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Estimating the Long-Run User Cost Elasticity

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Abstract

The user cost elasticity is a parameter of central importance in economics, with implications for monetary policy, macroeconomic models, tax policy, growth, and many other areas. If the supply curve for capital is upward sloping and shocks to demand are important (as they are likely to be over the business cycle), estimates of the user cost elasticity that rely on high-frequency movements in the variables will tend to be biased. This paper applies cointegration techniques to a small, open economy. The combination of exogeneity of user cost implied by the flat supply of capital curve for a small, open economy and appropriate correction for small sample bias yields an estimate of the long-run user cost elasticity which is about 75% larger (in absolute value) than the best existing estimate. In addition, the paper makes three further contributions: accounting for increases in depreciation (due to dramatic increases in computer use), estimating the long-run user cost elasticity for structures and the total capital stock, and disentangling the effects of capital goods prices, the real interest rate, and taxes.

Keywords: User cost elasticity, capital stock, investment
JEL codes: E22, E44, H25

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1. Introduction

Along the demand curve for capital, there is a negative relationship between user cost and the capital stock. This is the relationship that empirical researchers try to capture when estimating the user cost elasticity. But in the short run (i.e., at business cycle frequencies) there is some evidence that the demand curve for capital may shift in response to cyclical upswings and downturns. For example, it is well known that investment moves with output at business cycle frequencies and this may be part of the reason for the success of the accelerator model in traditional aggregate time series studies. If the supply curve for capital is upward-sloping in the short run, (as suggested, e.g., by Goolsbee (1998)), there will be a positive relationship between user cost and the capital stock along the supply curve in the short run. If, at business cycle frequencies, shifts in demand are relatively important, it is easy to understand why empirical studies have frequently failed to find a very strong negative relationship between user cost and the capital stock.

Shifts in the supply curve are probably due primarily to technological change and productivity shocks (which affect the price of investment goods and the real interest rate) and tax reforms. All of these tend to be relativity persistent, at least compared to the ups and downs in demand at business cycle frequencies. The relatively persistent nature of

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1 For a recent example of the success of the accelerator in aggregate data, see Figure 4 in Hassett and Hubbard (2002).
2 For empirical evidence on the persistence of the real interest rate, see Garcia and Perron (1996). A wide variety of evidence suggests that productivity shocks are highly persistent, perhaps best modeled as unit root processes.
supply shocks suggests that we should be able to estimate the user cost elasticity better by focusing on the long-run relationship between the capital stock and user cost.³

If shifts in the demand curve for capital are relatively common in the short run and the short-run supply curve is upward-sloping but the long-run supply curve is flat, we would expect much of the variation in user cost to be transitory. This is precisely what Kiyotaki and West (1996) find, and they argue that this is an important reason why estimates of the user cost elasticity tend to be low. In the presence of adjustment frictions, it doesn’t make sense for a firm to respond fully to transitory variation in user cost. Estimates based on short-run movements therefore tend to give a downward biased user cost elasticity. Kiyotaki and West (1996) provide evidence for Japan that shocks to user cost are less persistent than shocks to output. They estimate the long-run response to a 1% shock to the cost of capital is a 0.1% change in the cost of capital, while the long-run response to a 1% shock to output is a 1.1% change. This provides additional evidence – consistent with Goolsbee’s work – that much of the short-run variation in user cost is associated with transitory shifts in demand.⁴

There is a further problem with trying to estimate the user cost elasticity from short-run movements in investment. Existing economic theories make substantially different predictions about investment dynamics. For example, a pure neoclassical model without adjustment costs predicts that the capital stock will respond immediately to shocks to user cost. A q model with convex adjustment costs predicts that current

³ This is not to suggest that aggregate demand shocks may not have fairly persistent effects (e.g., in the presence of some combination of nominal and real rigidities), merely that persistent shifts in the supply curve for capital will be relatively more important in the long run than in the short run.

⁴ It may also help to explain why major tax reforms seem to be such good instruments for user cost, as shown by Cummins, Hassett, and Hubbard (1994, 1996), if major tax reforms involve changes in user cost that are much more persistent than the short-run variations induced by changes in demand at business cycle frequencies.
marginal $q$ is a sufficient statistic for investment. A model of irreversibility at the micro level suggests that the short-run aggregate user cost elasticity will not be constant; instead, it will vary over time depending on the sequence of previous shocks to user cost and the cross-sectional distribution of the gap between desired and actual capital stock. Models with some combination of fixed costs and irreversibility can have even more complicated dynamics. In contrast, a wide variety of theories -- Jorgensonian neoclassical, $q$, irreversibility, $(s,S)$ -- are based on the same steady-state relationship between user cost and the capital stock. Agreement on the steady-state capital stock and divergence on dynamics provide a strong argument for estimating the long-run user cost elasticity.

In view of the disadvantages of trying to estimate the user cost elasticity from short-run variation, this paper departs from the approach of the vast majority of previous empirical studies of user cost elasticity, which implicitly estimate the elasticity from the high frequency relationship between investment and fundamentals. Instead, this paper expands on the idea of Caballero (1994) to estimate the long-run user cost elasticity. Caballero’s insight -- widely employed in other areas of economics but largely neglected in estimating the user cost elasticity -- is to use a cointegrating relationship implied by economic theory.$^5$

The more exogenous the shocks to user cost, the better we should be able to do in estimating user cost elasticity. Many previous estimates of user cost elasticity have been based on U.S. data. Because of the size of its economy, the U.S. is a country in which changes in investment demand are more likely to affect user cost. In contrast, shocks to

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$^5$ For a very different approach to estimating the long-run user cost elasticity -- one that does not use cointegration -- see Chirinko, Fazzari, and Meyer (2001).
important components of user cost, such as the price of investment goods and the real interest rate, are more likely to be exogenous in a small, open economy, because they will be largely determined by world prices and the foreign real interest rate. One of the ways in which we further develop Caballero’s insight is by using data from a small, open economy to estimate the user cost elasticity. We choose Canada because even changes in tax parameters are more likely to be exogenous. Historically, Canadian corporate tax changes have tended to be driven by changes in U.S. tax policy rather than by domestic business cycles.6

Studies based on disaggregated data have many advantages, but they also have some disadvantages. As Hassett and Hubbard (2002, p. 1319) point out, even when micro studies find evidence that user cost affects the capital stock, this could reflect asset substitution, with little effect on the aggregate capital stock. As Mulkay, Hall, and Mauresse (1999, p. 4) note, each micro study covers only a fraction of the universe of firms or plants, leaving open the possibility of inadvertent sample selection bias.7 The potential importance of sample selection bias is highlighted by the work of Caballero, Engel, and Haltiwanger (1995), who find that the estimated user cost elasticity varies from -0.01 to -2.0 across U.S. SIC 2-digit industries.8 This paper uses aggregate data, specifically quarterly Canadian data over the period 1962:1 – 1999:4.

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6 For example, the last major reorganization of corporate tax rates and the ITC in Canada closely followed the 1986 U.S. tax reform.

7 For example, larger firms might react more quickly to changes in user cost, while smaller firms might more frequently be financed constrained and therefore respond more slowly to movements in fundamentals. Since most panels of firms are restricted to relatively large, publicly traded firms this could, in principle, bias the elasticity estimate.

8 One important motivation for the use of disaggregated data has been the hope that user cost would be exogenous. This hope has been called into question by the work of Goolsbee (1998), which provides evidence that the supply curve is upward sloping (i.e., not exogenous) in the short run, even at the micro level.
Most previous studies of user cost elasticity have assumed a constant depreciation rate. The growing importance of rapidly depreciating computer equipment calls this assumption into question. A further contribution of this paper is to examine the importance of changes in depreciation rates. Analysis of the Canadian data suggests substantial changes in the aggregate depreciation rate, especially for equipment, over the time period 1962-1999. Allowing for variation over time in the depreciation rate changes the user cost elasticity for equipment by about 15%.

For many public policy purposes, it would be useful to know the effects of individual components of user cost. For example, tighter monetary policy may push real interest rates up for an extended period (as in early 1980's). How much of an effect does this have on the capital stock? Is it the same as a corresponding proportional change in user cost due to changes in tax parameters or the real price of capital goods? We know very little about the relative importance of taxes, capital goods prices, and the real interest rate for long-run changes in the capital stock. This is the first paper to estimate the long-run elasticities of the capital stock with respect to taxes, capital goods prices, and the real interest rate.

