ALTERNATIVE POLICY RANKINGS IN A LARGE, OPEN ECONOMY WITH SECTOR-SPECIFIC MINIMUM WAGES

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The analysis by international trade theorists of factor market imperfections, and alternative policy rankings in the presence thereof, has distinguished between two major, polar types: (i) a distortionary wage differential between two sectors, while the wage is perfectly flexible in each sector; and (ii) a sticky (or minimum) wage which is however equal between the two sectors.

The analysis of the former class of distortions was pioneered by Everett Hagen [1958] and has subsequently been completed by Bhagwati-Ramaswami [1963], Kemp-Negishi [1969], and Bhagwati-Ramaswami-Srinivasan [1969].

The analysis of the second class of distortions was pioneered by Gottfried Haberler [1950] and has subsequently been completed for the traditional two-sector model of trade theory, by Brecher [1973].

The purpose of this paper is to analyze policy rankings in the presence of a yet different type of factor market imperfection, introduced in a pioneering paper by Harris and Todaro [1970] which combines specificity of wages (in one sector) with a resulting wage differential between the two sectors in an ingenious manner. In earlier papers [1973a] [1973b], we have analyzed the Harris-Todaro model, for this range of issues, in the context of a closed economy or a "small", open economy with given terms of trade. In this paper, we analyze definitively alternative policy rankings in the context of fully general assumption of a "large" country, which has monopoly power in trade.

Section I outlines the model. Section II briefly outlines the principal results of our analysis. Section III analyzes the policy instrument
defined by a wage subsidy in the sector with minimum wages. Section IV
discusses the policy instrument defined by a production tax-cum-subsidy.
Section V analyzes the policy instrument defined by a consumption
tax-cum-subsidy. Finally, Section VI discusses a tariff policy.

I. THE MODEL

The basic Harris-Todaro model consists of a set of relations
which can be stated as follows.

There are two commodities (A and M), produced in quantities
$X_A$ and $X_M$, using $L_A$ and $L_M$ units of labour, with strictly concave
production functions:

$$X_A \leq f_A(L_A)$$  (1)

$$X_M \leq f_M(L_M)$$  (2)

Next, with the fixed, overall labour supply assumed by choice of
units to equal unity, we have:

$$L_A + L_M \leq 1$$  (3)

$$L_A, L_M \geq 0$$  (4)

We now introduce foreign trade. Let $E$ denote net imports of
the agricultural good, exchanging for $g(E)$ of net imports of manufacturing.\(^2\)
We further assume that $g(0) = 0$, $g' > 0$, $g'' < 0$. This implies that the
marginal ($g'$) and average ($\frac{g}{E}$) terms of trade decline as $E$ increases and

\(^1\)Thus, implicitly, there is a second factor ($K_A$, $K_M$) which yields
the diminishing returns to labour input.

\(^2\)Since we do not wish to prejudge the question as to which commodity
will be imported, $E$ is allowed to take on negative values as well, in
which case $g(E)$ will also be negative. In such a case, agricultural goods
will be imported and manufactured goods will be imported.
the marginal is less than the average. The domestic consumption of the two commodities will then be:

\[ C_A = X_A - E \]  \hspace{1cm} (5)

\[ C_M = X_M + g(E) \]  \hspace{1cm} (6)

It is well known that if we now add a standard utility function:

\[ U = U[C_A, C_M] \]  \hspace{1cm} (7)

where \( U \) is concave with positive marginal utilities for finite \([C_A, C_M]\), free trade will be characterized by:

\[ \frac{U_1}{U_2} = \frac{f'_M}{f'_A} \]  \hspace{1cm} (8)

\[ \frac{U_1}{U_2} = \frac{g(E)}{E} \]  \hspace{1cm} (9)

together with (1) - (6) being satisfied (where \( U_1 \) and \( U_2 \) represent the partial derivatives of \( U \) with respect to \( C_A \) and \( C_M \) respectively and \( f'_i \) is the derivation of \( f_i \) with respect to its argument, \( i = A, M \)).

Figure 1 shows the production possibility curve BJ and the foreign offer curve PDC superimposed on it at P a la Baldwin. At the production point P, the price ratio faced by producers (i.e. the negative of the slope of PC) is the same as the marginal rate of substitution in production as represented by the (negative of the) slope of the tangent to the production possibility curve at P. At the consumption point C, this price ratio equals the marginal rate of substitution in consumption as represented by the (negative of the) slope of the indifference curve at C.

\[ ^3 \text{We rule out corner solutions by assuming } f'_A(0) = f'_M(0) = \infty. \]
The production and consumption points are so located that the price ratio equals the external terms of trade. The curve PDC corresponds to the graph of $g(E)$. 

