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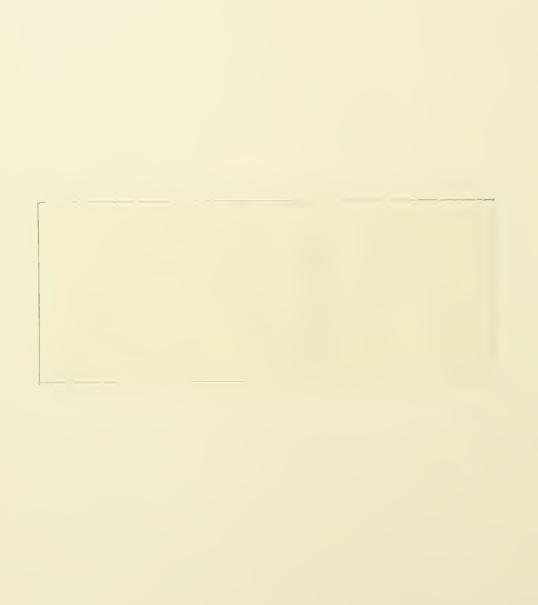
A COLLAPSE OF THE FIXED RATE REGIME
WITH A DISCRETE ALIGNMENT OF THE EXCHANGE RATE

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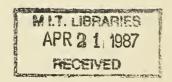
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#### ABSTRACT

A collapse of a fixed exchange rate regime is often characterized by the monetary authorities' loss of reserves in an astronomical rate and a discrete exchange rate alignment. The existing literature explains the former, but not the latter. This paper advances a mechanism of a collapse that explains the both. Uncertainty over the monetary authorities' decision on abandoning the fixed rate gives the exchange market intervention an unstable nature which ultimately manifests itself in a collapse with a loss of reserves in an astronomical rate. The uncertainty permits a discrete alignment by keeping arbitrage from equating the pre- and post-collapse exchange rate.



The collapse of the fixed exchange rate regime after overwhelming speculative attack is one of the most dramatic, spectacular, and so publicized economic phenomena. Nonetheless, no rigorous theoretical study of the collapse had been attempted until recently. Paul Krugman (1979), on the basis of Salant and Henderson (1978), initiated theoretical investigation of the collapse. He succeeded in explaining occurence and timing of speculative attack that terminates a fixed rate regime by very quickly depriving the monetary authorities of their reserves. Obstfeld (1984) extended Krugman's analysis to a plausible case in which the monetary authorities will reinstitute a fixed rate regime with a devalued exchange rate after a transition period of a floating regime that follows a collapse of a fixed rate. Flood and Garber (1984a) analyzed a collapse of a fixed rate regime under stochastic evolution of domestic credit. By so doing, Flood and Garber provided a reason for the forward premium that had been left unexplained by Krugman, though they were frequently observed before collapses. Obstfeld (Section I, 1986) corrected an important point in the Flood and Garber model. Obstfeld (Section II, 1986) showed that a collapse of the fixed rate regime could be a self-fulfilling event.

These contributions have laid the basic framework for the analysis of collapse, and provided important insights into the collapse. Yet, there is a prominent feature of the collapse that remains to be explained in a satisfying manner. Besides the monetary authorities' large and quick loss of reserves, a discrete exchange rate alignment immediately after a collapse frequently characterizes a collapse of a fixed rate regime. However, Krugman's pioneering work does involve the loss of

reserves, but does not the discrete alignment. Of course, cast in a discrete time (period) model, the Krugman model would imply a discrete change in the exchange rate after a collapse. However, such a discrete change does not capture observed discrete changes in the exchange rate associated with collapses.

In a self-fulfilling collapse model of Obstfeld (Section II, 1986), a collapse may occur much after the shadow exchange rate exceeded the predetermined fixed rate. Therefore, Obstfeld's model predicts the post-collapse exchange rate alignment which is comparable in terms of the magnitude to actual alignments associated with collapses. A collapse in his model is not an inevitable event, and is to be caused by possibly non-economic extrinsic incidents (such as sunspot) which private individuals believe to trigger a collapse. It is an interesting finding that an extraneous factor causes a collapse. However, following Krugman's original work, I restrict the scope of this paper to an inevitable collapse to be caused by internal logic.

 $<sup>0 &</sup>lt; (S_{T+1}-S)/S < (D_{T+1}-D)/\alpha S < \mu_{T+1}/\alpha S < \mu_{T+1}/M_T < \mu_{T+1}/D_T$  The fourth inequality follows from the positive home interest and equilibrium of the domestic money market. A realistic figure for  $\mu_{T+1}/D_T$  under inflationary credit policy would be less than 20% a year, and so less than 0.05% a day. As a collapse occurs in the matter of a day, a period in a collapse model is appropriately interpreted as a day in calender time. Then, the above inequality implies the Flood-Garber-Obstfeld model predicts less than 0.05% a day alignment of the exchange rate in the event of a collapse. This prediction is too small as a 10% devaluation often occurs in a day at the time of collapses.

This paper advances a mechanism of an inevitable collapse of the fixed rate regime that differs from the one in the literature. The mechanism is of interest, because a collapse that occurs by it involves a discrete exchange rate alignment as well as the monetary authorities' quick loss of reserves.

The model of this paper assumes that private individuals are uncertain over the timing of a collapse because the monetary authorities' decision on abandoning a fixed rate is not made according to a predetermined and well-defined principle such as the minimum reserves. Private individuals have the perceived probability on that decision. The probability depends upon the amount of reserves the authorities presently have.

The model shows that, if not terminated by depletion of reserves the monetary authorities are willing to use for defending a fixed rate, a fixed rate regime will be forced to collapse inevitably. The reason for the forced collapse in this paper is an unstable nature of the exchange market intervention. In response to an additional domestic credit, the monetary authorities must sell foreign money in the exchange market to keep the money market in equilibrium. Naturally, private individuals interpret the reduction in reserves as a signal of increased imminence of a collapse. Given an anticipation of home currency devaluation in the event of a collapse, then, the individuals think the prospect of capital gains from holding foreign assets becomes improved. Therefore, current intervention in the exchange market decreases subsequent demand for domestic money. Accordingly, the monetary authorities have to sell an ever increasing amount of foreign assets in response to successive

addition of domestic credit of the same magnitude. In the end, even the sale of foreign money in the infinite rate (per time) cannot keep the market in equilibrium. This means equilibrium of the money market has ceased to exist in the fixed rate regime. For this reason, the monetary authorities are forced to abandon the fixed rate regardless of their intention of defending the fixed rate further. Apparently, the forced collapse involves a loss of reserves in an astronomical rate just before a collapse.

The reason for a forced collapse differs from the reason for a collapse in Krugman (1979) and Flood and Garber (1984a) in which the regime collapses because the monetary authorities lose their reserves in speculative attack. If the authorities were willing (able) to use more reserves for defending the fixed rate, the regime could survive longer in their models. By contrast, such willingness does not postpone a forced collapse in this paper. I believe a forced collapse captures at least one of real collapses, the collapse of the 360 yen/dollar rate of August 1971, better than Krugman's and Flood and Garber's collapses.

The exchange rate changes discretely in the event of a collapse for the following reason. If individuals expect the exchange rate immediately after a collapse to be lower than, or equal to the precollapse fixed rate, the money market in the pre-collapse fixed rate regime is in equilibrium with the prospect of capital loss from holding foreign assets. On the other hand, the money market immediately after a collapse is in equilibrium with the prospect of capital gains from holding foreign bonds, because of an anticipation of continuing devaluation of the home currency in the post-collapse flexible rate

regime. Money supply just before a collapse being the same as that immediately after it, — then the post-collapse exchange rate must be expected to be higher than the fixed rate. This contradicts the initial supposition. Therefore, individuals expect the discrete home currency depreciation to accompany a collapse, and this expectation realizes when the fixed regime indeed collapses. As Obstfeld (1986) pointed out in criticizing Flood and Garber (1984), arbitrage does not work in the way Krugman (1979) envisages to equate the post and pre-collapse exchange rate if there is uncertainty over the timing of a collapse.

