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Efficient Unemployment Insurance

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Abstract

This paper argues that moderate unemployment insurance not only reduces the uncertainty faced by risk-averse workers but also improves efficiency and raises output. We develop a model in which the decentralized equilibrium is inefficient without unemployment insurance, because the labor market endogenously creates jobs that provide risk-averse workers with low unemployment risk and low wages. Essentially, the labor market offers its own version of insurance. In doing so, however, it inefficiently distorts resource allocation. This is because the flip side of workers facing low unemployment risk, is that firms are often unable to fill their vacancies. They respond to this high vacancy risk by reducing capital investments. Unemployment insurance makes workers accept more unemployment risk, thus inducing firms to create high wage, high capital jobs, and restoring efficiency. A by-product of this analysis is a very tractable general equilibrium model of search with risk-averse workers and incomplete insurance.

Keywords: capital investment, efficiency, risk-aversion, search, unemployment insurance, wage offer.

JEL Classification: D83, J64, J65.

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1 Introduction

The conventional wisdom is that there is a trade-off between maximizing net output and providing equity through unemployment insurance (UI). This paper argues, to the contrary, that moderate levels of UI enhance both efficiency and equity.\(^1\) We analyze the endogenous response of the labor market to the level of UI. In the absence of UI, there will be many low productivity, low wage jobs providing low unemployment risk to risk-averse workers. In other words, the labor market will offer its own version of insurance against unemployment risk. However, because these jobs will have low capital-labor ratios and low productivity, this decentralized allocation will neither maximize output nor offer an equitable distribution of consumption risk. As a result, UI increases both the level of output and the extent of risk-sharing.

To give the main flavor of our approach, consider the following example of employers hiring laborers. Prospective employers must rent equipment (\textit{e.g.} a bulldozer or shovels) before looking for workers. Contrast two situations: In the first, there is no UI, and so workers’ reservation wages are quite low. At this low wage, labor demand is high. But because there are many employers looking for workers, it is difficult for an employer to find a worker. In the second situation, there is UI, and so workers will only work at higher wages. Thus labor demand is lower, and so it is easier for each employer to locate workers. An important and largely ignored implication is that in the second situation, employers will be more willing to invest in expensive equipment (a bulldozer), whereas in the tight labor market of the first scenario, expecting that quite often they will be unable to hire a worker, they will only rent the cheapest equipment (a shovel). This reasoning suggests that because UI makes workers more choosy, the economy will achieve higher productivity and efficiency by inducing firms to invest in superior equipment. A key result of our paper is therefore that moderate levels of UI may improve efficiency by increasing the capital-labor ratio.

Our result is actually much stronger. We consider an economy where firms post wages, and workers, observing all wage offers, decide for which job to apply (see Peters, 1991; Montgomery, 1991; also Sattinger, 1990). In this framework, the equilibrium is efficient when workers are risk-neutral and there is no unemployment benefit (Moen, 1997; Shimer, 1996; Acemoglu and Shimer, 1997). In contrast, risk-averse workers are more willing to apply to low wages with low unemployment risk. Employers respond to this low ‘reservation wage’ by creating a large number of low wage jobs, reducing the unemployment risk faced by workers. However, the flip side of workers’ low unemployment risk is that employers face a high ‘vacancy’ risk. That is they face a lower probability of filling their vacancies, and so make smaller \textit{ex ante}

\(^1\)Throughout the paper we distinguish ‘efficiency’ and ‘optimality’. Following the older literature on the ‘efficiency-equity’ trade off, we define efficiency as net output maximization, and optimality as \textit{ex ante} expected utility maximization.
irreversible investments. As a result, risk-aversion reduces investment, net output and hence efficiency. Because workers’ risk-aversion is the source of inefficiency, UI is an appropriate policy instrument; it raises workers’ reservation wage, encourages them to apply to higher wages and induces firms to invest more. In fact, we show that there always exists a level of (incomplete) UI that restores efficiency in our economy.

The key contribution of this paper, the positive impact of UI on efficiency, is empirically plausible. It is well-known that European economies have much higher UI and have achieved higher rates of labor productivity growth than the U.S. economy (e.g. Houseman, 1995). Also, in line with our approach, it appears that the higher rate of labor productivity growth in Europe has been party due to more capital-intensive production. Similarly, Acemoglu (1997) shows that when U.S. states increase the generosity of their UI system, average labor productivity increases and the composition of jobs improves. Nevertheless, some care needs to be taken in thinking about the empirical predictions of our results. First, our model predicts that too much UI reduces output. Second, we do not attempt to explain why the private sector cannot offer the efficient level of UI. Here we defer to the existing literature on adverse selection (e.g. Rothschild and Stiglitz, 1976), which explains why a private insurance market may not develop.

Our model also offers a methodological improvement over the existing literature. We develop a full general equilibrium model of job search with risk-averse workers. Most existing models either assume risk-neutrality (Diamond, 1982; Mortensen and Pissarides, 1994) or are partial equilibrium (Mortensen, 1977). Some recent papers analyze search models with risk-aversion and incomplete unemployment insurance markets, but are forced to make strong assumptions about wage determination. For example, Andolfatto and Gomme (1996) and Gomes, Greenwood and Rebelo (1997) assume that wages are exogenously set at marginal product. Costain (1996) assumes that wages are a constant fraction of total output, and thus do not respond to changes in UI. In contrast, our analysis emphasizes the general equilibrium effects of risk-aversion on wages and productivity, which is key to our result, that UI can enhance efficiency. Second, despite the important general equilibrium interactions, our model is extremely tractable. In fact, in contrast to traditional search models which are often difficult to work with because most decisions are intertemporal, all of our results can be obtained in a static model. This makes the analysis quite straightforward and enables many extensions and applications.

Other papers have also pointed out that unemployment benefits may improve the allocation of resources in search models (Diamond, 1981; Acemoglu, 1997; Marimon and Zilibotti, 1997). However, all of these papers assume workers are risk-neutral, and so unemployment benefits are simply a subsidy to search. In contrast, in our model the main role of UI is to provide insurance. Also, capital investments, the most interesting margin in our paper, play no role in these other models. Finally, our work is related to the analysis of optimal UI in the presence of asymmetric information, including Shavell and Weiss (1979), Hansen and Imrohoroglu (1992), Atkeson and
Lucas (1995) and Hopenhayn and Nicolini (1997). The general equilibrium effects of UI and risk-aversion emphasized here do not feature in any of these papers, and so they predict that UI always reduces efficiency.

We start in the next section with a one period model that illustrates all of our main points. We characterize the equilibrium with general utility and production functions, and perform some basic comparative statics. Then in Section 3, we prove that with risk-neutral workers the equilibrium is efficient with no UI, but with risk-averse workers, efficiency is achieved for a strictly positive level of UI. In Section 4, we characterize the optimal level of UI. Section 5 extends our analysis to a dynamic setup. We establish that all of our results hold in this more complex environment, which demonstrates that our static model captures the important economic interactions. Section 6 discusses the robustness of these results along a number of other dimensions and offers some further extensions. Section 7 concludes. All proofs are contained in an appendix.

2 A Model of Job Search by Risk-Averse Agents

2.1 Preferences and Technology

Consider the following one period model. There is a continuum 1 of workers, each with wealth level A. Worker i has preferences described by the von Neumann-Morgenstern utility function:

\[ u(A + y_i) \]

where \( y_i \) is the net earnings of the worker. We assume that \( u \) is continuously differentiable, strictly increasing, and weakly concave.

There is also a larger continuum of potential firms, each with access to a production function \( f : (0, \infty) \rightarrow (0, \infty) \) that requires one worker and capital \( k > 0 \) to produce \( f(k) \) units of the unique consumption good of this economy. We assume that \( \lim_{k \to 0} f(k) = 0 \) and that \( f(k) \) is continuously differentiable, strictly increasing, and strictly concave. We also assume that \( \lim_{k \to 0} f'(k) > 1 \) and there exists \( \bar{k} \) such that \( f'(\bar{k}) \equiv 1 \), a weaker version of the familiar Inada conditions. Workers own a diversified portfolio of firms, and we will therefore assume that firms act as expected profit-maximizers. There is also free-entry of firms, so in equilibrium there is no aggregate profits.

Workers and firms come together via "search". At the beginning of the period, each firm \( j \) decides whether to be active or not. Each active firm, i.e. firm \( j \) such that \( j \in \mathcal{V} \), buys some capital \( k_j > 0 \) at marginal cost 1 and posts a wage \( w_j \).\(^2\)

\(^2\)Observe that firms choose their capital irreversibly before they hire a worker (Acemoglu, 1996, Acemoglu and Shimer, 1997). This implies that when buying (or renting) capital, they take into account the wages they will pay and how easy it will be to find a worker. Partially irreversible capital decisions are empirically plausible, since most equipment is purchased and many specific costs are incurred before workers are recruited.
Each worker observes all the wage offers and then decides where to apply. That is, each worker applies to one of the wages within the set \( \mathcal{W} = \{ w_j \text{ for all } j \in \mathcal{V} \} \). If the worker is hired, she earns the wage posted by that firm. Otherwise she is unemployed, and earns income \( y_i = z \), an unemployment benefit financed by lump-sum taxation.

An important variable in our analysis is the expected queue length at a job. Namely, there will be on average \( q_j \) workers applying to a firm posting the wage \( w_j \). We assume that the probability that a worker is hired by firm \( j \) is a continuously differentiable function \( \mu : \mathbb{R}_+ \cup \infty \to [0,1] \) of the expected queue length \( q_j \). The probability that firm \( j \) manages to hire a worker is then \( q_j \mu(q_j) \). We assume that: \( \mu(q) \) is decreasing and \( q \mu(q) \) is increasing; and \( \mu(0) = \lim_{q \to \infty} q \mu(q) = 1 \). Thus when there are many workers applying for a job, each worker has a low probability of employment, but the firm offering that job is likely to be able to hire someone. Symmetrically, if there are few workers applying for a job, workers are likely to be hired, but the firm is unlikely to hire someone. Peters (1991) and Montgomery (1991) describe similar models. However, this is more general, because we allow for risk-aversion, endogenous capital investments, and more general matching technologies. The first two generalizations are crucial for the results we will obtain.

Our formulation encompasses many reasonable possibilities. For example, following Peters (1991), we can think of each worker using identical mixed strategies when deciding where to apply. If in expectation \( q \) workers apply to each firm posting a wage of \( w \), then

\[
\mu(q) = \frac{1 - e^{-q}}{q}.
\]

This expression comes from the textbook urn-ball process. A large number of balls are independently thrown into a large number of urns. If the expected number of balls in an urn is \( q \), then with (approximately) probability \( e^{-q} q^n / n! \) there are \( n \) balls in the urn. If \( n \geq 1 \), one ball is selected at random. Then conditional on going into an urn that in expectation receives \( q \) balls, a ball is selected with probability

\[
\sum_{n=0}^{\infty} \frac{e^{-q} q^n}{n!} \frac{1}{n+1} = \frac{e^{-q}}{q} \left( \sum_{n=0}^{\infty} \frac{q^n}{n!} - 1 \right) = \frac{1 - e^{-q}}{q}
\]

Another alternative is the “frictionless” matching process,

\[
\mu^F(q) = \begin{cases} 
1 & \text{if } q < 1 \\
1/q & \text{if } q \geq 1
\end{cases}
\]

where the measure of matches is the smaller of the measure of workers applying for the wage and firms offering the wage. This matching process does not satisfy the differentiability assumption at \( q = 1 \), but serves as a useful limiting case. Our key results do not hold in this limit, which implies that matching imperfections are crucial to our analysis. However, our results do obtain under any smooth approximation to this frictionless matching process.
2.2 Definition of Equilibrium

Since queue lengths are crucial to utility and profits, they will be an important part of an allocation. We define an allocation as a tuple \( \{ \mathcal{W}, Q, K, U \} \) where \( \mathcal{W} \subseteq \mathbb{R}_+ \), \( Q : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \cup \infty \), \( K(w) \subseteq \mathbb{R}_+ \), and \( U \in \mathbb{R}_+ \). In words, an allocation consists of the set of wages offered, \( \mathcal{W} \), the queue length \( Q(w) \) of firms offering \( w \), the set of possible capital decisions \( K(w) \) of firms that offer wage \( w \), and a utility level for workers, \( U \). Note that parts of this allocation will not be observed in practice. For example, if \( w \notin \mathcal{W} \), then \( Q(w) \) or \( K(w) \) will not be observed. We include these as part of the allocation, because they will play a very important role in determining equilibrium behavior, as actions ‘off the equilibrium path’ often do. Also since matching probabilities are well-behaved even when \( Q(w) \to \infty \), \( Q(w) \) is defined onto the extended real line.

