) = x^s \quad (6.5)$$

If neither (6.4) nor (6.5) has a solution in the relevant ranges then the rigid norm dominates any system of giving partial discretion to either A or B. For example, consider the case of uniform distributions. For (6.4) to have a solution in the range of a it is necessary that

$$2\beta C < r_a \quad (6.6)$$

Likewise, for (6.5) to have a solution  $x^S$  we require that  $2\alpha C < r_a$ .

Hence if there is enough conflict, i.e. if

$$C > \max( r_a/2\beta, r_b/2\alpha ) \quad (6.7)$$

then the rigid norm is more efficient than any allocation of rights, even allowing for limits on rights.

While allowing A or B to choose  $x$  eliminates some inefficient rigidity, it also (absent side payments) introduces another inefficiency: the non-chooser's preferences are ignored. If there is a lot of conflict and little uncertainty, then the rigid norm is best. If the conflict is much more important to one side than to the other, however, then it is relatively unimportant to allow for the other's preferences, and so efficiency requires giving an almost unencumbered right to the more concerned person: if  $\alpha \gg \beta$ , then (6.4) will have a solution  $x^r \approx a_{\min}$ .

The analysis above leaves a question unanswered: Suppose that there is an interior solution  $x^r$  of (6.4). With general distributions, when we do not know that the locus of points in Figure 3 is convex, how do we know whether the limited rights system described by  $x^r$  (and satisfying the first-order condition (6.4) for efficiency) is better or worse than the rigid



norm? And what should be done if both (6.4) and (6.5) have solutions? The following proposition gives the answer.

Proposition 7: (i) If  $a < x^n$  with probability one, and if the function  $(E(x^* | a \leq y) - y)$  is decreasing in  $y$ , then it is better to enforce the norm  $x^n$  than to give A even a limited right to choose  $x$ .

(ii) If  $x^n$  is in the range of  $a$ , then there is a solution  $x^r < x^n$  to (6.4), and the limited rights system in which A chooses  $x$  subject to  $x \geq x^r$  is better than the rigid norm.

(iii) (6.4) and (6.5) cannot both have solutions in the relevant ranges.

Proof of Proposition 7: First, suppose that  $a_{\max} < x^n$ . By definition

$$x^n = E(x^*(a,b) | a \leq a_{\max}) \quad (6.8)$$

Thus

$$E(x^* | a \leq a_{\max}) - a_{\max} > 0 \quad (6.9)$$

which implies (by our assumption) that

$$E(x^* | a \leq y) - y > 0 \quad (6.10)$$

for all  $y \leq a_{\max}$ . Hence, if A were choosing  $x$  subject to a minimum level  $x^r < a_{\max}$ , it would be socially beneficial to increase  $x^r$  a little. Since this is true for all  $x^r < a_{\max}$ , it is better to use  $x^n$ .

On the other hand, suppose that  $x^n < a_{\max}$ . Then, since  $Eb > a_{\max}$ , the rule  $x = \max(x^n, a)$  is more efficient than the rigid  $x = x^n$  rule. But it is still not the optimum, since

$$E(x^* \mid a \leq x^n) < E(x^* \mid a \leq a_{\max}) = x^n \quad (6.11)$$

and so a (small) reduction in  $x^r$  below  $x^n$  is socially beneficial. This proves part (ii). Part (iii) follows from parts (i) and (ii).

When  $\alpha \gg \beta$ , then  $x^n \approx Ea$ , so that  $x^n$  is certainly in the range of  $a$ . Thus we see that when a decision is of special importance to one person, say A, he should be given rights to the decision, with relatively little ( $x^r \ll x^n \approx Ea$ ) restriction on the exercise of those rights. As  $\beta$  becomes closer to  $\alpha$ , A's rights should become more circumscribed, and eventually vanish.

Proposition 7 gives conditions for when A or B should be given some rights, and when instead it is best to enforce a rigid norm. We now try to understand when these conditions are likely to hold.

Suppose we begin with distributions of  $a$  and  $b$  such that, given  $\alpha$  and  $\beta$ ,  $a < x^n < b$  with probability one. Proposition 7 tells us to enforce a rigid norm given these distributions. Now suppose we increase  $\alpha$ . This makes  $x^n$  move towards  $Ea$ , and into the range of  $a$ . Proposition 7 now tells us to give A some discretion. Thus we confirm that, for given distributions, it becomes more appropriate to give A some rights as  $\alpha$  becomes large.

