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Convergence in International Output

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Summary

This paper proposes and tests new definitions of convergence and common trends for per capita output. We define convergence for a group of countries to mean that each country has identical long-run trends, either stochastic or deterministic, while common trends allow for proportionality of the stochastic elements. These definitions lead naturally to the use of cointegration techniques in testing. Using century-long time series for 15 OECD economies, we reject convergence but find substantial evidence for common trends. Smaller samples of European countries also reject convergence but are driven by a lower number of common stochastic trends.

1 Introduction

One of the most striking features of the neoclassical growth model is its implication for cross-country convergence. In standard formulations of the infinite-horizon optimal growth problem, various turnpike theorems show that steady-state per capita output is independent of initial output levels. Further, differences in microeconomic parameters will generate stationary differences in per capita output and will not imply different growth rates. Consequently, when one observes differences in per capita output growth across countries, one must either assume that these countries have dramatically different microeconomic characteristics, such as different production functions or discount rates, or regard these discrepancies as transitory.

Launched primarily by the theoretical work of Romer (1986) and Lucas (1988), much attention has been focused on the predictions of dynamic equilibrium models for long-term behavior when various Arrow-Debreu assumptions are relaxed. Lucas and Romer have shown that divergence in long-term growth can be generated by social increasing returns to scale associated with both physical and human capital. An empirical literature exploring convergence has developed in parallel to the new growth theory. Prominent among these contributions is the work of Baumol (1986), DeLong (1988), Barro (1991) and Mankiw, Romer, and Weil (1992). This research has interpreted a finding of a negative cross-section correlation between initial income and growth rates as evidence in favor of convergence.

The use of cross-section results to infer the long-run behavior of national output ignores valuable information in the time series themselves, which can lead to a spurious finding of convergence. First, it is possible for a set of countries which are diverging to exhibit the sort of negative correlation described by Baumol *et al.* so long as the marginal product of capital is diminishing. As shown by Bernard and Durlauf (1992), a diminishing marginal product of capital means that short-run transitional dynamics and long-run, steady-state behavior will be mixed up in cross-section regressions. Second, the cross-section procedures work with the null hypothesis that no countries are converging and the alternative hypothesis that all countries are, which leaves out a host of intermediate cases.

In this paper we propose a new definition and set of tests of the convergence hypothesis. Our research differs from most previous empirical work in that we test convergence in an explicitly stochastic framework. If long-run technological progress contains a stochastic trend, or unit root, then convergence implies that the permanent components in output are the same across countries. The theory of cointegration provides a natural setting for testing cross-country relationships in permanent output movements.

Our analysis, which examines annual log real output per capita for 15 OECD economies from 1900 to 1987, leads to two basic conclusions about international output fluctuations.[1] First, we find very little evidence of convergence across the economies. Per capita output deviations do not appear to disappear systematically over time. Second, we find that there is strong evidence of common stochastic elements in long-run economic fluctuations across countries. As a result, economic growth cannot be explained exclusively by idiosyncratic, country-specific factors. A relatively small set of common long-run factors interacts with individual country characteristics to determine growth rates.

Our work is related to studies by Campbell and Mankiw (1989), Cogley (1990), and Quah (1990) who have explored patterns of persistence in international output. Using quarterly post-1957 data, Campbell and Mankiw demonstrate that 7 OECD economies exhibit both persistence and divergence in output. Cogley, examining 9 OECD economies using a similar data set to the one here, concludes that persistence is substantial for many countries; yet at the same time he argues that common factors generating persistence imply that "long run dynamics prevent output levels from diverging by too much." Quah finds a lack of convergence for a wide range of countries on the basis of post-1950 data. Our analysis differs from this previous work in three respects. First, we directly formulate the relationship between cointegration, common factors, and convergence, which permits one to distinguish between common sources of growth and convergence. Second, we attempt to determine whether there are subgroups of converging countries and thereby move beyond the all or nothing approach of previous authors. Third, we employ different econometric techniques and data sets which seem especially appropriate for the analysis of long-term

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[1] The countries are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, the Netherlands, Norway, Sweden, the United Kingdom, and the United States.

growth behavior.