We can now define an equilibrium:

**Definition 1** An equilibrium is an allocation \( \{ \mathcal{W}^*, Q^*, K^*, U^* \} \) such that:

1. **[Profit Maximization]** \( Q^*(w) \mu(Q^*(w)) \left( f(k) - w \right) - k \leq 0 \) with equality if \( w \in \mathcal{W}^* \) and \( k \in K^*(w) \)

2. **[Optimal Application]** \( \forall w, \text{ if } u(A + w) - u(A + z) \leq U^* \text{ then } Q^*(w) = 0. \)
   
   Otherwise
   
   \[ \mu(Q^*(w)) \left( u(A + w) - u(A + z) \right) = U^*. \]

3. **[Maximal Utility]**

\[
U^* = \begin{cases} 
\sup_{w \in \mathcal{W}} \mu(Q^*(w)) \left( u(A + w) - u(A + z) \right) & \text{if } \mathcal{W} \neq \emptyset \\
0 & \text{if } \mathcal{W} = \emptyset 
\end{cases}
\]

The first part of the definition ensures that given the queue lengths associated with each wage offer, firms choose wages and capital investments that maximize their profits, and that in equilibrium these profits are equal to zero due to free-entry. The second part ensures optimal allocation decisions both on and off the equilibrium path for a given level of maximal utility, \( U^* \), imposing ‘subgame perfection’. If employment at a certain wage \( w \) gives less utility than \( U^* \), then no worker applies for it (i.e. \( Q^*(w) = 0 \)). Otherwise, \( Q^*(w) \) is chosen such that workers get expected utility \( U^* \) from applying to \( w \). Thus if a firm deviates, it recognizes that workers will apply for the wage until the queue length makes it no more desirable than alternative wages. Finally, the third part of the definition determines \( U^* \) as the utility level that a worker obtains from applying to an equilibrium wage.

It is important to note that the queue length function, \( Q^* \), contains two other pieces of useful information: first, if \( w^* \) is the unique equilibrium wage rate, the number of firms that enter the market is given by \( 1/Q^*(w^*) \); and secondly, the rate of unemployment of workers applying to a wage \( w^* \) is \( u(w^*) = 1 - \mu(Q^*(w^*)) \), i.e. the probability that this worker is turned down in favor of another applying to the same job. Clearly \( u(w^*) \) is increasing in \( Q(w^*) \); workers who apply to jobs with longer queues suffer a higher probability of unemployment.
2.3 Existence and Characterization

Our first result characterizes an equilibrium as a constrained optimization problem:

**Lemma 1** Let \( \{\mathcal{W}, Q, K, U\} \) be an equilibrium allocation. \( \forall w^* \in \mathcal{W}, q^* = Q(w^*), k^* \in K(w^*) \), the triple \( \{w^*, q^*, k^*\} \) solves:

\[
U = \sup_{w, q, k} \mu(q) \left( u(A + w) - u(A + z) \right) \\
subject to \quad q\mu(q) \left( f(k) - w \right) = k \\
w \geq z
\]

Also, if \( \{w^*, q^*, k^*\} \) solves this constrained optimization problem, then there exists an equilibrium \( \{\mathcal{W}, Q, K, U\} \) such that \( w^* \in \mathcal{W}, q^* = Q(w^*) \) and \( k^* \in K(w^*) \).

That the equilibrium allocation maximizes expected worker utility subject to firms making zero profits is fairly intuitive. In equilibrium, no worker will apply to a wage less than the unemployment benefit, \( z \), which establishes that (3) has to hold. The first constraint, (2), follows from the fact that firms are maximizing profits and have to make zero-profits in equilibrium. Finally, if an allocation did not maximize (1), there would be room for another firm to enter and entice workers to apply while making positive profits. In view of these results, with a slight abuse of terminology, we will sometimes refer to a triple \( \{w, q, k\} \) that solves (1) subject to (2) and (3) as an equilibrium.

The next lemma characterizes the profit maximizing capital choices as a function of wage offered, and shows that \( K(w) \) is a singleton.

**Lemma 2** Let \( \{\mathcal{W}, Q, K, U\} \) be an equilibrium allocation. Define \( K : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) implicitly by \( w \equiv f(K(w)) - K(w)f'(K(w)) \), increasing in \( w \). Then for all \( w \in \mathcal{W}, K(w) = \{K(w)\} \).

This lemma implies that, despite the search frictions, labor earns the residual output, after paying capital its marginal product (see Acemoglu and Shimer, 1997). This result is due to the fact that a firm offering wage \( w \) selects its capital stock to maximize its profit, and optimally earns zero profits.

We can now start to characterize the equilibrium. First, recall that \( f'(\bar{k}) = 1 \), and also let \( \bar{z} \equiv f'(\bar{k}) - \bar{k} \). Then:

**Proposition 1** An equilibrium \( \{\mathcal{W}, Q, K, U\} \) always exists.

1. \( \forall z > \bar{z}, \mathcal{W} = \emptyset \) in equilibrium, i.e. there are no active firms.
2. \( \forall z < \bar{z}, \mathcal{W} \neq \emptyset \); also \( \forall w \in \mathcal{W}, w > z \) and \( Q(w) < \infty \).

When UI is so generous that firms cannot offer wages high enough to attract workers and still make zero-profits, there is no activity in equilibrium, i.e. \( \mathcal{W} = \emptyset \). Thus an extremely high level of UI is definitely not beneficial. In all other cases, an equilibrium has some active firms offering wages greater than the unemployment benefit, \( w > z \), and attracting finite queue lengths. In the rest of the paper, we limit our attention to the interesting case, \( z < \bar{z} \).
2.4 Comparative Statics

We are now able to establish some important comparative statics.

Proposition 2 Consider two utility functions $u_1$ and $u_2$, and two unemployment benefits $z_1$ and $z_2$. Let $\{w_i, q_i, k_i\}$ be an equilibrium when the utility function is $u_i$ and the unemployment benefit is $z_i$, $i \in \{1, 2\}$.

1. If $z_1 < z_2$ and $u_1 \equiv u_2$, then $w_1 < w_2$, $q_1 < q_2$ and $k_1 < k_2$.

2. If $u_1$ is a strictly concave transformation of $u_2$ and $z_1 \equiv z_2$, then $w_1 < w_2$, $q_1 < q_2$ and $k_1 < k_2$.

This key result states that if agents become more risk-averse or UI is lower, wages, capital investments and queue lengths fall. The intuition for the effect of UI is rather straightforward: when $z$ increases, workers have less to lose by applying to a high wage job with a longer queue. Recognizing this change in preferences, firms willingly offer higher wages and attract longer queues. With these longer queues, firms are more likely to fill their vacancy, and so optimally make larger irreversible ex ante investments. Also, longer job queues correspond to a higher fraction of the workers not getting jobs. Thus with higher $z$, the unemployment rate will increase.

The more interesting and perhaps important part of the Proposition is the impact of risk aversion on equilibrium. The reason why wages and job queues fall as risk-aversion increases is that more risk-averse workers want more insurance. With $z$ given, the only way more insurance can be achieved is if the prize that a worker receives falls, but the probability of receiving the prize increases. That is, $w$ falls and $\mu(q)$, the hiring probability, increases. This is what we refer to as the market providing its own version of insurance to more risk-averse workers. Viewed in this light, the decline in wages and increase in hiring probabilities in response to less UI can also be interpreted as more ‘private insurance’. Again, in response to shorter queue lengths, firms optimally invest less. In the next section we will see that these changes have important efficiency consequences.

The proof of this proposition can be given graphically, although the appendix contains a more formal proof. Since $z < z$, Lemma 1 implies that an equilibrium allocation is a solution to the maximization of (1) subject to (2). As Lemma 2 showed, $k$ is an increasing function of $w$, independent of the utility function $u$ and the unemployment benefit $z$. Therefore, the constraint set (the set of points satisfying (2)) can be drawn in the $\{q, w\}$ space and the set remains unchanged when we vary the utility function or the level of insurance. We can therefore conduct our comparative statics simply by looking at how the indifference curves of the objective function change in this space. This function is independent of $k$, and it is straightforward to establish that when UI $z$ increases, the new set of indifference curves is everywhere flatter than the old set. In the same way, when we take a concave transformation of the utility function, i.e. consider a more risk-averse worker, the set of indifference curves become everywhere steeper. Thus, for both comparative static exercises, the
Figure 1: This picture depicts the effect of UI. The dashed curve is the constraint set, firms’ zero profit condition. The bold curve is a worker’s indifference curve with high UI. The solid curve is the worker’s indifference curve with low UI. High UI makes a worker relatively willing to accept large increases in queue lengths in return for small increases in potential wages.

new indifference curves and the old satisfy a single-crossing property. This ensures that the comparative statics are always unambiguous. Figure 1 draws the change in the indifference curves when UI increases. As the curves become flatter, the tangency between the indifference curves and the constraint set must shift to the right, to a point of higher wages and longer job queues.

With the frictionless matching technology represented by $\mu^F$ above, the comparative statics in Proposition 2 do not hold. For any degree of risk aversion and any unemployment insurance $z < \bar{z}$, the equilibrium allocation is $\{w, q, k\} = \{\bar{z}, 1, \bar{k}\}$. The reason is simple: workers gain nothing by applying for a job with queue length less than one, and firms gain nothing by maintaining a queue length greater than one. However, each possibility hurts the other party. This ensures that equilibrium queue lengths are always equal to one, and that there is no unemployment in equilibrium. UI and risk-aversion have no effect on the equilibrium allocation, as long as the unemployment benefit is less than the equilibrium wage. Instead, the model is completely neoclassical. Firms choose $k$ to maximize profit, $f(k) - k - w$. This implies $f'(k) = 1$, or $k = \bar{k}$. Free entry ensures that the maximized value of profits is zero, so wages are driven up to $f(\bar{k}) - \bar{k} \equiv \bar{z}$. These results can also be derived directly from the maximization problem described in Lemma 1, which is reassuring.
3 Efficient Unemployment Insurance

UI level $z^e$ is efficient if any decentralized equilibrium with UI equal to $z^e$ maximizes net output, $Y(q, k) \equiv \mu(q)f(k) - \frac{k}{q}$. To see why $Y(q, k)$ is equal to the economy's net output, recall that the number of jobs in the economy is equal to the fraction of workers who find a job, $\mu(q)$, times the measure of workers, which is normalized to 1. Each job produces $f(k)$ units of output, and so the economy's gross output is $\mu(q)f(k)$. To calculate net output, investment expenditures, including those by firms that do not manage to hire workers, must be subtracted from this amount. Each firm buys $k$ units of capital at cost normalized to 1, and in equilibrium there is a measure $\frac{1}{q}$ of active firms. More formally:

**Definition 2** UI $z^e$ is efficient if for all equilibria $\{W_{z^e}, Q_{z^e}, K_{z^e}, U_{z^e}\}$ with UI equal to $z^e$, $\forall w \in W_z$, all $q = Q_{z^e}(w)$ and $k \in K_{z^e}(w)$, maximize $Y(q, k)$.

Note that the efficient allocation has nothing to do with the distribution of output or consumption. In the next section, we characterize the socially optimal allocation, which maximizes workers' expected utility taking into account the cost of consumption variability to risk-averse agents.

In characterizing efficient allocations, we assume that the unemployment benefit $z$ is funded by a lump-sum tax or insurance premium $\tau(z)$, determined before agents decide where to apply. Formally, we assume that the government wishes to minimize taxes subject to a balanced budget constraint. Thus if the equilibrium queue length is $q$, the government choose $\tau(z) \equiv (1 - \mu(q)) z$. Agents' asset holdings are then equal to their initial assets $A_0$ minus taxes $\tau(z)$.

We begin with a key benchmark:

**Proposition 3** Suppose agents are risk-neutral and unemployment benefits $z$ are zero. Then any allocation is an equilibrium if and only if it is efficient.

