CAPITAL TAXATION IN A DYNAMIC GENERAL EQUILIBRIUM SETTING

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The familiar two-factor, two-commodity incidence model is extended to a dynamic setting in which the supply of capital is variable and the government can use money or bonds to balance its budget in addition to neutral lump sum taxation. The dynamic incidence effects of a sectoral tax on capital are qualitatively similar to the static incidence effects when the government balances its budget with neutral taxes, but are qualitatively different when the government uses money or bonds. In this case, while capital bears the burden of the tax in the short run, it is able to shift it in the long run.
1. Introduction

In analyzing the incidence effects of capital taxation, it is important to distinguish between static incidence and dynamic incidence. Static incidence analysis is concerned with the intratemporal general equilibrium effects of a (marginal) change in taxation upon relative factor and commodity prices, outputs, and ultimately upon welfare. Thus total factor supplies are assumed to be fixed, although intersectoral adjustments in factor allocations are assumed to take place.

As its name indicates, dynamic incidence analysis is concerned with similar intertemporal general equilibrium effects of a change in taxation. Because capital is a produced factor of production, it enters the general-equilibrium system not only as a factor input, but also as a produced output. Any change in the tax structure, inasmuch as it affects prices, income, interest rates and real wealth, will normally have an effect both on investment demand and the supply of savings and hence on the pattern of capital accumulation over time. Consequently, along the altered growth path of the dynamic system, the long-run dynamic incidence of any tax-structure change may be quite different from its short-run static incidence,\(^1\) and it is this long-run incidence that is ultimately the relevant concept.

\(^1\)Feldstein (1974, 1975) has analyzed the dynamic incidence of capital taxation in a single sector model and shown that labor may ultimately bear a large portion of capital taxation if savings are sensitive to the return to capital. Also see Grierson (1975).
The accepted general equilibrium analysis of sectoral capital taxation in the public finance literature is based on the model developed by Harberger (1962) and extended by Mieszkowski (1967), hereafter referred to as HM, that treats capital as a fixed factor of production. Using the standard 2-(fixed) factor, 2-commodity general equilibrium framework, HM show that a sectoral tax on capital will be borne by capital. From this Harberger (1962) inferred that the corporate income tax is not shifted. More recently, Musgrave (1973) and Aaron (1973, 1975) have used the HM analysis to argue that property taxes are primarily borne by owners of capital and not shifted forward to renters.2/ While the HM framework is indeed powerful and has formed the basis of general-equilibrium studies of tax incidence in a static setting, it must be extended in two significant ways to enable an in depth analysis of the dynamic aspects of capital taxation. First, the produced nature of capital must be made explicit, and specific savings and investment relationships must be introduced. Second, since the nature of the equilibrating adjustments not only depends upon the initial level of taxation but also upon the way in which the government balances its budget, the government budget constraint must be explicitly considered.3/

Consequently, in this paper we study the question of the incidence of capital taxation within the context of a general-equilibrium, two-sector model with the following characteristics: (1) capital is treated as a

2/ Musgrave qualifies this conclusion, however, by introducing noncompetitive price elements. With respect to the property tax, the analysis of Mieszkowski (1972) is also relevant.

3/ In a static setting, Vandendorpe and Friedlaender (1976) have shown that the government budget constraint can be ignored if there are no initial distorting taxes, but not if the initial tax structure is distorting.
produced factor of production; and (2) the government budget constraint is explicitly considered, and in addition to taxes, the government can also use money or bonds to balance its budget. 4/

Briefly, this paper takes the following form. In Section 2 we present such a model, while in Section 3 we develop the so-called "equations of change" that can be used to analyze questions of incidence. In Section 4 we present some simulation results that enable us to compare the dynamic incidence of sectoral capital taxation under alternative government equilibrating adjustments. Section 5 then gives a brief conclusion and some areas for future research.

2. The Model

The model utilized in the present paper is a hybrid micro-macro model. Following HM, the model is characterized by two factors and two commodities with production subject to constant returns to scale and perfect intersectoral factor mobility. While the HM model is static, however, our model is explicitly dynamic and treats capital as a produced factor of production and introduces money and bonds through the government budget constraint. 5/

The introduction of money and bonds permit us to analyze more realistic equilibrating adjustments on the part of government than that implied

4/ This model also permits initial taxes to be set at any arbitrary nonzero level. Since, however, the implications of an initial distorting tax structure have been extensively analyzed by Vandendorpe and Friedlaender (1976), they are not explicitly considered in this paper.

5/ Within a static context, this model also differs from that of HM in that it permits: (1) the existence of initial distorting taxes; (2) labor to be intra-temporally elastically supplied; and (3) the government to utilize labor and hence reduce the labor available to the private production sector.
by lump sum taxation and gives an otherwise micro model a distinctly macro flavor.\(^6\)

Following HM, we assume that the economy can be characterized by the outputs of a corporate sector \(X_1\) and a noncorporate sector \(X_2\), each of which is produced by the available stocks of labor \(L\) and capital \(K\) under constant returns to scale.\(^7\) Because of the constant-returns-to-scale assumption, a corporate profits tax is analytically equivalent to a sectoral tax on capital. Like HM, we can infer the incidence of a corporate profits tax by altering the tax rate on capital in the corporate sector. Following Jones' (1965, 1971) treatment of the two-factor, two-commodity model, the basic production equilibrium relationships consist of the full employment conditions,

\[
\begin{align*}
  a_{K1}X_1 + a_{K2}X_2 & = K \\
  a_{L1}X_1 + a_{L2}X_2 + L & = L
\end{align*}
\]

and the competitive zero profit conditions

\[
\begin{align*}
  a_{K1}r_1 + a_{L1}w_1 & = p_1 \\
  a_{K2}r_2 + a_{L2}w_2 & = p_2
\end{align*}
\]

\(^6\) For a similar treatment of a two-sector macro model see Foley and Sidrauski (1971). Our model differs from theirs, however, in two significant ways: first, the corporate sector is assumed to produce both consumer and investment goods; and second, factor and commodity taxes are assumed to be government control variables.