---

FIGURE 1

---
The Harris-Todaro problem of sector-specific rigid wages and resulting unemployment can now be readily introduced. Let the free trade solution above [at P and C in Figure 1] be:

\[ X^*, X_M^*, L_A^*, L_M^* \ (= 1-L_A^*), E^*, C_A^*, C_M^* \]. Assume now, however that there is an exogenously specified, minimum wage constraint in manufacturing, such that:

\[ w > \frac{w}{w} \]  

(10)

where \( w \) is the wage in manufacturing, in units of the manufacturing good (M).

For a competitive economy, this implies that

\[ f_M'(L_M^*) > \frac{w}{w} \]  

(10')

This constraint becomes binding, and P in Figure 1 is inadmissible, when:

\[ f_M'(I_M^*) < \frac{w}{w} \]

The competitive economy, when characterised by this wage constraint, will then experience unemployment of labour. We then have two options to characterise the labour market equilibrium in this situation: either assume that the wage in agriculture (A) will be equalised with the wage in manufacturing (M) despite the unemployment; or that the wage in agriculture will be equalised with the expected wage in manufacturing, the expected and the actual wage in manufacturing being different as the former would be defined as the latter weighted by the rate of employment

\[ \frac{L_M}{w} \frac{1-L_A}{1-L_A} \]

where \( L_M < (1-L_A) \) when there is unemployment.

The analysis of Harris-Todaro is based on the latter assumption, so that we can then write the equilibrium production conditions in competition and *laissez faire*, as follows:
\[ f'_M = \bar{w} \]  
(11)

\[ \frac{U_1}{U_2} f'_A = \bar{w} \frac{L_M}{1-L_A} \]  
(12)

\[ \frac{U_1}{U_2} = \frac{g(E)}{E} \]  
(13)

where we assume, in (12), that the production and consumption prices for the agricultural good are the same. Given \( \bar{w} \), we can solve (11), (12) and (13) for \( L_M \), \( L_A \), and \( E \) after setting \( X_A = f_A(L_A) \), \( X_M = f_M(L_M) \), \( C_A = f_A(L_A) - E \) and \( C_M = f_M(L_M) + g(E) \). The equilibrium production point corresponding to this situation of non-intervention, with unemployment will then lie, in Figure 1, along RK (where \( X_M \) and hence \( L_M \)
are fixed at the value that makes $f'_M = \bar{w}$ at $Q$. The consumption point will be at $F$.

The policy question that emerges then is: what alternative policies can be used in this model for intervention and what would be their impact on welfare and on unemployment?

II. THE BASIC RESULTS

In this model, there are a number of policy options which can be explored; however, many can be shown to be equivalent to one another or to combinations of other policies.

Thus, we will discuss the following policies:

(i) Non-intervention or laissez faire;
(ii) Wage subsidy in manufacturing (M);
(iii) Production subsidy to agriculture (A); and
(iv) Consumption subsidy to agriculture (A).

Note that, as a little reflection will show, the simple structure of the model implies that:

(v) A wage subsidy in agriculture is equivalent to policy (iii);
(vi) A uniform wage tax-cum-subsidy in all employment is a combination of policies (ii) and (iii); and
(vii) A tariff policy is a combination of policies (iii) and (iv).

We will eventually proceed to establish the following proposition, which we now illustrate with the usual diagrammatic techniques.

THEOREM 1: There exists a unique equilibrium corresponding to each wage subsidy $s$ to manufacturing in an interval $[0, \bar{s}]$. At $\bar{s}$, full employment is reached.

---

5 It is worth noting that the non-intervention equilibrium would lie along RK even if we assumed actual wages to be equalized between the two sectors.
Thus, imagine in Figure 2 that a laissez-faire production equilibrium is at Q as in Figure 1. Succeeding levels of wage subsidies should map out a locus of resulting production equilibria with H representing the full employment equilibrium, the wage subsidy that leads to it being $\bar{w}$. Consumption equilibrium is at C; the consumption, production and the international price-ratio are identically equal to HC; HDC is the foreign offer curve; and $W_H$ is the welfare level under this policy.

![Diagram](image)

**FIGURE 2**

---

6 That increasing wage subsidies in manufacturing should increase $L_M$ and hence, $X_M$ is obvious; however $L_A$ (and hence $X_A$) may either decrease or increase.
THEOREM II: A wage subsidy (in manufacturing) will exist which will improve welfare over laissez faire.

Thus, laissez faire (i.e. wage subsidy = 0) can be necessarily improved upon by some wage subsidy. Thus, in Figure 3, \((Q_o, C_o), (Q_1, C_1)\) and \((Q_H, C_H)\) represent the production \((Q)\) and consumption \((C)\) associated respectively with laissez-faire (0), a positive wage subsidy \((1)\) and the full-employment wage-subsidy \((H)\). (For visual clarity, the foreign offer curve \(g(E)\) is not shown in the Figure.) The welfare level \(U^1\) attained with the positive subsidy is greater than \(U^0\) attained in laissez-faire. In fact, any positive subsidy in some interval with zero as its left end point will be welfare-improving.

