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Bubbles and Capital Flow Volatility:
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Abstract

Emerging market economies are fertile ground for the development of real
estate and other financial bubbles. Despite these economies' significant growth
potential, their corporate and government sectors do not generate the financial
instruments to provide residents with adequate stores of value. Capital of-
ten flows out of these economies seeking these stores of value in the developed
world. Bubbles are beneficial because they provide domestic stores of value and
thereby reduce capital outflows while increasing investment. But they come at
a cost, as they expose the country to bubble-crashes and capital flow revers-
sals. We show that domestic financial underdevelopment not only facilitates
the emergence of bubbles, but also leads agents to undervalue the aggregate
risk embodied in financial bubbles. In this context, even rational bubbles can
be welfare reducing. We study a set of aggregate risk management policies to
alleviate the bubble-risk. We show that liquidity requirements, sterilization of
capital inflows and structural policies aimed at developing public debt markets
“collateralized” by future revenues, all have a high payoff in this environment.

JEL Codes: E32, E44, F32, F34, F41, G10

Keywords: Emerging markets, bubbles, excess volatility, crashes, capital flow
reversals, public debt market, financial underdevelopment, dynamic inefficiency.

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1 Introduction

Emerging market economies (EM’s) are plagued by episodes of bubble-like dynamics. These episodes begin with a “bubble” phase where credit, investment, asset prices, and capital inflows, all grow, and end with a bust phase when these variables collapse. Examples of these episodes include Argentina in the 1990’s, South East Asia in the mid 1990’s, Mexico in the early 1990’s, Chile, Mexico and other Latin American economies in the early 1980’s.

Academic theories of bubbles focus on two elements: the role of information in coordinating agents’ actions to grow and ultimately prick bubbles; and, the role of the macroeconomic environment in facilitating bubbles. We present a model that draws on the second element.

We first argue that an aggregate shortage of stores of value — which is the key element for bubble formation highlighted in the macroeconomics literature (i.e. dynamic inefficiency) — is prevalent in emerging markets. Poor investor protection, means that the corporate sector is unable to capitalize future earnings and provide stores of value to the economy. Fiscal and sovereign-default concerns also limit the ability of the government to issue reliable debt. These factors contribute to the “financial repression” that, for example, McKinnon (1973) has argued to be a prominent aspect of EM’s financial systems. The limited investment outlets, such as poor banking systems and conglomerates with severe corporate governance problems, receive investment flows despite their deficiencies. Real estate investment, which is one of the best protected investment vehicles in EM’s, serves as prominent store of value as well. Finally, where possible, agents actively seek high quality stores of value abroad by purchasing developed economies’ safe assets.¹

In this context, EM’s present a fertile macroeconomic environment for the emergence of bubbles. Starting from this premise, we develop a simple overlapping generations (OLG) model of stochastic bubbles. In the model, the absence of an adequate quantity of high-quality domestic financial instruments to store value induces domestic agents to seek this financial service abroad through systematic capital outflows. These outflows are costly for EM’s because they divert resources that may otherwise be spent growing the domestic economy; foreign interest rates are low relative to

¹In this light, the surge in demand for U.S. assets since the late 1990s, is a symptom of the shortage of high-quality stores of value in EM’s following the crash in local bubbles during that period.
these economies' growth potential and marginal product of capital. For reasons akin to those found in the closed economy literature on dynamic inefficiency, the gap between low external returns and high domestic growth rates creates a space for rational bubbles on unproductive local assets to arise. For concreteness, we refer to these as Real Estate Bubbles. They are a response to agents' demand for more profitable store of value instruments.

In the classical OLG model, the emergence of these rational bubbles is unambiguously good as they complete a missing “intergenerational” market (see, Samuelson 1958 and Tirole 1985). This is the case, at least in an ex-ante sense, even if these bubbles can crash as in Blanchard and Watson (1982) and Weil (1987). In contrast, in the emerging markets setup we describe, the presence of fragility in bubbles can render them socially undesirable, despite their service as a store of value.

We modify the standard OLG model in two ways to address the emerging markets issues that we are interested in, and to arrive at our results. First, we introduce an investment, rather than consumption, related demand for a store of value. Each generation of agents has an entrepreneurial and a banking sector. Investment projects arise in the entrepreneurial sector, when agents are old. Young agents demand some liquidity in order to fund these future investment projects. Second, and more importantly, we follow Caballero and Krishnamurthy (2001) by modelling constraints in the domestic and international capital market. At the margin, domestic entrepreneurs need to import international goods in order to undertake investment projects. However, the domestic capital market is segmented from the international capital market, and international investors do not lend to domestic agents against their investment projects. To facilitate trade with international investors, we endow each generation of domestic agents with a limited amount of international goods/collateral. The international endowment places a ceiling on how many goods the country can import for investment projects. We assume that only the domestic banking sector has the capability to lend to the entrepreneurial sector.

The international goods endowment of the generations grows at a high rate, capturing the idea that an EM grows fast and will be able to import more goods in the future. The high growth rate of this endowment creates some space for the development of a bubble. As in the standard analysis of OLG models, welfare may improve if the old sell a bubble asset to the next generation, collecting their international goods endowment in exchange, and so on. We study an economy with a stochastic bubble that provides the required liquidity to young agents, but may burst at any date.
Our main result can be understood by considering the liquidity demand of young bankers. These agents purchase a portfolio of the bubble asset and international liquidity (i.e. foreign bank account) in order to be in a position to lend to the entrepreneurial sector, when old. However, if the entrepreneurial sector cannot commit to repay all loans, in general the banker receives a return that is lower than the marginal product of entrepreneurial investment. In particular, if the bubble crashes both bankers and entrepreneurs are unable to sell their real estate holdings for the international endowment of the next generation. In this event, the investment in the entrepreneurial sector is constrained by the ex-ante portfolio share of international liquidity chosen by bankers and entrepreneurs. However, since the banker does not share fully in the return of entrepreneurial investment, the banker has a lower incentive to store the international goods that are required, at the margin, to finance all entrepreneurial investment projects in the crash, and more of an incentive to chase the higher return promised by the bubble. We show that welfare is often improved by reducing investment in the bubble.