\(^7\) We refer to \(K\) as the number of physical units of capital, intratemporally fixed at time \(t\). Since we assume that capital goods are only produced by the corporate sector, on the production side of the model, the concept of physical capital units is clear. On the demand side of the model, however, the term capital will need a careful definition in the context of consumption, investment, and savings.
where $K$ and $L$ are the amounts of labor and capital supplied at time $t$; $X_j$ represents the output of commodity $j$; $r_j$ and $w_j$ represent the cost (inclusive of factor taxes and depreciation) of employing respectively one unit of capital or labor in sector $j$; $p_j$ is the producer-price of commodity $j$; $a_{fj}$ is the amount of factor $f$ used per unit output of commodity $j$ (that is, $a_{kj} = k_j/x_j$ and $a_{Lj} = L_j/x_j$); and $L_g$ represents the exogenously determined demand for labor on the part of the government.

All variables are to be understood as dated at time $t$, unless otherwise specified.

Because of the possibility of distorting factor taxes, $r_1$ (or $w_1$) need not be equal to $r_2$ (or $w_2$). Indeed, if $r$ and $w$ represent the rental and wage rates respectively received by capital and labor, which are assumed to be equal in both sectors, then we can define the symbols $t_{fj}$, $\tau_{fj}$, and $T_{fj}$ by

$$w_j = w + \tau_{Lj} = w(1 + \tau_{Lj}) = wT_{Lj}$$

$$r_j = r + \tau_{Kj} + p_1\delta = r(1 + \tau_{Kj}) + p_1\delta = rT_{Kj} + p_1\delta$$

Thus $t_{fj}$ represents the specific tax imposed on factor $f$ in sector $j$, and $\tau_{fj}$ represents the ad valorem tax rate. The term "tax coefficient" will be used for the symbol $T_{fj}$ ($\equiv 1 + \tau_{fj}$); it is assumed that $\tau_{fj} > -1$.

We assume that the output of sector 1 (the corporate sector) can be used both as an investment good and as a consumption good, while the

---

8/ It should be noted that $a_{fj}$ is a function of the factor cost ratio $r_j/w_j$, defined by the unit isoquant and the cost-minimizing condition.

9/ The subscript $j$ will always refer to commodities (sectors) 1 and 2 while the subscript $f$ will always refer to factors $K$ and $L$. Unless clarity demands it, this range of the subscripts $f$ and $j$ will not be repeated every time.
output of sector 2 (the noncorporate sector) can be used only as a different consumption good. Consequently we shall value capital at \( p_1 \), the production cost of investment goods. Thus the term \( p_1 \delta \) represents unit depreciation costs in money terms. We assume that capital depreciates at a rate \( \delta \) that is independent of the industry in which the machine is used.

While the stock of machines, \( K \), is intratemporally fixed, we assume that the labor supply, \( L \), depends upon population, \( N \), and the real wage, \( \omega \); i.e., \( L \) is assumed to be wage elastic within the period.

\[
L = L(N, \omega)
\]

The real wage, \( \omega \), represents the purchasing power of the money wage, \( w \); hence we define

\[
\omega = \frac{w}{q}
\]

where \( w \) represents the money wage and \( q \) is some index of the general price level to be defined precisely within the context of the demand side of the model, which will be discussed below.

If we denote by \( q_j \) the consumer price of commodity \( j \), \( t_j \), \( r_j \), and \( \tau_j \) are defined by

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10/ This assumption is made as a compromise between tractability and realism of the model.

11/ We implicitly assume that depreciation is deductible. If not, then

\[
r_j = (r + \delta p_1) T_{kj} \]

The assumption that each industry has the same depreciation rate is made for expository simplicity and could easily be relaxed.
\[ q_j = p_j + t_j = p_j (1 + \tau_j) = p_j T_j; \]  
(6)
i.e., \( t_j \), \( \tau_j \), and \( T_j \) are respectively the specific tax, the ad valorem tax rate, and what we shall label the "tax coefficient" on commodity \( j \).

Since the model contains a monetary sector we shall be able to determine some measure of absolute prices. To this end we define

\[ q = \alpha_1 q_1 + \alpha_2 q_2. \]  
(7)

Expression (7), a (bilinear) function in the money prices \( q_j \) and some (as of yet unspecified) constants \( \alpha_j \), can be interpreted as the cost of the commodity basket \(( \alpha_1, \alpha_2 )\) at prices \( q_1 \) and \( q_2 \), or alternatively (if \( \alpha_1 \) and \( \alpha_2 \) were chosen so that \( \alpha_1 + \alpha_2 = 1 \)), as a weighted average of the consumer prices \( q_1 \) and \( q_2 \).

We assume that the government utilizes the output of both sectors and that (as previously indicated) investment goods are only produced by the corporate sector. Hence the aggregate demand for the output of sector 1 consists of government demand \((G_1)\), consumption demand \((C_1)\), and investment demand \((I)\), while the aggregate demand for the output of sector 2 consists of government demand \((G_2)\) and consumer demand \((C_2)\).

We may therefore write the market clearing equations as

\[ \text{[12]} \]

\[ \text{[12]} \] Setting \( \alpha_1 = 1 \) and \( \alpha_2 = 0 \) amounts to the customary procedure of choosing good one as the numéraire. The present more general definition in terms of arbitrary \( \alpha_1 \) and \( \alpha_2 \) seems preferable in the present context and the appropriate choice of \( \alpha_1 \) and \( \alpha_2 \) will be discussed in Section 3 when the model is differentiated.
\[ X_1 = C_1 + G_1 + I \quad (8a) \]
\[ X_2 = C_2 + G_2 \quad (8b) \]

We assume that consumers base their spending and savings decisions on relative prices \((q_1/q_2)\), real disposable income \((y)\), and real asset holdings \((W)\) and thus write the aggregate demand functions \(^{13/}\)

\[ C_j = C_j(q_1/q_2, y, W) \quad (9) \]

and

\[ S = S(y, W) \quad (10) \]

where \(C_j\) represents the consumption of good \(i\) and \(S\) represents real savings.

We assume that there are three assets in this economy: physical capital \((K)\); the outstanding stock of money \((M)\); and the stock of outstanding perpetuities \((b)\), each of which has a coupon rate of one dollar. Hence we can define real wealth as

\[ W = \frac{M(t)}{q} + \frac{b(t)}{q_i b} + \frac{P_1 K(t)}{q} \quad (11) \]

where \(i_b\) represents the rate of interest in the bond market. We value \(K\) capital at the price of the capital goods industry, but deflate all money assets by the consumer price index.