Intuitively, when there is no UI, risk-neutral agents maximize the expected value of their wages. This is efficient, because the sum of wages is equal to net income. This result is a generalization of Moen (1997) and Shimer (1996) to the case in which firms choose their physical capital investments (see Acemoglu and Shimer; 1997). Intuitively, despite the search frictions, competition drives the wage rate to the true marginal product of labor.

The next result follows immediately from Propositions 2 and 3.

**Proposition 4** Suppose agents are risk-averse and unemployment benefits $z$ are zero. Then any equilibrium allocation is inefficient.

Proposition 2 implies that in an equilibrium with risk-averse workers, wages, queue lengths, and capital investments are strictly smaller than in any equilibrium with
risk-neutral workers and the same UI. Proposition 3 states that the efficient allocation is an equilibrium with risk-neutral workers. Because changes in risk-aversion do not affect what is efficient, together these results imply Proposition 4.³

As noted above, with risk-averse agents, the market provides its own version of insurance with lower wages, shorter job queues, and lower unemployment risk. The important point is that this insurance is inefficient. The flip side of low unemployment risk, is that firms face a high ‘vacancy’ risk, and so make small irreversible capital investments. Thus workers have low productivity. A different way of expressing this intuition is to relate it to the efficiency results in search models with bargaining (e.g. Diamond, 1982; Hosios, 1991). In these models, if relative bargaining strengths dictate that workers receive too small a fraction of the output that they produce, unemployment is inefficiently low. In our model, risk-averse workers without access to UI prefer to receive a small fraction of output, and firms cater to that preference by offering low wages.

Fortunately, the inefficiencies created by risk-aversion can easily be reversed through moderate UI. This is because, while risk-aversion makes workers more willing to accept large wage cuts in return for small reductions in unemployment risk, UI makes workers less willing to accept that same trade-off. An appropriate level of UI therefore undoes the ‘distortionary’ impact of risk-aversion, restoring efficiency.

**Proposition 5** Suppose agents are risk-averse, and let an equilibrium with unemployment benefit $z$ be $\{w_z, q_z, k_z\}$. Denote an efficient allocation by $\{w^e, q^e, k^e\}$. Let $z^e \in (0, w^e)$ satisfy

$$w^e = \frac{u(w^e + A_0 - \tau(z^e)) - u(z^e + A_0 - \tau(z^e))}{u'(w^e + A_0 - \tau(z^e))}$$

where $\tau(z^e) = (1 - \mu(q^e))z^e$ is the balanced budget tax rate if queue lengths are $q^e$. Then $z^e$ decentralizes the efficient allocation, i.e. $\{w_{z^e}, q_{z^e}, k_{z^e}\} = \{w^e, q^e, k^e\}$.

If workers have constant absolute risk aversion, $u(c) = \frac{1-e^{-c}}{\theta}$, the efficient UI given by Proposition 5 has a simple form: $z^e = w^e - \log(1 + \theta w^e) / \theta$. This is increasing in the coefficient of absolute risk aversion $\theta$, and is equal to 0, thus yielding no UI, when workers are risk-neutral, i.e. $\theta = 0$; and $z^e = w^e$, giving complete insurance, when workers are infinitely risk-averse ($\theta \to \infty$).

The technical intuition behind these result can once again be given graphically (see Figure 2). Returning to $(q, w)$ space, recall that the constraint set, representing firms’ free entry and profit maximization conditions, is invariant to workers’ risk aversion and to UI. On the other hand, a more risk-averse worker has steeper indifference curves when faced with the same level of UI. Thus the point of tangency

³Note that since the comparative statics results of Proposition 2 do not hold under the frictionless matching technology $\mu^F$, Proposition 4 does not apply either. In fact, in this frictionless case, the allocation continues to be efficient even in the presence of risk-aversion. Reassuringly, our model again embeds the conventional frictionless model.
between the risk-averse worker's indifference curve and the constraint set must lie below the risk-neutral worker's point of tangency. Also, a worker enjoying more UI has a tangency point that lies above the tangency point of a worker with less UI. Thus an appropriate amount of UI can undo any effect of risk-aversion. Then since the risk-neutral worker's point of tangency is the efficient point \((q^e, w^e)\) when there is no UI, a risk-averse worker's point of tangency is efficient with moderate UI.\(^4\)

In Figure 3, we parameterize the model and plot the equilibrium values of the key endogenous variables as a function of the unemployment benefit, \(z\). The efficient level of unemployment insurance is \(z^e = 0.117\), compared with an efficient wage rate \(w^e = 0.179\), or a replacement ratio of 65%. The efficient output level is approximately 79% higher than the output level without UI, an enormous difference. Naturally, the magnitude of these results strongly depends on the parameters. In particular, with a lower degree of risk-aversion or less room for self-insurance (i.e., higher non-labor income, \(A_0\), which in the example is of the same order of magnitude as labor income), \(z^e\) would be lower and the difference between the laissez-faire equilibrium and the output maximizing allocation would be smaller.

Finally, we consider how the efficient level of UI changes with risk-aversion.

---

\(^4\)Proving Proposition 5 is slightly more subtle than this intuition suggests. We must prove that the risk-averse worker's indifference curve through \((q^e, w^e)\) does not intersect the constraint set else. We do this by showing that it lies above the risk-neutral indifference curve, which itself lies above the constraint set by Proposition 3.
Figure 3: This figure shows the effect of unemployment benefits on endogenous variables. The parameterization of the model is as follows: \( u(c) = \frac{1-c^{-5}}{5} \); \( f(k) = k^{1/2} \); \( \mu(q) = \frac{1-e^{-q}}{q} \); and \( A_0 = 0.1 \).

Proposition 6 Let \( u_1 \) and \( u_2 \) be twice differentiable utility functions with non-increasing absolute risk aversion.\(^5\) Assume that when all workers have utility \( u_i \), unemployment benefit \( z^e \) decentralizes the efficient allocation \( (w^e, q^e, k^e) \). Then if \( u_1 \) is a strictly concave transformation of \( u_2 \), \( z^e_1 > z^e_2 \).

4 Optimal Unemployment Insurance

The previous section characterized the output-maximizing or efficient level of UI. However, welfare depends not only on the level of output, but also on its distribution. For this reason, we also analyze the policy of a benevolent social planner who wishes to maximize the expected utility of a representative worker. We first characterize the planner's policy given a large set of policy tools (proof in the text):

Proposition 7 Suppose a planner is constrained by the same search technology as the decentralized economy, but can choose the measure of active firms and the wage and capital level of all firms. Then she opens \( 1/q^e \) firms, each with capital \( k^e \), and sets the wage and unemployment benefit to satisfy \( w = z = Y(q^e, k^e) \).

In words, the planner maximizes total output and divides it equally among all workers, providing full insurance to each. The optimality of this program is trivial —

\(^5\)This assumption is important. If agents have increasing absolute risk-aversion, an economy in which agents are more risk-averse may have lower efficient UI. This is because the financing of lower UI requires lower taxes, which implies that the agents behave as if they are less risk-averse, and therefore choose higher wages and queue lengths. Since increasing absolute risk aversion is considered empirically implausible, focusing on decreasing or constant absolute risk-aversion is not very restrictive.
the planner can never do better than maximizing net output and redistributing it equally. Also, this program is feasible as workers are indifferent between applying to one of the vacancies and remaining unemployed. The ‘second best’ is attainable, because the planner has enough instruments to eliminate moral hazard considerations (see Section 6). The only noncompetitive feature is the presence of search and coordination frictions, and the planner chooses queue lengths and capital levels optimally in light of these constraints.

While Proposition 7 provides a useful benchmark, a more interesting exercise is to ask what level of UI maximizes workers’ expected utility, given that entry decisions, investment, and wage offers are determined in equilibrium. To this end, we define optimal UI analogously to efficient UI:

**Definition 3** UI $z^*$ is optimal if it solves

$$
\max_z \mu(q)u(A - \tau(z) + w) + (1 - \mu(q))u(A - \tau(z) + z)
$$

subject to $w \in \mathcal{W}_z$ and $q \in Q_z$ for any equilibrium $\{\mathcal{W}_z, Q_z, K_z, U_z\}$ with UI $z$.

Optimal UI differs from Efficient UI in that the former takes workers’ risk-aversion into account, while the latter simply involves maximizing the average worker’s consumption, without worrying about distributional issues. Also, Optimal UI differs from the constrained efficient allocation characterized in Proposition 7, in that it requires that firms make profit maximizing entry and capital investment decisions, while in Proposition 7 the planner regulated these margins directly.

We are currently unable to prove the following conjecture:

**Conjecture 1** Optimal UI $z^*$ exists. For all $u$ strictly concave, $z^* \in (z^e, \bar{z})$.

Intuitively, one would expect that at the point of efficiency, a further increase in $z$ would lead to a second-order loss of net output. One would also expect it to increase the income of unemployed workers and to decrease the after-tax income of employed workers. Unfortunately, these conjectures are difficult to establish because a decrease in the unemployment benefit does not lead to a mean-preserving spread of the outcomes. Instead, equilibrium search behavior implies that it less likely that a worker winds up unemployed when UI is lower. Although we are unable to prove Conjecture 1, simulations support it. For example, in Figure 3 on page 12, the optimal level of unemployment insurance is $z^* = 0.139$, while the efficient level is $z^e = 0.117$. Similarly, the optimal replacement ratio $w^*/z^*$ is 71%, somewhat higher than the efficient replacement ratio of 65%. Note however that the change in output and expected utility in moving from $z^e$ to $z^*$ appears to be small.

We can prove that $z^* < w^* < \bar{z}$, i.e. optimal insurance is incomplete. This is because the choice of capital investments and wages is subject to moral hazard. With higher UI, workers wish to apply to riskier jobs, and firms oblige by creating these. If the government (or an insurance agency) could monitor what jobs a worker
applied for (e.g. banned workers from applying to jobs with \( w > z \)) or simply regulate job creation as in Proposition 7, full insurance would be possible. In the absence of this type monitoring, equilibrium wages are always strictly larger than the unemployment benefit, as long as \( z < \bar{z} \). And when \( z = \bar{z} \), the economy collapses.

5 Dynamics With CARA

Job search decisions are forward looking. To capture these issues of intertemporal optimization, most search models are dynamic. However, we have derived all of our results in a static model, and we view our ability to do this as an advantage of our approach. Still, it is important to demonstrate that this simpler setting is not omitting essential economic interactions that arise in a dynamic model. For this reason, we now consider a dynamic extension of our economy. We show that the analysis becomes considerably more complex, but all of our results continue to hold.

We also assume in this section that workers have constant absolute risk aversion (CARA). Although our results do in fact generalize to a dynamic environment with arbitrary preferences, CARA is a useful simplification. With other utility functions, a worker’s wealth affects her preferences over wages and queue lengths. As a result, a worker recognizes that her consumption decision today affects the probability distribution of jobs in the future. Since this determines her budget constraint, this feeds back and affects her consumption choice today. CARA allows us to avoid these issues, because a worker’s application decision is independent of her wealth. Section 6 discusses the implications of non-CARA preferences in more detail.

5.1 Preferences and Technology

Consider an infinite horizon economy in discrete time. Each worker makes her consumption and job search decisions in an effort to maximize her CARA utility function:

\[
U = \sum_{t=0}^{\infty} \beta^t \frac{1 - e^{-\theta c_t}}{\theta}
\]

where \( c_t \) is her consumption at time \( t \) and \( \beta < 1 \) is the discount factor. Similarly, firms maximize the expected present value of profits, discounted at the constant gross interest rate \( R > 1 \).

The search and production technologies are generalizations of the static model. At the start of a period, firms may be either inactive, vacant, or have a filled job. Similarly, workers may either be unemployed or employed. Then inactive firms have an opportunity to create a vacancy by buying \( k > 0 \) units of perfectly durable capital at a unit cost conveniently normalized to \( \frac{R}{R-1} \). After this, newly created and existing vacancies post wages in an effort to attract applicants. Unemployed workers see the menu of wages, and decide where to apply. If the expected number of applicants or queue length is \( q \), a worker’s probability of employment is \( \mu(q) \) and
a firm’s probability of hiring is \( q\mu(q) \). If this in fact occurs, the firm has a filled job and the worker is employed. Finally, all employed workers and filled jobs, new or old, produce \( f(k) \) units of output. If the firm cannot find a worker, its capital remains idle, and so the opportunity cost of not hiring a worker is increasing in the firm’s capital stock. This is the end of the period. In particular, we assume for simplicity that a productive relationship between the worker and firm never ends.\(^6\)

We assume that firms post flat-wage contracts, rather than more complex wage profiles (Shimer, 1996; Acemoglu and Shimer, 1997). As long as firms cannot pay money to rejected job applicants (see Section 6.4), one can show that more complex contracts are no better than flat-wage contracts.