During the growth phase of the bubble, the economy sustains high levels of investment. Entrepreneurs and bankers are able to sell their bubble asset for international liquidity with which they finance investment projects. Capital inflows during this period are high as agents are actively borrowing against the international collateral of the country. Domestic credit grows as bankers are flush with resources to lend to the entrepreneurial sector. When the bubble crashes, entrepreneurs and bankers are unable to trade for the international liquidity of other agents. Capital flows reverse, domestic credit and investment falls.\footnote{This is in stark contrast with the canonical OLG model, in which bubbles crowd out private investment, so that the bubble and private investment are negatively correlated. See Caballero, Farhi and Hammour (2004) and Ventura (2004) for models that also exhibit positively correlated bubbles and investment.} Thus, our model successfully reproduces bubble dynamics in emerging economies.

An important policy discussion in emerging markets (and developed ones) concerns managing the risks created by a bubble. In our model, rational bubbles arise endogenously as a result of the economy’s dynamic inefficiency but can be welfare reducing, because the private sector underestimates the costs of fragility.

There are two types of policies that can improve upon this outcome. The first are short-term risk-management policies that aim at discouraging excessive reallocation of liquidity and savings toward local real estate. Banking regulations such as inter-
national liquidity requirements reduce bubbles. These policies are similar to those discussed in Caballero and Krishnamurthy (2004) in the context of overborrowing and the dollarization of liabilities problems. A more novel liquidity policy in our model is sterilization of capital flows. We show that if the government has sufficient capacity to tax current endowments, then by issuing one-period government debt and using the proceeds to directly invest in international liquidity, the government can insure the private sector against the fragility of the bubble. In the crash, the government injects its international liquidity to offset the insufficient liquidity of the private sector.

The second type of policies are more directly linked to the source of bubbles in these economies. They are aimed at improving the supply of domestic financial assets whose prices are not governed by bubble dynamics. We show that if a government has sufficient credibility and commitment to securitize future taxes, then it can issue enough public debt (a sort of collateralized Diamond, 1965, debt) to crowd out the bubble while solving the dynamic inefficiency problem.3

The paper is related to several strands of literature. It builds on the work on rational bubbles in general equilibrium, and in particular on Tirole (1985) and on the segmented markets version in Ventura (2004). However, unlike the conclusion of these papers and that of much of the literature, bubbles may be socially inefficient in our model. This insight informs most of our discussion.

Saint-Paul (1992) also develops a model where rational bubbles may be socially inefficient. However his context is entirely different from ours, as in his case the inefficiency arises from bubbles crowding out physical capital in an endogenous growth model with positive capital spillovers. More closely related to our paper in terms of the focus on fragility, is that of Caballero, Farhi and Hammour (2004) where bubbles may increase the chance of a crash in a multiple equilibria economy. In their context too, the reason why this is potentially inefficient is an externality in capital accumulation. Instead, in our model the welfare implications stem from the riskiness of the bubble itself and from the private sector’s distorted perception of the aggregate risk of their choices.

In the literature on liquidity provision, Holmstrom and Tirole (1998) note that financial constraints lead to too little stores of value. In this context, they show that government debt can improve on the allocation of the private sector. Hellwig and

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3Of course, this solution may be fragile in itself, as expectations over the government’s ability to pledge future taxes may be variable. But this is uncertainty of a very different nature, unless it in itself can feedback into the governments ability or commitment to deliver on each of these fronts.
Lorenzoni (2004) study an economy where agents have limited commitment in repaying debt contracts. They show that in economies which are dynamically inefficient, long term debt can be sustained even under this limited commitment constraint.

In terms of the source of the pecuniary externality and private undervaluation of international liquidity, the mechanism is similar to that in Caballero and Krishnamurthy (2001, 2003). However, the focus on these papers is on the externality and not on the possibility of bubbles, their efficiency properties, and solutions to manage bubbles. Moreover, in the current paper the main problem is not one of insufficient country-wide international liquidity during crises but one of significant capital outflows due to a coordination failure.

Finally, our emphasis on preventive policies to manage bubble-risk rather than ex-post interventions contrasts with the views expressed by, e.g., Dornbusch (1999) and Bernanke (2002) for developed economies, where ex-ante and ex-post interventions take a more balanced role. The main reason for our emphasis on ex-ante policies is that while developed economy policymakers can count on access to resources during a bubble-crash, EM's governments and central banks often find themselves entangled in the crisis itself.

The paper is organized as follows. Section 2 describes the structure of the economy, while Section 3 introduces bubbles. Section 4 establishes the excessive volatility of the market outcome with bubbles. Section 5 and 6 discuss aggregate risk management and financial market development policies, respectively. Section 7 concludes.

2 A Productive Dynamically Inefficient Emerging Economy

The emerging economy is populated by an OLG of risk-neutral domestic agents that produce and consume only when old. There are two types of goods, international and domestic goods. The domestic goods are perishable, but the international goods can be saved abroad at the world interest rate of $r^*$. Domestic goods and international goods are perfect substitutes in the domestic consumers' preferences. While for international investors, only the international goods are tradeable and offer consumption value. These assumptions effectively limit foreign investors' participation in domestic markets.

Each agent is endowed with some date $t$ goods and some date $t+1$ goods. Agents
are born at date $t$ with an endowment of $W_t$ international goods. When old, agents receive an endowment of $RK_t$ domestic goods ($R > 1$). We can think of the latter endowment as the returns from a domestic plant of size $K_t$ that the agent is born with. Thus, endowments at $(t, t+1)$ for an agent born at time $t$ are $(W_t, RK_t)$.

At date $t+1$, one-half of the agents of generation $t$ become entrepreneurs, and one-half become bankers. The entrepreneurs receive an investment opportunity, which produces $RI_{t+1}$ units of domestic goods for an investment of $I_{t+1}$ units of international goods. This production occurs instantaneously. The bankers do not receive any direct investment opportunities, but may be in a position to lend to the investing entrepreneurs (see below). We note that agents are ex-ante identical, while ex-post there is heterogeneity.