Total nominal disposable income is defined as

\(^{13/}\)See Samuelson (1956).
\[ Y = wL + rK + b + H \]  \hspace{1cm} (12)

where \( H \) is the net lump sum transfer (tax) imposed by the government.

Total real disposable income is thus defined by

\[ y = Y/q \]  \hspace{1cm} (13)

Obviously equations (12) and (13) satisfy the consumer budget constraint

\[ Y = \sum q_j C_j + qS \]  \hspace{1cm} (14)

We now turn to the markets for the assets of money, bonds, and capital. As currently specified, the model yields two interest rates: the rate of interest on bonds \( (i_b) \) and the rate of return on physical capital \( (i_k) \), defined by\(^{14}\)

\[ i_k = \frac{r - \delta p_1}{p_1}. \]  \hspace{1cm} (15)

If all returns were subjectively certain so that there were no uncertainty, wealth owners would only hold bonds and capital if they yielded the same rate of return. The two assets would be perfect substitutes and equilibrium would require that \( i_k = i_b = i.\(^{15}\) \) In the real world, however, uncertainty certainly exists and \( i_k \neq i_b \). Thus, in terms of empirical relevance, it is probably desirable to permit uncertainty and to let \( i_k \) differ from \( i_b \).

\(^{14}\) Equation (15) assumes that there is no direct taxation of corporate income and hence neglects the relationships between taxes, depreciation, and investment credits, given by Hall and Jorgenson (1971) in deriving an expression for the cost of capital.

\(^{15}\) Strictly speaking, this is only true if there is no expected inflation.
The existence of money and bonds introduces a distinctly macro element into an otherwise micro model. Their introduction is important for two reasons, however. First, they permit us to express the model in absolute rather than relative prices and thus to derive measures of the absolute as well as the relative price effects of the corporate profits tax. Second, they extend the permissible range of the government's equilibrating adjustments in response to a given tax change. The incidence and output effects of a given tax change are quite sensitive to the assumed equilibrating adjustment. Since the government typically adjusts the monetary base or the supply of assets in response to a change in taxes or expenditures, it is important to permit these adjustments for insights into the actual incidence of the corporate profits tax.

We assume that the supply of nominal money (M) is determined by the government, while the demand for real money as a proportion of wealth depends upon the ratio of real disposable income to wealth \( \frac{y}{W} \), and the rates of return on bonds \( i_B \) and capital \( i_K \). Note that we also assume that money and bonds are held by the public as stores of wealth for future consumption. Hence, we measure the real value of money and bonds in terms of \( q \), the price of the composite consumption good. Equilibrium in the money market requires that the money supplied equals the money demanded: hence,

---

16/ Foley and Sidrauski (1971) also introduce financial variables into their model. They, however, assume that the economy can be characterized by an investment goods and consumption goods sector and, thus, stay within the standard framework of a two-sector growth model.

17/ We assume that there are static expectations and, hence, no expected capital gains on money, bonds, or capital.
\[ \frac{M(t)}{q} = W \cdot \mu_M (y/W, i_K, i_b) \] (16)

We also assume that the government determines the number of bonds outstanding, each of which is assumed to be a perpetuity with a yield of one dollar. Since bonds are a substitute for money, the demand for bonds as a proportion of wealth must also depend upon the ratio of real disposable income to wealth \((y/W)\), and the rate of returns on bonds and stocks \((i_b \text{ and } i_K)\). In equilibrium, the demand for bonds must equal their supply; hence

\[ \frac{b(t)}{q_i_b} = W \cdot \mu_b (y/W, i_K, i_b) \] (17)

From Walras' Law, we know that if \(n-1\) markets are in equilibrium, the \(n\)th market must also be in equilibrium. In this analysis we, therefore, do not utilize an explicit expression for the asset market for capital. Although the asset market for capital does not appear explicitly in the analysis, it is important to realize that it plays a very important role since it determines the investment demand equation. Thus although we do not state it explicitly, there is an implied investment demand that depends upon interest rates (and hence the rental return to capital), income, and wealth.

Equilibrium requires that savings plus taxes equals government expenditures plus investment. Thus,

\[ p_1 I - p_1 s_K(t) + G = qS + T \] (18)
where \( G \) and \( T \) respectively represent government expenditures and tax revenues.

Equilibrium also requires that total government receipts (funds from taxes plus changes in the stock of money and bonds) must equal total government expenditures (purchases of goods and service plus transfers). In this model we assume that the government purchases commodities from both sectors at producers prices and hires labor at the wage rate. In addition, the government makes lump sum transfers (taxes), and pays premiums on any bonds outstanding. Thus, total government expenditures are given by

\[
G = \sum p_j G_j + wL_g + H + b(t)
\]  

(19a)

We assume that the tax instruments available to the government are commodity taxes and sector factor taxes. Thus, total tax revenues are given by

\[
T = \sum t_j C_j + \sum f_j v_{fj}
\]  

(19b)

where \( v_{fj} \) represents the amount of factor \( f \) employed in sector \( j \).

If the budget is always balanced, \( G = T \) and the government budget constraint adds no dynamic element. If, however, the tax receipts and expenditures are not always in balance, the government can balance its
budget by issuing money or bonds. Hence, the government budget constraint will generally contain dynamic elements and can be written as\(^{18/}\)

\[
G - T = M(t+1) - M(t) + \frac{b(t+1) - b(t)}{i_b}
\] (19c)

Whether the government's budget is balanced or not, the growth of the capital stock will always introduce a dynamic element into the model. The expression for the growth of the capital stock is given by

\[
K(t+1) - K(t) = I(t) - SK(t)
\] (20)

Equations (1) – (20) form a full specification of the model, which we assume to be in initial equilibrium. To analyze the incidence of capital taxation, we will specify a change in capital taxation and analyze the general-equilibrium response of the system to this change in a way specified below.

3. **Incidence Analysis**

3.1. **General Methodology**

Within the context of any dynamic model, there are three levels at which incidence can be analyzed: comparative statics, comparative

\(^{18/}\) As Hansen (1973) has indicated, this form of the government budget constraint assumes that the government controls both the money supply and supply of bonds, as well as the level of taxes and expenditures. In the U.S., however, the functions of the Treasury and the Federal Reserve Board are in fact dichotomized and the government in effect has two constraints: one for the Fed and one for the Treasury. Since the impact of tax and expenditure changes differs with the form of the budget constraint employed, for empirical estimation we will probably want to pursue the implications of both kinds of budget constraint.
dynamics, and a full solution of the dynamic system.