Workers face the dynamic budget constraint:

\[
A_{t+1} = R(y_t + A_t - \tau_t - c_t)
\]

where \( y_t \) is the gross income of the individual at time \( t \), \( \tau_t \) is the lump-sum tax and \( c_t \) is consumption at time \( t \). We also impose the transversality condition:

\[
\lim_{s \to \infty} R^{-s} A_{t+s} = 0
\]

which rules out Ponzi-game schemes. We further simplify our analysis by assuming that this is a small open economy with an interest rate equal to the rate of time preference, \( R\beta = 1 \). Thus workers’ equalize the expected marginal utility of consumption across time, as can readily be verified from the Euler equation.

We focus our analysis on steady state equilibria in which wages, capital stocks, queue lengths, and unemployment rates are constant. In order to maintain a steady state, we assume that the labor force \( L_t \) grows at rate \( \delta \); all the \( \delta L_{t-1} \) new workers are unemployed at the start of period \( t \). As a result, in steady state the entire economy grows at rate \( \delta \) as well. Let \( E(A, w) \) be the lifetime utility of a worker who is employed at wage \( w \) and has assets \( A \) at the start of the time period. Similarly, denote the expected value of an unemployed worker with assets \( A \) by \( U(A) \), assuming she follows an optimal strategy at this and all future dates. Let \( J(A, w, q) \) be the expected value of an unemployed worker if she applies for wage \( w \) and the associated queue length \( q \) in this period, and then follows an optimal consumption rule thereafter.

**Definition 4** A steady state equilibrium is an allocation \( \{ W, Q, K, U, E, J \} \) such that:

\(^6\)This formulation, rather than a more standard one in which there are random separations, is adopted in order to simplify the analysis. With random separations, it is not optimal for firms to offer workers constant wages. Instead, since firms are risk-neutral, they would optimally bear the risk of future random separations. This could be done, for example, by paying workers large signing bonuses, and then holding them to their reservation wage. If one allows for such contracts, the risk of separations becomes irrelevant, and only the hiring risk remains. Thus this assumption is not important for our analysis.
1. [Profit Maximization]

\[
\frac{Q(w')\mu(Q(w'))(f(k') - w')}{1 - \beta(1 - Q(w')\mu(Q(w')))} - k' \leq 0
\]

with equality if \( w' \in \mathcal{W} \) and \( k' \in \mathcal{K}(w) \)

2. [Optimal Consumption Policy] Workers choose consumption to maximize their present discounted utilities, \( J(A, w, q) \) and \( E(A, w) \), given their application decisions. That is, they maximize (5) subject to (6) and (7), taking the stochastic path of income as given.

3. [Optimal Application] \( \forall w', \text{ if } J(A, w', 0) \leq U(A) \text{ for all } A, \text{ then } Q(w') = 0. \) Otherwise \( Q(w') = \sup q' \text{ such that for some } A, \)

\[
J(A, w', q') = U(A)
\]

4. [Maximal Utility] For all \( A, \)

\[
U(A) = \begin{cases} 
\sup_{w \in \mathcal{W}} J(A, w, Q(w)) & \text{if } \mathcal{W} \neq \emptyset \\
0 & \text{if } \mathcal{W} = \emptyset 
\end{cases}
\]

This is a generalization of the definition of equilibrium in the static model, taking into account that the value functions must be determined as part of the equilibrium. To understand the expression for firm profits, observe that free entry implies that the expected present value a vacant firm’s profits is \( k/(1 - \beta) \), the cost of creating a new vacancy. Then in steady state, if \( w \in \mathcal{W}, q = Q(w) \) and \( k \in \mathcal{K}(w), \)

\[
\frac{k}{1 - \beta} = q\mu(q)\frac{f(k) - w}{1 - \beta} + (1 - q\mu(q))\beta\frac{k}{1 - \beta}
\]

The value of a vacant firm comes from the possibility that it creates a job, which gives it profit \( f(k) - w \) for the infinite future, and the continuation value if it fails to create a job, discounted until next period.

If there is a unique queue length \( q \) in steady state, then the equilibrium unemployment rate \( u \) at the start of a period is \( \frac{k}{\delta + \mu(q)} \). This is because in steady state, the unemployed population at the end of period \( t \), \( uL_t \), is equal to the number of new births, \( \delta L_{t-1} \), plus a fraction \( 1 - \mu(q) \) of the workers who were unemployed at the start of the previous period, \( uL_{t-1} \). Then use the growth equation \( L_t \equiv (1 + \delta)L_{t-1} \). Note that the end of period unemployment rate is different and equal to \( (1 - \mu(q))u \).

5.2 Optimal Consumption Decisions and Value Functions

We look for an equilibrium in which all unemployed workers have the same preferences over queue lengths and wages, regardless of their asset level (i.e. a value
function $J(A, w, q)$ such that the utility maximizing choice of $w$ and $q$, $(w^*, q^*) \in \arg\max_{q,w} J(A, w, q)$ is independent of $A$. Once we find this value function $J$, by Blackwell’s theorem, there will be no other function that is a solution to the individual optimization, thus we will have completely characterized the equilibria.

**Lemma 3** 1. An employed worker who starts a period with assets $A_t$, pays taxes $\tau$ in every period, and earns wage $w$ in every period, consumes

$$c^e_t = w + (1 - \beta)A_t - \tau$$

so $A_{t+1} = A_t$. Her lifetime utility is

$$E(A_t, w) = \frac{1}{1 - \beta} \frac{1 - e^{-\theta(w+(1-\beta)A_t-\tau)}}{\theta}$$

(10)

2. An unemployed individual who starts a period with assets $A_t$, pays taxes $\tau$ in every period, applies for a job offering wage $w$ and queue length $q$ in every period, and earns benefit $z$ in every period that she fails to get a job, consumes

$$c^u_t = \beta\phi + (1 - \beta)(A_t + z) - \tau$$

where $\phi \in (z, w)$ is implicitly defined by

$$1 = \mu(q)e^{-\theta(w-\phi)} + (1 - \mu(q))e^{-\theta(1-\beta)(z-\phi)}$$

(11)

so $A_{t+1} = A_t + z - \phi$. Her lifetime expected utility is

$$J(A_t, w, q) = \frac{1}{1 - \beta} \cdot \frac{1 - e^{-\theta(\phi+(1-\beta)A_t-\tau)}}{\theta}$$

(12)

Because $R\beta = 1$, employed workers consume their current net income and the interest from their savings, maintaining a constant asset level. Unemployed workers have a decreasing consumption level while they are unemployed. Upon finding a job, their consumption jumps up.

Lemma 3 confirms that an unemployed worker’s asset level does not affect her preference over wages and queue lengths. Unemployed workers choose $w$ and $q$ in order to maximize $J(A_t, w, q)$; from equation (12), this is clearly equivalent to maximizing the shift term in the consumption function, $\phi$, subject to constraint (11), and that optimization problem is independent of the worker’s asset level. This result depends heavily on workers having CARA preferences, which ensure that there are no wealth effects. For this reason, we do not concern ourselves with characterizing the distribution of asset holdings, although this could easily be done.
5.3 Characterization of Equilibrium

The next lemma proves that as in the static case, an equilibrium is the solution to a maximization problem:

**Lemma 4** Let \( \{W, Q, K, U, E, J\} \) be an equilibrium allocation. Then \( \forall w^* \in W, q^* = Q(w^*) \), and \( k^* \in K(w^*) \), \( \{w^*, q^*, k^*, \phi^*\} \) solves:

\[
0 = h(\phi^*) \equiv \max_{w,q,k} \mu(q) \frac{1 - e^{-\theta(w - \phi)}}{\theta} + (1 - \mu(q)) \frac{1 - e^{-\theta(1-\beta)(z - \phi)}}{\theta}
\]

subject to

\[
q \mu(q) \left( f(k) - w \right) \frac{1 - \beta (1 - q \mu(q))}{1 - \beta (1 - q \mu(q))} = k
\]

\( w \geq z \)

\( E \) and \( J \) are defined by (10) and (12) respectively, and \( U(A) = J(A, w^*, q^*) \). Also, if \( \{w^*, q^*, k^*, \phi^*\} \) solves this program, then there is an equilibrium \( \{W, Q, K, E, J, U\} \) such that \( w^* \in W, q^* = Q(w^*) \) and \( k^* \in K(w^*) \), \( E \) and \( J \) are defined by (10) and (12) and \( U(A) = J(A, w^*, q^*) \).

The optimization in Lemma 4 is equivalent to choosing \( w \) and \( q \) to maximize \( J(A, w, q) \) for each possible value of \( \phi \), and then finding the level of \( \phi \) that satisfies (11). To see this, note that (12) expresses \( J \) as a function of \( \phi \) alone, which gives \( \phi \) as a one-to-one function of presented discounted utility when unemployed. Thus maximizing the utility of an unemployed worker is equivalent to maximizing \( \phi \) subject to two constraints. These constraints are: first, firms must make zero profits, as in Lemma 1 of the static problem. We impose this as a constraint in the usual fashion. Second, \( \phi \) must satisfy equation (11). Rearranging this constraint yields

\[
0 = \mu(q) \frac{1 - e^{-\theta(w - \phi)}}{\theta} + (1 - \mu(q)) \frac{1 - e^{-\theta(1-\beta)(z - \phi)}}{\theta} \equiv h(\phi)
\]

which gives the condition that \( h(\phi) = 0 \) in Lemma 4. The remainder of the proof closely follows that of Lemma 1 and is thus omitted. Note again that assets do not appear in Lemma 4, thanks to CARA preferences, and therefore, as discussed above, preferences over wages and queue lengths are independent of wealth, and the equilibrium is independent of the distribution of assets.\(^7\)

In light of this proposition, and similar to our informal terminology in the static model, we will refer to a tuple that \( \{w^*, q^*, k^*, \phi^*\} \) solves the above maximization problem as an equilibrium.

The next proposition proves existence of an equilibrium:

---

\(^7\)This is a feature of our search environment, not shared by the search and bargaining models of Diamond, Mortensen, and Pissarides. In particular, in those models even when workers have CARA preferences and there is incomplete insurance, the equilibrium wage of a worker will depend on his asset level. This is because what matters here is the preferences of workers over lotteries, whereas with bargaining what matters is the marginal utility of consumption, which always varies with wealth when agents are risk-averse.
Proposition 8 There exists a unique $\phi^*$ that solves (13) subject to (14) and (15).

Since $\phi^*$ is a one-to-one function of the level of utility when unemployed, there is a unique maximal value of unemployment. Note that the uniqueness of $\phi^*$ does not guarantee that $w^*$ and $q^*$ are unique, because as in the static model, the maximization problem is not strictly quasiconcave. However, this does not reduce our ability to perform comparative static.

5.4 Comparative Statics

The static results of Proposition 2 completely generalize to this environment.

Proposition 9 Consider two CARA utility functions parameterized by coefficients of absolute risk aversion $\theta_1$ and $\theta_2$, and two unemployment benefits $z_1$ and $z_2$. Let \{w_i, q_i, k_i, \phi_i\} be an equilibrium when risk aversion is $\theta_i$ and the unemployment benefit is $z_i$, $i \in \{1, 2\}$.

1. If $z_1 < z_2$ and $\theta_1 \equiv \theta_2$, then $w_1 < w_2$, $q_1 < q_2$ and $k_1 < k_2$.

2. If $\theta_1 > \theta_2$ and $z_1 \equiv z_2$, then $w_1 < w_2$, $q_1 < q_2$ and $k_1 < k_2$.

This proposition is once more proved using revealed preference arguments along the lines of Proposition 2, although $\phi$ complicates the story considerably. Figure 1 on page 8 again gives the intuition. An increase in $z$ makes workers’ indifference curves everywhere flatter, and therefore shifts the point of tangency to the right, raising $w$, $q$, and $k$. Also, $\phi$ is higher when $z$ is higher, implying that, taking taxes as given, unemployed workers are better off when unemployment benefits are higher. This further increases their willingness to apply for high wages. Similarly, when workers are more risk-averse, wages are lower, queues are shorter, and firms invest less. Also, unemployed workers consume less, because they are more worried about the possibility of low consumption in the future.