We are centrally interested in how the young agent saves his international goods to finance investment (directly or indirectly) when old. For this section, we assume that the young agent saves all of his international goods abroad at the world interest rate of $r^*$. Let $W'_t$ denote the international goods available at $t+1$ to a member of generation $t$, so that:

$$W'_t = (1 + r^*)W_t.$$

At each date there is a domestic financial market, where entrepreneurs with investment opportunities borrow from bankers. The financial market is instantaneous in the same sense as production is instantaneous. At date $t+1$, an entrepreneur with an investment opportunity borrows $l_{t+1}$ international goods from a banker, offering to repay $p_{t+1}l_{t+1}$ domestic goods when production is complete. We impose a collateral constraint on this loan:

$$p_{t+1}l_{t+1} \leq \psi RK_t$$

where $\psi$ parameterizes the tightness of the collateral constraint.\footnote{The collateral constraint we have imposed requires that loans be collateralized by $\psi RK_t$. Any output from the new investment of $l_{t+1}$ is not considered collateral. This latter assumption is different than the standard credit constraints model, which would require that}

$$p_{t+1}l_{t+1} \leq \psi R(K_t + I_{t+1})$$

where again $\psi < 1$ parameterizes the tightness of the collateral constraint. Assuming that the borrower saturates this collateral constraint, yields:

$$I_{t+1} = \frac{1}{1 - \frac{\psi R}{p_{t+1}}} \left( \frac{\psi R}{p_{t+1}}K_t + W'_t \right)$$

where the first term is the familiar "equity" multiplier that arises in models of credit-constraints.
Our model assumes that the domestic capital market is segmented. Neither foreign investors nor the young of generation $t+1$ participate in the loan market. Within the logic of the model, this occurs because loans are repaid in perishable domestic goods, which have no consumption value to either generation $t + 1$ or foreign investors. More generally, we think that foreign investors have limited participation in local debt markets because of lack of local knowledge, and fear of sovereign expropriation/selective default. The latter concern is studied extensively in the sovereign debt literature (e.g. Bulow and Rogoff, 1989) which motivates a country-level debt limit. Likewise, we think of the bankers of generation $t$ as having more experience dealing with the entrepreneurs of generation $t$.

Equilibrium in the loan market requires that,

$$1 \leq p_{t+1} \leq R,$$

since at a price below one, lenders will not lend, and at a price above $R$ borrowers will not borrow.

The supply of loans is $W_t'/2$ and the constrained demand for loans is $\frac{\psi RK_t}{2p_{t+1}}$. We assume throughout that $\psi R < 1$ and we normalize quantities and set $K_t = W_t$. Utilizing these conditions, we find that in equilibrium:

$$p_{t+1} = 1,$$

so that bankers, in equilibrium, have international goods that are not lent to the entrepreneurs.

On average, emerging market economies grow fast. We capture this feature by assuming that the endowments $(W_t, RK_t)$ grows at a rate $g$ such that:

$$g > r^*.$$  

Substituting this expression and solving for the amount borrowed, yields:

$$l_{t+1} = \frac{1}{\frac{p_{t+1}}{\psi R} - 1} (K_t + W_t')$$

As the domestic capital market is segmented, the aggregate supply of loans comes from the resources of bankers. In total, the loan supply is $W_t'/2$ and the aggregate demand for loans is $l_{t+1}/2$. Thus,

$$p_{t+1} = \max[1, \psi R(2 + K_t/W_t')] = \max[1, 3\psi R]$$

which is very similar to the expression we derive under our assumption. Our assumption simplifies some of the algebra later in the paper without distracting from the substance of the results.
Thus, our basic economy behaves as a dynamically inefficient economy, in the sense that store of value (rather than the marginal product of capital) has a lower return than the rate of growth of the endowment. In this context, dynamic inefficiency means that the country is lending (or leaving) too many international goods abroad.\textsuperscript{5} At each time $t$ there is a capital outflow of $W_t$ and an inflow of $(1 + r^*)W_{t-1}$, thus the net outflow is:

$$NetOutflow_t = W_t - (1 + r^*)W_{t-1} = (g - r^*)W_{t-1} > 0.$$ 

The positive outflow is highly inefficient for a resource-scarce emerging market economy.\textsuperscript{6} It also hints at the presence of a large latent demand for a higher return store of value instrument among domestic agents.

### 3 Real Estate Bubbles

Young agents need a store of value to finance investment opportunities when these arise. In the basic structure we have outlined, the (saving side of the) economy is dynamically inefficient and the only store of value is lending international goods abroad at the safe but low world interest rate of $r^*$.

The familiar Samuelson (1958) solution to dynamic inefficiency is for all generations to enter a social contract: Rather than lending $W_t$ abroad, the young transfer their $W_t$ endowment of international goods to the old, and when old, receive the $W_{t+1}$ endowment of the new young, and so on. Each generation, effectively, receives a return $g > r^*$ on its international goods endowment.\textsuperscript{7} However, in an OLG model there is no mechanism for the old and young to write such contracts. Thus, in the context of emerging markets that concerns us here, the OLG assumption captures the dimension of financial underdevelopment that limits contracts beyond a small groups of contemporaneous market participants.

We suppose that in this context an unproductive and irreproducible asset ("real estate," for short) is traded domestically. Since the asset is unproductive, any positive price must be a pure bubble. We denote this price by $B_t$.

A positive price on the bubble asset is necessarily fragile, as the asset retains value only if current generations expect that future generations will demand the asset. We

\textsuperscript{5}This is the analog to the corporate-cash-flow empirical concept of dynamic inefficiency proposed by Abel et al (1989) for the context of closed economies.

\textsuperscript{6}It is only matter of relabeling to translate these excessive outflows into depressed inflows.

\textsuperscript{7}And the first generation receives an additional return.
capture this fragility by assuming that as of time $t$, with probability $\lambda$ the coordination across generation ends, and the young of $t+1$ choose to save their international goods abroad instead of purchasing the bubble asset. In this case, the bubble crashes to a value of zero.