As implied by the name, a comparative-static analysis of incidence is purely static and is concerned with the intratemporal effects of a change in the tax structure upon relative prices, output, etc. Thus, in static incidence analysis, a change in the tax structure is postulated to take place at a given time period, and the changes in relative prices are determined that restore the private and government sectors to equilibrium within that time period. This kind of analysis permits us to determine the short-run incidence of a tax and is the kind usually employed in incidence analysis, e.g., Harberger (1962) and Meiszkowski (1967).

A comparative-dynamic analysis of incidence is concerned with the comparison of different steady-state equilibria of the dynamic system under different tax structures. Thus, a once and for all change in the tax structure is postulated, and the new steady state values of factor prices, commodity prices, and outputs are derived. By comparing the new steady state values of factor and commodity prices with their initial steady state values, we can determine the long-run incidence effects of the tax change. In this context we can either deal with levels or changes. In the levels case, we would have to derive a full solution to the model for a given tax structure, and then change the tax structure in a specified way and derive a solution for the ensuing periods. This approach was used by Feldstein (1974) in analyzing the incidence of capital taxation in a single sector model. This approach is attractive since it enables one to compare the solution values of the endogenous variables over the period of analysis. It is rather unwieldy, however, since it not only requires a full specification of all of the initial values of the relevant
variables, but also a full specification of all of the functions and their relevant parameters. Moreover, in so far as nonlinearities are present, this approach is computationally difficult.

As a more tractable alternative, we can analyze the time path of the change in the variables in the model. While this approach yields essentially the same results as a full solution to the model, it requires considerably less information and is computationally easier to handle since all of the functional forms are linear in percentage changes. We will, thus, postulate a given change in the tax structure and analyze the changes in the relevant endogenous variables that occur over time.

3.2. The Equations of Change

Analytic efforts to determine the short-run incidence effects of taxation within the context of a static, two-sector general equilibrium model assume that the economy is in initial equilibrium and involve the derivation of expressions in terms of the exogenous government tax changes for the endogenous change in the rental-wage ratio and the endogenous government adjustment (usually considered to be neutral lump sum taxation) necessary to restore equilibrium in the private and government sectors, following an exogenous change in one or more tax parameters. Thus, once the change in taxation is postulated, it is straightforward (if algebraically tedious) to derive the change in the rental wage ratio, and hence in output and ultimately in welfare, resulting from this tax change. 19/

19/ See, for example, Harberger (1962), Mieszkowski (1967), Vandendorpe and Friedlaender (1976).
In a dynamic setting, however, analytic solutions are not generally feasible for two reasons. First, the weights attached to the percentage changes in the relevant variables are no longer constant but are time-dependent variables; and second, the equilibrating stock variables are also time-dependent. Hence the system is inherently nonlinear instead of linear. Thus when we consider questions of dynamic incidence, simulation experiments are necessary in all but the simplest of models.

We now differentiate the model outlined in Section 2 to derive a system of equations that can be used to analyze questions of dynamic tax or expenditure incidence. Since Jones (1965) and Vandendorpe and Friedlaender (1976) have derived these equations in considerable detail, we will generally present these equations without a detailed derivation.

We begin by differentiating the factor market equilibrium conditions (1a and 1b) to obtain

\[
\lambda_{K1} \hat{X}_1 + \lambda_{K2} \hat{X}_2 + \lambda_{K1} \hat{a}_{K1} + \lambda_{K2} \hat{a}_{K2} = \hat{K}(t)
\]
(21a)

\[
\lambda_{L1} \hat{X}_1 + \lambda_{L2} \hat{X}_2 + \lambda_{L1} \hat{a}_{L1} + \lambda_{L2} \hat{a}_{L2} = \hat{L} - \lambda_{Lg} \hat{L}_g
\]
(21b)

where \( \lambda_{Kj} = K_j/K; \quad \lambda_{Lj} = L_j/L; \quad \lambda_{Lg} = L_g/L \)

The hat over a variable denotes the percentage change in that variable. Since all changes in the endogenous variables occur within any given period \( t \), we do not give them a time subscript. Similarly, since the share variables denote the equilibrium value of the relevant variables prior to the change in the government or stock variables in period \( t \), we do not give them a time subscript. To highlight the role of the
stock variables, however, we do give them an explicit time subscript.\(^{20/}\)

Although the stock variables do not change intratemporally in response to the changes in the government variables, they do change intertemporally and thus become the exogenous variables in all periods after the initial government change.

From the definitions of \(a_{fj}\) we derive

\[
\hat{K}_j = \hat{a}_{Kj} + \hat{X}_j \\
\hat{L}_j = \hat{a}_{Lj} + \hat{X}_j
\]

(22a) (22b)

Jones (1965) has shown that the change in factor utilization can be expressed by

\[
\hat{a}_{Kj} = -\theta_{Lj} \hat{a}_{Lj} (\hat{r}_j - \hat{w}_j) \\
\hat{a}_{Lj} = \theta_{Kj} \hat{a}_{Kj} (\hat{r}_j - \hat{w}_j)
\]

(23a) (23b)

and that \(\theta_{Kj} \hat{a}_{Kj} + \theta_{Lj} \hat{a}_{Lj} = 0\), \(j = 1, 2\)

---

\(^{20/}\)To make sure the timing relationships are clear, we can rewrite equation (21a) as:

\[
\lambda_{K1}(t-1) \hat{X}_1(t) + \lambda_{K2}(t-1) \hat{X}_2(t) + \lambda_{K1}(t-1) \hat{a}_{K1}(t) + \lambda_{K2}(t-1) \hat{a}_{K2}(t) = \hat{K}(t) \text{ where}
\]

\[
\lambda_{Kj}(t-1) = K_j(t-1)/K(t-1) \text{ and similarly for } \lambda_{Lj} \text{ and } \lambda_{Lg} . \text{ Note that } K_j \text{ and } K \text{ denote the equilibrium values of } K_j \text{ and } K \text{ prior to the exogenously determined change in the government variable. Thus, } \lambda_{Kj}(t-1) \text{ denotes the value of } \lambda_{Kj} \text{ prior to the change in the exogenous variable in period } t \text{ and } \hat{X}_j(t-1) \text{ denotes the intraperiod change of } \hat{X}_j \text{ in response to the change in the exogenous variable in period } t. \text{ Of course, in the initial period, } \hat{K}(t) = 0.\]
where $\theta_{Lj} = \omega_j a_{Lj}/p_j$; $\theta_{Kj} = r_j a_{Kj}/p_j$; and

