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AN ANALYSIS AND EVALUATION OF FINAL OFFER ARBITRATION

Henry S. Farber*

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massachusetts
institute of
technology

50 memorial drive
cambridge, mass. 02139

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ABSTRACT

Final offer arbitration (FOA) and conventional arbitration (ARB) are compared as mechanisms for settling labor disputes. It is recognized that the parties can manipulate their final offers under FOA in order to achieve a more desirable outcome, and the equilibrium pair of final offers is derived as a function of the uncertainty about the arbitrator's award and the relative risk preferences of the parties. Contract zones are derived for both FOA and ARB, and it is shown that the FOA induced contract zone is not unambiguously larger than that induced by ARB. This is contrary to the claims made by supporters of FOA. The quality of negotiated and arbitrated settlements are compared for the two procedures, and it found that both types of settlements are likely to be of lower quality under FOA than under ARB. It is concluded that FOA is not superior to ARB as a dispute settlement procedure because it promotes neither more negotiation nor higher quality settlements.

I. Introduction

Government prohibition of strikes by public employees (particularly public safety workers; i.e., policemen and firemen) has led to a search for alternative mechanisms for settling disagreements in negotiating labor contracts. In essence, this search is for an alternative way not only to impose disagreement costs on the parties so that they have an incentive to reach agreement but also to yield acceptable (to the public as well as to the parties) settlements both when the parties agree voluntarily and when they do not.

Many states utilize arbitration (ARB) to settle labor disputes involving public safety employees.[1] ARB operates by having a neutral third party impose a settlement in the event that one is not reached voluntarily. Farber and Katz (1979) argue that the major costs imposed by this procedure are related to uncertainty about what the arbitrator will do. If the parties are risk averse then there will be a set of outcomes (a contract zone) which are preferred by both parties to ARB. The larger is this contract zone the more likely it is that the parties will be able to reach a voluntary agreement.

It has been argued that ARB does not impose sufficient costs of disagreement because the uncertainty of ARB is reduced through a tendency for arbitrators to compromise the demands of the parties.[2] The result is a "chilling" of bargaining and excessive reliance on the arbitrator to impose a settlement. This is consistent with the analysis of Farber and Katz (1979) where it is shown that as the uncertainty concerning the arbitrator's behavior goes to zero the contract zone disappears.

Largely in response to this criticism of ARB an alternative dispute settlement mechanism has emerged. Final offer arbitration (FOA) differs from conventional arbitration in that the arbitrator is not permitted to compromise the demands of the parties. The formal mechanism of FOA is that in the event of a disagreement each party submits a final offer and the arbitrator chooses one final offer or the other.[3] The chosen offer then becomes the award. Stevens (1966) has argued that FOA ". . . generates just the kind of uncertainty . . . that is well calculated . . . to compel them [the parties] to seek security in agreement."

In order to investigate the validity of this claim for FOA a model of arbitrator behavior and of the formation of the final offers is developed. It is assumed that the arbitrators in ARB and FOA are identical in that they form a notion of a fair settlement.[4] In ARB the arbitrator imposes this fair settlement. In FOA the arbitrator is not permitted to impose the fair settlement, but it is assumed that he chooses the final offer which is closest to the fair settlement. The common factor between the two types of arbitration is that it is the uncertainty of the parties concerning what the arbitrator thinks is fair which encourages negotiated settlements.[5]

It is obvious that the parties can manipulate the arbitration outcome in FOA by manipulating the final offers. This action has no analogue in conventional arbitration. Given the FOA decision rule for selection of the award, each party faces a fundamental tradeoff in setting its final offer: In submitting a more "reasonable" final offer a party is gaining some probability that its offer will be selected while giving up some utility if its offer is selected.

In the next section this model is developed in more detail. Assuming that the parties are expected utility maximizers and recognizing that the expected utility of each party depends on the final offers of both parties, the Nash equilibrium set of final offers is derived. The dependence of the final offers on the risk preferences of the parties and on the uncertainty about the arbitrator's notion of the fair settlement is investigated. More specifically, it is shown that the more risk averse party submits a more reasonable final offer in that it is closer to the mean of the parties' prior distribution of the fair settlement. Hence, the more risk averse party's final offer has a higher probability of being chosen than that of the less risk averse party.[6] It is also shown that as the uncertainty about the fair award disappears the final offers converge to the certain fair award.

The contract zone of potential settlements which are preferred (in expected utility) by both parties to impasse and the concommitant resort to FOA is derived. It is shown that the final offers are more extreme than the limits of the contract zone they generate. This suggests both that arbitrated awards are likely to be of poor quality relative to the negotiated settlements because they cannot reflect potential negotiated settlements and that FOA may actually discourage negotiation. Next, it is shown that as the uncertainty concerning the fair award disappears the contract zone also disappears. It is also shown that both the center of the contract zone and the average arbitration award are skewed toward the less risk averse party. Finally, it is shown that if the difference between the risk preferences of the parties is sufficiently pronounced, the contract zone may not contain the mean fair award. These results suggest that negotiated settlements under FOA may be "biased" relative to the mean fair award.

Section III contains a brief summary of the analogous modus operandi of conventional arbitration. A contract zone is created in a manner parallel to that in FOA. To the extent that the ability of the parties to negotiate their own settlement is directly related to the size of the contract zone, it is shown that FOA does not provide an unambiguously larger incentive to negotiation than ARB. This is a direct result of the fact that the parties can effectively mitigate their risk in FOA through manipulation of their final offers while in ARB there is no analogous mechanism.

Comparison of the FOA and the ARB induced contract zones also yields the conclusion that, while both contract zones are skewed in favor of the less risk averse party, the center of the FOA contract zone is more favorable to the less risk averse party than the center of the ARB contract zone. This suggests that settlements negotiated in an FOA environment may be more favorable to the less risk averse party than the settlements which would have been negotiated in an ARB environment.

In Section IV the arbitrated awards under a final offer scheme are compared to the arbitrated awards under conventional arbitration. It is shown, not surprisingly, that the awards are likely to be more extreme on average in a final offer scheme than in a conventional scheme. While this is exactly what the proponents of final offer arbitration had in mind as the incentive to negotiation, it is argued that disadvantages associated with the extreme arbitration awards may outweigh any advantage final offer arbitration has in promoting negotiated settlements.

Section V contains a summary of the results as well as the conclusions which can be drawn from the analysis.

II. A Model of the Final Offer Arbitration Process

The decision required of an arbitrator in a final offer scheme is that he choose one of the two final offers. It is assumed that the mechanism which generates the final offers does not affect the arbitrator's decision process. Further, it is assumed that there is a single dimension along which there is disagreement and that the offers are quantifiable in that dimension.[7] The problem is formalized as one of pie-splitting; i.e., there is some quantity to be distributed to the two parties (a and b), and the bargaining (and subsequent disagreement) takes place over the share going to each party.

Let y represent the share of party a which implies that $1-y$ is the share of party b. It is assumed that the arbitrator forms a notion of what is a "fair" split of the pie from his (both objective and subjective) perceptions of the situation. Let y_f represent the arbitrator's notion of the fair share for party a. It is assumed that y_f is <u>not</u> a function of the final offers. In other words, the decision criterion of the arbitrator is exogenous to behavior of the parties.

Given y_f it is assumed that the arbitrator chooses the final offer which is closest to y_f. Let y_a and y_b represent the final offers of party a and party b respectively.[8] The offer of party b will be chosen if

$$|y_f - y_b| < |y_a - y_f| , \tag{1}$$

and the offer of party a will be chosen if the inequality does not hold.

Assuming that $y_a > y_b$ in equilibrium, it is possible to drop the absolute values from equation (1) which yields the result that the offer of party b will be chosen if[9]

$$y_f - y_b < y_a - y_f \quad . \tag{2}$$

Rearranging terms and noting that y_f is a random variable as far as the parties are concerned yields the result that

$$Pr(ch\ b) = Pr(y_f < \frac{y_a + y_b}{2}) \tag{3}$$

where Pr(ch b) is the probability that the arbitrator chooses b's final offer. Intuitively, equation (3) implies that b's final offer will be chosen if the average final offer of the two parties is larger than what the arbitrator thinks is fair. Analogously, a's final offer will be chosen if the average final offer is smaller than what the arbitrator thinks is fair.