This paper is organized as follows. Section II briefly summarizes previous estimates of user cost elasticity. Section III describes the data. Section IV discusses cointegration and presents simple OLS estimates of the long-run user cost elasticity based on the cointegrating relationship. Section V explains the potential bias in OLS estimates when adjustment frictions are important, illustrates how the DOLS estimator can overcome this potential bias, and presents DOLS elasticity estimates (which appear to be the first DOLS estimates of user cost elasticity, in the strict sense of implementing the
DOLS estimator proposed by Stock and Watson (1993), a point that turns out to make a difference). Section VI compares DOLS elasticity estimates with estimates based on the one-sided summation procedures used by Caballero (1994). Section VII expands on the existing literature by providing cointegration-based estimates of user cost elasticity for structures and the total capital stock. Section VIII examines the effect of changes in the depreciation rate on the estimated user cost elasticity. Section IX estimates the separate effects of taxes, the real interest rate, and capital goods prices on the capital stock. Section X concludes.

II. Previous Estimates of the User Cost Elasticity

A. Short-Run⁹:

There is considerable variation in estimates of the short-run user cost elasticity. In his survey of the literature, Chirinko (1993, p. 1906) concludes that “the response of investment to price variables tends to be small and unimportant relative to quantity variables.” In a more recent survey, Hassett and Hubbard (2002) suggest that the user cost elasticity is probably between -0.5 and -1. Turning to individual studies, Cummins and Hassett (1992) estimate an elasticity of slightly more than -1, using firm-level U.S. data. (The convention in this paper will be to use “more” and “larger” to refer to user cost elasticities that are greater in absolute value.) Clark (1993) finds an estimated elasticity of -0.01 using aggregate U.S. data; the elasticity estimate is higher (about -0.2) when Clark uses only tax changes (rather than full user cost) to estimate the elasticity. Using aggregate data for Japan, Kiyotaki and West (1996) estimate a user cost elasticity
of -0.05 to -0.07. Chirinko, Fazzari, and Meyer (1999) obtain a preferred elasticity of about -0.25 using firm-level U.S. investment data. Using U.S. data disaggregated by type of asset, Goolsbee (2000) finds that a 10% investment tax credit raises investment by about 4 to 5%.

There are several possible explanations for the variation in estimates of the short-run user cost elasticity. First, in models with non-convex adjustment costs, the short-run user cost elasticity is time-varying and history dependent, as discussed by Caballero (1999). Caballero, Engel, and Haltiwanger (1995) and Caballero and Engel (1999), among others, provide evidence of the importance of time variation in the response of the capital stocks to fundamentals. Estimates of the short-run elasticity can vary depending on the time period over which they are estimated. This applies to estimates based on both aggregate and disaggregate data.

Second, a number of the more recent studies use disaggregate data. If the short-run elasticity varies depending on firm size, across industries, or based on the degree to which firms face finance constraints, elasticity estimates can vary depending on the particular sample of micro units for which the researcher has data.¹⁰

B. Long-Run

Caballero (1994) is the pioneering study that uses a cointegrating relationship between user cost and the capital stock to estimate the long-run elasticity. Caballero (1994) uses quarterly aggregate U.S. data for equipment over the period 1957:1 – 1987:4

¹⁰ The term “long-run” in reference to user cost elasticity estimates is used here to refer to cointegration-based estimates; all other estimates are referred to as “short-run”. This usage follows Caballero (1999, Sections 2.2.1 and 2.2.2.)

¹⁰ As discussed elsewhere in this paper, Caballero, Engel, and Haltiwanger (1995) find considerable variation across industries, even for the long-run elasticity.
and an econometric procedure motivated by the work of Stock and Watson (1993). He obtains an estimated elasticity of about -0.9.

Caballero, Engel, and Haltiwanger (1995) use plant-level, U.S. data for equipment over the period 1972-88 to estimate user cost elasticities for 20 two-digit SIC industries. Their estimates range from -0.01 for transportation to -2.0 for textiles. Consistent with the issues raised in the introduction to this paper, they find short-run elasticities are much smaller (around 10% of the corresponding long-run elasticity).11

III. Data

This paper uses Canadian aggregate data for the period 1962:1 to 1999:4. In particular, the investment data is non-residential, gross, real, fixed capital formation (seasonally adjusted) for the business sector from the National Income and Expenditure Accounts. In the Canadian data, investment is divided into equipment and non-residential structures. To form a series for total investment, we sum equipment and structures.

The capital stock is calculated by the perpetual inventory method using a depreciation rate of .13 for equipment and .06 for structures. The total capital stock is the sum of the capital stocks for equipment and structures. The robustness of the results to depreciation rates is explored in Section VIII below.

The cost of capital is calculated as follows

\[ \bar{R}_t = (i_t + \delta + \gamma - \pi_t^k) \left( \frac{1 - \frac{\xi_t - \eta_t}{1 - \tau_t}}{1 - \tau_t} \right) \frac{p_t^k}{p_t^r} \]
where \( i \) is the nominal interest rate, \( \delta \) is set at .13 for equipment, .06 for structures, and .08 for total, \( \gamma \) is a fixed risk premium (set at 6%), \( \eta^{K} \) is the rate of inflation for capital goods, \( \zeta \) is the present value of depreciation allowances, \( u \) is the investment tax credit rate, \( \tau \) is the corporate tax rate, \( p^{K} \) is the price of capital goods, and \( p^{Y} \) is the price of output.\(^{12}\) For discussions and derivations of the user cost of capital, see, e.g., Hall and Jorgenson (1967), Auerbach (1983) or Hassett and Hubbard (2002). Although there is no time subscript on \( \delta \), we explore the issue of variable depreciation rates in Section VIII.

The cost of capital is calculated for each type of capital -- equipment, structures, and the total capital stock.

Output is matched to the investment data, which are for the business sector, by subtracting government expenditures from GDP.

A more detailed description of the data is contained in the Data Appendix, which provides Statistics Canada series numbers and discusses details such as the calculation of the present value of depreciation allowances.

A brief overview of the investment data may be of interest. Figure 1 presents the investment/capital ratio for equipment for the 1961-1999 sample period. Recessions (as defined by Statistics Canada) are shaded. Investment booms are associated with business cycle expansions in the 1960s, late 1970s, mid to late 1980s, and late 1990s. Large drops in investment are associated with the recessions of the early 1980s and early 1990s, both of which were relatively long and severe recessions in Canada. Figure 2 shows the investment/capital ratio for structures. Here again, there are upswings in investment

\(^{11}\) One paper is difficult to categorize. Chirinko, Fazzari, and Meyer (2001) are sensitive to the problems with estimating the short-run elasticity but do not use a cointegrating relationship. Instead, using firm-level U.S. data, they average the key variables over several years and obtain an elasticity estimate of -0.4.
during the 1960s, the late 1970s, and, to a lesser extent, the expansion of the 1990s, and large declines in investment coincide with the recessions of the early 1980s and early 1990s.

IV. Cointegration and SOLS

Suppose that

\[ k_i = \alpha_0 + \alpha_k R_i + z_i \]
\[ \Delta R_i = u_{2i} \]

where \( k \) is the log capital/output ratio, \( R = \ln \bar{R} \) is the log of user cost, and \( z \) and \( u_2 \) are stationary.\(^{13}\) The variables \( k \) and \( R \) will then be cointegrated.

Cointegration between \( k \) and \( R \) is a good description of the data. First, ADF tests suggest a unit root in \( k \), separate tests showing a unit root in \( k \) for equipment, structures, and the total capital stock. Second, ADF tests show a unit root in \( R \). Because prices and taxes are different for equipment and structures, there are three user cost measures – for equipment, structures, and the total capital stock. ADF tests show a unit root in each case. Third, standard cointegration tests reject the null hypothesis of no cointegration between \( k \) and \( R \) - once again for equipment, structures, and the total capital stock.\(^{14}\)

Cointegrating regressions are often estimated by Static (SOLS) OLS; i.e., the estimation of an equation like (2) by OLS. Under the strong assumption of no serial correlation in either \( z_i \) or \( u_{2i} \), the usual \( t \) and \( F \) statistics can be used to test hypotheses.

\(^{12}\) Since \( \delta \) and \( \gamma \) enter additively, the variation in \( \delta \) considered in Section VIII can also be interpreted as variation in \( \gamma \).