If the bubble does not crash, it grows at the rate $r^b$. We focus on the case where the interest rate is at the highest possible level consistent with rational expectations (this is the only case where the bubble does not vanish asymptotically in the absence of a crash), so that:

$$r^b = g.$$ 

Thus the expected return from investing in the bubble is

$$\hat{r}^b = g - \lambda(1 + g)$$

and

$$\mathbb{E}\left[\frac{B_{t+1}}{B_t}\right] = 1 + \hat{r}^b = (1 - \lambda)(1 + g) < 1 + g. $$

If $\lambda$ is not too high and the spread $\Delta \hat{r}^b \equiv (g - r^*)$ is sufficiently large, agents invest some of their international goods in the bubble. This will hold true as long as:

$$\hat{r}^b - r^* = (1 - \lambda)\Delta \hat{r}^b - \lambda(1 + r^*) > 0. $$

Recall that the young at date $t$ are endowed with $(W_t, RK_t)$. They divide $W_t$ into holdings of the bubble asset and saving in the international market. We denote the share of bubble assets in the portfolio by $\alpha_t$, and now write the international goods held by generation-$t$ at date $t+1$ as:

$$W_t' = W_t (1 + r^* + \alpha_t(\bar{r} - r^*))$$

where $\bar{r}$ is either $g$ (no crash) or $-1$ (crash).

At date $t + 1$, an entrepreneur with an investment opportunity enters into two transactions. First, he sells his bubble asset to the next generation to receive the return $\tilde{r}$. Next, he borrows $l_{t+1}$ at price $\tilde{p}_{t+1}$ from the bankers, to yield total output of,

$$RK_t + RW_t' + (R - \tilde{p}_{t+1})l_{t+1},$$

where

$$l_{t+1} \leq \frac{\psi R}{\tilde{p}_{t+1}} K_t.$$
As long as \( \tilde{p}_{t+1} < R \), it pays for the entrepreneur to borrow as much as possible. Assuming this case for now and substituting in the maximum loan size gives total output of,

\[
RK_t + RW'_t + (R - \tilde{p}_{t+1}) \frac{\psi R}{\tilde{p}_{t+1}} K_t.
\]

A banker at date \( t + 1 \) collects international goods by selling his real estate assets to the next generation and then lends these goods, along with any savings from date \( t \) international lending, to the investing entrepreneurs. As a result, the bankers receive total goods of,

\[
RK_t + W'_t \tilde{p}_{t+1}.
\]

Rolling back to date \( t \) agents are equally likely to find themselves as entrepreneurs or bankers at date \( t + 1 \). Thus the decision problem is to choose a portfolio of the bubble asset to solve:

\[
\max_{0 \leq \alpha_t \leq 1} E_t \left\{ RK_t + W'_t \frac{R + \tilde{p}_{t+1} + \psi R}{2} (R - \tilde{p}_{t+1}) \frac{\psi R}{\tilde{p}_{t+1}} K_t \right\}.
\]  

(2)

At an interior solution (we make parameter assumptions so this is the case), the first order condition is:

\[
(1 - \lambda) \frac{\Delta r^b}{1 + r^*} (R + p^B_{t+1}) - \lambda (R + p^C_{t+1}) = 0
\]

(3)

where \( p^B_{t+1} \) and \( p^C_{t+1} \) represent the equilibrium price of loans when the bubble survives and crashes, respectively.

In words, the young agent gets an excess return of \( \Delta r^b \) if the bubble does not crash, which happens with probability \( 1 - \lambda \). He trades this off against the loss \((-1)\) if the bubble does crash. When old, the agent is either an entrepreneur, in which case the returns from the bubble funds investment that yields \( R \); or, the agent is a banker, in which case the returns from the bubble are used to lend to the entrepreneur at the price \( \tilde{p}_{t+1} \). Thus returns on the bubble are valued by the agent at \( R + \tilde{p}_{t+1} \).

The supply of funds from bankers is at most \( W'_t \), while the demand for funds from entrepreneurs is at most \( \frac{\psi R}{\tilde{p}_{t+1}} K_t \). Thus, market clearing in the loan market yields for the no-crash state,

\[
p^B_{t+1} = \max \left\{ 1, \frac{\psi R}{1 + r^* + \alpha_t \Delta r^b} \right\}.
\]

Note that, as in the previous section, when \( \psi R < 1 \) we find \( p^B = 1 \).

For the crash state, market clearing yields:

\[
p^C_t = \max \left[ 1, \min \left\{ \frac{\psi R}{(1 + r^*)(1 - \alpha_t)}, R \right\} \right].
\]  

(4)
We now consider the equilibrium determination of $p^C_t$ and $\alpha^p_t$ (where the superscript $p$ stands for private). First note that agents' decision problems are the same across each period that the bubble does not crash. Thus, without loss of generality, we drop time subscripts.

Denote $\alpha^p(p^C)$ as the solution to the agents decision problem, (2), given $p^C$. Denote $p^C(\alpha)$ as the solution to the market clearing condition, (4), given choices of $\alpha$. We are looking for a fixed point $(\alpha^p, p^C)$ that solve both (2) and (4).

We begin the characterization by considering $\alpha^p(p^C)$. Given the linearity of the program, the solution is a step function. Note that if $p^C = 1$, then the derivative of the objective with respect to $\alpha$ is:

$$(1 - \lambda)\frac{\Delta r^b}{1 + r^*}(R + 1) - \lambda(R + 1) > 0$$

where the last inequality follows by condition (1). In words, if $p^C = 1$, the bubble asset dominates saving in the international bank account, so that $\alpha = 1$. We make an adjoining assumption by considering the case where $p^C = R$ (the highest value possible). At this price, the derivative of the objective is:

$$(1 - \lambda)\frac{\Delta r^b}{1 + r^*}(R + 1) - \lambda 2R.$$ 

We assume that

$$(1 - \lambda)\Delta r^b - \lambda(1 + r^*)\frac{2R}{1 + R} < 0 \quad (5)$$

which can hold along with condition (1) as long as $R > 1$ and $rac{(1+R)\Delta r^b}{\Delta r^b+R(2(1+r^*)+\Delta r^b)} < \lambda < \frac{\Delta r^b}{\Delta r^b+1+r^*}$. Under condition (5), if $p^C = R$, saving in the international bank account dominates saving in the bubble asset.

Conditions (1) and (5) guarantee an interior solution to our problem. The solution is illustrated in Figure 1. The solid line depicts $\alpha^p(p^C)$, the step function solution to the agents decision problem. Note that under conditions (1) and (5) there exists a unique value of $p^C$ that lies strictly between zero and one, whereby the first order condition is satisfied with equality. The s-shaped dashed curve in the figure is the solution to the market clearing condition, $p^C(\alpha)$. The unique equilibrium is at point $\alpha^p$.