The spirit of our analysis has much in common with work by Pesaran, Pierse and Lee (1993) and Lee, Pesaran, and Pierse (1992), who study multisector output persistence for the US and UK respectively. These papers derive methods to measure the long run effects of a shock originating in one sector on all sectors in the economy. While the focus of that research has been on measuring persistence rather than convergence, a useful extension of the current paper would be the application of the multivariate persistence measures to international data to both provide additional tests of convergence as well as to provide a framework for measuring the sources of growth.

The plan of the paper is as follows: Section 2 provides definitions of convergence and common trends using a cointegration framework. Section 3 outlines the test statistics we use. Section 4 describes the data. Section 5 contains the empirical results. The evidence from the cross-country analysis argues against the notion of convergence for the whole sample. Alternatively there do appear to be groups of countries with common stochastic elements.

2 Convergence in stochastic environments

The organizing principles of our empirical work come from employing stochastic definitions for both long-term economic fluctuations and convergence. These definitions rely on the notions of unit roots and cointegration in time series.

We model the individual output series as satisfying Equation 2.1:

$$a(L)Y_{i,t} = \mu_i + \varepsilon_{i,t} \tag{2.1}$$

where $a(L)$ has one root on the unit circle and $\varepsilon_{i,t}$ is a mean zero stationary process. This formulation allows for both linear deterministic and stochastic trends in output. The interactions of both types of trends across countries can be formalized into general definitions of convergence and common trends.

Definition 2.1. *Convergence in per capita output*

Log per capita outputs in countries 1,...,p converge if

1. $Y_{i,t}, \ldots, Y_{p,t}$ *satisfy Equation 2.1,*
2. $\mu_i = \mu_j \; \forall \, i, j,$
3. $Y_{i,t}, \ldots, Y_{p,t}$ *are cointegrated with a cointegrating matrix,* β', *such that*

$$\beta' = [I_{p-1}, \; -\bar{e}_{p-1}] \tag{2.2}$$

where I_{p-1} *is a* $p-1$ *identity matrix and* $\bar{e}_{p-1}$ *is a* $p-1 \times 1$ *vector of ones.*

Definition 2.2. *Common trends in per capita output*

Long-run log per capita outputs in countries 1,...,p are determined by common trends if

1. *the individual output series,* $Y_{i,t}, \ldots, Y_{p,t}$, *satisfy Equation 2.1,*

2. $\mu_i = \mu_j \ \forall \ i, j$,

3. $Y_{1,t}, \ldots, Y_{p,t}$ *are cointegrated.*

The first definition gives us a formal definition of convergence. If countries are to attain the same long-run growth rates with output levels separated only by a stationary difference, then they must satisfy Definition 2.1. Each series must contain the same time trend and be cointegrated with every other series with the cointegrating vector (1,-1).

However, if a group of output series does not satisfy Definition 2.1, but instead satisfies the weaker Definition 2.2, then output levels will be cointegrated but the stochastic trends for the group will not be equal. It will remain true that permanent shocks will be related across countries. This is the natural definition to employ if we are interested in the possibility that there are a small number of stochastic trends affecting output which differ in magnitude across countries.

The role of linear deterministic trends in our analysis is straightforward. If countries' outputs contain linear trends, then long-run levels and growth rates will be equal only if those trends are identical across all countries. Thus both convergence and common in any group require that all countries have the same linear trend. In practice we will find that output for all countries in our sample is well modeled by a stochastic trend.[2]

Our definition of convergence is substantially different from that employed by Baumol *et al.* who have defined convergence to mean that there is a negative cross-section correlation between initial income and growth, thereby inferring long-run output behavior from cross-section behavior. Our analysis studies convergence by directly examining the time series properties of various output series, which places the convergence hypothesis in an explicitly dynamic and stochastic environment.

One potential difficulty with the use of unit root tests to identify convergence is the presence of a transitional component in the aggregate output of various countries. Time series tests assume that the data are generated by an invariant measure, i.e. the sample moments of the data are interpretable as population moments for the underlying stochastic process. If the countries in our sample start at different initial conditions and are converging to, but are not yet at a steady state output distribution, then the available data may be generated by a transitional law of motion rather than by an invariant stochastic process. Consequently, unit root tests may erroneously accept a no convergence null. Simulations in Bernard and Durlauf (1992) suggest that the size distortions are unlikely to be significant.

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[2] Results are available upon request from the authors.