\(^{13}\) This relationship can be obtained by solving the firm’s problem (under the consumption of Cobb-Douglas technology) for the frictionless capital stock and relaxing the unit user cost elasticity constraint. See, e.g., Caballero (1999, p. 816-821).
Once the assumption of no serial correlation is relaxed, the SOLS t statistics must be adjusted, specifically by multiplying by \( \frac{s_t}{\lambda^*_t} \), where

\[
(3) \quad s_t^2 = (T - n)^{-1} \sum_{t=1}^{T} (k_t - \hat{\alpha}_0 - \hat{\alpha}_0 R_t)^2
\]

is the variance of the residuals from OLS estimation (T being the sample size and n=2 being the number of estimated coefficients) and \( \lambda^*_t \) is constructed from a regression of the residuals \( \hat{z}_t \) on q lags of the residuals:

\[
(4) \quad \hat{z}_t = \phi_1 \hat{z}_{t-1} + \phi_2 \hat{z}_{t-2} + \ldots + \phi_q \hat{z}_{t-q} + e_t
\]

where the order of this autoregression is chosen by selecting the value of q from the set \( \{1, 2, \ldots, T^{1/3}\} \) so as to minimize the Bayesian information criterion (BIC). Specifically:

\[
(5) \quad \lambda_t^* = \frac{\hat{\sigma}_1}{1 - \hat{\phi}_1 - \hat{\phi}_2 - \ldots - \hat{\phi}_q}
\]

where \( \hat{\sigma}_1 \) is the standard deviation of \( \hat{e}_t \):

\[
(6) \quad \hat{\sigma}_1^2 = (T - q)^{-1} \sum_{t=q+1}^{T} \hat{e}_t^2
\]

This is the procedure used to calculate all SOLS t- statistics reported in subsequent sections.

The SOLS estimate of user cost elasticity is -0.82 (with a standard error of 0.20).

This is considerably larger -- about twice as big -- as the only other comparable aggregate estimate of the long-run user cost elasticity, the Caballero (1994) SOLS estimate, which is for a large economy; i.e., one which is big enough to have an effect on world prices.

\[\text{Using aggregate U.S. data for equipment, Clark (1993) tests whether k and R are cointegrated (imposing the assumption that } \alpha_R = 1 \text{) and rejects the null hypothesis of no cointegration.}\]
The larger estimate is consistent with the idea that user cost should be more exogenous in a small, open economy.

V. Small Sample Bias and DOLS

A. An Intuitive Explanation of Small Sample Bias

Asymptotically, SOLS yields consistent estimates of the coefficients in the cointegrating regression. In the presence of adjustment frictions, though, SOLS will tend to produce biased estimates in samples of the size normally available for aggregate time series estimation. Analytical results in Caballero (1994) show that SOLS could be downward biased (i.e., biased towards 0) by 50 to 60% for a sample of 120 observations and 70 to 80% for a sample of 50 observations, if adjustment frictions are large.

To explain the intuition for the SOLS bias, it will be helpful to ignore the constant term. Let $k^*$ be the frictionless capital stock (measured in logs and normalized by the log of output) and let it be a linear function of user cost:

$$k^*_t = \alpha R_t \quad (7)$$

Adjustment frictions (broadly defined) will cause a gap $z_t$ between the actual capital stock $k_t$ and the frictionless capital stock. Thus the actual capital stock will be equal to the frictionless capital stock plus $z_t$:

$$k_t = \alpha R_t + z_t \quad (8)$$

In the presence of adjustment frictions, $k^*$ will typically fluctuate more than $k$, since $k$ will respond only slowly and partially to shocks. Since $k$ is a sum of the random variables $k^*$ and $z$.

$$\text{var}(k) = \text{var}(k^*) + \text{var}(z) + 2\text{cov}(k^*, z) \quad (9)$$
so the variance of k can be smaller than the variance of k* only if \( \text{cov}(k^*, z) \) is negative.

However, the OLS estimates of k* and z (i.e., \( \hat{k}^* = \hat{\alpha}_R R \) and \( \hat{z} = k - \hat{\alpha}_R R \)) are orthogonal by construction, which implies \( \text{var}(\hat{k}^*) \) is less than \( \text{var}(k) \). In order to achieve this, OLS will tend to bias the estimate of \( \alpha \) toward 0.\(^{15}\) Monte Carlo simulations by Caballero (1994) show that if the actual capital stock responds sluggishly to shocks, the OLS estimate can be biased downward by a factor of two or more.

**B. Dynamic OLS**

The necessary condition for unbiased SOLS estimation of \( \alpha_0 \) and \( \alpha_R \) is that \( z_t \) be uncorrelated with \( u_{2s} \) for all \( s \) and \( t \).\(^{16}\) This strong condition arises because it is only under this condition that \( R \) will be uncorrelated with the error term \( z \) since:

\[
\text{cov}(R_t, z_t) = \text{cov}(R_0 + \Delta R_1 + \Delta R_2 + \ldots + \Delta R_t, z_t) \\
= \text{cov}(u_{21} + u_{22} + \ldots + u_{2t}, z_t)
\]

One solution to the problem of small sample bias in SOLS is the DOLS estimator proposed by Stock and Watson (1993). Dynamic OLS (DOLS) addresses the problem of finite sample bias by replacing the original error term \( z \) by a new error term \( v \), which is constructed to be orthogonal to \( R \).\(^{17}\) The intuition is straightforward. OLS projects the dependent variable onto the space spanned by the right hand side variables. The remaining variation in the dependent variable is orthogonal to the right hand side variables. Suppose \( z \) were projected onto the space spanned by all leads and lags of \( \Delta R \)

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\(^{15}\) This argument follows Caballero (1994, 1999).

\(^{16}\) Early statements of SOLS bias, in a more general context, can be found in Bannerjee et al (1986) and Stock (1987).

\(^{17}\) In addition to addressing the SOLS bias, DOLS has other attractive properties. In the most general case considered by Stock and Watson (1993), DOLS is asymptotically efficient (when interpreted semiparametrically). Perhaps more important, in Monte Carlo simulations of the most general case (Case C) considered by Stock and Watson (1993), DOLS has the lowest RMSE among a set of estimators of cointegrating regressions.
(which is equivalent to the space spanned by \( u_2 \)). The error term \( v_t \) from this regression will be orthogonal to \( R_s \) since:

\[
\text{(11)} \quad \text{cov}(R_s, v_t) = \text{cov}(R_0 + \Delta R_1 + \ldots + \Delta R_s, v_t) = 0
\]

The last equality follows from the fact that \( v_t \) is orthogonal to all leads and lags of \( \Delta R_t \) by construction.

As noted above, the assumptions required for SOLS estimation are unlikely to be satisfied in estimating the user cost elasticity, primarily because of the slow adjustment of the capital stock to shocks and the resulting correlation between shocks to user cost \( (u_2) \) and the gap \( (z) \) between the frictionless capital stock and the actual capital stock. The preferred estimation procedure is therefore DOLS (rather than SOLS). Specifically, the empirical specification used below is:

\[
\text{(12)} \quad k_t = \alpha_0 + \alpha_R R_t + \sum_{s=-p}^{p} \beta_s \Delta R_{t-s} + \epsilon_t
\]

Results are presented below for many possible choices of \( p \).\(^{18}\) A common way of choosing \( p \) is by minimizing the BIC, and this is the option emphasized below when we focus on a single \( p \) to highlight the key results.

Adjusted DOLS t statistics are calculated as follows:

\[
\text{(13)} \quad t = \frac{\hat{\alpha}_t}{SE(\hat{\alpha}_t)} \frac{s}{\hat{\lambda}^*}
\]

\(^{18}\) There are both analytical results and Monte Carlo evidence that relatively high values of \( p \) are the most effective in reducing bias in aggregate data. Among the alternative values of \( p \) he considers, the Monte Carlo evidence in Caballero (1994, p.56) suggests that bias is smallest for \( p=25 \) when \( T=120 \), where \( T \) is the sample size. In contrast, Caballero, Engel and Haltiwanger (1995) set \( p=5 \) (like Caballero using only lags, rather than both leads and lags as in DOLS estimation) and report that their results are fairly robust to variation in \( p \) (footnote 19, p.15), perhaps because the cross-sectional dimension in their panel data makes the effective sample size large.
where $\hat{\alpha}_i$ is the estimate of $\alpha_i$, $i=0$ or R, $SE(\hat{\alpha}_i)$ is the standard error for $\hat{\alpha}_i$, and $s$ is the standard deviation of the regression residual:

$$s^2 = (T-n)^{-1} \sum_{t=1}^{T} \hat{\epsilon}_t^2$$

where $T$ is the sample size and $n$ is the number of parameters estimated in the original equation with leads and lags of $\Delta R_t$. Note that this formula is essentially the same as the formula for $s_T$ above, except that, in calculating the degrees of freedom, account must be taken here of the coefficients on the lagged differences. For the DOLS estimation, $n=[(2p+1)+g+1]$, where $g$ is the number of estimated regression coefficients (apart from the coefficients on the lagged differences) excluding the constant (so in (11), $g=1$) and there is a constant.