In summary, under (1) and (5), in each period that the bubble does not crash, agents hold a fraction $0 < \alpha^p < 1$ of their portfolio in the bubble asset. If the bubble crashes, the banker's (gross) return in the loans market is $1 < p^C < R$, while if the bubble does not crash, its return is $p^B = 1$. The banker's return of $\bar{p}$ is the central ingredient in the welfare statements we make next.
Notes: The figure illustrates the equilibrium determination in the crash state. The solution to the decision problem for agents traces out the solid-line step function. If agents expect higher returns on lending international liquidity in the crash-state, they cut back on purchase of bubbles, and increase their holdings of international liquidity. The $s$-shaped dashed curve traces out the market clearing condition as a result of these choices. If all agents hold more of the bubble asset, then in equilibrium there will be less international liquidity if the bubble crashes and $P^C$ will be higher. $\alpha^p$ denotes the equilibrium.

4 Excess Volatility

If $\lambda = 0$, and the bubble is not fragile, then its existence is unambiguously beneficial (Samuelson 1958).

4.1 Crash and Credit Crunch

However, it seems extreme to assume that a coordination-dependent financial instrument will always be stable. When $\lambda > 0$, the economy trades higher capital inflows (or lower net outflows) while the bubble is in place, for the possibility of a sudden reversal in capital flows and the consequent crash in the domestic real estate market.
Consider the expression for total output at date \( t \):

\[ U = RK_t + W'_t \left( \frac{R - \tilde{p}_{t+1}}{2} - \frac{R - \tilde{p}_{t+1}}{2} \psi R \right) K_t. \]

If there is no crash, then \( W'_t \) is high and \( \tilde{p}_{t+1} = p^B = 1 \). Output in this case is,

\[ U^B = \frac{R + 1}{2} W_t \left( 1 + r^* + \alpha_t \Delta r^b \right) + \frac{1}{2} (R - 1) \psi RK_t + RK_t. \]

Starting from this level, if we consider the effect at date \( t + 1 \) of a fall in \( W'_t \), there are two effects that arise.

First, the direct effect is that the investing entrepreneur’s wealth falls, curtailing his date \( t \) investment and reducing output. In the case of the crash, wealth falls to \((1 - \alpha_t)W_t(1 + r^*)\).

Second, the banker has less funds to lend to the investing entrepreneur at date \( t + 1 \). As real estate falls in value, banks are unable to collect as much international goods from the next generation, and so they cut back on loan supply. If the fall in the supply of funds is sufficiently large, the result is a domestic “credit-crunch”: loan interest rates rise causing investment and output to fall. This credit crunch occurs at the point at which entrepreneurial investment is constrained by the quantity of international goods of the current generation of bankers and entrepreneurs. At this point, since the supply of loanable international goods is limited, banks’ loan interest rates rise above one.

At date \( t \), agents choose \( \alpha_t \) and this leads to variation in \( W'_t \). We define a “small crash” as a circumstance where \( \alpha_t \) is small, so that the bubble crash leads to a small enough fall in \( W'_t \) that only the first effect arises. In this case,

\[ U^{C,S} = \frac{R + 1}{2} W_t(1 - \alpha_t)(1 + r^*) + \frac{1}{2} (R - 1) \psi RK_t + RK_t. \]

However, if \( \alpha_t \) is sufficiently large then the contraction in loan supply leads to a rise in \( \tilde{p}_{t+1} \) above one, resulting in the domestic credit crunch. We refer to this event as a “large crash.” From Figure 1, we see that the threshold between the small and large crash case is where \( \alpha_t = \alpha^S \).

In the large crash, \( \tilde{p}_{t+1} \) rises above one to \( \frac{\psi R}{(1 + r^*)(1 - \alpha_t)} \). We can substitute and find that,

\[ U^{C,L} = RW_t(1 - \alpha_t)(1 + r^*) + RK_t. \]

\(^8\)The complete expression for the output in the case of both large and small crash is:

\[ U^C = \text{Rmin} \left\{ (1 - \alpha_t)W_t, \frac{(1 - \alpha_t)W_t + \psi RK_t}{2} \right\} + \text{max} \left\{ \frac{(1 - \alpha_t)W_t - \psi RK_t}{2}, 0 \right\} + RK_t. \]
This last expression is intuitive. In the large crash, all of the international goods of the bankers and entrepreneurs are being invested at date \( t + 1 \). Each of these invested goods produces a gross return of \( R \), which in addition to the endowment of \( RK_t \), yields the expression for \( U^{C,L} \).

Finally, the crash in the bubble asset leads to a permanent loss of domestic stores of value. The next generations invest all of their endowment abroad causing capital outflows.\(^9\)

### 4.2 Overexposure to Large Crashes

Although bubbles lead to the possibility of crashes, they do provide the benefit of increasing growth while they last. The next question we address is whether agents in the economy make this risk/return trade-off optimally from society’s point of view. We show that the private sector’s choices lead to overexposure to large crashes.

Consider the choice of \( \alpha_t \) that maximizes expected aggregate output (which is equal to the generation \( t \)'s welfare), when \( \alpha_t \) is such that the economy generates a large crash:

\[
\max_{\alpha^g \leq \alpha_t \leq 1} \lambda U^{C,L} + (1 - \lambda)U^B \tag{6}
\]

The derivative of this function with respect to \( \alpha_t \) is:

\[
(1 - \lambda)(R + 1)\frac{\Delta r^b}{1 + r^*} - 2\lambda R.
\]

But at the optimum for private agents, the first order condition for the large crash case from (3) is:

\[
(1 - \lambda)(R + 1)\frac{\Delta r^b}{1 + r^*} = \lambda(R + p_{t+1}^C).
\]

When \( p_{t+1}^C = R \) it is evident that these two first order conditions coincide, and the private sector’s choice is constrained efficient. However, if \( p_{t+1}^C < R \), then the social planner’s solution to the program in (6) yields an \( \alpha_t \) that is strictly smaller than \( \alpha^p \).\(^{10}\)

The distortion in the agents’ decision can be most easily understood by considering the banker’s portfolio decision. At date \( t + 1 \), the value of international goods in

\(^9\)An example of this phenomena in practice may be the behavior of EM’s following the EM crises of the late 1990s. These economies turned around from being significant net borrowers before the crises, to become substantial net lenders to the developed world, and the US in particular.