This paper is organized as follows. Section I presents a model of the fixed exchange rate regime collapse. Attention is focused upon calculation of the expected rate of capital gain. Section II shows the fixed rate regime inevitably collapses. Section IV gives concluding remarks. Throughout this paper, the exchange rate means the price of foreign currency in terms of home currency.

Unlike the Krugman's model, no change in money supply occurs at the instant of a collapse in this paper.

#### I. The Model

The model of this paper is based upon the one developed by Flood and Garber (Section 2, 1984).

#### The Flexible Exchange Rate Regime

Flood and Garber introduced the following simple linear model of the flexible exchange rate:

(1) 
$$M(t)/P(t) = a_0 - a_2i(t)$$
,

$$(2) M(t) = R(t) + D(t),$$

(3) 
$$\dot{D}(t) = \mu, \mu > 0,$$

$$(4) P(t) = P*S(t).$$

(5) 
$$i(t) = i^* + \dot{S}(t)/S(t)$$
.

Notations in the above are the same as in Flood and Garber (1984). M(t), P(t), and i(t) respectively represent domestic money supply, the domestic price level, and the domestic interest rate at time t. Equation (1) then specifies equilibrium of the home money market.  $a_0$  and  $a_1$  are positive coefficients. In equation (2), D(t) and R(t) are the amount of domestic credit and the book value of foreign assets held by the monetary authorities (reserves) at t. Equation (2) means the money supply identity under the 100% reserve system, or the definition of the monetary

base. In the latter case, (1) must be interpreted as equilibrium of demand and supply of the monetary base. Equation (3) gives evolution of domestic credit over time. With P\* and S(t) being the given foreign price level and the exchange rate at time t, equation (4) represents purchasing power parity. Variable i\* is the given foreign interest rate. Under the assumptions of perfect substitutability between foreign and domestic bonds and perfect capital mobility, equation (5) is the arbitrage condition between foreign and domestic bonds.

Let  $\alpha = a_1 P^*$  and  $\beta = a_0 P^* - a_1 P^*i^*$ . Naturally,  $\alpha > 0$ . Following Flood and Garber, I assume  $\beta > 0$ . Given reserves R, the exchange rate at time t under perfect forsight in a floating exchange rate regime is given by

$$\tilde{s}(t) = \frac{\alpha \mu}{\beta^2} + \frac{R+D(t)}{\beta}$$
.

 $\widetilde{S}(t)$  defined by (6) is similar to, yet slightly different from Flood and Garber's shadow exchange rate which is defined as  $\alpha\mu/\beta^2$  +  $(R(t)+D(t)/\beta$ . If the monetary authorities, at time s, stop supporting the predetermined fixed rate when they have R(s) reserves, the exchange rate afterwards, say at t, is  $\alpha\mu/\beta^2$  +  $(R(s)+D(t))/\beta$  according to (6).

#### The Fixed Exchange Rate Regime

Let us turn to the fixed exchange rate regime. In Krugman (1979), Flood and Garber (1984), and Obstfeld (1985), private individuals know in advance and for certainty that the monetary authorities will abandon the fixed rate when the reserves reach some pre-determined, known minimum level. By contrast, I assume in this paper that private individuals do not have the exact knowledge about circumstances that will lead the

authorities to decide to abandon the fixed rate. This may be not because of sheer ignorance of the minimum reserves on the part of private individuals, but rather because the monetary authorities are not dictated by the objective, unambiguous and pre-determined rule, such as the minimum reserves, that triggers abandonment of the fixed rate. The monetary authorities abandon the fixed rate simply when they come to be somehow convinced it is now necessary.

Accordingly, private individuals are assumed to have probabilistic belief with regard to the monetary authorities' decision on abandonment of a fixed rate. Specifically, F(R,t) represents the private individuals' perceived probability that the monetary authorities with given R amount of reserves will abandon the fixed exchange rate in less than, or equal to t time. As the amount of reserves the authorities have is an important factor determining the monetary authorities ability to defend the fixed rate, and hence viability of the fixed rate, the amount of reserves is relevant to private individuals' perception of the probability.

I assume F(R,0)=0 for any  $R>\overline{R}$ . Without this assumption, the probability that the monetary authorities abandon the fixed rate in a finite time, and, in particular, an instant later becomes one. This contradicts our purpose of introducing subjective uncertainty over the authorities' decision. Economically, F(R,0)=0 for  $R>\overline{R}$  captures private individuals' view that the fixed rate will be abandoned only after consultation between the finance minister, the governor of the central bank, and so forth as far as R is not so small. The consultation

Possibly,  $\lim_{t\to\infty} F(R,t) < 1$ ; namely, it is probable that the authorities with R reserves will never abandon the fixed rate.

is necessary because abandoning a fixed rate is not decided automatically according to some pre-determined and well-defined rule as was already mentioned.

I also assume F(R,t)=1 for any  $t\geq 0$  and  $R\leq \overline{R}$ . That is, private individuals believe that the monetary authorities for certainty abandon the fixed rate immediately after their reserves reach  $\overline{R}$ .  $\overline{R}$  is an extremely low level so that no consultation is necessary for deciding on the abandonment.

From the viewpoint of private individuals, it is always possible that the monetary authorities abandon the fixed rate within a few seconds, and hence that the exchange rate changes in a few seconds. Therefore, under our assumtpion, the expected exchange rate matters in individuals' decisions, even under the fixed rate regime. Let us now calculate the expected exchange rate.

Let t denote the present time. Let  $s \ge t$ . Suppose the monetary authorities with R(s) reserves abandon the fixed rate at time s, and the economy then moves to a flexible rate system at time s, the exchange rate at time t' (> s > t) will be, by (6),  $\alpha\mu/\beta^2 + (R(s) + D(t'))/\beta$ . Let S(s,t') denote this exchange rate.

Let  $t_k = t + k\delta$  (k=0,1,2,...n), and  $\delta = (t'-t)/n$ . The probabilities that the monetary authorities abandon the fixed rate between  $t_k$  and  $t_{k+1}$  (k=0,1,2,...,n-1) are appropriately

$$\begin{array}{l} k-1 \\ \Pi \left[1-F(R(t_i), \delta)\right] \cdot F(R(t_k), \delta) \text{ for } 1 \leq k < n-1, \\ i=0 \end{array}$$

$$F(R(t_0), \delta)$$
.

The probability that the monetary authorities do not abandon the fixed rate by t' is approximately

$$\Pi \left[1 - F(R(t_i), \delta)\right].$$
i=0

Let f(R) be  $\partial F(R,0)/\partial t$  for  $R > \overline{R}$ . Then,

$$F(R, \delta) = F(R, 0) + \delta \circ f(R)$$
  
=  $\delta \circ f(R)$ 

Further, by the Taylor expansion,  $e^{-x} = 1 - x$ . Hence, the above three probabilities can be approximated by

$$\begin{aligned} &\overset{k-1}{\mathbb{H}} \left( 1 - \delta f(R(t_{\underline{i}})) \right) \, \delta f(R(t_{\underline{k}})) \\ &\overset{\cdot}{=} \, \delta f(R(t_{\underline{k}})) \, \exp \left[ - \delta \, \sum_{\underline{i} = 0}^{k-1} f(R(t_{\underline{i}})) \right] \\ &\overset{\cdot}{=} \, \delta f(R(t_{\underline{k}})) \, \exp \left[ - \int_{\underline{t}}^{t_{\underline{k}}} f(R(s)) \mathrm{d}s \right] \quad \text{for } 1 \leq k \leq n-1, \\ &\delta f(R(t_{\underline{0}})), \end{aligned}$$

and 
$$\exp \left[-\int_{t}^{t} f(R(s))ds\right]$$
.