C. DOLS Empirical Results

Table 1 presents estimates of the long-run user cost elasticity for many different values of $p$. At the value of $p$ where the BIC is minimized, the DOLS estimate of the user cost elasticity is -1.64. This is about 75% larger than the only previous comparable estimate of aggregate user cost elasticity, the Caballero (1994) estimate that accounts for small sample bias. Again, the larger estimate is consistent with the idea that user cost should be more exogenous in a small, open economy. In addition, this estimate is based on DOLS as proposed by Stock and Watson (1993) while the Caballero estimate uses a procedure which is inspired by DOLS but is not the same as the Stock and Watson (1993) DOLS estimator, a point to which we will return in the next section.

As noted above, there are analytical results and Monte Carlo evidence suggesting that large values of $p$ may be required to correct small sample bias if adjustment frictions are large. In Table 1, estimates of the elasticity increase as $p$ rises, reflecting the
reduction in small sample bias. By about $p=18$ or 20, the elasticity estimate stabilizes. Interestingly, this is also consistent with previous Monte Carlo results. Caballero (1994, Table 3) shows that after the elasticity estimate gets very close to the true elasticity, adding further lagged differences of the right-hand-side variables has little effect on the estimated elasticity. If we suppose that -1.64 is the true elasticity (as suggested both by the fact that this is the estimated elasticity at the $p$ that minimizes the BIC and that the elasticity estimate stabilizes at this value), then, for example, at $p=5$, the elasticity estimate is biased towards 0 by about 34%, roughly in the range obtained by Caballero (1994) in Monte Carlo simulations of the case where adjustment frictions are large.

VI. One-Sided Summation Procedures and DOLS

DOLS can quickly use up degrees of freedom, especially if $p$ is large. As noted above, both analytical results and Monte Carlo simulations have shown that large values of $p$ may be necessary to overcome the SOLS small sample bias. In an effort to mitigate this problem, Caballero (1994) does not actually use DOLS estimation. Instead, he uses either lags of the first difference of user cost or leads of the first difference of user cost (i.e., one-sided summations in place of the two-sided summation in equation (12)). He conjectures that whenever adjustment costs are large, leads would not have as big an effect as lags in reducing small sample bias. In U.S. data, he found that estimates of the user cost elasticity were smaller in absolute value using leads than using lags.

Table 2 presents a comparison of bias corrections for the Canadian data.\textsuperscript{19} The results are consistent with Caballero’s conjecture: the point estimate of the user cost

\textsuperscript{19}To conserve space, each entry in Table 2 reports the user cost elasticity for the value of $p$ that minimizes the BIC.
elasticity is larger in absolute value using lags than it is using leads. The magnitude of the correction for small sample bias that results from using the one-sided summation with leads only is comparable to that found in U.S. data by Caballero (1994). The point estimate of the user cost elasticity using a one-sided summation with leads only is about 22% higher in absolute value than the SOLS estimate in Canadian data and about 14% higher in U.S. data.

This paper presents the first aggregate estimates of the user cost elasticity using DOLS as proposed by Stock and Watson (1993), so it is useful to compare DOLS with the one-sided summation procedure using lags. In Monte Carlo simulations, Caballero (1994) finds that, if adjustment frictions are large, the one-sided procedure (using lags) can yield an economically significant bias even when the length of the one-sided summation is long. For example when adjustment frictions are large, the estimated elasticity is about 20% below the true elasticity even when 25 lags are used in the one-sided summation [Caballero (1994), Table 3]. The results are similar in the actual data for Canada. If we assume the true elasticity is -1.64, then the one-sided procedure (using lags only) is biased downward by about 27%. This suggests that, where feasible, the DOLS procedure proposed by Stock and Watson (1993) may be preferable to the one-sided summation procedure, especially if there is reason to believe that adjustment frictions may be large.
VII. Structures and the Total Capital Stock

A. Structures

The existing literature does not appear to contain any cointegration-based estimates of the user cost elasticity for structures. As noted in Section IIB, Caballero (1994) and Caballero, Engel, and Haltiwanger (1995) only examine equipment.

The empirical results for structures are markedly different than those for equipment. To begin with, the point estimate from SOLS is essentially 0 (0.02, with a standard error of 0.07). There is little evidence that this is the result of small sample bias.

Table 3 presents DOLS estimates for a variety of values of p. For the value of p which maximizes the BIC, the DOLS point estimate of the user cost elasticity is virtually identical to the SOLS estimate (.02, with a standard error of .04). Moreover, the value of p which minimizes the BIC is 1, apparently suggesting that little bias correction is required. As the first column of Table 4 illustrates, the results are essentially the same using the one-sided summation procedures.

The DOLS results for structures illustrate a point made by Caballero (1994). In his empirical work, there is a pattern of estimated user cost elasticities becoming more negative with larger values of p. Caballero pointed out that this pattern is not a mechanical artifact of the correction he uses for small sample bias nor, more generally, of the DOLS procedure proposed by Stock and Watson (1993). The results in Table 3 vividly illustrate this point: the point estimates of the user cost elasticity do not uniformly become more negative as p increases.

B. The Total Capital Stock

The elasticity estimates for the total capital stock are similar to those for structures. The SOLS estimate is -0.06 with a standard error of 0.12. As shown in Table
5, the DOLS estimate is 0.10 with a standard error of 0.28. The value of \( p \) which
minimizes the BIC is 1 and the elasticity estimate is stable around 0 for all values of \( p \).
The one-sided summation estimators yield similar results. Thus, in both an economic and
a statistical sense, the user cost elasticity for the total capital stock is 0.

One feature of the results for the total capital stock which distinguishes them from
the results for structures is that the DOLS standard errors are much larger. This may
reflect the fact that the total capital stock is aggregated over two types of capital –
equipment and structures – that have very different user cost elasticities. In fact, this
highlights one of the lessons of the results for equipment, structures, and the total capital
stock: aggregation over types of capital (specifically, equipment and structures) can
disguise the role of user cost.

VIII. Depreciation Rates and User Cost Elasticity

The past several decades have witnessed an accelerating use of computers and
related technology. Computers depreciate rapidly, much more rapidly than many other
types of capital. This could have important implications for measuring the quantity of the
capital stock. The standard approach is to use the perpetual inventory method to
construct the capital stock, assuming a constant depreciation rate.

Table 6 considers two types of modifications in the use of the depreciation rate to
construct the capital stock. The first modification maintains the assumption of a constant
depreciation rate but constructs the capital stock using higher and lower depreciation
rates than have been assumed in the base case. Specifically, the base case assumes a
depreciation rate of 13% for equipment. Table 6 presents estimates of user cost elasticity
for a low case depreciation rate of 10% and a high case depreciation rate of 16%. For structures, the low case depreciation rate is 3%, the base case 6%, and the high case 9%. For the total capital stock, the corresponding depreciation rates are used to construct the equipment and structures capital stock, and the two capital stocks are summed. The results of the first type of modification in the depreciation rate are useful to document but unexciting. These variations in the depreciation rate make virtually no difference in the estimated user cost elasticity.

The second type of modification is more radical: a variable depreciation rate is constructed based on National Income and Expenditure Account data for the capital consumption allowance. Essentially, the depreciation rate is calculated by dividing the capital cost allowance by the capital stock. This depreciation rate is then used in the perpetual inventory formula. Details are provided in the data appendix.

The use of a variable depreciation rate makes some difference in the estimated user cost elasticity. Specifically, for equipment, the absolute value of the point estimate of the user cost elasticity is reduced by about 15% -- from -1.64 to -1.42. For structures and the total capital stock, the use of a variable depreciation rate makes little difference in the estimated user cost elasticity. The fact that a variable depreciation rate makes more difference for equipment is not surprising; it is the equipment capital stock that has been primarily affected by the accelerating use of rapidly depreciating computers and related devices.

IX. The Separate Effects of Taxes, Interest Rates, and Capital Goods Prices

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20 Since the depreciation rate is a component of user cost, the corresponding change in δ is also made in equation (1), the formula for $R_c$. 

For many public policy purposes, it would be useful to know the effect of individual components of user cost. For example, it is frequently suggested that the corporate tax rate be lowered or that depreciation be accelerated in order to encourage capital formation. The usual procedure in the analysis of such a change is to estimate (or frequently simply assume) the user cost elasticity and then calculate the implied change in the capital stock from a given change in one of the components of user cost. But if the capital stock is not equally responsive to all components of user cost, the resulting estimate of the change in the capital stock could be misleading.