THEOREM III: The full-employment wage subsidy \(s\) may not be the "second-best" wage subsidy and may be inferior even to laissez-faire.

In Figure 3, the welfare level \(U^H\) associated with \(s\) is lower than \(U^0\), the laissez-faire welfare level; and, a fortiori, it is lower than the welfare attainable at a "second-best" wage subsidy. In Figure 4, on the other hand, \(U^H\) exceeds \(U^0\) as well as the welfare levels (not shown in the Figure) associated with any subsidy between 0 and \(s\). Thus, in this case, \(s\) (the full-employment subsidy) is also the "second-best" optimum subsidy.

THEOREM IV: There exists a unique production subsidy which will enable full employment to be reached and which is also the "second best" production subsidy.

Figure 4 illustrates the full employment production subsidy as the difference between the absolute values of the slopes of \(Q_p C_p\) (which is the price-ratio in consumption \(= \frac{U^1}{U^2}\) and \(Q_p Q'_p\) (i.e. the
tangent to the production possibility curve at $Q_p$, defining the producer's price "ratio" or alternatively the domestic rate of transformation). It is clear that the welfare level $U^P$ associated with consumption $C_p$ corresponding to production at $Q_p$ attainable with the full-employment production subsidy exceeds the laissez-faire level $U^0$. In fact, it exceeds the welfare levels (not shown in the figure) associated with the production subsidies between zero and the full employment subsidy. Thus, the full employment subsidy is the "second-best" optimum subsidy.

THEOREM V: The "second-best" wage subsidy (to manufacturing) and production subsidy (to agriculture) cannot be ranked uniquely.

In figure 4 the welfare level $U^H$ achieved with the "second-best" wage subsidy (which happens there to equal the full employment wage-subsidy) exceeds $U^P$, the welfare level associated with the "second-best" production subsidy. On the other hand, in Figure 5, $U^P$ (the welfare level associated with the "second-best" production subsidy) is greater than $U^1$.

THEOREM VI: There exists a unique consumption subsidy which will enable full employment to be reached and which is also the "second best" consumption subsidy.

In Figure 4 the production point associated with the full-employment consumption subsidy is $Q_p$, (which also is the production point associated with the full employment production subsidy). The consumption point is $C_c$ where $Q_pQ_p'$, the tangent to the production possibility curve, intersects the offer curve drawn through $Q_p$. The consumption subsidy is the difference between the absolute value of the slope of $Q_pQ_p'$ and that of the indifference curve through $C_c$. 
It is seen from Figure 4 that the welfare attained at $C_C$ exceeds the laissez-faire level $U^0$. Indeed, it exceeds the welfare level associated with any consumption subsidy between 0 and the full-employment level.

**THEOREM VII:** The "second best" wage subsidy (to manufacturing) and the "second best" consumption subsidy (to agriculture) cannot be ranked uniquely.

In Figure 4, the welfare level $U^H$ associated with the "second-best" wage subsidy exceeds that at $C_C$ associated with the "second best" consumption subsidy. In Figure 5, on the other hand, the welfare level $U^L$ at the "second best" wage subsidy is less than that at $C_C$.

**THEOREM VIII:** The "second best" production and consumption subsidies cannot be ranked uniquely.

As noted earlier, the production points associated with "second best" production and consumption subsidies to agriculture are identical: namely, $Q_p$ in Figure 4 and 5. However, the two consumption points $E_p$ and $E_C$ are different. In Figure 4, $C_p$, the consumption point associated with the "second-best" production subsidy yields higher welfare than $C_C$. In Figure 5, on the other hand, $C_U$ yields higher welfare than $C_p$.

**THEOREM IX:** A tariff (or trade subsidy) policy may not improve welfare but can improve employment,

The ambiguous result for a tariff (or trade subsidy) policy follows from the fact that a tariff has both a production and a consumption effect; the former is favourable but the latter need not be (as shown in Section VI). On the other hand, employment can always be improved.
FIGURE 3
FIGURE 4
THEOREM X: The first-best optimum can be reached if, in addition to the monopoly-power-in-trade tariff, a combination of a production tax-cum-subsidy and wage subsidy (to manufacturing) or any equivalent thereof (including a uniform wage subsidy on employment of labour in both sectors) is provided.

The combination of a suitable production tax-cum-subsidy plus an appropriate wage subsidy in manufacturing, or its equivalents such as a uniform wage subsidy in all employment, will yield the first-best optimum, when also combined with an appropriate tariff to exploit the postulated monopoly power in trade (as discussed in Section VII).