The $Pr(y_f < \frac{y_a + y_b}{2})$ is simply the cumulative distribution function of y_f $(F(\frac{y_a + y_b}{2}))$ with an associated density function $f(\frac{y_a + y_b}{2})$. This distribution function represents the parties' prior information concerning the uncertain value of y_f. It is assumed that the parties have identical prior distributions.[10]

In order to focus on the role of relative risk preferences in the analysis of FOA, both parties are assumed to have utility functions which exhibit constant absolute risk aversion. These are written as

$$U_a(y) = 1 - e^{-\delta_a y} \tag{4}$$

and

$$U_b(z) = 1 - e^{-\delta_b z} . \tag{5}$$

The parameters δ_a and δ_b represent the absolute risk aversion of a and b respectively. It is assumed that both parties are at least slightly risk averse ($\delta_a, \delta_b > 0$).

The expected utilities of the parties from use of FOA are derived by noting that $\Pr(\text{ch } b) = F(\frac{y_a + y_b}{2})$ and by using the utility functions specified above. The expected utilities are

$$E(U_a) = [1 - F(\frac{y_a + y_b}{2})](1 - e^{-\delta_a y_a}) + F(\frac{y_a + y_b}{2})(1 - e^{-\delta_a y_b}) \tag{6}$$

and

$$E(U_b) = [1 - F(\frac{y_a + y_b}{2})](1 - e^{-\delta_b (1-y_a)})$$

$$+ F(\frac{y_a + y_b}{2}) (1 - e^{-\delta_b (1-y_b)}). \tag{7}$$

A natural definition of the equilibrium pair of final offers for this problem is that of a Nash equilibrium. Given that the parties are manipulating their respective final offers so as to maximize their respective expected utilities, the Nash equilibrium set of final offers is that pair of final offers which has the property that neither party can achieve a higher expected utility by changing its final offer. Analytically, party a sets its final offer (y_a) so as to maximize its expected utility ($E(U_a)$) conditional on party b's final offer (y_b). Party b sets its final offer in an analogous fashion. The pair of final offers which simultaneously satisfy both of these conditional maximization problems is the Nash equilibrium.

Differentiating $E(U_a)$ and $E(U_b)$ with respect to y_a and y_b respectively, setting these derivatives equal to zero, and rearranging terms yields

$$0 = \delta_a[1 - F(\frac{y_a + y_b}{2})]e^{-\delta_a y_a} + \frac{1}{2}f(\frac{y_a + y_b}{2})[e^{-\delta_a y_a} - e^{-\delta_a y_b}] \tag{8}$$

and

$$0 = \delta_b F(\frac{y_a + y_b}{2})e^{-\delta_b(1-y_b)}$$

$$+ \frac{1}{2}f(\frac{y_a + y_b}{2})[e^{-\delta_b(1-y_b)} - e^{-\delta_b(1-y_a)}]. \tag{9}$$

Equations (8) and (9) implicitly determine the optimal final offers of the parties.

It is straightforward to solve both equation (8) and equation (9) for $\frac{1}{2}f(\frac{y_a + y_b}{2})$ which after some algebra yields the relationship

$$\frac{F(\frac{y_a + y_b}{2})}{1 - F(\frac{y_a + y_b}{2})} = \frac{\frac{1}{\delta_b}[e^{\delta_b(y_a-y_b)} - 1]}{\frac{1}{\delta_a}[e^{\delta_a(y_a-y_b)} - 1]}. \tag{10}$$

This is a particularly interesting relationship when it is noted that the left hand side is simply the odds that the arbitrator chooses b's final offer and that the right hand side is simply a function of the risk preferences of the parties. Noting that it was demonstrated above that $(y_a - y_b)$ is non-negative and ignoring temporarily the case where $(y_a - y_b)$ equals zero, this relationship can be analyzed.[11]

It is clear from equation (10) that if party a and party b are equally risk averse $(\delta_a = \delta_b > 0)$ then the odds that party b's offer is chosen equals unity. Hence, in the case where the parties are equally risk averse y_a and y_b are set equidistant from the mean of their common prior distribution on y_f and the offers will be chosen with equal probability. More formally, if $\delta_a = \delta_b$ then $\frac{y_a + y_b}{2} = \bar{y}_f$ where $\bar{y}_f$ is the mean of the prior distribution on y_f.

The numerator and the denominator of the right hand side of equation (10) both have the same functional form with the numerator being a function of δ_b and the denominator being a function of δ_a. As long as δ_b and $(y_a - y_b)$ are both positive, the numerator is a monotonically increasing function of δ_b. Similarly, the denominator is the <u>same</u> monotonically increasing function of δ_a as long as δ_a and $(y_a - y_b)$ are both positive. Thus, if $\delta_b > \delta_a$ then the numerator is larger than the denominator. In other words if party b is more risk averse than party a $(\delta_b > \delta_a)$ then the equilibrium pair of final offers has the property that y_b is closer to $\bar{y}_f$ than is y_a, and hence it is more likely that the arbitrator will choose b's final offer than a's final offer. Symmetry assures that the analogous result holds if a is the more risk averse party. Simply put, the more risk averse party submits a "more reasonable" final offer in order to reduce the probability of a bad outcome.

In order to solve equations (8) and (9) explicitly for the equilibrium pair of final offers, a particular parameterization of the prior distribution on y_f is specified. It is assumed that y_f has a uniform distribution over the range s where s is a subset of the unit interval over which bargaining takes place. This simple distribution has the advantage of being tractable while still being rich enough to capture the aspects of the arbitrator's behavior relevant to the FOA problem.

The parameter s $(0 \leq s \leq 1)$ is in essence a measure of the uncertainty of the parties concerning the arbitrators notion of y_f. If $s = 1$ this implies that the parties have no information (a flat prior) about what the arbitrator thinks is fair. This represents the case of maximum uncertainty. If $s < 1$ then some part of the range of outcomes (0 - 1) is ruled out as a potential value for y_f. This reflects increasing information about what the arbitrator thinks is fair. In the extreme, if $s = 0$ this implies that the parties know exactly what the arbitrator thinks is fair and, hence, there is no uncertainty.

Noting that $\bar{y}_f$ represents the mean of this uniform distribution, the probability density function of y_f is

$$f(y_f) = \frac{1}{s} \qquad \bar{y}_f - \frac{s}{2} \leq y_f \leq \bar{y}_f + \frac{s}{2} \tag{11}$$

and is zero otherwise.[12] The cumulative distribution function of y_f is

$$F(y_f) = 0 \qquad y_f \leq \bar{y}_f - \frac{s}{2}\ ;$$

$$F(y_f) = \frac{1}{2} + [y_f - \bar{y}_f]\frac{1}{s}$$

$$\bar{y}_f - \frac{s}{2} \leq y_f \leq \bar{y}_f + \frac{s}{2}\ ; \tag{12}$$

$$F(y_f) = 1 \qquad y_f \geq \bar{y}_f + \frac{s}{2}\ .$$

Proceeding with the explicit derivation of the final offers, equations (8) and (9) are solved for $[1 - F(\frac{y_a + y_b}{2})]$ and $[F(\frac{y_a + y_b}{2})]$ respectively which yields after some algebra

$$[1 - F(\frac{y_a + y_b}{2})] = \frac{1}{2\delta_a} f(\frac{y_a + y_b}{2})[e^{\delta_a(y_a - y_b)} - 1] \tag{13}$$

and

$$F(\frac{y_a + y_b}{2}) = \frac{1}{2\delta_b} f(\frac{y_a + y_b}{2})[e^{\delta_b(y_a - y_b)} - 1]. \tag{14}$$

Noting that [1 - Pr(ch b)] and [Pr(ch b)] sum to unity and using the definition of $f(y_f)$ in equation (11), addition of equations (13) and (14) yields

$$1 = \frac{1}{2s} [\frac{1}{\delta_a}(e^{\delta_a(y_a - y_b)} - 1) + \frac{1}{\delta_b}(e^{\delta_b(y_a - y_b)} - 1]. \tag{15}$$

On rearrangement of terms this results in

$$[2s + \frac{1}{\delta_a} + \frac{1}{\delta_b}] = \frac{1}{\delta_a} e^{\delta_a(y_a - y_b)} + \frac{1}{\delta_b} e^{\delta_b(y_a - y_b)} . \tag{16}$$

This equation determines $y_a - y_b$ as a function of the parameters of the model (δ_a, δ_b, s). However, a closed form solution cannot be derived in the general case. Nevertheless, some interesting conclusions can be drawn from this implicit expression.

