The Economics of Congestion and Pollution
An Integrated View
by
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I. Generic Congestion, Customary Congestion and Pollution

Increasing popular and professional attention is being given to two types of phenomena characterized by substantial externalities—urban congestion and environmental pollution. Both represent the unkind rub of human activities on one another, where there is no intermediation of a market to enable affected parties to confront their tormentors. Examination of the phenomena reveals that they have much in common beyond the sheer fact of externalities. The structure of the externalities is similar, and an exploitation of this similarity throws light on why they are both becoming critical problems in our society, what their consequences are, and what kinds of measures can be used to mitigate their damage.

In the present paper we propose to treat congestion and pollution problems as members of a single class, integrating the distinctive characteristics of the two together as aspects of one phenomenon. We shall first discuss the nature of this more abstract configuration of socio-economic interaction, and then display its ramifications in a mathematical model which treats both positive and normative considerations.

Highway traffic jams, queues, crowding of beaches and parks and museums, and air and water and noise pollution are all forms of social congestion. Common

1. The illuminating works of Allen V. Kneese on environmental pollution, as for example his, "Air Pollution—General Background and Some Economic Prospects," chapter in Harold Wolozin (ed.) The Economics of Air Pollution, Norton (N.Y., 1966), p. 34, recognize this similarity.
to all of them is that more than one agent are attempting to share a type of service that is not furnished in a separable unit earmarked for each user. They are consuming in common form a "public good"—whether it be a highway or beach or volume of air or a water-course. In all, the presence of other users adversely affects the quality of services which that public good renders to each. Quality deterioration may be revealed in terms of the length of time, safety or psychological tension of an auto trip, the size of available area of picnic or swim space, the level of aesthetic disfigurement of a setting by litter or noise, the degree of obstructedness or average viewing time, of paintings, the amount of eye and lung or ear irritation in the air, or the odor, taste, bacterial count or fish population in a body of water. In all of these, moreover, quality deterioration does not set in as soon as more than one user attempt to share the public good in question. Depending on the nature of the public good, a differing but rather wide range of users may be accommodated with no perceivable deterioration of quality. Each good has a "capacity", or threshold, beyond which interference effects first become noticeable and then increase disproportionately. Finally, while some part of the "capacity" of the public good may be natural, or given by Nature as a "free good" (this does not apply to museums), deliberate human action can either increase that capacity or mitigate the quality impairment stemming from any given level of socio-economic interaction. Thus, low flow augmentation procedures can enhance the waste assimilative powers—the "capacity"—of a river, or sewerage treatment plants can decrease the extent to which any given flow of effluents overwhelms it. Various forms of recycling affect air pollution, widening a highway affects traffic flow, rationing devices influence the obstructiveness of a given crowd in a swimming pool or museum.

This composite of public good sharing, and the policy variability of medium
capacity and rate of interference by-product for each unit flow of interaction is the basic characteristic of the generic "congestion" process which we shall treat. Under this broad head the more conventionally conceived "congestion" and "pollution" take somewhat specialized forms.

What distinguishes congestion from pollution in terms of the more inclusive concept is chiefly the relationship among the generators and victims of "interference". If highway traffic is the classic example of congestion, then the central interpersonal distributive fact about it is that all users are using the medium (the public good) in much the same way, each is damaging service quality for both others and himself, and the ratio of self-other damages is approximately the same for all users. It would be difficult to separate users into abusers and victims. Congestion is not looked on as a process in which important real income redistribution occurs: some benefitting by imposing damages upon others. The whole user group loses homogeneously by their self-imposed interaction.

The essence of pollution, on the other hand, is that there are some users who do abuse the medium—the polluters—whilst others are relatively passive victims of such abuse—the "public". In these processes users differ among themselves in how they use the medium. Some users employ rivers as sewers for noxious materials, others—downstream—simply want to drink the water. Some use the atmosphere similarly as a medium for noxious waste disposal, others simply want to breathe it. Jet planes make the noise, housewives are forced to submit to it. Thus, pollution often lends itself to a distinction between destructive and constructive uses of a medium, between guilty and innocent parties. Significant—whether morally or in scope—income redistribution is a key aspect of the process.