III. WAGE SUBSIDY IN MANUFACTURING

Let us now consider the wage subsidy as the policy intervention in this economy. Denoting by \( s \) the subsidy per unit of labour employed in manufacturing, we find that the equilibrium is now characterized by:

\[
\begin{align*}
\frac{U_1}{U_2} f'_M &= \bar{w} - s \quad (14) \\
\frac{U_1}{U_2} f'_A &= \bar{w} \frac{L_M}{1 - L_A} \quad (15) \\
\frac{U_1}{U_2} &= \frac{g(E)}{E} \quad (16)
\end{align*}
\]

Equation (14) assumes that each worker in manufacturing receives remuneration \( \bar{w} \), of which only \( \bar{w} - s \) is paid by the employer and \( s \) by the state out of some form of non-distortionary taxation. With the consumer and producer price of the agricultural good assumed to be identical, and equal to \( \frac{U_1}{U_2} \), we then have the actual wage in agriculture being
equated to the employment-rate-weighted (i.e. expected) wage in manufacturing in equation (15).

Existence of Unique Laissez-faire Equilibrium with Unemployment:

Given $\bar{w}$, $s$, concavity of $f_M$ and the assumption that $f'_M \neq \infty$ as $L_M \to 0$, equation (14) uniquely determines $L_M$ provided $0 \leq s \leq \bar{w}$. Equations (15) and (16) together determine $L_A$ and $E$. We now proceed to show that, in fact, the solution for $L_A$ and $E$ are unique and satisfy the constraints: $L_A + L_M \leq 1$ as well as the non-negativity constraints on $L_A$ and the two consumption levels $C_A$, $C_M$.

With the average terms of trade $\frac{g(E)}{E}$ denoted by $\phi(E)$, our assumptions on $g$ imply that $\phi > 0$ and $\phi' < 0$ for all $E$. Substituting for $U_1$ from (16) in (15), we get:

$$
\phi(E) f'_A = \bar{w} \frac{L_M}{1-L_A} \quad (15')
$$

Let us denote by $L_M(s)$, the unique value of $L_M$ satisfying (14). Then the non-negativity constraint and the constraint that total employment does not exceed available labour, restricts the range of feasible values of $L_A$ to the interval $[0, 1-L_M(s)]$. For any feasible $L_A$, the non-negativity constraint on $C_A$ requires that $E$ should not exceed $f_A(L_A)$. The non-negativity constraint on $C_M$ requires that $E$ should equal or exceed $g^{-1}(-f_M(L_M(s)))$ where $g^{-1}$ is the inverse of function $g$. A unique $g^{-1}$ exists, given that $g$ is monotonically increasing. Thus the feasible values of $E$ are restricted to the interval $[g^{-1}(-f_M(L_M(s))), f_A(L_A)]$.

Now consider (16). The right hand side is a decreasing function of $E$. 

-16-
Next, given \( L_A \), the left-hand side is an increasing function of \( E \) provided \( U_{12} > 0 \) since \( \frac{\partial}{\partial E} \left( \frac{U_1}{U_2} \right) = \frac{(-U_{11} + U_{12} g') U_2 - (-U_{21} + U_{22} g')}{U_2} > 0 \).

As \( E \) approaches its lower limiting value of \( g^{-1}(-f_M(L_M(s))) \), \( C_M \to 0 \) and as it approaches its upper limiting value of \( f_A(L_A) \), \( C_A \to 0 \).

Now if we assume that the marginal utility \( U_1(U_2) \) of agricultural good (manufactured good) tends to \( \infty \) as its consumption \( C_A(C_M) \) tends to zero, the left hand side of (16) increases from zero to \( +\infty \) as \( E \) increases from its lower to upper limiting value and hence, given \( s \), for any feasible \( L_A \) there exists a unique \( E \) denoted by \( E(L_A, s) \) which satisfies (16). This is shown in Figure 6.

![Figure 6](image)
It is easily seen that \( \frac{\partial E(L_A,s)}{\partial L_A} > 0 \). For, given \( s \) (and hence \( X \)) and a feasible \( E(\text{and hence } C) \), \( C \) increases as \( L \) increases resulting in a decrease in \( \frac{U_1}{U_2} \) (given our assumption of \( U_{12} \geq 0 \)).

Thus, as \( L \) increases, the graph of the left hand side of (16) shifts to the right while the graph of the right hand side stays put, resulting in a larger value for the \( E \) at which the two graphs intersect. The reader can readily verify, using a similar argument, that \( \frac{\partial E}{\partial s} < 0 \).

Let us now substitute the function \( E(L_A,s) \) for \( E \) in (15'). Then, for any given \( s \), both sides of (15') are functions of \( L_A \) only. The left hand side of (15') is then a decreasing function of \( L_A \) since

\[
\frac{\partial}{\partial L_A} (\phi f_A') = \phi \frac{\partial E}{\partial L_A} + \phi f_A'' < 0 \quad \text{because } \phi > 0, \phi' < 0, \frac{\partial E}{\partial L_A} > 0 \text{ and } f_A'' < 0.
\]

The right hand side is an increasing function of \( L_A \). Further, as \( L_A \to \infty \), the left hand side (i.e. \( \phi f_A' \)) also \( \to \infty \), while the right hand side takes the value \( wL_M(s) \).