Let  $\overline{S}$  be the predetermined fixed exchange rate. Then, the expected exchange rate at time t' is approximately

$$\sum_{k=0}^{n-1} \delta \cdot S(t_{k+1}, t') f(R(t_k)) exp(-\int_{t}^{t_k} f(R(s))ds)$$

$$+\overline{S} \exp \left[-\int_{t}^{t'} f(R(s))ds\right].$$

Hence, the expected exchange rate at time t' is

$$\int_{t}^{t'} S(s,t') f(R(s)) \exp \left[-\int_{t}^{s} f(R(\tau)) d\tau\right] ds$$

+ 
$$\overline{S} \exp \left[-\int_{t}^{t} f(R(s)) ds\right]$$
.

Let E(t,t') denote the above formula for the expected exchange rate.

Because of risk neutrality and perfect capital mobility,

$$E(t,t')e^{i*(t'-t)} = \overline{S} \exp\left[\int_{t}^{t'} i(s)ds\right]$$

equivalently,

$$\frac{E(t,t')-\overline{S}}{\overline{S}}=\exp\left[\int_{t}^{t'}i(s)ds-i*(t'-t)\right]-1$$

must hold. Dividing the RHS of the above equality by t'-t, and letting t'-t tend to zero give i(t)-i\*. On the other hand,

$$\frac{E(t,t')-\overline{S}}{(t'-t)\overline{S}} = \frac{1}{(t'-t)\overline{S}} \int_{t}^{t'} S(s \cdot t') f(R(s)) \exp\left[-\int_{t}^{s} f(R(\tau)) d\tau\right] ds$$

$$+ \frac{1}{t'-t} \left[ \exp \left[ - \int_{t}^{t'} f(R(s)) ds \right] - 1 \right].$$

Then,

$$\lim_{t'\to t} \frac{E-\overline{S}}{(t'-t)\overline{S}} = f(R(t)) S(t,t)/\overline{S} - f(R(t))$$

= 
$$f(R(t))(S(t,t)-\overline{S})/\overline{S}$$
.

Accordingly,

(7) 
$$i(t) = i^* + f(R(t)) \frac{S(t,t) - \overline{S}}{\overline{S}}$$

must hold instead of (5) as the arbitrage condition between foreign and domestic bonds. Equation (7) implies the forward premium exists even in the fixed exchange rate regime.

Note that the expected rate (per time) of capital gain from holding foreign bonds ( $\lim_{t\to 0} (E-\overline{S})/(t'-t)\overline{S}$ ) can be defined in the present model,  $t'\to t$  though a change in the exchange rate between pre- and post-abandonment ( $S(t,t)-\overline{S}$ ) can be discrete. This is because the probability of the monetary authorities' abandoning the fixed rate in an infinitesimal interval (dt) is negligible and dependent on the length of the interval. Then, the expected magnitude of capital gains from holding foreign bonds (the magnitude of a change in the exchange rate times the probability) in an infinitesimal interval is also negligible, and dependent upon the

length of the period. Then, the expected rate (per time) of capital gain can be defined.

Equation (1) (the equilibrium condition of the money market) is still right for the fixed rate regime. Then, in view of (6) and (7), equilibrium of the domestic money market under the fixed rate is specified as:

$$M(t) = a_0 P \overline{S} - a_1 P \overline{S} [i + f(R(t)) {\frac{\alpha \mu}{\beta^2} + \frac{M(t)}{\beta} - \overline{S}}/\overline{S}].$$

Let  $\theta = \alpha \mu / \beta - \beta \overline{S}$ . (2) still applies in the fixed rate regime. Then, the above equilibrium condition becomes

(8) 
$$M(t) - \beta \overline{S} = -\frac{\alpha}{\beta} f(M(t) - D(t))(M(t) + \theta).$$

Following Flood and Garber, I assume  $\beta$  is positive. Domestic credit evolves according to (3) regardless of the exchange rate regimes. Hence, (8) and (3) govern evolution of the fixed rate regime over time.

#### II. A Collapse of the Fixed Exchange Rate Regime

Let g(M) be defined as  $M - \beta \overline{S} = -\frac{\alpha}{\beta} f(M-g(\mu))(\mu+\theta)$ . Function g(M) gives a solution of (8) if it exists. This function shows the amount of domestic credit necessary to maintain M amount of money supply. By implication, this function indicates equilibrium reserves, namely, the amount of reserves that, given domestic credit, keeps the money market in equilibrium in the fixed rate regime.

I impose the following assumption on  $f(\cdot)$ :

Assumption 1: 
$$f'(\cdot) < 0$$
,  $\lim_{R \to \infty} f(R) = 0$ , and  $\lim_{R \to \overline{R} + 0} f(R) = \infty$ .

This assumption would be a natural one. Intuitively speaking, the assumption means first that the "instantaneous" perceived probability of abandonment (f(R)dt) decreases as reserves held by the monetary authorities increase, second that the probability becomes zero if the authorities have the infinite amount of reserves, and third that the probability becomes one when the amount of reserves reaches a very small number.

Now I characterize the  $g(\cdot)$  function under Assumption 1. Suppose  $M-\beta\overline{S} \geq 0$  for some M for which  $g(\cdot)$  is defined. By (8),  $\alpha > 0$ ,  $\beta > 0$  and  $f(\cdot)$ ,  $M + \theta \leq 0$ . Then,  $\beta \overline{S} \leq -\theta$ . However,  $\beta \overline{S} + \theta = \alpha \mu/\beta > 0$ . A contradiction. Accordingly,  $-\theta < M < \beta \overline{S}$  for such M that g(M) is defined. By Assumption 1, one can uniquely solve (8) for D for any M between  $-\theta$  and  $\beta \overline{S}$ , in other words, g(M) uniquely exists for  $-\theta < M < \beta \overline{S}$ . It is also easily seen  $-\overline{R} + M > g(M)$ .

By (8), 
$$\lim_{M\to 0} f(M-g(M)) = -\beta(M-\beta\overline{S})/\alpha(M+\theta) = \infty$$

By Assumption 1, then, M - g(M) tends to  $\overline{R}$  when M tends to  $-\theta$  from the right. Therefore,  $\lim_{M \to -\theta + O} g(M) = -\theta - \overline{R}$ . Function g(M) as the solution of  $M \to -\theta + O$ (8) is defined only for  $-\theta < M < \beta \overline{S}$ . However, on account of the above limiting property, I extend the domain of g(M) to  $-\theta \le M < \beta \overline{S}$  by defining  $g(-\theta) = -\theta - \overline{R}$ .

When M approaches  $\beta \overline{S}$  from the left, f(M-g(M)) must approach zero. Then, by Assumption 1, M-g(M) tends to infinity in that event. It follows that g(M) goes to negative infinity as M approaches  $\beta \overline{S}$ .

One can show g(M) is continuous in  $-\theta \leq M < \beta \overline{S}$  (see Appendix for its proof). Further, it has been seen that g(M) tends to negative infinity as M tends to  $\beta \overline{S}$ . Then, g(M) has the maximum in  $-\theta \leq M < \beta \overline{S}$ . Let  $D^*$  be the maximum value of g(M), and let g(M) reach it at  $M = M^*$ . Let  $R^*$  denote  $M^* - D^*$ .

For the time being, I assume the following:

Assumption 2:  $-\theta < M^* < \beta \overline{S}$  and function g(M) has only one peak and no trough in  $-\theta \leq M < \beta \overline{S}$ .