3 Output relationships across countries

3.1 Econometric methodology

In order to test for convergence and common trends, we employ multivariate techniques developed by Phillips and Ouliaris (1988) and Johansen(1988)

Let $y_{i,t}$ denote the log per capita output level of country i and $Dy_{i,t}$ the deviation of output in country i from output in country 1, i.e. $y_{1,t} - y_{i,t}$. Y_t is defined as the $n \times 1$ vector of the individual output levels, ΔY_t as the first difference of Y_t, DY_t as the $(n-1) \times 1$ vector of output deviations, $Dy_{i,t}$, and ΔDY_t the first differences of the deviations.

The starting point for the empirical work is the finding that the individual elements of the per capita output vector are integrated of order one. It is then natural to write a multivariate Wold representation of output as

$$\Delta Y_t = \mu + C(L)\epsilon_t. \tag{3.1}$$

As shown by Engle and Granger (1987), if the p output series are cointegrated in levels with r cointegrating vectors then $C(1)$ is of rank $p-r$ and there is a vector ARMA representation. A first test for the number of linearly-independent stochastic trends has been developed by Phillips and Ouliaris (1988) who analyze the spectral density matrix at the zero frequency. A second test is due to Johansen (1988, 1989) who estimates the rank of the cointegrating matrix.

For a vector of output series, convergence and common trends impose different restrictions on the zero frequency of the spectral density matrix of ΔY_t, $f_{\Delta Y}(0)$. Convergence requires that the persistent parts be equal; common trends require that the persistent parts of individual output series be proportional. In a multivariate framework, proportionality and equality of the persistent parts corresponds to linear dependence, which is formalized as a condition on the rank of the zero-frequency spectral density matrix. From Engle and Granger (1987), if the number of distinct stochastic trends in Y_t is less than n, then $f_{\Delta Y}(0)$ is not of full rank. If all n countries are converging in per capita output, then $f_{\Delta DY}(0)_{i,i} = 0$ $\forall i$, or equivalently, the rank of $f_{\Delta DY}(0)$ is 0. On the other hand, if several output series have common persistent parts, the output deviations from a benchmark country must all have zero-valued persistent components.

Spectral-based procedures devised by Phillips and Ouliaris permit a test for complete convergence as well as the determination of the number of common trends for the 15 output series. The tests make use of the fact that the spectral density matrix of first differences at the zero frequency will be of rank $q \leq n$ where q is the number of linearly-independent stochastic trends in the data and n is the number of series in the sample. This reduction in rank is captured in the eigenvalues of the zero frequency of the spectral density matrix. If the zero frequency matrix is less than full rank, $q < n$ then the number of positive eigenvalues will also be $q < n$. The particular Phillips-Ouliaris test we employ is a bounds test that examines the smallest $m = n - q$ eigenvalues to determine if they are close to zero. We use two critical values for the bounds test, $C_1 = 0.10\frac{m}{n}$ and $C_2 = 0.05$. These critical values assess the average of the m smallest eigenvalues in comparison to the average of all the eigenvalues. The first critical value, C_1, is $m \times 10\%$ of the average root. The second critical value, C_2, corresponds to 5% of the total variance.

For the Johansen tests we impose some additional structure on the output series. We assume that a finite-vector autoregressive representation exists and rewrite the output vector process as,

$$\Delta Y_t = \Gamma(L)\Delta Y_t + \Pi Y_{t-1} + \mu + \epsilon_t \tag{3.2}$$

where

$$\Gamma_i = -(A_{i+1} + \ldots - A_k), \qquad (i = 1, \ldots, k-1),$$

and

$$\Pi = -(I - A_1 - \ldots - A_k).$$

Π represents the long-run relationship of the individual output series, while $\Gamma(L)$ traces out the short-run impact of shocks to the system. We are interested only in the long-run relationships, and thus all the tests and estimates of cointegrating vectors come from the matrix, Π, which can be written as

$$\Pi = \alpha\beta' \tag{3.3}$$

with α and β, $p \times r$ matrices of rank $r \leq p$. β is the matrix of cointegrating vectors, as $\beta' Y_{t-k}$ must be stationary in Equation 3.2. However, β is not uniquely determined; a different choice of α satisfying Equation 3.3 will produce a different cointegrating matrix. Regardless of the normalization chosen, the rank of Π is still related to the number of cointegrating vectors. If the rank of Π equals p, then Y_t is a stationary process. If the the rank of Π is $0 < r < p$, there are r cointegrating vectors for the individual series in Y_t and hence the group of time series is being driven by $p-r$ common shocks. If the rank of Π equals zero, there are p stochastic trends and the long-run output levels are not related across countries. In particular, from Definition 2.1, for the individual output series to converge there must be $p-1$ cointegrating vectors of the form (1,-1) or one common long-run trend.