Next, with  $\alpha$  and  $\beta$  given, consider a mean-preserving

enlargement of the range of  $a$ . If carried far enough, this will make the range of  $a$  cover  $x^n$ ; thus we see that, as  $a$  becomes more dispersed<sup>16</sup>, it becomes more attractive to give some rights to A.

We conclude that limits on rights are generally desirable, and that a surprisingly simple rule tells us when A or B should be given some rights. The optimal allocation of limited rights may differ from the optimal allocation of absolute rights: that is, it may be better to give A absolute rights than to give B absolute rights, but it may be best to give B limited rights<sup>17,18</sup>

## 7. Individual Payoffs.

We now calculate individual payoffs in the rules considered above. Choices among rules are not based solely on efficiency, partly because choices are not made in Rawlsian ignorance, and partly because the distribution of surplus has effects when agents anticipate how conflicts will be resolved. (On this, see Grossman and Hart.) Accordingly, we now calculate and show in Figure 3 the individual expected payoffs from each of the rules considered.

First, consider the payoffs from the first-best choice  $x^*(a,b)$ . Since  $x^* = \alpha a + \beta b$ , we have

$$\begin{aligned} u^A &= -\alpha(\alpha a + \beta b - a)^2 = -\alpha\beta^2(b - a)^2; \\ Eu^A &= -\alpha\beta^2 E(b - a)^2 \\ &= -\alpha\beta^2 (C^2 + \text{var}(a) + \text{var}(b)) \end{aligned} \quad (7.1)$$

and likewise

$$Eu^B = -\beta\alpha^2 (C^2 + \text{var}(a) + \text{var}(b)) \quad (7.2)$$

Next, consider the system in which B bribes A and A has the right to choose  $x$ : the first system considered in Section 3.

$$u^A(a,b) = p(a,b) - \alpha(x(a,b) - a)^2 \quad (7.3)$$

Differentiating with respect to  $a$ , with  $b$  fixed,

$$\partial/\partial a(u^A(a)) = \partial p/\partial a(a,b) - 2\alpha(x - a)(\partial x/\partial a(a,b) - 1) \quad (7.4)$$

Using (3.2), we can simplify this to:

$$\partial u / \partial a = 2\alpha(x - a) \quad (7.5)$$

We also have the initial condition  $u^A(a_{\min}) = 0$  for all  $b$ , so  $u^A(a, b)$  is determined. A tedious calculation<sup>19</sup> gives us

$$Eu^A = \alpha\beta C r_a + \alpha r_a^2 (1 - 5\alpha)/6 \quad (7.6)$$

$Eu^B$  can then be calculated from (7.6), using the expression (3.11) for the loss relative to  $x^*(a, b)$ : we obtain<sup>20</sup>:

$$\begin{aligned} Eu^B = & -\alpha\beta C(C + r_a) - \alpha\beta r_b^2/12 \\ & - \alpha(3 - 7\alpha)r_a^2/12 \end{aligned} \quad (7.7)$$

Naturally, we can write down similar expressions for the payoffs in the opposite system in which B has the rights and A bribes him.

Next, we turn to the liability systems analysed in Section 4. If A has to pay B the amount  $\beta(x - Eb)^2$  required to make whole the presumptively average B, then he will set:

$$x = x^*(a, Eb) = \alpha a + \beta Eb \quad (7.8)$$

so

$$(x - a) = (\alpha - 1)a + \beta Eb = \beta(Eb - a) \quad (7.9)$$

$$(x - b) = \alpha a + \beta Eb - b = \alpha(a - Eb) + (Eb - b) \quad (7.10)$$

Hence expected payoffs before the liability payment are

$$-\alpha E(x - a)^2 = -\alpha\beta^2 (C^2 + \text{var}(a)) \quad (7.11)$$

$$-\beta E(x - b)^2 = -\beta\alpha^2 C^2 - \beta\alpha^2 \text{var}(a) - \beta \text{var}(b) \quad (7.12)$$

and after the payment of  $\beta E(x - Eb)^2 = \beta\alpha^2(C^2 + \text{var}(a))$  from A to B,