I will abandon this assumption later, but, for the time being, I use this assumption as it facilitates the exposition. Assumption 2, for instance, holds for f(R) = c/(R-R) with  $0 < c \le 1$ . The preceding characterization of  $g(\cdot)$  and Assumption 2 allow one to depict  $g(\cdot)$  as in Figure 1.

Let the amount of domestic credit at time t be  $\overline{\text{OE}}$  in Figure 1. Then, either point A or B in the figure is equilibrium of the domestic

money market. I preclude B as equilibrium, because it is an unstable equilibrium.

In the fixed exchange rate regime, the variable equilibrating the money market is the amount of reserves the monetary authorities have; the monetary authorities' sales and purchases of foreign money in the exchange market keep the money market in equilibrium at the predetermined exchange rate. A natural and realistic postulate to make on the rule for the monetary authorities' exchange market intervention is to sell (buy) foreign money in response to an excess demand (supply) of foreign exchanges, in other words, in response to an excess supply (demand) of domestic money. This paper therefore postulates the monetary authorities conduct the intervention following this rule. Given this postulate of the authorities' behavior, only equilibrium that is stable under the intervention rule is realizable. Hence, one can preclude unstable equilibria from consideration under the postulate.

Point B is not a stable equilibrium under the intervention rule. This is seen as follows. On account of (8), a stable equilibrium under the intervention rule means  $d(-\frac{\alpha}{\beta}f(R)(M+\theta) - M+\beta\overline{S})/dR < 0$ , namely,  $\frac{\alpha}{\beta}f'(R)(M+\theta) + \frac{\alpha}{\beta}f(R) + 1 > 0.$  By the definition of  $g(\cdot)$ ,  $g'(M) = (1+\frac{\alpha}{\beta}f+\frac{\alpha}{\beta}(M+\theta)f')/\frac{\alpha}{\beta}(M+\theta)f'$ . At B, g'>0, and hence  $1+\frac{\alpha}{\beta}f+\frac{\alpha}{\beta}(M+\theta)f'<0$ . Therefore, B is not a stable equilibrium. By contrast, one can see that A is a stable one under the intervention rule.

The above result implies that the money market under the fixed rate regime stays on the declining portion of the g(M) curve in Figure 1. As D(t) increases according to (3), the economy monotonically moves towards C along the declining portion of the g(M) curve as arrows in Figure 1 shows.

If the monetary authorities cease to support the fixed rate before their reserves reach R\*, the fixed exchange rate regime then terminates. If the authorities do not do so, the economy eventually reaches C. When the economy reaches C, equilibrium does not exist any longer. The fixed rate regime must terminate here. Let  $t^* = (D^*-D(0))/\mu$ . We have obtained the following proposition:

Proposition 1: Under Assumptions 1 and 2, the fixed exchange rate regime has equilibrium only as far as D(t) is no greater than  $D^*$ , in other words, until  $t^*$ . Under the fixed exchange rate regime, that is, for  $0 \le t \le t^*$ , reserves change according to:

$$\dot{R}(t) = \frac{1-g'(M(t))}{g'(M(t))} < 0$$
 for  $t < t^*$ ,

and

$$\hat{R}(t^*) = \lim_{t \to t^*} \frac{1 - g'(M(t))}{g'(M(t))} = -\infty$$

Needless to say, the formula for  $\mathring{R}(t)$  and its sign in the proposition is obtained by differentiating (8) and using g' < 0 at a stable equilibrium.  $\mathring{R}(t^*) = -\infty$  obtains because  $g'(M(t^*)) = g'(M^*) = 0$ .

Let me explain implications of the proposition. First, it means that the fixed exchange rate regime inevitably collapses by t\* as far as domestic credit continues to expand. If the monetary authorities stop the exchange market intervention before their reserves reach R\*, the regime collapses at that time. I refer to this collapse as a collapse due to reserve depletion, because it is caused by the monetary

authorities' unwillingness to use more reserves for intervention.

Otherwise, a fixed rate regime collapses at t\* when the amount of reserves reaches R\*. Because there no longer exists equilibrium of the domestic money market under a fixed rate regime, the monetary authorities are forced to abandon the fixed rate at this time, even if they are still willing to sell more foreign money in order to support the fixed rate. I refer to this type of collapse as a forced collapse.

Second,  $\mathring{R}(t^*) = -\infty$  in the proposition implies that, just before a forced collapse, the authorities sell foreign money in an astronomical rate in response to speculative attack (see Figure 2). As a result, the authorities lose a large portion of their reserves in a very short time just before a forced collapse (see Figure 2). By contrast, in Krugman's model, a discrete loss of reserves occurs simultaneously with a collapse.

Third, while the existing literature fails to capture discrete exchange rate alignments which one often observes in the event of collapses, the proposition predicts a discrete alignment both in a forced collapse and in a collapse due to reserve depletion. Let t be the timing of a collapse. Since the fixed rate regime terminates at C or the declining portion of the g(•) curve, M(t) +  $\theta$  > 0, and hence  $\beta \left[\frac{R(t)+D(t)}{\beta} + \frac{\alpha\mu}{\beta^2} - \overline{S}\right] > 0.$  Therefore, as  $(R(t)+D(t))/\beta + \alpha\mu/\beta^2$  is the post-collapse exchange rate at t, the model of this paper predicts a discrete home currency devaluation in the event of a collapse.

To outside observers, a collapse in Krugman's model is a completely sudden event. However, one may say that, in reality, a collapse finally occurs after an interval of continuing crisis. A crisis continues to increase its momentum, ending up with the final collapse of a fixed rate

under overwhelming speculative attack. As Figure 2 shows, a forced collapse of this paper may capture this feature of a collapse.

Let me make a comment on a possible criticism of the present model. We have found that a fixed rate regime inevitably collapses when reserves reach  $R^*$ , a level higher than  $\overline{R}$  at which private individuals are assumed to expect a collapse to occur for certainty. For this feature of the model, some may criticize that expectations in this paper are not formed in a consistent manner with what will happen. This criticism is based upon the supposition that private individuals can know the value of  $R^*$  and hence that they can revise  $\overline{R}$  in such a way that  $\overline{R}$  does not diverge from  $R^*$  systematically. Without inside information on the monetary authorities' decision making, however, private individuals cannot learn the value of  $R^*$  in relation to  $\overline{R}$ , and hence are not in a position to keep  $\overline{R}$  from diverging from  $R^*$ .

One may say that, overwhelming speculative attack  $(R = -\infty)$  being an indication of a forced collapse, private individuals can distinguish a forced collapse from one due to reserves depletion, and hence they can learn the value of  $R^*$ . However, without Assumption 2 (a provisional assumption for expository convenience), as will be seen later, overwhelming specualtive attack occurs well before reserves reach  $R^*$ , and the authorities may abandon the fixed rate in that event. Hence, speculative attack is not an indication of a forced collapse, and a collapse accompanied by speculative attack does not necessarily reveal the value of  $R^*$ .

If the monetary authorities (in effect) stick to a fixed rate with a devalued rate even after a collapse rather than move to a flexible rate regime, one can infer the collapse was a forced one and can learn the value of R\*. However, I assume here that the authorities move to a full floating rate regime after a collapse.

#### Intuitive Account

Let me explain economic intuitions behind the collapse of the fixed rate regime and the discrete exchange alignment in this paper.

The fixed rate regime collapses if the monetary authorities becomes unable (unwilling) to tolerate further depletion of their reserves that will be caused by continuing domestic credit expansion. There is nothing surprising in this kind of collapse. This collapse is not a focus of this paper. This paper focuses on the forced collapse; the monetary authorities are forced to stop the exchange market intervention, even if they are willing to use more reserves in supporting the fixed rate. Let me accordingly concentrate on a forced collapse by supposing that the monetary authorities do not decide to stop intervention by t\*.