User cost can be decomposed into three components which capture the effects of interest rates, taxes, and the price of capital goods.

\[
\ln R_i = \ln(i, + \delta + \gamma - \pi'_i) + \ln \left( \frac{1 - z_i - u_i}{1 - \tau_i} \right) + \ln \left( \frac{p'_i}{p'} \right) \\
= R'_i + R^f_i + R^p_i
\]

where \( R'_i \) is the component of user cost associated with the real interest rate (including a risk premium and depreciation), \( R^f_i \) is the component incorporating all of the tax terms, and \( R^p_i \) is the component which incorporates the price of investment goods (normalized by the price of output). In the empirical work in this section, we relax the restriction that the elasticity of the capital stock with respect to each component of user cost is the same. Instead, we estimate several different specifications which are designed to let the data speak as flexibly as possible about the importance of taxes, the interest rate, and capital goods prices for long-run movements in the capital stock.

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21 See Clark (1993, p. 319) for a comparable decomposition of user cost, specifically into a relative price term (corresponding to the combined price and interest rate terms in this paper) and a tax term. He does not, however, include both terms in any of his regressions.
The first approach is the \textit{full decomposition}. This replaces $R_i$ on the right hand side of (12) with $R_i', R_i^r$, and $R_i^p$, allowing a separate coefficient for each.

\begin{equation}
    k_i = \alpha_o + \alpha_i R_i' + \alpha_i R_i^r + \alpha_i R_i^p + \sum_{s=-p}^{p} \beta_i^s \Delta R_{r,s-1} + \sum_{s=-p}^{p} \beta_i^s \Delta R_{r,s-1} + \sum_{s=-p}^{p} \beta_i^s \Delta R_{r,s-1} + \epsilon_i
\end{equation}

As in the previous estimations, DOLS is being used to avoid small sample bias.

Table 7 shows a clear and interesting pattern. There is no evidence that increases in the real interest rate discourage capital formation. Although this is surprising from the point of view of economic theory, it is consistent with much previous empirical work. For example, when Caballero, Engel, and Haltiwanger (1995, footnote 18, p. 14) and Cummins and Hassett (1992, p. 245) estimate user cost elasticity, they assume a constant real interest rate. They therefore rely on variation in the tax and price components to estimate user cost elasticity. The results in Table 7 suggest that this may not make much difference to the estimated elasticity. In the same vein, Cummins, Hassett, and Hubbard (1994, footnote 47, p. 25) report that they obtain virtually identical results if they assume a fixed interest rate. Clark (1993) also uses a constant interest rate and reports that using time-varying measures of the interest rate yield uniformly low elasticity estimates.

Nor is there any evidence that the tax parameters have a statistically significant effect on the capital stock. This result is the same regardless of whether we look at equipment, structures, or the total capital stock.

On the other hand, there is evidence that the price of capital goods has a statistically significant and economically important effect on equipment capital. The estimated elasticity is close to one (-.87, with a standard error of .03). There is weaker (but still statistically significant) evidence that capital goods prices affect the total capital
stock. For the total capital stock, the elasticity with respect to the price of capital goods is about one-third.

Previous simulation results suggest that large values of $p$ may be required to adequately deal with the problem of small sample bias if adjustment frictions are important. DOLS estimation uses many degrees of freedom. In the full decomposition, there are three right hand side variables, and the maximum value of $p$ is set at 15 in Table 7. It is possible that the elasticity estimates could be biased towards zero if too low a value of $p$ is used.\footnote{How important setting the maximum value of $p$ at 15 is in the Canadian data is debatable. As Table 1 shows, if the maximum value of $p$ were restricted to 15, the estimated user cost elasticity for equipment would decrease by about 10\%.}

A second approach is the \textit{single components} specification, in which only one component of user cost is included on the right hand side of the regression. Table 8 presents estimates of the single components specification. In a few cases, the standard errors are substantially larger, but the main results are similar to those for the full decomposition. In no case is there evidence that increases in the real interest rate significantly reduce the capital stock. There is also no evidence that the tax parameters have a statistically significant effect on the equipment or structures capital stock.\footnote{The point estimate of the elasticity of the total capital stock with respect to the tax component of user cost is greater than one in absolute value and significantly different from zero. Although it is tempting to treat this as evidence for the importance of taxes, it seems prudent to exercise some caution in view of the rest of the results.} As in the full decomposition, there is evidence that capital goods prices affect the equipment capital stock. Again, the elasticity is close to one (-.88, with a standard error of .04). There is also evidence that the price of capital goods significantly affects the total capital stock; the elasticity is about one-third (-.31, with a standard error of .02).
The full decomposition has the disadvantage that it requires setting the maximum value of $p$ at a level that might be too low. The single component specification also has a disadvantage: by omitting the other components of user cost, it may be subject to omitted variable bias. A third approach provides a compromise between the two previous approaches. The partial decomposition specification separates out one component of user cost at a time, combining the two remaining components into a single term. By successively changing the component which appears on its own, it is possible to estimate a separate elasticity for each component of user cost while including all the components and increasing the maximum value of $p$.

Results for the partial decomposition specification are presented in Table 9. The first specification separates out the interest component (combining the tax and capital goods price components). The estimated elasticities of all the capital stocks (equipment, structures, and total) with respect to the real interest rate component are essentially zero.

The second specification separates out the tax component. Here, the results are somewhat mixed. In a statistical sense, none of the estimated user cost elasticities are different from zero, but the point estimates for equipment and the total capital stock are close to one (-.85 for equipment and -.80 for total, respectively). Unfortunately, the standard errors on the tax component are large, so it is difficult to make strong inferences.

The third specification separates out the capital goods price component. The results are consistent with those for both the full decomposition and the single component specification. For equipment, the elasticity with respect to the price of capital goods is close to one; for the total capital stock, it is around one-third.
A further indication of the strong role of the price of capital goods, especially for the equipment capital stock, comes from a closer examination of the coefficients on the combined terms. For example, in the specification which separates out the interest component, the remaining component combines the price of capital goods and tax parameters. The estimated elasticity with respect to this combined term is close to one. A similar result holds for the combined term in the second specification (which separates out the tax component). Again, the combined term includes the price of capital goods and the estimated elasticity is close to one. By contrast, in the third specification, the combined component includes the real interest rate and the tax parameters – and the estimated elasticity with respect to the combined component is close to zero.

X. Conclusion

The quantity of capital is determined by supply and demand. If the supply curve is upward sloping and shocks to demand are important (as they are likely to be over the business cycle), estimates of the user cost elasticity (the demand elasticity) that rely on high-frequency movements in the variables will tend to be biased. This has long been viewed as a fundamental problem in estimating user cost elasticity and an important potential explanation for the low elasticities frequently found in empirical studies. Although he didn’t motivate it in this way, Caballero (1994) proposed an approach that could go a long way towards addressing this problem. Essentially, by using cointegration techniques, it should be possible to estimate the long-run user cost elasticity, thus reducing the bias.
This paper makes a logical extension. In a small, open economy, user cost will be largely exogenous. Equivalently, the supply curve will be flat. This extension makes a substantial difference, increasing the estimated user cost elasticity by about one-third. These results are consistent with other evidence that the supply curve for capital is upward sloping in the U.S..

The econometric procedure also makes a difference in the estimated user cost elasticity. Previous analytical results and Monte Carlo simulations show that ignoring small sample bias in simple OLS cointegration procedures can have a large effect on estimates when adjustment frictions are important. Work on actual data in this paper reinforces the importance of this point. The estimated user cost elasticity is about 50% higher when a correction is made for small sample bias (about -1.2 versus approximately -0.8 without any correction for small sample-bias). Moreover, there is evidence that the exact nature of the small sample bias correction is important. Previous Monte Carlo simulations show that a one-sided summation procedure can be substantially biased if adjustment frictions are important. In fact, the estimated elasticity is about 25% smaller if a one-sided procedure is used instead of DOLS as described by Stock and Watson (1993).

The combination of exogeneity of user cost implied by the flat supply of capital curve for a small, open economy and appropriate correction for small sample bias yields an estimate of the long-run user cost elasticity which is about 75% larger (in absolute value) than the best existing estimate. The best existing estimate is about -0.9 (for a large, open economy). Using the same econometric procedure applied to a small, open economy yields an estimate of about -1.2. Using data from a small, open economy and
the DOLS estimation technique described by Stock and Watson (1993) yields an estimate of about -1.6.