$\sigma_j$ represents the elasticity of factor substitution in sector $j$ defined as

$$\sigma_j = \frac{\hat{a}_{Kj} - \hat{a}_{Lj}}{\hat{r}_j - \hat{\omega}_j}$$

Differentiation of the zero profit conditions (2a and 2b) yields

$$\hat{p}_j = \theta_{Kj} \hat{r}_j + \theta_{Lj} \hat{\omega}_j,$$  \hspace{1cm} (24)

while differentiation of the factor cost equations (3a and 3b) yields

$$\hat{r}_j = \rho_{Kj} (\hat{r} + \hat{T}_{Kj}) + \rho_{\delta j} (\hat{p}_j + \hat{\delta})$$  \hspace{1cm} (25a)

$$\hat{\omega}_j = \hat{\omega} + \hat{T}_{Lj}$$  \hspace{1cm} (25b)

where $\rho_{Kj} = r_{T_{Kj}}/r_j$; $\rho_{\delta j} = p_1 \delta/r_j$

When we differentiate the labor supply and real wage equations (4 and 5) we obtain

$$\hat{L} = \varepsilon_{L\omega} (\hat{\omega} - \hat{q}) + \varepsilon_{LN} \hat{N},$$  \hspace{1cm} (26)

where $\varepsilon_{L\omega}$ and $\varepsilon_{LN}$ respectively represent the elasticity of the labor supply with respect to the real wage and population.

Turning now to the price equations, differentiation of equation (7) yields, for initial weights $a_j$,

$$\hat{q} = \frac{\alpha_1 q_1}{\lambda a_j q_j} \hat{q}_1 + \frac{\alpha_2 q_2}{\lambda a_j q_j} \hat{q}_2.$$  \hspace{1cm} (27)
If we choose the weights $\alpha_1$ and $\alpha_2$ to be equal to the initial level of consumption $C_1$ and $C_2$, then we can write equation (27) as

$$\hat{q} = \gamma_{C1}' \hat{q}_1 + \gamma_{C2}' \hat{q}_2$$

where $\gamma_{Cj}' = q_j C_j / \sum_j q_j C_j$, i.e., the share of consumption expenditures spent on commodity $j$. By equation (6) we also know that

$$\hat{q}_j = \hat{p}_j + \hat{T}_j.$$  \hspace{1cm} (28)

We now differentiate the market clearing equations to obtain

$$\hat{X}_1 = f_{C1} \hat{C}_1 + f_{G1} \hat{G}_1 + f_{I} \hat{I}$$  \hspace{1cm} (29a)

$$\hat{X}_2 = f_{C2} \hat{C}_2 + f_{G2} \hat{G}_2$$  \hspace{1cm} (29b)

where $f_{Cj}$ and $f_{Gj}$ represent the share of $C_j$ and $G_j$ in the total output sector $j$ and $f_{I}$ represents the share of $I$ in the output of sector 1 (e.g., $f_{C1} \equiv C_1 / X_1$, etc.).

Instead of using equations (29a) and (29b) separately, it is useful to combine them into a general equilibrium supply equation that equates the substitution elasticity of supply with the substitution elasticity of aggregate demand and write

$$\hat{X}_1 - \hat{X}_2 = f_{C1} \hat{C}_1 - f_{C2} \hat{C}_2 + f_{G1} \hat{G}_1 - f_{G2} \hat{G}_2 + f_{I} \hat{I}$$  \hspace{1cm} (29c)

21/ The percentage changes in the price of the composite commodity ($\hat{q}$) also equals the percentage change in the Laspeyres price index, measured in terms of the base period.
We now differentiate the consumer demand equation (9) and obtain

\[ \dot{C}_j = \varepsilon_j(p_1 - p_2) + \varepsilon_j(\dot{T}_1 - \dot{T}_2) + \varepsilon_{yj}(\dot{Y} - \dot{q}) + \varepsilon_{wj} \dot{W} \]  
\[ (30) \]

where \( \varepsilon_j \) represents the elasticity of consumer demand for commodity \( j \) with respect to \( q_1/q_2 \); \( \varepsilon_{yj} \) represents the elasticity of consumer demand for commodity \( j \) with respect to real disposable income; and \( \varepsilon_{wj} \) represents the elasticity of consumer demand for commodity \( j \) with respect to real wealth.

Differentiation of the expressions for real wealth, eq. (11), yields

\[ \dot{W} = A_M \dot{M}(t) + A_B (\dot{b} - \dot{i}_b) + A_K (\dot{K} + \dot{p}_1) - \dot{q} \]  
\[ (31) \]

where

\[ A_M = \frac{M(t)}{qW} \]
\[ A_B = \frac{b(t)}{i_b} qW \]
\[ A_K = \frac{p_1 K}{qW} ; \]

while differentiation of the expressions for disposable income, eqs. (13) and (14) yields

\[ \dot{Y} = \gamma_k (\dot{r} + \dot{K}(t)) + \gamma_L (\dot{L} + \dot{w}) + \dot{H} + \dot{b} \]  
\[ (32a) \]

where

\[ \gamma_k \equiv \frac{rK(t)}{Y} \]
\[ \gamma_L \equiv \frac{wL}{Y} \]
\[ h = \frac{H}{Y} \]
\[ \beta = \frac{b(t)}{Y} ; \]
and

$$\dot{y} = \gamma_{c_j}(\dot{c}_j + \dot{p}_j + \dot{T}_j) + \gamma_s(\dot{q} + \dot{S})$$  \hspace{1cm} (32b)$$

where $\gamma_{c_j} = q_j c_j / Y$ and $\gamma_s = qS / Y$.