Consider \( s = 0 \). Then \( L_M(0) \) satisfies \( f_A' = \bar{w} \). By assumption, \( L_M^* \) (the laissez-faire without the minimum wage constraint value of \( L_M \)) results in \( f_A' < \bar{w} \) and hence \( L_M^* > L_M(0) \).

This means that \( L_A^* = 1-L_M^* < 1-L_M(0) \). Thus if we set \( L_A = 1-L_M(0) \), the following hold true:

\[
(1) \quad f_A'(1-L_M(0)) < f_A'(L_A^*); \quad (11) \quad E\{1-L_M(0),0\} > E\{1-L_M^*,0\};
\]

\[
(11) \quad \phi[E\{1-L_M(0),0\}] < \phi[E\{1-L_M^*,0\}] \quad \text{Thus } \phi f_A' \text{ (left hand side of (15') evaluated at the largest feasible value of } L_A (\text{given } s = 0), \text{ i.e. at } 1-L_M(0), \text{ is less than its value evaluated at } L_A = 1-L_M^* \). \text{ But at}
\]
Thus we have established the existence of a unique laissez-faire equilibrium with unemployment.

Existence of Unique Equilibrium for each value of s in \([0, \bar{s})

Now as s is increased, for any given \(L_A\) the left hand side of

\[(15')\] increases since \(\frac{\partial}{\partial s} (\phi_f') = \phi'_f \frac{\partial E}{\partial s} > 0\), and hence its graph shifts to
the right. The right hand side also increases since \( L_M(s) \) increases with \( s \). Thus, its graph shifts to the left, with its value at \( L_A = 1-L_M(s) \) always equaling \( \bar{w} \). Hence the two graphs continue to intersect at a unique \( L_A \) in the interval \([0, 1-L_M(s)]\) as \( s \) increases up to a maximum value \( \bar{s} \), when this value of \( L_A \) equals its upper bound \( 1-L_M(\bar{s}) \), and full employment is reached. This is shown in Figure 8.

\begin{center}
\includegraphics[width=\textwidth]{figure8.png}
\end{center}

\begin{center}
FIGURE 8
\end{center}

For values of \( s > \bar{s} \), no equilibrium exists. Thus, we have shown the existence of a unique equilibrium for each value of \( s \) in \([0, \bar{s}]\).

**Impact on Welfare of Change in** \( s \):

Let us now evaluate the change in welfare, i.e. \( \frac{du}{ds} \), as \( s \) increases.
After some manipulation, the following can be derived:

\[
\frac{dE}{ds} = \left[ \phi_f'' - \frac{\bar{w}L_M}{(1-L_A)^2} \right] \frac{\partial L_M}{\partial U_2} \left( \frac{U_1}{U_2} \right) + \frac{\bar{w}}{1-L_A} \frac{\partial}{\partial L_A} \left( \frac{U_1}{U_2} \right) \Bigg|_D \tag{17}
\]

\[
\frac{dL_A}{ds} = - \phi_f' \frac{\partial}{\partial L_M} \left( \frac{U_1}{U_2} \right) + \left\{ \phi_f'' - \frac{\bar{w}L_M}{(1-L_A)^2} \right\} \frac{\partial}{\partial E} \left( \frac{U_1}{U_2} \right) \Bigg|_D \tag{18}
\]

\[
\frac{dU}{ds} = U_2 \frac{N}{D} \tag{19}
\]

where

\[
D = \phi_f' \frac{\partial}{\partial L_M} \left( \frac{U_1}{U_2} \right) + \left\{ \phi_f'' - \frac{\bar{w}L_M}{(1-L_A)^2} \right\} \left\{ \phi_f' - \frac{\partial}{\partial E} \left( \frac{U_1}{U_2} \right) \right\} \tag{20}
\]

\[
N = \left[ -\phi \left( \phi_f'' - \frac{\bar{w}L_M}{(1-L_A)^2} \right) - \phi_f' \left( f_f' \right)^2 - \frac{\bar{w}f_f'g_f'}{L_M} \right] \frac{\partial}{\partial L_M} \left( \frac{U_1}{U_2} \right) + \left[ \frac{g_f'}{(1-L_A)^2} + \frac{f_f'}{f_f'} \left( \phi_f'' - \frac{\bar{w}L_M}{(1-L_A)^2} \right) \right] \frac{\partial}{\partial L_A} \left( \frac{U_1}{U_2} \right) \tag{21}
\]