\(^{10}\)It is clear that the first order conditions differ between the private and planner programs when \( p_{t+1}^C < R \). We can assert that \( \alpha_t \) is strictly smaller than \( \alpha^p \) because conditions (1) and (5) guarantee that \( \alpha^p \) is at an interior, so that at least one of the first order conditions is valid.
the crash-state to the banker is proportional to $p^C_{t+1}$. However, the social value of international goods in the crash-state is proportional to $R$, since these goods are always used by entrepreneurs to undertake investment projects.\footnote{When $p^C_{t+1} = 1$, some international goods stored by the bankers/entrepreneurs go unused by the entrepreneur. For this reason, the undervaluation argument only applies in the case of large crashes.} To the extent that $p^C_{t+1} < R$, the banker has less of an incentive to ensure that he has sufficient international goods to lend to entrepreneurs in the crash-state. Moreover, recall that

$$p^C_{t+1} = \min \left\{ \frac{\psi R}{(1 - \alpha_t)(1 + r^*)}, R \right\},$$

so that less domestic financial development, captured by smaller values of $\psi$, lowers $p^C_{t+1}$ and increases this distortion.

Ex-ante, the low return on lending to entrepreneurs translates into a lack of prudence in the date $t$ portfolio decisions. Bankers chase higher returns by investing excessively in the risky real estate bubble, rather than retaining some international liquidity to be in a position to lend to the entrepreneurial sector. Lack of financial development, in the dimension of tighter domestic collateral constraints, overexpose the economy to the risky bubble.

### 4.3 Welfare Maximizing Choice

We conclude this section by deriving the welfare maximizing portfolio share.

For each generation that the bubble does not crash, the planner chooses $\alpha$ to solve:

$$\max_{0 \leq \alpha \leq 1} \lambda U^C + (1 - \lambda) U^B. \tag{7}$$

There are two cases to consider in deriving the solution. If $\alpha \leq \alpha^S$, from Figure 1 we note that $p^C = 1$ and the economy avoids the credit crunch. In this case, $U^C = U^{C,S}$. If $\alpha > \alpha^S$, $p^C > 1$ and the economy enters the credit crunch if the bubble crashes. Thus, $U^C = U^{C,L}$.

We note that under (5), the first order condition that maximizes (7) when $U^C = U^{C,L}$, indicates choosing the lowest possible value of $\alpha$. On the other hand, it follows from (1) that in the range where $U^C = U^{C,S}$, the first order condition indicates choosing the highest possible value of $\alpha$. Thus, the welfare maximizing $\alpha$ must lie at the boundary where $\alpha$ is equal to $\alpha^S$.

In words, a social planner chooses an $\alpha$ that is as large as possible so as to avoid the credit crunch situation. This choice maximizes the intergenerational transfers
afforded by the bubble, while avoiding the credit crunch where international goods are scarce. We also note that the stark result of avoiding the credit crunch completely is due to the linearity of our model, and conditions (1) and (5). On the other hand, the model does highlight the novel aspect of our welfare analysis of bubbles.

5 Aggregate Risk Management Policies

We now focus on the case where the rational real estate bubbles are welfare reducing and consider a variety of corrective government policies that implement \( \alpha^p = \alpha^S \). In practice, a bubble crash in a developed economy leads the central bank to inject liquidity into the banking sector to offset the credit crunch. While such a policy may be helpful in our model, in practice it is unlikely to be readily available to governments in EM’s. In a bubble crash, the credibility and liquidity of the central bank are likely to be low, so that the central bank is in the same position as the banking sector. Thus, we focus in this section on ex-ante policies that aim to reduce the exposure to bubble risk.

5.1 Liquidity Requirement

One solution to the risk problem we have raised is for all agents of generation \( t \) to enter into a contract requiring that each agent hold a fraction \( (1 - \alpha^S) \) of their wealth in foreign reserves.

The most natural interpretation of such a social contract is in terms of liquidity requirements on the banking system. Such requirements are common in many emerging markets. For example, during the period of Argentina’s currency board banks were required to hold significant dollar-reserves.

But such a solution requires some policing and enforcement. Consider the incentive for one agent to deviate from holding \( 1 - \alpha^S \) when all other agents are holding \( 1 - \alpha^S \). From Figure 1, we see that \( p_C = 1 \) at \( \alpha^S \) (i.e. at \( \alpha^S \), even if the bubble bursts, the credit crunch is avoided). But, from the same figure, the solution to the agent’s decision problem at \( p_C = 1 \) is \( \alpha = 1 \). An agent will prefer to invest only in

\[ \text{For example, when } \lambda \text{ goes toward zero, intuition suggests that the optimal } \alpha \text{ will rise. However, } (5) \text{ is violated in this case.} \]

\[ \text{See Caballero and Krishnamurthy (2005) for a model of (ex-post) extreme events interventions in economies with developed (complete) financial markets.} \]
the bubble asset.\textsuperscript{14}

We can also see this by considering the program in (2). The utility gain for an agent to choose $\alpha = 1$ versus $\alpha = \alpha^S$, when $p^c = 1$ is,

$$W_t(1 - \alpha^S)(\hat{r}_B - r^*) > 0,$$

where the last inequality follows from (1).

Intuitively, if all agents are holding sufficient international liquidity, a crash avoids the credit crunch. Thus, a deviating agent values the bubble asset purely for the expected return it offers and does not assign any additional cost to the bubble crashing. This leads the agent to invest all of his wealth in the bubble asset.

If portfolio decisions are costly to observe, a liquidity requirement will be costly to impose. If the costs are high enough, the economy will revert to the equilibrium we have described earlier where $\alpha = \alpha^p$.

5.2 Capital Inflow Sterilization

An alternative policy to implement the optimal investment in the bubble is capital inflow sterilization. As we show next, this sort of policy avoids the policing issues of the liquidity requirement, but instead requires that the government having credible powers of taxation.