Suppose domestic credit is increased. This creates an excess supply of domestic money through two effects. First, additional domestic credit increases supply of money. Second, with the increased supply of domestic money, the exchange rate that would result in the event of a collapse of the fixed rate regime becomes higher than otherwise; a higher post-collapse exchange rate implies a higher expected rate of capital gains from holding foreign bonds; through arbitrage, a higher expected rate of capital rain raises the domestic interest rate; then, demand for domestic money decreases. For these two effects respectively on supply and demand, additional domestic credit creates an excess supply of domestic money.

It is the amount of the monetary authorities' reserves, in other words, their exchange market intervention that brings the domestic money market into equilibrium in the fixed exchange rate regime. Therefore,

the monetary authorities must sell foreign money and absorb domestic money to eliminate the excess supply created by the additional domestic credit. Thus, domestic credit expansion necessarily reduces the amount of reserves.

Private individuals naturally suppose that the monetary authorities' ability to maintain the fixed rate any further, and hence future viability of the fixed rate regime should depend upon the amount of reserves the authorities presently have. Hence, when reserves have decreased, private individuals interpret the decrease as a signal of increased imminence of a collapse. Given expectations of capital gains from holding foreign bonds in the event of a collapse (I will turn to this later), the increased imminence means an improved prospect of getting capital gains from foreign bonds. This prospect, through arbitrage, translates itself into a higher home interest, and then decrease demand for domestic money. Thus, current decrease in reserves has the effect of decreasing subsequent demand for domestic money.

Accordingly, current credit expansion not only creates a current excess supply of domestic money, but also has the effect of decreasing future demand for domestic money. Thus, as credit expansion continues, the monetary authorities have to intervene in the market in an ever increasing magnitude. In other words, the exchange market intervention is an unstable, cumulative process. Then, as the monetary authorities continue to sell reserves in an effort to offset continuing domestic credit expansion, the rate (per time) of reserve sale ultimately becomes infinite in the end (Point C in Figure 1). Put in another way, the monetary authorities will in the end become unable to keep the domestic

money market in equilibrium in whatever large rate they sell reserves in the exchange intervention. What is happening in this situation is that the effect of a decrease in reserves upon the prospect of capital gains from foreign assets outweighs its direct effect of reducing money supply. Private individuals relinquish whatever domestic money the monetary authorities try to pump up.

The exchange market intervention in the end ceases to be an equilibrating variable, and hence equilibrium no longer exists under the predetermined fixed rate. The monetary authorities are then forced to abandon the fixed rate regardless of their intention of defending the fixed rate further. The fixed rate regime is forced to collapse here. The economy moves to a flexible rate regime. The inherent unstable nature of the exchange market intervention due to its effect on expectations inevitably drives a fixed rate regime towards a collapse.

The forced collapse in this paper differs from a collapse in Krugman's and Flood and Garber's models in mechanisms bringing about collapses. The fixed rate regime collapses in their models, because the monetary authorities lose their reserves. Indeed, if the authorities are willing (able) to use more reserves, the regime could survive longer. By contrast, in the model of this paper, a forced collapse occurs at the same timing even if the authorities decide to use more reserves for the intervention.

There is at least one episode of a collapse which fits into the model of this paper better than Krugman's. It is the collapse of the 360 yen = 1 dollar rate of August 1971 after overwhelming speculative attack. At that time, it is not the U.S. government, but the Japanese government that sold yen to keep the U.S. dollar from depreciating. Then, it is

inconceivable that speculative attack deprived the defending monetary authorities of yen funds for the intervention. Even if the amount of the funds were limited for some reasons, one cannot think that the Japanese government ran out of the funds at that time. For, even after the collapse, the Japanese government continued to intervene in the market in an effort to keep yen from appreciating substantially against dollar. If the government had lost their funds for the intervention in the way of Krugman's model, it could not have intervened in the market after the collapse.

A discrete change in the exchange rate after a collapse is a main subject of this paper. A discrete change occurs in both a forced collapse and a collapse due to reserves depletion. The reason for a discrete alignment of the exchange rate in the event of a collapse is as follows. Suppose that the exchange rate in the event of a collapse is expected to be less than or equal to the fixed rate. If the fixed rate regime collapses in an instant, those holding foreign bonds now incur capital losses. Then, the domestic money market under the pre-collapse fixed rate regime is in equilibrium with the prospect of capital loss from holding foreign bonds. On the other hand, an anticipation of domestic credit expansion to continue even after a collapse of the fixed rate generates an anticipation of continuing home currency depreciation in the post-collapse floating exchange rate regime. Then, the domestic money market immediately after a collapse of the fixed rate regime is in equilibrium with the prospect of capital gains from foreign bonds. Supply of domestic money just before and immediately after a collapse are the same, but on account of the difference in the prospect of capital gains, demand for it is smaller immediately after a collapse than just

before it. Then, the post-collapse exchange rate must be higher than the pre-collapse fixed rate. This contradicts the initial supposition.

Therefore, home currency must be expected to, and does depreciate discretely in the event a collapse.

Private individuals are aware that a discrete alignment occurs in the event of a collapse, but, they are uncertain over the timing of the collapse. Then, as Obstfeld (1986) criticizes the second model of Flood and Garber (1984a), arbitrage does not close the gap between the pre- and post-collapse exchanges rate in the way Krugman (1979) considers, unless individuals have an agreement on the timing of a collapse, or unless an individual or a body of colluded individuals is large enough to be able to buy up entire reserve alone. Under uncertainty over the timing of a collapse, arbitrage equates only the foreign interest rate plus the expected rate of capital gain from foreign bonds with the domestic interest rate.

#### A Forced Collapse without a Discrete Alignment

In Assumption 2, g(M) is assumed to reach its maximum at M larger than  $-\theta$ . I now assume only the latter part of Assumption 2:

Assumption 2': g(M) has only one peak, and no trough.

Under Assumption 2', it is possible that g(M) defined for  $-\theta \le M < \beta \overline{S}$  may reach its maximum at  $-\theta$ , namely  $M^* = -\theta$ . Figure 3 depicts this rather exceptional case. The economy moves towards point C in the figure, and the fixed rate regime collapses there. In this case,

the amount of reserves at the time of a forced collapse is  $\overline{R}$ ; that is, this is the case in which  $\overline{R}$  equals  $R^*$ .

The reason for this collapse is the same as before; equilibrium no longer exists when the economy reaches C. In this case, however, a discrete exchange rate alignment is not associated with the collapse. For, M\* = -0 means the post-collapse exchange rate is the same as the pre-collapse fixed rate. Economic intuition behind this is as follows. Just before a collapse, private individuals feel a collapse is almost certain. This high subjective instantaneous probability of collapse compensates for a prospect of negligible capital gains from foreign bonds in such a way that the expected rate of capital gains from holding foreign bonds before a collapse equals the post-collapse expected rate of home currency depreciation. With no change in supply of money between just before and immediately after a collapse, then, the exchange rate immediately after a collapse should be the same as the pre-collapse fixed rate.

Speculative attack  $(\mathring{R} = -\infty)$  may occur in this rather exceptional collapse, but generally does not.

#### A Collapse after Several Crises

I will now discard Assumption 2 entirely. Without the assumption, one can depict g(M) as in Figure 4.

Suppose the initial amount of domestic credit to be  $D_A$  in Figure 4. Then, point A gives the initial equilibrium of the domestic money market. Domestic credit expands according to  $\dot{D}=\mu$ . The economy stays on the declining portion of the g(M) curve, and monotonically moves upwards on the curves as domestic credit expands. If the monetary authorities

abandon the fixed rate with reserves larger than  $M_B - D_B$ , the fixed rate regime then terminates before the economy reaches B. Otherwise, the economy reaches B in a finite time.