Now, if we assume \( U_{12} > 0 \) then:

\[
\frac{\partial}{\partial L_M} \left( \frac{U_1}{U_2} \right) > 0, \ \frac{\partial}{\partial L_A} \left( \frac{U_1}{U_2} \right) < 0, \ \frac{\partial}{\partial E} \left( \frac{U_1}{U_2} \right) = - \frac{1}{f_f'} \frac{\partial}{\partial L_A} \left( \frac{U_1}{U_2} \right) + \frac{g_f'}{f_f'} \frac{\partial}{\partial L_M} \left( \frac{U_1}{U_2} \right) > 0.
\]
Further, $\phi > 0$, $\phi' > 0$, $f'_A > 0$, $f''_A < 0$, $f'_M > 0$ and $U_2 > 0$. Hence $D > 0$. It is seen that $\frac{dE}{ds} < 0$ i.e. the next export of the agricultural commodity decreases as the wage subsidy to manufacturing increases. However, the signs of $\frac{dL_A}{ds}$ and $\frac{dU}{ds}$ are in general indeterminate. But, using the fact that the marginal terms of trade $g'$ is by assumption less than the average terms of trade $\phi$, we can show that

$$N > \left[ -\phi^2 f''_A - \phi' f'_A - \frac{s \phi w L_M}{(1-L_A)^2 f'_M} \right] \frac{\partial}{\partial L_M} \left( \frac{U_1}{U_2} \right)$$

$$+ \left[ \frac{\phi f'_M f''_A}{f'_A} + \frac{s w L_M}{(1-L_A)^2 f'_A} \right] \frac{\partial}{\partial L_A} \left( \frac{U_1}{U_2} \right) + \phi' f'_M f''_A + \frac{s w L_M \phi'}{(1-L_A)^2}$$

In the above inequality, all terms involving $s$ explicitly are negative and the rest are positive. When $s = 0$, the terms involving $s$ drop out making $N$ and hence $\frac{dU}{ds} > 0$ at $s = 0$. This means that welfare can be increased over its laissez-faire level by giving any positive wage subsidy in an interval. It is also clear that the full employment wage subsidy need not be the second-best optimum subsidy.

IV. PRODUCTION SUBSIDY

We now consider the policy of subsidising production in agriculture. To do this, we rewrite the critical equilibrium conditions as follows:

$$f'_M = \bar{w} \quad (22)$$

$$\pi p f'_A = \frac{\bar{w} L_M}{1-L_A} \quad (23)$$

$$\frac{U_1}{U_2} = \phi(E) \quad (24)$$
where $\pi_p$ is the producer's price of the agricultural good, the production subsidy being $\frac{\pi_p - \phi}{\phi}$ per unit.

Now (22) determines $L_M$ uniquely as $L_M(0)$ (its *laissez-faire* value). The feasible values of $L_A$ then lie in the interval $[1, 1-L_M(0)]$.

Equation (24) is the same as (16) when $s = 0$ and hence, for any feasible $L_A$, there exists a unique $E(L_A, 0)$ which satisfies (24) and clearly $\frac{\partial E}{\partial L_A} > 0$. Now the left hand side of (23) is a decreasing function of $L_A$ (for any given $\pi_p$) and the right hand side is an increasing function of $L_A$. We have already seen that when $\pi_p$ is at its *laissez-faire* value, the graphs of the two sides intersect at a unique $L_A(0)$ such that $D < L_A(0) < 1-L_M(0)$. Now as we increase $\pi_p$ continuously above its laissez-faire value, thus increasing the rate of production subsidy, the graph of the left hand side of (23) shifts to the right and continues to intersect the right hand side (which does not shift) at a feasible value of $L_A$ until $\pi_p$ reaches a value $\pi_p^*$ at which the intersection occurs at $L_A = 1-L_M(0)$. At this point, full employment is attained; and for values of $\pi_p > \pi_p^*$ no equilibrium exists. This is illustrated in Figure 9.
It is also clear that as $\pi$ increases, $L_A$ increases and hence $X_A$ increases, i.e. $\frac{dL_A}{d\pi_p} > 0$ and $\frac{dx_A}{d\pi_p} = f_A\frac{dL_A}{d\pi_p} > 0$. It can thus be shown that:

$$\frac{dU}{d\pi_p} = \left[ \frac{\phi f_A'}{f'_M} \frac{\partial}{\partial L_M} \left( \frac{U_1}{U_2} \right) - g' \frac{\partial}{\partial L_A} \left( \frac{U_1}{U_2} \right) \right] \frac{dL_A}{d\pi_p}$$

$$\frac{1}{f_A} \frac{\partial}{\partial L_A} \left( \frac{U_1}{U_2} \right) + \frac{g'}{f'_M} \frac{\partial}{\partial L_M} \left( \frac{U_1}{U_2} \right) - \phi' > 0 \quad (25)$$
Hence, clearly the second best optimum production subsidy is the full employment subsidy (which is the maximum, feasible subsidy).