The legal and ethical characterization of the distinction are controversial.
Is a pulp processor destructive because of what he puts into the river, or because of what other upstream users are putting in at the same time, or because of what a downstream user is taking out? If no other upstream discharger existed, or if there were no downstream user, his actions would hurt no one. Who has property rights in the river? Is it a question of who came first? Even if it were, this would not resolve the issue, since it is not the sheer existence of upstream and downstream users that causes the difficulty but the scale of their interaction. Up to a certain scale upstream use causes no quality deterioration. It is the marginal increment which crosses the threshold that becomes noticed. But then the damage is produced by all the upstream users, not just the marginal one. Can a destructive-non-destructive dichotomization of uses rest on scale considerations?

Economists like Coase and Buchanan have leaned on this kind of consideration to argue an essential legal symmetry for different uses. They accept any status quo pollution and suggest that it is ethically and allocationally equivalent whether victims offer payments to polluters to desist or the state forces polluters to pay compensation to victims for the right to continue. Only income distribution is affected.

One can recognize that real pollution depends on primacy, on scale of polluter use, and on the presence and scale of victim use, without wishing to abandon the distinction between destructive and non-destructive uses. At any time, given the contemporary context of the nature and volume of potential users of a medium, some uses are at least potentially damaging to other conceivable users. There is an important asymmetry between those who spew gases into the air and those who only want to breathe it. The former do at least potential ill to the latter, but the latter do not do damage to the former. If this kind of asymmetry be granted, then it is not the case that neutrality (symmetry) of
property rights is allocationally neutral. For if external diseconomies against others can be expected to lead to bribes by victims to desist, then the production of negative externalities becomes a valid by-product of primary production. Profitability is enhanced whenever any firm can select from among its input and/or output alternatives those which cause substantial damage to third parties. Resource use will tend to become specialized toward much-augmented third-party interference. The new legal industry of selling protection against disturbance will be highly profitable. So long as the distinction between negative and positive externalities is maintained, such an adaptation of resource use must be deplored.

Our treatment of pollution does maintain the distinction. There is a unique direction of service quality impairment, and some uses involve a higher rate of "potential impairment" than others. Whether "potential impairment" eventuates as actual impairment depends on concrete situations—the scale considerations.

"Generic congestion" subsumes both customary congestion and pollution as special cases of a general phenomenon: (1) pure congestion is the case where all users generate identical rates of quality interference per unit of activity and share equally in the resulting quality impairment; (2) pure pollution is the case where some users generate very high rates of per unit interference while others generate zero rates, and only the latter experience quality impairment; (3) the general case is where all users both generate impairment and share in it, but they differ from one another in both respects. The variety of both abuse and victimization prevents an easy or complete categorization of users into guilty and innocent. Generic congestion, by enabling us to study a multitude of patterns of relative generation and sharing of damages,
can throw light on a greater range of allocational, distributional and public policy issues than the customary congestion and pollution concepts. Indeed, conventional terminology makes it difficult to understand most observable real-world phenomena, because it obtrudes extreme case insights into situations that typically have elements of both extremes. Real world cases are neither pure congestion nor pure pollution. Policy recommendations based on these polar concepts are likely to be deficient to the extent that they fail to take account of the mixed characteristics.

In the remainder of the paper we shall briefly present a model of generic congestion to give some idea of the facets which can be explored.

II. A Model of Generic Congestion

A. Preliminaries

In order to place the model in more than a partial equilibrium context we assume that there are only two commodities in the system, X and Z. X is subject to generic congestion but Z is not. The nature of the generic congestion is as follows. Government provides a public good-type capacity from which it sells shared services to the population. A variety of forms of crowding and interactive interference characterize the actual consumption of these services. The population contains three different types of users. All three share equally the results of congestion, but they contribute to it unequally. There is a low polluter, a middle polluter, and a high polluter group. Each generates a low, middle or high rate of interference for every unit of X he consumes: groups \( L_{1low}, L_{2middle}, L_{3high} \); \( L_{i} \) \( \in \) \( L_{j} \); member \( i \) of group \( j \); the total population is divided among the groups in the fraction \( n_{1}, n_{2}, n_{3} \) respectively.