First note that $(y_a - y_b)$ is <u>not</u> a function of the mean $(\bar{y}_f)$ of the prior distribution on y_f. Thus, while the final offers are a direct function of $\bar{y}_f$, the relationship between the final offers is strictly a function of the risk preferences of the parties and of the level of uncertainty.[13]

The second result is that totally differentiating equation (16) with respect to s and $(y_a - y_b)$ yields the comparative statics result that

$$\frac{d(y_a - y_b)}{ds} = \frac{2}{e^{\delta_a(y_a - y_b)} + e^{\delta_b(y_a - y_b)}} \tag{17}$$

which is strictly positive. In other words as the uncertainty increases the final offers diverge. As the uncertainty is reduced to zero $(s \to 0)$, equation sixteen reduces to

$$\frac{1}{\delta_a} + \frac{1}{\delta_b} = \frac{1}{\delta_a} e^{\delta_a(y_a - y_b)} + \frac{1}{\delta_b} e^{\delta_b(y_a - y_b)}. \tag{18}$$

It is clear that this expression requires that $(y_a - y_b) = 0$. Thus, as the uncertainty is reduced the final offers tend to converge to a single point $(\bar{y}_f)$.

A third result is that an increase in the risk aversion of either party causes a reduction in the difference between the final offers. To illustrate this suppose that δ_a increases. Note that the first term in the brackets in equation (15) $(\frac{1}{\delta_a}(e^{\delta_a(y_a - y_b)} - 1))$ is a monotonically increasing function of δ_a. Total differentiation of equation (15) with respect to δ_a and $(y_a - y_b)$ yields the result that $\frac{d(y_a - y_b)}{d\delta_a} < 0$. From considerations of symmetry an increase in δ_b will have the same qualitative effect on $(y_a - y_b)$.

These properties and some others can be clearly illustrated using the special case of the model where the parties have equal risk aversion

($\delta_a = \delta_b = \delta$). This assumption ensures that the final offers have equal probabilities of being chosen ($F(\frac{y_a + y_b}{2}) = .5$ and $\frac{y_a + y_b}{2} = \bar{y}_f$), but it should have no systematic effect on the difference between the final offers ($y_a - y_b$). The advantage of this assumption is that ($y_a - y_b$) can be solved for explicitly as a function of δ and s. Letting $\delta_a = \delta_b = \delta$ and solving equation (16) for ($y_a - y_b$) yields

$$y_a - y_b = \frac{1}{\delta} \ln(s\delta + 1). \tag{19}$$

If there is no uncertainty ($s = 0$) then the final offers converge to a single point ($y_a = y_b = \bar{y}_f$).[14] Alternatively, in the case of maximum uncertainty (least information, $s = 1$) $y_a - y_b$ becomes $\frac{1}{\delta}\ln(1 + \delta)$. Table 1 contains values of $y_a - y_b$ computed from equation (19) for various levels of uncertainty (s) and risk aversion (δ). Reading across any row of table 1 clearly illustrates that the difference between the final offers increases with increasing uncertainty.

It is clear from Table 1 that for any level of uncertainty the difference between the final offers becomes very small as the risk aversion increases. In fact, applying L'Hospital's Rule to equation (19) yields the result that $\lim_{\delta \to \infty} (y_a - y_b) = 0$. As the risk aversion decreases the difference between the final offers increases. In the limit application of L'Hospital's Rule once again equation (19) yields the result that $\lim_{\delta \to 0} (y_a - y_b) = s$. In other words, in the absence of risk aversion the final offers tend to be at the extreme ends of the range of the prior distribution on y_f.

Returning to the general case ($\delta_a \neq \delta_b$), equations (10) and (15) implicitly determine $\frac{y_a + y_b}{2}$ and $y_a - y_b$. It is straightforward to then

TABLE 1:

Difference Between Final Offers for Various Levels of Uncertainty and Risk Aversion[a]

δ \ s	0	.125	.25	.5	1.
.1	0.	.124	.247	.488	.953
1	0.	.118	.223	.406	.693
2	0.	.112	.203	.346	.549
4	0.	.101	.173	.275	.403
8	0.	.086	.137	.202	.275
100	0.	.026	.033	.039	.046

s = measure of uncertainty
δ = absolute risk aversion (assumed equal for the two parties)

[a]Derived from equation (19).

derive the equilibrium pair of final offers from their average and difference.[15] In order to get a feel for how assymetry in the risk preferences of the parties affects the final offers and hence the arbitrator's choices, Table 2 contains equilibrium pairs of final offers as well as the associated probabilities of choice for some illustrative values of δ_a, δ_b, and s.

Two things are readily apparent from Table 2. First, as the assymetry in risk preferences grows, the equilibrium of final offers adjust so that the probability of choice of a particular award deviates substantially from .5. In addition, this deviation is larger in the case with more uncertainty (s = .5). The second result is that as party a becomes more risk averse its equilibrium final offer moves much closer to $\bar{y}_f$ (.5). However, party b's equilibrium final offer moves relatively little. At low levels of δ_a, y_b becomes more extreme as δ_a increases while at higher levels of δ_a the final offer of b becomes more reasonable as δ_a increases.

A contract zone of voluntary settlements preferred by both parties to use of FOA can be derived from the equilibrium pair of final offers and the utility functions. The lower limit of the contract zone is the minimum settlement party a will accept rather than use FOA. This is the share which yields party a a level of utility equal to its expected utility under FOA. Denote this certainty equivalent share for party a by y_{ca}. It is derived by equating $U_a(y_{ca})$ with $E(U_a)$ from equation (6) which yields

$$1 - e^{-\delta_a y_{ca}} = [1 - F(\frac{y_a + y_b}{2})](1 - e^{-\delta_a y_a})$$

$$+ F(\frac{y_a + y_b}{2})(1 - e^{-\delta_a y_b}). \qquad (20)$$

TABLE 2:

Equilibrium Final Offers and Choice Probabilities

for Differential Risk Aversion Case[a]

$s = .25,\ \delta_b = 1$

δ_a	y_a	y_b	$\frac{y_a + y_b}{2}$	Pr(ch b)
1	.612	.389	.5	.5
2	.600	.388	.494	.477
4	.581	.389	.485	.442
8	.555	.394	.474	.397

max y_f = .625
min y_f = .375

$s = .5,\ \delta_b = 1$

δ_a	y_a	y_b	$\frac{y_a + y_b}{2}$	Pr(ch b)
1	.703	.297	.5	.5
2	.668	.296	.482	.464
4	.616	.301	.458	.415
8	.552	.312	.432	.364

max y_f = .75
min y_f = .25

[a]The numbers were derived by solving equations (10) and (15) numerically for y_a and y_b under the assumption that $\bar{y}_f = .5$.

Subtraction of 1 from both sides and multiplication of the resulting expression by $-e^{\delta_a y_a}$ yields

$$e^{\delta_a(y_a - y_{ca})} = [1 - F(\frac{y_a + y_b}{2})] + F(\frac{y_a + y_b}{2}) e^{\delta_a(y_a - y_b)} . \tag{21}$$

The assumption that $\delta_a > 0$ and the fact that the equilibrium final offers have the property that $y_a \geq y_b$ imply that the right hand side of equation (21) is greater than one. Hence, the certainty equivalent share for party a is less than a's final offer $(y_{ca} \leq y_a)$. Algebraic manipulation of equation (21) yields the relationships

$$1 - e^{\delta_a(y_a - y_{ca})} = F(\frac{y_a + y_b}{2})[1 - e^{\delta_a(y_a - y_b)}] \tag{22}$$

and

$$1 - e^{\delta_a(y_a - y_{ca})} \geq 1 - e^{\delta_a(y_a - y_b)} \tag{23}$$

noting that $0 \leq F(\frac{y_a + y_b}{2}) \leq 1$ and that $(1 - e^{\delta_a(y_a - y_b)}) < 0$. Further manipulation results in the inequality

$$e^{\delta_a(y_a - y_{ca})} \leq e^{\delta_a(y_a - y_b)} . \tag{24}$$

It is clear from this inequality that the certainty equivalent share for party a is greater than b's final offer $(y_{ca} \geq y_b)$.