Two test statistics proposed by Johansen to test the rank of the cointegrating matrix are derived from the eigenvalues of the MLE estimate of $\hat{\Pi}$. If $\hat{\Pi}$ is of full rank, p, then it will have no eigenvalues equal to zero. If, however, it is of less than full rank, $r < p$, then it will have $p-r$ zero eigenvalues. Looking at the smallest $p-r$ eigenvalues the statistics are

$$trace = T \sum_{i=r+1}^{p} \hat{\lambda}_i \approx -2ln(Q;r,p) = -T \sum_{i=r+1}^{p} ln(1-\hat{\lambda}_i) \tag{3.4}$$

and

$$maximum\ eigenvalue = T\hat{\lambda}_{r+1} \approx -2ln(Q;r,r+1) = -Tln(1-\hat{\lambda}_{r+1}) \tag{3.5}$$

The *trace* statistic tests the null hypothesis that the rank of the cointegrating matrix is r against the alternative that the rank is p. The *maximum eigenvalue* statistic tests the null hypothesis that the rank is r against the alternative that the rank is $r+1$. Critical values for the asymptotic distributions of both statistics are tabulated in Osterwald-Lenum (1992).

4 Data

The data used in the empirical exercise are annual log real GDP per capita in 1980 international dollars. The series run from 1900-1987 for 15 industrialized countries with the GDP data drawn from Maddison (1989) and the population data from Maddison (1982). Population for 1980-1987 comes from IFS yearbooks.

The population data as published in Maddison (1982) are not adjusted to conform to current national borders, while the GDP data are adjusted. Failure to account for border changes can lead to large one time income per capita movements as population is gained or lost. For example, GDP per capita in the UK jumps in 1920 without a correction for the loss of the population of Ireland in that year. To avoid these discrete jumps we adjust the population to reflect modern borders.[3] The GDP data set also has a few minor problems. The year-to-year movements during the two world wars for Belgium and during WWI for Austria are constructed from GDP estimates of neighboring countries.

5 Empirical results on convergence and cointegration

In testing for convergence and common trends, we use three separate groupings of countries: all 15 countries together, the 11 European countries and finally a subset of 6 European

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[3] This type of gain or loss affects Belgium, Canada, Denmark, France, Italy, Japan, and the UK at least once. If territory, and thus population, are lost by country X in year T_1, we adjust earlier years by extrapolating backward from T_1 using the year-to- year population changes of country X.

countries which exhibit a large degree of pairwise cointegration.[4] Results from the Phillips-Ouliaris procedures on convergence and common trends are in Tables 1 and 2 respectively. Results using the Johansen methods are in Table 3.

We initially test for convergence in the 15 country group by performing the Phillips-Ouliaris bounds tests on the first difference of output deviations, ΔDY_t, having subtracted off the US output. If countries converge, then we would expect to find 1 distinct root in output levels and no roots in the deviations from a benchmark country. If idiosyncratic trends dominate for every country, then we would expect to find n distinct roots for n countries in the levels. If the number of significant roots lies between these extremes, this indicates the presence of common trends in international output. As an alternative measure of the number of common trends, we look at the cumulative percentage of the sum of the roots. If the first $p < n$ largest roots contribute 95% or more of the sum, then we conclude that there are p important common stochastic trends for the block.[5]

Table 1 presents the Phillips-Ouliaris bounds tests for convergence and the cumulative sums of the eigenvalues for the groups mentioned above.[6] If the lower bound on the largest root is greater than the critical level we can cannot reject the no convergence null. Additionally, if the largest root acounts for less than 95% of the total variance we conclude that there is more than one stochastic trend for the group. Table 2 presents two different tests for the number of common trends in each group. First, if the upper bound is less than the critical value for a given p, we can reject the null hypothesis that there are p or more distinct roots. If the lower bound is greater than the same critical value then we cannot reject the hypothesis that there are at least p distinct roots. We also look for the number of eigenvalues that account for 95% or more of the total variance.