While we have spoken about the consequences of congestion as quality impairment, we shall facilitate the treatment by supposing that each user wishes to consume X at a standard quality and must pay to have any quality damage offset. So the cost of quality impairment to any individual is the amount he has to pay to undo the adverse effects of congestion. The greater the damage the greater must be the cost of offsetting it.

B. Cost Functions

Commodity \( Z \) is produced under competitive conditions of constant costs.
Since we shall speak about the cost to each $L^i$ of consuming a standard quality of $X$, we speak also of the cost to each $L^i$ of consuming $Z$.

(1) \[ C^i_z = a \cdot z_i \quad \text{so} \quad \frac{C^i_z}{z_i} = a = p^Z \]

where $C^i_z$ is the total cost to $L^i$ of consuming $z_i$ units of $Z$

$a$ is a constant

$p^Z$ is the price of $Z$

We treat $Z$ as the numeraire good. Therefore, let $p^Z = 1$.

Commodity $X$ is also produced under conditions of constant cost, but
congestion interferences add equally to the consumption costs for all $L^i$:

(2) \[ C^i_x = C_x(X^i_1, Q) \]

where $C^i_x$ is the total cost to $L^i$ of consuming $X^i_1$ units of $X$

$Q$ is the degree of generic congestion

(3) \[ Q = \frac{1}{K} \sum_{j=1}^{3} \sum_{i \in L_j} W_j X_{ji} = \frac{T}{K} \]

where $K$ is the quantity of the assimilating medium—the capacity of the public good associated with consumption of $X$

$X_{ji}$ is the amount of $X$ consumed by $L^i \in L_j$

$W_j$ is the interactive disturbance created by each unit of consumption of $X$ by any member of $L_j$

$T$ is the total interactive disturbance by the whole population

(3) shows that the size of congestion externalities depend on (a) total $X$, (b)
the distribution of $X$ among $L_1$, $L_2$, $L_3$ (c) the capacity of the assimilative medium. Notice that each $L^i$ adversely affects himself and others, the relative amounts depending on his membership in one of the polluter groups.

Some properties of $C^i$ and $Q$ (we henceforth omit subscript $X$ in $C^i_x$):

- $\frac{\partial C^i}{\partial x_i}$ is non-linear: (a) $\frac{\partial C^i}{\partial Q_i} = 0$ for $Q \leq Q_o$, (b) $\frac{\partial^2 C^i}{\partial Q^2} > 0$ for $Q > Q_o$, $Q > Q_o$
where \( Q \) is the threshold value of \( Q \) beyond which congestion effects occur.

\( K \) can be influenced by explicit public investment: social assimilation investment: \( I \) (for example, highway width or general sewerage treatment plants). Where \( I = 0, K = K_J \), the medium capacity given by nature (e.g. air and water as original "free goods", or unimproved earth surface for transportation). \( I \) is shown as annual costs—as though they represent either a non-durable resource use requiring annual replacement or the annual carrying charge of a durable investment. By this treatment we can compare annual cost with the annual value of the services rendered by the investment.

(4) \( K = K_J + K(I) \)

\( W_j \) is the interactive disturbance rate specific to each \( L_j \). Its value can be affected, however, by specific investments undertaken by the particular pollution groups: group treatment investment: \( I_j \). These differ from social assimilative investments in being focused on the particular externalities generated by each member of the group—e.g. like smoke or fluid effluent recycling by each plant—rather than being facilities that deal with an aggregate of interactive effects from various sources.

(5) \( W_j = W(j, I_j) \)

\( a) \quad W_1 (I_1=0) < W_2 (I_2=0) < W_3 (I_3=0) \)

\( b) \quad \frac{\partial W_j}{\partial I_j} < 0 \quad c) \quad \left| \frac{\partial^2 W_j}{\partial I_j^2} \right| < 0 \)

Group investment decreases its own rate of pollution, but this is a decreasing effect. No \( I_j \) totally wipes out \( W_j \).