This analysis bounds y_{ca} between the final offers of the parties:

$$y_b \leq y_{ca} \leq y_a . \tag{25}$$

The symmetry between a and b implies that the certainty equivalent share for party b (y_{cb}) which is derived from the relationship $(U_b(1-y_{cb}) = E(U_b))$

$$1 - e^{-\delta_b(1-y_{cb})} = [1 - F(\frac{y_a + y_b}{2})](1 - e^{-\delta_b(1-y_a)})$$

$$+ F(\frac{y_a + y_b}{2})(1 - e^{-\delta_b(1-y_b)}) \qquad (26)$$

is bounded in the same way:

$$y_b \leq y_{cb} \leq y_a \; . \qquad (27)$$

Since y_{ca} represents the minimum share that party a will accept voluntarily rather than utilize FOA and y_{cb} represents the maximum share that party b will give voluntarily to party a rather than utilize FOA, they are the lower and upper bounds respectively of the contract zone.

The interesting conclusion to be drawn from inequalities (25) and (27) is that the final offers are more extreme than the limits of the contract zone. In other words if the parties for some reason do not agree on a point in the contract zone, they retrench in a sense to more extreme positions in FOA. This has two implications. First, the arbitrated awards will be of low quality in the sense that they cannot reflect potential negotiated settlements. The second implication is due to the possibility that in the context of a particular bargaining situation it may be difficult to retreat from a bargaining position when formulating the final offers. Such a retreat is necessary if the final offers are more extreme than the limits of the contract zone, and to the extent that there is a difficulty in retreating either the final offers will not be the equilibrium pair derived above or the parties will be reluctant to concede as much as they "should" in bargaining. The latter response will result in an increased resort to FOA that is due strictly to the structure of the FOA game, and it may defeat the major justification for FOA that it provides an incentive for the parties to negotiate their own agreements.

It is argued below that the ability of FOA to encourage negotiated settlements is a function of the size of the contract zone (Z) created by FOA. This contract zone is bounded by y_{cb} and y_{ca} and it has width (derived from equations (20) and (26))

$$Z = (y_{cb} - y_{ca}) = \frac{1}{\delta_b} \ell n[1 - F(\frac{y_a + y_b}{2})(1 - e^{-\delta_b(y_a - y_b)})]$$

$$+ \frac{1}{\delta_a} \ell n[1 - F(\frac{y_a + y_b}{2})(1 - e^{\delta_a(y_a - y_b)})]. \qquad (28)$$

The center of the contract zone (CCZ) is

$$CCZ = \frac{y_{cb} + y_{ca}}{2} = \frac{y_a + y_b}{2}$$

$$+ \frac{1}{2\delta_b} \ell n[F(\frac{y_a + y_b}{2}) + (1 - F(\frac{y_a + y_b}{2}))e^{\delta_b(y_a - y_b)}]$$

$$- \frac{1}{2\delta_a} \ell n[F(\frac{y_a + y_b}{2}) + (1 - F(\frac{y_a + y_b}{2}))e^{\delta_a(y_a - y_b)}]. \qquad (29)$$

These expressions are quite complex and analyses of their comparative statics properties is difficult because of the inability to derive closed form solutions for $y_a - y_b$ and $\frac{y_a + y_b}{2}$ in the general case. Nonetheless, three conclusions are relatively straightforward. First, there will be a positive contract zone as long as both parties are risk averse just because risk averse agents are willing to give up some expected share E(y) in order to avoid the risk of FOA. In more formal terms it must be true that

$$y_{ca} < E(y) \tag{30}$$

and

$$1 - y_{cb} < 1 - E(y) \tag{31}$$

where

$$E(y) = F(\frac{y_a + y_b}{2})y_b + [1 - F(\frac{y_a + y_b}{2})]y_a \tag{32}$$

and represents the expected share under arbitration. Inequalities (30) and (31) yield on rearrangement the ordering $y_{ca} < E(y) < y_{cb}$ which implies that $y_{cb} > y_{ca}$ and that $Z = y_{cb} - y_{ca} > 0$.

The second conclusion is that as the uncertainty disappears ($s = 0$), the contract zone also will shrink to zero. It was shown above that as s goes to zero the final offers converge ($y_a - y_b = 0$). Hence, at this point Z in equation (28) is trivially zero.

Third, note that the size of the contract zone (Z) is not a function of $\bar{y}_f$ because, as was shown above, neither $y_a - y_b$ nor $F(\frac{y_a + y_b}{2})$ is a function of $\bar{y}_f$. On the other hand, the location of the contract zone (CCZ) is a direct function of $\bar{y}_f$ because the average final offer $(\frac{y_a + y_b}{2})$ is a direct function of $\bar{y}_f$ (see footnote 13). Thus, a shift in the mean of the prior distribution of y_f will shift the location of the final offers and the contract zone by the same amount $(\frac{dy_a}{d\bar{y}_f} = \frac{dy_b}{d\bar{y}_f} = \frac{dCCZ}{d\bar{y}_f} = 1)$, but the size of the contract zone will be unchanged.

The fact that there is always a positive contract zone suggests that the parties will always be able to negotiate their own agreements without resort to arbitration. However, the guarantee of a positive contract zone is a result of the assumption that the parties have identical prior distributions on y_f. If this assumption is relaxed (in a relatively simple fashion) to the extent that party a and party b can have prior distributions with different means ($\bar{y}_{fa}$ and $\bar{y}_{fb}$ respectively) and if the parties don't realize that their prior distributions are different then the parties solve different equilibrium final offer-minimum acceptable settlement problems.

These solutions differ only to the extent that $\bar{y}_{fa}$ and $\bar{y}_{fb}$ differ.[16] Where $\bar{y}_f$ is different for the two parties, the lower limit of the contract zone is a function of $\bar{y}_{fa}$ while the upper limit of the contract zone is a function $\bar{y}_{fb}$. The size of the contract zone in this more general case is $Z + (\bar{y}_{fb} - \bar{y}_{fa})$ where Z is defined in equation (28).[17] Clearly, if $\bar{y}_{fb} = \bar{y}_{fa}$ then the size of the contract zone reduces to Z. However, if the parties have relatively optimistic expectations about the arbitrator ($\bar{y}_{fa} > \bar{y}_{fb}$) then the contract zone is smaller than that derived in the case of identical priors.[18]

If $\bar{y}_{fb}$ is sufficiently smaller than $\bar{y}_{fa}$ then there will not be a positive contract zone. More formally, there will be a contract zone only if $Z > \bar{y}_{fa} - \bar{y}_{fb}$. Thus, in a world of imperfect information about the arbitrator where $\bar{y}_{fa}$ and $\bar{y}_{fb}$ can differ, a larger contract zone in the identical priors case (Z) makes it more likely that differences in $\bar{y}_f$ will not eliminate the actual contract zone. Since a positive contract zone is a precondition for a negotiated settlement, the size of the contract zone in the identical priors case is a good indicator of the ability of FOA (or any arbitration scheme) to overcome differences in expectations and induce negotiated settlements.

Note that in this simple model the difference in expectations cannot persist over time because each party will recognize that the final offers of the other party is not what was expected from their equilibrium calculations. Thus, in a steady state the prior distributions will converge, and there will be a positive contract zone. Of course, conditions may vary over time to such an extent that $\bar{y}_{fa}$ and $\bar{y}_{fb}$ never converge because the parties gauge the arbitrator's response to changing conditions differently.