The multivariate results from the Johansen trace and maximum eigenvalue statistics on convergence and cointegration are presented in Tables 3a and 3b for a VAR lag length of 2.[7] The two statistics give different estimates of the cointegrating vectors; the maximum eigenvalue test is often not significant for any number of cointegrating vectors. Test results are presented for null hypotheses on number of common trends ranging from 1 to 15.

The evidence on convergence is quite striking. For all test statistics and in all three samples the convergence hypothesis fails. The direct convergence test in Table 1 cannot reject the no-convergence null for both critical levels as the largest eigenvalue is statistically

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[4] The 6 European countries are Austria, Belgium, Denmark, France, Italy and the Netherlands. Bernard (1991) finds cointegration in 10 of 15 pairs.

[5] Cogley (1990) uses a similar measure.

[6] K, the size of the Daniell window was chosen to be $T^{0.6}$, or 27 for our sample.

[7] Reducing the lag length increased the number of trends somewhat.

different from zero for all three groupings. Additionally, both the trace and the maximum eigenvalue statistics reject convergence in every group in Table 3a.

Having failed to find evidence for convergence, or a single long-run trend, we turn to the test for the number of common trends. The Phillips-Ouliaris statistics for the fifteen country sample and critical value C_1 $(= \frac{m}{n})$ reject the null hypothesis that there are 7 or more distinct roots and cannot reject the null that there are at least 4 distinct roots. With the alternative critical value of 5% of the sum of the eigenvalues,C_2, we again reject for 7 or more distinct roots but now cannot reject for at least 5. This leads us to posit that there is a large common stochastic component over the sample. The six largest roots account for 96.7% of the total, coinciding with the results from the test statistics. On the other hand, the largest root accounts for barely 50% and the largest two roots for about 75% of total variance, which argues against the existence of just a single common factor, as is required for convergence. The Johansen trace statistic rejects 12 or fewer cointegrating vectors at the 5% level and 13 or fewer at the 10% level for the entire fifteen country sample. This implies that there are only two or three long-run shocking forces for the entire group. The maximum eigenvalue statistic does not reject for any number of trends.

Taking all 11 of the European countries as a group, we reject the null hypothesis that there are 6 or more trends and cannot reject that there are at least 4 trends with the C2 statistic and that there are at least 3 trends with the C1 statistic. The Johansen trace statistic rejects 5 or more trends, while the maximum eigenvalue test again cannot reject for any number of cointegrating vectors. These results suggest that there are on the order of 4-5 long-run processes driving output in the European countries.

Turning to the results for the six European countries, we reject the null that there are are 4 or more distinct trends with both the C_1 and C_2 critical values and cannot reject the null that there are at least 3, again with both values. 97.8% of the sum comes from the three largest eigenvalues. Using MLE statistics, the smaller six European country group rejects 2 or more trends with the trace statistic and 5 or more with the maximum eigenvalue test.

6 Conclusions

This paper attempts to answer empirically the question of whether there is convergence in output per capita across countries. We first construct a stochastic definition of convergence based on the theory of integrated time series. Time series for per capita output of different countries can fail to converge only if the persistent parts of the time series are distinct. Our analysis of the relationship among long-term output movements across countries reveals little evidence of convergence. Virtually all of our hypothesis tests cannot reject the null hypothesis of no convergence. On the other hand, we find evidence that there is substantial cointegration across OECD economies. The number of integrated processes driving the 15 countries' output series appears to be on the order of 3 to 6. Our results therefore imply that there is some set of common factors which jointly determines international output growth.

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Table 1a. Phillips-Ouliaris Bounds Tests for Convergence								
Bounds Tests**								
All Countries			6 European Countries			11 European Countries		
Trends	Lower	Upper	Trends	Lower	Upper	Trends	Lower	Upper
<1	1.60+	3.09	<1	0.68+	1.32	<1	0.29+	0.46

If the upper bound is below the critical value for the largest root, reject null of no convergence.
If the lower bound is above the critical value for the largest root, cannot reject null of no convergence.
\+ Significant for critical value of 0.05.
** These statistics are calculated on the vector of first differences of $GDPi - GDPk$. For all countries, the US is subtracted off. For the 6 European countries, France is subtracted off. For the 11 European countries, France is subtracted off.