C. Demand Functions

In order to isolate congestion issues, we assume that all \( L_i \) have the same income \( \bar{Y}(t) \) and the same tastes for \( X \) and \( Z \). Then, since every \( L_i \in L_j \) has the
same income, tastes and, by (3), faces the same cost of consuming \( X \), \( X_{ij} = \bar{X}_j \) (where \( \bar{X}_j \) is per capita consumption of \( X \) in group \( L_j \)). The demand for \( X \) and \( Z \) in \( L_j \) are:

(6) \[
\bar{X}_j = X \left( \frac{\bar{Y}}{P^Z}, \frac{P_j}{P^Z} \right) \quad \text{or, since } P^Z = 1, \quad \bar{X}_j = X(\bar{Y}, P_j)
\]

(7) \[
\bar{Z}_j = Z \left( \frac{\bar{Y}}{P^Z}, \frac{P_j}{P^Z} \right) \quad \text{or } \quad \bar{Z}_j = Z(\bar{Y}, P_j)
\]

where \( P^Z \) is the market price of \( Z \) to everyone

\( P_j \) is the market price of \( X \) to members of \( L_j \)

\[
\frac{\partial \bar{X}_j}{\partial P_j} < 0 \quad \frac{\partial \bar{Z}_j}{\partial P_j} > 0 \quad \frac{\partial \bar{X}_j}{\partial \bar{Y}} > 0 < \frac{\partial \bar{Z}_j}{\partial \bar{Y}}
\]

In the present model we shall subsequently examine the consequences of having government impose a congestion charge on each group per unit of \( X \) consumed by it. This congestion charge, \( \psi_j \), will be an amount necessary to make the price facing each group equal the marginal resource cost to the whole population resulting from that group's incremental consumption of \( X \). As explained below, it supplements the group's own cost with those overall system costs resulting from its contribution to congestion and with whatever group investments have been undertaken to decrease its congestion impact.

(8) \[
P_j = C_j \frac{\bar{X}_j}{\bar{Y}} + \psi_j \bar{Y} \quad \text{where } C_j = \sum_{k=1}^{i} C_{ij} \]

\[
\psi_j = \frac{\partial C_j}{\partial \bar{X}_j} \left( \frac{\partial C_j}{\partial \bar{Y}} = \frac{\partial}{\partial \bar{Y}} \left( \sum_{k=1}^{j} C_k \right) \right)
\]

D. Total Demand: The Effect of Population Growth

(9) \[
L(t) = L_0 e^{rt} \quad \text{where } L(t) \text{ is the total population at time } t, \ L_0 \text{ is the initial population at time } t, \ r^0 \text{ is the rate of growth,}
\]
Since we assume the relative distribution among $L_1, L_2, L_3$ constant
and in fractions $n_1, n_2, n_3$:

\[ N_j(t) = n_j L_0 e^{rt} \quad j = 1, 2, 3. \]

Then the total demand for $X$ by $L_j$ is:

\[ \sum_{i \in L_j} X_{ji}(t) = \bar{X}_j(t) N_j(t) \]

And total $X$ demanded at $t$:

\[ X(t) = \sum_j \bar{X}_j(t) N_j(t) \]

Consequently $T(t)$ is given by:

\[ T(t) = \sum_j W_j \bar{X}_j(t) n_j L_0 e^{rt} = L_0 e^{rt} \sum_j W_j \bar{X}_j(t) n_j \]

Since $\bar{X}_j(t) < \bar{X}_j(t_0)$ only if $T(t) > T(t_0)$, then, as population increases over time so does total interactive disturbance and congestion: $\dot{T}(t) > 0 < Q(t)$. (Of course, since as a result prices rise, $X$ rises slower than population growth alone would warrant.)

E. Total Demand: The Effect of Income Growth

As with population, let us examine the consequence of an upward drift of per capita income.

\[ \bar{Y}(t) = \bar{Y}_0 e^{yt} \quad \text{(similar definitions as for (9))} \]

So $\bar{X}_j$ is a function of time because of both population and income growth.