Returning to the case of identical prior distributions on y_f and in order to demonstrate another, more subtle, conclusion that the contract zone is skewed toward settlements more favorable to the less risk averse party, Table 3 contains the bounds, width, and center of the contract zone for various levels of risk aversion and uncertainty. Note that if the parties are equally risk averse ($\delta_a = \delta_b$) the center of the contract zone is $\bar{y}_f = .5$. This is clear both from equation (29) ($\frac{y_a + y_b}{2} = \bar{y}_f$ if $\delta_a = \delta_b$) and from Table 3. Further, it is apparent from Table 3 that as party a is progressively more risk averse than party b the center of the contract zone represents a progressively smaller share for a, and as a result the more risk averse party will be at a disadvantage in bargaining.[19] It is also true that the more risk averse party receives a lower arbitral award on average ($E(y) < .5$ for $\delta_a > \delta_b$).

TABLE 3:

Contract Zones Induced by FOA for Various Levels of Uncertainty and Risk Aversion[a]

$s = .25,\ \delta_b = 1$

δ_a	y_{ca}	y_{cb}	Z	CCZ	E(y)
1	.494	.507	.012	.5	.5
2	.488	.504	.017	.496	.499
4	.478	.501	.023	.489	.496
8	.466	.494	.028	.480	.491

$s = .5,\ \delta_b = 1$

δ_a	y_{ca}	y_{cb}	Z	CCZ	E(y)
1	.480	.520	.041	.5	.5
2	.461	.512	.051	.487	.495
4	.437	.497	.060	.467	.485
8	.410	.471	.061	.440	.465

y_{ca} = certainty equivalent share for a (equation (20))

y_{cb} = b's certainty equivalent share for a (equation (26))

$Z = y_{cb} - y_{ca}$ = size of contract zone (equation (28))

$CCZ = \frac{y_{cb} + y_{ca}}{2}$ = center of contract zone (equation (29))

E(y) = expected arbitration award (equation (32))

[a]These numbers were derived assuming $\bar{y}_f = .5$.

Another conclusion which can be drawn from Table 3 is that, holding risk preferences fixed, as the uncertainty increases (s = .25 to s = .5) the center of the contract zone represents a smaller share for the more risk averse party. The average arbitration awards are also less favorable to the more risk averse party when uncertainty increases. These results suggest that the FOA procedure for dispute settlement has an asymmetric effect on both the negotiated and the arbitrated outcomes which is related to the relative risk preferences of the parties. Whether or not the risk preferences of the parties differ is an empirical issue, but to the extent that they do differ FOA cannot be considered neutral in its impact on the parties.

The results in Table 3 also suggest that the center of the contract zone is less favorable to the more risk averse party than the average arbitration award under FOA. This has an important implication for the evaluation of "bias" in FOA schemes. It may be true that the average arbitrated settlements under FOA are systematically more favorable to one party or the other than negotiated settlements.[20] The fact that CCZ $< E(y)$ when $\delta_a > \delta_b$ suggests that the more risk averse party will be the party which does "better" with arbitrated awards than with negotiated settlements. The temptation is great to conclude that the arbitration procedure is biased in favor of the more risk averse party. However, the reality is just the opposite because both the average arbitrated and average negotiated settlements are skewed against the more risk averse party relative to $\bar{y}_f$. The apparent paradox is simply due to the fact that the average arbitration award is skewed less than the negotiated settlements.

It is interesting to note that increased asymmetry in the risk preferences of the parties can shift the contract zone sufficiently so that it no longer includes the mean of the distribution of the arbitrator's notion of the fair outcome ($\bar{y}_f = .5$).[21] To the extent that $\bar{y}_f$ represents a desirable negotiated outcome, FOA can yield poor quality negotiated settlements.[22] Conventional arbitration does not have this property of excluding $\bar{y}_f$ from the contract zone.[23]

In order to investigate the determinants of the size of the contract zone (Z) induced by FOA the simplifying assumption that $\delta_a = \delta_b = \delta$ is made once again. Equation (28) simplifies on substitution from equation (19) for $(y_a - y_b)$ under this assumption to

$$Z = \frac{1}{\delta}\{2\ell n(\frac{1}{2}) + \ell n[\frac{(s\delta + 2)^2}{(s\delta + 1)}]\} . \tag{33}$$

Differentiation of Z with respect to s yields

$$\frac{\partial Z}{\partial s} = \frac{s\delta^2}{(s\delta + 1)(s\delta + 2)} > 0 \tag{34}$$

which implies that increasing the uncertainty unambiguously increases the size of the contract zone. This is illustrated in Table 4 which contains sizes of contract zones that are induced by FOA for various levels of uncertainty and risk aversion under the assumption that $\delta_a = \delta_b = \delta$.

The derivative of Z with respect to δ is

$$\frac{\partial Z}{\partial \delta} = \frac{s}{\delta}[\frac{s\delta}{(s\delta + 1)(s\delta + 2)} - \frac{1}{s\delta}\ell n[\frac{(s\delta + 2)^2}{(s\delta + 1)}] - \frac{1}{s\delta}\ell n(\frac{1}{4})] \tag{35}$$

TABLE 4:

Contract Zones Induced by FOA for Various Levels of Uncertainty and Risk Aversion Where There is Equal Risk Aversion[a]

δ \ s	0	.125	.25	.5	1.
.1	0.	.000	.002	.006	.023
1	0.	.003	.012	.041	.118
2	0.	.006	.020	.059	.144
4	0.	.010	.029	.072	.147
8	0.	.015	.036	.073	.128
100	0.	.014	.019	.026	.032

[a]Computed from equation (33).

which is not one signed. This is evident from the lack of monotonocity exhibited by Z with respect to changes in δ for any level of uncertainty in Table 4. In other words the contract zone is not unambiguously larger where the parties are more risk averse. At relatively low levels of risk aversion an increase in δ will increase the contract zone, but if there is a high degree of risk aversion an increase in δ will tend to shrink the contract zone. The intuitive explanation for this is that as the degree of risk aversion increases the equilibrium pair of final offers tend to converge. This pair of final offers bounds the contract zone so that of necessity as the degree of risk aversion becomes large the contract must shrink along with $y_a - y_b$.[24]

III. A Comparison of FOA and Conventional Arbitration (ARB)

In order to compare FOA and ARB it is necessary to develop a model of ARB analogous to the model developed in the previous section for FOA.[25] The appropriate comparison is to assume that the arbitrators under FOA and ARB are identical in that they have the same notion of a fair settlement (y_f). In addition, it is assumed that the parties hold the same prior distribution on y_f whether the dispute settlement procedure is FOA or is ARB. The parties preferences (and risk preferences) are also assumed not to be affected by the dispute settlement procedure. The sole difference between the two models is the way the arbitrator uses the common y_f. In FOA the arbitrator chooses the final offer closest to y_f, and that final offer becomes the settlement. This was analyzed above in detail. In ARB the arbitrator actually imposes y_f as the settlement.

The first obvious difference between FOA and ARB is that the expected arbitral settlements (E(y)) are different. In FOA the expected arbitral settlement is the probability weighted average of the final offers contained in equation (32). In conventional arbitration the expected arbitral settlement is simply $\bar{y}_f$.[26] It is clear from Table 3 that if the parties are not equally risk averse then $E(y)_{FOA} \neq \bar{y}_f$ and that $E(y)_{FOA}$ is less favorable to the more risk averse party. Hence, arbitration awards under FOA are biased against the more risk averse party relative to arbitration awards under conventional arbitration.

The contract zone under ARB is derived analogously to that for FOA by finding the certainty equivalent shares for the parties which yield utility levels equal to the expected utilities from use of the arbitration procedure. For party a this equality is

$$1 - e^{-\delta_a y_{ca}} = \int_{\bar{y}_f - \frac{s}{2}}^{\bar{y}_f + \frac{s}{2}} \frac{1}{s}[1 - e^{-\delta_a y}]dy, \tag{36}$$

and for party b this equality is

$$1 - e^{-\delta_b(1-y_{cb})} = \int_{\bar{y}_f - \frac{s}{2}}^{\bar{y}_f + \frac{s}{2}} \frac{1}{s}[1 - e^{-\delta_b(1-y)}]dy. \tag{37}$$

The specific density function for y_f used in these equations is the same uniform prior distribution used in the analysis of FOA and defined in equation (11). Solving equation (36) and (37) for y_{ca} and y_{cb} respectively yields

$$y_{ca} = \bar{y}_f + \frac{1}{\delta_a} \ell n[\frac{\delta_a s}{e^{\delta_a s/2} - e^{-\delta_a s/2}}] \tag{38}$$

and

$$y_{cb} = \bar{y}_f - \frac{1}{\delta_b} \ell n[\frac{\delta_b s}{e^{\delta_b s/2} - e^{-\delta_b s/2}}]. \tag{39}$$

The expressions in brackets are bounded between zero and one for all positive values of δ_s. Hence, the size of the contract zone,

$$Z_{ARB} = y_{cb} - y_{ca} = -\frac{1}{\delta_b} \ell n[\frac{\delta_b s}{e^{\delta_b s/2} - e^{-\delta_b s/2}}]$$

$$- \frac{1}{\delta_a} \ell n[\frac{\delta_a s}{e^{\delta_a s/2} - e^{-\delta_a s/2}}], \tag{40}$$

is positive as long as there is some uncertainty and as long as at least one party is risk averse. The $\lim_{s\to 0} Z_{ARB} = 0$ so that the contract zone disappears as the uncertainty disappears just as in FOA.