Furthermore, differentiation of the savings function, eq. (10), yields

$$\dot{S} = \varepsilon_{Sy}(\dot{Y} - \dot{q}) + \varepsilon_{Sw}\dot{W}$$  \hspace{1cm} (33)$$

where $\varepsilon_{Sy}$ and $\varepsilon_{Sw}$ respectively refer to the elasticity of saving with respect to real income and wealth. Note that in this formulation, any interest-elasticity of savings comes through the wealth term.

Turning to the money markets, we differentiate the definition of $i^*_K$, given in eq. (15), and obtain

$$\dot{r} = \pi_i \dot{i}^*_K + \pi_\delta \dot{\delta} + \dot{p}_1$$  \hspace{1cm} (34)$$

where $\pi_i = i^*_K p_1 / r$; $\pi_\delta = p_1 \delta / r$.

We now consider the asset market equilibrium conditions and differentiate eqs. (16) and (17) to obtain

$$\dot{M}(t) = \varepsilon_{my}\dot{y} + (1-\varepsilon_{my})\dot{W} + (1-\varepsilon_{my})\dot{q} + \varepsilon_{m_b}\dot{i}_b + \varepsilon_{m_K}\dot{i}_K$$  \hspace{1cm} (35a)$$

where $\varepsilon_{my}$, $\varepsilon_{m_b}$, and $\varepsilon_{m_K}$ respectively refer to the elasticities of demand for money with respect to $y/W$, $i_b$, and $i_K$.

Furthermore

$$\dot{b}(t) = \varepsilon_{by}\dot{y} + (1-\varepsilon_{by})(\dot{W} + \dot{q}) + (1+\varepsilon_{b1_b})\dot{i}_b + \varepsilon_{b1_K}\dot{i}_K$$  \hspace{1cm} (35b)$$
where $\varepsilon_{by}$, $\varepsilon_{bi}$, and $\varepsilon_{biK}$ respectively refer to the elasticities of demand for bonds with respect to $y/W$, $i_b$ and $i_K$.

We now turn to the savings-investment equilibrium condition (18), which states that the money value of savings less net investment must equal the value of the government deficit, $G-T$. Since, however, the differential of $G-T$ enters the government budget constraint, it is useful to consider it explicitly. From eqs. (19a) and (19b) we readily see that

$$dG - dT = \left[ \sum_j \varepsilon_{pj} dG_j + \sum_j \varepsilon_{Cj} dp_j + \sum_j w dj + \sum_j \lambda_j d\lambda + \sum_j d\theta + \sum_j db(t) \right]$$

$$- \left[ \sum_j \varepsilon_{Cj} dt_j + \sum_j \varepsilon_{pj} dv_f + \sum_f \sum_j v_f j dt_f \right]$$

(36)

The first term in brackets on the left hand side represents the change in expenditures, while the second term in brackets on the left hand side represents the change in revenues.

We can simplify equation (36) by making use of the following relationships

$$\sum_f \sum_j v_f j dt_f = \sum_j \varepsilon_{Xj} dp_j - (K d r + L d w)$$

(37a)

$$\sum_f \sum_j v_f j dv_f = \sum_j dp_j \sum_j \varepsilon_{Xj} - (w d L + r d K(t))$$

(37b)

$$\sum_j \varepsilon_{Cj} dt_j = \sum_j \varepsilon_{Cj} dp_j - \sum_j \varepsilon_{Xj} dp_j - \sum_j \varepsilon_{pj} dt_j - \sum_j \varepsilon_{pj} dv_f - \sum_f \sum_j v_f j dt_f$$

(37c)

$$\sum_j \varepsilon_{Cj} dt_j = \sum_j \varepsilon_{Cj} dp_j - \sum_j \varepsilon_{Xj} dp_j + \sum_j \varepsilon_{pj} dp_j + Idp_{1}$$

(37d)

---

See Vandendorpe and Friedlaender (1976) for a full discussion.
By substituting equations (32a) and (37a-d) into equation (36), collecting terms, expressing the resulting expressions in percentage terms, and dividing through by disposable income $Y$, we obtain

$$\frac{dG - dT}{Y} = \gamma_{G1} \hat{C}_1 + \gamma_{G2} \hat{C}_2 + \gamma_{Gw}(\hat{L}_g + \hat{w}) - \gamma_{X1} \hat{X}_1 + \gamma_{X2} \hat{X}_2 + \gamma_{It} \hat{I}_t \hat{I} - (\gamma_{C1} + \gamma_{I}) \hat{P}_1 - \gamma_{C2} \hat{P}_2 - \gamma_{C1T} \hat{T}_1 - \gamma_{C2T} \hat{T}_2 + \hat{Y} \tag{38}$$

where

$$\gamma_{Gj} = \frac{q_j G_j}{Y}$$
$$\gamma_{Gw} = \frac{wL_g}{Y}$$
$$\gamma_{Xj} = \frac{q_j X_j}{Y}$$
$$\gamma_{Cj} = \frac{q_j C_j}{Y}$$
$$\gamma_{I} = \frac{p_I}{Y}$$
$$\gamma_{It} = \frac{p_{It} I}{Y}$$

If we then differentiate the expression for savings less investment and divide by disposable income we obtain

$$\frac{d[qS - (p_I - p_dK(t))]}{Y} = \gamma_S (\hat{q} + \hat{S}) - \gamma_I (\hat{p} + \hat{I}) + \gamma_D (\hat{p} + \hat{d} + \hat{K}) \tag{39}$$

where

$$\gamma_S = qS/Y; \quad \gamma_I = p_I/Y; \quad \gamma_D = p_dK(t)/Y$$

Equating eq. (39) and (38) and collecting terms we finally obtain

$$\left(\gamma_I + \gamma_{It} \right) \hat{I}_t = \gamma_S (\hat{S} + \hat{q}) + \Sigma \gamma_{Xj} \hat{X}_j + (\gamma_{C1} + \gamma_{D}) \hat{P}_1$$
$$+ \gamma_{C2} \hat{P}_2 + \Sigma \gamma_{Cj} \hat{T}_j - \hat{Y} - \gamma_{(\omega)} (\hat{g} + \hat{\omega})$$
$$+ \gamma_{D} (\hat{d} + \hat{K(t)}) - \Sigma \gamma_{Gj} \hat{G}_j \tag{40}$$
If the government maintains its budget balance by adjusting lump sum taxes \( \hat{H} \) we can use equations (21a & b), (22a & b), (23a & b), (24), (25a & b), (26), (27a), (28), (29c), (30), (31), (32a), (33), (34), (35a & b), (38) and (40) to solve for the following variables in terms of the exogenous changes in the government variables: \( \hat{X}_j, \hat{\alpha}_f, \hat{K}_j, \hat{L}_j, \hat{p}_j, \hat{r}_j, \hat{w}_j, \hat{r}, \hat{w}, \hat{L}, \hat{q}, \hat{q}_j, \hat{C}_j, \hat{\dot{W}}, \hat{\dot{Y}}, \hat{\dot{S}}, \hat{\dot{i}}_b, \hat{\dot{i}}_K, \hat{\dot{i}}_L \) and \( \hat{H} \). In all, there are a total of 31 equations and 31 unknowns.