Assuming $W_j = 0$,

\[ \dot{T}(t) = \sum_j \frac{\partial T}{\partial \bar{X}_j} \bar{X}_j + \sum_j \frac{\partial T}{\partial N_j} N_j \]

\[ \dot{\bar{X}}_j = \frac{\partial \bar{X}_j}{\partial \bar{Y}} \dot{\bar{Y}} + \frac{\partial \bar{X}_j}{\partial (P_j)} \dot{P}_j \]

$\bar{X}_j$ tends to increase over time as per capita income rises, but tends to decrease only as $P_j$ increases due to growing congestion. Thus, a fortiori,
\( T(t) > 0 \). Thus, \( \dot{Q}(t) > 0 \) also. As a result, \( X_j(t) \) will rise also, but due to the growing congestion, will rise slower than warranted by the pure income effect, especially as congestion becomes more and more serious.

This indicates that a system which begins with no congestion problem— with air and water and beaches uncongested, unpolluted "free goods"—will gradually move toward greater and greater congestion of all such natural media by the sheer growth and affluence of the society, especially when these occur in a context of greater spatial concentrations (urbanization) (since such growing concentration increases each \( W_j \) as it increases interdependence). The problem creeps up on the society that is doing nothing differently, only more and better—creeps, and then gallops.

\textbf{F. Optimal Resource Allocation between X and Z}

\textbf{1. Optimal Conditions in Exchange and Production}

For each \( L_i \in L_j \), an optimal budgetary allocation between \( X \) and \( Z \) requires:

\[
\frac{P_i}{P^2} = MRS^i, \text{ or } P_j = MRS^i
\]

where \( MRS^i \) is \( L_i \)'s marginal rate of substitution between \( X \) and \( Z \).

For all \( i, k \), optimal production-consumption allocation requires:

\[
MRS^i = MRS^k = \frac{MC_X}{MC_Z} = \frac{MC_X}{a}
\]

(since average cost of \( Z \) is an invariant \( a \), so too is marginal cost)

\( MC_X \) is the marginal cost of consuming \( X \) (at standard quality). In our model this includes the various disposal, queueing and other interactive interference procedures that characterize congestion. The extent of these procedures determines the terms on which everybody can enjoy his own \( X \), since is helps determine \( c^i \).
But these terms depend on who gets the additional (marginal) \( X \). The three pollution groups differ in their efficiency in handling \( X \). Group \( L_1 \) is the most efficient, \( L_2 \) next and \( L_3 \) least: \( \delta C_1 < \delta C_2 < \delta C_3 \). If all groups face the same price for \( X \), intra-industry optimality requires that all of it go to \( L_1 \), and none to \( L_2 \) or \( L_3 \).

2. Efficiency under Zero Congestion Charges

Assume that \( \Psi_j = 0 \), for all \( j \). Then there is a high probability that no group will find it worthwhile to make a group treatment investment. Since most of the impact of each group's interactive disturbance is on others rather than on itself, any such investment largely benefits those others. Profitability for self-interest would require that a very small portion of the project's overall benefits be large enough to exceed the whole cost of the investment. Benefit to the investor from such a project is given as:

\[
(19) \quad \Pi^i_j = \frac{\partial C_i}{\partial \Psi_j} = \left( \frac{\partial C_i}{\partial \Psi_j} \right) \cdot \left[ 1 - \frac{\partial \Psi_j}{\partial X_j} \cdot \frac{\partial X_j}{\partial \Psi_j} \right]
\]

Benefits to the population as a whole is given as:

\[
(20) \quad \Pi_j = L_j \Pi^i_j
\]

With this disparity it is most doubtful that there exists an \( I_j^i \) such that \( \Pi^i_j \geq I_j^i \). Thus, with \( \Psi_j = 0 \) we can expect also that \( I_j = 0 \) for all \( j \).

If this is so, then all individuals face the same \( P_j = C_i \). Consequently, all have the same incentive to consume any marginal resource transfer from \( Z \), i.e., all will bid equally to consume an extra unit of \( X \). As a result, any marginal resources flowing to \( X \) are likely to be divided among the three groups in the same proportions as their percentage of the population: \( n_1, n_2, n_3 \). So the average of the marginal costs of consuming \( X \) that follows a marginal resource transfer
from \( Z \) is \( \sum_j \delta C_j \), which exceeds \( \min (\delta C_1, \delta C_2, \delta C_3) \) (where \( \delta C_j = \sum_k \frac{\partial C}{\partial x_j} \)).