V. CONSUMPTION SUBSIDY

We now consider the policy of subsidising the consumption of agricultural goods. To do this, we must rewrite the equilibrium conditions as follows:

\[ f'_M = \frac{-w}{w} \]  
\[ \phi(E) f'_A = \frac{w L_M}{1-L_A} \]  
\[ \pi_c = \frac{U_1}{U_2} \]

where \( \pi_c \) is the consumer's price of agricultural good, the consumption subsidy being \( \frac{\phi(E) - \pi_c}{\pi_c} \) per unit.

Now consider (28). For any given \( L_A \), the right hand side is an increasing function of \( E \). Further, as \( E \) tends to its lower limiting value of \( g^{-1} \{ -f_M (L_M(0)) \} \), \( \frac{U_1}{U_2} \) tends to zero; and as \( E \) tends to its upper limiting value of \( f_A(L_A) \), \( \frac{U_1}{U_2} \rightarrow \infty \). Hence, for any positive \( \pi_c \), there exists a unique \( E \) denoted by \( E(L_A, \pi_c) \) that satisfies (28). It is also clear that

\[ \frac{\partial E(L_A, \pi_c)}{\partial L_A} > 0 \quad \text{and} \quad \frac{\partial E(L_A, \pi_c)}{\partial \pi_c} > 0. \]

Substituting \( E(L_A, \pi_c) \) for \( E \) in (27), we then find, for given \( \pi_c \), that the left hand side of (27) is a decreasing function of \( L_A \) while the right hand side is an increasing function.
We have already seen (from Section IV when \( \pi_c \) equals its laissez-faire value, the graphs of the two sides of (27) will intersect at a unique \( L_A(0) \), satisfying \( 0 < L_A(0) < 1-L_M(0) \). Furthermore, as we decrease \( \pi_c \), thus increasing the rate of consumption subsidy, the graph of the left hand side will shift to the right, while the graph of the right hand side stays put. Hence, until \( \pi_c \) reaches a value \( \bar{\pi}_c \), the two graphs will intersect at a feasible value of \( L_A \); and at \( \bar{\pi}_c \), they will intersect at \( L_A = 1-L_M(0) \). And, for any lower value of \( \pi_c \), there is no equilibrium. This is shown in Figure 10.

![Figure 10](image-url)
As the graph indicates, $L_A$ equilibrium (and hence $X_A$) increases as $\pi_c$ decreases, i.e. $\frac{dL_A}{d\pi_c} < 0$ and $\frac{dx_A}{d\pi_c} < 0$. It can thus be shown that

$$\frac{dE}{d\pi_c} = \left[ \frac{-gL_M}{(1-L_A)^2} - \phi' \right] \frac{dL_A}{d\pi_c} > 0$$  \hspace{1cm} (29)$$

$$\frac{dU}{d\pi_c} = U_2 \left[ \phi' \frac{dL_A}{d\pi_c} + (g' - \phi) \frac{dE}{d\pi_c} \right] < 0$$ \hspace{1cm} (30)$$

This means that, as $\pi_c$ decreases from its laissez-faire value to its full employment value $\pi_c$, welfare increases. Thus the full employment subsidy is also the second best consumption subsidy.

VI. TRADE TARIFF (SUBSIDY)

Let us now consider a tariff policy. The equilibrium will now be characterized by:

$$f'_M = \bar{w}$$ \hspace{1cm} (31)$$

$$\phi(E)(1+t)f'_A = \frac{-gL_M}{1-L_A}$$ \hspace{1cm} (32)$$

$$\frac{U_1}{U_2} = \phi(E)(1+t)$$ \hspace{1cm} (33)$$

where $t$ is the ad valorem tariff rate. If the agricultural commodity is exported (imported), i.e. $E$ is positive (negative), then $t$ represents an export subsidy (import duty).

As earlier, $L_M$ is uniquely determined at $L_M(0)$ by (31). From the argument of Section III, it follows that for any given $t$ and $L_A$
in the feasible range \( \{0, 1-L_M(0)\} \), there exists a unique feasible \( E(L_A, t) \) that satisfies (33). It is also clear that:

\[
\frac{\partial E}{\partial L_A} = \frac{-\frac{\partial E}{\partial L_A} \left( \frac{U_1}{U_2} \right)}{-\frac{\partial E}{\partial L_A} \left( \frac{U_1}{U_2} \right) - \phi'(1+t)} > 0
\]

\[
\frac{\partial E}{\partial t} = \frac{\phi}{\frac{\partial E}{\partial L_A} \left( \frac{U_1}{U_2} \right) - \phi'(1+t)} > 0
\]