Central banks often respond to capital inflows by sterilizing. They sell government debt proportionate to the quantity of the capital inflow. The practice is conventionally seen as an attempt to reduce the monetary expansion created by the capital inflow, but note that, as we show next, it has power even in the absence of a monetary friction. If the sterilization is large enough to provide the private sector with alternative non-bubble assets, it has the potential of reducing investment in real estate bubbles. Government debt crowds out the bubble, raising interest rates in the process.

Suppose that the government issues one-period debt with face value of $G_t$ at date $t$ at interest rate $r^G_t$. In addition, it raises taxes at the rate of $\tau_t$ on the international goods endowment of generation $t$. The revenue from the debt sale and the tax are invested at the international interest rate of $r^*$. Finally, the debt is repaid at date

\textsuperscript{14}Jacklin (1987) makes a related point in the context of the Diamond and Dybvig (1983) model. He argues that the Diamond-Dybvig bank is not coalition incentive-compatible if agents can make side-trades.
$t + 1$, so that the government balances its budget:

$$
\left( r_s W_t + \frac{G_t}{1 + r_t^G} \right) (1 + r^*) - G_t = 0.
$$

(8)

Agents purchase $\alpha_{G,t}$ of these bonds with their international endowment ("a capital inflow") at interest rate $r_t^G$. Then, the wealth equation for generation $t$ becomes,

$$
W_t' = \tilde{W}_t \left( 1 + r^* + \alpha_{G,t}(r_t^G - r^*) + \alpha_t(\tilde{r} - r^*) \right)
$$

where $\tilde{r}$ is either $g$ (no crash) or $-1$ (crash), and,

$$
\tilde{W}_t = W_t(1 - \tau_s).
$$

The decision problem for an agent is now:

$$
\max_{0 \leq \alpha_{G,t} + \alpha_t \leq 1} E_t \left\{ R K_t + W_t' \frac{R + \tilde{p}_{t+1}}{2} + \frac{R - \tilde{p}_{t+1} \psi R}{\tilde{p}_{t+1}} K_t \right\}.
$$

We note that government bonds provide the same stable cash-flows as investing the international endowment abroad. Thus, as long as $\alpha_{G,t} + \alpha_t < 1$, government bonds and international liquidity are perfect substitutes and $r_t^G$ must be equal to $r^*$. From the government’s budget constraint, in this case $\tau_s = 0$. Any sales of government debt are accommodated by a reduction in the international liquidity of the private sector, and the sterilization has no effect.

For the sterilization to have any effect, it must be sufficiently large. Again denoting by $\alpha^p$ the portfolio share that the private sector’s optimally chooses to invest in the bubble, the sterilization has any effect only if,

$$
G_t > G_t^* \equiv (1 - \alpha^p)W_t(1 + r^*).
$$

Let us consider such "large" sterilizations. For these cases, the private sector reduces its international liquidity holdings to zero and only holds government debt and the bubble asset:

$$
W_t' = \tilde{W}_t \left( 1 + r_t^G + \alpha_t(\tilde{r} - r_t^G) \right).
$$

The private agents’ first order condition, at an interior solution for $\alpha_t$, is now:

$$
(1 - \lambda)(g - r_t^G)(R + p_{t+1}^B) - \lambda(R + p_{t+1}^C)(1 + r_t^G) = 0.
$$

At the welfare maximizing solution we know that $p^B = p^C = 1$. Rewriting, and solving for $r_t^G$, we obtain

$$
r_t^G = (1 - \lambda)g - \lambda = \tilde{r}^b.
$$
If the government sells debt that raises \((1 - a^s)W_t\) resources at date \(t\), agents purchase both this debt as well as the welfare-maximizing amount of the bubble asset. Since any investment in the government debt reduces investment in the bubble asset one-for-one, the interest rate on the debt has to rise to \(r^b\), the expected return on the bubble.

However, in implementing the optimal portfolio share, the government has to raise some taxes. Using the government’s budget constraint, (8), we can solve to find that,

\[
\tau_s = (1 - a^s) \frac{r^b - r^*}{1 + r^*}.
\]

A higher return on the bubble or a lower international interest rate, raise the required taxes.

The need to collect taxes to support the sterilization raises a concern. If the government is limited in its ability to raise taxes, then it can only implement small sterilizations (or sterilizes with bonds that have a low effective return, in the eyes of the public). But, small sterilizations have no effects. Thus, effective sterilization requires a government with sufficiently large tax powers.

6 Public Debt Market Development

Sterilization has the potential to solve the inefficiency stemming from agent’s undervaluation of the systemic risk generated by bubbles. However, the optimal solution is not to eliminate bubbles altogether, but to reduce their amount to \(a^s\) in Figure 1. The bubble is the endogenous market solution to the dynamic inefficiency created by domestic financial underdevelopment. Sterilization, as we describe it, is merely an instrument to reduce the fragility created by this market solution. It is not a substitute for the missing “intergenerational” contract. Can we find mechanisms that both reduce fragility and alleviate dynamic inefficiency? We turn to answering this question next.

The reason our sterilization policy does not solve the dynamic inefficiency problem is that the interest rate spread of \(r^G - r^*\) is financed with taxes on the same generation. Thus, on net, there is no intergenerational transfer associated with sterilization. This is in sharp contrast to Diamond’s (1965) perpetually rolled-over public debt, which does represent a solution to the dynamic inefficiency problem. However, Diamond’s public debt is a bubble in itself and hence raises the same fragility concerns
surrounding the real estate bubble. If the next generation does not roll-over the debt, the bubble crashes.

We next show that public debt that is supported by future rather than current taxes achieves a compromise: Such debt implements intergenerational transfers (like Diamond), as well as reduces fragility risk (like our sterilization policy). Of course, pledging future taxes requires credibility and commitment. We thereby establish a natural connection between a government’s credibility and ability to raise taxes and the extent to which the government can solve the economy’s dynamic inefficiency problems, while limiting the fragility costs of the bubble.