5.5 Efficient Unemployment Insurance

The analysis of Efficient UI also parallels the static model.

Proposition 10 Suppose agents are risk-neutral and unemployment benefits $z$ are zero. Then an allocation is an equilibrium if and only if it is efficient.

We prove this by showing that risk-neutral workers with no unemployment benefits attempt to maximize their labor income, which is identical to net output.

Once again, an immediate corollary of Propositions 9 and 10 is that if agents are risk-averse and there is no UI, then any allocation is inefficient. However, it is again possible to decentralize the efficient allocation:
Proposition 11 Suppose agents are risk-averse, and let the equilibrium with unemployment benefit \( z \) be \( \{w_z, q_z, k_z, \phi_z\} \). Denote an efficient allocation by \( \{w^e, q^e, k^e\} \). Let \( z^e \) be the unique solution to

\[
\frac{1 - \beta (1 - \mu(q^e))}{1 - \beta} \cdot \frac{e^{-\theta(w^e - \phi^e)} + e^{-\theta(1-\beta)(w^e - \phi^e)}}{\theta e^{-\theta(w^e - \phi^e)}}
\]

Then \( z^e \) decentralizes the efficient allocation, i.e. \( \{w_{z^e}, q_{z^e}, k_{z^e}\} = \{w^e, q^e, k^e\} \).

6 Discussion

This section discusses some of the assumptions that are important for our results. We also analyze the robustness of our model by considering several generalizations.

6.1 Worker Heterogeneity

We have assumed that all workers have the same level of assets, the same utility function, and receive the same unemployment benefit. All of these are unrealistic, but served to simplify our analysis. A more general case is that workers differ according to all these three features. All of our results would hold in this more general case. Suppose that there are \( s = 1, 2, ..., S \) types of workers, with fraction \( \lambda_s > 0 \) of type \( s \). Assume that type \( s \) has utility function \( u_s \), after-tax asset level \( A_s \) and unemployment benefit \( z_s \). Differences in the attitude towards risk and in the initial asset endowment are likely to be important in most analyses, while differences in the unemployment benefit might differing self-insurance possibilities due to different household structures. Letting \( U \) now be a vector in \( \mathbb{R}^S \), we establish the following generalization of Lemma 1:

Lemma 5 Let \( \{W, Q, K, U\} \) be an equilibrium allocation. Then, \( \forall w^* \in \mathcal{W}, q^* = Q(w^*), k^* \in K(w^*) \), the triple \( \{w^*, q^*, k^*\} \) solves:

\[
U_s = \sup_{w,q,k} \mu(q) (u_s(A_s + w) - u_s(A_s + z_s))
\]

subject to

\[
q \mu(q) (f(k) - w) = k
\]

\[
w \geq z_s
\]

for some \( s \in \{1, 2, ..., S\} \). Also, if a vector \( \{(w'_s, q'_s, k'_s)\}_{s=1}^{S} \) solves this constrained optimization problem for each \( s \in [0, S] \), then there exists an equilibrium \( \{\mathcal{W}', Q', K', U'\} \) such that \( w'_s \in \mathcal{W}', q'_s = Q'(w'_s) \) and \( k'_s \in K'(w'_s) \) for all \( s = 1, 2, ..., S \).

\footnote{This analysis can be generalized to a countable number of types.}
Thus all of our results would generalize to a case of worker heterogeneity across multiple dimensions. Intuitively, the equilibrium of this model is an amalgam of the equilibrium of $S$ separate models populated by homogeneous agents. Since workers observe all the posted wages, they are able to find the ‘right’ wage for them. The proof of the lemma is otherwise analogous to Lemma 1, and is omitted.

This result shows that it is conceptually easy to generalize our results to dynamic environments with general utility functions, in which past application decisions and idiosyncratic shocks imply that workers have different asset levels at the start of a period. Nevertheless, monitoring the complex wealth dynamics is quite hard. To avoid these complications, Section 5 only analyzed the case of CARA preferences where the distribution of assets did not affect equilibrium application decisions.

This result also leads to a very intuitive empirical prediction. If there is a group of workers who have better unemployment insurance, they will apply for higher wage jobs with longer queues and they will suffer longer unemployment spells. This prediction is verified empirically (Ehrenberg and Oaxaca, 1976; Katz and Meyer, 1990).

6.2 A less restrictive information structure

The assumption that workers observe all posted wages is very stringent, and much stronger than we require. There are two reasons that we make this assumption: first, because it simplifies the analysis; and second, because it emphasizes how close our model is to a competitive economy. The only imperfections are embedded in the matching function $\mu$. Acemoglu and Shimer (1997) prove that with risk-neutral workers, if every worker observes the wage offered by (at least) two independently drawn firms, the equilibrium allocation is qualitatively and quantitatively unaffected. It is straightforward to extend the results in that paper to the risk-averse case. Intuitively, if a single firm considers deviating from the equilibrium allocation $\{w, q, k\}$, it knows that any worker who observes its wage, will also observe the wage offered by a non-deviating firm. Since the non-deviator earns zero profits, either the deviator must offer a high wage, which gives it negative profits, or it must offer a low wage, in which case it cannot attract the worker, giving it negative profits. Building on this argument, we can prove that our results are robust to more general information structures.

6.3 Other Forms of Moral Hazard

Our model recognizes that one type of moral hazard is amplified by UI. This is the possibility that workers search for high wages, which are associated with high unemployment risks. However, we ignore the type of moral hazard that is emphasized in the literature on optimal UI (Shavell and Weiss, 1979; Hopenhayn and Nicolini, 1997). That is the possibility that insured workers will not spend much effort searching for job. Omitting this source of moral hazard is an important simplifying assumption. If search is an extremely unattractive endeavor, then the efficient level of UI may in fact be zero or even negative. However, if this effect is moderate, it
will simply reduce the efficient unemployment benefit. Thus we expect that moderate unemployment insurance will reduce search intensity, but will also increase the capital intensity of production sufficiently, so as to raise net output.

6.4 Restrictions on Private Contracts

We have assumed that firms may only pay money to a worker whom it employs. One could also imagine that firms guarantee applicants ‘interview payments’ in the event that they apply for a job and are rejected. For example, consider an equilibrium allocation \( \{w, q, k\} \), and suppose a firm deviates and offers a wage \( w' \) to any worker it employs and an interview payment \( w' - z \) to any worker who applies for the job and is rejected, and therefore collects an unemployment benefit \( z \). Thus a worker applying for this job is guaranteed utility \( u(A + w') \), where \( A \) is the post-tax asset level. Then if

\[
\begin{align*}
    u(A + w') & \equiv \mu(q)u(A + w) + (1 - \mu(q))u(A + z),
\end{align*}
\]

workers are indifferent about applying for this job. In particular, the equilibrium expected queue length \( q \) may prevail for this job as well. Since the firm is taking on workers’ risk, this contract reduces the expected wage payment, assuming the hiring probability remains \( q\mu(q) \).\(^9\) Firms have a very uncertain wage bill, but since they are risk-neutral, they are willing to bear this additional risk. Thus we are placing a real restriction on private contracts by assuming that firms can only pay workers whom they employ. However, we feel that this restriction is very natural. In practice, unemployed workers are worse off than employed workers, which is a prediction of our baseline model but is not implied by a model with large interview payments. Also payments to applicants are not observed in reality. More importantly, many realistic extensions of our setup would immediately rule out interview payments.\(^{10}\)

\(^9\)Because workers are risk-averse,

\[
\begin{align*}
    \mu(q)u(A + w) + (1 - \mu(q))u(A + z) & < u(A + \mu(q)w + (1 - \mu(q))z)
\end{align*}
\]

Since the left hand side equals \( u(A + w') \) by definition, algebraic manipulation implies

\[
q\mu(q)w > q(\mu(q)w' + (1 - \mu(q))(w' - z))
\]

\( q\mu(q)w \) is the expected labor costs on the equilibrium path. The firm hires a worker with probability \( q\mu(q) \), in which case it pays the wage \( w \). The right hand side is the expected labor costs under the alternative plan. In expectation the firm receives \( q \) applications. In expectation, a fraction \( \mu(q) \) of the workers are employed and get a wage \( w' \), while the remainder are unemployed and get \( w' - z \) as an interview payment. This establishes that the expected wage payment is lower.

\(^{10}\)For example, in a world with heterogeneous utilities of leisure, a high interview payment firm would attract many workers who do not want to work, increasing its wage bill.
6.5 Government versus Private Insurance

We have assumed that UI is provided by the government and financed by lump-sum taxation. This assumption is of obvious empirical relevance. Still, one would like to understand whether a private insurance sector could perform the government’s function. To fully answer this question, we must model a competitive insurance sector. However, even without doing this, one can recognize that there are likely to be a number of obstacles. The first is adverse selection. A private company that offers UI may first attract workers who for some reason are more likely to be unemployed, and this will not be profitable. Furthermore, Rothschild and Stiglitz (1976) show that with heterogeneity, efficiency may require some cross-subsidization of different workers. Free entry implies that private insurance companies are unable to achieve this.

There is another reason that private insurance is problematic in the version of our model where workers only observe finitely many wage offers. Recall that in this case, none of our other results are affected (see section 6.2), but now consider an insurer who provides full insurance to a small group (zero measure) of workers. In contrast to our baseline specification where each worker observed all wages, firms would not create separate jobs for these workers. This because each worker observes a finite number of wage offers, and so there will be close to zero probability that any worker observes the wage offer of any particular firm, thus firms would not change their wage and queue lengths. However, if all workers tried to take advantage of such an opportunity, then firms would realize that they have to pay increasingly higher wages, and the economy would thus be driven towards the point where it breaks down, $z = \bar{z}$. This problem does not arise if workers observe every wage offer. In that case, the market will separately cater for the workers who have better insurance, as Lemma 5 demonstrated.

7 Conclusion

The conventional wisdom maintains that there is a trade-off between efficiency and the insurance provided by UI. We show that the endogenous response of the labor market to risk aversion and UI may reverse this wisdom. Increasing UI makes workers more choosy and encourages them to apply to higher wage jobs. This induces firms to invest more in capital, and increases output. The distinctive feature of our model is that it is the risk-aversion of workers that is responsible for the inefficient job composition and low equilibrium output. Therefore, UI is precisely the right tool to deal with this market failure. In fact, we establish that there exists a unique level of UI which restores efficiency.

Central to our results is the assumption that a firm must make an irreversible investment before it attempts to hire labor. As a result, if workers are less willing to accept unemployment risk, firms must accept greater vacancy risk, and so there is less investment. While we believe that this mechanism is important, other plausible
mechanisms would lead to the same qualitative results. For example, more capital intensive technologies may require more specialized skills. Then if workers do not know whether their talents are suitable for a job until after they apply, they will shy away from more capital intensive sectors when unemployment is associated with low consumption for a number of periods. Similarly, if the efficient technology involves idiosyncratic risk that could lead to future unemployment, as is the case with start-up companies, then risk-averse workers will tend towards more stable, low productivity technologies in the absence of UI.

Our analysis contains methodological improvements over the existing search literature. We solve a dynamic general equilibrium model with risk-aversion, incomplete insurance markets, and endogenous wages and productivity. To our knowledge, this is the only paper to accomplish these tasks. The real advantage to our framework, is that all the results can be obtained using a static model. Because the additional complexity of intertemporal optimization is unnecessary to capture the essence of this search model, it can easily be applied to other problems in search theory.
References


8 Appendix: Proofs

Proof of Lemma 1: First we prove that an allocation that is not a solution to this maximization problem is not an equilibrium, and then we show that any solution to the maximization problem is an equilibrium by construction.