Substituting \( E(L_A, t) \) for \( E \) in (32), we then see that the left hand side is a decreasing function of \( L_A \) while the right hand side is an increasing function of \( L_A \). We know that, when \( t = 0 \), the graphs of the two sides intersect at a unique \( L_A(0) \) in \( \{0, 1-L_M(0)\} \). As we increase \( t \) above zero, the graph of the left hand side shifts to the right while that of the right hand side stays put, so that the two graphs continue to intersect at an \( L_A \) in the feasible range until \( t \) reaches a value \( \bar{t} \) when the intersection occurs at \( L_A = 1-L_M(0) \), thereby attaining full employment. For \( t > \bar{t} \), there is no equilibrium. This is shown in Figure 11.
$w^{(0)}$
$\phi(E)(1+t)f_A'$ when $t = \bar{t}$

$\phi(E)(1+t)f_A'$ when $t = 0$

$\bar{w}_{L_L}(0)$

$1-L_L(0) \rightarrow L_A$

FIGURE 11
Furthermore, as $t$ increases, equilibrium $L_A$ increases as is evident from Figure 11. It can then be shown that:

\[
\frac{dL_A}{dt} = \frac{\phi f' A \frac{\partial}{\partial E} \left( \frac{U_1}{U_2} \right)}{\left\{ \phi'(1+t) \frac{\partial}{\partial E} \left[ \frac{U_1}{U_2} \right] \right\} \left\{ \phi(1+t)f'' A - \frac{\bar{w}L_M(0)}{(1-L_A)^2} \right\} + f'_A \phi'(1+t) \frac{\partial}{\partial L_A} \left( \frac{U_1}{U_2} \right)} > 0 \quad (34)
\]

\[
\frac{dE}{dt} = \frac{\phi \left[ -f'_A \frac{\partial}{\partial L_A} \left( \frac{U_1}{U_2} \right) + \frac{\bar{w}L_M(0)}{(1-L_A)^2} - \phi(1+t)f'' A \right]}{\left\{ \phi'(1+t) \frac{\partial}{\partial E} \left[ \frac{U_1}{U_2} \right] \right\} \left\{ \phi(1+t)f'' A - \frac{\bar{w}L_M(0)}{(1-L_A)^2} \right\} + f'_A \phi'(1+t) \frac{\partial}{\partial L_A} \left( \frac{U_1}{U_2} \right)} > 0 \quad (35)
\]

\[
\frac{dU}{dt} = \left[ \phi(1+t) - g' \right] \left\{ \phi(1+t)f'' A - \frac{\bar{w}L_M(0)}{(1-L_A)^2} \right\} + f'_A \frac{\partial}{\partial L_A} \left( \frac{U_1}{U_2} \right) \phi'(1+t) \frac{\partial}{\partial E} \left( \frac{U_1}{U_2} \right) \frac{dL_A}{dt} \frac{dA}{dt} \quad (36)
\]

Now, clearly (36) shows that the change in welfare $\frac{dU}{dt}$ is the sum of two terms, consisting of the production effect $\phi(1+t)f'_A$ $\frac{dL_A}{dt}$ and a consumption (or trade) effect. While the production effect of the tariff is unambiguously positive, the consumption effect is of indeterminate sign, and it is not possible to determine its sign even at $t = 0$. Hence we cannot in general assert anything about the welfare effect of a tariff. However, as we say earlier, $L_A$ and hence employment $(L_A + L_M(0))$ increases monotonically as the tariff is increased and full employment is reached at $\bar{t}$. 
VII. OPTIMAL POLICY INTERVENTION

We may now briefly state the combination of policies which would yield the first-best optimum in this model.

Thus, let \( t^* \) be the optimal tariff and \( s^* \) the optimal wage subsidy in all employment, which would obtain at the optimal equilibrium. We would then be meeting the constraints of the model as follows:

\[
\frac{f'_M}{M} = \bar{w} - s^* \quad (37)
\]

\[
\phi(E)(1+t^*)\frac{f'_A}{A} = \bar{w} - s^* \quad (38)
\]

\[
\phi(E)(1+t^*) = \frac{U_1}{U_2} \quad (39)
\]

and

\[
\phi'(E) = \frac{U_1}{U_2} = \frac{f'_M}{f'_A} \quad (40)
\]

The diagrammatic counterpart of this optimal equilibrium is in Figure 12 where the optimal wage subsidy is supposed, along with the optimal tariff, to lead to production at \( P^* \) (tangent to production price-ratio \( \pi^*_p \)), consumption at \( C^* \) [tangent to identical consumption price-ratio \( \pi^*_c = \pi^*_p = \phi(E)(1+t^*) \)] and international terms of trade \( \phi(E) \) equal to \( P^*C^* \). The utility function is then maximized at value \( U^* \).

It is, of course, readily seen that the uniform wage subsidy \( s^* \) could equally well be given equivalently as wage subsidy to manufacturing alone as rate \( s^* \) plus a suitable production subsidy to agriculture; and so on. To derive other equivalences, the reader has only to refer to our earlier discussion of this subject in Section II.
REFERENCES


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