When the economy reaches E, there are two possibilities to happen: the monetary authorities, forced to sell foreign money in the infinite rate, may abandon the fixed rate; or they are stubborn enough to pass  $\overline{\text{BE}}$  amount of reserves to private agents instantaneously when the economy reaches B. A termination of a fixed rate regime in the first possibility has an appearance of a forced collapse. However, it is not a forced collapse, but a collapse due to reserves depletion. The fixed rate regime terminates at B, because the monetary authorities are unwilling to use more reserves to defend the fixed rate. If they use more reserves, the domestic money market can move to E, another equilibrium under the fixed rate, and continues to have equilibrium with a larger domestic credit. This is the second possibility.

In the second possibility, the monetary authorities defend the regime by passing the  $\overline{BE}$  amount of foreign assets to private individuals (in return, reducing domestic money stock by  $\overline{BE}$  instantly). A discrete change in domestic money stock transfers the economy from B to E instantly. At B, the fixed rate regime was brought to a brink of a collapse, but the monetary authorities succeeded in withstanding the crisis, and defending the regime.

After the crisis at B, the economy moves towards F, another crisis, unless the monetary authorities stop selling foreign money before the economy reaches F. At F, the monetary authorities face speculative attack again, and sell foreign money in an astronomical rate again. The regime can survive this crisis as well if the authorities pass the FG

amount of foreign assets to private individuals instantly. The economy instantly moves from F to G, and begins to move upwards towards C according to domestic credit expansion. Finally, the economy reaches C. The fixed regime cannot withstand this crisis. The fixed rate regime is forced to collapse here as it is under Assumption 2.

Even if Assumtpion 2 does not hold, the fixed rate regime collapses inevitably. However, without the assumption, it may occur after several crises are overcome.

Without Assumption 2, one can imagine more complicated cases than the one depicted by Figure 4. In the case of Figure 5, for example, there are multiple possible paths heading for a collapse. With the  $\overline{DO}$  amount of initial domestic credit, an economy may start with point A, or A' or A', and end up with C or C' or C'. A more complicated case than these is an economy starting with A, moving from B to E', and ending up with C'. In all these complicated cases, the same thing happen: the fixed rate regime inevitably collapses. Unless the economy ends up with C', the final moment of the fixed rate regime is accompanied by overwhelming speculative attack and a discrete alignment of the exchange rate.

We have obtained a sufficient condition for a collapse of a fixed rate regime accompanied by a discrete alignment of the exchange rate and speculative attack:

Proposition 2: Suppose a fixed rate regime does not collapse due to reserves depletion. If g(M) does not have a local maximum at  $-\theta$ , the fixed rate regime collapses with a discrete exchange alignment and speculative attack.

## III. Conclusion

This paper has studied the circumstances in which private individuals are uncertain over the timing of the monetary authorities' decision to abandon the fixed rate. In that circumstances, private individuals evaluate imminence of the monetary authorities' abandoning the fixed rate on the basis of the amount of reserves the authorities presently have. The dependence of the perceived probability of a collapse on the amount of reserves gives an unstable nature to the exchange market intervention. This nature in the end manifests itself in a collapse of a fixed rate regime accompanied the monetary authorities' loss of their reserves in an astronomical rate. The uncertainty over the timing of a collapse keeps arbitrage from equating the pre- and post-collapse exchange rate with each other. This allows a discrete alignment to accompany a collapse of a fixed rate regime.

## APPENDIX

I will prove the following proposition in this appendix.

<u>Lemma</u>: Function g(M) is continuous for  $-\theta \leq M < \beta \overline{S}$ .

<u>Proof</u>: Let  $D_n = g(M_n)$ , and  $\lim_{n \to \infty} M_n = M^0$ . Suppose  $-\theta < M^0 < \beta \overline{S}$ .

Let M" and M' be numbers such that  $-\theta < M' < M^O < M'' < \beta \overline{S}$ . By (8) and  $f'(\cdot) < 0$ , for sufficiently large n,

$$\overline{\mathbb{R}} < f^{-1} \left( \frac{\beta}{\alpha} \{ -1 + \frac{\theta + \beta \overline{S}}{\mathbb{N}^{'} + \theta} \} \right) < \mathbb{M}_{n} - \mathbb{D}_{n} < f^{-1} \left( \frac{\beta}{\alpha} \{ -1 + \frac{\theta + \beta \overline{S}}{\mathbb{N}^{''} + \theta} \} \right).$$

Then, since  $\{M_n\}$  is bounded,  $\{D_n\}$  is also bounded. Accordingly, some of subsequences of  $\{D_n\}$  converge. Let  $\{D_n\}$  denote such a subsequence. Let  $\lim_{k \to \infty} D_k = D^0$ . By the above inequality,  $\lim_{k \to \infty} (M_n - D_n) > \overline{R}$ .

Then, by (8) and continuity of f(R) for  $R > \overline{R}$ ,

$$M^{O} - \beta \overline{S} = -\frac{\alpha}{\beta} f(M^{O} - D^{O})(M^{O} + \theta),$$

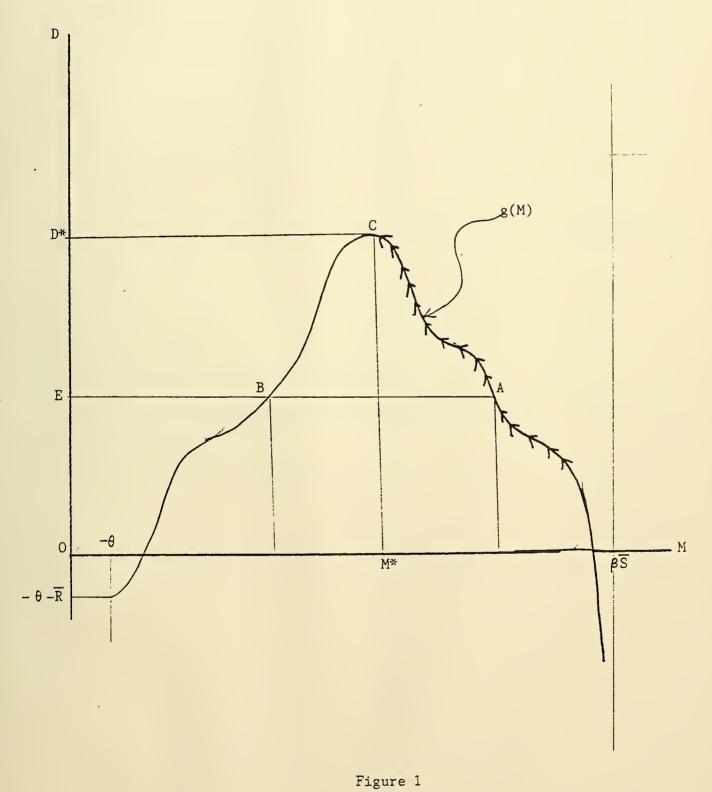
in other words,  $\overline{D}^O = g(\overline{M}^O)$ . This holds for any converging subsequence of  $\{D_n\}$ . Hence,  $g(\overline{M})$  is continuous in  $-\theta < \overline{M} < \beta \overline{S}$ . Finally,  $g(\overline{M})$  is continuous at  $\overline{M} = -\theta$  by the definition of  $g(-\theta)$ . (Q.E.D.)