1. Consider a triple \((\hat{w}, \hat{q}, \hat{k})\) that does not solve this maximization problem. That is, either \(\{\hat{w}, \hat{q}, \hat{k}\}\) does not satisfy constraint (2), or (3), or it satisfies both constraints, but there exists another triple \(\{w', q', k'\}\) in the constraint set that achieves a higher value of (1). We will prove that if \(\hat{w} \in W, \hat{q} = Q(\hat{w})\), and \(\hat{k} \in \mathcal{K}(\hat{w})\), then \(\{W, Q, \mathcal{K}, U\}\) is not an equilibrium. First, if \(\{\hat{w}, \hat{q}, \hat{k}\}\) does not satisfy (2), then the 'profit maximization' condition of equilibrium is trivially violated. Second, if \(\hat{w} < z\), then from the 'optimal application' part of the definition of equilibrium, \(Q(\hat{w}) = 0\). But if \(\hat{k} \in \mathcal{K}(\hat{w})\), then \(\hat{k} > 0\) (by the restriction that all active firms have \(k\) strictly positive), and this violates 'profit maximization'. Third, suppose that \(\{\hat{w}, \hat{q}, \hat{k}\}\) and \(\{w', q', k'\}\) both satisfy the constraint, but the latter achieves a higher value of the objective. Then by the definition of \(Q(w')\),

\[
\mu(\hat{q}) (u(A + \hat{w}) - u(A + z)) = \mu(Q(w')) (u(A + w') - u(A + z))
\]

Also, \(q' < Q(w')\), since \(\mu\) is a decreasing function. Then:

\[
Q(\hat{w})\mu(Q(\hat{w}))(f(\hat{k}) - \hat{w}) - \hat{k} = q'\mu(q')(f(k') - w') - k'
\]

\[
< Q(w')\mu(Q(w'))(f(k') - w') - k'
\]

which implies that \(\{w', Q(w'), k'\}\) offers higher profits than \(\{\hat{w}, \hat{q}, \hat{k}\}\), and completes the proof.

2. Consider a triple \(\{w^*, q^*, k^*\}\) that maximizes (1) subject to (2) and (3), then we will construct an equilibrium \(\{W, Q, \mathcal{K}, U\}\) where \(W = \{w^*\}\), \(q^* = Q(w^*)\), and \(\mathcal{K}(w^*) = \{k^*\}\). Define \(U = \mu(q^*) (u(A + w^*) - u(A + z))\). Then for all \(w'\), let \(Q(w')\) be the unique solution to

\[
\mu(Q(w')) (u(A + w') - u(A + z)) = U
\]

or \(Q(w') = 0\) if there is no solution to (20). Then, by construction, all three equilibrium requirements, profit maximization, optimal allocation and maximal utility, are satisfied.\(\blacksquare\)

Proof of Lemma 2: \(k \in \mathcal{K}(w)\) must maximize firms’ profit, \(q\mu(q)(f(k) - w) - k\). Otherwise, \(\{w, q, k\}\) could not be a solution to (1) subject to (2) and (3). This implies the necessary and sufficient first order condition:

\[
q\mu(q)f'(k) = 1
\]
Using (2), we get that for all \( k \in \mathcal{K}(w) \), \( w = f(k) - kf'(k) \). Since \( f'' < 0 \), this defines a unique \( k = K(w) \) with \( K'(w) > 0 \). Hence, \( \mathcal{K}(w) \) is a singleton, equal to \( \{K(w)\} \). ■

Proof of Proposition 1: We will first prove that if \( z > \bar{z} \), there is an equilibrium with no active firms. Then we prove that if \( z < \bar{z} \), an equilibrium with active firms exists.

1. Suppose \( z > \bar{z} \). Since \( w \geq z \) in any equilibrium, (2) implies \( f(k) - z \geq k \). Since \( z > \bar{z} \equiv f(\bar{k}) - \bar{k} \), this implies \( f(k) - k > f(\bar{k}) - \bar{k} \), which is impossible. Thus the constraint set is empty, and there is no equilibrium with active firms (i.e. \( \mathcal{W} \neq \emptyset \)). Consider the no-activity allocation: \( \mathcal{W} = \emptyset, U = 0 \), and all three equilibrium requirements are satisfied. Thus, there is a unique no-activity equilibrium.

2. Suppose \( z < \bar{z} \). The constraint set is nonempty and compact (with \( q \) defined on the extended real line), and the maximand, (1), is well-defined and continuous at all points over the constraint set. Thus a maximum exists. Also, the maximum is strictly positive, since reversing the previous argument shows that there are \( w > z \) that satisfies the constraints. Finally, since the value of the objective is zero when \( q = \infty \) or \( w = z \), both desired inequalities follow. ■

Proof of Proposition 2: First, note that the constraint set (2) is independent of both the utility function of workers and the level of unemployment benefit. Therefore, in both cases \( \{w_i, q_i, k_i\} \) satisfy (2) for \( i = 1, 2 \).

1. From Proposition 1, we have the following ‘revealed preference’ result:

\[
\begin{align*}
\mu(q_1) \left( u(A + w_1) - u(A + z_1) \right) & \geq \mu(q_2) \left( u(A + w_2) - u(A + z_1) \right) \quad (22) \\
\mu(q_2) \left( u(A + w_2) - u(A + z_2) \right) & \geq \mu(q_1) \left( u(A + w_1) - u(A + z_2) \right) \quad (23)
\end{align*}
\]

Multiplying (22) with (23) and simplifying, we obtain:

\[
\begin{align*}
(u(A + w_1) - u(A + z_1)) (u(A + w_2) - u(A + z_2)) & \geq (u(A + w_1) - u(A + z_2)) (u(A + w_2) - u(A + z_1))
\end{align*}
\]

Simplifying, we find \( (u(A + z_1) - u(A + z_2)) (u(A + w_1) - u(A + w_2)) \geq 0 \). Then since \( z_1 < z_2 \), we find \( w_1 \leq w_2 \). From \( w = f(k) - kf'(k) \), \( k_2 \geq k_1 \). Then, from (21), \( q_2 \geq q_1 \). And both inequalities are strict if and only if \( w_2 > w_1 \). Finally, that all these inequalities are strict follows from the fact that the constraint set and the objective function are smooth, and so small changes in parameters will necessarily perturb the solution (see Edlin and Shannon, 1996).
2. Using an analogous revealed preference argument,

\[
(u_1(A + w_1) - u_1(A + z)) (u_2(A + w_2) - u_2(A + z)) 
\geq (u_2(A + w_1) - u_2(A + z)) (u_1(A + w_2) - u_1(A + z))
\tag{24}
\]

Suppose \( w_1 > w_2 > z \). Then by the intermediate value theorem, there exists a \( \lambda \in (0, 1) \) with

\[
u_2(A + w_2) \equiv \lambda u_2(A + w_1) + (1 - \lambda) u_2(A + z)
\tag{25}
\]

Since \( u_1 \) is a strictly concave transformation of \( u_2 \),

\[
u_1(A + w_1) > \lambda u_1(A + w_1) + (1 - \lambda) u_1(A + z)
\tag{26}
\]

Eliminating \( u_2(A + w_2) \) and \( u_1(A + w_2) \) from (24) using (25) and (26) respectively, we obtain:

\[
\lambda (u_1(A + w_1) - u_1(A + z)) (u_2(A + w_1) - u_2(A + z)) 
\geq \lambda (u_2(A + w_1) - u_2(A + z)) (u_1(A + w_1) - u_1(A + z))
\tag{27}
\]

Since both sides of (27) are exactly the same expression, this gives a contradiction. Hence \( w_2 \geq w_1 \). The rest of the argument follows as in Part 1 to establish \( w_2 > w_1, q_2 > q_1, \) and \( k_2 > k_1 \). ■

**Proof of Proposition 3:** Let \( u(x) = ax + b \). Then we can rewrite the objective (1) of the constrained optimization problem with \( z = 0 \) as \( \max_{w, q, k} \mu(q) w \). Use constraint (2) to eliminate \( w \) from this expression; the resulting unconstrained maximization is equivalent to the output maximization problem, \( \max_{q, k} Y(q, k) \). ■

**Proof of Proposition 4:** Proposition 3 states that any efficient allocation is an equilibrium with risk-neutral workers. Since a risk-averse utility function is a strictly concave transformation of a risk-neutral utility function, Proposition 2 states that the capital investments are lower and queue lengths are shorter in any equilibrium with risk-averse workers than in any equilibrium with risk-neutral workers. The result follows by combining these statements. ■

**Proof of Proposition 5:** First observe that \( z^e \) is well defined by equation (4). The right hand side is a continuous function of \( z \). Also, concavity of \( u \) implies

\[
u(A_0 - \tau(0)) < u(w^e + A_0 - \tau(0)) - u'(w^e + A_0 - \tau(0)) w^e
\]

so the right hand side of (4) is bigger than \( w^e \) when \( z \) is zero, and it is equal to 0 when \( z = w^e \). Then the intermediate value theorem implies that this equation implicitly defines at least one value of \( z^e \in (0, w^e) \).

Next, compare two constrained optimization problems as in Lemma 1:
(i) A risk-neutral worker with \( z = 0 \);

(ii) A risk-averse worker with \( z = z^e \) defined in the statement of the Proposition.

The constraint set defined by (2) is identical in the two problems. Also, according to Proposition 3, the solution to problem (i) is the efficient allocation \( \{w^e, q^e, k^e\} \). We will use this fact, that the risk-neutral worker weakly prefers \( \{w^e, q^e\} \) to any other feasible allocation, in order to prove that the risk-averse worker strictly prefers \( \{w^e, q^e\} \) to any other feasible allocation, which proves the Proposition.

The risk-neutral worker weakly prefers \( \{w^e, q^e\} \) to another feasible allocation \( \{w', q'\} \):

\[
\mu(q^e)w^e \geq \mu(q')w'
\]

Let \( A = A_0 - \tau(z^e) \) be the risk-averse worker’s after-tax asset level. Also, one can confirm that \( (u(w + A) - u(z^e + A))/w \) is strictly quasi-concave, achieving its maximum at \( w^e \), by equation (4). In particular, since \( w' \neq w^e \),

\[
\frac{u(w^e + A) - u(z^e + A)}{w^e} > \frac{u(w' + A) - u(z^e + A)}{w'}
\]

Multiplying (28) and (29),

\[
\mu(q^e)(u(w^e + A) - u(z^e + A)) > \mu(q')(u(w' + A) - u(z^e + A))
\]

That is, the risk-averse worker strictly prefers \( \{w^e, q^e\} \) to \( \{w', q'\} \).

**Proof of Proposition 6:** In order to find a contradiction, assume \( z^e_2 \geq z^e_1 \). Since both unemployment benefits decentralize the efficient allocation, the fraction of workers who are unemployed is the same at both allocations; thus \( \tau(z^e_2) \geq \tau(z^e_1) \) as well. Then since \( u_2 \) satisfies (by Proposition 5)

\[
u_2(A_0 - \tau(z^e_2) + w^e) - u_2(A_0 - \tau(z^e_2) + z^e_2) = \nu'_2(A_0 - \tau(z^e_2) + w^e)w^e
\]

and has non-increasing absolute risk-aversion, \( u_2 \) is a (weakly) concave transformation of \( \tilde{u}_2 \) (see Claim 1 below), defined by workers paying \( \tau(z^e_2) - \tau(z^e_1) \) less taxes:

\[
\tilde{u}_2(A_0 - \tau(z^e_1) + w^e) - \tilde{u}_2(A_0 - \tau(z^e_1) + z^e_1) \equiv \tilde{u}'_2(A_0 - \tau(z^e_1) + w^e)w^e
\]

Next, since \( u_1 \) is a strictly concave transformation of \( u_2 \), it is a strictly concave transformation of \( \tilde{u}_2 \) as well. Then equation (4) defines that \( z^e_1 \) satisfies

\[
u(\tilde{u}_2(A_0 - \tau(z^e_1) + w^e)) - \nu(\tilde{u}_2(A_0 - \tau(z^e_1) + z^e_1)) \equiv \nu'((\tilde{u}_2(A_0 - \tau(z^e_1) + w^e))(\tilde{u}'_2(A_0 - \tau(z^e_1) + w^e))w^e
\]

Also, strict concavity of \( \nu \) implies that the function lies below its tangent line:

\[
u(\tilde{u}_2(A_0 - \tau(z^e_1) + w^e)) - \nu(\tilde{u}_2(A_0 - \tau(z^e_1) + z^e_1)) > \nu'(\tilde{u}_2(A_0 - \tau(z^e_1) + w^e))(\tilde{u}_2(A_0 - \tau(z^e_1) + w^e) - \tilde{u}_2(A_0 - \tau(z^e_1) + z^e_1))
\]

30
Combining these previous two statements,

\[ \tilde{u}_2'(A_0 - \tau(z^e_1) + w^e)w^e > \tilde{u}_2(A_0 - \tau(z^e_1) + w^e) - \tilde{u}_2(A_0 - \tau(z^e_1) + z^e_1) \]

Finally, eliminate \( \tilde{u}_2(A_0 - \tau(z^e_1) + w^e) - \tilde{u}_2(A_0 - \tau(z^e_1) + w^e)w^e \) using this inequality and (30):

\[ \tilde{u}_2(A_0 - \tau(z^e_1) + z^e_2) < \tilde{u}_2(A_0 - \tau(z^e_1) + z^e_1) \]

This implies \( z^e_2 < z^e_1 \), a contradiction. ■

Claim 1 Let \( u \) be a twice differentiable utility function with non-increasing absolute risk aversion. Assume \( a < \tilde{a} \). Then \( u(a + w) \equiv \psi(u(\tilde{a} + w)) \) for some (weakly) concave function \( \psi \).