As long as the government budget remains in balance through taxes, there is no change in the stock of money or bonds. Hence, the only dynamic equation is given by the relationship between the capital stock and investment.

\[
\dot{K}(t+1) = (\phi_K - \psi_D \phi_I) \dot{K}(t) - \psi_D \phi_I \dot{\hat{I}}(t)
\]  
(41)

where \( \phi_K = K(t)/K(t+1) \)

\( \phi_I = I(t)/K(t+1) \)

\( \psi_D = \delta K(t)/I(t) \)

Thus, once the change in investment has been determined in response to the exogenous changes in the government sector, equation (41) determines the change in the capital stock. This then is treated as an exogenous variable in the subsequent period, and the system then determines a new equilibrium in response to this change.

If the government uses bonds or money to balance its budget then \( \hat{H} \) drops out as an endogenous variable and equation (38) drops out as an intratemporal equation. In this case we can write

\[
\frac{dG}{Y} - \frac{dT}{Y} = \gamma_m \dot{\hat{M}}(t+1) - \gamma_m \ddot{M}(t) + \gamma_b \dot{\hat{b}}(t+1) - \gamma_b \ddot{b}(t) - (\gamma'_b - \gamma'_b) \dddot{\hat{b}}_b
\]  
(42)
where $\gamma'_m = \frac{M(t+1)}{Y(t)}$

$\gamma'_m = \frac{M(t)}{Y(t)}$

$\gamma'_b = \frac{b(t+1)}{i_b(t)}Y(t)$

$\gamma'_b = \frac{b(t)}{i_b(t)}Y(t)$

Thus, equation (42) is only used if the government changes the stock of money or bonds to balance the budget. In this case, equation (42) is used to determine the stock adjustment of money or bonds in the same way that equation (41) is used to determine the stock adjustment of capital. Thus if the government balances the budget with money or bonds, the change in the endogenous variables comprising $(dG - dT)/Y$ (see equation (36)) in period $t$ determines $\hat{M}(t+1)$ or $\hat{b}(t+1)$, which are treated as being exogenous in period $(t+1)$. Thus, in period $(t+1)$ the system reacts to the values of $\hat{K}(t+1)$ and $\hat{M}(t+1)$ or $\hat{b}(t+1)$, determined by equations (41) and (42).

4. Simulation Analysis of the Impact of Capital Taxation

The model outlined above is quite general and can be used to analyze a number of incidence questions. While the focus of this paper is on the incidence of sectoral capital taxation, we can also analyze the incidence effects of alternative forms of equilibrating adjustments in the government budget constraint. Thus by letting the government balance its budget by using either neutral head taxes, bonds, or money, we can obtain some
insights into the question of the "burden" of the national debt.\textsuperscript{23}

To analyze the incidence of a sectoral capital tax, we assume a
once and for all increase in the capital tax coefficient in sector 1
and thus set $\hat{T}^1 = .10$ in period one and $\hat{T}^1 = 0$ in subsequent periods.
We then analyze the general equilibrium response of the system to the
exogenously given tax change under three different endogenous adjustments
in the government budget constraints: head taxes ($\hat{H}$), money ($\hat{M}$) and
bonds ($\hat{b}$). With the exception of the specified change in $\hat{T}^1$ and the
intertemporally determined (induced) change in the stock variables, we will
assume that all other exogenous variables remain constant. Thus changes in
the endogenous variables can be attributed to the direct change in the
tax coefficient and the induced changes in the relevant assets.

We assume an initial equilibrium with its associated share variables,
and then postulate a given change in $\hat{T}^1$ in period one. This leads to
equilibrating changes in all of the endogenous variables in period one,
and thus new share variables to be used in period 2. Moreover, because
levels of savings and investment have changed, there will be a change
in the capital stock. Similarly, if the budget is balanced through bonds
or money, there will be a change in these stock variables. Thus the changes
in the stock variables (capital, bonds, and/or money) become the exogenous
variables that drive the system in subsequent periods. Thus the first
period response of the system reflects the direct effect of the tax change
and corresponds to the usual static analysis. The response of the system
in subsequent periods reflects the effect of the asset changes that were

\textsuperscript{23}See Diamond (1965) for a full discussion of the burden of the debt.
induced by the initial tax change. These subsequent effects, of course, represent the dynamic incidence effects. The sum of the first period and subsequent effects therefore represents the net incidence effects of the change in sectoral capital taxation.

We can assume that the relevant elasticities remain constant throughout the period. These are given in Table 1. Table 2 gives the initial share variables, which, of course, change throughout the period of analysis.

Tables 3-5 illustrate the incidence effects of setting $T_{KL} = .10$ in the first period and zero thereafter. The first column shows the direct impact of the tax change, while columns (2)-(5) show the response of the system to equilibrating changes in the capital stock alone (Table 3), the capital stock and money (Table 4), and the capital stock and bonds (Table 5).