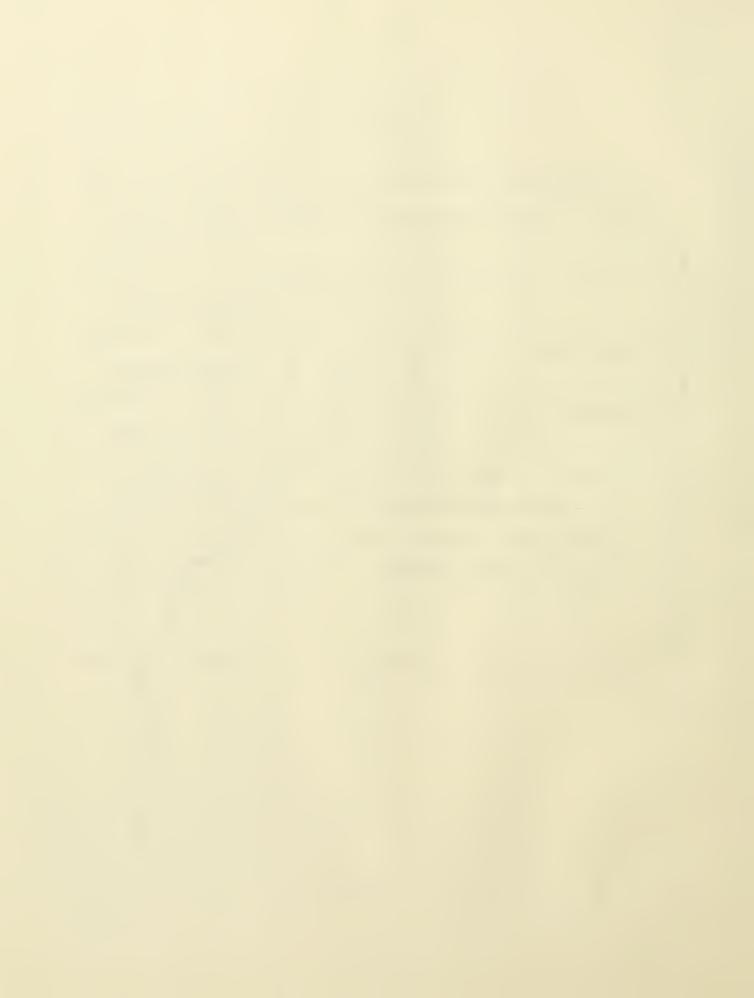
## REFERENCES

- Flood, R.P. and P.M. Garber, "Collapsing Exchange Rate Regimes: Some
  Linear Examples," Journal of International Economics, 17, (August
  1984a), pp. 1-13.
- \_\_\_\_\_ and \_\_\_\_\_, "Gold Monetization and Gold Dicipline," <u>Journal of</u>

  Political Economy, 92, (February 1984b), pp. 90-107.
- Krugman, P., "A Model of Balance-of-Payments Crises," <u>Journal of Money</u>
  Credit and Banking, 11, (August 1979), pp. 311-25.
- Obstfeld, M., "Balance-of-Payments Crises and Devaluation," <u>Journal of</u>
  <u>Money, Credit and Banking</u>, 16, (May 1984), pp. 208-17.
- ----, "Rational and Self-Fulfilling Balance-of-Payments Crises,"

  American Economic Review, 76, (March 1986), pp. 72-81.
- Salant, S.W. and D.W. Henderson, "Market Anticipations of Government Policies and the Price of Gold," <u>Journal of Political Economy</u>, 86 (August 1978), pp. 627-48.