It was demonstrated for FOA that the center of the contract zone is skewed toward settlements more favorable to the less risk averse party. This is also true of ARB. The center of the contract zone in ARB is

$$CCZ_{ARB} = \frac{y_{ca} + y_{cb}}{2} = \bar{y}_f + \frac{1}{2} \{ \frac{1}{\delta_b} \ell n[\frac{e^{\delta_b s/2} - e^{-\delta_b s/2}}{\delta_b s}] + \frac{1}{\delta_a} \ell n[\frac{\delta_a s}{e^{\delta_a s/2} - e^{-\delta_a s/2}}] \} . \tag{41}$$

It is clear that, as in FOA, when the parties are equally risk averse ($\delta_a = \delta_b = \delta$) the center of the contract zone is simply the mean of the distribution of the fair award ($\bar{y}_f$). In order to compare the degree of skewness of the centers of the FOA and ARB contract zones, Table 5 contains the contract zones and their centers for various degrees of uncertainty and risk aversion for both FOA and ARB. The striking result is that, for all of the chosen values of δ_a, δ_b, and s, the FOA contract zone is skewed against the more risk averse party by a larger amount than the ARB contract zone. This result both supports the conclusion of the last section that FOA may have an adverse qualitative impact on negotiated settlements and suggests that conventional arbitration will mitigate this adverse impact.[27]

The results contained in Table 5 illustrate that the contract zone induced by FOA is not unambiguously larger than that induced by ARB.

TABLE 5:

Contract Zones and Their Centers Induced by FOA and by ARB for Various Levels of Uncertainty and Risk Aversion[a]

$s = .25,\ \delta_b = 1$

δ_a	Z_{FOA}	Z_{ARB}	CCZ_{FOA}	CCZ_{ARB}
1	.012	.005	.5	.5
2	.017	.008	.496	.499
4	.023	.013	.489	.496
8	.028	.023	.480	.491

$s = .5,\ \delta_b = 1$

δ_a	Z_{FOA}	Z_{ARB}	CCZ_{FOA}	CCZ_{ARB}
1	.041	.021	.5	.5
2	.051	.031	.489	.495
4	.060	.051	.467	.485
8	.061	.085	.440	.468

$s = 1,\ \delta_b = 1$

δ_a	Z_{FOA}	Z_{ARB}	CCZ_{FOA}	CCZ_{ARB}
1	.118	.083	.5	.5
2	.132	.122	.457	.480
4	.131	.190	.401	.446
8	.110	.281	.346	.401

Z_{FOA} = size of final offer arbitration contract zone (equation (28));
Z_{ARB} = size of conventional arbitration contract zone (equation (40));
CCZ_{FOA} = center of FOA contract zone (equation (29));
CCZ_{ARB} = center of ARB contract zone (equation(41)).

[a]Derived under the assumption that $\bar{y}_f = .5$.

The advantage of FOA in creating a contract zone seems to be larger when there is little uncertainty and/or risk aversion. In order to examine this issue more carefully it is assumed once again that the parties are equally risk averse ($\delta_a = \delta_b = \delta$). Substitution into equation (4) yields the result that

$$Z_{ARB} = - \frac{2}{\delta} \ell n\left[\frac{\delta s}{e^{\delta s/2} - e^{-\delta s/2}} \right]. \qquad (42)$$

Table 6 contains contract zones computed from equation (42) for various values of uncertainty and risk aversion. These are compared with the analogous contract zones for FOA computed from equation (33) and contained in Table 4 and the difference between Z_{FOA} and Z_{ARB} is contained in Table 7.

The results contained in Table 6 illustrate that, as with FOA, the ARB induced contract zone increases with increasing uncertainty. However, unlike FOA, the ARB induced contract zone is also unambiguously larger where there is more risk aversion. This difference arises because ARB contains no mechanism for very risk averse parties to mitigate the risk analogous to the mechanism of moderation of final offers available in FOA.

The comparison of FOA and ARB induced contract zones which is summarized in Table 7 yields the conclusion that FOA is relatively more successful than ARB in creating a contract zone where there is little risk aversion and/or uncertainty. Conversely where there is more risk aversion and/or uncertainty conventional arbitration is relatively more successful than FOA in creating a contract zone. Thus, the major justification for FOA, that it induces more negotiated settlements than ARB, is not necessarily the case.[28]

TABLE 6:

Contract Zones Induced by Conventional Arbitration for Various Levels of Uncertainty and Risk Aversion[a]

δ \ s	0	.125	.25	.5	1.
.1	0.	.000	.001	.002	.008
1	0.	.001	.005	.021	.083
2	0.	.003	.010	.041	.161
4	0.	.005	.021	.081	.298
8	0.	.010	.040	.149	.480
100	0.	.074	.186	.422	.908

[a]Computed from equation (42).

TABLE 7:

Difference Between Size of FOA and ARB Induced Contract Zones for Various Levels of Uncertainty and Risk Aversion[a]

δ \ s	0	.125	.25	.5	1.
.1	0.	.000	.001	.004	.015
1	0.	.002	.007	.020	.035
2	0.	.003	.010	.018	-.017
4	0.	.005	.008	-.009	-.151
8	0.	.005	-.004	-.076	-.352
100	0.	-.060	-.167	-.396	-.940

[a]This difference is $Z_{FOA} - Z_{ARB}$ and it is computed from Tables 4 and 6.

IV. Relative Quality of Arbitration Awards in FOA and ARB

It was concluded above that the average arbitration award under ARB is simply $\bar{y}_f$. This represents a desirable outcome to the extent that $\bar{y}_f$ represents an average neutral party's notion of a "fair" outcome. The results contained in Table 3 were used to conclude that the average arbitration award under FOA is biased against the more risk averse party relative to $\bar{y}_f$. Thus, unless risk aversion is considered to be a valid criterion for arbitration award, FOA will yield arbitrated settlements which are inferior on average to those from ARB.[29]

A second criterion for judging the relative quality of arbitration awards is the probability of getting extreme awards. Extreme awards may be inferior because they will impose an undue sacrifice on one party and an undue gain on the other party. This can reduce the long run viability of such settlements and undermine the underlying procedure in the long run.

In order to compare FOA and ARB on this ground note that under FOA the awards are constrained to be either y_a or y_b (the final offers) while under conventional arbitration the arbitrator can compromise. Thus, to the extent that the final offers are extreme FOA must yield extreme arbitral awards. This is not the case under ARB. In formal terms the probability that a conventional arbitration award would be less extreme then a FOA award by the same arbitrator is simply the probability that the fair award (y_f) is between the final offers. This probability is

$$\Pr(y_b < y_f < y_a) = \int_{y_b}^{y_a} \frac{1}{s}\, dy = \frac{y_a - y_b}{s} . \tag{43}$$

Assuming once again that the parties are equally risk averse, substitution from equation (19) for $y_a - y_b$ yields the result that

$$\Pr(y_b < y_f < y_a) = \frac{1}{s\delta}\,\ell n(s\delta + 1). \tag{44}$$

For positive values of $s\delta$ this probability is a monotonically decreasing function of $s\delta$ with $\lim_{s\delta \to 0} \Pr(y_b < y_f < y_a) = 1$ and $\lim_{s\delta \to \infty} \Pr(y_b < y_f < y_a) = 0$. The two procedures are equally likely to yield an award no more extreme than the final offers ($\Pr(y_b < y_f < y_a) = .5$) when $s\delta = 2.51$. If $s\delta > 2.51$ then FOA has a higher probability of yielding more reasonable settlements, and when $s\delta < 2.51$ ARB is superior in this regard.