Dramatic increases in the use of rapidly depreciating computer equipment raise questions about the appropriateness of standard techniques for constructing the capital stock in empirical studies of user cost elasticity. The standard technique is to use the perpetual inventory method, using the assumption of a constant discount rate. In the Canadian data, changing the depreciation rate used in constructing the capital stock makes virtually no difference in the estimated user cost elasticity, so long as a constant depreciation rate is used. However, allowing for a variable depreciation rate reduces the estimated user cost elasticity for equipment by about 15%.

A further novel feature of this paper is that it offers cointegration-based estimates of the user cost elasticity for structures, an apparent gap in the existing literature. The elasticity estimates for structures are strikingly different than those for equipment. The estimated elasticity is insignificantly different from 0 in both an economic and statistical sense. There is little evidence that this is the result of small sample bias.

One of the lessons from this paper is that aggregation over types of capital (specifically, equipment and structures) can hide the role of user cost. When we estimate the user cost elasticity for the total capital stock in the economy, the estimated elasticity is close to 0. This masks the fact that the long-run response of equipment capital to the user cost for equipment is large, according to the estimates presented in Sections IV, V, and VI.

Another lesson is that a different kind of “aggregation” can obscure the effects of fundamentals on the capital stock, namely “aggregation” over components of user cost.
This paper appears to be the first to estimate the long-run elasticity of capital with respect to individual components of user cost: capital goods price, interest rate, and tax. 

Estimation of the effect of each of these components on the capital stock yields several interesting results.

First, there is a strong result that the price of capital goods has an economically and statistically significant effect on equipment capital. This result is precisely estimated and robust to variation in the econometric specification. In contrast, the elasticity of structures capital with respect to capital goods prices is close to zero. Again, this result is robust to a variety of econometric specifications. The total capital stock is affected by the price of capital goods, but to a substantially smaller extent than equipment capital.

The elasticity of both equipment and structures capital with respect to the real interest rate is close to zero. This result is robust to several econometric specifications.

The evidence on the role of tax parameters is mixed. In most of the econometric specifications, the elasticity of both equipment and structures capital with respect to tax parameters is close to zero. There are exceptions, however. Depending on the details of the econometric specification, the point estimate of the elasticity of both equipment and total capital stock is sometimes substantial. In most cases, the estimates are imprecise and it is impossible to reject the null hypothesis that the elasticity is zero. Overall, the data provide only weak evidence of a role for tax parameters in determining the capital stock.
Data Appendix

Capital stock is calculated by the perpetual inventory method, specifically using the following formula

\[ K_{t+1} = (1 - \delta) K_t + I_{t+1} \]

The annual depreciation rates are set at .13 for equipment and .06 for structures.\(^{24}\) (Note that since the investment data are quarterly, the depreciation rate used in the formula above is a quarterly rate.)

To construct the initial values of the capital stock, we use the current dollar measures of the capital stock (D818267 for equipment, D818265 for building construction, and D818266 for engineering construction\(^ {25}\)) for 1960 and deflate using the appropriate price index for investment goods produced by the System of National Accounts Division of Statistics Canada (D15605 for equipment and D15604 for structures). For example, for equipment we multiply the current dollar capital stock for 1960 by the ratio (equipment price index for 1992)/(equipment price index for 1960).

The initial capital stock for structures is the sum of the building construction and engineering construction capital stocks, constructed as just described.

The investment data are business sector, non-residential, gross, real, fixed capital formation (seasonally adjusted) from the National Income and Expenditure Accounts,

\(^{24}\) In the specifications which are based on variable depreciation rates, we divide the capital cost allowance by the previous year’s capital stock to obtain asset- and time-specific depreciation rate series. Specifically, for equipment, we divide the capital cost allowance for equipment (D834915) by the equipment capital stock (D834919). (Both the capital stock and the capital cost allowance are in constant dollars and are reported at annual frequency.) The Investment and Capital Stock Division of Statistics Canada divides structures into two categories - building construction and engineering construction. We calculate depreciation rates for building construction and engineering construction, using the respective capital cost allowances (D834913 and D834914) and capital stocks (D834917 and D834918) and then create a weighted average depreciation rate for structures using the proportions of capital each year (as measured by series D834917 and D834918) for the two categories as the weights.

\(^{25}\) Statistics Canada divides structures into “building construction” and “engineering construction.”
which is divided into non-residential structures (D14854) and equipment (D14855). Since the quarterly investment data are reported at annual rates, we divide by 4 to obtain investment in a given quarter.

Our measure of total capital stock is simply the sum of our measures of equipment and structures capital stock.

The cost of capital is calculated as follows

\[ \tilde{R}_t = (i_t + \delta + \gamma - \pi^K_t) \left( \frac{1 - z_t - u_t}{1 - \tau} \right) \frac{p^K_t}{p^K_t} \]

where \( i_t \) is the nominal interest rate, \( \delta \) is set at .13 for equipment, .06 for structures, and .08 for total, \( \gamma \) is a fixed risk premium (set at 6%), \( \pi^K_t \) is the rate of inflation for investment goods, \( z \) is the present value of depreciation allowances, \( u \) is the investment tax credit rate, \( \tau \) is the corporate tax rate, \( p^K_t \) is the price of investment goods, and \( p^Y_t \) is the price of output. \( \tilde{R} \) is expressed as an annual rate, so \( i_t, \delta, \gamma, \pi^K_t \) are all expressed as annual rates. The corporate tax rate is the combined federal and Ontario (provincial) tax rate on income other than small business or manufacturing income.

The nominal interest is a three month T-bill rate (B14060). The interest rate data is monthly and starts in 1962. In order to transform it into quarterly data we took the corresponding three-month average for each of the quarters. For example, 1962:1 = (Jan/62 + Feb/62 + Mar/62)/3, where the dates refer to the interest rate for that date. The following points were missing from the original series: February 1970, November 1970, February 1971, April 1973. In these cases we constructed the quarterly data by obtaining the average of the interest rates for the two months that were available for each of those quarters. For example, the interest rate for 1970:1 = (Jan/70 + Mar/70)/2.
The corporate tax and investment tax credit rates are drawn from *Finances of the Nation* (previously *The National Finances*), published by the Canadian Tax Foundation, various issues, supplemented for the period since 1996 by personal communication with the Department of Finance. Because the investment tax credit applies only to equipment, \( u = 0 \) for structures. When we examine the total business sector capital stock, we multiply the statutory ITC rate for each quarter by the ratio of equipment investment to the sum of structures and equipment investment for that quarter.

The present value of depreciation allowances (per dollar of investment) is calculated as follows [Hayashi 1982, p. 221-222]:

\[
z_t = \tau \sum_{n=1}^{T} D(n,t)(1 + i)^{-n}
\]

where \( D(n,t) \) is the depreciation allowance at time \( t \) for an asset at age \( n \) and \( T \) is the asset life for tax purposes. In general, depreciation allowances in Canada are based on the declining balance method. Essentially, the Canadian declining-balance method sets \( D(n,t) \) equal to the depreciation rate for that class of assets divided by two times the purchase cost in the first year (with the idea that, on average, assets are purchased halfway through the year) and the depreciation rate times the remaining undepreciated value of the asset in subsequent years. Thus,

\[
D(1,t) = .5\delta_{i}^{T}
\]

\[
D(n,t) = (1 - .5\delta_{i}^{T})(1 - \delta_{i}^{T})^{n-2} \delta_{i}^{T}, n \geq 2
\]

---

26 The ITC was in place from 1962 to 1985 with two exceptions (October 10, 1966 to March 9, 1967 and April 19, 1969 to August 15, 1971). We assign the ITC rate based on the rate that prevailed for the majority of a given quarter (e.g., setting the ITC rate to zero for the last quarter of 1966 and the first quarter of 1967).