This has two implications.

First, it means that whatever resources flow from \( Z \) to \( X \) are inefficiently distributed within \( X \) (as noted in the last section). Members of \( L_2 \) and \( L_3 \) consume too much relative to \( L_1 \). Second, when \( L^i \) experiences incremental consumption he pays \( \frac{\partial C_i}{\partial x_i} \) (marginal private cost). But his extra consumption adds to everyone else's cost as well: \( \sum_j \frac{\partial C}{\partial x_j} \) (i.e. marginal social cost less \( \frac{\partial C_i}{\partial x_i} \)). Thus, the marginal social cost substantially exceeds the marginal private cost: \( \delta C_j > \frac{\partial C}{\partial x_j} \). Consequently, resource flow between \( Z \) and \( X \) is determined by an aggregate demand for \( X \) which is systematically biased upward above the socially optimal amount, because each buyer faces a price lower than the marginal social cost of supplying it. Too much \( X \) is consumed, too little \( Z \).

Optimal allocation would be arrived at where each \( L^i \) were faced with the true \( \delta C_j \) as his cost of consuming extra \( X \). Then members of \( L_1, L_2 \) and \( L_3 \), faced with differentially higher prices of \( X \) in the same order, would cut their consumption differentially in that order. The lesser per capita consumption in \( L_2 \) and \( L_3 \) relative to \( L_1 \) would be determined at those points where the resulting higher marginal valuation of \( X \) equalled the differentially higher prices.

By comparison, the suboptimal situation shows that there is both a distributional and an allocational distortion. The distribution issue underscores what might be called the customary pollution aspect: heavy polluters making too much use of a scarce medium relative to light polluters (or victims). The allocational issue underscores the customary congestion aspect: everyone's activity damaging the quality of others' and his own prospects.
3. Congestion Charges

Let us now examine the consequences of having the government impose congestion charges on everyone so that each $L^i$ pay a price for consumption of $X$ which reflects the true marginal social cost of that consumption. The total revenues collected are subsequently redistributed equally among the same $L^i$.

Since each $L_j$ is homogeneous, charges will differ only with respect to group. The charge $\psi_j$ is set so that each $L^i$ pays $P_j = \frac{C_i}{x_j} + \psi_j + I_j$ and

\[(21) \quad \psi_j = \Delta C_j - \frac{C_i}{x_j} = \sum_k \frac{W_k}{x_j} \frac{\partial C_k}{\partial x} - \frac{C_i}{x_j} \quad (\text{from (2) and (3)}) \]

The result of this has been discussed in the last section. Every $L^i$ faces the true marginal social cost of his consumption of $X$. His relative demand between $X$ and $Z$ is undistorted. So aggregate demand for $X$ is undistorted. Less $X$ will be consumed, more $Z$, and the distribution of $X$ will show $x_1 > x_2 > x_3$, because, by (21), $P_1 < P_2 < P_3$. Since generic congestion in our model is shared equally by all, the relative price changes among $L_1$, $L_2$ and $L_3$ result in a real income redistribution. All are benefitted by an increase in $Z$ and a decrease in congestion cost associated with the total consumption of $X$. But $L_3$ pays most per capita for these benefits in its members' sacrifice of $X$ consumption, $L_2$ pays next most, and $L_1$ least, for the same benefits. At the least there is a change in relative income distribution. But there is likely to be an absolute change too. Members of $L_3$ may actually be worse off after the resource shift than before, since they pay much more in charges than they receive in per capita refund. Members of $L_2$ may gain or lose on balance from the shift, but probably not by much in either direction (depending, of course, on the technological, taste and relative numbers characteristics of the actual situation). If per capita refund is nearly as great as their average charge, they will gain because of the lower congestion cost in $X$. Member of $L_1$ probably gain substantially—both because of a refund which greatly exceeds the charge, and because of the lessened congestion in $X$: gains which, by the Compensation Principle, are more than sufficient to make it possible to pay off the losers to make them no worse off than before. Congestion charges damage the "polluters" and help the "victims".
G. Group Treatment Investment