Suppose the tax base of the government is $\tau W_t$. However, because of credibility concerns the market discounts the future tax revenues at the rate $(1 - \rho)$ such that the pledgeable tax revenues are:

$$(1 - \rho)^s \tau W_{t+s} \quad s \geq 0.$$  

We assume that at $t$ the government issues the maximum debt it can, collateralized by these future (potential) taxes. With such a debt structure, a high $\rho$ can be interpreted as very short-term maturity structure of debt, while a low $\rho$ corresponds to a long-term maturity structure of debt. Let the value of the stock of outstanding government debt be denoted by $D_t$. The value satisfies,

$$\frac{D_t}{W_t} = (1 - \rho)(1 + g) \left\{ \tau + \frac{1}{1 + r_t} E_t \frac{D_{t+1}}{W_{t+1}} \right\}.  \tag{9}$$

Assume, momentarily, that the stock of government debt, $D_t$, is sufficient to satisfy all of agents’ store of value demand, that agents use this as the only saving instrument, and that the government maintains the practice of issuing the maximum collateralized debt at each point in time. Then, $D/W$ is equal to one at all dates and the interest rate is constant.\footnote{\(D/W\) is always equal to one, while the tax base is equal to $\tau W_t$. There is no contradiction between these statements because, in equilibrium, the taxes are not levied; they simply serve as collateral for the debt. If all of the taxes of $\tau W_t$ were levied, then ratio of debt to endowments would shrink toward zero over time.} Let $r^D$ denote this equilibrium interest rate, which from (9) satisfies:

$$\frac{r^D - g}{1 + g} = \tau (1 - \rho) (1 + r^D) - \rho.$$  

It is apparent from this expression that if the government has enough credibility so that $\rho$ is small relative to $\tau$, then the dynamic inefficiency can be completely removed
as the $r^D$ that solves the above equation exceeds $g$.\(^6\) To implement this constrained efficient equilibrium, the government issues enough public debt to bring $r^D$ exactly to $g$.

While the pledgability of future taxes is critical, the government actually never needs to collect taxes to pay for this debt, as it can fully finance the expiring debt with new debt. That is, the collateralized debt behaves as Diamond’s debt, but is not bubbly since it is fully collateralized by taxes.

To see the role of collateralization, consider one of the bonds in the stock of outstanding government debt at time $t$. Suppose this bond matures at date $T > t$ and is collateralized by $(1 - \rho)\tau W_T$ revenues. Then, at date $T - 1$, the young agent of generation $T - 1$ will buy up to $(1 - \rho)\tau W_T$ face value of this bond, since he knows that regardless of the behavior of the young agent of generation $T$, the bond will be fully repaid. Anticipating this behavior by generation $T - 1$, the young at generation $T - 2$ also buy the bond, and so on. Since every bond in the stock of government debt is collateralized by a specific, dated, tax revenue, the argument applies to the entire stock of government debt.

The same argument does not apply to Diamond’s debt solution, or the real estate bubble. In these cases, the young of generation $T - 1$ buys the bubble only because they believe that the young of generation $T$ will do likewise. Of course, such a coordination dependent asset is fragile.

Collateralized public debt also dominates the real estate bubble as a savings vehicle since:

$$r^D - \bar{r}^b = r^D - g + \lambda(1 + g) > r^D - g > 0.$$ 

The collateralized debt eliminates the bubble, and its fragility.

Note that in the limit case where the government is fully credible, so that $\rho = 0$, then even an infinitesimal tax base is enough to implement the first best with stable public debt.\(^7\) In the more realistic case of a government with limited credibility, the government may be unable to generate enough reliable store of value instruments to

\(^6\)For smaller values of $\tau$ relative to $\rho$ for which the government cannot issue debt such that $D/W$ equals one, there is still room for improvement on the market bubble outcome. For example, if parameters allow for a $D/W = 1 - \alpha_S < 1$, then the government can opt for the sterilization solution. But, in contrast to the solution in the previous section, the sterilization leverages future taxes and not current taxes.

\(^7\)This limit result is akin to the point made in McCallum (1987) and extended in Caballero and Ventura (2002), that well defined property rights even over an infinitesimal share of an economy’s growing output, can eliminate the possibility of bubbles.
fully crowd out the bubble. When $\rho = 1$, the government can only sterilize but is unable to implement intergenerational redistributions without incurring risks similar to those of the market bubble.

7 Final Remarks

One view of emerging markets crises describes normal times as periods with significant capital inflows, which are suddenly interrupted by liquidity crises.\(^\text{18}\)

This paper highlights a different view. Normal times are those with net capital outflows. These normal periods are occasionally interrupted by speculative bubbles, which can crash. Moreover, we show that in many instances these bubbles, while rational, are socially inefficient since they introduce excessive aggregate fragility.

Both views stem from some form of financial underdevelopment. However, the bubble-view reflects a more primitive domestic financial market, where residents find limited trust-worthy local assets.

Ingredients of both views probably coexist and vary in relative importance across economies and times. During the mid 1990s, for example, South East Asia received large amounts of capital inflows, which financed both productive investments and fed Real Estate bubbles. In this context, the series of crises that started with Thailand and soon spread to the rest of the region, had elements of both a liquidity crisis and a crash in local bubbles. Over time, the liquidity crunch disappeared, but the bubble-assets were not recreated. The latter phenomenon has played out in much of the Emerging Market world as well as in some newly industrialized economies.\(^\text{23}\)

From the point of view of our analysis, the disappearance of the bubble assets is an important factor behind the acceleration of the capital inflows to the US — and hence the worsening of the US current account deficit — since the Asian/Russian crises. Along the same lines, despite the attention that China’s dollar-reserves accumulation and its potential reversal has received, the real danger may lie elsewhere. Today, Chinese savers are forbidden from accessing US instruments directly. A normalization of capital controls may lead to a crash in their buoyant real estate market and increased capital outflows toward the US.

It goes without saying that ours is a highly stylized characterization of emerging market dynamics. For practical purposes, we do not mean to highlight the multiple

equilibria nature of bubbles, or even the extreme form of rationality we have required on agents. They simply capture the high volatility in emerging markets and the role of speculation and expectations. Moreover, although the bursting of bubbles is exogenous in our model, one could extend it and link the bubble bursting to fundamentals; for instance, any domestic or external factor that raises the relative appeal of foreign assets, could crash the bubble in our model. Bubbles for us represent highly volatile domestic financial assets which are not backed by solid fundamentals, sound government policy, and institutions. They have the potential to yield high returns if domestic markets, rich in funds but poor on quality assets, choose to speculate on them, but they are also susceptible to large drops if fundamentals turn sour.
References


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