Table 1b. Cumulative Percentage from p Largest Eigenvalues						
	All Countries		6 European Countries		11 European Countries	
	Trends	Cumulated %	Trends	Cumulated %	Trends	Cumulated %
Largest	1	0.74	1	0.69	1	0.69
	2	0.88	2	0.89	2	0.89
	3	0.93	3	0.98	3	0.95
	4	0.96	4	1.00	4	0.97
	5	0.97	5	1.00	5	0.98
	6	0.98			6	0.99
	7	0.99			7	1.00
	8	0.99			8	1.00
	9	1.00			9	1.00
	10	1.00			10	1.00
	11	1.00				
	12	1.00				
	13	1.00				
Smallest	14	1.00				

Table 2a. Phillips-Ouliaris Bounds Tests for Cointegration

All Countries			6 European Countries			11 European Countries		
Trends	Lower	Upper	Trends	Lower	Upper	Trends	Lower	Upper
15	0.00	0.00	6	0.00	0.00	11	0.00	0.00
14	0.00	0.00	5	0.01	0.01	10	0.00	0.00
13	0.00	0.00	4	0.02	0.03^{*+}	9	0.00	0.01
12	0.00	0.00	3	0.06^{*+}	0.11	8	0.01	0.01
11	0.00	0.01	2	0.23	0.40	7	0.01	0.02
10	0.01	0.01				6	0.02	0.03^{*+}
9	0.01	0.01				5	0.03	0.06
8	0.01	0.02				4	0.06^{+}	0.10
7	0.03	0.04^{*+}				3	0.12^{*}	0.19
6	0.04	0.07				2	0.30	0.46
5	0.07^{+}	0.10						
4	0.11^{*}	0.17						
3	0.19	0.29						
2	0.39	0.57						

Table 2b. Cumulative Percentage from p Largest Eigenvalues

	All Countries		6 European Countries		11 European Countries	
	Trends	Cumulated %	Trends	Cumulated %	Trends	Cumulated %
Largest	1	0.52	1	0.69	1	0.63
	2	0.76	2	0.91	2	0.84
	3	0.87	3	0.98	3	0.92
	4	0.92	4	0.99	4	0.95
	5	0.95	5	1.00	5	0.98
	6	0.97	6	1.00	6	0.99
	7	0.98			7	0.99
	8	0.99			8	1.00
	9	0.99			9	1.00
	10	1.00			10	1.00
	11	1.00			11	1.00
	12	1.00				
	13	1.00				
	14	1.00				
Smallest	15	1.00				

If the upper bound is below the critical value, reject null of P or more distinct roots. If the lower bound is above the critical value, cannot reject null of at least P distinct roots.

$\star$ Significant at $0.10m/n$, n is the number of countries, m is the number of roots $= 0$.

$+$ Significant at 5% of the sum of the roots.

Table 3. Multivariate Tests for Convergence and Cointegration
(VAR lag length = 2)

Table 3a. Convergence‡

All	European 6	European 6
62.00*	53.84*	31.89*

Table 3b. Cointegration

	All			European			European 6	
Trends	**Trace**	**Max Eig**	**Trends**	**Trace**	**Max Eig**	**Trends**	**Trace**	**Max Eig**
> 14	553.99	71.00	> 10	312.18	60.46	> 5	102.95	40.54*
> 13	482.99	68.62	> 9	251.72	53.06	> 4	62.41	23.27
> 12	414.37	62.00	> 8	198.66	51.06	> 3	39.14*	17.55
> 11	352.37	59.38	> 7	147.60	38.29	> 2	21.59	14.02
> 10	292.99	48.89	> 6	109.31*	34.80	> 1	7.57	7.00
> 9	244.10	41.07	> 5	74.51	22.49	> 0	0.57	0.57
> 8	156.82	34.66	> 4	52.02	18.07			
> 7	162.59	36.03	> 3	33.95	15.95			
> 6	126.56	32.61	> 2	18.00	9.16			
> 5	93.95	31.23	> 1	8.84	7.55			
> 4	62.71	24.52	> 0	1.28	1.28			
> 3	38.19*	17.22						
> 2	20.97	10.51						
> 1	10.46	9.08						
> 0	1.38	1.38						

* Rejects at 5%.

‡ Distributed $\chi^2(p-1)$ where p is the number of countries.