$$Eu^A = -\alpha\beta C^2 - \alpha\beta \text{var}(a) \quad (7.13)$$

$$Eu^B = -\beta \text{var}(b) \quad (7.14)$$

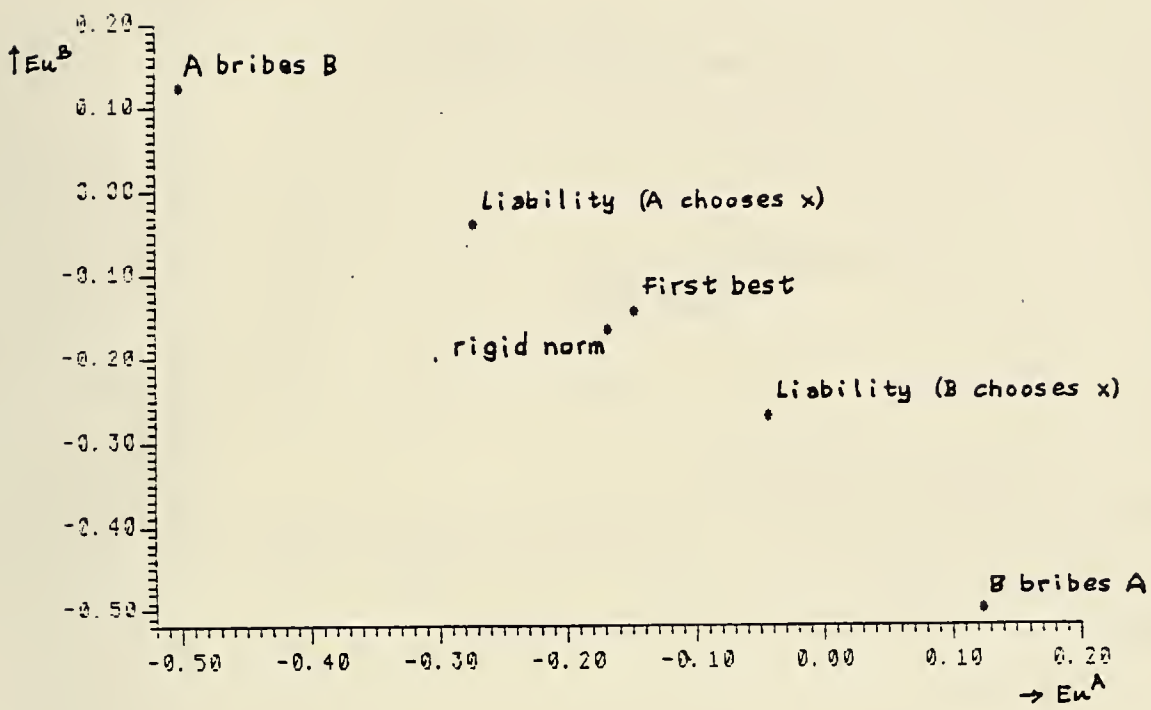
Under the rigid norm,  $x = \alpha Ea + \beta Eb$ , so that:

$$Eu^A = -\alpha\beta^2 C^2 - \alpha \text{var}(a) \quad (7.15)$$

$$Eu^B = -\alpha^2\beta C^2 - \beta \text{var}(b) \quad (7.16)$$

In Figure 3, we have chosen various values for the parameters  $C$ ,  $\alpha$ , and  $r_b$  (normalizing  $r_a$  to be 1), and we have shown  $Eu^A$  and  $Eu^B$  under the various rules considered: rigid norm, rights to A, rights to B, liability to A, liability to B, and the first best allocation  $x^*(a,b)$ . We see that, for example, the liability rules do not necessarily Pareto dominate the property rules, despite their greater efficiency (Proposition 3), and the first best payoffs do not dominate the property payoffs. In the cases where there is a lot of conflict, the only visible relationship of Pareto comparability is that the first best may dominate the rigid norm; but even this does not always happen.