When \( y_j = 0 \) for all there is little incentive for individuals to undertake group treatment investment to lessen the externality effect of their consumption. All \( I_j \) is likely to be zero. But imposition of congestion charges changes this. \( y_j \) is a positive function of \( W_j \) (by (21)), and the size of \( W_j \) can be decreased by such investments. \( y_j \) and \( I_j \) are substitute outlays, then, and each individual will undertake such investment as decreases \( y_j \) by more than the cost of the investment. In effect, he determines his optimal \( I_j \) by minimizing \( P_j \) with respect to \( I_j \). \( I_j \) represents a "treatment cost" per unit of \( X \) consumed by each member of \( L_j \). So it enters the unit price of \( X \) facing each such member:

\[
P_j = \delta C_j + I_j
\]

The condition for the optimal \( \hat{I}_j \) is therefore given as:

\[
\frac{1}{K} \sum_k \frac{\delta C_k}{\delta Q} \frac{\partial y_j}{\partial I_j} = -1 \quad \text{or} \quad \frac{\partial y_j}{\partial I_j} = - \frac{K}{\sum k \frac{\delta C_k}{\delta Q}}
\]

The left-hand term in the first version represents the impact on marginal social costs of a marginal investment. Investment is more favored the higher its impact on \( W_j \), the more strongly increments in congestion increase the necessary offset costs to achieve standard consumption quality, the larger the overall population, or the lesser the assimilative capacity of the common medium. The first is a technological consideration. The others essentially denote the several elements comprising the seriousness of the congestion problem. Investment is favored the more serious is current congestion and the more effectively it can decrease the per unit contributions to that condition. It should be noted that each \( I_j \) is a substitute for the only other policy variable implied in (23), \( I \) (social assimilation investment),
since the latter increases $K$ and therefore decreases the payoff to each $I_j$.

The only term which can reveal which groups are likely to invest
is $\frac{dW_j}{dI_j}$. This is subject to technological possibilities. We may surmise,
however, some properties of the relationship. The larger is $W_j$ the easier
it probably is to effect a unit decrease in it, since critical mass phenomena
may make several types of recycling economical or may make available a greater
variety of ameliorative procedures. Thus, we assume

\begin{equation}
\frac{dW_j}{dI_j} \bigg|_{I_j=0} > \frac{dW_k}{dI_k} \bigg|_{I_k=0}
\end{equation}

Since $W_3 > W_2 > W_1$, this suggests that greater polluters are more likely to
substitute treatment investment for congestion penalties than lesser polluters.

It is even conceivable that the treatment will be carried far enough to wipe
out the difference in observed $W_j$ between two adjacent groups (the group
will, however, remain distinct because the originally higher group
will have achieved the lower $W_j$ only by a greater $I_j$ than that of the lower
group). Whatever the actual pattern of $I_j$ among groups, the presence of the
congestion charges (in the context of equal incomes, tastes and congestion
sharing) will guarantee that the total resources spent on treatment will
be efficiently spent: it will buy the largest total decrease in congestion
cost for the population: i.e.

\begin{equation}
\sum_j \delta c_j \frac{dI_j}{dI_j} N_j - \sum_j dI_j N_j = H_j(I^*) \max \text{ for } \sum_j dI_j N_j = I^*_j
\end{equation}

where $H_j(I^*_j)$ is the aggregate cost improvement resulting from a given total of
$I^*_j$ in resources being used for $I_j$ by individual users as part of their utility
maximizing decisions, less the amount $I^*_j$: i.e., an aggregate private investment
profit.
H. Social Assimilation Investment