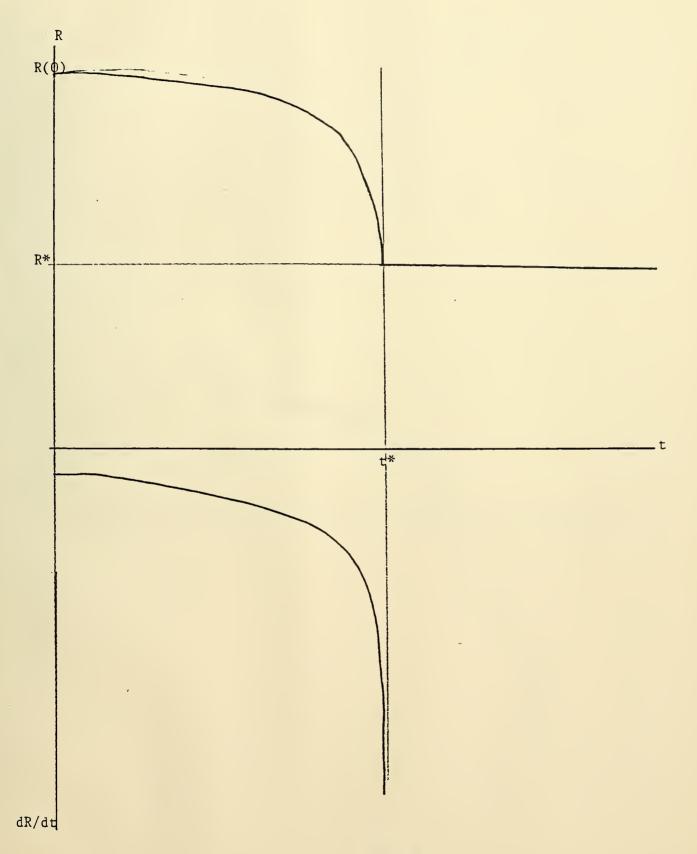


Figure 2



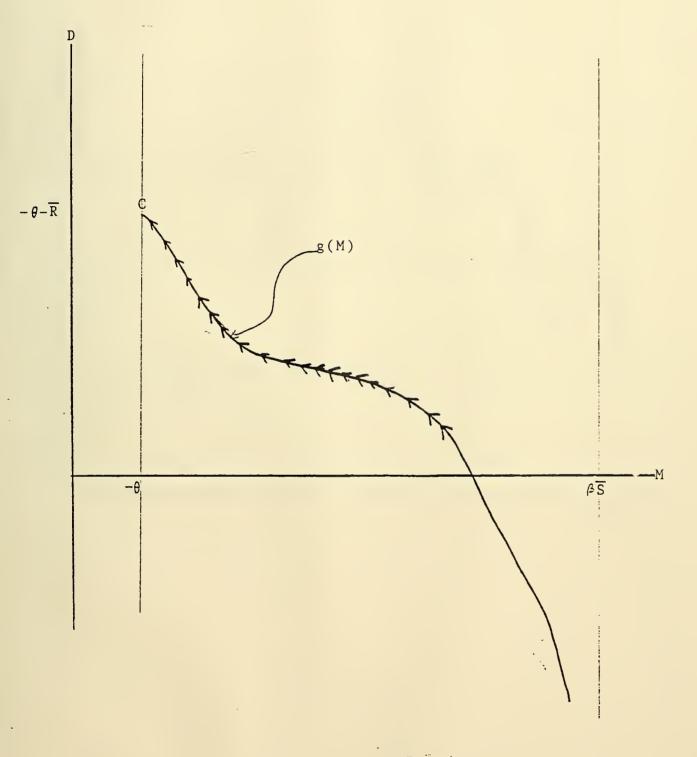
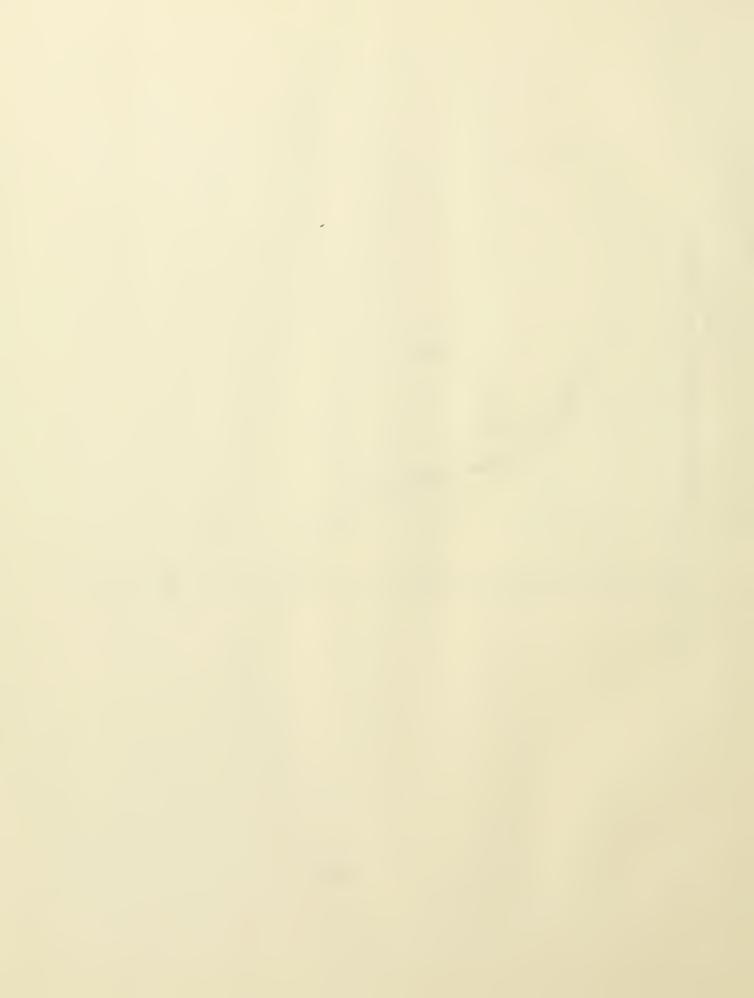
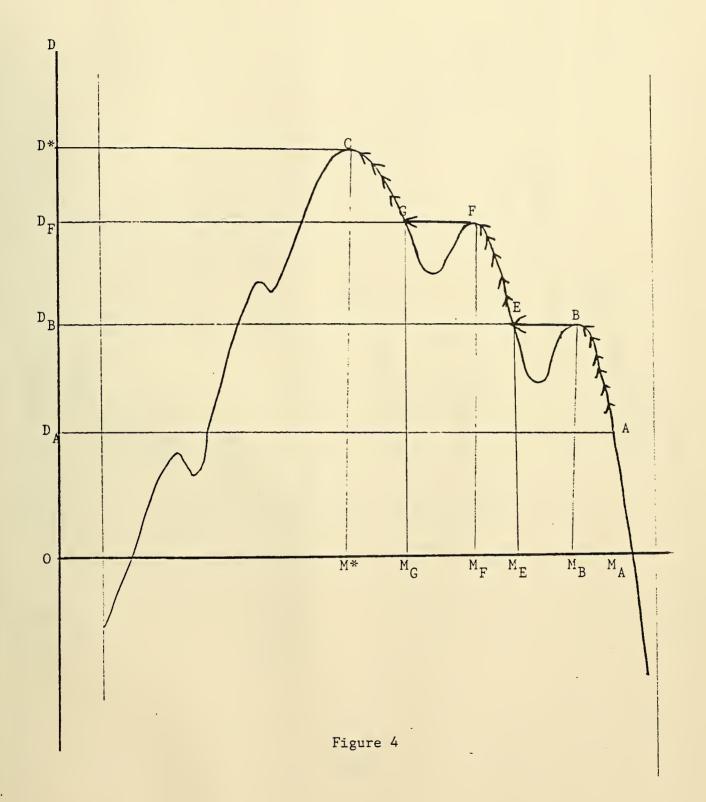
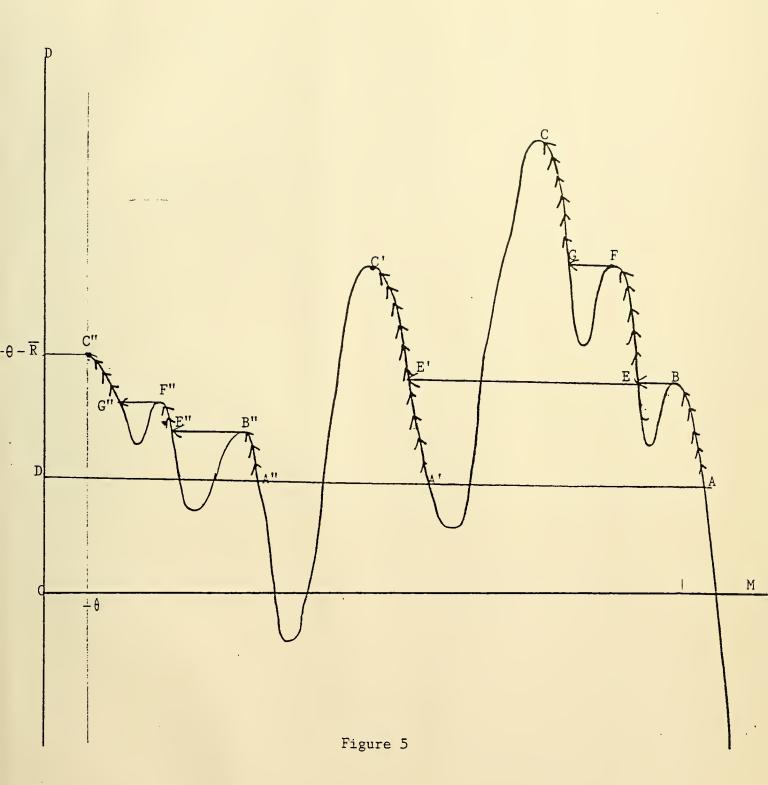


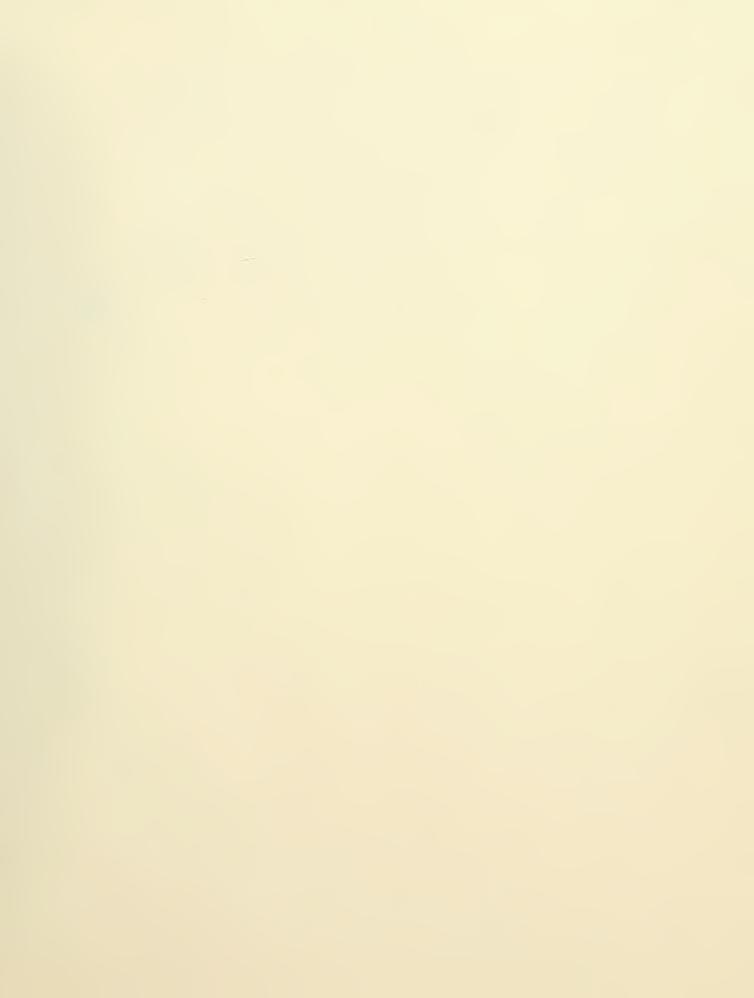
Figure 3















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