case 1  
 $r(b) = 1$   $c = 1$   $\alpha = .5$   
 $\alpha = .5$



case 3  
 $r(b) = 4$   $c = 3$   $\alpha = .5$   
 $\alpha = .5$

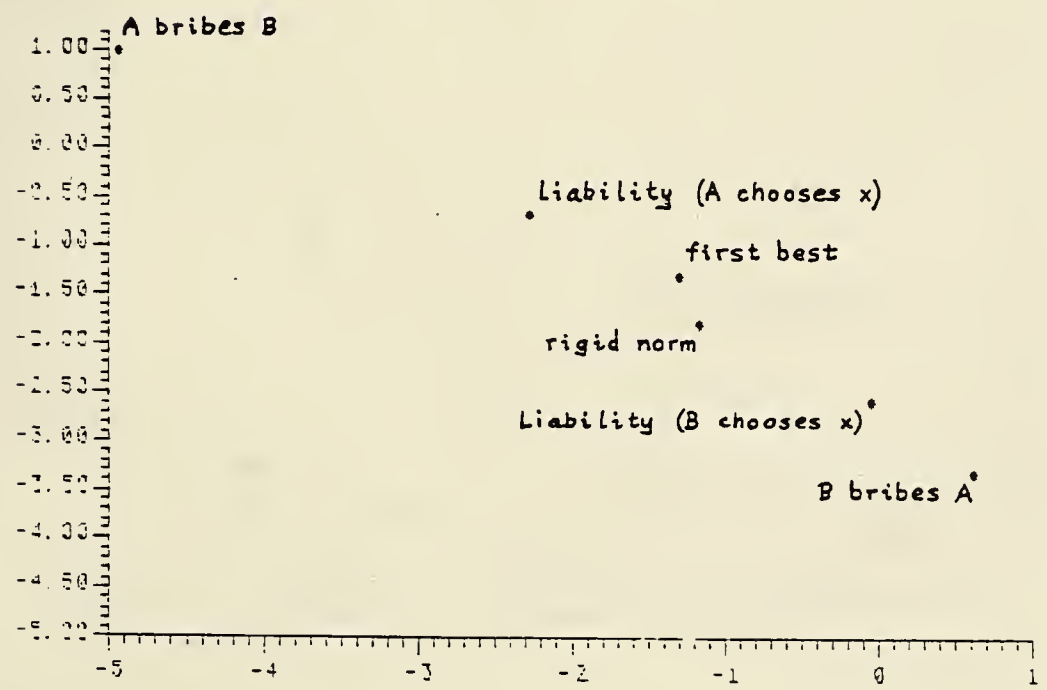
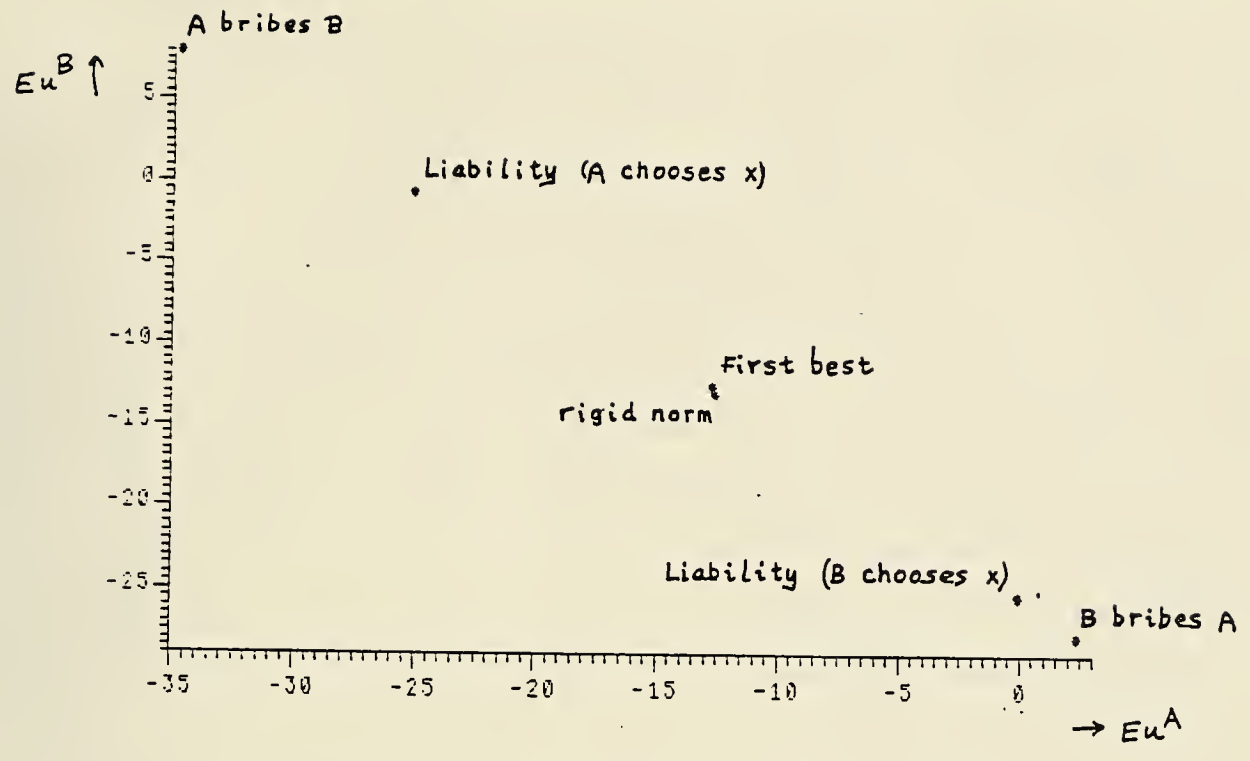