Social assimilation investment \( I \), unlike the private projects of \( I_{ij} \), is carried out by the collectivity of \( L \)--the government. It decreases the overall system costs of congestion by increasing the assimilative capacity of the common medium which is shared by users of \( X \). Multiple user treatment plants, road capacity enlargement, low flow augmentation, are examples of this type of investment. We have already noted that \( I \) is a substitute for \( I_{ij} \). It is also a substitute for more production of \( X \) or \( Z \). Its justification therefore depends upon traditional cost-benefit analysis. This consists in comparing the aggregate cost savings from decreased congestion less the loss of net cost saving from private treatment investment that is excluded by the public investment--the project's benefits--with the project's costs (i.e., the opportunity cost in terms of \( Z \)). The marginal profit from an increment of such investment is:

\[
\frac{\partial \Pi}{\partial I} = \left[ \frac{3 \Sigma C^k}{3I} - \frac{3 (\Pi_{ij})}{3I} \right] - 1 = -\frac{3}{K^2} \frac{\partial K}{\partial I} \frac{3 \Sigma C^k}{3I} - \frac{3 (\Pi_{ij})}{3I} - 1
\]

The second term is complex, since it shows the effect of a changing \( K \) on the cost impact of each marginal \( I_{ij} \), and the effect of this on the optimal \( I_{ij} \). It is therefore difficult to characterize the properties of \( \frac{\partial \Pi}{\partial I} \) in general.

More precise specification of the components is necessary for deeper analysis. With a precise specification we can, by setting \( \frac{\partial \Pi}{\partial I} = 0 \), find the optimal \( \hat{I}_{-1} \) and for each user thereby, the optimal set of private investments, \( \hat{I}_1, \hat{I}_2, \hat{I}_3 \). This in turn establishes the set \( \hat{W}_{ij} \) and so the set \( \hat{W}_j \). All of these together determine the marginal social costs for different users of \( X \), and for \( X \) as a whole relative to \( Z \). Thus, the overall allocation of resources and distribution of income are determined.

The most obvious qualitative result suggested by (23) and (26) is an asymmetry between \( I \) and the set \( (I_{ij}) \). The former is, but the latter are not, a positive function
of $T$ (aggregate interactive disturbance). Since $\dot{T}(t) > 0$, then we can expect also (27) $\dot{I}(t) > 0$

I will be increasingly justified by population and income growth. The profitability of each $I_j$ will not be directly enhanced by a growing $T$, but to whatever extent this growth in $T$ stems from population growth there will be more total users, each one of whom has an unchanged incentive for $I_j$.

However, the growing $I$ and thereby the ever-increasing $K$ will systematically decrease the attractiveness of private treatment expenditures. Growth may well tend to favor a gradual switch from private to public investment, assuming that decreasing returns to $I$ (i.e., the size of $\frac{dK}{dT}$) do not set in substantially.

What could mitigate this trend is for incremental public investment to have so disastrous an effect upon the profitability of private investment that benefit-cost considerations justify only small increases relative to population growth (which tends to increase $\sum_{j} I_j N_j$—total private investment). But precise characteristics of (26) and the set (23) are necessary to determine this.

One subsidiary result is that public investment, unlike private, does not require the presence of congestion charges to justify its existence. As with the latter, the presence of charges affects the opportunity costs of undertaking investment. But the direction of effect is reversed. Congestion charges, by making private investment attractive, make the opportunity cost of public investment higher, because public investment tends to displace private. In the absence of charges there would be no private investment and, therefore, none to be displaced by public. Public investment would be considerably larger. The resulting intensity of congestion is difficult to compare under the two situations. It will generally depend on the effectiveness of individuals vs. multi-user methods of controlling congestion. A rough guess is that some forms of congestion are easier to control individually than collectively, so the situation that can find room for both—i.e. with charges — will use resources more efficiently in this regard.
I. Conclusion

Only the briefest conclusion can be given here. A system with congestion charges will differ from one without in a variety of dimensions. Relative prices between X and Z, and among different users, will be affected. But so too will be the amount of private and public investment to control congestion. While no definitive propositions can be made at this level of generality once the complex interaction of all these dimensions is allowed, the no-charge system is likely to display the following characteristics relative to the more efficient charge system:

1. too much consumption of X relative to Z;
2. too much consumption of X by $L_3$ especially and $L_2$ relative to $L_1$;
3. not enough private investment and hence too high congestion generation rates per unit of private activity;
4. too much relative dependence on public investment to carry the burden of control against a growing congestion problem;
5. in sum, more congestion at any time, and a more rapid worsening of the problem over time.