Proof. Differentiate the definition of \( \psi \) twice with respect to \( w \):

\[
\begin{align*}
    u'(a + w) &= \psi'(u(\tilde{a} + w))u'(\tilde{a} + w) \\
    u''(a + w) &= \psi''(u(\tilde{a} + w))u'(\tilde{a} + w)^2 + \psi'(u(\tilde{a} + w))u''(\tilde{a} + w)
\end{align*}
\]

Use the first relationship to eliminate \( \psi' \) from the second one:

\[
\frac{-u''(a + w)}{u'(a + w)} = \frac{-\psi''(u(\tilde{a} + w))u'(\tilde{a} + w)^2 + \psi'(u(\tilde{a} + w))u''(\tilde{a} + w)}{u'(a + w)}
\]

Since \( a < \tilde{a} \) and \( u \) has non-increasing absolute risk aversion, \( -\frac{u''(a + w)}{u'(a + w)} \geq -\frac{u''(\tilde{a} + w)}{u'(\tilde{a} + w)} \).

Thus \( -\frac{\psi''(u(\tilde{a} + w))u'(\tilde{a} + w)^2}{u'(a + w)} \geq 0 \), or \( \psi'' \leq 0 \). ■

Proof of Lemma 3: For a worker with asset \( A_0 \) at time \( t = 0 \) and employed at the wage \( w \) in all future periods, the budget constraint (6) takes the form:

\[
\sum_{t=0}^{\infty} R^{-t}(c_t + \tau) = A_0 + \sum_{t=0}^{\infty} R^{-t}w
\]

The standard Euler equation is:

\[
\beta e^{-\theta c_t} = R^{-1} \lambda
\]

where \( \lambda \) is the multiplier on the budget constraint. Therefore, using the fact that and \( \beta R = 1 \):

\[
c_t = c^e = w + \frac{R - 1}{R} A_0 - \tau = w + (1 - \beta) A_0 - \tau
\]

and \( A_t = A_0 \). Hence the value function of a worker employed at wage \( w \) having assets \( A_t \) is given by (10).
Next consider an unemployed worker with asset $A_0$ at time $t = 0$, who plans to apply to wage $w$ with queue length $q$ in all future periods that she is unemployed, and who faces a net UI $z - \tau$ in all periods of unemployment. The unemployed worker looks for a job, and then chooses his consumption. If he finds a job at time $t$, we know from above that he will consume $w + (1 - \beta)A_t - \tau$. If he fails to find a job, he earns net benefit $z - \tau$, and consumes $c_t^w$. This leaves him next period with $A_{t+1} = R \cdot (A_t + z - c_t^w)$. Let us conjecture a linear consumption rule as a function of assets, with marginal propensity to consume out of assets equal to $1 - \beta$. This conjectured consumption function can be written as: $c_t^w = \beta \phi + (1 - \beta)(A_t + z - \tau)$, for some unknown parameter $\phi$. Under this consumption rule, his assets at the start of next period are $A_{t+1} = A_t + z - \phi$. Now, the Euler equation can be written as:

$$e^{-\theta(\beta \phi + (1 - \beta)(A_t + z - \tau))} = \mu(q)e^{-\theta(w + (1 - \beta)A_{t+1} - \tau)} + (1 - \mu(q))e^{-\theta(\beta \phi + (1 - \beta)(A_{t+1} + z) - \tau)}$$

where $A_{t+1} = A_t + z - \phi$. Multiplying both sides of this by $e^{\theta(\beta \phi + (1 - \beta)(A_t + z) - \tau)}$, we obtain (11), which implicitly defines $\phi \in (z, w)$ as an endogenous function of $w, q$ and $z$. Note that since $\phi > z$, unemployed individuals spend more than they earn, so $A_{t+1} < A_t$.

Using this consumption rule, the value function, $J(w', q', A_t)$ can be written as:

$$J(w', q', A_t) = \mu(q')E(w', A_t) + (1 - \mu(q')) \left( \frac{1 - e^{-\theta(\beta \phi + (1 - \beta)(A_t + z) - \tau)}}{\theta} + \beta V(A_t + z - \phi) \right)$$

(31)

Note that this equation is written for any $(w', q')$, not necessarily for the same $(w, q)$ that solves (11). We have already solved for $E(w', A_t)$ in equation (10). Now conjecture the form of $J$ as in (12) and substitute this into (31). This yields:

$$J(w', q', A_t) = \frac{1}{1 - \beta} \left( \mu(q) \frac{1 - e^{-\theta(w + (1 - \beta)A_t - \tau)}}{\theta} + (1 - \mu(q)) \frac{1 - e^{-\theta(\beta \phi + (1 - \beta)(A_t + z) - \tau)}}{\theta} \right)$$

(32)

Now eliminate $\phi$ using its definition in equation (11). The resulting expression reduces to the definition of $V$ in (12), confirming our conjecture. ■

**Proof of Proposition 8:** To begin, note that the constraints (14) and (15) do not depend on $\phi$. It is therefore sufficient to establish that there exists a unique value of $\phi$ such that $h(\phi) = 0$. We will do some in three steps, first showing that $h$ is monotonically decreasing, second that it is continuous, and third that it starts above 0 and ends below it.

1. $(h$ is decreasing) That is, if $\phi_1 < \phi_2$ then $h(\phi_1) > h(\phi_2)$. Let $(w_1, q_1, k_1)$ solve the constrained maximization problem for $\phi_1$ and $(w_2, q_2, k_2)$ solve the problem
for \( \phi_2 \). Then

\[
\begin{align*}
   h(\phi_1) & = \mu(q_1) \frac{1 - e^{-\theta(w_1 - \phi_1)}}{\theta} + (1 - \mu(q_1)) \frac{1 - e^{-\theta(1-\beta)(z - \phi_1)}}{\theta} \\
   & \geq \mu(q_2) \frac{1 - e^{-\theta(w_2 - \phi_1)}}{\theta} + (1 - \mu(q_2)) \frac{1 - e^{-\theta(1-\beta)(z - \phi_1)}}{\theta} \\
   & > \mu(q_2) \frac{1 - e^{-\theta(w_2 - \phi_2)}}{\theta} + (1 - \mu(q_2)) \frac{1 - e^{-\theta(1-\beta)(z - \phi_2)}}{\theta} = h(\phi_2)
\end{align*}
\]

the first inequality follows by revealed preference, while the second inequality follows because \( \phi_2 > \phi_1 \).

2. \((h \text{ is continuous})\) Take \( \phi_1 < \phi_2 \). Consider the following string of inequalities:

\[
0 \geq h(\phi_2) - h(\phi_1)
\]

\[
\begin{align*}
   & \equiv \left( \mu(q_2) \frac{1 - e^{-\theta(w_2 - \phi_2)}}{\theta} + (1 - \mu(q_2)) \frac{1 - e^{-\theta(1-\beta)(z - \phi_2)}}{\theta} \right) \\
   & - \left( \mu(q_1) \frac{1 - e^{-\theta(w_1 - \phi_1)}}{\theta} + (1 - \mu(q_1)) \frac{1 - e^{-\theta(1-\beta)(z - \phi_1)}}{\theta} \right) \\
   & \geq \left( \mu(q_1) \frac{1 - e^{-\theta(w_1 - \phi_2)}}{\theta} + (1 - \mu(q_1)) \frac{1 - e^{-\theta(1-\beta)(z - \phi_2)}}{\theta} \right) \\
   & - \left( \mu(q_1) \frac{1 - e^{-\theta(w_1 - \phi_1)}}{\theta} + (1 - \mu(q_1)) \frac{1 - e^{-\theta(1-\beta)(z - \phi_1)}}{\theta} \right) \\
   & = \mu(q_1) \frac{e^{-\theta w_1}}{\theta} \left( e^{\theta \phi_1} - e^{\theta \phi_2} \right) + (1 - \mu(q_1)) \frac{e^{-\theta(1-\beta)z}}{\theta} \left( e^{\theta(1-\beta)\phi_1} - e^{\theta(1-\beta)\phi_2} \right)
\end{align*}
\]

The first inequality follows by monotonicity. Then we simply expand \( h(\phi_2) - h(\phi_1) \). The second inequality follows by revealed preference, that \((w_2, q_2, k_2)\) yields a higher payoff than \((w_1, q_1, k_1)\) under \( \phi_2 \). The last equality is just algebraic manipulation. Now this final expression is negative, since \( \phi_1 < \phi_2 \). However, it is arbitrarily close to zero — and so \( h(\phi_2) - h(\phi_1) \) is arbitrarily close to zero — if \( \phi_2 - \phi_1 \) is sufficiently small.

3. \((h(\phi) = 0 \text{ for a unique } \phi)\) Observe that \( h(z) > 0 \) and \( h(w_1) < 0 \). Thus continuity and the intermediate value theorem imply that \( h(\phi) = 0 \) for some \( \phi \). Monotonicity guarantees uniqueness. \( \blacksquare \)

**Proof of Proposition 9:** We first establish the comparative static results with respect to \( z \) and then in the second part with respect to \( \theta \).

1. Suppose \( z_2 > z_1 \). We will first show that \( \phi_1 < \phi_2 \), which will then allow us to prove the desired inequalities, \( w_2 > w_1, q_2 > q_1, \) and \( k_2 > k_1 \). Assume to the
contrary that \( \phi_1 \geq \phi_2 \), in order to reach a contradiction.

\[
\begin{align*}
\quad h_2(\phi_2) & \equiv \mu(q_2) \frac{1 - e^{-\theta(w_2 - \phi_2)}}{\theta} + (1 - \mu(q_2)) \frac{1 - e^{-\theta(1-\beta)(z_2 - \phi_2)}}{\theta} \\
\quad & \geq \mu(q_1) \frac{1 - e^{-\theta(w_1 - \phi_2)}}{\theta} + (1 - \mu(q_1)) \frac{1 - e^{-\theta(1-\beta)(z_2 - \phi_2)}}{\theta} \\
\quad & > \mu(q_1) \frac{1 - e^{-\theta(w_1 - \phi_1)}}{\theta} + (1 - \mu(q_1)) \frac{1 - e^{-\theta(1-\beta)(z_1 - \phi_2)}}{\theta} \\
\quad & \geq \mu(q_1) \frac{1 - e^{-\theta(w_1 - \phi_1)}}{\theta} + (1 - \mu(q_1)) \frac{1 - e^{-\theta(1-\beta)(z_1 - \phi_1)}}{\theta} \equiv h_1(\phi_1)
\end{align*}
\]

The first inequality follows by revealed preference because \( \{w_2, q_2, k_2, \phi_2\} \) is the equilibrium, therefore when they expect \( \{w_2, q_2, k_2, \phi_2\} \) in all future periods, unemployed workers prefer to apply to \( w_2 \) with queue length \( q_2 \) rather than \( \{w_1, q_1\} \). The second inequality is true because \( z_2 > z_1 \). The third inequality is true if \( \phi_2 \leq \phi_1 \). But since \( h_2(\phi_2) = h_1(\phi_1) = 0 \), we have shown that \( 0 > 0 \), a contradiction. Therefore, \( \phi_2 > \phi_1 \).