Figure 3

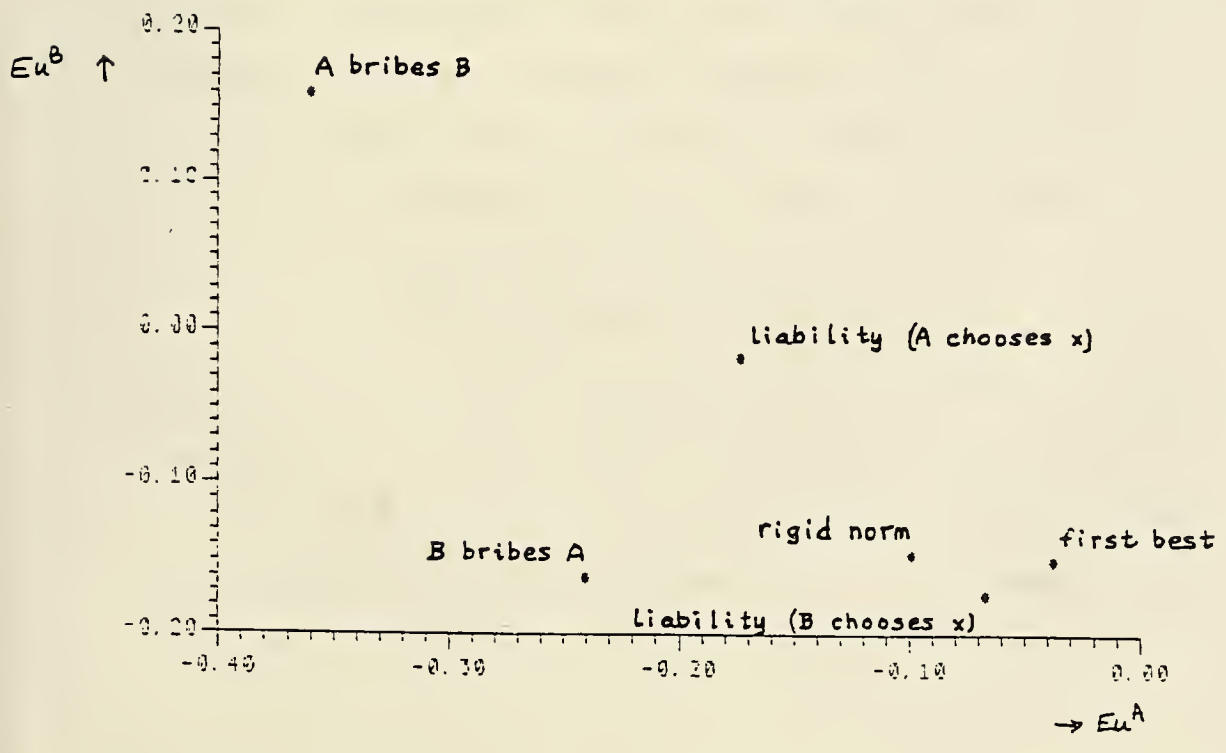




case 4  
 $r(b) = 4$   $c = 10$   $\alpha = .5$



case 5  
 $r(b) = 1$   $c = 1$   $\alpha = .8$   
 $\alpha = .8$





## 8. Related Work

Weitzman (1981) discussed the ex-ante design of a contract between a buyer and a seller. The buyer does not know what his value of the good will be, and the seller does not know his cost function. The two parties have to choose one of three forms of contract: at a fixed price, the quantity may be fixed, or may be chosen ex-post by the seller, or may be chosen ex-post by the buyer. Weitzman shows how the efficient form of contract depends on the variance and covariance of the uncertain parameters, and on the slopes of the supply (marginal cost) and demand (marginal value) curves. If we think of our  $x$  as his "quantity", this would correspond to the agents jointly assigning rights and choosing a linear bribe-schedule ex-ante. This relates to our discussion of liability rules (Section 4).