Table 8 contains probabilities that conventional arbitration awards are more reasonable than FOA awards for various levels of uncertainty and risk aversion. The results suggest that unless there is both a high level of uncertainty and an extremely large (and empirically unlikely) degree of risk aversion conventional arbitration will generally yield an award which is less extreme than the final offers.

Two comments are in order regarding this analysis. First, y_a and y_b are endogenous and may not reflect an appropriate base for measuring extremity of awards. This is particularly true when there is a large difference in the risk preferences of the parties because in this case the final offers are skewed on the basis of the relative risk preferences.[30] However, when the parties are equally risk averse, as in the numerical example in Table 8, the final offers are equidistant from $\bar{y}_f$ and the use of y_a and y_b as a basis of reasonableness is acceptable.

A second comment is that the analysis highlights an inherent paradox in the use of arbitration procedures to settle public sector labor disputes.

TABLE 8:

Probabilities that ARB Yields a Settlement Between the Final Offers for Various Levels of Uncertainty and Risk Aversion[a]

δ \ s	0	.125	.25	.5	1
.1	1	.992	.988	.976	.953
1	1	.944	.892	.812	.693
2	1	.896	.812	.692	.549
4	1	.808	.692	.550	.403
8	1	.688	.548	.404	.275
100	1	.208	.132	.078	.046

[a] $\Pr(y_b < y_f < y_a) = \frac{y_a - y_b}{s} = \frac{1}{s\delta} \ell n(s\delta + 1).$

To the extent that arbitration procedures rely on risk aversion to create a contract zone, uncertainty about y_f is crucial to maintain a contract zone but at the same time it also yields more extreme arbitration awards. Where uncertainty is very high FOA may yield more reasonable arbitration awards than ARB (Table 9), but ARB may yield a larger contract zone (Table 8). Where there is less uncertainty FOA has the advantage in maintaining a contract zone, but ARB has the advantage in promoting reasonable arbitration awards.

V. Summary and Conclusions

Is final offer arbitration a superior dispute settlement procedure? The analysis presented in this study suggests that it is not. This conclusion rests on the evidence derived from the model of FOA presented in Section II and from a comparison of FOA outcomes with outcomes under conventional arbitration. The FOA and ARB procedures were compared in three areas. The first is a comparison of their relative ability to promote voluntarily negotiated settlements which is essentially a comparison of the size of the contract zones induced by the two procedures. The second is a comparison of the relative quality of negotiated settlements under the two procedures. The final criterion concerns the relative quality of the arbitrated settlements under the two procedures. In no area was FOA clearly superior to conventional arbitration, and a strong case can be made for preferring conventional arbitration to FOA.

The evidence (summarized in Table 7) on the relative size of the contract zones induced by FOA and ARB, and hence on their relative ability to promote negotiated agreements, shows that FOA induces a slightly larger contract zone than ARB at low levels of uncertainty and where there is little risk aversion. However, if there is a high degree of uncertainty and if there is a fair degree of risk aversion then conventional arbitration has the advantage. The degree of risk aversion of the parties is an unresolved empirical issue, but what little evidence there is suggests that a value for δ of two to four is not unreasonable.[31] At this level of risk aversion FOA has little or no advantage over conventional arbitration in creating a contract zone.

The relative quality of negotiated settlements in environments which contain FOA and ARB was judged on the basis of the bounds and location of the contract zones induced by the two procedures. Conventional arbitration has the property of always creating a contract zone which includes the mean of the prior distribution of the arbitrator's notion of the fair settlement ($\bar{y}_f$). On the other hand, FOA induced contract zones may not include $\bar{y}_f$ if the difference in the risk preferences of the parties is substantial.[32] To the extent that $\bar{y}_f$ is a desirable outcome on average, FOA is at a disadvantage if the relative risk preferences are such that $\bar{y}_f$ is ruled out as a possible negotiated outcome.

The contract zones induced by both procedures are skewed against the more risk averse party relative to $\bar{y}_f$. However, the FOA induced contract zone is more skewed than the contract zone induced by conventional arbitration.[33] There is no reason to believe that relative risk preferences are a valid normative criterion for judging negotiated outcomes so that both procedures (and any other which relies primarily on uncertainty to encourage bargaining) are subject to the criticism that settlements negotiated in their presence will show a systematic bias on the basis of a spurious consideration. However, FOA is a worse offender than conventional arbitration in this regard.

The relative quality of arbitrated settlements was judged on the basis of the average arbitrated settlement and the probability of extreme awards. It was shown that the average arbitrated settlement under FOA is biased relative to $\bar{y}_f$ against the more risk averse party. This was due to the facts that the final offers themselves are relatively unfavorable to the more risk averse party and that the average arbitrated settlement is simply a weighted average of the final offers.[34] Conventional arbitration does not suffer from this defect because the arbitrator is free to choose any award he wishes. Thus, the average award under ARB is exactly $\bar{y}_f$.

The evidence summarized in Table 8 suggests that for most reasonable parameter values arbitrated settlements under conventional arbitration are likely to be more moderate than the equilibrium final offers under FOA. Since the final offers are the only arbitration awards possible under FOA, it is true that arbitration awards under ARB are likely to be less extreme than arbitration awards under FOA for most parameter values.

The weight of these arguments suggests that FOA is not a superior alternative to conventional arbitration. In fact, conventional arbitration may be preferable. Any slight advantage which FOA has in creating a contract zone under conditions of little uncertainty are offset by the lower quality of both negotiated and arbitrated settlements under FOA. Effort would be better spent in maintaining the uncertainty in conventional arbitration, perhaps by reducing pre-award communication between the parties and the arbitrator, rather than by shifting from ARB to FOA.[35] A more fundamental change than shifting from FOA to ARB is necessary to remove the relative risk preference bias from the settlements negotiated under either arbitration procedure.

FOOTNOTES

[1]These include Alaska, Maine, Minnesota, New York, Oregon, Pennsylvania, Rhode Island, Washington, and Wyoming.

[2]See Feuille (1975).

[3]Some variant of this procedure is used to settle public employee labor disputes in Connecticut, Iowa, Massachusetts, Michigan, New Jersey, and Wisconsin. FOA is also used to resolve salary disputes involving major league baseball players. See Dworkin (1976).

Where more than one issue is in dispute two major variants of FOA procedures have evolved. The first is "whole package" final offer arbitration where each side submits a final offer covering all of the issues in dispute and the arbitrator picks one final offer package or the other. The second variant is "issue by issue" FOA where each side submits separate final offers for each issue in dispute. The arbitrator is then free to fashion a compromise by awarding some issues to one party and the rest to the second party. The analysis in this study deals strictly with the single issue case, and hence this distinction is not relevant. See Stern, et al. for a discussion of some of the general issues involved in FOA.

[4]The arbitrator's notion of a fair award will vary across bargaining units and over time. It may depend on bargaining outcomes of comparable workers elsewhere, the rate of change of prices, the financial health of the employer, etc. Many arbitration statutes provide a long list of factors which the arbitrator is supposed to consider in fashioning an award.

[5]See Farber and Katz (1979) for a detailed analysis of conventional arbitration.

[6]This suggests that a tendency for final arbitrators to choose union (or employer) offers consistently more frequently is not evidence of "bias" on the part of the arbitrator. It merely reflects the quality of the final offers and indirectly the relative risk preferences of the parties. Somers (1977) erroneously uses evidence that during a given period in Massachusetts unions "won" 67 percent of FOA's to conclude that ". . . unions have a distinct advantage under the Massachusetts procedure" (p. 199).

[7]Crawford (1979a, 1979b) analyzes a final offer arbitration scheme where there is disagreement over a number of distinct issues. This raises some interesting issues of the <u>ex post</u> pareto efficiency of arbitrated awards. However, in most of his analysis he neglects the uncertainty concerning the fair settlement which is given central importance in the present study.

[8]y_b represents the share which the final offer of party b yields to party a.