Table 3 permits us to analyze the impact of capital taxation with a neutral head tax adjustment. Column (2) shows the short-run static incidence effects with a fixed capital stock and is analogous to the traditional static incidence analysis. The results of this simulation experiment are consistent with the results of Harberger (1962) and Mieszkowski (1967) and indicate that capital bears virtually the entire burden of the tax in the short run. Thus the increase in the tax causes capital and labor to move from sector 1 to sector 2 and leads to a rise in the real wage and a fall in the real return to capital. The tax also causes a fall in the price level. Consequently although money disposable income falls, real disposable income rises as does savings and investment. Hence the capital stock rises as does the labor force.
Table 1

Elasticity Values used in Simulation Analyses

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>$\sigma_1$</td>
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<tr>
<td>$\sigma_2$</td>
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</tr>
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Table 2

Initial Values of Share Variables Used in Simulation Experiments

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<td>$\mu_{D}$</td>
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Table 3

Cumulative Response of Selected Endogenous Variables to
a Change in Capital Taxation in Sector 1, Head Taxes

Endogenous ($\tilde{T}_{K1} = .10$ in period 1, 0 thereafter)

<table>
<thead>
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<th>Endogenous Variable</th>
<th>Initial Change in period 1</th>
<th>Subsequent Cumulative Changes in Periods</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>2-5</td>
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</tr>
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Table 4
Cumulative Response of Selected Endogenous Variables to a Change in Capital Taxation in Sector 1, Money Endogenous

\( \hat{I}_{K1} = .10 \text{ in period 1; 0 otherwise} \)

<table>
<thead>
<tr>
<th>Endogenous Variable</th>
<th>Initial Change in period 1</th>
<th>Subsequent Changes in Periods 2-5</th>
<th>Cumulative Changes in Periods 2-10</th>
<th>Cumulative Changes in Periods 2-15</th>
<th>Cumulative Changes in Periods 2-20</th>
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Table 5
Cumulative Response of Selected Endogenous Variables to a Change in Capital Taxation in Sector 1, Bonds Endogenous
($T_{K1} = .10$ in period 1; 0 otherwise)

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<th>Subsequent Cumulative Changes in Periods</th>
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The change in the capital stock then leads to further adjustments in the economy, which tend to counteract the impact of the initial tax change. Subsequent adjustments cause the rate of investment to fall relative to the growth of the labor force and thus the return to capital rises relative to that of labor.

The magnitude of these counteracting adjustments is quite small, however. By the end of the 20th period, capital has only recovered some six percent of its initial losses in absolute terms. Relative to labor, however, it has fared somewhat better. Initially, \( r-\hat{w} = -0.097 \), while in periods 2-20, \( r-\hat{w} = 0.008 \). Thus the cumulative impact is still that \( r-\hat{w} = -0.089 \), which indicates that capital still bears the brunt of the tax. Thus in a world of neutral lump sum adjustments, the dynamic incidence effects do not appear to differ substantially from the static incidence effects.

Of course, neutral lump sum tax adjustments are not available in the real world where the government has to balance its budget by other means; and when the government balances its budget using bonds or money the story is quite different from the one in which the government uses neutral taxes.

Table 4 shows the incidence effects of setting \( T_{K1} = 0.10 \) when the government balances the budget using money and Table 5 shows the incidence effects when the government uses bonds. Since neither money nor bonds can change within the first period columns (1) of both tables are the same and differ from column (1) of Table 3 in that no offsetting transfers are assumed. Nevertheless, the results of column (1) in all three tables are qualitatively similar, and indicate that, in the short run, capital
bears the burden of a sector tax on capital.

When money or bonds adjust endogenously, in subsequent periods the system must not only react to the changes in the capital stock, but also to the changes in the stock of money or bonds. In the case where the government uses money to balance its budget, $\hat{r} - \hat{w} = -0.087$ initially, indicating that capital bears the brunt of the tax increase. In subsequent periods, however, the return to capital rises relative to that of labor. Thus in periods 2-20, the cumulative change of $\hat{r} - \hat{w} = 0.066$ indicates that capital has regained much of its position relative to labor.

The results are qualitatively similar where the government balances the budget by buying bonds. The initial $\hat{r} - \hat{w} = -0.087$, while during periods 2-20, the cumulative $\hat{r} - \hat{w} = 0.05$. Thus in the long run, capital bears considerably less of the tax than it does in the short run.

A comparison of the incidence effects of capital taxation financed by money and bonds is instructive, and yields some insight into the burden of the debt, which considers the incidence effects of increased government expenditure under bond or money finance. Because bond finance reduces capital formation, income (and presumably utility) are lower under bond finance than under money finance. In this case, we postulate an increase in distorting taxes, and hence a reduction in money or bonds. Thus we would expect more capital formation under bond finance, which indeed occurs. With money finance, there is a cumulative increase of $\hat{I} = 0.0615$, while under bond finance there is a cumulative increase of $\hat{I} = 0.0677$. However, the cumulative changes in real income are virtually the same in both cases; for period 1-20, under money finance, $\hat{Y} - \hat{q} = 0.010$, while under bond finance $\hat{Y} - \hat{q} = 0.008$. Thus in terms of income changes, the impacts of money and bond finance appear to be qualitatively similar.
5. Summary and Conclusions

Perhaps the most striking finding of this paper is the differential incidence effects of sectoral capital taxation when the government uses neutral head taxes to balance its budget and when the government uses either money or bonds to balance its budget. In the first case, the long-run incidence effects are quite similar to the short-run incidence effects and indicate that capital bears the brunt of an increase in a sectoral capital tax. Thus this analysis corroborates the static incidence analysis of Harberger and Mieszkowski.

When the government uses either money or bonds to balance its budget, however, the conclusions change substantially. While capital still bears the tax in the short run, in the long run, it is able to shift a major portion of the tax to labor. In the case of money finance, capital ultimately bears only some 20 percent of the tax burden, while in the case of bond finance, capital ultimately bears somewhat less than 40 percent of the tax burden. Hence under the more realistic assumptions concerning budgetary balance, we find that labor shares a major portion of the tax with capital.

Thus this paper not only illustrates the importance of analyzing the incidence of capital taxation in a dynamic framework, but also the importance of considering a range of equilibrating adjustments on the part of government. Since the government typically cannot use lump sum adjustments to balance its budget, it is important to analyze tax incidence under a range of realistic budgetary adjustments. It is clear that the incidence effects under realistic budgetary adjustments may be quite different from those under lump sum tax adjustments.
It would, of course, be rash to draw any firm conclusions about the incidence of the corporate profits tax (or any form of capital taxation) from this highly simplified analysis. Nevertheless, it does indicate that a static framework that assumes lump sum tax adjustments may lead to quite different incidence conclusions than a dynamic framework that assumes adjustments in the stock of money or bonds. Consequently, until we know more about the general equilibrium characteristics of the economy, it is probably well to remain agnostic concerning the incidence effects of capital taxation.
REFERENCES