Grossman and Hart (1984) considered the design of contracts when the parties know that there are events that (for some reason) they cannot explicitly contract on. They show how the efficient allocation of "ownership", by which they mean control over the outcome in these events, depends on the payoff functions. They emphasise the possibility of efficient renegotiation ex-post, so that private information is not important. Inefficiencies result from the fact that the owner gets a relatively high payoff in the default or threat outcome in the background of the ex-post renegotiations. This prospect can adversely affect incentives to invest in the relationship ex-ante.

Holmström (1984) discussed delegation as a solution to the principal-agent problem. He showed that, if the agent has relevant information and if the principal's and the agent's preferences over decisions in different states of the world are sufficiently alike, then it may be useful simply to delegate the decision. He considered restricting

the agent's freedom of choice, and showed that that is generally helpful. Our analysis in Sections 5 and 6 can be thought of as a delegation problem: society has to delegate the choice of  $x$  to an agent, with more or less freedom. In our context, the agent's relevant information is his own preferences, but that is not necessary.

D'Aspremont and Gerard-Varet (1979) considered public decisions with incomplete information. They showed that, in problems like ours, the first-best outcome is attainable using a mediator. Equivalently, if the agent (say, B) who does not have a right to choose  $x$  is constrained to choose one of a carefully selected menu of bribe schedules, then he will choose one, say  $P(.;b)$ , which will induce A to choose  $x = x(P(.;b); a) = x^*(a, b)$ . We do not consider such schemes because we are more interested in evaluating commonly used rules than in characterising the optimum. Also, the assumption that B is constrained in his choice of bribe schedule is often implausible in the absence of a mediator - and a mediator is often absent. In this work, as in Farrell, 1985, I am concerned with natural mechanisms without a mediator.

## 9. Conclusions

In conflicts, there is no assurance that an arbitrary allocation of property rights will yield efficient outcomes. Negotiation is not perfectly efficient with incomplete information; and that is an essential part of the problem of conflict (otherwise society could easily enforce the efficient outcome without decentralization), negotiation is not perfectly efficient. Using a simple single-offer bribe model, we showed how the allocation of rights matters for efficiency. In a symmetric conflict, it is best to enforce a rigid norm and not allocate rights at all. If rights are to be assigned, it should (in our parametrization) be to the party who has less at stake and whose preferences are more predictable.

Rather than allocating rights, it may be better to have a court set a schedule of damage payments. This can be better even though the court knows nothing that is not common knowledge to the two parties, and even though the only source of inefficiency in bargaining is incomplete information.

For a conflict without side payments, we showed that if a decision is especially important to one person, then it is more efficient to give him the right to take the decision than to give it to the other person, or to enforce a rigid norm. In this sense, the Pareto principle is entirely consistent with rights, even in a world in which the Coaseian argument, based on side payments, does not apply. In symmetric conflicts, however, especially if there is a great deal of conflict and little private information, a rigid norm is more efficient. For efficiency, it is sometimes but not always desirable to allocate rights. We also showed that, when a rights system is used, it always pays to restrict the rights at least somewhat; and we characterised the extent to which this is desirable.

In this paper, I have shown, first, that the Coase theorem does not extend to a world with private information. Secondly, I have argued that one view of rights is consistent with the Pareto principle when there are externalities. Some decisions should be allocated to particular people simply because they care the most or have the most information about them. Finally, I have clarified the conditions under which allocation of rights will produce more efficient outcomes than some feasible alternatives. From a utilitarian viewpoint, rights are sometimes a good institution, but they are by no means the best feasible solution to all conflicts.



Footnotes.

1. See for instance Peter Cramton (1984).

2. See Amartya Sen, (1970).

3. We do not mean to claim that all rights are decentralization devices for utilitarian ethics. None the less, it is plausible that many are. One test of this view against a strict entitlement view is whether rights change as the importance of externalities changes (over time, as the result of changing population density, new technology, etc.). We see many cases in which rights do indeed become more or less restricted, or actually change hands.

4. See (e.g.) Partha Dasgupta, Peter Hammond and Eric Maskin (1979).

5. There is also an "individual rationality" constraint, since A is not obliged to accept B's offer at all. We require that, for all a,  

$$-\alpha (x - a)^2 + p(a) \geq 0.$$

In this model, the effect of this is merely to determine the level of the bribe schedule; since it is the slope of the bribe schedule that determines x, we ignore this in the text.