27 A detailed discussion of the basic structure and historical information on rates is available in Buckwold (1990), p. 93-105. The rates were verified by direct personal contact with the Department of Finance.
where $\delta^T$ is the depreciation rate for tax purposes. The present value of depreciation allowances will therefore be:

$$z_i = \frac{\tau_i \delta_i^T}{2} + (1 - .5\delta_i^T)\tau_i \delta_i^T (1 + i_i)^{-1} + \frac{(1 - .5\delta_i^T)\tau_i \delta_i^T}{1 + i_i} \left( \frac{1 - \delta_i^T}{1 + i_i} \right)^{-1} + ...$$

$$= \frac{\tau_i \delta_i^T}{2} + (1 - .5\delta_i^T)\tau_i \delta_i^T (1 + i_i)^{-1} \sum_{t=0}^{\infty} \left( \frac{1 - \delta_i^T}{1 + i_i} \right)^t$$

$$= \frac{\tau_i \delta_i^T}{2} \frac{(1 - .5\delta_i^T)\tau_i \delta_i^T}{i_i + \delta_i^T}$$

For asset class 29 (the primary category for equipment between May 8, 1972 and 1987, inclusive), three-year straight-line depreciation at rates 25%/50%/25% was applied, so the present value of depreciation allowances was:

$$z_i = .25\tau_i + \frac{.5\tau_i}{1 + i_i} + \frac{.25\tau_i}{(1 + i_i)^2}$$

For structures, the rate was 5% before 1988 and has been 4% since then. The standard rate for equipment was 20% before 1972, 40% from 1988 to 1989 inclusive, 30% for 1990, 25% from 1991 until February 26, 1992, and 30% from February 26, 1992 on.

For total investment, $z$ is a weighted average of the corresponding $z$'s for equipment and structures, with the weights corresponding to the proportions of equipment and structures investment in a given quarter.

The price indexes for business sector structures and equipment and software investment are D15604 and D15605, respectively, and (D15603) for the total. The price index for output is the GDP deflator at market prices (D15612).

We attempt to match output as closely as possible to the investment data, which are for the business sector. To do this, we subtract government expenditures – net
government current expenditure on goods and services (D14848), government gross fixed capital formation (D14849), and government inventories (D14850) – from GDP (D14872).
References


Table 1
Estimates of User Cost Elasticity

Equipment

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Standard errors are in parentheses. Estimation is by DOLS. See equation (12) for the precise specification. p is the number of leads and lags of first differences of the right hand side variable (user cost) used in DOLS estimation. BIC is the Bayesian Information Criterion.
Table 2

A Comparison of Bias Corrections

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Parameter Estimates (Standard Errors)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOLS</td>
<td>-.82</td>
</tr>
<tr>
<td></td>
<td>(.20)</td>
</tr>
<tr>
<td>One-Sided Summation (Lags only)</td>
<td>-1.20</td>
</tr>
<tr>
<td></td>
<td>(.17)</td>
</tr>
<tr>
<td>One-Sided Summation (Leads only)</td>
<td>-1.00</td>
</tr>
<tr>
<td></td>
<td>(1.38)</td>
</tr>
<tr>
<td>DOLS</td>
<td>-1.64</td>
</tr>
<tr>
<td></td>
<td>(.08)</td>
</tr>
</tbody>
</table>

Standard errors are in parentheses. The estimation technique is shown on the left. “One-Sided Summation (Lags only)” refers to an estimator in which the two-sided summation in equation (12) is replaced by a one-sided summation in which s runs from 0 to p. “One-Sided Summation (Leads only)” refers to an estimator in which the two-sided summation in equation (12) is replaced by a one-sided summation in which s runs from −p to 0. Elasticities are reported for the value of p that minimizes the BIC.
Table 3

Estimates of User Cost Elasticity

<table>
<thead>
<tr>
<th>Structures</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>BIC</td>
<td>Elasticity</td>
<td>P</td>
<td>BIC</td>
<td>Elasticity</td>
<td>P</td>
</tr>
<tr>
<td>1</td>
<td>-5.90391</td>
<td>0.025 (0.036)</td>
<td>11</td>
<td>-5.23701</td>
<td>0.019 (0.028)</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>-5.81824</td>
<td>0.028 (0.035)</td>
<td>12</td>
<td>-5.18460</td>
<td>0.016 (0.038)</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>-5.73122</td>
<td>0.030 (0.037)</td>
<td>13</td>
<td>-5.13793</td>
<td>0.014 (0.033)</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>-5.65948</td>
<td>0.031 (0.052)</td>
<td>14</td>
<td>-5.09128</td>
<td>0.012 (0.033)</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>-5.62333</td>
<td>0.031 (0.028)</td>
<td>15</td>
<td>-5.03356</td>
<td>0.010 (0.039)</td>
<td>25</td>
</tr>
<tr>
<td>6</td>
<td>-5.54806</td>
<td>0.030 (0.030)</td>
<td>16</td>
<td>-4.95397</td>
<td>0.010 (0.033)</td>
<td>26</td>
</tr>
<tr>
<td>7</td>
<td>-5.46965</td>
<td>0.028 (0.030)</td>
<td>17</td>
<td>-4.87014</td>
<td>0.010 (0.038)</td>
<td>27</td>
</tr>
<tr>
<td>8</td>
<td>-5.41869</td>
<td>0.026 (0.026)</td>
<td>18</td>
<td>-4.78208</td>
<td>0.009 (0.040)</td>
<td>28</td>
</tr>
<tr>
<td>9</td>
<td>-5.36370</td>
<td>0.023 (0.027)</td>
<td>19</td>
<td>-4.69480</td>
<td>0.010 (0.041)</td>
<td>29</td>
</tr>
<tr>
<td>10</td>
<td>-5.30376</td>
<td>0.021 (0.036)</td>
<td>20</td>
<td>-4.59829</td>
<td>0.008 (0.040)</td>
<td>30</td>
</tr>
</tbody>
</table>

Standard errors are in parentheses. Estimation is by DOLS. See equation (12) for the precise specification. p is the number of leads and lags of first differences of the right hand side variable (user cost) used in DOLS estimation. BIC is the Bayesian Information Criterion.
Table 4

A Comparison of Bias Corrections:

Structures and the Total Capital Stock

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Parameter Estimates (Standard Errors)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Structures</td>
</tr>
<tr>
<td>SOLS</td>
<td>.02</td>
</tr>
<tr>
<td></td>
<td>(.07)</td>
</tr>
<tr>
<td>One-Sided Summation (Lags only)</td>
<td>.03</td>
</tr>
<tr>
<td></td>
<td>(.04)</td>
</tr>
<tr>
<td>One-Sided Summation (Leads only)</td>
<td>.01</td>
</tr>
<tr>
<td></td>
<td>(.57)</td>
</tr>
<tr>
<td>DOLS</td>
<td>.02</td>
</tr>
<tr>
<td></td>
<td>(.04)</td>
</tr>
</tbody>
</table>

Standard errors are in parentheses. The estimation technique is shown on the left. “One-Sided Summation (Lags only)” refers to an estimator in which the two-sided summation in equation (12) is replaced by a one-sided summation in which s runs from 0 to p. “One-Sided Summation (Leads only)” refers to an estimator in which the two-sided summation in equation (12) is replaced by a one-sided summation in which s runs from −p to 0. Elasticities are reported for the value of p that minimizes the BIC.
Table 5