The rest of the lemma follows by a standard revealed preference argument similar to the ones we used before. We include the proof for completeness. Making convenient simplifications,

\[
\begin{align*}
\quad & \mu(q_1) \left( -e^{-\theta w_1} + e^{-\theta((1-\beta)z_1 + \beta \phi_1)} \right) \geq \mu(q_2) \left( -e^{-\theta w_2} + e^{-\theta((1-\beta)z_1 + \beta \phi_1)} \right) \\
\quad & \mu(q_2) \left( -e^{-\theta w_2} + e^{-\theta((1-\beta)z_2 + \beta \phi_2)} \right) \geq \mu(q_1) \left( -e^{-\theta w_1} + e^{-\theta((1-\beta)z_2 + \beta \phi_2)} \right)
\end{align*}
\]

which together imply

\[
\left( e^{-\theta((1-\beta)z_1 + \beta \phi_1)} - e^{-\theta((1-\beta)z_2 + \beta \phi_2)} \right) \left( e^{-\theta w_1} - e^{-\theta w_2} \right) \geq 0
\]

Since \((1-\beta)z_1 + \beta \phi_1 < (1-\beta)z_2 + \beta \phi_2\), the first term is positive. Then the second term must be positive as well, so \( w_1 \leq w_2 \). Since the problem is smooth, the inequality is in fact strict. The remainder of the inequalities follow from the firms’ constraints, as given by (8).

2. In this case, we will prove that \( \theta_1 > \theta_2 \) implies \( \phi_1 \leq \phi_2 \), that is, more risk-averse workers consume less when they are unemployed. Assume to the contrary that \( \phi_1 \geq \phi_2 \) to derive a contradiction:

\[
\begin{align*}
\quad h_2(\phi_2) & \equiv \mu(q_2) \frac{1 - e^{-\theta_2(w_2 - \phi_2)}}{\theta_2} + (1 - \mu(q_2)) \frac{1 - e^{-\theta_2(1-\beta)(z_2 - \phi_2)}}{\theta_2} \\
\quad & \geq \mu(q_1) \frac{1 - e^{-\theta_2(w_1 - \phi_2)}}{\theta_2} + (1 - \mu(q_1)) \frac{1 - e^{-\theta_2(1-\beta)(z_2 - \phi_2)}}{\theta_2} \\
\quad & > \mu(q_1) \frac{1 - e^{-\theta_1(w_1 - \phi_2)}}{\theta_1} + (1 - \mu(q_1)) \frac{1 - e^{-\theta_1(1-\beta)(z_2 - \phi_2)}}{\theta_1} \\
\quad & \geq \mu(q_1) \frac{1 - e^{-\theta_1(w_1 - \phi_1)}}{\theta_1} + (1 - \mu(q_1)) \frac{1 - e^{-\theta_1(1-\beta)(z_1 - \phi_2)}}{\theta_1} \equiv h_1(\phi_1)
\end{align*}
\]

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The first inequality is once more revealed preference. The second inequality follows because \( \frac{1-e^{-\theta c}}{\theta} \) is decreasing in \( \theta \), and strictly so for \( c \neq 0 \); this is easily confirmed by differentiation. The third inequality follows if \( \phi_1 \geq \phi_2 \). But this string of inequalities again implies \( 0 > 0 \).

We establish the remaining claims in the standard fashion, following closely the proof of Proposition 2. Revealed preference implies:

\[
\mu(q_1) \left( -e^{-\theta_1 w_1} + e^{-\theta_1 ((1-\beta)z + \beta \phi_1)} \right) \geq \mu(q_2) \left( -e^{-\theta_1 w_2} + e^{-\theta_1 ((1-\beta)z + \beta \phi_2)} \right)
\]

\[
\mu(q_2) \left( -e^{-\theta_2 w_2} + e^{-\theta_2 ((1-\beta)z + \beta \phi_2)} \right) \geq \mu(q_1) \left( -e^{-\theta_2 w_1} + e^{-\theta_2 ((1-\beta)z + \beta \phi_2)} \right)
\]

Multiplying and simplifying:

\[
\begin{align*}
\left( -e^{-\theta_1 w_1} + e^{-\theta_1 ((1-\beta)z + \beta \phi_1)} \right) \left( -e^{-\theta_2 w_2} + e^{-\theta_2 ((1-\beta)z + \beta \phi_2)} \right) \\
\geq \left( -e^{-\theta_1 w_2} + e^{-\theta_1 ((1-\beta)z + \beta \phi_2)} \right) \left( -e^{-\theta_2 w_1} + e^{-\theta_2 ((1-\beta)z + \beta \phi_2)} \right)
\end{align*}
\]

(33)

Assume in order to find a contradiction that \( w_1 > w_2 \). Since \( w_2 > \phi_2 > z \), we obtain that \( w_1 > w_2 \geq (1 - \beta)z + \beta \phi_2 \). Then by the intermediate value theorem, there exists a \( \lambda \in (0, 1) \) such that

\[-e^{-\theta_2 w_2} \equiv \lambda \left( -e^{-\theta_2 w_1} \right) + (1 - \lambda) \left( -e^{-\theta_2 ((1-\beta)z + \beta \phi_2)} \right)\]

Also, since \( \theta_1 > \theta_2 \), \( -e^{-\theta_1 c} \) is a concave transformation of \( -e^{-\theta_2 c} \). Thus

\[-e^{-\theta_1 w_2} > \lambda \left( -e^{-\theta_1 w_1} \right) + (1 - \lambda) \left( -e^{-\theta_1 ((1-\beta)z + \beta \phi_2)} \right)\]

Eliminating \( e^{-\theta_2 w_2} \) and \( e^{-\theta_1 w_2} \) from inequality (33) yields

\[
\lambda \left( -e^{-\theta_1 w_1} + e^{-\theta_1 ((1-\beta)z + \beta \phi_1)} \right) \left( -e^{-\theta_2 w_1} + e^{-\theta_2 ((1-\beta)z + \beta \phi_2)} \right) \\
\geq \left( -\lambda e^{-\theta_1 w_1} - (1 - \lambda) e^{-\theta_1 ((1-\beta)z + \beta \phi_2)} + e^{-\theta_1 ((1-\beta)z + \beta \phi_1)} \right) \times \\
\left( -e^{-\theta_2 w_1} + e^{-\theta_2 ((1-\beta)z + \beta \phi_2)} \right)
\]

Because \( -e^{-\theta_2 w_1} > -e^{-\theta_2 ((1-\beta)z + \beta \phi_2)} \), simplifying gives:

\[-e^{-\theta_1 ((1-\beta)z + \beta \phi_1)} \geq -e^{-\theta_1 ((1-\beta)z + \beta \phi_2)}\]

or \( \phi_1 > \phi_2 \), which is a contradiction. This proves \( w_1 \leq w_2 \). The remaining steps are standard. ■

Proof of Proposition 10: Take the limit of the constrained maximization problem described in Lemma 4 as \( \theta \) converges to 0. Manipulating equation (13) yields

\[
\phi \frac{1}{1 - \beta} = \max_{w, q, k} \frac{\mu(q)}{1 - \beta (1 - \mu(q))} \left( \mu(q) \frac{w}{1 - \beta} + (1 - \mu(q))z \right)
\]

\[
= \max_{w, q, k} \sum_{t=0}^{\infty} (\beta (1 - \mu(q)))^t \left( \mu(q) \frac{w}{1 - \beta} + (1 - \mu(q))z \right)
\]

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This is easily shown to equal the expected present discounted value of labor and unemployment income accruing to a worker who earns unemployment benefit \( z \), but is hired with probability \( \mu(q) \), at which point she earns wage \( w \). Thus risk-neutral workers maximize their expected labor income if and only if \( z = 0 \). Since other sources of income, including profits and unemployment benefits net of tax contributions, always sum to zero, we have shown that workers maximize net output in the economy if and only if \( z = 0 \). □

**Proof of Proposition 11:** Define \( \phi^e \equiv \phi^e_{z^e} \) for convenience. Recall that \( \phi^e \) is defined by (16) and

\[
0 = \mu(q^e) \frac{1 - e^{-\theta(w^e - \phi^e)}}{\theta} + (1 - \mu(q^e)) \frac{1 - e^{-\theta(1-\beta)(z^e - \phi^e)}}{\theta} \tag{34}
\]

Given \( w^e \) and \( q^e \), one can eliminate \( z^e \) from (16) and (34), and solve the remaining equation first for \( \phi^e \) and then for \( z^e \). This proves uniqueness of the solution, although the algebra is not illuminating.

The bulk of the proof involves showing that if \( z^e \) satisfies these equations, then the unique equilibrium is \( \{w^e, q^e, z^e\} \). Observe that

\[
-(1 - \beta)e^{-\theta(w - \phi)} + e^{-\theta(1-\beta)(z - \phi)} - \beta
\]

is a strictly quasiconcave function of \( w \), maximized at the unique \( \hat{w} \) satisfying

\[
\hat{w} \theta(1 - \beta)e^{-\theta(\hat{w} - \phi)} \equiv -(1 - \beta)e^{-\theta(\hat{w} - \phi)} + e^{-\theta(1-\beta)(z - \phi)} - \beta \tag{36}
\]

On the other hand, (34) implies

\[
\mu(q^e) = \frac{e^{-\theta(1-\beta)(z^e - \phi^e)} - 1}{e^{-\theta(1-\beta)(z^e - \phi^e)} - e^{-\theta(w^e - \phi^e)}} \tag{37}
\]

Use this to eliminate \( \mu(q^e) \) from (16):

\[
w^e = \frac{-e^{-\theta(w^e - \phi^e)} + e^{-\theta(1-\beta)(z^e - \phi^e)} - \beta(1 - e^{-\theta(w^e - \phi^e)})}{(1 - \beta)\theta e^{-\theta(w^e - \phi^e)}} \tag{38}
\]

Comparing (36) and (38), clearly \( \hat{w} \equiv w^e \). Thus by our choice of \( z^e \), \( w^e \) maximizes (35).

Since a risk-neutral worker who receives no UI will choose \( \{w^e, q^e\} \) in preference to any other feasible pair \( \{w', q'\} \),

\[
\frac{\mu(q')w'}{1 - \beta(1 - \mu(q'))} < \frac{\mu(q^e)w^e}{1 - \beta(1 - \mu(q^e))} \tag{39}
\]

Also, according to the previous paragraph,

\[
\frac{-(1 - \beta)e^{-\theta(w' - \phi^e)} + e^{-\theta(1-\beta)(z^e - \phi^e)} - \beta}{w' (e^{-\theta(1-\beta)(z^e - \phi^e)} - 1)} < \frac{-(1 - \beta)e^{-\theta(w^e - \phi^e)} + e^{-\theta(1-\beta)(z^e - \phi^e)} - \beta}{w^e (e^{-\theta(1-\beta)(z^e - \phi^e)} - 1)}
\]

\[
\text{36}
\]
where we have multiplied both denominators by $e^{-\theta(1-\beta)(z^e - \phi^e)} - 1 > 0$. Alternatively, rewrite this as

$$\frac{-(1 - \beta)e^{-\theta(w' - \phi^e)} + e^{-\theta(1-\beta)(z^e - \phi^e)} - \beta}{w'(e^{-\theta(1-\beta)(z^e - \phi^e)} - 1)} < \frac{1 - \beta - \frac{1 - e^{-\theta(w^e - \phi^e)}}{e^{-\theta(1-\beta)(z^e - \phi^e)} - e^{-\theta(w^e - \phi^e)}}}{w^e \frac{e^{-\theta(w^e - \phi^e)} - 1}{e^{-\theta(1-\beta)(z^e - \phi^e)} - e^{-\theta(w^e - \phi^e)}}}$$

Now eliminate the fractions from the right hand side of this inequality using (37) which gives:

$$\frac{-(1 - \beta)e^{-\theta(w' - \phi^e)} + e^{-\theta(1-\beta)(z^e - \phi^e)} - \beta}{w'(e^{-\theta(1-\beta)(z^e - \phi^e)} - 1)} < \frac{1 - \beta(1 - \mu(q^e))}{w^e \mu(q^e)} \leq \frac{1 - \beta(1 - \mu(q'))}{w' \mu(q')}$$

where we have applied (39). Finally, simplifying these inequalities yields

$$\mu(q'') \left(1 - e^{-\theta(w' - \phi^e)}\right) + (1 - \mu(q')) \left(1 - e^{-\theta(1-\beta)(z^e - \phi^e)}\right) < 0$$

Combining this with (34), we have proved that the risk-averse worker strictly prefers $\{w^e, q^e\}$ to $\{w', q'\}$. ■