6. Readers familiar with the income-tax literature will recognise this result.

7. This too is familiar from the income-tax literature.

8. In a previous version of this paper I assumed that B offers a linear bribe schedule  $P(x) = p(x - Ea)$  (which A is obliged to accept). The variables a and b are independent but can have any distributions with finite variances. In that formulation, when  $\alpha = \beta$ , if there is enough conflict on average ( $Eb - Ea$  large) then the rigid norm is the best policy. When there is less conflict, rights should be assigned to the party with the more predictable preferences. If  $\alpha \gg \beta$ , then it is better to give rights to A. See Farrell (1984).

9. It is sometimes suggested that courts do not estimate damages, but rather award only objectively verifiable damages, and thus systematically under-award. In the present model that is represented as making A pay a  $b_{\min}$ -type's damages, so that A will set

$$\begin{aligned} x &= x^*(a, b_{\min}) \\ &= \alpha a + \beta b_{\min} \end{aligned}$$

so that

$$\begin{aligned} x^* - x &= \beta (b - b_{\min}) \\ L &= \beta^2 E(b - b_{\min})^2 \\ &= \beta^2 r_b^2 / 3 \quad \text{if } b \text{ is uniformly distributed.} \end{aligned}$$

This is the same as the loss (3.12) from giving B a property right in x. This result will not generalise: although the present derivation is

general up to the last line, the derivation of (3.12) is not.

10. In this model, this is the most efficient award scheme for the court to use, given its ignorance of B's private information  $b$ . To see this, note that A's decision cannot take  $b$  into account, so that the choice of  $x$  enforced by the liability rule can depend only on  $a$ . Given that, the choice  $x = x^*(a, E_b)$  is optimal for each  $a$ .

11. This assumes that A is liable for all variations in payoff to the hypothetical  $E(b)$ -type B. If not, for instance if A's payment to B would be zero for a range of  $x$ -values, then A's calculation (and ours) is much more involved.

$$12. \quad x^*(a,b) - x(a,b) = \beta (b - E_b),$$

whence

$$L = E(x^* - x)^2 = \beta^2 \text{var}(b).$$

13. Polinsky, having reported this argument, goes on to discuss issues of distribution. It is not entirely clear whether he believes the argument, distribution aside.

14. The clearest cases are when the parties literally cannot communicate (drivers approaching an intersection, for instance). In other cases, there may be important difficulties and imperfections in writing and enforcing side-payment contracts. For example, suppose "A" represents a large and nebulous collection of people, say those harmed by a certain source of pollution controlled by B.

15. It may or may not be better still to give some rights to B.

16. This may seem puzzling: the range of  $a$  can be expanded arbitrarily by spreading a very small amount of probability mass. Surely such a spread ought not to change the efficient rule by much; but Proposition 7 seems to suggest that it should. The resolution is this. If we expand the range of  $a$  by spreading only a little probability mass, then indeed it becomes good to give A some limited rights. However, if very little of the distribution of  $a$  is above  $x^n$ , then  $x^r$  will be very close to  $x^n$ . Thus the efficient rule will insist on a minimum standard that is almost  $x^n$ , but allow for A to exceed  $x^n$  when he wishes to do so.

17. For example, suppose that  $\alpha \geq \beta$ , so that absolute rights should be assigned to A rather than to B. Suppose however that  $x^n$  is in the range of  $b$  (and hence of course not in the range of  $a$ ). Then the best limited-rights system is to assign limited rights to B.

18. What about limiting rights when there are side payments? There are two possible sources of efficiency gain from doing this. First, given the bribe schedule, it may be desirable to prevent A from choosing unreasonably low values of  $x$ . Secondly, prohibiting low values of  $x$  will change B's incentives in designing his bribe schedule.

Suppose for instance that we insist that  $x$  be no smaller than  $x^*(a_{\min}, b_{\min})$ . Then certainly we get an efficiency gain given the bribe schedule: either the limitation makes no difference, or else it



creates an improvement. What will the knowledge of the limitation do to B's incentives? In the model of Section 3, since B's calculation is entirely from the top down, the answer is nothing. In general, perhaps one might expect that not having to worry about preventing very low choices of  $x$  might lead B to be less inclined inefficiently to shade his bribe schedule. However, I have not analysed this question.

19. First, we calculate that

$$x - a = (2\alpha - 1)(a - a_{\min}) + \beta(b - Eb) + \beta C + (\beta/2 - \alpha)r_a$$

Then we can integrate the differential equation (7.5), and get

$$u = \alpha(2\alpha - 1)(a - a_{\min})^2 + (\beta(b - Eb) + \beta C + (\beta/2 - \alpha)r_a)(a - a_{\min})$$

Taking expectations and regrouping terms gives (7.6).