Estimates of User Cost Elasticity

Total Capital Stock

<table>
<thead>
<tr>
<th>p</th>
<th>BIC</th>
<th>Elasticity</th>
<th>p</th>
<th>BIC</th>
<th>Elasticity</th>
<th>p</th>
<th>BIC</th>
<th>Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-4.85960</td>
<td>0.103</td>
<td>11</td>
<td>-4.21845</td>
<td>0.068</td>
<td>21</td>
<td>-3.85589</td>
<td>0.082</td>
</tr>
<tr>
<td></td>
<td>(0.281)</td>
<td></td>
<td></td>
<td>(0.380)</td>
<td></td>
<td></td>
<td>(0.097)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-4.78533</td>
<td>0.118</td>
<td>12</td>
<td>-4.14717</td>
<td>0.050</td>
<td>22</td>
<td>-3.87563</td>
<td>0.085</td>
</tr>
<tr>
<td></td>
<td>(0.277)</td>
<td></td>
<td></td>
<td>(0.350)</td>
<td></td>
<td></td>
<td>(0.127)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-4.72386</td>
<td>0.130</td>
<td>13</td>
<td>-4.06798</td>
<td>0.038</td>
<td>23</td>
<td>-3.88086</td>
<td>0.125</td>
</tr>
<tr>
<td></td>
<td>(0.382)</td>
<td></td>
<td></td>
<td>(0.340)</td>
<td></td>
<td></td>
<td>(0.214)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-4.67261</td>
<td>0.133</td>
<td>14</td>
<td>-3.99468</td>
<td>0.029</td>
<td>24</td>
<td>-3.80158</td>
<td>0.157</td>
</tr>
<tr>
<td></td>
<td>(0.388)</td>
<td></td>
<td></td>
<td>(0.281)</td>
<td></td>
<td></td>
<td>(0.253)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-4.62781</td>
<td>0.130</td>
<td>15</td>
<td>-3.93082</td>
<td>0.024</td>
<td>25</td>
<td>-3.70810</td>
<td>0.167</td>
</tr>
<tr>
<td></td>
<td>(0.187)</td>
<td></td>
<td></td>
<td>(0.389)</td>
<td></td>
<td></td>
<td>(0.243)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-4.54767</td>
<td>0.126</td>
<td>16</td>
<td>-3.85940</td>
<td>0.030</td>
<td>26</td>
<td>-3.61391</td>
<td>0.155</td>
</tr>
<tr>
<td></td>
<td>(0.240)</td>
<td></td>
<td></td>
<td>(0.350)</td>
<td></td>
<td></td>
<td>(0.247)</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-4.48964</td>
<td>0.115</td>
<td>17</td>
<td>-3.81283</td>
<td>0.045</td>
<td>27</td>
<td>-3.52162</td>
<td>0.131</td>
</tr>
<tr>
<td></td>
<td>(0.232)</td>
<td></td>
<td></td>
<td>(0.236)</td>
<td></td>
<td></td>
<td>(0.234)</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-4.43469</td>
<td>0.109</td>
<td>18</td>
<td>-3.77927</td>
<td>0.062</td>
<td>28</td>
<td>-3.46422</td>
<td>0.081</td>
</tr>
<tr>
<td></td>
<td>(0.351)</td>
<td></td>
<td></td>
<td>(0.132)</td>
<td></td>
<td></td>
<td>(0.214)</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-4.37353</td>
<td>0.095</td>
<td>19</td>
<td>-3.72924</td>
<td>0.075</td>
<td>29</td>
<td>-3.43514</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(0.284)</td>
<td></td>
<td></td>
<td>(0.178)</td>
<td></td>
<td></td>
<td>(0.166)</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-4.30245</td>
<td>0.078</td>
<td>20</td>
<td>-3.73183</td>
<td>0.086</td>
<td>30</td>
<td>-3.40898</td>
<td>-0.082</td>
</tr>
<tr>
<td></td>
<td>(0.327)</td>
<td></td>
<td></td>
<td>(0.126)</td>
<td></td>
<td></td>
<td>(0.185)</td>
<td></td>
</tr>
</tbody>
</table>

Standard errors are in parentheses. Estimation is by DOLS. See equation (12) for the precise specification. p is the number of leads and lags of first differences of the right hand side variable (user cost) used in DOLS estimation. BIC is the Bayesian Information Criterion.
Table 6

Variation in the Specification of the Depreciation Rate

<table>
<thead>
<tr>
<th></th>
<th>Low</th>
<th>Base</th>
<th>High</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equipment</td>
<td>-1.65(0.08)</td>
<td>-1.64(0.08)</td>
<td>-1.64(0.08)</td>
<td>-1.42(0.07)</td>
</tr>
<tr>
<td>Structures</td>
<td>0.06(0.02)</td>
<td>0.02(0.04)</td>
<td>0.00(0.05)</td>
<td>0.06(0.02)</td>
</tr>
<tr>
<td>Total</td>
<td>0.13(0.57)</td>
<td>0.10(0.28)</td>
<td>0.10(0.19)</td>
<td>0.11(0.28)</td>
</tr>
</tbody>
</table>

Standard errors are in parentheses. Low, Base, and High refer to the depreciation rate used in constructing the capital stock under the assumption of a constant depreciation rate. The depreciation rate is shown in square brackets for equipment and structures. The total capital stock is constructed by summing the equipment and structures capital stocks, using the depreciation rate for the indicated case (e.g., for the Low case, δ=.10 is used for equipment, δ=.03 for structures, and total capital is the sum of structures and equipment capital). Estimation is by DOLS. See equation (12) for the precise specification. Elasticities are reported for the value of p that minimizes the BIC.
Table 7

Estimated Elasticities with Respect to Components of User Cost

Full Decomposition

<table>
<thead>
<tr>
<th></th>
<th>( R' )</th>
<th>( R^t )</th>
<th>( R^p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equipment</td>
<td>0.04</td>
<td>0.13</td>
<td>-0.87</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.10)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Structures</td>
<td>0.00</td>
<td>0.06</td>
<td>-0.22</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.46)</td>
<td>(0.84)</td>
</tr>
<tr>
<td>Total</td>
<td>0.03</td>
<td>0.25</td>
<td>-0.33</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.49)</td>
<td>(0.15)</td>
</tr>
</tbody>
</table>

\( R' \) is the component of user cost associated with the real interest rate. \( R^t \) is the component of user cost which incorporates tax parameters. \( R^p \) is the component of user cost involving the relative price of capital goods. Standard errors are in parentheses. Estimation is by DOLS. See equation (16) for the precise specification. Elasticities are reported for the value of \( p \) that minimizes the BIC.
Table 8

Estimated Elasticities with Respect to Components of User Cost

<table>
<thead>
<tr>
<th>Single Components</th>
<th>$R^f$</th>
<th>$R^t$</th>
<th>$R^p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equipment</td>
<td>.92</td>
<td>-.27</td>
<td>-.88</td>
</tr>
<tr>
<td></td>
<td>(.61)</td>
<td>(1.45)</td>
<td>(.04)</td>
</tr>
<tr>
<td>Structures</td>
<td>.03</td>
<td>.14</td>
<td>-.42</td>
</tr>
<tr>
<td></td>
<td>(.05)</td>
<td>(.43)</td>
<td>(.68)</td>
</tr>
<tr>
<td>Total</td>
<td>.17</td>
<td>-1.58</td>
<td>-.31</td>
</tr>
<tr>
<td></td>
<td>(.06)</td>
<td>(.32)</td>
<td>(.02)</td>
</tr>
</tbody>
</table>

$R^f$ is the component of user cost associated with the real interest rate. $R^t$ is the component of user cost which incorporates tax parameters. $R^p$ is the component of user cost involving the relative price of capital goods. Standard errors are in parentheses. Estimation is by DOLS. See equation (16) for the precise specification. Elasticities are reported for the value of $p$ that minimizes the BIC.
Table 9  
Estimated Elasticities with Respect to Components of User Cost

Partial Decomposition

<table>
<thead>
<tr>
<th></th>
<th>Equipment</th>
<th>Structures</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R'$</td>
<td>.02</td>
<td>.01</td>
<td>.06</td>
</tr>
<tr>
<td></td>
<td>(.40)</td>
<td>(.06)</td>
<td>(.08)</td>
</tr>
<tr>
<td>$R'^p$</td>
<td>-.80</td>
<td>-.02</td>
<td>-.21</td>
</tr>
<tr>
<td></td>
<td>(.23)</td>
<td>(.31)</td>
<td>(.09)</td>
</tr>
<tr>
<td>$R^t$</td>
<td>-.85</td>
<td>-.01</td>
<td>-.80</td>
</tr>
<tr>
<td></td>
<td>(.51)</td>
<td>(.41)</td>
<td>(.86)</td>
</tr>
<tr>
<td>$R'^p$</td>
<td>-1.05</td>
<td>.02</td>
<td>.08</td>
</tr>
<tr>
<td></td>
<td>(.15)</td>
<td>(.05)</td>
<td>(.19)</td>
</tr>
<tr>
<td>$R^p$</td>
<td>-.87</td>
<td>-.18</td>
<td>-.28</td>
</tr>
<tr>
<td></td>
<td>(.04)</td>
<td>(.73)</td>
<td>(.09)</td>
</tr>
<tr>
<td>$R'^{-}t$</td>
<td>.04</td>
<td>.00</td>
<td>.05</td>
</tr>
<tr>
<td></td>
<td>(.07)</td>
<td>(.06)</td>
<td>(.07)</td>
</tr>
</tbody>
</table>

$R'$ is the component of user cost associated with the real interest rate. $R'^p$ is the combined component of user cost associated with tax parameters and the price of capital goods. $R^t$ is the component of user cost which incorporates tax parameters. $R'^p$ is the combined component of user cost associated with the real interest rate and the price of capital goods. $R^p$ is the component of user cost involving the relative price of capital goods. $R'^{-}t$ is the combined component of user cost associated with the real interest rate and tax parameters. Standard errors are in parentheses. Estimation is by DOLS. See equation (16) for the precise specification. Elasticities are reported for the value of $p$ that minimizes the BIC.
Figure 1
I/K: Equipment

I is investment and K is the capital stock. Shaded areas mark recessions in Canada (as defined by Statistics Canada). Data sources are described in the data section and the data appendix.
Figure 2

I/K: Structures

I is investment and K is the capital stock. Shaded areas mark recessions in Canada (as defined by Statistics Canada). Data sources are described in the data section and the data appendix.