[9]The equilibrium will be described below, but note now that the Nash equilibrium set of final offers must have the property that a's final offer exceed b's final offer to a. ($y_a > y_b$). If this were not true then one party or the other could change its offer so that it had both a higher probability of being chosen and yielded a higher utility if it were chosen. Thus, it could not be an equilibrium. For example, suppose $y_a < y_b < \bar{y}_f$ where $\bar{y}_f$ is the mean of the parties' prior distribution of y_f. This can not be an equilibrium set of y_a and y_b because party a can make itself unambiguously better off by increasing y_a at least enough so that $y_a = y_b < \bar{y}_f$.

[10]To the extent that the prior distributions of the parties differ the following analysis is more complicated. The effect of relaxing the assumption of identical priors to a certain extent is discussed below. See Farber and Katz (1979) for a general discussion of how divergent priors affect the analysis of conventional arbitration.

[11]It will be shown below that $y_a = y_b$ in the special cases where $\delta_a = \delta_b = 0$ or where there is no uncertainty regarding y_f.

[12]Note that if $\bar{y}_f$ is not .5 then s cannot equal one because this would imply a positive density outside the unit interval. The shortcoming of the uniform distribution in not admitting skewness causes this problem. However, it is not a gross distortion to assume that as the uncertainty of the parties increase to the limit the mean of the distribution moves to the center of the interval. It is shown below that $\bar{y}_f$ plays a neutral role in that a shift in $\bar{y}_f$ causes equal shifts in y_a, y_b, and the location of the contract zone but no change in Pr(ch b) or the size of the contract zone. Strictly speaking these results are due to the lack of skewness, but they do illustrate that $\bar{y}_f$ is not a parameter that is central to the model. Hence, useful information can be derived around the point $\bar{y}_f = .5$, and the analysis continues on this basis.

[13]In order to show that the final offers are a direct function of $\bar{y}_f$, the definition of $F(y_f)$ in equation (12) is used with equation (14) for $\frac{y_a + y_b}{2}$ to yield

$$\frac{y_a + y_b}{2} = \bar{y}_f - \frac{s}{2} + \frac{1}{2\delta_b}\left(e^{\delta_b(y_a - y_b)} - 1\right).$$

Since $y_a - y_b$ is not a function of $\bar{y}_f$, a shift in $\bar{y}_f$ shifts the average final offer by the same amount. In fact, each final offer shifts by the same amount as $\bar{y}_f$ $(\frac{dy_a}{d\bar{y}_f} = \frac{dy_b}{d\bar{y}_f} = 1)$ because

$$y_a = \frac{y_a + y_b}{2} + \frac{1}{2}(y_a - y_b)$$

and

$$y_b = \frac{y_a + y_b}{2} - \frac{1}{2}(y_a - y_b).$$

[14]When $\delta_a = \delta_b = \delta$, $\frac{y_a + y_b}{2} = \bar{y}_f$ and $y_a - y_b = \frac{1}{\delta} \ell n(s\delta + 1)$. Thus, the final offers (y_a and y_b) can be solved for explicitly in terms of s and δ:

$$y_a = \frac{y_a + y_b}{2} + \frac{1}{2}(y_a - y_b) = \bar{y}_f + \frac{1}{2}(\frac{1}{\delta} \ell n(s\delta + 1))$$

and

$$y_b = \frac{y_a + y_b}{2} - \frac{1}{2}(y_a - y_b) = \bar{y}_f - \frac{1}{2}(\frac{1}{\delta} \ell n(s\delta + 1)).$$

Note that when $y_a - y_b = 0$, $y_a = y_b = \bar{y}_f$.

[15]See footnote 13.

[16]Recall that $\frac{\partial y_a}{\partial \bar{y}_f} = \frac{\partial y_b}{\partial \bar{y}_f} = \frac{\partial y_{ca}}{\partial \bar{y}_f} = \frac{\partial y_{cb}}{\partial \bar{y}_f} = 1$. Thus, the final offers and the limits of the contract zones will differ between the two solutions by exactly the difference between $\bar{y}_{fa}$ and $\bar{y}_{fb}$.

[17]In computing Z, $F(\frac{y_a + y_b}{2})$ and $(y_a - y_b)$ in equation (28) are those from <u>either</u> of the two solutions and not the actual outcomes.

[18]By the same reasoning the final offers will be farther apart when $\bar{y}_{fa} > \bar{y}_{fb}$ than when $\bar{y}_{fa} = \bar{y}_{fb} = \bar{y}_f$. y_a will be a function of $\bar{y}_{fa}$ and y_b will be a function of $\bar{y}_{fb}$ so that

$$y_a - y_b = (y_a - y_b)_0 + (\bar{y}_{fa} - \bar{y}_{fb})$$

where $(y_a - y_b)_0$ is the difference between the final offers in the case of identical priors.

[19]Strictly speaking, the determination of the precise outcome within any contract zone is beyond the scope of this study. However, it is reasonable that movements in the center of the contract zone are suggestive of movements in the average negotiated settlements as the parameters of the model changes.

[20]This comparison must be done after appropriately standardizing for differences in conditions between bargaining relationships. Presumably they would be accounted for by differences in $\bar{y}_f$.

[21]If the pie is split approximately evenly, $\delta_a = 4$ implies a relative risk aversion of two while $\delta_b = 1$ implies a relative risk aversion of one half. If party a is thought to be the union, a relative risk aversion of two is in line with Farber's (1978) estimate of the relative risk aversion for the United Mine Workers. No estimates of the risk aversion of public sector employers are available, but Farber and Katz (1979) argue that public sector employers are likely to be less risk averse than the unions.

[22]This turns the standard argument that the parties know better than the arbitrator on its head, but this becomes crucial when it is realized that the potential arbitrator's behavior is what controls in negotiations in any case.

[23]This is demonstrated in the next section.

[24]It is important to keep in mind that the degrees of risk aversion are not control variables. Policy makers may be able to manipulate the level of uncertainty (s), but δ is a utility function parameter that is determined exogenously. The above discussion merely serves to illustrate some of the properties of the model.

[25] Farber and Katz (1979) develop this model in detail. For that reason the following analysis is rather terse.

[26] More formally, $$E(y)_{ARB} = \int_{\bar{y}_f - s/2}^{\bar{y}_f + s/2} y \frac{1}{s} dy = \bar{y}_f.$$

[27] An additional piece of support for this conclusion is that the contract zone in conventional arbitration must include the mean fair award $\bar{y}_f$. This is clear from equations (38) and (39) by noting that the expressions in brackets are less than one. It was shown above (see Table 3) that the FOA induced contract zone does not necessarily include $\bar{y}_f$.

[28] A qualification to this conclusion is based on the fact that as the parties gain information over time about y_f from previous experience with the arbitration schemes the uncertainty is reduced (s falls). It may be true that the information about y_f contained in an ARB award is greater than the information contained in a FOA award. Thus, FOA may be better than ARB at maintaining a given level of uncertainty over time. To the extent that this is true FOA will have a long run advantage over ARB in maintaining a contract zone. However, it must be noted that the structure of most public sector labor dispute settlement procedures (including FOA and ARB) both encourage communication with arbitrators (or neutral "factfinders" at an earlier stage) and provide a standard set of criteria which the arbitrators are supposed to use in making an award (i.e., in choosing y_f). These characteristics of the laws tend to shrink the uncertainty about y_f and may discourage real collective bargaining.

[29] Certainly none of the relevant statutes include risk aversion as a criterion to be considered by the arbitrator in forming the award. It is the arbitration processes themselves which induce this consideration.

[30] See Table 2.

[31] See Friend and Blume (1975) and Farber (1979). See also footnote 18.

[32] See Table 3 and Farber and Katz (1979) for a discussion of why public sector employers might be less risk averse then the unions. See also footnote 21. Empirical work in this area would be very valuable for evaluating these issues.

[33] See Table 5.

[34] See Table 3.

[35] This includes neutral factfinding reports which must give the parties a good idea of what the arbitrator will think is fair and hence shrink the uncertainty. This will discourage real bargaining and even negotiated settlements will reflect this information reducing the scope of bargaining. Perhaps skillful mediation <u>without</u> factfinding is a useful pre-impasse intervention.

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