20. Expected welfare in the first-best is

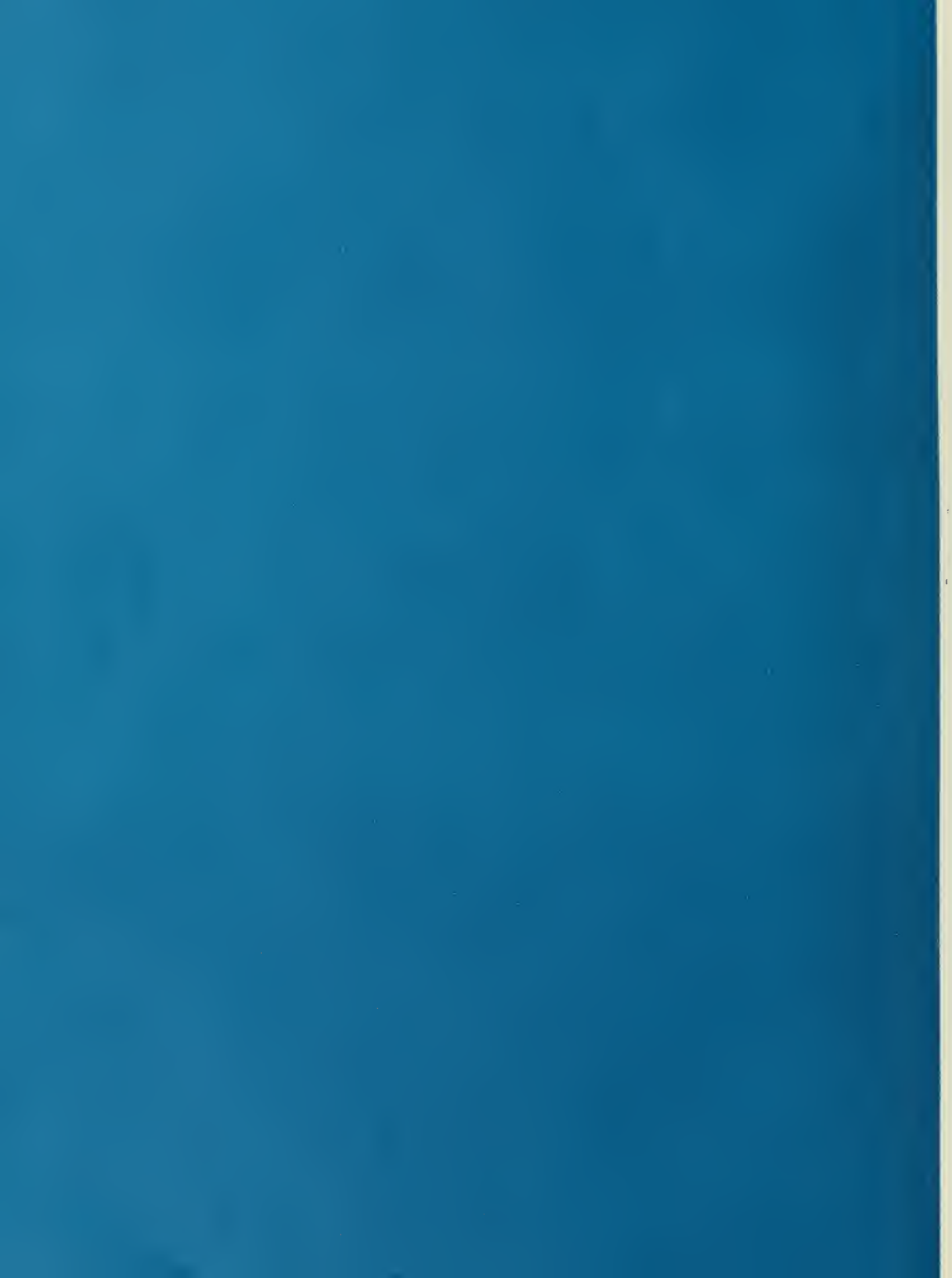
$$- \alpha\beta C^2 - \alpha \text{var}(a) - \beta \text{var}(b)$$

Substitute in for the variance terms, and then subtract  $L(R^A)$  as given in (3.11) to get an expression for expected welfare when A has the right to choose  $x$ . Now subtract (7.6) from that to get B's expected payoff.

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